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The Effect of Competition among Brokers on the Quality and Price of Differentiated Internet Services

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Abstract

Price war, as an important factor in undercutting competitors and attracting customers, has spurred considerable work that analyzes such conflict situation. However, in most of these studies, quality of service (QoS), as an important decision-making criterion, has been neglected. Furthermore, with the rise of service-oriented architectures, where players may offer different levels of QoS for different prices, more studies are needed to examine the interaction among players within the service hierarchy. In this paper, we present a new approach to modeling price competition in (virtualized) service-oriented architectures, where there are multiple service levels. In our model, brokers, as the intermediaries between end-users and service providers, offer different QoS by adapting the service that they obtain from lower-level providers so as to match the demands of their clients to the services of providers. To maximize profit, players, i.e. providers and brokers, at each level compete in a Bertrand game while they offer different QoS. To maintain an oligopoly market, we then describe underlying dynamics which lead to a Bertrand game with price constraints at the providers’ level. Numerical simulations demonstrate the behavior of brokers and providers and the effect of price competition on their market shares.

Keywords: Service-oriented architecture, Quality of Service (QoS), oligopolistic competition, service differentiation, Bertrand competition, price constraints.

I. INTRODUCTION

In today’s highly competitive Internet service market, service providers, in order to survive, should offer their customers more flexibility in their quality-of-service (QoS) / price offerings to meet a variety of customer needs and application constraints. Clearly, any successful solution for a service provider to stay in the market, not only depends on supporting new and updated technologies, but also involves economic aspects. However, pricing the services of the network, even without considering quality differentiation, is a challenging problem that involves different issues. There have been many studies that attempted to address these issues with or without considering differentiated QoS. Pricing approaches include Paris Metro pricing [29], congestion pricing [7, 15], rate-reliability pricing [19], and fairness pricing [18]. On the other hand, with the rise of service-oriented architectures, such as computational clouds and recursive networks [36], and network virtualization such as CABO [10] and also with the increasing number of service brokerage companies such as Google in “Project Fi” [14], there is a need for more advanced solutions that manage the interactions among service providers at multiple levels. The ultimate goal in service-oriented architectures and network virtualization is to decouple the services offered by network providers from those of service providers which yield the layered structure of the network. Also, brokers as the intermediaries between clients and lower-level providers, play a key role in improving the efficiency of service-oriented structures by matching the demands of clients to the services of providers. They can downgrade or upgrade a service by sharing it among customers or by combining several services to satisfy customers’ demand. For example, in “Project Fi”[14], Google offers a flat data rate of $10 per gigabyte of data that is provided by either T-Mobile or Sprint, i.e., Google selects the best network provider based on factors such as coverage and performance, thus adding flexibility and providing the best service to its customers. Furthermore, Project Fi customers can manage their costs based on their monthly needs. This
is in contrast to network providers, e.g., T-mobile and Sprint, which offer their customers fixed data plans regulated by a static contract.

In this study we propose a multi-layer network market in which service brokers and service providers compete at different levels in an oligopoly to maximize their profit. In our settings, brokers can pay a cost to upgrade or downgrade the service that they buy from (lower-level) providers so as to offer a new service to the market (customers). The broker incurs costs when adapting a lower-level service as it expends resources to either enhance the service extended to its customers (e.g., by employing delay-jitter reduction or capacity allocation techniques over a best-effort service) or degrade it (e.g., by multiplexing client demands over a guaranteed service). We consider the competition among providers and among brokers separately, while brokers impose some preference constraints on (infrastructure, cloud or service) providers. We also consider conditions that may lead to a monopoly market and study how players act under such conditions. We model service quality differentiation after Hotteling’s location model — [17], where firms compete and price their products in only one dimension, geographic location. In our model, brokers and (lower-level) providers compete and price their services based on the quality of the service that they offer. Our numerical results show that more service differentiation generally yields more profit for all players. However, besides quality differentiation, the cost that brokers undergo also plays an important role and they should forgo maximum differentiation to reduce the cost, which leads to higher profit. Also, as the number of brokers increases, the market gets more competitive and prices drop further.

Our contributions in this paper are as follows: we model a two-layered network market in which providers and brokers offer differentiated services and compete in a Bertrand game at each layer following the Hotteling’s location model. We also analyze the actions of players under a monopoly setting. In Section II we review related work, and then in Section III, we develop a game-theoretic model by characterizing the competitive behavior of players with one another at each level of the service hierarchy. Section IV presents our analysis and numerical results when only the interaction of two brokers is captured in the model, while in Section IV-C we show how our model is useful to capture interactions among more brokers, dissecting two more case studies where we consider three and four brokers. Finally, in Section V we conclude the paper.

II. RELATED WORK

Game theory has been applied to a wide range of networking problems to capture the interaction of (selfish or cooperating) players seeking a maximum value for their (private) utility. The assumption is that every step (or move) toward the maximization of such utility impacts the utility of other players in the model (or game). Given the connectivity nature of a network of agents, a wide range of networking mechanisms have been modeled with game theory; from the physical ISO-OSI layer with transmission power utility games [5] or spectrum sharing [2, 35] to Medium Access Control [37] to routing and packet forwarding [4, 30], both in wireless [23] and wired [28] scenarios.

