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A study of the value of formal analysis in problem solving

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Thesis

A STUDY OF THE VALUE OF FORMAL ANALYSIS
IN PROBLEM SOLVING

Submitted by

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CHAPTER I

STATEMENT OF THE PROBLEM

The purpose of this study was to discover, through testing and interviewing, whether children who had been taught to formally analyze arithmetic problems used this method of formal analysis or some other method of solving problems.

Before proceeding further, the meaning of "formal analysis" must be clarified.

The formal analysis method consists of analyzing an arithmetic problem by following a series of steps in reasoning such as the following:

1. Read the problem.
2. What is given?
3. What is asked?
4. What process must be used?
5. Solve the problem.

These steps often vary in number and in the form of statement but the above steps are typical of those which are used. Other additional steps which are sometimes included are to have the pupil estimate a probable answer, determine whether the answer is reasonable, and check the answer.

After using this step procedure it is expected that pupils will arrive at the correct solution of a problem. Much time and effort are spent in classrooms on problem work
whereby the correct solutions are worked out with either oral or written responses to the steps in the analysis method. After thorough training in this particular method in the fourth, fifth and sixth grades, it is generally assumed that pupils will solve arithmetic problems by using this analytical plan.

It was the purpose of this study, therefore, to select two heterogeneous groups of children who had been trained in this analysis method and to determine whether they actually followed this procedure of solving problems when not required to do so. A comparison was to be made of the results of a group test in arithmetic problems in which the formal analysis method was used, and an oral test of the same problems in which pupils used their own methods of reasoning.

The procedures used in this investigation consisted of testing and interviewing thirty-eight sixth grade children in order to discover whether they used the step method they had been taught or some other method of solving problems.
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CHAPTER II
CHAPTER II

REVIEW OF THE LITERATURE AND RESEARCH IN PROBLEM SOLVING

Introduction

As children learn to perform the operations of addition, subtraction, multiplication and division they are given additional practice in solving simple one-step problems. The purpose of the problem-solving activity is to provide situations in which particular processes are required and which give practice in the recognition of these processes. As a result, it is expected that pupils will be able to solve the problems which they encounter in school activities and in their after school experiences.

In order to clarify the term "problem" it must be differentiated from an "example" in arithmetic. "The term problem in arithmetic means a quantitative situation described in words in which a definite question is raised, but for which the arithmetical operation to be performed is not indicated." \(^1\)

An arithmetic example would have the arithmetical operation to be performed indicated, as:

\[
8 \times 9 = 72 \quad 12 \div 4 = 3 \quad 9 + 7 = 16 \quad 11 - 5 = 6
\]

Undoubtedly all pupils have at some time or other experienced difficulty with written problems in arithmetic. Regardless of their skill and accuracy in the fundamental operations their success in problem solving is not insured. Many investigations have centered around the various causes of difficulty and failure encountered by children in their solution of these problems. Some important causes of failure that have been studied are: inability to read, lack of skill in the fundamental processes, lack of mental capacity, lack of a general and technical vocabulary, physical defects and lack of a proper method or technique for attacking the problems.

Information on problem solving.

Failure in problem solving is often due to the inability to understand the conditions of the problem as stated. Because a teacher might caution her pupils to "read the problem carefully" it does not signify that the situation will be any clearer. Frequently those who read with apparent facility comprehend only a partial amount of the material which they have read.

A study by Estaline Wilson² points out the possibility of doing in arithmetic what has been done for improving silent

reading ability. She suggests that various reading devices be employed to increase problem solving ability, such as re-stating the problem in the form of a story or dramatizing the situation.

Reading of verbal problems calls for certain reading skills and these skills should be regarded as a "composite of specific skills rather than as a generalized ability." 3

As an approach to improving this ability much stress is given to the meanings of terms within problems, both general and mathematical. Unless the vocabulary of the problem is familiar to the pupil this unfamiliarity will interfere in the process of solution. In order to determine whether improvement in specific mathematical vocabulary would lead to an improvement in the solution of problems, Johnson 4 undertook an experiment in 1941 with 898 pupils in twenty-eight seventh-grade classes. Practice exercises were designed for use in the experimental classes to develop a meaningful understanding of vocabulary beyond that which was provided by the textbook itself. The control classes relied entirely upon the textbook itself.


and regular class discussion periods for their learnings of these mathematical terms. At the beginning of the experiment both groups were equivalent with respect to all of the abilities measured by the Analytical Scales of Attainment. Learnings measured by the vocabulary and problem tests prepared by the writer after fourteen weeks of practice indicated that statistically significant differences were in favor of the experimental group. Emphasis placed on adequate building of an arithmetic vocabulary should probably be one of the considerations in improving work in problem solving.

One investigation concludes that the best material for use in teaching problem solving in arithmetic is to have problems selected by pupils themselves and which are from their own environment.\(^5\)

Problems involving familiar childlike situations were found to be more accurately solved by pupils of the fifth grade level, than problems outside of their own experiences.\(^6\)

In 1933, Grace Kramer\(^7\) conducted an experiment in an

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effort to investigate the effect of certain factors on children's success in problem solving. The factors considered were as follows:

1. Problem interest
2. Sentence form of the problem
3. Style of the problem
4. Vocabulary of the problem

As a result of her study she found that children preferred the interrogative sentence form, a simplified statement of the facts essential for solution and familiar vocabulary. She also noted that interesting problems failed to induce more successful arithmetical thinking. "This evidence also satisfies the general findings in that the forms causing greatest difficulty are declarative, with details, and in unfamiliar vocabulary."\(^8\)

In determining the "interest" factor involved in a learning activity, Herbert Bowman\(^9\) attempted to determine the relationship between expressed preference or interest for problems in arithmetic and achievement on these problems by pupils of junior high school level. After a thorough study of textbooks and standardized tests to obtain a representation of

\(^8\) Ibid., p. 63.

problem types more commonly found, he selected five types:

1. Problems based upon adult activities
2. Problems based upon children's activities
3. Problem settings in field of science
4. Problems stated in nature of a puzzle
5. Problems of pure computation only

Two test forms were administered, each consisting of twenty-five problems. At the end of each paper containing the five types of problem situations, the pupil indicated the problem he liked best. All the problems on one page required like operations for solution and as nearly as possible used the same digits. In this way preference was based upon the problem situation only and not affected by the difficulty of the problem. The tests were administered to 564 pupils of grades seven, eight and nine.

The most significant result was that the pupils of high ability performed equally as well on all types of problems and indicated no decided preference for any particular type. There was a tendency for the pupil to select as his preference that problem which he felt more capable of solving successfully. Bowman concludes that: "We seemed to be justified in inferring that the expectation of greater success leads an individual to prefer one problem over others, and that belief in success causes preference rather than that preference is a cause of
successful performance."^{10/}

Of the selected factors briefly discussed here it may be added that each single factor can be broken down into separate elements which contribute to the main disability. These elements in themselves need to be located for they represent areas of particular weaknesses which must be strengthened. Several such elements are suggested in a study by Hansen.^{11/} Those which are related to successful achievement in problem solving involve an understanding of arithmetical processes, clear number concepts and skill in mathematical computation and reasoning.

**Literature and studies on the improvement of problem solving.**

A practical type of investigation was that made by Clifford Woody^{12/} who attempted to diagnose the difficulties involved in the solution of problems. This study illustrates a method which might be employed in trying to ascertain the various causes of a child's inability to solve verbal problems in arithmetic. Certain types of general information about the

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^{10/} Ibid., p. 275.


learner were obtained in relation to health status, socio-
logical and cultural background, school record, subject-
history, and attitude of the pupil. This information was an
aid in adopting instruction to the pupil's needs—not only in
the subject under consideration but in other subjects as well.
A battery of educational and mental tests were administered
and the errors found in responses were classified. Pupils'
methods of work and study were ascertained through observation
of behavior in attempts at solving problems. Diagnosing a
pupil's difficulties in this manner would provide the necessary
information needed to begin specific remedial instruction in
specific areas.

Brueckner states that:

The diagnostic study of pupil difficulty in problem
solving should be regarded as an integral part of the
Teaching-learning situation and should be conducted on
the same basis as diagnosis in computation. This means
that whenever the need arises, the activities of the
pupil and his methods of work in solving real or verbal
problems should be scrutinized carefully to discover
the nature of the difficulties present. The methods of
diagnosis are similar to those used for analyzing the
nature of deficiencies in computation, including testing,
observation, analysis of oral and written responses, and
interviews. 13/

In order to determine why children make the mistakes
that they do and in order to answer the question, "Why, after

13/ Leo J. Brueckner and Foster E. Grossnickle,
How to Make Arithmetic Meaningful. (Philadelphia: The John C.
years of training, presumably intended to establish correct habits in the use of fundamental processes and number facts, do many pupils persist in error and improper habits of reasoning?" Chase undertook a diagnosis of problem difficulties. This investigation aimed at diagnosing and treating the difficulties of seventeen cases selected from fifty-four students doing unsatisfactory work in arithmetic, yet having normal or above normal intelligence. For each of the seventeen cases a detailed history was prepared by the homeroom teacher and then a battery of tests in fundamentals and reasoning was given to ascertain specific causes of difficulty. These tests were analyzed and persistent errors were isolated and incorporated into verbal problems which were used later in an oral examination. This examination was conducted to determine why the pupil made these errors and to find out what was wrong in his reasoning by close observation of his working habits. An analysis of reasoning difficulties was made, a diagnosis projected and remedial treatment selected and planned for each individual case.

After a remedial period of about three months the final test results showed definite improvement, some cases having made over three years' gain in specific abilities in which

they were deficient. The causes of difficulty varied in each case but each showed improvement in response to individual remedial treatment.

Brueckner suggests that increased use be made of problematic situations which arise naturally in school activities. In this way number processes will be taught in concrete applications. Yet textbook problems of the traditional type seldom present problematic situations which are based upon real experiences. Actually it would seem that the problem is subordinated to arithmetical computations.

