Firm-level frictions in macroeconomics

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ESSAYS ON FIRM-LEVEL FRICTIONS IN MACROECONOMICS

by

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ABSTRACT

This dissertation consists of three essays on firm-level frictions and their aggregate implications. The first two chapters show that inter-firm lending plays an important role in business cycle fluctuations. In Chapter I, I theoretically investigate the role of supplier credit relationships in propagating and amplifying small shocks using a stylized model of inter-firm trade and lending. I build a network model of the economy in which trade in intermediate goods is financed by supplier credit. In the model, a financial shock to one firm affects its ability to make payments to its suppliers. The credit linkages between firms then transmit financial shocks across firms, amplifying their effects on aggregate output.

In Chapter II, I embed this mechanism into a more general macroeconomic framework to study empirically the role that inter-firm credit plays in the business cycle. To calibrate the model, I construct a proxy of inter-industry credit flows from firm- and industry-level data. I find that the credit network of the US accounts for 22 percent of the fall in GDP occurring from an aggregate financial shock. Finally, I use a structural factor approach to estimate the shocks which affected US industrial production (IP) industries from 1997-2013. I find that most aggregate volatility in IP
was driven by aggregate liquidity shocks and idiosyncratic productivity shocks, and that the credit network of IP industries generated 17 percent of observed aggregate volatility. During the recent recession, three-quarters of the drop in aggregate IP was due to an aggregate financial shock.

Chapter III presents a theoretical investigation of the long-run relationship between international trade and unemployment. I develop and analyze a static general equilibrium model with labor market frictions and heterogeneous firms in which firms can engage in cross-border hiring by employing labor domestically or from abroad. This chapter outlines the conditions on the model parameters under which unemployment rises or falls after trade liberalization, and demonstrates that models in the literature which ignore cross-border hiring likely underestimate the upward force of trade liberalization on unemployment.
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### List of Abbreviations

- $b_d$ ............. Unit search cost for matching with domestic workers
- $b_f$ ............. Unit search cost for matching with foreign workers
- $b_i$ ............. Firm $i$’s bank loan
- $B_i$ ............. Firm $i$’s bank pledgeability of revenue
- $c_i$ ............. Consumption of good $i$
- $C$ ............. Aggregate consumption
- $f_e$ ............. Fixed entry cost for domestic market
- $f_f$ ............. Fixed entry cost for foreign labor market
- $f_x$ ............. Fixed export cost
- $F$ ............. Foreign country
- $G$ ............. Distribution of firm productivity
- $H$ ............. Home country
- $J$ ............. Set of firms
- $l^i_d$ ............ Measure of domestic workers hired by firm $i$
- $l^i_f$ ............ Measure of foreign workers hired by firm $i$
- $l^i_x$ ............ Collateralizability of accounts receivable
- $M$ ............. Number of firms
- $n_i$ ............. Firm $i$’s employment
- $N$ ............. Labor supply
- $p_i$ ............. Price of good $i$
- $Q$ ............. Aggregate consumption index
- $x_{ij}$ ............ Amount of good $j$ used by firm $i$
- $y_i$ ............. Firm $i$’s output
- $Y$ ............. Aggregate output
- $\alpha$ ........... Collateralizability of accounts receivable
- $\chi_i$ ........... Tightness of firm $i$’s constraint
- $\eta_i$ ........... Share of labor in firm $i$’s production
- $\eta$ ............ Upper bound on elasticity of substitution
- $\gamma$ ........... Firm’s share of revenue in Nash bargaining
- $\omega_{ij}$ ........ Share of good $j$ in firm $i$’s production
- $\Omega$ ........... Firm $i$’s labor wedge
- $\phi_i$ ........... Firm $i$’s labor wedge
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<td>Firm $i$’s labor wedge</td>
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<tr>
<td>$\varphi_x$</td>
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<td>$\theta_{ij}$</td>
<td>Firm $i$’s pledgeability of revenue to supplier $j$</td>
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<td>$\tau_{ij}$</td>
<td>Trade credit loan from firm $j$ to $i$</td>
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<td>$\tau$</td>
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Chapter 1

Inter-Firm Credit and the Propagation of Financial Distress

1.1 Introduction

The recent financial crisis and ensuing recession have underscored the importance of external finance for the real economy. Generally, firms obtain most of their short-term external financing from their suppliers, in the form of delayed payment terms for their purchases. In spite of its importance, the aggregate implications of these lending relationships remain poorly understood.

In this chapter, I show that inter-firm lending can transmit financial distress across firms and amplifying its aggregate effects. To this end, I introduce supplier credit into a network model of the economy and analytically show that the credit network of an economy amplifies the effects of financial shocks. I provide intuition with a stylized model in which trade in intermediate goods is financed by supplier credit. In this model, a shock to one firm’s liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects.

The credit linkages that I model take the form of trade credit, or delayed payment terms, that suppliers of intermediate goods often extend to their customers. Trade credit is the single most important source of short-term external finance for firms, and facilitates most inter-firm trade. In the US, trade credit was three times as
large as bank loans and fifteen times as large as commercial paper outstanding on the aggregate balance sheet of non-financial corporations in 2012. In most OECD countries, trade credit accounts for more than half of firms short-term liabilities and more than one-third of their total liabilities. All of these facts point to the presence of strong credit linkages between non-financial firms.

An important feature of trade credit is that it leaves suppliers exposed to the financial distress of their customers. A notable example of this is the US automotive industry in 2008, when the Big Three automakers (Chrysler, Ford, and General Motors) faced an acute shortage of liquidity. While Ford did not require a bailout from the US government itself, it requested one on behalf of its competitors, fearing that a bankruptcy by Chrysler or General Motors would transfer the liquidity shortage to their common suppliers, as the money owed to them could not be paid until they exited bankruptcy. This episode suggests that when firms play a dual role of supplier and creditor, shocks may not only affect trade directly, but may also affect the availability of liquidity to finance trade.

There is growing evidence to suggest that this intuition is empirically relevant. A number of studies - including Boissay and Gropp (2012), Jacobson and von Schedvin (2015), and Raddatz (2010) - have found that firm- and industry-level trade credit linkages propagate financial shocks from firms to their suppliers. In spite of this evidence, the macroeconomic implications of trade credit have been largely overlooked in the literature. I therefore develop a framework for understanding how inter-firm trade and credit interact in response to credit conditions.

I consider an economy similar to that of Bigio and La’O (2015), in which firms are organized in a production network and trade intermediate goods with one another. Limited enforcement problems require firms to make cash-in-advance payments to their suppliers before production takes place. However, firms can delay part of these
payments by borrowing from their suppliers. I assume that, to obtain this credit, a firm can credibly pledge some fraction of its revenue to repay its suppliers. Importantly, this implies that the cash-in-advance payments collected by each firm are endogenous to the model, and depend on the prices of its customers’ goods. As it turns out, endogenous changes in firms’ cash-in-advance constraints are crucial for how the economy behaves in response to shocks.

When one firm is hit with an adverse shock to its cash on hand, there are two channels by which other firms in the economy are affected. First is the standard input-output channel, which has been the focus of studies such as Acemoglu et al. (2012) and Bigio and LaO (2015): the shocked firm cuts back on production, reducing the supply of its good to its customers. Second is a new credit linkage channel in which the shock disrupts the cash flow from firms to their suppliers. That is, when the shocked firm cuts back on production, the price of its good rises. This increases the collateral value of its receivables, allowing the firm to reduce the cash-in-advance payments it makes to its suppliers. With less cash on hand, these suppliers may themselves be forced to cut back on their own production. Thus, the credit linkages propagate the shock up the production chain, amplifying the fall in aggregate output.

**Related Literature**

This paper contributes to several strands of the literature. A growing literature examines how network effects across firms can generate aggregate fluctuations from idiosyncratic shocks. Much of this literature builds on the multi-sector real business cycle model of Long and Plosser (1983). Most notably, these include Acemoglu et al. (2012), Shea (2002), Dupor (1999), Horvath (1998), Horvath (2000), and Acemoglu et al. (2015). These studies all focus on the role of input-output linkages between firms. Input-specificity in the production of intermediate goods prevents firms from easily
switching suppliers or customers in response to productivity shocks. However, most of this literature do not model how trade in intermediate goods is financed. Indeed, most abstract away from financial frictions, leaving no role for financial factors in aggregate fluctuations.

A notable work to which this paper is closely related is that of Bigio and La’O (2015), who examine the role of financial frictions in the context of an input-output network. They find that the input-output structure of an economy is an important determinant of the aggregate impact of firm-level financial shocks. However, they do not explicitly model any credit relationships between firms; the cash-in-advance constraints faced by firms are therefore fixed exogenously. In contrast, I explicitly model these credit relationships, in turn endogenizing these constraints. Moreover, I show that the structure of the credit network of an economy is also an important determinant of the aggregate impact of financial shocks.

There is a growing literature on origins of trade credit and its implications for the transmission of shocks. A question which was once the focus of this literature is why trade credit exists when there are financial intermediaries who specialize in lending? Burkart and Ellingsen (2004) attribute this to a monitoring advantage that suppliers have over banks which allows them to lend to a customer when a bank will not. They find that, in a setting in with a moral hazard problem, the optimal trade credit contract between supplier and customer is one in which the credit limit is proportional to the customer’s revenue. In addition, they assert that firms will grant credit to their customers, even when they themselves are liquidity constrained, because trade credit can be collateralized for bank borrowing. Their results are line with evidence of trade credit use presented in Petersen and Rajan (1997). A number of studies have looked at how trade credit relationships transmit financial distress across trading firms. For example, Boissay and Gropp (2013) find evidence that firms pass over a fifth of their
liquidty shocks to their firms via their trade credit linkages: an increase in the default probability by one firm increases its suppliers chance of defaulting by 0.2%. Jacobson and von Schedvin (2015) both use firm-level data to show that firms pass a significant fraction of their financial shocks to their suppliers via trade credit lending. Barrot (2015) examines data on trucking firms in France and finds that delayed payment terms are associated with greater financial distress. Raddatz (2010) shows that, even controlling for input-output linkages, greater intensity of trade credit use linking two industries increases their correlation in output growth. In my appendix, I follow a strategy similar to that of Raddatz (2010) to test my model’s implications for the comovement of sectoral output growth, using industry- and firm-level data. My empirical findings are in line with the literature: even when accounting for the (direct and indirect) input-output linkages between two industries, a model-based measure of the credit linkages between industry-pairs increases the cross-sectional correlation of their output growth. Furthermore, I utilize a theoretical framework to assess the aggregate implications these findings.

A growing empirical literature tries to evaluate the origins of aggregate fluctuations by measuring the contribution of idiosyncratic versus aggregate shocks. In this context, the seminal work of Gabaix (2011) examined the role of granularity in the firm size distribution. Taking cue from the network literature, a few studies have incorporated input-output linkages as a mechanism by which idiosyncratic shocks may account for larger portion of fluctuations. Broadly speaking, there are two approaches: a more structural approach (e.g. Horvath (2000)) and a more statistical approach. Foerster et al. (2011) and Stella (2014) bridge these approaches using structural and factor approaches together; they account for the effects of input-output linkages in propagating idiosyncratic shocks. My empirical approach follows the same methodology, but uses additional data on industry employment growth to
decompose the data into an additional source of fluctuations - financial shocks. The presence of credit linkages between firms implies a greater role for financial shocks in driving the business cycle. I show that failing to account for the credit linkages created by trade credit underestimates the importance of idiosyncratic shocks, and over-attributes aggregate volatility to aggregate productivity shocks. I also explicitly estimate the contribution of the production and credit networks US industrial production in generating aggregate volatility.

Jermann and Quadrini (2012) evaluate the importance of financial shocks by explicitly modelling the tradeoff between debt and equity financing. They find these shocks explain about half of observed aggregate volatility in the US. My paper, which excludes equity financing, produces empirical results in line with theirs by accounting for the importance of trade credit in financing firms’ working capital.