Aside from modeling multi-agent protocol behaviors and the various resource allocation mechanisms, markets and pricing equilibrium have further exemplified the synergy between game theory (and economics) and networked (cloud) systems [21, 31]. In particular, network economics has been a very active research area in which both pricing and market regulation strategies have been studied widely. However the exponential growth of Internet services in hierarchical (i.e., multi-layer) markets requires a deeper study of new market features that will become available. One of the earliest work on layered networks, [13] identifies and discusses some difficult economic problems related to resale and complexity of competition among multiple owners of physical networks. The authors study some integrated and unintegrated telecommunication companies and the services that they offer to create differentiated products to cover their costs. The paper does not suggest any specific architecture or policies for pricing, but discusses the need for a full economic model that features oligopolistic competition among a few large companies that invest in the physical infrastructure as well as firms at the virtual network level.

Our work is inspired by Zhang et al. [39] and Nagurney and Wolf [27]. They propose an economic model for the interaction and competition among service providers, network providers and users. Both
studies develop a two-stage (Stackelberg) game, where service providers compete in a Cournot game,
and network providers compete in a Bertrand game. Stackelberg games have also been proposed in other
networking contexts, e.g., in Wi-Fi scenarios [38]. In our framework, providers at all levels compete in
Bertrand games (i.e., competition on price). Although our work shares the same two-level game with [27,
39], Zhang et al. [39] study a market with two service providers and two network providers offering
the same level of service quality. In [27] the authors consider a market with more than two providers
for service and network, in which network providers offer different levels of service quality. In their
model, the market is managed through the demand-price functions, which depend on both quality and
quantity. Our model considers a market where at the providers’ level, players can offer different qualities
of service, while at the brokers’ level, players can upgrade or downgrade the service to optimize their
profit. Semret et al. [34] also consider a retail market where, for each network, three types of players
interact: a service provider, a broker and end-users. However, their main contribution is a decentralized
auction-based bandwidth pricing for differentiated Internet services; the authors show that Progressive
Second Price (PSP) provides a stable pricing in a market where service providers receive most of the
profits and the brokers’ profit margin is small.

Pricing for single-level games has been studied even more extensively than two-level games. He and
Walrand [16] consider a self-regulated service model, where market demand determines the service quality.
In their model, there is a single Internet Service Provider (ISP) who offers two classes of service with
different prices to manage congestion. The authors show that when the price does not match the service
quality, the system may end up in an equilibrium similarly to the Prisoner’s Dilemma game. Shetty
et al. [35] also compare the revenue of a monopolist operator with and without service differentiation.
They show that the revenue is higher when an operator offers two different services. Both Li et al. [20]
and Fulp and Reeves [11] provide a traffic-sensitive pricing scheme for differentiated network services.
The focus of [11] is on maximizing the profit of the service provider who buys a differentiated service
connection from domain brokers and sells it to users; the focus of [20] instead is to provide economic
incentives to users so as to maintain a given level of traffic load.

Different game-theoretic models for differentiated service markets of users and service providers have
also been proposed [1, 22, 26]. In [26], the authors propose a game-theoretic model where service providers
compete with duration-based contracts for differentiated service. Instead, [1] considers a joint price-quality
market with a Stackelberg game where providers are leaders and users are followers. In their model,
providers consider the migration of users when setting their price and quality. Finally, the authors in [22]
study a congestion-prone market with usage-based pricing. They propose a model for users’ preference
over their value and sensitivity to congestion, and based on such model they characterize the market share
and optimal price for providers.

Our model considers multi-layer differentiated service games where the service obtained from the lower
level can be upgraded or downgraded, and hence can be sold to the higher level provider. In our analysis,
we apply price constraints when players’ optimal price would lead to losing market share, and we also
give insights on how players should then update their price.

III. MODEL AND SOLUTION

In this section, we present our model and analysis of a two-level game configuration and focus on
the competition among providers and brokers and what emerges as pricing of their services. Figure 1
illustrates the game-theoretic model: At the lower level, we have two service providers, while at the
higher level, we have \( m \geq 2 \) service sellers or brokers that deal directly with users. Note that owning
a network infrastructure is expensive, and only a few large companies can afford its cost. There are however
many brokerage companies. Our model’s goal is limited to analyzing and understanding the dynamics of a
market in a formal economic setting. To this aim, we start by considering only two network / lower-level
providers in a simple oligopoly market competition. The exclusion of more complex relationships that
may exist in real markets keeps our model tractable while still producing interesting results and insights.
To model service quality differentiation, we adopt Hotteling’s location model [6, 17] which introduced the idea of modeling difference between products as differences in a product’s location in a product space. The idea is widely used for both location problems [24, 32, 33] and quality differentiation [8, 9, 25] in network studies. In Hotteling’s model [17], there are two firms selling identical goods along a street. Customers are assumed to be uniformly distributed in the space, and the transport cost is a linear function of their distance to the selected firm. A consumer selects the firm that minimizes her cost of transportation to buy the product. Hotteling concluded that two firms would locate close to each other near the center. Later, D’Aspremont et al. [6] changed the utility function from linear to quadratic form, where firms choose to maximize their distance to the opposite player, and there is equilibrium for price competition. Brenner [3] extended the game with the quadratic cost function to more than two firms. He has shown that for more than two firms, the “principle of maximum differentiation” does not hold, and corner firms would benefit from moving marginally toward the market center. In our work, we model service quality differentiation after Hotteling’s product differentiation, where customers have different preference for service quality that is modeled by their willingness to pay for that quality. We start by presenting our notation and some basic settings, then we discuss some analytical and numerical results.