In a study of textbook problems in a recent arithmetic series, Dexter found that out of four volumes equaling 990 pages and 2416 problems, there was, "not one truly functional problem unit, and only seven that could be characterized as based upon real experiences." 

Oftentimes problems are grouped under processes or topics, or stress particular number combinations. These number combinations or processes occur in the textbook problems as disguised drill, deemed essential by the author.

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A recent analysis of two-step problems was conducted by Barone17/ to determine the frequency of various process combinations. He found that each textbook series presents these combinations in varying degrees. On the whole each series placed strongest emphasis on almost identical combinations, and in the majority of cases emphasis was placed on process combinations which cause the least percentage of errors.

If the purpose of these problems is to give added drill on combinations, it would seem more profitable to provide examples to be solved which embody these processes and omit problems of doubtful value.

After a survey of written problems in arithmetic textbooks in 1938 Marguerite DuBois18/ concludes that the growing tendency is to discard the traditional problem and replace it with dynamic problems in connection with social and business situations. This would be putting problem work on a functional basis and would aim at making arithmetic more meaningful to the child.

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An illustration of direct experience in putting arithmetic problem work on a functional basis was the method used by Irene Cummings.\(^{19}\) She used the informational unit of work with a group of high school girls. The problem chosen was that of selecting, hiring and equipping a two-room apartment. They discussed rents, budgets, salaries, leases, bank checks, furnishings and the like. The development of the unit was based upon the situation as it would occur in reality. No drill was given except that which was necessary for the execution of the unit. As a result the pupils were made aware of the manner in which arithmetic functions in their daily lives and problem work was put on a functional basis. Teaching was based on reality.

Instead of ten problems solved in one period, a class may require ten periods for the solution of one problem. Ordinary routine teaching methods must be enhanced by increased pupil-activity and purposeful problem situations. The manipulation of numbers as stressed in the traditional type of problems will assume a secondary place in significant problem units. Wilson\(^{20}\) states that there has been no essential

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change in the written problems of arithmetic textbooks and proposes that the isolated written problem be completely abandoned. Recent advances in arithmetic as outlined by Hartung\(^{21}\) show that gradually we are nearing a more functional program, and that a movement is on to teach for meaning and understanding of arithmetic. The progress is slow, but decidedly sure.

Methods of teaching arithmetic problems.

Various methods have been devised for improving pupil's procedures with verbal arithmetic problems. Many investigations have been undertaken to scientifically study and determine that method which is the most successful in the teaching of problems. These methods are called by various names but in general they may be summarized as follows:

1. The formal analysis or conventional method
2. The analogies method
3. The graphic or diagrammatic method
4. The individual method

That method which is most commonly used is generally referred to as the "conventional method" or the method of "formal analysis". According to this method the pupil is

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given a series of process steps to use as an aid toward the solution of problems. These steps vary in number, in the form of statement and in the order in which they are presented. Oftentimes they are put in the form of questions which the pupil is to ask himself and to answer for himself. A typical outline of these steps follows:

1. Read the problem.
2. What is given?
3. What is asked?
4. What process must I use?
5. Solve the problem.

An analysis was made by Paul Hanna to discover which procedure was the most frequently recommended in teaching pupils to solve problems. Out of ten fourth-grade textbooks examined, six proposed a method similar to the conventional analysis. Three of the ten did not suggest a definite formal analysis, but treated each type of problem independently and with no general technique.

Eight out of the ten textbooks examined for the seventh grade level used the conventional method or some variation of it. Only one text gave no general procedure. Nine courses of study were examined for the same purpose as the textbooks. Five of these gave no general method or technique of problem solving. Three gave the conventional analysis and one a slight variation of it.

Out of sixteen references in professional literature twelve suggested the step method as a technique of teaching
problem solving. The four remaining references used no general technique. The author concludes that, "Considering the methods of problem solving suggested in textbooks, courses of study and professional literature, it is quite clear that the conventional method is most widely recommended."\textsuperscript{22/}

To determine the relationship between ability to solve problems and the ability to make formal analysis, Washburne carried on an extensive study. An experimental group was given careful training in the various steps of formal analysis over a period of two weeks. During this time the control group simply solved problems with no special technique. Testing results at the end of two weeks showed that, "those who had not been taught formal analysis did as well as those who had been trained in such analysis -- in many cases, better."\textsuperscript{23/}

Washburne also found that, "there was little or no relation between ability to solve the problem and ability to take any of the other steps. The children analyzed a problem correctly and solved it incorrectly or solved it correctly and analyzed it incorrectly just as often, or almost as often, as

\textsuperscript{22/} Paul R. Hanna, A Study of the Relative Effectiveness of Three Methods of Problem Solving, Bureau of Publications, Teachers College, Columbia University, New York, 1929.

they both solved and analyzed it either correctly or incorrectly."

In other words the ability to make the formal analysis as taught in schools had practically no relation to ability to solve problems.

Following this study Washburne and Osburne investigated the results and relative merits of three methods of training children to solve problems. Groups involved were approximately equal in ability in problem solving, arithmetic fundamentals, intelligence and chronological age. Three methods of teaching were used.

One group was taught to solve problems without any special technique. The child was to generalize for himself as a result of doing many problems.

The second group was trained in analyzing problems with a specific technique such as the following:

1. Read the problem carefully
2. Determine what is to be found
3. Determine what elements in the problem will help find the answer
4. Decide what process to use
5. Estimate roughly the magnitude of the result
6. Solve the problem

The chief aim here was not the solving of the largest possible number of problems, but to provide thorough training

24/ Ibid.

in the technique of analyzing each problem before solving it.

The third group was to be trained to see analogies or similarities between written problems that were difficult and corresponding oral problems that were easy. By noting likenesses in problem situations they would generally apply the same methods of solution.

The general conclusions drawn from this experiment indicated that the children who used no special technique for solving problems, but simply solved many problems, surpassed those who had been taught definite techniques of attack. For the superior half of the children training in the seeing of analogies appeared slightly superior to training in formal analysis. The method of analysis appeared to be superior to analogy for the lower half. The most effective method was to give many problems without any special technique and then to help each child with any particular difficulty encountered.

As an initial step in detecting the sources of failure in problem solving, George Spache\(^{26}\) developed an arithmetic reasoning test which attempted to measure in five sections the pupils' abilities:

1. ability to recognize and understand the facts given.
2. ability to decide what facts are to be found in solving the problem.
3. ability to choose the appropriate arithmetical computations to be employed.
4. ability to estimate a probable answer.
5. ability to execute the solution.

The test was constructed to differentiate among pupils who showed poor ability in these areas. Through preliminary testing problems of appropriate difficulty were selected.

The test was given to a total of 158 pupils of grades five and six. The highest and lowest scoring 27 percent of the population was found and an item analysis conducted to determine the discriminatory value of the five sections.

Results showed that sections 1, 2, and 4 appeared to be the most effective in distinguishing between the two groups, while sections 3 and 5 were not as discriminatory. Mr. Spache concludes that, "If these data may be taken at face value, they would seem to imply that, in so far as success on this test is concerned, there is greater difference between high-and-low scoring pupils in their abilities to read and understand problems and to estimate the probable answers than there is in choice of the correct steps and the actual solution."²⁷/

A study of the effectiveness of three methods of problem solving was undertaken by Paul Hanna²⁸/ to compare experimentally the values and limitations of each. Careful drill was given to 1000 children from the fourth and seventh grades for six weeks on selected problems of two or more steps.

²⁷/ Ibid., p. 443.

Hanna used three different methods of problem solving and then attempted to measure the difference of the gains attending these three methods, along with measuring their effects with children of three levels of arithmetic ability.

Identical standardized forms were used in the initial and final testing, including the New Stone Reasoning Test in Arithmetic and the Stanford Achievement Test in Arithmetic Reasoning.

The first method used was what he called the "Dependencies Method", a procedure otherwise known as the "graphic" or "diagrammatic" method. This aims at a logical analysis of factors within the problem which in turn are dependent upon other factors and so on until the pupil has sifted out the essential facts. The following problem will illustrate this method of reasoning backward.

Jane had 7 ribbons. She bought 3 more and then gave 2 of them to her sister. How many ribbons did Jane have left?

The pupil must think what is asked for, as-

I am to find the number of ribbons Jane has left. To find how many ribbons she has left I must know the total number of ribbons she had and the number of ribbons she gave away. To find the total number of ribbons she had, I must know the number she had at first and the number she bought.
These facts may be represented graphically as follows:

\[
\begin{align*}
\text{total number} & \quad \text{number at first (7)} \\
\text{number left} & \quad \text{number she bought (3)} \\
& \quad \text{number given away (2)}
\end{align*}
\]

The necessary computations are then made after an analysis is completed in this manner and the data recorded.

The second method (conventional-formula) directed pupils to follow particular thought patterns of four steps similar to those already stated as the steps of formal analysis, and to record the facts necessary to complete the four steps in the formula.

The third method allowed children to use whatever procedure they desired to solve problems, thus constituting the method used by the control group in this experiment. This is commonly known as the "individual" method which is simply the name given to the procedure used by children when left to their own devices.

The results showed that the pupils using an individual method excelled in both speed and accuracy those using either the conventional-formula or the dependencies method. When compared with the individual or dependencies methods, the conventional-formula method of solving problems was found to give the least gain in ability.

As one author puts it, "Superior pupils apparently can devise efficient techniques of problem-solving, and they
should not be taught a single, set technique."

Somewhat conflicting results were obtained by Newcomb who claimed that pupil difficulties were due to faulty methods of attack and attempted to discover the best method. Each pupil in the experimental group was required to solve a number of problems over a period of six weeks on solution sheets which provided spaces for written analysis of the problems. The control group solved problems in the usual manner. His results showed that the experimental group had a superiority over the control group in both speed and accuracy.

In opposition to Newcomb's findings were those made by Monroe and Englehart, who carried out a controlled experiment at the fifth grade level, using twenty-six classes in thirteen schools. Out of almost 600 cases, 181 pairs of pupils were obtained matched on intelligence quotients and chronological ages. The control group followed the usual instructions for solving problems and the experimental group

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was asked to define terms, restate the problems, and so on making a complete and thorough analysis. Final test results revealed no significant differences in favor of either group, although it was noted that children of inferior intelligence profited most by the special instruction.

