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Abstract
Forwarding in DTNs is a challenging problem. We focus on the specific issue of forwarding in an environment where mobile devices are carried by people in a restricted physical space (e.g., a conference) and contact patterns are not predictable. We show for the first time a path explosion phenomenon between most pairs of nodes. This means that, once the first path reaches the destination, the number of subsequent paths grows rapidly with time, so there usually exist many near-optimal paths. We study the path explosion phenomenon both analytically and empirically. Our results highlight the importance of unequal contact rates across nodes for understanding the performance of forwarding algorithms. We also find that a variety of well-known forwarding algorithms show surprisingly similar performance in our setting and we interpret this fact in light of the path explosion phenomenon.

1. INTRODUCTION
Effective forwarding in a delay tolerant network (DTN) is strongly dependent on particular aspects of the setting, such as the size of the physical space involved, and the degree to which contact and mobility patterns are predictable. We are concerned with understanding forwarding in a setting that has not been extensively studied to date: human-carried mobiles moving in a restricted space with high node and contact density. This situation corresponds to individuals at conferences, around office spaces, and in social settings. Networks in these environments are examples of Pocket Switched Networks (PSNs), in which both mobility and multihop forwarding can be used to support communication.

We believe that in order to design effective forwarding algorithms for PSNs, it is necessary to start by understanding the opportunities for forwarding that exist. While some work has already studied the performance of various forwarding algorithms in PSNs, there is little understanding to date on the nature of the forwarding problem in such settings. In particular, little is known about the kinds of paths (making use of both mobility and multiple hops) that exist in PSNs.

In this paper we study the paths that are available for forwarding in PSNs. We use as examples of the PSN setting measurements of contacts between Bluetooth-enabled Intel iMotes at recent Infocom and CoNext conferences. We develop an efficient method to enumerate all paths of interest between nodes (iMotes) and apply it to these traces to capture all paths of interest for a set of randomly generated messages.

The most striking property of the collection of paths we obtain is the presence of a phenomenon we term path explosion. Path explosion refers to the case in which, once the first path reaches the destination the number of subsequent paths grows rapidly with time, so there exist many near-optimal paths. We find that path explosion occurs for the large majority of messages in our datasets.

To explore this effect we build an analytic model describing how paths are created in a homogeneously mixing population and we use it to show how path explosion arises. However our homogeneous model does not explain all aspects of the phenomena observed in our data. In order to understand the nature of forwarding paths more accurately, we show that it is critically necessary to take into account the different contact rates exhibited by different nodes in the population. We find that it is useful to characterize the source and destination as either high contact rate (‘in’) nodes or low contact rate (‘out’) nodes. We show that when the contact rates of the source and destination nodes are taken into account, that the empirical properties of paths and the path explosion process can be understood more completely.

Once we understand the nature of paths available for forwarding, it is easier to interpret the behavior of forwarding algorithms. Using our traces, we evaluate a range of different forwarding algorithms that are appropriate for DTNs. We find the surprising result that forwarding algorithms with very different strategies nonetheless show remarkably similar performance in terms of message delivery success rate and average message delivery delay. In fact we find that the particular choice of routing algorithm does not have a strong
impact on delay or success rate for our data. We show that these results can be understood in the context of the path explosion phenomenon. Furthermore, understanding the presence of path explosion, and the particular details of how it occurs between combinations of ‘in’ and ‘out’ nodes, allows us to understand how efficient message delivery can take place in these settings.

2. RELATED WORK

Our work studies the set of paths available for forwarding in Pocket Switched Networks. While little work to date has studied the forwarding paths themselves, researchers are beginning to recognize that the movements of individuals carrying mobile nodes create contact patterns that are not well modeled using uniform or homogeneous assumptions [1, 14].

Nonetheless, to date most forwarding algorithms have been analyzed under the assumption that contact rates between nodes are homogeneous [5, 6, 16, 17]. Likewise, the most common mobility model used for forwarding evaluation is random waypoint [2] in which all nodes’ speeds and directions are drawn from the same distributions. In contrast, our work shows that it is the differences in behavior of nodes in the network that is key to understanding the performance of forwarding in our setting. The closest work to our paper deals with the heterogeneity of inter-contact time distributions and its implications for routing [4]. The authors show that if the distributions of inter-contact times are homogenous and exponential then no routing scheme will be able to make practical use of past contact information. However if not one can exploit heterogeneity to improve performance.

One of the main results in our work is that for our datasets, a wide variety of forwarding algorithms perform equally well in terms of success rate and delay. Among the forwarding algorithms we consider, we use epidemic forwarding [18] to study the path explosion phenomenon as well as to obtain upper bounds on performance. Among the factors we consider in comparing other forwarding algorithms are the use of past contact history, as proposed in [10, 9, 12, 7]. Another factor we consider is whether the algorithm bases decisions on single-hop performance or multi-hop performance. The authors of [10] propose basing forwarding decisions on Minimum Estimated Expected Delay (MEED) where the delay between nodes is calculated using past contact history over a sliding window. In contrast, Grossglauser et al [5] use only the last encounter time between nodes as a basis for forwarding decisions. We consider both these alternatives in our evaluation.

Many papers have looked at the performance of forwarding algorithms based on wireless measurements, but few have looked at forwarding in a dense environment with hard-to-predict contact patterns. Exceptions are [8, 3], which use data similar to ours. However those papers are concerned with studying forwarding algorithms rather than the nature of forwarding paths themselves, which is the focus of our work.

3. DATA

As described in our introduction our study is based on empirically collected data sets. These data sets were collected using short-range Bluetooth enabled devices (iMotes [3]) carried by individuals. These devices perform an inquiry-scan every 120 secs and log each response to the inquiry as a contact. A contact record includes the MAC address of the device that responded, as well as the start time and the end time of the contact. More details about the devices and the datasets, including synchronization issues can be found in [15].

Initial analysis of the data showed that contact activity varied considerably over the course of each day. The analyses in our paper are based on the assumption of approximately constant contact rates over time. Therefore we selected four 3-hour periods for which visual inspection indicated that contact rates were relatively stable. This can be verified in Figure 1 which shows the time series of total number of contacts over all nodes (totals calculated over 1 minute bins) in each of the 3-hour periods described below. There is some noticeable drop off from 5:30 to 6:00 pm in two datasets. To reduce the influence of this drop-off, and to reduce edge-effects at the end of the simulation, we only generate messages during the initial 2 hours, so each message has at least 1 hour during which it is delivered. Finally, when a node A contacts node B, we assume that B and A can exchange data in both directions; we do not consider the effect of asymmetry in this paper.

