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The Roman system of mathematics

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THE ROMAN SYSTEM OF MATHEMATICS

by

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THE ROMAN SYSTEM OF MATHEMATICS
INTRODUCTION

"In summo apud illos (i.e. Graecos) honore geometria fuit; itaque nihil mathematicis illustriis. At nos metiendi ratiocinandique utilitate huius artis terminavimus modum."

("Geometry was in high esteem with them i. e. the Greeks, therefore none were more honorable than mathematicians. But we the Romans have confined this art to bare measuring and calculating.")

Perhaps no words in Roman literature more forcibly and accurately describe the attitude of the Roman toward the whole science of mathematics, and the place of mathematics in Roman life as well as the limitation of any Roman theoretical development of the science. These words from the mouth of a notable Roman also bring to the reader's mind the realization that in the field of mathematics and science as in other phases of life there was reflected the difference between the Greek and the Roman mind, and the contrast in the everyday needs and aims of life of these two ancient peoples.

The Greeks were concerned with the development of man's mind, and the fashioning of his soul; they were inclined toward speculation; they sought to create "beauty for beauty's sake." The Romans whether nature or man was being considered were interested only in what would be useful in practical life. Even literature and art were evaluated in terms of utility. Life for the Romans was seriously, painfully utilitarian.

1 Cic., "Tus. Dis.", 1. 2. 5.
What was useful in Greece was beautiful, but what was beautiful in Italy had to be useful. Knowledge for its own sake did not interest the Romans. They cultivated neither philosophy nor science spontaneously and they were quite indifferent "to the purely disinterested speculative virtue which the Pythagoreans and Platonists exalted in mathematical research and despised pure science."^2 A mixture of suspicion and contempt for pure science was inherent in the Roman because of his narrow, rustic horizon. There probably never was and never could have been a scientific circle in Rome. If a person did dabble in science even for his own amusement, he might do so amid the ridicule of his associates. Among the Romans it appears that there was a general feeling that in the field of scientific endeavor the ultimate limits would soon be reached so no real call was felt to join in the search. Even Archimedes of Syracuse who was equally great in both theory and application failed to inspire the Romans to imitate him.

The present high development of modern mathematics is indebted for its foundations along the theoretical and practical lines to Greece and Rome respectively. In these fields of mathematical endeavor the one people was the complement of the other. The "Imperium Romanum" characterized by the practical Roman spirit, a lack of intellectual curiosity and imagination, and an indifference to the free activities of the mind cast a cold blight over the scientific life of the ancient world.

^2Arnold Reymond, "Histoire des sciences exactes et naturelles dans l'antiquite greco-romaine".
There was no longer a continuation of what the Greeks had done in mathematical theory but an endeavor to make science tangible to meet utilitarian needs. Since many of these technical applications, engines of war, the pulley, the crane, the lever, etc. had been anticipated in the Hellenistic age, few of them were distinctly of Roman invention. As Rome borrowed the theory of mathematics, she likewise borrowed but extended and perfected its applications.

Rome in exalting the power of the Roman State gave her attention to developing a great military and commercial commonwealth. Here was a real and vital need not for pure mathematics but for the applied side of it. There was need of a wholesome water supply for the people, of introducing hygienic measures in the form of drains, of rescuing large tracts of land from overflowing rivers, and draining marshy land. There was need of creating a network of roads to connect remote parts of the empire, of building strong walls and fortifications, of constructing machines of warfare, of laying out camps for soldiers, for surveying the locations of cities in recently acquired lands. There was an ever present need of means of accurate computing and calculating in the financial and commercial life throughout the evergrowing empire.

**Roman Mathematics**

**Arithmetic**

It was to meet these practical needs that arithmetic with great emphasis on securing perfection in calculation and finger reckoning was given an important place in the Roman schools.
Special attention also seems to have been paid to the representation of numbers by signs. Quintilian states the value of such training in his "Institutio Oratoria."

"In geometria partem fatentur esse utilem teneris aetatisibus. Agitari namque animos et acui ingenia et celeritatem percipiendi venire inde concedunt, sed prodesse eam non ut ceteras artes, cum perceptae sint, sed cum discatur, existimant: ea vulgaris opinio est. Nec sine causa summi viri etiam impensam huic scientiae operam dixerunt. Nam cum sit geometria divisa in numeros atque formas, numerorum quidem notitia non oratori modo, sed cuicumque saltem primit litteris erudito necessaria est. In causis vero vel frequentissime versari solet; in quibus actor, non dico, si circa summas trepida, sed si digitorum saltem incerto aut indecoro gestu a computatione dissentit, iudicatur indoctus. Illa vero linearis ratio et ipsa quidem cadit frequenter in causas (nam de terminis mensurisque sunt lites), sed habet maiorem quandam aliam cum arte oratoria cognatione. Iam primum ordo est geometriae necessarius; nonne et eloquentiae?

Ex prioribus geometria probat insequentia, ex certis incerta; nonne id in dicendo facimus? Quid? illa propositarum quaestionum conclusio non fere tota constat syllogismis? Propter quod plures invenias, qui dialecticae similem quam qui rhetoricae fateantur hanc artem. Verum et orator etiam raro non tamen nunquam probabit dialectic…quid autem magis oratio quam probationem petit? Falsa quoque veris similia geometrica ratione deprehendit."

("As regards geometry it is granted that portions of this science are of value for the instruction of children, for admittedly it exercises their minds, sharpens their wits and generates quickness of perception. But it is considered that the value of geometry resides in the process of learning, and not as with other science in the knowledge thus acquired. Such is the general opinion. But it is not without good reason that

3 Quintilian, "Institutio Oratoria", 1. 10, 34-49.
4 Geometry here includes all mathematics.
some of the greatest men have devoted special attention to this
science. Geometry has two divisions; one is concerned with the
numbers, the other with figures. Now knowledge of the former is
a necessity not merely to the orator, but to anyone who has had
even an elementary education. Such knowledge is frequently
required in actual cases, in which a speaker is regarded as
deficient in education, I will not say if he hesitates in making
a calculation, but even if he contradicts the calculation which
he states in words by making an uncertain or inappropriate
gesture with his fingers. Again linear geometry is frequently
required in cases, as in lawsuits about boundaries and measure-
ments. But geometry and oratory are related in a yet more
important way than this. In the first place logical development
is one of the necessities of geometry. And is it not equally a
necessity for oratory? Geometry arrives at its conclusions from
definite premises, and by arguing from what is certain proves
what was previously uncertain. Is not this just what we do in
speaking? Again are not the problems almost entirely solved by
the syllogistic method, a fact which makes the majority assert
that geometry bears a closer resemblance to logic than to
rhetoric? But even the orator will sometimes, though rarely,
prove his point by formal logic.... And what is the aim of
oratory if not proof? Again oratory sometimes detects false-
hoods closely resembling the truth by the use of geometrical
methods...."
According to present usage "arithmetic" means that part of mathematics which deals with the science of numbers and the art of computation by figures. The Greeks used the term to apply only to the theory of numbers—as the study of primes and the properties of square numbers. They gave the name "logistic" to the practical use of numbers which would include methods of writing and of computing. The latter, "logistic", was looked upon by ancient scholars as plebeian, and the former, "arithmetic", was considered an intellectual pursuit worthy of the attention of philosophers. Plato states "Arithmetic is of two kinds, one of which is popular and the other philosophical."

Since logistic was not ranked as one of the mathematical sciences, it was not advocated in the schools of philosophy; and being so humbly regarded was chiefly taught by slaves as a part of the trade of the commercial apprentice. No treatise on the subject has come to us from classical times.

Both Greece and Rome inherited certain notions of numbers from prehistoric ages which served as the beginnings of ancient arithmetic. These notions especially involved superstitions regarding the "mystery of numbers." Since the even numbers after the first were capable of separation into other numbers, they were considered feminine and earthly. On the other hand, the odd numbers were considered masculine and divine except for three, five, and seven which being looked on by primitive people as mysterious potent for good or evil were put in a class by themselves.5

The number scripts of the ancient world show evidences of number and sex. Perhaps this fact should not be too surprising when we consider that primitive man was preoccupied with his own fertility, and that of his flocks. The prominence given to three was probably suggested by the male organs of generation, an idea which has survived in many languages where three trines are used as a symbol of potency as in "thrice armed is he." In the early Phoenician and Sumerian systems of numeration, the grouping of signs in threes is commonly found. The interval of three also prevails in the Roman numerals as: I II III, X XX XXX, C CC CCC. This characteristic, however, proved to be an inconvenience as it hampered the development of a system of calculation when more civilized man came to need a means of computing.

The classical writers developed beliefs relative to numbers which were independent of superstition. Having searched into the properties of numbers, these writers separated them into digits (digiti, fingers); joints (articuli), and composites (compositi) but which also was a relic of the early symbolism. Geometric forms were associated with numbers. The square represented three plus one or a square with a side equal to two while a square whose side equaled three represented three plus one plus five. In other words the sum of the first \( n \) odd numbers was \( n^2 \) since this fact might be deducted from the related geometrical figure. For example, five might be represented by a kind of right triangle in which the upright part was conceived to resemble the pointer set up in a

Hogben, "Mathematics for the Million."
sun dial. It was called a "gnomon" from the Greek word meaning "know" since it is from the pointer's shadow that one might "know" the time of day or even the seasons. The result was that odd numbers were called "gnomons." Triangular numbers were also recognized as \( \cdot \cdot = \text{one plus three} \).\( \cdot : \cdot = \text{one plus two plus three} \). A triangular number is defined as "the sum of the arithmetical series of the natural numbers beginning with unity."\(^7\) There were also pentagonal, hexagonal oblong numbers as well as solid ones which were represented by a cube. As an example of the latter if a cube has an edge equal to three units, the volume will contain three times three, times three or twenty-seven cubic units. Thus was the number twenty-seven related to the solid and called a cube. In a similar way it was possible to represent pyramidal numbers by piling up spheres.

**Geometry**

Geometry was of interest to the Romans in the original meaning of the word, land measuring. Such was true in pre-Grecian times with the Egyptian and Babylonian surveyors in whose hands surveying reached its highest development for the era in the measurement of tangible objects. The Greeks always working for the love of science developed the theoretical side of demonstrative geometry. The Romans never loving science for its own sake were ready to reap the fruits of the discoveries of the Greeks and enjoy the utilitarian profit which they might obtain from them. Cato voiced the Roman point of view when he said of Greek thought in general that it was good because it was

\(^7\)Our Debt to Greece and Rome Series. "Mathematics," Smith.
profitable to have a notion of it but useless to immerse oneself in it. Vitruvius also would have one know the general theory of any art or science, but reserve the practice and thorougher knowledge to the specialist. With knowledge limited to such a degree in a definite field, yet with a broad program of subjects he says regarding the education of an architect:

"Geometrica autem plura praesidia praestat architecturae; et primum ex euthygrammis circini tradit usum, e quo maxime facilius aedificiorum in areis expediuntur descriptiones normarumque et librationum et linearum directiones. Item per opticon in aedificiis ab certis regionibus caeli lumina restre ducentur. Per arithmeticon vero sumptus aedificiorum consummantur, mensurarum rationes geometricis rationibus et methodis inveniuntur."

("Mathematics again furnishes many resources to architecture. It teaches the use of rule and compass and thus facilitates the laying out of buildings on their sites by the use of set squares, level and alignments. By optics, in buildings, lighting is duly drawn from certain aspects of the sky. By arithmetic, the cost of building is summed up; the methods of mensuration are indicated; while the difficult problems of symmetry are solved by geometrical rules and methods.")

The easier parts of Nichomachus were taught in Roman schools and instruction in geometry was limited to a few practical rules, but which were carried on to a high degree of excellence in the field of surveying. To simplify the work of surveyors important geometrical theorems were collected into a large work of which fragments are preserved in the "Codex

8Vit., "De Archit.", I. 1.
Arceriamus." Romans wishing to know about the subject of mathematics went to Alexandria which was well located for a scientific center, undisturbed by war, the pride of its rulers, and where in commerce East met West, or ambitious scholars might go to other eastern places which drew inspiration from Alexandria. It was from this center of learning that specialists were frequently summoned to Rome to carry out technical or scientific projects.

**Roman Mathematicians**

The history of mathematics does not contain the name of a single Roman who was a mathematician in the true sense of the word. In Rome there was no Aristotle, Erastosthenes, Euclid, Hipparchus, or Ptolemy. There were mathematicians in Rome but they were Greek, and their pupils pursued the subject only from a utilitarian point of view. There was no school that caused the science of mathematics to make progress or develop to any degree worthy of note. The importance of Rome in the history of theoretical mathematics lies in the fact that Rome served as a stepping stone by which the subject was able to pass on to the curriculum of Medieval Europe.

Such men as Pliny the Elder, Varro, Vitruvius, Hyginus, Seneca, Censorinus, and Boethius are worthy of mention not because they were true scientists or mathematicians, but because their extant works serve as a compendium, as it were, of the scientific knowledge and beliefs of their day.

Pliny the Elder (23–79 A.D.) wrote the only Roman contribution to natural science—namely, his "Naturalis Historia."
The work contains twenty thousand "noteworthy observations" in thirty-seven books. The whole forms a repertory of the knowledge of the ancients with emphasis, as might be expected from the pen of a Roman, on the utility of the various subjects treated. In the middle ages, Pliny was received with unquestioning belief and served as a store-house of scientific knowledge. He is no longer considered a scientific authority because of his lack of precision, but he does hold an important place of value to the historian of science.

Along with Pliny in the class of Roman encyclopaedists who may be placed among the scientific spirits though not true discoverers of truth there may be classed Varro (B.C. 116-27). In the nine books of his "Disciplinae" he produced the earliest encyclopaedia of the liberal arts which included grammar, dialectic, rhetoric, geometry, arithmetic, astrology, music, medecine, and architecture.

Vitruvius wrote not as a man of letters, a rhetorician or a philosopher but as an architect, engineer and craftsman who was proud of his profession. "De Architecturae", a ten book treatise on architecture deals with temples, houses, aqueducts, sun dials, and engines of war. Vitruvius' skill in the latter field was appreciated by an "Imperator Caesar" who was probably Augustus. Vitruvius owed much to the lost "Disciplinae" of Varro. He is frequently quoted by the Elder Pliny and by Frontinus, since his work was not only a comprehensive treatise on architecture, but an encyclopedia of the technical and scientific knowledge of his day and a valuable compendium of the
at 10:00 a.m. on the date and place of the meeting.

The agenda for the meeting shall include:

1. Approval of the minutes of the previous meeting.
3. Discussion of the financial report.
4. Reports of the various committees.
5. Approval of the budget for the next fiscal year.
6. Consideration of any scheduled new business items.

The meeting shall conclude at 11:00 a.m. on the date and place of the meeting.

The minutes of the meeting shall be distributed to all members of the association and any guests in attendance.

The association shall maintain a record of all meetings, including the date, time, location, and agenda items.