1.2 Stylized Model: Vertical Production Structure

In this section introduce and analyze the stylized model to build intuition. The simplicity of the production structure of the economy and preferences of the household permits closed-form expressions for equilibrium variables. I will later generalize both the production structure and preferences.

There is one time period, consisting of two parts. At the beginning of the period, contracts are signed. At the end of the period, production takes place and contracts are settled. There are three types of agents: a representative household, firms, and a bank. There are $M$ goods, each produced by a continuum of competitive firms with constant returns-to-scale in production. We can therefore consider each good as being produced by a representative, price-taking firm. Each good can be consumed by the household or used in the production of other goods.
1.2.1 Representative Household

The representative household supplies labor competitively to firms and consumes a final consumption good. It has preferences over consumption $C$ and labor $N$ given by $U(C, N)$.

$$U(C, N) = \log C - N$$

Later I will generalize the preferences. Let $w$ denote the competitive wage earned from working, and $\pi_i$ the profit earned by firm $i$. The household chooses how much to work and how much of each good to consume to maximize its utility subject to the following budget constraint.

$$C = wN + \sum_{i=1}^{M} \pi_i \quad (1.1)$$

The household’s optimality condition is given by

$$\frac{V'(N)}{U'(C)} = w \quad (1.2)$$

This equates the competitive wage with the marginal rate of substitution between labor and consumption.

1.2.2 Firms

There are $M$ firms who each produce a different good. Suppose for now that firms are arranged in a supply chain, where each firm produces an intermediate good for one other firm. The last firm in the chain produces the consumption good, which it sells to the household. Firms are indexed by their order in the supply chain, with $i = M$ denoting the producer of the final good. The flow of goods is depicted below.

$$\begin{align*}
\text{Firm 1} & \to \text{Firm 2} \to \cdots \to \text{Firm M} \to \text{Household}
\end{align*}$$
Firms are price-takers. The production technology of firm $i$ is Cobb-Douglas over labor and intermediate goods.

$$x_i = \begin{cases} 
  z_i n_i^{\eta_i} & \text{for } i = 1 \\
  z_i n_i (1-\eta_i) & \text{for } i > 1 
\end{cases}$$

Here, $x_i$ denotes firm $i$'s output, $n_i$ its labor use, and $x_{i-1}$ its use of good $i-1$. Parameter $z_i$ denotes firm $i$'s total factor productivity, $\eta_i$ the share of labor in its production, and $\omega_{i,i-1}$ the use of good $i-1$ in firm $i$'s production. Let $p_s$ denote the price of good $s$. The value of the sales from firm $s$ to firm $c$ is then $p_s x_{cs}$.

Limited enforcement problems create a need for ex ante liquidity. The household cannot force any debt repayment. Therefore, firm $i$ must pay the full value of wage bill, $wn_i$, up front to the household before production takes place. In addition, each firm $i$ must pay for its intermediate goods purchases, $p_{i-1} x_{i-1}$ up front to its supplier. Thus, firms are required to have some funds at the beginning of the period before any revenue is realized.

Firm $i$ can delay payment to its supplier by borrowing some amount $\tau_{i-1}$ from its supplier. This represents the trade credit loan given from $i-1$ to $i$. In addition, each firm can obtain a cash loan $b_i$ from the bank. The net payment that firm $i-1$ receives from its customer at the beginning of the period is therefore $p_{i-1} x_{i-1} - \tau_{i-1}$.

Firm $i$'s cash-in-advance constraint takes the form

$$wn_i + p_{i-1} x_{i-1} - \tau_{i-1} \leq b_i + p_i x_i - \tau_i$$

This constraint states that the amount of cash that firm $i$ is required to have in order to employ $n_i$ units of labor and purchase $x_{i-1}$ units of intermediate good $i-1$, is bounded by the amount of cash that firm $i$ can collect at the beginning of the period. Note that trade credit appears on both sides of the cash-in-advance: a loan from its
supplier increases firm i’s liquidity, but a loan to its customer reduces its liquidity by reducing the cash-in-advance payment it collects. There is therefore a one-to-one relation between the amount of cash-in-advance a firm can collect from its customer and the size of the trade credit loan it gives its customer.

Firms face borrowing constraints on the size of loans they can obtain from their suppliers and the bank. In particular, firm i can obtain the loan $b_i$ from the bank at the beginning of the period by pledging a fraction $B_i$ of its total end-of-the-period revenue $p_i x_i$, and a fraction $\alpha$ of its accounts receivable $\tau_{i+1}$, where $\alpha \in (0, 1]$. Thus, firm i faces a bank borrowing constraint of the form

$$b_i \leq B_i p_i x_i + \alpha \tau_i$$

Parameters $B_i$ and $\alpha$ provide an exogenous source of liquidity to each firm, and represent the severity of the agency problem between firm i and the bank. I will later show that $\alpha$ parameterizes the degree of substitutability between bank credit and cash-in-advance payments from customers. I assume that $b_i$ is chosen by firm i, which implies that these bank borrowing constraints will bind in equilibrium as each firm obtains the maximum bank loan possible.

In addition, firm i can pledge a fraction $\theta_{i,i-1}$ of its end-of-the-period revenue to repay its supplier. Then the trade credit loan is bounded by the collateral value of firm i’s output

$$\tau_{i-1} \leq \theta_{i,i-1} p_i x_i$$

The parameter $\theta_{i,i-1}$ is a reduced-form representation of the limited ability of firms to delay payment to their suppliers. The precise limited enforcement problem which produces this borrowing constraint is described in detail in Appendix A1. Because firms can collateralize their trade credit (accounts receivable) for bank borrowing (i.e.
\( \alpha > 0 \), they find it optimal to borrow as much as possible from suppliers and the bank. Hence, both the trade credit and bank borrowing constraints bind for all firms in equilibrium. A detailed derivation of this result is given in Appendix A2.\(^1\)

Given the binding borrowing and constraints, we can now re-write firm \( i \)'s cash-in-advance constraint as

\[
wn_i + p_{i-1}x_{i-1} \leq \chi_i p_i x_i \tag{1.3}
\]

where

\[
\chi_i \equiv \frac{b_i}{p_i x_i} + \frac{\tau_{i-1}}{p_i x_i} + 1 - \frac{\tau_i}{p_i x_i}
\]

This constraint says that a firm’s expenditure on inputs is bounded by the value of its liquid funds. The variable \( \chi_i \) describes the tightness of firm \( i \)'s cash-in-advance constraint, and will play a key role in the mechanism of the model. The tightness of a firm’s cash-in-advance constraint is comprised of the firm’s debt-to-revenue ratio and its cash-to-revenue. These describe how much of the firm’s revenue is financed by debt, and how much of its revenue is collected as a cash-in-advance payment, respectively. Notice that \( \chi_i \) is decreasing in \( \frac{\tau_i}{p_i x_i} \), the amount of \( i \)'s output sold on credit: the more credit that \( i \) gives its customer, the less cash it collects at the beginning of the period.

---

\(^1\)Even if firms could not collateralize their accounts receivables, perfect competition amongst firms in an industry would imply that supplier credit borrowing constraints bind in equilibrium, under some benign assumptions. Suppose that the pledgeability of revenue is common across all firms within an industry. Also suppose that before signing trade and debt contracts, firms anticipate that they may be liquidity-constrained in equilibrium. Finally, suppose also that firms can compete not only in price but also in the length of payment terms. Then competition amongst suppliers in industry \( i \) will bid up the trade credit to the maximum that each firm in \( i \) can offer their customers in \( i + 1 \). This maximum is pinned down by the moral hazard problem at \( \theta_{i+1},p_{i+1}x_{i+1} \). (More than this, and customers would simply pocket the loan.) Thus, in equilibrium, firms in a given industry will all sell at the same price and offer the maximum trade credit terms, implying that borrowing constraints bind. In this setting, the allocation of liquidity (cash) across firms would be \textit{ex ante} efficient (at the beginning of the period), but \textit{ex post} inefficient (at the end of the period).
We can re-write $\chi_i$ using firms’ binding borrowing constraints to replace $\tau_i$ and $b_i$.

$$\chi_i = B_i + \theta_{i,i-1} \left[ \begin{array}{c} \text{debt/revenue ratio} \\ \text{cash/revenue ratio} \end{array} \right] + 1 - (1 - \alpha)\theta_{i+1,i} \frac{p_{i+1}x_{i+1}}{p_i x_i}$$

(1.4)

Notice that the firm’s debt-to-revenue ratio is fixed, because firms collateralize their end-of-period revenue for borrowing.

Crucially, equation (4) shows that $\chi_i$ is an equilibrium object - it is an endogenous variable which depends on the firm’s forward credit linkage $\theta_{i+1,i}$ and the revenue of its customer. Hence, changes in the price of its customer’s good affect the tightness of firm $i$’s cash-in-advance. Note also that the dependence of $\chi_i$ on prices $p_i$ and $p_{i+1}$ means that changes a shock will have general equilibrium effects on each $\chi_i$. This a key difference with Bigio and La’O (2015), in which the tightness of each firm’s cash-in-advance is an exogenous parameter because there is no inter-firm lending. Here, the endogeneity of $\chi_i$ will be a critical determinant of how the economy responds to shocks.

Firm $i$ chooses its input purchases to maximize its profits, subject to its cash-in-advance.

$$\max_{n_i, x_{i-1}} p_i x_i - wn_i - p_{i-1} x_{i-1}$$

s.t. $wn_i + p_{i-1} x_{i-1} \leq \chi_i p_i x_i$

Firms take the tightness $\chi_i$ of their constraints as given when choosing inputs.\textsuperscript{2} The solution of each firm’s given in detail in Appendix A3. Firm $i$’s optimality condition equates the ratio of expenditure on each type of input with the ratio of their share of

\textsuperscript{2}The firm’s decisions of how much to borrow and lend, i.e. $b_i$, $\tau_i$, and $\tau_{i-1}$, are already embedded in $\chi_i$ at this point. (See equation (4)). Therefore, given its choice of how much to borrow and lend, the firm’s choice of $n_i$ and $x_{i-1}$ is independent of $\chi_i$. 
production.

\[
\frac{wn_i}{p_{i-1}x_{i-1}} = \frac{\eta_i}{\omega_{i,i-1}(1 - \eta_i)}
\]

This condition pins down the ratio of expenditure on each input. However, the constraint will limit the firm’s total expenditure on both inputs.

If firm i’s cash-in-advance is not binding in equilibrium, then it simply maximizes its profit function. Its optimal level of expenditure on each input is determined by a condition which equates the marginal cost of the input with its marginal revenue product. The firm’s expenditure on labor is therefore given by

\[
wn_i = \eta_ip_ix_i , \quad p_{i-1} = \omega_{i,i-1}(1 - \eta_i)\frac{p_ix_i}{x_{i-1}}
\]

If, on the other hand, the constraint is binding in equilibrium, then the amount of liquidity \(\chi_ip_ix_i\) that firm i has limits how much the firm can spend on both inputs. In particular, firm i’s expenditure on labor and good \(i - 1\) is given by

\[
wn_i = \chi_i\eta_ip_ix_i , \quad p_{i-1} = \chi_i\omega_{i,i-1}(1 - \eta_i)\frac{p_ix_i}{x_{i-1}}
\]

I show in Appendix A3 that firm i’s cash-in-advance (3) binds in equilibrium if and only if \(\chi_i < 1\). Combining the two cases (constrained and unconstrained) yields

\[
w = \phi_i\eta_i\frac{p_ix_i}{n_i} , \quad p_{i-1} = \phi_i\omega_{i,i-1}(1 - \eta_i)\frac{p_ix_i}{x_{i-1}} \tag{1.5}
\]

where \(\phi_i \equiv \min \{1, \chi_i\}\) describes firm i’s shadow value of funds.\(^3\) \(\phi_i\) is strictly less than one if and only if firm i’s cash-in-advance is binding in equilibrium. Equations (5) says that, if binding, the cash-in-advance inserts a wedge \(\phi_i < 1\) between the marginal cost and marginal benefit of each input, representing the distortion in the firm’s input

\(^3\)More precisely, the shadow value of funds of firm i is given by \(\frac{1}{\phi_i} - 1\).
use created by the constraint. A tighter cash-in-advance (lower $\chi_i$) corresponds to a greater distortion, and lower output.