Empirical results in this paper are based on the following data sets:

Infocom 2006: This data set consists of logs taken during the conference Infocom 2006. There are 98 nodes in total; with 20 nodes are stationary nodes distributed within the conference venue and the remaining nodes are distributed to conference participants who carried them for 4 days from 24th April 2006 to 28th April 2006. The specific time periods we study are 25 April (Tuesday) 9AM-12PM and 3PM-6PM on the same day.

Conext 2006: We collected similar data sets at Conext 2006 which was held from 4 Dec 2006 to 7 Dec 2006. 78 nodes were distributed to conference participants and the remaining 20 nodes were placed around the conference venue. The data sets we use are 4 Dec(Monday), 9AM-12PM and 3PM-6PM.

We replicated portions of our analyses on datasets taken at Infocom 2005 [3]. All of our results in this
paper were consistent with those analyses. However we omit details for lack of space.

4. PATH EXPLOSION

A forwarding algorithm solves a decentralized search problem—it searches for a short path between a source and destination node, starting at a given point in time. This path exists in space and time, and its duration is the amount of time between message generation and message delivery. In order to understand how hard the search problem is for PSN settings, we start by studying the solution space.

Given a set of nodes $\mathcal{M}$ with $|\mathcal{M}| = N$, and a continuous time index $0 \leq t < t_{max}$, we define a path as a sequence of tuples:

$$((x_1, t_1), (x_2, t_2), \ldots, (x_k, t_k))$$

where for all $i$, $t_i \leq t_{i+1}$, $x_i \in \mathcal{M}$, and

$$\ldots, (x_i, t_i), (x_{i+1}, t_{i+1}), \ldots$$

may be present only if $x_i$ is in contact with node $x_{i+1}$ at time $t_{i+1}$. Each tuple in the sequence is a hop and the length of a path is the number of hops it contains.

We assume that communication occurs via messages which are transmitted in whole from node to node in zero elapsed time. Messages travel along paths, i.e., they are only transmitted between nodes that are in contact. For any source $\sigma \in \mathcal{M}$ and destination $\delta \in \mathcal{M}$ a successful message delivery beginning at time $t_1$ can occur if there is a path

$$((\sigma, t_1), (x_2, t_2), \ldots, (\delta, t_k)).$$

The most basic goals of a forwarding algorithm are, given $\sigma, t_1$, and $\delta$: (a) to find such a path if it exists; and (b) to find the path with shortest-achievable duration if more than one path exists. We use $\mathcal{P}_A(\sigma, \delta, t_1)$ to denote the fact that forwarding algorithm $A$ can find a path from $(\sigma, t_1)$ to $(\delta, t_k)$ for some $t_k < t_{max}$. If forwarding algorithm $A$ finds at least one path, we denote the duration of the shortest-found path $(t_k - t_1)$ as $T_A(\sigma, \delta, t_1)$.

Our concern centers on the performance of forwarding algorithms in the PSN setting. We define the performance of forwarding algorithm $A$ as the average delay

$$D_A = E[T_A(\sigma, \delta, t_1) | \mathcal{P}_A(\sigma, \delta, t_1)]$$

and the success rate

$$S_A = E[1{\mathcal{P}_A(\sigma, \delta, t_1)}]$$

when $\sigma$ and $\delta$ are chosen uniformly at random over $\mathcal{M}$.

Figure 1: Time Series of Total Contacts (a) Infocom 06 9-12 (b) Infocom 06 3-6 (c) Conext 06 9-12 (d) Conext 06 3-6
and \( t_1 \) is chosen uniformly at random over \([0, t_{\text{max}}]\).

Note that our formulation of the problem does not consider the costs of forwarding: for example, power or storage consumption. Reasonable secondary goals of a forwarding algorithm might be to minimize costs. However our focus is on understanding what alternatives in terms of paths are available for use by a forwarding algorithm. After obtaining an understanding of what paths are available, it is easier to concentrate on minimizing costs.

### 4.1 Path Enumeration

As discussed in Section 2, many previous studies of forwarding algorithms have looked at forwarding performance in specific settings. However, our work takes a different approach by first empirically characterizing the set of paths that are available for use by forwarding algorithms. To accomplish this properly, there are a number of specific considerations.

First, it is important to specify the characteristics that are expected of all forwarding algorithms, because these define the set of paths of interest. We assume that under any reasonable forwarding algorithm, a node holding a message for a destination node will deliver that message whenever it encounters the destination. We call this the assumption of minimal progress. We also restrict our attention only to loop-free paths, i.e., paths in which no node appears more than once. We make this restriction because if looping paths are considered, then an arbitrarily large number of paths may be generated simply by following a loop a varying number of times. For example, if two nodes are in contact, many distinct paths can be created simply by passing a message back and forth. Enumerating such cases does not expose truly distinct forwarding paths. Furthermore, such paths are not usually relevant in practice, although they may occur in the normal operation of some forwarding algorithms. To the extent that looping paths do occur in normal operation it is not a concern, because disregarding such looping paths means that our counting results are conservative—so in fact even more paths may be present than our results indicate. For a similar reason, we assume nodes have infinite buffers and we do not consider paths that may be created because a node is forced to drop a message. Thus, once a node receives a message it holds the message forever.

A path respects minimal progress and loop avoidance if it does not contain more than one instance of any node, and the destination appears only at the end of the path (if at all). Furthermore, the assumption of minimal progress combined with unlimited message holding implies a more subtle condition, which we call first preference. Consider a path

\[
P = ((\sigma, t_1), \ldots, (x_i, t_i), \ldots, (x_j, t_j), (\delta, t_k)).
\]

If it also happens that \( x_i \) encounters the destination \( \delta \) at time \( t' < t_k \), then the following is also a path:

\[
P' = ((\sigma, t_1), \ldots, (x_i, t_i), (\delta, t')).
\]

Note that no forwarding algorithm respecting minimal progress would take path \( P \) rather than path \( P' \). Thus path \( P \) is not a first preference path. Any path respecting loop avoidance and minimal progress (including first preference) is a valid path.

Second, in moderately-large datasets (such as those we work with) a complete enumeration of all valid paths with a given \((\sigma, \delta, t_1)\) is prohibitively expensive. Therefore a key element of our approach is efficient enumeration of the most important paths. Our strategy for tackling this problem has two steps:

1. Define the problem as one of path enumeration on a space-time graph (defined below) and
2. at each time step, use dynamic programming to maintain the (up to) \( k \) shortest valid paths reaching each node. By ‘shortest’ we mean the path with the least number of hops.

