Government guarantee, inflation-linked bonds, and investment with ambiguity and learning

Huang, Peng
BOSTON UNIVERSITY
GRADUATE SCHOOL OF ARTS AND SCIENCES

Dissertation

GOVERNMENT GUARANTEE, INFLATION-LINKED BONDS,
AND INVESTMENT WITH AMBIGUITY AND LEARNING

by

PENG HUANG

B.S., Nanjing University, 2004
M.A., State University of New York – Albany, 2009

Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
2015
Approved by

First Reader

Jianjun Miao, PhD
Professor of Economics

Second Reader

Adam Zawadowski, PhD
Assistant Professor of Finance

Third Reader

Zhongjun Qu, PhD
Associate Professor of Economics
Acknowledgments

I would like to express my sincere gratitude to my main advisor Zvi Bodie (on medical leave), Jianjun Miao and Adam Zawadowski for guidance and continuous encouragement throughout my doctoral research. In addition, I would like to thank Larry Kotlikoff, Zhongjun Qu, Pierre Perron and Stefania Garetto for the great support. Finally, I would like to thank my family, especially my wife Wei Jiang, for the love, support, and constant encouragement I have gotten over the years.
The broad aim of this dissertation is to explain some puzzling phenomena in Finance and Macroeconomics, focusing on the role of (1) government guarantee, (2) Inflation-Linked Bonds (ILBs) and (3) models with ambiguity and learning. I explore their quantitative and qualitative influences on consumption, investment and financing decisions.

The first chapter analyzes the effect of government guarantee cost as a new incentive to issue ILBs. During a political decision or reform process, a government typically has to provide some form of compensation to avoid noncompliance. The cost of this guarantee could be significant, and issuing ILBs instead of providing this guarantee would be a way for the government to avoid this cost. The model with this new feature provides a mechanism to explain why some countries issue ILBs with low inflation and also justifies why countries typically only issue small amounts of ILBs compared to nominal bonds, neither of which practices is well explained by the previous literature.

The second chapter introduces ambiguity and learning into a portfolio-choice model to explain some puzzling stylized facts, especially the hump-shaped share in risky asset in relation to age. I find that the ambiguity over labor income will make the agent not invest much in risky assets at the beginning of the working life. As the agent approaches retirement, there are two partially offsetting effects. First, the learning mechanism gradually solves the uncertainty. Second, the value of the bond position implicit in his human capital decreases. Eventually, this second effect prevails, and hence explains the hump-shaped
stock allocation of the agents life-cycle profile.

The third chapter discusses the role of ILBs in Chinas pension reform. Firstly it reviews the current problems of Chinas pension structure so as to find ways to improve the conditions of Chinese pensioners. In particular, I argue that the government could issue ILBs. Then by conducting a simulation using Chinas macro and financial data, I show that the ILBs will provide the investor with a significantly better risk-return trade-off.
## Contents

1 Concerns about Debt Service and Government Guarantee Cost: Revisiting the Inflation Issue

1.1 Introduction .............................................. 1

1.1.1 Main Incentives for Issuing ILBs .................. 3

1.1.2 Main Disincentives for Issuing ILBs ................. 5

1.2 Model ..................................................... 7

1.2.1 The Economy: a Non-Commitment Problem ............. 7

1.2.2 Definition of Markov Perfect Equilibrium (MPE) ....... 12

1.3 Quantitative Analysis .................................... 12

1.3.1 Empirical Findings .................................. 12

1.3.2 Benchmark Calibration and Numerical Solution ....... 15

1.3.3 Equilibrium Characterization and Discussion ........... 17

1.3.4 Comparative Statics and Robustness Check .............. 20

1.3.5 Stylized Facts Analysis Country by Country .......... 21

1.3.6 Extension of Baseline Model ......................... 23

1.4 Conclusion .............................................. 25

1.5 Appendix ................................................. 27

1.5.1 Appendix 1.A: Tables and Figures .................... 27

1.5.2 Appendix 1.B: Numerical Computation ................ 38

1.5.3 Appendix 1.C: Proof of Proposition 1 ................ 39

1.5.4 Appendix 1.D: Pricing Government Guarantee Costs using Martingale Pricing Theory ............ 40
## 2 Life-Cycle Investment with Ambiguity and Learning

2.1 Introduction ......................................................... 42
2.2 Model ................................................................. 46
  2.2.1 Utility ............................................................ 46
  2.2.2 Labor Income Process ........................................ 49
  2.2.3 The Kalman Filtering Learning .............................. 49
  2.2.4 Financial Market ............................................. 52
  2.2.5 Liquid Wealth and Individual Constraints ............... 52
  2.2.6 The Individual’s Optimization Problem .................. 53
2.3 Calibration ......................................................... 54
2.4 Results Analysis and Robustness Check .......................... 55
2.5 Conclusion ......................................................... 57
2.6 Appendix ........................................................... 59
  2.6.1 Appendix 2.A: Figures ....................................... 59
  2.6.2 Appendix 2.B: Numerical Computation .................... 64

## 3 China’s Pension Reform and Inflation-Linked Bonds

3.1 Introduction ......................................................... 65
3.2 Review of China’s Pension-Reform Structure ..................... 66
  3.2.1 Demographic Trend and the Aging Population of China .... 66
  3.2.2 Current Chinese Pension System ............................ 68
  3.2.3 Problems of China’s Pension system ........................ 73
  3.2.4 Proposals for China’s Pension System ..................... 79
3.3 Impact of Inflation-Linked Bonds on the Pension System: Case Study .... 83
  3.3.1 Review of Indexed Bonds in the Global Market ............ 83
  3.3.2 Current Inflation-Protection Strategies in China ........... 85
  3.3.3 Case Study of Hong Kong’s iBond ......................... 86
3.3.4 Empirical Study of China Mainland: Role of Inflation-Linked Bonds in Asset Allocation ........................................... 87
3.3.5 Role of Inflation-Linked Bonds in Providing Correct Incentives in Pension Reform: A Lesson from Chile ................... 92
3.4 Conclusion ........................................................................... 93
3.5 Appendix ............................................................................ 94

Bibliography .............................................................................. 97

Curriculum Vitae ........................................................................ 107
List of Tables

1. Incentives and Disincentives for Issuing ILBs .............................................................. 3
2. Comparison with Two Other Economies ..................................................................... 8
3. Timing of ILBs and Government Guarantees ............................................................ 13
4. Calibrated Parameters ............................................................................................... 15
5. Equilibrium Characterization .................................................................................. 17
6. Comparison between Calibrated and Estimated Guarantee Cost .......................... 27
7. Countries issuing ILBs ............................................................................... 27
8. ILBs Issuance and DB to DC Transition ................................................................. 28
9. Calibrated Guarantee Cost (Percent of GDP) ......................................................... 30
10. Age structure change of China in the past Ten years ........................................ 66
11. Historic Investment Performance of NSSF ........................................................... 78
12. Descriptive Statistics of Asset Return .................................................................... 89
13. Correlation Matrix for Different Assets ................................................................. 89
14. Non-indexed Bonds and Stock (Annual) ............................................................... 94
15. Indexed Bonds and Stock (Annual) ....................................................................... 94
16. Improvement of Sharpe Ratio (Annual) ............................................................... 94
17. Non-Indexed Bonds and Stock (Monthly) ............................................................ 95
18. Indexed Bonds and Stock (Monthly) ....................................................................... 95
19. Improvement of Sharpe Ratio (Monthly) ............................................................... 95
20. Non-Indexed Bonds and Stock (Quarterly) ........................................................... 96
21. Indexed Bonds and Stock (Quarterly) .................................................................... 96
22. Improvement of Sharpe Ratio (Quarterly) ............................................................ 96
List of Figures

1. Countries Issuing ILBs(1991-2013) ................................................................. 2
2. Timing of Dynamic Game Each Period .......................................................... 9
3. US Outstanding Real and Nominal Debt (2000-2012) ..............................15
4. A Baseline Time Pattern for ILBs Issuance ..............................................19
5. Government Total Expenditure ................................................................... 29
6. Comparative Statistics: $GR_H$ ................................................................... 31
7. Comparative Statistics: $GR_L$ ................................................................... 32
8. Comparative Statistics: $g_H$ ..................................................................... 33
9. Comparative Statistics: $g_L$ ..................................................................... 34
11. Comparative Statistics: Discount Factor .................................................... 36
12. Comparative Statistics: Total Debt .............................................................. 37
13. Life-Cycle Profile of Risky Assets Holdings (US Data) ............................. 44
14. Data Vs Baseline Model ............................................................................ 59
15. Average Consumption and Labor Income ............................................... 60
16. Different Degrees of Risk Aversion ............................................................ 61
17. Different Degrees of Ambiguity Aversion .................................................. 62
18. Different Levels of Wealth ....................................................................... 63
20. China’s Old-Age Dependency Ratio Projection (2015-2100) .................... 68
Chapter 1

Concerns about Debt Service and Government Guarantee: Revisiting the Inflation Issue

1.1 Introduction

Over the last half century or so, finance academics have increasingly proposed the issuance of Inflation-Linked Bonds (ILBs), instruments that provide a safe asset in a “real” sense, and that perfectly protect purchasing power for the long-term investor. James Tobin (1963) made one of the most convincing arguments in favor of inflation-indexed debt on behalf of households:

“...markets do not provide, at any price, a riskless way of accumulating purchasing power for the future, whether for old age, or for college education or for heirs. ...Meanwhile we force savers to take risk, even if they would gladly pay for the privilege of avoiding it. ...No private institution can fill this gap. No insurance company or pension fund could assume the risk of offering purchasing power escalation to its creditors without similarly (inflation) escalated securities in which to invest at least some of their funds.”

The “modern era” of ILBs started with the issue of UK index-linked Gilts in 1981, and over the last couple of decades, more than 20 countries have introduced ILBs (see Figure 1). The most recent include Denmark (2012), India (2013), and the Philippines (forthcoming).

---

1 Hereafter, I will call these bonds ILBs. Sometimes I also use indexed bonds or real debt. All these notations are equivalent in this paper.

2 Some countries have issued ILBs earlier, such as Brazil (1964), Israel (1955) and Argentina (1972). The documented earliest ILBs were issued by the State of Massachusetts in 1790.
There is a large literature discussing the advantages of ILBs, including completing financial markets, extracting information regarding expected inflation, lowering future inflation expectations, the cost of debt financing, and so forth. Both lowering inflation expectations and the cost of debt financing have been considered as the main incentives for governments to issue ILBs.\(^3\) However, it seems puzzling that a lot of countries have not yet decided to issue ILBs.\(^4\) Obviously a careful analysis needs to be conducted on the incentives and disincentives of issuing ILBs versus nominal bonds. Next, I will briefly discuss the incentives and disincentives of ILBs versus nominal bonds as well as the related literature. Empirical findings for these (dis)incentives will be discussed in Section 1.3.1.

**Figure 1: Countries Issuing ILBs(1991-2013)**\(^5\)

---

3For more general details about ILBs, see two excellent summaries by Campbell and Shiller (1996) and Campbell, Shiller, and Viceira (2009).

4A related “puzzle” is why the private financial sector rarely either provides this asset class or recommends it to their customers. The most plausible explanation is that the low volatility of ILBs is not favored by the finance industry. This is also confirmed by several financial professionals including fixed income fund managers.

5See Table 6 in Appendix A for complete country information.
Table 1: Incentives and Disincentives for Issuing ILBs

<table>
<thead>
<tr>
<th>Incentive</th>
<th>Disincentive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowering cost of debt financing and inflation expectation</td>
<td>Losing option of using surprise inflation</td>
</tr>
<tr>
<td>Completing financial market</td>
<td>(Bright side) hedging negative fiscal shock</td>
</tr>
<tr>
<td>Extracting information of expected inflation</td>
<td>(Dark side) repudiating debt</td>
</tr>
<tr>
<td>Avoiding potential government guarantee cost</td>
<td></td>
</tr>
</tbody>
</table>

1.1.1 Main Incentives for Issuing ILBs

1.1.1.1 Avoiding Government Guarantee cost

I first refer to this incentive since it has not been considered by previous literature and is one of the main contributions of this paper. This proposed incentive is based on empirical findings. Typically when facing political pressures in some structural change or reform, a government has to provide some form of government guarantee to ease the pressure, a guarantee which could be very costly.\(^6\) Hence, sometimes the government issues ILBs instead, and thus avoids the potential government guarantee cost they would have to pay otherwise.\(^7\) In this paper, I show that this incentive plays a significant role in explaining the timing of ILB issuance and why only small amounts of ILBs are issued compared to nominal bonds. A good example is what happens during the transition from the traditional pay-as-you-go Defined-Benefit (DB) to a fully-funded Defined- Contribution (DC), during which a main issue is that the contributor will bear all the investment risk. In order to gain support and avoid the evasion/non-compliance problem during this transition, the government generally gives an individual some form of compensation as an incentive to move to the new DC system. A typical compensation is the nominal return guarantee on

\(^6\)Hence, the government guarantee cost issue is actually a two-fold concern: whether the government will neglect this off-balance-sheet item and whether this cost could be correctly calculated.

\(^7\)In Section 1.3.1, I will discuss in detail how this incentive is supported by the observed facts.
the DC portfolio,\(^8\) such as that instituted in Belgium, Hungary, Poland, and Switzerland.\(^9\) Alternatively, some countries choose to issue ILBs (see Table 7 in Appendix A).\(^10\) Pennacchi (1999) proposes a method to value such government guarantees by using contingent claim analysis. Smetters (2002) provides the quantitative results on the unfunded government guarantee cost during the DB to DC transition, in which he shows that this cost in the new system could be very high and render the transition from an unfunded to a funded system meaningless. Lucas and McDonald (2006, 2009) analyze the government guarantee cost of government-sponsored enterprises (GSE). They argue that the government might have underestimated the guarantee cost, which is an important reason that the financial crisis has deepened. My research on government guarantee in this paper is more related to Pennacchi (1999) and Smetters (2002).\(^11\)

1.1.1.2 Lowering Cost of Debt Financing and Future Inflation Expectation

Issuing nominal bonds only will lead to incentive problems since people will worry about debt repudiation through surprise inflation. Hence, lenders will charge an inflation risk premium which increases the government’s borrowing cost. Issuing real debt will eliminate this cost. Also ILBs will lower the future inflation expectations since their presence reduces the incentive for the government to inflate in order to collect seigniorage revenues (see Campbell and Shiller 1996, Roush, Dudley, and Ezer 2008, and Campbell, Shiller, and Viceira 2009). Although the results of this literature are arguably best interpreted as more supportive of this incentive, it is fair to say that this literature has not produced an unequivocal verdict. Recent research by Fleckenstein, Longstaff and Lustig (2013)

---

\(^8\) The only documented country to provide real guarantee during this transition is Uruguay, which guarantees a minimum annual real rate of return of 2%.

\(^9\) See Bodie and Merton (1993) for a discussion on pension guarantees.

\(^10\) Empirical findings in the real world show that, during the transition from DB to DC, the government typically either provides the nominal guarantee or gives the individual access to the indexed bonds. To the best of my knowledge, no country has provided both at the same time, which would seem too generous from the perspective of government. Also, some countries switch between these two policies, and I will discuss how this interesting phenomenon can be explained by the model.

\(^11\) The government guarantee literature is not limited to social security privatization or GSE as discussed here. There is also literature discussing government guarantee during an economic/banking/currency crisis (see Moore 1997).
develop an arbitrage strategy to show that the (nominal) Treasury bonds are almost always overvalued relative to Treasury Inflation Protected Securities (TIPS), and they question “why US Treasury leaves billions of dollars on the table by issuing securities that are not as highly valued by the market as nominal Treasury bonds.” Although it is the area of classical asset pricing theory that their results intend to challenge, it also serves as an evidence that the current literature cannot well explain why a government chooses to issue ILBs,\textsuperscript{12} and the introduction of a government guarantee incentive in this paper is partly motivated by these findings.

1.1.1.3 Other Incentives

Other main incentives include the role of completing the financial market with a real risk-free asset and extracting information regarding future expected inflation by the yield spread between nominal bonds and ILBs (see Price 1997, and D’Amico, Kim, and Wei 2007). Since incomplete market and information are beyond the scope of this paper, these incentives are not included here but might be included in a future model.

1.1.2 Main Disincentives for Issuing ILBs

1.1.2.1 “Dark Side” Disincentive

Despite the government’s being reluctant to admit it publicly, the reason why the overwhelming majority of bonds are issued in nominal rather than real term is closely related to the option of diluting the real value of debt, or, more bluntly speaking, debt repudiation. This is related to the literature on the (partial) default on sovereign debt through generating inflation. Calvo (1988) studies a model in which debt repudiation is possible either openly or through inflation. He shows cases in which the existence of government bonds generates multiple equilibria. Cole, Dow, and English (1995) consider two types of government and analyze Perfect Bayesian Equilibria in which lenders’ actions are supported by beliefs

\textsuperscript{12}Some scholars also discuss the liquidity premium of ILBs, which offsets the advantage of saving the cost due to the inflation risk premium (see Roush 2007, and Shen 2009).
about the borrower's type. Grossman and Van Huyck (1988) discuss default by classifying sovereign debt default into two kinds: excusable default (for example, due to hedging the negative fiscal shock) and non-excusable (for example, pure debt repudiation). This disincentive is also considered as the Achilles' heel for the literature, proposing that the government should issue ILBs without taking this point into consideration.

1.1.2.2 “Bright Side” Disincentive

Barro (1979) develops a theory of “optimal” public finance and proposes that the government should use debt to smooth distortionary taxation over time. Lucas and Stokey (1983) show that in the environment without capital, the government will have an incentive to inflate away its nominal liabilities, unless the government could avoid all distortionary tax, or prices are predetermined. Since neither assumption seems realistic, this paper is considered as an evidence that issuing indexed debts is the dominant strategy over issuing nominal debt. Instead, Bohn (1988) argues that nominal debt, rather than real debt, should be used for that purpose. In his study, he shows that default through inflation on nominal bonds will improve the welfare in the environment with regard to distortionary tax and discretionary policy. The intuition is that if the debt is nominal, then the government could realize capital gains through high inflation in bad states where the gains are very valuable. Bohn’s paper is the first one to provide bright-side justification for nominal bonds. Alfaro and Kanczuk (2006) analyze the effects of this disincentive for ILBs quantitatively. Diaz-Gimenez et al. (2008) focus on comparing economies with real vs. nominal debt, with and without commitment, and evaluate the welfare implications of these different institutional arrangements. Martin (2009) conducts a positive analysis on nominal government debt. Similar to Martin’s paper, this paper also considers the case of a benevolent government that cannot commit to future policy choice. However, in addition

---

13 Also see Calvo and Guidotti (1993) and Kumhof (2004) for discussion on inflation tax.
14 Aiyagari et al. (2002) provide a microfoundation for Barro’s model in a general equilibrium framework with incomplete market.
15 Similarly, Barro (1997) discusses the optimal management of nominal and indexed debt assuming the government has commitment technology; the result suggests little role for nominal bonds.
to the tools of distortionary tax and inflation on nominal debt, I will also allow government
to use real debt (ILBs) for financing. Further, since I focus on the choice between nominal
and real debt, I consider a government that needs to finance a constant debt, which is an
alternative setting for financing constant expenditure as in Martin (2009).