The association shall also maintain a record of all actions taken during the meetings, including any votes taken.

The association shall ensure that all meetings are held in accordance with the rules and regulations governing associations in this jurisdiction.
writings on the subject by numerous Greek architects.

C. Julius Hyginus wrote "De Astrologia," also called "Poetica Astronomia" which written in four books is partly astronomical and mathematical, partly mythological and philosophical in its character. The value of the work though carelessly written lies in the information it furnishes relative to ancient astronomy.

L. Annaeus Seneca (3 B.C.-65 A.D.) was undoubtedly the most brilliant figure of his time as he embodied all the leading characteristics of the age with which he was in thorough harmony. Among his numerous works are the extant seven books entitled "Naturales Quaestiones." During the Middle Ages these were used as a standard textbook of physical science. Seneca's work contains more proof of actual knowledge than do the corresponding parts of Pliny's "Naturalis Historia." There is, however, a lack of scientific value because of the author's eclecticism and mode of treatment.

Censorinus flourished under Maximus and Gordianus about A.D. 238. On the occasion of the birthday of his wealthy friend Q. Cerellius Censorinus composed "De Die Natali." It is a small work which contains valuable information on topics of a chronological, mathematical, and cosmographical nature. While the style of the author is good, it is not free of the blemishes characteristic of his time. The subject matter is incomplete and probably largely compiled from Varro and a lost work of Suetonius. Censorinus also composed a work on geometry, but it is not extant.
Anicius Manlius Severinus Boethius (475-526 A.D.), a Christian by faith was called the "bridge from antiquity to modern times," since he gave Medieval Europe some glimpse of the intellectual life of the old world. Boethius was the last of the Romans of note to study in Greece. His "De Institutione Arithmetica Libri Duo" includes much of the work of Euclid, Nicomachus, and Theon of Smyrna. It is generally agreed that his works contain little that was original, but being superior to the other writers who lived just after the fall of Rome his arithmetic was looked on by his countrymen as representing the highest type of scholarship.

For Boethius all things in the world were either discrete (multitudes) or continuous (magnitudes). The former were represented by numbers or their ratios by music. The latter, at rest are treated by geometry, those, in motion by astronomy. These four subjects arithmetic, music, geometry, and astronomy comprised the "quadrivium" of which Boethius says in the introduction to his arithmetic:

"Inter omnes priscae auctoritatis viros qui Pythagora duce purore mentis ratione viguerunt, constare manifestum est, haud quemquam in philosophiae disciplinis ad cumulum perfectionis evadere, nisi cui talis prudentiae nobilitas quodam quasi quadrivio vestigatur, quod recte intuentis sol-lertiam non latebit."

("By all men of old reputation who following Pythagora's reputation have distinguished themselves by pure intellect it has always been considered settled that no one can reach the highest perfection of philosophical doctrines, who does not
Finger Reckoning

As has been previously mentioned, one of the primary aims in the teaching of arithmetic in the Roman schools was to secure perfection in finger reckoning or the representation of numbers by means of the fingers. It is frequently mentioned by the Latin writers, and was apparently of great importance as a commercial device where digital communication might overcome linguistic difficulties.

The idea of finger notation is primitive, and evidences of its usage date back to the earliest peoples. Nearly all the number systems, ancient and modern, are based on finger symbolism. It was found among the ancient Egyptians, Babylonians, Greeks, Romans, and the European nations of the Middle Ages. Today it is used in nearly all the Eastern Nations, among the Africans, Eskimos, South Sea Islanders, and Chinese. Among the lower races the systems based on the human anatomy are quinary, and vigesimal; but among the higher nations the former is too scanty, and the latter too cumbersome, hence the decimal system is preferred. People, however, do not consistently adhere entirely to any one system. The Romans, as was true of all who used the decimal scale, show evidences of the fact that the quinary system when extended to higher mathematics usually ran into the decimal or vigesimal system.

9Boethius, "De Institutione Arithmetica Liber Duo.", Liber Primus, Proemium.
null
The Greeks and Romans developed a system of finger notation whereby the smaller numbers from one to one hundred were represented by eighteen different positions of the fingers of the left hand. The numbers from one hundred to ten thousand were indicated by the right hand. The left hand applied to different parts of the body was used to express numbers from ten thousand to ninety thousand; and the right hand was similarly used for larger numbers.

An account of the gestures used in finger reckoning has been preserved by Bede "quum dicis decem, unguem indicis in medio figes artu pollicis. Quum dicis viginti summitatem pollicis inter medios indicis et impudici artus immittes. Quum dicis triginta, ungues indicis et pollicis blando coniunges amplexu---. Quum dicis quadraginta, interiora pollicis lateri vel dorso indicis superduces, ambobus dumtaxat erectis.... Quum dicis sexaginta, pollicem curvatum indice circumflexo diligenter a fronte praeceinges." ("When you mean ten, you will press the nail of the first finger against the middle joint of the thumb. When you mean twenty, you will place the tip of the thumb between the middle joints of the first and middle fingers. When you mean thirty, you will join the nails of the first finger and the thumb in a light circle. When you mean forty, you will place the lower part of the thumb against the side or back of the index finger, keeping both extended. When you mean sixty you will enclose the curved thumb by bringing the first finger around on the front."\(^{10}\)

\(^{10}\)(a) Bede, p. 122.
(b) For diagrams and explanations see Karpinski, "The History of Arithmetic."
The manner of representing the number of days of the year was shown by a certain statue of Janus of which Macrobius speaks. "Inde et simulacrum eius plerumque fingitur manu dextra C C C et sinistra sexaginta et quinque numerum tenens ad demonstrandum anni dimensionem." ("Commonly his statue is made, holding the number three hundred upon the right hand, and sixty-five upon the left, for the purpose of showing the length of the year.")¹¹

Of the same statue Pliny says "praeter Ianus geminus a Numa rege dicatus, qui pacis bellique argumento colitur digitis ita figuratis ut C C C L X V dierum non aut per significationem anni, temporis et aevi esse deum indicent; signa quoque Tuscania per terras dispersa, quae in Etruria facitata non est dubium." (And then besides King Numa dedicated the statue of the two-faced Janus; a deity who is worshipped as presiding over both peace and war. The fingers too are so formed as to indicate three hundred sixty-five days or in other words, the year; thus denoting that he is the god of time and duration.")¹²

Juvenal refers to finger representation of numbers in the words--"Felix nimirum qui tot per saecula mortem, distulit atque suos iam dextra computat annos". ("Happy was he indeed who postponed the hour of his death so long, and finally numbers his years upon his right hand.")¹³

¹¹Macrobius., "Saturnalia.", 1. 9. 10.
¹²Pliny., "Nat. His.", XXXIV. 7. 33.
The improbability of confounding the signs for sixty and forty is pointed out by Apuleius. He was accused of marrying a woman of sixty who was in reality but forty. In his refutation he points out the falsity of the accusation by proving that a mistake could not have been made due to a confusion of the finger gestures.

"Si triginta annos pro decem dixisses, posses videri computationis gestu errasse, quos circulare debueris digitos adperisse cum vero quadraginta, quae facilius ceteris porrecta palma significantur, ea quadraginta tu dimidio auges, non potes digitorum gestu errasse, nisi forte triginta annorum Pudentillam ratus binos cuiusque anni consules numerasti." ("If you should have said thirty years instead of ten, you could seem to have made the mistake in your counting gesture of pressing together the fingers which you should have made a circle of. But when you take forty which is more easily expressed with the hand outstretched, and increased it by one half then you cannot seem to have made a mistake in the showing of your fingers unless of course thinking Pudentilla was thirty years old you counted twice the consuls of each year.")\(^{14}\)

\(^{14}\) Apul. "Apologia." C. 89.
Ovid in speaking of the reckoning of the Romans says,

"Annum erat, decimum cum luna
reciperat orbem;
hic numerus magno tunc in honore
fuit;
seu quia tot digitii, per quos
numerare solemus
-- -- -- -- -- --
seu quod adusque decem numero
crescente venitur
principium spatiiis sumitur inde
novis."

("A year was counted when the moon has returned to the full for the tenth time; that number was then in great honor, whether because that is the number of the fingers by which we are wont to count...or because the numerals increase up to ten, and from that we start a fresh round.")

Numerous other Latin writers might be cited to indicate the universal use of finger symbolism in Roman life. It was a system which starting with the primitive man had expanded to meet the needs of a somewhat more complex civilization. This "stagnation" of reckoning and computing due to a cumbersome notation system is well stated by Dantzig: "The long period of nearly five thousand years saw the rise and fall of many a civilization, each leaving behind it a heritage of literature, art, philosophy, and religion. But what was the net achievement in the field of reckoning, the earliest art practiced by man? An inflexible numeration so crude as to make progress well nigh impossible, and a calculating device so limited in scope that even elementary calculations called for the services of an expert...Man used these devices for thousands of years without

\[^{15}\text{Ovid, "Fasti", III. 121-26.}\]
contriving a single worthwhile improvement in the instrument, without contributing a single important idea to the system.... Even when compared with the slow growth of ideas during the dark ages, the history of reckoning presents a peculiar picture of desolate stagnation.\textsuperscript{16}

Roman Numerals

Roman numerals were alphabetic in character as is shown by the forms employed today. Their origin is uncertain and authorities themselves vary and disagree on this point. There is an especial difference of opinion as to the source of letters used to represent certain numerals as C for one hundred, D for five hundred, M for one thousand, etc. Cantor states that the resemblance between the symbols and the letters of the Etruscan alphabet cannot be pronounced accidental.\textsuperscript{1}

Before the Roman ascendancy the Etruscans who ruled in Rome about 500 B.C. used numeral signs which resembled the symbols used later by the Romans. The following table shows the similarity between the two systems of number representation\textsuperscript{2}.

<table>
<thead>
<tr>
<th>Number</th>
<th>Etruscan Symbol</th>
<th>Roman Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( V )</td>
<td>( V )</td>
</tr>
<tr>
<td>10</td>
<td>( X )</td>
<td>( X )</td>
</tr>
<tr>
<td>50</td>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>100</td>
<td>( C )</td>
<td>( C )</td>
</tr>
<tr>
<td>1,000</td>
<td>( M )</td>
<td>( M )</td>
</tr>
</tbody>
</table>

\textsuperscript{16}Dantzig, "Number."

\textsuperscript{1}Moritz Cantor, "Vorlesungen über Geschichte der Mathematik", Vol. 1.
That the Roman numerals were probably derived from the Etruscans may be based on the following noteworthy peculiarities common to both:

1. The lack of zero.
2. The subtractive principle whereby a value may be diminished by placing before it one of the lower order, as IV for four.
3. The multiplicative effect of a bar over numerals, as XXX for 30,000.

There appears to be a general argument that in earliest times a number, if represented by a symbol, was indicated by the requisite number of strokes. These may have been merely "tally" marks or may have represented the fingers as in the Egyptian hieroglyphics where the symbols for one, two, three are one, two, three fingers. In later hieratic writing these symbols were reduced to straight lines with additional symbols added for ten and one hundred.

The oldest extant Egyptian and Phoenician writing repeats the symbol for unity as many times up to nine as was necessary and so on. There are no specimens of Greek numeration of a similar kind in existence, but there is every reason to believe the testimony of Iamblichus that the above mentioned method was the one by which the Greeks first expressed numbers in writing. Thus it may be assumed that the symbols for one through nine in the Roman numerals derived their origin, as in the case of earlier peoples, from the use of single strokes in writing, and being representative of finger notation. Throughout Roman history this system of representing numbers was current. The question
arises as to the origin of the use of symbols for and the way of representing numbers larger than nine. It is upon this point that we find equally authoritative sources disagreeing. Imaginative historians have been unusually active in this field and scholars of mathematics have not failed to advance possible working hypothesis. The lack of positive information on the subject, and the lack of consistency among the Romans themselves in using the symbols as well as the contracted forms of lapidary characters, renders any one theory inadequate, and at the same time gives basis to support the various theories advanced.

A dominating feature of the notation systems of antiquity is the addition and subtraction of symbols. Thus were numbers expressed by a few signs, and these symbols combined by addition or by addition combined with multiplication or by subtraction. Alexander Von Humboldt says "Summations by juxtaposition one finds everywhere among the Etruscans, Romans, and Egyptians; the subtraction or lessening forms of speech among the Romans in undeviginti (unus de viginti) for nineteen, undeoctoginta for seventy-nine, duo de quadraginta for thirty-eight."\(^3\)

In none of these systems do we find the all important principle of position or principle of local value such as we have in our notation now in use. Having missed this principle the ancients had no use for a symbol to represent zero, and thus were far from reaching an ideal system of notation. Here both the Greeks and Romans failed to achieve what the Hindus

\(^3\)Cajori, "A History of Mathematical Notations."
Von Humboldt, Alexander, "Crelles die reine und angewandte mathematik."
did in the fifth or sixth century after Christ. To the latter we are indebted for a great contribution to mathematical science; namely, the development of the use of zero and principle of local value. The older notations merely recorded an answer to an arithmetical computation while the Hindu (wrongly called the Arabic) notation assists in actually performing the computation itself. For example, the product of seven hundred twenty-three and three hundred sixty-four would be indicated in the Hindu notation by 723×364 with each symbol having value by position; in the Roman notation the same would be indicated by DCCXXIII and CCLXIV with each symbol having absolute value. In the latter case little or no help is offered as regards the actual computation. An abacus would be needed for the calculation and then the result might be represented by Roman numerals.

The principle of addition is the older principle as applied in systems of notation. Its use in Roman notation is clearly shown in such numerals as II, XII, CC, MDC, etc. Equally conspicuous is that of subtraction by which a symbol is placed before one of greater value indicative that its value is to be subtracted from the greater as in IV, IX, XL.

Mommsen gives the following points in regard to the subtractive method:

1. "Not merely one number but many in order could be subtracted, so that IIX is just as correct as IX.
2. "Only the numerals I, X, C were as a rule used in subtraction; seldom the sign for one thousand and never for five, fifty and five hundred.

<table>
<thead>
<tr>
<th>C.I.L. I</th>
<th>1166</th>
<th>C.CX</th>
<th>C.X ( \text{VIII} ) S</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.I.L. I</td>
<td>1143</td>
<td>C.X</td>
<td>C.XVC</td>
</tr>
<tr>
<td>C.I.L. I</td>
<td>536 dating 567/187</td>
<td>C.CXVC</td>
<td></td>
</tr>
<tr>
<td>C.I.L. I</td>
<td>198 dating 631/123</td>
<td>C.DL</td>
<td></td>
</tr>
<tr>
<td>C.I.L. VI 1243 e.f.c.</td>
<td>( \text{L} ) C ( \infty ) L X</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. "The number one is subtracted as a rule only before V and X, and only by exception before L and the higher numbers.