This strategy allows us to determine the optimal path reaching the destination, that is, the path with shortest achievable duration under any forwarding algorithm. We denote the duration of the optimal path \( T(\sigma, \delta, t_1) \) and note that it is the minimum duration path found by epidemic forwarding, i.e.,

\[
T(\sigma, \delta, t_1) = \min_A T_A(\sigma, \delta, t_1) = T_{\text{Epidemic}}(\sigma, \delta, t_1).
\]

This strategy also allows us to determine each subsequent valid path that reaches the destination, up until the point at which \( k \) or more valid paths reach the destination in a single timestep.

To organize the search process, we convert the sequence of node contacts into a space-time graph, which is a special kind of directed weighted graph. Our use of the space-time graph is based on [13].

Time is discretized in increments of \( \Delta \). In all our work we use \( \Delta = 10 \) sec. Vertices in the space-time graph are pairs \((x_i, T)\) with \( x_i \in M \) and \( T = c\Delta \) for \( c \in \{1, 2, \ldots, \lceil t_{\text{max}}/\Delta \rceil \} \). Edges in the space-time graph come in two kinds:

1. There is an edge from vertex \((x_i, T)\) to \((x_j, T)\) iff node \( x_i \) was in contact with node \( x_j \) at any time during \([T - \Delta, T)\). Such an edge has weight zero.
2. There is an edge from vertex \((x_i, T)\) to \((x_i, T + \Delta)\) for all \( x_i \in M \). Such edges have unit weight.

For example, consider a network with three nodes. Nodes 1 and 2 are in contact during the first timestep, while all three nodes are in contact with each of the others during the second timestep. Then the corresponding
space-time graph is as shown in Figure 2. The horizontal edges have weight zero and the vertical ones have weight 1.

Given a space-time graph discretized by ∆ and a message defined as (σ, δ, t₁), we enumerate shortest paths using dynamic programming. The algorithm uses as its data structure an $N \times k$ array of paths, denoted $P$. It maintains the following invariant: at any given timestep $T = c\Delta$ the entry $P_{ij}$ is the $j^{th}$ shortest path reaching from $(σ, t₁)$ to $(x_i, t'')$ for some $t'' \in [T - \Delta, T)$. The algorithm is given in Figure 3.

4.2 Observations

Using the algorithm described in the previous subsection we are able to enumerate paths for a given message with $(σ, δ, t₁)$ up to the time when $k$ paths reach the destination in a single timestep. For any given message, we will use $T_n$ to denote the time at which the $n^{th}$ path reaches the destination, in order of increasing delivery time.

Our results show that for many messages, the duration of the optimal path can be quite long (thousands of seconds). However the most striking aspect of our results is that in the vast majority of cases, once the first path reaches the destination a very large number of additional paths reach the destination soon thereafter — typically tens or a few hundreds of seconds later. We refer to this phenomenon generically as path explosion.

To make the notion concrete, we define the time that path explosion occurs to be the time by which 2000 paths in total have reached the destination, i.e., $T_{2000}$. We would like to point out that there is nothing sacrosanct about the number 2000; we believe that by enumerating 2000 paths (; a large number of paths) we can develop a concrete picture of the set of paths available. Note that we can always identify this time accurately (to within an error of ∆) as long as we set $k$ in our algorithm to be 2000 or greater. We refer to the ‘time to explosion’ ($T_E$) as the elapsed time between the arrival of the first path and the arrival of the 2000th path, i.e., $T_E = T_{2000} - T_1$. We would also like to point out that there is a possibility that some messages might not have 2000 paths, but may still have a fairly large number of paths between them. We do not consider such messages.

![Figure 2: Example space-time graph.](image)

1. Let $T = c\Delta$ such that $t₁ \in [T - \Delta, T)$.
2. Let $s = i$ such that $x_i = σ$.
3. Let $P_{ij} = \emptyset$ for all $i,j$.
4. Let $P_{11} = (σ, T)$.
5. While (true)
   (a) Let $X = \emptyset$.
   (b) For $i = 1, \ldots, N$
      i. For $j = 1, \ldots, k$
         • If $P_{ij} \neq \emptyset$
            A. Let $X' = \text{all distinct extensions of path } P_{ij} \text{ to vertices reachable from } (x_i, T + \Delta)$ via paths of zero weight.
            B. Let $X = X \cup X'$.
      (c) Remove the invalid paths from $X$.
      (d) Output all paths in $X$ reaching $δ$.
      (e) If there are $k$ or more paths in $X$ reaching $δ$, stop.
      (f) For $i = 1, \ldots, N$
         i. Select the (up to) $k$ shortest paths from $X$ that terminate at node $x_i$ and place them in $P_{ij}, j = 1, \ldots, k$.
   (g) Let $T = T + \Delta$.

![Figure 3: Algorithm for k shortest paths enumeration.](image)

In Figure 4(a) we show the CDFs for optimal path duration in datasets Infocom ’06 9-12 and Infocom ’06 3-6. The figure shows that optimal path duration can be quite long; in fact, a large fraction of messages (over 25%) require over 1000 seconds before the first path reaches the destination. On the other hand, Figure 4(b) shows CDFs for the time to explosion. Here the story is quite different: almost half the messages see an explosion with little or no delay, and 97% of them have $T_E$ less than or equal to 150 secs.

These figures show the surprising fact that optimal path duration can be an order of magnitude or more greater than time to explosion. That is, in many cases, the first path takes a long time to reach the destination, after which many paths reach the destination in relatively short order. In a phrase, path explosion means that shortly after the optimal path reaches the destination, there are a large number of nearly-optimal paths to the destination.

A closer look at the path explosion phenomenon is shown in Figure 5, which is also based on dataset Infocom ’06 9-12. Each point in the figure corresponds to $(T_1, T_E)$ for a single message $(σ, δ, t₁)$ (note the difference in scale on the $x$ and $y$ axes). The figure shows that there is no clear relationship between optimal path
duration and time to explosion; there are many cases in which the optimal path reaches the destination quickly but the path explosion occurs comparatively late, and there are many cases in which the optimal path reaches the destination late while the path explosion occurs quickly thereafter.

It is difficult to characterize how the number of successful paths grows in time, in part because path explosion generally occurs quite rapidly and our time measures are rather coarsely discretized. However we can get a rough sense of how the explosion occurs if we look at the slowest cases. In Figure 6 we examine all paths of the messages for which the time to explosion $T_E$ was 150 seconds or greater, again using dataset Infocom '06 9-12. Note that although for each of these messages, the 2000th message arrives at least 150 seconds after the optimal time, nonetheless many messages still arrived during the period between $T_1$ and $T_{2000}$. The figure shows that the number of paths arriving over time grows in an approximately exponential fashion.