Through its dependence on the tightness $\chi_i$ of the firm’s constraint, $\phi_i$ is endogenous to the model. The credit relationships between firms also imply that firms’ shadow value of funds $\phi_i$ are interdependent. To see this, first recall firm $i+1$’s optimality condition for its intermediate good (5),

$$p_i = \phi_{i+1} \omega_{i+1,i}(1 - \eta_{i+1}) \frac{p_{i+1} x_{i+1}}{x_i} \tag{1.6}$$

This says that the firm $i+1$ chooses its level of intermediate good use $x_i$ to equate the marginal cost of the good $p_i$ with the marginal revenue product, times times the wedge $\phi_{i+1}$ created by its cash-in-advance. Re-arranging this and replacing $\frac{p_{i+1} x_{i+1}}{p_i x_i}$ in (6) yields $\phi_i$ as an increasing function of $\phi_{i+1}$.

$$\phi_i = \min \left\{ 1, \frac{B_i + \theta_{i+1,i-1} - (1 - \alpha) \theta_{i+1,i}}{\phi_{i+1} \omega_{i+1,i}(1 - \eta_{i+1})} \right\}$$

The positive relationship between $\phi_i$ and $\phi_{i+1}$ is a consequence of the fact that firms collateralize their revenue to borrow from suppliers. A tighter constraint of firm $i+1$ implies that every firm upstream of $i+1$ also has a tighter constraint.

1.2.3 Equilibrium

I close the model by imposing labor and goods market clearing conditions:

$$N = \sum_{i=1}^{M} n_i, \quad C = Y \equiv x_M$$

**Definition of Equilibrium**: An equilibrium is a set of prices $\{p_{i\epsilon I}, w\}$, quantities $x_i, n_i, \tau_{i\epsilon I}$ that

i) maximize the representative household’s utility, subject to its budget constraint
ii) maximize each firm’s profits subject to its cash-in-advance, bank borrowing, and supplier borrowing constraints

iii) clear goods markets and the labor market.

Let $\tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,i-1}$ denote firm $i$’s share in total intermediate good use, and $\tilde{\eta}_i \equiv \eta_i \tilde{\omega}_i$ denote firm $i$’s share of labor in aggregate output. Let $\bar{Y}$ denote the equilibrium aggregate output that would prevail in a frictionless economy (la Acemoglu et al. (2012)), given by

$$\bar{Y} \equiv \prod_{i=1}^{M} \tilde{\eta}_i \tilde{\omega}_i$$

$\bar{Y}$ is log-linear in each firm’s productivity $z_i$ and depends on technology parameters $\eta_i$ and $\omega_{i,i-1}$ for all $i$. This is equivalent to an Acemoglu et al. (2012) economy in which firms are organized in a vertical production network and face no financial constraints.

In Appendix A3, I show that for my economy, a closed-form expression for equilibrium aggregate output $Y$ is log-linear in the unconstrained aggregate output and some aggregation of each firm’s shadow value of funds.

$$Y = \bar{Y} \Phi \quad \Phi \equiv \prod_{i=1}^{M} \phi_i \sum_{j=1}^{i} \tilde{\eta}_j$$  (1.7)

The expression $\Phi$ describes how the shadow values aggregate in the input-output network. Thus, equilibrium aggregate output equals $\bar{Y}$ if and only if $\phi_i = 1$ for all $i$ - i.e. if no firm’s cash-in-advance is binding in equilibrium.\(^4\) $\Phi$ captures the aggregate liquidity available to all firms in the economy for trade in inputs. Therefore, (7)

\(^4\)Note that although $Y$ is log-linear in each $\phi_i$, it is not globally log-linear in $\chi_i$. (This is reflected in the kink in $\phi_i$ at $\chi_i = r_i$.) Why is $Y$ not globally log-linear in $\chi_i$? The cash-in-advance creates a kink in the policy function for employment $n_i$ at the point at which the cash-in-advance is no longer binding, i.e. at $\chi_i = r_i$. This kink carries over to $Y$ in aggregation. The kink implies: i) $Y$ is not differentiable with respect to $\phi_i$ at $\phi_i = 1$; ii) the left derivative of $Y$ with respect to $\chi_i$ is strictly positive at $\chi_i = r_i$, and the right derivative is zero; iii) $Y$ is not globally log-linear in $\chi_i$. 
says that equilibrium aggregate output is constrained by the aggregate liquidity in the economy at the beginning of the period. Notice that through $\tilde{\eta}_j$, firms who are further downstream have a higher share of total employment through the use of intermediate goods, and therefore have a higher impact on aggregate liquidity.

To summarize the equilibrium, the cash-in-advance constraints faced by firms induces a wedge on their production, which depends on the tightness of their constraints. But in a setting where firms share liquidity via trade credit, these wedges depend endogenously on the prices of downstream goods and the structure of the credit network. In the next section, I explore the implications of this endogenous relationship between wedges and prices for how aggregate output responds to firm-level shocks.

At this stage, it is worth discussing how this economy compares to that of Bigio and La’O (2015). The novelty of Bigio and La’O (2015) is to show how wedges aggregate in an input-output network. However, in Bigio and La’O (2015), all payments between firms are settled at the end of the period after production takes place. As a result, there is no role for trade credit; and $\chi_i$ and $\phi_i$ are fixed exogenously. As I show in the next section, the endogeneity of the wedges means that the economy behaves qualitatively very differently in response to local shocks.

1.2.4 Aggregate Impact of Firm-Level Shocks

I now examine the response of aggregate output to firm-level liquidity and productivity shocks. I model a liquidity shock to firm $i$ by a change in $B_i$, the fraction of firm $i$’s revenue that the bank will accept as collateral for the bank loan. Consider a marginal fall in $B_i$ given by $dB_i$. This is a reduced-form way to capture a reduction in the supply of bank credit to firm $i$.\(^5\)

\(^5\)In the general network model in the following section, each firm sells some portion of its output directly to the household. In this setting, one could alternatively interpret the fall in $B_i$ as a failed
The fall in $B_i$ directly affects the amount of cash firm $i$ can raise at the beginning of the period, causing firm $i$’s cash-in-advance to tighten.

$$\frac{d\chi_i}{dB_i} = 1 > 0$$

If firm $i$’s cash-in-advance is not binding in equilibrium (i.e. if $\chi_i < 1$), then the marginal drop in liquidity does not affect firm $i$’s output.

$$\frac{d\phi_i}{dB_i} = \begin{cases} 
1 > 0 & \text{if } \chi_i < 1 \\
0 & \text{otherwise}
\end{cases}$$

On the other hand, if firm $i$’s constraint is binding in equilibrium, then the tighter cash-in-advance forces the firm to cut back on production, as it no longer has sufficient beginning-of-the-period funds to finance its original input purchases. This is represented by an increase in firm $i$’s shadow value of funds, i.e. a decrease in $\phi_i$. Since the drop in firm $i$’s output is a contraction in the supply of good $i$, the price $p_i$ of the good rises.

In the absence of any linkages with other firms, the effects of the shock would be contained to firm $i$. However, the firm is linked to other firms via input-output linkages $\omega_{cs}$ and credit linkages $\theta_{cs}$, which transmit the shock to other firms. Indeed, the rise in the price $p_i$ of firm $i$’s good affects other firms in the economy in two ways.

**Network Effects: Standard Input-Output Channel**

The first channel, which I call the *standard input-output channel*, arises from the input-output linkages between firm $i$ and the other firms in the production network, and is the standard channel analyzed in the input-output literature, including Ace-payment by final consumer. In either case, these are idiosyncratic shocks to the firm’s liquidity, and are not well-represented by a change in its productivity or technology.
This channel affects only firms downstream of firm \( i \). The reduction in firm \( i \)'s output increases the price \( p_i \) of good \( i \). This acts as a supply shock to the customer downstream (firm \( i + 1 \)), who is now faced with a higher unit cost of its intermediate good. In response, firm \( i + 1 \) cuts back on its use of both good \( i \) and labor.\(^6\) Its output falls, and the price of its owns good \( p_{i+1} \), rises. This, in turn, acts as a supply shock to firm \( i + 2 \), and so on. Thus, as a result of the shock to firm \( i \), all firms downstream experience a supply shock to their intermediate goods, and cut back on labor as a result.

In this way, the initial liquidity shock to firm \( i \) is propagated downstream by the input-output linkages between firms. The effect of the shock on aggregate output is amplified because each time that a firm reduces its output, it cuts back on its employment. The resulting fall in labor demand reduces the wage and therefore reduces the household’s demand for the consumption good (and aggregate output). Thus, by propagating the shock from firm to firm, the input-output linkages cause a greater fall in aggregate demand for labor, thereby amplifying the initial effect of the shock on aggregate output.

Note that this channel is ultimately driven by the input specificity in each firm’s production technology, as each downstream firm is unable to offset the supply shock by substituting away from using good \( i \) in their production, and each upstream firm is unable to offset the demand shock by finding other customers for its good.

With Cobb-Douglas production, the price and quantity effects of the shock exactly offset, leaving firm \( i \)'s demand for inputs unchanged. As a result, upstream firms are unaffected by the input-output channel.\(^7\)

\(^6\)Firm \( i + 1 \)'s optimality condition for its use of intermediate good \( i \) implies that a higher \( p_i \) will cause the firm to reduce \( x_{i+1} \) in response to the increase marginal cost of the good. This amounts to reducing \( x_i \), its use of the intermediate good. The other optimality condition pins down the ratio of expenditure on each input, implying that the fall in \( x_i \) also causes the firm to reduce its employment \( n_i \).

\(^7\)See Acemoglu et al. (2015) for a detailed discussion.
Network Effects: Credit Linkage Channel

In addition to the standard input-output channel, there is a new channel of propagation - which I call the credit linkage channel - in which the shock disrupts the flow of cash-in-advance payments upstream. This channel describes the endogenous responses in the tightness $\chi_j$ of other firms’ cash-in-advance constraints. Recall that when firm $i$ cuts back on production, the price $p_i$ of its good rises. This rise in price increases the collateral value of firm $i$’s revenue, which means that firm $i$ can reduce the cash-in-advance payment it makes to its supplier $i-1$. Thus, with less cash on-hand, the supplier $i-1$ is now faced with a tighter cash-in-advance itself.

More precisely, there are three effects on $\chi_{i-1}$, the tightness of $i-1$’s constraint. Recall the expression for $\chi_{i-1}$.

$$
\chi_{i-1} \equiv B_{i-1} + \theta_{i-1,i-2} + 1 - (1 - \alpha)\theta_{i,i-1} \frac{p_i x_i}{p_{i-1} x_{i-1}} \quad (1.8)
$$

First, the increase in $p_i$ reduces $\chi_{i-1}$ due to the lower cash-in-advance payment received from firm $i$, as discussed above. Second, the fall in firm $i$’s output increases the ratio $\frac{x_i}{x_{i-1}}$ due to the decreasing returns to $x_{i-1}$ (since $(1 - \eta_i) < 1$). And third, the fall in $i$’s demand reduces the price $p_{i-1}$ of good $i-1$. Each of these effects reduces the amount of cash that firm $i-1$ has per unit of its revenue, and so the shock to $i$ unambiguously tightens firm $i-1$’s cash-in-advance constraint. Notice from (8) that these effects are increasing in $i-1$’s downstream credit linkage $\theta_{i,i-1}$. In this manner, the credit linkages between $i$ and $i-1$ transmit the liquidity shock upstream by reducing the cash flow $i$ makes to its supplier.

