The determination of refractive indices of dynamic gaseous media by a scanning grid (Thesis)

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The Determination of Refractive Indices of Dynamic Gaseous Media by a Scanning Grid

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Thesis

THE DETERMINATION OF REFRACTIVE INDICES OF DYNAMIC GASEOUS MEDIA BY A SCANNING GRID

By

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INTRODUCTION

The desire to obtain quantitative measurements of the pressure distributions in dynamic gaseous media has recently been increased. Considerations of transonic and supersonic flight, velocities, in particular, are presently hampered by lack of means for experimental verification of certain theoretical developments. Whereas it is not possible to introduce physical equipment for measurement into a dynamic medium without seriously altering its pressure distributions at high velocities of flow, consideration has been given to optical methods.

Earlier than this, Topler and others had used an optical method for acoustic determinations. Essentially the compressive wave front generated by a highly damped spark circuit was photographed, the high density wave front manifesting itself in the form of a dark band by virtue of refracting the rays of light passing through it to other parts of the field. Some modification of this was made by Mach, notably supplying collimated light and intercepting it with an edge at the focal point of a collecting mirror. An application to the problem in question was made by Schardin. While the sensitivity of these latter methods was considerably greater, the critical value of data so collected is questionable due to the necessity of an intermediate step in which photograph film characteristics are inseparably involved.

More recently, applications of interferometric methods were placed under consideration. Reports of a successful Mach-Zehnder Interferometer method have been noted by Sobel. At the outset, an interferometric method is forbidding in respect economic and labor considerations.
Whether such method is sufficiently critical for the purpose intended is an unanswered question.

An investigation of these previous methods suggests that one may exist which optically permits of quantitative results with the comparative simplicity of the early Schlieren method, and with sufficient accuracy to render usable data. Such a method would be independent of the characteristics of photograph film, yet make use of film for recording of the pressure distribution. In addition, optical alignments must be kept to a minimum and be of the order of 100,000 Angstrom units to permit use of sufficiently large optical components to investigate a field of appreciable size. A possible method satisfying these conditions which comes to mind is not unlike that used by Ronchi for the qualitative investigation of an optical surface. An illustration of the optical alignment is shown in Figure 1.

It is the purpose of this paper to arrive at the operating conditions of such an optical alignment and ascertain its suitability for the measurement of refractive indices of the various parts of the flow field. It is understood that having so proceeded, it is possible by some means, such as that due to Lorentz, to pass directly to the density of the media at the corresponding point and hence the pressure, viz. the pressure distribution of the field.
GENERAL OPTICAL PROPERTIES

If an arrangement such as Figure 1 is used, certain theoretical simplifications become apparent immediately. The wave front entering the medium in question will be a plane, and accordingly in respect an x axis be in phase at a given value for all values of y. Temporarily no restriction will be placed on the continuity of the medium other than specifying that it is two dimensional being completely uniform within its boundaries along the x axis. This is quite commonly put into experimental practice in the "two dimensional wind tunnel" so the limitation so imposed is not prohibitive. Again in practice the angle included at the focal point of either mirror is not in excess of ten degrees in a plane, and accordingly throughout the angle deviations of the refracted rays detectable by the mirror will be considered small.

In general, light emitted by a high intensity, poly-chromatic source placed at the focal point of the collimating mirror will produce the illuminating source. The plane wave front proceeding from the collimator will arrive at the discontinuity representing the boundary of the dynamic medium (confined) under consideration and thence through the second boundary discontinuity. Some alteration of the wave front in the gaseous medium is assumed. At any rate, those rayes unrefracted by virtue of having passed through the medium at points of no refraction, will continue to the collecting mirror, pass through the focal point of this mirror and be subtended on a screen or plate at the conjugate distance for this mirror of the medium. Those particular rays subject to refraction will generally be both displaced and refracted in the
medium, and accordingly will not pass through the focal point of the collecting mirror, but will necessarily be subtended by the screen.

