2014

Quality, information and certification

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http://hdl.handle.net/2144/15136

Boston University
BOSTON UNIVERSITY
GRADUATE SCHOOL OF ARTS AND SCIENCES

Dissertation

QUALITY, INFORMATION AND CERTIFICATION

by

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2014
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This dissertation is dedicated to my wife Emily,

to my parents Rita and Pietro,

and to my sister Bianca.
Acknowledgments

I am extremely grateful to my advisers Andy Newman, Sambuddha Ghosh and Albert Ma for support, guidance and encouragement. I thank Dilip Mookherjee, Juan Ortner, Bart Lipman, Francesco Decarolis for helpful comments. All errors are my own.
QUALITY, INFORMATION AND CERTIFICATION

(Order No. )

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Boston University, Graduate School of Arts and Sciences, 2014

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ABSTRACT

This dissertation consists of three chapters that study issues in Corporate Finance and Industrial Organization related to the behavior of markets with asymmetric information. The first two chapters study the economics of credit rating agencies; the third chapter examines a process of social learning about product quality. Chapter 1 models the effect of rating agency competition on the quality of rated securities. I compare equilibria across a regime of competition between two rating agencies and a monopolistic regime. In both regimes, all available agencies are hired in equilibrium, so under competition more ratings are observed. However, competing agencies do not fully internalize the return of a reputation for being honest. Whenever strategic agencies are not very concerned about their reputation, competition can induce more issuer effort than monopoly. Otherwise, a monopolistic agency induces more effort. Chapter 2 analyzes the effect of the Cuomo Plan, a much-discussed regulation that prohibits issuers of residential mortgage-backed securities from making payments to rating agencies contingent on the assigned ratings. I construct a certification model which consists of the following features: (i) an issuer privately informed about her security’s quality can hire a rating agency to assign a rating; (ii) the agency can observe, at a cost, a private signal correlated with the quality of the security; (iii) an undeserved favorable rating reduces the agency’s future revenues.
I show that the Plan has an effect on the informative content of the rating only if the agency’s signal is not too costly. In this case, the Plan ensures that the rating is more informative; otherwise the Plan has no effect. In chapter 3, I study the pricing strategy of a monopolistic firm in a market characterized by consumers with heterogeneous preferences and private information about the product quality. Consumers purchase sequentially and observe the history of purchasing decisions, prices, and consumers’ preferences. I characterize the conditions under which the monopolist gains when consumers learn the true quality, and, which pricing strategy ensures that learning takes place.
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Chapter 1

Certifier Competition and Product Quality

1.1 Introduction

The recent financial crisis put credit rating agencies in the public spotlight.\textsuperscript{1} Favorable and undeserved ratings were blamed for encouraging issuers of structured products to sell extremely low quality securities. Unlike issuers of other financial assets such as corporate bonds, issuers of structured products could design their securities to meet the requirements of the rating procedures.\textsuperscript{2} A recent report by the U.K. Financial Services Authority highlights this issue:

\begin{quote}
While a corporate bond issuer can make only limited adjustments to its balance sheet to improve its rating ... an originator of a structured credit product has an incentive and flexibility to design them in such a way as to obtain maximal ratings.\textsuperscript{3}
\end{quote}

In these markets, however, undeserved ratings not only left investors uninformed, but also encouraged issuers to originate and securitize high-risk loans.

The lack of competition in the credit rating market was blamed for the low quality of the rating process. Regulators in the United States and the European Union have argued that increased competition among rating agencies is desirable. A recent SEC report illustrates the position of the U.S.

\textsuperscript{1}Ashcraft \textit{et al.} (2011) and Benmelech and Duglosz (2009b) provide detailed accounts of the role of credit rating agencies in the recent financial crisis.

\textsuperscript{2}See the Coburn Levin Senate Report, part V, section B, and Benmelech and Duglosz (2009a) for a description of the rating process of a structured finance product.

\textsuperscript{3}Fennell and Medvedev (2011).
Congress:

In enacting the Rating Agency Act, Congress found that “the 2 largest credit rating agencies [Moody’s and S&P] serve the vast majority of the market, and additional competition is in the public interest.”

Will an increase in the number of available rating agencies lead to more informative ratings? Will informative ratings induce socially efficient choices of investment in product quality? To answer these questions, I construct a two-period certification model that captures the relationship between credit ratings and the quality of rated products under different structures of the credit rating market. In every period, an issuer can exert effort to increase the quality of her security. Quality can be any characteristic of the security that affects its value, such as expected return or riskiness. Neither the issuer’s effort nor the security’s quality can be observed by the buyers. However, the issuer can hire rating agencies to observe a signal correlated with the security’s quality and then assign a rating. Rating agencies are long-lived agents. They can be strategic or committed to honesty. Strategic agencies face a classic trade-off: inflate their ratings and increase current revenues, or rate honestly and preserve a reputation for honesty. In this context, I compare equilibria across a regime of competition between two rating agencies and a monopolistic regime.

If two agencies are available in the market, I show that the issuer hires both agencies. This result matches the empirical evidence for structured finance ratings. Benmelech and Duglosz (2009b) show that over the period from 2004 to 2007 most structured finance tranches were receiving more than one rating. Rating agencies are not textbook competitors selling substitute goods. Rather, they are experts with competing opinions.

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4SEC (2012).
When reputation incentives are weak, that is, when rating agencies heavily discount future revenues, I show that competing agencies can provide more informative ratings than a monopolistic agency. As a result, in the competition regime the issuer has a stronger incentive to invest in the quality of her security. When reputation provides a strong discipline, a monopolistic agency induces more investment in quality.5

Even when reputation motives are weak and strategic agencies are likely to assign undeserved ratings, under the competition regime buyers have the opportunity to compare independent ratings. The presence of a low rating, for example, makes a high rating look suspicious. This opportunity to compare ratings ensures that buyers are better informed and has an indirect effect on the issuer’s incentive to invest in quality. At the same time, a monopolistic agency has stronger incentives to maintain a reputation for honesty in order to induce the issuer to invest in quality. When buyers expect more effort from the issuer, they are willing to pay a higher price for a security with favorable ratings. As long as the rating fee is proportional to the expected quality of the security, a higher security price allows the agency to request a higher fee from the issuer. If multiple agencies rate the same security, each and every agency’s reputation determines the issuer’s decision of effort. Competing agencies do not fully internalize the effect of their reputation. Ultimately, this externality reduces the competitors’ incentives to maintain a reputation for honesty.

My results imply that competition might increase the incentive to invest in the quality of a security in markets where rating agencies are only weakly disciplined by reputation motives. One

5Monopoly and competition might also differ in the total amount of information that all agencies obtain. I abstract from this issue and assume that in the two regimes the total amount of information observed by the rating agencies is constant.
such example are markets where a large number of new securities are issued in a short amount of time, as in the case of asset-backed securities in the years preceding the financial crisis. In these markets, many ratings are assigned before the investors acquire the information necessary to evaluate the ratings’ quality. At the same time, competition among rating agencies is not desirable in every rating market. In markets where the volume of ratings is constant over time, competition might be detrimental to the quality of the ratings and the investments in securities’ quality.

In an extension of my model, I characterize the equilibria in the case of competition between rating agencies observing identical signals of quality. When the signals are closely correlated with the quality of the security, conditionally independent signals result in stronger incentives to invest in quality than identical signals.

This paper presents the first model to simultaneously consider rating agencies’ reputational incentives and issuers’ investments in the quality of their securities. These issues have, so far, been considered separately. The certification models for products of endogenous quality developed by Albano and Lizzeri (2001) and Donaldson and Piacentino (2012) abstract from the reputational incentives of certifiers. Both papers conclude that a monopolistic regime in the market for certification induces inefficient amounts of investment in product quality. Under a monopolistic regime, firms under-invest in product quality, anticipating that any potential increase in revenues would be captured by the rating fees. A regime of competition would ensure lower certification fees and could mitigate this sort of hold-up problem. In my model, investment in a security’s quality takes place only after the agencies set their fees. As a result, this inefficiency is not present.
A growing body of literature in quality certification considers competition among strategic certifiers. The focus is on certifiers concerned with their reputation for rating honestly. Unlike my model, this literature considers the certification of products of exogenous quality. In Bouvard and Levy (2012), firms can hire more than one rating agency to rate their security. The authors are concerned with the possibility of rating agencies developing different reputations among firms and buyers. They find, in line with my results, that reputation provides a weaker discipline for raters under competition than under monopoly. Strausz (2005), Lo (2010), and Camanho and Deb (2012) limit the firms to hiring, at most, one rating agency. All these models predict that monopoly always ensures more informative ratings than competition among certifiers.

In the model developed in this paper, reputational incentives are always desirable. A strategic agency that worries about its reputation will honestly report the signal privately observed. Reputational incentives are not always socially desirable. In Mariano (2011), rating agencies improve their reputation for expertise by disregarding private information and assigning ratings based on public information; competition among agencies exacerbates the inefficiency and results in informational losses.

In my model, the issuer does not gain from hiding unfavorable ratings as opposed to other models in which different assumptions give rise to rating shopping. Rating shopping refers to the issuers’ strategy to cherry-pick the most favorable ratings. In Skreta and Veldkamp (2009) and Bolton et al. (2012), issuers hide unfavorable ratings to deceive nave investors. In Sangiorgi et al. (2009), issuers publish only the most favorable ratings to attract investors who are legally required to invest only in high-rated securities. Rating shopping does not take place in my model because
buyers are considered to be fully rational, and cannot be systematically deceived in equilibrium.

Finally, the informativeness of the signal observed in my model by the rating agencies is exogenous. The incentives for the raters to acquire information has been considered, among others, in Bizzotto (2012), Bouvard and Levy (2012), and Kashyap and Kovrijnykh (2013).

The theoretical literature on rating agencies is matched by a growing number of empirical studies. These papers address questions including whether credit ratings influence the price of rated products, and the effect of new agencies on the quality of ratings by incumbents. Using a sample of residential mortgage-backed securities issued before the financial crisis, Ashcraft et al. (2012) show that credit ratings did influence prices. Becker and Milbourn (2011) measure the effect of the sequential entry of Fitch in separate corporate-ratings markets on the quality of the ratings from incumbent agencies. They show that when a new competitor enters the market, the informative content of the incumbents’ ratings is reduced. They explain their finding by suggesting that reputational incentives decrease when competition increases. Xia (2012) considers the entry of Egan-Jones Rating Company, an investor-paid rating agency, in the market for corporate rating, and comes to the opposite conclusion. This study shows that the ratings of the incumbents became more strict and more responsive to information after the entry of a new competitor. Doherty et al. (2012) study the entry of S&P in the market for insurance ratings and focuses on the ratings of the new agency. They show that S&P set higher standards than incumbent agencies for securities that received the same rating.

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6 A comprehensive review of the theoretical literature on credit rating agencies is beyond the scope of this section; White (2010), Jeon ad Lovo (2013) and Dranove and Jin (2010) provide comprehensive reviews of the subject.
The rest of the paper is structured as follows. Section 2 presents the monopoly case. The competition case is discussed in Section 3. Section 4 compares the different equilibria obtained in Section 2 and 3. Section 5 discusses an extension. Section 6 concludes. All proofs are contained in the appendix.

1.2 Monopoly

In this section, I characterize the equilibria in a market with a single rating agency. In the next sections, I will compare the equilibria in a competition regime to the monopoly benchmark.

1.2.1 The Model

I characterize the perfect Bayesian equilibria of a two-period game. In every period, a new issuer has an indivisible unit of a security of quality \( q_t \in \{ B, G \} \) with \( N \geq 2 \) potential buyers. A monopolistic rating agency is the only agent active in both periods. The security is worthless to the issuer and the agency. To the buyers, the security is worth 1 if \( q_t = G \) and 0 otherwise. The security’s quality is endogenous and unobservable.

At the beginning of period \( t \in \{ 1, 2 \} \), the rating agency announces its fee \( \phi^M_t \geq 0 \) to rate the security. The issuer decides whether to hire the rating agency. The agency is hired to assign a rating \( r^M_t \in \{ g, b \} \) identical to its signal of quality to be observed later on. Only after deciding whether to hire the agency does the issuer choose an effort \( e_t \in [0, 1] \). Let the issuer’s effort cost be \( c(e_t) : [0, 1] \rightarrow \mathbb{R}_+ \). Effort ensures \( Pr\{q_t = G\} = e_t \) and satisfies \( c \in C^1, c', c'' \geq 0, \) and \( c'' > 0, \) for all \( e_t \in [0, 1] \).\(^7\) Moreover, \( c(0) = c'(0) = 0 \) and \( c'(1) > 1 \). Once the security’s quality is realized, the

\(^7\)The unusual assumption \( c'' \geq 0 \) ensures the uniqueness of the issuer’s optimal choice of effort.
rating agency and the issuer observe at no cost a signal $s_t^M \in \{g, b\}$ correlated with $q_t$ as follows:

$$Pr\{s_t^M = g|q_t\} = \begin{cases} 1 & \text{if } q_t = G, \\ \pi & \text{if } q_t = B, \end{cases}$$

for some $\pi \in (0, 1)$. This order of actions is particularly suited to describe the rating process of structured finance products. These products are often modified (for example, by adding credit enhancements) after one or more agencies are hired to rate them. In general, the order describes any market in which the certifiers can commit to their certification fees before the sellers decide how much to invest in quality of their products.

The rating agency can be one of two types. With probability $\mu_1 \in (0, 1)$, the rating agency is an honest type that always assigns a rating identical to the signal of quality. With probability $1 - \mu_1$, the agency is strategic. Upon observing a $b$ signal, a strategic agency can request a monetary bribe $\beta_t^M > 0$ to renege on the original contract. If the issuer pays the bribe, then $r_t^M = g$. Let $h_t^M \in [0, 1]$ denote the probability that a strategic agency does not request a bribe upon observing a $b$ signal. The initial reputation $\mu_1$ satisfies the following condition.

**Assumption 1.** $\mu_1 > \frac{c''(0) - 1 + \pi}{c''(0)(1 + \pi)/(1 - \pi) - 1}$.

In equilibrium, Assumption 1 ensures that in every period the issuer has an incentive to exert a strictly positive amount of effort. This assumption rules out equilibria in which the issuer exerts no effort.
Finally, the issuer can decide whether to publish the rating or conceal it. The potential buyers only observe the rating, or the lack thereof, and the rating fee. They simultaneously bid for the security. Let bid$(i, r_t^M, \phi_t^M)$ denote buyer $i$’s bid. The winning bid determines the security’s price $p(r_t^M, \phi_t^M | \mu_t^M, h_t^{M^*})$. At the end of every period, the quality of the security is observed by all the agents. Figure 1.1 summarizes the timeline.

The issuer and buyers active in the second period observe the rating assigned as well as the quality of the security in the first period. $\mu_2(r_t^M, q_t)$ denotes their updated belief about the agency’s type.

The equilibrium concept of Perfect Bayesian Equilibrium does not restrict beliefs out of the equilibrium path. Nevertheless, I impose a restriction on out-of-equilibrium beliefs to rule out equilibria in which buyers arbitrarily pay no attention to the rating.

**Assumption 2.** Upon observing an out-of-equilibrium rating fee $\phi_t^M$, the issuer’s and buyers’ beliefs about the agency’s type are identical to their prior beliefs. Moreover, upon observing an out-of-equilibrium rating, buyers hold beliefs consistent with the effort choice $(e_t^*)$ and with the agency’s rating strategy $(h_t^{M^*})$.

---

8If a rating is not published, I use the notation $r_t^M = \emptyset$. 

---
The payoff functions complete the model. The payoff of issuer $I_t$ amounts to

$$U^I_t = p(r_t, \phi^M_t|\mu_t, h^M_t) - I^\phi_t \phi^M_t - A^\beta_t I^\beta_t \beta^M_t - c(e_t),$$

where $I^\phi_t, I^\beta_t \in \{0, 1\}$ denote, respectively, the issuer’s decision to hire the agency and to pay a bribe, while $A^\beta_t \in \{0, 1\}$ denotes the agency’s decision to request a bribe. If buyer $n$ purchases the security, the payoff amounts to

$$U^n_t = \mathbf{1}_{\{q_t = G\}} - p(r^M_t, \phi^M_t|\mu^M_t, h^M_t).$$

If the buyer does not purchase the security, $U^n_t = 0$. Finally, the rating agency’s payoff consists of a discounted sum of fees and bribes:

$$U^M = u^1_1(\phi^M_1, \beta^M_1, A^\beta_1) + \delta u^2_2(\phi^M_2, \beta^M_2, A^\beta_2),$$

for some $\delta > 0$. I allow the discount factor to be larger than 1 because the second period is a reduced form of all future periods in which the rating agency is active.

### 1.2.2 The Equilibria

I first consider the equilibrium of a single-period game, in which a strategic agency rates honestly with some exogenous probability $\bar{h} \in [0, 1)$. Buyers form their beliefs by observing the rating fee and the rating, or the lack thereof. The price paid for the security depends on the buyers’ beliefs about its quality as described in the next lemma.

**Lemma 1.** The price of the security equals its expected value:
\[ p(g, \phi_I^M | \mu_t, \bar{H}) = \frac{e^*_I(\mu_t, \bar{H})}{e^*_I(\mu_t, \bar{H}) + (1 - e^*_I(\mu_t, \bar{H})) Pr(e^*_I = g | \phi_I^M g = B, \mu_t, \bar{H})}, \]  
\[ p(b, \phi_I^M | \mu_t, \bar{H}) = p(\emptyset, \phi_I^M | \mu_t, \bar{H}) = 0, \]  

where \( e^*_I(\mu_t, \bar{H}) \) denotes the equilibrium effort of the issuer.

An unfavorable rating and a lack of rating lead to the same price, so the issuer is indifferent about the publication of an unfavorable rating.\(^9\)

The issuer decides how much effort to devote to a security’s quality by weighing a deterministic cost and a stochastic benefit. If the agency is not hired, the issuer has no reason to exert costly effort. If instead the agency is hired, effort increases the probability of obtaining a favorable rating without incurring the cost of a bribe. The expected amount of the bribe determines the issuer’s choice of effort. In equilibrium, if a \textit{strategic} agency requests a bribe, it sets the highest bribe that the issuer is willing to pay. Therefore, by Lemma 1, the bribe amounts to

\[ B^*_I(\mu_t, \bar{H}) = p(g | \mu_t, \bar{H}). \]

In equilibrium, the bribe is paid whenever it is requested. If the issuer pays to obtain a rating for the security, her payoff equals

\[ U_I^f = (e_t + (1 - e_t) \pi) p(g | \mu_t, \bar{H}) - c(e_t) - \phi_I^M. \]  

(1.2)

The issuer’s choice of effort maximizes the utility defined in (1.2):

\[ e^*_I(\mu_t, \bar{H}) = e((1 - \pi) p(g | \mu_t, \bar{H})), \text{ where } e(\cdot) := e^{-1}(\cdot). \]  

(1.3)

\(^9\)As in equilibrium the security price does not depend on the rating fee, I drop \( \phi_t \) from the argument of \( p(r, \phi_t) \) in the rest of the section.
The issuer’s effort and security price are mutually consistent. Assumption 1 implies that there is at most one unique positive level of effort, and a corresponding price, which satisfy the two equations.

**Lemma 2.** A pair \( e^*_{t}(\mu_t, \bar{\mu}) \), \( p(g|\mu_t, \bar{\mu}) > 0 \) that satisfy (1.1) and (1.3) exists for any \( \bar{\mu} \in [0,1] \) iff

\[
\mu_t(\phi_t) > \mu^M := 1/(1 - \pi) - 1/\mu''(0).
\] (1.4)

These \( e^*_{t}(\mu_t, \bar{\mu}) \) and \( p(g|\mu_t, \bar{\mu}) \) are unique. \( e^*_{t}(\mu_t, \bar{\mu}) = p(g|\mu_t, \bar{\mu}) = 0 \) satisfy (1.1), (1.3) for any \( \mu_t(\phi_t) \in [0,1] \).

Lemma 2 states that the presence of a rating agency with a high reputation is necessary but not sufficient to ensure an issuer’s effort. Nevertheless, the next assumption rules out equilibria in which the issuer and the buyers coordinate on \( e^*_{t}(\mu_t, \bar{\mu}) = p(g|\mu_t, \bar{\mu}) = 0 \).

**Assumption 3.** Whenever the agency’s reputation satisfies (1.4), the issuer and the buyers coordinate on \( e^*_{t}(\mu_t, \bar{\mu}) > 0 \) and \( p(g|\mu_t, \bar{\mu}) > 0 \).

A rating is valuable because it allows a profitable investment in quality. The monopolist agency, regardless of its type, extracts the entire surplus generated by investment in effort with its rating fee.\(^{10}\)

**Lemma 3.** In equilibrium, the rating agency, regardless of its type, requires the highest fee that the issuer is willing to pay:

\[
\phi^M_{t}(\mu_t, \bar{\mu}) = (e^*_{t}(\mu_t, \bar{\mu}) + (1 - e^*_{t}(\mu_t, \bar{\mu}))\pi) p(g|\mu_t, \bar{\mu}) - c(e^*_{t}(\mu_t, \bar{\mu})).
\]

Lemma 3 implies that the rating fee in equilibrium does not reveal any information about the type of the agency and concludes the description of the single-period equilibrium. Before consider-

\(^{10}\)I rule out equilibria in which the agency follows a weakly dominated strategy and requests a fee larger than what the issuer is willing to pay for a rating.
ing the complete two-period model, I illustrate the effect of the expected honesty \( \bar{h} \) on the investment in effort for a quadratic cost of effort.

**QUADRATIC COST EXAMPLE.** Let \( c(e) = e^2 \). Figure 1.2 shows equilibrium price and effort for \( \bar{h} = 0 \) and \( \bar{h} = 1 \).

![Figure 1.2: Effort choices and prices for \( \pi = 1/5 \) and \( \bar{H} = 9/10 \)](image)

The intersections of the two curves correspond to the equilibria of the static game. For larger \( \bar{h} \) the equilibrium with effort is characterized by a more effort and a higher price. The honesty of the agency determines the price that buyers are willing to pay for any expected level of effort. Indirectly, it also determines the equilibrium choice of effort.

I can now characterize the probability that a strategic agency will request a bribe. A strategic agency does not care about its reputation after the last period, and so requests a bribe whenever \( s_2^M = b \). In the first period a strategic agency is faced with a trade-off: to collect a bribe or to assign an unfavorable rating in order to maintain a good reputation. In the last period the reputation determines the expected payoff for a strategic agency. This payoff is
\[ u^*_2(\mu^*_2, h^*_2) = \phi^*_2(\mu^*_2, h^*_2) + Pr\{s_2 = b|e^*_2\} \beta^*_2(\mu^*_2, h^*_2), \text{ where } h^*_2 = 0. \]

If the agency is hired in the first period, its reputation is updated to \( \mu^*_2(r^*_M|h^*_1) \). The updated reputation equals

\[ \mu_2(r^*_M|h^*_1) = \begin{cases} \frac{\mu_1}{\mu_1 + (1-\mu_1)h^*_1} := \mu^b(h^*_1) & \text{if } r^*_M \in \{b, \emptyset\}, \\ \frac{\mu_1 \pi}{\pi + (1-\pi)(1-\mu_1)(1-h^*_1)} := \mu^s(h^*_1) & \text{if } r^*_M = g \text{ and } q_1 = B, \\ \mu_1 & \text{if } r^*_M = g \text{ and } q_1 = G. \end{cases} \]  

Assumption 1 ensures that \( \mu^s(0) > \mu^M \). As \( \mu^s(0) \) is the worst possible reputation, in every period, positive effort can be sustained in equilibrium. Proposition 4 characterizes the equilibria of the entire game.

**Proposition 4.** In equilibrium, the rating agency is hired in every period. In the last period \( h^*_2 = 0 \), while in the initial period \( h^*_1 = 0 \) iff \( \delta < \delta^M \), where

\[ \delta^M := \frac{\beta^M_1(\mu_1, 0)}{u^*_2(\mu^b(0), 0) - u^*_2(\mu^s(0), 0)}. \]

For \( \delta > \delta^M \), \( h^*_1 \) satisfies the implicit function

\[ \delta u^*_2(\mu^b(h^*_1), 0) = \beta^M_1(\mu_1, h^*_1) + \delta u^*_2(\mu^s(h^*_1), 0). \]

For a small discount factor (\( \delta \leq \delta^M \)), reputation motives do not discipline a strategic rating agency. In contrast, for \( \delta > \delta^M \), a strategic rating agency rates honestly with a positive probability.

The following corollary describes how the rating strategy depends on \( \delta \).

---

11If in equilibrium the agency is not hired in the first period, then \( \mu^*_2(r^*_M|h^*_1) = \mu_1 \).
Corollary 1. \( \partial h_1^{M*} / \partial \delta > 0 \) for \( \delta > \overline{\delta}^M \) and \( \lim_{\delta \to \infty} h_1^{M*} = 1 \). Moreover \( \partial \overline{\delta}^M / \partial \mu_1 > 0 \).

For larger values of \( \delta \), the strategic type mimics more closely the honest type. Corollary 1 also states that a strategic agency is more likely to request a bribe in the first period if the agency has a better reputation. I characterize the equilibrium strategies of the issuer and the buyers in case of quadratic cost function.

Quadratic Cost Example. Figure 1.3 shows the probability that a strategic agency does not request a bribe upon observing \( s_1^M = b \). As stated in Corollary 1, for larger values of \( \delta \), a strategic agency more closely mimics the ratings of the honest type. Figure 1.4 shows the equilibrium level of the issuer’s effort as a function of the rating agency’s discount factor. As \( h_1^{M*} \) increases, the corresponding level of effort increases.

Figure 1.3: \( h_1^{M*} \) as a function of \( \delta \), for \( \mu_1 = 9/10 \) and \( \pi = 1/5 \)  
Figure 1.4: \( e_1^* \) as a function of \( \delta \), for \( \mu_1 = 9/10 \) and \( \pi = 1/5 \)
1.3 Competition

In this section, I present the equilibria in a market with two rating agencies that rate simultaneously and non-cooperatively.

1.3.1 The Model

In the competition regime, two rating agencies, denoted $A^I$ and $A^{II}$, are present on the market. The types of the agencies are independent, and every agency is an honest type with probability $\mu_i$. The two agencies act simultaneously and the sequence of actions is identical to the monopoly model. The issuer can hire one agency or both, and the agencies observe two conditionally independent signals, $s^I_t$ and $s^{II}_t$, distributed as the monopoly one. Figure 1.5 describes the timeline. Assumption 1, 2, and 3 hold also under competition, and the payoffs are defined as in the monopoly case.