Thus, $\chi_{i-1}$ unambiguously falls in response to the shock to $i$. Faced with a tighter constraint, firm $i-1$ may have to further cut back on its output, represented by a rise in its wedge (i.e. a fall in $\phi_{i-1}$). If it does indeed further cut back production, than it also cuts back on employment. This reduces the demand for labor faced by
the household, which in turn reduces the wage it earns. In this manner, the initial effect of the shock is amplified. In addition, firm $i - 1$ in turn passes the shock on to its own suppliers and customers via both channels. This is discussed in the next section.

Note the role that $\alpha$ plays in mitigating the transmission of the shock via the credit linkage channel. The higher that $\alpha$ is, i.e. the more that firm $i - 1$ can collateralize its trade credit $\tau_{i,i-1}$, the less that $\chi_{i-1}$ falls in response to the shock to i. Although $i - 1$ receives a smaller cash-in-advance payment from its customer, it can collateralize a higher fraction of its trade credit to obtain more credit from the bank. This reduces the loss in liquidity that it suffers due to the smaller cash payment. Therefore, $\alpha$ parameterizes the degree to which each firm can substitute lost cash-in-advance payments for a higher bank loan. The value of $\alpha$ does not effect the qualitative results of the model, but may have a quantitative effect, which I explore later on.

**Interaction Between Channels**

Importantly, the two channels of propagation interact to amplify the impact of the initial shock to firm $i$, illustrated in the diagram below. The input-output (credit linkage) channel is represented by blue (red) arrows. The effects begin with line (1), when the initial liquidity shock to firm $i$ triggers supply effects to other firms in the network via the standard input-output channel. The initial impact of the shock is amplified by the input-output linkages between firms. In the absence of the credit linkage channel, the aggregate effect of the shock would be limited to this top line.

However, each firm’s constraint reacts endogenously to the initial shock through changes in cash-in-advance payments, implying that the aggregate impact of the shock is actually much larger. Indeed, the fall in $\phi_i$ causes firm $i - 1$ to receive less cash-
in-advance, pushing down $\phi_{i-1}$. This is equivalent to a second liquidity shock to firm $i-1$, causing it to further reduce production. This extra drop in firm $i-1$’s output again propagates to other firms in the network via input-output linkages, causing a larger drop in aggregate output, represented by line (2). In turn, firm $i-1$’s reduced cash payment to its supplier yields yet more supply effects, and so on, causing the initial effect of the shock to be amplified as it is transmitted upstream.

In this manner, the credit linkages between firms trigger the standard input-output channel at every level of production, increasing the total downstream supply effects faced by each firm. Thus, a firm-level liquidity shock to in my model is isomorphic to an aggregate liquidity shock to all firms in a model with fixed constraints, e.g. Bigio and La’O (2015). I explore this point in further detail in the quantitative analysis.

**Impact of Firm-Level Shock on Aggregate Output**

I now formalize the network effects of the shock on aggregate output. Recall from (7) that equilibrium aggregate output is log-linear in each firm’s wedge

$$\log Y = \log \bar{Y} + \log \Phi$$

Then the elasticity of aggregate output with respect to firm $i$’s bank borrowing $B_i$ is given by

$$\frac{d \log Y}{d B_i} = \frac{d \log \Phi}{d B_i}$$

$\bar{Y}$ depends only on technology parameters and the productivity of each firm. The liquidity shock to $i$ therefore affects aggregate output only via $\Phi$, which represents the aggregate liquidity available to all firms. Indeed, if no firm’s cash-in-advance binds
in equilibrium, then a marginal change in any firm’s liquidity has no impact on any firm’s output.

In Appendix A5, I show that the effect of $B_i$ on aggregate liquidity can be decomposed as follows

$$\frac{d \log \Phi}{dB_i} = \sum_{j=1}^{M} \bar{v}_j \frac{d \log \phi_j}{dB_i}$$

(1.9)

The terms $\frac{d \log \phi_j}{dB_i}$ capture how the liquidity shock to firm $i$ affects the shadow value of funds of every other firm $j$ in the network via the credit linkage channel of propagation. The terms $\bar{v}_j$ map these changes in each firm’s constraint into aggregate output, and capture the standard input-output channel. $\bar{v}_j$ depends on the share of labor in aggregate output of each firm.

$$\bar{v}_j = \sum_{k=1}^{j} \bar{\eta}_k$$

The decomposition given by (9) will allow me to quantify the aggregate effects of each channel later on.

Proposition 1 and its corollary, below, constitute the main theoretical result of the paper: firm-level shocks are amplified by the credit network of the economy. Intuitively, stronger credit linkages imply that in response to increases in collateral value, suppliers increase their lending by more, and therefore receive less cash-in-advance; as a result, aggregate liquidity dries up faster in response to shocks. Firms have to cut back on employment and production by more, amplifying the impact of the shock on aggregate output. Notice also that the aggregate impact of a firm-level shock depends on its location in network: shocks to different firms will propagate differently depending on the input-output and credit linkages between firms. Indeed, how central the shocked firm is in both the production and credit networks of the
economy will ultimately determine a shock’s aggregate impact.

**Proposition 1:** \( \frac{d \log \phi_j}{dB_i} \geq 0 \) and is weakly increasing in \( \theta_{ij} \) for all firms \( i \) and \( j \).

**Proof:** See Appendix A5.

Proposition 1 states that a drop in firm \( i \)’s liquidity \( B_i \) causes other firms \( j \) to experience an adverse liquidity shock as well, and that the size of this effect is increasing in the downstream credit linkages between firms, as I discussed in the description of the credit linkage channel. A corollary of this proposition shows how this in turn affects aggregate output.

**Corollary:** \( \frac{d \log Y}{dB_i} \geq 0 \) and is weakly increasing in \( \theta_{jk} \) for all firms \( i, j, \) and \( k \).

**Proof:** This follows from Proposition 1 and (7)

In the absence of the credit linkage channel, i.e. if the wedges \( \phi_j \) were fixed as in Bigio and La’O (2015), we would have \( \frac{d \log \phi_j}{dB_i} = 0 \) for all \( j \neq i \), and (9) would reduce to \( \bar{v}_i \). However, since \( \frac{d \log \phi_j}{dB_i} \geq 0 \) for all \( j \), the endogenous response of the wedges amplifies the aggregate impact of the shock. In addition, the size of this amplification depends on the structure of credit linkages between the firms, \( \theta_{ij} \).

Now consider a productivity shock to firm \( i \), represented by a fall in \( i \)’s total factor productivity (TFP) \( z_i \). What is the effect on aggregate output? Recall the closed-form expression (7) for aggregate output

\[
Y = \bar{Y} \Phi
\]

where
\[
\tilde{Y} \equiv \prod_{j=1}^{M} \tilde{\eta}_j \tilde{\omega}_j \\
\Phi \equiv \prod_{j=1}^{M} \phi_j \sum_{k=1}^{\tilde{n}_k}
\]

As it turns out, the aggregation of firm wedges \( \Phi \) is independent of \( z_i \). To see this, first recall that \( \phi_M = \min\{1, \chi_M\} \), where \( \chi_M = \theta_{M,M-1} + B_M \). Firm M’s shadow value of funds is thus independent of all \( z_i \). Next, recall that \( \phi_{M-1} = \min\{1, \chi_{M-1}\} \), where

\[
\chi_{M-1} = \theta_{M,M-1} + B_M + 1 - (1 - \alpha) \frac{\theta_M}{\phi_M \omega_{M,M-1}(1 - \eta_M)}
\]

Thus, \( \phi_{M-1} \) is also independent of all \( z_i \). Continuing recursively, it follows that all wedges \( \phi_j \) are independent of TFP \( z_i \). Intuitively, changes in a firm’s TFP do not affect the severity of agency frictions between the firm and its creditors, and therefore they do not affect the tightness of its cash-in-advance.

Since \( z_i \) enters only in \( \tilde{Y} \), we have

\[
\frac{d \log Y}{d z_i} = \frac{\tilde{\omega}_i}{z_i}
\]

Recall that \( \tilde{\omega}_i \equiv \prod_{j=i+1}^{M} \omega_{j,j-1} \) represents firm \( i \)'s share in total intermediate good use. A fall in firm \( i \)'s productivity affects its demand for intermediate goods and its supply of good \( i \). This is the standard input-output channel at work. However, productivity shocks don’t affect firms’ cash-in-advance constraints \( \phi_j \). Therefore, the credit network plays no role in propagating productivity shocks.

Because liquidity shocks directly affect firm wedges while productivity shocks do not, productivity and liquidity shocks will have differential effects on a firm’s output and employment. In the empirical part of the paper, I will use these differential effects to separately identify liquidity and productivity shocks from the data.

To summarize, three main insights emerge from the model. First, when firms are
suppliers of intermediate goods as well as the creditors who finance the transactions of these goods, firm-level shocks can endogenously generate large changes in the aggregate liquidity available for trade in intermediate goods. This creates a multiplier effect which amplifies the aggregate effects of firm-level shocks. Second, the aggregate impact of these shocks depends on structure of the credit network, i.e. how firms borrow from and lend to one another.

But what precisely is the role of the credit network? Until now, the structure of the networks was assumed to be a straight line, shedding little light on its exact role in generating aggregate fluctuations. And is this mechanism quantitatively relevant? To answer these questions requires a model incorporating more features of the economy which can be taken to the data. To this end, I return to the general network framework in the next section.
Chapter 2

Financial Shocks in a Credit Network Economy

2.1 Introduction

In this chapter, I build on the insights from Chapter I and show that inter-firm lending plays an important role in business cycle fluctuations. To this end, I introduce supplier credit into a network model of the economy and quantitatively show that the credit network of an economy amplifies the effects of financial shocks. I then use my framework to empirically shed light on the origins of observed business cycle fluctuations in the US.

My approach involves three steps. First, I embed the mechanism in Chapter I into a more general macroeconomic framework, similar to that of Acemoglu et al. (2012) and Bigio and La’O (2015). Second, I calibrate the model to assess the quantitative importance of the propagation mechanism. For this, I construct a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level input-output data. I find that the credit network of the US economy accounts for 22 percent of the fall in GDP following an aggregate financial shock. Finally, I investigate which shocks drive the US business cycle when we account for the credit linkages between industries. To do so, I use a structural factor approach to estimate the contribution of productivity and financial shocks to observed aggregate fluctuations in US industrial production (IP) from 1997-2013, and
find that these fluctuations were driven primarily by aggregate financial shocks and idiosyncratic productivity shocks. During the Great Recession, productivity shocks played a minimal role; rather, most of the drop in aggregate IP was driven by an aggregate financial shock.

Next, I evaluate the quantitative relevance of the mechanism. I first generalize the model to capture more features of the economy. In order to calibrate the model, I then construct a proxy of inter-industry trade credit flows by combining firm-level balance sheet data from Compustat with industry-level input-output data from the Bureau of Economic Analysis. With this, I produce a map of the credit network of the US economy at the three-digit NAICS level of detail. I calibrate the model to match this proxy and the input-output matrices of the US. I also allow for substitutability between cash-in-advance payments and bank credit, so that firms can partially offset a loss in customer payments with increased bank borrowing. I set this parameter to match firm-level evidence from Omiccioli (2005) on how much Italian firms collateralize their trade credit for bank borrowing.

In response to a one percent financial shock to every industry in the US, I find that GDP falls by 2.9 percent. The credit linkage channel accounts for nearly a quarter of this drop, reflecting the quantitative importance of the propagation mechanism. Compared to a model without inter-firm lending, the credit network of the economy amplifies the fall in GDP by 28 percent. I then explore which features of the US economy contribute to this amplification. In particular, I find that industries which are important suppliers of intermediate goods to the rest of the economy are also more vulnerable to nonpayment by their customers - a feature which exacerbates the impact of financial shocks.