These refracted rays, unless sufficiently refracted to entirely avoid collection by the mirror, will go undetected; masked by the general illumination of the field. It is at this point that a consideration of a detecting means is advisable. A scanning grid of suitable ruling, i.e., five to twenty-five, is shown in Figure 2. Such a grating is readily attainable by producing opaque lines on a transparent material. It will suffice at this point in the development to note that the grating is placed anywhere within the region of fifteen centimeters fore or aft of the focal point of the collecting mirror. It is necessarily, accordingly, that the grating and extent of rulings be sufficiently large so that the entire wave front pass through the grid at the grid location. How variations in the location of the grid affect the pattern and sensitivity will develop naturally in the discussion. It should be noted that if the grid is placed at the focal point of the collecting mirror, the apparatus degenerates into the Schlieren type of alignment in that one line of the grid becomes effectively and edge owing to the inordinately small surface of the wave front at this point.
OPTICS OF THE COLLECTING MIRROR

It is quite obvious that all optical phenomena of interest originate at the medium and affect the optical arrangement at all subsequent points in the light path. Accordingly, temporarily foregoing the nature of the wave front in the medium, we may generally hypothesize the following:

1. The impinging wave front on the medium boundary is plane.

2. Distortion of the wave front in the medium by some means will generally result in
   
a) An angular refraction of the emerging rays of the wave front.
   
b) A linear displacement in either the x or y direction or both of the various rays.

On these hypotheses we may investigate the effect of the collecting mirror on the impinging wave front.
This is most expeditiously carried out by investigating the effect on a single ray by the medium. Confining the investigation temporarily to a single plane in space and subjecting the ray to the general hypotheses given for its deviation, this ray will no longer impinge on the collecting mirror at the point predicted by the absence of the medium. According to the hypotheses, the ray will be displaced in the plane some distance, will be refracted to some new angle, and will be again displaced in the plane in its course from the medium to the mirror a distance proportional to the angle of refraction. Referring now to Figure 3 wherein $\alpha$ is the angle of refraction, $\beta/2$ the angle intercepted by radius of curvature with the axis of symmetry the effect of the refraction may be investigated. From the geometry of the mirror and Snells law, it is apparent that

$$\alpha = \beta - \epsilon$$

(1)

Referring to Figure 4, a simplification of the previous figure, and extending the rays to the screen which is placed at the conjugate distance for the media two rays are shown. The undeviated ray impinging at the same point as the deviated ray passes through the focal point and is intercepted by the screen at a distance $z$ from the axis of symmetry. The refracted ray impinging at this point does not generally pass through the focal point and is intercepted on the screen at a distance $y$ from the axis of symmetry. Accordingly,

$$z = \left( \gamma - f \right)/\beta$$

(2)

$$f = \left( \gamma - f - \alpha \right)\epsilon$$

(3)
This presumes that the angles are sufficiently small subject to conditions previously stated. Combining (2) and (3)

\[ z - y = (\gamma - \epsilon)(\beta - \epsilon) + \alpha \varepsilon \]  

(4)

and using (1) arrive at

\[ z - y = (\gamma - \epsilon) \lambda + \alpha \varepsilon \]  

(5)

Referring to Figure 3 again, and using the law of sines for small angles

\[ \frac{\alpha}{\lambda} \approx \frac{f}{\varepsilon} \]  

(6)

This latter step presumes that the focal length is equal to the distance from the impingement point to the focal point. Accordingly, a substitution reveals that

\[ z - y = \lambda \varsigma \]  

(7)

Substituting the conjugate object distance \( c \) for \( b \), finally the refracted angle

\[ \varsigma = (c - \epsilon)(z - y) / f_c \]  

(8)

Though the refraction angle is thus detectable, it is necessary to determine the effect of the displacement in the plane by virtue of the angle in the path from the medium to the mirror. The magnitude of displacement is \( \varsigma c \). To investigate the effect of displacement alone, assume a displacement of this magnitude by some means so that no angle of refraction is produced. Referring to Figure 5, the angle included between the initial ray and displaced ray is \( g \).