The rest of the paper is organized as follows: The model is introduced in Section 1.2.
Section 1.3 presents the characterization of equilibrium with discussion, comparative statics
and extension of baseline model. Section 1.4 concludes.

1.2 Model

1.2.1 The Economy: a Non-Commitment Problem

The economy consists of a government and a representative individual. These two players
play a stochastic dynamic game. I assume that both government spending and government
guarantee follow an exogenous, stochastic process. Unlike a Ramsey solution such as Chari
and Kehoe (1993) discuss, I consider the case in which government cannot get access to
the commitment technology. The nominal liability will be a source of time-inconsistency
since government will have incentive to inflate away some nominal liabilities. Hence, it is
difficult to convince people that government will keep its promise on the inflation target,
especially during a period of high government expenditures, such as war. I assume the real
level of total debt to be constant, which means debt cannot be used to smooth taxes. As
Martin (2009) points out, the government debt over GDP displays a mean-reversion patten
which suggests the existence of a stable long-run level of debt over GDP. Accordingly, a
government could only use distortionary labor tax and surprise inflation to balance the
budget. Finally, I assume there is no access to lump-sum tax. Under this framework, I
consider Markov perfect equilibrium (MPE) for this problem and in the setup there are
two levels for government spending and government guarantee shocks, high level and low
level, following the Markov process of common knowledge with the transition probabilities
given by
\[
\Psi_g = \begin{bmatrix}
\psi_g & 1 - \psi_g \\ 1 - \psi_g & \psi_g
\end{bmatrix}
\]

\[
\Psi_{GR} = \begin{bmatrix}
\psi_{GR} & 1 - \psi_{GR} \\ 1 - \psi_{GR} & \psi_{GR}
\end{bmatrix}
\]

Specifically, the high-level shock at period \( t \) remains at the high level at period \( t+1 \) with probability \( 1 - \psi \), and transitions to the low level with probability \( \psi \). Similarly, the low-level shock at \( t \) remains a low level at \( t+1 \) with probability \( 1 - \psi \), and transitions to the high level with probability \( \psi \).

Table 2: Comparison with two other economies:

<table>
<thead>
<tr>
<th></th>
<th>Pareto</th>
<th>Ramsey</th>
<th>MPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commitment or not</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Lump-tax or not</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Figure 2 below shows the timing of the game: At the beginning of each period, the government inherits amounts of nominal and indexed bonds equal to \( b_{t-1}, d_{t-1} \) with the interest rate of \( i_{t-1}, p_{t-1} \). Then, the nature reveals the government expenditure and guarantee cost levels. After observing these two shocks, the government decides whether to issue ILBs (\( d_t \)) and whether to partially default by generating surprise inflation (\( \theta_t \)). Based on this decision, the individual decides how large of a portion to invest on nominal bonds, hence how much to charge for next-period nominal bonds, \( i_t \) (the individual will charge higher \( i_t \) if the government does not issue indexed bonds or he will have a higher inflation expectation next period), which, in turn, determines the other endogenous variables.
1.2.1.1 Government’s Problem

The government’s problem is to finance exogenous government expenditures in the least distortionary manner. A key assumption in this paper is that the government has no access to the lump-sum tax.

$$U = E \sum_{t=1}^{\infty} \beta^t u(c_t, L_t)$$

$$St : g_t + (1 + \rho_{t-1})d_{t-1} + \frac{p_t-1}{p_t}(1 + i_{t-1})\frac{B_{t-1}}{p_{t-1}} + 1_{\{d_t=0\}}GR_t = \tau_t w_t L_t + b_t + d_t \quad (1)$$
where \( \frac{p_t}{p_{t-1}} = (1 + \pi_t)(1 + \theta_t), \quad b_t = \frac{B_t}{p_t}, \quad u(c_t, L_t) = c_t - \omega_1 \frac{L_t^{1+1/\omega_2}}{1 + 1/\omega_2} \)

\( \{d_t = 0\} \) is an indicator function which equals 1 if \( d_t \neq 0 \) and equals 0 if \( d_t = 0 \). The characteristic of an indicator function plays an important role to explain why a government only issues small amounts of ILBs compared to nominal bonds. In addition to the notations in Figure 2 above, \( p_t \) is the price level, the inflation \( \frac{p_t}{p_{t-1}} \) consists of two parts, the regular inflation rate, \( \pi_t \), and surprise inflation \( \theta_t \). \( B_t \) is the nominal value of nominal bonds.

1.2.1.2 Individual’s Problem

\[ U = E \sum_{t=1}^{\infty} \beta^t u(c_t, L_t) \]

\( St : \ c_t + I_t + b_t + d_t = \)

\[ (1 - \tau_t) w_t L_t + MP_t \cdot k_t + (1 + \rho_{t-1}) d_{t-1} + \frac{p_{t-1} - \rho_t}{p_t} (1 + i_{t-1}) \frac{B_{t-1}}{p_{t-1}} + \{d_t = 0\} GR_t \quad (2) \]

where \( u(c_t, L_t) = c_t - \omega_1 \frac{L_t^{1+1/\omega_2}}{1 + 1/\omega_2} \)

\( \{d_t = 0\} \) is an indicator function which equals 1 if \( d_t \neq 0 \) and equals 0 if \( d_t = 0 \). \( I_t \) is investment, \( w_t \) is the real wage, \( k_t \) is capital and \( MP_t \) is the marginal product of capital. \( \omega_1 \) and \( \omega_2 \) are two parameters representing labor disutility and labor labor intertemporal elasticity in utility function.
1.2.1.3 Optimality Condition

For the individual,

Euler equation for consumption:\(^{16}\)

\[ 1 + \rho_t = 1/\beta \quad (3) \]

Euler equation for labor with competitive firm’s maximization problem:

\[ L_t = \left[ \frac{(1 - \tau_t)(1 - \alpha)k^\alpha}{\omega_1} \right]^{1/(\alpha+1/\omega_2)} \quad (4) \]

Nominal interest rate rule:\(^{17}\) is:

\[ 1 + i_t = (1 + \pi_t^e)(1 + \rho_t) \quad (5) \]

Since this is a leader-follower dynamic game as previously described, the government will make its optimal choice by taking the optimal strategy of the individual into consideration. Due to the complexity of the government’s problem (leader in this dynamic game) and discrete-choice setting (for issuing indexed bonds or not), it is impossible to get the analytical result. The numerical solution will be provided and discussed in the next section. Considering the limitation of the numerical solution such as the sensitivity to the change of parameters, the comparative statics are conducted to check whether the numerical solution is robust to the change of parameters, and furthermore, whether it is consistent with the basic intuition and matches the stylized facts on the timing of issuing indexed bonds.

\(^{16}\)This assumption that consumption enters linearly in the period utility function has the advantage of simplifying the determination of the equilibrium interest rates without affecting the tax-smoothing motive of the model.

\(^{17}\)To make this model solvable, I will feed into the nominal interest rate rule by making the nominal interest rate be a function of the inflation expectation and real interest rate. The inflation expectation will increase with surprise inflation rate and decrease with the amounts of ILBs. Empirical findings show the inflation expectation is more sensitive to the issuance of ILBs (see Campbell and Shiller 1996, Campbell, Shiller and Viceira 2009).
1.2.2 Definition of Markov Perfect Equilibrium (MPE)

As described above, this model is a non-commitment stochastic dynamic game where the two random shocks evolve according to the Markov process of common knowledge, so I restrict my attention to the Markov perfect equilibrium (MPE).

The state of economy is \( \{ g_t, GR_t, b_{t-1}, d_{t-1} \} \). Given the state of economy, I could write the government’s problem recursively as

\[
V(g_t, GR_t, b_{t-1}, d_{t-1}) = \max_{\{\theta_t, b_t, d_t\}} [u(c_t, L_t) + \beta \cdot EV(g_{t+1}, GR_{t+1}, b_t, d_t)]
\]

A Markov perfect equilibrium is a set of state variables, a value function \( V \) and a policy function \( \{\theta_t, b_t, d_t\} \), such that given the state of the economy, \( \{\theta_t, b_t, d_t\} \) solve the government’s problem above. Note that in this leader-follower dynamic game, the government optimizes its problem by taking into consideration the optimal response of the individual. In other words, given the individual’s strategy and the state, the government maximizes utility. Also given the government’s strategy and the state, the private sector optimally chooses its labor choice, consumption, etc. Notice that since I feed into the interest rate rule so this is a partial equilibrium framework rather than a general equilibrium framework, this is also why I do not need to consider the market clearing condition of bond markets. This is a necessary setting for the model to be solvable.

1.3 Quantitative Analysis

1.3.1 Empirical Findings

1.3.1.1 ILBs and Potential Government Guarantee

It has been argued that ILBs should serve as an important asset class in pension management. The aversion to inflation risk of pensioners is expected to rise as time goes on. In the event that an individual loses part of his or her financial assets (in real terms) at a mature age, it would be more difficult to find ways (such as finding a job) to compensate for that
loss in real income. If aversion to inflation is dependent on the ability to find a job (i.e., use one’s human capital as a hedge), then aversion to inflation should rise with age. As an illustration, suppose a typical pensioner at age 60 is expected to live for approximately 20 more years, and with an average inflation rate of 2% over that period, this pensioner receiving a nominal annuity would see his or her income lose one-third of its value over this period. If he or she could live to be 100 years old, more than half of the real purchasing power of his or her annuity would have evaporated. Hence, ILBs are of particular interest to the pensioner in the DC system since keeping purchasing power is the main objective for pension investment.18

Table 3: Timing of ILBs and Potential Government Guarantee19

<table>
<thead>
<tr>
<th>Country</th>
<th>ILBs Issue Year</th>
<th>DB to DC Transition Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>2003</td>
<td>2002</td>
</tr>
<tr>
<td>Australia</td>
<td>1985</td>
<td>1985</td>
</tr>
<tr>
<td>Australia</td>
<td>1993</td>
<td>1992</td>
</tr>
<tr>
<td>Belgium</td>
<td>2004</td>
<td>2003</td>
</tr>
<tr>
<td>Denmark</td>
<td>2012</td>
<td>2011</td>
</tr>
<tr>
<td>Hungary</td>
<td>1996</td>
<td>1998</td>
</tr>
<tr>
<td>Germany</td>
<td>2006</td>
<td>2004</td>
</tr>
<tr>
<td>India</td>
<td>2013</td>
<td>2009</td>
</tr>
<tr>
<td>Japan</td>
<td>2004</td>
<td>2001</td>
</tr>
<tr>
<td>Mexico</td>
<td>1999</td>
<td>1997</td>
</tr>
<tr>
<td>Poland</td>
<td>1992</td>
<td>1991</td>
</tr>
<tr>
<td>Sweden</td>
<td>1994</td>
<td>1994</td>
</tr>
<tr>
<td>UK</td>
<td>1975</td>
<td>1975</td>
</tr>
</tbody>
</table>

Source: OECD Data Lab, IMF World Economic Outlook

As Table 3 shows, quite a few countries have introduced the DC system and ILBs in the same period, which accounts for 46% of the total cases of ILBs’ issuance. Hence, it is hard to interpret this as a coincidence of time. Also some governments make ILBs only accessible to pensioners/individuals rather than institutional investors. Huang (2013) conducts a

18For investment and inflation issues in social security, see Bodie (1995, 2001) and Abel (2001a, 2001b).
19See Appendix A for complete information.
case study in Hong Kong’s iBond program. A total of 155,835 valid applications from individuals were received for over 13 billion HK dollars (about 1.669 billion U.S. dollars) in the principal amount for the bonds, which exceeded the planned issue amount of 10 billion HK dollars. Among all the valid applications for the iBond, those who have applied for 44 units (HK$10,000 for one unit) or less will be allocated the full amount applied for, and there are 150,869 such applications. In addition, there are 4,966 valid applications that have applied for more than 44 units. Each of them will be allocated 44 units first, and 1,344 of them will be allocated one additional bond unit based on ballot results. The elderly, retirees and the middle class comprise the majority of iBond subscribers, with an average subscription amount of HK$70,000 to HK$100,000. The overall subscription rate was 30%.

1.3.1.2 Inflating Away Nominal Liabilities During Recession

From Figure 1 in Section 1.1, there are two obvious time gaps during which there is no country issuing ILBs: 2000-2002 and 2008-2010. A common feature with these two periods is the world-wide recession due to the stock market crash. It is reasonable to conjecture that the governments need to inflate away some nominal liabilities during the recession and provide a buffer to the economy and budget. Another support is from the relative amounts of nominal and real debt in the US Treasury bond market. The detrended nominal bonds and Treasury Inflation Protected Securities (TIPS) reach their peak (trough) and trough (peak) at the same time during economic recession (boom) (see Figure 3 below).
1.3.2 Benchmark Calibration and Numerical Solution

Table 4: Calibrated Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.95</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>8.71</td>
<td>labor disutility parameter</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0.5</td>
<td>labor intertemporal elasticity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>capital share in Cobb-Douglas</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.05</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>$(B + D)/y$</td>
<td>0.58</td>
<td>total debt/output</td>
</tr>
<tr>
<td>$g_H/y$</td>
<td>0.51</td>
<td>high level government spending over output</td>
</tr>
<tr>
<td>$g_L/y$</td>
<td>0.27</td>
<td>low level government spending over output</td>
</tr>
<tr>
<td>$GR_H/y$</td>
<td>0.34</td>
<td>high level guarantee cost over output</td>
</tr>
<tr>
<td>$GR_L/y$</td>
<td>0.02</td>
<td>low level guarantee cost over output</td>
</tr>
<tr>
<td>$\psi_g$</td>
<td>0.95</td>
<td>government expenditure persistency</td>
</tr>
<tr>
<td>$\psi_{GR}$</td>
<td>0.95</td>
<td>government guarantee persistency</td>
</tr>
</tbody>
</table>

The first five parameters are taken from the standard business-cycle literature.\textsuperscript{20} The

\textsuperscript{20}For calibration parameters, see Lucas (2000), Domeij and Floden (2006), Alfaro and Kanczuk (2006),
total debt, the two levels of both government spending and government expenditure persistence are calibrated using country-specific data from IMF World Economic Outlook. (This benchmark calibration uses the average level of all countries studied in this paper, and Section 1.3.5 provides the analysis using data country by country). Following convention, after getting the $g/y$ time series, I detrend the series and calculate the mean, standard deviation and estimate the autoregressive coefficients. See Figure 5 in Appendix A for country-specific government spending.

For the calibration value of guarantee cost, i.e., $GR_H/y$, it is difficult to calibrate these values since no data are available. I temporarily take them as the free parameters and choose them to match the stylized patterns. Later I will use the Martingale pricing theory to estimate the guarantee cost and compare that with the calibrated value here.\(^{21}\)

Several assumptions are needed to make MPE computable. First I assume the real level of total debt to be a constant ratio over GDP, which means debt cannot be used to smooth taxes. As Martin (2009) points out, the government debt over GDP displays a mean-reversion pattern which suggests the existence of a stable long-run level of debt over GDP. Further, I set capital to be constant. This implies that similar to debt, capital also cannot be used to smooth taxes. This greatly reduces the calculation burden and is a necessary step to make equilibrium computable. As Grossman and Han (1999) point out, this assumption is much less restrictive than it seems. These authors show that, when government can save after defaulting, contingent debt (or capital) does not allow for any additional tax smoothing. In contrast, contingent service may engender more tax smoothing than the one already attained through savings. Grossman and Van Huyck (1988) and Alfaro and Kanczuk (2006) make a more restrictive assumption by fixing the real levels of capital, indexed and nominal debt to be constants over time.\(^{22}\) Here I have

---
\(^{21}\)In Section 1.3.6, as a special case, I will price the government guarantee cost in the scenario of the transition from the traditional Defined-Benefit to Defined-Contributions system.

\(^{22}\)Alternatively, Cole and Kehoe (2000) assume that a risk-neutral household with no access to international bonds chooses the level of capital. Hence in their case, debt is used neither for production nor for
relaxed their assumption by allowing the changing amount of real and nominal debt each period while keeping the sum unchanged. These assumptions will reduce the number of state variables to three, \( \{g_t, GR_t, d_{t-1}\} \).23 With concerns about multiple equilibria in MPE, I follow previous literature to set two levels for each exogenous state variable and hence I consider the discrete-choice problem for each period. There are 8 states in the economy:

- **State 1**: \( \{GR_H, g_H, no\, ILBs\, last\, period\} \); **State 2**: \( \{GR_H, g_L, no\, ILBs\, last\, period\} \);
- **State 3**: \( \{GR_L, g_H, no\, ILBs\, last\, period\} \); **State 4**: \( \{GR_L, g_L, no\, ILBs\, last\, period\} \);
- **State 5**: \( \{GR_H, g_H, ILBs\, last\, period\} \); **State 6**: \( \{GR_H, g_L, ILBs\, last\, period\} \);
- **State 7**: \( \{GR_L, g_H, ILBs\, last\, period\} \); **State 8**: \( \{GR_L, g_L, ILBs\, last\, period\} \),

where “\( GR \)” represents guarantee, “\( g \)” represents government spending, and subscript “\( H \)” and “\( L \)” represent high and low level, respectively.

### 1.3.3 Equilibrium Characterization and Discussion

Using the benchmark calibration, the equilibrium characterization is as follows:24

<table>
<thead>
<tr>
<th>( d_t )</th>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
<th>State 5</th>
<th>State 6</th>
<th>State 7</th>
<th>State 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>( \theta_t )</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>( \tau_t )</td>
<td>0.286</td>
<td>0.109</td>
<td>0.144</td>
<td>0.113</td>
<td>0.29</td>
<td>0.1</td>
<td>0.136</td>
<td>0.105</td>
</tr>
<tr>
<td>( \lambda_t )</td>
<td>0.288</td>
<td>0.317</td>
<td>0.311</td>
<td>0.316</td>
<td>0.287</td>
<td>0.318</td>
<td>0.312</td>
<td>0.317</td>
</tr>
</tbody>
</table>

The characteristic of MPE is that the optimal decision will depend only on the payoff relevant states (shown above) rather than the whole time history.25 In the baseline model, the derived MPE indicates that the government will issue ILBs in State 2 and State 6.

---

23 When real level of total debt is constant, once the number of indexed bonds \( d_{t-1} \) is determined, the number of nominal bonds \( b_{t-1} \) will be fixed.

24 A potential concern about MPE is the possibility of multiple equilibria. By a reasonable range of initial guess about value function, this issue is not found in the numerical calculation. See Calvo (1988) for a discussion of multiple equilibria.

25 The detail of algorithm to solve the Markov perfect equilibrium (MPE) is provided in Appendix B.
Both states share the feature of a low government spending level and a high government guarantee level.