Monroe's study seems to uphold the findings of Clark and Vincent who state that "formal analysis should be considered a tool or ally to be used only when the solution is not otherwise possible." Clark and Vincent compared the results of the conventional and graphical methods of teaching and found that the latter method held an advantage over the former. It was also recognized that pupils found the correct answers to problems without formulating the process by which they arrived at the correct solutions. This may indicate the probability that a technique of analysis retards progress.

Concerning the method of formal analysis Spitzer claims that:

Although the steps in the formal-analysis method of problem solving are sound, many teachers have been disappointed by their experience with the procedure. Pupils fail to do the steps in order; they take so much time to produce evidence that they have used each step that they lose interest; and so much attention is focused on the first steps in the procedure that the later steps are often carelessly done. Perhaps the

difficulty experienced in teaching the formal-analysis method is due primarily to the mechanics of the procedure used in getting pupils to carry out the steps. On the other hand, it must be admitted that few adults when they solve a problem, consciously follow in sequence the steps in the formal-analysis method. Frequently, the adult uses a sort of intermingling of steps and has in mind no distinct and clearly recognizable progression. To ask children, therefore, to apply the steps in a fixed sequence in the solution of problems may not be in harmony with the best adult practice.\(^{33}\)

To simplify the transition of the mechanics of arithmetic to facility in the process of solving verbal problems, Washburne\(^ {34}\) experimented with children of the second, fourth and sixth grades. Children of each grade were divided into equivalent groups. The first group was to be taught a number process through the use of verbal problems, while the second group was to be taught the same process without relation to concrete situations. When the mechanics had been mastered in this second group concentration was then to be centered on problem work.

At the end of a six weeks drill period, a test was administered to both groups. This test included problem solving involving the newly learned process, and the mechanics of the process. Washburne concluded that there was no apparent


difference between the results of teaching by the two methods, for the children learned both the mechanics of the process and the problem solving equally well either way.

A comprehensive survey of investigations and experimental studies on the improvement of problem solving was undertaken by Johnson. On the basis of the studies reported he found that there was still room for doubt concerning the superiority of any one method over others, though he concludes that "systematic and persistent training in any reasonable procedure for attacking problems is bound to result in improvement."^35/

As a further aid to problem solving one professional textbook^36/ suggests that particular words or phrases within a problem be used as indicators of the fundamental operations to be used. This text further suggests that these words be taught to children as specific cues in the solution of problems.

The use of cues was studied by McEwen^37/ who endeavored to discover the effect of cue words in problem solving, by

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testing and interviewing 202 children at various grade levels. The written tests contained ten selected cues used to detect the extent to which children would respond to the cues in solving the problem. The interviews revealed just how children determined from the wording of the problems what process to use. McEwen's evidence shows that particularly in the lower grades cues are decided factors in influencing children's solutions, but that they are frequently misleading. As grade level advanced, it was noticed that the reliance upon cues was not as great. He also noted that some children arrived at their answers by a process of elimination, and that children low in problem solving achievement were more responsive to the verbal cues than were those of superior ability within the same grade.

In making an analysis of pupils' errors in problem solving one author found evidence of what seemed to be purely random manipulation of the numbers within a problem. The conditions of the problem were not grasped and no definite plan of reasoning formulated for solution.\(^{38}\) Obviously then many pupils solve problems by a trial-and-error procedure, relying upon cue words or the form of the numbers within the problem.

Summary.

Such is the evidence obtained by educators which pertains to pupils' difficulties in solving problems and to various methods of teaching children to solve problems. Some of the material presented is not in agreement; yet, basically each study has for its purpose the ultimate goal of improving the work in the field of the written problem. One writer suggests that problem situations be such that they are familiar and within the experience range of the child. Problems that present unfamiliar situations are usually hampered by the element of vocabulary. Preliminary training in the mathematical vocabulary of the problem is recommended by one author who found that an understanding of terms contributed to success in the solution. Another investigation maintains that problems should be selected by pupils themselves and be within their own environment. The use of informational units of work has recently been stressed as an illustration of putting arithmetic problem work on a functional basis.

In the attempts to improve problem solving educators advocate the use of various methods of attack as an aid toward the solution of problems. As a result many studies have been undertaken to compare experimentally the values of these various methods taught to children. Yet there is little evidence in favor of any one method. Information concerning the method of "formal analysis" has proven that it is often
ineffective; nevertheless it has been found that it is the method most frequently used. Several studies have been made to compare the merits of the analysis method with other methods, yet no attempt has been made to determine whether pupils actually make use of this analysis technique which they have learned. It would seem important therefore that such a study be undertaken and in a school system where the formal analysis method is taught. For this reason, the present study has tried to find out the use which children make of the formal analysis method after they learn how to use it.
CHAPTER III
CHAPTER III

PLAN OF PROCEDURE

Introduction

This investigation was primarily concerned with discovering whether children who were taught to formally analyze arithmetic problems actually used this method of analysis when solving problems. According to the formal analysis method the pupil follows a series of steps in reasoning to find the solution to a problem. The steps often vary in number and in the form of statement but usually appear in the following order:

1. Read the problem.
2. What is given?
3. What is asked?
4. What process must be used?
5. Solve the problem.

After analyzing a problem in this fashion it is expected that pupils will arrive at the correct solution.

The general plan in this investigation was to select children who had been trained in this analysis method and through testing and interviewing determine whether they used the step method or some other method of solving problems.
Administration of Tests

Description of the population.

Two heterogeneous groups of children from two sixth grades were used; the first group consisted of twenty-two pupils and the second group of sixteen pupils. The children in the first group had been given quite thorough training in analyzing problems. The sixteen pupils of the second group had been given an unusual amount of intensive drill in this method, including extra problem solving work. Actually more children in each group were tested but some of the results could not be used because of absences on one of the test days.

Information concerning the analysis test.

The test used in this study was an arithmetic reasoning test for grades four to six adapted from the revised edition by George Spache, Chappaquay, New York. Permission to use the test for this investigation was obtained by Robert L. Burch, Professor of Education, Boston University.

Spache had previously refined the test so that the problems were of appropriate difficulty. A copy of the test will be found in the appendix.

Description of the tests.

The arithmetic problem test consisted of thirteen problems involving the four fundamental processes. Each
problem in the test was to be solved according to five steps in the formal analysis plan as follows:

1. What does the problem tell you?
2. What must you find?
3. What must you do?
4. Guess which answer is closest to the right answer.
5. Do the problem.

The pupil indicated the answers to parts one through four by circling the correct response out of a choice of four responses. For step number five the pupil did the actual work of computing the problem. The most important factor considered in the results of the analysis test was whether the pupil selected the correct process to be used after analyzing the problem. The following sample problem will illustrate this factor.

Jim delivers newspapers 5 days a week. If he delivers 14 papers each day how many papers will he deliver in 5 days?

A. B.

14 14
x5 x5
70 papers

(process selected) (example computed)

Once the process has been decided upon, as in part A, the "problem" then becomes an "example", as in part B.

Results from the analysis test were to be compared with the results of a similar test given during the interview. Hereafter the formal analysis test will be referred to as the
"analysis test" to distinguish it from the "oral test" used in the interview. The oral test contained the original thirteen problems but not the steps one through four. A copy of the oral test is included in the appendix.

Method of administering the tests.

The analysis test was given in two parts to the twenty-two children of Group I on March 2 and 3, 1948. The examiner began the testing program on March 2 by explaining that the class was to answer some questions about five arithmetic problems. Three pages of the test were distributed and specific instructions given for indicating name, date, age, grade and school. A sample problem was provided on page one which was read silently by the pupils and orally by the examiner. This sample problem was then solved in the same manner as the remaining problems were to be solved, i.e. by selecting the correct response to each step of the analysis and computing the answer. The pupils were then given an opportunity to ask any questions which they might have had about the way the work was done for the different parts in the sample problem. They were told that no questions could be asked after work had started on the problems. It was suggested that they attempt to answer every question yet not spend too much time on any one part. When the pupils started solving the problems the examiner walked around the room to be sure they were following directions correctly. Test papers were collected as soon as
they were finished.

On the following day at the same hour the remaining eight problems of the analysis test were administered. Directions were repeated and the pupils told to do the work on these problems as they had done on the others the day before.

Two weeks later the twenty-two pupils were personally interviewed and given the "oral test" to discover how they would solve these same problems when not told to follow the analysis method. For this interview the problems were printed on four sheets and no steps were included. It was felt that the results from these interviews would directly reveal those factors which influenced the pupils' method of solving problems and also show whether they would apply that method of solution in which they had had specific training in the classroom.

For the sixteen pupils of group II, a different procedure was followed. The oral test, containing the thirteen problems without the analysis steps, was given as a group test to these sixteen pupils before they were interviewed. In other words, the "oral test" became a written test for these pupils and two weeks later it was administered again as an oral test. In both situations the pupils were not using a specified method of solving problems, but rather were to determine their own technique.
Because the second group had been given extra problem solving work in the formal analysis method it was expected that the reasoning methods of these pupils might be significantly different in comparison with the reasoning methods of the first group. Both testing results of group II were to be compared in order to discover the variability of pupil scores on the same test when they used their own method of solving problems.

Plan of the interview.

When the thirty-eight children of groups I and II were interviewed they were given the thirteen problems of the original test with a space provided after each problem for the necessary written work required in solution. (See copy in appendix.) The length of each interview was usually from thirty to forty-five minutes.

Previous to interviewing these two groups of pupils, three representative pupils were chosen to be interviewed in order to determine the length of the interview, the time to allow for work on the problems, and the manner of questioning. These three pupils were selected by their classroom teacher and judged by her to be superior, average, and below average.

