Growth in arithmetic understandings developed during the course 'Methods in teaching arithmetic.'

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http://hdl.handle.net/2144/13067

Boston University
BOSTON UNIVERSITY
SCHOOL OF EDUCATION

Thesis

GROWTH IN ARITHMETIC UNDERSTANDINGS
DEVELOPED DURING THE COURSE "METHODS IN TEACHING ARITHMETIC"

Submitted by

Marcella M. Moran
(B.B.A., Northeastern University, 1953)

In Partial Fulfillment of Requirements for
the Degree of Master of Education

1955
ACKNOWLEDGMENT

The writer wishes to express her appreciation to Dr. Weaver for his continual aid and constant patience.
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CHAPTER I

THE PROBLEM

Purpose.-- It was the purpose of this study to:

1. Determine the mathematical understandings possessed by a group of college students at the beginning of a methods course in the teaching of arithmetic.

2. Compare the level of mathematical understanding possessed by this group with a similar group previously studied by Cristiani, et al.¹ as they begin the course in methods of teaching arithmetic.

3. Determine if the students increase their mathematical understandings during the course in methods of teaching arithmetic. The amount of the increase and the areas in which the increases occurred are to be determined.

4. Determine the degree of difficulty and the discriminating power of each item in Dr. Vincent Glennon's "Test of Basic Mathematical Understandings."

Justification.-- In recent years arithmetic instruction has placed more and more emphasis upon the development of important mathematical understandings. In our attempt to provide meaningful teaching and learning experiences, we have come to realize that all too often the teacher herself

¹Vincent A. Cristiani, Nicholas J. Giacobbe, and Joseph G. Thibeault, A Study of Mathematical Understanding Possessed by Prospective Elementary School Teachers, Master's Thesis, Boston University, School of Education, Boston, 1954.
represents one of the greatest obstacles to the fulfillment of our objectives. In many instances she herself does not possess the necessary background of essential mathematical understandings.

This problem must be attacked at both the in-service and pre-service levels. Our concern in the present investigation, however, centers only around the latter level. A pioneer investigation conducted and reported by Glennon\(^1\) showed undergraduate students in State Teachers Colleges to be willfully weak in their background of mathematical understandings. An investigation conducted here at the Boston University School of Education by Cristiani\(^2\) and others just last year confirmed the condition elsewhere as reported by Glennon.

Considerable attention has been directed in the professional literature to the need for a background course in content mathematics prior to the professional or methods course in the teaching of arithmetic as one approach to the solution of this problem. At the present time such a course, truly appropriate in terms of its content, is provided in all too few of our training programs. In those situations in which there is no background course in mathematics that is prior to the professional course, our greatest hope lies in devoting needed attention to content background in the methods course itself. This is the approach which has been taken, of necessity, as far as the Boston University School of Education is con-

\(^1\)Vincent J. Glennon, A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels, Doctor's Dissertation, Harvard University, Graduate School of Education, Cambridge, 1948.

\(^2\)Vincent A. Cristiani, et al., op. cit.
cerned. Unfortunately, however, there has been no objective measurement to date of the effectiveness of this combined content-methods course in raising the level of mathematical understanding of undergraduate students in the elementary education training program. This need for evaluating the effectiveness of one aspect of our existing course in "Methods of Teaching Arithmetic" at the undergraduate level provides the basic justification for the present investigation.

Scope and Limitations.-- The present investigation involved 67 undergraduate students who were enrolled in the course, "Methods of Teaching Arithmetic," at the Boston University School of Education during the first semester of the 1954-55 academic year. This group consisted mainly of juniors, with a few seniors, sophomores, and full-time graduate students. None had previous teaching experience in the elementary school. Although their previous mathematical experiences and background varied, none of the students had had previous instruction in the types of mathematical understandings to be studied.

The basic data for this study were obtained through the use of "A Test of Basic Mathematical Understandings" constructed by Dr. Vincent J. Glennon and used with his permission. This test was first administered to the group at the outset of the course, before any pertinent instruction was given, and again during the last session of the course. The validated instrument tested the students' understandings relating to the four fundamental processes of addition, subtraction, multiplication, and division as used with whole numbers, common fractions, and decimal fractions. On both occasions this test was administered and supervised by the instructors in the course.
For administration reasons, a time limit of 65 minutes was imposed. This total working time was divided proportionately among the five sections of the test.
CHAPTER II
REVIEW OF RELATED LITERATURE AND RESEARCH

Fundamental to this study are the statements made concerning the lack of arithmetic understandings of prospective and in-service teachers. The teaching of these understandings is necessary if children are to learn arithmetic. One can teach no more than he knows; therefore, teachers must develop a workable knowledge of arithmetic. Teachers Colleges and Schools of Education should aid in this.

Educators have put much emphasis on the lack of background training in mathematics in the admission requirements of colleges. Summarized by Weaver,\(^1\) the research by Grossnickle, by Layton, and by Rhoads shows the extent of this weakness:

"Grossnickle's chief sources of data included general professional literature on the subject, catalogs from liberal arts colleges and universities which have departments or colleges of education which prepare teachers for the elementary school, and replies from a questionnaire sent to accredited teachers colleges. Layton's data were drawn from official certification rules and regulations for teachers of mathematics Grades I through XII, for each of the states and the District of Columbia; and from replies to questionnaires submitted to each state certification officer and to nationally recognized specialists in mathematics. Rhoads secured her data from normal school and teachers college catalogs; from letters of inquiry sent to the superintendent of schools, or public instruction, in each state and the District of Columbia; and from previous research and general professional literature on the subject.

All three studies pointed to the extremely low requirements in the subject matter of mathematics for those preparing to teach in the elementary schools. Both Grossnickle and Rhoads reported that three

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fourths of the teachers colleges require no credits in secondary-school mathematics for admission. Furthermore, Layton reported that only ten states require any work in background mathematics at the college level for certifying applicants without teaching experience. The mean requirements were reported as .52 semester hours. Paralleling this condition, Grossnickle reported that only one out of 62 departments of education in liberal arts colleges, and only three out of 18 colleges of education require any work in background mathematics as part of the training program for elementary-school teachers. The mean requirements were reported as .1 semester hours for the departments of education and .5 semester hours for the colleges of education. The condition in the state teachers colleges is slightly better, but not encouragingly so. Between one third and two thirds of the state teachers colleges require work in background mathematics, depending upon the different general or specialized curriculums offered. The mean requirements ranged between 1.2 semester hours and 2.0 semester hours. Generally speaking, both Grossnickle and Rhoads found that during the past 25 years, whereas the overall training period for teachers in the elementary school has increased from a mode of two years to a mode of four years, the amount of required work in background mathematics has decreased dangerously. Both Grossnickle and Layton recommend six semester hours of work in background mathematics as a minimum requirement for elementary-school teachers, regardless of curriculum level of specialization.

The concern of various authorities in the field of mathematics to the need for increased preparation of teachers of arithmetic has been evidenced in many professional articles. Most of them feel that Teachers Colleges and Schools of Education should stress mathematical understandings in their courses in the teaching of arithmetic. Quotations from some of the writings of experts in the field of mathematics will serve as evidence of this.

Discussing the training of teachers of arithmetic, Wren lists six objectives which he feels are basic to the preparation of competent arithmetic teachers. "No teacher can hope to lead a student to a level of functional competence higher than that which the teacher himself has attained."

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The Second Report of the Commission of Post-War Plans\(^1\) stated that:

"...prospective teachers must learn how to develop meanings, understandings, generalizations, a pure grasp of relationships, and the like.... A well-prepared teacher of mathematics should have adequate training so that he can meet all classroom situations with that assurance which can be based only on wide knowledge and rich background."

In his article, "A Crucial Aspect of Meaningful Arithmetic Instruction," Weaver\(^2\) lists three problems of the pre-teacher and in-service teacher of arithmetic:

1. They must recognize the necessity of meaningful instruction as a prerequisite of functional competence.
2. They must have an understanding of meanings to be developed, both from the level of experience and maturity of the pupil being taught and from that of the teacher herself.
3. They must be conscious of the psychological and methodological aspects of a meaningful instructional program.

Both Schaaf\(^3\) and Newsom\(^4\) have stated in articles that a large number of elementary school teachers are unprepared to teach arithmetic.

"In 1951, Wilburn and Wingo\(^5\) wrote of the importance of providing prospective elementary school teachers with a better understanding of arithmetic."

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"One thing which is probably needed is for teacher-training institutions to pay more attention to providing prospective elementary school teachers with a better understanding of arithmetic and the number system."

All of the writings cited here emphasize the need for improving the preparation of teachers of arithmetic. This training should stress content material, especially the basic arithmetic understandings. Dr. Vincent J. Glennon\(^1\) strikingly pointed up this need in his doctor's dissertation.

A summary of his findings are the following:

"The Teachers College freshman (144 students were tested on an 80-item test understands 44% of the basic understandings tested. These understandings are basic to the computational processes commonly taught in grades one through six!"

The Teachers College senior (172 students were tested understands about 43% of these basic mathematical understandings!"