Now with this result, by specifying exogenous shocks of government spending and guarantee shock for each period in advance, I could simulate a typical time pattern of ILBs issuance. As Figure 4 shows, in period 1, both government spending and guarantee cost are on their low level, and there is no incentive to issue ILBs. At period 2, the government expenditures jump to the high level, the reason for which could be a negative technology or fiscal shock. There is strong disincentive to issue ILBs during this period. At period 3, the economic condition is still not improved and simultaneously the government announces the social security privatization plan, i.e., there is a high level government guarantee shock starting from period 3. Notice that although the government guarantee is high in period 3, the government does not choose to issue ILBs to waive this cost at this period. Instead, it chooses to generate the surprise inflation. The reason is that the economic condition is still bad, and government spending level is still high. However, there is no free lunch: what follows the surprise inflation is the higher inflation expectation, and hence higher borrowing cost on the nominal bond. Then when the government enters into period 4, the government-spending level goes back to a low level, which signals the improvement of the economic situation. Now it is the best timing for the government to issue the ILBs. In addition, there is no surprise inflation in this period and the nominal interest rate also decreases due to the lower inflation expectation for the next period. The government will keep issuing the indexed bonds until it is hit by another high level government spending shock. This is a typical pattern for some countries that I will discuss below in Section 1.3.5.

\[26^\text{Note that in the production function I do not consider the technology term since it could be “internalized” into the government expenditure shock. For example, if there is a negative technology shock, the output will decrease and government expenditure will increase correspondingly, which is also consistent with the facts.}\]
Figure 4: A Baseline Time Pattern for ILBs Issuance
1.3.4 Comparative Statics and Robustness Check

State 1: \{GR_H, g_H, no ILBs last period\};  
State 2: \{GR_H, g_L, no ILBs last period\};  
State 3: \{GR_L, g_H, no ILBs last period\};  
State 4: \{GR_L, g_L, no ILBs last period\};  
State 5: \{GR_H, g_H, ILBs last period\};  
State 6: \{GR_H, g_L, ILBs last period\};  
State 7: \{GR_L, g_H, ILBs last period\};  
State 8: \{GR_L, g_L, ILBs last period\};

In addition to solving the model, the comparative statics are conducted to check the robustness of the equilibrium. I check the comparative statics with respect to the parameters of two levels of government spending and government guarantee, the ratio of total debt over GDP, the time discount factor, and the number of ILBs (given that the government issues them). The result shows that the MPE is robust to the change of parameters. Here I will list the comparative statics result regarding the change of government guarantee parameters. A “rank” on these 8 states could be constructed to show which state is more likely to issue the indexed bonds. As Figure 6 in Appendix A shows, when the high level of government guarantee \(GR_H\) is low, the government will not issue indexed bonds in any state, then as \(GR_H\) increases, the first state to issue the indexed bonds is state 2, i.e., \(\{2\}\). The features of state 2 include (a) low level of government spending, (b) high level of government guarantee cost, and (c) no indexed bonds in the last period (hence the inflation expectation for this period is high). This result is quite intuitive: The feature (a) is very clear since if government spending is instead at the high level, the government will have more pressure on balancing the budget so will tend to use surprise inflation to finance the government expenditure with the motive of distortionary tax smoothing rather than issuing ILBs. For (b), holding all else unchanged, the government wants to issue ILBs since by doing so it could avoid the guarantee cost especially when the guarantee cost is high, as in the transition from DB to DC. (c) has already been discussed by the previous literature. As Campbell and Shiller (1996) argue, an important motivation to issue ILBs
is to lower the inflation expectation. As the $GR_H$ keeps increasing, the set of states in which the indexed bonds are issued evolves as the following order: \( \{2\} \Rightarrow \{2,6\} \Rightarrow \{1,2,6\} \Rightarrow \{1,2,5,6\} \Rightarrow \{1,2,4,5,6\} \). Then, keeping $GR_H$ unchanged, we continue to increase $GR_L$.

As Figure 7 in Appendix A shows, the set of states to issue indexed bonds keeps enlarging: \( \{1,2,4,5,6\} \Rightarrow \{1,2,4,5,6,8\} \Rightarrow \{1,2,3,4,5,6,8\} \Rightarrow \{1,2,3,4,5,6,7,8\} \). So the “rank” of states which are more likely to issue indexed bonds is 2>6>1>5>4>8>3>7. This is consistent with the intuition. As I discussed before, empirical findings show that a government is more likely to issue ILBs when (1) the potential guarantee cost is high, (2) government spending is low, and (3) people have higher inflation expectations. This is exactly what the rank of states reveals here. Another interesting comparative statics is regarding the change of the relative ratio of ILBs over nominal bonds. From the data of US Treasury bond and TIPS market, I set the benchmark ratio to be 8%. Holding all else unchanged, increasing this \textit{ex ante} fixed ratio will make a government less likely to issue ILBs in the states where it chooses to issue ILBs. This provides a justification for why countries typically only issue small amounts of ILBs compared to nominal bonds. The intuition is clear: the substitution role of ILBs for government guarantee and the according indicator function set-up in the budget constraint (1) make government have an incentive to only issue a small rather than a large amount of ILBs. Of course, this ratio could not be infinitely small since another role of ILBs, lowering inflation expectation and decreasing financing cost, also has an effect when a government makes its decision.

1.3.5 Stylized Facts Analysis Country by Country

1.3.5.1 High Prior Inflation, No Government Guarantee Shock

This category includes Mexico (1989), China Mainland (1988,1992),\textsuperscript{27} Canada (1991), Italy (1983), and Argentina (1972). During the issuance of ILBs for these countries, there is no government guarantee shock, and the main motivation of issuing ILBs is the prior high

\textsuperscript{27}Rigorously speaking, what the Chinese government provided in 1988 and 1992 were inflation-indexed banking savings which could be considered as a substitute for non-marketable ILBs.
inflation. They issue ILBs in order to lower the future inflation expectation and debt financing cost for nominal bonds. For Argentina (1972) and Italy (1983), the timing is more interesting since there is a one- or two-year lag between the year with highest inflation rate and year of issuing ILBs. The reason is that the economy is in recession when the inflation hits the peak, so these two countries choose to postpone the issuance of ILBs when economic conditions improve.

1.3.5.2 High Inflation with Government Guarantee Shock

Several countries fall into the category with high prior inflation and potential government guarantee cost, such as Chile (1981), Poland (1992), Mexico (1999), and India (2013). In this scenario, the best strategy is undoubtedly to issue ILBs. However, one thing worth noting is that the level of government spending is important when a government ponders the decision to issue ILBs. When the government spending level is high, for example, the economy is bad, then some will postpone the issuance of ILBs, such as Mexico (1999) and India (2013) did.

1.3.5.3 Low Inflation with Government Guarantee Shock

The previous literature fails to explain why quite a few countries issue ILBs when there is no fear of high inflation. Instead, the introduction of concerns about government guarantee cost could well explain what happened for Australia (1985, 1993), Sweden (1994), Austria (2003), Belgium (2004), Japan (2004), Germany (2006), HongKong (2011) and Denmark (2012). When taking a careful look at the countries in Section 1.3.5.2 and 1.3.5.3, we will find the countries in Section 1.3.5.3 are mainly developed economies, while those in Section 3.4.2 are mainly less developed. It makes sense since the government guarantee cost of developed countries is generally higher than that of developing countries. That is why the developed countries are more likely to issue ILBs during a low inflation environment when they are hit by a government guarantee shock.
1.3.5.4 Switch between Issuing ILBs and Providing Nominal Government Guarantee

The model introduced in this paper could match the most interesting empirical findings: some countries switch between issuing ILBs and providing a nominal government guarantee as Figure 4, above, shows. As with Poland, the country was hit by a government guarantee shock since there was a transition from DB to DC in 1991, and the country was experiencing the recession (negative GDP growth in 1991). Therefore, Poland announced it would provide a government guarantee on the DC portfolio, but in 1992, when the economic condition improved, the government switched back to issuing ILBs since the government guarantee cost was high and the need to use the nominal bond to collect inflation tax was less urgent. Similar stories happened in Mexico (1997-1999), Hungary (1996-1998), and Belgium (2003-2004).

1.3.6 Extension of Baseline Model

1.3.6.1 Extension with Portfolio Choice and Price Government Guarantee in Scenario of DB to DC Transition

For the baseline model, the government guarantee cost is an exogenous, lump-sum cost for simplicity. It would be more realistic to endogenize this cost since the guarantee cost typically depends on an individual’s risk-taking behavior in the DC account. For example, if the individual invests more on the risky asset, then the guarantee cost on this portfolio will be higher. A good way to think about it is to model this guarantee cost as a put option. When the volatility of the underlying asset (here it is money put into the DC portfolio) increases, the value of the put option (here it is the government guarantee cost) will be higher. Keeping this in the mind, the individual’s budget constraint will become:

$$c_t + I_t + b_t + d_t + eq_t = (1 - \tau^L_t - \tau^S_t)w_tL_t + MP_t * k_t + (1 + p_{t-1})d_{t-1} + \frac{p_{t-1}}{p_t}(1 + i_{t-1})b_{t-1} + \max\left\{\frac{p_{t-1}}{p_t}(1 + i_{t-1})b_{t-1} + \frac{p_{t-1}}{p_t}(1 + ER_t)eq_{t-1}, \left(\frac{p_{t-1}}{p_t}b_{t-1} + \frac{p_{t-1}}{p_t}eq_{t-1}\right) * (1 + NGR_t)\right\}$$  \ (6)
The last term of the budget constraint represents that the individual will receive the higher of a DC investment return and a nominal guarantee rate of return.

\[ d_t = \gamma_t \tau_t^S w_t l_t \quad \text{and} \quad b_t = (1 - \gamma_t)(1 - \eta_t)\tau_t^S w_t l_t \quad \text{and} \quad eq_t = (1 - \gamma_t)\eta_t \tau_t^S w_t l_t \] (the government decides how much is invested in indexed bonds, and individual choose the allocation between nominal bonds and stock).

Hence, the government guarantee cost

\[
GR_t = \max \{0, \left( \frac{p_{t-1}}{p_t} B_{t-1} \frac{B_{t-1}}{p_{t-1}} e_{eq_{t-1}} \right) \ast (1 + NGR_t) - \\
\left( \frac{p_{t-1}}{p_t} (1 + i_{t-1}) \frac{p_{t-1}}{p_t} e_{eq_{t-1}} (1 + ER_t) \right) \} \quad (7)
\]

Note that this guarantee value equals zero if the government chooses \( \gamma_t = 1 \)

**Proposition 1:** The individual will invest all the money on the most risky asset in the guaranteed DC account. Proof: See Appendix C.

This result is intuitive and this free-rider problem has been discussed by previous literature, as in Smetters (2002). Proposition 1 will greatly simplify the equilibrium characterization and make the calculation of the government guarantee cost on the DC portfolio possible.

I use Martingale pricing theory to price the potential government guarantee cost. The details of the algorithm is provided in Appendix D. The results are provided in Table 9 below.

As Table 9 shows, it would be interesting to compare this estimated guarantee cost with calibrated cost. The baseline result shows that the guarantee cost calculated by Martingale pricing theory is generally lower than the calibrated guarantee cost. However, the order of magnitude is preserved. The possible explanation is that the estimated cost is the cost from social security privatization, but there could be government guarantee cost from other sources, such as the financial or political crisis. However, I would like to point out

\footnote{See Moore (1997) for a discussion of government guarantee cost from other sources.}
that this estimate cost is the sensitivity to the parameters. The change of parameters of
duration of government guarantee $n$ and/or growth rate of wage $g$ might reverse the policy
of issuing ILBs or not. It also leaves room for countries to reconsider their ILB issuance
policy.

Table 9: Comparison between Calibrated and Estimated Guarantee Cost (Percent of GDP)

<table>
<thead>
<tr>
<th>Country</th>
<th>Calibrated</th>
<th>Estimated, Baseline Case $n=20, g=0$</th>
<th>Estimated, $n=30, g=0$</th>
<th>Estimated, $n=20, g=0.03$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria (2003)</td>
<td>0.43</td>
<td>0.32</td>
<td>0.48</td>
<td>0.54</td>
</tr>
<tr>
<td>Australia (1993)</td>
<td>0.42</td>
<td>0.26</td>
<td>0.39</td>
<td>0.44</td>
</tr>
<tr>
<td>Belgium (2004)</td>
<td>0.44</td>
<td>0.27</td>
<td>0.41</td>
<td>0.45</td>
</tr>
<tr>
<td>Chile (1981)</td>
<td>0.37</td>
<td>0.26</td>
<td>0.39</td>
<td>0.44</td>
</tr>
<tr>
<td>Denmark (2012)</td>
<td>0.43</td>
<td>0.31</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>Hungary (1996)</td>
<td>0.33</td>
<td>0.29</td>
<td>0.44</td>
<td>0.49</td>
</tr>
<tr>
<td>Germany (2006)</td>
<td>0.42</td>
<td>0.25</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>India (2013)</td>
<td>0.27</td>
<td>0.23</td>
<td>0.35</td>
<td>0.39</td>
</tr>
<tr>
<td>Japan (2004)</td>
<td>0.39</td>
<td>0.25</td>
<td>0.38</td>
<td>0.42</td>
</tr>
<tr>
<td>Mexico (1999)</td>
<td>0.26</td>
<td>0.22</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>Poland (1992)</td>
<td>0.3</td>
<td>0.23</td>
<td>0.35</td>
<td>0.39</td>
</tr>
<tr>
<td>Sweden (1994)</td>
<td>0.44</td>
<td>0.29</td>
<td>0.44</td>
<td>0.49</td>
</tr>
<tr>
<td>UK (1975)</td>
<td>0.39</td>
<td>0.19</td>
<td>0.29</td>
<td>0.32</td>
</tr>
</tbody>
</table>

1.4 Conclusion

This paper introduces concerns about government guarantee cost to help a dynamic model
to better match countries’ timing of issuing ILBs. ILBs’ substitution role for government
guarantee provides a mechanism to explain why some countries issue ILBs when there is
no fear of high inflation and why some countries choose to issue ILBs during structural
change or reform such as social security privatization. This also gives an explanation why
countries typically only issue small amounts of ILBs compared to nominal bonds. Finally,
the difference between calibrated and estimated government guarantee cost also leaves room for a government to reconsider the policy of issuing ILBs.
1.5 Appendix

1.5.1 Appendix 1.A: Tables and Figures

Table 6: Countries issuing ILBs

<table>
<thead>
<tr>
<th>Country</th>
<th>ILBs Issue Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1972</td>
</tr>
<tr>
<td>Australia(^{29})</td>
<td>1985, 1993</td>
</tr>
<tr>
<td>Austria</td>
<td>2003</td>
</tr>
<tr>
<td>Belgium</td>
<td>2004</td>
</tr>
<tr>
<td>Brazil</td>
<td>1964</td>
</tr>
<tr>
<td>Canada</td>
<td>1991</td>
</tr>
<tr>
<td>Chile</td>
<td>1981</td>
</tr>
<tr>
<td>China (Hong Kong)</td>
<td>2011</td>
</tr>
<tr>
<td>China (Mainland)(^{30})</td>
<td>1950, 1988, 1992</td>
</tr>
<tr>
<td>Denmark</td>
<td>2012</td>
</tr>
<tr>
<td>Finland</td>
<td>1945-1968</td>
</tr>
<tr>
<td>France</td>
<td>1998</td>
</tr>
<tr>
<td>Germany</td>
<td>2006</td>
</tr>
<tr>
<td>Greece</td>
<td>2003</td>
</tr>
<tr>
<td>Hungary</td>
<td>1996</td>
</tr>
<tr>
<td>Iceland</td>
<td>1955</td>
</tr>
<tr>
<td>India</td>
<td>2013</td>
</tr>
<tr>
<td>Israel</td>
<td>1955</td>
</tr>
<tr>
<td>Italy</td>
<td>1983, 2003</td>
</tr>
<tr>
<td>Japan</td>
<td>2004</td>
</tr>
<tr>
<td>Korea (South)</td>
<td>2007</td>
</tr>
<tr>
<td>Mexico</td>
<td>1989, 1999</td>
</tr>
<tr>
<td>Poland</td>
<td>1992</td>
</tr>
<tr>
<td>Sweden</td>
<td>1994</td>
</tr>
<tr>
<td>Thailand</td>
<td>2011</td>
</tr>
<tr>
<td>Turkey</td>
<td>2007</td>
</tr>
<tr>
<td>UK(^{31})</td>
<td>1975, 1981</td>
</tr>
<tr>
<td>US</td>
<td>1997</td>
</tr>
</tbody>
</table>

\(^{29}\)Superannuation system (DC) in Australia was introduced in 1985 and became compulsory in 1992.
\(^{30}\)Inflation-linked Banking Deposit was provided in 1950, 1988 and 1992
\(^{31}\)Non-marketable ILBs in 1975 and marketable ILBs was first introduced in 1981
Table 7: ILBs Issuance and DB to DC Transition

<table>
<thead>
<tr>
<th>Country</th>
<th>ILBs Issue Year</th>
<th>DB to DC Transition Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>2003</td>
<td>2002</td>
</tr>
<tr>
<td>Australia</td>
<td>1985</td>
<td>1985 (Superannuation system was first introduced)</td>
</tr>
<tr>
<td>Australia</td>
<td>1993</td>
<td>1992 (Superannuation system became compulsory)</td>
</tr>
<tr>
<td>Belgium</td>
<td>2004</td>
<td>2003</td>
</tr>
<tr>
<td>Denmark</td>
<td>2012</td>
<td>2011</td>
</tr>
<tr>
<td>Hungary</td>
<td>1996</td>
<td>1998</td>
</tr>
<tr>
<td>Germany</td>
<td>2006</td>
<td>2004</td>
</tr>
<tr>
<td>India</td>
<td>2013</td>
<td>2009 (NPS)</td>
</tr>
<tr>
<td>Japan</td>
<td>2004</td>
<td>Oct, 2001</td>
</tr>
<tr>
<td>Mexico</td>
<td>1999</td>
<td>1997</td>
</tr>
<tr>
<td>Poland</td>
<td>1992</td>
<td>1991</td>
</tr>
<tr>
<td>Sweden</td>
<td>1994</td>
<td>1994 (Legislated)</td>
</tr>
<tr>
<td>UK</td>
<td>1975</td>
<td>1975</td>
</tr>
</tbody>
</table>
Figure 5: Country-Specific Government Spending

Government Total Expenditure (Percent of GDP)

Units: % of GDP

Source: International Monetary Fund
Table 8: Calibrated Guarantee Cost (Percent of GDP, limited to the countries with available data)