4. "The subtractive method has the object of saving space. It is therefore inadmissible unless room is gained so VII not IIIX; LXX not XXXC. It predominates in cases when an essential simplification is obtained and therefore particularly in the numbers eighty and ninety, and occurs more in the carelessly made private inscriptions than in the precise monumental writing.

5. "The numeral or numerals to be subtracted were placed before the last numeral of the addition series so that XIIIX = 18 was written instead of IIIXX and CCCX = 380 instead of XXCCCC."^4

In regard to the subtractive principle Adrian Cappelli makes the statement:

"The well-known rule that a smaller number placed to the left of the larger shall be subtracted from the latter as \( \text{D} \) \( \text{C} \) \( \text{D} \) \( \text{C} \) = 4,000, etc. was seldom applied by Romans and during the entire middle ages finds only a few instances of it."^5

^4 Cajori, "A History of Mathematical Notations."
Mommsen, Hermes, XXII. 603.
(b) Hervas, "Arithmetica della Nazioni," (1786) P. 11, 16.
minutes now. Here is the presentation of the session on the
research methods used and the results obtained in the study.
I believe these findings will be valuable to us, as they have implications for our own investigations and may guide us in understanding the nature of the relationship between these two
factors. Further research is needed to confirm these results and to
explore the potential implications for medical practice.

These findings suggest that the use of cognitive-behavioral therapy is associated with improved mental health outcomes for patients with depression. The benefits of this approach are particularly evident in long-term studies, where the positive effects are sustained over time.

It is important to note, however, that the effectiveness of cognitive-behavioral therapy may vary depending on the individual patient and the specific circumstances of the treatment. Therefore, it is essential to tailor the interventions to the needs of each patient to achieve optimal results.

In conclusion, the findings presented here highlight the potential of cognitive-behavioral therapy in improving mental health outcomes for patients with depression. Further research is needed to confirm these findings and to explore the mechanisms underlying this relationship. In the meantime, clinicians may consider incorporating these approaches into their treatment protocols to enhance patient outcomes.
However, the subtractive principle is found on some early tombstones and on a signboard of 130 B.C. where at the crowded end of a line eighty-three is written *XXCIII* instead of *LXXXII*. One finds even four units placed before ten as *IIIIX* for six in rare Roman inscriptions. The principle of subtraction has passed over into the graphics of numbers when the group signs for five, ten, and even their multiples are placed to the left of the characters they modify as *IV* and *IΔ* for four, and *XL*, and *XT* for forty.

The later Roman notation contains sporadic occurrences of the principle of multiplication. According to this principle *VM* does not stand for 995, but for 5,000. One finds in Pliny:⁶ *LXXXIII.M* for 83,000; *XCII.M* for 92,000; *CX.M* for 110,000.

Although variations and other symbols are found, the usual numerals used in Roman notation were: *I* (1), *II* (2), *III* (3), *IV* or *IΔ* (4), *V* (5), *VI* (6), *VII* (7), *VIII* or *IIX* (8), *VIIII* or *IX* (9), *X* (10), *XI* (11), *XII* (12), *XIII* (13), *XIII* or *XIV* (14), *XV* (15), *XVI* (16), *XVII* (17), *XVIII* or *XIX* (18), *XVIII* or *XX* (19), *XX* (20), *XXI* (21), ..., *XXVIII* or *XXIX* (28), *XXVIII* or *XXIX* (29), *XXX* (30), ..., *XXXX* or *XL* (40), ..., *L* (50), *LX* (60), ..., *LXX* (70), *LXXX* or *XC* (80), ..., *LXXXX* or *XC* (90), ..., *XCIIX* or *ICC* (98), *XCIX* or *IC* (99), *C* (100), ..., *CXIII* or *CXXIV* (124), ..., *CC* (200), ..., *CCXXX* (230), ..., *CCC* (300), ..., *CCCC* (400), ..., *IC* (500), ..., *IIC* (600), ..., *IC* (700), ..., *IICC* (800), *IICCCC* (900), ..., *CIIC* (1000), ..., *CIIC* *XXXV* (1235), ..., *CIIC* (2000), ...

⁶Pliny, "Nat. Hist.", VII. 26; XXXIII. 3; IV praef.
Certain of the Roman numerals for smaller numbers deserve special consideration. The symbol for two with the earlier position of line of differentiation is regularly seen in //++ when sestertius is denoted; // occurs very frequently, particularly when indicating successive consularships and duumviri ( // vir); also at the beginning of lines // is found.

For four either IV or IIII is common since the former, the subtractive form, belongs to the writing of the lower class. It first appeared on vessels of San Caesareo to date the seventh century of the city. IIII is the more usual form and has the following variations: ++, //, ///// while quattuorviri is devoted by ||||, or ///// vir.

The origin of the symbol for five being somewhat linked with that for ten is elsewhere discussed. Mention, however, may be made that the usual symbol V is found on Pompeian tablets on its side, and that in African inscriptions ///// appears.

For six VI is most common but ///// also is found. In the case of eight, the subtractive form (IX) is of rare occurrence with the additive form VIII preferred. The same is true of nine with its rare subtractive form IX.

According to Mommsen the Roman number symbols I, V, X, represent the finger, the open hand with thumb extended, and

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7 Roby, "A Grammar of the Latin Language from Plautus to Suetonius."
double the hand respectively. This theory is a generally accepted one in view of the fact that it is consistent with the symbolic numeral representation of earlier civilizations than the Roman.

Zangemeister proceeds from the standpoint that decem is related to decussare meaning a ‖ or an oblique crossing and argues that every straight or curved line drawn across the symbol of a number in the decimal system multiplies that number by ten. Representations of one, ten, one thousand, five, five hundred found on monuments support his assertion.

Some Latinists maintain that the three Greek aspirate letters which the Romans did not require, viz. ψ, θ, φ, i.e. χ (ch), θ(th), φ(ph) were used to represent the numbers ten, fifty, and one hundred. From θ viz. ω came the symbol for ten and the half of it for five.

Karpenski and others reason that the ten symbol is supposed to be derived from some form of crossing out nine units as ‖‖‖‖‖‖‖‖‖‖‖‖ . This view is confirmed by the symbol for twenty ‖‖ , ‖‖ (twenty-five), ‖‖ (thirty), ‖‖ (thirty-five) ‖‖ (forty) found in some inscriptions.

There is also the explanation given that the Roman sign for ten was formed by two V's with their vertices placed together. This theory led to the use of "decussis" to denote the intersection of two lines in the form of a cross, "regula fititur in primo decussis puncto."8

8Vit., "DeArchit.", 10, 11.
A variation in X for ten is a transverse cut in the middle of the letter when denarius is denoted ¥.

Fourteen represented by XIII is more common than XIV. Eighteen and nineteen are indicated by the additive forms XVIII and XVIII respectively although for the former XIX is found. Similar is the case of twenty-eight (XXVIII) with the form XXIX appearing.

The sign for forty-five is regularly XLV but with the exception VL. For fifty ↓ is the original form while ↓ appears down to the Augustan period. ↓ is seen in the Lex Repetundarum (123 B.C.) and during the first and second centuries A. D. to represent numbers from fifty to one hundred the additive forms of L and the necessary X's are usual. XXXXX is found, however, and [xxx], [xxv] are worthy of notice.

C for one hundred is found on earlier monuments but it could not have been adopted from the word "centum" as the earlier Latin initial of centum was K. Thus it would seem that the symbol was used and a later explanation was made.

Wordsworth derives C for centum, M for mille, L for quin- quaginta from the three letters of the Chalcidian alphabet. He states, "The origin of this notation, I believe, quite uncertain or rather purely arbitrary, though of course we observe that the initials of mille and centum determine the final shape taken by the signs which at first were very different in form."
Cantor states "Accidental appears the relationship with the later Roman signs I, V, X, L, C, M, which from their resemblance to letters transformed themselves by popular etymology into these very letters."\[13]\n
Following the previously mentioned theory that the three Greek aspirate letters were used by the Romans to represent numbers, we find that \( \Upsilon \) was written \( \underline{L} \) and abbreviated into \( L \) as a symbol for fifty, that \( \Theta \) from a false notion of its origin was made like the initial letter of centum, while \( \Phi \) was assimilated to the ordinary letters \( C\Omega \). The half of \( \Phi \) was \( D \) and was taken to be one half of one thousand or five hundred. \( D \) marked by a transverse into \( D \) is found in the Lex Repetundarum and in many inscriptions of the empire. The remaining Roman numerals were formed by indicating multiplication by doubling the signs. If \( \Phi \) represented one thousand then \( \Theta \) represented ten thousand, and \( \Omega \) one hundred thousand.

As to halves if \( D \) represented five hundred then half of each of the other symbols indicated half the value as \( D \) for five thousand, \( D \) for fifty thousand. In as much as in ancient times, Roman numerals did not go beyond one hundred thousand, it was necessary to repeat the symbol for one hundred thousand to indicate higher amounts. Inscriptions on the Columna Rostrata show the sign for one hundred thousand repeated as many as thirty times. At a later period a sign \( \Theta \) indicating quingenta milia is found for the repeated forms. In the time

\[13\] Moritz Cantor, "Vorlesungen uber Geschichte der Mathematik," Vol. I.
of Caesar and Cicero methods of indicating large numbers varied but in general writers employed the words in full. \( \Phi \), the oldest symbol for one thousand was not maintained, but appeared in the more common form modified often into \( \sim \) or \( \times \).

According to Priscan, the celebrated Latin Grammarian of about 500 A.D., the symbol \( \Phi \), used for one thousand, originated from the ancient Greek sign \( \varepsilon \) for the same number. The symbol \( \Phi \) was modified by connecting the sides by curved lines to eliminate confusion with the Roman symbol for ten. As late as 1593 \( \Phi \) is found used by C. Dasypodius, the designer of the famous Strassbourg cathedral.

The symbol \( \Phi \) for one thousand was a / enclosed in parenthesis or a cursive writing of the symbol \( \Phi \) although this style of parenthesis to us more nearly resembles a \( C \) made forward on the left of the stroke and backward on the right. \( \Phi \) represented one half the value or five hundred. Priscan states "quinque milia per \( \Pi \) et duos in dextera parte apostrophos \( \Pi \Pi \), decem milia per supra dictam formam additis in sinistra parte contrariis duabus notis quam sunt apostrophi \( \Pi \Pi \Pi \Pi \)." Through Priscan it is established that this notation is at least as old as 500 A.D., and probably was much older.

Cicero in the fifth against Verres expresses 3600 by \( \Phi \Phi \Phi \Phi \Phi \).

Previous to the time of Augustus the symbol \( \Lambda \) for one thousand is found, and from the second century on \( \Lambda \) is found although usually in the combination \( \Lambda \cdot P \) (milia passuum).

14Priscan, "De Figuris Numerorum," (Lipsiae, 1859), Vol. III.
\( \text{\( \overline{\text{M}} \) was never used by the Romans as a numeral but always as an abbreviation. In the time of Hadrian the sign \( \overline{\text{I}} \) regularly denoting one hundred thousand was strangely used to indicate one thousand.}

Combinations of signs were used to denote higher numbers for 4100, \( \overline{\text{C}} \) for 3220 \( \overline{\text{C}} \text{XIDCLXI} \) for 21,661. By exception the additional thousands were denoted by the subtractive forms: thus \( \text{\( \overline{\text{C}} \text{X} \)} (4,000), \( \overline{\text{D}}, \overline{\text{I}}, \overline{\text{L}}, \overline{\text{D}}, (\text{\( \overline{\text{L}} \))}, \text{\( \overline{\text{D}} \)} (10,000); \( \overline{\text{D}}, \overline{\text{I}}, \overline{\text{X}}, \overline{\text{C}} \) (5,000); \( \overline{\text{C}}, \overline{\text{X}} \) (100,000); \( \overline{\text{D}}, \overline{\text{C}} \) (50,000).

Any number could be augmented in value a thousand times either by enclosing it with two hooks \( \overline{\text{C}} \overline{\text{C}}, \) or by a horizontal line placed above it. Thus \( \overline{\text{C}} \text{X} \overline{\text{O}} \) or \( \overline{\text{C}} \) denoted ten thousand.

Pliny uses \( \overline{\text{C}} \text{LIVIM} \) for 156,000,000. Aelius Lampridius (fourth century A.D.) says in one place "\( \text{\( \overline{\text{C}} \text{X} \text{X} \)} \) equitum Persarum fudimus; et mox \( \overline{\text{X}} \) in bello interemimus" to express the numbers 120,000 and 10,000 respectively. From the time of Hadrian inscriptions are found which indicate thousands by strokes placed on top and also on the sides, but in general this method was employed for hundred thousands as \( \overline{\text{X}} \text{L} \text{X} \text{X} \text{D} \) \( \text{\( \overline{\text{C}} \))} \) for 1,180,600. In the more recent practice the strokes sometimes occur only on the sides as in \( \overline{\text{X}} \text{I} \text{D} \text{C} \cdot \text{X} \text{C} \). Naturally the variety of uses for which the horizontal and vertical lines were used caused confusion as is well illustrated by Suetonius:

"Observavit ante omnis Liviam Augustam, cuius et vivae\( ^{15} \text{C.I.L. X 1273} \).
gratia plurimum valuit et mortuæ testamento paene ditatus est; sestertium namque quingentiis praecipium inter legatarios habuit, sed quia notata non perscripta erat summa, herede Tiberio legatum ad quingenta revocante ne haec quidem accepit." ("He showed marked respect to Livia Augusta to whose favor he owed great influence during her lifetime and by whose last will he almost became a rich man; for he had the largest bequest among her legatees, one of fifty million sesterces. But because the sum was designated in figures and not written out in words, Tiberius, who was her heir reduced the bequest to five hundred thousand, and Galba never received even that amount.")

Occasionally a letter was placed over another to indicate their product thus $\frac{D}{M}$ would express 500,000. The multiplier was sometimes written as an exponent as $\frac{Ll}{C}$ was used to express three hundred. In expressing very large numbers points were sometimes interposed as Pliny writes XVI.XX.DCCCXXIX for 1,620,829.

In the later centuries of the Roman empire the smaller letters of the alphabet seem to have been used in imitation of the numeral systems of the Greeks. The letters a, b...i represented the nine digits 1...9; the next series h...r expressed 10, 20, 30...90; the remaining letters t...z represented 100...500. To represent the rest of the hundreds it was necessary to employ capitals or other characters and 600 ...900 were designated by I, V, hi, hu. This mode of notation

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16 Suetonius, "Galba", V.
never received any degree of accuracy and was confined mostly to the use of those foreign adventurers from Greece, Egypt and Chaldea who preyed on the credulity of the Romans by pretended skill in astrology.

It may be observed that there were numerous classes of numerals, and variations in their form. In the compound numbers thirteen through nineteen, the smaller is usually prefixed to the larger without "et" as "septem decem" or "septem decim". The order is sometimes reversed in cardinals and ordinals. Often in the former "et" is inserted especially if the larger number comes first e.g. decem septem, decem et septem, septem et decem. With the numbers twenty-one to ninety-nine the rule is that either the larger should precede the smaller number without "et" or the smaller precede the larger with "et" e.g. "viginti quattuor" or "quattuor et viginti".