Having shown that the credit network of the economy can generate quantitatively significant aggregate fluctuations, I then ask whether it does in the data. In the
empirical part of the paper, I evaluate how much of observed fluctuations in output can be attributed to the credit network of the economy. I also investigate which shocks drive the US business cycle when we account for the effects of credit linkages in propagating financial shocks across industries.

To this end, I use quarterly output and employment data on US industrial production industries over 1997-2013, from the Federal Reserve Board’s Industrial Production Indexes and the Bureau of Labor Statistics’ Quarterly Census of Employment and Wages, respectively. With this data, I use a structural factor approach similar to that of Foerster et al. (2011) to estimate shocks which hit these industries over my sample period. This approach involves two stages.

In the first stage, I use a log-linear approximation of the model to back-out the productivity and financial shocks to each industry required for the model to match the fluctuations in the output and employment data. The model is able to separately identify these shocks because each type of shock has differential effects on industry output and employment. Namely, productivity shocks affect the ratio of output to employment through Cobb-Douglas production functions. On the other hand, financial shocks do not affect production functions. Rather, they work through cash-in-advance constraints and thereby affect industries’ first-order conditions. In the second stage, I use standard factor methods to decompose each of these quarterly productivity and financial shocks into an aggregate component and an idiosyncratic component. In all, I thus estimate four types of shocks: aggregate and idiosyncratic productivity shocks, and aggregate and idiosyncratic financial shocks.

To gauge the external validity of the structural factor analysis, I compare the aggregate financial shocks estimated using the model with a measure of the risk-bearing capacity of the US financial sector - namely, the excess bond premium of Gilchrist and Zakrajsek (2012). There turns out to be a fairly strong negative correlation between
the two time series (approximately negative one-half), suggesting that the aggregate financial shocks picked up by the model are indeed reflective of changes in the supply of credit from the financial sector to the IP industries.

With the estimated liquidity and productivity shocks at hand, I perform a variance decomposition of aggregate IP to estimate how much of aggregate volatility was driven by each of these shocks. I find that, before the Great Recession, aggregate fluctuations were driven primarily by idiosyncratic productivity shocks and aggregate financial shocks. Moreover, the credit network of US IP played a quantitatively important role in propagating these financial shocks, generating at least 17 percent of observed aggregate volatility over this period.

During the Great Recession, however, productivity shocks seemed to have played little role. Rather, three-quarters of the peak-to-trough drop in aggregate IP can be attributed to an aggregate financial shock to these industries. In addition, I show that idiosyncratic financial shocks to the three most systemically important IP industries - oil and coal manufacturing, chemical manufacturing, and auto manufacturing - accounts for between 9 and 27 percent of the drop in aggregate IP during the recession. Furthermore, the credit and input-output linkages between industries played a significant role in propagating these industry-level shocks across the economy.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for such shocks. This paper, however, finds a minimal role for aggregate productivity shocks in the US business cycle, but a vital one for aggregate financial shocks. Hence, my results suggest that when one accounts for the effects of credit linkages between industries, aggregate financial shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle. This finding is in line both with growing evidence on the importance of
the financial sector for real activity, and with standard interpretations of the causes of the Great Recession. Thus, by explicitly accounting for the propagation generated by credit linkages, this paper captures the importance of financial shocks for aggregate fluctuations in the real economy.

2.2 Generalized Model

I now generalize the mode of the previous chapter to capture more features of the economy. Specifically, I generalize the network structure of the economy and the household preferences. I assume that each of the \( M \) goods can be consumed by the representative household or used in the production of other goods. The household’s total consumption \( C \) is Cobb-Douglas over the \( M \) goods

\[
C \equiv \prod_{i=1}^{M} c_i^{\beta_i}
\]

The household has Greenwood-Hercowitz-Huffman (GHH) preferences given by

\[
U(C, N) = \frac{1}{1-\gamma} \left( C - \frac{1}{1+\epsilon} N^{1+\epsilon} \right)^{1-\gamma}
\]

where \( \epsilon \) and \( \gamma \) respectively denote the Frisch and income elasticity of labor supply. Quantitatively similar results will hold for preferences which are additively separable in aggregate consumption \( C \) and labor \( N \). The household maximizes its utility subject to (1), the household budget constraint. This yields optimality conditions equating the ratio of expenditure on each good with the ratio of their marginal utilities, and equating the competitive wage with the marginal rate of substitution between aggregate consumption and labor.

\[
\frac{p_i c_i}{p_j c_j} = \frac{\beta_j}{\beta_i}, \quad N^{1+\epsilon} = C
\]
Each firm can trade with all other firms. Firm \(i\)'s production function is again Cobb-Douglas over labor and intermediate goods.

\[
x_i = z_i^{\eta_i} n_i^{\eta_i} \left( \prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i}
\]

Here, \(x_i\) denotes firm \(i\)'s output and \(x_{ij}\) denotes firm \(i\)'s use of good \(j\). Since \(\omega_{ij}\) denotes the share of \(j\) in \(i\)'s total intermediate good use, I assume \(\sum_{j=1}^{M} \omega_{ij} = 1\), implying that each firm has constant returns to scale. The input-output structure of economy can summarized by the matrix \(\Omega\) of intermediate good shares \(\omega_{ij}\).

\[
\Omega \equiv \begin{bmatrix}
\omega_{11} & \omega_{12} & \omega_{13} & \cdots & \omega_{1M} \\
\omega_{21} & \omega_{22} & \omega_{23} & & \\
\omega_{31} & \omega_{32} & \omega_{33} & & \\
& & & \ddots & \\
\omega_{M1} & & & & \omega_{MM}
\end{bmatrix}
\]

This matrix describes the structure of the production network. Note that the production network is defined only by technology parameters. As we will see, the presence of financial frictions will distort inter-firm trade in equilibrium. Hence, \(\Omega\) describes how firms would trade with each other in the absence of frictions.

Each firm’s cash-in-advance takes the same form as in the stylized model, with the exception that each firm has \(M\) suppliers and \(M\) customers instead of just one of each. Firm \(i\) is required to pay its wage bill \(w_n_i\) and its intermediate good purchases \(p_s x_{is}\) from each supplier \(s\) in advance. It receives a loan \(b_i\) from the bank and a trade credit loan \(\tau_{ci}\) from each supplier.

\[
w_n_i + \sum_{s=1}^{M} (p_s x_{is} - \tau_{is}) \leq b_i + \sum_{c=1}^{M} \tau_{ci}
\]

This is simply a generalization of the input-output structure in the stylized model. In that case, the \(\Omega\) would be given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.

\[^{1}\text{This is simply a generalization of the input-output structure in the stylized model. In that case, the }\Omega\text{ would be given by a matrix of zeros, with one sub-diagonal of ones, reflecting the vertical production structure and the constant returns to scale technology of firms.}\]
Each firm faces a borrowing constraint each of its suppliers, to which it can pledge fractions \( \theta_{is} \) of its revenue in return for the loans. The borrowing constraints take the form

\[
\tau_{is} \leq \theta_{is} p_i x_i
\]

The structure of the credit network between firms can be summarized by the matrix of \( \theta_{ij} \)'s.

\[
\Theta = \begin{bmatrix}
\theta_{11} & \theta_{12} & \theta_{13} & \cdots & \theta_{1M} \\
\theta_{21} & \theta_{22} & \theta_{23} & & \theta_{2M} \\
\theta_{31} & \theta_{32} & \theta_{33} & & \ \ \\
& & \ddots & & \theta_{M1} \\
\theta_{M1} & & & \cdots & \theta_{MM}
\end{bmatrix}
\]

Henceforth, I refer to this matrix as the credit network of the economy. As we will see, the structure of this network will play an important role in determining the aggregate impact of idiosyncratic shocks.

Each firm can also borrow \( b_i \) from the bank by pledging \( B_i \) of its revenue and \( 1 - \alpha \) of its accounts receivable \( \sum_{c=1}^{M} \tau_{ci} \), so that its bank borrowing constraint takes the form

\[
b_i \leq B_i p_i x_i + \alpha \sum_{c=1}^{M} \tau_{ci}
\]

\( \alpha < 1 \) parameterizes the substitutability of cash-in-advance payments and bank credit. If \( i \)'s customer \( c \) reduces its cash-in-advance payment to \( i \) by one dollar, then \( i \) experiences a net loss in liquidity of \( 1 - \alpha \) dollars; it loses 1 dollar in cash, but is able to borrow \( \alpha \) more dollars from the bank. Thus, it is able to partially substitute the lost cash payment with more bank credit. \( \alpha = 0 \) corresponds to the case when the two are not substitutable, and \( \alpha = 1 \) to the case when they are fully substitutable. The choice of \( \alpha \) will have an effect on the quantitative predictions of
the model, which I discuss later on.

Each firm chooses the size of the loan to obtain from each creditor, so that the borrowing constraints bind in equilibrium. Plugging the binding borrowing constraints into firm $i$’s cash-in-advance yields a constraint on $i$’s total input purchases

$$wn_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i$$

where $\chi_i$ denotes the tightness of $i$’s cash-in-advance.

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1 - \alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

Note that $\chi_i$ is again an equilibrium object, depending on the prices customers’ goods $p_c$ and forward credit linkages $\theta_{ci}$ for all $c$.

Firms choose labor and intermediate goods to maximize profits subject to their cash-in-advance. This yields optimality conditions of the same form, equating the ratio of expenditure on each good with the ratio of their marginal revenue products.

$$\frac{wn_i}{p_j x_{ij}} = \frac{\eta_i}{(1 - \eta_i) \omega_{ij}}$$

Again, the cash-in-advance of firm $i$ inserts a wedge $\phi_i$ between the marginal cost and marginal revenue product of each input

$$n_i = \phi_i \eta_i \frac{p_i}{w} x_i$$

$$x_{ij} = \phi_i (1 - \eta_i) \omega_{ij} \frac{p_i}{p_j} x_i \quad (2.1)$$

where the wedge is determined by the firm’s shadow value of funds.

$$\phi_i = \min \{1, \chi_i\} \quad (2.2)$$

Note that the wedge is still an equilibrium object, depending on collateral value of each customer’s output and forward credit linkages. Endogenous wedges imply
equilibrium will take same form, and will respond in qualitatively the same way as previously.

Market clearing conditions for labor and each intermediate good are given by

\[ N = \sum_{i=1}^{M} n_i \quad x_i = c_i + \sum_{c=1}^{M} x_{ci} \]

The richness of the model afforded by the general network structure and household preferences will allow me to take the model to the data and examine quantitatively the role of the credit network in generating aggregate fluctuations. The equilibrium conditions take the same form as in the stylized model, and the economy will behave in qualitatively the same way in response to shocks as in the stylized model. However, the general network structure precludes a closed-form solution.

### 2.2.1 Relationship Between Firm Influence and Size

A well-known critique of standard input-output models such as Acemoglu et al. (2012) is that a sufficient statistic for a firm’s influence is its share of total sales in the economy. In other words, the size of a firm as measured by its share of aggregate sales is sufficient to determine the aggregate impact of a shock to sector \( i \), and one does not need to know anything about the underlying input-output structure of the economy. All relevant information is captured by the sales share. As a result, an idiosyncratic shock to any firm is isomorphic to an aggregate TFP shock weighted by each firm’s share of total value-added. This feature makes it difficult to claim that the origin of aggregate fluctuations is idiosyncratic rather than aggregate shocks, using this class of frictionless models.

Bigio and La’O (2015), however, show that this isomorphism breaks down when the economy has frictions. In particular, the impact on economic aggregates of an idiosyncratic shock to sector \( i \) depends on the underlying input-output structure of
the economy, and cannot be summarized by the sector’s share of aggregate sales.