<table>
<thead>
<tr>
<th>Country</th>
<th>ILBs Year</th>
<th>Calibrated Guarantee Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1972</td>
<td>N/A</td>
</tr>
<tr>
<td>Australia</td>
<td>1985, 1993</td>
<td>N/A, 42%</td>
</tr>
<tr>
<td>Austria</td>
<td>2003</td>
<td>43%</td>
</tr>
<tr>
<td>Belgium</td>
<td>2004</td>
<td>44%</td>
</tr>
<tr>
<td>Brazil</td>
<td>1964</td>
<td>N/A</td>
</tr>
<tr>
<td>Canada</td>
<td>1991</td>
<td>41%</td>
</tr>
<tr>
<td>Chile</td>
<td>1981</td>
<td>38%</td>
</tr>
<tr>
<td>China (Hong Kong)</td>
<td>2011</td>
<td>32%</td>
</tr>
<tr>
<td>China (Mainland)</td>
<td>1950, 1988, 1992</td>
<td>N/A, 25%, 22%</td>
</tr>
<tr>
<td>Denmark</td>
<td>2012</td>
<td>43%</td>
</tr>
<tr>
<td>Finland</td>
<td>1945-1968</td>
<td>N/A</td>
</tr>
<tr>
<td>France</td>
<td>1998</td>
<td>42%</td>
</tr>
<tr>
<td>Germany</td>
<td>2006</td>
<td>42%</td>
</tr>
<tr>
<td>Greece</td>
<td>2003</td>
<td>39%</td>
</tr>
<tr>
<td>Hungary</td>
<td>1996</td>
<td>33%</td>
</tr>
<tr>
<td>Iceland</td>
<td>1955</td>
<td>N/A</td>
</tr>
<tr>
<td>India</td>
<td>2013</td>
<td>26%</td>
</tr>
<tr>
<td>Israel</td>
<td>1955</td>
<td>N/A</td>
</tr>
<tr>
<td>Italy</td>
<td>1983, 2003</td>
<td>33%, 43%</td>
</tr>
<tr>
<td>Japan</td>
<td>2004</td>
<td>39%</td>
</tr>
<tr>
<td>Korea (South)</td>
<td>2007</td>
<td>34%</td>
</tr>
<tr>
<td>Mexico</td>
<td>1989, 1999</td>
<td>21%, 26%</td>
</tr>
<tr>
<td>Poland</td>
<td>1992</td>
<td>30%</td>
</tr>
<tr>
<td>Sweden</td>
<td>1994</td>
<td>44%</td>
</tr>
<tr>
<td>Thailand</td>
<td>2011</td>
<td>29%</td>
</tr>
<tr>
<td>Turkey</td>
<td>2007</td>
<td>31%</td>
</tr>
<tr>
<td>UK</td>
<td>1975, 1981</td>
<td>N/A, 33%</td>
</tr>
<tr>
<td>US</td>
<td>1997</td>
<td>34%</td>
</tr>
</tbody>
</table>
Figure 6: Comparative Statics (GRₜᵣ)
Figure 7: Comparative Statics ($\mathbf{GR}_L$)

State 1: $\mathbf{GR}_H$, $\mathbf{g}_H$, no ILBs last period

State 2: $\mathbf{GR}_H$, $\mathbf{g}_L$, no ILBs last period

State 3: $\mathbf{GR}_L$, $\mathbf{g}_H$, no ILBs last period

State 4: $\mathbf{GR}_L$, $\mathbf{g}_L$, no ILBs last period

State 5: $\mathbf{GR}_H$, $\mathbf{g}_H$, ILBs last period

State 6: $\mathbf{GR}_H$, $\mathbf{g}_L$, ILBs last period

State 7: $\mathbf{GR}_L$, $\mathbf{g}_H$, ILBs last period

State 8: $\mathbf{GR}_L$, $\mathbf{g}_L$, ILBs last period
Figure 8: Comparative Statics ($g_H$)

State 1: GRH, $g_H$, no ILBs last period

State 2: GRH, $g_L$, no ILBs last period

State 3: GRL, $g_H$, no ILBs last period

State 4: GRL, $g_L$, no ILBs last period

State 5: GRH, $g_H$, ILBs last period

State 6: GRH, $g_L$, ILBs last period

State 7: GRL, $g_H$, ILBs last period

State 8: GRL, $g_L$, ILBs last period
Figure 9: Comparative Statics ($g_L$)

State 1: $GR_H$, $g_H$, no ILBs last period

State 2: $GR_H$, $g_L$, no ILBs last period

State 3: $GR_L$, $g_H$, no ILBs last period

State 4: $GR_L$, $g_L$, no ILBs last period

State 5: $GR_H$, $g_H$, ILBs last period

State 6: $GR_H$, $g_L$, ILBs last period

State 7: $GR_L$, $g_H$, ILBs last period

State 8: $GR_L$, $g_L$, ILBs last period
Figure 10: Comparative Statics (the ratio of ILBs over nominal bonds)

State 1: GR\(_H\), g\(_H\), no ILBs last period

State 2: GR\(_H\), g\(_L\), no ILBs last period

State 3: GR\(_L\), g\(_H\), no ILBs last period

State 4: GR\(_L\), g\(_L\), no ILBs last period

State 5: GR\(_H\), g\(_H\), ILBs last period

State 6: GR\(_H\), g\(_L\), ILBs last period

State 7: GR\(_L\), g\(_H\), ILBs last period

State 8: GR\(_L\), g\(_L\), ILBs last period

The ratio of ILBs over nominal bonds
Figure 11: Comparative Statics (discount factor)
Figure 12: Comparative Statics (total debt)

State 1: GR\textsuperscript{H}, g\textsuperscript{H}, no ILBs last period

State 2: GR\textsuperscript{H}, g\textsuperscript{L}, no ILBs last period

State 3: GR\textsuperscript{L}, g\textsuperscript{H}, no ILBs last period

State 4: GR\textsuperscript{L}, g\textsuperscript{L}, no ILBs last period

State 5: GR\textsuperscript{H}, g\textsuperscript{H}, ILBs last period

State 6: GR\textsuperscript{H}, g\textsuperscript{L}, ILBs last period

State 7: GR\textsuperscript{L}, g\textsuperscript{H}, ILBs last period

State 8: GR\textsuperscript{L}, g\textsuperscript{L}, ILBs last period

Total debt level
1.5.2 Appendix 1.B: Numerical Computation

Algorithm of solving MPE: I use value function iteration to solve this discrete-choice model. Alternatively, the policy function iteration could be conducted with the concern of shortening the computation time and double-checking the result. I try both and there is no significant improvement of computation time for the second approach and the result is the same.

Step 1: Calibration the values of parameters from previous literature and data.

Step 2: Define the states of economy (8 states in this case). Specify the reward function u, the transition probability matrix P.

Step 3: Define the convergence tolerance (I set it as 10^-6), the maximum number of iterations (I set it as 1000) and an initial guess for v(s) for each state s (I set it as zero and also try different numbers to check the possibility of multiple equilibria).

Step 4: Update the v(s) for each state s using the Bellman operator defined above, until the value function in all states converges.
1.5.3 Appendix 1.C: Proof of Proposition 1

Proof of Proposition 1: In the scenario of a government guarantee on a DC portfolio, there is no access to ILBs. Then, from the budget constraint (6), in order to maximize the utility, the individual will maximize the value of potential guarantee government promise to provide.

Case 1. The nominal guaranteed rate of return $NGR_t \geq i_t$, assume that the individual will put a positive portion $x$ on the riskless bond, then the individual could always decrease $x$ and increase the portion invested on the risky stock to $y$ such that the sum of these two terms does not change, but the expected utility will increase. The option value based on the new portfolio will also be increased. Therefore, a positive number invested in a non-stock asset can never be optimal, which completes the proof.

Case 2. The nominal guaranteed rate of return $NGR_t < i_t$, the rate of return offered by riskless nominal bonds (here since there are no ILBs, it is easier to consider all terms as nominal rather than real). This case is trickier. In this case, although a rational agent has incentive to hold bonds, he should hold all the bonds in his non-mandatory portfolio. In other words, the agent should still hold equity in the guaranteed DC portfolios. An implicit assumption is that there are no short-sale constraints in the non-mandatory portfolio.
1.5.4 Appendix 1.D: Pricing Government Guarantee Costs using Martingale Pricing Theory

I use the Martingale pricing theory (or called option-based approach) to value the government guarantee. The basic idea is that the guarantee can be valued as a put option. I assume the evolution of instantaneous interest rate of return \( dS/S \) is given by:

\[
dS_t/S_t = \mu_t dt + \sigma_t dW_t \tag{1D}
\]

\( \mu_t \) is the expected rate of return for the DC portfolio, \( \sigma_t \) is the standard deviation for the rate of return on the portfolio, and \( dW_t \) is the standard Wiener process. For simplicity, I assume \( \sigma_t \) to be a constant and there exists a nominal risk-free asset that pays a nominal rate of return equal to \( r \). Also I assume a guarantee of a minimum fixed nominal rate of return equal to \( m \). The current value on a DC portfolio is denoted by \( S \). If this guarantee starts at date 0 and ends at date \( \tau \), then its value can be calculated by the standard Black-Scholes put option formula with an exercise price \( X = Se^{m\tau} \). So the value of the guarantee \( GR \),

\[
GR = Xe^{-r\tau}N(-d_2) - SN(-d_1) = Se^{m\tau}e^{-r\tau}N(-d_2) - SN(-d_1) = S[e^{(m-r)\tau}N(-d_2) - N(-d_1)] \equiv Sh(\tau) \tag{2D}
\]

where \( d_1 = (r-m+(1/2)\sigma^2\tau/\sigma\sqrt{\tau}) \) and \( d_2 = d_1 - \sigma\sqrt{\tau} \).

Besides, it is not difficult to calculate the guarantee that begins at some future year \( y \) and lasts for \( \tau \) periods, since the guarantee value is proportional to \( S \). Denote the current date as 0 and the current value of this guarantee as \( GR(0,y,\tau) \). Using the risk-neutral measure approach, the value of guarantee could be calculated by

---

32 See Pennacchi (1999), and Lucas et al. (2006) for the details of this approach.

33 Under the risk-neutral measure, the expectation is computed under the assumption that the rate of
This setting could also be relaxed to incorporate the growth of DC contribution as well as the multiple-year guarantee. If the nominal growth rate of new contribution is \( g \), then \( \hat{E}_0[S(y)] = Se^{r\gamma + gy} \). Substituting this into equation (3D), the guarantee value for the period \( y \) to \( y + \tau \) at date 0 is

\[
GR(0, y, \tau) = e^{-r\tau}h(\tau)\hat{E}_0[S(y)] \quad (3D)
\]

Taking one step further, if this guarantee lasts for \( n \) consecutive years, then the guarantee value, \( GR_n \), is

\[
GR_n = Sh(1) \sum_{y=0}^{n-1} e^{gy} \quad (5D)
\]
Chapter 2

Life-Cycle Investment with Ambiguity and Learning

2.1 Introduction

During the last few decades, a bunch of datasets such as Survey of Consumer Finances (SCF) or Panel Study of Income Dynamics (PSID) have documented a significant increase in the propensity of households to invest in stock, probably due to the growing prevalence of defined-contribution plans. Simultaneously, there has been a surge of research interest in household’s life-cycle financial investment decision problem. However, the existing life-cycle models seem still not satisfactory, of which the prediction is inconsistent with the stylized facts. As Figure 1 shows, one unsolved puzzle is that the empirical studies from data show that the share of portfolio invested in risky assets tend to be “hump shape” in age (See Poterba and Samwick 2001, Ameriks and Zeldes 2001, Faig and Shum 2002).

The early studies such as Merton (1969) and Samuelson (1969) show that a life-cycle agent should hold a constant fraction of the wealth in the risky asset through his or her life. Some subsequent studies have tried to reconcile the theory and stylized facts. One strand is to focus on how the labor income affects the portfolio choice. Early papers such as Bodie, Merton, and Samuelson (1992) treat the discounted labor income as riskless income, which even reinforce the puzzle since such assumption suggests that the allocation toward stocks should start high (close to 100%) early in life and decline over a person’s

---

1Data source: Guiso et al. (2001). “Direct and indirect stockholding” means the shares held directly, mutual funds, investment accounts, retirement accounts. “Risky financial assets” means direct and indirect stockholding, plus corporate, foreign and mortgage-backed bonds.
age as human capital depreciates. Some researches use micro data to calibrate the individual labor income process, such as Campbell et al. (2001). But only (counterfactually) high correlations between shock to labor income and stock return can explain why the young people hold such a low portion of risky assets. Some other models have to resort to some assumptions such as high participation costs in stock, etc, which also seems not quite substantial (See Abel 2001a, Vissing-Jorgensen 2002, Cocco, Gomez, and Maenhout 2005, Gomes and Michaelides 2005, Alan 2006 ). In addition, even if some qualitative evidence could be provided to some degree, but the quantitative match is quite poor (See Campbell et al. 2001). Benzoni, Collin-Dufresne and Goldstein (2007) impose a cointegration relationship between aggregate labor income and aggregate dividends on market portfolio and find that such a relation leads to a large reduction in optimal stock holdings for a typical risk-averse investors. By modelling human capital as risky asset, they find that the optimal portfolio choice for the young investor is to take a substantial short position in the risky portfolio, in sharp contrast to much of the previous literature.

More recently, another strand of literature has looked for explanations from non-standard preference, some of which also combine the learning mechanism. More specifically, this strand of literature is mainly on applications of ambiguity aversion preferences to the study of the portfolio choice problem. The earlier study on this literature typically applied either the multiple-prior approach or robust control approach (See Epstein, Wang 1994, Anderson, Hansen, and Sargent 2003, and Boyle, Garlappi, Uppal, and Wang 2010), but do not introduce learning. Epstein and Schneider (2007) build a model in which ambiguity (on the mean stock return) can be reduced over time through learning. Campanale (2011) also applies this theoretical framework to quantitatively rationalize the moderate stock market participation rates.
Figure 1: Life-Cycle Profile of Risky Assets Holdings (US Data)
In this paper, we build a dynamic model in which an investor is concerned about ambiguity over labor income and averse to the model uncertainty. Regarding the model framework, the closest paper is Campanale (2011), of which the main focus is on participation rate rather than the portfolio choice in this paper. Relative to Campanale (2011) who uses multiple-priors utility model, we use the generalized recursive ambiguity utility model. As discussed in Ju and Miao (2012), this is a generalized utility model which includes some other models of ambiguity as special cases, e.g., the recursive utility model of Epstein and Zin (1989), the recursive smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2009), the recursive multiple-priors model of Epstein and Schneider (2003), Campanale (2011), as well as the robust control model of Hansen and Sargent (2007).

Another difference between our paper and Campanale (2011) is that we introduce ambiguity and learning on labor income instead of stock return. Including Campanale (2011), there has been some previous paper considering the ambiguity on stock return, which moves a step in the right direction towards matching the empirically observed pattern. Considering both the nature and the significance of future flow of labor income in the life-cycle portfolio framework, it is reasonable to consider people could perceive labor income process to be ambiguous. And to the best of our knowledge, no paper has yet considered ambiguity on labor income. Hence it makes our exploration towards this direction potentially meaningful and interesting in this field.

For labor income process, we adopt the “heterogeneous income profiles” (HIP) process, departing from the traditional “restricted income profiles” (RIP) process. Guvenen (2007) points out that a potential important reason why the RIP setting has been overwhelmingly used given the weak empirical support from labor income data is that this process could generate the consumption behavior consistent with stylized facts (See Deaton and Paxson

---

2In addition to the stock market participation rate, Campanale (2011) also tries to simulate the conditional shares but is unable to generate hump-shaped age-portfolio profile.

3The HIP refers to a labor income process, in which individuals are subject to shocks with modest persistence while facing life-cycle profiles that are individual-specific and vary significantly across the population.

4The RIP refers to a labor income process, in which individuals are subject to large and very persistent shocks while facing similar life-cycle income profiles.
Considering there was no relevant study of the economic agent’s behavior when he or she faces the HIP process, Guvenen (2007) fills this gap and study the life-cycle consumption behavior, and compare the effects of two labor income settings. Guvenen (2007) shows that these two models display some similarities. In addition, HIP performs better in explaining substantial risk in consumption inequality over the life cycle, and generates steeper consumption profiles for people with higher education, neither of which are generated by RIP model.

By adding the features described above, our model is able to create hump-shaped portfolio holdings in relation to age. As we will discuss in detail later, introducing learning plays the key role in the success to explain this stylized fact. The mechanism is intuitive: when there exists ambiguity over labor income, the future flow of labor is better described as risky in contrast to the riskless asset assumed by many of previous literature. Hence the agent will not invest so much in risky assets at the beginning of a working life. As the agent approaches retirement, there are two partially offsetting effects. First, the learning mechanism gradually solves the uncertainty and leads human capital to take on more riskless features. Second, the residual value of future labor income shrinks, since the agent has fewer years left to work, and therefore the value of the bond position implicit in his human capital decreases. Eventually, this second effect prevails, and hence explains the hump-shaped stock allocation of his or her life-cycle profile.

The rest of the paper proceeds as follows. Section 2.2 presents the description of the model. Section 2.3 conducts calibration, main findings of analysis as well as robustness check. Section 2.4 concludes.

2.2 Model

2.2.1 Utility

The standard rational expectation model assume the investor have an intertemporally additive expected utility function. The implicit assumption behind this setting is that the
agent know which model is the correct model. However, many argue that this assumption is too strong. That is the motivation to introduce ambiguity into the model. Intuitively, the consideration of ambiguity will drastically reduce the demand for the risk asset. That is why this utility class has successfully explained the high risk premium puzzle.

Suppose the agent is not sure whether he is picking up the right model, it is natural he would also consider all the other alternative models. We need to give a framework how the agent compare all the possible models. Here, we assume the agent has ambiguity over the probability distribution on state space $S$. This uncertainty is described by an unobservable parameter $z$ in the space $Z$.

The agent has a prior $\mu_0$ over the parameter $z$. Each parameter $z$ gives a probability distribution $\pi_z$ over the full state space. The posterior $\mu_t$ and the conditional likelihood can be obtained by Bayes’ learning mechanism.