In ordinals and distributives exceptions to both usages occur e.g. "quadragesimun et sextum", "sexto tricesimo", "quin- quagena et singula" "quinos vicenos", etc. However, sometimes in cardinals and distributives the conjunction is sometimes inserted even when the larger precede e.g. "viginti et quisque" (Cic.) "quadraginta et quisque" (Livy, etc.).

From a hundred and one up the larger number is usually put first either without or with a conjunction (except in distributives) e.g. "ducentos (et) quadraginta et quattuor", quingentesimun (et) quinquagesimum (et) octavum, duceni septuageni, centies (et) quadragies. With a conjunction the smaller (cardinal or ordinal) number sometimes is found
preceding e.g. "quinquaginta et ducenta", "septimum et quinquagesimum ac centesimum." Also "ducentos et mille", and "mille et ducentos" are found.

For 18, 19, 28, 29, etc. the subtractive forms e.g. "duodevigenti", "undevigenti", "undetrigesimus", etc. are most common. The compound forms are also found e.g. "decem octo", "decem et octo" (frequently), "octo decim" (rarely), "novem et triginta", "quinquaginta octo", "triginta novem" (Liv.), "octava decimo" (Tac.), "octoni deni" (Liv.).

Distributives indicate that the number belongs to each of several persons or things as "Caesar et Ariovistus denos comites ad colloquim adduxerunt" ("Caesar and Ariovistus took ten companions each to the conference"). If "singuli" is expressed with the persons, etc. the cardinal number may be used with the things numbered e.g. "singulis denarii trecenti imperabantur" ("each was required to pay three hundred pence"). In this use "terni" not "trini" is found. Distributives are used in expressions of multiplication e.g. "bis bina" (twice two). The distributive numerals in such expressions as "decies centena millia" do not mean a million to each person but a hundred thousand taken each of ten times. They are found in nouns which have no singular e.g. "bina castra" (the two camps). In such usage "uni" not "singuli", "trini" not "terni" would be used. Poets use distributives as merely equivalent to cardinals e.g. "centum quoi brachia dicunt centenasque manus" ("a hundred hands in all not a hundred in each arm"). Post-Augustan writers use "trinus" not "ternus". The poets also sometimes
use the distributives in the singular e.g. "centauri corpore bino" (a double body), "centenaque arbore fluctum veberat assurgens" (with an hundred-fold shaft, i.e. a hundred oars). Each other is expressed by "alterni" e.g. "alternis diebus" (every second day).

Multiplicative adjectives are formed with the suffix "plex" (fold) viz. simplex, sescuplex (1 1/2 fold), duplex, triplex, quadruplex, quincuplex, septemplex, decemplex, centuplex.

Others in "plus" are generally used in the neuter only to denote a magnitude twice, etc. as great as another. These are simplus sescuplus, duplus, triplus, quadruplus, octuplus.

Another series (Front. "De Aqua." 26-62) is binarius (containing two), tenarius, quaternarius, quinarius, senarius, septenarius, octonarius, novenarius, denarius, duodenerarius, vicenarius, duodenarius, vicenarius ("lex quina vicenaria", Plaut.) tricenarius, quadragenarius, quinquagenarius, sexagenarius, non-agenarius centenarius, ducenarius, trecenarius, quadringenarius, quingenarius, septingenarius, octingenarius, millenarius.
### List of Roman Numerals

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I</td>
<td>unus-a-um&lt;sup&gt;18&lt;/sup&gt;</td>
<td>primus-a-um</td>
<td>singuli-ae-a</td>
<td>semel</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>duo-ae-o&lt;sup&gt;19&lt;/sup&gt;</td>
<td>secundus-a-um</td>
<td>bini</td>
<td>bis</td>
</tr>
<tr>
<td>3</td>
<td>III</td>
<td>tres, tria</td>
<td>tertius-a-um</td>
<td>terni (or trini)</td>
<td>ter</td>
</tr>
<tr>
<td>4</td>
<td>III or IV</td>
<td>quattuor</td>
<td>quartus-a-um</td>
<td>quaterni</td>
<td>quater</td>
</tr>
<tr>
<td>5</td>
<td>V</td>
<td>quinque</td>
<td>quintus</td>
<td>quini</td>
<td>quinquiens&lt;sup&gt;21&lt;/sup&gt;</td>
</tr>
<tr>
<td>6</td>
<td>VI</td>
<td>sex</td>
<td>sextus-a-um</td>
<td>seni</td>
<td>sexiens</td>
</tr>
<tr>
<td>7</td>
<td>VII</td>
<td>septem</td>
<td>septimus-a-um</td>
<td>septeni</td>
<td>septiens</td>
</tr>
<tr>
<td>8</td>
<td>VIII or I X</td>
<td>Octo</td>
<td>octavus</td>
<td>octoni</td>
<td>octiens</td>
</tr>
<tr>
<td>9</td>
<td>VIII or IX</td>
<td>novem</td>
<td>nonus</td>
<td>noveni</td>
<td>noviens</td>
</tr>
<tr>
<td>10</td>
<td>X</td>
<td>decem</td>
<td>decimus</td>
<td>deni</td>
<td>deciens</td>
</tr>
<tr>
<td>11</td>
<td>XI</td>
<td>undecim</td>
<td>undecimal</td>
<td>undeni</td>
<td>undeciens</td>
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<td>12</td>
<td>XII</td>
<td>duodecim</td>
<td>duodecimus</td>
<td>duodeni</td>
<td>duodeciens</td>
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<td>13</td>
<td>XIII</td>
<td>tredecim</td>
<td>tertius decimus</td>
<td>terni deni</td>
<td>terdecies</td>
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<td>14</td>
<td>XIII or XIV</td>
<td>quattuordecim</td>
<td>quartus decimus</td>
<td>quaternideni</td>
<td>quaterdecien</td>
</tr>
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<td>XV</td>
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<td>quintus decimus</td>
<td>quini deni</td>
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<td>Roman Numeral</td>
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<td>(Continued)</td>
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<td>16</td>
<td>XVI</td>
<td>sedecim</td>
<td>seni deni</td>
<td>sedeciens</td>
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</tr>
<tr>
<td>17</td>
<td>XVII</td>
<td>septemdecim</td>
<td>septenii denii</td>
<td>septiens deciens</td>
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<td>XVIII or XIX</td>
<td>duodeviginti</td>
<td>duodevicensimus (^{22})</td>
<td>duodeviciens (?)</td>
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</tr>
<tr>
<td>19</td>
<td>XVIII or XIX</td>
<td>undeviginti</td>
<td>undevicensimus</td>
<td>undeviciens (?)</td>
<td></td>
</tr>
<tr>
<td>20</td>
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<td>viginti</td>
<td>vicenii</td>
<td>vicens</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>XXI</td>
<td>unus et viginti</td>
<td>unus (more rarely primus) et vicensimus</td>
<td>vicien singuli</td>
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<td>22</td>
<td>XXII</td>
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<td>alter et vicensimus</td>
<td>vicien bini</td>
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</tr>
<tr>
<td>28</td>
<td>XXVIII or XXIX</td>
<td>duodetrigintia</td>
<td>duodetricensimus</td>
<td>duodetriici</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>XXVIII or XXIX</td>
<td>undetrigintia</td>
<td>undetricensimus</td>
<td>undetricieni</td>
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</tr>
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<td>triginta</td>
<td>tricenii</td>
<td>tricien</td>
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</tr>
<tr>
<td>40</td>
<td>XXXX or XL</td>
<td>quadranginta</td>
<td>quadriageni</td>
<td>quadragiensi</td>
<td></td>
</tr>
<tr>
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<td>L</td>
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<td>quinquageni</td>
<td>quinquagieni</td>
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</tr>
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<td>sexageni</td>
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<td>septuageni</td>
<td>septuagien</td>
<td></td>
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<tr>
<td>80</td>
<td>LXXX or XXC</td>
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<td>octogeni</td>
<td>octogiens</td>
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<td>Latin</td>
<td>English</td>
<td>English</td>
<td>English</td>
<td>English</td>
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<tr>
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<td>---------</td>
<td>---------</td>
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<td>90</td>
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<td>nonaginta</td>
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<td>nonageni</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or XC</td>
<td></td>
<td></td>
<td>nonagiens</td>
<td></td>
</tr>
<tr>
<td>98</td>
<td>XCIIX</td>
<td>octo et nonaginta</td>
<td>duodecentensimus</td>
<td>duodecenteni</td>
<td></td>
</tr>
<tr>
<td>or IIC</td>
<td></td>
<td></td>
<td></td>
<td>duodecentiens</td>
<td></td>
</tr>
<tr>
<td>99</td>
<td>XCIX</td>
<td>undecentum</td>
<td>undecentensimus</td>
<td>undecenteni</td>
<td></td>
</tr>
<tr>
<td>or IC</td>
<td></td>
<td></td>
<td></td>
<td>undecentiens</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>C</td>
<td>centum</td>
<td>centensimus</td>
<td>centeni</td>
<td></td>
</tr>
<tr>
<td>101</td>
<td>CI</td>
<td>centum (et) unus</td>
<td>centensimus primus</td>
<td>centeni singuli</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>centiens semel</td>
<td></td>
</tr>
<tr>
<td>124</td>
<td>CXXIII</td>
<td>centum viginti quattuor</td>
<td>centensimus vicen-simus quartus</td>
<td>centeni vicini quaterni</td>
<td></td>
</tr>
<tr>
<td>or CXXIV</td>
<td></td>
<td></td>
<td></td>
<td>centiens viciens quater</td>
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</tr>
<tr>
<td>200</td>
<td>CC</td>
<td>ducenti-aed-a</td>
<td>ducentensimus</td>
<td>duceni23</td>
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</tr>
<tr>
<td>230</td>
<td>CCXXX</td>
<td>ducenti (ae-a) triginta</td>
<td>ducentensimus tricensimus</td>
<td>ducenti triceni</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>CCC</td>
<td>trecenti-aed-a</td>
<td>trecentensimus</td>
<td>trecenteni</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>CCC</td>
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<td>quadrungentensimus</td>
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<td>500</td>
<td>L</td>
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<td>quingentensimus</td>
<td>quingeni</td>
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</tr>
<tr>
<td>600</td>
<td>LCC</td>
<td>sescenti-aed-a</td>
<td>sescentensimus</td>
<td>sescenti</td>
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<td>septingentensimus</td>
<td>septingeni</td>
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</tr>
<tr>
<td>800</td>
<td>LCCCC</td>
<td>octingenti-aed-a</td>
<td>octingentensimus</td>
<td>octingeni</td>
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</tr>
<tr>
<td>900</td>
<td>LCCCCC</td>
<td>nongenti-aed-a</td>
<td>nongentensimus</td>
<td>nongeni</td>
<td></td>
</tr>
<tr>
<td>1,000</td>
<td>CD</td>
<td>mille</td>
<td>millensimius</td>
<td>singula millia</td>
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</tr>
<tr>
<td>1,235</td>
<td>CDCCXXXV</td>
<td>mille ducenti(ae-a) triginta quinque</td>
<td>millensimus ducen-tensimus tricensimus quintus</td>
<td>singula millia</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>milliens ducentiens</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ducentiens triceni</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>quina</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>quinquiens</td>
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### List of Roman Numerals (Continued)

<table>
<thead>
<tr>
<th>Numerals</th>
<th>English Equivalent</th>
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<tbody>
<tr>
<td>2,000</td>
<td>duo millia</td>
</tr>
<tr>
<td>4,000</td>
<td>quattuor millia</td>
</tr>
<tr>
<td>5,000</td>
<td>quinque millia</td>
</tr>
<tr>
<td>6,000</td>
<td>sex millia</td>
</tr>
<tr>
<td>10,000</td>
<td>decem millia</td>
</tr>
<tr>
<td>20,000</td>
<td>viginti millia</td>
</tr>
<tr>
<td>50,000</td>
<td>quinquaginta millia</td>
</tr>
<tr>
<td>100,000</td>
<td>centum millia</td>
</tr>
<tr>
<td>500,000</td>
<td>quingenta millia</td>
</tr>
<tr>
<td>100,000,000</td>
<td>decies centum millia</td>
</tr>
</tbody>
</table>

18 Unus is declined in the plural only when used with nouns whose plural denotes a singular as unae litterae (one epistle), or when used as a pronoun.  
19 Duo, tres, are declined while the other cardinals to centum are undeclined also mille when an adjective. As a substantive it has a declinable plural in media. The other cardinals, all ordinals and distributive numbers are declinable adjectives.  
20 The ordinals not the cardinals are used in giving dates--1938 is anno millesimo nongentesimo treciesimo octavo.  
21 In adverbs "ens" instead of the later "es" is found in the "Monumentum Ancyranum."  
22 Following the "Monumentum Ancyranum" the ordinal terminations "ensimus" is used instead of the older "ensumus" and the later "esimus."  
23 Some good M.S.S. rarely give other forms for the distributives of hundreds e.g. ducenteni, quadrincenteni, etc. These are mentioned by Priscan.  
24 The "Monumentum Ancyranum" gives millia not milia.
Roman Fractions

A striking feature in Roman arithmetic was the partiality for duodecimal fractions. Just why this system was used instead of the decimal system except that it was probably of Etruscan origin and for this reason was adopted in Rome is not known. An answer may lie in the fact that in everyday affairs the divisions of a unit into two, three, four or six equal parts is commonest and the duodecimal fractions give easier representation of these parts. Equally indefinite also is any information regarding the time, place or manner of origin of these fractions. Although the advantage of twelve was recognized by early peoples, the Romans were first of the later peoples to take advantage of using the duodecimal system.

The Babylonians had shown a predilection for sixty since the number of factors of it permits of easy fractional expressions as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{12}$ etc. These fractions known as sexagesimal fractions derive their name from the Latin sexagesimus (sixtieth from sexaginta meaning sixty). In the middle ages they were called "physical or astronomical fractions." To distinguish them from those used by merchants, the latter were called "fractiones vulgares" whence the name "vulgar fractions" of England and "common fractions" of America. Because of the large number of factors of one hundred twenty, the Greek Astronomers took one hundred twenty units for the diameter of their standard circle or sixty units for the radius. Since three was the value commonly used for $\pi$ by the
ancients, the Greeks were led to take three times one hundred twenty or three hundred sixty as the number of units in a circumference. They next followed the Babylonian system and divided each of these units into sixtieths and these again into sixtieths and so on. This circular unit the Greeks called "moira" meaning position or degree. The Latins translated this as "gradus" meaning step, and it appears in English as "degree". The sixtieth of a "moira" in Greek was known as a "prota hexekosta" or "lepta" and in Latin, as "pars minuta prima." The next lower division into sixtieths was in Greek a "deutera herekosta" and in Latin "pars minuta secunda".