1.3.2 The Equilibria

I first consider a single-period model in which the strategic agencies rate honestly with probabilities $\Pi := [\Pi^I, \Pi^{II}] \in [0,1]^2$. When two rating agencies are available, the issuer can hire one, two or none. The next lemma ensures that in equilibrium both rating agencies are hired.
Lemma 5. In equilibrium, the issuer hires both rating agencies. Accordingly, two favorable ratings are necessary to obtain a positive price:

\[
p(g,g,\Phi_t|M_t,\overline{H}) = \frac{e^*_i(\Phi_t)}{e^*_i(\Phi_t) + (1 - e^*_i(\Phi_t)) \Pr\{R_t = [g,g] | q_t = B,M_t,\overline{H}\}},
\]

\[
p(R_t,\Phi_t|M_t,\overline{H}) = 0 \text{ if } r_i \in \{0,b\} \text{ for some } i \in \{I,II\}.
\]

Multiple features of my model ensure that the issuer will not hire only a single agency. First of all, an additional rating cannot decrease the price paid for the security because the issuer can hide any unfavorable ratings. Moreover, Assumptions 2 and 3 ensure that two favorable ratings are a stronger signal that \(q_t = G\) than a single favorable rating. Finally, the rating agencies can observe the quality signal at no cost. This extreme assumption captures the low marginal cost of the rating process. Consider an hypothetical equilibrium in which the issuer is expected to exert effort and hire one agency. Agencies would compete to be hired and lower their fees. Fees would be low enough to make it convenient for the issuer to hire a second agency.

As the issuer is expected to hire both agencies, she is indifferent about publishing unfavorable ratings. If an unfavorable rating is published, buyers infer that \(q_t = B\). If less than two ratings are published, the buyers correctly infer that the issuer is hiding one or more unfavorable ratings, and therefore \(q_t = B\). This result contrasts with the models of rating shopping developed by Bolton et al. (2012) and Skreta and Veldkamp (2009)). In these models, hiding bad ratings is profitable because some buyers are nave and do not suspect that only the best ratings are published. To my knowledge, Bouvard and Levy (2012) is the only other model in which a seller can hire more than one rating agency, and buyers are fully rational. In their model all the raters are hired if the cost for rating

\[12\text{As in monopoly, the security price does not depend on the rating fee, so I drop } \Phi_t \text{ from } p(R_t,\Phi_t).\]
agencies to obtain a quality signal is small enough.\footnote{The other models of competition between rating agencies (Strausz (2005), Lo (2010), Camanho and Deb (2012), and Donaldson and Piacentino (2012)) exogenously limit the issuer to hiring only one agency.}

A security price larger than zero requires two favorable ratings. Accordingly, the issuer bribes agency $A_i$ only if the other agency observes a favorable signal ($s_{it}^{-i} = g$), or if the two agencies can be bribed at a total cost not exceeding the value of the favorable ratings ($\beta_{I}^t + \beta_{II}^t \leq p(g, g|M_t, \overline{H})$).

As a result, a strategic agency faces a trade-off. A small bribe is paid even if the issuer needs to bribe the other agency at the same time, while a high bribe is paid only if the other agency observes a favorable signal. The next lemma characterizes the choice of bribes.

**Lemma 6.** Either both agencies request “high” bribes $\beta_{I}^{ts} = \beta_{II}^{ts} = p(g, g|M_t, \overline{H})$ or they both request “low” bribes $\beta_{I}^{ts}, \beta_{II}^{ts}: \beta_{I}^{ts} + \beta_{II}^{ts} = p(g, g|M_t, \overline{H})$.

In equilibrium, the rating agencies coordinate their bribes. A “low” bribe $\beta_i^t < p(g, g|M_t, \overline{H})$ maximizes $A_i$’s expected revenue only if the other agency requests a low bribe $\beta_{i}^{-1} = p(g, g|M_t, \overline{H}) - \beta_i^t$. Moreover, low bribes are the best response only if every agency believes its competitor is strategic with a large enough probability. Rating agencies learn more about each others’ types than buyers and issuer do. Let $\mu_{i}(i)$ denote $A_i$’s belief about the competitor’s type, at the beginning of period $t$.

The next lemma provides a necessary condition for low bribes to be selected in equilibrium.

**Lemma 7.** Bribes $\beta_{I}^{ts}, \beta_{II}^{ts}: \beta_{I}^{ts} + \beta_{II}^{ts} = p(g, g|M_t, \overline{H})$ are selected in equilibrium only if

$$\mu_{i}(i) \leq \frac{1 - 2\pi}{1 - \pi}.$$
\[
U^I = (e^* + (1 - e^*)\pi^2)p(g, g|M_t, H) - c(e^*) - \phi^I_t - \phi^{II}_t. 
\] (1.7)

The choice of effort maximizes the expected payoff of the issuer:

\[
e^*_t(\Phi_t) = e((1 - \pi^2)p(g, g|M_t, H)). \tag{1.8}
\]

As stated in the next lemma, Assumption 1 ensures that an equilibrium with positive effort exists for any \(H\). As both ratings are necessary, I only consider equilibria in which the two rating agencies set identical rating fees.

Assumption 4. I consider only equilibria in which \(\phi^I_t = \phi^{II}_t \forall t\).

With their fees, the rating agencies can extract the entire surplus generated by the issuer’s effort, as stated in the next lemma.

Lemma 8. In the single-period game’s equilibrium, the issuer exerts positive effort. The rating agencies require the highest fee that the issuer is willing to pay:

\[
\phi^I_t = \phi^{II}_t = [(e^* + (1 - e^*)\pi^2)p(g, g|M_t, H) - c(e^*)]/2. \tag{1.9}
\]

In Appendix A, I characterize the equilibrium of the one-period game in which the rating agencies coordinate on “low” bribes. The next lemma states that the low-bribes equilibrium is characterized by less effort than the high-bribes one.

Lemma 9. For any \(H \in [0, 1)^2\), the equilibrium level of effort is lower if rating agencies coordinate on “low” bribes than in the case of high bribes.
Low bribes induce lower effort for two reasons. First, the issuer can simultaneously bribe the two agencies only if the bribes are low. Therefore, when agencies request low bribes, a low-quality security has a larger probability of receiving two favorable ratings than in the case of high bribes. This results in a lower price for a security that receives two favorable ratings than in case of agencies requesting high bribes. A lower expected price, in turn, reduces the issuer’s incentive to invest in quality. Moreover, obtaining favorable ratings when the signals are unfavorable is cheaper under low bribes. By reducing the difference between the payoffs following favorable and unfavorable signals, low bribes reduce even more the incentive to invest in effort.

**QUADRATIC COST EXAMPLE.** Figure 1.6 shows the equilibrium price following two favorable ratings and the issuer’s effort, when agencies coordinate on high and low bribes.

![Figure 1.6: Equilibrium effort choice and market price for π = 1/8 and μ_1^I = μ_2^H = 7/10](image)

When the agencies are expected to request low bribes: (i) the buyers pay a lower price for any expected \( e^* \) and (ii) the issuer exerts less effort for any expected price.

Consider the equilibria of the two-period game. By Lemma 7, low bribes are mutually consistent only if the agencies’ reputations satisfy \( \mu_i^{-1(i)} \leq (1 - 2\pi)/(1 - \pi) \). In general, low bribes ensure
larger payoffs than high bribes if every agency believes that the competitor is strategic with a high probability. The next assumption allows me to focus on equilibria in which agencies coordinate on low bribes if their reputations for honesty are low.

**Assumption 5.** I only consider equilibria in which the rating agencies coordinate on low bribes in period $t$ iff their reputations satisfy $\mu_t^{II} \leq \mu$ for some $\mu \in [0, (1-2\pi)/(1-\pi)]$.

In the rest of the section, I characterize the equilibrium in which the rating agencies request high bribes in both periods. Then, I show that this equilibrium ensures more effort in the first period than any equilibrium which involves low bribes. I start characterizing the equilibrium with high bribes from the rating decisions of a strategic agency. In the last period, a strategic agency requests a bribe whenever possible, while in the first period a strategic agency faces a trade-off between a better reputation and a bribe. The reputations are updated to $\mu_i^2(R_1, q_1)$. The reputation in the second period satisfies

$$
\mu_i^2(R_1, q_1) = \begin{cases} 
\frac{\mu_1}{\mu_1 + (1-\mu_1)h_i^*} := \mu_i^{i,b}(H_i^*) & \text{if } r_1^i = b \text{ and } r_{-i} = g, \\
\frac{\mu_1(\pi + (1-\pi)(1-\mu_1)(1-h_i^*))}{\pi + (1-\pi)(1-\mu_1)\sum_{t=2}^{\infty}(1-h_t^*)} := \mu_i^{i,g}(H_i^*) & \text{if } r_1^i = r_{-i} = g \text{ and } q_1 = B, \\
\mu_1 & \text{otherwise.}
\end{cases}
$$

(1.10)

Because the two agencies rate simultaneously, a competitor’s rating provides information about an agency’s type. For example, the second-period agents interpret a rating $r_1^i = g$ for a security of

---

14 Under competition, agencies learn more about each other than other agents do, but their strategies in the second period only depend on an agency’s reputation among issuer and buyers.
quality $q_1 = B$ differently depending on the rating assigned by the other agency. As the issuer bribes
an agency only if the other agency receives a signal $g$, a favorable rating is interpreted as an honest
mistake if the other agency publishes an unfavorable rating $r_{1-i} = b$. If instead the other agency also
assigns a favorable rating, then the rating $r_1 = g$ could be the result of a mistake or a bribe.

Assumption 1 ensures that the agencies’ reputations are sufficiently high to support effort in
every period. Therefore, in the second period a strategic-agency’s payoff amounts to

$$u^s_2(M_2) := \phi_2^i + (1 - e_2^i)\pi (1 - \pi)\beta_2^i = \left[(e_2^i + (1 - e_2^i)\pi (2 - \pi))p(g,g|M_2,0,0) - c(e_2^i)\right]/2.$$

The strategic-agency’s payoff in the second period depends on the reputations of both agencies. A
monopolistic agency that accepts a bribe in the first period only lowers its own expected revenues.
In competition, a lower reputation for either agency lowers the revenues of both agencies. The next
proposition characterizes the equilibrium in which the rating agencies coordinate on high bribes in
both periods.

**Proposition 10.** If in equilibrium the rating agencies coordinate on high bribes, every issuer hires
both rating agencies. For every $i \in \{I, II\}$, the probability that a strategic agency rates honestly is $h_2^i = 0$ and

$$h_1^i = \begin{cases} 0 & \text{if } \delta \leq \delta^c, \\ h^c(\delta) & \text{otherwise,} \end{cases}$$

where $\delta^c := \frac{p(g,g|M_1,0,0)}{u_2^s(\mu^{s}(0),\mu_{1}) - u_2^s(\mu^{s}(0),\mu_{1}-s(0,0))}$ and $h^c(\delta)$ is defined by the implicit function:

$$\delta = \frac{p(g,g|M_1, (h^c(\delta), h^c(\delta)))}{u_2^s(\mu^{i,b}(h^c(\delta)), \mu_{1}-i) - u_2^s(\mu^{i,s}(h^c(\delta), h^c(\delta)), \mu -i,s(\delta), h^c(\delta))}.$$ (1.11)
In equilibrium the two strategic-type agencies follow the same rating strategy in both periods. Similarly to the monopoly case, for a low discount factor a strategic agency is not disciplined by the threat of losing its reputation, while for larger discount factors, the strategic-type mimics the honest type, as described in the next corollary.

**Corollary 2.** $\partial h_i^*/\partial \delta > 0$ if $\delta \geq \delta^c$, and $\lim_{\delta \to \infty} h_i^* = 1$.

The next lemma is composed of two parts. First, it states that in every equilibrium in which the rating agencies coordinate on low bribes in some period, they will coordinate on low bribes in the first period. Moreover, the strategic agencies coordinate on low bribes only if they strictly prefer to request a bribe. This is the case because an agency that is indifferent between a low bribe and an honest rating should strictly prefer to request a higher bribe. As a result, in any equilibrium that involves low bribes the issuer’s effort in the first period is lower than in the case of high bribes.

**Lemma 11.** If in equilibrium the rating agencies coordinate on low bribes in the second period, they also coordinate on low bribes in the first period. Therefore, in any equilibrium in which agencies coordinate on low bribes, $h_i^L = h_i^H = 0$ and the first-period effort $e_1^*$ is weaker than in the equilibrium in which agencies request high bribes in every period.

I proceed to compare equilibria in the regime of competition among rating agencies and in the monopoly regime.

### 1.4 Comparing Equilibria

In this section, I compare the expected quality of the first-period security under monopoly and competition. The last-period security is not considered, as the last period amounts to a modeling device to account for reputational concerns of the rating agencies.
A different number of rating agencies can result in different amounts of information generated. My model has little to say about the process through which rating agencies collect their information. In fact, I even assume away any cost to obtain a signal correlated with quality. Along this line, I consider different market structures while holding constant the overall amount of information available to the rating agencies. I compare the effort choice of the issuer when a monopolistic agency observes two signals of quality and when each of two competing agencies observes a single signal.

The monopolistic regime and the regime of competition differ in the number of ratings assigned. Proposition 4 and Lemma 5 ensure that in both regime the issuer requests a rating from every available rating agency. The regimes also differ in the reputational concerns of the rating agencies. These two dimensions can be considered in turn. Lemma 12 focuses on the effect of increasing the number of agencies, while abstracting from their reputational concerns. This lemma compares equilibria under monopoly and under competition for an exogenously-given probability that a strategic agency requests a bribe.

**Lemma 12.** Assume that every agency has a probability $\bar{\pi}$ to be an honest type, and each strategic agency rates honestly with probability $\bar{\eta}$. Two competing agencies, observing a single signal each and requesting high bribes, $\beta_i^l = \beta_i^H = p(g, g_i)$, induce more issuer’s effort than a monopolistic agency observing two signals.

Figure 1.7 gives an intuitive explanation. Under competition a strategic agency is less likely to be bribed. In fact, under competition the issuer agrees to pay a bribe to agency $A_i$ only if agency $A^{-i}$ observes a favorable signal, that is, $s_{-i}^r = g$. When both agencies observe an unfavorable signal,
however, their ratings reveal the signals to the buyers, whether or not either agency requested a bribe. In contrast, a bribe is paid whenever it is requested under the monopoly regime.

<table>
<thead>
<tr>
<th>Signals</th>
<th>(b,b)</th>
<th>(b,g) or (g,b)</th>
<th>(g,g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopoly rating</td>
<td>(b)</td>
<td>(g)</td>
<td>(g)</td>
</tr>
<tr>
<td>Competition ratings</td>
<td>(b,b)</td>
<td>(b,g) or (g,b)</td>
<td>(g,g)</td>
</tr>
</tbody>
</table>

Figure 1.7: Ratings for exogenous $\bar{\mu}$ and $\bar{h}$

The next proposition compares monopoly and competition assuming that Assumption 1 holds for $\pi = \bar{\pi}^2$.

**Proposition 13.** Let competing agencies coordinate on high bribes. If $\pi > 1/3$, there is a unique discount factor $\delta^*$ at which a monopolistic agency and two competing agencies both ensure the same effort in the first period. The monopolistic agency ensures a higher effort iff $\delta > \delta^*$.

Proposition 13 is the main result of the paper. It states that monopoly can induce more effort than competition with high bribes if the reputational incentives are strong. It also gives a sufficient condition to ensure that, for a large enough discount factor, the monopolist is indeed more informative than the two competitors. Intuitively, if the signal is often wrong (large $\pi$) the reputation update of the monopolist is larger than the reputation update of the competitors. At the same time, the monopolist’s reputational motives are stronger because the payoff of the monopolist is more dependent on its reputation than the payoffs of competitors.

**QUADRATIC COST EXAMPLE.** Figure 1.8 describes the equilibrium choice of effort for a quadratic
cost function.

![Figure 1.8: Effort in the first period for $\mu_1 = 15/16$ and $\pi = 1/2$](image)

Effort choice under monopoly is lower for low values of $\delta$. In particular if $\delta < \min\{\delta^M, \delta^C\}$, in both regimes a strategic agency requests a bribe whenever possible. In this case, Lemma 12 ensures that the buyers are more informed and the issuer has more incentive to invest in effort under rating agency competition. As the issuer’s choice of effort is a continuous function of the agencies’ discount factor, competition induces more effort than monopoly for any $\delta$ below a threshold.

### 1.5 Extension: Identical Signals

Will an issuer exert more effort when competing agencies observe identical signals of quality or when they observe conditionally independent signals of quality? Should policy makers incentivize standardization in the methods used to evaluate financial products? On the one hand, the presence of a competitor observing the same signal of quality can deter an agency from assigning undeserved ratings. On the other hand, agencies following different procedures might have stronger reputational incentives. In this section, I consider competition among rating agencies which observe the same
signals of quality. I proceed then to compare equilibria in a regime of competition where agencies
observe identical signals, and where agencies observe conditionally independent signals.

Consider the single-period game with exogenous rating strategies $\overline{H}$ and quality signals $s^I_t = s^H_t := s_t$. Lemma 5 does not depend on the correlation between the signals observed by the rating
agencies. As a result, two ratings are necessary to ensure a price larger than zero even if the rating
agencies observe the same signal.

A security may receive two favorable ratings if each of the agencies observes a favorable signal.
Two favorable ratings can also be the result of two bribes. When two agencies observe the same
signal, the issuer either bribes both agencies, or neither of them. The issuer will pay the bribes only
if $\beta^I_t + \beta^H_t \leq p(g, g|M_t, \overline{H})$. I only consider symmetric equilibria, in which agencies request bribes
$\beta^I_t = \beta^H_t = p(g, g|M_t, \overline{H})/2$. The next lemma provides a sufficient condition to ensure that there is
an equilibrium with positive effort.

**Lemma 14.** A pair $p(g, g|M_t, \overline{H}) > 0$ and $e^* > 0$ that satisfies (1.1) and
$$e^*_t(\Phi_t) = e(1 - \pi)p(g, g|M_t, \overline{H})$$
(1.12)
exists for any $\overline{H}$ iff
$$(1 - \mu^I_t)(1 - \mu^H_t) < 1/c''(0) - \pi/(1 - \pi).$$
(1.13)
This pair is unique. $e^*_t = p(g, g|M_t, \overline{H}) = 0$ satisfy (1.1) and (1.12) for any $M_t$.

Lemma 15 compares the equilibria of a single period game under competition with different
signal structures. I hold constant the overall amount of information received by the rating agencies
in the two regimes. This amounts to assuming that under both regimes two signals of quality are
generated.
Lemma 15. In the equilibrium of the single period game, the issuer exerts more effort when agencies receive identical signals than when agencies receive conditionally independent signals if

$$\frac{\sum_i(1 - \mu_i t)(1 - h_i)}{\prod_j(1 - \mu_i)(1 - h_i)} \geq \frac{1 + \pi}{\pi}.$$ 

When agencies observe conditionally independent signals, they either coordinate on high or low bribes, as described in Section 4. If agencies coordinate on low bribes, the issuer has less incentive to exert effort than in the regime of identical signals of quality. Consider the equilibria in which the agencies coordinate on high bribes. Figure 1.9 compares the ratings in the two informational regimes. When the two signals are different, buyers are more likely to observe an unfavorable rating if each agency observes both signals. In contrast, if the two signals are identical and unfavorable, agencies that observe independent signals are more reliable: as the agencies coordinate on high bribes, they cannot be bribed at the same time. As a result, in case of two unfavorable signals, the agencies can be bribed only in the regime of identical signals.

<table>
<thead>
<tr>
<th>Signals</th>
<th>(b,b)</th>
<th>(b,g) or (g,b)</th>
<th>(g,g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratings for identical signals</td>
<td>(b,b)</td>
<td>(g,g)</td>
<td>(b,b)</td>
</tr>
<tr>
<td>Ratings for independent signals and high bribes</td>
<td>(b,b)</td>
<td>(b,g) or (g,b)</td>
<td>(g,g)</td>
</tr>
</tbody>
</table>

Figure 1.9: Ratings for exogenous $\overline{H}$ under different informational regimes

Consider the two-period game. In the last period, the reputation of the rating agencies among
the other agents is updated to $\mu_i^2(R_1, q_1)$. In this case, the second period reputation satisfies

$$
\mu_i^2(R_1, q_1) = \begin{cases}
\frac{\mu_1}{p_r\{R_1=(b,b)\}} & : \mu_i^1(H_1^r) \quad \text{for } r_1^I = r_1^H = b, \\
\frac{\mu_1 \pi}{p_r\{R_1=(g,g)\}} & : \mu_i^1, g(H_1^r) \quad \text{for } r_1^I = r_1^H = g \text{ and } q_1 = B, \\
\mu_1 & \text{otherwise.}
\end{cases}
$$

(1.14)

The payoff of a strategic agency in the second period depends on the reputation of both agencies. The agency’s reputation among the issuer and the buyers determines, respectively, the choice of effort and the willingness to pay for the security. The belief about the type of the competitor determines the expected probability of receiving a bribe. The continuation payoff equals

$$
\phi_i^2 + (1 - e_2)(1 - \pi)(1 - \mu_i^2) \beta_i^2/2.
$$

The next proposition characterizes the equilibrium of the game.

**Proposition 16.** In equilibrium, for every $i \in \{I, II\}$,

$$
\bar{\delta}_i = \begin{cases}
0 & \text{if } \delta \leq \bar{\delta}_i, \\
h_i^1(\delta) & \text{otherwise}.
\end{cases}
$$

$\bar{\delta}_i$ and $h_i^1(\delta)$ are defined, respectively, by:

$$
\bar{\delta}_i = \frac{p(g, g|M_1, 0, 0)}{u_2^i(\mu_i^1, h_i^1(\delta), 0) - u_2^i(\mu_i^1, h_i^1(\delta), 0)},
$$

$$
\delta = \frac{p(g, g|M_1, h_i^1(\delta), 0)}{u_2^i(\mu_i^1, h_i^1(\delta), h_i^1(\delta), 0) - u_2^i(\mu_i^1, h_i^1(\delta), h_i^1(\delta), 0)}.
$$

---

\[\text{Agencies can learn about each others' type more than the other agents do, but in equilibrium the belief about the type of the competing agency will not be determinant for the choice of action of each agency.}\]
As in the regime of conditionally independent signals, there is a threshold discount factor \( \delta^s \).

If and only if \( \delta < \delta^s \), a strategic agency strictly prefers to obtain a bribe in the first period. Note that \( \delta^s \) and \( h_1^s(\delta) \) are defined by the indifference condition of a strategic rating agency. The next corollary is the equivalent of Corollary 2.

**Corollary 3.** If \( \delta > \delta^c \), then \( \partial h_1^s / \partial \delta > 0 \) and \( \lim_{\delta \to \infty} h_1^s = 1, \forall i \).

Proposition 17 compares the equilibrium effort under monopoly and under competition. I consider only the case of issuers endowed with a quadratic cost function.

**Proposition 17.** Let \( c(e_t) = e_t^2 \). If every competing rating agency observes both signals, there is a unique \( \delta^{**} < \delta^c \) for which a monopolistic agency and competing agencies induce the same effort choice in the first period. The issuer exerts more effort in the monopoly regime iff \( \delta > \delta^{**} \).

The competition regime ensures more effort than monopoly for low discount factors. Proposition 17 ensures that this is the case regardless of the signal structure. Figure 1.10 shows a numerical example.

---

**Figure 1.10:** First-period effort under monopoly and competition (identical signals)

The next lemma compares the threshold discount factors obtained by comparing effort under
competition for the two signal structures and effort under monopoly.

**Lemma 18.** Let $c(e_t) = e_t^2$. Then $\delta^{**} > \delta^*$ iff $\mu_1 > \frac{1 - 2\pi}{1 + \pi}$.

Identical-signal competition ensures more effort than monopoly for a larger set of $\delta$ than independent-signal competition for large values of $\mu_1$ and $\pi$. Identical signals ensure that each agency observes more information. If the agencies are likely to be honest, identical signals ensure more informative ratings than independent signals.

### 1.6 Conclusion

My paper represents the first attempt to simultaneously consider rating agencies’ reputational incentives and issuers’ investments in the quality of their securities. I show that competition among agencies can ensure more investment in a security’s quality than monopoly whenever the rating agencies have weak reputational incentives. Reputational incentives can be weak for many reasons. For example, while rating new or complex securities, rating agencies are likely to make many honest mistakes. As a result, inflated ratings will pass unnoticed and will not hurt the agencies’ reputations for rating honestly. If the number of new securities to be rated is very high, the reputational incentives can be weak because many ratings are issued before the returns of the securities are observed. The markets for asset-backed securities in the years preceding the financial crisis are an example of market in which a large volume of new and complex securities were rated in a short amount of time, and rating agencies were likely to lack reputational concerns. Therefore, my model supports the regulators’ claims that increasing rating competition is in the public interest in these markets.

Other certification models with endogenous product quality (Albano and Lizzeri (2001) and
Donaldson and Piacentino (2012) conclude that competition always ensures more investment in security quality than monopoly does. These models differ from mine primarily because their certifiers do not have reputational incentives. Models that do consider reputational incentives, such as Strautz (2005), Lo (2010) and Camanho and Deb (2012), conclude that a monopolistic agency always ensures more information for investors than competing agencies. Unlike my model, these papers consider ratings for products of exogenous quality and limit the issuer to hiring at most a single rating agency.

My analysis could be extended to consider competition between more than two rating agencies. In my setting, the issuer hires all the available rating agencies, regardless of their number. The analysis, however, becomes significantly more complicated when more than two rating agencies are available. This is because strategic agencies can coordinate their bribes in many different ways, and the number of possible equilibria increases quite rapidly.

In my model, the structure of the securities market is exogenously determined. After agencies announce their rating fees, a single issuer decides on her effort. It would be interesting, however, to explicitly model the issuer’s decision to enter the securities market, and therefore consider the effect of the rating fees on the structure of the securities market. In fact, the rating agencies could potentially demand low fees in order to encourage multiple issuers to enter the market, or high fees that result in a monopoly in the securities market. Ultimately, this extension could provide conditions under which external certifiers should be expected to induce a socially desirable market structure for rated products.
Chapter 2

Contingent Payments and Certification Quality

This is a major overhaul of the system ... it is a dramatic change.

- Andrew Cuomo on the Cuomo Plan. June, 2008.¹

This feels cosmetic to me, ... getting paid for just showing up doesn’t strike me as a good model or incentive structure.

- Professor Lawrence White on the Cuomo Plan. June, 2008.¹

2.1 Introduction

In 2011, the United States Senate produced a report on the causes of the recent financial crisis. An entire section of this report was devoted to the role of credit rating agencies, which were faulted for deliberately overlooking factors that induced lower ratings for structured financial products.² The following excerpt is illustrative:

Despite the increasing number of ratings issued each year and record revenues as a result, neither Moody’s nor S&P hired sufficient staff or devoted sufficient resources to ensure that the initial rating process and the subsequent surveillance process produced accurate credit ratings.³

²Coburn Levin Report (2011), Section V. For a complementary account of the role of credit rating agencies in the recent financial crisis, see Benmelech and Dlugosz (2009).
In 2008, Andrew Cuomo, who at the time served as Attorney General of New York, reached an agreement with the three largest rating agencies to address these issues. The agreement was known as the Cuomo Plan. The Plan prohibited rating agencies from receiving compensation based on the issuer agreeing to publish an assigned rating. The Plan imposed a fee-for-service compensation scheme, requiring that issuers pay whenever they requested a rating. My model studies the effect of the policy on the informational content of ratings. The model has the following features:

- an issuer privately observes the quality (high or low) of her security,
- a monopolistic rating agency, decides whether to:
  - observe a signal correlated with the quality of the security, and
  - truthfully report this information.