We consider the following recursive ambiguity utility function: \[^5\]

$$V_t(C) = [C_t^{1-\rho} + \delta \{v^{-1}E_{\mu_t}v \circ u^{-1}E_{\pi_z,t}[u(V_{t+1}(C))]\}^{1-\rho}]^{\frac{1}{1-\rho}}, V_{T+1} = 0 \quad (1)$$

and $u$ and $v$ are given by

$$u(c) = \left(\frac{1-v}{1-\gamma}\right), \gamma > 0 \text{ and } \gamma \neq 1, \quad (2)$$

$$v(c) = \left(\frac{1-v}{1-\eta}\right), \gamma > 0 \text{ and } \gamma \neq 1, \quad (3)$$

$u$ and $v$ are increasing functions and $\mu$ is a subjective prior over the set $\prod$ of probability measures on $S$, which follows the literature standard. $\delta \in (0,1)$ is the subjective discount factor, $1/\rho$ is the elasticity of intertemporal substitution, and $\gamma$ and $\eta$ are the coefficients of constant relative risk aversion and ambiguity aversion, respectively. For illustration purposes, we define $\phi = v \circ u^{-1}$, then attitudes towards ambiguity are characterized by the function form of $\phi$, while attitudes towards risk are characterized by the function form

\[^5\]See Klibanoff, Marinacci, and Mukerji (2005), Ju and Miao (2012) for this utility class.
of \( u \). Ambiguity is characterized by properties of the subjective set of measures \( \Pi \). The agent displays risk aversion if and only if \( u \) is concave, while he reveals ambiguity aversion if and only if \( \phi \) is concave. Intuitively, an ambiguity-averse agent prefers consumption plan that is more robust to the potential variation in probabilities. This distribution represents the uncertainty about the ex ante utility evaluation of \( C \), \( E_\pi u(C) \) for all \( \pi \in \Pi \). When \( \phi \) is in linear form, the agent is ambiguity neutral and the smooth ambiguity utility function reduces to the standard expected utility function. If \( \gamma = \eta \), then the agent is ambiguity neutral and (3) reduces to the standard Epstein-Zin utility model. In this case, the posterior \( \mu_t \) and the likelihood distribution \( \pi_{z,t} \) can be reduced to a predictive distribution, which is the key idea underlying the Bayesian analysis. The agent is ambiguity averse if and only if \( \gamma < \eta \). Hence, this model achieves a separation between ambiguity and ambiguity aversion.

If we take a snapshot at one period, the utility setting with ambiguity preferences over consumption is shown as below:

\[
v^{-1} \left( \int_\Pi v \circ u^{-1} \left( \int_S u(C) d\pi \right) d\mu(\pi) \right), \quad \forall C : S \rightarrow R_+ \quad (4)
\]

If the agent’s ambiguity aversion goes to infinity, (1) is reduced to the recursive multiple-priors model of Epstein and Schneider (2007):

\[
V_t(C) = \min_z \left[ C_t^{1-\rho} + \delta E_{\pi_{z,t}} [V_{t+1}^{1-\gamma}(C_{t+1})] \right]^{1/(1-\gamma)} \] \( (5) \)

Finally, the utility function defined in (1) is also connected with robust control, as studied by Hansen and Sargent (2007). Specifically, the agent recognizes the possibility of model misspecification and accounts for it in his decision. Our framework could be considered as a special case of taking considerataion of model misspecification, since in the general robust control framework, we need not restrict model misspecification to uncertainty re-

---

\(^6\)This setting is ordinally equivalent to the smooth ambiguity model of Klibanoff, Marinacci, and Mukerji (2005):
\[
\int_\Pi \phi \left( \int_S u(C) d\pi \right) d\mu(\pi) \equiv E_\mu \phi(E_\pi u(C)).
\] They adopt the power-exponential specification to model the feature of ambiguity aversion.
Regarding a particular parameter. The agent knows the distribution $\pi_z$ for each possible model. The agent is ambiguous about which is the right model specification and he has a subjective prior $\mu_0$ over alternative models.

### 2.2.2 Labor Income Process

In the year before retirement $t \leq T_1$, I assume the investor i’s log labor income is given by:

$$y_{i,t} = \alpha_i + \beta_i t + z_{i,t}$$  \hspace{1cm} (6) \\
$$z_{i,t} = \lambda z_{i,t-1} + u_{i,t}$$  \hspace{1cm} (7)

where $\beta_i$ is the individual specific labor income growth rate with variance $\sigma_{\beta_i}^2$, $u_{i,t}$ is the innovation to the AR(1) process.

Following Guvenen (2007), the labor income process we consider here is “heterogeneous income profiles” (HIP) process, where individuals are subject to shocks with modest persistence while facing life-cycle profiles that are individual-specific and vary significantly across the population (that is $\lambda$ is significantly less than 1 and $\sigma_{\beta_i}^2$ is large). An alternative setting is called “restricted income profiles” (RIP) process, individuals are subject to large and very persistent shocks while facing similar life-cycle income profiles (i.e., $\lambda$ is close to 1 and $\sigma_{\beta_i}^2$ is zero). We choose the first one because of two reasons: (1) there are more empirical evidence from labor income data in favor of HIP; (2) The consumption behavior generated in response to the HIP process is consistent with important empirical facts such as within-cohort consumption inequality over the life cycle and the consumption growth parallels income growth over the life cycle. As Guvenen (2007) discusses, the fit to consumption data from HIP is much better than that of RIP.

### 2.2.3 The Kalman Filtering Learning

In line with the standard literature, the stochastic component of income is modeled as an AR(1) process plus a purely transitory shock. The process for log earnings, $y_{i,t}$, of individual i who is t years old is given by
\[ y_{i,t} = \alpha_i + \beta_i t + z_{i,t} + \epsilon_{i,t} \quad (8) \]

\[ z_{i,t} = \lambda z_{i,t-1} + u_{i,t} \quad (9) \]

\[ z_{i,0} = 0 \quad (10) \]

We assume that \( \alpha_i \) is known and \( \beta_i \) is distributed across individuals with zero mean, and variances of \( \sigma_{\alpha}^2 \). For the innovations \( u_t \) and \( \epsilon_t \), we assume them to be with zero mean, and variances of \( \sigma_u^2 \) and \( \sigma_\varepsilon^2 \), and also independent of each other and over time.

We assume the agent is ambiguous about his labor income growth rate \( \beta \). To embed the learning process into a life-cycle model, we need to be specific about what individual knows about \( \beta \). So we assume there is some prior belief about his income growth. This prior could incorporate some relevant information unavailable to the econometrician. A rational agent will refine these initial beliefs over time by incorporating the information revealed by subsequent series of labor income realizations. We introduce Kalman filter learning into the model. An important step is that we need to specify which components of income are observable. We assume that individuals observe only total income, \( y_{i,t} \), and not its components separately. As Guvenen (2007) discusses, if both \( y_{i,t} \) and the stochastic component, \( z_{i,t} + \epsilon_{i,t} \), were observable, then individual’s income growth rate \( \beta \) would be revealed in just one period.

In this standard Kalman learning framework, the “state equation” describes the evolution of the vector of state variables that is unobserved by the agent:

\[
\begin{bmatrix}
\beta_{i,t+1} \\
S_{i,t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & \lambda
\end{bmatrix}
\begin{bmatrix}
\beta_{i,t} \\
S_{i,t}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
u_{i,t+1}
\end{bmatrix}
\quad (11)
\]

Equation (11) expresses the observed log labor income as a linear function of the underlying hidden state and a transitory innovation:
\( y_{i,t} = \alpha_i + [t \ 1] \begin{bmatrix} \beta_i \\ z_{i,t} \end{bmatrix} + \varepsilon_{i,t} = \alpha_i + H'_t S_{i,t} + \varepsilon_{i,t} \quad (12) \)

Both shocks are assumed to be i.i.d. Gaussian distribution and are independent of each other. The covariance matrix of \( i_{i,t} \) and the variance \( \varepsilon_{i,t} \) are denoted by \( Q \) and \( R \) respectively. The prior belief over \( (\beta_i, z_{i,1}) \) is modeled by a multivariate Normal Distribution with mean \( \hat{S}_{i,1|0} \equiv (\hat{\beta}_{i,1|0}, \hat{z}_{i,1|0}) \) and variance-covariance matrix:

\[
P_{1|0} = \begin{bmatrix} \sigma^2_{\beta,1|0} & 0 \\ 0 & \sigma^2_{z,1|0} \end{bmatrix} \quad (13)\]

The agent’s belief about the unobserved state vector \( S_{i,t} \) has a normal posterior distribution with a mean vector \( \hat{S}_{i,t|t} \) and covariance matrix \( P_{t|t} \). \( \hat{S}_{i,t+1|t} \) and \( P_{t+1|t} \) denote the one-period-ahead forecasts of these two variables and their evolutions via Kalman learning are given by

\[
\begin{align*}
\hat{S}_{i,t|t} &= \hat{S}_{i,t|t-1} + P_{t|t-1} H'_t [H'_t P_{t|t-1} H_t + R]^{-1} * (y_{i,t} - H'_t \hat{S}_{i,t|t-1}) \\ \hat{S}_{i,t+1|t} &= F \hat{S}_{i,t|t} \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1} H'_t [H'_t P_{t|t-1} H_t + R]^{-1} * H'_t P_{t|t-1} \\ P_{t+1|t} &= F P_{t|t} F' + Q
\end{align*} \quad (14-17)\]

As a characteristic of Kalman filter learning, the posterior variance of \( \beta_i \) is monotonically decreasing over time, hence, the belief on \( \beta_i \) become more concentrated around the true values after each new observation.\(^7\) Finally, the conditional labor income based on the updated belief is normally distributed:

\[
y_{i,t+1|t} \sim N(H'_t \hat{S}_{i,t+1|t}, H'_t P_{t+1|t} H_{t+1} + R) \quad (18)\]

\(^7\)This could be shown from equation (16) mathematically.
2.2.4 Financial Market

Without loss of generality, we assume the agent could invest two assets: a riskless bond with real return $R_f$, and a risky stock with real return $R_t$. The excess return return on the risky asset, $R_{t+1} - R_f$, is given by

$$R_{t+1} - R_f = \mu + \eta_{t+1} \quad (19)$$

where the innovation $\eta_{t+1}$, is assumed to be i.i.d. and normally distributed with zero mean and variance $\sigma^2_{\eta}$. We allow innovations to excess returns to be correlated with innovations to the shock to labor income $u_t$, and the correlation coefficient is denoted by $\rho_{\eta u}$.

There is a mandatory social security system for retirement. During working life the individual must save a fraction $\theta$ of the current labor income into the system for retirement. Hence the disposable labor income is given by

$$y_{d,i,t} = (1 - \theta) y_{i,t} \quad (20), \text{ for } t \leq T_1$$

During working life, the individual cannot consume or borrow against the money which has been put into social security system. As age $T_1$ retirement wealth is converted into a riskless annuity, so that the individual receives the annuity value corresponding to $W^{R}_{i,t}$ in each of retirement years. We assume that the individual is forced to hold retirement wealth in riskless assets, which is typical for social security systems.\(^8\)

2.2.5 Liquid Wealth and Individual Constraints

We denote cash on hand in period $t$ by

---

\(^8\)Previous literature also considers the scenario in which retirement wealth could be invested in risky assets. See Campbell et al. (2001)
\[ X_{i,t} = W_{i,t} + (1 - \theta) * y_{i,t} \quad (21) \]

\( W_{i,t} \) represents liquid wealth of investor \( i \) at date \( t \). Liquid holdings of bonds and stocks are denoted by \( B_{i,t} \) and \( S_{i,t} \) respectively. We impose non-negative consumption, borrowing and short-sales constraints:

\[ C_{i,t} \geq 0 \quad (22) \]
\[ B_{i,t} \geq 0 \quad (23) \]
\[ S_{i,t} \geq 0 \quad (24) \]

The timing for each period of a household’s working life \((t \leq T_1)\) is as follows: the investor starts the period with liquid wealth \( W_{i,t} \) and retirement wealth \( W_{i,t}^{R} \). Then labor income \( y_{i,t} \) is realized. The choice variables include consumption and asset allocation between risky and riskless assets. We let \( \psi_{i,t} \) denote the portfolio share of the risky asset. Next-period liquid and retirement wealth are then given by

\[ W_{i,t+1} = [1 + \psi_{i,t} R_{t+1} + (1 - \psi_{i,t}) R_f][W_{i,t} + (1 - \theta) * y_{i,t} - C_{i,t}] \quad (25) \]
\[ W_{i,t+1}^{R} = [1 + R_f][W_{i,t}^{R} + \theta * y_{i,t}] \quad (26) \]

After retirement \((t > T_1)\), retirement wealth no longer accumulates. Instead, it provides riskless annuity \( A(W_{i,s}^{R}) \). After-tax labor income \((1 - \theta) * y_{i,t} \) in the above two formulas is now replaced by \( A(W_{i,s}^{R}) \).

### 2.2.6 The Individual’s Optimization Problem

The individual’s problem is to maximize the recursive ambiguity utility subject to all constraints discussed above during working-life and retirement phase. The control variables are \( C_{i,t} \) and \( \psi_{i,t} \) at each date \( t \). The state variables are time \( t \), cash on hand \((X_{i,t})\), retirement wealth \( W_{i,t}^{R} \), and the last period’s forecast of the true state in the current period,
The problem is to solve for the policy rules as functions of the state variables. Note during retirement, pension income is constant and there is source for neither uncertainty nor learning, so the problem will simplify significantly.

This problem cannot be solved analytically, and we will resort to standard numerical methods for the dynamic optimization. We use backward induction method for this finite-period problem and discretize the state-space and choice variables to address the continuous variable issue. The Gaussian quadrature is used to calculate the expectation operator. The details of algorithm has been provided in the appendix.

2.3 Calibration

Now we describe the parameterization of this life-cycle model and it is summarized in Table 1 below. Individuals enter the labor market at age 20, retire at 65 and die at 80. We set the discount factor to be 0.96. For the recursive ambiguity preference parameters \( \rho, \gamma, \) and \( \eta, \) we set \( \rho = 0.5, \gamma = 5, \) and \( \eta = 80 \) as benchmark parameter, following Chen, Ju and Miao (2013). For benchmark intercept and growth rate of wage, we set \( \alpha = 1.5 \) and \( \beta = 0.012. \)

The other HIP process parameters are also taken from Guvenen (2007), including the variance of income growth rate \( (\sigma_\beta^2 = 0.00038) \), the autocorrelation coefficient of the stochastic component of income, \( z_{t,t} (\lambda = 0.82) \), the variance of innovation to \( z_{t,t} (\sigma_u^2 = 0.025) \), the variance of purely transitory shock to income is \( (\sigma_z^2 = 0.032) \). In addition, the following parameters are taken from the standard portfolio-choice literature: riskless real interest rate \( r = 2\% \), equity risk premium \( \mu = 4\% \), standard deviation of innovations to risky asset \( \sigma_\eta^2 = 0.157 \), correlation between income and stock return shock \( \rho_{\eta u} = 0.516 \), and the social security tax rate \( \theta = 0.1 \).

Also we assume that individuals could only observe the total income \( y_t \) and stochastic component \( (z_t + \varepsilon_t) \) is unobservable. The motivation behind this assumption is that if the stochastic component \( (z_t + \varepsilon_t) \) were also observable, then it will only take one period to

---

9As discussed in Guvenen (2007), the intercept term is a scaling parameter and the growth rate \( \beta \) is set to the mean growth of log income in the Panel Study of Income Dynamics (PSID)
learn the value of wage growth rate $\beta$ and there is no further learning after the first period. Of course, to introduce nontrivial learning, we could allow either $z_t$ or $\varepsilon_t$ to be separately observable. However, there is no good reason why one component is observable and the other is not.

Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.96</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>5</td>
<td>relative risk aversion</td>
</tr>
<tr>
<td>$\eta$</td>
<td>80</td>
<td>ambiguity aversion parameter</td>
</tr>
<tr>
<td>$1/\rho$</td>
<td>2</td>
<td>elasticity of intertemporal substitution</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.012</td>
<td>benchmark growth rate of labor income</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.5</td>
<td>intercept of labor income</td>
</tr>
<tr>
<td>$\sigma^2_{\beta}$</td>
<td>0.00038</td>
<td>variance of labor growth rate</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.82</td>
<td>autocorrelation coefficient of $z_{i,t}$</td>
</tr>
<tr>
<td>$\sigma^2_{\nu}$</td>
<td>0.025</td>
<td>variance of innovation to AR(1) process</td>
</tr>
<tr>
<td>$\varepsilon^2$</td>
<td>0.032</td>
<td>variance of purely transitory shock to income</td>
</tr>
<tr>
<td>$r$</td>
<td>0.02</td>
<td>riskless real interest rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.04</td>
<td>equity risk premium</td>
</tr>
<tr>
<td>$\sigma^2_{\eta}$</td>
<td>0.157</td>
<td>standard deviation of innovations to risky asset</td>
</tr>
<tr>
<td>$\rho_{\eta\nu}$</td>
<td>0.516</td>
<td>correlation between income and stock return shock</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.1</td>
<td>social security tax rate</td>
</tr>
</tbody>
</table>

2.4 Results Analysis and Robustness Check

As Figure 2 in Appendix shows, the share on risky asset starts at a low value and increases until age 55, after which it goes down and the decrease accelerates after the retirement age 65. Basically, the baseline model prediction is in line with data, although the quantitative match is not perfect, especially after the retirement age. Most importantly, the model achieves one of the main goals of this paper: to generate the hump-shaped portfolio strategy over the life-cycle and provide an intuitive mechanism for that, which is not well explained by previous literature. Figure 3 plots the consumption and labor income, which is the averaged results of 1000 households receiving random draws of the stochastic variables.

The model mechanism is described as follows: most previous models attribute riskless-
ness qualities to the future flow of labor income, which suggests people should invest more on risky asset when they are young. Here we assume there is uncertainty about the growth rate of labor income, so the future discounted flow of labor is a uncertain financial source for individuals. Hence the individuals will not invest too much at the beginning of their working life (or even want to take a short position in stock if possible). As the agent approaches retirement, there are two partially offsetting effects. First, the learning mechanism gradually solve the uncertainty and make human capital take on more bond-like features. Second, as the agent get older, the value of the his human capital decreases. Eventually, this second effect dominates, which explains why the agent starts to reduce her investment strategy.

Besides, one thing worth noting is the Kalman filter learning mechanism. We introduce Kalman filter as learning mechanism, unlike Campanale (2011) which assumes learning velocity as an exogenous variable\textsuperscript{10}. One concern is that typically Kalman filter learning will resolve (a large fraction of) prior uncertainty very quickly, but it is not the case for the baseline model. First, as discussed before, we assume that individuals could only observe the total income $y_t$ and stochastic component ($z_t + \varepsilon_t$) is unobservable. This assumption will rule out the scenario that the uncertainty will resolve in one period by learning, which does not make economic sense. Also, there are two more reasons for the slow learning. The first reason is that the contribution to income from the slope parameter is small when individual are young.\textsuperscript{11} The second reason is due to moderate persistence of income shocks in HIP process.\textsuperscript{12} For more detailed discussion about learning speed regarding HIP process, see Guvenen (2007).

Figure 4 in appendix documents the sensitivity of the results to changes in risk aversion coefficient. As expected, a less risk-averse agent will invest more in risky asset. More

\textsuperscript{10}Specifically, Campanale (2011) specifies three parameters for learning processes: the long run ambiguity, the initial ambiguity and the learning speed.

\textsuperscript{11}This argument could be confirmed when we adjust the relative contribution between intercept and slope parameter for robustness check in the following section.

\textsuperscript{12}Guvenen (2007) shows that the speed of learning is not a monotone function of persistence: as $\lambda$ increases up to 0.85, the learning speed slows down but then speeds up again. In particular, learning is fastest when the income shock follows a unit root process.
importantly, compared to the baseline case, the portfolio-choice patterns over the life cycle do not change qualitatively with different degrees of risk aversion.