In summary, we find that once paths are enumerated, a number of surprising properties emerge. While the optimal path duration can be long, there is a path explosion effect that occurs relatively quickly after the first path reaches the destination. However, there is no clear relationship between optimal path duration and time to explosion. Finally, it appears that the explosion process is roughly exponential in time. We seek to explain these observations in the following sections, and we study the implications of these observations in later sections of the paper.

### 5. PROPERTIES OF PATHS

In this section we use models and data analysis to understand the questions raised in Section 4.2. In particular the next two subsections ask: (1) How and why does path explosion occur? and (2) What determines the relationship between optimal path duration and time to explosion? To answer the first question we develop an analytic model making the assumption of homogeneity – equal contact rates among all nodes. To answer the second question we incorporate considerations of unequal contact rates (inhomogeneity).

#### 5.1 A Homogeneous Model for Path Explosion

As a first step in trying to understand the path explosion phenomenon, we use a highly simplified model. We consider a setting in which nodes contact other nodes randomly and at constant rate. For a given message, we count the number of paths arriving at each node over time. The model is not restricted to counting just valid paths (which would add considerable complexity) but we seek qualitative results which are presumably insensitive to such details. The main result we show in this section is that the number of paths arriving at any node (such as the destination) grows exponentially in time, with rate given by the (homogeneous) contact.
rate between nodes.

Our results use known techniques from the study of epidemic process [11, 19]; however we extend those tools to study the set of all forwarding paths, instead of just the first path obtained to each destination.

5.1.1 Assumptions

Our model makes two assumptions, which derive from the uniformity of the setting.

- Poisson contacts: We associate with each node \( x_n \) a process of contact opportunities with all other nodes that is a homogeneous Poisson process with intensity \( \lambda \).

- Homogeneity: whenever a contact opportunity occurs for node \( x_i \), the contacted node \( x_j \) is chosen independently from the past, and uniformly among the \( N \) nodes in \( \mathbb{M} \). Note that this model does not include variability of contact rates among pairs, an issue we take up in Section 5.2.

Without loss of generality we study forwarding paths for a message created by a source (denoted by \( \sigma \)) at time \( t = 0 \).

5.1.2 Evolution and limit for large networks

Our model is adapted from the analysis of population dynamics. The state \( S_n(t) \) of a node \( x_n \) at time \( t \) is the number of paths from the source that reach \( x_n \) before \( t \). When \( x_n \) has an opportunity to contact \( x_m \), the following transition occurs at \( x_m \):

\[
S_m(t) \leftarrow S_m(t) + S_n(t).
\]

For any \( k \geq 0 \), we denote

\[
U_k(t) = \# \{ x_n \in \mathbb{M} \mid S_n(t) = k \},
\]

for the number of nodes with state \( k \). To focus on the evolution of the population of nodes as a whole we study the evolution of the collection of variables

\[
U(t) \in \mathbb{N}^k, \quad U(t) = \{ U_k(t) \mid k \geq 0 \}.
\]

Note that we have \( \sum_{k \geq 0} U_k(t) = N \), so \( U(t)/N \) represents the empirical density of nodes in each states. If one does not differentiate between nodes in the same state, then \( U(t) \) entirely characterizes the system at time \( t \).

Note that when node \( x_n \) having state \( i \neq 0 \) contacts node \( x_m \) having state \( j \), the collection of variables \( U \) is modified as follows.

\[
U_{i+j} \leftarrow U_{i+j} + 1 \quad \text{and} \quad U_{j} \leftarrow U_{j} - 1.
\]

We denote by \( \lambda_{n,m} \) the contact rate between node \( x_n \) and node \( x_m \). For any fixed \( i > 0 \) and \( j \geq 0 \), transitions of the type \( (i, j) \) as defined above occur with the following rate:

\[
\beta_{i,j} = \sum_{n \neq m} \{ \lambda_{n,m} \mid n \neq m \text{ s.t. } S_n(t) = i, S_m(t) = j \}.
\]

Proposition 1. The rate of transition \( \beta_{i,j} \) is a function of the density of nodes in each state \( U(t)/N \).

\[
\beta_{i,j} = N \cdot \lambda \frac{U_i(t) U_j(t)}{N} = N \tilde{\beta}_{i,j} \left( \frac{U(t)}{N} \right)
\]

Proof. Let us fix an \( n \) such that \( S_n = i \). Then we have

\[
\sum_{x_m \mid S_m(t) = j} \lambda_{n,m} = \frac{U_j(t)}{N}.
\]

since we have assumed that the contact process for each node \( x_n \) has rate \( \lambda \) and is homogeneous with regard to other nodes.

Summing for all nodes \( x_n \) having state \( i \) we obtain

\[
\beta_{i,j} = U_i(t) \lambda \frac{U_j(t)}{N} = N \cdot \lambda \frac{U_i(t) U_j(t)}{N}.
\]

We can describe the process \( U(t) \) taking on values in \( \mathbb{N}^k \) as a Markov jump process, with transitions indexed by \( \{ (i, j) \mid i > 0, j \geq 0 \} \).

The rate of transition \( (i, j) \) is a function \( \tilde{\beta}_{i,j} \) only of the density \( U(t)/N \) of nodes having states \( i \) and states \( j \), multiplied by \( N \). In other words, the process \( U(t) \) may be written as

\[
U(0) + \sum_{i > 0, j \geq 0} (e_{i+j} - e_i) \cdot \int_0^t \mathcal{N}_{i,j} \left( N \tilde{\beta}_{i,j} \left( \frac{U(s)}{N} \right) \right) ds
\]

where, for all \( i > 0, j \geq 0, \mathcal{N}_{i,j} \) denotes a Poisson counting process with intensity 1, and \( e_k \) denotes the infinite vector with all entries null except for 1 at position \( k \).

When one considers the density process \( U(t)/N \) in the case where \( N \) is large, Kurtz’s limit theorem [11] shows...
that one can replace the Poisson counting process in the above expression by the process mean. This means that the trajectory of the density process for large \( N \) closely approaches the solution of a deterministic ordinary differential equation.