These interviews were conducted by the writer in a very small quiet room which contained a desk, a table, and three chairs. The pupil to be interviewed came to this room and worked at the table where he was supplied with several
sharpened pencils and the necessary test materials. The interviewer tried to establish rapport between the examiner and the subject by telling the pupil that he could help the examiner find out how children solved problems by solving these problems out loud. The examiner said further: "Today you are to think out loud as you solve each problem. Read the problem out loud and then tell me what you are thinking as you go about finding the answer. Tell me just what you are going to do and your reasons for doing it. As you tell me these things I will write them down on these little cards. The first problem is a sample problem for you to do so that you will understand how it is to be done."

As the pupil worked his responses were tabulated for each problem. If at any time the pupil hesitated and could not explain his method of reasoning the examiner asked such questions as:

1. How did you know that you had to add?  
   (subtract, multiply, divide)

2. Was there anything in the problem that made you decide on the process?

3. What makes you think that is the correct way to solve the problem?

If the subject paused at all during his work or indicated that he was thinking yet not reporting his thinking he was reminded to think out loud and tell everything that came into his mind about the problem. After the last problem had been completed the examiner then asked the following questions
and recorded the words of the pupil.

1. As you did this work were you thinking of the steps you have been taught to use in solving problems?

2. Do you use the steps at all in doing problem work?

3. Is there any particular step that you use often?

4. What do you find most helpful to you in solving problems?

At no time during the interview did the observer show any signs of approval or disapproval of any remarks offered by the pupil. These questions were asked to determine whether the pupil used the method of formal analysis and to discover just how he went about solving problems.

Justification of the interview.

Results on the group test permitted conclusions to be drawn concerning pupils' success on selecting the correct process to be used in solving problems when following the formal analysis method. Nevertheless, the interviews directly revealed children's procedures used successfully in choosing the process when not required to formally analyze problems. The interviews made possible a closer observation and diagnosis of children's mental processes as they operated in solving problems.

The information recorded during the interview may be of some assistance in locating causes of disability encountered in problem solving and may indicate the advantages or disadvantages of the method of formal analysis.
CHAPTER IV

INTERPRETATION OF RESULTS

Introduction

It is evident from the preceding chapter that the pupils in both groups had not been influenced by the writer in their selection of their methods of reasoning used in solving the problems of the oral test. Nevertheless, it was reasonable to suppose that the pupils interviewed in group I would solve these problems, which they had seen before, by using the formal analysis method. The reasons for this assumption were:

1. That the pupils had already used the method of formal analysis in solving the same problems.

2. That the pupils had been trained to solve problems by this method in the fourth, fifth, and sixth grades.

3. That the pupils were aware of the fact that the interviewer was well acquainted with the method of problem solving that they should have used and they might pretend to use formal analysis to please the interviewer.

The second and third assumption also applied to the pupils of group II, because they had been given extra drill in the method of analysis and they too might have tried to satisfy the interviewer.
Comparisons of test results for Group I.

Table I shows the distribution of process scores for group I on the formal analysis test and the distribution and improvement of process scores on the oral test in which pupils used their own methods of reasoning. The highest possible score was thirteen.

**TABLE I**

COMPARISONS OF PROCESS SUCCESSES FOR THE ANALYSIS TEST AND ORAL TEST FOR PUPILS IN GROUP I

<table>
<thead>
<tr>
<th>Test Scores</th>
<th>Analysis Test</th>
<th>Oral Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>22</strong></td>
<td><strong>22</strong></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td><strong>9.36</strong></td>
<td><strong>11.45</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td><strong>2.7</strong></td>
<td><strong>1.3</strong></td>
</tr>
<tr>
<td><strong>SE</strong>mean</td>
<td><strong>.6</strong></td>
<td><strong>.3</strong></td>
</tr>
<tr>
<td><strong>SE</strong>diff</td>
<td></td>
<td><strong>.7</strong></td>
</tr>
<tr>
<td><strong>CR</strong></td>
<td></td>
<td><strong>3</strong></td>
</tr>
</tbody>
</table>
The analysis test for group I showed how successful pupils were in selecting the correct processes for solution of the problems after reasoning through the steps in the formal analysis method. The results of the oral test showed how successful they were in selecting the correct processes when reasoning out loud by their own methods. The mean scores for the two tests have been presented in Table I.

One may assume that because the analysis test was given first and the oral test was administered two weeks later, the advantage in favor of the oral test might have been due to practice and to the use of the oral procedure. However, group II had the problems without formal analysis as a written test and later as an oral test. In the written test the mean for the boys was 12.0 and in the later oral test their mean was 12.7. For the girls the written test mean was 11.9 and the oral test mean was 11.8. (See Tables A-1 and A-2 in the appendix). The number of cases is small, so these results may not be typical, but the improvement for individual pupils was never more than one score unit, and the scores were frequently a score unit smaller. Thus, there is evidence that when pupils determine their own technique of solution they do as well in the earlier written test as in the later oral test and practice effect is relatively unimportant. This was not true when the earlier test was the analysis test. In fact, the critical ratio reported in Table I reveals that a real difference at
better than the one percent level of confidence was found in favor of the oral testing. The table of probability for small samples was used in obtaining the level of statistical significance.

The following method was employed in determining the critical ratio for the two tests when the tests are symbolized by A (analysis test) and O (oral test).

\[
CR = \frac{M_O - M_A}{SE_{diff}}
\]

As previously stated, after the "process" had been chosen the problem then became an "example". Although this study does not stress accuracy on computations it was felt that the number of correct answers in both tests would be an interesting factor to record. An analysis according to intelligence and sex was made. Intelligence quotients were secured from the homeroom teacher. Tables II and III show the results of these two tests by comparing the number of times the correct processes had been selected by the boys and by the girls, and the number of times the correct answers had been derived for each test. Pupils have been arranged according to intelligence quotients in descending order.

In determining the difference in the number of processes correct for the analysis and oral test for the boys, a critical
TABLE II

RESULTS OF ANALYSIS TEST AND ORAL TEST FOR BOYS IN GROUP I

<table>
<thead>
<tr>
<th>Pupil Number</th>
<th>I.Q.</th>
<th>Number of processes correct</th>
<th>Number of answers correct</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analysis Test</td>
<td>Oral Test</td>
</tr>
<tr>
<td>1</td>
<td>115</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>111</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>109</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>105</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>97</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>88</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>87</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>73</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>68</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>92</td>
<td>120</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td>9.2</td>
<td>12.0</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td>3.1</td>
<td>1</td>
</tr>
<tr>
<td>SE mean</td>
<td></td>
<td>1.0</td>
<td>.3</td>
</tr>
<tr>
<td>SE diff</td>
<td></td>
<td>1.</td>
<td>1.5</td>
</tr>
<tr>
<td>CR</td>
<td></td>
<td>2.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>
ratio of 2.7 was found. This ratio indicates a real difference at the two percent level of significance in favor of the oral test. The difference in the number of answers correct for the analysis and oral test for the boys shows a critical ratio of 1.8 which is significant at about the ten percent level of confidence in favor of the oral test. This level of significance is relatively low but is in the same direction as the other differences. The results of Table II indicate that the scores of every boy were the same or better on the oral test -- not only in the selection of the correct process for solution but in the computation of answers. One boy selected thirteen correct processes for the oral test. Another boy computed eleven correct answers for the analysis test and eleven correct answers for the oral test. These boys had the only scores which did not change.
There was a tendency for boys with intelligence quotients below 100 to improve their scores on the oral test almost as much as boys with intelligence quotients above 100.

Table III compares the results of the analysis test and oral test for the girls. It will be noted that the girls were slightly higher on the analysis test, although the boys were higher on the oral test. Unlike the oral scores for the boys, which remained the same for two pupils and improved for all other pupils, the oral scores for the girls lowered in two cases, remained the same in three cases and improved in all other cases. In relation to intelligence, girls whose intelligence quotients were 100 or above improved equally as well on the oral test as those whose intelligence quotients were below 100.

In determining the difference in the number of processes correct for the girls on the analysis and oral test a critical ratio of two was attained in favor of the oral test at almost the five percent level of significance. The difference in the number of answers correct for the two tests shows a critical ratio of 1.9 which is significant at almost the ten percent level of confidence.
TABLE III

RESULTS OF ANALYSIS TEST AND ORAL TEST
FOR GIRLS IN GROUP I

<table>
<thead>
<tr>
<th>Pupil Number</th>
<th>I.Q.</th>
<th>Number of processes correct</th>
<th>Number of answers correct</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analysis Test</td>
<td>Oral Test</td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>115</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>106</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>105</td>
<td>11</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>99</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>98</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>88</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>9</td>
<td>88</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>79</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>78</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>114</strong></td>
<td><strong>132</strong></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td><strong>9.5</strong></td>
<td><strong>11.0</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
<td></td>
<td><strong>2.4</strong></td>
<td><strong>1.4</strong></td>
</tr>
<tr>
<td><strong>SE</strong> mean</td>
<td></td>
<td><strong>.7</strong></td>
<td><strong>.4</strong></td>
</tr>
<tr>
<td><strong>SE diff</strong></td>
<td></td>
<td><strong>.8</strong></td>
<td></td>
</tr>
<tr>
<td><strong>CR</strong></td>
<td></td>
<td><strong>2.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Tables II and III indicate that although pupils often selected the correct process for solution in both tests the majority of pupils did not have a corresponding number of correct answers. This would substantiate the fact that in a study of problem solving it is important to note the processes selected by children in solving problems. Oftentimes a teacher will determine a pupil's reasoning ability by his final answer to the example of the problem and not by his selection of the correct process.

Although the number of cases is small, the results of the analysis and oral test suggest that when pupils are left to their own devices in solving problems they generally do as well, if not better, than when they are following the analysis method.

Problem solving procedures of pupils in Group I.

In order to determine just how these pupils did solve the problems in the oral test their methods were recorded by the interviewer for each problem in the test. Pupils' methods of solving the oral problems indicate to a certain extent how they decided upon the processes necessary to compute the answer.