In this model, the quality signal can be high or low, with the agency having the option to assign a high rating or refusing to rate the product at all. A high rating can be denied only if the agency observes a low signal. If the agency assigns a high rating to a low-quality security, it incurs a reputation cost, which is meant to capture the potential damage from a loss of credibility. Moreover, before the policy is introduced, the rating agency can require an ex ante fee, to be paid to request a rating, and an ex post fee, to be paid only if a high rating is assigned.

The rating agency’s willingness to obtain a signal and to rate honestly will determine the issuer decision to request a rating. This potential effect is described by an analyst in her evaluation of the

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\(^4\)The agreement was signed by Standard & Poor’s, Moody’s and Fitch.

\(^5\)The argument that the value of ratings derives from the rating agency’s desire to maintain a good reputation is commonly accepted but not immune to critiques. Hunt (2008) and Partnoy (2001) propose an alternative explanation, based on the legal value attributed to the rates of Nationally Recognized Statistical Rating Organization (NRSRO) by the financial regulatory system.
consequences of Moody’s procedures:

I am worried that we are not able to give these complicated deals the attention they really deserve, and that they [Credit Suisse, the seller] are taking advantage of the light review...  

In this work, I demonstrate that by prohibiting contingent ex post fees, the Cuomo Plan incentivizes the agency to obtain the quality signal and dissuades it from assigning a high rating upon observing a low signal. Nevertheless, in equilibrium, the rating agency obtains the costly signal less often as a result of the Plan. This is the case because a fee that is independent of the rating obtained reduces the incentive to submit a low-quality security for rating. The reason is straightforward: a low-quality security is less likely to receive a high rating than a high-quality one. The issuer prefers an ex ante to an ex post fee regardless of the quality of the security, but the low-quality issuer finds it particularly convenient to pay the fee ex post.

As the issuer’s reaction to the policy depends on the quality of the security, the expected quality of securities submitted for certification is thus affected by the policy. The average quality of a security submitted for rating increases as less low-quality securities are submitted, and as a consequence the agency has a weaker incentive to collect information. Overall, even if the agency is less informed as a result of the policy, buyers are more informed because a low-quality security is less likely to receive a high rating.

As might be expected, the Cuomo Plan has an impact on the rating process only if the reputational cost is not already sufficient to discipline the agency. When the reputational cost is high,

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6Coburn Levin (2011), page 305.
it is optimal for the agency to require the entire fee ex ante to discourage issuers of low-quality securities from requesting a rating.

Less intuitive is the second characterization of the impact of the policy. In the context of low reputational cost, the effect of the Cuomo Plan depends on the cost of the quality signal:

- If the cost to obtain the signal is so high that in the absence of the policy the agency would not have any incentive to obtain a signal of quality and therefore would just blindly assign a high rate, then the policy has no effect.

- If, on the other hand, there is a low cost of effort, the Plan then reduces the probability of a low-quality security receiving a high rating, as described above.

Based on this assessment, it is clear therefore that while the Cuomo Plan is likely to reduce information asymmetries in markets in which rating agencies have weak reputational incentives, it is less likely to induce rating agencies to spend the resources necessary to evaluate complex financial products. To the extent that complex financial products are also characterized by the largest information asymmetries between issuers and investors, the Plan might be ineffective when informative ratings are most needed.

The remainder of the paper is structured as follows. Section 2 contains a review of the literature. Section 3 introduces the model. Section 4 discusses the equilibria. Section 5 provides a conclusion. All the proofs are contained in the Appendix.
2.2 Review of the Literature

The literature on the Cuomo Plan thus far (Bolton et. al (2012), Kovbasyuk (2010), Bouvard and Levy (2009)) does not consider how the procedures followed by rating agencies to assign their ratings affect the decisions of sellers endowed with products of different quality. Bolton et al. (2012) show that non-contingent fees would dissuade rating agencies from collecting costly information on security quality. This conclusion, however, directly contradicts what my model suggests primarily because different assumptions are being made. In the model proposed by Bolton et al. (2012), buyers punish a rating agency for lying about the signal observed but not for assigning a rating based on a misleading signal. In my model, however, buyers are not able to distinguish the two cases.

According to Kovbasyuk (2011), the Plan can have opposite effects on the quality of ratings. This effect depends on whether or not the contract between issuer and rating agency can be observed by the investors. In my model, contracts are non-observable. Kovbasyuk (2011) shows that in this case the Plan reduces the incentive to assign high ratings to low-quality securities. Bouvard and Levy (2012) consider a rating market in which contingent fees are banned. They show that rating agencies, regardless, prefer to inflate their ratings. Rating inflation is motivated by the desire to attract low-quality issuers.

More generally, recent theoretical research on market credit ratings has taken on a broad focus. The role of reputation for honesty in disciplining rating agencies is considered in Strausz (2005), Mathis et al. (2009), and Frenkel (2010). Mariano (2012) argues that reputational motives do not
necessarily ensure more reliable ratings: rating agencies improve their reputation for expertise by disregarding their private information and assigning ratings based on public information. Fahri et al. (2009), Skreta and Veldkamp (2009), Sangiorgi et al. (2009) focus on credit shopping, that is, the possibility of the issuers to cherry-pick the most favorable ratings. Unsolicited ratings are considered in Fulghieri et al. (2010). Pagano and Volpin (2009) and Fahri et al. (2009) study the transparency of rates and Damiano et al. (2008) considers the role of coordination among raters working for the same credit rating agency. White (2010) and Dranove and Jin (2010) provide comprehensive reviews on the subject.

The theoretical literature on credit ratings is matched by a limited number of empirical studies. A notable example is Ashcraft et al. (2011), who study whether credit ratings affect the market price of rated products. They show that in a sample of residential mortgage-backed securities, issued in the years preceding the financial crisis, ratings did have an influence on prices.

2.3 The Model

The model I have developed is a game with four players: an issuer, a rating agency, and two buyers. The issuer owns a unit of a security of quality \( q \in \{H, L\} \). The security is worth 1 to the buyers if \( q = H \) and \(-1\) if \( q = L \). For the other agents the security has no value. The issuer is privately informed about \( q \). The other agents know that

\[ Pr \{ q = H \} = \alpha \in (0, 1). \]

The issuer cannot credibly communicate the quality of the security to the buyers, but she can request a rating. In order to request a rating, the issuer needs to commit to pay a schedule of non-negative
fees set by the rating agency. The issuer pays an ex ante fee $\phi_I$ to request a rating, and an ex post fee $\phi_R$, only in case a high rating is assigned.

The rating agency can observe a signal $\theta = q$ with probability $e$ at a cost $c(e) = Ce$ where $C \geq 0$. The rating agency can either assign a high rating or no rating. The agency can refuse to issue a high rating only if it receives a private signal $\theta = L$.\footnote{If the agency observes the quality, this is equivalent to a very simple model of quality revelation in which the expert can either reveal $q \in \{L\}$ or $q \in \{L,H\}$.} If the agency assigns a high rating to a security of quality $L$, it incurs a reputation cost $\rho > 0$. I model reputation as an exogenous cost, as in Bolton et al. (2012). The other players know that $Pr\{q = H\} = \alpha \in (0,1)$ and observe a public signal $\psi \in \{L,H\}$ distributed as follows:

$$Pr\{\psi = H|q = H\} = 1 - \beta,$$

$$Pr\{\psi = H|q = L\} = 0.$$

and observe a public signal $\psi \in \{L,H\}$ distributed as follows:

$$Pr\{\psi = H|q = H\} = 1 - \beta,$$

$$Pr\{\psi = H|q = L\} = 0.$$

Hence, the expected value of the security for a signal $\psi$, denoted as $V_\psi$, is:

$$E(q|\psi = H) := V_H = 1,$$

$$E(q|\psi = L) := V_L = \frac{\alpha \beta - (1-\alpha)}{\alpha \beta + (1-\alpha)} < 1.$$

I assume that the security has a non-negative expected value for the buyers even if $\psi = L$.

**Assumption 1.** $V_L > 0$. 
The time-line of the game is depicted in Figure 1.

![Figure 2.1: Timeline](image)

The equilibrium concept is sequential equilibrium. Buyers only observe the public signal and whether the security is certified. Their bids are given as follows:

\[ b_i : \{ \text{cert}, no \text{ cert} \} \times \{ L, H \} \to \mathbb{R}_+ \cup \{0\}, \text{ for } i \in \{1, 2\}. \]

Let \( 1_L = 1 \) iff \( q = L \). If a buyer gets the security, her payoff equals:

\[ U_{Bi}(b_i, q) = 1 - 21_L - b_i, \]

and if he does not get the security, \( U_{Bi}(b_i, q) = 0 \). In equilibrium, buyers bid the expected value of the security, as long as it is non-negative. If \( \psi = H \), the quality of the security is known, and the certificate has no informative content. If instead \( \psi = L \), the expected value of the security might depend on the presence of the certificate. For \( \psi = L \), I define the expected value of the security as \( \bar{v}_c \) or \( \bar{v}_{nc} \), depending on whether the certificate is issued or not, and I denote the corresponding bids as \( b_c \) and \( b_{nc} \), respectively.\(^8\) If \( v_c \) and \( v_{nc} \) are defined in equilibrium, then:\(^9\)

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\(^8\)E.g. \( \bar{v}_c \equiv P\{q = H|cert, \psi = L\} - P\{q = L|cert, \psi = L\} \) and \( b_c \equiv b_i(cert, L) \).

\(^9\)I.e. whenever certification takes place respectively w.p. > 0 or < 1.
The issuer choice to request a certificate depends on the certification fees:

$$r_q : \mathbb{R}_+^2 \to \{0, 1\} \text{ for } q \in \{H, L\}.$$ 

I assume, without loss of generality, that the security is sold to one of the buyers that make the highest bid. The payoff to the issuer is:

$$U_S(r_q, \phi_I, \phi_R, b) = -r_q(\phi_I, \phi_R)(\phi_I + 1_{\text{cert}}\phi_R) + b,$$

where $1_{\text{cert}} = 1$ iff certification takes place, and $b$ equals the highest bid if a bid is made, and $b = 0$ otherwise.

The rating agency’s strategy is given by the fee structure $(\phi_I, \phi_R) \in \mathbb{R}_+^2$, a choice of effort $e : \mathbb{R}_+^2 \to [0, 1]$, and a function $c : \mathbb{R}_+^2 \to [0, 1]$, which defines the probability of certification when $\theta = L$. The agency’s payoff when the issuer demands the certificate is:

$$U_C(\phi_I, \phi_R, e) = \begin{cases} \phi_I - Ce + \phi_R - 1_{\text{L}}, & \text{if the certificate is issued,} \\ \phi_I - Ce, & \text{otherwise.} \end{cases}$$

If the issuer does not request the certificate, her payoff equals 0.

### 2.4 The Equilibria

The issuer demands the certificate only if it ensures higher bids for her security. As a consequence, there exists an equilibrium in which buyers are not willing to bid enough if the certificate is obtained, and therefore the issuer does not request the certificate.
Lemma 19. There exists an equilibrium in which certification does not take place.

In this section, I characterize the parameter values for which this equilibrium is not unique.

The General Case

The equilibria for $C > 0$ are presented here. A preliminary observation is that the rating agency incurs the cost, $e$, of the private signal only if the information is a determinant of the decision to certify.

Lemma 20. For $C > 0$, if the rating agency chooses $e > 0$, the certificate is not assigned upon observing $\theta = L$.

Lemma 2 implies that the informative content of the certificate is determined by the rating agency’s choice to observe $\theta$. The decision depends on $C$ and $\rho$. The rating agency does not get the signal if the cost is too large compared to the loss from losing its reputation.

Proposition 21. If $C > (1 - \alpha)\rho$, the rating agency does not acquire the private signal. An equilibrium with certification exists iff $\rho \leq \beta V_L / (1 - \alpha)$ and in this equilibrium the certificate is not informative as the security is always certified, regardless of its quality.

It might seem surprising that an issuer demands a certificate which provides no information to the buyers, but the certification equilibrium can be sustained by low bids out of the equilibrium path.10

The rest of the section considers the alternative case, $C \leq (1 - \alpha)\rho$.

I characterize the strategies of issuer and certifier by considering the certification game, defined as the game between issuer and certifier, for a given pair of bids.

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10This equilibrium is similar to the no-disclosure equilibrium described in Lizzeri (1999). The certifier does not reveal anything, but all the seller types request a certificate.
Proposition 22. In the certification game, certification takes place only if $\beta (b_c - b_{nc}) \geq C$. In any equilibrium with certification, the certifier announces a fee structure $(\phi_I, \phi_R)$ that satisfies: 

$$\phi_I + \phi_R = \beta (b_c - b_{nc}).$$

The contingent fee satisfies:

- if $\beta (b_c - b_{nc}) < \rho$, $\phi_R = 0$,
- if $\beta (b_c - b_{nc}) > \rho$, $\phi_R \geq \rho - C/\alpha$.

The certifier sets the sum of the two fees to extract the expected gain from certification of the high-quality issuer. The ex post fee determines the participation of the issuer of a low-quality security - $r_L$. For a large ex post fee, i.e., $\phi_R > \rho - C/(1 - \alpha)$, the rating agency does not collect any information ($e = 0$), certifies regardless of the quality; as a consequence $r_L = 1$. For a small ex post fee ($\phi_R \leq \rho - C/(1 - \alpha)$), the decision to get a private signal depends on $r_L$. $r_L$ in turn depends on $e$.

Figure 2.2 shows the effect of a reduction of the ex post fee, on the participation choice of the low-quality issuer and the rating agency’s effort. The reduction in the ex post fee is matched by an identical increase in the ex ante fee. As a result, the total fee $\phi_I + \phi_R$ is unchanged.

A lower $\phi_R$ increases the incentive to get a private signal, but in equilibrium the certifier gets the signal less often. In fact, an unchanged or increased level of effort cannot be part of an equilibrium. A low-quality issuer, expecting a better informed certifier, would not request the certificate. As a result, the signal would not provide any information to the certifier.

With regards to Result 2, for $\beta (b_c - b_{nc}) < \rho$, the expected payoff of the rating agency decreases if the low-quality issuer demands the certificate. Therefore, the certifier tries to reduce the proba-
Figure 2.2: Effect of a reduction of $\phi_R$, for $\phi_R \leq \rho - C/(1 - \alpha)$

probability of this event, demanding the entire fee ex ante: $\phi_R = 0$. Below I refer to this equilibrium as the case of a careful certifier. On the contrary, for $\beta(b_c - b_{nc}) > \rho$, the certifier sets $\phi_R \geq \rho - \frac{C}{1 - \alpha}$ to ensure $r_L = 1$. This case can be called the careless certifier. Finally, for $\beta(b_c - b_{nc}) = \rho$, any $r_L$ and, therefore, any ex post fee ensures the same payoff to the certifier: I refer to this case as an indifferent certifier. The next Proposition describes the equilibria of the game.

**Proposition 23.** In equilibrium, certification takes place only if $C \leq \frac{\beta \rho}{2\beta + \rho}$, and three types of equilibria exist for different values of $\rho$ and $C$:

1. If $\rho > f_2(C)$, the certifier is careful.
2. If $\rho \in [f_1(C), f_2(C)]$, the certifier is indifferent.
3. If $\rho < f_1(C)$, the certifier is careless.\(^{11}\)

Figure 2.3 shows the intervals described in Result 3.

$C > \beta \rho/(2\beta + \rho)$ is an extreme and not very relevant scenario: there exists no equilibrium with certification because the fees collected would not compensate the cost of getting the signal or losing

\(^{11}\)$$f_1(x) \equiv \frac{1}{2} \left( \beta - \frac{x}{1 - \beta} (\frac{a \beta + 1 - \alpha}{\alpha (1 - \alpha)}) + \sqrt{\left( \beta - \frac{x}{1 - \beta} (\frac{a \beta + 1 - \alpha}{\alpha (1 - \alpha)}) \right)^2 + 4 x \beta \left( \frac{\beta}{1 - \alpha} - \frac{1}{\alpha} \right)} \right),$$

$$f_2(x) \equiv \frac{\beta + \sqrt{\beta^2 - 4 \beta x}}{2 - \beta x}.$$
the rating agency’s reputation. If \( C \leq \beta \rho / (2 \beta + \rho) \) instead, there are equilibria with certification, in which the informative content of the certificate depends on the size of \( \rho \) and \( C \).

If the reputation cost is low (area 3 in Fig. 2), in any equilibrium with certification the rating agency is careless. This implies that it is demanding a high share of the fee contingent on certification (\( \phi_R \geq \rho - C / (1 - \alpha) \)), thereby inducing the issuer of a low-quality security to request a certificate. The rating agency obtains the same payoff from certifying the security for any quality or from collecting a signal and certifying only the high-quality security. As a result, there exists a continuum of equilibria characterized by a different informative content of the signal - measured by \( \tilde{w}_c \).
Equilibria in which the certificate is more informative - i.e., $\tilde{v}_c$ is larger - exists for larger $\rho$. In fact, for a low reputation cost, if buyers expect the certifier to provide reliable information, their willingness to pay for a certified security would induce the certifier to certify regardless of the quality.

For intermediate values of $\rho$ (area 2), i.e., $\rho = \beta(b_c - b_{nc})$, any choice of $\phi_R$ ensures the same payoff for the certifier. For higher values of $\rho$, the certifier demands the entire fee before assigning the certificate ($\phi_R = 0$) to minimize the participation of the issuer of a low-quality security. Figure 2.4 below summarizes these observations and shows how $\tilde{v}_c$ depends on the size of $\rho$.

![Figure 2.4: $\tilde{v}_c$ as a function of $\rho$](image)

In any equilibrium $r_H = 1$ and $c = 0$, and, therefore, the ex ante total welfare is equal to:

$$W^1(r_L, e) = \alpha - (1 - \alpha)r_L(1 - e)(1 + \rho) - eC.$$ (2.1)
The ex ante total welfare is the sum of the expected gain or loss of the buyer from obtaining the security, and the reputation, as well as the screening cost for the rating agency. Note that when there are equilibria characterized by different $\tilde{v}_c$ for a given level of $\rho$ (as happens for $\rho \in [C/(1 - \alpha), f_1(C)]$), equilibria with higher $\tilde{v}_c$ ensure a higher total welfare. Higher $\tilde{v}_c$ is, in fact, equivalent to a lower probability that a low-quality security is certified and, therefore, sold.

In equilibrium the marginal cost of effort equals the marginal private benefit of the rating agency, but is smaller than the social benefit, that is:

$$C = (1 - \alpha)(\rho - \phi_R) < (1 - \alpha)(\rho + 1).$$

Therefore, in equilibrium, the rating agency chooses a level of effort lower than the socially optimal one.

**Effect of the Policy**

The result below defines the cap on $\phi_R$ that maximizes the informative content of the certificate.

**Proposition 24.** The optimal cap on the ex post fee is $\overline{\phi_R} = 0$. Introducing a cap has an effect on $\tilde{v}_c$ only if $\rho \leq f_2(C)$ and $C \leq (1 - \alpha)\rho$. In this case, a regulation imposing $\phi_R = \overline{\phi_R} = 0$ increases the informative content of the certificate and ensures $\tilde{v}_c = (\rho - 2C)/\rho$ and $\tilde{v}_{nc} = -1$.

If $C > (1 - \alpha)\rho$, the rating agency has no incentive to get the private signal for any $\phi_R$, and therefore the regulation has no effect. In particular, the equilibrium presented in Result 2 in which the certifier exerts no effort and publishes a certificate that has no informative content does not disappear. Notice that for a high cost $C$ inducing the certifier to obtain the private signal is not necessarily welfare enhancing. As an extreme case, if $C > (1 - \alpha)(\rho + 1)$, it is socially optimal for
the certifier to exert no effort even if $r_L = 1$.

The policy has no effect either if $\rho > f_2(C)$. This should not surprise, as Proposition 23 shows that for large $\rho$ ($\rho \geq f_2(C)$), in any equilibrium with certification, the rating agency chooses $\phi_R = 0$ even in the absence of the regulation, in order to reduce the participation of the low-quality issuer.

If $\rho < f_2(C)$, imposing a cap ensures a $\tilde{v}_c$ higher than in any equilibrium described in the last subsection. The effect of the policy is the result of two components. Eliminating the contingent fee increases the incentive of the rating agency to get the signal, for any given probability of participation of the low-quality issuer. At the same time, it reduces the issuer’s incentive to demand the certificate. The result is a new equilibrium with less effort on the part of the rating agency and a lower probability that the low-quality issuer requests the certificate, exactly as represented in Fig. 2. The two components have opposite effects on $\tilde{v}_c$, but the second component outweighs the first, and therefore the policy results in an overall increase of $\tilde{v}_c$.

The regulation strictly increases social welfare. This can be seen by rewriting (2.1) in terms of $\tilde{v}_c$ as show below:

$$W^1(r_L,e) = \alpha - \frac{\alpha(1-\tilde{v}_c(e,r_L))(1+\rho)}{1+\tilde{v}_c(e,r_L)} - eC. \quad (2.2)$$

The regulation increases welfare as it increases $\tilde{v}_c(e,r_L)$, and therefore increases the expected gain for the buyers and reduces the expected reputation loss while at the same time reducing the cost of effort $eC$.

**Costless Private Signal**
I consider here a rating agency that can find out the quality of the security at no cost: $C = 0$. In this case, the agency always assigns the certificate to a high-quality security, while for a low-quality one the decision depends on the reputation cost and the ex post fee.

**Proposition 25.** When $C = 0$, for $\rho > \beta$ in equilibrium $\bar{v}_c = 1$, while for $\rho \leq \beta$ an interval of $\bar{v}_c \leq 1$ can be sustained in equilibrium: $\bar{v}_c^\rho(x) \equiv$

$$
\bar{v}_c \in \begin{cases} 
\left[\frac{\rho}{\beta}, \bar{v}_c^\rho(\rho)\right] & \text{if } \beta V_L \leq \rho < \beta, \\
\left[V_L, \bar{v}_c^\rho(\rho)\right] & \text{if } \rho < \beta V_L.
\end{cases}
$$

where $\bar{v}_c^\rho(\rho) := \frac{(2\alpha - 1 + (\alpha\beta + 1 - \alpha)x/\beta + }{+\sqrt{[1 - 2\alpha - (\alpha\beta + 1 - \alpha)x/\beta]^2 - 4x(\alpha - (1 - \alpha)/\beta)/2}}.$

A high reputation cost (i.e., $\rho > \beta$) induces the rating agency assign the certificate only if $\theta = H$. For a lower reputation cost, the low-quality security receives the certificate with positive probability. A continuum of $\bar{v}_c$ can be supported, and for a larger $\rho$ more informative equilibria survive, as $\frac{\partial \bar{v}_c^\rho(x)}{\partial x} > 0$.

As in equilibrium $r_H = 1$ and wlg. $e = 1$, the ex ante total welfare is given by:

$$
W^2(r_L,c) = \alpha - (1 - \alpha)(1 + \rho)c r_L.
$$

Where $c$ is the probability that a certificate is issued for $\theta = L$.

**Effect of the Policy**

The lemma below describes the effect of the policy.

**Lemma 26.** If an upper bound $0 \leq \bar{\theta}_R < \rho$ is imposed, in equilibrium $\bar{v}_c = 1$. 
The cap directly affects the incentive to assign the certificate upon observing $\theta = L$: the rating agency will deny a certificate if and only if the ex post fee is smaller than the reputation cost. Therefore, it is enough to impose a cap on the contingent fee that is low enough to ensure that only high-quality securities are certified. The policy has the expected impact on total welfare; it increases total welfare by preventing the sale of the low-quality security, and therefore ensures an ex ante total welfare: $W^2(r_L, c) = \alpha$.

This result holds for any $\rho > 0$ and is in line with the results of Bolton et al. (2012) and Kovbasyuk (2011).

### 2.5 Conclusion

A one-period model with a single rating agency is presented here. The agency is paid by the issuer and has the incentive to certify only securities of high-quality. The rating agency also has the incentive to exert effort to discover the quality of the security only to the extent that it needs to maintain a reputation for honesty among uninformed buyers of the security. I then analyze the effect of a regulation that requires payments from the issuer to the rating agency to be independent of the agency’s choice to issue the certificate. In this model, I allow a more general policy of putting a non-negative cap on the amount of contingent payments. I show that the cap that ensures the highest informative content of the certification, or, in other words, minimizes the probability that a low-quality product receives the certificate, is the lowest possible cap: the complete elimination of contingent payments.

I also show that the policy has no effect when the cost of finding out the quality of the product is
high, compared to the cost of losing reputation \((C > (1 - \alpha)\rho)\). In that case, regardless of the policy, in the unique equilibrium in which certification takes place, the certificate is assigned regardless of the quality of the security and the policy does not reduce rating inflation.

When instead the cost of screening the product is relatively low \((C \leq (1 - \alpha)\rho)\), the policy has an effect when the reputational incentive is also low. The policy increases the agency’s incentive to obtain a precise signal of the security’s quality - for a given probability that the issuer of a low-quality security requests the certificate. The policy also reduces the incentive of a low-quality issuer to request the certificate. In equilibrium, eliminating contingent payments at the same time reduces the probability that a low-quality product is brought to the rating agency and the agency’s effort. Overall, the first effect has a stronger impact on the probability that a low-quality security gets the certificate, and therefore, the policy reduces the inflation of ratings.

There are two dimensions along which the model could be further elaborated to get fruitful insights. First, allowing competition in the market for certification would have the effect of changing the profits of issuer and certifier(s) and, at the same time, affect the incentives of the certifier to exert effort, as well as the incentive for the issuer to demand one or more certificates, depending on the quality of her security. Second, introducing more types of securities could also prove interesting. It could be interesting to consider the issuer’s participation decision, conditional on the quality of her security, and the agency’s choice of screening, when there is a continuum of securities’ types.
Chapter 3

Social Learning Among Heterogeneous Consumers

3.1 Introduction

Information about the popularity of consumer products is more readily available than ever. Lists of best-selling books and most viewed movies have provided consumers with information about the purchasing choices of other consumers for decades. But in 2014, sites like Amazon provide rankings of every product’s sales, and in almost any other market, from cars to college education, data on consumers’ choices are easy to obtain. To the extent that other consumers’ decisions can be observed, these decisions may convey valuable information about a product’s quality. If the sales of a product can be observed by the entire market, how should firms price their products?

We model a market in which a monopolistic firm sells a product of unobservable binary quality to a sequence of consumers with heterogeneous preferences. Our firm can set a new price every time a consumer enters the market. Consumers have limited private information about the product’s quality, but can observe the purchasing decisions of all their predecessors, as well as the history of prices. For theoretical simplicity, we abstract from salient features of the markets described above, and we assume that consumers also observe the preference types of their predecessors. In this setting, each consumer’s action might affect the public belief about the quality of the product.
When a consumer’s action reveals her private information, we say that social learning takes place. Whenever there is sufficient uncertainty about the product’s quality, the firm gains, *ex ante*, from social learning. The price of the product affects the probability of social learning. We provide conditions under which the firm chooses a price higher or lower than the static optimal one in order to increase the probability of social learning.