Unlike the Greeks the Romans began with concrete fractions, which became abstract as they were applied to other measurements. Each duodecimal subdivision had its own name and symbol so that Roman fractions became a complicated system.
In place of the straight lines (—) occur also curved ones (««).

The Roman "as" weighing one pound was divided into twelve "unciae" which were again divided and subdivided into twelve parts until we find 1/144 of an uncia represented. The abstract fraction 11/12 was called deuna (de - uncia i.e. one less uncia) while the various fractions had their individual names and signs, not all of these were used to the same extent. Since \( \frac{1}{6} \) plus 1/3 equals five sixths, there was commonly used in ordinary life semis et triens in place of decunx.

\[ \text{TABLE OF ROMAN FRACTIONS} \]

<table>
<thead>
<tr>
<th>Name of Fraction</th>
<th>Value as part of an &quot;As&quot;</th>
<th>Value as part of an &quot;Uncia&quot;</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>As</td>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Deunx</td>
<td>11/12</td>
<td>11</td>
<td>( s = - or : : : )</td>
</tr>
<tr>
<td>Dextans (decuns)</td>
<td>5/6</td>
<td>10</td>
<td>( s = - or : : : )</td>
</tr>
<tr>
<td>Dodans</td>
<td>3/4</td>
<td>9</td>
<td>( s = - or )</td>
</tr>
<tr>
<td>Bes</td>
<td>2/3</td>
<td>8</td>
<td>( s = or - s - or S )</td>
</tr>
<tr>
<td>Septunx</td>
<td>7/12</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Semis</td>
<td>1/2</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Quincunx</td>
<td>5/12</td>
<td>6</td>
<td>( \frac{1}{2} or )</td>
</tr>
<tr>
<td>Triens</td>
<td>1/3</td>
<td>4</td>
<td>( \frac{1}{3} or )</td>
</tr>
<tr>
<td>Quadrans</td>
<td>1/4</td>
<td>3</td>
<td>( \frac{1}{4} \text{ or } )</td>
</tr>
<tr>
<td>Sextans</td>
<td>1/6</td>
<td>2</td>
<td>( \frac{1}{6} \text{ or } )</td>
</tr>
<tr>
<td>Sescunx (Sescuncia)</td>
<td>1/8</td>
<td>1/2</td>
<td>( \frac{1}{8} \text{ or } )</td>
</tr>
<tr>
<td>Uncia</td>
<td>1/12</td>
<td>1</td>
<td>( \frac{1}{12} \text{ or } )</td>
</tr>
<tr>
<td>Semuncia</td>
<td>1/24</td>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>Binal sextulae</td>
<td>1/36</td>
<td>1/3</td>
<td>( \cup )</td>
</tr>
<tr>
<td>or Duella</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sicilicus</td>
<td>1/48</td>
<td>1/4</td>
<td>( \frac{1}{4} \text{ or } )</td>
</tr>
<tr>
<td>Sextula</td>
<td>1/72</td>
<td>1/6</td>
<td>( \frac{1}{6} \text{ or } )</td>
</tr>
<tr>
<td>Dimidia sextula</td>
<td>1/144</td>
<td>1/12</td>
<td>( \frac{1}{144} \text{ or } )</td>
</tr>
<tr>
<td>Scriptulum or</td>
<td>1/288</td>
<td>1/24</td>
<td>( \frac{1}{288} \text{ or } )</td>
</tr>
<tr>
<td>Scripulum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Siliqua</td>
<td>1/1728</td>
<td>1/144</td>
<td>( \frac{1}{1728} \text{ or } )</td>
</tr>
</tbody>
</table>

\[ ^{25} \text{a) Egbert, "Introduction to the Study of Latin Inscriptions."} \]
\[ ^{25} \text{b) "Cajori", Vol. I, p. 36 from Friedlein.} \]
The signs for the various fractions are derived from the initial letters of the particular words as $S$ for semis; $\Sigma$ and the later $\Sigma$ for semuncia and sembella; $\sigma$ for sextula; $T$ for terruncius. The $\Sigma$ of the sicilicus and $\sigma$ of scriptulum are from the rounded forms of the Greek sigma. $\Upsilon$ for binal sextulae is due to the repetition of $\sigma$ the symbol for sextula.

The fact that the foot as well as the pound was divided into twelfths each called an "uncia" or a twelfth part an impression perhaps an unfortunate one has been left on our system of weights and measures. The English words inch and ounce are both derived from "uncia", the scruple of the apothecary's weight comes from the Latin "scrupulum", 1/288 of an as of 1/24 uncia. Counting commercially by twelves or the Latin duodecim has given us our "dozen".

The Romans did not entirely confine themselves to duodecimal fractions or even to simplified equivalents namely, 1/2, 1/3, 1/4, 1/6, etc. With the introduction of the nummus and sestertius corresponding to the old as became the major unit of silver measuring. The libella became the minor unit corresponding to the uncia, but as the system was decimal the libella was equal to a tenth of the sestertius.

The following table shows this fractional system with its subdivisions and symbols.26

<table>
<thead>
<tr>
<th>Value</th>
<th>Symbol</th>
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</thead>
<tbody>
<tr>
<td>Sestertius</td>
<td>$S$</td>
</tr>
<tr>
<td>Semis</td>
<td>$\Sigma$</td>
</tr>
<tr>
<td>Libella</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>(Singula) Sembella</td>
<td>$\Upsilon$</td>
</tr>
<tr>
<td>(Sicilicus) Terruncius</td>
<td>$T$</td>
</tr>
</tbody>
</table>

26 ibid.
There are various ways of expressing fractions. All fractions with one for a numerator are denoted by ordinal numbers with or without pars e.g. for one half "dimidium" (not "dimidia") or "dimidia pars", for one third "tertia" or "tertia pars", for one fourth "quarta", etc. Fractions with a numerator less by one than the denominator are denoted by the cardinal with partes e.g. two thirds "duae partes", three fourths "tres partes", four fifths "quattuor partes" five sixths "quinque partes". All fractions with twelve or its multiples for a denominator are denoted by the parts of an "as" which is taken as the whole and is equal to twelve "unciae". Hence "heres exasse" (heir to the whole inheritance"), "ex triente" (to one third). Other fractions not expressible by one of the above methods are denoted by the cardinal for a numerator and the ordinal for the denominator e.g. four sevenths "quattuor septimae", seven ninths "septem nonae".

Some fractions are denoted by resolution into their components e.g. three fourths "dimidia et quarta", two thirds "pars dimidia et sexta", four ninths "pars tertia et nona", ten twenty-firsts "pars tertia et septima". Sometimes further division is resorted to e.g. one tenth "dimidia quinta", "dimidia" is used for "sexta", dimidia quarta for "octava". "Sesqui" (one and a half) is used only in compounds.
THE ABACUS

It would require but a superficial study of Roman numerals to demonstrate how ill adapted they were for even the simplest calculations. True it is that a scarcity of writing materials rendered the performing of even the simple operations difficult. However, had there been a great abundance of such equipment, mathematical computation would not have been facilitated so long as it was necessary to use the cumbersome system of Roman notation.

To us number symbols are symbols with which mathematical processes can actually be performed; to the ancients number symbols were merely labels which recorded the results of processes performed by some mechanical aid. Even the most advanced minds of ancient Greece lacked completely our present day conception of number symbols—a conception which centuries later the Hindus made possible by the introduction of zero to the number scale. This invention which was the most revolutionary in all the history of mathematics, made possible the performing of the simple operations of addition and subtraction without mechanical aids, and also any number regardless of size might be expressed by number symbols. No longer was number representation handicapped by the limited number of letters of the alphabet; no longer was it necessary to invent new signs such as the Roman X, C, M, in order to indicate successive multiplications by ten.
Having pointed out the difficulties encountered in calculation, because of the ancient notation, let us consider how these difficulties were met. Several millennia before the Christian era the Chinese and Egyptians possessed an instrument of calculation known to us as the abacus. Certain Etymologists consider the word to be related to the Semitic word "abq" meaning dust. Others maintain that it originated from the first three letters of the Greek alphabet. An early type of abacus well known in both Greek and Roman schools, which served a somewhat similar purpose as our blackboard, as well as a counting board, actually was a board or kind of table covered with dust on which one could trace figures, draw columns, and work with pebbles. Cicero refers to Archimedes having used this type of abacus in the words "Ex eadem urbe humilem homunculum a pulvere et radeo excitabo, qui multis annis post fuit, Archimedes." ("I will summon up from his dust board and stylus a humble and obscure man of the same city called Archimedes who lived many years after.")

Priority in the use of the abacus cannot be ascribed definitely to any one nation, Egyptian, Babylonian or Phoenician since no abacus or description of such have come to us from these peoples. Herodotus in speaking of the Egyptians states that they wrote characters and reckoned with pebbles going from right to left, while the Greeks went from left to right. Some time before Plato there developed a line abacus

1Cic., "Tus. Dis.," V. 23, 64.
of several types which are mentioned by various Greek writers. In general they consisted of a table on which parallel lines were ruled, one for units, the next for tens and so on. On these lines or in the spaces between objects were placed to indicate the number of units, tens, etc. of the number to be represented. Stone after stone might be put in the right hand partition until they amounted to ten when it would be necessary to take them all out and instead place one stone in the next section. When the stones in this division amounted to ten, it would then be necessary to take out the ten and put one stone in the third partition, and so on.

The fact that the different spaces on the abacus indicated different values is obvious from Polybius' words, "All men are subject to be elevated and again depressed by the most fleeting event; but this is particularly the case with those who frequent the palaces of kings. They are like stones upon the abaci, which according to the calculator are at one time the value of a small copper coin and immediately afterward are worth a talent of gold. Thus courtiers at the monarch's nod may suddenly become either happy or miserable."  

A Greek abacus of uncertain date was found at Salamis in 1846. It is probably the type used by bankers if any conclusion

2Smith, "Dictionary of Greek and Roman Antiquities, Amer.Ed.(1843)
3Polybius, V. 26.
can be drawn from its arrangement and characters. This abacus is a slab of marble about 1.5m. long and 0.75m. wide. It is ruled with incised lines and has characters to represent the talent, certain fractions of the obol and the numbers 1, 5, 10, 50, 100, 500, 1,000, 5,000.

The Greek abacus influenced Eastern Europe in the Armenian "choreb", the Russian "stchoty" and the Turkish "coulba" since they all possess the distinctive feature of the Greek abacus; namely, the ten line division of the board. The Russians sometimes speak of their "stchoty" as a Chinese abacus, thus partially confirming the sometimes expressed hypothesis that it was introduced from China. While the method of stringing the beads is Chinese the general form is unquestionably Greek. In China, where the complete system of computation including weights, measures, etc. are decimal, an instrument called the "swanpan" is considerably used and with surprising rapidity.

The Romans may have inherited the abacus from the Etruscans but authorities more generally agree that they were indebted to the Greeks. Since the Greek computers do not seem to have recognized the "five space" or "five line," it would appear that the Romans developed the idea. According to this plan each line was broken so that a counter in the upper part of the partition was equivalent to five units of the order in which the column stood. At the same time each counter in the lower section of the partition represented one unit of the same order.
This arrangement was probably related to and may have been based on the Roman use of V, L, and D for five, fifty, and five hundred respectively.

The Latin writers mention three types of abacus used in Rome—the sand board or wax tablet, the marked table for counters, and a table with grooves in which counters were free to slide, or with beads so fastened that they could be moved back and forth in their respective lines.

In addition to the previously mentioned quotation from Cicero, elsewhere in referring to the dusting board, he uses the words "eruditum attigisse pulverem" ("clever at handling the sand.") Persius apparently had both forms of calculating board in mind when he derides the buffoon "qui abaco numeros et secto in pulvere metas scit risisse" ("who sniggers at the figures on the abacus or the ridges of furrowed sand.")

These two later varieties of Roman abacus are depicted from antique monuments one by Ursinus, and a second by Marcus Velserus. In the former the numbers are represented by flattish perforated beads ranged on parallel wires, while in the latter the numbers are signified by small round counters moving in parallel grooves. These instruments each contain seven capital divisions expressing in regular order units, tens, hundreds, etc., and many shorter divisions of five times the

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4 Cubberly, "A Brief History of Education", p. 34.
5 Cic., "Tus. Dis.", V. 23, 64
relative value of the larger ones. With four beads on each of the long grooves or wires, and one on each corresponding short one it is evident that any number up to ten millions might be represented.

The late Roman instrument with counters sliding in grooves was never generally adopted in northern Europe. Some time between the fall of Rome and the twelfth century the line abacus came into common use. In this form fives, fiftys, five hundreds and so on were represented by counters in the space between the lines.

While there were three main types of Roman abacus, they appear to have been more elaborate than those of many races. Some marked ounces, half ounces, quarter ounces, and thirds of an ounce. The addition of fractions having denominators four or twelve was rendered possible by a type of abacus with two marginal grooves or wires, one with four beads for the addition of fractions having denominators four, and the second wire or groove with twelve beads for adding fractions with twelve as a denominator. With the Roman progress in luxury and refinement the abacus came to be made of metal, brass, and sometimes silver instead of wood. The counters too rose from common materials and pebbles to being made of ivory or even of silver.

Since training in the use of the abacus constituted an essential part of the education of the Roman youth, he went to school carrying his reckoning board and a box of counters or
"calculi." The word "calculus" seems to have been carried over from the Greek word "psephos" meaning pebble and came to mean a computing disk. The form "calculi" is the diminutive plural of "calx" or limestone of which marble is a species. From this word came "calculare" meaning to compute or calculate. Those who computed in the capacities of keepers of accounts and arithmetic teachers were known as "calculatores" or "numerii", literally "pebblers" or "coiners". Isodorus Hispalensis explains the derivation of the word "calculator" as a "calculus, id est apellis minutis, quos antiqui in manu tenentes numeros componebant". ("from pebbles, that is little stones which the ancients holding in their hands put together as numbers.")

In Roman families of importance there was a "calculator" or account keeper who is frequently called by the name "dispensator" or "procurator." Martial classes the "calculator" with the "notarius" or writing master when in appealing to schoolmasters to deal gently with their pupils in summer he uses the words,

"Ludi magister, parce simplici turbae; sic te frequentes audiant capillati et delicatae diligat chorus mensae, nec calculator nec notarius velox maiore quisquam circulo coronetur."

("Schoolmaster, be indulgent to your simple scholars; if you would have many a long-haired youth resort to your lectures, and the class seated round your critical table love you. So may no teacher of arithmetic or of swift writing be surrounded by a greater ring of pupils.")

