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Kant, infinity and his first antinomy

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KANT, INFINITY AND HIS FIRST ANTINOMY

by

JAMES WILLIAM LINCOLN

B.A., University of Illinois at Springfield, 2011
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Approved by

First Reader ____

Manfred Kuehn, PhD
Professor of Philosophy

Second Reader ____

Judson C. Webb, PhD
Professor of Philosophy

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ABSTRACT

Kant's antinomies skillfully juxtapose two arguments which expose the dangerous propensity for human reasoning to stretch beyond the conditioned and into the transcendental ideas of the unconditional. Kant believes this is a natural process and affirms the limits of pure reason in so much as they should prevent us from believing that we can truly know anything about the unconditional. His first antinomy addresses the possibility that a belief in a beginning in time or that a belief in no beginning in time is dubious at best. This thesis will focus on this first antinomy and critically assesses it in set theoretic terms. It is this author's belief that the mathematical nuances of infinite sets and the understanding of mathematical objects bear relevance to the proper interpretation of this antinomy. Ultimately, I will argue that Kant's argument in the first antinomy is flawed because it fails to account for infinite bounded sets and a conceptualization of the infinite as a mathematical object of reason.

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Section 1 - Introduction

Kant's chapter on the antinomies serves two purposes[1]. The first is intended to illustrate the problematic nature of transcendental ideas. Secondly, they serve as a secondary proof for time as a form of perception. This manuscript is exclusively concerned with them in the former sense and thus as road signs for reason. Kant denies the objective reality of cosmological ideas by strategically showing that reason contradicts itself when it considers them[2]. These types of ideas are cosmological concepts which describe unconditioned ideas and are identified with what presents an ultimate explanation for some idea[3]. They are unconditioned totalities that don't depend on some instance of an idea[4]. Kant describes human reasoning as possessing the propensity to move from the conditioned to the unconditioned within the framework that "if the conditioned is given, the entire sum of conditions, and consequently the absolutely unconditioned is also given"[5]. This synthetic process means that we move from an instance of an idea to an encompassing totality of that. Kant believes this is a natural process which creates transcendental ideas; ones which transcend experience towards the unconditional ideas of reason[6]. He sees this as a blatant transgression to the limits of experience and thus believes that reason's employment should thereby be limited to things a posteriori[7].

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1 Gardner p. 234 - 235
[2] Gardner p. 234 - 235
[3] Gardner p. 216-217
[4] Gardner p. 217
[5] Gardner p. 218
[6] Gardner p. 217
[7] Gardner p. 218-219

He suggests that the antinomies, as a whole, embody this truth. The cosmological ideas are “categories extended to the unconditioned” and his categories “are not fitted to such employment[8].” The logical contradiction that arises out of the antinomies, in Kant’s view, supports this claim. In this paper, I shall focus on Kant’s first instance of the “antinomical” project and its conclusions about time. His arguments for and against the possibility of a beginning in time seem, to me, the most unsettling part of the antinomies. What is disconcerting is not Kant’s final suggestion, namely that we cannot empirically know transcendental ideas, but rather the fact that his conception and methodological use of infinity is dubious from a mathematical perspective. We should question the apparent contradiction generated by the opposition of his thesis and antithesis. Specifically, this should cause us to doubt the mechanics used to formulate his first antinomy and by extension should reinvest our faculties in an effort to investigate the possibility that the concept of infinity can be accessed; even if only in as a mathematical object. This is because his thesis and antithesis concerning time turn on mathematical hinges.

From this pivot point, any argumentative flaw should preclude our ability to accept Kant’s conclusion because any misapplication of infinity in his thesis or antithesis compromises his conclusion. I believe that the concepts of “infinity” and “series” are pivotal components to Kant’s discussion of time. This is, of course, not unintentional. Kant divides the antinomies into two types, the mathematical and dynamical, the first two being mathematical, while the later are classified are dynamical[9]. His exploration of time,

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[8] Kant (2007) p. 386
[9] Gardner p. 234 - 235

as a part of the first antinomy, focuses on our conception of time in the past as a series of moments. Thus, a mathematical investigation of his work is more than appropriate because of its relationship to mathematical concepts like infinity and the series whether they be conceived in Kant or more contemporary thinkers. Apropos of that, this piece will elucidate ideas about series, the finite and the infinite in a mathematical and Kantian framework. Along the way I will highlight several papers on this very subject and show, that, given a proper mathematical conception of the infinite, we must accept that Kant's thesis and antithesis describe traversing of an infinite number of moments in both cases. This, however, allows for the possibility for a reasonable conception of the past as infinite without Kant's proclaimed contradiction for reason and, in the mathematical sense, allows us to access the infinite though mathematical reasoning. From this, we can gain access to the experience of the infinite as only a finite being can. Namely, as a recursive definition which can encapsulate potential infinitely as a mathematical object of reason. This object represents an understanding of infinity as a method.