The following tabulation points out the procedures of boys and girls in solving addition, subtraction, multiplication, and division problems. The numbers recorded under boys and
girls total more than twenty-two because some children used more than one kind of attack on the problems.

Addition problems.

No.3  Betty had $3\frac{1}{2}$ apples. Ellen gave her $2\frac{1}{2}$ more. Do you know how many apples Betty had then?

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

---depended on cue words, "how many", and "more" to signify addition.

1 ---figured the answer would be bigger and that meant addition.

1 ---said she had to find what Betty had "altogether" which meant addition.

1 ---felt that multiplication would give too big an answer and that addition was better.

No.5  Jake's father has a farm in Westchester County. Last week he sold five loads of hay. The loads weighed 2.5 tons, 1.9 tons, 2.4 tons, 2.8 tons, and 1.7 tons. How many tons of hay did he sell?

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

---knew this was an addition problem because there were so many numbers no other process was possible.

3 ---said the words "how many" meant addition.

1 ---selected the word "and" in the problem to be a cue word because and meant addition.

2 1 ---reasoned that you had to find out what was sold "altogether" and though the problem didn't actually say that, nevertheless that would mean addition.
Subtraction problems.

No. 1  Jack has a stick $12\frac{1}{2}$ feet long. If he uses a piece $4\frac{1}{2}$ feet long for a fishing pole, how much will he have left?

Boys  Girls

8  10  ----used cue word "left" and "how much will he have left?" to find the process.

1  ----said, "If he uses some and has some left over he will have a remainder and remainder is a cue word for subtraction."

1  ----said, "You've taking part of the stick off so your answer will be smaller. You always get a smaller answer by subtracting."

1  ----said, "He's taking away a piece and take away is subtraction."

1  ----didn't know why she used subtraction but knew it had to be the subtraction process.

No. 6  In New York the longest day of the year from sunrise to sunset is 15.13 hours. The shortest day is 9.22 hours of daylight. How many more hours of daylight are there on the longest day of the year?

Boys  Girls

8  11  ----depended on cue words, "how many more" to mean subtraction.

2  ----said you were finding the difference between the two days and difference meant subtraction.

1  1  ----didn't know why they selected the subtraction process but felt it was the correct one.

1  ----reasoned the answer would be a small one and addition or multiplication would make it larger. Therefore subtraction seemed the "only sensible thing to do."
null
boys  girls
1 ----depended on the word "more" to mean addition.

**Multiplication problems.**

---

**No.2** Last year Jane's father bought 4000 gallons of gas each time he went to the gas station. He went to buy gas for his trucks 20 times. How many gallons of gas did he buy last year?

boys  girls
5  4 ----selected the word "times" as the cue word for multiplication, or "20 times" to mean they should multiply by 20.
2  3 ----selected the cue words "how many" to mean multiplication.
1  3 ----decided it "told about one and asked about many" which meant multiplication.
2 ----knew they had to multiply but didn't know why.
1 ----decided the answer would have to be large and multiplication was the only process to use.
1 ----eliminated subtraction and division because the answer would be large but felt that multiplication would be better than addition because it was much shorter.
1 ----grasped at the word "year" which seemed to convey the idea that she should divide twelve into 4000.
No. 7  Miss Lake told the fourth grade class that last year each child paid $9.15 for their lunches during the year. This year there are 250 children in the grades. How much will all the children pay for their lunches during this year, if they spend as much as they did last year?

Boys  Girls
10  12 ——reasoned the problem to be multiplication.

2  2 ——felt the answer was going to be large which would eliminate the subtraction and division processes.

1 ——said, "You can only do two things to make your answer more, addition or multiplication. In this problem you can't add money and children so you must multiply."

1 ——said, "You can't add this because one number has a dollar sign and the other number hasn't."

1  2 ——felt you could get your answer by multiplication quicker than by adding $9.15 250 times, though it could be done by addition.

1  1 ——said it told about one price and asked about many which indicated the multiplication process.

2  2 ——selected multiplication as the process but actually guessed and couldn't explain why they had chosen the process.

No. 9  Jane's mother had \( \frac{1}{4} \) of a pie left from dinner. She gave Jane and Nancy each \( \frac{1}{2} \) of what was left. What fraction of the whole pie did each girl get?

Boys  Girls
5  1 ——thought that the word "of" in the problem meant multiplication.
boys  girls

---got the answer mentally by dividing \( \frac{1}{2} \) in half. When they tried the division on paper they decided it was the wrong process. They tried other processes until arriving at the correct one.

1 --- said, "Take \( \frac{1}{2} \) from \( \frac{1}{2} \) because she took half of it away."

2 --- decided "left" was a cue word for subtraction.

1 --- drew a picture of the pie and marked off the sections. They found the correct answer in this way but could not do the actual computation.

1 --- decided the process was division.

2 --- couldn't solve the problem.

1 --- thought it was multiplication but then felt that multiplication would make the answer "more."

1 --- skipped the problem.

1 --- knew the answer but could not compute it though she tried all the processes.

1 --- thought it might be addition because "gave" sometimes meant addition.

No. 12   Douglas saw a map in his geography book on which 1 inch represented 600 miles. How many miles are represented on this map by a line \( 14\frac{1}{2} \) inches long?

boys  girls

4      5 --- reasoned "it told about one and asked about many," which meant multiplication.

2      3 --- thought the words "how many" were cue words for multiplication.
2 boys | girls
---|---
said the word "by" was a cue word for multiplication.

1 ----used subtraction because $14 \frac{1}{2}$ was "smaller" than 600.

1 ----reasoned that, "The answer is going to be big and the only thing to make it big would be multiplication.

1 ----used addition.

1 ----couldn't solve the problem.

2 ----knew it was multiplication but couldn't tell why.

10 9 ----selected the correct process.

Division problems.

No. 4 Clara's family wants to buy a second-hand washing machine. They plan to pay for it in 24 weeks. How much will they pay each week, if the machine costs $39.98?

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
s----selected the cue word "each" to mean division.

3 said they were paying a "part" of all that money each time and to get a part of anything it was necessary to divide.

2 3 ----knew it was division but didn't know why.

1 ----felt that any other process would make the answer too big and it wouldn't seem sensible.

1 ----decided that Clara's family was dividing up their money therefore she should divide also.

1 ----figured that if he used multiplication the answer would be bigger and they would be losing money instead.
boys    girls
1      ---decided you could get the answer by addition
       if you already knew how much they paid in
       one week. (He meant to add the amount 24
       times and see if it totaled $39.98. He
       realized that was not what was asked in the
       problem.)

1      1 ---tried multiplication first and then sub-
       traction but felt their answer was still
       too big to be sensible. They finally tried
       division.

1      2 ---guessed it was division.

1      1 ---couldn't solve the problem.

No.8    Mr. Brown owns 1.89 acres of land.
       This land is marked off into 9 equal lots.
       How much of an acre does each lot contain?

boys    girls
3      3 ---felt it was division because Mr. Brown was
       dividing up his land and therefore they had
       to divide also.

2      ---noticed the similarity of this problem to
       problem No. 4 as requiring the same process.

4      4 ---relied on the words "each" and "equal" to
       signify division.

1      1 ---thought the word "into" in the problem could
       mean division also.

1      1 ---said, "It tells about all the land and asks
       about part of it. When you find a part of
       something it's division."

1      1 3 ---used the division process but couldn't say
       why they had selected it.

2      2 ---felt the problem "told about many and asked
       about one" which meant division.
No. 10  It costs Hugh $942.50 to feed his cows and chickens for a year.  How much does he spend a month on the average for feed?

boys  girls
8  8 ----divided because the word "average" meant division and the word "month" told them to use twelve as the divisor because there are twelve months in a year.
3  3 ----used "month" as the cue word and used thirty as the divisor because there are thirty days in a month.
1  1 ----selected the correct process but seemed to guess at it.  They couldn't explain why they had solved it by division.
1  1 ----thought it might be division but decided that there was only one number, the dividend, and no divisor. Said, "There's only one number in this problem so you can't do anything with it."

No. 11  A plane was reported flying over an airport at noon. Forty-five minutes later it was 135 miles farther west. What was its average rate of flying per minute?

boys  girls
8  6 ----selected the correct process because the word "average" signified division.
Boys     Girls

2     4  ---admitted that finding an average required
addition too but in this situation there was
"nothing to add."

2  ---used sixty as the divisor because sixty
seconds equal a minute.

2  ---thought it was multiplication.

1  ---used division because he thought any other
process would give a bigger answer and
division would give the only "sensible"
answer.

---

No. 13  The Browns are planning a trip to
Mexico. Frank knows that his father
usually drives about 35 miles an
hour. How many hours will the family
be driving if they are 3748 miles
from Mexico?

---

Boys    Girls

2     4  ---selected the correct process but had difficulty
reasoning the problem.

3     2  ---decided the answer would be less and it would
be either division or subtraction. They felt
by using subtraction the answer would still
be too big.

1  ---said the word "from" meant subtraction but she
still thought she should divide.

2     2  ---used multiplication noticing the words "how many"
which to them made this problem similar to
No. 12.

1  ---used subtraction because the word "from" was a
cue word for subtraction.

4     1  ---just knew it was division but couldn't tell why.

3  ---reasoned that you could tell by division how
many 35's are in 3748 and that would be the
number of hours.
Pupil replies to questions asked.

The following is a summary of the replies of boys and girls in Group I recorded during the interview period which followed the oral test. When the interviewer asked the pupils about the steps of the formal analysis, not one of them reported that he had used this procedure to solve the problems in the oral test. The following are the responses of the girls:

1. "I don't always use the steps. I only think of the steps when I can't figure out the problems. Even though I use the steps sometimes my answer is still wrong."

2. "I don't remember the steps. I can tell what to do from the sense of the problem."

3. "I don't follow all the steps. I just look for what is asked."

4. "I never use the steps. I just do the problems in my own way."

5. "Sometimes I use the steps when I can't figure it out. Sometimes I can figure problems easier without using the rules."

6. "I just read the problem and I can tell more by the words what to do, than when I use the steps."