Between those students tested who had completed a course in the "Psychology and Teaching of Arithmetic" and those seniors who had not taken the course, there was no significant difference in their mathematical understandings. Although this course "made some significant contribution to the professional preparation of the prospective teachers investigated," this work caused "no significant change in their level of mathematical understanding, at least of the types tested."

A study by Orleans\(^2\) showed basically the same results:

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"...prospective teachers have difficulty in verbalizing their thoughts when they try to explain arithmetic concepts and processes," but also

"...that there are few arithmetic concepts and processes that can be readily explained by a large percentage of prospective teachers."

It is significant to observe that both Glennon and Orleans make similar statements on the basis of their research findings.

"One aspect of the needed redirection of the training of teachers of arithmetic seems to lie in the professional training offered in the teachers colleges and schools of education.... Little emphasis is placed on the professional study of arithmetic as a science of numbers, -- as a system of related ideas -- as a series of number relationships.... Understanding--this is the frontier of needed redirection in the training of teachers of arithmetic."

"If the understanding of arithmetic possessed by teachers is to be increased, teacher-training institutions must make this one of their goals. The teacher-education institution may have only an indirect influence on the program of number work in the schools, but they can directly influence the prospective teacher's knowledge and understanding of arithmetic and his preparation for his responsibilities in getting children to learn about numbers."

The results of the study by Cristiani, et al. further proves the need for instruction of pre-teachers in the basic mathematical understandings:

"1. Based on the administration of Glennon's Test of Basic Mathematical Understandings, the group of 92 students at the outset of the course, Methods of Teaching Arithmetic as a whole had a relatively low level of understanding of important quantitative concepts commonly taught in the elementary grades. The mean number of items correct on the Glennon Test was but 56% of the total number of items.


3Vincent A. Cristiani, et al., op. cit.
2. ...the performance of the group as a whole was especially poor in such concepts as multiplication and division of fractions, numerical understandings of size and place value of decimals, and an understanding of the processes involved in multiplying by a two- or-more-figured multiplier.

3. The results ... clearly point to the need for systematic instruction in basic mathematical understandings as part of the pre-service training of elementary school teachers."

With this evidence as background, the present study sought to investigate the growth in arithmetic understandings of a group of students in Boston University's School of Education who received instruction in basic mathematical concepts during the course, "Methods of Teaching Arithmetic."
CHAPTER III

PROCEDURE

In September, 1954 and in January, 1955 the Glennon "Test of Basic Mathematical Understandings" was administered to the students in the course "Methods of Teaching Arithmetic" given at Boston University's School of Education. For this study, only those students who were tested twice were used. These numbered 69.

Having had its validity and reliability previously determined through a research study, the test was used with only two changes made by Professor J. Fred Weaver with the permission of Dr. Glennon, the author.

The test, consisting of 80 items, is grouped into five sections as follows:

Section I: The Decimal System of Notation.
Section II: Basic Understandings of Integers and Processes.
Section III: Basic Understandings of Fractions and Processes.
Section IV: Basic Understandings of Decimals and Processes.
Section V: Basic Understandings of the Rationale of Computation.

A time limit of 65 minutes was put on the test with proportionate time limits put on each section. During the first administration of the test 40 per cent of the students completed the 80 items on the test. During the second administration 77 per cent of the students completed all of the items.

For ease in scoring, each paper had the following tabulation done for each section of the test and the test as a whole: items right, items wrong, items omitted. For each section of the test and for the test as a whole,
each student's raw score was represented by the number right.

Comparative frequency distributions were set up for the raw scores on each section and on the test as a whole as obtained by the group of college students tested in 1953 and the 1954 group tested twice. From these, the means and standard deviations were derived for each group as shown in Tables I to VI.

The distribution of the gains and losses in score from the pre-test to the end-test as taken by the 1954 group of college students on each section of the test and on the whole test are shown in Tables VII and VIII together with their means and standard deviations.

In order to determine the difficulty level and discriminating power of the items of the test, Fan's Item Analysis Table was used. These indexes are shown in Tables XIII to XVII.

Using the pre-test results of the 1953 and 1954 groups, the writer obtained the t ratio of significance between means for each section and for the test as a whole. These data are shown in Table IX.

The t ratio of significance of the differences in mean score for each section and for the test as a whole from the 1954 pre-test and end-test are shown in Table X.

The relationship between the gains on the test when taken by the 1954 group at the end of the course in methods of teaching arithmetic and their scores on the test when taken at the beginning of the course are shown for each section and for the test as a whole in Table XI.

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The relationship between the per cent of gain in the end-test over the results in the pre-test is shown for each section and the whole test in Table XII.
CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

The basic data from the administration of Glennon's "Test of Basic Mathematical Understandings" is presented below (Tables I-VI) in the form of comparative frequency distributions of the raw scores (Number of items correct) on each section of the test and on the test as a whole for the pre-test of the 1953 group and the pre-test and end-test of the 1954 group.
### TABLE I

Distribution of Scores on *A Test of Basic Mathematical Understandings, Section I, "The Decimal System of Notation".*

<table>
<thead>
<tr>
<th>Number Right</th>
<th>Pre-Test 1953 Group</th>
<th>Pre-Test 1954 Group</th>
<th>End-Test 1954 Group</th>
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</thead>
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<td>2</td>
<td>7</td>
<td>13</td>
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<tr>
<td>14</td>
<td>10</td>
<td>5</td>
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</tr>
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<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>N</td>
<td>92</td>
<td>69</td>
<td>69</td>
</tr>
</tbody>
</table>

\[ M_{1953} = 10.61 \quad M_{1954} = 10.88 \quad M_{1954} = 12.61 \]

\[ \sigma_{1953} = 2.62 \quad \sigma_{1954} = 2.91 \quad \sigma_{1954} = 1.75 \]
These data show that the mean number of items correctly done by the 1953 pre-test group and the 1954 pre-test is approximately the same with the 1953 group having 71% (10.61) of the 15 items correct and the 1954 pre-test group having 73% of the 15 items correct.

The results of the end-test taken by the 1954 group show an increase of 16% over their pre-test. The mean number of items correct in the end-test (12.61) represents approximately 84% of the 15 items in this section of the test.
TABLE II

Distribution of Scores on A Test of Basic Mathematical Understandings, Section II: "Basic Understandings of Integers and Processes"

<table>
<thead>
<tr>
<th>Number Right</th>
<th>Pre-Test 1953 Group</th>
<th>Pre-Test 1954 Group</th>
<th>End-Test 1954 Group</th>
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<td>9</td>
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<tr>
<td>13</td>
<td>17</td>
<td>11</td>
<td>20</td>
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<tr>
<td>N</td>
<td>92</td>
<td>69</td>
<td>69</td>
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</tbody>
</table>

\[
\begin{align*}
M &= 11.38 \\
\sigma &= 1.77 \\
M &= 11.22 \\
\sigma &= 2.09 \\
M &= 12.52 \\
\sigma &= 1.75
\end{align*}
\]

These data show that the mean number of items correctly done by the 1953 pre-test group and the 1954 pre-test group is approximately the same with the 1953 group having 76\% (11.38) of the 15 items correct and the 1954 pre-test group having 75\% (11.22) of the 15
items correct in this section of the test.

The results of the end test taken by the 1954 group show an increase of approximately 12% over their pre-test score. The mean number of items correct in the end-test (12.52) represents approximately 83% of the 15 items in this section of the test.
\textbf{TABLE III}

\textit{Distribution of Scores on A Test of Basic Mathematical Understandings, Section III: "Basic Understandings of Fractions and Processes".}

<table>
<thead>
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\[ M = 7.03 \quad \text{and} \quad \sigma = 2.65 \]
\[ M = 7.32 \quad \text{and} \quad \sigma = 2.42 \]
\[ M = 9.88 \quad \text{and} \quad \sigma = 2.31 \]

The mean number of items correctly done by the 1953
pre-test group and the 1954 pre-test group is approximately the same with the 1953 group having about 47% of the 15 items correct and the 1954 pre-test group having about 49% of the 15 items correct in this section of the test. An increase in the mean of approximately 35% over the 1954 pre-test is shown by the 1954 end-test mean (9.88). This represents a total of 66% of the items.
<table>
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<td>6</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>N</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE IV

Distribution of Scores on A Test of Basic Mathematical Understanding, Section IV: "Basic Understandings of Decimals and Processes"
Distribution of Scores on A Test of Basic Mathematical Understanding, Section IV: Basic Understandings of Decimals and Processes.

\[ M = 9.27 \quad M = 9.12 \quad M = 11.70 \]
\[ \sigma = 4.28 \quad \sigma = 3.86 \quad \sigma = 3.61 \]

The mean number of items correctly done by the 1953 pre-test group and the 1954 pre-test group is approximately the same with the 1953 group having about 64% (9.27) of the 20 items correct and the 1954 group having 46% (9.12) of the items in this section correct.

Between the pre-test and the end-test, the 1954 group showed a gain of 28% over their previous mean (9.12). The end-test mean (11.70) represents approximately 59% of the 20 items in this section of the test.
### TABLE V

Distribution of Scores on A Test of Basic Mathematical Understandings, Section V: "Basic Understandings of the Rationale of Computation".