It is prudent, however, to begin with a short sketch of Kant's antinomy about time. His thesis is an argument with four premises which purports us to conclude that there is a beginning in time. His antithesis is an argument consisting of five premises which lead to the conclusion that there is no beginning in time. Stated succinctly:

Time has a Beginning (Thesis)

P_T1) Time has no beginning

P_T2) If time has no beginning then up to every moment an eternity has elapsed, namely an infinite series has been completed.

Time has no Beginning (Antithesis)

P_A1) Time has a beginning

P_A2) If something has a beginning then there was a time preceding it when it was "not."

P_A3) There must have been a preceding time

	when everything was "not."
P_T3) A series is completed through successive synthesis	P_A4) If everything is "not" then no coming to be is possible.
P_T4) It is impossible for this infinite series to have passed, i.e. it is impossible to complete an infinite successive synthesis	P_A5) If no coming to be is possible then there is no ability to start a series of time which leads to this moment.
C_T) Therefore, our assumption must be false and time must have a beginning.	C_A) Therefore, our assumption must be false and time must have no beginning.

Table 1: Outline of Kant's First Antinomy[10]

Ultimately, Kant affirms that a conception of the infinitude or finitude of time which quantify the ontological status of a beginning or non-beginning of time, as unconditioned ideas, must be inaccessible to experience because pure reason pushes beyond what we can know a posteriori in the search for the unconditioned[11]. Its non-experiential character results from the fact that the infinitude or finitude of time can only be derived by reason. As Kant would say, the "unconditioned is never to be met within experience, but only in the idea"[12].

From this, Kant concludes that reason's treatment of transcendental ideas creates an unavoidable contradiction which is indicative of the search for absolute totality demanded by reason[13]. Furthermore, reasonable beings naturally ascend from a condition to the unconditional to consider whether the series does or does not cease[14]. Thus, we necessarily think of time as having completely elapsed to a given moment because absolute totality is demanded[15]. I believe that this conception of time formulates itself

[10] Kant (2007) p. 396 - 397
[11] Kant (2007) p. 386
[12] Kant (2007) p. 386 [B436]
[13] Kant (2007) p. 387 [A410]
[14] Kant (2007) p. 387 [B437]
[15] Kant (2007) p. 387 [A410]

around the mechanics required to complete a series of moments and touches upon the ontology of the infinite.

Section 2- Kant's conception of a series as the unconditioned

Overall, I'm proposing that Kant's time is consistent with contemporary mathematical terminology used to define a series. Kant's time describes a formal condition for internal sense[16]. In this way, his perception of a series is formally conditioned by time[17]. We naturally think of time as a series of moments in succession. This is the dominant characteristic of time in Kant's undertaking. It is one that, as Lawrence Friedman would say, conforms to the rules of a series thereby consisting "at least in part in the comparing or relating of the elements of the series" and that the convergence of Kant's series of time persists as a "bringing together," a successive synthesis, of that series to the present moment[18]. The moments of time in sequence are "coming together" to converge at the limit of the present moment and can't do otherwise. Time, as a sequence of moments, can therefore be quantitatively analyzed. We will return to this notion soon, but I urge the reader to keep it in mind.

In terms of a sequence, a series is "a function whose domain is either a natural number" or infinity, i.e. all the natural numbers[19]. The former constitutes a finite sequence and the later an infinite. To illustrate the mathematical conception of a series, consider the following examples. The set natural numbers, denoted $\boldsymbol{N} = \{0, 1, 2, 3, \dots\}$, is a series, denoted $\boldsymbol{N} = <0, 1, 2, 3, \dots>$. The former notation signifies the form of the natural numbers as a set and the latter as a series. The difference between the two is that, as a set, the group of natural numbers need not be ordered, while, in a series, its members

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[16] Friedman p. 381
[17] Kant (2007) p. 388
[18] Friedman p. 381
[19] Jech p. 46

become ordered in some particular sequence[20]. So when given set ***N*** we are well within the definition to denote it as $N = \{ 2, 1, 3, 1000, \ldots\}$ even though it is not intuitively understood that it is the set of natural numbers. In terms of a series, if we change the order of its representation we structurally transform it into a new series. This means that $\langle 0, 1, 2, 3 \rangle$ is not identical to $\langle 2, 3, 1, 0 \rangle$. In this regard, we rightly expect a series to be a kind of ordered set[21].