Again for small angles,

\[ \varsigma = g (\epsilon - \gamma) \]  

(9)
In the condition of refraction, this displacement is of magnitude
\[ f q = \Delta c \quad (10) \]
and by substitution
\[ l = \frac{\Delta c}{f} \left( b - f \right) \quad (11) \]
A further substitution to express \( b \) in terms of its conjugate distance yields
\[ l = \Delta c \left( \frac{c}{c - f} - 1 \right) \quad (12) \]
in which \( l \) is the distance displaced on the screen. It is immediately noted that (8) and (12) are equal in magnitude. Unfortunately, the sum effect, then, of the refraction which tends to displace the ray on the screen towards the axis of symmetry and the consequent lateral displacement of this angle which tends to displace the ray away from the axis of symmetry is undetectable. That is to say, where it is possible by some means to detect ray displacements on the screen, in so far as the geometry of rays after the medium is concerned, there would be no effect available for detection.

So far, the actual displacement in the medium, itself, has been neglected. Accordingly, the effect of the lateral displacement within the medium will be the solely detectable effect in any ray deviation. Making use of Figure 5 again, and calling \( d \) the lateral displacement the medium offers the ray in question results in
\[ l' = \frac{d}{f} \left( b - f \right) \quad (13) \]
specifically the effect of any lateral displacement alone.
It should be understood that the actual effect on the ray is the sum effect of all three changes noted and discussed, it is, however, only this latter displacement which manifests itself by a detectable ray displacement. Although the discussion has been confined to one plane, it is equally true for any plane attainable by rotation of the plane about the axis of symmetry. This latter consideration is quite in accordance with the geometry of the optical arrangement.

The question immediately arises of an actual detection of the ray deflection. Generally the overall illumination on the screen provided by undeviated rays would mask any effect produced by the alteration of a small bundle of rays. At any rate, under these conditions detection becomes a matter of relative intensity between the deviated rays and general illumination. It is upon this principle that the Schlieren apparatus is derived. Such a process involves the interspersion of an edge at the focal point, i.e., the singular point through which all collimated rays are said to intersect, and presumably the edge uniformly diminishes the overall illumination of the field. Any rays deviated from the focal point intensify the illumination of the field at the point of interception of the screen, provided the deviation is away from the edge. Deviation towards the edge admits of the converse. In any event, photographic recording of this effect then becomes a matter of great exactness in control for processing or film effects considerably alter the effect of relative intensities. The insertion of the scanning grid proposed, see Figure 1, at some point near the focal point eliminates this effect.
If the grid is inserted at some point near the focal point, and the medium absent, there will be reproduced a shadow image of the grid on the screen. The magnification evidently resultant of this grid is equal to the area ratio of the wave front on the screen and on the grid. The introduction of the medium will generally result in deviation of the rays as described. Confining the discussion to a single ray which normally passes through the grid and assuming a deviation perpendicular to a grid line an alteration of the normal shadow pattern is expected. If the ray is displaced on to the next adjacent line, no illumination will be available at the original position of the ray, and if a considerable number of rays suffer this deviation, the appearance of the shadow is that the grid line has increased in thickness at this point. If, however, the deviation of the normal rays is parallel to the lines of the grid, no great alteration of the normal pattern is expected since the lines will not appear to thicken, deviation being masked by general illumination. Accordingly, the thickening of the grid lines depends upon the deviation normal to the lines, i.e. a vertical line grid detects horizontal deviation and horizontal grid lines detect vertical displacement. A matrix grid consequently detects both simultaneously.

It should be noticed that the measurable deviation on the screen (or film) in no way depends on the film and processing characteristics in practical arrangements. Also, the position of the grid and grid number is not critical for the deviation is predicated by the geometry of the alignment before the insertion of the grid. That is, whereas changes in
grid positioning alter the magnification of the effect and grid number alter the number of regions of the field detectable, the relative thickening is fixed. The former two conditions are chosen in accordance with the situation.