7. "I usually forget the steps. When I read a problem I can tell whether the answer is going to be more or less. Then I can tell if it will be addition and multiplication, or subtraction and division. Sometimes the little words help you to decide what to do."

8. "If the problems look hard I think of the rules, and look for little words. But if it isn't hard I just have to read it and common sense tells me what to do."

9. "I don't remember the steps. I just read it and do it the way I think it should be done."

10. "I don't use the rules. I watch for little words that will help me."
11. "I don't use the rules. It's easier to figure out if the teacher underlines the little words that will help me."

12. "I don't use the rules. Just by reading it you know what to do. The words help you."

The responses of the boys were as follows:

1. "I think of the steps sometimes, especially, 'What is asked?' and 'Is the answer reasonable?' I look for words to help me too."

2. "The steps make it easier, but sometimes I forget what they are. If I remember them I use them. I look for words too. They help a lot sometimes."

3. "I read the problem and look for hints. If there aren't any I'll try the rules. If the rules don't work I skip the problem."

4. "I don't follow the steps unless I'm told to. When you read it you can find out what to do."

5. "I know the rules but I don't think of them when I'm doing a problem because when you solve a problem you're really doing what the rules tell you to do even though you might not be thinking of them. If I get stuck I look for little words."

6. "If it's easy I don't use the steps. They help me if it's a hard problem. Usually I just use part of the rules, especially, 'What is asked?'"

7. "I seldom use steps. I usually go on my own judgment. If you use common sense you know what to do. If it's a hard one I try the steps or else I do what I think it is and leave it. I just look at the numbers or start looking for hints in the words."

8. "I don't use the rules unless the teacher tells us to go by them."

9. "I use the rules if I have to because sometimes they're in the book and we have to use them. You can tell what to do by the words and you don't need the rules."

10. "I just read it and think what process I should do. If it's more than three numbers I add. If it says 'left' I subtract. 'How much more', I multiply. Like that!"
Problem solving procedures of pupils in Group II.

The problem solving methods of the sixteen pupils in the second group were recorded in the same manner as the methods of the pupils in the first group. These sixteen children had been given extra problem solving work by their homeroom teacher in which the formal analysis method had been stressed. Because they had been so intensively drilled in the analysis method it was expected that they would solve the problems in the oral test by using the analytical plan.

The following tabulation points out the procedures of these boys and girls in solving addition, subtraction, multiplication, and division problems.

Addition problems.

No. 3 Betty had $3\frac{1}{2}$ apples. Ellen gave her $2\frac{1}{2}$ more. Do you know how many apples Betty had then?

<table>
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<th>boys</th>
<th>girls</th>
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<tbody>
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<td>4</td>
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</table>

said it was necessary to find what Betty had "in all" and that signified addition.

depended on cue words "how many" and "more" to mean addition.

decided "more" was a cue word that meant subtraction.

No. 5 Jake's father has a farm in Westchester County. Last week he sold five loads of hay. The loads weighed 2.5 tons, 1.9 tons, 2.4 tons, 2.8 tons, and 1.7 tons. How many tons of hay did he sell?
1. knew this was an addition problem because with so many numbers no other process was possible.

1. reasoned that you only need two numbers for multiplication, subtraction and division and he could tell what process to use for this problem "just by looking at it."

3. said the cue words "how many" meant addition.

3. knew they had to find what was sold "altogether" and that always meant addition.

**Subtraction problems.**

---

**No.1**  Jack has a stick $12\frac{1}{2}$ feet long. If he uses a piece $4\frac{1}{2}$ feet long for a fishing pole, how much will he have left?

---

9. said "left" was a cue word for subtraction and they could tell from that one word what process to use.

1. said, "When you take anything away it gives you less, and less is subtraction."

---

**No.6**  In New York the longest day of the year from sunrise to sunset is 15.13 hours. The shortest day is 9.22 hours of daylight. How many more hours of daylight are there on the longest day of the year?

---

6. depended on the words, "how many more" as cue words for subtraction because these three words were usually in subtraction problems.
boys  girls
2  ---reasoned that actually the problem was asking for the difference between the length of the two days and difference meant subtraction.

1  ---said it was a division problem because "you're told about many hours and asked about one day."

1  ---didn't know what made him decide to use subtraction but he felt it was the correct process to use.

Multiplication problems.

No.2 Last year Jane's father bought 4000 gallons of gas each time he went to the gas station. He went to buy gas for his trucks 20 times. How many gallons of gas did he buy last year?

boys  girls
6  6  ---decided it "told about one and asked about many" which meant multiplication.

2  1  ---said "how many" often meant multiplication.

2  1  ---selected the word "times" as the cue word for multiplication, or "20 times" to mean they should multiply by 20.

1  ---said the word "of" in the problem meant multiplication.

No.7 Miss Lake told the fourth grade class that last year each child paid $9.15 for their lunches during the year. This year there are 250 children in the grades. How much will all the children pay for their lunches during this year, if they spend as much as they did last year?
said it "told about one and asked about many" which indicated the multiplication process.

decided that because the problem said "how much will all the children pay" she knew she had to multiply because her answer was going to be big.

Jane's mother had \( \frac{1}{4} \) of a pie left from dinner. She gave Jane and Nancy each \( \frac{1}{2} \) of what was left. What fraction of the whole pie did each girl get?

felt that the word "of" in the problem meant multiplication.

decided "left" was a cue word for subtraction.

didn't know what process to use so drew a diagram of a pie and found the answer by sectioning off parts.

knew that \( \frac{1}{2} \) of \( \frac{1}{4} \) was \( \frac{1}{8} \) but couldn't figure it mathematically on paper.

decided that \( \frac{1}{3} \) was being taken away for each girl so it was a subtraction problem.

Douglas saw a map in his geography book on which 1 inch represented 600 miles. How many miles are represented on this map by a line 14\( \frac{1}{2} \) inches long?

agreed that the problem "told about one inch and asked about many inches" which indicated multiplication.

thought that the words "how many" meant multiplication.
### Division problems.

**No. 4** Clara's family wants to buy a second-hand washing machine. They plan to pay for it in 24 weeks. How much will they pay each week, if the machine costs $39.98?

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<th>boys</th>
<th>girls</th>
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</table>

---

decided that because the problem "told about many weeks and asked about one week" it was a division problem.

selected the word "each" to mean division.

---

**No. 8** Mr. Brown owns 1.89 acres of land. This land is marked off into 9 equal lots. How much of an acre does each lot contain?

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<th>boys</th>
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</table>

used the division process because the problem "told about many lots and asked about each lot, or one lot."

said, "Divide, because you're going to separate the lots so you have to separate the amount of land by dividing too."

thought that the word "into" in the problem meant division.

said, "Contain means how many times it will go into it, so I'd divide."

---

**No. 10** It costs Hugh $942.50 to feed his cows and chickens for a year. How much does he spend a month on the average for feed?

<table>
<thead>
<tr>
<th>boys</th>
<th>girls</th>
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<td>10</td>
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</table>

---
boys    girls
8       4      ----divided because the word "average meant
division.
2       5      ----selected the division process because the
problem "told about many months and asked about one month."

No.11   A plane was reported flying over an
airport at noon. Forty-five minutes
later it was 135 miles farther west.
What was its average rate of flying per
minute?

boys    girls
8       6      ----selected the correct process because "average"
meant division.
1       1      ----said that the word "per" was a cue word for
division, especially if it came near the end
of the problem.
1       1      ----used division because the problem "told about
many minutes and asked about one minute."

No.13   The Browns are planning a trip to
Mexico. Frank knows that his father
usually drives about 35 miles an hour.
How many hours will the family be
driving if they are 3748 miles from
Mexico?

boys    girls
2       3      ----decided to use multiplication because the
problem "told about one hour and asked about
many hours."
4       3      ----just knew it was division but couldn't tell
why.
boys  girls

1 thought "how many" was a cue word for multiplication.

1 said, "You could say '35 miles per hour' instead of 'an hour', and per means division."

Pupils replies to questions asked.

The following is a summary of the replies of boys and girls in Group II recorded during the interview period which followed the oral test. When the interviewer asked the pupils about the steps of the formal analysis six pupils reported that they sometimes used the steps for "hard" problems, yet not one of the sixteen pupils analyzed the problems in the oral test by this method.

The following are the responses of the girls:

1. "Sometimes I use the steps for hard problems. I use cue words more often. If I use the steps I just look for what is asked."

2. "I think mostly of cue words. After I read the problem I find the words that will help me decide what process to use. Then I look to see if my answer is reasonable."

3. "It's easier to use the steps in solving problems than it is to use cue words. But I only thought of what the problem asked for in these problems."

4. "I use the steps if the cue words don't help me in hard problems. I see what is given and what is asked."

5. "I usually use the steps for hard problems, or easy ones that are tricky. These problems weren't too hard though and I didn't need to use the steps."

6. "Sometimes I use the steps if I don't know what the problem is, but I didn't use them now. The cues work faster and give it to you right away. But I think you need both steps and cues to work problems."
7. "I have the steps in my notebook but I don't use them. Sometimes I can make the problem out by myself without even using cue words."

The following are the responses of the boys:

1. "I didn't use the steps for these problems. I would have used them if I had thought about them."

2. "I get mixed up when I follow the steps. The cues are better because they're quicker in telling you what to do."

3. "You can tell what to do in a problem by the words quicker than to think of all the steps."

4. "I think cue words help you more than the rules, but if you didn't have rules you might just pick out the numbers and do anything."

5. "I use cue words mostly, but sometimes I think of the steps. I don't remember all of them."

6. "The cues only tell you what process to use, but sometimes the rules help you to find what's asked and what's given. They remind you to label and check your answer too."

7. "If you know all the cue words you don't have to think how to solve it. A cue word tells you your process to use right away. Steps waste time because you have to read it two or three times."

8. "I look for cue words or helps mostly. If I use the steps I look for what is given and what is asked."