In a similar setting, Bose *et al.* (2006) and Bose *et al.* (2008) have studied the pricing strategy of a monopolistic firm when consumers have *identical* preferences and only differ in their private information. We extend their analysis to consider a market in which consumers have heterogeneous preferences. While it is clear that consumers are heterogeneous, the need to consider explicitly this heterogeneity should be justified. First of all, we show that the results in Bose *et al.* (2008), discussed below, do not generalize to the case of heterogeneous preferences. Moreover, our setting with heterogeneous consumers provides an example of a model of social learning in which rational herds and informational cascades, two phenomena which have attracted a lot of attention in the recent literature, do not take place. Finally, we provide a framework that can be extended to the case of imperfect competition.

In Bose *et al.* (2008), a monopolist selling a product of unobservable, binary quality always gains *ex ante* from social learning and the probability of social learning is maximized by a price (weakly) higher than the static-profit-maximizing price. In our setting, the monopolist does not necessarily gain from social learning. When consumers believe that the product quality is likely to be high, the monopolist static profits are a concave function of the product’s reputation, and the monopolist prefers to avoid any social learning. Moreover, high prices do not necessarily trigger
social learning. Intermediate prices generate the largest probability of social learning. High prices make most types of consumers unwilling to purchase, regardless of their private signal. Low prices, instead, make many types of consumers willing to buy for any realization of their private signal. Intermediate prices result in the largest set of consumer types that are willing to buy if and only if they observe a private signal correlated with high quality.

Finally, for the set of parameters that we consider, in equilibrium each private signal is inferred from the consumer’s action with positive probability. *Informational cascades*, that is, situations in which consumers disregard their private signal and social learning comes to a complete halt, are typical of models with binary signals and homogeneous consumers, but do not take place in our setting. Our assumptions on the heterogeneity of preferences ensure that the monopolist firm never sets a *pooling* price for which all consumers types, or none at all, buy the product. In equilibrium there is always an interval of types that buy only if they observe a private signal correlated with high quality.

The seminal papers in the literature on social learning, Banerjee (1992) and Bikhchandani et al. (1992), have considered the process of social learning in the context of fixed prices.\footnote{Chamley (2003) provides a comprehensive review of the literature on social learning.} Bose et al. (2006) is the first attempt to consider the dynamic pricing problem of a monopolist in the context of social learning.

The interaction between price dynamics and social learning has been considered before in [?]. The authors consider a financial market in which privately informed traders act sequentially and the asset price adjusts to incorporate all public information available. If the uncertainty is unidimen-
sional, the equilibrium price prevents informational cascades. In case of multiple dimensions of uncertainty instead, there is the possibility of herding.

The results presented here are a preliminary step towards an analysis of the optimal pricing strategies of competing firms that can influence the process of social learning with their prices. Caminal and Vives (1996) have considered the question in a ”large” market in which every consumer type is present and active in each period. Their paper gives an information-based explanation for the focus of firms on market shares and they assume that the history of consumers’ decisions is observable, but the history of prices is not.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 characterizes the equilibria and presents some comparative statics. Section 4 concludes.

3.2 The Model

Our model extends the framework used in Bose et al. (2008). In an infinitely repeated game, a firm sells a product of quality $Q \in \{H, L\}$ to a sequence of short-lived consumers. $Q$ is determined once and for all, at the beginning of the game. The good is equally likely to be of high or low quality:

$$Pr(Q = H) = 1/2.$$ 

While this distribution is common knowledge, the actual quality is not known to anyone, including the firm. At each period $t \in \{1, 2, \ldots\}$, the firm sets a price $p_t$, a new consumer $t$ arrives in the market and observes a private signal $s_t \in \{h, l\}$, correlated with the product quality:

---

2Also in Bose et al. (2008) the firm ignores the quality of its product. This assumption rules out the possibility of using the price as a signal of quality.
\[ Pr(s_t = h|Q = H) = Pr(s_t = l|Q = L) = \alpha \in \left( \frac{1}{2}, 1 \right). \]

The consumer buys at most one unit of the good, and exits the market at the end of the period. Let \( a_t \in \{0, 1\} \) describe the action of the consumer: \( a_1 = 1 \) if the consumer purchases the good, \( a_t = 0 \) otherwise. The payoff of the consumer active in period \( t \) equals

\[ U^C_t(a_t) = a_t(\theta_t 1_H - p_t), \]

where \( 1_H = 1 \) if \( Q = H \), and \( 1_H = 0 \) otherwise and \( \theta_t \sim U[0, 1] \) is the consumer’s valuation of the product. This valuation is public information, and is observed only once the consumer enters the market. In particular, the parameter is observed after the firm sets its price. We assume observable preferences, as in our reference model (Bose et al. (2008)), in order to provide a clean characterization of the new results due to the introduction of consumer heterogeneity.\(^3\)

The firm has no cost to produce the good and its payoff is

\[ U^F(p_t) = \sum_{t=1}^{\infty} \delta^{t-1} u^F(p_t), \text{ where } u^F(p_t) := a_t p_t. \]

The discount factor satisfies \( \delta \in [0, 1) \). As in Bose et al. (2008), it is crucial that the action of each consumer is observed by the consumers who subsequently enter the market. The sequence of actions in period \( t \) is as follows:

1. The firm and consumer \( t \) observe the preference parameters of past consumers, their purchasing decisions, as well as the history of prices. We denote the public history as \( h_t := \{ p_s, \theta_s, a_s \}_{s=1,t} \), with \( h_1 = \emptyset \), and the set of all possible histories as \( H_t \).

\(^3\)As discussed in the conclusion, we left the model with unobservable preferences for future work.
2. The firm sets a price \( p_t : H_t \rightarrow \mathbb{R}^+ \).

3. Consumer \( t \) observes the private signal \( s_t \) and takes an action \( a_t \in \{0, 1\} \).

Let \( \lambda_t \) denote the \textit{public} belief about the probability that the quality is \( H \) at the beginning of period \( t \), that is \( \lambda_t = Pr(Q = G|h_t) \), where \( \lambda_1 \) is exogenous. We denote with \( \phi(\lambda_t) \) the probability that signal \( s_t = h \) is observed:

\[
\phi(\lambda_t) := Pr(s_t = H|\lambda_t) = \lambda_t \alpha + (1 - \lambda_t)(1 - \alpha).
\]

The consumer’s \textit{private} belief about quality is a function of the private signal and the public belief, and is denoted as follows:

\[
\lambda_t^+ := Pr(Q = H|\lambda_t, s_t = h) = \frac{\alpha \lambda_t}{\phi(\lambda_t)}, \\
\lambda_t^- := Pr(Q = H|\lambda_t, s_t = l) = \frac{(1 - \alpha) \lambda_t}{1 - \phi(\lambda_t)}.
\]

We begin the next section with the PBE of a static setting. We then proceed to characterize the equilibrium in a two period setting, and compare it with the case of homogeneous preferences. We conclude with a few numerical examples of equilibria in the infinite period game.

### 3.3 The Equilibrium

We assume that, when indifferent, a consumer buys the product. Therefore consumer \( t \) purchases if and only if

\[
p_t \leq \theta_t E(Q|\lambda_t, s_t). \tag{3.1}
\]
We first characterize the equilibrium in the single period setting.

### 3.3.1 Single Period Game

The next lemma characterizes the monopolist pricing strategy, and the corresponding probability that the product is sold, in the single period game.⁴ ⁵

**Lemma 27.** If \( \lambda < \frac{3\alpha-2}{2\alpha-1} \) (condition 1), the monopolist sets \( p(\lambda) = p^*(\lambda) := \frac{\lambda^+}{2} \) and the product is purchased only if the consumer’s preference parameter satisfies \( \theta \geq 1/2 \) and the consumer observes a private signal \( s = h \). If condition 1 is not satisfied, then \( p(\lambda) = p^{**}(\lambda) := \frac{\lambda^+\lambda^-}{2(\lambda^++\phi(\lambda)\lambda^-\lambda^+)} \). At this price, the consumer purchases regardless of her private signal iff \( \theta \geq \theta^L \), and purchases only if \( s = H \) in case \( \theta \in [\theta^H, \theta^L) \), where

\[
\theta^H := \frac{p^{**}(\lambda)}{\lambda^+}, \quad \text{and} \quad \theta^L := \frac{p^{**}(\lambda)}{\lambda^-}.
\]

The monopolist either sets a high price \( p^*(\lambda) \), and sells only in case the consumer has a high valuation and she observes a favorable signal, or it sets a lower price \( p^{**}(\lambda) < p^*(\lambda) \). For this price, a consumer with high valuation buys regardless of her signal, and a consumer with intermediate valuation buys only if \( s = h \).

Depending on the precision of the private signal, the monopolist adopts one of two alternative pricing regimes. If \( \alpha < 2/3 \) (we will refer to this case as the imprecise signal regime), the private signal is not very informative and therefore the consumer’s willingness to pay does not vary much with her signal. In this setting, the monopolist sets \( p(\lambda) = p^{**}(\lambda) \) for any public belief \( \lambda \in [0, 1] \).

Figure 3.1a shows the monopolist’s payoff from setting \( p^*(\lambda) \) and \( p^{**}(\lambda) \), while Figure 3.2a shows how the optimal price - \( p^{**}(\lambda) \) - depends on the public belief.

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⁴ In the single period game, we drop the time subscript.

⁵ Lemma 27 resembles the first scenario described in Bose et al. (2008), section 4.2.
If instead $\alpha > 2/3$ (the precise signal regime), the consumer’s willingness to pay varies considerably with her private signal. As a result, for a low public belief, the firm prefers to set $p(\lambda) = p^*(\lambda)$ and sell only if the consumer observes $s = H$, while for a high public belief, the firm chooses $p^{**}(\lambda)$. Figures 3.1b and 3.2b describe, respectively, the value function and the optimal price. In the precise signal regime, the price does not increase monotonically with the public belief $\lambda$.

![Figure 3.1: Static value function as a function of public belief](image1)

(a) Imprecise signal, $\alpha = 3/5$

(b) Precise signal, $\alpha = 3/4$

Figure 3.1: Static value function as a function of public belief

![Figure 3.2: Static price as a function of public belief](image2)

(a) Imprecise signal, $\alpha = 3/5$

(b) Precise signal, $\alpha = 3/4$

Figure 3.2: Static price as a function of public belief

In the next subsection, we take the first step towards characterizing the dynamic game: we describe the process of social learning.
3.3.2 The Process of Social Learning

In the dynamic version of the game, the public belief about the product’s quality evolves as a result of the observed consumers’ decisions, their preferences and the prices. The public belief is updated as follows

If \( \lambda_t^- < p_t \leq \lambda_t^+ \):

\[
\lambda_{t+1} = \begin{cases} 
\lambda_t^+ & \text{if } a_t = 1, \text{ and } \theta_t \in \left[ \frac{p_t}{\lambda_t^-}, 1 \right], \\
\lambda_t^- & \text{if } a_t = 0, \text{ and } \theta_t \in \left[ \frac{p_t}{\lambda_t^+}, 1 \right], \\
\lambda_t & \text{otherwise,}
\end{cases}
\]

If \( p_t \leq \lambda_t^- \):

\[
\lambda_{t+1} = \begin{cases} 
\lambda_t^+ & \text{if } a_t = 1, \text{ and } \theta_t \in \left[ \frac{p_t}{\lambda_t^-}, \frac{p_t}{\lambda_t^+} \right], \\
\lambda_t^- & \text{if } a_t = 0, \text{ and } \theta_t \in \left[ \frac{p_t}{\lambda_t^-}, \frac{p_t}{\lambda_t^+} \right], \\
\lambda_t & \text{otherwise.}
\end{cases}
\]

Although we have not yet characterized the equilibrium in the two period setting, it is immediately clear that, in equilibrium, the consumer buys if and only if condition 3.1 is satisfied. We make the innocuous assumption that an out-of-equilibrium action does not have any informative content.

As the consumer’s preference parameter is observable, the learning process is straightforward. If the consumer is a type that purchases if and only if she observes \( s_t = h \), then the private signal can be inferred from her action. Otherwise, her action has no informative content. Therefore, the probability that the market can infer the private signal corresponds to the probability that the consumer makes her decision contingent on the signal observed. Figure 3.3 shows how the probability to infer the private signal depends on the price set by the firm.

If \( p_t < \lambda_t^- \) learning takes place if and only if \( \theta_t \in \left[ \frac{p_t}{\lambda_t^-}, \frac{p_t}{\lambda_t^+} \right] \) and therefore a higher price results in a higher probability that the signal can be inferred. If \( \lambda_t^- < p_t \leq \lambda_t^+ \), the private signal can be inferred if and only if \( \theta_t \in \left[ \frac{p_t}{\lambda_t^+}, 1 \right] \). A higher price results in a lower probability that the private
signal is inferred.

In the next subsection, we characterize the equilibrium pricing strategy in the two period game.

### 3.3.3 Two Period Game

In the two period setting, we do not limit $\delta$ to be smaller than 1. In the second period, the monopolist faces the same incentives as in the static model described in subsection 3.1, and therefore follows the pricing strategy described in Lemma 27.

Let $v_2(\lambda_2)$ denote the monopolist’s expected payoff in the second period as a function of the public belief, that is $v_2(\lambda_2^i) := E(u^F(p(\lambda_2))|\lambda_2 = \lambda_2^i)$, where $p(\lambda)$ is defined in Lemma 27. Moreover, let $Ev_2(\lambda_1)$ be the expected value of $v_2(\lambda_2)$ if the firm and the second-period consumer can infer the private signal observed in the first period

$$Ev_2(\lambda_1) := \phi(\lambda_1)v_2(\lambda_1^+) + (1 - \phi(\lambda_1))v_2(\lambda_1^-).$$
Whenever $E v_2(\lambda_1) > v_2(\lambda_1)$, the monopolistic firm has an incentive to enhance social learning. The next lemma characterizes how the incentive to generate social learning depends on the public belief.

**Lemma 28.** There exists two thresholds, $\bar{\lambda} := \max \{ \frac{3\alpha - 2}{4\alpha - 2}, 0 \}$, and $\bar{\lambda} \in (\bar{\lambda}, 1)$, such that:

1. If $\lambda_1 \leq \bar{\lambda}$ or $\lambda_1 = \bar{\lambda}$, the firm is indifferent, ex ante, to social learning: $E v_2(\lambda_1) = v_2(\lambda_1)$,

2. If $\lambda_1 \in (\bar{\lambda}, \bar{\lambda})$, the firm (strictly) prefers, ex ante, if social learning takes place: $E v_2(\lambda_1) > v_2(\lambda_1)$,

3. If $\lambda_1 \in (\bar{\lambda}, 1)$, the firm (strictly) prefers, ex ante, if social learning does not take place: $E v_2(\lambda_1) < v_2(\lambda_1)$.

The interval $[0, \bar{\lambda}]$ is not empty if and only if $\alpha > 2/3$. In this case, for a low initial reputation, the firm anticipates to charge $p^*(\lambda_2)$ in the second period, whether or not social learning takes place.

For this pricing strategy, the revenues are a linear function of the public belief, and therefore the firm is locally risk neutral. If instead the firm expects to charge $p^{**}(\lambda_2)$ then for low values of $\lambda_2$ the firm revenues are a convex function of the public belief, while for high enough $\lambda_1$, the revenues are concave.

In the first period, the monopolist follows one of two pricing strategies, depending on the precision of the private signal. The next subsections describe these strategies.

**Case I. Imprecise signal regime.**

We first characterize the equilibrium for $\alpha < 2/3$. In the first period, the monopolist chooses the price that maximizes the discounted sum of current and future payoffs. The next lemma defines the pricing strategy in the first period.
Lemma 29. Let $\alpha < 2/3$. Define $p_1^{*\ast}(\lambda)$ as:

$$p_1^{*\ast}(\lambda) := p^{*\ast}(\lambda)\left(1 + \delta\left(\frac{1}{\lambda^-} - \frac{1}{\lambda^+}\right)(E v_2^{\ast\ast}(\lambda) - v_2^{\ast\ast}(\lambda))\right)$$ (3.2)

where $v^{**}(\lambda') := E(uF(p^{**}(\lambda))|\lambda = \lambda')$. In the first period, the monopolist charges

$$p_1(\lambda_1) := \begin{cases} 
p_1^{**}(\lambda_1) & \text{if } p_1^{**}(\lambda_1) \in [0, \lambda_1^-] \\
0 & \text{if } p_1^{**}(\lambda_1) < 0 \\
\lambda_1^- & \text{if } p_1^{**}(\lambda_1) > \lambda_1^- 
\end{cases}$$

$\alpha < 2/3$ implies $\bar{\lambda} = 0$, and therefore for any public belief $\lambda_1 < \bar{\lambda}$, the monopolist gains - in expectation - if social learning takes place. If instead $\lambda_1 > \bar{\lambda}$, the monopolist is better off if less social learning takes place. Figure 3.4 shows the optimal price in the first period, as well as the static optimal price. For $\lambda_1 < \bar{\lambda}$, in the first period, the monopolist sets a price larger that the static optimal price: $p_1(\lambda_1) > p^{**}(\lambda)$, while for $\lambda_1 > \bar{\lambda}$, the price in the first period is smaller than the static optimal price $p_1(\lambda_1) < p^{**}(\lambda_1)$. In case of $\delta$ large enough, Figure 3.4 shows that for low values of the public belief $\lambda_1$ the monopolist charges the price that ensures the largest possible probability of social learning: $p_1 = \lambda_1^-$. 
Figure 3.4: Pricing strategy in the first period - imprecise signal (\(\alpha = .65, \delta = 100\))

Case II. Precise signal.

We characterize here the equilibrium for \(\alpha > 2/3\). In the second period the monopolist sets \(p_2(\lambda_2) = p^*(\lambda_2)\) if \(\lambda_2 < (3\alpha - 2)/(2\alpha - 1)\), and \(p_2(\lambda_2) = p^{**}(\lambda_2)\) otherwise. The next lemma characterizes the choice of price in the first period.

Lemma 30. Let \(\alpha > 2/3\), then, in the first period, the price satisfies:

1. If \(\lambda_1 \leq \frac{3\alpha - 2}{4\alpha - 2}\), then \(p_1(\lambda_1) = p^*(\lambda_1)\).

2. If \(\lambda_1 \in \left(\frac{3\alpha - 2}{4\alpha - 2}, \frac{3\alpha - 2}{2\alpha - 1}\right]\), then

\[
p_1(\lambda_1) := \begin{cases} 
p_1^*(\lambda_1) & \text{if } p_1^*(\lambda_1) \in [\lambda_1^-, \lambda_1^+] \\
\lambda_1^- & \text{if } p_1^*(\lambda_1) < \lambda_1^- \\
\lambda_1^+ & \text{if } p_1^*(\lambda_1) > \lambda_1^+ 
\end{cases}
\]

Where \(p_1^*(\lambda_1) := p^*(\lambda_1) + \frac{\delta}{2\phi(\lambda_1)}(v_2(\lambda_1) - Ev_2(\lambda_1))\).

3. If, instead, \(\lambda_1 > \frac{3\alpha - 2}{2\alpha - 1}\), then
\[
p_1(\lambda_1) := \begin{cases} 
  p_1^{**}(\lambda_1) & \text{if } p_1^{**}(\lambda_1) \in [0, \lambda_1^-] \\
  0 & \text{if } p_1^{**}(\lambda_1) < 0 \\
  \lambda_1^- & \text{if } p_1^{**}(\lambda_1) > \lambda_1^-
\end{cases}
\]

Moreover if \(\frac{3\alpha-2}{2\alpha-1} < \lambda_1 < \frac{\bar{\alpha}}{\alpha-2}\), then \(p_1(\lambda_1) > p^{**}(\lambda_1)\), while for \(\lambda_1 > \bar{\lambda}\) then \(p_1(\lambda_1) < p^{**}(\lambda_1)\).

Figure 3.5 represents the optimal price in the first period, as well as the optimal static price. For low values of the public belief, the monopolist is indifferent to the possibility of generating social learning. Therefore, in the first period, the price equals the static optimal price. For intermediate values of \(\lambda\) - that is \(\lambda \in \left[\frac{3\alpha-2}{2\alpha-1}, \bar{\lambda}\right]\) - the monopolist strictly prefers to generate social learning. To generate social learning, it sets a price \(p_1\) that is lower than the static optimal price for low values of the public belief \((\lambda_1 \in \left[\frac{3\alpha-2}{2\alpha-1}, \frac{3\alpha-2}{4\alpha-2}\right])\) and a price higher than the static optimal price for larger values of the public belief \((\lambda_1 \in \left[\frac{3\alpha-2}{4\alpha-2}, \bar{\lambda}\right])\). For even larger values of the prior belief, the monopolist prefers if social learning does not take place, and therefore sets a price lower than the static optimal one.

The pricing strategy of the monopolist can be explained with the relation between price and probability of social learning. The probability of social learning increases as the price moves closer to \(\lambda^-\). Whenever the monopolist prefers a larger probability of social learning, it sets a price higher than the static optimal price \(p(\lambda)\), if the latter is smaller than \(\lambda^-\), but it sets a price lower that the static optimal price if \(p(\lambda) > \lambda^-\).

In the next subsection, we characterize the pricing strategy in a market in which consumers have identical preferences.
3.3.4 Homogeneous Preferences

In this section we assume $\theta_t = 1$ in every period. The next lemma characterizes the equilibrium price in the single period setting.$^6$

\textbf{Lemma 31.} Let $\theta = 1$. In the single period setting, if $\lambda < \frac{\alpha + \alpha^2 - 1}{2\alpha^2 - \alpha}$ (condition 2), the monopolist sets $p(\lambda) = \lambda^+$ and the product is purchased only if the consumer observes $s = H$. If condition 2 is not satisfied, then $p(\lambda) = \lambda^-$. At this price, the consumer buys regardless of the signal.

As in the case of heterogeneous preferences, there are two potential optimal prices. Price $p(\lambda) = \lambda^-$ can be called the \textit{pooling} price: at this price the consumer buys regardless of the private signals.

Price $p(\lambda) = \lambda^+$ is instead a \textit{separating} price: the consumers buys only if she observes signal $s = h$.

If the signal is imprecise ($\alpha < \frac{\sqrt{5} - 1}{2}$), the \textit{pooling} price $p(\lambda) = \lambda^-$ maximizes the revenues. For a precise signal, the \textit{separating} price is optimal for low priors that satisfy condition 2. If the reputation does not satisfy condition 2, then the firm chooses the \textit{pooling} price.

$^6$Lemma 31 is a two-period version to the first scenario described in Bose et al. (2008), section 4.2.
We consider now the two period setting. The social learning process is straightforward: if \( p_1 \in [0, \lambda_i^-] \), then in the first period the consumer buys regardless of the signal observed and her private information cannot be inferred. If instead \( p_1 \in (\lambda_i^-, \lambda_i^+) \) the consumer buys only if the signal is \( s_1 = h \), and therefore her signal can be perfectly inferred. As discussed in the introduction, in case of homogeneous preferences, a higher price generates more information for future consumers.

In the second period, the firm adopts the optimal static pricing strategy. The next two lemmas characterize the optimal pricing strategy in the first period, for different precisions of the private signal.

**Lemma 32.** Let \( \theta = 1 \) and \( \alpha \leq \frac{\sqrt{5} - 1}{2} \). Then in the first of two periods the firm sets:

\[
p_1(\lambda) = \begin{cases} 
\lambda^- & \text{if } \delta < \ddelta(\alpha, \lambda) \\
\lambda^+ & \text{if } \delta \geq \ddelta(\alpha, \lambda)
\end{cases}
\]

Where \( \ddelta(\alpha, \lambda) := \frac{(\alpha^2 + \lambda - 2a\lambda)(1-a(1+\lambda)+a^2(-1+2\lambda))}{((-1+2\alpha)^3(-1+\lambda)^2\lambda)} \).

If the signal is uninformative, the firm anticipates setting the pooling price in the second period. Whenever the firm sets the pooling price, its payoff is a convex function of the public belief. Therefore, the firm gains, in expectation, if social learning takes place. As a result, for a discount factor large enough, the firm will set the separating price. The next lemma characterizes the pricing strategy for the case of informative signal.

**Lemma 33.** Let \( \theta = 1 \) and \( \alpha \geq \frac{\sqrt{5} - 1}{2} \). Then in the first of two periods the firm sets:

\[
\begin{itemize}
  \item \( p_1(\lambda) = \lambda_i^+ \) if \( \lambda_i \leq \frac{\alpha + \alpha^2 - 1}{2\alpha^2 - \alpha} \)
  \item otherwise: \( p_1(\lambda_i) = \begin{cases} 
\lambda_i^- & \text{if } \delta < \ddelta(\alpha, \lambda_i) \\
\lambda_i^+ & \text{if } \delta \geq \ddelta(\alpha, \lambda_i)
\end{cases}
\end{itemize}
\]
Where \( \tilde{\delta}(\alpha, \lambda) := \frac{-1 + \alpha(1 + \alpha + \lambda - 2\alpha\lambda)}{(2\alpha - 1)(\lambda - 1)} \).

Similarly to the case of heterogeneous consumers, for informative private signals and low reputation the monopolist is indifferent to social learning and therefore sets the static optimal price. For higher values of the public belief, the firm sets the *separating* price if the discount factor is large enough.

In the setting with homogeneous consumers the firm is either indifferent to social learning or strictly prefers if social learning takes place, and social learning takes place if the firm sets the high *separating* price. It should be noted that in the infinite game considered in Bose et al. (2008) the firm strictly prefers if social learning takes place for any value of the parameters.

In the next subsection we discuss a few numerical examples of the equilibrium in the infinite horizon setting.

### 3.3.5 Infinite Horizon Game

To characterize the pricing strategy in the infinite period setting, we define the value function \( V(\lambda_t) \) as the discounted sum of present and expected future payoffs: \( V(\lambda_t) = E(\sum_{i=t}^{\infty} u^F(p_e^i)) \), where \( p_e^i \) is the equilibrium price in period \( t \). In each period, the monopolist chooses a price that belongs to one of two intervals: either \( p_t \in [\lambda_t^-, \lambda_t^+] \) or \( p_t \in [0, \lambda_t^-] \). We define a different value function for each case.

**Case 1** \((\lambda_t^- < p \leq \lambda_t^+)\). Let the value function for this case be denoted by \( V^+(\lambda_t) \).

\[
V_1(\lambda_t) = \max_{\lambda^- \leq p \leq \lambda^+} \{ \phi(\lambda_t)p(1 - \frac{p}{\lambda_t^+}) + \delta(\frac{p}{\lambda_t^+})(V(\lambda_t) - EV(\lambda_t)) \}
\]
Where $EV(\lambda) := \phi(\lambda)V(\lambda^+) + (1 - \phi(\lambda))V(\lambda^-)$. The optimal price in this case is

$$p_2(\lambda_t) := \begin{cases} 
\bar{p}^*(\lambda_t) & \text{if } \bar{p}^*(\lambda_t) \in [\lambda^-_t, \lambda^+_t] \\
\lambda^-_t & \text{if } \bar{p}^*(\lambda_t) < \lambda^-_t \\
\lambda^+_t & \text{if } \bar{p}^*(\lambda_t) > \lambda^+_t
\end{cases}$$

where:

$$\bar{p}^*(\lambda_t) = p^*(\lambda_t) + \frac{\delta \phi(\lambda_t)}{2}(V(\lambda_t) - EV(\lambda_t))$$

Hence, if $V(\lambda_t)$ is convex in the interval $[\lambda^-_t, \lambda^+_t]$, then $\lambda^-_t \leq \bar{p}^*(\lambda_t) \leq p^*(\lambda_t)$. And if $V(\lambda_t)$ is concave, $p^*(\lambda_t) \leq \bar{p}^*(\lambda_t) \leq \lambda^+_t$.