A. Model Description

Let us consider a system with a continuum of customers, $m$ service sellers (brokers), denoted by $B_i$, $i = 1, \ldots, m$, and two service providers, $S_j$, $j = 1, 2$. We assume that customers have different preference for quality (utility) described by:

$$\theta q - p$$

where $\theta$ is the customer’s marginal willingness to pay for quality $q$, and $p$ is the price of service. There is a distribution of $\theta$ among customers. For simplicity, we assume that $\theta$ is uniformly distributed on an interval $\theta \in [\theta_{\min}, \theta_{\max}]$ and $\theta_{\max} > 2 \theta_{\min}$. Customers seek a broker that maximizes their utility.

Both brokers and service providers can offer services with different qualities. The service quality offered by brokers is denoted by $q_i$ and is in an interval $q \in [q_{\min}, q_{\max}]$, and the quality offered by service providers is denoted by $Q_j$. Also, we assume that brokers and service providers compete in an imperfectly competitive market. Furthermore, we assume that there is no supply constraint and so there are enough resources to meet each demand. We also assume that each broker buys just from one of the service providers that is more economical. Without loss of generality, we assume that $B_1$ buys from $S_1$.
and $B_m$ buys from $S_2$ (unless as we note in Section III-F, the market does not support this assumption) and other brokers choose the provider with the lower cost.

We assume that providers, and brokers, compete with each other in a Bertrand game. In a Bertrand game, players compete on the price based on their cost and other market information. We analyze the market at the Nash equilibrium point where the market is in steady state and no player intends to change her price.

B. Demand Distribution

Brokers first choose the quality of service that they will provide to customers, then they compete on prices. If the brokers choose the same quality, then the customers decide only based on the price, and this leads to a Bertrand competition with identical goods, whose prices should be set equal to costs, and no one makes profit. Thus the brokers should choose to offer different service qualities to make profits. Without loss of generality, we assume that $q_m > ... > q_2 > q_1$, and also $Q_2 > Q_1$. Therefore, customers with a high willingness to pay for quality will buy from $B_m$, while customers with a low willingness will buy from $B_1$.

For simplicity, first let’s assume that we have two brokers, $B_1$ and $B_2$. We can characterize the demand for each broker by identifying the customers who are indifferent between the two differentiated qualities. The indifferent customers, denoted by $\theta^*$, satisfy:

$$\theta^* q_1 - p_1 = \theta^* q_2 - p_2 \Leftrightarrow \theta^* = \frac{p_2 - p_1}{q_2 - q_1}$$

(1)

Having uniformly distributed $\theta$, the demand for each broker, $B_1$ and $B_2$, is given by:

$$D_1(p_1, p_2) = \frac{\theta^* - \theta_{min}}{\Delta \theta} = \frac{1}{\Delta \theta} \left( \frac{p_2 - p_1}{q_2 - q_1} - \theta_{min} \right)$$
$$D_2(p_1, p_2) = \frac{\theta_{max} - \theta^*}{\Delta \theta} = \frac{1}{\Delta \theta} \left( \theta_{max} - \frac{p_2 - p_1}{q_2 - q_1} \right)$$

(2)

where $\Delta \theta = \theta_{max} - \theta_{min}$.

For more than two brokers, we can generalize equation (1) to find indifferent customers $\theta_i^*$ between any two brokers $B_i$ and $B_{i+1}$:

$$\theta_i^* q_i - p_i = \theta_{i+1}^* q_{i+1} - p_{i+1} \Leftrightarrow \theta_i^* = \frac{p_{i+1} - p_i}{q_{i+1} - q_i}$$

(3)

Consequently, the demand for each broker can be found by:

$$D_1(p_1, p_2, ..., p_m) = \frac{\theta_1^* - \theta_{min}}{\Delta \theta}$$
$$D_i(p_1, p_2, ..., p_m) = \frac{\theta_i^* - \theta_{i-1}^*}{\Delta \theta} \quad 1 < i < m$$
$$D_m(p_1, p_2, ..., p_m) = \frac{\theta_{max} - \theta_{m-1}^*}{\Delta \theta}$$

(4)

Note that in the above equations $D_i$’s assume values in the interval $[0, 1]$. This means that if for broker $B_i$ the demand $D_i$ is negative, then $B_i$ is “out of the market”; more precisely, we can rewrite the demand function as:

$$D_i = \min \left\{ \max \left\{ 0, \frac{\theta_i^* - \theta_{i-1}^*}{\Delta \theta} \right\}, 1 \right\} \quad \text{for } 1 \leq i \leq m$$

(5)
C. Brokers’ Profits

Now that we have the demand distribution, we can calculate broker $i$’s profit, assuming that converting $Q_j$ to $q_i$ (whether to upgrade or downgrade the service) has a marginal cost $c_i$:

$$\Pi_i = p_i D_i - \frac{q_i D_i}{Q_j} r_j - c_i D_i (Q_j - q_i)^2$$  \hspace{1cm} (6)

where $r_j$ is the price of service that broker $B_j$ pays to service provider $S_j$ and $\frac{q_i D_i}{Q_j}$ is the amount of service that $B_j$ needs to buy to supply its own market. We assume here that the cost to the broker, $c_i$, for converting the service quality it gets ($Q_j$) to that it offers its customers ($q_i$), is proportional to the square of the difference in quality, $(Q_j - q_i)$. Intuitively, the cost increases more rapidly as the service quality increases, or alternatively, there is a diminishing return in service quality as more resources are allocated and cost increases. We henceforth, for simplicity, assume that $c_i = c$.