Ordered sets have been given a structure in some framework. They are reflexive, antisymmetric and transitive[22]. In the context of the "less than or equal to relation", this means that:

1. Each member is identical within the framework to itself. (Reflexivity: $a \leq a$)[23]
2. If "a" and "b" are members of an ordered set then if "a" stands in relation to "b" and "b" in relation to "a" that "a" and "b" are identical.
   (Antisymmetry: If $a \leq b$ and $b \leq a$ then $a = b$)[24]
3. If "a," "b" and "c" are members and if "a" stands in relation to "b" and "b" in relation to "c" then "a" stands in relation to "c." (Transitivity: If $a \leq b$ and $b \leq c$ then $a \leq c$)[25]

Strictly ordered sets, however, are neither reflexive nor antisymmetric. They are more rigid relations which are asymmetric and transitive. Thus under the "less than" relation, to be a strictly ordered set means:

[20] Jech p. 1 & 46
[21] Jech p. 33 - 34
[22] Jech p. 34
[23] Jech p. 33
[24] Jech p. 33
[25] Jech p. 33

1. $a < b$ and $b < a$ are never both true (Asymmetry)[26]

2. If "a," "b" and "c" are members and if "a" stands in relation to "b" and "b" in relation to "c" then "a" stands in relation to "c." (Transitivity: If $a < b$ and $b < c$ then $a < c$)[27]

Kant's conception of time, in my assessment, fits within the context of a strictly ordered set. The moment before now is described as such because it comes before and can't come after or at the same time as this moment. This naturally describes asymmetry. Furthermore, the sequence of time describes a transitive set because any moment stands in rigid relation to other moments within the series. It is locked in its position along the sequence of moments. I am confident in affirming this relationship because Kant suggests that the past constitutes the present insomuch as the past is not present in itself but as a foundation for the possibility of my present experience[28]. Therefore, this strict relationship between past and present can only be correlated to a strictly ordered series.

To elaborate further, a strictly ordered series is grounded in its enumeration by the natural numbers. In abstract notation a series $\boldsymbol{A}$ can be of the form $\boldsymbol{A} = <a_0, a_1, a_2, a_3, \ldots., a_n, a_{n+1}, \ldots>$ where the subscript "n" is a member of the sequence of natural numbers[29]. It should be noted that finite series exists such that the sequence is limited by a finite natural number "n". An infinite series is not restricted to this condition such that it will continue to "n+1", "n+2" and so on.

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[26] Jech p. 33
[27] Jech p. 33
[28] Kant (2007) p. 442 [A495]
[29] Jech p. 34

Given this overview of the mathematical characterization of the series, we can see that when Kant says reason demands an absolute totality is demanded is the absolute completeness of the conditions of a series' possibility. Stated simply, reason is looking for an absolute complete synthesis of the series, i.e. a complete enumeration. The competed series, in this sense, is the unconditioned as we defined earlier[30]. However, the unconditioned for Kant can have two conceived notions in this series of moments[31]. In one sense it consists *in* the entire series and in another the absolute unconditioned *is part* of the series[32]. Either infinity is constituted in the totality of the series of moments or as a member of the series of moments.

In the former alternative, all members of the sequence are conditioned and only its totality is unconditioned[33]. This means that members of the series form conditioned instances of moments in time and that the totality of them is the infinitude of time's completed series of moments. From this conception there is no end to the series, i.e. it is infinite, and for the past we see that the regressive enumeration is never ending. Kant does say that this regress is never completed and "can only be called potentially infinite[34]." To illustrate, consider the series $\boldsymbol{S} = <\ldots.\ S_{n+1},\ S_n,\ \ldots\ S_1,\ S_0>$. Its enumeration when "n" is a natural number is clearly the series of natural numbers, $\boldsymbol{N} = <\ldots 3, 2, 1, 0>$. Therefore, the series $\boldsymbol{S}$ has no limit and it will never be completed as we count backwards. To speak in mathematically equivalent terms, I would suggest that this