7 His. Iso. "Origines," 10. 43.
8 Mart. "Epig.," X. 62.
Numerous are the Latin literary references to the computing disks used, although none describe the abacus itself. Juvenal probably refers to the tablets upon which the computing was done by counters when he says:

"Computat et cevet. Ponatur calculus, adsint cum tabula pueri; numera sestertice quinque omnibus in rebus numerentur deinde labores." ("He reckons up and still acts the wanton. Let us settle our accounts! Send for the slaves with my account book (tablets). Reckon up five thousand sesterces in all! Then count up your services." )

That counters were made of brass or bronze is proved by the fact that both Cicero and the earlier Lucilius speak of them as "aera". Juvenal mentions the use of ivory: "Adeo uncia nobis est eboris, nec tessellae, nec calculus ex hoc materia." ("I do not possess a single ounce of ivory, neither is counter for reckoning of this material.")

Counters made of glass and known as "abaculi" were used as Pliny remarks: "Fragmenta teporata agglutinantur tamen, rursus tota fundi non queunt, praeterquam abrupta sibimet in guttas, veluti cum calculi fiunt quos quidem abaculos appellant, aliquos etiam pluribus modis versiculares." (When broken, too, glass admits of being joined by the agency of heat; but it cannot be wholly fused without being pulverized into small fragments, as we see done in the process of making small chequers ("calculi") known as "abaculi."

10 Cic.: "Phil. Frag." V. 59.
12 Pliny, "Nat. hist.", XXXVI. 67.
There is no satisfactory record as to how the Greeks and Romans before the Christian era performed the operations of multiplication and division. These processes were evidently carried out by a series of successive additions and subtractions. In division the abacus would be used to represent the remainder resulting from the subtraction from the dividend of the divisor or of a convenient multiple of the divisor. The process was complicated and difficult and consequently division with large numbers must have been beyond the powers of the ordinary computor. This difficulty was somewhat obviated by the use of arithmetical tables from which the required sum, difference, or product of two numbers could be obtained. In the case of multiplication the abacus evidently served for the simpler problems while multiplication tables were employed for more difficult computations.

From Boethius who calls the abacus "apices" we learn that multiplication and division were probably performed by use of complements. Boethius uses the term "differentia" to represent the complement of the divisor or the difference between the given divisor and the next complete ten or hundred. Thus for the divisors 7, 84, 213 the differential are 3, 6, and 87, respectively. The essential characteristics of this complementary division are seen from the following example put in modern form:
\[
\begin{align*}
257 &= 257 = 10 + \frac{60 \times 57}{20-6} = 10 + \frac{117}{20-6} \\
117 &= 5 \times \frac{30 + 17}{20-6} = 5 + \frac{47}{20-6} \\
47 &= 2 \times \frac{12 - 7}{20-6} = 2 + \frac{19}{20-6} \\
19 &= 1 + \frac{5}{14} \\
257 &= 18 \times \frac{5}{14}
\end{align*}
\]

Multiplication was performed by beginning with the highest order in each factor and forming the sum of the partial products. Thus (in modern form) the product of 126 and 237 might be thus effected:

\[
\begin{align*}
126 \times 237 &= (100 + 20 + 6) (200 + 30 + 7) \\
&= 20,000 + 3,000 + 700 \\
&+ 4,000 + 600 + 140 \\
&+ 1,200 + 180 + 42 \\
&= 29,982
\end{align*}
\]

Thus may it be safe to assume that by a somewhat similar way did the earlier Romans perform these operations, which are among the simplest in modern mathematics. The use of the abacus as a mechanical means of computation was inherited by the Romans from some earlier people, probably either the Greeks or Etruscans. The Romans in turn handed it on to Southern Europe where this method of calculating was in general use from the beginning of the Christian era to the fourteenth century.
ROMAN APPLICATIONS OF MATHEMATICS.

Astronomy

Emphasis has already been made on the fact that Roman interest in mathematics centered on the application of the science to practical uses. There was taught in the Roman school, usually by a Greek, calculation, finger reckoning, and skill in the use of the abacus. With the growth of Rome from a primitive settlement to a far reaching empire there developed an unwieldy system of notation. All of which the Roman felt was worthy of consideration in so far as it had some utilitarian value.

In the field of applied mathematics, astronomy first was a genuine interest in Rome. As might be expected, the Romans lacked a general interest in the science of astronomy or "astrologia" as it was called—a term which was applied to both astronomy and astrology. The latter was an Eastern science practised by the Chaldaei who in one period were called "mathematici". "Astrologia" was placed among the liberal arts. Varro, in his lost encyclopedic treatise on nine such "artes liberales", assigned one book to astronomy. Pliny in his "Natural History" described the structure of the universe, but this work is an outline of a non-scientific character. Seneca ever remained an amateur, who interpreted the heavens from a preacher's point of view, and employed others to gather astronomical facts.

In discussing the subject of education, Quintilian felt that applied mathematics in astronomy had value in proving that
the fate of mankind was predetermined: "Quid quod se eadem
gemetria tollit ad rationem usque mundi: in qua cum siderum
certos constitutosque cursus numeris docet discimus nihil esse
inordinatum atque fortuitum; quod ipsun nonnumquam pertinere ad
oratorem potest." ("But geometry (i.e. mathematics) soars
still higher to the consideration of the system of the universe:
for by its calculations it demonstrates that courses of the
stars are fixed and ordained, and thereby we acquire the knowl-
edge that all things are ruled by order and destiny, a con-
sideration which may at times be of value to an orator.")¹

The facility with which the Greeks demonstrated numerous
theories relative to the geometrical movements of the heavenly
bodies must have dazed even the more thoughtful and intelligent
Romans. These various theories were supported by geometrical
proofs rather than evidences from physics since the latter
science as compared with mathematics remained undeveloped in
the ancient world. It is therefore impossible to tell which of
these theories advanced corresponded with physical facts. The
Greeks and few Romans who attempted astronomical speculation
had to be content with their individual choice of theories.

There was the theory of concentric spheres revolving about
the earth as a center, which has its present day survival in the
phrase "music of the spheres". This theory, which was later
accepted with modifications by Aristotle and his school, was
attributed to Eudoxus of Cnidus who died about 355 B.C. He

¹Quint., "Inst. Orat.", I. 10. 46.
assumed that there were no less than twenty-seven of the hollow spheres, of which three accounted geometrically for the movements of the sun, three for the movements of the moon, four for each of the five planets and one for the fixed stars.

Heraclides Ponticus, a pupil of Plato was of the belief that Venus and Mercury revolved about the sun, while the sun and the rest of the planets revolved about the earth rotating on its axis. Other astronomers assumed that each of the five planets had the type of motion accepted by Heraclides for the nearest only, moving, that is, along the circumference of its own circle whose center was in motion along the circumference of a larger circle. This complete system of epicycles or circles upon circles was developed by Apollonius of Perge in Pamphylia, the famed mathematician and "father of conic sections". This system was accepted by Pliny and Ptolemy. There was also a theory of eccentric circles applied to the superior planets Mars, Jupiter, and Saturn. In the latter system each planet revolved about a point other than its center with the point of revolution in each case on a line between earth and sun.

The heliocentric system places the sun in the center and the planets and earth revolving about it, with the earth at the same time turning on its axis. This theory was propounded by Aristarchus, but had to wait centuries to be proved correct by Copernicus. This explanation of the plan of the universe seems to have roused least interest among the Romans and probably seemed least probable to them since only one ancient
astronomer, Seleucus, adopted it about 150 B.C. Since it is mentioned in an extant book of Archimedes (B.C. 287-212), it is possible that his machine invented to show the motions of the heavenly bodies was built to illustrate the hypothesis that the earth revolves about the sun. Cicero was interested in the Sicilian scientist and refers to the two globes of Archimedes which were taken to Rome by Marcellus, when Syracuse was destroyed in 212 B.C. In the "Tusculani Disputationes" he mentions the "more beautiful" globe of the two which Marcellus had placed in the temple of "Virtus" and which was more widely known by the people:

"Nam cum Archimedes lunae solis quinque errantium motus in sphaeram illigavit, effecit idem quod ille qui in Timaeo mundum aedificavit Platonis deus, ut tarditate et celeritate dissimillimos motus una rereret conversio." (For when Archimedes fastened on a globe the movements of moon, sun, and five wandering stars, he, just like Plato's God who built the world in the "Timaeus" made one revolution of the sphere control several movements utterly unlike in slowness and speed.)

Cicero is more detailed, when he speaks of the other globe made by Archimedes: "admirandum esse inventum Archimedi, quod excogitasset, quem ad modum in dissimillimis motibus inaequalibilis et varios cursus servaret una conversio. Hanc sphaeram Gallus cum moveret fiebat, ut soli luna totidem conversionibus in aere illo, quod diebus in ipso caelo, succederet, ex quo et in sphaera solis fieret eadem illa defectio et incideret luna

tum in eam metam quae esset umbra terrae cum sol e regione."

("The invention of Archimedes deserved special admiration because he had thought out a way to represent accurately by a single device for turning the globe those various and divergent movements with their different rates of speed. And when Gallus moved the globe, it was actually true that the moon was always as many revolutions behind the sun on the bronze contrivance as would agree with the number of days it was behind it in the sky. Thus the same eclipse of the sun happened on the globe as would actually happen, and the moon came to the point where the shadow of the earth was at the very time when the sun...out of the region...")

Cicero neglects to state whether the earth or the sun occupied the central position in the "sphaera". It would appear, however, that since the theory of the sun as a center was not accepted generally, that had such been the case with Archimede's globe, Cicero would have mentioned the fact.

The "sphaera" of the Sicilian attracted the interest of Ovid sufficiently for him to say:

"Arte Syracosia suspensus in aere clauso
stat globus, immensi parva figura poli,
et quantum a summis, tantum secessit ab imis
terra; quod ut fiat, forma rotunda facit."

("There stands a globe hung by Syracusan art in closed air, a small image of the vast vault of heaven, and the earth is equally distant from the top and bottom that is brought about by its round shape.")

3 Cic. "De Rep.", I. XIV. 22.
4 Ovid, "Fasti", VI, 277
Such were the various theories relative to the order of the universe which the Romans inherited from the Greeks. Roman interest in astronomical observations was not directed toward further investigation of these hypotheses, but toward the measurement of time as it affects the calendar. Like many of the ancients they could not appreciate the difference between the sidereal and the tropical year— the latter being shorter because of the procession of the equinoxes or the annual movement from east to west of the equinoctial points along the ecliptic.

Early Rome was an agricultural community and for the operations of the farmers it was imperative that there be some means of recognizing the approach of the seasons as dependent on the course of the sun. Before the official calendar became a trustworthy guide, the Italian peasant observed the risings and settings of certain constellations as Arcturus and the Pleiades and so divided the year into seasonal sections. Even after the calendar was reduced to order, the tiller of the soil clung to this system. It is safe, therefore, to presume that the early Romans kept their official calendar in fair agreement with the sun's course.

The calendar under the control of the Pontifices was regulated by pure empiricism and was often complicated by arbitrary interference for political motives. The chief difficulty arose from the lack of concordance between the months which were governed by the course of the moon, and the
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year governed by that of the sun. Twelve lunar months of twenty-nine and one half days give three hundred fifty-four days for the year, while the mean solar year contains three hundred sixty-five days, five hours, forty-eight minutes, and forty-nine seconds; so there is a difference of ten or eleven days. The Romans had adopted the plan of fixing the year at three hundred fifty-five days and intercalating, every second year, between February and March, a month which sometimes had twenty-two days and sometimes twenty-three. The result was that the complete cycle of four years contained two short years and two increased years with the average year consisting of three hundred sixty-six days while the calendar was a month ahead every thirty years. No doubt, this is the reason why Manius Ancilius Glabrio, who defeated Antiochus in 191 B.C., on his return from his expedition to Greece and Asia authorized the Pontifices to make intercalations or not to make them as they saw fit. The Pontifices, however, used this right to shorten or prolong magistracies, or to damage or favor the farmers of the public taxes.

The resultant confusion of the calendar lasted until the year 46 B.C. when Julius Caesar entrusted to the Greek mathematician, Sosigenes the task of re-establishing concordance between the civil year and the solar year, and of laying down the rules which the Pontifices should observe in the future in order to maintain this concordance. At that time the calendar was two long months behind; so two intercalary months of sixty-
seven days altogether were inserted between November and December, although the year had been given an extra month of twenty-three days before March. The year of forty-six B.C. called by Macrobius the "annus confusionis" contained four hundred forty-five days.

Suetonius describes the confusion of the old calendar and Caesar's reform:

"Conversus hinc ad ordinandum rei publicae statum fastos correxit iam pridem vitio pontificum per intercalandi licentiam adeo turbatos, ut neque messium feriae aestae neque vindemiaria autumno competerent; annumque ad cursum solis accommodavit, ut trecentorum sexaginta quinque dierum esset et intercalario mense sublato unus dies quarto quoque anno intercalaretur. Quo autem magis in posterum ex Kalendis Ianuariis novis temporum ratio congrueret, inter Novembrem ac Decembrem mensem interiecit duos alios; fuitque is annus, quo haec constituebantur, quindecim mensium cum intercalario, qui ex consuetudine in eum annum inciderat."

("Then turning his attention to the reorganization of the state, he reformed the calendar, which the negligence of the pontiffs had long since so disordered, through their privilege of adding months or days at pleasure, that the harvest festivals did not come in summer nor those of the vintage in the autumn; and he adjusted the year to the sun's course by making it consist of three hundred and sixty-five days abolishing the intercalary month and adding one day every fourth year."
Furthermore, that the correct reckoning of time might begin with the next Kalends of January, he inserted two other months between those of November and December; hence the year in which these arrangements were made was one of the fifteen months, including the intercalary month, which belonged to that year according to the former custom."

To prevent a return of the error, Sosigenes suggested that every year ten days should be added to the lunar year of three hundred and fifty-five days. Caesar decided to distribute these extra days among the lunar months of twenty-nine days, three of them receiving two more days, and four getting one more each. There was still a difference of a quarter of a day between the lunar year and the solar year, so every four years another day had to be added to the end of February, in the place formerly occupied by the intercalary months.

The authority of Caesar carried through the reform of Sosigenes, but the Pontifices could not understand it as they made a mistake over the intercalation of the supplementary day. The wording of the decree ran "quarto quoque anno" and the Pontifices interpreted this as "at the beginning of each fourth year", i.e. every three years. This gave rise to an error which was only corrected by Augustus in 8 A.D. Caesar's reform also established in Rome a stellar calendar, fixing the rising and setting of the stars on every day of the calendar, according to observations made at Alexandria; so they were not completely

6Suet., "Cae.", 40.
accurate in Rome. This change was based, not on empericism but on astronomical science which had exactly calculated the mean solar year. The solar year was incorrectly reckoned at three hundred and sixty-five days, six hours, instead of the exact measurement of three hundred and sixty-five days, five hours, forty-eight minutes and forty-nine seconds, so there was an annual difference of eleven minutes, which in one hundred and twenty-nine years produced one day in advance. This discordance was obviated by the Gregorian reform in 1582. Today the Orthodox peoples, who have kept the Roman calendar of Julius Caesar, are thirteen days ahead of our calendar in consequence of the accumulation of the annual difference of eleven minutes.