As we mentioned earlier, we assume that the broker with lowest quality ($B_1$) buys from the lower quality provider ($S_1$) and the highest quality broker ($B_m$) buys from the higher quality provider ($S_2$). This is the only valid assumption to have an oligopoly market at the level of service providers. For simplicity, again let’s assume that we have two Brokers. If $B_1$ prefers to buy from $S_2$, then the cost of buying from $S_2$ must be less than the cost of buying from $S_1$:

$$\frac{q_1}{Q_2} r_2 + c(q_1 - Q_2)^2 < \frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2$$

Given $(q_1 - Q_2)^2 > (q_1 - Q_1)^2$, we have $\frac{r_2}{Q_2} < \frac{r_1}{Q_1}$. Thus, given $\frac{r_2}{Q_2} < \frac{r_1}{Q_1}$, and also $(q_2 - Q_2)^2 < (q_2 - Q_1)^2$, we deduce that it is less costly for broker $B_2$ to buy from $S_2$ as well, which results in a monopoly market at the level of service providers. The same logic applies if $B_2$ prefers to buy from $S_1$, and also for general case with $m$ brokers.

Now that we have the profit function for brokers, we can find the optimal price for them. In the first stage, given the service prices $r_j$, and service qualities $Q_j$, the brokers compete in a Bertrand game with differentiated goods. We present the results for case $m = 2$, but all of results can be calculated for general cases with more than two brokers. Plugging Equation (2) into Equation (6), and solving $\partial \Pi_i / \partial p_i = 0$ for achieving Nash equilibrium, leads to:

$$p_1 = \frac{1}{3}((q_2 - q_1)(\theta_{\text{max}} - 2\theta_{\text{min}}) + \frac{2q_1 r_1}{Q_1} + \frac{q_2 r_2}{Q_2} + 2c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2)$$ \hspace{1cm} (7)

$$p_2 = \frac{1}{3}((q_2 - q_1)(2\theta_{\text{max}} - \theta_{\text{min}}) + \frac{q_1 r_1}{Q_1} + \frac{2q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 + 2c(q_2 - Q_2)^2)$$ \hspace{1cm} (8)

Now we have the brokers’ prices, $p_1$ and $p_2$, as a function of the brokers’ and providers’ service qualities, and providers’ prices $r_j$’s. The next step is to plug them into $D_i$’s to make demand function as a function of $r_j$’s:

$$D_1 = \frac{1}{3\Delta \theta} (\theta_{\text{max}} - 2\theta_{\text{min}}) + \frac{\frac{q_2 r_2}{Q_2} - \frac{q_1 r_1}{Q_1} - c(q_1 - Q_1)^2 + c(q_2 - Q_2)^2}{3\Delta \theta (q_2 - q_1)}$$ \hspace{1cm} (9)

$$D_2 = \frac{1}{3\Delta \theta} (2\theta_{\text{max}} - \theta_{\text{min}}) + \frac{\frac{q_1 r_1}{Q_1} - \frac{q_2 r_2}{Q_2} + c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{3\Delta \theta (q_2 - q_1)}$$ \hspace{1cm} (10)

Now, $D_1$ and $D_2$ are dependent on service providers’ prices $r_j$’s, which shows the interaction between the two layers. Providers’ prices are part of the cost for brokers and in turn affect the price of brokers and consequently the demands of both brokers and providers. In the next subsection we show how to find the optimal $r_j$’s.
D. Providers’ Profits

At this stage, we have the total demand served by (service sold by) each broker. To have an imperfectly competitive market at the level of service providers, the combination of their price and quality should be such that each broker prefers a different service provider. Assuming $B_1$ prefers $S_1$ and $B_2$ prefers $S_2$, the following inequalities should hold for $B_1$ and $B_2$, respectively:

$$\frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2 < \frac{q_1}{Q_2} r_2 + c(q_1 - Q_2)^2$$

$$\frac{q_2}{Q_2} r_2 + c(q_2 - Q_2)^2 < \frac{q_2}{Q_1} r_1 + c(q_2 - Q_1)^2$$

(11)

These constraints ensure that broker $B_1$ chooses provider $S_1$ and $B_2$ chooses $S_1$, as the cost is lower than that of getting service from the other provider. Later we will discuss the situation when one of these constraints is violated.

For general case, we assume that the $k$ first brokers choose provider $S_1$ and from broker $B_{k+1}$ to broker $B_m$ choose $S_2$. Constraint (11) should be hold for $B_k$ and $B_{k+1}$ instead of $B_1$ and $B_2$.