30 Kant (2007) p. 391 [B444]
31 Kant (2007) p. 391
32 Kant (2007) p. 391
33 Kant (2007) p. 391 [B445]
34 Kant (2007) p. 392 [B446]

conception of the infinite fits within the boundaries of a countable set. An infinite series, as a type of set, is countable if the number of members a set has is equivalent to the set of all natural numbers or some finite number[35]. This means that the members of an infinite set can be put into a one-to-one correspondence with the natural numbers while finite sets are put into a one-to-one correspondence with a finite number. When we mathematically analyze Kant's unconditioned in the first sense we are creating an isomorphism between a series and the natural numbers. Consequently, the series is countable but inexhaustible and the unconditioned exists as a totality of the inexhaustible enumeration in accord with the natural numbers. It is the totality of the successive synthesis.

In the latter sense, the unconditioned is a part of the sequence with all other members as subordinate to it[36]. Under this conception, there exists a first member of the series, and, in terms of time, a beginning to the world[37]. In mathematical terms, this conception of the unconditioned can only be pictured as a bounded series with a beginning along at some earlier moment and an "end" at the present moment. What is unconditioned in this series is its beginning.

Clearly, the unconditioned in the first sense refers to the thesis and in the latter sense to the antithesis. It is here that the unsettling characteristic of Kant's first antinomy emerges. On the one hand, Kant is suggesting that an actual infinite amount of time describes an unconditioned and that an infinite enumeration can only be potential[38]. On

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[35] Jech p. 74
[36] Kant (2007) p. 391 – 392 [A418]
[37] Kant (2007) p. 392 [B446]
[38] Kant (2007) p. 393 [A418]

the other hand, I believe that Kant assumes that the bounded set of a past with a beginning is a finite set because he believes a finite set can be completely enumerated in successive synthesis as a totality thus satisfying an absolute completeness of the conditions of their possibility[39]. Given the aforementioned mathematical conception of Kant's unconditioned, I am forced to question his application of the infinite series because his antithesis seems compatible with the conception of an infinite bounded series.

This, in my assessment, fatally undermines Kant's enterprise. He assumes that the thesis describes an infinite series and the antithesis describes a finite series. The former is conceived as such because he believes it impossible to constitute the totality of an infinite past in such a way to constitute the present[40]. In terms of the latter, Kant believes that the describe series has termini which allows the comprehension of the totality of that series[41]. However, I believe it is possible that they both describe infinite series. This assertion rests on our understanding of an infinite series, the unconditioned and infinite bounded series. The following is intended to clarify these ideas.

[39] Kant p. 390 [A416]
[40] Kant p. 396-397
[41] Kant p. 396-397

Section 3 - The Infinite Series and the Unconditioned

Defining an infinite set, and by extension an infinite series, is made possible by Georg Cantor's work on the infinite. In his authoritative theory, an infinite set "is a class which can be put into one-to-one correspondence with a part of itself"[42]. This means that you can take any set or series and find a subset with the same number of members as the whole set. Some who hold this view criticize Kant on the grounds that he confuses the infinite and the indefinite[43]. However, I would suggest that, while Kant could not have access to Cantor's work, we can see a conception of the infinite his *Inaugural Dissertation* which is consistent with it in so much as we are working the first transfinite number, namely omega as denoted by $\boldsymbol{\omega}$.

Kant suggested that he sees, in the infinite, a measure to which "nothing is greater[44]". It is thus hard to see how Cantor's conception of the infinite can bring about grounds for criticism because if a subset of itself is the same size as the original set there remains nothing greater in size than that set. The cardinality, the number of members within a set, is the same. Put in set theoretical terms, Kant's conception is consistent with the mathematical convention that there exists an infinite subset of an enumerable set that is also enumerable[45]. To illustrate this, we should consider the series of natural numbers, $\boldsymbol{N}$ = <0, 1, 2, 3, ….>. An infinite subset of that series, say the even natural number $\boldsymbol{N}^{2n}$ = <2, 4, 6, 8, ….>, is also enumerable by the natural numbers. If a set is enumerable by a natural number it is defined as countable but if it is enumerable by all the natural

[42] Fried p. 214
[43] Cobb p. 690
[44] Kant (2011) Location 592
[45] Jech p. 74

numbers it is infinitely countable[46]. This is reflected in the terminology used by Kant in his thesis because he specifically says that "up to every moment and eternity has elapsed[47]." This can be translated into strict mathematical terms to mean that a subset of the infinite series of the past can also be countably infinite.