<table>
<thead>
<tr>
<th>NUMBER RIGHT</th>
<th>Pre-Test 1953 Group</th>
<th>Pre-Test 1954 Group</th>
<th>End-Test 1954 Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>13</td>
<td>4</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>92</td>
<td>69</td>
<td>69</td>
</tr>
</tbody>
</table>

$M = \bar{X}_{1953} = 6.55$  
$M = \bar{X}_{1954} = 6.71$  
$M = \bar{X}_{1954} = 9.93$

$\sigma = 3.15$  
$\sigma = 2.72$  
$\sigma = 2.79$
The means attained by the 1953 pre-test group (6.55) and the 1954 pre-test group (6.71) represent approximately 4.4% and 45% of the fifteen items in this section of the test. A gain of 48% was made between the means of the 1954 pre-test (6.71) and the mean of the end-test (9.93). In the end test, approximately 66% of the 15 items in this section of the test were done correctly by the group.
<table>
<thead>
<tr>
<th>NUMBERS RIGHT</th>
<th>Pre-Test 1953 Group</th>
<th>Pre-Test 1954 Group</th>
<th>End-Test 1954 Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>77-79</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>74-76</td>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>71-73</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>68-70</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>65-67</td>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>62-64</td>
<td>1</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>59-61</td>
<td>7</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>56-58</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>53-55</td>
<td>6</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>50-52</td>
<td>7</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>47-49</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>44-46</td>
<td>13</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>41-43</td>
<td>12</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>38-40</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>35-37</td>
<td>11</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>32-34</td>
<td>10</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>29-31</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>26-28</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>23-25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-22</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>17-19</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| N             | 92                  | 69                  | 69                  |
Distribution of Scores on A Test of Basic Mathematical Understandings: Whole Test.

\[ M = 44.84 \quad M = 45.26 \quad M = 56.65 \]
\[ \sigma = 11.02 \quad \sigma = 11.21 \quad \sigma = 9.53 \]

The mean of the 1953 pre-test group (44.84) represents 56% of the items on the test were correctly done. In the 1954 pre-test group, the mean (45.26) shows that 57% of the test items were correctly done. The increase in mean of the end-test (56.65) shows that 71% of the test items were correctly done.
TABLE VII

Distribution of Gains and Losses from Pre-Test to End-Test for 1954 Group on Each Section of A Test of Basic Mathematical Understandings.

<table>
<thead>
<tr>
<th>GAIN/LOSS</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>-11</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-10</td>
<td></td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-9</td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>-8</td>
<td>2</td>
<td></td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>-7</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-6</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>-5</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>-4</td>
<td>6</td>
<td>3</td>
<td>8</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>-3</td>
<td>8</td>
<td>12</td>
<td>14</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>-2</td>
<td>12</td>
<td>16</td>
<td>13</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>7</td>
<td>9</td>
<td>7</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>14</td>
<td>16</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>10</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

N = 69 69 69 69 69 69

M = 1.72  M = 1.30  M = 2.57  M = 2.58  M = 3.22
σ = 2.41  σ = 1.87  σ = 2.36  σ = 3.07  σ = 2.43
From the above means, one can note that the overall gain in score between the pre-test of the 1954 group and the end-test of the same group was small to moderate. No outstanding improvement was made. Section V showed the largest mean gain (3.22) with Section II showing the least (1.30). This may well be due to the fact that many students during the pre-test did not complete Section V (despite the fact that time was provided for all students to work on this section) thus giving it the lowest mean of the pre-test and therefore the largest possible gain.
TABLE VIII

Distribution of Gains and Losses from Pre-Test to End-Test for 1954 Group on the Whole Test of A Test of Basic Mathematical Understandings.

<table>
<thead>
<tr>
<th>GAIN/LOSS</th>
<th>FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>31 to 33</td>
<td>1</td>
</tr>
<tr>
<td>28 to 30</td>
<td>0</td>
</tr>
<tr>
<td>25 to 27</td>
<td>0</td>
</tr>
<tr>
<td>22 to 24</td>
<td>3</td>
</tr>
<tr>
<td>19 to 21</td>
<td>4</td>
</tr>
<tr>
<td>16 to 18</td>
<td>10</td>
</tr>
<tr>
<td>13 to 15</td>
<td>13</td>
</tr>
<tr>
<td>10 to 12</td>
<td>13</td>
</tr>
<tr>
<td>7 to 9</td>
<td>11</td>
</tr>
<tr>
<td>4 to 6</td>
<td>5</td>
</tr>
<tr>
<td>1 to 3</td>
<td>4</td>
</tr>
<tr>
<td>2 to 0</td>
<td>3</td>
</tr>
<tr>
<td>5 to -3</td>
<td>2</td>
</tr>
</tbody>
</table>

\[M = 11.39\]  \[\sigma = 6.76\]

On the pre-test of the 1954 group, the mean score was 45.26 which represents 57% of the 80 items on the test that were correctly done. The mean gain of 11.39 increases this
per cent of correct items to 71% of the 80 items on the test.
TABLE IX

$t$ Ratio of significance of the scores for each section of the test and the test as a whole using the 1953 and 1954 pre-test scores.

<table>
<thead>
<tr>
<th>Part</th>
<th>1953 Pre-Test (N=92)</th>
<th>1954 Pre-Test (N=69)</th>
<th>Diff. in M.</th>
<th>S.E.</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$M$</td>
<td>$\sigma$</td>
<td>$m$</td>
<td>$\sigma$</td>
<td>$0.27$</td>
</tr>
<tr>
<td>Part I</td>
<td>10.61</td>
<td>2.62</td>
<td>10.83</td>
<td>2.91</td>
<td>0.27</td>
</tr>
<tr>
<td>Part II</td>
<td>11.38</td>
<td>1.77</td>
<td>11.22</td>
<td>2.09</td>
<td>0.16</td>
</tr>
<tr>
<td>Part III</td>
<td>7.03</td>
<td>2.65</td>
<td>7.32</td>
<td>2.42</td>
<td>0.29</td>
</tr>
<tr>
<td>Part IV</td>
<td>9.27</td>
<td>4.28</td>
<td>9.12</td>
<td>3.86</td>
<td>0.15</td>
</tr>
<tr>
<td>Part V</td>
<td>6.55</td>
<td>3.15</td>
<td>6.71</td>
<td>2.72</td>
<td>0.16</td>
</tr>
<tr>
<td>WHOLE TEST</td>
<td>44.84</td>
<td>11.02</td>
<td>45.26</td>
<td>11.21</td>
<td>0.42</td>
</tr>
</tbody>
</table>

To be significant at the 5% level with 159 degrees of freedom, the $t$ ratio must be at least 1.98; and to be significant at the 1% level, $t$ must be at least 2.61. Since the $t$ ratios for each section and for the test as a whole are considerably less than these amounts, the differences in scores on the pre-tests of the 1953 and 1954 groups can be attributed largely to chance.
TABLE X

Table of significance of differences in score for each section and the test as a whole from the 1954 Pre-Test and End-Test.

<table>
<thead>
<tr>
<th>Part</th>
<th>1954 Pre-Test (N=69)</th>
<th>1954 End-Test (N=69)</th>
<th>Diff. per Tab. VII&amp;VIII</th>
<th>SE</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
<td>σ</td>
<td>m</td>
<td>σ</td>
<td>m</td>
</tr>
<tr>
<td>Part I</td>
<td>10.88</td>
<td>2.91</td>
<td>12.61</td>
<td>1.75</td>
<td>1.72</td>
</tr>
<tr>
<td>Part II</td>
<td>11.22</td>
<td>2.09</td>
<td>12.52</td>
<td>1.75</td>
<td>1.30</td>
</tr>
<tr>
<td>Part III</td>
<td>7.32</td>
<td>2.42</td>
<td>9.88</td>
<td>2.31</td>
<td>2.57</td>
</tr>
<tr>
<td>Part IV</td>
<td>9.12</td>
<td>3.36</td>
<td>11.70</td>
<td>3.61</td>
<td>2.58</td>
</tr>
<tr>
<td>Part V</td>
<td>6.71</td>
<td>2.72</td>
<td>9.93</td>
<td>2.79</td>
<td>3.22</td>
</tr>
<tr>
<td>WHOLE TEST</td>
<td>45.26</td>
<td>11.21</td>
<td>56.65</td>
<td>9.53</td>
<td>11.39</td>
</tr>
</tbody>
</table>

To be significant at the 5% level with 68 degrees of freedom, the t ratio must be at least 1.67, and to be significant at the 1% level, t must be at least 2.38. In that the t ratio for each section and for the test as a whole are highly significant, the gains cannot be accounted for on the basis of chance and very reasonably may be attributed to the instruction given in the course, Methods of Teaching Arithmetic.
TABLE XI

Correlation Between Gain on the End-Test and the Initial Status of the 1954 Group.