In this stage of the game, service providers compete in another Bertrand game. The profit of each provider is defined as:

$$U_1 = \sum_{i=1}^{k} \frac{D_i q_i}{Q_1} (r_1 - f_i) - eQ_1^2$$

$$U_2 = \sum_{i=k+1}^{n} \frac{D_i q_i}{Q_2} (r_2 - f_i) - eQ_2^2$$

(12)

where $eQ_j^2$ is the cost of providing quality $Q_j$, $r_j$ is the service price and $f_i$ represents some general cost (fee). After plugging Equations (9) and (10) into the providers’ profit, we obtain quadratic equations in $r_j$.

To obtain the optimal solution (Nash equilibrium), we solve $\partial U_j / \partial r_j = 0$ which for two brokers yields:

$$r_1 = \frac{2f_1}{3} + \frac{f_2 q_2 Q_1}{3 q_1 Q_2} + \frac{Q_1}{3 q_1} \times [c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2 - (q_1 - q_2)(4\theta_{\text{max}} - 5\theta_{\text{min}})]$$

$$r_2 = \frac{2f_2}{3} + \frac{f_1 q_1 Q_2}{3 q_2 Q_1} + \frac{Q_2}{3 q_2} \times [c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2 - (q_1 - q_2)(5\theta_{\text{max}} - 4\theta_{\text{min}})]$$

By substituting $r_j$’s in Equations (7) and (8), we get the final values for $p_i$’s only as functions of user preferences and service qualities:

$$p_1 = \frac{1}{9} \left(5c(q_1 - Q_1)^2 + 4c(q_2 - Q_2)^2\right) + \frac{4f_2 q_2 Q_1 + 5f_1 q_1 Q_2}{9 Q_1 Q_2} + \frac{1}{9} (q_2 - q_1)(16\theta_{\text{max}} - 20\theta_{\text{min}})$$

$$p_2 = \frac{1}{9} \left(4c(q_1 - Q_1)^2 + 5c(q_2 - Q_2)^2\right) + \frac{5f_2 q_2 Q_1 + 4f_1 q_1 Q_2}{9 Q_1 Q_2} + \frac{1}{9} (q_2 - q_1)(20\theta_{\text{max}} - 16\theta_{\text{min}})$$

We obtain the final values for $D_i$’s from Equations (9) and (10):

$$D_1 = \frac{1}{9\Delta\theta} (4\theta_{\text{max}} - 5\theta_{\text{min}}) + \frac{c(q_1 - Q_1)^2 - c(q_2 - Q_2)^2}{9\Delta\theta(q_1 - q_2)} + \frac{-f_2 q_2 Q_1 + Q_2 f_1 q_1}{9\Delta\theta(q_1 - q_2) Q_1 Q_2}$$

$$D_2 = \frac{1}{9\Delta\theta} (5\theta_{\text{max}} - 4\theta_{\text{min}}) + \frac{c(q_2 - Q_2)^2 - c(q_1 - Q_1)^2}{9\Delta\theta(q_1 - q_2)} + \frac{f_2 q_2 Q_1 - Q_2 f_1 q_1}{9\Delta\theta(q_1 - q_2) Q_1 Q_2}$$
E. Positive Utility

In the previous setting we assumed that customers buy service from either \( B_1 \) or \( B_2 \), even if their utility is negative. Here we solve a game with only positive utility customers, i.e., customers whose value of \( \theta q - p \) is positive. Therefore, customers with zero utility provide a lower bound on \( \theta \) (we call it \( \theta_0 \)), which can be found by solving \( \theta q_1 - p_1 = 0 \). Thus \( \theta_{\min} \) is replaced by \( \frac{\theta_0}{q_1} \):

\[
D_1(p_1, \ldots, p_m) = \frac{\theta^*_1 - \theta_0}{\Delta \theta} = \frac{1}{\Delta \theta} \left( \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1} \right)
\]

As in our previous setting, this is a two-stage Bertrand game, and the Nash equilibrium for each game is found by replacing the \( D_i \)'s into the profit functions and solving \( \partial \Pi_i / \partial p_i = 0 \) and \( \partial U_i / \partial r_i = 0 \). We discuss the difference between this positive utility game and the previous (unconstrained utility) game later in Section IV.

F. Game with Constraints

At the lower level of service providers, the constraints (11) are not considered while calculating the equilibrium points. Therefore, in some situations, one of the constraints might be violated. Let us assume that after finding \( r_i \)'s, the constraint for \( B_1 \) is violated, i.e., \( \frac{q_1}{Q_1} r_1 + c(q_1 - Q_1)^2 \geq \frac{q_2}{Q_2} r_2 + c(q_1 - Q_2)^2 \). This means that, under this condition, for broker \( B_1 \), it incurs more cost to buy service from provider \( S_1 \) than provider \( S_2 \); so if provider \( S_1 \) does not change its price, \( B_1 \) will get service from \( S_2 \), and this situation leads to a monopoly market at the providers' level.