In light of this, I think we should reassess what it means to traverse an infinite series. Kant's conception of the past in his thesis, as a totality, is an actual infinity as opposed to the potential infinity of the future. Paul Carus, and others, might argue that it is presumptuous to take what we know in finite terms and push it to an infinite conception of the totality of things. Kant could be confusing a finite perception of time for an infinite if he tries to use it to experientially grasp an infinite series of moments in time. This could mean that he confuses the infinite with the indefinite and that he sees the indefinite as something we can't experience; however, in my assessment he departs from that logic because I believe that he doesn't try to move from a finite conception of an elapsed series to an infinite conception. Kant clearly intends his thesis to describe the infinite and his antithesis as finite. Because of this, he demonstrates that he understand the difference between the finite, the infinite and the indefinite. The problem of Kant's traversable infinite series is not a problem of an "indefinite object" because infinity is a definite unconditioned idea for Kant.

Instead, the critical problem is one of experience verses mathematical necessity. Josiah Royce would articulate this problem by saying that an intuition of the Kantian type

[46] Jech p.74
[47] Kant (2007) p. 397

merely holds characteristics belonging to the human experience and that such finite experiences cannot leap into mathematical truisms about infinity even though both are definite objects of experience[48]. The infinite, is thus a topic of the unconditional suited exclusively to the mathematical discipline which deals with the consequences of infinity thus making it possible to subject them to exact treatment in relation to experience within reason[49]. And so, when we ask ourselves if it is possible to experience infinity we should be surprised when the answer I'm providing requires a mathematical paradigm which pulls a mathematical object into Kant's manifold of intuition generating the concept clearly and distinctly.

Furthermore, as I made clear earlier, Kant has walked away from experience and is dealing with ideas of pure reason. Royce's response would suggest that we may only be capable of experiencing the infinite as a concept of mathematical necessity but it remains accessible as a mathematical recursive construction because "forms of thought are unquestionably the forms of mathematical science"[50]. A simple illustration of this idea is the possibility of a set of past experiences because we structure experience in terms of temporal sequences. Moreover, Kant believes "mathematical cognition has this particularity: it must first exhibit a concept in intuition, and do so a priori, in an intuition that is not empirical but pure[51]." Furthermore, he says that "arithmetic attains it concepts of number by the successive addition of units in time[52]" and therefore the form of Kant's

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[48] Royce p. 200
[49] Royce p. 202
[50] Royce p. 206
[51] Kant (2001) p. 23 [281]
[52] Kant (2001) p. 29

time series gains meaning through the conception of enumeration because our intuition of time is enumerated by a successive addition of units in time. We thus “experience” the infinite as a method of enumeration and as a pure mathematical construction.

This is a powerful notion indeed. Carus, and others, argue that infinity is not an object in sensible terminology. However, I believe it makes sense to suggest that infinity is a mathematical object, consistent with the Kantian notion of time, and our execution of the inexhaustible enumeration of the infinite, through a recursive definition, is the “sensible” object which sustains the present moment. Mathematical recursive definitions can define the contents of Kant’s series. In terms of our past time set $\boldsymbol{S} = <\ldots S_{n+1}, S_n, \ldots S_1, S_0>$, where “n” is a member of the natural numbers, a recursive definition of Kant’s first antinomy would look something like this:

Kant’s Thesis	*Kant’s Antithesis*
S_0 = the current moment S_{n+1} = the moment just before moment S_n S_∞ = the unconditional idea of every moment that occurred before the present	S_0 = the current moment S_1 = the moment just before moment S_0 S_n = the beginning of time

Table 2: The First Antinomy in Recursive Form

This is a controversial suggestion but one, in my assessment, that is consistent with Kant’s notion of infinity. His thesis is contingent on the idea that the world today, as an end of a series, is impossible to reach because we can’t complete the series to this moment. However, a recursive definition allows us to know any point along the infinite regress of the series. When conceiving the past as an infinite regress, recall series $\boldsymbol{S}$, we can define any moment from the present to the past as some enumerated member of that set $\boldsymbol{S}$. In any way we conceive it, I can know the location of any moment in the past as a definite moment as soon as I define it in correspondence with some "n". The limit of this conception is an infinite number, known as a transfinite number and, in this case, the number ω, which is itself an inexhaustible limit. Transfinite numbers are those that are larger than all finite numbers[53]. This completion of a "limit" is intrinsically different from that found in Kant's antithesis. By limit, I do not mean a beginning to the series. In the antithesis the limit of the regress is the beginning of time. As a mathematical conception, the conception of "*limit"* in Kant's first antinomy suggests a process of the enumeration for each past moment. Stated otherwise, choosing a point in the past to define a series to the present moment as a subset of the whole part or, as Kant does, choosing the first member of the series if one exists.