<table>
<thead>
<tr>
<th>SECTION</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-0.790</td>
</tr>
<tr>
<td>II</td>
<td>-0.565</td>
</tr>
<tr>
<td>III</td>
<td>-0.538</td>
</tr>
<tr>
<td>IV</td>
<td>-0.473</td>
</tr>
<tr>
<td>V</td>
<td>-0.412</td>
</tr>
<tr>
<td>WHOLE TEST</td>
<td>-0.750</td>
</tr>
</tbody>
</table>

All the above coefficients in Table XI are highly significant statistically, since r must reach a value of only 0.309 to be significant at the 1% level when df = 67.

The fact that all coefficients are negative is indicative of the tendency for students who had the highest scores on the pre-test to make the least amount of gain on the end-test; and conversely, for those who received the lowest scores on the pre-test to make the greatest gains on the end-test.

This observation is not surprising, since those who made high scores on the pre-test had less opportunity to gain on the end-test than those who made low scores on the pre-test. Conversely, those who received low
scores on the pretest had greater opportunity to gain on the end-test than those who made high scores on the pretest.

Because of this condition, another method was used to investigate the relation between pre-test scores and gains on the end-test. The basis for this method can be illustrated best by a simple example involving the performance of two students.

<table>
<thead>
<tr>
<th>STUDENT</th>
<th>Pre-Test Score</th>
<th>End-Test Score</th>
<th>Actual Gain</th>
<th>Possible Gain</th>
<th>% of Possible Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>68</td>
<td>18</td>
<td>30</td>
<td>60%</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>55</td>
<td>25</td>
<td>50</td>
<td>50%</td>
</tr>
</tbody>
</table>

In this illustration we see that Student A had a higher pre-test score and lower actual gain on the end-test than Student B. However, when the gain made by each student is compared with the maximum gain he might have made (difference between pre-test score and total items in test, or section), we see a somewhat different picture. Student A had the higher pre-test score and also the higher per cent of possible gain.

Table XII which follows summarizes the relationships found in this way between pre-test scores and gains expressed as per cents of possible gains.
Correlation Between the Per Cent of the Possible Gain on the End-test and the Initial Status of the 1954 Group

<table>
<thead>
<tr>
<th>Section</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-0.206</td>
</tr>
<tr>
<td>II</td>
<td>0.095</td>
</tr>
<tr>
<td>III</td>
<td>-0.050</td>
</tr>
<tr>
<td>IV</td>
<td>-0.257</td>
</tr>
<tr>
<td>V</td>
<td>-0.016</td>
</tr>
<tr>
<td>Whole Test</td>
<td>-0.321</td>
</tr>
</tbody>
</table>

When the test sections are considered separately, in only one instance (Section IV) was there a significant correlation. Here the r of -0.257 exceeded the value of ±0.237 needed for significance at the 5 per cent level. For the test as a whole, however, there was a highly significant correlation (-0.321, which exceeds the value of ±0.309 needed for significance at the 1 per cent level). Thus, for Section IV and for the test as a whole, there was a significant tendency for those with highest initial score to make lowest per cent of gain. Although statistically significant, the relationship is not a strong one.
Item Analysis Showing Indices of Difficulty and Discrimination
for Each Test Item

Using Fan's **Item Analysis Table** to determine the difficulty level and discriminating power of each item, the writer tabulated two indices of difficulty (p and $\Delta$) and one index of discrimination (r).

Cristiani, et al.\(^1\) described the procedure for the interpretation of the results attained through the use of Fan's table.

"In the Tables which follow, (p) represents the proportion of the total population passing an item in the highest 27% and the lowest 27% of the total scores; (\(\Delta\)) represents a statistic related to p in such a manner that \(\Delta\) increases with increasing item difficulty, and that equal increments in \(\Delta\) represent equal increments of difficulty; and (r) represents the biserial correlation between success or failure on each test item and total score.

In interpreting the data presented in Tables XIII-XVII, values of 8.0 and below are considered to be indicative of very easy items which have been mastered well by the group tested. \(\Delta\) values of 18.0 and above are to be considered indicative of very difficult items which have not been mastered by the group tested. Furthermore, r values of .40 and above have been interpreted as good levels of discrimination. Items for which r is between .30 and .39 are interpreted as being questionable in their discriminating power. Finally, items for which r fell below .30 discriminate poorly and merit serious study with a view toward revision."

If no value was indicated in Fan's table, the symbol (*) was placed to the right of the number to show that the number was approximated.

---

\(^1\) Vincent A. Cristiani, et al., *op. cit.*, p. 27.
TABLE XIII

Item Analysis Showing Indices of Difficulty and Index of Discrimination of Each Test Item of Section I (the Decimal System of Notation) of the Test

<table>
<thead>
<tr>
<th>Item Number</th>
<th>1953 p</th>
<th>1954 p</th>
<th>Δ</th>
<th>1953 Δ</th>
<th>1954 Δ</th>
<th>1953 r</th>
<th>1954 r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00*</td>
<td>&gt; .95*</td>
<td>Δ &lt; 6.4*</td>
<td>&lt; 6.5*</td>
<td>.00*</td>
<td>&gt; .43*</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.98*</td>
<td>7.95*</td>
<td>Δ &lt; 6.4*</td>
<td>&lt; 6.3*</td>
<td>r &lt; 0.00*</td>
<td>&gt; .26*</td>
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<td>.79</td>
<td>&gt; .85*</td>
<td>9.7</td>
<td>&lt; 8.9*</td>
<td>.40</td>
<td>&gt; .67*</td>
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<td>4</td>
<td>.79</td>
<td>.81</td>
<td>9.7</td>
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<td>.54</td>
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</tr>
<tr>
<td>5</td>
<td>.90*</td>
<td>.81</td>
<td>7.8*</td>
<td>9.5</td>
<td>.57</td>
<td>.54</td>
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<tr>
<td>6</td>
<td>.71</td>
<td>.79</td>
<td>10.7</td>
<td>9.7</td>
<td>.43</td>
<td>.59</td>
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<tr>
<td>7</td>
<td>.83</td>
<td>7.89*</td>
<td>9.2</td>
<td>Δ &lt; 8.1*</td>
<td>.51</td>
<td>&gt; .60*</td>
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</tr>
<tr>
<td>8</td>
<td>.78</td>
<td>.56</td>
<td>9.8</td>
<td>12.4</td>
<td>.28</td>
<td>.25</td>
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<td>.81</td>
<td>10.9</td>
<td>9.5</td>
<td>.46</td>
<td>.54</td>
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</tr>
<tr>
<td>10</td>
<td>.62</td>
<td>.64</td>
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<td>11.6</td>
<td>.58</td>
<td>.55</td>
<td></td>
</tr>
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<td>.58</td>
<td>.74</td>
<td>12.2</td>
<td>10.4</td>
<td>.64</td>
<td>.50</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>.56</td>
<td>.64</td>
<td>12.4</td>
<td>11.6</td>
<td>.25</td>
<td>.35</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>.60</td>
<td>.63</td>
<td>12.0</td>
<td>11.7</td>
<td>.00</td>
<td>.66</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>.48</td>
<td>.60</td>
<td>13.2</td>
<td>11.9</td>
<td>.16</td>
<td>.33</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>.39</td>
<td>.42</td>
<td>14.1</td>
<td>13.8</td>
<td>.50</td>
<td>.64</td>
<td></td>
</tr>
</tbody>
</table>

For the most part, the understandings measured by items 1, 2, 5, and 7 gave little or no difficulty to the 1953 or 1954 pre-test groups.

Items 1, 2, 8, and 14 discriminated poorly between those who had high and low scores on the test as a whole for both pre-test groups.

In addition, items 12 and 13 discriminated poorly with the 1953 pre-test group.
### TABLE XIV

Item Analysis Showing Indices of Difficulty and Index of Discrimination of Each Test Item of Section II
(Basic Understandings of Integers and Processes) of the Test