To find an optimal point that also meets the constraints (11), provider \( S_1 \) should set its price such that \( r_1 < \frac{Q_1}{q_1} \left( \frac{q_2}{Q_2} r_2 + c(q_1 - Q_2)^2 - c(q_1 - Q_1)^2 \right) \). In response, provider \( S_2 \) updates its price by plugging \( r_1 \) into \( \partial U_2 / \partial r_2 = 0 \) which leads to \( r_2 = f(r_1) \), i.e., \( r_2 \) as a function of \( r_1 \). Thus, \( S_1 \) can replace the \( r_2 \) with \( f(r_1) \) in its inequality to calculate an optimal price that satisfies the constraint:

\[
r_1 = \frac{Q_1}{q_1} \left( \frac{q_1}{Q_2} f(r_1) + c(q_1 - Q_2)^2 - c(q_1 - Q_1)^2 \right) - \epsilon \quad \epsilon > 0
\]

In this stage of the game, \( S_1 \) should find a positive value for \( \epsilon \) that maximizes its profit. By substituting \( r_1 \) and \( r_2 \) as functions of \( \epsilon \), \( U_1 \) is a decreasing quadratic function of \( \epsilon \). Solving \( \partial U_1 / \partial \epsilon = 0 \) results in optimal \( \epsilon \). If \( \epsilon < 0 \), it can be replaced with a small positive number close to zero. Since \( U_1 \) is decreasing with respect to \( \epsilon \), any other positive value larger than the chosen \( \epsilon \) leads to less profit. Clearly, the new set of prices for the service providers is an equilibrium point for the game, since it maximizes the revenue of both providers while meeting the constraints so each service provider does not lose its market (i.e., one of the two brokers stays as its customer); therefore neither of the service providers has an incentive to change its price independently.

IV. Numerical Analysis

In this section we present some numerical results to illustrate the effect of choosing different qualities of service by brokers. We consider settings with two, three and four brokers. We also study the positive game model for two brokers. We show in detail how the best strategy for any broker is to choose a quality level that maximizes quality differentiation with other brokers. Also, when there are more brokers, the higher competition leads to more reasonable prices and a lower probability of a monopoly market.

A. Two Brokers

For the two brokers case, we consider a setting where \( \theta_{\max} = 1.5 \), \( \theta_{\min} = 0.2 \), \( c = 0.1 \), and \( f_i = 0.01265 \times Q_i^{1.5} \). The service quality of the providers are set to \( Q_1 = 20 \) and \( Q_2 = 45 \). For the brokers, \( q_2 \) varies between 30 and 60, and we set \( q_1 \) to different values such that it is less than, equal to, or larger than \( Q_1 \) to see how the market changes under different conditions.
Examine a first set of plots shown in Figures 2 and 3, we note that the total demand constitutes the whole market. So, when the demand for one broker/provider side decreases, the demand for the other side increases and vice versa. But this is not the case for prices and profits – they increase or decrease together. Furthermore, observing the behavior for different values of $q_1$, we see from the brokers’ and providers’ price plots, when broker $B_1$ downgrades the lower-level service obtained from its provider $S_1$ (i.e., $q_1 < Q_1$), all brokers and providers can offer their service at higher prices and make more profit. Similarly, by comparing the behavior for higher values of $q_2$, where $q_2 > Q_2$, with that for lower values where $q_2 < Q_2$, we observe that a better strategy for broker $B_2$ is to upgrade the lower-level service that it obtains from $S_2$ (i.e., $q_2 > Q_2$). This happens because upgrading $q_2$ or downgrading $q_1$ leads to a larger gap between $q_1$ and $q_2$, therefore the two sets of broker and provider can offer more differentiated services at higher prices. In fact it follows the maximum differentiation principle. In this setting, since we have the minimum number of brokers to compete, it’s more likely that monopoly situations happen. For example, for $q_1 = 29$, the market exhibits abnormal behavior when the gap between $q_1$ and $q_2$ is small, while the gap between providers’ qualities and brokers’ qualities is large. Specifically, the market approaches a monopoly where $S_2$ and $B_2$ have a small market share when broker $B_2$ is downgrading the service obtained from its service provider $S_2$, i.e., $q_2 < Q_2$. Observing the results when the values of $q_2$ are close to 30, we note that here, although the providers’ game is a monopoly at some points where $S_2$’s price $r_2 = 0$, the brokers’ game is not, and $B_2$ can have a small share of the market $D_2$ while it gets service from provider $S_1$. This is because when the gap between $q_1$ and $q_2$ is not significant, most
of the customers prefer the cheaper service provided by broker $B_1$. When the market is a monopoly, the provider or broker who remains in the market can increase its price to a value such that the other competitor cannot enter the market even if it lowers its price to equal its cost, thus there is no way for the competitor to make profit and is prevented from entering the market.

On the other hand, for $q_1 = 29$, when broker $B_2$ is upgrading the service quality obtained from $S_2$, i.e., $q_2 > Q_2$, as the gap between $q_2$ and $Q_2$ gets larger, $S_2$ starts to decrease its price to cover the cost of the quality upgrade for $B_2$ so as not to lose its market share. Since the value of $q_1$ is somewhere between $Q_1$ and $Q_2$, it is more economical for $B_1$ to buy service from $S_2$ rather than $S_1$ at the optimal prices, i.e., the optimal price of $S_1$ violates constraints (11) and it should update its price $r_1$ as we explained in Section III-F. Consequently, $S_2$ should also update its price. Since there is a substantial gap between $q_1$ and $q_2$, both providers can compete in the market.