However, by choosing a limit of the enumeration to be infinite we can be assured that it is supported by a recursive definition which describes the process of enumeration. Therefore, it is reasonably acceptable, in my assessment, to allow the recursive process to provide a foundation for the proposition that the past is infinite. Kant suggests that "an infinite given magnitude is impossible" and he may be sensibly right; however he fails to recognize that when he says "the successive synthesis of units required for the enumeration of a quantum can never be complete" [54] he allows us to construct a recursive definition of the successive synthesis of that past. This serves as a mathematical model for the infinite object of our past as an unconditional idea. Furthermore, as we will discuss shortly, this simultaneously applies to the thesis and antithesis allowing us to

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[53] Jech p.115
[54] Kant (2007) p. 400

accept the possibility that the past as infinite and as a mathematical object subject to human experience through mathematical reason. The infinite limit of our enumeration allows for the mathematical object to exist transcendentally in this sense, while the recursive definition of the process to mediate an impression on our manifold of intuition of that mathematical object. This is a position which I believe is also supported by Ray H. Dotterer and Smith.

Smith explicitly points out in *Infinity and the Past* that "the collection of events cannot add up to an infinite collection in a finite amount of time, but they do so add up in an infinite amount of time[55]." This is in line with my reasoning, because I believe it presupposes an infinite enumeration. Smith also suggests that an infinite number of events can be separated from the present by an infinite number of distances[56]. He clarifies this point by saying that "it is quite possible for there to be an infinite number of events that have really occurred such that each of the events is separated from the present event by a finite number of intermediate events[57]." This describes, in ordinary language, a recursive definition for the enumeration of past events from the present and thus supports an infinite, recursively definite, regressive synthesis.

Dotterer concludes, in *The Definition of Infinity*, that the notion of totality of the infinite, by any conception, "cannot be regarded as a something that is actually existing, but only as a scheme or plan that is in process of realization" and that this inexhaustible

[55] Smith p. 72
[56] Smith p. 64
[57] Smith p. 64

series “is nevertheless a definite and thinkable unity[58].” Dotterer, in my assessment, is spot on. He does however highlight the commonly argued notion that one-to-one correspondences are problematic in infinite sets and because my argument tends to use of this method for comparing any two infinite series, it is necessary to address that concern.

[58] Dotterer p. 300

Section 4 - Correspondence and Cardinality for Infinite Sets

Briefly put, Dotter explains that one-to-one correspondences for infinite sets are problematic because when they are made it is not inconceivable to suggest that a many-to-one correspondence is possible as well[59]. This is a problem outlined by E.R. Emmet in nearly identical terms. It is simpler to illustrate the problem with an example. We know that the even numbers can be put into a one-to-one correspondence with the odd. However, they can also be put into a two-to-one correspondence as well. Consider the following:

Even Number	Odd number in one-to-one correspondence	Odd numbers in one-to-two correspondence
2	1	1, 3
4	3	5, 7
6	5	7, 9
8	7	11, 13
…	…	…

Table 3: Correspondence Breakdown

The table above illustrates that in a one-to-one correspondence even and odd numbers are enumerated congruently. However, while in a one-to-two correspondence it could be suggested that even numbers outnumber odd by factor of two. Dotter suggests that we should elucidate the ambiguities of isomorphisms in order to understand that true nature of correspondence in terms of the infinite[60]. This suggests that a finite collection works fine with one-to-one correspondence, but that in an infinite setthese standards

[59] Dotterer p. 295
[60] Dotterer p. 296

break down[61]. I believe that the problem illustrated above is a misapplication of correspondence because the very fact that a one-to-one correspondence is possible is a stronger assertion for enumeration than any "larger" correspondence.

To clarify, consider the aforementioned pairing activity. You have two infinite groups, both of which can be enumerated by the natural numbers. When comparing sets enumerated by the natural numbers, within $\boldsymbol{\omega}$**,** this problem disappears because these finite sets are predictable and exhaustible. For infinite sets, a recursive definition allows us to match case to case recursive instances from one set to another. In my assessment, the mere ability to compare infinite sets with cardinality equal or less than $\boldsymbol{\omega}$ is enough to accept correspondence as an accurate measuring because their structures preserve the process of correspondence. While, it is true that a two-to-one or a one-to-two correspondence is possible, this is so only because each set is inexhaustible.