<table>
<thead>
<tr>
<th>Item Number</th>
<th>1953 p</th>
<th>1954 p</th>
<th>1953 Δ</th>
<th>1954 Δ</th>
<th>1953 r</th>
<th>1954 r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>Δ&lt;6.4</td>
<td>&lt;6.4</td>
<td>.00*</td>
<td>0.00</td>
</tr>
<tr>
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<td>.98*</td>
<td>&gt;.95*</td>
<td>Δ&lt;6.4</td>
<td>&lt;6.4*</td>
<td>r&lt;0.00*</td>
<td>&lt;0.00*</td>
</tr>
<tr>
<td>3</td>
<td>.90</td>
<td>&gt;.95*</td>
<td>7.9</td>
<td>&lt;6.3*</td>
<td>-.09</td>
<td>&gt;.26*</td>
</tr>
<tr>
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<td>8.9</td>
<td>9.9</td>
<td>.28</td>
<td>.17</td>
</tr>
<tr>
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<td>.60</td>
<td>.56</td>
<td>12.0</td>
<td>12.4</td>
<td>.24</td>
<td>.41</td>
</tr>
<tr>
<td>6</td>
<td>.92*</td>
<td>.80</td>
<td>7.4*</td>
<td>9.6</td>
<td>.53*</td>
<td>.24</td>
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<td>.84</td>
<td>.78</td>
<td>9.0</td>
<td>10.0</td>
<td>.48</td>
<td>.59</td>
</tr>
<tr>
<td>8</td>
<td>.74</td>
<td>.56</td>
<td>10.4</td>
<td>12.4</td>
<td>.15</td>
<td>.33</td>
</tr>
<tr>
<td>9</td>
<td>.65</td>
<td>.63</td>
<td>11.4</td>
<td>11.7</td>
<td>.43</td>
<td>.66</td>
</tr>
<tr>
<td>10</td>
<td>.83</td>
<td>.70</td>
<td>9.2</td>
<td>10.9</td>
<td>.51</td>
<td>.46</td>
</tr>
<tr>
<td>11</td>
<td>.10*</td>
<td>&gt;.22*</td>
<td>18.2*</td>
<td>&gt;16.1*</td>
<td>.57*</td>
<td>&gt;.75*</td>
</tr>
<tr>
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<td>.56</td>
<td>.57</td>
<td>12.4</td>
<td>12.3</td>
<td>.41</td>
<td>.49</td>
</tr>
<tr>
<td>13</td>
<td>.96*</td>
<td>&gt;.95*</td>
<td>Δ&lt;6.3*</td>
<td>&lt;6.4*</td>
<td>r&gt;.26*</td>
<td>0.00</td>
</tr>
<tr>
<td>14</td>
<td>.86</td>
<td>&gt;.89*</td>
<td>8.7</td>
<td>&lt;8.1*</td>
<td>.44</td>
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<tr>
<td>15</td>
<td>.63</td>
<td>.66</td>
<td>11.7</td>
<td>11.4</td>
<td>.47</td>
<td>.52</td>
</tr>
</tbody>
</table>

The understandings measured by Items 1, 2, 3, and 13 gave both groups little or no difficulty.

For the 1953 pre-test group, Item 6 also gave no difficulty.

For both groups, Items 1, 2, 3, 4, 8, and 13 discriminated poorly between the high and low scores on the test as a whole.

With the 1954 group, Item 6 also discriminated poorly.
TABLE XV

Item Analysis Showing Indices of Difficulty and Index of Discrimination of Each Test Item of Section III (Basic Understandings of Fractions and Decimals) of the Test

<table>
<thead>
<tr>
<th>Item Number</th>
<th>1953 p</th>
<th>1954 p</th>
<th>1953 Δ</th>
<th>1954 Δ</th>
<th>1953 r</th>
<th>1954 r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.86*</td>
<td>.83</td>
<td>8.7*</td>
<td>9.2</td>
<td>.65*</td>
<td>.51</td>
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<tr>
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<td>.46</td>
<td>.62</td>
<td>13.4</td>
<td>11.8</td>
<td>-.20</td>
<td>.04</td>
</tr>
<tr>
<td>3</td>
<td>.96*</td>
<td>&gt;.95*</td>
<td>Δ&lt;6.3*</td>
<td>&lt;6.3*</td>
<td>r&gt;.26*</td>
<td>&gt;.26*</td>
</tr>
<tr>
<td>4</td>
<td>.46</td>
<td>.52</td>
<td>13.4</td>
<td>12.8</td>
<td>.20</td>
<td>.40</td>
</tr>
<tr>
<td>5</td>
<td>.69</td>
<td>.69</td>
<td>11.0</td>
<td>11.0</td>
<td>.58</td>
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<td>.39</td>
<td>.63</td>
<td>14.1</td>
<td>11.7</td>
<td>.42</td>
<td>.38</td>
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<tr>
<td>7</td>
<td>.41</td>
<td>.34</td>
<td>13.9</td>
<td>14.7</td>
<td>.53</td>
<td>.22</td>
</tr>
<tr>
<td>8</td>
<td>.21</td>
<td>&lt;.05*</td>
<td>16.2</td>
<td>&gt;19.6*</td>
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<td>&lt;.08*</td>
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<td>.08</td>
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<td>18.6*</td>
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<td>11.1</td>
<td>.74</td>
<td>.71</td>
</tr>
<tr>
<td>11</td>
<td>.36</td>
<td>.54</td>
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<td>12.6</td>
<td>.55</td>
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</tr>
<tr>
<td>12</td>
<td>.46</td>
<td>.21</td>
<td>13.4</td>
<td>16.3</td>
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<tr>
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<td>.19</td>
<td>.14</td>
<td>16.5</td>
<td>17.4</td>
<td>.54</td>
<td>.22</td>
</tr>
<tr>
<td>14</td>
<td>.63</td>
<td>&gt;.77*</td>
<td>11.7</td>
<td>&lt;10.1*</td>
<td>.77</td>
<td>&gt;.76*</td>
</tr>
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<td>.50</td>
<td>.47</td>
<td>13.0</td>
<td>13.3</td>
<td>.44</td>
<td>.63</td>
</tr>
</tbody>
</table>

Item 3 gave little or no difficulty to both pre-test groups, thus showing that the understanding measured was known by most of the groups. For both groups, Item 9 was also very difficult. In addition, Item 8 was extremely difficult for the 1954 group.

Items 2, 3, and 8 discriminated poorly between those students in both
groups making high and low scores on the test as a whole. For the 1953

group, Item 4 also discriminated poorly. For the 1954 group, Items 7, 9,
11, and 13 also discriminated poorly.
<table>
<thead>
<tr>
<th>Item Number</th>
<th>1953 p</th>
<th>1954 p</th>
<th>1953 Δ</th>
<th>1954 Δ</th>
<th>1953 r</th>
<th>1954 r</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.86*</td>
<td>.81</td>
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<td>.65</td>
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</tr>
<tr>
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<td>.70</td>
<td>.63</td>
<td>10.9</td>
<td>11.7</td>
<td>.46</td>
<td>.11</td>
</tr>
<tr>
<td>3</td>
<td>.53</td>
<td>.50</td>
<td>12.7</td>
<td>13.0</td>
<td>.63</td>
<td>.59</td>
</tr>
<tr>
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<td>.39</td>
<td>14.2</td>
<td>14.1</td>
<td>.58</td>
<td>.42</td>
</tr>
<tr>
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<td>.45</td>
<td>.29</td>
<td>13.5</td>
<td>15.3</td>
<td>.60</td>
<td>.43</td>
</tr>
<tr>
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<td>.37</td>
<td>.24</td>
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<td>15.9</td>
<td>.47</td>
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</tr>
<tr>
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<td>.37</td>
<td>14.8</td>
<td>14.3</td>
<td>.49</td>
<td>.38</td>
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<td>11.2</td>
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<tr>
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<td>.57</td>
<td>13.2</td>
<td>12.3</td>
<td>.56</td>
<td>.56</td>
</tr>
<tr>
<td>10</td>
<td>.39</td>
<td>.38</td>
<td>14.1</td>
<td>14.2</td>
<td>.50</td>
<td>.21</td>
</tr>
<tr>
<td>11</td>
<td>.65</td>
<td>.54</td>
<td>11.4</td>
<td>12.6</td>
<td>.43</td>
<td>.44</td>
</tr>
<tr>
<td>12</td>
<td>.36</td>
<td>.29</td>
<td>14.4</td>
<td>15.3</td>
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<tr>
<td>13</td>
<td>.69</td>
<td>.56</td>
<td>11.0</td>
<td>12.4</td>
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<td>.41</td>
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<tr>
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<td>.43</td>
<td>14.6</td>
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<tr>
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<td>12.2</td>
<td>12.8</td>
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<td>.48</td>
</tr>
<tr>
<td>16</td>
<td>.11*</td>
<td>&lt;.14*</td>
<td>17.9*</td>
<td>&gt;17.3*</td>
<td>.60*</td>
<td>&gt;.65*</td>
</tr>
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<td>.25</td>
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<td>15.7</td>
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<td>.36</td>
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<td>.34</td>
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<td>14.6</td>
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<tr>
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</tr>
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<td>.42</td>
<td>.54</td>
<td>14.0</td>
<td>12.6</td>
<td>.61</td>
<td>.29</td>
</tr>
</tbody>
</table>
As a whole, the items in this section were difficult for both groups tested.