**B. Positive Utility Results**

We now consider the case of positive utility competition. Intuitively, we expect to see some restriction on the prices for all brokers and providers, otherwise they lose part of the market for which the utility $(\theta q - p)$ is negative. Thus it is a compromise between price and demand. The numerical results confirm this intuition. Comparing the prices of brokers and providers under positive utility and unconstrained utility, for the same conditions, shows that the highest prices under positive utility are below half of the
Also in this positive utility game, whether brokers upgrade or downgrade the service obtained from their providers, the behavior is different from that in the unconstrained utility game. Specifically, since the positive utility market is more sensitive to prices, a smaller gap between the service quality offered by the broker and the quality it gets from its provider yields more profit. Furthermore, while for both brokers, upgrading the service obtained from lower-level providers (and in turn, selling a higher quality service to customers) is generally more profitable, $B_2$ makes more profit when $B_1$ downgrades the obtained (lower-level) service, and $B_1$ makes slightly more profit when $B_2$ upgrades the obtained (lower-level) service.

Unlike the unconstrained utility game, if profit increases for one player, profit decreases for the other player. Another interesting observation from these plots is when the market is a monopoly: while there are conditions under which broker $B_1$ or $B_2$ can lose their market share, service provider $S_1$ can manage to stay in the market under all conditions.

C. Results with Three and Four Brokers

In this section we extend our setting to three and four brokers to see if the maximum differentiation principle holds for more brokers. We assume that two brokers offering the lowest and the highest quality of service to users are already in the market and define the range of feasible quality. We then let the other
one or two brokers enter the market with a quality level chosen in such range. After fixing a quality level, the third (and fourth) broker obtains service from the (lower-level) provider that minimizes the quality difference between them. This in turn minimizes the broker’s cost in providing service to its customers. As in previous case studies with only two brokers, we show results at the equilibrium of the game by identifying indifferent customers between available service qualities. We also apply all constraints on the providers’ level to have an oligopoly market.

1) Three Brokers: We simulate the game with $\theta_{\text{min}} = 1$, $\theta_{\text{max}} = 70$, two providers $S_1$ and $S_2$ with $Q_1 = 30$ and $Q_2 = 60$, and three brokers, $B_1$, $B_2$ and $B_3$, with qualities $q_1$, $q_2$ and $q_3$, respectively. We assume that the quality levels of $B_1$ and $B_3$ are fixed and we let the quality of broker $B_2$ change in the interval $(q_1, q_3)$. Broker $B_2$ chooses the service provider with least quality difference to reduce its (service conversion) cost. Given the above settings, we observe a tipping point for the quality of broker $B_2$ ($q_2$): for $q_2 < 45$, $B_2$ chooses provider $S_1$ and for $q_2 > 45$ $B_2$ chooses provider $S_2$; for the frontier value of $q_2 = 45$, although there is no quality differentiation between the two (lower-level) providers, we observe that downgrading the service has less cost than upgrading it, therefore $B_2$ chooses to get its service from $S_2$. The jump in profit at $q_2 = 45$ in Figure 6 is because of $B_2$’s switching provider.

As we can see in Figure 6, for brokers $B_1$ and $B_3$, which have been already in the market, it is more profitable if broker $B_2$ chooses to offer a quality with the maximum difference from their quality, while for broker $B_2$ it is more profitable to have maximum difference with both $B_1$ and $B_3$. As we have observed in the case of two brokers, it is not advisable to choose a quality of service similar to that of other providers.
Intuitively, this is because the more difference in the service quality that they offer customers, brokers are more likely to serve customers at a higher price. We note this by observing that the optimal quality for broker $B_2$ is the average of the other fixed brokers’ qualities ($q_1$ and $q_3$). For example, in the first plot from left in Figure 6, the optimal $q_2 = 50$, which is obtained from $(q_1 + q_3)/2 = (10 + 90)/2$.

We also change the fixed service qualities of brokers $B_1$ and $B_3$ toward the optimal quality for $B_2$ to see how the market changes and compare such results with those of Hotelling’s location model with more than two firms [3]. As we can see in Figure 6, unlike the Hotelling’s model [3] where corner firms have a tendency to move toward internal firms, here all brokers make less profit when quality differentiation decreases. However, for broker $B_3$, its market share increases (Figure 7), though the effect of dropping the price (Figure 8) is more pronounced than the extra share of the market. Therefore, unlike the Hotelling’s location model, for three firms, the market follows the maximum differentiation principle and brokers make more profit when their service qualities are more different from each other. In the following setting we study four brokers to see if this pattern repeats.

2) Four Brokers: In this setting, we simulate a scenario with two brokers, $B_1$ and $B_4$ already in the market and offering fixed service qualities $q_1 = 10$ and $q_4 = 90$, respectively, and two other brokers, $B_2$ and $B_3$, that enter the market later. Without loss of generality, we assume that $q_2 < q_3$. Figure 9 shows the changes in profit for brokers $B_2$ and $B_3$. We omit the results for $B_1$ and $B_4$ since they follow the same pattern as in the previous case study with three brokers, i.e., the more differentiation between their qualities and those we set for $B_2$ and $B_3$, the higher is their profit. This means that such brokers are not the decision makers in this situation.