II Refraction within the Medium boundaries

Up to this point, all refraction has been hypothesized. It is necessary to correlate the refraction within the boundaries of the medium with the resultant effect shown by the grid.

Referring to illustration 6, some continuous distribution of refractive index in the y and z directions is assumed, but in conformance with the initial conditions, ie. a two dimensional flow, along the x axis complete uniformity. Two small sections of refracted wave front of area \( da \) are shown looking along the z axis. The extremity rays are indicated for this wave front segment. Now for the included angle

\[
\frac{\delta s_1}{\delta s_2} = \frac{R + \delta R}{R} \quad (14)
\]

and, also, by Huyghens principle which presumes equal phase differences between successive wave fronts,

\[
\frac{\delta s_1}{\delta s_2} = \frac{v}{v - \delta v} \quad (15)
\]

which combine into

\[
\frac{R + \delta R}{R} = \frac{v}{v - \delta v} \quad (16)
\]

Since \( v \) is \( f(\mu) \) so that

\[
\mu = \frac{c}{v} \quad (17)
\]

we get by differentiation

\[
d\nu = -\frac{c}{\mu^2} d\mu \quad (18)
\]
and by replacing the velocity in (16) arrive at
\[ 1 + \frac{\Delta R}{R} = \mu \left/ \mu + d\mu \right. \]  \hspace{1cm} (19)

It is desired to eliminate \( R \) in terms of \( y \). Now since the index of refraction is postulated as some continuous function of \( y \), and \( R \) is shown a function of the refractive index then \( R \) is \( f(y) \). Calling \( \angle \) the angle of refraction of the ray,
\[ y = R \cos \angle \]  \hspace{1cm} (20)

or by differentiating
\[ dy = dR \cos \angle \]  \hspace{1cm} (21)

A substitution of (20) and (21) into (19) evolves
\[ \frac{dy}{d\mu} = \mu \left/ \mu + d\mu \right. - 1 \]  \hspace{1cm} (22)

We have at hand a means of evaluating the refractive index in terms of its effect on the radius of curvature of the wave front, measuring this latter in terms of its projection on the \( y \) axis. A direct integration of this expression in (22) is not possible until the relation between the change of refractive index with \( y \) is known, but we have at hand a means for numerical step by step integration. The displacement of the ray is given by the differential projection increment of \( R \) on the \( y \) axis. This displacement is again determinable in the apparatus through relation (13). Accordingly, knowing the refractive index at a given point \( y \), measured downward, and positioning the grid so the grid lines are perpendicular to the \( y \) axis and along the \( z \) axis a measureable line thickness distribution
will be observed. The screen image may be divided up into two squares and from the value known for \( y \) and refractive index for a given box, proceed through a numerical integration to each adjacent box. In this way, a complete distribution of refractive indices in the flow field in the \( y \) direction is obtainable. By rotating the grid through ninety degrees, and proceeding in the same manner the distribution in the \( z \) axis is determinable. In this manner, the combination of both distributions results in the whole flow field distribution.

III Additional Notes
Throughout the boundaries of the flow field have been considered distinct. Actually the boundary layer effect of the flow and the glass behind the flow field, looking towards the collecting mirror, introduce deflection. In practice these are made small by making the flow field considerably wider (in the \( x \) direction) than the glass. The exact effect of the boundary is indeterminable, but a correction factor for the glass may be applied.

1 It is tacitly assumed that \( y \) is replaced by \( z \) in (22) and \( dy \) replaced by \( dz \).
IV Numerical Computation

The most suitable means of application of the scanning grid is best illustrated by carrying out a specific application numerically. For this purpose, a vertical gradient is proposed in Figure 7. For examination of this gradient a horizontal orientation of grid lines is necessary. The following conditions imposed upon the system are here tabulated.