In the 1953 group, Items 15 and 30 discriminated poorly between those students making high and low scores on the test as a whole. Items 2, 6, 10, and 20 distinguished poorly between the high and low scorers in the 1954 group.
### TABLE XVII

**Item Analysis Showing Indices of Difficulty and Index of Discrimination of Each Test Item in Section V**  
(Basic Understandings of the Rationale of Computation) of the Test

<table>
<thead>
<tr>
<th>Item Number</th>
<th>1953 P</th>
<th>1954 P</th>
<th>1953 Δ</th>
<th>1954 Δ</th>
<th>1953 r</th>
<th>1954 r</th>
</tr>
</thead>
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<td>&gt;14.0*</td>
<td>-.12*</td>
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</tr>
<tr>
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<td>.23</td>
<td>&lt; .20*</td>
<td>15.9</td>
<td>&gt;16.4*</td>
<td>.32</td>
<td>&lt; .00</td>
</tr>
<tr>
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<td>.43</td>
<td>.36</td>
<td>13.7</td>
<td>14.2</td>
<td>.56</td>
<td>.58</td>
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<td>.68</td>
<td>.63</td>
<td>11.1</td>
<td>11.7</td>
<td>.18</td>
<td>.38</td>
</tr>
<tr>
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<td>.54</td>
<td>.56</td>
<td>12.6</td>
<td>12.4</td>
<td>.20</td>
<td>.41</td>
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<tr>
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<td>.61</td>
<td>.62</td>
<td>11.9</td>
<td>11.7</td>
<td>.50</td>
<td>.30</td>
</tr>
<tr>
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<td>.09</td>
<td>.20</td>
<td>18.3</td>
<td>16.4</td>
<td>.30</td>
<td>.24</td>
</tr>
<tr>
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<td>.89</td>
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<td>14.1</td>
<td>.35</td>
<td>.69</td>
</tr>
<tr>
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<td>13.9</td>
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<td>.37</td>
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<tr>
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<td>.62</td>
<td>.58</td>
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<td>12.2</td>
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</tr>
<tr>
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<td>.10*</td>
<td>&lt; .08*</td>
<td>18.2*</td>
<td>&gt;18.6*</td>
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<td>&gt;.53*</td>
</tr>
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<td>12</td>
<td>.45</td>
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<td>.71</td>
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<td>.69</td>
<td>.61</td>
<td>11.1</td>
<td>11.9</td>
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</tr>
<tr>
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<td>.65</td>
<td>10.8</td>
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<td>.75</td>
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<td>13.7</td>
<td>13.2</td>
<td>.49</td>
<td>.40</td>
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</tbody>
</table>

All of the items in this section of the test gave both groups moderate to a great deal of difficulty. Items 7 and 11 were exceptionally difficult.

For both groups, Items 1, 2, 7, and 10 distinguished poorly between those making high and low scores on the test as a whole. In the 1953 group, Items 4, 5, 8, and 9 also discriminated poorly.
CHAPTER V

CONCLUSIONS

Purpose I

This study was undertaken to determine whether the students in the course, "Methods of Teaching Arithmetic," developed increased mathematical understandings during the period of instruction given.

1. Comparing the results of the two administrations of Glennon's Test of Basic Mathematical Understandings, the writer found that the students developed increased mathematical understandings as a result of the course in "Methods of Teaching Arithmetic."

2. Growth in the understandings tested by each section of the test and by the test as a whole was highly significant.

3. The greatest gains were made in the final section of the test which was not completed by many during the pre-test.

Purpose II

This investigation sought to study the mathematical understandings possessed by students at the outset of a pre-service course in "Methods of Teaching Arithmetic."

1. In the pre-test, the group showed that their understanding of the concepts tested in Glennon's Test of Basic Mathematical Understandings was low. Of the 80 items on the test, the group had only 57 per cent of these correct.

2. A few of the items presented no difficulty to the group. For the
group as a whole, the greatest weaknesses were in the following:

a. the multiplication and division of fractions
b. numerical understandings of size and place values of decimals
c. processes involved in multiplying by a two or more figured multiplier.

Purpose III

This study was to show the comparison between the mathematical understandings of two groups of college students at the outset of the course, "Methods of Teaching Arithmetic."

1. Although varying slightly, the means attained by the 1954 group were not significantly different from those of the 1953 group.
2. From the data in this study, it can be stated that the mathematical understandings of pre-teachers is low.
3. The areas of mathematical weakness were the same for both groups.
4. Most of the items which discriminated poorly for one group did so with the other group.
5. Most of the items which were extremely difficult for one group were difficult also for the other group.

Purpose IV

Through this study, the degrees of difficulty and the discriminating power of each item in Dr. Glennon's Test of Basic Mathematical Understandings were to be investigated.

1. Most of the items discriminated adequately between those students who had high scores on the test as a whole and those who had low scores on the test as a whole.
2. Very few items discriminated so poorly that they should be eliminated or changed.

3. Very few items caused extreme difficulty to those tested.

4. On the whole, the test items had adequate discriminatory power and were varied as to difficulty.
Suggestions for Further Study

1. An analysis could be made of those items which caused extreme difficulty for the groups tested. The purpose of this analysis would be to determine the common types of errors made by the students on the individual items. These misconceptions could be cleared up in the Methods of Teaching Arithmetic course.

2. A comparative study could be done of the test results as obtained from the group tested by Dr. Glennon and the combined results of the two groups tested at Boston University. Norms could be established for use with future groups, entering the course Methods of Teaching Arithmetic.

3. Test results could be grouped according to the amount of mathematical training each student has had. This could lead to a revision in the prerequisites for the course, Methods of Teaching Arithmetic.

4. A third administration of the test could occur when the students have completed student teaching. The results would show the changes in arithmetic understandings which resulted from the actual teaching of arithmetic.

5. "A comparison might be made between the difficulty rankings of the 80 test items in the present investigation with those in Glennon's study.

6. Determine the extent to which the pre-service students' mastery of mathematical understandings influences the progress in arithmetic made by children during student teaching or actual professional service."¹

¹Vincent A. Cristiani, et al., op. cit., p. 45.
BIBLIOGRAPHY
BIBLIOGRAPHY


This is a test to see how well you understand arithmetic. You do not have to do any written work to find the answers. In fact, you will not be permitted to work out any written computations whatsoever.

The test is divided into five parts:
I. The Decimal System of Notation.
II. Basic Understandings of Integers and Processes.
III. Basic Understandings of Fractions and Processes.
IV. Basic Understandings of Decimals and Processes.
V. Basic Understandings of the Rationals of Computation.

Read each statement or question carefully and decide which of the suggested answers is the correct one. Then write the letter for this answer on the proper line on the answer sheet. All answers are to be recorded in this way on the separate answer sheet. MAKE NO WRITTEN MARKS WHATSOEVER ON ANY OF THE TEST SHEETS.

Sample Item

Which of the following numbers has the largest value?
A. 23   B. 9   C. 35   D. 45   E. 11

Since D is the correct answer, you would write the letter D on the proper line on the answer sheet.

Try each example but do not stay too long on any one statement or question. If you cannot find the answer you may go on to the next example and come back to the one which you omitted if time permits.

You may go all the way through the test without stopping. When you finish the examples in one section, go right on to the next section.

In Section III you will find shaded diagrams similar to the one at the right of this page. This diagram should be read as 3/4 (i.e., three-fourths). Read all diagrams in this way. Remember: The value of the fraction is indicated by the white or unshaded part of the diagram.

When you are told to do so, begin at the top of the next page and proceed thru the test in the manner which has been indicated.

Remember: DO NO WRITTEN WORK TO FIND THE ANSWERS. Make no written marks on any of the test sheets. Record the letter of your choice for each correct answer on the proper line on the answer sheet.

Note: This test has been copyrighted (1947) by Dr. Vincent J. Glennon, School of Education, Syracuse University. The test has been reproduced, and is being used, by permission of the author.
Section I. The decimal system of notation.

1. If you rearranged the figures in the number 43,125 which of the following arrangements would give the smallest number?
   A. 54,321       B. 21,345       C. 12,345
   D. 14,532       E. 13,245

2. If you rearranged the figures in the number 53,429 which of the following arrangements would give the largest number?
   A. 95,324       B. 95,432       C. 59,432
   D. 95,234       E. 95,243

3. Which of the following has a 3 in the hundreds' place?
   A. 23,069       B. 86,231       C. 49,563
   D. 39,043       E. 12,304

4. In the number 2,222 the 2 on the left represents a value how many times as large as the 2 on the right?
   A. 1 (same value) B. 10           C. 100
   D. 200          E. 1,000

5. About how many tens are there in 6542?
   A. 6.5         B. 65\frac{1}{2}     C. 65\frac{1}{2}
   D. 6,540       E. 65,000

6. If the figures in 23,469 were rearranged, which of the following would place the smallest figure in the tens' place?
   A. 46,932       B. 96,432       C. 69,234
   D. 34,629       E. 92,346

7. In the number 7,255 the 5 on the left represents a value how many times as large as the 5 on the right?
   A. 1 (same value) B. 2            C. 5
   D. 10           E. 100

8. Which of the following statements best tells why we write a zero in the number 4,039 when we want it to say "four thousand thirty-nine"?
   A. Because the number would say "four hundred thirty-nine" if we wrote a zero in some other place.
   B. Writing a zero helps us to read the number.
   C. Writing a zero tells us to read the hundreds' figure carefully.
   D. Because the number would be wrong if we left out a zero some place.
   E. Because we use zero as a place-holder to show that there is no amount to record in that place.