**Case 2 ($p \leq \lambda^-_t$).** Let the value function for this case be denoted by $V^{**}(\lambda_t)$.

$$V_2(\lambda_t) = \max_{0 \leq p \leq \lambda^-_t} \left\{ p(1 - \frac{1}{\lambda^-_t}) + \phi(\lambda_t)(\frac{1}{\lambda^-_t} - \frac{1}{\lambda^+_t})p) + \delta(\frac{p}{\lambda^-_t} - \frac{p}{\lambda^+_t})(EV(\lambda_t) - V(\lambda_t)) \right\}$$

The optimal price in this case is

$$p_2(\lambda_t) := \begin{cases} 
\bar{p}^{**}(\lambda_t) & \text{if } \bar{p}^{**}(\lambda_t) \in [0, \lambda^-_t] \\
\lambda^-_t & \text{if } \bar{p}^{**}(\lambda_t) > \lambda^-_t \\
0 & \text{if } \bar{p}^{**}(\lambda_t) < 0
\end{cases}$$

where:

$$\bar{p}^{**}(\lambda_t) = p^{**}(\lambda_t)(1 + \frac{\delta}{2}(\frac{1}{\lambda^-_t} - \frac{1}{\lambda^+_t}))(EV(\lambda_t) - V(\lambda_t))$$

Hence, if $V(\lambda_t)$ is concave in the interval $[\lambda^-_t, \lambda^+_t]$, $\bar{p}^{**}(\lambda_t) \leq p^{**}(\lambda_t)$. And if $V(\lambda_t)$ is convex, $p^{**}(\lambda_t) \leq \bar{p}^{**}(\lambda_t) < \lambda^-_t$. 
Given these two cases, now we can get the general value function:

\[ V(\lambda_t) = \max \{ V_1(\lambda_t), V_2(\lambda_t) \} \]

If \( V_1(\lambda_t) > V_2(\lambda_t) \), the monopolist charges \( p_1(\lambda_t) \). If \( V_1(\lambda_t) < V_2(\lambda_t) \), the monopolist charges \( p_2(\lambda_t) \). And if \( V_1(\lambda_t) = V_2(\lambda_t) \), the firm is indifferent between the two prices. We provide a few numerical examples of value functions.

Value Function Iteration

![Value Function Iteration Graphs](image)

(a) \( \delta = 0.2, \alpha = 0.6 \)  
(b) \( \delta = 0.2, \alpha = 0.8 \)  
(c) \( \delta = 0.8, \alpha = 0.6 \)  
(d) \( \delta = 0.8, \alpha = 0.8 \)

**Figure 3.6: Iterated Value Functions**

We consider four different sets of parameter values and in each of the cases compare the iterated
values of $V_1$ and $V_2$. The value function is the maximum of $V_1$ and $V_2$ for each value of $\lambda$. These cases are shown in Figure 3.6.

When the private signals are imprecise ($\alpha = 0.6$ in our examples), it is profitable to charge $\bar{p}^{**}$ for a large interval of values of $\lambda$. This is the case because the price $\bar{p}^*$ ensures lower current revenues than $p^*$ but higher expected future revenues (as the future consumers learn faster about the quality of the product). Imprecise private signals, however, limit the scope of social learning.

As the monopolist becomes more patient ($\delta$ is higher), the difference between $V_1(\lambda_t)$ and $V_2(\lambda_t)$ becomes smaller for intermediate value of $\lambda_t$. A high value of $\delta$ means a higher weight on future payoffs. So the potential optimal prices $\bar{p}^*$ and $\bar{p}^{**}$ move closer to $\lambda_t^-$, which is the price that ensures the largest probability of social learning.

### 3.4 Conclusion

We consider the optimal pricing strategy of a monopolist that faces consumers with heterogeneous preferences. Consumers can observe the purchasing decisions and the preferences of previous consumers, as well as the prices posted in the past by the firm. Our paper belongs to a subset of the literature on social learning that considers how firms strategically adjust the price of their product to influence the process of social learning.

We fully characterize the equilibrium in a two period setting and we present numerical examples for the infinite period setting. Heterogeneous preferences ensure that information cascades do not take place and the private signal observed by a consumer is inferred in equilibrium with a positive probability. Our equilibrium differs from the case of homogeneous consumers in two important
respects. First, the monopolist does not always gain from social learning. If consumers believe the product to be of high quality with a large enough probability, the monopolist loses in expectation from generating social learning. Moreover, higher prices do not correspond necessarily to a larger probability of social learning. In our setting, intermediate prices generate the largest probability of social learning. The firm might choose a price higher or lower than the static optimal price, depending on the initial public belief and the quality of the private signal observed by each consumer.

The next step in our research is to fully characterize the infinite period model. The infinite period setting would allow an easier comparison with the previous literature. We are also interested in describing the process of social learning in a setting where the preferences of previous consumers cannot be observed. Observable preference types simplify the process of social learning, while at the same time keeping the crucial feature due to the heterogeneity in preferences: the process of social learning depends on the size of the interval of types that decide their purchase based on the signal observed. At the same time, however, the assumption of observable preference types seems quite unrealistic, especially in the infinite period setting.

On a longer horizon, we plan to use our framework to characterize the equilibrium in a market with social learning and competition between two firms. Our setting with heterogeneous preferences could be easily adapted to consider a setting of competition between firms selling horizontally differentiated products.
Appendix

Appendix to Chapter 1

A) Low-Bribes Equilibrium

I first consider the single-period game for a given $\bar{H}$. I focus on equilibria in which the rating agencies request identical bribes $\beta^I_t, \beta^II_t : \beta^I_t + \beta^II_t = p(g, g|M_t, \bar{H})$. The price of the security is defined by Lemma 5. If the issuer hires the two agencies, she ensures an expected payoff equal to

$$U^I = (e^*_t + (1 - e^*_t)(\pi^2 + (1/2)\pi(1 - \pi)\Sigma_i(1 - \mu^i_t)(1 - \bar{H}_t))p(g, g|M_t, \bar{H}) - c(e^*_t) - \phi^I_t - \phi^{II}_t.$$  

(3.3)

The optimal effort choice maximizes the issuer’s expected payoff

$$e^*_t(\Phi_t) = e((1 - \pi)(1 + \pi - (\pi/2)\Sigma_i(1 - \mu^i_t)(1 - \bar{H}_t))p(g, g|M_t, \bar{H})).$$  

(3.4)

Assumption 1 ensures that an equilibrium with positive effort exists for any $\bar{H} \in [0, 1]^2$. In equilibrium, the rating agencies extract all the surplus generated by the issuer’s effort, as described in the next lemma.

Lemma 34. An equilibrium with positive effort exists for any $\bar{H} \in [0, 1]^2$. In equilibrium, the rating agencies require the highest fee that the issuer is willing to pay
\( \phi_t^I = \phi_t^{II} = (1/2)(e^* + (1 - e^*)((\pi^2 + (\pi(1 - \pi)/2)\Sigma_i(1 - \mu_i^I)(1 - \bar{h}))p(g, g|\bar{M}, \bar{H}) - c(e^*)). \)

**B) Proofs**

**Proof of Lemma 1.** Assumptions 2 and 3 imply that all buyers hold identical beliefs on and out-of the equilibrium path. Therefore, in any PBE:

\[ |i_t: bid_i^E = Pr\{q_t = G|r_t, \mu_t, \bar{h}, \phi_t^M\}| \geq 2 \quad \forall r_t, \mu_t, \bar{h}, \phi_t^M, \text{ and} \]

\[ bid_i^E \leq Pr\{q_t = G|r_t, \mu_t, \bar{h}, \phi_t^M\} \quad \forall i_t, r_t, \mu_t, \bar{h}, \phi_t^M. \]

By construction, \( Pr\{q_t = B|r_t^M = b\} = 1 \) therefore \( p(b, \phi_t|\mu_t, \bar{h}) = 0, \forall \phi_t, \mu_t \). \( p(g, \phi_t|\mu_t, \bar{h}) \) is defined by Bayes Rule, and as \( p(g, \phi_t|\mu_t, \bar{h}) \geq p(b, \phi_t|\mu_t, \bar{h}) \) the rate is hidden only if \( r_t = b \) or if \( r_t = g \) and \( p(g, \phi_t|\mu_t, \bar{h}) = 0. \) Moreover, if the agency is not hired, then \( e^*_t = 0; \) therefore \( p(\emptyset, \phi_t|\mu_t, \bar{h}) = 0 \) \( \forall \phi_t, \mu_t \). □

**Proof of Lemma 2.** (1.1), (1.3) hold at the same time iff \( p(g|\mu_t, \bar{h}) = f(e((1 - \pi)p(g|\mu_t, \bar{h}))), \) where \( f(x) := x/(x + (1 - x)a), \) and \( a := \pi + (1 - \pi)(1 - \mu_t)(1 - \bar{h}). \) As \( a \in (0, 1) \) and \( e((1 - \pi)p(g|\mu_t, \bar{h}))(0, 1), \) then \( f''(x) < 0 \) (i). Moreover, \( e'' \geq 0 \) and \( e'' > 0 \) imply

\[ \partial^2 e((1 - \pi)p)/\partial p^2 = -c''(e((1 - \pi)p))(1 - \pi)^2/(c''(e((1 - \pi)p)))^3 \leq 0 \quad (3.5) \]
(i) and (3.5) imply

\[ \partial f(e((1 - \pi) p(g, \cdot))) / \partial p(g, \cdot) > 0 > \partial^2 f(e((1 - \pi) p(g, \cdot))) / \partial^2 p(g, \cdot) \]  

(3.6) implies \( |\{x : x = f(e((1 - \pi)x))\}| \leq 2 \). \( 0 = f(e((1 - \pi)0)) \) implies that \( p(g|\mu, \overline{h}) = e^* = 0 \) satisfy (1.1) and (1.3) and that there is at most a unique pair \( p(g|\mu, \overline{h}) > 0 \) and \( e^* > 0 \) that satisfies (1.1) and (1.3). As \( f(e((1 - \pi)1)) \leq 1 \), then a necessary and sufficient condition for the existence of a unique \( x \in (0, 1) \) s.t. \( x = f(e((1 - \pi)x)) \) is

\[ \partial f(e((1 - \pi)x)) / \partial x|_{x=0} > 1 \]  

(3.7)

If (3.7) does not hold, then \( x > f(e((1 - \pi)x)) \forall x \in (0, 1) \). (3.7) holds for all \( \overline{h} \in [0, 1] \) iff

\[ \mu_t > 1/(1 - \pi) - 1/c''(0) \]

Let

\[ \overline{\phi}_t := (e^*_t + (1 - e^*_t)\pi)p(g|\mu_t, \overline{h}) - c(e^*_t). \]

I rule out pathological equilibria in which some type of rating agency requests \( \phi^*_t > \overline{\phi}_t \). In any equilibrium in which \( \mu(\phi_t) = \mu_t \forall \mu_t \), both types of agency set \( \phi^*_t = \overline{\phi}_t \). If instead in equilibrium \( \mu(\phi'_t) \neq \mu_t \) for some \( \phi'_t \), by Assumption 2, \( \phi'_t \) must be on the equilibrium path for some type of agency. But this implies that there is at least another \( \phi''_t \) on the equilibrium path s.t. \( \mu(\phi''_t) \neq \mu(\phi''_t) \).

WLG let \( \mu(\phi'_t) > \mu(\phi''_t) \) (i). If the honest type strictly prefers \( \phi''_t \) over \( \phi'_t \), then \( \mu(\phi'_t) = 0 \), which contradicts (i). If the honest type is indifferent or strictly prefers \( \phi'_t \) over \( \phi''_t \) then the strategic type
strictly prefers $\phi_t'$ over $\phi_t''$ because $\beta(\mu(\phi_t'), h) > \beta(\mu(\phi_t''), h)$, which implies that either $\mu(\phi_t'') = 1$ (which contradicts (i)) or $\phi_t''$ is out of the equilibrium path, which is also a contradiction □

**Proof of Proposition 4.** First of all $h^*_2 = 0$. In the last period a strategic agency has no incentive to maintain a reputation for honesty. To characterize the equilibrium, I need to pin down $h^*_1$ and to show that it is optimal for the rating agency - regardless of its type - to set a fee in each period s.t. the issuer is willing to hire the agency. For the moment, I assume that the latter condition is satisfied and I pin down $h^*_{1M}$. For $s_1 = b$, an honest rate ensures a continuation payoff $\delta u^M_2(\mu^b(h^*_1))$. If instead a bribe is paid, the continuation payoff is $p(g|\mu_1, h^*_1) + \delta u^M_2(\mu^s(h^*_1))$.

Step 1. $p(g|\mu, h^*_1)$ satisfies the implicit function

$$F(p(g|.), \mu, h^*_1) := p(g|.) - f(e((1 - \pi)p(g|.) )) = 0$$

where $f(.)$ is defined in the Proof of 2. As shown in Lemma 2,

$$F_p(p^s(g|\mu_1, h^*_1), \mu_1, h^*_1)|_p > 0 \quad \forall \mu_1, h^*_1.$$

Moreover, the Implicit Function Theorem ensures that

$$\partial p(g|\mu_1, h^*_1)/\partial \mu_k = -F_{\mu}(p(g|.), \mu_1)/F_p(p(g|.), \mu_k) > 0.$$

Step 2. Note that

$$\partial (p(g|\mu_1, h^*_1))/\partial h^*_1 > 0 \quad (i), \quad \partial \mu^s(h^*_1)/\partial h^*_1 > 0 \quad (ii)$$

$$\partial u^s_2(\mu_2)/\partial \mu_2 = (\partial p(g|\mu_2, 0)/\partial \mu_2)(1 - (1 - \pi)p(g|\mu_2, 0)\partial e((1 - \pi)x)/\partial x|_{x=p(g|\mu_2, 0)}) > 0 \quad (iii)$$
(iii) holds as $\frac{\partial p(g|\mu_2, 0)}{\partial \mu_2} > 0$ by Step 1 and

$$p(g|\cdot)\partial e((1 - \pi)x)/\partial x|_{x = p(g|.)} < e((1 - \pi)p(g|.) \quad (iv)$$

$e((1 - \pi)p(g|.) < 1$. (iv) in turn, holds as $\partial^2 e((1 - \pi)x)/\partial x^2 < 0$ and $e(0) = 0$. (i), (ii), and (iii) imply

$$\partial (p(g|\mu_1, h_1^*) + \delta u_2^M(\mu^b(h_1^*))/\partial h_1^* > 0.$$  

Step 3. $\partial u_2^b(\mu^b(h_1^*))/\partial h_1^* < 0$: as $\partial \mu^b(h_1^*)/\partial h_1^* < 0$ and $\partial u_2^M(\mu_2)/\partial \mu_2 > 0$ by in step 2.

Step 4. From steps 2 and 3, either:

$$p(g|\mu_1, h_1^*) + \delta u_2^M(\mu^b(h_1^*)) > \delta u_2^M(\mu^b(h_1^*)), \forall h_1 \in [0, 1)$$

in which case the agency strictly prefers to request a bribe and $h_1^{M*} = 0$, or there is a unique $h_1^{M*} \in [0, 1]$ s.t.

$$p(g|\mu_1, h_1^{M*}) = \delta (u_2^b(\mu^b(h_1^{M*})) - u_2^b(\mu^b(h_1^{M*})))$$

in which case $h_1^M = h_1^{M*}$. Let $\overline{\delta}^M$ be defined by

$$p(g|\mu_1, 0) + \overline{\delta}^M u_2^M(\mu^b(0)) = \overline{\delta}^M u_2^M(\mu^b(0)).$$

As $u_2^M(\mu^b(0)) < u_2^M(\mu^b(0))$, for $\delta < \overline{\delta}^M h_1^M = 0$, while for $\delta \geq \overline{\delta}^M h_1^M = h_1^{M*}$ holds.

Step 5. All is left to show is that it is optimal to set a fee s.t. the issuer is willing to hire the
agency. In the last period, Lemma 3 ensures that the agency - regardless of its type - will set a fee equal to the highest willingness to pay of the issuer. In the first period, assume that each type sets a fee s.t. it is not optimal to hire an agency; then the payoff of a honest type equals $u^C_2(\mu_1) = (e^* + (1 - e^*)\pi)p(g|\mu_1, 0) - c(e^*)$. By deviating to $\bar{\phi}_1 := (e^*_1 + (1 - e^*_1)\pi)p(g|\mu_1, h^*_1) - c(e^*_1)$, the strategic type has a profitable deviation. In fact $\bar{\phi}_1 > 0$ and $E(u^C_2(\mu_1)) > \mu_1(i)$. (i) is the case because

\begin{align*}
a) & E(\mu_2) = e^*\mu_1 + (1 - e^*)\pi\mu^b(h^{M*}) + (1 - e^*)(1 - \pi)\mu^g(h^{M*}) > \\
& > e^*\mu_1 + (1 - e^*)(\pi + (1 - \pi)(1 - \mu^b_1)(1 - h^*_1))\mu(h^{M*}) + \\
& + ((1 - e^*)(1 - \pi)(1 - \mu_1(1 - h^{M*}_1))\mu^g(h^{M*}) > \mu_1.
\end{align*}

b) Define

\[ \bar{u}^C_2(\mu_2) := (e^*_2(\mu_1) + (1 - e^*_2(\mu_1))\pi)p(g|\mu_1, 0) - c(e^*_2(\mu_1)), \]

then $\partial^2 \bar{u}^C_2(\mu_2)/\partial \mu_2 > 0$. But $\bar{u}^C_2(\mu_2) \geq \bar{u}^C_2(\mu_2)$ and $\partial(\bar{u}^C_2(\mu_2) - \bar{u}^C_2(\mu_2))/\partial \mu_2 < 0$ iff $\mu_2 < \mu_1$,

therefore $\partial^2 \bar{u}^C_2(\mu_2)/\partial \mu^2_2|_{\mu_2=\mu_1} > 0$.

As in the first period fees s.t the issuer is not willing to hire the agency can be ruled out, Lemma 3 ensures that the agency - regardless of its type - will set a fee equal to the issuer’s highest willingness to pay □

**Proof of Corollary 1.** Let $F(h^*_1, \delta) := \delta - p(g|\mu_1, h^*_1)/(u^M_2(\mu^b(h^*_1)) - u^M_2(\mu^g(h^*_1)))$. By the Implicit Fun. Theorem:

\[ \partial h^*_1/\partial \delta = -\frac{\partial F(h^*_1, \delta)/\partial \delta}{\partial F(h^*_1, \delta)/\partial h^*_1} \text{ and } \partial F(h^*_1, \delta)/\partial \delta > 0 \text{ while} \]

\[ \partial F(h^*_1, \delta)/\partial h^*_1 = \frac{(dp|\mu_1, h^*_1)/dh^*_1)(u^M_2(\mu^b(h^*_1)) - u^M_2(\mu^g(h^*_1)))(dp|\mu_1, h^*_1)/(u^M_2(\mu^b(h^*_1)) - u^M_2(\mu^g(h^*_1)))+}{(u^M_2(\mu^b(h^*_1)) - u^M_2(\mu^g(h^*_1)))^2} \]
\[-(du_2(\mu^b(h_1^*))/\partial \mu^b(h_1^*)/(\partial u_2^M(h_1^*)) < 0\]

The last inequality holds as: \(\partial p(g|\mu_1,h_1^*)/\partial h > 0\), \(du^M_2/d\mu_2 > 0\), and \(\partial \mu^b(h_1^*)/\partial h_1^* > 0\)

\[> \partial \mu^b(h_1^*)/\partial h_1^*\]

To show \(\lim_{\delta \rightarrow 0} h_1^*\) it is enough to show that \(u_2^M(\mu^b(h_1^*)) - u_2^M(\mu^\phi(h_1^*)) = 0\) iff \(h_1^* = 1\). The if part holds as \(\mu^b(1) = \mu^\phi(1)\) the only if part holds as: \(h_1^* < 1 \rightarrow \mu^b(h_1^*) > \mu^\phi(h_1^*)\) and as shown above \(\partial u_2^M/\partial \mu_2 > 0\). Finally, \(\partial \delta^M/\partial \mu_1 > 0\) as \(\partial p(g|\mu_1,0)/\partial \mu_1 > 0\) and

\[\partial (u_2^M(\mu^b(0)) - u_2^M(\mu^\phi(0))) / \partial \mu_1 = \partial (1 - u_2^M(\mu^\phi(0))) / \partial \mu_1 < 0\] \(\square\)

**Proof of Lemma 5.** In equilibrium, suppose that following fees \(\Phi_i\) the issuer is expected to hire only one agency and \(e^\phi(\Phi_i) > 0\). Then by Assumption 3 \(e^\phi(\Phi_i) > 0\) and \(p(g,\Phi_i|.) > 0\) must be mutually consistent as in monopoly. Assumption 3 requires that also \(p(g,g,\Phi_i|.)\) must also be mutually consistent with the effort choice. Suppose \(p(g,g,\Phi_i|.) \leq p(g,\Phi_i|.)\), then the effort choice is identical whether the agency hires one or two agencies. But for identical effort, it must be the case that \(p(g,g,\Phi_i|.) > p(g,\Phi_i|.)\) as \(\mu^I, \mu^H > 0\). Therefore, it must be the case that \(p(g,g,\Phi_i|.) > p(g,\Phi_i|.)\).

Suppose there is an equilibrium in which the issuer hires only one agency. Then either the fees \(\phi_i^I\) and \(\phi_i^H\) are such that the issuer is indifferent to hire either of the agencies and \(\phi_i^I > 0\) and \(\phi_i^H > 0\), in which case every agency has an incentive to charge an \(\varepsilon\)-smaller fee, or \(\phi_i^I \geq 0\) and \(\phi_i^H \geq 0\) with at least one weak inequality holding as an equality; then any agency charging \(\phi_i^I = 0\) could deviate to a strictly positive, small enough fee and ensure a larger profit. If in equilibrium the issuer does not hire the agencies, he exerts no effort, \(p(r_i|M_i,H_i) = 0\) if \(r_I\) and the Lemma holds. If in equilibrium the issuer hires the two agencies, if \(p(R_i|M_i,H_i) > 0\) for \(r_i = \emptyset\) for some \(i \in \{I,II\}\), then the issuer
Proof of Lemma 6. If $\beta_i^{-i} \geq p(g,g,\Phi_i)$, then $\beta_i^i$ is paid only if $\beta_i^i \leq p(g,g,\Phi_i)$ and $s^{-i} = g$. So $\beta_i^i = p(g,g,\Phi_i)$ maximizes $A'$'s expected payoff. If instead $\beta_i^{-i} < p(g,g,\Phi_i)$, the payoff of $A_i$ is maximized by:

$$
\beta_i^i = \begin{cases} 
p(g,g,\Phi_i) - \beta_i^{-i} & \text{if } (\pi + (1 - \pi)(1 - \mu_i^{-i})(1 - h_i^{-i}))(p(g,g,\Phi_i) - \beta_i^{-i}) \geq \pi p(g,g,\Phi_i), \\
p(g,g,\Phi_i) & \text{otherwise.}
\end{cases}
$$

So, in equilibrium either $\beta_i^{i*} = \beta_i^{H*} = p(g,g,\Phi_i)$ or $\beta_i^i + \beta_i^i = p(g,g,\Phi_i)$ $\square$

Proof of Lemma 7. (1.6) and (1.8) hold at the same time iff

$$
p(g,g,M_i,H) = f^H(e((1 - \pi^2)p(g,g,M_i,H))),
$$

where $f^H(x) := x/(x + (1 - x)a^H)$, and $a^H := \pi^2 + \sum_{j \neq i} (1 - \pi)\pi(1 - \mu_j^i)(1 - h_i^j)$. Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an $x \in (0,1)$ :

$$
x = f^H(e((1 - \pi^2)x)) \text{ is: } \partial f^H(e((1 - \pi^2)x))/\partial x|_{x=0} > 1 \text{ (i). (i) holds for every } H \in [0,1]^2 \text{ iff }$

$$
e'(0)(1 - \pi^2)/\pi^2 + \pi(1 - \pi)(2 - (\mu_i^i + \mu_i^H)) > 1
$$

which is equivalent to $\leftrightarrow \mu_i^i + \mu_i^H > (2 - \pi)/(1 - \pi) - (1 + \pi)/(\pi^2(0)) \square$

Proof of Lemma 8. Let $\overline{\phi}_i := (e_i^* + (1 - e_i^*)\pi) p(g,g,M_i,H) - c(e_i^*)$. I rule out pathological equilibria
in which some type of rating agency requests \( \phi_t^* > \phi_t \). In any equilibrium in which \( \mu(\phi_t) = \mu_t \forall \mu_t \)

Assumption 5 ensures that both types of agency set \( \phi_t^* = \phi_t / 2 \). If instead in equilibrium \( \mu^i(\phi_t^i) \neq \mu_t^i \) for some \( \phi_t^i \), by Assumption 2 \( \phi_t^i \) must be on the equilibrium path for some type of agency \( A^i \). But this implies that there exists at least another \( \phi_t'' \) on the equilibrium path s.t. \( \mu^i(\phi_t') = \mu^i(\phi_t'') \). WLG let \( \mu^i(\phi_t^i) > \mu^i(\phi_t'') \) (i). If the honest type strictly prefers \( \phi_t'' \) over \( \phi_t' \), then \( \mu^i(\phi_t^i) = 0 \), which contradicts (i). If the honest type is indifferent or strictly prefers \( \phi_t' \) over \( \phi_t'' \), then the strategic type strictly prefers \( \phi_t' \) over \( \phi_t'' \) because \( \beta(M(\phi_t'), H) > \beta(M(\phi_t''), H) \) which implies that either \( \mu^i(\phi_t'') = 1 \) (contradicting (i)) or \( \phi_t'' \) is out of the equilibrium path which is also a contradiction \( \square \)

**Proof of Lemma 9.** Using the definitions in the proofs of Lemma 7 and Lemma 24

\[
a^H < a^L \rightarrow f^H(e((1 - \pi^2)p(g, g|M_t, \bar{H}))) > f^L(e((1 - \pi^2)p(g, g|M_t, \bar{H}))), \forall M \in (0, 1)^2, \bar{H} \in [0, 1)^2,
\]

(i) Moreover:

\[
f^L(e((1 - \pi^2)p(g, g|M_t, \bar{H}))) > f^L(e((1 - \pi^2)(1 + \pi - (\pi/2)\Sigma_{I,II}(1 - \mu^i_t)(1 - \bar{H}^i)p(g, g|M_t, \bar{H})))\forall M \in (0, 1)^2, \bar{H} \in [0, 1)^2. (ii)
\]

Therefore, if \( p^H > 0 \) and \( p^L > 0 \) and \( p^H = f^H(e((1 - \pi^2)p^H)) \) while

\[
p^L = f^L(e((1 - \pi^2)(1 + \pi - (\pi/2)\Sigma_{I,II}(1 - \mu^i_t)(1 - \bar{H}^i)p^L)),
\]

then \( p^H > p^L \). Therefore, the equilibrium price following two favorable ratings is larger if the agencies coordinate on high bribes. Also, the equilibrium effort is larger if agencies coordinate on
high bribes, as

\[ e((1 - \pi^2)p) > e((1 - \pi)(1 + \pi - (\pi/2)\Sigma_{t,\Pi}(1 - \mu^i_t)(1 - \bar{H}_t^i)p) \quad \forall p \]

Proof of Proposition 10. First of all \( h_2^i = h_2^H = 0 \). In the last period a strategic agency has no incentive to maintain a reputation for honesty. To characterize the equilibrium, I need to pin down \( h_1^s, h_1^H \) and to show that it is optimal for the rating agency - regardless of its type - to set a fee in each period s.t. the issuer is willing to hire the agency. For the moment, I assume that the latter condition is satisfied and I pin down \( h_1^s \). In equilibrium, \( \beta_1^i \) is paid if \( s_1^i = g \), in which case a strategic agency faces a trade-off between demanding a bribe and obtaining \( p(g, g|M_1, H_1^i) + \delta u_2^s(\mu^{i,s}(H_1^i), \mu^{i,s}(H_1^s)) \) and rating honestly to obtain \( \delta u_2^s(\mu^{i,b}(h_1^s), \mu_1) \).