My model shows that when the constraints faced by firms depends endogenously on their credit relationships and the prices of downstream goods, knowing the input-output structure of the economy is no longer sufficient to measure the aggregate impact of a shock to a sector or firm \( i \). How a liquidity shock propagates to other firms depends on the credit linkages between them. Therefore, to know how shocks propagate in my economy, one needs to know the underlying structure of credit linkages between firms. Thus, the aggregate impact of an idiosyncratic shock depends on the structure of the input-output network, and the structure of the credit network.

2.2.2 Solving the General Model

The equilibrium of the general model is the solution to system of \( M^2 + 5M + 2 \) nonlinear equations in the same number of unknowns, listed in Appendix A6. For any set of model parameters

\[
\left\{ \left\{ z_i, B_i, \eta_i, \beta_i, \{ \theta_{ij}, \omega_{ij} \}_{j \in I} \right\}_{i \in I}, \alpha, \epsilon, \gamma \right\}
\]

there is a unique solution to the system. Since the model is one period, the behavior of the system in response to shocks can be modeled by comparative statics. In particular, I am interested in the change in the economy that results form a perturbation of one or more of the model parameters \( \{ B_i, z_i \}_{i \in I} \), representing liquidity and productivity shocks, respectively. I therefore log-linearize the system of nonlinear equations around a point \( \{ B^*_i, z^*_i \} \). In the quantitative analysis, I calibrate this point (and the remainder of the parameters), to match data for the US economy. I thus obtain a log-linear approximation for the response of the equilibrium variables to firm-level liquidity and productivity shocks.

It is worth clarifying one point about productivity shocks. It turns out from
the Cobb-Douglas specification of firm production functions that the equilibrium is already log-linear in each $z_i$. Therefore, the log-linearized response of the equilibrium variables to a change in $z_i$ is independent of the level of $z_i$. Therefore, I do not need to calibrate the parameters $\{z_i\}_{i\in I}$ to approximate a response in the economy to a productivity shock. Indeed, when one log-linearizes the equilibrium system around $\{B_i^*, z_i^*\}$, $z_i^*$ drops out of the log-linear equations.

2.3 Quantitative Analysis

Having established analytically that the credit network of the economy can amplify firm-level shocks, I now ask whether this mechanism is quantitatively significant for the US, and examine more carefully the role that the structure of the credit network plays. But before these questions can be addressed, I need disaggregated data on trade credit flows in order to calibrate the credit network of the US economy.

Unfortunately, data on trade credit flows at any level of detail is scarce. While accounts payable and receivable are generally observable at the firm-level from Compustat, flows of trade credit between firm- or industry-pairs is not. In order to overcome this paucity of data, I construct a proxy of industry-level trade credit flows from industry-level input-output data and firm-level balance sheet data, which I now describe.

2.3.1 Mapping the US Credit Network

The purpose of this section is to construct a proxy for trade credit flows $\tau_{ij}$ between industries $i$ and $j$, from which I can later calibrate the structural parameters $\theta_{ij}$. To build my proxy, I use two sources of data: input-output tables from the Bureau of Economic Analysis (BEA) and Compustat North America over the sample period 1997-2013. The BEA publishes annual data on commodity use by industry (Uses by
Commodity Table) at the three-digit level of the North American Industry Classification System (NAICS). At this level, there are 58 industries, excluding the financial sector. From this data, I observe annual trade flows between each industry-pair, which corresponds to $p_jx_{ij}$ in my model for every industry pair $\{i, j\}$. The BEA also publishes an annual Direct Requirements tables at the same level of detail, which indicate for each industry the amount of a commodity that is required to produce one dollar of that industry’s output. These values are quite stable over my sample period. In constructing my proxy, and also in calibrating the model later, I use the input-output tables of the median year in my sample, 2005.

Compustat collects balance-sheet information annually from all publicly-listed firms in the US. The available data includes each firm’s total accounts payable, accounts receivable, cost of goods sold, and sales in each year of the sample. Therefore, while I cannot identify from whom each firm receives trade credit or to whom it extends credit, I observe the total stock of trade credit and trade debt that it has in any year.

To construct the proxy of trade credit flows, I partly follow the strategy of Raddatz (2010). I begin with the observation that a trade credit loan from supplier to customer is typically a fraction of the value of the sale from supplier to customer.\footnote{This has been documented empirically in various studies including Petersen and Rajan (1997).} I therefore assume that the trade credit from industry $j$ to industry $i$ is proportional to the value of the sale.

\[ \tau_{ij} = q_{ij}p_jx_{ij} \]

Here, $q_{ij}$ denotes the fraction of $i$’s purchase from $j$ made on trade credit. The value of the total purchase $p_jx_{ij}$ is directly observable from the BEA input-output tables.
So to construct the proxy for $\tau_{ij}$, it remains to construct an estimate of $q_{ij}$ for each industry-pair.

Appendix A14 describes this procedure in detail. The idea is as follows. Using data on firms’ accounts payable and accounts revenue from Compustat, I first construct measures of each industry’s payables financing and receivables lending. These respectively describe, on average, how intensively firms in the industry borrow from their suppliers and lend to their customers. Then to proxy $q_{ij}$, i.e. how much of $j$’s sales to $i$ are made on credit, I take a weighted average of industry $i$’s payables financing and industry $j$’s receivables lending. My weighting scheme minimizes the mean squared errors in the observed accounts payables of each industry.

Given my proxy $\hat{q}_{ij}$, inter-industry trade credit flows are then proxied as

$$\hat{\tau}_{ij} = \hat{q}_{ij}p_jx_{ij}$$

As such, I have a map of the credit network of the US economy at the three-digit NAICS level of detail, with which I can quantitatively evaluate the model.\(^3\)

### 2.4 Calibration

With proxy for trade credit flows at hand, I calibrate the general model of Section 6 to match data on the US economy. My calibration strategy involves using the BEA input-output tables to calibrate technology parameters, and my proxy to calibrate the financial parameters. In this section, I describe this strategy in detail.

\(^3\) I omit the financial sector from my analysis.
2.4.1 Technology Parameters

I calibrate technology parameters $\eta_i$ and $\omega_{ij}$ to match the BEA input-output tables of the median year in my sample, 2005. At the three-digit level, I have 58 industries after excluding financial industries. From firm $i$’s optimality conditions (10), we can write the firm’s total expenditure on inputs as

$$wn_i + \sum_{j=1}^{M} p_j x_{ij} = \left( \eta_i + [1 - \eta_i] \sum_{j=1}^{M} \omega_{ij} \right) \phi_i p_i x_i$$

$$= \phi_i p_i x_i$$

where the second equality holds due to the constant returns to scale of $i$’s production technology. This implies that

$$\phi_i = \frac{wn_i + \sum_{j=1}^{M} p_j x_{ij}}{p_i x_i} \quad (2.3)$$

The right-hand side of (12) is directly observable from the BEA’s Direct Requirements table. Therefore I calibrate $\phi_i$ to match industry $i$’s direct requirements of all commodities and labor.

Looking through the lens of the model, the observed input-output tables reflect both technology parameters and distortions created by the liquidity constraints. My calibration strategy respects this feature. In particular, I calibrate technology parameters using firm $i$’s optimality conditions for each input and my calibrated $\phi_i$’s

$$\eta_i = \frac{wn_i}{\phi_i p_i x_i} \quad \omega_{ij} = \frac{p_j x_{ij}}{(1 - \eta_i) \phi_i p_i x_i}$$

Again the ratios $\frac{wn_i}{p_i x_i}$ and $\frac{p_j x_{ij}}{p_i x_i}$ are directly observable from the Direct Requirements tables for every industry $i$ and $j$. 
2.4.2 Financial Parameters

I calibrate the parameters $\theta_{ij}$, representing severity of agency problems between industry $j$ and $i$, to match my proxy of inter-industry trade credit flows $\hat{\tau}_{ij}$. Industry $i$’s binding borrowing constraints pin down its level of borrowing from each of its suppliers $j$.

$$\theta_{ij} = \frac{\tau_{ij}}{p_i x_i}$$

Industry $i$’s total revenue $p_i x_i$ is directly observable from the Uses by Commodity tables. (Recall that I use the input-output tables for year 2005). I then use this and my proxy for trade credit $\hat{\tau}_{ij}$ to calibrate $\theta_{ij}$.

To calibrate $B_i$, the parameters reflecting the severity of agency problems between each industry and the bank, recall the definition of $\phi_i$ given by (11), which depends on the technology parameters (calibrated as described above) and the tightness $\chi_i$ of each industry’s cash-in-advance, where

$$\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1 - \alpha) \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}$$

The total revenue of each industry $p_i x_i$ is observable from the Uses by Commodity tables, and $\phi_i$ and $\theta_{is}$ for all $s$ were calibrated as described above. I therefore use (13) and (11) to back out $B_i$ for each industry.

Appendix A12 plots the calibrated matrix $\Theta$, which represents the credit network of the US economy at the three-digit NAICS level of detail. The matrix is relatively sparse in areas in which industries do not engage in much trade. Also firms within the same industry are lend to and borrow from one another more intensively, as represented by the red diagonal.

To identify which industries take a more central role in the credit network, I define
the credit out-degree ($COD_i$) and credit in-degree ($CID_i$) of industry $i$ as

\[ COD_i \equiv \sum_{c=1}^{M} q_{ci} \quad CID_i = \sum_{s=1}^{M} q_{is} \]

These two measures respectively measure how much trade credit an industry provides the rest of the economy, and how much it receives from the rest of the economy. An industry with a high credit out-degree (credit in-degree) makes a high fraction of its total sales (intermediate goods purchases) on credit, *ceteris paribus*. A few industries take particularly central positions in the credit network of the US: the technical services and oil and gas extraction industries provide the rest of the economy with a lot of credit, while the oil and gas auto manufacturing absorb a large amount of credit from the rest of the economy. Figure 2 plots the distribution of the credit out- and in-degrees of the US.

![Distribution of Credit Out-Degrees](a)

![Distribution of Credit In-Degrees](b)

**Figure 2.1:** Distribution of trade credit use by industry: (a) trade credit lending; and (b) trade credit borrowing.

While there is significantly more variation in the credit in-degrees of industries (standard deviation 2.24) than the credit out-degrees (standard deviation 1.48), the distribution of the former is skewed right.
2.4.3 Remaining Parameters

It remains to calibrate the Frisch and income elasticity parameters $\epsilon$ and $\gamma$, and $\alpha$ which parameterizes the substitutability of cash-in-advance payments and bank credit. I follow the standard literature and set $\epsilon = 1$ and $\gamma = 2$. Omiccioli (2005) examines how firms collateralize their trade credit for bank borrowing for a sample of Italian firms, and finds that the median firm in the sample collateralizes about 20 percent of its accounts receivable. I therefore set $\alpha = 0.2$.

2.5 Quantitative Results

With my model calibrated to match the US economy, I am in a position to examine the quantitative response of the economy to industry-level and aggregate productivity and liquidity shocks. I first ask how much aggregate fluctuations does the credit network of the US economy generate?

2.5.1 Aggregate Liquidity Shock

In order to answer this, I perform the following exercise. Suppose that the economy is hit with a one percent aggregate liquidity shock: each industry $i$’s liquidity falls by one percent. By how much does aggregate output $Y$ fall?