**Proposition 2.** As \( N \) goes to infinity, if we assume that \( U(0)/N \to u(0) > 0 \), then we have for all \( K \geq 0 \):

\[
\sup_{0 \leq s \leq t, 0 \leq k \leq K} \left| \frac{U_k(s)}{N} - u_k(s) \right| \to 0 \text{ a.s.}
\]

where \( u : [0; \infty[ \to \mathbb{R}^N \) is the solution of the ODE

\[
u(t) = u(0) + \sum_{i=0, j \geq 0} (c_{i+j} - e_j) \cdot \int_0^t \beta_{i,j}(u(s)) \, ds
\]

**Proof.** A priori the process \( U(t)/N \) evolves in a space with an infinite number of dimensions \( \mathbb{N}^N \), hence Kurtz’s limit theorem [11] does not immediately apply to it. However, one can consider for any \( K > 0 \) a threshold process where all nodes in states \( \{ k \mid k > K \} \) are collapsed into a collection of nodes in a single sink state. This threshold process has a finite number of dimensions, and satisfies all the assumptions of the Kurtz limit theorem. To complete the proof we note that the threshold process defines exactly the same dynamics for states in \( \{ 0, \cdots, K \} \) as in the infinite process. \( \square \)

**Proposition 3.** For any \( k \geq 0 \), we have

\[
u_k(t) = u_k(0) + \lambda \int_0^t \sum_{i=0}^k u_i(s) u_{k-i}(s) - u_k(s) \, ds \tag{1}
\]

**Proof.** From the definition of \( u \) found in Proposition 2, we obtain that \( u_k(t) - u_0(t) \) may be written for all \( k \geq 0 \) as:

\[
\sum_{i=0, j \geq 0, i+j=k} \int_0^t \beta_{i,j}(u(s)) \, ds - \sum_{i>0, j=k} \int_0^t \beta_{i,j}(u(s)) \, ds
\]

which may be rewritten as:

\[
\sum_{i=1}^k \int_0^t \beta_{i,k-i}(u(s)) \, ds - \sum_{i \geq 0} \int_0^t \beta_{i,k}(u(s)) \, ds
\]

\[
\sum_{i=1}^k \lambda u_i(s) u_{k-i}(s) \, ds - \sum_{i \geq 0} \lambda u_i(s) u_k(s) \, ds
\]

After adding \( \lambda u_0(s) u_k(s) \) to each of the above terms and using \( \sum_{i \geq 0} u_i(s) = 1 \) the proposition is proved. \( \square \)

In other words, from a non trivial initial condition, the average number of nodes with exactly \( n \) paths in a large network evolves according to a deterministic equation.

### 5.1.3 Solution for ODEs, moments

We introduce for all \( x \in \mathbb{R} \) the series \( \phi_x : t \mapsto \sum_{k \geq 0} x^k u_k(t) \). After multiplying each equation in (1) by \( x^k \) and summing all of them we obtain

\[
\frac{d\phi_x}{dt} = \lambda (\phi_x^2 - \phi_x) .
\]

Note that this equation may be rewritten as:

\[
\lambda \cdot dt = \frac{d\phi_x}{\phi_x(\phi_x - 1)} = \frac{d\phi_x}{\phi_x - 1} + \frac{d\phi_x}{\phi_x} .
\]

This ordinary differential equation can be solved for all \( x \in \mathbb{R} \). One needs to distinguish between two cases, depending on the initial value taken by \( \phi_x \).

- Assume \( x^0 < \phi_x(0) < 1 \) (for instance, by choosing \( x \) in \([0;1]\)). We know that \( \phi_x \) and \( \phi_x - 1 \) remains positive. The previous equation may then be written as:

\[
\ln(1 - \phi_x(t)) - \ln(\phi_x(t)) - \lambda t \text{ is constant.}
\]

Hence, \( \phi_x(t) = \frac{\phi_x(0)}{\phi_x(0) - (\phi_x(0) - 1)e^{\lambda t}} \).

- On the other hand, assume \( \phi_x(0) > 1 \) (for instance, by choosing \( x > 1 \)). We know that \( \phi_x \) and \( \phi_x - 1 \) remains positive. The previous equation may then be written as:

\[
\ln(\phi_x(t) - 1) - \ln(\phi_x(t)) - \lambda t \text{ is constant.}
\]

Hence, \( \phi_x(t) = \frac{\phi_x(0)}{\phi_x(0) - (\phi_x(0) - 1)e^{\lambda t}} \).

The first case above allows us to derive closed form expression for the evolution with time of the mean number of paths per node (see below), as well as the other moments. In the second case, for any \( x > 1 \) the series \( \phi_x \) becomes infinite in finite time \( T_{\phi_x}(x) = \frac{1}{\lambda} \ln \left( \frac{\phi_x(0)}{\phi_x(0) - 1} \right) \).

In other words, if the initial distribution for the number of paths per node is light-tailed for a given coefficient, it loses this property within a finite time.

**Mean number of paths:**

According to the definition of \( \phi_x \) we can compute the expected number of paths for a node as follows:

\[
E[S_n(t)] = E \left[ \left( \frac{1}{N} \sum_{m \in M} S_n(t) \right) \right] = \sum_{k \geq 0} k \cdot u_k(t)
\]

thus \( E[S_n(t)] = \left. \frac{\partial \phi_x(t)}{\partial x} \right|_{x=1} = E[S_n(0)] e^{\lambda t} \).

The last equation is obtained from (2) since

\[
\frac{\partial \phi_x(t)}{\partial x} (x) = \frac{\partial \phi_x(0)}{D(x, t)} (x) - \frac{\phi_x(0) \cdot \partial \phi_x(0)}{D(x, t)^2} (x) \cdot (1 - e^{\lambda t}) .
\]
with \( D(x, t) = \phi_x(0) + (1 - \phi_x(0)) \cdot e^{\lambda t} \), so \( D(1, t) = 1 \) for all \( t \).

**Second moment for the number of paths.**

It follows from the properties of \( \phi_x \) that:

\[
E[S_n(t)(S_n(t) - 1)] = E \left[ \frac{1}{N} \sum_{m \in M} S_m(t)(S_m(t) - 1) \right] = \sum_{k \geq 0} k(k - 1) \cdot \nu_k(t) = \left. \frac{\partial^2 \phi_x(t)}{\partial^2 x} \right|_{x=1}.
\]

From the above expression of \( \frac{\partial \phi_x(t)}{\partial x} \), one can obtain the second derivative in \( x \). Again the expression simplifies when \( x = 1 \) and it implies:

\[
E \left[ (S_n(t))^2 \right] = \left( E \left[ (S_n(0))^2 \right] + 2(e^{\lambda t} - 1)E [S_n(0)]^2 \right) e^{\lambda t}.
\]

If \( \forall [X] \) denotes the variance of a random variable \( X \), then \( \forall [S_n(t)] = \forall [S_n(0)] e^{\lambda t} + \forall [S_n(0)] (e^{2\lambda t} - e^{\lambda t}) \).

Note that the variance of the number of paths increases exponentially fast with time \( t \) as \( E [S_n(0)] e^{2\lambda t} \), even if it is null when \( t = 0 \).