9. "I only follow the rules for hard problems, and use cues most of the time."
CHAPTER V
CHAPTER V

SUMMARY AND CONCLUSIONS

Introduction

The purpose of this study was to discover, through testing and interviewing, whether children who had been trained to formally analyze arithmetic problems used this step method or some other method of solving problems.

The test used in this investigation contained thirteen arithmetic problems which were to be solved according to the formal analysis method. This test was administered to a group of twenty-two sixth-grade children who had been trained to formally analyze problems. Two weeks later the pupils of this group were individually tested on the same problems without the analysis steps in order to discover whether they would use the analysis method which they had been taught to use or some other method of solving problems.

A second group of sixteen sixth-grade children were given the same problems without the analysis steps and later orally tested on these problems in order to compare the difference in scores in both tests when pupils used their own techniques of solution. These sixteen children had been given an intensive amount of drill in formal analysis and it seemed
logical to suppose that they would use this method in solving the problems of the oral test.

Summary of Findings

Although this study involved a small sample, nevertheless it was possible to investigate in detail the problem solving procedures of thirty-eight pupils and to deduce certain facts from the data secured during the oral testing period and the personal interviews.

It was particularly noted by the examiner that the pupils in this study failed to use the formal analysis method of reasoning which had been emphasized in the arithmetic program. It was also evident that the pupils in this study failed to follow definite techniques of attack in solving problems.

Comparative results from analysis test and oral test.

From the comparisons made of the results obtained from the two problem tests which were used in this investigation, (one test in which formal analysis was used and another test in which formal analysis was not used) it may be noted that:

1. The twenty-two pupils of group I in this study performed equally as well, if not better, when left to their own methods of reasoning, than they did when solving problems through an analytical approach.

2. There was a tendency for pupils with intelligence quotients below 100 to improve their scores on the oral test almost as well as pupils with intelligence quotients above 100.
3. The girls in group I did better on the analysis test than the boys in group I although the boys did better than the girls on the oral test.

**Pupils' methods of solving problems.**

The following observations seem worthy of note in summarizing the problem solving methods of the pupils in the two groups in this study.

1. The children in this study did not use the formal analysis method of solving problems.

2. The children in this study relied upon cue words within the problem as a means of choosing the process for solution.

3. Some children in this study selected irrelevant cues in determining the choice of operations.

4. The children in this study were influenced by the form of the problem in selecting the process for solution. (Many numbers mean addition, etc.)

5. Several children in this study tended to guess at the process or to arrive at the solution through a process of elimination.

6. Many children in this study were unable to report on their reasons for selecting certain processes.

7. The children in this study who had been given extra training in formal analysis showed no important differences in their problem solving techniques when compared with the children who had the regular classroom training in formal analysis.

**Pupils' reactions to questions concerning formal analysis.**

Children's responses during the interview period concerning the method of formal analysis indicate that:

1. Six out of the thirty-eight children in this study occasionally follow the steps of formal analysis in solving difficult problems.
2. Most children rely on cue words only in solving problems.

3. The following steps of the formal analysis were perhaps used most frequently.

   What is given?  What is asked?

4. Although children have been given thorough training in the method of formal analysis it is no indication that they will use the method in solving problems.

5. Children often forget the steps of formal analysis.

6. Children tend to follow the formal analysis method of problem solving only when told to do so.

Conclusions.

1. Training children to use a technique of analysis in solving problems is not a guarantee that they will use it.

2. An extensive program of practice in the formal analysis method of problem solving does not result in better methods of reasoning.

3. Children seem to prefer to use a simple, quick method of solving problems rather than an involved procedure.

4. Some pupils are able to solve problems without explaining the processes by which they arrived at the solution.

5. Children may do better work in solving problems when not retarded by a method of analysis.

6. Children tend to respond to cue words rather than to the essential facts of the problem.

Limitations of the study.

1. The number of pupils involved will not permit too broad an application of the generalizations which were drawn.
2. The oral testing technique does not get at every thought process involved in problem solving.

3. Practice effect from one test to the other could not be completely controlled.

4. Pupils involved were all at the sixth grade level.

Suggestions for further research.

Following are some suggestions for further research.

1. Use the procedures of this investigation with a larger population and at various grade levels.

2. Study the procedures of children shortly after they have been introduced to the method of formal analysis.

3. Investigate the possibility that one or two of the steps of formal analysis might be helpful.
BIBLIOGRAPHY
BIBLIOGRAPHY


McEwen, Noble Ralph, "The Effect of Selected Cues in Children's Solutions of Verbal Arithmetic Problems." Unpublished Doctor's dissertation, Graduate School of Arts and Sciences, Duke University Library, 1941.


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# TABLE A-2

RESULTS OF WRITTEN TEST AND ORAL TEST
FOR GIRLS IN GROUP II

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APPENDIX B
V.P.S.
ARITHMETIC REASONING TEST GRADES IV - VI
(Adapted from Revised Edition by George Spache)

Problem: Sample:
Tom told us that he took a bus last summer to go to camp. The driver told Tom that he drove 232 miles every day, 6 days a week. How far did he drive in a week?

What does the problem tell you? Put circle around a, b, c, or d.

What must you find? Put circle around a, b, c, or d.

---

Problem: 1. Jack has a stick 12.5 feet long. If he uses a piece 2 feet long for a fishing pole, how much will he have left?

What does the problem tell you? Put circle around a, b, c, or d.

What must you find? Put circle around a, b, c, or d.

---

Date_______ Pupil's name________ Grade_______
Age_______ Birth date________ Teacher_______
School_______ (Name)_______ (City)_______ (State)_______ Examiner_______

What must you do? Guess which answer is closest to the right answer? Put circle around a, b, c, or d.

What must you do? Put circle around a, b, c, or d.

What must you do? Put circle around a, b, c, or d.

Now do the problem. Put your work below. Put a circle around the answer.

---

What must you do? Add 232 and 6

What must you do? Multiply 232 by 6

What must you do? Divide 232 by 6

What must you do? Divide 6 by 232

---

232

x 6

1392
### Problem 2

Last year Jane's father bought 4000 gallons of gas each time he went to the gas station. He went to buy gas for his trucks 20 times. How many gallons of gas did he buy last year?

What does the problem tell you? Put circle around a, b, c, or d.

- a. How much gas Jane's father bought each time
- b. How much gas Jane's father sold
- c. How much gas Jane's father used in a year
- d. How much gas Jane's father bought each time he went to the gas station

What must you find? Put circle around a, b, c, or d.

- a. The number of gallons he bought each time
- b. The number of times he bought gas last year
- c. The number of gallons of gas bought last year
- d. The number of gallons of gas Jane's father bought each time he went to the gas station

What must you do? Put circle around a, b, c, or d.

- a. Subtract 20 from 4000
- b. Divide 4000 by 20
- c. Multiply 4000 by 20
- d. Add 4000 and 20

Guess which answer is closest to the right answer? Put a circle around a, b, c, or d.

- a. About 3900
- b. About 8000
- c. About 80,000
- d. About 4020

Now do the problem. Put your work below. Put a circle around the answer.

---

### Problem 3

Betty had \( \frac{3}{2} \) apples. Ellen gave her \( \frac{2}{3} \) more. Do you know how many apples Betty had then?

What does the problem tell you? Put circle around a, b, c, or d.

- a. How many apples Betty had after Ellen gave her some
- b. How many apples Betty had before Ellen gave her some
- c. How many apples Betty ate
- d. How many apples Ellen had

What must you find? Put circle around a, b, c, or d.

- a. The number of apples Betty had before Ellen gave her some
- b. The number of apples Betty had after Ellen gave her some
- c. The number of apples Ellen gave to Betty
- d. The number of apples Betty and Ellen had

What must you do? Put circle around a, b, c, or d.

- a. Add 3 and 2 and \( \frac{1}{2} \)
- b. Divide \( \frac{3}{2} \) by \( \frac{2}{3} \)
- c. Add \( \frac{3}{2} \) and \( \frac{2}{3} \)
- d. Subtract \( \frac{2}{3} \) from \( \frac{3}{2} \)

Guess which answer is closest to the right answer? Put a circle around a, b, c, or d.

- a. About 5
- b. About 1
- c. About 10
- d. About 3

Now do the problem. Put your work below. Put a circle around the answer.
4. Clara's family wants to buy a second-hand washing machine. They plan to pay for it in 24 weeks. How much will they pay each week, if the machine costs $39.98?

What does the problem tell you? Put a circle around a, b, c, or d
a. How much they will pay each week
b. How much the washing machine costs
c. How much profit the dealer made
d. How many months they will have to pay

What must you find? Put circle around a, b, c, or d
a. The total cost of the washing machine
b. The charge for paying for it by the week
c. The number of payments
d. The amount paid each week

What must you do? Put circle around a, b, c, or d
a. Add $39.98 and 24
b. Multiply $39.98 by 24
$c. Divide $39.98 by 24$
d. Subtract 24 from $39.98

What must you do? Guess which answer is closest to the right answer? Put a circle around a, b, c, or d
a. About $1.50
b. About $15
c. About $5
d. About 60

Now do the problem. Put your work below. Put a circle around the answer

5. Jake's father has a farm in Westchester County. Last week he sold five loads of hay. The loads weighed 2.5 tons, 1.9 tons, 2.4 tons, 2.8 tons, and 1.7 tons. How many tons of hay did he sell?

What does the problem tell you? Put a circle around a, b, c, or d
a. How many tons of hay he sold
b. How many tons of hay in each load
b. How much hay he sold
b. How much hay he raised

What must you find? Put circle around a, b, c, or d
a. The number of tons of hay he sold
b. The number of loads in each load
c. The selling price of the hay
d. The number of tons in each load

What must you do? Put circle around a, b, c, or d
a. Subtract 1.7 from 2.8
c. Subtract 1.7 from 2.8
b. Add 2.5, 1.9, 2.4, 2.8, and 1.7
c. Add 2.5, 1.9, 2.4, 2.8, and 1.7, then multiply by $4.00
d. Add them all and divide by 5 to find the average

Guess which answer is closest to the right answer? Put a circle around a, b, c, or d
a. About 1
b. About 12
$c. About 28.00$
d. About 3

Now do the problem. Put your work below. Put a circle around the answer

Date Name Grade School
### Problem 6
In New York, the longest day of the year from sunrise to sunset is 15.13 hours. The shortest day is 9.22 hours of daylight. How many more hours of daylight are there on the longest day of the year?