Step 1. \( p(g, g|M_i, H_1^i) \in C^1 \) is increasing in \( \mu_i \), and \( h_1^s \forall i \) (analogous to Proof of Prop. 4).

Step 2. \( \partial (p(g, g|M_i, H_1^i) + \delta u_2^s(\mu^{i,s}(H_1^i), \mu^{i,s}(H_1^s))) / \partial h_1^i > 0 \). This is the case as \( \partial \mu^{i,b}(h_1^s) / \partial h_1^i > 0 \), and

\[ \partial u_2^s(M_2, 0, 0) / \partial \mu_2^i = \]

\[ = (\partial p(g, g|M_2, 0, 0) / \partial \mu_2^i) \partial ((1/2)((e_2 + (1 - e_2)\pi(2 - \pi))p(g, g|.) - c(e_2))/\partial p(g, g|.) = \]

\[ = (1/2)(\partial p(g, g|M_2, 0, 0) / \partial \mu_2^i) \times \]

\[ \times ((e_2 + (1 - e_2)\pi(2 - \pi) - \partial e((1 - \pi^2)x) / \partial x_{x=p(g, g|M_2, 0, 0)p(g, g|.)}) > 0. \]

The inequality holds as \( \partial p(g, g|M_2, 0, 0) / \partial \mu_2^i > 0 \) (step 1), and by the Proof of Prop. 4 for \( x = \)
\[ p(g, g|M_2, 0, 0) : \]
\[ \partial (e(1 - \pi^2)x)/\partial x < e((1 - \pi^2)x). \]

Step 3. \( \partial u_2^i(\mu_2^i(h_1^*)^*, \mu_1)/\partial h_1^i + \partial u_2^i(\mu_2^i(h_1^*)^*, \mu_1)/\partial h_1^{-i} = \partial u_2^i(\mu_2^i(h_1^*)^*, \mu_1)/\partial h_1^i < 0. \)

This is the case as \( \partial \mu^i_{lb}(h_1^*)/\partial h_1^* < 0. \)

Step 4. From steps 2 and 3, in the first period the two agencies will choose the same \( h_1^* \). Suppose not and \( h_1^I > h_1^H \) (i), then \( h_1^I > 0 \) implies

\[ p(g, g|M_1, H_1^i) + \delta u_2^H(\mu^I_{ls}(H_1^*), \mu^I_{ls}(H_1^*)) = \delta u_2^I(\mu_2^I_{lb}(h_1^*)) \]

but then (i) implies: \( u_2^I(\mu_2^I_{lb}(h_1^*)) < u_2^H(\mu_2^H_{lb}(h_1^*)) \) and therefore

\[ \delta u_2^H(\mu_2^H_{lb}(h_1^*)) > p(g, g|M_1, H_1^i) + \delta u_2^H(\mu^I_{ls}(H_1^*), \mu^I_{ls}(H_1^*)) = \]
\[ = p(g, g|M_1, H_1^i) + \delta u_2^H(\mu^I_{ls}(H_1^*), \mu^I_{ls}(H_1^*)) \]

Therefore, either

\[ p(g, g|M_1, H_1) + \delta u_2^i(\mu_{ls}(H_1), \mu_{-ls}(H_1)) > \delta u_2^i(\mu_{lb}(h_1^i)) \forall h_1^i \in [0, 1), \]

in which case each agency strictly prefers to request a bribe and \( h_1^I = h_1^H = 0 \), or there is a unique value \( h_1^i(\delta) \) s.t.

\[ p(g, g|M_1, H_1) + \delta u_2^i(\mu_{ls}(H_1), \mu_{-ls}(H_1)) = \delta u_2^i(\mu_{lb}(h_1^i)), \]
for $h_1^I = h_1^{II} = h^c(\delta)$. Let $\overline{\delta}^c$ be defined by

$$ p(g,g|M_1,[0,0]) + \overline{\delta}^c u_2^i(\mu^{i,g}(0,0),\mu^{-i,g}(0,0)) = \overline{\delta} u_2^i(\mu^i(0)). $$

For $\delta < \overline{\delta}^c$ $h_1^I = 0$, while for $\delta \geq \overline{\delta}^c$ $h_1^{II} = h^c(\delta)$.

Step 5. All is left to show is that is it optimal to set a fee s.t. the issuer is willing to hire the agency. In the last period, Lemma 8 ensures that the agency - regardless of its type - will set a fee equal to the highest willingness to pay of the issuer. I rule out equilibria in which both agencies set a rating fee higher than the fee defined in Lemma 8. In the first period, assume that each type of agency $A^i$ sets a fee s.t. it is not optimal to hire an agency: then the payoff of an honest agency equals $u_2^i(\mu_1) = (e^{*} + (1-e^{*})\pi)p(g,g|M_1,[0,0]) - c(e^{*})$. By deviating to $\overline{\phi}_1 := (1/2)((e^{*} + (1-e^{*})\pi)p(g,g|M_1,H^*_1) - c(e^{*}))$ the strategic type has a profitable deviation (the proof is identical to the proof of Proposition 4, step 5).

Proof of Corollary 2. Let

$$ F(h^c(\delta),\overline{\delta}) := \overline{\delta} - p(g,g|M_1,h^c(\delta),h^c(\delta))/(u_2^i(\mu^{i,h}(h^c(\delta)),\mu_1) - u_2^i(\mu^{i,g}(h^c(\delta),h^c(\delta)),\mu^{-i,g}(h^c(\delta),h^c(\delta))))), $$

(1.11) implies that $F(h_1^I(\delta),\overline{\delta}) = 0$. By the Implicit Function Theorem,

$$ \frac{\partial h^c(\delta)}{\partial \overline{\delta}} = -\frac{\partial F(h^c(\delta),\overline{\delta})/\partial \overline{\delta}}{\partial F(h^c(\delta),\overline{\delta})/\partial h^c(\delta)}, $$

where $\partial F(h^c(\delta),\overline{\delta})/\partial \overline{\delta} = 1 > 0$, and
\[ \partial F(h^c(\delta), \delta)/\partial h^c(\delta) = -\frac{\partial p(g, g|\mu)}{\partial h^c(\delta)} u_2^1(\mu^b, \mu_1) - u_2^1(\mu^s, \mu_1) + \]
\[ \frac{\partial p(g, g|\mu)}{\partial h^c(\delta)} - \frac{\partial u_2^1(\mu^b, \mu_1)}{\partial h^c(\delta)} - \frac{\partial p(g, g|\mu)}{\partial h^c(\delta)} - \frac{\partial u_2^1(\mu^s, \mu_1)}{\partial h^c(\delta)} < 0 \]

The second inequality holds as: \( \partial p(g, g|\mu)/\partial h^c > 0 \) and \( \partial u_2^1/\partial \mu_1 > 0 \), \( \partial u_2^1/\partial \mu_2^{-i} > 0 \) and 
\[ \partial \mu^{-i}_s/\partial h^s > 0 > \partial \mu^{i,b}/\partial h^s. \]
Therefore \( \partial h^c(\delta)/\partial \delta > 0 \). To show \( \lim_{\delta \to \infty} h^c(\delta) = 1 \) it is enough to show that 
\[ u_2^1(\mu^{i,b}, \mu_1) - u_2^1(\mu^{i,s}, \mu_1^{-i}) = 0 \text{ iff } h^i_1 = h^{i,-i}_1 = 1. \]
The if part holds as \( \mu^{i,b} = \mu^{i,s} = \mu_1 \) for \( h^i_1 = h^{i,-i}_1 = 1. \) The only if part holds as: \( h^c(\delta) < 1 \to \mu^{i,b}(h^c(\delta)) > \mu^{i,s}(h^c(\delta), h^c(\delta)) \) and \( \mu^{i,s}(h^c(\delta), h^c(\delta)) < \mu_1. \)

Moreover \( \partial p(g, g|\mu)/\partial \mu_1, \partial p(g, g|\mu)/\partial \mu_1 > 0 \) and as shown above \( \partial u_2^1/\partial p(g, g|\mu) > 0 \square \)

Proof of Lemma 11. For the first part of Lemma 11, assume that rating agencies select \( \beta_1^i = \beta_1^H = p(g, g|M_1, H_1) \). This implies that \( \mu_1 > \bar{\mu}. \) But if \( q_1 = G \) then \( \mu_2^i = \mu_2^H = \mu_1 \), while if \( q_1 = B \) for

\[ R_1 = (b, b) \mu_2^{i,-i} = \mu_1 \text{ so } \mu_2^{i,-i} > \mu_1 \forall l, \text{ for } r_1^i = g, \text{ } r_1^i = b \text{ } r_1^{i*} = \mu_1/(1 - \mu_1) > \mu_1, \]
while for \( R_1 = (g, g) \mu_2^{i,-i} = \mu_1 \) or \( \mu_2^{-i} = \mu_1 \). Therefore \( \max\{\mu_2^{i,H}, \mu_2^{H} \} \geq \mu_1 \), therefore by Assumption 6 if rating agencies select \( \beta_1^i = \beta_1^H = p(g, g|M_1, H_1) \), then \( \beta_1^{i*} = \beta_2^{H*} = p(g, g|M_2, H_2) \).

For the second part of the theorem in equilibrium, rating agencies choose bribes \( \beta_1^i, \beta_1^H : \beta_1^i + \beta_1^H = p(g, g|M_1, H_1) \) only if they strictly prefer to receive a bribe, that is only if \( h^i_1 = h^{i,H}_1 = 0. \)

This is the case because an agency \( A_i \) that is indifferent between honest rating and the bribe \( \beta_1^i = p(g, g|M_1, H_1)/2 \) will strictly prefer to request bribe \( \beta_1^i = p(g, g|M_1, H_1) \square \)

Proof of Lemma 12. To prove the lemma, I first prove an intermediate result:

Result: For any \( \bar{\pi} \in [0, 1] \), two signals with \( \pi = \bar{\pi} \) have the same informative content of a single
signal with $\pi = \overline{\pi}^2$.

Proof: Let $s^1, s^2$ be conditionally independent signals with $\pi = \overline{\pi}$ while $s^3$ is a signal with $\pi = \overline{\pi}^2$ for some $\overline{\pi} \in [0, 1]$. Then $Pr\{s^1, s^2\} = (g, g) | q = G \} = Pr\{s^3 = g | q = G \} = 1$, and $Pr\{s^1, s^2\} = (g, g) | q = B \} = Pr\{s^3 = g | q = B \} = \overline{\pi}^2$. So receiving signals $(s^1, s^2) = (g, g)$ has the same informative content as signal $s^3 = g$. Moreover signals $(s^1, s^2) \neq (g, g)$ have the same informative content of signal $s^3 = b$ as $Pr\{(s^1, s^2) \neq (g, g) | q = G \} = Pr\{s^3 = b | q = G \} = 0$ In monopoly $e_t = e^*(1 - \pi^2)p(g)$ and

$$p(g) = e/e + (1 - e)Pr\{r = g | q = B, \mu, \nu \}.$$

In competition, if rating agencies coordinate on $\beta^i_1 = \beta^i_1$ then $e_t = e^*((1 - \pi^2)p(g))$ and

$$p(g, g) = e/e + (1 - e)Pr\{R = (g, g) | q = B, M_i, \overline{\mu} \}.$$

The lemma holds as $Pr\{r^M = g | q = B, \overline{\mu}, \overline{\nu} \} < Pr\{r^M = g | q = B, \overline{\mu}, \overline{\nu} \} \forall \overline{\mu}, \overline{\nu}$

**Proof of Proposition 13.** Step 1. $e^M_1 = e^C_1$ iff $h^M_1 = f(\pi, h^C_1)$, where $f(\pi, h^C_1) := \frac{1 - \pi}{1 + \pi} + \frac{\pi}{1 + \pi} h^C_1$.

Step 2. $\mu^g(f(\pi, h^C_1)) < \mu^{g, g}(h^C_1, h^C_2) \forall \pi, h^C_1$ as: $\mu^g(f(\pi, h^C_1)) = \mu^g(\pi, h^C_1) = (\mu_1 \pi^2 / y^C(M, H)) < (\mu_1 \pi(1 - \pi)(1 - \mu_1)(1 - h^C_1))/y^C(\mu^l_1, h^l_1, h^l_2)) = \mu^{g, g}(h^C_1, h^C_2)$.

Where $y^M(\mu, h) := \pi^2 + (1 - \pi^2)(1 - \mu)(1 - h)$ and $y^C(\mu^l, h^l_1, h^l_2) := \pi^2 + \pi(1 - \pi)\Sigma_{l, l'}(1 - \mu^l_1)(1 - h^l_1)$.

Step 3. If $\pi > 1/3$ then $\mu^b(f(\pi, h^C_1)) > (\mu^{g, g}(h^C_1, h^C_2) + \mu_1)/2$ for $h^C_1 \in (\max(0, (2\mu_1 \pi - (1 -$
π)/(2μ1π − 2π), 1).

Step 4: π > 1/3 is sufficient to ensure \( \partial u_2^M(p(g|μ_2))/\partial μ_2 > \partial u_2^C(p(g, g|μ_2))/\partial μ_2 |_{p(g|μ_2)=p(g,g|M_2)} \) ∀p(g|μ_2)

\( \partial u_2^M(p(g|μ_2))/\partial μ_2 = (\partial u_2^M(p(g|)))/\partial μ_2 |_p(p(g)|)/\partial y_2^M(\partial y_2^M/\partial μ_2^M) = \)

\( = -(1 − (1 − π^2)p(g|)(\partial e_2/\partial b))(\partial p(g|)/\partial y_2^M)(1 − π^2). \)

\( \partial u_2^C(p(g,g|M_2^C))/\partial μ_2^C = (\partial u_2^C(p(g,g|M_2^C))/\partial μ_2^C + (\partial u_2^C(p(g,g|M_2^C))/\partial μ_2^C) = \)

\( = (\partial u_2^C(p(g,g|M_2^C))/\partial μ_2^C)(\partial p(g,g|)/\partial y_2^C)((\partial y_2^C/\partial μ_2^C + (\partial y_2^C/\partial μ_2^C)) = \)

\( = -(1/2)e_2 + (1 − e_2)(2π − π^2)) − π(1 − π)p(g, g|)(\partial e_2/\partial p)(\partial p(g, g|)/\partial y_2^C)(2π(1 − π)) \)

Note that \( -(\partial p(g|)/\partial y_2^M) = -(\partial p(g, g|)/\partial y_2^C) > 0. \) So:

\( \partial u_2^M(p(g|μ_2))/\partial μ_2 > \partial u_2^C(p(g, g|μ_2))/\partial μ_2^C \leftrightarrow (1 − (1 − π^2)b(\partial e_2/\partial p))(1 + π) > (1/2)(e_2 + (1 − e_2)π(2 − π)) - π(1 − π)e_2)2π. \) This condition holds ∀e_2 ∈ [0, 1] if π > 1/3.

Step 5. \( u_2^M(μ^b(f(π,h_1^C)))-u_2^M(μ^a(f(π,h_1^C)) = \)

\( \int_{μ^a(f(π,h_1^C))}^{μ^b(f(π,h_1^C))} (\partial u_2^M/\partial p(g|μ_2^M))(\partial p(g|μ_2^M)/\partial y_2^M(\partial y_2^M/\partial μ_2) dμ_2 > \)

\( > \int_{μ^a(f(π,h_1^C))}^{μ^b(f(π,h_1^C))} (\partial u_2^C/\partial p(g, g|μ_2^C))(\partial p(g, g|μ_2^C)/\partial y_2^C(\partial y_2^C/\partial μ_2) dμ_2. \)

By step 4, the inequality holds for π > 1/3. By steps 2 and 3, the following inequality holds for h_1^C ∈ (max{0, (1 − π − 2μ_1π)/2π(1 − μ)}, 1).

\( \int_{μ^a(f(π,h_1^C))}^{μ^b(f(π,h_1^C))} (\partial u_2^C/\partial p(g, g|μ_2^C))(\partial p(g, g|μ_2^C)/\partial y_2^C(\partial y_2^C/\partial μ_2) dμ_2 > \)
\[
\int_{\mu S \to (h^C_1, h^C_2)}\left(\partial u_2^C / \partial p(g, g|\mu_2^C)\right)\left(\partial p(g, g|\mu_2^C) / \partial y_2^C\right)\left(\partial y_2^C / \partial \mu_2\right) d\mu_2
\]

\[
= u_2^C(\mu_{S}^{0}, \mu_1) - u_2^C(\mu_{S}^{0}, \mu_{S}^{0}).
\]

Step 6: By Corollary (2) for \(\delta\) large enough \(h^C_1 \in (\max\{0, (1 - \pi - 2\mu_1, 2) / 2\pi(1 - \mu_1)\}, 1)\).

By step 5, \(h^C_1 \in (\max\{0, (1 - \pi - 2\mu_1, 2) / 2\pi(1 - \mu_1)\}, 1)\) and \(\pi > 1 / 3\) ensure \(h^M_1 > f(\pi, h^C_1)\) and therefore \(e^M_1 > e^C_1\).

As \(h^C_1 \in [0, 1]\) is increasing in \(\delta\) and every \(h^C_1 \in [0, 1]\) is chosen in equilibrium for some \(\delta\), and for \(\delta < \min\{\overline{\delta}^M, \overline{\delta}^C\} e^M_1 < e^C_1\).

By Corollary (2) and Corollary (1) \(e^M_1\) and \(e^C_1\) are continuous functions of \(\delta\). Therefore there is a unique \(\delta^*\) s.t. \(e^M_1 = e^C_1\) and \(e^M_1 > e^C_1\) iff \(\delta > \delta^*\) □

**Proof of Lemma 14.** (1.1) and (1.12) hold at the same time iff

\[
p(g, g|M_t, \overline{H}) = f^L(e((1 - \pi)p(g, g|M_t, \overline{H}))),\text{ where } f^L(x) := x / (x + (1 - x)\alpha^S),\text{ and }
\]

\[
\alpha^S := \pi + (1 - \pi)\Pi_{L,H}(1 - \mu^l)(1 - \overline{H}).
\]

Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an \(x \in (0, 1) : x = f^L(e((1 - \pi)x))\) is:

\[
\partial f^L(e((1 - \pi)x)) / \partial x |_{x=0} > 1 \text{ (i). (i) holds for every } \overline{H} \in [0, 1]^2 \text{ iff } (1 - \mu^l)(1 - \mu^H) < 1 / c''(0) - \pi / (1 - \pi) \Box
\]

**Proof of Lemma 15.** Whether agencies observe independent signals (and demand high bribes) or correlated signals, the issues sets \(e^* = e((1 - \pi^2)p(g, g|M_t, \overline{H}))\). Therefore, effort is larger with identical signals iff:
\[ Pr\{R_t = [g, g]|M_t, \overline{H}_t, e^*, \text{indep}\} > Pr\{R_t = [g, g]|M_t, \overline{H}_t, e^*, \text{ident}\} \leftrightarrow \]
\[ \pi^2 + \pi(1-\pi)\Sigma_i (1-\mu_i^t)(1-\pi) > \pi^2 + (1-\pi^2)\Pi_i (1-\mu_i^t)(1-\pi) \leftrightarrow \]
\[ \Sigma_i (1-\mu_i^t)(1-\pi)/\Pi_i (1-\mu_i^t)(1-\pi) > (1+\pi)/\pi. \]

For \( \mu_i^t = \mu_i^{II} := \mu_i^t \) and \( \pi^t := \pi^t \) the condition reduces to: \( (1-\mu_i)(1-\pi) < 2\pi/(1+\pi) \)

**Proof of Proposition 16.** The only equilibrium strategies left to define are \( h_1^t, h_1^{II} \). I focus on equilibria in which \( h_1^t = h_1^{II} \) (I will refer to them as \( h^{e^*} \)). I show that if \( \delta \leq \delta^{e^*} \), for a \( \delta^{e^*} \) defined below, \( h^{e^*} = 0 \). If instead \( \delta > \delta^{e^*} \), there is a unique \( h^{e^*} \) such that for \( h_1^{e^*} = h_1^{II} = h^{e^*} \) a strategic agency is indifferent to request a bribe or not. In equilibrium \( \beta_i^t \) is paid only if \( A^t \) is also requesting a bribe, in which case a strategic agency faces a tradeoff between demanding a bribe and obtaining a continuation payoff equal to \( p(g, g|.) + \delta u_2^t(\mu_1^{is}, \mu_2^{js}) \) and rating honestly and getting \( \delta u_2^t(\mu_1^{ib}, \mu_1^{ib}) \).

**Step 1.** \( p(g, g|M_t, H^e_t) \in C^1 \) is increasing in \( \mu_i^t, \mu_i^{II}, h_i^{e^*} \) and \( h_i^{e^*} \) (analogous to proof of Prop. 4)

**Step 2.** \( \partial p(g, g|.) + \delta u_2^t(\mu_1^{is}, \mu_2^{js})/\partial h_1^t + \partial (p(g, g|.) + \delta u_2^t(\mu_1^{is}, \mu_2^{js}))/\partial h_1^{II}|_{h_1^{II} = h_i^{e^*}} > 0: \)

I prove this inequality in two intermediate steps.

(1.14) ensures \( \partial \mu^{is}/\partial h_1^t > 0 \) \( \forall i, j \in \{I, II\} \), and

\[ \partial u_2^t(M_2, 0, 0)/\partial \mu_2^t = \left( \frac{\partial p(g, g|M_2, 0, 0)}{\partial \mu_2^t} \right) \frac{\partial (1/2)((e_2 + (1-e_2)(\pi + (1-\pi)(1-\mu(i)))p(g, g|.) - e(e_2))}{\partial p(g, g|.)} = \]
\[ (\partial p(g, g|M_2, 0, 0)/\partial \mu_2^t)(e_2/2 + ((1-\pi)/(1-\pi)(1-\mu(i)))+ \]
\[ -\delta e^*((1-\pi^2)x)/\partial x|_{x=p(g, g|M_2, 0, 0)}(1-\mu(i))(1-\pi)p(g, g|.) > 0 \]

The inequality holds as \( \partial p(g, g|M_2, 0, 0)/\partial \mu_2^t > 0 \), and \( \partial e^*((1-\pi^2)x)/\partial x|_{x=p(g, g|M_2, 0, 0)}p(g, g|.) < e_2 \)

(see Proof of Prop. 4).
Finally, $\partial p(g, g|\mathcal{M}_1, h^c, h^c) / \partial h^i_1 > 0 \forall i \in \{I, II\}$.

Step 3. $\partial u^i_1(\mu^i_1) / \partial h^i_1 + \partial u^i_2(\mu^i_2, \mu^i_1) / \partial h^i_1 < 0$: the inequality holds as $\partial \mu^i / \partial h^i_1 < 0 = \partial u^i_2(\mu^i_2, \mu^i_1) / \partial h^i_1$.

The rest of the proof is identical to step 2).

Step 4. Analogous to Step 4 in Proof of Prop. 4 □

Proof of Corollary 3. The Proof follows the same steps of the Proof of Corollary 2 □

Proof of Proposition 17. Assume that there is a $\delta^{**}$ such that for that value of the discount factor, monopoly and competition ensure that same effort in the first period.

Step 1. Same effort requires: $e^M_1 = e^C_1 \leftrightarrow p(g|\mu_1, h^M_1) = p(g|\mu_1, h^C_1, h^i_1)$ \leftrightarrow

$h^M_1 = f^S(\mu_1, h^C_1) := 1 - (1 - \mu_1)(1 - h^i_1)^2$.

Step 2. Same effort implies same reputation updates, that is: $\mu^g(\mu_1, h^C_1) = \mu^g(h^C_1, h^i_1)$ and $\mu^b(\mu_1, h^C_1) = \mu^b(h^C_1, h^i_1)$.

This is the case as: $\mu^g(\mu_1, h^C_1) = \mu_1 \pi^2 / (\pi^2 + (1 - \pi^2)(1 - \mu_1)(1 - h^M_1)) = \mu_1 \pi^2 / (\pi^2 + (1 - \pi^2)(1 - \mu_1)^2(1 - h^i_1)^2) = \mu^g(h^C_1, h^i_1)$.

$\mu^b(\mu_1, h^C_1) = \mu_1 / (1 - (1 - \mu_1)(1 - h^M_1)) = \mu_1 / (1 - (1 - \mu_1)^2(1 - h^i_1)^2) = \mu^b(h^C_1, h^i_1)$.

Step 3.

$Pr\{r_1 = g|q_1 = B, h_t = 0\} := y^M(\mu_t, 0) = \pi^2 + (1 - \pi^2)(1 - \mu_1) > \pi^2 + (1 - \pi^2)(1 - \mu_1)^2 = Pr\{R_1 = (g, g)|q_1 = B, H_t = (0, 0)\} := y^M(\mu_t, 0), \forall \mu_t < 1$.

And $\frac{\partial y^M(\mu_2, 0) / \partial \mu_2}{\partial y^C(\mu_2, 0) / \partial \mu_2} = \frac{1}{2(1 - \mu_2)} > 1 \leftrightarrow \mu_2 > 1/2$. By Assumption 1, $\mu_2 > \mu^M := 1/c''(0) = 1/2$. 
Step 4. By Steps 2 and 3, \( h_1^{M*} = f^S(\mu_1, h_1^{C*}) \rightarrow y^M(\mu^b, 0) - y^M(\mu^g, 0) > y^C(\mu^i_b, 0) - y^C(\mu^i_g, 0) \) and \( y^M(\mu^b, 0) > y^C(\mu^i_b, \mu^i_b, 0, 0) \) and \( y^M(\mu^g, 0) > y^C(\mu^i_g, \mu^i_g, 0, 0) \).