### 5.2 The Effects of Inhomogeneity

The analysis leading to Equation 4 sheds considerable light, but does not explain all the phenomena noted in Section 4.2. The analytic results confirm that path explosion can occur and that it should be exponential in nature. However, the analysis in Section 5.1 would also predict that optimal paths should be short, and that path explosion should occur immediately after the first path reaches the destination. These two predictions are not borne out in the data, as already shown in Section 4.2. To explain these phenomena, we need to examine the model assumptions.

The most unrealistic aspect of our analytic model is the assumption that all nodes contact each other at the same rate. In fact, prior work has shown that per-node contact rates in datasets like ours can be highly variable across different nodes. For example, the authors in [8] show that the distributional tails of inter-contact times for such data sets approximately follow a power law.

In Fig. 7 we plot the CDF for the total number of contacts each node has over each three hour period (Infocom '06 9-12, 3-6, Conext 9-12, and 3-6). We refer to the number of contacts a node makes per unit time as the node’s contact rate or just rate. The CDFs suggest that the distribution of contact rates can be approximated as uniform over the range \((0, \text{max})\) where \( \text{max} \) varies depending on the dataset. The key observation is that some nodes have rates quite close to zero, yielding extremely large average inter-contact times. In fact, it is clear that the population consists of some nodes that are quite frequently in contact with a large number of other nodes, while there are also some nodes that rarely encounter other nodes at all.

Our analytic results suggest that for a message generated at time 0, the number of paths reaching node \( i \) will grow proportional to \( e^{\lambda t} \) for a population in which all nodes have the same rate \( \lambda \).

To discuss the case where nodes have different rates, we introduce some notation. We define the random variable \( T(\sigma, \delta) \) to be equal to \( T(\sigma, \delta, t) \) when \( t_i \) is chosen uniformly at random in \([0, t_{\text{max}}]\). Furthermore we no longer assume equal contact rates, so we define \( \lambda_i \) to be the contact rate of \( x_i \in \mathbb{M} \), i.e., \( \lambda_i = \sum_k \lambda_i.k \). Finally we define the expected time for the first path as

\[
H = E[t \mid S_i(t) = 1].
\]

In the case of the homogeneous model in Section 5.1,

\[
H = (t \mid E [S_i(0)] e^{\lambda t} = 1) = \frac{\ln N}{\lambda}
\]

since \( E [S_i(0)] e^{\lambda t} = 1/N \).

To understand the case when nodes have different rates, we reason as follows. Assume that at time \( t_0 \) a message is held by a node \( x_i \) having contact rate \( \lambda_i \). Then at minimum, we expect a path explosion to occur starting at \( t_0 \) with rate \( \lambda_i \) among the subset of nodes with contact rates greater than or equal to \( \lambda_i \). That is, we can infer a lower bound on \( E [S_j(t - t_0)] \) proportional to \( e^{\lambda_j(t-t_0)} \) for all \( j \) such that \( \lambda_j \geq \lambda_i \). This subset path explosion means that the number of paths arriving at nodes with rate \( \lambda_i \) or greater grows at least as fast as \( e^{\lambda_j t} \).

Now if \( \lambda_i \) is relatively small compared to typical contact rates, then initially the subset path explosion may proceed very slowly. However if at time \( t_1 > t_0 \), \( x_i \) encounters a node \( x_j \) with contact rate \( \lambda_j > \lambda_i \), then a more rapid path explosion beginning at \( t_1 \) will occur with rate \( \lambda_j \) among nodes with contact rates \( \lambda_j \) or greater.

Thus if the source is a low-rate node, then there will be some initial time before it encounters a high-rate node, at which point a high-rate path explosion occurs. This initial time is related to \( H \) and so we argue that the time until high-rate path explosion occurs is on the order of \( 1/\lambda_i \). Thus in this case, \( T_{\text{E}} \) will tend to be larger than typical.

Furthermore, if the destination is a low-rate node, then path explosion may not be able to reach a high rate. Thus in this case, \( T_{\text{E}} \) will tend to be large.

This model suggests that a critically important role is played by the contact rates of the source and destination nodes. Then we can make the following hypotheses concerning the relative sizes of \( T_1 \) and \( T_{\text{E}} \) for four situations:

- First, when \( \lambda_s \) is high and \( \lambda_d \) is high, then path explosion begins immediately and at high rate. So
both $T_1$ and $T_E$ will tend to be small.

- Next, when $\lambda_\sigma$ is high and $\lambda_\delta$ is low, then path explosion begins immediately but at a low rate. So $T_1$ will tend to be small but $T_E$ will tend to be large.

- Next, when $\lambda_\sigma$ is low and $\lambda_\delta$ is high, then there is a significant period before path explosion begins (on the order of $1/\lambda_\sigma$) but, once begun, path explosion proceeds at high rate. So $T_1$ will tend to be large but $T_E$ will tend to be small.

- Finally, when $\lambda_\sigma$ is low and $\lambda_\delta$ is low, then both $T_1$ and $T_E$ will tend to be large.

To explore whether these hypotheses hold in our data, we separated nodes in each dataset into two equal-sized groups. The in set are those nodes with contact rates greater than the median rate. The out set are those nodes with contact rates lower than the median rate. Since rate distribution is approximately uniform (as shown in Figure 7), the median rate in each case is approximately half of the maximum rate.

Then each message $(\sigma, \delta, t_0)$ can be placed into one of four cases: in-in, where $\lambda_\sigma$ and $\lambda_\delta$ are both high; in-out, where $\lambda_\sigma$ is high and $\lambda_\delta$ is low; out-in, where $\lambda_\sigma$ is low and $\lambda_\delta$ is high; and out-out, where $\lambda_\sigma$ and $\lambda_\delta$ are both low.

Using this labeling, we separate the points in Figure 5 into four groups, which are plotted separately in Figure 8. Each plot has the same format as Figure 5, except that only a subset of points are included.

The figures suggest that our hypotheses hold to a certain extent. In particular,

- For all the in-in messages, the optimal path duration $T_1$ is very low and the time to explosion $T_E$ is less than 150 seconds.

- For the in-out messages, the optimal path duration $T_1$ is similar to the in-in messages, i.e., small. However the time to explosion $T_E$ is much more variable and can take on relatively large values.

- For the out-in case, the optimal path duration $T_1$ tends to be larger than in the first two cases, while the time to explosion $T_E$ is relatively small (as in the in-in case).

- Finally, for the out-out case, both $T_1$ and $T_E$ can take on large values.

Results for other datasets are not shown due to lack of space, but generally showed similar behavior.