Ultimately, this problem is reconcilable because the existence of a "smallest" type of correspondence serves to represent the very least pair-wise organization of the elements of two sets. An illustrative analogy would be the difference between the height of a building and every height above it. We say that a building is x feet tall because the lowest measure that corresponds to the building's height is x. We wouldn't say that the building is $x + y$ feet tall because there is a height taller than it. In the same way, I am suggesting that we wouldn't say a one-to-one correspondence between two infinite sets

[61] Dotterer p. 297

and a two-to-one correspondence is grounds to describe the cardinality of the set in terms of the latter correspondence.

This topic is far too extensive to completely explore in this thesis, but for the sake of moving forward I will accept one-to-one correspondences as a justifiable means for measuring the size of sets because of the aforementioned, albeit informal, analysis. This means that we can reasonably compare Kant's series of time in his thesis to his antithesis. However, as I stated earlier the nature of his antithesis is such that it seems compatible with an infinitely bounded series. Before I continue to that comparison, I first need to explain that inference.

Section 5 - Infinite Bounded Series

Instinctively, when we consider Kant's antithesis on time, we conceive a bounded series with a beginning and an end. In this context it is described by its beginning and end as bounds. For simplicities sake, we will work exclusively with a closed bounded series. This means that all bounded series from here forward will include their beginning and ending elements[62]. Kant suggests that this strictly ordered set is easily traversed because there is a start and the successive synthesis of moments in the series can be completed. This allows us to move from the beginning to the end of the series. However, there exists the same problematic inclusion of infinitude in the antithesis as there is in the thesis if we understand that an infinitely bounded set is compatible with the antithesis—a problem which is caused by the fact that Kant does nothing to address the natural density of moments.

To elucidate this problem, we must first understand that a bounded series can be exemplified in many ways. We can consider Kant's antithesis to be the most general example. But to break it down, consider the series of moments that existed from one minute ago to this moment. This example describes a series of moments within the set [-1,0] along a timeline[63]. Kant's antithesis and [-1,0] each have a beginning, an end and would, by Kantian standards, be described as a traversable series. Secondly, we must understand the concept of density. Density, in mathematical terms, describes a series that has an element between any two other elements of itself[64]. For example, our bounded set

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[62] Jech p. 182
[63] [-1,0] is a mathematical form for the set containing the numbers between -1 and 0, including -1 and 0.
[64] Jech p. 83

[-1,0] has as a member -0.5 minutes ago. It also has -0.75 minutes ago, -0.625 minutes ago, and so on. It is therefore necessarily conceivable for bounded series to be infinite when they are dense. Kant's thesis and antithesis can be conceived using this framework. In mathematical terms, the densely bounded set [-n, 0] can represent time's series as described in the antithesis, and the bounded set [-∞, 0] can represent time's series as described in the thesis; both of which are infinite series[65].

Now in terms of the argument Kant presents in his antithesis, we may wonder what consequences exist when we apply a densely bounded set of moments to it. I assert that it would then describe an infinite series that is not traversable in the same way Kant believes his thesis is not traversable. This is because a conception of the density of moments is isomorphic to the rational numbers, an infinitely countable set. Such an argument, in practical terms, consistently rounds irrational numbers into rational numbers with varying degrees of precision. I believe that demarcating moments in a dense way requires no more precision than this because we can make a moment's enumeration number more precise by expanding its decimal notation. This also attributes uniqueness to these moments since they remain rigidly asymmetric as described earlier. Concerning ourselves with irrational numbers is neither helpful to our example nor to the conception of moments as dense. Furthermore, if we consider irrational numbers we gain little because each set becomes uncountable and the "traversing" of the two sets would persist as a legitimate problem. It is clear that I can traverse a moment ago to a moment now in spite of the density of those moments so, again, an irrational division of moments is

[65] Jech p. 82-83

counter intuitive to the successive synthesis of time. Its uncountable nature precludes its application.

Such a conception of time, wherein between any two moments there exists another moment, is comparable to the density of rational numbers. Rational numbers are dense in that between any two there is another rational number. Moments in time, in my assessment, behave similarly. Furthermore, considering Kant’s conception of the infinite, as to which nothing greater can be measured, does just as much justice to our conception of moments as it does to our enumeration of rational numbers. This is because we can construct the density of moments by taking the average of two moments in time to find the moment exactly half way between them and can do so indefinitely. Nothing will be greater in size than the series of dense moments in the antithesis nor can anything be greater in size than the infinite number of moments in the thesis.