Step 5. Let \( F(y, p) := p - \frac{(1 - \pi^2)p/2}{(1 - \pi^2)p/2 + (1 - \pi^2)p/2y} \), then by the Implicit Function Theorem \( p = p(y) \) and \( p'(y) = \frac{1 - 2/(1 - \pi^2)}{(1 - y)^2} < 0 \) and

\[
p''(y) = \frac{1 - 2/(1 - \pi^2)}{(1 - y)^2} < 0.
\]

By step 4, for identical effort in the first period,

\[
p(g|\mu^b, 0) - p(g|\mu^g, 0) > p(g, g|\mu^i_b, \mu^i_b, 0, 0) - p(g, g|\mu^i_g, \mu^i_g, 0, 0).
\]

Step 6. For \( \mu^{ij} = 0 \): \( u_2^C(p_2) = (p_2 - c(e_i))/2 = u_2^M(p_2)/2 \).

As utility \( u_2^C(p_2) = u_2^M(p_2)/2 \) and \( d^2u_2^M(b_2)/d^2b_2 = -(1 - \pi^2)/2 < 0 \), by step 5:

\[
u_2^i(\mu^i_g(\mu_c, \mu_c), \mu^i_g(\mu_c, \mu_c), 0) - u_2^i(\mu^i_b(\mu_c, \mu_c), \mu^i_b(\mu_c, \mu_c), 0) < 0.
\]

As for identical effort in the first period \( \beta_1^C = \beta_1^M/2 \), then if the monopolist is indifferent to offer a bribe, the competitors strictly prefer to offer a bribe.

Step 7. This in turn implies that, for any \( \delta > \delta^C \), monopoly induces more effort than competition.

This is the case because if competitors are indifferent to lying, or to being honest for \( h^{C*} \), then the monopolist must choose in equilibrium an \( h_1^{M*} > f^S(\mu_1, h_1^{C*}) \). The continuity of \( h_1^{M*}(\delta) \) ensures that there is a unique \( \delta^{**} \) s.t. \( e_1^M = e_1^C \). Moreover, \( e_1^M > e_1^C \rightarrow \delta > \delta^{**} \)

Proof of Lemma 18. First, I show \( \delta^* < \delta^C \). At first, I assume there is a \( \delta^* \) for which \( e_1^M = e_1^C \), and I show that for \( \delta = \delta^* \) if in equilibrium a (strategic) monopolist rating agency is indifferent to request a bribe then a (strategic) competing rating agency which strictly prefers to request a bribe.
Therefore, $\delta^*$ is unique and the monopoly ensures a higher level of effort in the first period iff $\delta > \delta^*$. I use this observation and the continuity of $h^*_1$ w.r.t. $\delta$ to show that $\delta^*$ exists and satisfies $\delta^* < \delta^C$.

Step 1. $e^M_t = e^C_t \leftrightarrow h^*_1 = f(h^C_1) := (1 - \pi + 2\pi h^C_1)/(1 + \pi)$.

Step 2. condition 1 implies

$$h^M_1 = f(h^C_1) \rightarrow y^M(\mu^s(h^M_1), 0) - y^M(\mu^b(h^M_1), 0) > y^C(\mu^s, h^C_1, 0) - y^C((\mu^s, h^C_1) + \mu_1)/2, 0).$$

Where $y^M(\mu_t, h_t) := \pi^2 + (1 - \pi^2)(1 - \mu_t)(1 - h_t)$ and

$$y^C(\mu^l_t, \mu^u_t h^C_t, h^M_t) := \pi^2 + \pi(1 - \pi)\Sigma_{II}(1 - \mu^l_t)(1 - h^C_t).$$

This is the case as

$$h^M_1 = f(h^C_1) \rightarrow \mu^b - \mu^s > (\mu^b + \mu_1)/2 - \mu^s, \forall \mu_1 \geq \max \{0, g(h^C_1, \pi)\} \text{ (condition 2).}$$

Where $g(h^C_1, \pi) := (-2 + 4h^C_1 + 3\pi - 4h^C_1\pi - \pi^2 + 2h^C_1\pi^2)/(-4 + 4h^C_1 + 4\pi - 4h^C_1\pi - 2\pi^2 + 2h^C_1\pi^2).$ Note that $\partial g(h^C_1, \pi)/\partial \pi < 0$, $\partial g(h^C_1, \pi)/\partial h^C_1 < 0$ and $g(h^C_1, \pi) < 1/2$, $\forall h^C_1, \pi$.

Assumption 2 ensures that condition 2 holds.

$$\mu^b(h^M_1) - \mu^s(h^M_1) > (\mu^s, h^C_1, h^C_1) + \mu_1)/2 - \mu^s, (h^C_1, h^C_1) \rightarrow$$

$$(1 - \pi^2)(\mu^b(h^M_1) - \mu^s(h^M_1)) > 2\pi(1 - \pi)((\mu^s, h^C_1, h^C_1) + \mu_1)/2 - \mu^s, (h^C_1, h^C_1) \rightarrow$$

$$y^M(\mu^s(h^M_1), 0) - y^M(\mu^b(h^M_1), 0) > y^C(\mu^s, h^C_1, 0) - y^C((\mu^s, h^C_1) + \mu_1)/2, 0).$$

Step 3. $h^M_1 = f(h^C_1) \rightarrow y^M(\mu^s(h^M_1), 0) > y^C(\mu^s, h^C_1, 0)$.

3.a $y^M(\mu_t, 0) > y^C(\mu_t, \mu_t, 0, 0) \forall \mu_t < 1.$
3.b $\mu^g(h^C_1) = \mu_1 \pi^2/y^M(\mu_1, f(h^C_1)) = \\
= \mu_1 \pi^2/y^C(\mu_1, h^C_1) < (\mu_1 \pi + (1 - \pi)(1 - \mu_1)(1 - h^C_1))/y^C(\mu, h) = \\
= \mu^{g,s}(h^C_1, h^C_1).

Step 4. $p(g|\mu, h) = k(y^M(\mu_1, h_1))$ and $p(g, g|M, H) = k(y^C(\mu_1, \mu_1, h_1^G, h_1^H))$ where $k(x) := (1 - (2/(1 - \pi^2))x)/(1 - x).

So the price is decreasing and concave in $y$.

Therefore, by Steps 2 and 3: $p(g|\mu^b(h^M_1), 0) - p(g|\mu^s(h^M_1)) > \\
> p(g, g|\mu^{b,s}(h^C_1, h^C_1)) - p(g, g|\mu^{s,s}(h^C_1, h^C_1))$.

Step 5. The utility in the second period can be expressed as a function of the price in case of favorable ratings

$u^M_2(p) := e((1 - \pi^2)p) + (1 - e((1 - \pi^2)p)\pi^2)p - c(p) + \\
+ (1 - e((1 - \pi^2)p)(1 - \pi^2)p = p - c(e((1 - \pi^2)p)),$

$u^C_2(p) := (1/2)(e((1 - \pi^2)p) + (1 - e((1 - \pi^2)p)\pi^2))p - c(e((1 - \pi^2)p)) + \\
+ (1 - e((1 - \pi^2)p)\pi(1 - \pi)p,$

where $e((1 - \pi^2)p) = (1 - \pi^2)p/2$. Then

$$\partial u^M_2(p|g,.)/\partial p(g,.) > \partial u^C_2(p(g,|g))./\partial p(g,g,.) > 0$$

$\partial u^M_2(p)/\partial p = 1 - (1 - \pi^2)^2p/2 > 0$ and $d^2 u^M_2(p)/dp^2 = -(1 - \pi^2)^2/2 < 0.$

$\partial u^C_2(p)/\partial p = (1 - \pi^2)/2 ((1 - \pi^2)/2 - \pi(1 - \pi)) p + (1 - (1 - \pi^2)p/2)\pi(1 - \pi) > 0.$

So $\partial u^M_2(p)/\partial p > \partial u^C_2(p)/\partial p \forall p$.
Step 6. By steps 4 and 5 $e_1^M = e_1^C$ implies

$$p(g|\mu^b(h_1^M), 0) = p(g, g|\mu^{b, g}(h_1^C, h_1^C))$$

and if $p(g|\mu_1, h_1^M, 0) \geq \delta(u_2^M(\mu^b(h_1^M)) - u_2^M(\mu^g(h_1^M)))$ then

$$p(g, g|\mu^{b, g}(h_1^C, h_1^C)) > \delta(u_2^C(\mu^{g, b}) - u_2^C(\mu^{g, g})).$$

This implies that for $\delta \geq \delta^C e_1^M > f(h_1^C)$. As $e^M$ is a continuous function of $\delta$ and $e^M < e^C$ for $\delta = 0$, then it must be the case that $\delta^* < \delta^C$

As $\delta^* < \delta^C$ and $\delta^{**} < \delta^C \delta^{**} > \delta^*$ iff $f^S(0) > f(0)$ ($f^S(0)$ is defined in the Proof of Proposition 17) which is equivalent to $\mu_1 > (1 - 2\pi)/(1 + \pi)$ $\square$

**Proof of Lemma 34.** (1.6) and (3.4) hold at the same time iff

$$p(g, g|M, \overline{H}) = f^L(e((1 - \pi)(1 + \pi - (\pi/2)\Sigma_{I, II}(1 - \mu^I_j)(1 - h^I) + (1 - \pi^2)\Pi_{I, II}(1 - \mu^I_j)(1 - h^I))) p(g, g|M, \overline{H})),$$

where $f^L(x) := x/(x + (1 - x)a^L)$, and

$$a^L := \pi^2 + \Sigma_{I, II}(1 - \pi)\pi(1 - \mu^I_j)(1 - h^I) + (1 - \pi^2)\Pi_{I, II}(1 - \mu^I_j)(1 - h^I).$$

Applying the same steps followed in Lemma 2, the necessary and sufficient condition for the existence of an $x \in (0, 1): x = f^L(e((1 - \pi)(1 + \pi - (\pi/2)\Sigma_{I, II}(1 - \mu^I_j)(1 - h^I)x)))$ is:

$$\partial f^L(e((1 - \pi)(1 + \pi - (\pi/2)\Sigma_{I, II}(1 - \mu^I_j)(1 - h^I)x))) / \partial x |_{x=0} > 1$$ (i). (i) holds for every $\overline{H} \in [0, 1]^2$ iff

$$\mu^I_j + \mu^I_j \mu^I_j (1 - \pi) c''(0)/(c''(0) + \pi) > c''(0)/(c''(0) + \pi)(1 - \pi) - (1 - \pi)/(c''(0) + \pi)$$
Let
\[ \tilde{\phi}_t := (e^*_t + (1 - e^*_t) (\pi + \pi (1 - \pi)) \Sigma_{i,II} (1 - \mu^t_i) (1 - \tilde{h}^t_i) / 2) p(g, g|M_t, \Pi) - c(e^*_t). \]

I rule out pathological equilibria in which some type of rating agency requests \( \phi^*_t > \tilde{\phi}_t \). In any equilibrium in which \( \mu(\phi_t) = \mu_t \forall \mu_t \) Assumption 5 ensures that both types of agency set \( \phi^*_t = \tilde{\phi}_t / 2 \). If instead in equilibrium \( \mu^t(\phi'_t) \neq \mu^t_t \) for some \( \phi'_t \), by Assumption 2 \( \phi'_t \) must be on the equilibrium path for some type of agency \( \Lambda^t \). But this implies that there exists at least another \( \phi''_t \) on the equilibrium path s.t. \( \mu^t(\phi'_t) \neq \mu^t(\phi''_t) \). W.l.g. let \( \mu^t(\phi'_t) > \mu^t(\phi''_t) \) (i). If the honest type strictly prefers \( \phi''_t \) over \( \phi'_t \), then \( \mu^t(\phi'_t) = 0 \) which contradicts (i). If the honest type is indifferent or strictly prefers \( \phi'_t \) over \( \phi''_t \), then the strategic type strictly prefers \( \phi'_t \) over \( \phi''_t \) because \( \beta(M(\phi'_t), \Pi) > \beta(M(\phi''_t), \Pi) \), which implies that either \( \mu^t(\phi''_t) = 1 \) which contradicts (i) or \( \phi''_t \) is out of the equilibrium path, which is also a contradiction □

**Appendix to Chapter 2**

**Proof of Lemma 19.** There exists an equilibrium in which buyers bid: \( b_c \leq b_{nc} = V_L \) and the issuer chooses \( r_q = 0 \) for any \( q \) and any schedule of fees. In this equilibrium, certification does not take place □

**Proof of Lemma 20.** For a schedule of fees \( \phi_I, \phi_R \) the agency chooses \( e(\phi_I, \phi_R) > 0 \) only if she expects the reputation cost from rating with no information to be higher than the cost of effort. The expected reputation cost depends on her beliefs over the participation choice of the issuer, denoted as \( r^I_H(\phi_I, \phi_R) \) and \( r^L_H(\phi_I, \phi_R) \). Choosing \( e(\phi_I, \phi_R) > 0 \) is optimal whenever the following condition
holds:

\[(1 - \alpha) r^*_H (\rho - \phi_R) / (\alpha r^*_H + (1 - \alpha) r^*_L) \geq C > 0 \rightarrow \rho - \phi_R > 0.\]

Therefore, whenever \(e(\phi_I, \phi_R) > 0\) is optimal, \(c(\phi_I, \phi_R) = 0\) is the only seq. rational choice \(\Box\)

**Proof of Proposition 21.** Assume:

\[C > (1 - \alpha) \rho, \quad (3.8)\]

and consider the case \(\beta V_L \geq (1 - \alpha) \rho\). In the unique set of equilibria with certification the agency announces \(\phi_I + \phi_R \equiv \phi = \beta (b_c - b_{nc})\) and chooses \(e = 0\): as the issuer chooses \(r_q = 1\) for any \(q\) her expected payoff is:

\[U_C = \beta (b_c - b_{nc}) - (1 - \alpha) \rho \equiv \overline{U}.\]

where \(b_c = V_L\) and \(b_{nc} \in [0, b_c - \frac{1 - \alpha}{\beta} \rho]\). For these strategies issuer and buyers have no profitable deviations and neither does the agency: if \(\phi > b_c - b_{nc}\), then \(U_C = 0\), as \(r_H = r_L = 0\), if instead \(\phi \in (\beta (b_c - b_{nc}), b_c - b_{nc}]\), then \(U_C = (1 - \alpha)(b_c - b_{nc} - \rho) < \overline{U}\), by Assumption 1 as \(r_H = 0\) and \(r_L = 1\). Finally, if she announces \(\phi < \beta (b_c - b_{nc})\), then \(U_C \leq \phi - (1 - \alpha) \rho < \overline{U}\), as \(r_H = 1\) and \(r_L \in [0, 1]\). For \(r_H = r_L = 1\) \((3.8)\) implies that \(e = 0\) is the unique seq. rat. choice and if the agency is expected to choose \(e = 0\), \(r_L = 1\) is the unique seq. rat. choice, while \(r_H = 1\) is the unique choice in equilibrium: if \(r_H < 1\) the equilibrium is not defined as the agency can get a payoff indefinitely close to \(\overline{U}\) by announcing:

\[\phi = \beta (b_c - b_{nc}) - \varepsilon \text{ for } \varepsilon \to +0.\]

Finally, in equilibrium it can not be the case that \(r_H\) and \(r_L\) are s.t. the agency finds it seq. rational to set \(e > 0\), as in that case the agency obtains a payoff \(U_C < \overline{U}\) and an equilibrium is not defined by
the argument presented in the last paragraph for the $r_H < 1$ case.

Therefore, for $\beta V_L - (1 - \alpha)\rho \geq 0$ in the unique equilibrium with certification $\tilde{v}_c = V_L$ and $\tilde{v}_{nc}$ is not defined. A corollary of this proof is that for $\beta V_L < (1 - \alpha)\rho$ there is no equilibrium with certification: in equilibrium the agency announces $\phi \geq b_c - b_{nc}$, and induces:

$$r_H = r_L = 0 \text{ and } b_c < b_{nc},$$

where $b_{nc} = \tilde{v}_{nc} = V_L$, while $\tilde{v}_c$ is not defined $\square$

Proof of Proposition 22. With an abuse of notation, I refer to $r_H \in [0,1]$ as a strategy in which the agency participates w. p. $r_H$ for $q = H$ (and similarly for $r_L$). Following $\phi_I, \phi_R : \phi_I + \phi_R > \beta(b_c - b_{nc})$, $r_H = 0$ and the maximum expected payoff for the agency equals $U_1 \equiv \max\{0,(1 - \alpha)(b_c - b_{nc} - \rho)\}$. Let the agency announce:

$$\phi_I, \phi_R : \phi_I + \phi_R \leq \beta(b_c - b_{nc}).$$

Consider first the case $\phi_R \leq \rho - C/(1 - \alpha)$.

If $\phi_I + \phi_R < \beta(b_c - b_{nc})$, the set of seq. rat. choices of the issuer are $r_H = 1$ and $r_L = r(r_H, \phi_R)$, if $\phi_I > 0$, or $r_L \in [r(r_H, \phi_R), 1]$ if $\phi_I = 0$. Where:

$$r(x,y) \equiv \alpha x C/(1 - \alpha)(\rho - C - y)$$

The choice of effort is

$$e = f(\phi_I, \phi_R, b_c - b_{nc}) \text{ if } r_L < 1, \quad e \in [0, f(\phi_I, \phi_R)] \text{ if } r_L = 1.$$ 

Where: $f(x,y,z) \equiv 1 - \frac{x}{z-y}$. The agency expected payoff is: $U_2(\phi_I, \phi_R) \equiv \alpha(\phi_I + \phi_R) + \frac{\alpha C(\phi_I + \phi_R - \rho)}{(\rho - C - \phi_R)}$

if $\phi_I > 0$, and some $x \leq U_2(\phi_I, \phi_R)$ if $\phi_I = 0$. Note that:
\[
\sup_{\phi_r < \beta(b_c - b_{nc}) - \phi_R} U_C = \overline{U_2}(\phi_R) \equiv U_2(\beta(b_c - b_{nc}) - \phi_R, \phi_R), \forall \phi_R.
\]

If \( \phi_I + \phi_R = \beta(b_c - b_{nc}) \), for \( \phi_I > 0 \) there is a continuum of sets of seq. rat. choices characterized by:

\[
r_H \in [0, 1], r_L = r(r_H, \phi_R) \quad \text{and} \quad e = f(\phi_I, \phi_R, b_c - b_{nc}).
\]

The agency payoff is given by \( U_C = r_H \overline{U_2}(\phi_R) \).

If instead \( \phi_I = 0, r_H \in [0, 1], r_L = [r(r_H, \phi_R), 1] \) and \( e = f(0, \phi_R, b_c - b_{nc}) = 1 \) and the payoff satisfies \( U_C \leq r_H \overline{U_2}(\phi_R) \).

Let instead \( \phi_R > \rho - \frac{C}{1-\alpha} \). If \( \phi_I < \beta(b_c - b_{nc}) - \phi_R \), the set of seq. rat. choices are: \( r_H = 1, \ r_L = 1 \) and \( e = 0 \). The payoff is

\[
U_3(\phi_I, \phi_R) = \phi_I + \phi_R - (1-\alpha)\rho.
\]

Note that: \( \sup_{\phi_r \not< \beta(b_c - b_{nc}) - \phi_R} U_3(\phi_I, \phi_R) = \overline{U_3}(\phi_R) \equiv U_3(\beta(b_c - b_{nc}) - \phi_R, \phi_R), \forall \phi_R \).

If instead \( \phi_I = \beta(b_c - b_{nc}) - \phi_R \), there is a continuum of sets of seq. rat. choices characterized by

\[
r_H \in [0, 1] \quad \text{and} \quad r_L = \min \{1, r(r_H, \phi_R)\}.
\]

The effort choice is: \( e = 0 \) if \( r(r_H, \phi_R) > 1, e = f(\phi_I, \phi_R, b_c - b_{nc}) \) if \( r(r_H, \phi_R) < 1 \) and \( e \in [0, f(\phi_I, \phi_R, b_c - b_{nc})] \) otw. For any given \( \phi_R \) the largest expected payoff is obtained for \( r_H = 1 \) and equals \( \overline{U_3}(\phi_R) \). If in a seq. equlilibrium the agency announces with pos. prob. fees that satisfy (1), it must be the case that \( \phi_I + \phi_R = \beta(b_c - b_{nc}) \) and \( r_H = 1 \) for those fees, as in any other case the agency has a profitable deviation. Note that

\[
\overline{U_2}(\rho - \frac{C}{1-\alpha}) = \overline{U_3}(\phi_R), \forall \phi_R \geq \rho - \frac{C}{1-\alpha}, \text{ and } \overline{U_2}(\rho - \frac{C}{1-\alpha}) > U_1
\]

by Assumption 1. Moreover, \( \overline{U_2}(\phi_R) \) is continuous, differentiable and \( \overline{U_2}(\phi_R) > (\leq) 0 \) iff \( \beta(b_c - b_{nc}) > (\leq) \rho \).

Therefore in any equilibrium with certification \( \phi_I = \beta(b_c - b_{nc}) - \phi_R \), and

- if \( \beta(b_c - b_{nc}) > \rho: \phi_R \in [\rho - C/(1-\alpha), \beta(b_c - b_{nc})], \)
• if $\beta(b_c - b_{nc}) = \rho$: $\phi_R \in [0, \beta(b_c - b_{nc})]$,

• if $\beta(b_c - b_{nc}) < \rho$:

  - $\phi_R = 0$ if $U_2(0) \geq 0$ ($\leftrightarrow \beta(b_c - b_{nc}) \geq C$),

  - $\phi_R > b_c - b_{nc} \rightarrow r_H = r_L = 0$ and no certification otherwise $\square$

**Proof of Proposition 23.** Define

$$v_c(x, y) \equiv \frac{a\beta-(1-\alpha)r(x)(1-f(\beta y-x,x,y))}{a\beta+(1-\alpha)r(x)(1-f(\beta y-x,x))},$$

where $f(.)$ and $r(.)$ are defined as in the Proof of Result 22. $v_c(\phi_R, b_c - b_{nc})$ is the expected value of a certified security when $\psi = L, r_H = 1, r_L = r(1, \phi_R), \phi_l + \phi_R = \beta(b_c - b_{nc})$ and $e = f(\phi_l, \phi_R, b_c - b_{nc})$. Note that for

$$\phi_R \in [0, \beta(b_c - b_{nc})] \cap [0, \rho - \frac{C}{1-\alpha}],$$

$v_c(\phi_R, b_c - b_{nc})$ is continuous and differentiable. From Result 22, for $\beta(b_c - b_{nc}) \in [C, \rho), \phi_l = \beta(b_c - b_{nc}), \phi_R = 0$ and therefore

$$\bar{v}_c = v_c(0, b_c - b_{nc}) = (\rho - 2C)/C,$$

and $\bar{v}_{nc} = -1$. An equilibrium with $\beta(b_c - b_{nc}) \in [C, \rho)$ exists iff

$$(C, \rho) \in \text{set}_1 \equiv \{(x_1, x_2) \in \mathbb{R}_+^2 : (\beta(x_2 - 2x_1)/x_2) \in [x_1, x_2]\}.$$  

This condition is equivalent to:

1) $\beta(\rho - 2C/\rho) < \rho \leftrightarrow \rho \notin [\beta - \sqrt{\beta^2 - 8C\beta}] / 2, f_2(C)],$

2) $\beta(\rho - 2C/\rho) \geq C \leftrightarrow C \leq \beta \rho / (2\beta + \rho).$

Where $f_2(C) \equiv (\beta + \sqrt{\beta^2 - 8C\beta}) / 2$. As $\partial v_c(x, \rho / \beta) / \partial x < 0$, an equilibrium in which $\beta(b_c - b_{nc}) = \rho$ exists iff:
(C, ρ) ∈ set₂ = { (x₁, x₂) ∈ ℝ⁺₂ : β(x₂ - 2x₁/x₂) ≥ x₂ }

or equivalently ρ ∉ [(β - √(β² - 8Cβ))/2, f₂(C)]. In particular, \( \tilde{v}_c = \max \{ \rho / \beta, V_L \} \), and \( \tilde{v}_{nc} = -1 \) whenever \( \tilde{v}_c > V_L \) (for \( \tilde{v}_c = V_L \), \( \tilde{v}_{nc} \) is not defined). If \( \rho / \beta ∈ (v_c(\rho - C/(1 - \alpha), \rho / \beta), v_c(0, \rho / \beta)] \), in equilibrium the agency announces \( \phi_R : v_c(\rho, \rho / \beta) \), \( e = f(\rho - \phi_R, \phi_R, \rho / \beta) \) and \( r_L = r(1, \phi_R) \).

If instead \( \rho / \beta ∈ (V_L, v_c(\rho - C/(1 - \alpha), \rho / \beta)] \), in equilibrium \( \phi_R = \rho - C/(1 - \alpha) \), \( r_L = 1 \) and \( e \) satisfies

\[
(\alpha\beta - (1 - \alpha)(1 - e))/(\alpha\beta + (1 - \alpha)(1 - e)) = \rho / \beta.
\]

If \( \rho / \beta ≤ V_L \), then \( \phi_R = \rho - C/(1 - \alpha) \), \( e = 0 \) and \( b_{nc} = V_L - \rho / \beta \).

From Proposition 22, for \( \beta(b_c - b_{nc}) > \rho \) the agency sets \( \phi_I = \beta(b_c - b_{nc}) - \phi_R \) and \( \phi_R ≥ \rho - C/(1 - \alpha) \). If \( \phi_R > \rho - C/(1 - \alpha) \), then \( \tilde{v}_c = V_L \) and \( \tilde{v}_{nc} \) is not defined. When instead \( \phi_R = \rho - C/(1 - \alpha) \), either \( \tilde{v}_c = V_L \) and \( \tilde{v}_{nc} \) is not defined, or \( \tilde{v}_c ∈ (V_L, v_c(\rho - C/(1 - \alpha), b_c - b_{nc})] \) and \( \tilde{v}_{nc} = -1 \). Wlg I consider only equilibria in which \( \phi_R = \rho - C/(1 - \alpha) \). Note that \( \partial v_c(x, y)/\partial y < 0, \forall x, y, \) and

\[ 1 > v_c(\rho - C/(1 - \alpha), 1) \]  

Therefore. iff \( \rho / \beta < v_c(\rho - C/(1 - \alpha), \rho / \beta) \) there exists a unique \( x ∈ (V_L, 1) \cap (\rho / \beta, 1) \) that satisfies \( x = v_c(\rho - C/(1 - \alpha), x) \). Denoting \( \tilde{v}_2 \) this unique value, then

\[ \tilde{v}_2 = \tilde{v}(\rho - C/(1 - \alpha)) \], where:

\[ \tilde{v}(x) = (2α - 1 + (\alphaβ + 1 - \alpha)x/\beta + \sqrt{(1 - 2α - (\alphaβ + 1 - \alpha)x/\beta)^2 - 4x(\alpha - (1 - \alpha)/\beta)})/2. \]

\( \rho / \beta < v_c(\rho - C/(1 - \alpha), \rho / \beta) \) is a necessary and sufficient condition for an equilibrium in which \( \beta(b_c - b_{nc}) > \rho \) to exist. Under Assumptions 1 and 2, the condition is equivalent to: \( \rho < f_1(C) \), where

\[ 7 \partial v_c(x, y)/\partial y < 0 \text{ if } v_c(x, y) ≥ 0 \text{ which is ensured } \forall x, y \text{ by Assumption 1.} \]
\[ f_1(C) \equiv (\beta - \frac{C}{1-\beta} \left( \frac{a \beta + 1 - \alpha}{\alpha(1-\alpha)} \right)) + \sqrt{\left( \beta - \frac{C}{1-\beta} \left( \frac{a \beta + 1 - \alpha}{\alpha(1-\alpha)} \right) \right)^2 + \frac{4C \beta}{1-\beta} \left( \frac{\beta}{1-\alpha} - \frac{1}{\alpha} \right)}/2. \]

Let \[ \text{set}_3 \equiv \{(x_1, x_2) \in \mathbb{R}^2_+ : x_2 < f_1(x_1)\} \]. If \((C, \rho) \in \text{set}_3\), for any \(z \in [V_L, \tilde{v}_2] \cap (\rho/\beta, \tilde{v}_2]\) there exists an equilibrium ensuring \(\tilde{v}_c = z\). If \(\tilde{v}_c > V_L, \tilde{v}_{nc} = -1\), otherwise \(\tilde{v}_{nc}\) is not defined.