9. Which of the following has a 4 in the ten thousands' place?
   A. 423,102       B. 643,112       C. 438,116
   D. 374,912       E. 763,420
10. If the figures in 86,473 were arranged differently, which of the following would place the largest figure in the thousands' place?
   A. 73,648  B. 38,467  C. 76,483
   D. 87,643  E. 86,734

11. In the number 3,914 the 4 on the right represents a value how many times as large as the 4 on the left?
   A. 1/10  B. 1/2  C. 5  D. 1 (same value)  E. 10

12. In the number 5,492 the 4 represents a value how many times as large as the 2?
   A. 2  B. 10  C. 20  D. 100  E. 200

13. About how many hundreds are there in 34,820?
   A. 34  B. 35  C. 350  D. 3,500  E. 35,000

14. Which of the following methods is the best for determining the value of a figure in a number? for example, the value of the 7 in 3748.
   A. Its position in the number.
   B. Its value when compared with other figures in the number.
   C. Its value in the order from 1 to 9.
   D. Its value when compared with the whole of the number.
   E. Its position in the number and its value.

15. In the number 7,843 the 4 represents a value how many times as large as the 8?
   A. 1/10  B. 1/20  C. 1/2  D. 2  E. 20

(Go right on to Section II)
Section II  Basic understandings of integers and processes.

1. If you had a bag of 365 marbles to be shared equally by 5 boys, which would be the quickest way to determine each boy's share?
   A. counting  B. adding  C. subtracting  D. multiplying  E. dividing

2. When a whole number is multiplied by a whole number other than 1, how does the answer compare with the whole number multiplied?
   A. larger  B. smaller  C. same  D. 10 times as large  E. can't tell

3. When a whole number is divided by a whole number other than 1, how does the answer compare with the whole number divided?
   A. larger  B. smaller  C. same  D. one-half as large  E. can't tell

4. Which of the following is the quickest way to find the sum of several numbers of the same size?
   A. by counting  B. by adding  C. by subtracting  D. by multiplying  E. by dividing

5. If the zeros in the two numbers in this example were left off, how would the answer be changed?
   A. The answer would be ten times as large.  B. The answer would be one hundred times as large.
   C. The answer would be one-tenth as large  D. The answer would be one-hundredth as large.
   E. The answer would not change.

6. Here is an example in subtraction in which letters have been used instead of figures. Which statement is true:
   A. AFGB and CXU added together equal TWMY.
   B. CXU and TWMY added together equal AFGB.
   C. AFGB and TWMY added together equal CXU.
   D. TWMY subtracted from CXU equals AFGB.
   E. CXU subtracted from TWMY equals AFGB.

7. How would the answer to this example be changed, if a zero were added (annexed) to the right of each number?
   A. The answer would be ten times as large.
   B. The answer would be one hundred times as large.
   C. The answer would not change.
   D. Cannot tell until you add both ways.
   E. The answer would be one thousand times as large.

8. Adding (annexing) two zeros to the right of a whole number has the same effect as:
   A. Adding ten to the number.
   B. Adding one hundred to the number.
   C. Multiplying the number by ten.
   D. Multiplying the number by one hundred.
   E. Dividing the number by one hundred.
9. What would be the effect on the answer if you added (annexed) two zeros to 439 and took away the zero from 450?
   A. The answer would be ten times as large.
   B. The answer would be one hundred times as large.
   C. The answer would remain the same.
   D. The answer would be one-tenth as large.
   E. The answer would be one-hundredth as large.

10. Crossing off a zero from the right side of a number has the same effect as:
    A. Subtracting ten
    B. Subtracting one hundred
    C. Multiplying by ten
    D. Multiplying by one
    E. Dividing by ten

11. What would be the effect on the answer if you added (annexed) two zeros to 92 and changed 4500 to 450?
    A. The answer would be ten times as large.
    B. The answer would be one-tenth as large.
    C. The answer would be one hundred times as large.
    D. The answer would be one-hundredth as large.
    E. The answer would be one-thousandth as large.

12. Which one of the following methods could be used to find the answer to this example?
    A. Multiply 17 by the quotient.
    B. Add 17 six hundred twelve times. Answer would be the sum.
    C. Subtract 17 from 612 as many times as possible. Answer would be number of times you were able to subtract.
    D. Add 612 seventeen times. Answer would be the sum.
    E. Multiply 17 by 612. Answer would be the product.

13. If the numbers in a large addition example were changed so that the top number was placed at the bottom and the bottom number was placed at the top, how would the answer be affected?
    A. Answer would be larger.
    B. Answer would be smaller.
    C. Answer would not change.
    D. Could not do the example.
    E. Cannot tell until you add both ways and compare.

14. How would the example be affected if you put the 29 above 4306?
    A. The answer would be larger.
    B. The answer would be smaller.
    C. The answer would be the same.
    D. Cannot tell until you multiply both ways.
    E. You cannot do the example when the large number is on the bottom and the small number on top.

15. What would be the effect on the answer if you added (annexed) two zeros to 39?
    A. The answer would be one hundred times as large.
    B. The answer would be one-hundredth as large.
    C. The answer would be one-thousandth as large.
    D. The answer would not change.
    E. You could not do the example.

(Go right on to Section III)
Section III. Basic understandings of fractions and processes.

1. Which of the following fractions is the largest?
   A. 1/7   B. 5/7   C. 3/7   D. 11/7   E. 6/7

2. Which of these statements best tells why we cannot say that the unshaded parts of this picture represent 5 "eighths"?
   A. Because more than 5/8 of it is unshaded.
   B. Because the unshaded parts are not together.
   C. Because all the unshaded parts are not the same size.
   D. Because less than 5/8 of it is unshaded.
   E. Because the parts are not the same shape.

3. Which of the following fractions is the smallest?
   A. 1/9   B. 1/5   C. 1/2   D. 1/7   E. 1/3

4. Which picture shows how the result would look if you divided the numerator and denominator of 10/8 by 2?

5. When a whole number is multiplied by a common (proper) fraction, how does the answer compare with the whole number?
   A. larger   B. smaller   C. same   D. cannot tell   E. half as large

6. Which picture shows how the result would look if you divided the numerator of this fraction by 2?

7. Which picture best shows the example, 4 x 2/3?

8. When a common (proper) fraction is divided by a common fraction, how does the answer compare with the fraction divided?
   A. larger   B. smaller   C. same   D. cannot tell   E. twice as large
9. Which picture shows how the result would look if you multiplied the numerator and denominator of $\frac{3}{5}$ by $2$?

- A. 
- B. 
- C. 
- D. 
- E. 

10. Which picture shows how the result would look if you multiplied the denominator of this fraction by $2$?

- A. 
- B. 
- C. 
- D. 
- E. 

11. When a whole number is divided by a common (proper) fraction, how does the answer compare with the whole number?

- A. larger
- B. smaller
- C. same
- D. cannot tell
- E. varies

12. Which picture looks like this example: $3 \div \frac{1}{2}$?

- A. 
- B. 
- C. 
- D. 
- E. 

13. Which sentence best tells why the answer is larger than the $5$?

$$\frac{5}{\frac{3}{4}} = 6\frac{2}{3}$$

- A. Because inverting the divisor turned the $\frac{3}{4}$ upside down.
- B. Because multiplying always makes the answer larger.
- C. Because the divisor $\frac{3}{4}$ is less than $1$.
- D. Because dividing by proper and improper fractions makes the answer larger than the number divided.
- E. Inverting a fraction puts the larger number on top.

14. Which sentence is shown by this picture?

- A. Fractions with common denominators may be added.
- B. The value of a fraction is changed if a number is subtracted from the numerator and denominator.
- C. Dividing the numerator and denominator of a fraction by the same number does not change the value of the fraction.
- D. Fractions with the same denominators are equal.
- E. Fractions with the same numerators are equal.

15. When a common (proper) fraction is multiplied by a common fraction, how does the answer compare with the fraction multiplied?

- A. larger
- B. smaller
- C. same
- D. cannot tell
- E. varies
Section IV. Basic understandings of decimals and processes.

1. How should you write the decimal, "eighty and eight hundredths"?
   (A) .8008  (B) 80.008  (C) 80.08  (D) 80.008  (E) 8008.08

2. How should you read this decimal: .0309?
   A. Three and nine hundredths.
   B. Three hundred nine thousandths.
   C. Three hundred nine ten-thousandths.
   D. Thirty-nine thousands.
   E. Three hundred nine hundredths.

3. Which decimal tells how long line Y is when compared with line X?
   line X _______  line Y ________
   (A) .5  (B) .625  (C) 1.25  (D) 7.5  (E) 33

4. About how many tenths are there in 1.25?
   (A) .13  (B) 1.3  (C) 13  (D) 125  (E) 1250

5. About how many hundredths are there in .635?
   (A) 1/2  (B) 6.35  (C) 63.5  (D) 635  (E) 6350

6. What would be the effect on the answer if you dropped the zero from 23.909?
   A. The answer would have the same value.
   B. The answer would be one-tenth as large.
   C. The answer would be ten times as large.
   D. You would point off three places.
   E. It would be the same as subtracting zero from the answer.

7. How would the answer be changed if you changed 6.5 to .65 and 84.5 to 84.5?
   A. The answer would be the same.
   B. The answer would be ten times as large.
   C. The answer would be one hundred times as large.
   D. The answer would be one-tenth as large.
   E. The answer would be one-hundredth as large.