To gauge the maximum effect that the credit network can generate, I first compute the fall in $Y$ for $\alpha = 0$. This corresponds to the case in which industries cannot substitute lost cash-in-advance payments for more bank credit. The propagation of liquidity shocks is strongest for this case. I then allow for substitutability by setting $\alpha$ to its baseline calibrated value of 0.2, in order to have a more conservative estimate of the aggregate impact of the shock.
Table 2.1: Aggregate response for \( \alpha = 0 \)

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>With credit linkage channel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shut-Off</td>
<td></td>
</tr>
<tr>
<td>Pct. Fall in Y</td>
<td>3.70</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Results for \( \alpha = 0 \)

I find that, under this specification, aggregate output falls by 3.7 percent. This is represents a large aggregate effect of the shock. To assess how much of this drop in aggregate output is generated by the propagation of shocks via the credit network, I perform the same exercise, shutting down the credit linkage channel. I leave the detailed technical explanation of how I do this to Appendix A8. The intuitive explanation is as follows. Recall that in the model, changes to firm \( i \)'s wedge \( \phi_i \) come either from the direct liquidity shock \( \tilde{B}_i \) to firm \( i \), or from shocks to other firms being transmitted to \( i \) via its credit linkages. In shutting down the credit linkage channel, I impose that changes in the wedges come only from direct liquidity shocks to each firm. In this way, the credit linkages play no role in propagating shocks. With the credit linkage channel shut down, I compute the response in aggregate output to the same aggregate shock, and compare it to the baseline case. The results are summarized in Table 1.

The effect of the credit linkages in propagating the shocks throughout the network increase the response in aggregate output to the shock by 1.38 percentage points. Put differently, the credit network accounts for 31.4 percent of the fluctuation in aggregate output in response to an aggregate liquidity shock. These are quantitatively significant results, suggesting that the credit network of the US can play an important role in generating aggregate fluctuations in GDP from liquidity shocks.
Table 2.2: Aggregate response for $\alpha = 0.2$

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>With credit linkage channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shut-Off Pct. Fall in $Y$</td>
<td>2.92</td>
<td>2.28</td>
</tr>
</tbody>
</table>

Results for $\alpha = 0.2$

Next, I perform the same exercise for $\alpha = 0.2$, allowing for substitutability between bank credit and cash-in-advance payments. Table 2 reports the results.

Even in this more conservative case, the aggregate impact of the shock is quite large: $Y$ falls by 3.15 percent. Although the amplification generated by the credit network falls substantially, it is still quantitatively relevant. The credit linkages between industries produce a larger drop in $Y$ by 0.64 percentage points. Put differently, the credit network of the US accounts for 22 percent of the drop in GDP in response to the aggregate liquidity shock. Therefore, even allowing for firms to substitute lost payments with increased bank borrowing does not substantially diminish the effect of credit linkages in generating aggregate fluctuations. The remainder of the paper uses $\alpha = 0.2$.

Which industries are most vulnerable to the aggregate liquidity shock? Put differently, which experience the largest drop in output?

![Vulnerable Industries: Aggregate Liquidity Shock](image)

**Figure 2.2:** Most vulnerable industries.
Figure 3 plots the five most vulnerable and five least vulnerable industries. The figure indicates that there is a large degree of heterogeneity in the response of industries to the aggregate liquidity shock. While the output of the construction industry falls by less than 1 percent, that of oil and gas mining falls by over 16 percent.

### 2.5.2 Industry-Level Liquidity Shocks

Next, I ask which industries are most systemically important in the economy, and how this relates to their position in the credit network. I measure the systemic importance of industry $i$ by the elasticity of aggregate output with respect to its liquidity $B_i$.\footnote{Recall that in the general model precludes analytical expressions for this elasticity. I therefore compute these numerically.} A higher elasticity implies that an industry-level liquidity shock to $i$ has a larger impact on aggregate output.

Figure 4 shows a bar graph of the ten most systemically important industries in the US. The blue bars show the elasticity of aggregate output with respect to each industry’s liquidity, or the percentage drop in $Y$ following a 1 percent drop in $B_i$.

The red bars show the contribution of the full credit network to each elasticity, which is computed by subtracting the drop in $Y$ that occurs with credit linkage channel shut off, from the total drop in $Y$. To shut off the credit linkage channel, I impose that each industry’s wedge $\phi_i$ changes only in response to a direct liquidity shock to that industry, and not endogenously through credit linkages with other industries. This gives the drop in aggregate output that would occur in the absence of credit linkages, i.e. if the wedges of industries did not respond endogenously to changes in prices. This is explained in more detail in Appendix A9. In this way, I numerically measure by how much the industry-level shock is amplified by the credit network.

Two results emerge from this exercise. First, the model implies that an industry-
level liquidity shock can have a strong impact on US GDP. For example, although
the technical services industry accounts for only .069 percent of US GDP, a one
percent liquidity shock this industry causes a fall in GDP of .19 percent, due to its
input-output and credit linkages with other industries. This is an enormous response
in aggregate output. In the absence of any linkages, the elasticity of GDP to this
industry's liquidity would be equal to its share of GDP; i.e. GDP would fall by only
.069 percent in response to this shock. Therefore, the network effects generated by
input-output and credit linkages greatly amplify the aggregate impact of the industry-
level shock.

Second, the credit network of the US plays a quantitatively significant role in
amplifying these industry-level shocks. On average, between one fifth to one half of
the fall in GDP in response to an industry-level shock is due to the role of credit
linkages in propagating the shock across the network. Consider again a one percent
liquidity shock to the technical services industry. In the absence of credit linkages,
US GDP would fall by only .16 percent in response this shock. Therefore, the credit
network accounts for about one fifth of this industry’s systemic importance. (The
remainder of the amplification is then caused, of course, by the input-output linkages).

Figure 2.3: Most systemically important industries.
2.5.3 What Features of the US Economy Contribute to Amplification?

What features of the US economy contribute to the amplification of liquidity shocks? As it turns out, in the US, industries which are important suppliers of intermediate goods (e.g. manufacturing industries) happen to also be important providers of credit to the rest of the economy. This means that these industries are more vulnerable to nonpayment by customers. As a result of this property, liquidity shocks to industries downstream will have a larger aggregate impact.

To see this, recall the definition of an industry’s credit out-degree (COD), reproduced below, which summarizes how much credit the industry provides the rest of the economy. Industries with a higher credit out-degree are more vulnerable to nonpayment by their customers: they are more constrained ceteris paribus, and have extended more credit to their customers. Now define an industry’s production out-degree (POD) as

\[
COD_i \equiv \sum_{c=1}^{M} q_{ci} \quad POD_i \equiv \sum_{c=1}^{M} \omega_{ci}
\]

The production out-degree of an industry summarizes how important it is as a supplier of intermediate goods to the rest of the economy. A higher production out-degree means that the goods produced by this industry are widely used by other industries.

Figure 5 shows a scatter plot of industries’ credit and production out-degrees, with a fitted least-squares line. There is a fairly strong positive correlation between the two measures (correlation 0.35), indicating that industries which take a more central position in the economy’s input-output network also take a more central position in the credit network, on average.\(^5\)

\(^5\)While my paper is agnostic about the source of this correlation, one could conjecture explanations for a positive correlation. For example industries which produce more ‘basic’ goods, such as mining or manufacturing industries, may have some advantage over others to enforce debt repayment due to the nature of the goods. Although the question of why important suppliers provide more credit is an important and interesting one, it is beyond the scope of this paper.
To see why this positive correlation increases the sensitivity of GDP to liquidity shocks, consider the following example. The metal manufacturing industry has a high production out-degree - many other industries use metal products intensively. One of its most important customers is auto manufacturing. Metal manufacturing also has a high credit out-degree - these firms sell a lot of output on credit. Suppose that firms in auto manufacturing are hit with a liquidity shock; i.e. banks reduce the supply of credit to these firms. These firms will then reduce their cash-in-advance payments to their suppliers in metal manufacturing (via the credit linkage channel). Because these suppliers are quite dependent on receiving these payments from their customers in auto manufacturing, they experience a sharp tightening of liquidity. This forces them to cut back on production. As a result, other industries experience a sharp drop in the supply of metal goods. Because the rest of the economy uses metal products intensively, this fall in production has a large impact on aggregate output. Thus, the liquidity shock to firms in auto manufacturing will have larger aggregate impact because their suppliers in metal manuf are both more vulnerable to nonpayment and are important suppliers of goods to rest of economy.

**Figure 2-4**: Production out-degree and trade credit use by industry.

The plot indicates that there is a strong positive relationship between the credit out-degree of an industry and its systemic importance. The correlation between the
two measures is 0.6. On average, a one standard deviation increase in an industry’s credit out-degree corresponds to an increase of 0.13 percentage points in the elasticity of $Y$ with respect to its liquidity, or 0.59 standard deviations. Put differently, a one percent liquidity shock to an industry will reduce GDP by 0.13 percentage points more than the same shock to an industry which provides less credit to the economy by one standard deviation. Therefore, there is a strong association in the model between an industry’s systemic importance and how important that industry is in providing credit to the rest of the economy.

2.5.4 Summary of Quantitative Analysis

The quantitative analysis showed that i) the credit linkages between US industries play a quantitatively significant role in amplifying aggregate and industry-level liquidity shocks, even when allowing for substitutability between bank credit and cash payments and ii) the systemic importance of an industry depends on how important for the economy its suppliers are in providing credit.

Therefore an understanding of the role that credit linkages play in propagating idiosyncratic shocks introduces a new notion of the systemic importance of firms or industries based on their place in the credit network. The effects of these linkages are quantitatively important. Therefore, by overlooking the importance of credit linkages between nonfinancial firms, the literature has missed an important determinant of what makes an industry or firm systemically important.

2.6 Empirical Analysis

Now that I have established the role that the credit network plays in propagating shocks, and shown that it can play a quantitatively significant role in generating fluctuations in aggregate output by amplifying liquidity shocks, I turn to the empirical
analysis. I ask, in light of the credit linkages we observe between industries in the US, what role did the credit network of the US play during the Great Recession? How much of observed aggregate volatility can be accounted for by liquidity versus productivity shocks? Have idiosyncratic or aggregate shocks played a more important role in US business cycles? The answers to these questions depends on the nature and magnitude of the shocks that hit the economy over this period. Therefore, to answer these questions, I first need to estimate shocks.

My empirical strategy follows a structural factor analysis approach, similar to that of Foerster et al. (2011), on US industrial production industries at the three-digit NAICS level. In all, I allow for four types of shocks: aggregate liquidity and productivity shocks, and industry-level (idiosyncratic) liquidity and productivity shocks. This approach involves a two-step procedure for estimating each type of shock. First, I use the model to back-out the liquidity and productivity shocks which hit each industry each quarter, using data on each industry’s output growth and employment growth. Second, I use dynamic factor methods to decompose these shocks into an aggregate component and an industry-level component. I then feed these estimated shocks into the model to estimate the role of each type of shock, and the credit network of US manufacturing industries, in generating observed aggregate volatility.

2.6.1 Data

From the Federal Reserve Board’s Industrial Production Indexes, I observe the growth rate in output of all manufacturing and mining industries at the three-digit NAICS level, at the quarterly frequency. There are 23 such industries at this level of detail. From the Bureau of Labor Statistics’ Quarterly Census of Employment and Wages, I observe the number of workers employed by all industries at the three-digit NAICS
level. For each dataset, I take 1997 Q1 - 2013 Q4 as my sample period. I seasonally-adjust and de-trend each series.

Looking through the lense of the model, these observed quarterly fluctuations may be driven by:

1. Industry-level liquidity or productivity shocks
2. Aggregate liquidity or productivity shocks
3. Credit and input-output linkages which propagate these shocks

The answers to my questions of interest depend on the relative importance of each of these in driving fluctuations. Since my calibrated model tells me how much industry j’s output or employment changes in response to a liquidity or productivity shock to i, I use the model to control for the effect of credit and input-output linkages in propagating shocks across industries. To identify aggregate versus industry-level components of the estimated shocks, I use standard dynamic factor methods. The only remaining challenge is to identify how much fluctuations are driven by changes in productivity versus changes in liquidity.

Most of the literature takes one of two extreme positions on the source of fluctuations: they are assumed to be driven either entirely by productivity shocks (as in Foerster et al. (2011) and Acemoglu et al. (2012)) or entirely by liquidity shocks (as in Bigio and La’O (2015)). By making use of both employment and output data, I make a weaker assumption and allow for both types of shocks. In the next section, I first describe how I back-out shocks using this data and my model. I then discuss how my model is able to separately identify liquidity and productivity shocks from the data on output and employment.