How the life-cycle portfolio choice changes with ambiguity aversion is shown in Figure 5 in appendix. The more ambiguity-averse agent will invest less in risky asset, especially at the earlier age when there is greater uncertainty. As learning from past wage resolves the uncertainty, the behavior will get closer to the baseline model.

As shown in Figure 6 in appendix, the patterns of risky asset holding over different levels of wealth is perhaps one of the most interesting findings. We use the level of intercept of income to measure the wealth since it is simple and intuitive. In addition, we try to compare people with different wealth levels since the beginning of their working life and there is no uncertainy about this.\footnote{That is why we do not use the different level of income growth rate or other measure to represent how “wealthy” individuals are, which will greatly complicate the analysis and add difficulty in providing intuitive explanation.} In this setting, there is no uncertainy about wealth level and the portfolio choice will differ since the first year of working life. At the first year of working life, the wealthier people will invest more in risky asset. The intuition is clear: they have more discounted cash flow and they know this without any uncertainty. This means they have more “bond-like” future cash flow, and they intend to invest more risky asset now. Also, the more wealthy people will learn faster than the less wealthy people, since the contribution to income from the slope parameter is larger. In addition, faster learning will make the more wealthy people reach peak allocation in risky asset at the earlier age than the less wealthy people.

2.5 Conclusion

In this paper, we have studied the implications of ambiguity and learning in a life-cycle portfolio choice model with labor income following a HIP process. The model is able to generate hump-shaped investment behavior over the life-cycle. Also another contribution is that we use Kalman filter learning mechanism so we do not need to resort to exogenous learning speed or initial ambiguity used by previous literature. One limitation is that the
model prediction of shares in risky asset after retirement drop much quickly than what is revealed in the data. This limitation is due to the nature of this model, which splits the life span into two stages: working life and retirement life. In addition, it would be interesting to augment this model with other economic decisions, such as labor supply and retirement decision.
2.6 Appendix

2.6.1 Appendix 2.A: Figures

Figure 2: Data Vs Baseline Model
Figure 3: Average Consumption and Labor Income
Figure 4: Different Degrees of Risk Aversion ($\gamma = 2, 5, 10$)
Figure 5: Different Degrees of Ambiguity Aversion ($\eta = 60, 80, 100$)
Figure 6: Different Levels of Wealth ($\alpha = 0.3, 1.5, 5$)
2.6.2 Appendix 2.B: Numerical Computation

The problem could only be solved numerically. We use the standard backward induction method to solve this finite-period dynamic optimization and compute by discretizing the state variables and the choice variables. For expectation operation, we compute by using Gaussian quadrature method (i.e., using Gaussian quadrature to approximate the distributions of innovations to labor income and the return of risky asset, See Tauchen and Hussey 1991). Then the decision rules are computed from the agent’s dynamic programming problem. In the last period T, the investor consumes all the wealth: $C_t = W_t$, there is no portfolio choice. In every period prior to T, and for each possible combination of state variables, we compute the value associated with each level of consumption and conditional share on risky asset. This value is the sum of current-period utility and the discounted continuation value. For the points which do not lie on the grid, we use cubic spline interpolation. Following standard method, the combinations of the choice variables which are not allowed by constraints are assigned a very large negative value to ensure they are never optimal. Repeat the above procedure until we reach $t=0$.

After the decision rules are determined, simulation are conducted to compute the lifecycle profiles for 1000 random draws of the stochastic variables and the simulation is repeated for 50 times and the results are the average of these 50 repetitions. Increasing grid points or number of random draws does not change the result much.
Chapter 3

China’s Pension Reform and Inflation-Linked Bonds

3.1 Introduction

Due to the effects of the “one-child” policy, an aging population has become a more serious problem for China. Accordingly, problems associated with the current Chinese pension system have drawn increasing attention in recent years. There is little doubt that the original Pay-As-You-Go (PAYGO) system is financially unsustainable. Following other countries facing similar problems, China is trying to switch from a PAYGO Defined Benefit (DB) system to fully funded Defined Contribution (DC) system. But doing so will give rise to a new problem: Workers will have to bear all the investment risk in the new system. Furthermore, distorted incentives in the system have led to the failure of previous pension reforms. Recently, many have come out proposing new pension reforms to address these pressing questions.

In this paper, I will argue that the government could issue Inflation-Linked Bonds (ILBs), which can reduce the investment risks and provide important protection against inflation — the main concern for pension investment. Although there are no such assets in China mainland, it is noticeable that Hong Kong has already issued ILBs in July 2011. Besides, drawing on the lessons of Chile’s pension reform thirty years ago, indexed bonds (called recognition bonds in Chile) might also provide better incentives to manage the pension system transition. I find, with the help of this kind of bonds, Chile conducted a
successful pension reform in 1980s since it boosted people’s confidence on government and hence on that pension reform. Case studies of Hong Kong and Chile in this paper will help to understand the impact of inflation-linked bonds on China.

There will be three parts in this chapter as shown below: Section 3.1 provides a comprehensive review of China’s current pension structure, problems, and possible proposals. Section 3.2 first analyzes the benefits of indexed bonds from international experiences. A case study of Hong Kong’s newly issued index bonds follows. The paper then presents an empirical study using the Chinese Mainland’s actual macro and finance data. The paper focuses on whether ILBs could improve the investment risk-return tradeoff. In other words, it examines whether ILBs could reduce investment risk and achieve target retirement income. A case study on Chile is also done from the perspective of incentive mechanism. Section 3.3 concludes.

3.2 Review of China’s Pension-Reform Structure

3.2.1 Demographic Trend and the Aging Population of China

Table 1: Age structure change of China in the past 10 years

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0-14 years old</td>
<td>16.60%</td>
<td>22.89%</td>
</tr>
<tr>
<td>15-59 years old</td>
<td>70.14%</td>
<td>66.78%</td>
</tr>
<tr>
<td>60+ years old</td>
<td>13.26%</td>
<td>10.33%</td>
</tr>
</tbody>
</table>

(Source: China’s Statistical Yearbook, 2010)

The problem of China’s aging population is obvious, and the United Nations in 2010 forecast that this problem would deteriorate over the next few decades (see Table 1 and Figure 1). Several reasons contribute to this phenomenon, including the steep decline of
the birth rate since China implemented the one-child policy in the 1980s, the fact that replacement-level fertility has been occurring since the early 1990s; and the increasing life expectancy and continually declining mortality rates in middle-aged and elderly people. China’s demographic dividend will decline after the working-age population, those aged 20 to 60, peaks at around 70% in 2012 to 2015. Some worry that China may have an insufficient labor force in the future to sustain economic growth. A more imminent issue is that increasing old-age dependency ratio due to aging population will jeopardize the current pension system in China (See Figure 2). Noticing the potential problem in the future, China has initiated series of pension reform since 1980s, which I will discuss in detail in the next section.

Figure 1: China’s Median Age Projection (2015-2100)
3.2.2 Current Chinese Pension System

3.2.2.1 Current Structure

In 1997, China announced that it would deviate from the pure PAYGO system by establishing fully funded individual pension accounts. Most academic literature refers to this step as a milestone in China’s pension reform. However, 1997 system largely failed as funds in most individual accounts were “borrowed” by the government to finance the benefits payments of the basic pension plan. The Chinese pension system is a three pillar-system that includes the basic social pooling pension, or mandatory PAYGO benefit; the individual account, which is a mandatory fully funded contribution; and the voluntary annuity.

- Basic Social Pooling Pension (mandatory, PAYGO, defined benefit)
- Contribution: About 20% of payroll (depending on city), 100% responsibility of employer,
- Benefits: Based on specific formula involving years of contribution, individual average wage, and social average wage,

\(^1\)The old-age dependency ratio is the ratio of the population aged 65 years or over to the population aged 20 to 64. The ratio is presented as number of dependents, per 100 persons of working age (20 to 64).
- Minimum 15 years of contributions to qualify,
- Indexation rules do not apply, but, since 2006, in practice benefits tend to increase by 10% each year.

- Individual Account (mandatory, “nominally” fully funded, defined contribution)
  - Contribution: 8% of individual wages, which are subject to lower and upper bounds, that is, 60% and 300% of local average wages, 100% responsibility of employee (civil servants, especially government employees, stay in another pension system, See Caveats below for more details),
  - Benefit: - No clear rule about rate of return on this account,
  - “Nominally” fully funded but they are actually mingled with the social pooling pension and both are managed by local government together,
  - By 2009, at least 13 provinces have committed to full funding and prohibited “leakages” to the social pooling. But due to the heavy fiscal burden of legacy cost, they could sustain this only by depending on the fiscal transfer from the central government. Some of them have failed in fulfilling their commitment and allowed the leakages from fully funded system to social pooling again, such as Liaoning Province since four years ago, which was the first of the 13 provinces to commit to fully funded system.
- Voluntary pension (The main voluntary pension plan is enterprise annuity (EA), which is a fully funded, defined contribution plan, with complementary individual plans, much like 401(k) plans in the United States.)

Several Caveats:
- This three-pillar structure constitutes the major accomplishment of China’s pension reform and provides a good basis for continuing reform. This reform begins with the urban worker and does not yet cover workers in rural locations. The Chinese government places a priority on pension reform because it relates to the means of production in China. It is worth noting, however, that this priority has become weaker as mass industrialization oc-

---

2See Section “Legacy Cost” for more details.
3The rural pension program is designed and operated separately from the urban pension system. See Leisering et al. (2002) and Chen (2004) for further discussion.
occurs in China (see Leisering et al., 2002). If the Chinese industrialization process continues its rapid pace in this century, it is likely that most of the population will live in urban areas at some point in the future (see World Bank 1997). In Jan 17, 2012, China announced that people living on its towns and cities outnumbered those in the countryside, making it a predominantly urban nation for the first time in Chinese civilization.

- The basic pension plays a key role not only in redistribution by providing higher replacement rates for people with lower incomes but also in reducing poverty and providing insurance. The basic pension comprises four subcategories:

  (a) The Basic Old Age Insurance System (BOAIS), which covers urban workers. The “Basic Social Pooling Pension” is just one part of the two parts of BOAIS, which also includes individual accounts.

  (b) The rural pension system covers rural workers.

  (c) The pension plans of state organizations and public institutions cover public employees. In this system, the pension benefits are financed by government budgets without any contribution by workers. Some question whether this policy is fair, especially after China announced in June 2012 that it was considering deferring the retirement age. Now, the government is experimenting with including public sector employees\(^4\), mainly in the education and health sectors, in the mandatory urban system.

  (d) The Minimum Life Security System, or the urban Dibao,\(^5\) covers the individual who are not covered by (a), (b) or (c). Dibao provides resources to some poor elderly.

By 2008, these three government-sponsored and mandatory pension arrangements - BOAIS, the rural pension system, and state or organization pension benefits - has covered approximately 40% of the Chinese labor force. The three plans respectively covered 28.3, 7.2, and 5.2% of the workers. The Dibao system covered those individuals that the other three plans did not cover (see Impavido et al., 2009).

\(^4\)The public sector employees mentioned here does not include government employees. In China, the former is referred as “Shi Ye Dan Wei” and the later is referred as “Gong Wu Yuan”

\(^5\)Dibao was originally conceived for urban residents but now has also extended to rural residents.
3.2.2.2 Sources of Funding

There are three main sources of pension fund. The BOAIS represents the largest pool of assets in the pension system. The establishment of the National Social Security Fund as a demographic buffer has recently increased this pool of assets. The more recently established EA system and the complementary individual voluntary plans have also increased this pool.

(a) BOAIS (Social Pooling and Individual Account):

Social pooling and individual accounts now make up the largest pool of pension funds in the Chinese pension system. Official figures for 2008 (see Impavido et al., 2009) reported RMB 993.1 billion in total assets at the national level. In addition, approximately RMB 120 billion has been accumulated in the individual accounts in the 13 provinces that are working with fully funded system of individual accounts. Of this total, NSSF managed RMB 33 billion in 2008. Provinces, cities, and counties managed the other RMB 960.4 billion. Data on how much the individual accounts in the provinces other than those 13 provinces have accumulated are not publicly available.

(b) NSSF

The NSSF was established in 2000 as a national long-term strategic reserve fund to meet the future pension obligations of the BOAIS. The NSSF has a dual role as buffer in response to the expected peak of the Chinese demographic transition around 2030 and as a reserve fund for pension expenditures in case the BOAIS is unsustainable. Funding sources of NSSF include: (1) capital derived from reduction or transfer of state-owned shares; (2) fiscal allocation of the central government; (3) allocation from the lottery-based public welfare fund; (4) capital raised in other manners with approval of the State Council; and (5) investment proceeds and equity assets. The concentration of the NSSF sources of funds has decreased in the past few years, with the share of proceeds from initial public offerings (IPO) and lotteries increasing from 20% in 2001 to more than 50% in 2006.

6To improve standards by centralizing the fund, the NSSF manages the central government’s fiscal transfers to nine of these 13 provinces. The provinces do not need to pay NSSF the management fee, which the Ministry of Finance (MOF) budget covers. I will discuss NSSF in detail in the following section.
By the end of 2011, the total asset of the NSSF had reached RMB 868.8 billion. Notwithstanding the rapid growth, the Chinese NSSF is still smaller than that of reserve funds in other countries, such as Canada, Norway, Ireland, and New Zealand. Some has proposed that the government should put more funds into NSSF or a similar organization for more efficient fund management (see Zheng, 2011).

By regulation, Chinese state-owned enterprises must contribute 10% of their IPO proceeds to the NSSF at the time of their public offerings. As Bodie and Merton (1992) point out, it might be the best strategy to give shares of privatized SOEs to pension fund. First, selling shares of SOEs provides the Chinese government a financing channel for the transition cost of pension reform. Second, this transfer could minimize disruptions that large-scale privatization would otherwise cause. (Under ordinary circumstances, it might be difficult for the stock market to absorb large blocks of new stock.) Furthermore, it will improve the corporate management. From the international experiences, pension funds have played an important role in monitoring the management of the companies they invest in.

(c) Enterprise Annuity (EA)

The third main funding resource of the Chinese pension system is from the EA system. Hinz (2007) provides a comprehensive introduction for EA system in China.

The EA system is relatively new, but it has already grown considerably, especially in the recent several years. According to Zheng (2011), by the end of 2006, 24,000 Chinese enterprises have EA systems, covering 9.64 million employees. By the end of 2010, this number had increased to 37,100 enterprises covering 13.35 million employees. During the same period, the total assets in EA increased to RMB 280.9 billion (2010) from RMB 91 billion (2006).
3.2.3 Problems of China’s Pension system

3.2.3.1 Financing Problems

(a) Legacy Cost

The pensions of workers who retired before 1997, and the accrued pension entitlement of current workers for employment before 1997 represented China’s “legacy cost/transition cost.” Workers who retired or who enrolled in the old pension system before 1997 are entitled to significantly more generous benefits—75 to 80% of wages—than the benefits under the revised system (58.5%)\textsuperscript{7}. Because the old system was not funded and because the contribution rates in the revised system’s social pension had a lower replacement rate than the old system, the transition costs represent an unfunded liability.

The pension reforms that the government has initiated since 1997 have become stuck in difficulties over the legacy costs that the old system provided. Official estimates of the size of this unfunded liability are not available, but an estimate from the International Monetary Fund (IMF) placed it at 7% of the gross domestic product (GDP) as of 2003. The actual cost of funding the unfunded liabilities, however, could be higher than this total if parameters assumed in the plan are realized. For example, the cost could rise if the replacement rate increases or if life expectancy after retirement turns out to be higher than the plan assumed.

Many scholars, such as Feldstein (2006) and Dunaway et al. (2007), have suggested that Chinese government should separate the legacy problem from the problem of setting up a new pension system.

(b) Empty Individual Accounts

The problem of empty individual accounts relates closely to the legacy-cost problem. Although the 1997 reform proposed a well-designed multipillar system, it did not address

\textsuperscript{7}As Dunaway et al. (2007) points out, in practice, the effective replacement rate for the transition generation is estimated to be around 60% (Sin, 2005) or higher (Chen, 2004). Moreover, the ranks of the transition generation swelled by the early retirement and layoffs associated with the acceleration of SOE reform after 1997, with SOEs employment falling by one-third (35 million workers) during 1997 through 2005.
the problem of how to move from the current PAYGO system to a funded system and how to finance the legacy obligation. In fact, the government will need to pay not only for the implicit pension debt it owed to workers and retirees under the old system but also the borrowed funds from the individual accounts over the past years. Meanwhile, the Chinese government has not officially recognized the implicit pension debt and has not made a formal decision about how to pay for it. As a result of this policy vacuum—and, indeed, a lack of other legitimate financing sources—many local governments have moved funds in individual accounts to pay pensions to current retirees, and most individual accounts are “empty.” This situation has led to the erosion of people’s confidence and, in turn, noncompliance with and evasion of the pension reform. All of these factors lead to poor coverage. The Liaoning Pilot Program, which separates social pooling from individual accounts, addresses this problem. However, this problem does not extend to all of China, possibly because of local government’s pressure for financing.

(c) Financing Unsustainability

The current pension scheme will most likely be unsustainable in the future because the legacy costs do not account for much of what is necessary for universal coverage. In addition to the legacy costs for those who retired or enrolled before 1997, these costs also include the pensions of current enrollees, and the cost of covering those who currently lack coverage. Sin (2005) estimates the number as 141% of GDP for 2001, which approximates US$ 1.6 trillion.

From a long-term perspective, the increasingly rapid aging of the population with an associated increase in the dependency ratio (the ratio of retirees to workers) will constitute a serious financial challenge to the pension system. This trend will become a potential threat to the fiscal sustainability of the central government. In addition to the pressures from demographic changes, the government owes a large debt to individual accounts. The Ministry of Labor and Social Security (MOLSS\(^8\)) has estimated that the PAYGO system

\(^8\)The Ministry of Labor and Social Security (MOLSS) in 2008 merged with the Ministry of Personnel as the Ministry of Human Resources and Social Security (MHRSS).
borrowed a total of RMB 199 billion from individual accounts in 2000. It is difficult to finance the transition cost of this action, although the government has taken some steps, such as selling SOE assets.

Furthermore, the financial difficulties of the system coexist with the fact that wage growth has in recent years outstripped pension growth. From Dunaway et al. (2007), average wages in China grew by 15% annually from 2000 through 2005 in state-owned units and 12.5% in the manufacturing sector. In contrast, average (nominal) pensions grew by 8.5% annually. Due to the strong relative growth in wages, the effective replacement rate decreased from 75% of SOE wages in 2000 to 55% in 2005. On the one hand, the increase in average wages and, therefore, contributions relative to pension payouts might boost the finances of the system. On the other hand, the strong growth in wages would require strong growth in future pensions if the government hopes to achieve the replacement rates in the revised system.