8. Which seems to be the correct answer to this example:
   ten divided by five-tenths.
   (A) 1/2  (B) 2  (C) 10  (D) 20  (E) 50

9. Which decimal tells how long line Y is when compared with line X?
   line X _______  line Y ______
   (A) 1.25  (B) 1.50  (C) 2  (D) 2.40  (E) 2.50

10. Which of the following decimals has the largest value?
    (A) 30.3  (B) 30.03  (C) 30.0333  (D) 30.303  (E) 30.003

11. What would be the effect on the answer if you changed 368 to 3680 and 24 to 2.4?
    A. The answer would be smaller.
    B. It would not change the answer.
    C. It would be the same as adding a zero to the answer.
    D. The answer would be one-tenth as large.
    E. Cannot tell until you do the example both ways.
12. Which decimal has the smallest value?
   (A) .3 (B) .09 (C) .048 (D) .693 (E) .0901

13. How would the answer be affected if you moved the point one place to the left in both numbers?
   A. The answer would be one-tenth as large.
   B. The answer would be one-hundredth as large.
   C. The answer would be one hundred times as large.
   D. It would be the same as subtracting 100 from the answer.
   E. The answer would have the same value.

14. How would the answer be changed if you moved the point two places to the right in both numbers?
   A. The answer would have the same value.
   B. The answer would be ten times as large.
   C. You would point off differently.
   D. You cannot move the point in the top number two places.
   E. The answer would be 10,000 times as large.

15. How would the answer be affected if you moved the point one place to the right?
   A. The answer would be ten times as large.
   B. The answer would be 10 larger.
   C. The answer would be one-tenth as large.
   D. The answer would have a zero at the right.
   E. The value of the answer would be the same.

16. How would the answer be affected if you changed 7.3 to 73 and 1390 to 13.90?
   A. The answer would be one hundred times as large.
   B. The answer would be one-tenth as large.
   C. The answer would be one thousand times as large.
   D. The answer would be one-hundredth as large.
   E. The answer would be one-thousandth as large.

17. About how many tenths are there in .055?
   (A) 0 (B) 1/2 (C) 5 (D) 10 (E) 50

18. About how many thousandths are there in 16.5?
   (A) 1.7 (B) 17 (C) 170 (D) 1,700 (E) 17,000

19. Why is the answer smaller than the top number?
   A. Because 8 is more than .5
   B. Because you are finding how many .5's in 8.
   C. Because .5 is less than 8.
   D. When you multiply by a decimal the answer is always smaller than the top number.
   E. Because multiplying by .5 is the same as finding half of the number.

20. How would the answer be changed if you changed 1.47 to 147?
   A. You would get the same answer.
   B. The answer would be ten times as large.
   C. The answer would be one hundred times as large.
   D. The answer would be one-tenth as large.
   E. The answer would be one-hundredth as large.

(Go right on to Section V)
Section V  Basic understandings of the rationale of computation.

1. Why do we find a common denominator when adding fractions with unlike denominators?
   A. You cannot add together things that are different.
   B. It is easier to add fractions when they have a common denominator.
   C. The denominators have to be the same in order to add.
   D. We learned to add unlike fractions that way.
   E. So that all the fractions will have the same value.

2. When dividing by a decimal, why do we move the point to the right?
   A. Multiplying by a multiple of ten is a quick way of changing a decimal to a whole number.
   B. It places the decimal point in the quotient correctly.
   C. You can only divide by a whole number.
   D. To make the divisor equal to the dividend.
   E. It is easier to divide by a whole number than a decimal.

3. Which one of the following would give the correct answer to this example? 2.1 x 21
   A. The sum of 1 x 2.1 and 21 x 2.1
   B. The sum of 10 x 2.1 and 2 x 2.1
   C. The sum of 10 x 2.1 and 20 x 2.1
   D. The sum of 1 x 2.1 and 20 x 2.1
   E. The sum of 1 x 2.1 and 2 x 2.1

4. Which statement best tells why we "invert the divisor and multiply" when dividing a fraction by a fraction?
   A. It is an easy method of finding a common denominator and arranging the numerators in multiplication form.
   B. It is an easy method for dividing the denominators and multiplying the numerators of the 2 fractions.
   C. It is a quick, easy and accurate method of arranging two fractions in multiplication form.
   D. Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
   E. It is a quick method of finding the reciprocals of both fractions and reducing to lowest terms (cancelling).

5. Why do we move the second partial product one place to the left when we multiply by the 6?
   A. Because the answer has to be larger than 729.
   B. Because the six means six tens.
   C. Because 6 is the second figure in 68.
   D. Because we learned to multiply that way.
   E. Because the 6 represents a greater value than the 8 represents.

6. Which statement best tells why we arrange numbers in addition the way that we do?
   A. It is an easy way to keep the numbers in straight columns.
   B. It helps us to add correctly.
   C. It helps us add only those numbers in the same position.
   D. It helps us to carry correctly from one column to another.
   E. It would be harder to add if the numbers were mixed.
7. When you multiply by the \(4\) in \(48\) you will get a number that is how large compared with the final answer?
   A. One-twelfth as large.
   B. One-tenth as large.
   C. One-half as large.
   D. Five-sixth as large.
   E. Twice as large.

8. The answer to this example will be how large when compared with the 69?
   A. Twice as large.
   B. Sixty-nine times as large.
   C. One sixty-ninth as large.
   D. Eight hundred twenty-seven times as large.
   E. 1 as large.

9. Which statement best tells why it is necessary to 'borrow' in this example?
   A. Because the top number is smaller than the bottom number.
   B. You cannot subtract 92 from 67.
   C. You cannot subtract 9 tens from 6 tens.
   D. You cannot subtract 39 tens from 56 tens.
   E. You cannot subtract 9 from 6.

10. Which statement best tells why we carry 2 from the second column?
   A. The sum of the second column is 23 which has two figures in it. We have room for the 3 only, so we put the 2 in the next column.
   B. The sum of the second column is more than 20, so we put the 2 in the next column.
   C. Because we learned to add that way.
   D. The value represented by the figures in the second column is more than 9 tens, so we put the hundreds in the next column.
   E. If we do not carry the 2, the answer will be 20 less than the correct answer.

11. In this example you multiply by the 6, then by the 3. How do the two results (partial products) compare?
   A. The second represents a number one-half as large as the first.
   B. The second represents a number twice as large as the first.
   C. The second represents a number five times as large as the first.
   D. The second represents a number ten times as large as the first.
   E. The second represents a number twenty times as large as the first.

12. Which would give the correct answer to \(439 \times 563\)?
   A. Multiply \(439 \times 3; 439 \times 6; 439 \times 5\) - then add answers.
   B. Multiply \(439 \times 3; 439 \times 63; 439 \times 563\) - then add answers.
   C. Multiply \(563 \times 9; 563 \times 3; 563 \times 4\) - then add answers.
   D. Multiply \(563 \times 9; 563 \times 39; 563 \times 439\) - then add answers.
   E. Multiply \(439 \times 3; 439 \times 60; 439 \times 500\) - then add answers.
13. Which statement best explains the 4 in the answer?
   A. The 4 means that there are forty-eight 26's in \( \frac{48}{26/1248} \).
   B. The 4 in the answer means that there are four 26's in \( \frac{48}{26/1248} \).
   C. The 4 means that 2 goes into 12 four times, and 5 would be too large.
   D. The 4 means that there are at least forty 26's in \( \frac{48}{26/1248} \).
   E. The 4 means that the answer will come out even.

14. Here is an example in subtraction of mixed numbers in which it is necessary to "borrow." Which statement best explains the borrowing.
   A. You cannot subtract \( \frac{5}{8} \) from \( \frac{3}{8} \) so you take 1 from the 3 and put it in front of the \( \frac{3}{8} \) making \( \frac{5}{8} \).
   B. You cannot subtract \( \frac{5}{8} \) from \( \frac{3}{8} \) so you add the 3 and the 8 making \( \frac{11}{8} \).
   C. You cannot subtract \( \frac{5}{8} \) from \( \frac{3}{8} \) so you turn them around and subtract \( \frac{3}{8} \) from \( \frac{5}{8} \).
   D. You cannot subtract \( \frac{5}{8} \) from \( \frac{3}{8} \) so you take 1 from the 5 and add it to \( \frac{3}{8} \) making it \( \frac{4}{8} \).
   E. You cannot subtract \( \frac{5}{8} \) from \( \frac{3}{8} \), so you take 1 from the 5 and change it to \( \frac{8}{8} \); then add the \( \frac{8}{8} \) to \( \frac{3}{8} \) making \( \frac{11}{8} \).

15. Which statement best explains what happens when you reduce a fraction to lowest terms?
   A. The size of the terms and the value of the fraction become smaller.
   B. The value of the fraction does not change. The size of the part represented by the new denominator is smaller, and the number of parts represented by the new numerator is less.
   C. The value of the fraction does not change. The terms are smaller, but they represent more parts of larger size.
   D. The value of the fraction does not change, but the parts of the fraction represented by the new numbers become fewer in number and larger in size.
   E. The value of the fraction changes because the new numbers are smaller.

End