<table>
<thead>
<tr>
<th>What does the problem tell you?</th>
<th>Put a circle around a, b, c, or d</th>
<th>What must you find?</th>
<th>Put circle around a, b, c, or d</th>
<th>What must you do?</th>
<th>Put circle around a, b, c, or d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. how many more hours of daylight on the longest day</td>
<td>c. the hours of daylight on the longest day and shortest day</td>
<td>a. add 15.13 and 9.22</td>
<td>a. about 25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. how many hours of daylight on the longest and shortest days</td>
<td>b. the number of hours more of daylight on the longest day</td>
<td>b. subtract 15.13 from 24</td>
<td>b. about 135</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. how many hours of daylight on any day</td>
<td>c. the hours of daylight on the shortest day</td>
<td>c. multiply 15.13 by 9.22</td>
<td>c. about 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. how many hours in the longest day</td>
<td>d. the number of hours of daylight in a day</td>
<td>d. subtract 9.22 from 15.13</td>
<td>d. about 9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Problem 7
Mrs. Lake told the fourth grade class that last year each child paid $9.15 for their lunches during the year. This year there are 250 children in the grades. How much will all the children pay for their lunches during this year, if they spend as much as they did last year?

<table>
<thead>
<tr>
<th>What does the problem tell you?</th>
<th>Put a circle around a, b, c, or d</th>
<th>What must you find?</th>
<th>Put circle around a, b, c, or d</th>
<th>What must you do?</th>
<th>Put circle around a, b, c, or d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. how much the lunches will cost for all the children this year</td>
<td>a. the number of children buying lunches</td>
<td>a. divide 250 by $9.15</td>
<td>a. about 30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. how much the lunches cost last year</td>
<td>b. the cost of the lunches last year</td>
<td>b. multiply 250 by $9.15</td>
<td>b. about 250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. how many children ate lunch at school</td>
<td>c. the cost of the lunches for 250 children</td>
<td>c. subtract $9.15 from 250</td>
<td>c. about $2,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. how much each lunch costs</td>
<td>d. the cost of one lunch</td>
<td>d. add 250 and $9.15</td>
<td>d. about 250</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Date: ______________________ Name: ______________________ Grade: ______________________ School: ______________________
8. Mr. Brown owns 1.89 acres of land. This land is marked off into 9 equal lots. How much of an acre does each lot contain?

<table>
<thead>
<tr>
<th>Problem</th>
<th>What does the problem tell you?</th>
<th>Put a circle around a, b, c, or d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>how much land in each lot</td>
<td>a. the size of each lot</td>
</tr>
<tr>
<td>b.</td>
<td>how much land Mr. Brown owns</td>
<td>b. the number of lots</td>
</tr>
<tr>
<td>c.</td>
<td>how much Mr. Brown sold his land for</td>
<td>c. the size of all the land</td>
</tr>
<tr>
<td>d.</td>
<td>how many acres in each lot</td>
<td>d. the number of lots in an acre</td>
</tr>
</tbody>
</table>

9. Jane's mother had 1/4 of a pie left from dinner. She gave Jane and Nancy each 1/2 or what was left. What fraction of the whole pie did each girl get?

<table>
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<tr>
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<th>Put a circle around a, b, c, or d</th>
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</thead>
<tbody>
<tr>
<td>a.</td>
<td>How much of the piece that was left each girl got</td>
<td>a. how much pie was left</td>
</tr>
<tr>
<td>b.</td>
<td>how much of a piece each girl got</td>
<td>b. how much pie each girl got</td>
</tr>
<tr>
<td>c.</td>
<td>how much of the pie each piece was</td>
<td>c. how much pie Mrs. Brown ate</td>
</tr>
<tr>
<td>d.</td>
<td>how many pieces the pie had</td>
<td>d. how much of the pie was eaten</td>
</tr>
</tbody>
</table>

**Guess which answer is closest to the right answer? Put a circle around a, b, c, or d.**

<table>
<thead>
<tr>
<th>What must you do?</th>
<th>Put a circle around a, b, c, or d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. subtract 1.89 from 9</td>
<td>a. about 8</td>
</tr>
<tr>
<td>b. divide 9 by 1.89</td>
<td>b. about 6</td>
</tr>
<tr>
<td>c. add 1.89 and 9</td>
<td>c. about 9</td>
</tr>
<tr>
<td>d. divide 1.89 by 9</td>
<td>d. less than 1</td>
</tr>
</tbody>
</table>

**Guess which answer is closest to the right answer? Put a circle around a, b, c, or d.**

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<th>What must you do?</th>
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</tr>
</thead>
<tbody>
<tr>
<td>a. multiply 1/2 by 1/4</td>
<td>a. about 1</td>
</tr>
<tr>
<td>b. add 1/4 and 1/2</td>
<td>b. about 4</td>
</tr>
<tr>
<td>c. subtract 1/4 from 1/2</td>
<td>c. about 1/16</td>
</tr>
<tr>
<td>d. divide 1/4 by 1/2</td>
<td>d. about 1/8</td>
</tr>
</tbody>
</table>
10. It costs Hugh $942.50 to feed his cows and chickens for a year. How much does he spend a month on the average for feed?

What must you do?
Put circle around a, b, c, or d

What must you find?
Put circle around
a. b, c, or d

What does the problem tell you? Put a circle around a, b, c, or d

a. how much the feed costs in a year
b. how much it cost to run his farm
c. how much he spent each month
d. how much it cost to raise cows and chickens

e. the cost of the food for the cows and chickens
f. the cost of the farm for one year
g. the average cost of food each month
h. the number of animals he had to feed

11. A plane was reported flying over an airport at noon. Forty-five minutes later it was 135 miles farther west. What was its average rate of flying per minute?

What must you do?
Put circle around a, b, c, or d

What must you find?
Put circle around
a. b, c, or d

What does the problem tell you? Put a circle around a, b, c, or d

a. how far the airplane flew in 45 minutes
b. how fast the airplane could go
c. how fast the airplane was going
d. how far the airplane went that day

e. how far the airplane went in 45 minutes
f. how fast the airplane went

Guess which answer is closest to the right answer? Put a circle around a, b, c, or d

Date 

Name 

School
### Problem 12.
Douglas saw a map in his geography book on which 1 inch represented 600 miles. How many miles are represented on this map by a line 1\(\frac{1}{4}\) inches long?

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>a. Divide 600 by 1</td>
<td>a. The number of miles represented by a line one inch long</td>
<td>a. How many miles a line 1(\frac{1}{4}) inches long represents</td>
</tr>
<tr>
<td>b. Divide 600 by 1(\frac{1}{4})</td>
<td>b. The number of miles represented by a 600 mile long line</td>
<td>b. How many miles long the line was</td>
</tr>
<tr>
<td>c. Multiply 600 by 1</td>
<td>c. The number of miles represented by a line 1(\frac{1}{4}) inches long</td>
<td>c. How many inches long a line should be</td>
</tr>
<tr>
<td>d. Multiply 600 by 1(\frac{1}{4})</td>
<td>d. The number of miles represented by a one inch line</td>
<td>d. How many miles one inch represents</td>
</tr>
</tbody>
</table>

### Problem 13.
The Browns are planning a trip to Mexico. Frank knows that his father usually drives about 35 miles an hour. How many hours will the family be driving if they are 3748 miles from Mexico?

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<tr>
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</thead>
<tbody>
<tr>
<td>a. Multiply 3748 by 35</td>
<td>a. The speed of the car</td>
<td>a. How many miles they have traveled</td>
</tr>
<tr>
<td>b. Divide 3748 by 35</td>
<td>b. The number of miles to Mexico</td>
<td>b. How long they took to drive to Mexico</td>
</tr>
<tr>
<td>c. Subtract 35 from 3748</td>
<td>c. The number of hours of driving it took for the trip</td>
<td>c. How far they are from Mexico</td>
</tr>
<tr>
<td>d. Add 3748 and 35</td>
<td>d. The distance from New York to Mexico</td>
<td>d. How far it is from New York to Mexico</td>
</tr>
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</table>

Guess which answer is closest to the right answer? Put circle around a, b, c, or d.

<table>
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<th>Now do the problem. Put a circle around the answer</th>
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<tr>
<td>a. about 120,000</td>
<td>a. about 120,000</td>
</tr>
<tr>
<td>b. about 110</td>
<td>b. about 110</td>
</tr>
<tr>
<td>c. about 3700</td>
<td>c. about 3700</td>
</tr>
<tr>
<td>d. about 4000</td>
<td>d. about 4000</td>
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</table>
V.P.A.
ARITHMETIC REASONING TEST - GRADES IV - VI
(Adapted from Revised Edition by George Spache)

Date_________ Pupil's Name__________ Boy or Girl
Grade____ Age____ Birth date__________
School_________ (Name) (City) ___________
Teacher_________ Examiners__________

Problem

Sample
Tom told us that he took a bus last summer to go to camp. The driver told Tom that he drove 232 miles every day, 6 days a week. How far did he drive in a week?

\[
232 \times 6 = 1392
\]

2. Last year Jane's father bought 4000 gallons of gas each time he went to the gas station. He went to buy gas for his trucks 20 times. How many gallons of gas did he buy last year?

3. Betty had \(3\frac{1}{2}\) apples. Ellen gave her \(2\frac{1}{2}\) more. Do you know how many apples Betty had then?
Problem

4. Clara's family wants to buy a second-hand washing machine. They plan to pay for it in 24 weeks. How much will they pay each week, if the machine costs $39.98?

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13. The Browns are planning a trip to Mexico. Frank knows that his father usually drives about 35 miles an hour. How many hours will the family be driving if they are 3746 miles from Mexico?