As we observe in Figure 9, for broker $B_3$, whose quality is between $q_2$ and $q_4$, the optimal quality
q_3 \text{ value is one that yields maximum differentiation from both qualities } q_2 \text{ and } q_4, \text{ which is close to the average of } q_2 \text{ and } q_4. \text{ For broker } B_2 \text{ we expect instead that the optimal quality level is around } q_2 = 37, \text{ that is, the quality with maximum difference from } q_1 \text{ (10) and the optimal } q_3 \text{ (which equals 64 given maximum quality differentiation among all brokers). However, we observe that the optimal quality for } B_2 \text{ is at } q_2 = 45, \text{ when broker } B_2 \text{ switches from provider } S_1 \text{ to provider } S_2 \text{ and instead of upgrading the quality, downgrades the service that it obtains from provider } S_2 \text{ (recall that } Q_1 = 30 \text{ and } Q_2 = 60). \text{ To understand why } B_2 \text{ violates the maximum differentiation rule, we analyze the situations under both } q_2 = 37 \text{ and } q_2 = 45.

For } q_2 = 37, \text{ the optimal value for } B_3 \text{ is } q_3 = 56, \text{ and not the expected value of } q_3 = 64. \text{ To explain this situation, we should consider that in making profit, besides quality differentiation with other competitors (brokers), the cost of buying the lower-level service is also important. In this case, broker } B_3 \text{ makes more profit if it chooses } q_3 = 56 \text{ and downgrades the service it obtains from } S_2 \text{ (recall } Q_2 = 60) \text{ instead of choosing } q_3 = 64 \text{ and upgrading the service. Broker } B_3 \text{ can then offer a quality-price combination that attracts more customers, while because of the sufficient gap between } q_2 \text{ and } q_3, \text{ the competition on the price is not tough. However, in this situation, broker } B_2 \text{ is upgrading the service that it obtains from provider } S_1 \text{ (recall } Q_1 = 30) \text{ and to compete with broker } B_3, \text{ it cannot offer a high price, and the profit that it makes is relatively low.}

On the other hand, for } q_2 = 45, \text{ the situation is reversed. } B_2 \text{ downgrades the service that it obtains from provider } S_2, \text{ while } B_3 \text{ at its optimal point is upgrading the service. So the combination of quality-price of broker } B_2 \text{ attracts more customers which leads to making more profit. Therefore in this game, besides maximum quality differentiation, the cost that brokers undergo is also playing an important role}
and sometimes brokers should compromise on maximum differentiation to reduce cost.

Assuming rational players, i.e., the two new brokers pick the quality that maximizes their profit, we compare the price of the service that such brokers offer for the case studies of three and four brokers (Figures 8 and 10). In the case of three brokers, we observe that the optimal quality for broker $B_2$ is at $q_2 = 50$ while $q_1 = 10$ and $q_3 = 90$. The optimal price for brokers in this setting is $p_1 = 2092$, $p_2 = 3203$ and $p_3 = 5447$, respectively. When four brokers are playing the game, the optimal quality for broker $B_2$ is $q_2 = 45$ and for broker $B_3$ is $q_3 = 66$. In this situation, the optimal prices are $p_1 = 1746$, $p_2 = 2535$, $p_3 = 3442$ and $p_4 = 4869$. As we can see, the price of service with quality 10 and 90 drops from 2092 and 5447 to 1746 and 4869, respectively. So, when there are more competitors in the market, the gap between their service qualities decreases, the competition on the price becomes tougher and brokers should offer their services at lower prices to be able to attract customers and make profit.

V. CONCLUSION

In this paper, we developed a game-theoretic model that captures the interaction among players in a multi-level market. In our model, brokers, as the intermediaries between users and service providers, adapt the quality of the service that they get from lower-level providers so as to attract more customers and maximize their profit. The game consists of two service providers, two, three or four brokers, and users, though we study more extensively the case with two brokers. Numerical results show that the more differentiation between the quality of service offered by brokers, the higher is their profit. However in some situations, besides quality differentiation, cost plays an important role and forces brokers to compromise on quality differentiation with their competitors to reduce cost and make more profit. An interesting result
in the two brokers game is that although players compete for more profit, the competition only affects their market share; the profit increases for one player if it increases for the other one. But this is not the case for more brokers. When there are more than two brokers, the market is more competitive and brokers should offer their services at lower prices to be able to stay in the market.

We also studied situations where all brokers prefer to buy service from just one of the lower-level providers, \( i.e. \), the providers’ market is about to become a monopoly. We developed a Bertrand game with price constraints to keep the market as an oligopoly if possible. Moreover, for the two brokers game, we explored the case where customers buy the service only if the combination of price-quality has positive utility for them. In this situation, players (brokers) try to offer the service cheaper to attract more customers. Unlike the unconstrained utility game with two brokers, if profit increases for one player, profit decreases for the other player.

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Price of Brokers, $q_1 = 10$, $q_4 = 90$

![Price of Brokers Diagram]

Fig. 10. Price of each broker in a four-broker setting.


