2011-07-21

Asynchronous Auction for Distributed Nonlinear Resource Allocation Problem

Bangla, Ajay

http://hdl.handle.net/2144/1429

Boston University
Abstract 111

Resource Allocation Problems (RAPs) are concerned with the optimal allocation of resources to tasks. Problems in fields such as search theory, statistics, finance, economics, logistics, sensor & wireless networks fit this formulation. In literature, several centralized/synchronous algorithms have been proposed including recently proposed auction algorithm, RAP Auction. Here we present asynchronous implementation of RAP Auction for distributed RAPs.

What is RAP?

Notation:
- $G = (W, T, E)$: bipartite graph
- $W$: Set of $N$ sources
- $T$: Set of $M$ sinks
- $E$: Edges
- $x_i$: Qty of res at $i^{th}$ src
- $c_{ij}$: Gain on arc $(i, j)$
- $x_{ij}$: Allocation from $i$ to $j$
- $W_j = \{ i : (i, j) \in E \}$
- $z_j = \{ j : (i, j) \in E \}$
- $f_j$: Non-increasing convex utility

RAP Formulation:

$$\min \sum_{j \in T} f_j(z_j) \text{ s.t. } \sum_{i \in W_j} x_{ij} \leq x_i \quad \forall i \in W \quad z_j = \sum_{i \in W_j} c_{ij}x_{ij} \quad \forall j \in T \quad x \geq 0$$

Why RAP?

Search Theory
- Target hidden in one of $M$ regions with
  - Prior prob: $p_j$
  - Reward: $c_{ij}$
  - Area: $A_j$
- $N$ resources each with
  - area sweep per effort: $c_{ij}$
- For region $j$
  - Frac. area swept: $z_j = \sum_{i=1}^{N} c_{ij}x_{ij}/A_j$
  - Prob. of no det. $\approx \exp(-z_j)$

Distribution of search effort

RAP Auction

- Define $\epsilon$-CS for pair $\{x, p\}$ as $p_j \in [-f_j^+(z_j), -f_j^-(z_j)]$
- $x_{ij} > 0 \Rightarrow c_{ij}p_j \geq \max_{k \in \mathcal{T}} c_{ik}p_k - \epsilon$

Bidding Phase
- Pick source $i$ with surplus $> 0$.
- Compute $v_i = c_{ij}p_j$.
  - $(j_1 \text{ best sink & } j_2 \text{ sec. best})$
  - Bid price: $b_i = (v_{ij} - \epsilon)/c_{ij}$.
  - Bid surplus: $y_i = c_{ij} - b_i$.
  - Submit bid $(y_i, b_i)$ to $j_1$.
  - Update old bid prices
  - if $x_{ik} > 0$, set $b_k = (v_{kj} - \epsilon)/c_{ik}$

Allocation Phase
- Assume $j$ received bid
- Absorb as much of $y_i$ while preserving $\epsilon$-CS till either:
  - $p_j \downarrow b_i$
  - $y_i$ is absorbed

Async RAP Auction

Paradigm:
- Nodes are autonomous decision makers.
- Sources bid at arbitrary times based of outdated prices.
- Sinks accept bids, update and broadcast prices at arbitrary times.
- Outdated prices models memory or communication delays.
- No/min synchronization.

Notation:
- $p_j(t)$: price of sink $j$.
- $z_j(t)$: demand at sink $j$.
- $g_i(t)$: surplus at source $i$.
- $U(t)$: set of all sources with +ve surplus.
- $R(t)$: set of sources with a 'ready bid'.

Bidding Procedure
- If $i \in R(t)$
  - Compute $v_i = c_{ij}p_j(t_j(t))$.
    - $(j_1 \text{ best sink & } j_2 \text{ sec. best})$
    - Bid price: $b_i = (v_{ij} - \epsilon)/c_{ij}$.
    - Bid surplus: $y_i = c_{ij} - b_i$.
    - Update: $g_i(t+1) = 0$.
    - Submit bid $(y_i, b_i)$ to $j_1$.

Allocation Procedure
- If $p_j(t) \geq b_i + \epsilon/c_{ij}$
  - Accept as much of $y_i$ while preserving $\epsilon$-CS till either:
    - $p_j(t) \downarrow b_i$
    - $y_i \downarrow 0$
    - Intimate $i$ if $y_i \neq 0$.
    - Intimate srcs with flows reversed, else discard & intimate $i$.

Convergence
- Theorem 3: Async Auction terminates in finite time to $\epsilon$-CS satisfying $\{x^*, p^*\}$ and feasible $x^*$ under following assumptions:
  - $U(t) \neq \emptyset \Rightarrow R(t') \neq \emptyset$ for some $t' \geq t$.
  - For all $i, j$, and $t$, $\lim_{t \to \infty} \tau_{ij}(t) = \infty$.

Conclusions
- Total asynchronous algorithm: Autonomous agents collaborating.
- Near optimal solution in finite time.
- Works for any monotonic convex utility. Doesn't presuppose a particular cost function.
- Works even for non-differentiable and non-strictly convex utilities.