3.2.3.2 Widespread Noncompliance and Evasion

Noncompliance and evasion have emerged as widespread problems since the 1997 reform. According to the MHRSS, participating enterprises owed the system RMB 30.2 billion in social-security payments by the end of 1998. This number rose to RMB 38.3 billion by November 1999 and further increased to RMB 41.4 billion by the end of June 2000. According to the MHRSS, underreporting in employment and wages is one of the important reasons for massive pension system deficit in recent years is , which will reduce the pension contribution (Hu, 2001). For private companies and the self-employed, refusing to participate is the main form of noncompliance. These companies usually have a relatively young work force.

Evasion is the main barrier to expand coverage. According to the objective set by the State Council in January 1999, all workers in the urban business sector would be included in the system by the end of June 1999. However, by the end of 1999, the participation rate expanded from only 50.4% to 55.6%, and, by the end of 2000, the rate reached only 63.4%.
The major reason behind the noncompliance and evasion is lack of incentives for both enterprises and individuals in and out of the system. Pension reform places a high burden on enterprises. According to the State Council’s Document No. 26 (1997), the contribution rate of enterprises is 20% of total wages. However, the contribution rates of enterprises in 18 provinces in 1998 amounted to more than the suggested 20% (Liu, 2000). Taking into account the additional enterprise contribution for health care (6%), unemployment insurance (2%), work injury insurance (1%), maternity (0.8%), and, housing provident fund (usually, 5%-10%), the overall contribution by employers is approximately 40% of wages in most provinces. This serious financial burden weakens enterprises’ competitiveness and leads to broad contribution evasion (see Ma and Zhai, 2002).

For individuals, the most important disincentive is probably the low and unclear rate of return on contributions to individual accounts. This problem is largely due to the fact that the contributions to personal accounts are not invested. Individual accounts are “empty accounts” — money paid into personal accounts is used to pay current pensioners and thus the accounts become accounting tools, which means it is still a PAYGO system. The rates of return are determined administratively without a consistent and clear rule. The government always has incentive to set low rates to reduce future pension obligations. When the rates of return are lower than the opportunity cost of the capital, it is natural that people would rather invest the money on their own instead of investing in pension accounts. Diamond and Barr (2010) suggested there should be clear rule about rate of return on individual account, i.e., the system should switch from “empty account” to “notional account”. This approach could be the best one for China over the next five years, given that the fully funded system is unrealistic in the current situation.

The second disincentive for individuals is the large redistributive social pooling pillar.
Most pension models in the world recognize the necessity of a public pillar. Its function is to guarantee a minimum pension for those unlucky people who “due to factors beyond their control, will retire early with disabilities, die young and leave dependents, live longer than average and run out of resources, or earn very low lifetime incomes which are insufficient to support them both for their working and non-working lives” (James, 1996).

3.2.3.3 Poor Return of Pension Fund

The poor return of pension fund mainly refers to the low return of BOAIS, which is the largest part of pension pool. It is a widely recognized fact although very little information is publicly available on governance and fund management of BOAIS. The government—specifically the MHRSS—currently manages the money in BOAIS at the municipal level. According to regulations, pension funds accumulated in this system can only be invested in bank deposits and government bonds with the vast majority reported to be deposited in the banking system. Typically, funds are placed in term deposits but the situation varies significantly across provinces. Some provinces simply roll over one-year deposits, while in others more than 75% of funds are placed in term deposits. According to Trinh (2006), BOAIS are currently invested in government bonds and bank deposits with average real-yields of 2-3%.

This fund management of BOAIS gives rise to two problems. First having the same agencies manage both the PAYGO system and the individual accounts permits the transfer or “leakage” of funds from individual accounts to PAYGO pensions. Second, this kind of fund management fails to give fund managers proper incentive. If those managers intend to invest risky portfolio for higher expected return, on the one hand, they could not get corresponding bonus if the high return is realized, on the other hand, if the fund suffers a big loss, they will be harshly criticized and probably be fired.

In contrast, the return of NSSF which allows the investment on stock market is quite good. Since the foundation in 2000, the NSSF has achieved an average real annual return of 7.03 % by 2010.
Table 2 Historic Investment Performance of NSSF

<table>
<thead>
<tr>
<th>Year</th>
<th>Annual Return of NSSF</th>
<th>Annual Inflation in China</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>1.73%</td>
<td>0.73%</td>
</tr>
<tr>
<td>2002</td>
<td>2.59%</td>
<td>-0.77%</td>
</tr>
<tr>
<td>2003</td>
<td>3.56%</td>
<td>1.17%</td>
</tr>
<tr>
<td>2004</td>
<td>2.61%</td>
<td>3.90%</td>
</tr>
<tr>
<td>2005</td>
<td>4.16%</td>
<td>1.82%</td>
</tr>
<tr>
<td>2006</td>
<td>29.01%</td>
<td>1.47%</td>
</tr>
<tr>
<td>2007</td>
<td>43.19%</td>
<td>4.77%</td>
</tr>
<tr>
<td>2008</td>
<td>-6.79%</td>
<td>5.90%</td>
</tr>
<tr>
<td>2009</td>
<td>16.12%</td>
<td>-0.68%</td>
</tr>
<tr>
<td>2010</td>
<td>4.23%</td>
<td>3.33%</td>
</tr>
<tr>
<td>2011</td>
<td>0.85%</td>
<td>5.50%</td>
</tr>
<tr>
<td></td>
<td>8.44% (mean)</td>
<td>2.47% (mean)</td>
</tr>
</tbody>
</table>

(Source: Annual report of the National Council for Social Security Fund, 2001-2011)

Motivated by the sharp difference of return between the basic pension fund and the NSSF, in Dec 15 2011, Guo Shuqing, the chairman of Chinese Securities Regulatory Commission, proposed the fund from BOAIS plus House Provident Fund (RMB 4 trillion in total) should be invested into the Chinese stock market. The reason is simple: The original return was too low and could not beat inflation. Dai Xianglong, the chairman of NSSF, several days later (Dec 21, 2011) supported this proposal. An important argument is that the NSSF, which is allowed to invest in the stock market has realized a higher return in the 10 years since its foundation (Table 2). This proposal immediately activated a big debate in China over whether the basic pension fund should be invested in stock market. Section 3.2.4.2 provides more details about how to more efficiently manage the pension fund.

In addition, the cross-subsidization built into the consolidation of pension pools also leads to a disincentive to manage the pension fund. If surpluses will be taken away, then
there is no incentive for a municipality to generate a surplus; if deficits will be paid for by other municipalities, then there is no incentive to keep the deficits down. If consolidating the pools becomes inevitable, a rational choice for the management of pools is to spend the entire surplus and even create deficits before turning the pension fund over to an upper level for management. There are many ways to reduce the fund balances. They include reducing collection efforts, approving early retirement, and increasing retirement benefits. Early retirement presents a vivid example of the discretionary behavior of local governments when facing distorted incentives (see Zhao and Xu 2002 for more details).

3.2.4 Proposals for China’s Pension System

3.2.4.1 Financing Issues

(a) Legacy-Cost Issue

The legacy cost is a big problem in which the 1997 reform has become mired. China must find extra funding resources to finance this cost of its pension reform. From the past experience of other countries, China has four options: issue national, provincial, or local debt; raise new taxes or use general tax revenue; sell state assets; or issue special lotteries. For more details about the means of financing legacy cost and comparison (see Wang et al. 2004). Currently, the government prefers to finance the debt by using proceeds from state asset sales, probably because it is simple and also because China is currently privatizing to make its enterprises more competitive. Countries such as Chile and Israel have had similar experiences (see Bodie and Merton, 1992). A conservative estimate places the value of China’s SOEs assets more than US $1 trillion in 2001 (see James, 2002). Friedman et al. (2006) argue that China could easily finance its pension debt by selling SOE assets.

Chile’s approach of giving inflation-linked bonds that are redeemable at retirement in the pension reform 1980s has also been successful (See Section 3.3.5).

(b) Fund Financing
It is widely accepted that keeping a pure PAYGO system is unrealistic in view of the country’s demographic outlook. Almost all scholars agree that China’s pension system should be at least partially funded\(^{10}\). An attempt to establish a fully funded system failed in 1997, activating a debate over whether it is realistic to hold fully funded individual accounts, at least in the short run. Feldstein and Leibman (2006) suggest the establishment of investment-based prefund individual accounts and that there were serious drawbacks to considering a notional individual account system. First, this system is still PAYGO, and it deviates from the spirit of China’s pension reform in the last decade. Second, it will distort incentives and force individuals and enterprises into noncompliance and evasion. Third, an unfunded system, such as a notional defined contribution system, reduces national savings and, therefore, the size of the economy. In contrast, Diamond and Barr (2010) suggested that in the short run, i.e., the next five-year plan, China should set up the existing individual accounts as notional defined contribution accounts, along the lines of Sweden’s Inkomstpension, which is designed to combine the defined contribution scheme and the PAYGO system. Two points are worth noting among views on notional individual accounts: First, the notional individual accounts that most scholars mention and oppose are, more accurately, empty individual accounts because empty accounts have no clear interest rate. The notional individual account Diamond and Barr (2010) propose is different from the empty individual accounts\(^{11}\): “in detail each worker has an account in which is recorded the total of his cumulative contributions over the years, which are credited each year by the pension authority with an interest rate defined by law. That is, the recorded accumulation increases each year by the amount of the contributions during the year plus the product of the notional interest rate and the level of the accumulation at the end of the previous year.” (Diamond and Barr, 2010)

Second, Diamond and Barr do not oppose the fully funded system in the long run. They

\(^{10}\)However, the opinions on the degree of funding vary among scholars. Some think it should be reformed as fully funded as soon as possible (see Feldstein and Leibman, 2006).

\(^{11}\)The term “notional individual accounts” used by many other scholars is in fact the empty individual accounts.
supported the notional individual account because the fully funded individual accounts were not realistic in the short run, given China’s current economic and social circumstances. The 1997 reform introduced individual accounts as part of the pension reforms, with an expectation of full funding, but failed. The pilot of Liaoning Province and 12 other provinces, which is considered as the second try to build fully-fund individual accounts, failed again two years ago. Unless the government can provide correct incentives for individual, the fully fund system seems not to be a practical concern at least for now. This notional individual account allows for gradual evolution into a mix of funded and notional accounts, as Sweden has, or eventually a pure funded account.

Also, for the financing from social pooling, Feldstein (2006) suggest, instead of using payroll tax, social pooling should be financed with a broader-based tax, such as the value added tax (VAT). A huge advantage of general tax over a payroll tax is that a general tax makes evasion and noncompliance more difficult. Another advantage is that it would permit lower tax rates than possible with the payroll tax. The high payroll tax rate also drives noncompliance and evasion. Currently, payroll tax administration is inefficient. Tax evasion has produced a small tax base and high tax rates.

(c) Deferring Retirement

In general, the government could address financial sustainability by increasing the contribution rate, deferring the retirement age, or using both these approaches. On the one hand, further increasing the contribution rate seems infeasible because the current 28% contribution rate is high and has led to evasion and noncompliance. On the other hand, the mandatory retirement age in China is currently 60 for men, 50 for women, and 55 for women in civil service. This retirement age is earlier than that of most other countries, providing China some room to defer the retirement age. In an attempt to test the waters, the MHRSS has announced more than once in recent years that it would initiate research on deferring the retirement age. But the result of an online survey by www.sina.com.cn, the

---

12Wang et al. (2004) also suggest using a VAT to finance the legacy cost.
largest and most popular website in China, show that 90% respondents oppose deferring retirement age. Government employees constitute the only group supporting this deferral, probably because their contributions are the government’s responsibility. Blue-collar workers are most strongly opposed to this policy. Nevertheless, China will most likely defer the retirement age, and the only questions remaining are when and how\textsuperscript{13}.

### 3.2.4.2 How to More Efficiently Manage Pension Fund

As Section 3.2.3.3 points out, the BOAIS has a poor rate of return. The fund is now estimated to be approximately RMB 2 trillion. Since December 2011, several Chinese government officials have proposed that the BOAIS fund should be invested in the stock market because current management cannot guarantee that it can beat inflation. This proposal is problematic for several reasons.

First, the removal the strict regulation on the pension fund might put this money into a large amount of risk because of the risky characteristics of stock market itself and because the Chinese equity market lacks the maturity and stability of its counterparts in the developed countries, such as the United States and the United Kingdom. Although the NSSF historically has had a high rate of return, it is questionable whether this rate is sustainable.

Second, it is hasty to conclude that the only solution for the poor return is a move to the stock market. The most important objective for a pension fund is to beat inflation. Hence, a more sensible way is to let the government issue inflation-linked bonds, which should serve as the benchmark asset for individual/institutional pension investors.

\textsuperscript{13}\textsuperscript{13}The main concern for the government is whether to implement voluntary retirement or just to defer the retirement age, keeping mandatory retirement.
3.3 Impact of Inflation-Linked Bonds on the Pension System: Case Study

3.3.1 Review of Indexed Bonds in the Global Market

A half-century ago, finance academics started to propose the issuance of inflation-link bonds, which provide safe assets, perfectly protecting the purchasing power for long-term investors (James Tobin, 1963). The “modern era” of inflation-linked bonds started with the issuance of UK index-linked Gilts in 1981, followed by Australia (1985, 1993), Mexico (1989), Canada (1991), Poland (1992), Sweden (1994), the United States (1997), France (1998), Austria (2003), Greece (2003), Belgium (2004), Japan (2004), Germany (2006), South Korea (2007), Turkey (2007), Hong Kong (2011), Thailand (2011), Denmark (2012), and India (2013). From both theoretical and practical reviews of international experiences, the introduction of indexed bonds yields many benefits, even though they might be difficult to quantify. One thing worth noting is that the finance industry does not favor indexed bonds. Some investment professionals believe that the main reason is the low volatility of this asset, which gives institutional investors less opportunity to generate profit. Ironically, this property could be considered valuable for diversification purpose and a desirable property for households to do pension investment which have more controllable risks.

One main advantage of indexed bonds is to provide investors with a perfect hedge against inflation and real interest rate risk. This might be the earliest motivation for the government to issue Treasury Inflation Protected Securities (TIPS). James Tobin (1963) made one of the most convincing arguments in favor of inflation indexed debt on behalf of households:

“...markets do not provide, at any price, a riskless way of accumulating purchasing power for the future, whether for old age, or for college education or for heirs....Meanwhile we force savers to take risk, even if they would gladly pay for the privilege of avoiding it.....No private institution can fill this gap. No insurance company or pension fund could..."
assume the risk of offering purchasing power escalation to its creditors without similarly (inflation) escalated securities in which to invest at least some of their funds."

This asset class will complete the financial market by providing individuals a way to perfectly insure against inflation risk. It enables individual investors to choose how much inflation risk they want to hold, allowing for a more optimal allocation among investors with different risk preferences over inflation. Section 3.3.4 provides a quantitative analysis of this concept.

Another main advantage is that it is a good source for inferring the expected inflation, short-term real interest rate, and perceived probability of deflation. Surveys such as the University of Michigan Inflation Expectation Survey and Survey of Professional Forecasters (SPF) are also important sources of information, but these surveys have important drawbacks. First, there is a lag time between when these surveys are conducted and when they are available, and they are infrequently updated. Second, and more important, research in the field of behavioral economics suggests that biases due to framing are likely to make survey measures of long-run inflation expectations unreliable. As Mishkin (2010) comments in Campbell, Shiller, and Viceira (2009), the problem is that “when survey measures of short-run inflation expectations change, survey measures of long-run inflation expectations are likely to move with them, even if long-run expectations have not changed. This might happen because questions about both are asked at the same time, and the answer to the first question influences (“frames”) the response to the second, resulting in a spurious co-movement between the two.” Third, reliability about survey measures is also one reason that it might make more sense to trust expectations measures that are derived from financial market data. After all, people buying or selling securities are betting their money on them; they thus have a strong incentive to base their decisions on their true forecasts. Here, the inflation-indexed bond market provides more reliable information.
3.3.2 Current Inflation-Protection Strategies in China

To the best of my knowledge, the existing inflation-protection tools in China are limited to interest rate subsidies on fixed deposits and Treasury bonds, which the Chinese government has provided in 1950, 1988, and 1992-1993. However, these tools are unsatisfactory for several reasons. First, these tools are implemented “ex post”—that is, after the high inflation period—and it is unanticipated. It is unknown when the Chinese government will take this action and how much it will subsidize. So, it does not provide individuals the same hedge against inflation risk as TIPS does. Second, these ex post tools aggravate the speculation on Treasury future market. The famous 3.27 Treasury future incident in 1995 is one example of such speculation behaviors. In 1995, due to the high inflation between 1992 and 1994, there was speculation that Chinese government might provide interest rate subsidies on the Treasury bonds issued on 1992 (with three-year maturity). Guan Jinsheng, then considered the “godfather of the Chinese bond market,” did not believe the Chinese government had enough financial resources to subsidize the Treasury bonds. When the Ministry of Finance increased the T-bond payout, Guan wanted to guard against rising futures prices. So, within the last 8 minutes of trading on Feb. 25, 1995, his faction shorted bond futures worth RMB 1,460 billion (approximately US $180 billion), which represented one-third of China’s GDP in 1994. Guan then threw 7.3 million short contracts onto the market. Prices were decreased from RMB 155.75 down to RMB 147.4. This “flash crash” became one of the most famous scandals during the development of Chinese financial markets, and the trading on treasury futures was suspended later in 1995, not too long after the market was born in 1993.

Scholars are now proposing that the Chinese government should issue inflation-linked bonds. As a pilot program, Hong Kong issued these bonds in July 2011.
3.3.3 Case Study of Hong Kong’s iBond

Although the Chinese government has not yet issued inflation-linked bonds in the mainland, Hong Kong has recently issued the inflation-linked retail bond (called iBond), which could become part of a pilot program in China. Although Hong Kong and Mainland China have different types of economies, they share several common aspects, especially in individual characteristics, such as investment and consumption behavior and risk preference. Since the United Kingdom transferred sovereignty over Hong Kong to China in 1997, this phenomenon has been increasingly obvious, possibly due to more interaction and synergy between these two economies. Hong Kong issued the iBond on July 28, 2011, and listed on the Stock Exchange of Hong Kong the following day. The offered issue amount is HK $10 billion. The minimum investment is HK $10,000. The tax-free bond pays the investors at least one per cent interest every six months and matures in 2014. If inflation is higher than 1%, the bond will pay a rate linked to the consumer price index (CPI). According to a spokesman for the Hong Kong government, “Both the total number of applications and total application amount are at high levels as compared with those for recent retail bond offerings.”

(a) Primary Market of Hong Kong’s iBond

The Hong Kong government has announced the result of the subscription and allocation of the iBond: a total of 155,835 valid applications were received for more than HK $13 billion (approximately US $1.669 billion) in principal for the bonds, which exceeded the planned issue amount of HK $10 billion. Of the 150,869 valid applications, those who have applied for 44 or fewer units (HK $10,000 for one unit) will receive allocations of the full amount applied for. In addition, 4,966 valid applications exist for more than 44 units. Each of those will receive an allocation of 44 units first, and 1,344 will receive an allocation of one additional bond unit based on ballot results. The elderly, retirees, and the middle class comprise the majority of iBond subscribers, with an average subscription amount of HK $70,000 to HK $100,000. The oversubscription rate was 30%, which is significantly
higher than that of other nominal bonds with similar maturities.