Table 1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length Collector Mirror</td>
<td>100 cms</td>
</tr>
<tr>
<td>Mirror diameter</td>
<td>10 cms</td>
</tr>
<tr>
<td>Grid ruling</td>
<td>10 lines/cm</td>
</tr>
<tr>
<td>Conjugate object distance (c)</td>
<td>200 cms</td>
</tr>
<tr>
<td>Conjugate image distance (b)</td>
<td>200 cms</td>
</tr>
<tr>
<td>Grid position</td>
<td>10 cms</td>
</tr>
</tbody>
</table>

From this data, a scanning grid pattern resulting from the index gradient proposed may be evaluated. The diameter of the image at the conjugate distance is given by

\[
\text{Magnification} = \frac{c}{b}
\]

which for the data given is \( \frac{200}{200} = 1 \).

Subtended on the grid at its position is a circle of diameter approximately

\[
d'' = \alpha \times g
\]

wherein \( d'' \) is diameter of circle subtended on grid.

\( g \) grid to focal point distance along axis of symmetry of system.
for the system given this is

\[ d'' = \frac{\omega}{\lambda} \]  \hspace{1cm} (10)

\[ = \frac{\text{Mirror diameter}}{\text{Focal length}} \]  \hspace{1cm} (10) = 1

The projected grid diameter on the screen is given by

\[ \text{Grid Mag.} = \frac{\text{Grid area on screen}}{\text{Grid area subtended}} \]  \hspace{1cm} (25)

\[ = \frac{\omega (b-f)}{\omega (g)} \]

which, again, for data given is

\[ (100)/10 = 10 \text{ centimeters} \]

The grid ruling is proportionately expanded on the screen to one line per on the screen image.

Making use of (22), it is possible to compute the lateral displacement in the medium of the rays passing through the various positions along the y axis. These are tabulated in Table II

<table>
<thead>
<tr>
<th>( y )</th>
<th>( \mu )</th>
<th>( \Delta \mu )</th>
<th>( \Delta y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.000 cm</td>
<td>1.000</td>
<td>0.200</td>
<td>-.033 cm</td>
</tr>
<tr>
<td>4.000</td>
<td>1.200</td>
<td>&quot;</td>
<td>-.0571</td>
</tr>
<tr>
<td>3.000</td>
<td>1.400</td>
<td>&quot;</td>
<td>-.075</td>
</tr>
<tr>
<td>2.000</td>
<td>1.600</td>
<td>&quot;</td>
<td>-.0222</td>
</tr>
<tr>
<td>1.000</td>
<td>1.800</td>
<td>&quot;</td>
<td>-.100</td>
</tr>
<tr>
<td>0.000</td>
<td>1.000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

These computed values of \( \Delta y \) are transposed into relation (13) in which \( d \) and \( \Delta y \) are the same. Supplying the balance of date required from
Table I it is possible to compute $l'$ which is the displacement seen on the screen. This data is shown in Table III.

Table III - Computation of displacement seen on screen at conjugate distance 200 centimeters.

<table>
<thead>
<tr>
<th>$y$</th>
<th>$y = d$</th>
<th>$\frac{y - y}{d}$</th>
<th>$l'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-.833</td>
<td>1.0</td>
<td>-.833 cms.</td>
</tr>
<tr>
<td>4</td>
<td>-.571</td>
<td>&quot;</td>
<td>-.571</td>
</tr>
<tr>
<td>3</td>
<td>-.375</td>
<td>&quot;</td>
<td>-.375</td>
</tr>
<tr>
<td>2</td>
<td>-.222</td>
<td>&quot;</td>
<td>-.222</td>
</tr>
<tr>
<td>1</td>
<td>-.100</td>
<td>&quot;</td>
<td>-.100</td>
</tr>
<tr>
<td>6*10</td>
<td>0</td>
<td>&quot;</td>
<td>0</td>
</tr>
</tbody>
</table>

The various displacements of the grid lines on the screen are thus shown. Because of the homogeneous nature of the medium proposed the line thickening will be uniform and the net effect is a line displacement. Figure 8 shows the screen pattern of the grid before introducing the field distribution shown in Figure 8. The displacements computed are effected on introduction of this field and the displacement shown in Figure 9 superimposed on the normal position.