Surveying

A second undertaking of Julius Caesar in the field of applied mathematics, which was of great importance, was a general survey of the Empire. The actual execution of the scheme, however, which required nearly thirty years fell to Augustus.

Such a survey was necessary in the division of provinces, the demands of trade and the distribution of the fleets. This work was done by a regular service of skilled surveyors whose work was incorporated in the reports of the provincial governors and generals. From this mass of material a huge map was prepared and painted on the wall of the Porticus Vipsania on the Campus Martius. It was copied many times, and a probable medieval version of it which doubtless contains some revisions
Now more than ever, our nation, our world, is in need of unity, of shared values, of cooperation. We must work together to overcome the challenges we face. Whether it's economic stability, environmental issues, or social justice, the solutions lie in our collective efforts. Let us strive for a better future, for a world united by our common goals.
is the Peutinger Map. The topographical features, mountain ranges are represented in a conventional fashion. The map is really a diagram of the system of roads with cities marked, and distances between points indicated. "The map is merely a painted road-book. The Greeks, in a happy phrase, guided themselves by the stars to determine the surface of the earth; the Romans looked at the milestones along the roads." 

In the field of surveying the Romans achieved the highest degree of perfection of the ancient world. Considerable knowledge of the subject had come from Egypt to Greece. To this the Greeks added demonstrations of the accuracy of the empirical rules thus received and passed the whole body of knowledge on to Rome. Modern science has developed finer instruments, but the Romans with their much simpler methods achieved results which in many ways have never been surpassed. For instance, in running levels there has been in modern times little improvement in the solution of many ordinary problems. Certain extant fragments of the works of Hyginus show that the Roman method of laying out a field for purposes of mensuration was not very different from that of the present day.

As to the degree of accuracy attained by Roman surveyors, roads, bridges, aqueducts and public buildings furnish abundant evidence.

Two specific and striking examples were given some years ago in an article in "Nature".

7 Now in the Library in Vienna.
9 Vol. 88 p. 158, Nov. 30, 1911.
"In the "Zeitschrift fur Vermessungswesen" (Heft 21, 1911) Prof. E. Hammer discusses the precision with which the nations of antiquity were able to mark out lines on the surface of the earth with the means at their disposal. He investigated, first, that portion of the frontier of the Roman Empire which existed as a straight line about 80 kilometers long from near the River Rems in Wurttemburg to the district of Walldurn in Baden he investigates the question whether this line was laid down approximately straight by chance, or whether it was intended to be a straight line and special care was taken to arrive at this result. From a portion amounting to 29 kilometers of the whole length the mean error in position of a point on the boundary was found to be 12 meters which indicates a surprising accuracy in carrying such a line over rough ground while for a portion of it an even greater precision was attained. Further observations by Professor Leonhard on the remaining 50 kilometers of the boundary indicate the same accuracy is there maintained. The Romans must have fixed a few principal points in prominent positions by signals at night, and then interpolated intermediate points; the observed accuracy could never have been attained by prolonging a line."

"A second case is that of the amphitheatre at Pola laid

10 Surveyed, marked and fortified by order of the Emperor Domitian.
11 In surveying of U.S. public lands a lateral error of 10 meters in a 6 mile latitudinal section line is allowed.
12 Pola situated about 50 miles south of Trieste. Destroyed by Augustus but rebuilt at the request of his daughter Julia and renamed in the honor Pretas Julia. The amphit. referred to was built about 200 A.D. and is the only Roman amphit. where outer walls have been preserved into act.
out by a Roman architect land-surveyor, which Herr Hofrath A. Broch, an Austrian surveyor recently investigated using a plan on a scale of 1:250 he investigated the accuracy with which the amphitheatre as constructed approached an ellipse. Taking 12 points on the curve, their mean error in position from an ellipse was but 15 cm., in spite of the weathered surfaces of the stone contributing to the uncertainty. The axis of this ellipse were $2a=129.9$ m. and $2b=102.6$ m. or in the ratio of very near $9:7$, as in the case of many Roman amphitheatres."

Surveyors

Various terms were used to designate Roman surveyors. Generally they were known as "agrimensores", a word, which in later times was simplified to "mensores". There is also found the names "gromatici" from "groma", "finitores", "decempedatores". A "gromaticus" was properly a professor of surveying while a "geometres" was a teacher of the subject. Military surveyors were known as "metatores". During the empire surveyors formed a collegium, and sometimes were vested with judicial power. In the times of Theodosius and Valentinian these measurers of land, and fixers of boundaries were called "spectabiles" and "clarissimi". There were schools for training in this field which was doubtless the greatest Roman application of mathematical science.

The extant writings of the "agrimensores" consists chiefly of short treatises of about the second century after Christ by Frontinus, Siculus Flaccus, by two men each bearing the name of
Hyginus; and several short mathematical treatises of uncertain date by Balbus, Nipsus, a so-called Boethius, and others as well as extracts from official registers, lists, and descriptions of boundary stones; extracts from the Theodosian code and one title (X,1) of Justinian’s Digest; and an obscure and barbarous tract ("Casae Litterum") by a certain Innocentius. Among later writers to arouse modern interest in the subject are Niebuhr, Lachmann, Blume, Mommsen, and Rudorff.

Since the gromatici were in a way the successors to the augurs, the mode of their "limitatio" was derived from the old augurial method of forming a templum. This word was of Greek origin and simply meant a division. It came to mean the vault of heaven because of the fact that the directions were always ascertained according to the true cardinal points. The augur originally according to a fixed procedure marked out with the litusus a space where he pitched his tent. The ground space was a square or rectangle having its four sides turned to the different points of the compass with the front facing the west so that one on entering had his face to the east. At the inauguration of a king or a consul, the augur faced toward the south. In this case the latter person was considered the chief, and the direction in which he looked was the main direction. Thus in land surveying the augur faced the south, since from this position he would be looking in the same direction as did the Gods who were supposed to be in the north.
what time be commenced. It is therefore declared by the Assemblies of the Board which has been ever the case, that in all cases of 
the present day, when it shall be deemed necessary to make any changes or alterations in these rules and regulations, the 
provisions herein contained shall be subject to such changes or alterations as may be necessary to carry into effect the 
object of the same. And the said Board of Directors shall have power to make such rules and regulations as are necessary 
and proper for the management of said company. And the said Board of Directors shall have power to make such rules and 
regulations as are necessary and proper for the management of said company. The said Board of Directors shall have power 
to make such rules and regulations as are necessary and proper for the management of said company.
As a result of these ideas and directions in land-surveying, the main line was drawn from north to south. This line was called the "cardo" corresponding to the axis of the world or the pivot on which the heavens revolved. The lower extremity supposedly turned upon a pivot corresponding to that at the bottom of a door. The conception of these two principal points in geography and astronomy led to the application also of the same term to the east and west. Hence ancient writers speak of the four points of the compass as the "quattuor cardines orbis terrarum".

The "decumanus" (also spelled decimanus) was the line cutting the "cardo". It was so called since a cross resembling the numeral X was formed. These two lines were produced to the extremity of the plot of ground to be laid out and parallel to these were drawn other lines according to the size of the quadrangle required. The limits of these divisions were indicated by balks called "limites" which were left as highways, the ground for them being deducted from the land to be divided. As every sixth "limes" was wider than the others the square bordering upon this would lose "pro tanto".

The fields of activity of the Roman surveyor involved in public life the allotment of public lands, the laying out of camps and surveying of roads.
Public Lands

The public land was the result of the conquests of war, and surveying was naturally necessary in the matter of dealing with this confiscated land. In the case of cultivated land the assigning might be done in one or all of four different ways. The "agri quaestorii" were portions sold by the quaestors. In this case the "gromatici" measured and divided it by balks (limites) into square plots (laterculi) which measured ten actus each side and contained fifty nigera (thirty-one acres). As such a plot contained one hundred square actus, it was sometimes called a centuria.

By a second method of distribution the land might be given and assigned in full ownership to Roman citizens, in which case it was known as "agri dati adsignati". The surveyors measured and divided it by balks into centuries each containing two hundred nigera (one hundred twenty-five acres), and assigned it by lot. The surveying and distribution was done by a special commission of three, (five, or ten) men who were called III Viri A.D.A. \(^{13}\) meaning "agris dandis adsignandis". In general the amount granted each individual depended on the extent of land to be distributed and the number of citizens to share. The earliest assignments that are recorded were two nigera in which a person had life interest and then passed it on to his heirs. In this case lots of one hundred men formed one century (cent-vir-ia). Later the normal size lot was seven ingera. Land

\(^{13}\) Cic. "Agr.", II. 7.
null
was also given back to the former owners or let for rent to state contractors who sublet it. Hyginus mentions one of these leases running for as long as one hundred years.\textsuperscript{14}

Uncultivated land, pastures, woods, etc. were granted to old proprietors, the municipality, reserved for the state, or used for the purpose of a new Roman colony.

Camp Surveying

The "mensors" or military surveyors nightly marked the divisions of the camp. During the emperical period the general superintendence of the camp arrangements was in the hands of the "praefectus castrorum". Such is first mentioned during the time of Augustus.

In the matter of Castramentation a treatise of Hyginus\textsuperscript{15} gives the most trustworthy account. He states that the ordinary form of a camp was a rectangle the length of which was about a third greater than the width. The legions formerly posted within the camp were at this time encamped along the whole line of ramparts which had a width of sixty feet. A road (via sagularis) thirty feet wide running parallel to the ramparts separated them from the interior of the camp. The interior of the camp was earlier divided into two sections\textsuperscript{2}, later into three. The middle one of these lay between the "via principalis" which was sixty feet wide, and the "via quintana" which had a width of forty feet. This part was occupied by the "praetorium" and the troops of the guard. The auxiliary troops

\textsuperscript{14}H\textgreek{y}, "De Munitionibus Castrorum."

\textsuperscript{15}\textgreek{y}bid.
were stationed in the front part between the "via principalis" and the "porta praetoria", and the rear between the "via quintana" and the "porta decumana*. The "via praetoria" was also sixty feet wide and led from the "praetorium" and "forum" to the "porta praetoria".

Road Surveying

The vast Roman Empire was linked by a network of roads, the engineering of which have hardly been equalled in modern times. Roads built by the Etruscans and Greeks had long existed in Italy. According to tradition, the Romans learned road-building from the Carthaginians in Sicily. It was the latter who introduced the use of lime mortar in road construction, and it is in the use of this material that the latter built Roman roads as the Appian Way differed from the old roads of Latium and Etruria.

The engineering involved in the original construction of these highways cannot help but arouse admiration; as for example, the portion built on the side of the Alban Hills near Aricia, and the embankment from ten to thirteen feet high which runs for seventeen miles across the Pontine Marshes.

The time of Trajan (A.D. 98-117) marked the culmination of Roman engineering. It was at that time that Apollodorus of Damascus, the greatest engineer of the period, built the forum and aqueduct of Trajan, and in 108 built a permanent bridge across the Danube.
From a technical point of view the Augustan Age had been a great age, as one of application not of scientific creation. Admirable monuments, public works, a survey of the empire, and establishment of a calendar were created.

Surveying Instruments

Roman surveyors depended on the sun to give them true bearings, and two devices (essentially one in principle) were utilized for this purpose; namely, the sun-dial or horologium or solarium, and the gnomon or pole.

The horologium was first used by the ancients, dating back to the Babylonians, for determining the length of day and night. Authorities disagree as to whether the first solarium was brought to Rome by Papirius Cursor, twelve years before the war with Pyrrhus, or by M. Valerius Messala at the time of the First Punic War. These instruments were evidently in general use in Italy if any conclusion may be drawn from the number found by excavation. One of the simplest of these was discovered in 1741 on a hill of Tusculum among the ruins of an ancient villa.¹⁶

Once a sun-dial was correctly set, the points of the compass could be determined at least roughly. Since the dial was not possible to be generally employed as a surveying instrument.

¹⁶Stone, Edward, "Roman Surveying Instruments"

a) Zuzzeri, Gio. Luca in "D'una antica villa scoperta sul dosso del Tuscolo, e d'un antico orologia a sole."
b) Martini, G. H. in "Abhandlung Von den Sonnenuhren der Atten."
Vitruvius gives quite full directions as to the methods employed in constructing and setting up a sun-dial.\textsuperscript{14} He especially calls attention to the fact that a dial can be accurately set only by taking the equinoctial shadow. He also mentions a modified form of this instrument used in laying out the streets of a city.\textsuperscript{15}

The terms horologium, solarium, gnomon were used quite interchangeably. Originally the gnomon was the simplest type of horologium and probably the more ancient. It evidently was a very imperfect instrument, consisting of a staff or pillar which stood perpendicularly in a place exposed to the sun so that the length of its shadow might be easily ascertained. The shadow of the gnomon was measured by feet, which were probably marked on the place where the shadow fell.

The principle of the gnomon was used by Roman surveyors in establishing the true meridian. Ancient writers mention establishing such by means of ranging two plumb-lines or rods with the pole-star. This is probably due to the fact that in the age of Caesar and Cicero the star Polaris (Alpha of Ursa Minor) was about ten degrees farther away from the celestial pole than at the present time. So in place of ranging the plumb-lines or rods, the Roman surveyor would construct a huge sun-dial having a considerable area of level ground for its plate, and a tall pole in the place of the pin. This method or rather two modifications of it are described by Hyginus.\textsuperscript{16}

\textsuperscript{14}Vit., "De Archit.", IX. 7.  
\textsuperscript{15}Ibid I. 6.  
\textsuperscript{16}Thulin, "Corpus Agrimensorum Romanorum" (Hygini Gromatici Constitutio Limitum).
This method is still sometimes used.\textsuperscript{17}

Roman surveyors in making linear measurements used the pertica or decempeda, cords or ropes, and the hodometer. The pertica was a pole of seasoned wood protected at the ends by metal shoes or ferrules, and because it was usually ten feet long was also called the decempeda. This length, however, did vary to twelve, fifteen, and even seventeen Roman feet. It is possible that the seventeen foot pertica may have been the origin of the English rod, perch, or pole of the old arithmetic tables. These longer poles were used in subdividing public land where the soil was of inferior quality so that larger lots were assigned. Cicero,\textsuperscript{18} Horace,\textsuperscript{19} and other Roman writers make frequent mention of the decempeda.

Cords and ropes were used as longer measuring devices, and were probably as accurate as the old-fashioned Gunter's Chain or the Engineers Chain with their two hundred or more wearing surfaces and easily bent links. No mention is made of the various writers on the subject, of marking-pins in connection with either the pole or the cord, but something of that sort must have been used.