(b) Secondary Market of Hong Kong’s iBond

On July 29, 2011 (the second day of the issuance of the iBond and the first day listed on Stock Exchange of Hong Kong), the iBond closed at a price of HK $106.70 on the first day of trading, representing as much as 6.7% for those who had invested. The price has remained HK $106 to HK $108 in the five months since its issuance. Because of the iBond’s popularity in Hong Kong, it is reasonable to assume that inflation-linked bonds will be popular in China, as well.

3.3.4 Empirical Study of China Mainland: Role of Inflation-Linked Bonds in Asset Allocation

This section examines the role of inflation-linked bonds in asset allocation for Mainland China and will conduct portfolio simulations from Chinese macro and financial data.

3.3.4.1 Data Description

The CPI data (1980-2011)\textsuperscript{15} is from the IMF. The stock market data uses the Shanghai Composite Index (1991-2011)\textsuperscript{16} from DataStream. The asset allocation does not consider the nominal Treasury bond because because the available data sample is too small (2006-2011)\textsuperscript{17}. Instead, this paper uses the fixed bank CD (1991-2011) as the proxy of nominal bond. For the hypothetical inflation-linked bond in China mainland, this paper uses Hong Kong’s iBond as benchmark, which provides zero real return and perfectly compensates for the CPI. None of the rate of return series was continuously compounded.

Several caveats of the methodology:

\textsuperscript{15}I use inflation data after China’s economic reform in 1978, which is considered as the beginning of modernization. As a convention, I drop the first two years (1978 and 1979) to avoid the possible anomaly at the beginning of any reform.

\textsuperscript{16}Since the Shanghai stock exchange was set up in 1991, there could be concern that the first one or two years might generate abnormal returns. I consider both cases and find that dropping the first two years’ data (1991 and 1992) does not change the basic result of this paper.

\textsuperscript{17}The main limitation of the small sample (2006-2011) is for calculation of asset return correlations. In the earlier simulation ignoring this issue, I consider one-year Treasury bill in the asset allocation, and I find that the inclusion of one-year bill have little impact on the final result for inflation-linked bonds.
(a) There are two justifications for using bank certificates of deposit (CDs) as proxies of nominal treasury bonds in China. First, considering that the largest banks are all state-owned and that the maturities of CDs in China ranges from three months to five years, no significant difference exists between default premiums of treasury bonds and bank CDs of similar maturities. Second, due to investment tradition, Chinese people prefer CDs to nominal Treasury bonds for asset allocation in fixed income.

(b) The results-report section provides the comparison of result for two groups: Group 1 is a two-asset portfolio combining nominal CDs and stock, and Group 2 is also a two-asset portfolio including indexed bonds and stock. This paper ignores the optimal portfolio, which combines nominal bonds, real bonds, and stock. The main reason is that it is trivial to predict that the Sharpe ratio will improve for this case. Before the introduction of inflation-linked bonds, there are no risk-free assets, so no tangency portfolio exists in real terms. Beyond this trivial point, it is more meaningful to compare the two groups I mentioned above.

(c) This paper does not consider the issue of inflation-risk premiums for nominal bonds or liquidity premiums for indexed bonds. First, as the most feasible and cautious step, the author suggests the Chinese government to issue I-Saving-Bond-like indexed bonds as pilot rather than Treasury Inflation-linked Securities (TIPS) as this paper does. According to some Chinese government officials in the MOF, Chinese government has been very cautious about financial innovation since 2008 financial crisis. The U.S. government has since 1997 issued both I-Saving bonds and TIPS. The key difference is that people can sell TIPS in secondary markets; hence they contain more risk if the owners do not keep them until they reach maturity. Although I-Saving bonds only would mean that the Chinese government would lose some benefits, such as the ability to extract information of expected inflation, this approach makes sense because the first priority of financial innovation relates to investment safety. When the fixed income market becomes more mature, the Chinese government could consider introducing the secondary market for these bonds. Another point worth noting is that, if the inflation risk premium and the liquidity premium are
both positive\(^{18}\), they will weaken the effect of each other. Ideally, if these premiums were equal, they would cancel out each other’s effect. From the previous literature, the liquidity premium of indexed bonds dominates the inflation risk premium of nominal bonds. For example, D’Amico, Kim and Wei (2007) estimated that the liquidity premium in the ten-year TIPS yield was as large as 200 basis points during the initial period of issuance and then decreased and fluctuated to less than 50 bp by 2007. Similarly, Pflueger and Viceira (2012) gave an estimate of 30 bp during normal time and 150 bp during the early years of TIPS and the financial crisis of 2008 to 2009. Further, Grishchenko and Huang (2011) estimated the inflation risk premium to be 14-19 bp. In summary, the domination of liquidity premium over inflation premium means that the extension including these two risk premiums will make the indexed bonds more. To put it another way, the simulation result of this paper will be further improved to support the introduction inflation-linked bonds.

<table>
<thead>
<tr>
<th>Table 3: Descriptive Statistics of Asset Return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Standard deviation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4: Correlation Matrix for Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
</tr>
<tr>
<td>Stock</td>
</tr>
<tr>
<td>One-year fixed deposit</td>
</tr>
<tr>
<td>Indexed bonds</td>
</tr>
</tbody>
</table>

3.3.4.2 Simulation Procedure

Step 1: Calculate the mean, standard deviation and correlation among different assets.

Step 2: Based on the statistics in step 1, randomly generalize the asset return data for 35

\(^{18}\)Campbell et al. (2009) point out that the inflation-risk premium could be negative theoretically.
years, the average number of working years in China. Then calculate the Sharpe ratio with weights on stock and fixed income asset in two scenarios: with and without the hypothetical inflation-linked bonds. This study assume that stock returns are mean-reverting (see Campbell and Viceira, 1999) Step 3: Repeat Step 2 50,000 times to get the final result. Step 4: Perform a robustness check.

3.3.4.3 Results and Explanation

As we could see from Table 5 and 6 in appendix, except the portfolio with zero weight on stock, the portfolios with indexed bonds dominate those of portfolios with nominal bonds at all levels of weight. The optimal Sharpe ratio with indexed bonds is 0.653, compared to 0.553 without indexed bonds. One interesting point is that the Sharpe ratio of the portfolio purely invested in indexed bonds is smaller than that of the portfolio purely invested in nonindexed bonds. A natural question then arises: why is the optimal Sharpe ratio with indexed bonds much larger than that of portfolio without indexed bonds? The intuition is that the return of indexed bonds and stock is negatively correlated, which makes indexed bonds a good hedge asset, reducing portfolio variance\(^1\). Other countries, including the United States, have experienced similar empirical results, although no satisfactory explanation for these results has yet arisen.

3.3.4.4 Robustness check

The first step of the robustness check is to vary the data frequency—that is, to convert the annual data to monthly/quarterly data, of which the data availability is from January 1994 to November 2012.

As tables 8, 9, and 10 in appendix show, for monthly data, the portfolios with hypothetical indexed bonds dominate those of nominal bonds at all levels of weights for 10% to 100%. But the optimal Sharpe ratio with indexed bonds is 0.2629, beating 0.2460 without

\(^1\)The corresponding two-asset portfolio variance formula \(\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2\) clearly shows the advantage of having negative correlation, where \(\sigma\), \(w\) and \(\rho\) represent standard deviation, weight, and correlation, respectively.
indexed bonds. An interesting finding for the monthly data is that the optimal portfolio in two scenarios contains no stock. The possible reason is the high volatility of stock in month frequency compared to those of indexed/nonindexed fixed income asset. In addition, the previous finding from annual data indicates that the return of indexed bonds and stock correlates negatively. However, the finding is weak in monthly data, which indicates a correlation of -0.035 and is insignificant at the 10% level. But the main finding still holds that the introduction of indexed bonds will significantly improve the risk-return tradeoff over that of nominal fixed income asset.

Table 11, 12 and 13 in appendix give the results for quarterly data, which are basically consistent with the previous result, especially the improvement of Sharpe ratio. Quarterly data again confirms that stock is not favored for short-term investment due to the high volatility of short-term stock in China.

(b) Shenzhen Exchange Versus Shanghai Exchange, A Shares Versus B Shares

This empirical study uses the Shanghai Composite Index to calculate stock returns, giving rise to two issues. First, there are two stock exchanges: Shanghai and Shenzhen. Using the Shenzhen Composite Index does not significantly change the result, which is not surprising because the correlation between the two stock indexes is higher than 99%. Second, the Shanghai Composite Index combines both A and B shares. The key distinction is that A shares are denominated in renminbi and B shares are denominated in foreign currency (US dollars in Shanghai and Hong Kong dollars in Shenzhen). For a long time, the other main difference between the two, from a regulatory standpoint, was that the A-share market was closed to foreign investors, whereas the B share market was open only to foreigners. However, in 2001, the Chinese authorities tried to boost the B share market by opening it to individual Chinese investors. Also considering the huge size and volume differences between A and B shares, it is advisable to use Shanghai Composite index as proxy of stock return.
3.3.5 Role of Inflation-Linked Bonds in Providing Correct Incentives in Pension Reform: A Lesson from Chile

One use of the inflation-linked bonds in the pension system is to serve as compensation to those who choose to switch to the new pension system from the old one. Chile’s pension reform in 1981 provides an example for that. As two fast-growing economies, China and Chile have several similarities. Both have implemented deep economic reform since 1980s, with the similar economic structures of market-oriented economy characterized by a high level of foreign trade. Interestingly, Chile has also experienced highly volatile inflation, as China does. An important and perhaps among the key factors leading Chile’s pension reform success is that Chile issued inflation-protected recognition bonds with the objective of protecting the benefits of those who transferred from the old system to the new one.

The Chilean government compensated those who decided to switch with inflation-protected bonds, equal in value to their accrued benefits under the old system. These bonds were available to all people who had at least 12 months of coverage under the old system in a 60-month period ending October 1980. The amount for people with at least 35 years of contribution to the old system is 80% of their salary in the last 12 months before July 1979, with CPI indexing from the last month of such wages up to the month of entry into the new system. The result was then multiplied by an annuity purchase factor (10.35 for men and 11.36 for women). The bonds would be redeemable at retirement. Some people believe that the introduction of recognition bonds played a positive role in Chile’s pension reform (Myers, 1992). The data shows that 90% of Chile’s workers chose to move into the new system, even though some national trade-union leaders advised against it. This move is even “faster than Germans moving from East to West after the fall of the Berlin Wall” (Pinera, 1995).
3.4 Conclusion

This paper reviews the existing problems and reform proposals for China’s pension system and discusses the possibility of introducing inflation-linked bonds to China. With the switch from DB to DC system, one of the main concerns is the corresponding investment risk. I show that the introduction of indexed bonds will play an important role in pension asset-liability matching and will improve investment risk-reward tradeoff. In addition, it will provide correct incentives in pension reform and also help to develop China’s fixed income market, which is an important direction of China’s financial reform.

Finally, there could be several future extensions on this paper, including putting the ongoing privatization of SOEs into this framework, which might produce a kind of synergy with China’s pension reform. Also due to the incomplete financial market in China, there is good reason to believe that more financial innovations, such as swaps and tranches, as well as a broader international diversification, could further improve risk-reward tradeoffs for investment in China. Another interesting extension would be to calculate the potential cost of issuing inflation-linked bonds from the perspective of the government. This issue seems to be the main barrier for countries that are hesitating to issue indexed bonds. However, considering the significant unfunded cost of an implicit or explicit government guarantee, which could be modeled as a put option, during the transition from a DB plan to a DC plan, issuing inflation-linked bonds may be an attractive alternative to a government guarantee. It remains to be seen which option is preferable.
### 3.5 Appendix

#### Table 5 Non-indexed Bonds and Stock (Annual)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>0.017</td>
<td>0.039</td>
<td>0.062</td>
<td>0.085</td>
<td>0.108</td>
<td>0.130</td>
<td>0.153</td>
<td>0.176</td>
<td>0.198</td>
<td>0.221</td>
<td>0.244</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.032</td>
<td>0.071</td>
<td>0.129</td>
<td>0.189</td>
<td>0.249</td>
<td>0.310</td>
<td>0.371</td>
<td>0.432</td>
<td>0.494</td>
<td>0.555</td>
<td>0.616</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.514</td>
<td>0.553</td>
<td>0.482</td>
<td>0.449</td>
<td>0.431</td>
<td>0.420</td>
<td>0.412</td>
<td>0.406</td>
<td>0.402</td>
<td>0.399</td>
<td>0.396</td>
</tr>
</tbody>
</table>

#### Table 6 Indexed Bonds and Stock (Annual)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>0.027</td>
<td>0.049</td>
<td>0.070</td>
<td>0.092</td>
<td>0.114</td>
<td>0.135</td>
<td>0.157</td>
<td>0.179</td>
<td>0.200</td>
<td>0.222</td>
<td>0.244</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.064</td>
<td>0.075</td>
<td>0.123</td>
<td>0.181</td>
<td>0.242</td>
<td>0.303</td>
<td>0.365</td>
<td>0.428</td>
<td>0.490</td>
<td>0.553</td>
<td>0.616</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.418</td>
<td>0.653</td>
<td>0.568</td>
<td>0.507</td>
<td>0.470</td>
<td>0.446</td>
<td>0.430</td>
<td>0.418</td>
<td>0.409</td>
<td>0.402</td>
<td>0.396</td>
</tr>
</tbody>
</table>

#### Table 7 Improvement of the Sharpe Ratio (Annual)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement</td>
<td>-0.096</td>
<td>0.100</td>
<td>0.086</td>
<td>0.058</td>
<td>0.039</td>
<td>0.026</td>
<td>0.018</td>
<td>0.012</td>
<td>0.007</td>
<td>0.004</td>
<td>0</td>
</tr>
</tbody>
</table>
### Table 8 Non-indexed Bonds and Stock (Monthly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>0.0003</td>
<td>0.0013</td>
<td>0.0024</td>
<td>0.0034</td>
<td>0.0044</td>
<td>0.0054</td>
<td>0.0065</td>
<td>0.0075</td>
<td>0.0085</td>
<td>0.0095</td>
<td>0.0106</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0013</td>
<td>0.0127</td>
<td>0.0252</td>
<td>0.0378</td>
<td>0.0504</td>
<td>0.0630</td>
<td>0.0756</td>
<td>0.0882</td>
<td>0.1007</td>
<td>0.1133</td>
<td>0.1259</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.2460</td>
<td>0.1065</td>
<td>0.0940</td>
<td>0.0898</td>
<td>0.0877</td>
<td>0.0864</td>
<td>0.0855</td>
<td>0.0849</td>
<td>0.0845</td>
<td>0.0841</td>
<td>0.0838</td>
</tr>
</tbody>
</table>

### Table 9 Indexed Bonds and Stock (Monthly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>0.0024</td>
<td>0.0032</td>
<td>0.0041</td>
<td>0.0049</td>
<td>0.0057</td>
<td>0.0065</td>
<td>0.0073</td>
<td>0.0081</td>
<td>0.0089</td>
<td>0.0097</td>
<td>0.0106</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0092</td>
<td>0.0149</td>
<td>0.0260</td>
<td>0.0381</td>
<td>0.0505</td>
<td>0.0630</td>
<td>0.0755</td>
<td>0.0881</td>
<td>0.1007</td>
<td>0.1133</td>
<td>0.1259</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.2629</td>
<td>0.2176</td>
<td>0.1556</td>
<td>0.1275</td>
<td>0.1124</td>
<td>0.1030</td>
<td>0.0967</td>
<td>0.0921</td>
<td>0.0887</td>
<td>0.0860</td>
<td>0.0838</td>
</tr>
</tbody>
</table>

### Table 10 Improvement of the Sharpe Ratio (Monthly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement</td>
<td>0.0169</td>
<td>0.1110</td>
<td>0.0615</td>
<td>0.0377</td>
<td>0.0247</td>
<td>0.0166</td>
<td>0.0111</td>
<td>0.0072</td>
<td>0.0042</td>
<td>0.0019</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 11 Non-indexed Bonds and Stock (Quarterly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>.0007</td>
<td>.0036</td>
<td>.0066</td>
<td>.0096</td>
<td>.0126</td>
<td>.0156</td>
<td>.0186</td>
<td>.0215</td>
<td>.0245</td>
<td>.0275</td>
<td>.0305</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0039</td>
<td>0.0211</td>
<td>0.0418</td>
<td>0.0626</td>
<td>0.0835</td>
<td>0.1043</td>
<td>0.1252</td>
<td>0.1461</td>
<td>0.1670</td>
<td>0.1878</td>
<td>0.2087</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.1692</td>
<td>0.1729</td>
<td>0.1586</td>
<td>0.1535</td>
<td>0.1509</td>
<td>0.1493</td>
<td>0.1483</td>
<td>0.1475</td>
<td>0.1469</td>
<td>0.1465</td>
<td>0.1461</td>
</tr>
</tbody>
</table>

Table 12 Indexed Bonds and Stock (Quarterly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess return</td>
<td>0.0074</td>
<td>0.0097</td>
<td>0.0120</td>
<td>0.0143</td>
<td>0.0166</td>
<td>0.0189</td>
<td>0.0212</td>
<td>0.0236</td>
<td>0.0259</td>
<td>0.0282</td>
<td>0.0305</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0187</td>
<td>0.0288</td>
<td>0.0465</td>
<td>0.0659</td>
<td>0.0859</td>
<td>0.1062</td>
<td>0.1266</td>
<td>0.1471</td>
<td>0.1676</td>
<td>0.1881</td>
<td>0.2087</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.3936</td>
<td>0.3364</td>
<td>0.2580</td>
<td>0.2170</td>
<td>0.1934</td>
<td>0.1782</td>
<td>0.1678</td>
<td>0.1602</td>
<td>0.1544</td>
<td>0.1498</td>
<td>0.1461</td>
</tr>
</tbody>
</table>

Table 13 Improvement of the Sharpe Ratio (Quarterly)

<table>
<thead>
<tr>
<th>Bond weight</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
<th>10%</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improvement</td>
<td>0.2244</td>
<td>0.1635</td>
<td>0.0994</td>
<td>0.0635</td>
<td>0.0425</td>
<td>0.0289</td>
<td>0.0195</td>
<td>0.0127</td>
<td>0.0074</td>
<td>0.0033</td>
<td>0</td>
</tr>
</tbody>
</table>
Bibliography


Curriculum Vitae

Contact

Peng Huang
Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215, USA

Education

Nanjing University, B.S., Theoretical Physics, Nanjing, China, 2004
Boston University Ph.D. candidate, 2009 – present. (Dissertation Defense Approval Date: 09/05/2014)