In actual practice the reverse computation is performed in that $l'$ is the measureable quantity and we are seeking the various indices. It is necessary, as was pointed out, to ascertain the index of refraction in some auxiliary manner for some one point of the field in that we are seeking essentially the gradient of the refractive index.
ILLUSTRATIONS

Figure 1

Spherical Mirror (collector) — Flat

Field

Point Source

Sph. Mirror

Grid

Screen

Figure 2

Sample scanning grid: Approximate ruling of eight lines/inch; on transparent material

Figure 3

Collector Mirror
Figure 4

Collector Mirror  Screen

Figure 5

Collector Mirror

vel. Increasing magnitude

Wave front segments of area $dA$. 

Increasing magnitude
ABSTRACT OF THE THESIS

Investigations of aerodynamic phenomena in the transonic and supersonic ranges have generally necessitated a means of rapidly obtaining experimental data. Conventional methods used in the low speed range using auxiliary mechanical equipment are usually not suitable in that alterations of the normal flow are affected by the introduction of this equipment. Recently, interest has been shown in the possibility of obtaining such information through optical methods. Schardin has investigated quantitatively the Topler Striation or Schlieren apparatus. Recent reports indicate investigation of the use of a Mach-Zehnder type of Interferometer.

In this paper, a method is suggested whereby optical equipment is also used to obtain the refractive indices of the various regions of a flow field about a body of interest. An optical arrangement is given in which a scanning grid of comparatively small ruling is introduced near the focal point of a collector mirror. It is shown that a plane wave front impinging on the medium alters the normal optics of the collector mirror in the absence of the medium. Hypothesizing a refracting medium, a relation is given for the refraction angle of the outcoming ray and the deflection from normal position on a screen placed at conjugate distance to the medium. The displacement relation between the screen displacement and lateral displacement to impingement on the mirror by virtue of the alteration in the medium is also given. These two relations are identical, viz. \( \theta = \alpha \left( \frac{f}{c} - \frac{1}{\epsilon} \right) \) in which \( \theta \) is the screen displacement, \( \alpha \) the angle of refraction, \( f \) the focal length of the collector mirror.
and c the distance from collector mirror to medium. Though equal in magnitude, the two separate displacements are opposite in direction and hence a null effect is produced. Any displacement on the screen is accordingly attributed to the later displacement of the ray (or wave front) in the medium, itself. The relation derived is

\[ \ell' = \frac{\ell}{f} (f - d) \]

in which \( \ell' \) is the displacement seen and \( d \) the actual deflection in the medium, itself. It is shown that the effect is made independent of film characteristics by insertion of the grid. If the grid is positioned near the focal point the effect is to make visible these displacements normal to the direction of the grid lines. The positioning of the grid is not critical in that displacement is dictated by the medium and the optical elements of the system given. Any alteration of grid position serves only to magnify the effect. As the grid approaches the focal point, the system degenerates into a Schlieren type of apparatus in that only a single line of the grid becomes effective.

In an effort to relate the displacement to the various refractive indices of the medium a simplification to a two dimensional flow is made. It is assumed that complete uniformity in the flow field in refractive index along the normal path of light is permissible. The curvature imparted to the plane wave front is studied and it is concluded that under the conditions imposed a relation between the displacement in the medium in any direction and the refractive index is given by

\[ \frac{dq}{q} = \frac{a}{\mu + dq} - 1 \]

If \( q \) is a direction normal to the grid rulings, using the previous relation, a numerical point by point integration of the field may be performed. Knowing the index refraction of a particular point in the flow field, a reasonable consideration, it is possible to readily determine the
index of refraction of any point in the field. It is assumed that some means are used to pass from the refractive index of a given point to the density and hence (static) pressure at the point.

A numerical computation for a given index distribution is performed and the resultant pattern shown.
ACKNOWLEDGMENT

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