This is a very powerful idea because what it ultimately indicates is that the number of moments between now and a minute ago is the same as now and five minutes ago or between any beginning of time and now; and this is a problem for Kant because he submits, in his thesis, that you can’t traverse an infinite series of moments. Thus, Kant’s methodology becomes inconsistent because the world with a beginning in time and a world without a beginning in time are isomorphic with each other and to the natural numbers. If he says we can’t traverse time in his thesis then we can’t traverse it in his antithesis.

This means that in both arguments, you would need to traverse an infinite number of moments to reach the present moment. It could be suggested that Kant wouldn’t have had access to the mathematical concept of density in sets; however, I would respond to such an argument by noting the density of moments is a product of common sense judgment even if it isn't formalized. Thus, it is easy to see that the problem of traversing an infinitely bounded set is as problematic in Kant's own conception as traversing an infinite past to the present in an unbounded set. Thus, it makes more sense to consider the antithesis's bounded set as a subset of the series of time described in his thesis than as an independent description of time allowing us to accept the possibility of an infinite past that is a priori accessible to mathematical reasoning. Furthermore, it is through mathematical conception that we can have any reasonable experience of the recursive definition of an infinite past as a possibility.

Section 6 - Conclusion

Ultimately, I believe Kant's method in the antithesis and his description of infinity in the thesis does not contradict each other. If it did then a consistent use of "traversing an infinite set" would not be applicable to both the thesis and antithesis. Furthermore, from this conclusion it is reasonable to accept that a potential infinity transforms into a recursive definition from which we can have access to infinity as method. This method must allow for the traversing of an infinite series by a successive enumeration to an ever more continuous set of finite instances within a set of infinite cardinality for if it didn't then no forward motion of time would be perceivable in Kant's enterprise.

To challenge this, one need only show that Kant's conception of a bounded set of time cannot be dense, but I suspect that this is a herculean task, primarily because the idea that my experience of moments in time are "non-dense" is counter to the persistent existence of our intuitions. This scheme, best stated by Lawrence Friedman, is indicative of the very idea that the succession of moments is an uninterrupted continuum with each moment in relation, i.e. in a strictly ordered series, with another[66]. There are no times when the possibility of a moment does not stand in relation between two other moments.

The comparable relation of moments to others is pivotal. Friedman continues by saying "in that which does the comparing, there must be a different series wherein the comparison takes place and ultimately some series must be compared in something which

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[66] Friedman p. 381

is not at all a series or an extension, in order to avoid an infinite regress"[67]. This comparable notion of infinity as a mathematical object allows us to conceive a notion of an infinite regress as a recursively defined enumeration of past events which avoids logical contradiction. This atones reason of its contradiction because when we are "performing a synthesis of apprehension by representing a series" we do so in a unity, namely in a totality of the mathematical infinity as a mathematical object[68]. As Friedman concludes, we are then allowed to reasonably accept an infinite past as a special reference which is doing the grasping rather than referring to the grasped extension[69].

I believe it is important to put the previous statements in perspective. Infinity, in my view, is a mathematical object that saves us from the sensible infinite regress and focuses our attention on the regressive process. This is a challenge to Kant's Antinomy on Time because it reconciles the antithesis's method within the thesis's conception of an infinite time. If any presupposition has been made throughout my enterprise, I would suggest that it is the acceptance of Kant's premise that "no coming to be is possible from a time where everything is not[70]." However, I don't believe this handicaps my reasoning in any critical way. If anything, accepting this idea is required to traverse any strictly ordered series. You cannot move to the next member if there is no previous member. I would also suggest that such a premise supports the present moment because an infinite past would always support a successive moment. We can conclude then, that focusing on the process allows us to attain something sensible with which to view the relationship

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[67] Friedman p. 381 - 382
[68] Friedman p. 382
[69] Friedman p. 382 - 383
[70] Kant (2007) p. 397

between successive moments as comparable to an inexhaustible infinite, namely $\boldsymbol{\omega}$. In this light, the contradiction in Kant's first antinomy is reconciled with a mathematical object. An object which helps us access the transcendental idea of infinity by means of pure reason alone and apply it to the present possibility of experience as it stands on some conception of the past as an infinite series of moments in some well-ordered framework; either bound or unbounded.

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