Finally, I show that \(\text{set}_3 \subset \text{set}_2\), and therefore for any set of parameters for which an equilibrium with \(\beta (b_c - b_{nc}) > \rho\) exists there is no equilibrium with \(\beta (b_c - b_{nc}) < \rho\). Assume an equilibrium with \(\beta v_c > \rho\) exists. This implies \(\tilde{v}_2 > \rho/\beta\). Note also that:

\[ (\rho - 2C)/\rho = v_c(0, \rho/\beta) > v_c(\rho - C/(1 - \alpha), \rho/\beta) > v_c(\rho - C/(1 - \alpha), \tilde{v}_2) = \tilde{v}_2 > \rho. \]

So \(\beta \tilde{v}_2 > \rho\) implies \(\beta (\rho - 2C)/\rho > \rho\) or equivalently \(\text{set}_3 \subset \text{set}_2\).

**Proof of Proposition 24.** If \(C > (1 - \alpha)\rho\), in any equilibrium in which \(r_H > 0\) and/or \(r_L > 0\), the agency chooses \(e = 0\) regardless of \(\phi_R\). Therefore for any cap \(\tilde{\phi}_R \geq 0\) the equilibria are identical to the one described in Result 23.

Let instead \(C \leq (1 - \alpha)\rho\). If \(C > \beta \rho/(2 \beta + \rho)\) from the Proof of Result 22 and the Proof of Result 23 certification does not take place. Moreover for any set of bids, the fees that ensure the highest payoff for the agency while ensuring certification with positive probability are \(\phi_l = \beta (b_c - b_{nc})\) and \(\phi_R = 0\). An equilibrium with those fees does not exist as it would entail a negative expected payoff of the agency. Therefore imposing a cap has no effect.

If instead \(C < \beta \rho/(2 \beta + \rho)\), in equilibrium if \(\phi_R > \rho - C/(1 - \alpha)\), \(\tilde{v}_c = V_L\) and \(\tilde{v}_{nc} = -1\). If \(\phi_R \leq \rho - C/(1 - \alpha)\) the most informative equilibrium (which is also the unique for \(\phi_R < \rho - C/(1 - \alpha)\)) ensures \(\tilde{v}_{nc} = -1\) and \(\tilde{v}_c = v_c(\phi_R, \tilde{v}_c)\) (for the definition of \(v_c(\ldots)\) see the Proof of Result 23).
Therefore \( \tilde{v}_c \) is a continuous and differentiable function of \( \phi_R \). If \( \beta \tilde{v}_c \geq \rho - C \), then \( \partial \tilde{v}_c / \partial \phi_R \leq 0 \).

If instead \( \beta \tilde{v}_c < \rho - C \):

\[
\frac{\partial \tilde{v}_c}{\partial \phi_R} > 0 \text{ iff } \phi_R > \beta \tilde{v}_c - \sqrt{(1 - \beta)(\rho - C - \beta \tilde{v}_c)\tilde{v}_c}
\]

In equilibrium the agency sets \( \phi_R > 0 \) only if \( \beta (b_c - b_{nc}) \geq \beta \tilde{v}_c \geq \rho - C \rightarrow \beta \tilde{v}_c \geq \rho - C \).

Therefore imposing a cap \( \overline{\phi}_R = 0 \) maximizes \( \tilde{v}_c \) in equilibrium. And from Result 23 it ensures \( \tilde{v}_c = (\rho - 2C)/\rho \).

**Proof of Proposition 25.** As \( C = 0 \) it is wlg to consider only equilibria with \( e = 1 \). Let \( \rho > \beta \). On the equilibrium path the agency sets: \( \phi_I^*, \phi_R^* : \phi_I^* + \phi_R^* = \beta \), chooses \( c = 0 \) and the issuer chooses: \( r_H = 1 \), \( r_L = 0 \) and the buyers bid \( b_c = 1 \), \( b_{nc} = 0 \). The issuer has no profitable deviations and the expected payoff of the agency is \( \pi = \alpha \beta \). The agency cannot obtain a higher payoff by announcing different fees: setting \( \phi_I, \phi_R : \phi_I + \phi_R < \beta \) ensures \( \pi \leq \alpha (\phi_I + \phi_R) < \alpha \beta \), while choosing \( \phi_I, \phi_R : \phi_I + \phi_R > \beta \) ensures, by Assumption 1:

\[
\pi = \max \{0, (1 - \alpha)(1 - \rho)\} < (1 - \alpha)(1 - \beta) < \alpha \beta.
\]

This is the unique equilibrium as for \( \rho > \beta \) the agency cannot obtain more than \( \pi = \alpha \beta \) in equilibrium. Moreover, this is the unique equilibrium ensuring \( \pi = \alpha \beta \), and any set of strategies that ensures \( \pi < \alpha \beta \) cannot be an equilibrium: the agency can obtain profits \( \pi' > \pi \) by setting:

\( \phi_I + \phi_R = \beta - \varepsilon \), for \( \varepsilon = (\alpha \beta - \pi)/2\alpha \).

Let \( \rho \leq \beta \). An equilibrium with \( b_c - b_{nc} < \rho / \beta \) does not exist. For those bids the agency sets \( \phi_I + \phi_R = \beta (b_c - b_{nc}) \) and certifies only high-quality security in which case \( \tilde{v}_c = 1 \) and \( \tilde{v}_{nc} = -1 \rightarrow \tilde{v}_c \neq b_c \).
For $b_c - b_{nc} \geq \rho / \beta$, the unique strategies that can be part of an equilibrium are: for the agency to
set $\phi_I = \beta (b_c - b_{nc}) - \phi_R$ and $\phi_R \geq \rho$ and choose
\[
e \in [(\beta (b_c - b_{nc}) - \rho) / (b_c - b_{nc} - \rho), 1],
\]
if $\phi_R = \rho$ and $c = 1$, if $\phi_R > \rho$ and for the issuer to choose $r_H = r_L = 1$ for those fees. In equilibrium,
\[
\tilde{v}_c = f(c), \text{ where}
\]
\[
f(x) \equiv \left( \alpha \beta - (1 - \alpha) x \right) / (\alpha \beta + (1 - \alpha) x),
\]
and therefore $\tilde{v}_c \in [f((\beta (b_c - b_{nc}) - \rho) / (b_c - b_{nc} - \rho))]].$

In equilibrium, it must be the case that $\tilde{v}_c = b_c$ and whenever $\tilde{v}_c > f(1)$, $b_{nc} = 0$, therefore
\[
\tilde{v}_c \leq f((\beta \tilde{v}_c - \rho) / (\tilde{v}_c - \rho)), \text{ which is equivalent to } \tilde{v}_c \leq \tilde{v}^\ast (\rho). \text{ } \tilde{v}^\ast (x) \text{ is defined as in the Proof of Result 23. Any } \tilde{v}_c \in [\max \{V_L, \rho / \beta\}, \tilde{v}^\ast (\rho)] \text{ can be sustained in equilibrium, if the agency chooses the appropriate } c \in [0, (\beta (b_c - b_{nc}) - \phi_R) / (b_c - b_{nc} - \phi_R)] \Box
\]

**Proof of Lemma 26.** $\phi_R \leq \phi_I < \rho$ $\rightarrow$ in equilibrium $c = 0, \phi_I + \phi_R = \beta, r_H = 1$ and $r_L = 0$. Therefore,
\[
\tilde{v}_c = 1 \text{ and } \tilde{v}_{nc} = -1 \Box
\]

**Appendix to Chapter 3**

**Proof of Lemma 27.** Assume $p > \lambda^\ast$. At this price, the consumer purchases only if $s_t = H$. Then
firm profits are $\pi(p) = \phi(\lambda) p (1 - \frac{p}{\lambda^\ast})$, and the price that maximizes profits is $p^\ast (\lambda) = \frac{\lambda^\ast}{\frac{\lambda^\ast}{2} + \alpha^2}$. Note that $p^\ast (\lambda) = \frac{\lambda^\ast}{\frac{\lambda^\ast}{2} + \alpha^2} \geq \lambda^\ast$ iff: $\alpha \geq 2 - \sqrt{2}$ and $\lambda < \frac{2 - 4 \alpha + \alpha^2}{2 - 5 \alpha + 2 \alpha^2}$.

Assume instead that $p < \lambda^\ast$. Then, firm profits are $\pi(p) = p (1 - (1 - \frac{\phi(\lambda)}{\lambda}) + \frac{\phi(\lambda)}{\lambda} p)$, and the price that maximizes profits is $p^{**} (\lambda) = \frac{\lambda^\ast + \lambda^\ast}{2 (\lambda^\ast + \phi(\lambda) / (\lambda - \lambda^\ast))}$. Notice that $p^{**} < \lambda^\ast \leftrightarrow
\[ \alpha \leq \frac{1}{2} \sqrt{\frac{-4 + 12\lambda - 7\lambda^2}{(5 - 14\lambda + 8\lambda^2)^2}} + \frac{6 - 15\lambda + 8\lambda^2}{2(5 - 14\lambda + 8\lambda^2)}. \]

Comparing the profits for the two prices:

\[ U^F(p^*(\lambda)) > U^F(p^{**}(\lambda)) \iff \phi(\lambda)p^*(\lambda)(1 - \frac{p^*(\lambda)}{\lambda}) > p^{**}(\lambda)(1 - \frac{1 - \phi(\lambda)}{\lambda} + \frac{\phi(\lambda)}{\lambda^2})p^{**}(\lambda) \rightarrow \]

\[ \frac{\phi(\lambda)p^*(\lambda)}{2} > \frac{p^{**}(\lambda)}{2} \rightarrow \lambda^+ \geq \frac{1 + \phi(\lambda)}{\phi'(\lambda)} \lambda^- \text{ (condition 1). Condition 1 is equivalent to } \alpha > \frac{2}{3} \text{ and } \lambda < \frac{3\alpha - 2}{2\alpha - 1}. \] Condition 1 implies \( p^* \geq \lambda^- \) and if condition 1 does not hold, then \( p^{**} \leq \lambda^- \). So the monopolist sets \( p = p^* \) iff condition 1 is satisfied \( \square \)

**Proof of Lemma 28.** We first show four useful properties of the *static* game pricing strategy.

**Property 1:** If \( \lambda \) satisfies condition 1 in Lemma 1: \( v(\lambda) = v^*(\lambda) := \phi(\lambda)p^*(1 - \frac{p^*}{\lambda}) = \frac{\alpha + (1 - \alpha)(1 - \lambda)}{4}. \)

Therefore the payoff is a *linear* function of \( \lambda \).

**Property 2:** If \( \lambda \) does not satisfy condition 1, then:

\[ v(\lambda) = v^{**}(\lambda) := p^{**}(\lambda)(1 - \frac{1 - \phi(\lambda)}{\lambda} + \frac{\phi(\lambda)}{\lambda^2})p^{**}(\lambda) = \frac{1}{4} \frac{\lambda^+ + \lambda^-}{\lambda^+ + \phi(\lambda)(\lambda^- - \lambda^+)}. \]

\[ \frac{\partial^2 v^{**}(p^{**}(\lambda))}{\partial \lambda^2} > 0 \text{ if } \lambda < \frac{2}{3} \text{ or } \lambda \geq \frac{2}{3} \text{ and } \alpha > \frac{1}{2} + \frac{1}{2} \sqrt{\frac{3\lambda - 2}{6 - 9\lambda + 4\lambda^2}}. \] Therefore the payoff is a convex function of \( \lambda \) if \( \lambda < \frac{2}{3} \) or \( \lambda \geq \frac{2}{3} \) and \( \alpha > \frac{1}{2} + \frac{1}{2} \sqrt{\frac{3\lambda - 2}{6 - 9\lambda + 4\lambda^2}} \) and it is a concave function otherwise.

**Property 3:** as \( \frac{\partial^3 v^{**}(p^{**}(\lambda))}{\partial \lambda^3} < 0 \forall\lambda \), then whenever condition 1 holds strictly, \( v^*(\lambda) > v^{**}(\lambda) \), and whenever condition 1 does not hold, then \( v^*(\lambda) < v^{**}(\lambda) \).

**Property 4:** There exists a \( \hat{\lambda} \) that satisfies \( \hat{\lambda} \in (0, 1) \) and satisfies the following equation

\[ \hat{\lambda}^3 f_3(\alpha) + \hat{\lambda}^2 f_2(\alpha) + \hat{\lambda} f_1(\alpha) + f_0(\alpha) = 0, \]
where:

\[ f_3(\alpha) := 1 - 6\alpha + 14\alpha^2 - 16\alpha^3 + 8\alpha^4 \]

\[ f_2(\alpha) := -2 + 14\alpha - 38\alpha^2 + 48\alpha^3 - 24\alpha^4 \]

\[ f_1(\alpha) := 1 - 10\alpha + 31\alpha^2 - 42\alpha^3 + 21\alpha^4 \]

\[ f_0(\alpha) := 2\alpha - 8\alpha^2 + 12\alpha^3 - 6\alpha^4 \]

and \( \phi(\lambda)v_2^*(\lambda^+) + (1 - \phi(\lambda))v_2^*(\lambda^-) > v_2^*(\lambda) \) if \( \lambda < \hat{\lambda} \), and \( \phi(\bar{\lambda})v_2^*(\bar{\lambda}^+) + (1 - \phi(\bar{\lambda}))v_2^*(\bar{\lambda}^-) < v_2^*(\bar{\lambda}) \) if \( \lambda > \hat{\lambda} \). Moreover \( \hat{\lambda} \geq (3\alpha - 2)/(2\alpha - 1) \).

**Property 5:** if \( \alpha > 2/3 \), there exists a \( \hat{\lambda} \) that satisfies \( \hat{\lambda} \in (0, 1) \) and satisfies the following equation

\[
\hat{\lambda}^5 g_5(\alpha) + \hat{\lambda}^4 g_4(\alpha) + \hat{\lambda}^3 g_3(\alpha) + \hat{\lambda}^2 g_2(\alpha) + \hat{\lambda} g_1(\alpha) + g_0(\alpha) = 0,
\]

where:

\[ g_5(\alpha) := \alpha - 8\alpha^2 + 32\alpha^3 - 16\alpha^4 + 16\alpha^5 \]

\[ g_4(\alpha) := 1 - 13\alpha + 62\alpha^2 - 140\alpha^3 + 152\alpha^4 - 64\alpha^5 \]

\[ g_3(\alpha) := -5 + 47\alpha - 179\alpha^2 + 346\alpha^3 - 344\alpha^4 + 152\alpha^5 - 16\alpha^6 \]

\[ g_2(\alpha) := 8 - 67\alpha + 234\alpha^2 - 432\alpha^3 + 437\alpha^4 - 224\alpha^5 + 44\alpha^6 \]

\[ g_1(\alpha) := -5 + 40\alpha - 137\alpha^2 + 255\alpha^3 - 270\alpha^4 + 153\alpha^5 - 36\alpha^6 \]

\[ g_0(\alpha) := 1 - 8\alpha + 28\alpha^2 - 54\alpha^3 + 60\alpha^4 - 36\alpha^5 + 9\alpha^6 \]

and \( \phi(\lambda)v_2(\lambda^+) + (1 - \phi(\lambda))v_2(\lambda^-) > v_2(\lambda) \) if \( \lambda < \hat{\lambda} \), and \( \phi(\bar{\lambda})v_2(\bar{\lambda}^+) + (1 - \phi(\bar{\lambda}))v_2(\bar{\lambda}^-) < v_2(\bar{\lambda}) \)
\( v^*_2(\lambda) \) if \( \lambda > \hat{\lambda} \). Moreover \( \hat{\lambda} \geq (3\alpha - 2)/(2\alpha - 1) \).

To prove part 1 of the Lemma, note that \( \bar{\lambda} > 0 \) only if \( \alpha > 2/3 \). Moreover, \( \lambda_1 < \bar{\lambda} \) implies \( \lambda_1^+ \leq \frac{3\alpha - 2}{2\alpha - 1} \) and therefore \( \lambda_1^-, \lambda_1 \) and \( \lambda_1^+ \) satisfy condition 1 in Lemma 27. Property 1 in turn implies that whenever \( \lambda_1^-, \lambda_1 \) and \( \lambda_1^+ \) satisfy condition 1 in Lemma 27 then \( Ev_2(\lambda_1) = v_2(\lambda_1) \) and the firm is indifferent to whether social learning takes place.

To prove part 2 and 3 of the Lemma, we consider two cases separately. If \( \alpha \leq 2/3 \), then property 4 implies part 2 and part 3 (and \( \bar{\lambda} = \hat{\lambda} \)). If instead \( \alpha > 2/3 \), then property 5 implies parts 2 and 3 of the lemma \( \Box \)

**Proof of Lemma 29.** The price \( p^+_1(\lambda_1) \) that satisfies \( p_1 \in [0, \lambda^-] \) and maximizes the two period payoff of the firm satisfies:

\[
p^+_1(\lambda_1) := \arg\max_{\lambda_1 > p > 0} \left( 1 - \left( \frac{\phi(\lambda_1)}{\lambda^-_1} + \frac{1 - \phi(\lambda_1)}{\lambda^-_1} \right)p \right) + \left( \frac{p}{\lambda^-_1} - \frac{p}{\lambda^+_1} \right)(Ev^*_2(\lambda_1) - v^*_2(\lambda_1)).
\]

Which is equivalent to (3.2). Moreover, for any \( p' \in [\lambda^-, \lambda^+] \) price:

\[
p'' := \frac{p'(1 - 2\alpha + \alpha^2 - \lambda + 3\alpha\lambda - 2\alpha^2\lambda - a\lambda + \alpha^2\lambda)}{1 - 2\alpha - \lambda + 2\alpha\lambda}
\]

satisfies \( \forall \alpha \in [\frac{1}{2}, \frac{2}{3}] \) and \( \forall \lambda \in [0, 1] \):

1) \( p'' \in [0, \lambda^-] \)

2) \( p'' \) ensures the same probability of social learning as \( p' \): \( (\frac{1}{\lambda^-} - \frac{1}{\lambda^+})p'' = 1 - \frac{p'}{\lambda^+} \)

3) \( p'' \) ensures a strictly larger current payoff than \( p' \): \( p''(1 - (\frac{\phi(\lambda)}{\lambda^+} + \frac{1 - \phi(\lambda)}{\lambda^-})p'') > \phi(\lambda)p'(1 - \frac{p'}{\lambda^+}) \)
This is enough to show that $p_1(\lambda_1) = \min\{\lambda^-, \max\{p_1^*(\lambda_1), 0\}\}$ \[ \square \]

**Proof of Lemma 30.** If $\lambda_1 \leq \frac{1 - 4\alpha}{4\alpha^2 + 1 - 4\alpha}$ then $\lambda_1^+ \leq \frac{3\alpha - 2}{2\alpha - 1}$ and therefore $EV_2(\lambda_1) = V_2(\lambda_1)$. As a result, as $\lambda_1 < (3\alpha - 2)/(2\alpha - 1)$ then $p_1(\lambda_1) = p^*(\lambda_1)$. We consider the case $\lambda_1 > \frac{1 - 4\alpha}{4\alpha^2 + 1 - 4\alpha}$ in the rest of the proof.

The price $p$ that satisfies $p \in [\lambda^-, \lambda^+]$ and maximizes the two period payoff of the firm is:

$$
\bar{p}_1^*(\lambda_1) := \arg\max_{\lambda_1^- \leq p \leq \lambda_1^+} \left[ p(1-\frac{p}{\lambda_1^+}) + \left( \frac{p}{\lambda_1^-} - \frac{p}{\lambda_1^+} \right) (EV_2(\lambda_1) - V_2(\lambda_1)) \right].
$$

Which is equivalent to

$$
p_1^*(\lambda_1) = \frac{1}{2} \left[ \lambda_1^+ + \frac{\delta}{\phi(\lambda_1)} (V_2(\lambda_1) - EV_2(\lambda_1)) \right] \tag{3.9}
$$

as long as $\frac{1}{2} [\lambda_1^+ + \frac{\delta}{\phi(\lambda_1)} (V_2 - EV_2(\lambda_1))] \in [\lambda_1^-, \lambda_1^+]$.

The price $p$ that satisfies $p \in [0, \lambda_1^-]$ and maximizes the two period payoff of the firm satisfies:

$$
\bar{p}_1^{**}(\lambda_1) := \arg\max_{\lambda_1^- \leq p \leq 0} \left[ p(1-\frac{\phi(\lambda_1)}{\lambda_1^+} + \frac{1 - \phi(\lambda_1)}{\lambda_1^-}) + \left( \frac{p}{\lambda_1^-} - \frac{p}{\lambda_1^+} \right) (EV_2(\lambda_1) - V_2(\lambda_1)) \right].
$$

Which is equivalent to (3.2).

To determine the optimal price we note that

I) If $\lambda_1 < \frac{3\alpha - 2}{2\alpha - 1}$ then:
for any \( \tilde{p} \in [0, \lambda^-] \) we can define

\[
\tilde{p} := \frac{\alpha \lambda_1 - \alpha^2 \lambda_1 + \tilde{p}(1 - 2 \alpha - \lambda_1 + 2 \alpha \lambda_1)}{1 - 2 \alpha + \alpha^2 - \lambda_1 + 3 \alpha \lambda_1 - 2 \alpha^2 \lambda_1}
\]

\( \tilde{p} \) satisfies:

I.a) \( \tilde{p} \in [\lambda_1^-, \lambda_1^+] \)

I.b) \( \tilde{p} \) ensures the same probability of social learning as \( \hat{p} \): \( (\frac{1}{\lambda_1} - \frac{1}{\lambda_1^+}) \hat{p} = 1 - \frac{\tilde{p}}{\lambda_1} \)

I.c) \( \tilde{p} \) ensures a strictly larger payoff in period 1 than \( \hat{p} \): \( \hat{p}(1 - (\frac{\phi(\lambda_1)}{\lambda_1^+} + \frac{1}{\lambda_1} - \frac{\phi(\lambda_1)}{\lambda_1^+}) \hat{p}) < \phi(\lambda_1) \hat{p}(1 - \frac{\hat{p}}{\lambda_1^+}) \).

II) If \( \lambda_1 > \frac{3\alpha - 2}{2\alpha - 1} \) then:

for any \( \tilde{p} \in [\lambda^-, \lambda^+] \) price:

\[
\tilde{p} := \frac{\tilde{p}(1 - 2 \alpha + \alpha^2 - \lambda_1 + 3 \alpha \lambda_1 - 2 \alpha^2 \lambda_1 - \alpha \lambda_1 + \alpha^2 \lambda_1)}{1 - 2 \alpha - \lambda_1 + 2 \alpha \lambda_1}
\]

satisfies \( \forall \alpha \in \left[\frac{1}{2}, \frac{2}{3}\right] \) and \( \forall \lambda \in [0, 1] \):

II.a) \( \tilde{p} \in [0, \lambda^-] \)

II.b) \( \tilde{p} \) ensures the same probability of social learning as \( \hat{p} \): \( (\frac{1}{\lambda_1} - \frac{1}{\lambda_1^+}) \hat{p} = 1 - \frac{\tilde{p}}{\lambda_1} \)

II.c) \( \tilde{p} \) ensures a strictly larger current payoff than \( \hat{p} \): \( \hat{p}(1 - (\frac{\phi(\lambda_1)}{\lambda_1^+} + \frac{1}{\lambda_1} - \frac{\phi(\lambda_1)}{\lambda_1^+}) \hat{p}) > \phi(\lambda_1) \hat{p}(1 - \frac{\hat{p}}{\lambda_1^+}) \).

(I) and (II) are enough to show that if \( \lambda_1 < \frac{3\alpha - 2}{2\alpha - 1} \) then \( p_1 \) satisfies \( p_1 = \min[\max[\lambda^-, \hat{p}_1], 1] \), while if \( \lambda_1 > \frac{3\alpha - 2}{2\alpha - 1} \) then \( p_1 \) satisfies \( p_1 = \max[\min[\lambda^-, \hat{p}_1^*], 1] \), where \( \hat{p}_1^* \) is defined in (3.2) \( \square \)

Proof of Lemma 31. The product is purchased regardless of the private signal if \( p \leq \lambda^- \) and it is purchased only if \( s = H \) in case \( p \in (\lambda^-, \lambda^+] \). Therefore the optimal price satisfies \( p \in \{\lambda^-, \lambda^+] \).
Profits from setting \( p = \lambda^- \) are \( u^F(\lambda^-) = \lambda^- \) while from setting \( p = \lambda^+ \) are \( u^F(\lambda^+) = \alpha\lambda^+ \).

\( u^F(\lambda^+) > u^F(\lambda^-) \) iff condition 2 holds. ☐

**Proof of Lemma 32.** The payoff from setting \( p_1 = \lambda_1^+ \) is equivalent to \( u^F(\lambda_1^+) + \delta\phi(\lambda_1)u^F(\lambda_1|\lambda_2 = \lambda_1^+) + (1 - \phi(\lambda_1))u^F(\lambda_1^-|\lambda_2 = \lambda_1^-) \) (1)

The utility from setting \( p_1 = \lambda_1^+ \) is equivalent to \( u^F(\lambda_1^-)(1 + \delta) \) (2).

(1) \( \geq \) (2) iff \( \delta \geq \bar{\delta} \) ☐

**Proof of Lemma 33.** If \( \lambda_1 \leq \frac{\alpha + \alpha^2 - 1}{2\alpha^2 - \alpha} \), then the firm sets \( p_1(\lambda_1) = \lambda_1^+ \), as this price maximizes both the current and the expected payoff. In this case the firm is indifferent to whether social learning takes place, and therefore it sets the static optimal price, that is \( p_1(\lambda_1) = \lambda_1^+ \). Otherwise the proof is identical to the proof of Lemma 32, except that in this case (1) \( \geq \) (2) iff \( \delta \geq \bar{\delta} \) ☐
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