In summary, we have shown two key results in this section. First, we have used an homogeneous analytical model to show how path explosion occurs. We have shown that path explosion is an exponential function of both elapsed time and the rate $\lambda$ at which nodes come into contact. This addresses the first question about our empirical results, namely, how and why does path explosion occur? Second we have shed light on the other set of questions arising from our empirical results by considering the effects of inhomogeneity in rates. We have shown that by looking at the contact rates of the source and destination nodes, one can gain insight into why the optimal path duration and the time to explosion vary, and one can relate these quantities to the relative magnitudes of the two contact rates.

6. IMPLICATIONS FOR FORWARDING

In the previous sections we have developed insight into what sorts of paths are available for use by routing algorithms in settings such as ours. This has laid the groundwork for an understanding of how various forwarding algorithms perform in our setting. In this section we study the performance of a wide range of forwarding algorithms and relate their performance to the results developed in previous sections.
6.1 Experiments

We study forwarding by implementing different forwarding algorithms in a trace-driven simulator. We use as input to our simulator the contact traces described in Section 3. Each simulation therefore ran for simulated time of 3 hours. However, to avoid end-effects, only results from the first two hours are used.

For each trace and forwarding algorithm, we generate a set of messages \((\sigma, \delta, t_1)\) uniformly at random. We generate messages according to a Poisson process with rate one message per 4 seconds. As discussed in Section 4.1 we model nodes as having infinite buffers and nodes carry all the messages they receive until the end of the simulation. All our results are averaged over 10 simulation runs. Our metrics are success rate \(S\) and average delay \(D\) as defined in Section 4.1.

Our goal in this section is not to determine which forwarding algorithm is “best” but to compare the performance of a wide range of algorithms and gain insight into what properties of a forwarding algorithm yield good performance. Hence we choose a set of algorithms designed to span a range of design choices. The design choices include:

- Destination aware vs. Destination unaware. Destination aware algorithms take the choice of \(\delta\) into consideration in forwarding, while destination unaware algorithms do not.
- Single hop vs. Multi hop. Single hop algorithms use information about the most recent contact or next expected contact; multi hop algorithms take into account sequences of past or expected future contacts.
- Complete history vs. Recent history vs. Future knowledge. Complete history algorithms take into account the entire past history of other nodes when forwarding. Recent history algorithms taken into account only a limited amount of history (e.g., only the most recent encounter with the destination). Future knowledge algorithms make use of oracles that provide knowledge of future behavior of the nodes. Future knowledge algorithms are not practical but provide useful comparison cases.

To span these design choices we use the following forwarding algorithms. Some of these algorithms are known from the literature and others are modifications or extensions.

**Epidemic Forwarding (Flooding):** A node forwards all the packets it has to any node it meets, unless that node already has a copy of the packet [18].
already noted, Epidemic Forwarding achieves the best possible performance, so this algorithm yields upper bounds on success rate and average delay.

**FRESH**: Node $x_i$ forwards a message to $x_j$ upon contact if $x_j$ has contacted $\delta$ more recently than has $x_i$ [5].

**Greedy**: Node $x_i$ forwards a message to $x_j$ upon contact if $x_j$ has contacted $\delta$ more times since the start of the simulation than has $x_i$. Greedy uses the entire history of contacts in contrast to FRESH which uses only the most recent contact history.

**Greedy Total**: Node $x_i$ forwards a message to $x_j$ upon contact if $x_j$ has more total contacts (with all other nodes) than $x_i$. Thus Greedy Total is destination unaware, while FRESH and Greedy are destination aware. Greedy Total uses both past and future knowledge of internode contacts.

**Greedy Online**: Node $x_i$ forwards a message to $x_j$ upon contact if $x_j$ has more contacts (with all other nodes) since the start of simulation than does $x_i$. Greedy Online is destination unaware but uses only past knowledge, unlike Greedy Total.

**Dynamic Programming**: Node $x_i$ calculates the average delay between all pairs of nodes then finds the optimal path. This is based on the Minimum Expected Delay algorithm [9]. Dynamic Programming is based on both past and future knowledge of internode contacts.

### 6.2 Observations

#### 6.2.1 Similarity of Performance

The most striking aspect of our results is the similarity in performance of the various different forwarding algorithms. This is illustrated in Figure 9. For each data set and forwarding algorithm we plot the average delay vs success rate.

The figure shows that almost all forwarding algorithms show virtually identical performance. The exception is epidemic routing (shown using square symbols in the plots) which shows somewhat better performance than the others, since it always finds the optimal path if one exists.

While the plots in Figure 9 show the similarity of average delay, a more detailed view is given in Figure 10. This figure shows the entire distribution of delay. The figure shows that the different algorithms show quite similar distributions of delay as well.

To explore reasons behind the similar performance of the different algorithms, we first verify that message delivery is not ‘bursty’. That is, we confirm that the
Figure 10: Delay Distributions (a) Infocom 06 9-12 (b) Conext 9-12

Figure 11: Message Reception Times Infocom 9-12

The phenomenon is not simply due to many nodes all making contact at relatively infrequent times. An example scenario in the conference could be that participants convened at a common area after a session and it was during this time period that many packets were delivered. To confirm that this is not the case, we plot the cumulative totals of in Figure 11. We plot the cumulative totals of delivery times of all optimal and near-optimal paths. As can be seen the delivery rate is fairly uniform in time. This shows that there are no special periods of time in which most messages are delivered.

Likewise, the fact that epidemic routing does noticeably better than the other algorithms means that similarity in performance among algorithms does not occur because they are all finding the same, optimal paths. That is, in many cases there are better paths to be found than those chosen by most forwarding algorithms. However algorithms generally seem to find paths that are close to, though not exactly, optimal.

To understand the remarkably similar performance of the many different forwarding algorithms we examine which paths are being taken by the different algorithms. Some typical results are shown in Figure 12. Each plot in the figure corresponds to a particular message \((\sigma,\delta,t_1)\). The \(x\) axis measures the time since \(T_1\) for this particular message, and the \(y\) axis shows the number of paths that reach the destination over time. Furthermore, for each algorithm, its symbol is superimposed on the bar corresponding to the path chosen by the algorithm.

The two cases in the figure show the typical path explosion phenomenon — the number of paths arriving at the destination grows approximately exponentially with time. Furthermore, the plots show that the paths used by all the forwarding algorithms reach the destination early in the path explosion process.

For example, the left hand plot shows that for this particular message, FRESH and Greedy are able to find paths arriving at time \(T_1\) (i.e., optimal paths) while Greedy Online and Greedy Total find paths arriving in the next burst, 20 sec after \(T_1\). Finally Dynamic Programming finds a path arriving in the third burst at 90 seconds after \(T_1\). The situation is similar for the case of the message in the right hand plot.