The hodometer or "road measurer" was used by both Greeks and Romans. It was especially employed by engineers to measure quickly long distances; as for example, in laying off the base of a triangle along one bank of a river whose width was to be

\textsuperscript{17}Stone, "Roman Surveying Instruments."
\textsuperscript{18}a) Gillespie, William M., "Treatise on Surveying".
\textsuperscript{19}b) McCullough, Ernest, "Practical Surveying."
\textsuperscript{18}Cic., Phil. 13, 18, 37.
\textsuperscript{19}Hor., Car. II, 15, 14.
determined while the angles at each end were measured with the dioptra. Vitruvius gives full directions for the construction of such an instrument. 20

During the whole period of Roman history the most commonly used instrument for measuring or laying off horizontal angles was the groma. In form and function it resembled the Surveyor's compass of fifty or a hundred years ago, and the old-fashioned Surveyor's Cross used during the nineteenth century. Roman writers frequently mention the groma, but give no description of it. A rude representation of the instrument was found in 1852 on a tombstone near Iurea (the ancient Eporedia) in Piedmont. The inscription bears the name of a surveyor Lucius Aebutius Faustus. 21

Excavations at Pompeii in 1912 unearthed the office 22 of a Roman surveyor, Verus by name. The noted Italian archaeologist, Dr. Matteo della Corte, has succeeded in reconstructing a groma from eleven scattered pieces of bronze and iron, and adding wooden parts to replace those lost. 23

The dioptra was adapted to measuring both horizontal and vertical angles as well as being used for leveling. It must have been used by Roman surveyors as early as the beginning of the empire since Vitruvius, who served as a military engineer under Julius Caesar, mentions the instrument. It was evidently for taking accurate measurements as was a similar device used by the Greeks if any conclusion may be drawn from Fliny's

20 Vit., "De Archit.", X. 9.
21C.I.L. V. 6786.
22Reg. I. Ins. 6 Nr. 3.
statement.24

The ancients measured angles in terms of degrees only in connection with astronomical observations. Although the dioptra had some mark or attachment for setting right angles, there appears to be no record of such measurement being used for other purposes. Trigonometry had been developed before the Christian era to considerable extent by the Greeks, but it was used only as an aid in astronomy. For all kinds of terrestrial surveying both Greek and Roman surveyors used geometrical principles as in chain surveying of the present day. The Roman numerical notation was too complicated to permit any general use of involved arithmetical or trigonometrical formulas and tables.

The dioptra could be used for measuring altitudes since it could be revolved in either a horizontal or a vertical plane. For the same purpose another instrument, simple both in construction and use was used. We do not know what the Romans called it, but the Greek name was "lynchia" meaning "lampstand".

Ancient leveling instruments belonged in two classes: those depending on the principle that a line perpendicular to a vertical line is a horizontal line; and those depending on the principle that the surface of a liquid in repose is horizontal. Instruments coming in the first class involved the use of a plumb-line, while those in the second required a groove or a tube into which water was poured. In the latter group was an

24 Pliny, "Nat. His.", II, 69.
instrument known as the chorobates. This Vitruvius especially recommends for obtaining accuracy in leveling.\textsuperscript{25}

Technical Science

The statement that the present is the "age of technical science" would suggest that the creation and development of technical science had been reserved for the era in which we live. The statement would also imply that these sciences did not even exist in the remote past. On the contrary technical science has been defined as "an expression of human beings with its root in the very nature of things and having been inspired from time immortal by man's very existence".\textsuperscript{1}

The present has developed to a very high degree the use of certain natural forces, steam power, and electricity. Today there is an abundance of steel and iron; there is a carefully computed strength of materials needed and of pressures that must be withstood. In ancient times there was an abundance of time and slave labor.

The exploitation of a few easily recognizable physical laws resulted in a few simple machines and contrivances by the use of which admirable results were obtained especially in the line of design and durability. The ancients used simple devices such as the lever, inclined plane, wedge, pulley and toothed-wheel. These in mechanics are classified as "simple machines" since they cannot be analyzed into simple ones and yet can be combined into

\textsuperscript{25}Vit., "De Archit.", VIII. 5.
\textsuperscript{1}Neuburger, "The Technical Arts and Sciences of the Ancients."
compound machines. Vitruvius definition of a machine seems to us rather limited, "a machine is a connected construction of wood which gives very great advantages in raising heavy masses; it is set into action, artificially, namely by notation."

The theory of the lever had exercised the greatest minds of antiquity. Pliny ascribes its origin to Kinyras of Cyprus. However, this is quite generally considered a contribution to mythology and not to the history of technical science. The action of the lever was regarded by Aristarchus in the light of related arcs; and Archimedes discovered the law of levers by calculation. The Romans made numerous applications of the use of levers, the chief of which were in war machines, and in the theater to raise or lower planks of which isolated beams and panels give evidence.

For raising heavy loads the importance of the inclined plane was early recognized. The fact that it was used in the building of the Pyramids is doubtful although the opinion is held by many. It was used first in the form of a screw for the purpose of drawing water, and later, the single screw was used in the olive presses and the double screw for other purposes.

During early Roman times the compound pulley was used for all oil presses, theatrical machines, etc. In the Imperial Roman palaces this device was used for lifts and cranes. The so-called "hall of machines" on the Palatine shows niches at a depth of sixty feet, in which the lifts worked as well as the tubes and grooves through which the pulleys passed.

²Pliny, "Nat. His.", VII. 195.
The toothed wheel was chiefly used in measuring distances as in the hodometer which has been previously described.

The very existence of the states in antiquity depended on the arts and crafts; so the influence of technical science was far reaching throughout the whole civic and public life. The technical worker had high prestige and enjoyed great respect. Many ancient constructions still bear witness of this fact since in Rome hardly a bridge was not crowned by a sort of triumphal arch in honor of the builder. The greatest rulers drew technical scientists into their services and in some cases gave them a high position, and often provided special training for expert workers. Often in states and towns there were specially appointed technical officials.

Conclusion

The indifference of the Romans to scientific and mathematical theory is quite conclusively proved by their failure to advance to a higher degree the theory of technical science. Even in this branch of science in which the practical application would be constantly obvious, the Roman was not inspired to delve into research. Professor Whitehead has stated "No Roman ever lost his life because he was absorbed in the contemplation of a mathematical diagram", and it may also be added in any scientific investigation. The Greeks taught the nobility of scientific study; the Romans sought the "utilitas" of scientific study. The indifference of the Romans toward this branch of learning and their extreme utilitarianism rather than the barbarian
invasions stifled ancient science. The typical Roman is well characterized by the author who writes:

"Then she (Rome) had to fight hard for her life by work and war. Circumstances forced upon her childhood a sense of realities, rigid discipline, and long acquaintance with poverty. From these she may have derived in later years, her timidity of imagination, her habit of moral observation, her care for practical things and logic....Whatever his merits, the Roman never has the charm of the Greek. His breadth of conception does not make up for his lack of imagination. His ideal never rises from the ground. He has learned to calculate too much; his reasoning never loses sight of the action which will be its consequence. His art always has a utilitarian purpose, or else it betrays the effort of the scholar's application.

"Yet the Roman mind lacks neither activity, nor versatility, nor delicacy. It adheres to traditions which have been tested by use, but it is none the less eager for new experiences and ideals. It borrows institutions, ideas, and arts from neighbors with discernment. It does not copy slavishly, but stamps its own character on what it adopts. Even in the early centuries which we have been considering, the Romans' capacity for assimilation appears as highly developed as their originality. Rome triumphed over the whole of Italy, not because her people had remained the most primitive of all, but because, having become about equal to the others in civilization, it was also better disciplined and more energetic."

1Grenier, "Roman Spirit", III p. 82.
SUMMARY
The interest of the Romans in mathematics and science was purely utilitarian. The Romans were a practical people whose everyday life, civilization, and culture were characterized by a dominant sense of utility. What was useful was worthy of consideration. Rome almost slavishly borrowed from the civilization and culture of earlier peoples, particularly the Greeks, but upon what she thus acquired she placed her own stamp—usefulness. The Romans had neither time nor interest in pure theory or scientific research. On the contrary the Greeks possessed intellectual curiosity, a scientific spirit and a love of learning for its own sake. In the field of mathematics and science the Greeks developed the theory; the Romans preserved the theory and achieved the application.

Mathematics was taught in the Roman schools to meet practical ends. In arithmetic emphasis was placed on finger reckoning, and calculation by means of the abacus. The term "arithmetic" was applied to the science of numbers, and the art of computation by figures, while "logistica" was used to mean the practical use of numbers as in methods of writing and computing.

There were among the Greeks and Romans as was characteristic of ancient peoples certain rather superstitious beliefs regarding the "mystery of numbers." There is also found a tendency toward phallicism or an association between number and sex. Gradually, beliefs relative to numbers developed which
were independent of superstition, and we find a so-called geometrical representation of numbers having been developed.

Geometry was of interest to the Romans in so far as its theory might have a practical application, and this lay chiefly in the line of land measuring. Vitruvius expressing the characteristic Roman attitude, though perhaps more broadly than would the average of his countrymen, toward the general theory of any art or science stated that it was well for a person to know the general theory, but a thorougher knowledge and practice of it should be left to the specialist.

There were no Romans who might be considered true mathematicians. Those who were the nearest approach to being such and worthy of mention were Varro, Pliny the Elder, Vitruvius, Hyginus, Seneca, Censorinus, and Boethius.

Finger reckoning as previously stated was useful in the daily life of every Roman. This fact was especially true in the commercial life of the empire where linguistic difficulties might be encountered. The idea of finger reckoning dates back to primitive man. The system gradually developed through the ages until among the Romans there was a rather complete system, whereby not only small but large numbers might be indicated by means of the fingers and hands. In general numbers from one to one hundred were represented by various positions of the fingers of the left hand; numbers from one hundred to ten thousand, by the right hand; numbers from ten thousand to ninety thousand, by the left hand applied to various parts of the body; larger
numbers than ninety thousand, by the right hand likewise applied to different parts of the body. Numerous Latin writers give information relative to the general use of finger symbolism, as well as some of the positions used.

Roman numerals were alphabetic in character, and are of uncertain origin. Authorities on the subject disagree especially in regard to particular symbols V for five, X for ten as D for five hundred, M for one thousand, etc. Probably there was a decided influence on Roman notation from the numeral symbolism of the Etruscans. It is practically impossible to develop a working hypothesis regarding the origin of the Roman numerals since there is a lack of definite information, and of inconsistency among the Romans themselves in the use of the characters.

Ancient systems of notation were characterized by the principles of addition and subtraction of symbols. The important principle of position is nowhere found; so there was no use for a symbol for zero. When the Hindus in the fifth or sixth century developed the use of zero and principle of local value, they made one of the greatest contributions to the science of mathematics. From then on it became possible to actually perform operations by means of symbols; whereas in the past signs merely recorded results of computation.

The Roman symbols used to represent numbers varied greatly. During some periods of Roman history various symbols were used for the same number. In other cases, several forms for the same number were used during the same era. Again certain symbols are found almost exclusively in inscriptions. It is on these lasting
memorials and in the works of Latin writers, that very definite evidence as to how the Romans represented numbers has come down to the present day.

Roman fractions were in general duodecimal although the decimal system is found. The various divisions and subdivisions of the unit had their own names, as "semis" for one half, "quadrans" for one fourth, etc. Symbols for the various fractions were derived from the initial letter of the name for the particular fraction as "S" for "semis". There were various ways of expressing fractions: for some the cardinal was used; for others the ordinal, and for still others a combination of both the cardinal and ordinal is found.

The cumbersome Roman notation system necessitated a mechanical means for ordinary computation. The abacus was the device which served the ancients for performing the fundamental operations of addition, subtraction, division, and multiplication. It is not definitely known just how these last two processes were carried out. Ancient writers are definite as to the fact that the abacus was used but not regarding the methods employed. They give considerable information descriptive of counting disks, but not of the appearance of the abacus itself.

The history of the abacus dates far back among ancient peoples, and various types are still in use among certain present day people as the Russians and the Chinese. It is uncertain whether the Romans obtained the idea of this device from the Greeks or Etruscans.
The chief difference between the Greek and Roman abacus lay in the use of the "five-line" whereby a disk in the upper part of a division was equivalent to five units below in the same section.

There were three types of Roman abacus, the sand board or wax tablet, the table marked for counters and the table with grooves in which the counters could be moved back and forth in their respective lines. Training in masterful reckoning on the abacus was an important part of the mathematical education of the Roman boy. Proficiency in computing with the abacus was essential in the everyday life of the Roman from the keeping of household accounts to commercial transactions.

The Roman's interest in mathematics was practical. Astronomy was the first field in which there was an opportunity for applied mathematics. The Greeks presented the various theories relative to the universe and the movements of the heavenly bodies. As in all mathematical and scientific theory, the Romans were ready to borrow unquestionably. It is interesting to note that the scientific work of the Sicilian Archimedes drew the attention of numerous outstanding Romans, particularly Cicero.

The calendar presented the first opportunity for the Romans to make a practical use of astronomy. As a result of the pontifical control of the calendar, it had become decidedly out of harmony with the seasons. As a result, it was two full months behind when in 46 B.C. Julius Caesar employed a Greek
mathematician, Sosigenes, to bring about a concordance between the civil and the solar year as well as to establish definite rules relative to intercalation. Unfortunately, the mean solar year was incorrectly reckoned at exactly three hundred and sixty-five days instead of three hundred sixty-five days five hours, forty-eight minutes and forty-nine seconds. This variation produced a difference of one day in one hundred and twenty-nine years—a discordance which the Gregorian reform in 1582 removed.

A second field in which the Romans found an opportunity for applied mathematics, was that of surveying. Roman surveyors achieved a very high degree of proficiency, and accuracy, and their solutions of certain problems have not been surpassed even in modern times. These measurers of land were known by various terms as "agrimensores", "gromatici", "finitores", and "decempedatores". Such men as Frontinus, Siculus Flacus, two men by the name of Hyginus, Balbus, Nipsus, and a so-called Boethius are among those who furnish information on the subject of surveying. The chief fields of activity for the most part included the allotment of public lands, planning of camps, and surveying of roads.

It is surprising what exceptional achievements were possible when the crudeness of their instruments is taken into consideration. It was necessary to depend on the sun for bearings, and two devides the horologium and the gnomon were used for this purpose. For making linear measurements, we find the pertica or decempeda, cords or ropes, and the hodometer employed. The
groma was the means by which horizontal angles were laid off, while the dioptra was used for determining both horizontal and vertical angles.

The devices used in technical science were even simpler than those used in surveying. The applications of the lever, inclined plane, wedge, pulley and toothed-wheel were the "simple machines" by which the Romans achieved surprising results. The uses to which these were put varied from the use of the inclined plane in drawing water to the use of the principles of the lever in the theaters and war machines.

Mathematics was of value to the Romans wherever there was an opportunity for the application of the science. To go beyond that the Roman mind had no interest, desire or inclination. Had it been otherwise, Rome might have attained a much higher degree of proficiency and achievement in applied science and mathematics.
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*Hyginus

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*Macrobius

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*Ovid

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*Pliny, the Elder

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*Priscan
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