This figure suggests that different algorithms may have similar (though non-optimal) performance because of the large number of paths that reach the destination shortly after \(T_1\). When there are a large number of nearly-optimal paths, a forwarding algorithm may be able to find one of those paths relatively easily. That is, the existence of path explosion may be a factor that allows many algorithms to achieve somewhat similar performance.

If this hypothesis is correct, then the performance of forwarding algorithms should be strongly influenced by the way in which path explosion occurs. In Section 5.2 we found that path explosion occurs in different ways for different types of source-destination pairs, i.e., in-in, in-out, out-in, and out-out. Here we can put that insight to use.
If the path explosion effect is a major reason why different forwarding algorithms have similar performance, then algorithm performance should be fairly similar within pair types and quite different between pair types. For example, algorithm performance should be similar across all in-in pairs, but performance on in-in pairs should be quite different from that on in-out pairs.

To see whether this is the case, we separate our simulation results by pair type. The results are shown in Figure 13. The figure shows the average delay and success rate across the four pair types for each of the six forwarding algorithms.

The figure shows that success and delay depend primarily on the type of the source-destination pair as opposed to the type of algorithms used. All forwarding algorithms, with the exception of epidemic, again show similar performance.

However the figure also shows that, once messages are broken down by pair type, differences between forwarding algorithms start to emerge. In particular, we note a difference for algorithms making use of maximum information about contact patterns – i.e., Greedy Total and Dynamic Programming, which use both past and future information. These algorithms outperform others, but only in the case where one node is an ‘out’ node. That is, in the case where both nodes are ‘in’ nodes, then information about contact patterns is not particularly helpful. However, when one or both nodes are ‘out’ nodes, then maximum information about contact patterns is helpful.

Digging deeper, we see that Greedy Total performs particularly well when the source node is an ‘out’ node. Recalling the discussion in Section 5.2, when a source node has low rate $\lambda_\sigma$, then rapid path explosion does not occur until the message has been moved to a high rate node with $\lambda_i > \lambda_\sigma$. This is consistent with the strategy of Greedy Total. Note that Greedy Total is a destination-unaware strategy — it only seeks to move the message toward nodes with higher contact rates, i.e., those with $\lambda_i > \lambda_\sigma$.

### 6.2.2 Effective Forwarding

These results suggest some heuristics for effective forwarding in settings like pocket switched networks. In particular, they suggest that a forwarding algorithm will be successful if it causes path explosion to take place as quickly as possible. This is a somewhat different principle for forwarding than has typically been used in past proposals. That is, rather than seeking to find a short path to the destination directly, a forwarding algorithm may instead work to cause path explosion to occur as quickly as possible. This suggests forwarding toward high rate nodes preferentially, regardless of their relationship to the destination. We can use our enumeration of paths to check whether forwarding toward high rate nodes is an effective strategy. If it is, then the nodes making up the near-optimal paths should tend to increase in rate along the path. That is, hops along successful paths should tend to be from lower-rate nodes to higher-rate nodes.

We test this hypothesis by inspecting the contact rate of nodes appearing at each hop along near-optimal paths. Note that for different paths, message delivery will occur after different numbers of hops, so the set of nodes at any given hop may contain nodes at the beginning, middle, or end of their respective paths. Nonetheless this aggregate view can tell us if any general trend is present. The results are shown in Figure 14, which plots the mean of node contact rates along the near-optimal paths for the Infocom 2006 9-12 dataset. Error bars are plotted corresponding to a 99% confidence interval on the mean; in many cases the confidence intervals are
Figure 13: Performance of Forwarding Algorithms on Different Types of Source-Destination Pairs: (a) Average Delay (b) Success Rate.

Figure 14: Mean rates of Intermediate Nodes

too small to see. The figure shows that for the first three hops of near-optimal paths, the contact rates of the nodes tends to increase. After a slight drop-off at hop 4, the mean contact rate then levels off. The reasons for the drop at hop 4 are not clear, but it may be because by that point most messages have been delivered and sample sizes are small.

While Figure 14 gives an aggregate view of nodes on forwarding paths, it does not indicate whether individual paths tend to progress from lower-rate toward higher-rate nodes. To verify that this is indeed the case, we look at the ratio of rates $r = \lambda_j / \lambda_i$ for consecutive nodes $i, j$ along near-optimal paths. If our hypotheses is correct, then these ratios will tend to be greater than 1. We plot the results as box-and-whiskers plots in Figure 15. These plots show the distribution of $r$ values for successive hops. For each hop, the box shows the limits of the 25 and 75 percentiles of the distribution and the median is marked. The figure shows that nearly all of the first hops are to nodes with higher rate than the source. Likewise, the second and third hops tend to be toward higher-rate nodes as well.

The picture that emerges from the results in this paper gives a better view of the nature of node mobility, contact patterns, and forwarding performance in a conference type setting. In broad terms, the picture is as follows. Connection patterns between nodes lead to an exponential path explosion effect whose rate depends on the contact rates of nodes. Since contact rate varies considerably across nodes, path explosion occurs much faster among the higher rate nodes than the lower rate nodes. In this setting successful forwarding relies on moving the message toward high contact rate nodes so that path explosion can occur quickly. Once path explosion occurs among high contact rate nodes, contact between one of them and the destination leads to message delivery.

7. CONCLUSION

Forwarding in pocket switched networks is challenging because of the irregular and unpredictable contact patterns between nodes. In order to understand the nature of the forwarding problem, this paper has taken the novel approach of enumerating and characterizing the set of forwarding paths that are available in such settings. Our main result is the observation that for most messages there is a path explosion effect, meaning that while the optimal path may be long, there are usually a large number of nearly-optimal paths to the destination. We support this result both analytically and empirically. We present a simple model of homogeneous
inter-node contact and show that path explosion is to be expected in that case. And we show empirically that path explosion does occur in our data, although in a more complicated manner due to varying contact rates across nodes. Our empirical results focus attention on the difference between high contact rate ('in') nodes and low contact rate ('out') nodes.

These results help us understand why different forwarding algorithms show quite similar performance. As a result of path explosion, different forwarding algorithms often find paths to the destination of nearly-equal duration.

Our study leaves open a number of important questions. Our understanding of the inhomogeneous case is not as precise as that of the homogeneous case; more sophisticated analytic models would be useful in that regard. Furthermore, we have not considered the cost of forwarding, and so there may be good reasons to prefer one algorithm over another even if they show similar performance. It is possible that an understanding of the path explosion phenomenon may help in understanding the relative costs of different forwarding algorithms.

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8. REFERENCES