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6Hours worked is not directly available at this level of industry detail and this frequency. However, I will check that hours worked and number workers employed are correlated at lower frequencies and lower levels of industry detail.
2.6.2 Identification of Liquidity Shocks versus Productivity Shocks

What allows the model to identify productivity shocks and liquidity shocks separately? In other words, how does the model attribute an observed fall in industry $i$’s output $x_{it}$ and employment $n_{it}$ to a liquidity shock rather than productivity shock? In the model, productivity and liquidity shocks have differential effects on labor and employment.

Productivity shocks work through the Cobb-Douglas production functions, and directly affect the amount of labor employed per unit of output produced. Liquidity shocks, on the other hand, do not affect production functions. Rather they work by affecting the tightness of industry cash-in-advance constraints, and show up as changes in industry wedges (first-order conditions). The model uses these differential effects to identify the source of fluctuations in observed output and employment.

In short, the model essentially backs-out productivity shocks from industry-level Solow residuals, or unexplained changes in output given changes in the factors of production. Similarly, it backs-out liquidity shocks from unexplained changes in the ratios of each industry’s labor expenditure to revenue. In equilibrium, this ratio is equal to the labor share of production (a constant) times the wedge. Because the model can track how a liquidity shock or productivity shock to one industry spills over to other industries via their credit and input-output linkages, the model can back out exactly how much of a change in an industry’s output and employment is coming from spillover effects versus a direct shock, and can identify the industry which was shocked. A more detailed discussion of this identification procedure is relegated to Appendix A10.

\footnote{See equation (5).}
2.6.3 Using the Model to Back Out Shocks from the Data

Recall that the model yields a system of log-linear equations describing the (first-order approximated) elasticity of each equilibrium variable to the liquidity $B_i$ and productivity $z_i$ of each industry $i$. Suppose that the static model is extended to be a repeated cross-section. Then these same equations describe the evolution of the equilibrium variables that occurs each period in response to liquidity and productivity shocks, to a first-order approximation. I obtain a closed-form solution for this evolution, which is derived in the Appendix.

Let $X_t$ and $N_t$ denote the $M$-by-1 dimensional vectors of industry output and employment growth at time $t$, $\tilde{x}_{it}$ and $\tilde{n}_{it}$, respectively. And let $B_t$ and $z_t$ similarly denote the $M$-by-1 dimensional vectors of industry liquidity and productivity growth (i.e. shocks) at time $t$, $\tilde{B}_{it}$ and $\tilde{z}_{it}$, respectively. The closed-form solutions for $X_t$ and $N_t$ yield

$$X_t = G_X B_t + H_X z_t$$

$$N_t = G_N B_t + H_N z_t$$

These respectively describe how each industry’s output and employment changes each period in response to the liquidity and productivity shocks to every industry. Here, the $M$-by-$M$ matrices $G_X$, $G_N$, $H_X$ and $H_N$ are functions of the economy’s input-output and credit networks $\Omega$ and $\Theta$, and capture the effects of the input-output and credit linkages in propagating either type of shock across industries, as was described in the theoretical analysis. The elements of these matrices depend only on the model parameters, and therefore take their values from my calibration.

I construct $X_t$ and $N_t$ for US industrial production industries (at the three-digit
NAICS level) from the output and employment data described above. Let $\hat{X}_t$ and $\hat{N}_t$ denote these observed fluctuations. I then have a system of $2M$ equations in as many unknowns for each quarter, and can invert the system to back-out shocks $B_t$ and $z_t$ each quarter from 1997 Q1 to 2013 Q4.

$$B_t = G_N^{-1} \left( \hat{N}_t - H_N z_t \right)$$

$$z_t = Q^{-1} \hat{X}_t - Q^{-1} G_X G_N^{-1} \hat{N}_t$$

where

$$Q \equiv H_X - G_X G_N^{-1} H_N$$

Thus, I construct liquidity and productivity shocks as the industry-level fluctuations in output and employment, filtered for the effects of credit and input-output linkages in propagating them from industry to industry.

Figure 6 shows the time series of the estimated liquidity and productivity shocks which hit the US auto manufacturing industry each quarter over the sample period.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{US_Auto_Manufacturing_Shocks.pdf}
\caption{Estimated liquidity and productivity shocks to auto manufacturing industry.}
\end{figure}
From the figure, we can see that the changes in auto manufacturing’s liquidity and productivity both fluctuate moderately around zero for most of the sample period. Between 2007 and 2009, the liquidity available to this industry took a sharp drop for a number of consecutive quarters, reaching up to a 25 percent decline. Over this period, the industry’s output and employment experienced a large drop attributable to changes in the labor wedge of the industry. Given the credit linkages, the model is able to trace how much of the drop in the wedge is due to a direct liquidity shock to auto manufacturing versus shocks to other industries being transmitted to it. The blue line plotted in the figure reflect the direct liquidity shocks experienced each quarter by the industry.

In addition, the TFP of the industry seems to have not fluctuated greatly over this recessionary period; in fact, it increased slightly. These features broadly hold across most industries in industrial production. The aggregate effects of these features and their interpretation will be discussed in subsequent sections.

2.6.4 Dynamic Factor Analysis

Next, I decompose the changes in industry liquidity and productivity, $B_t$ and $z_t$, into an aggregate and industry-level shock. I assume that each may be described by a common component and a residual idiosyncratic component.

\[
B_t = \Lambda_B F^B_t + u_t
\]

\[
z_t = \Lambda_z F^z_t + v_t
\]

Here, $F^B_t$ and $F^z_t$ are scalars denoting the common factors affecting the output and employment growth of each industry, respectively, at quarter $t$. I interpret these factors as aggregate liquidity and productivity shocks, respectively. The $M$-by-1 vectors
\( \Lambda_B \) and \( \Lambda_z \) denote the factor loadings, and map the aggregate shocks into each industry’s liquidity and productivity shocks. Together, \( \Lambda_B F_B^t \) and \( \Lambda_z F_z^t \) comprise the aggregate components of \( B_t \) and \( z_t \).

The residual components, \( u_t \) and \( v_t \), unexplained by the common factors, are the idiosyncratic or industry-level shocks affecting each industry’s liquidity and productivity growth. I assume that \( (F_B^t, u_t) \) and \( (F_B^t, u_t) \) are each serially uncorrelated, \( F_B^t, u_t, F_z^t \), and \( v_t \) are mutually uncorrelated, and the variance-covariance matrices of \( u_t \) and \( v_t \), \( \Sigma_{uu} \) and \( \Sigma_{vv} \), are diagonal.

I suppose further that the factors follow an AR(1) process such that

\[
F_B^t = \gamma_B F_B^{t-1} + \psi_B^t
\]

\[
F_z^t = \gamma_z F_z^{t-1} + \psi_z^t
\]

Here, \( \psi_B^t \) and \( \psi_z^t \) are independently and identically distributed. Hence, I have two dynamic factor models; one for the liquidity shocks \( B_t \) and one for the productivity shocks \( z_t \).

Use standard methods to estimate the model. To predict the factors, I use both a one-step prediction method and Kalman smoother. The Kalman smoother yields factors which explain more of the data. Since it utilizes more information in predicting the factors, I use this method as my baseline. All subsequent reported results used the factors predicted using a Kalman smoother.

Figure 7 plots the time series for the estimated liquidity shocks and their aggregate components for the auto manufacturing industry over the full sample period. The aggregate component explain most of the liquidity shocks suffered by auto manufacturing. These features are fairly representative of those in other industries. A similar decomposition for the productivity shocks to auto manufacturing is given in
Appendix A13.

Figure 2-6: Liquidity shocks to auto manufacturing and their aggregate component.

2.7 Aggregate Liquidity Shocks and the Excess Bond Premium

To gauge the external validity of the structural factor analysis, I compare the aggregate liquidity shocks estimated using the model with a measure of the risk-bearing capacity of the US financial sector - namely, the excess bond premium of Gilchrist and Zakrajsek (2012). If the aggregate shocks estimated using my model are indeed picking up liquidity shocks to each industry, then the two time series should exhibit a negative correlation. The correlation between these two time series turns out to be -0.51.\(^8\) This is suggestive evidence that the aggregate liquidity shocks picked up by the model are indeed reflective of changes in the supply of credit from the financial sector to the IP industries.

From Figure (8), we can see that, toward the end of 2007, the model picks up a huge aggregate liquidity shock, of about -20 percent. Simultaneously, the excess

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\(^8\)This finding is robust to using different methods to estimate the factors (e.g. Kalman smoother, one-step prediction procedure). The correlation ranges between -0.44 and -0.51.
bond premium exhibits a huge spike, suggesting that the estimated aggregate liquidity shock captures the severe credit crunch that occurred during the financial crisis.

![Figure 2.7: Excess bond premium and estimated aggregate liquidity shock.](image)

2.8 Empirical Results

I now present and discuss the empirical results using the shocks estimated in the previous sections.

2.8.1 Aggregate Volatility Before the Great Recession

In this section, I use the shocks estimated in the previous section to estimate how much of observed volatility in aggregate industrial production from 1997Q1:2013Q4 can be explained by each type of shock. In addition, I estimate the contribution of the credit network of the US industrial production industries to aggregate volatility. What follows is a brief summary of the procedure; a more detailed description is given in Appendix A11.

Let the variance-covariance matrix of industry output growth $X_t$ be denoted by $\Sigma_{XX}$. In addition, let $\bar{s}$ denote the $M$-by-1 vector of industry shares of aggregate
output during the median year of my sample, 2005. Since these shares are close to constant across the quarters in my sample, the volatility of aggregate industrial output - henceforth aggregate volatility - can be approximated by \( \sigma^2 \), where

\[
\sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s}
\]

The factor model described above implies the following identities for the variance-covariance matrices of output growth \( X_t \) and those of the shocks \( B_t \) and \( z_t \).

\[
\Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X'
\]

\[
\Sigma_{BB} = \Lambda_B \Sigma_{FF} B + \Sigma_{uu} \quad \Sigma_{zz} = \Lambda_z \Sigma_{FF} z + \Sigma_{vv}
\]

The fraction of observed aggregate volatility generated by aggregate liquidity shocks can be computed as the ratio of volatility generated by the aggregate component of \( B_t \) to \( \sigma^2 \).

\[
\frac{\bar{s}' G_X \left( \Lambda_B \Sigma_{FF} B \right) G_X' \bar{s}}{\sigma^2}
\]

I estimate the above variance-covariance matrices \( \Sigma_{BB} \) and \( \Sigma_{zz} \) using the estimated liquidity and productivity shocks \( B_t \) and \( z_t \). Similarly, I estimate the variance-covariance matrices of the factors and idiosyncratic shocks using the predicted factors from my factor estimation, imposing that \( \Sigma_{uu} \) and \( \Sigma_{vv} \) are diagonal matrices. I find that, for the full sample period 1997Q1:2013Q4, aggregate volatility in industrial production is about 0.19\%.

\[\text{Footnote: This is roughly in line with the findings of Foerster et al. (2011). If I compute growth rates and aggregate volatility using the same scaling conventions as they, I find aggregate volatility to be about 9.35 compared to their 8.8 for 1972-1983 and 3.6 for 1984-2007. The higher volatility that I get comes from including the Great Recession in my sample period.}\]

The results of this analysis are summarized in Table (3). The results indicate...
that, before the Great Recession, aggregate volatility was driven primarily by aggregate liquidity shocks and idiosyncratic productivity shocks; aggregate liquidity shocks account for nearly a half of aggregate volatility. On the other hand, there appears to be only a minor role for aggregate productivity shocks in generating aggregate fluctuations, accounting for only about 13 percent. Nevertheless, idiosyncratic productivity shocks still play an important role, accounting for a quarter of aggregate volatility. Note that idiosyncratic productivity shocks do not average out precisely because of the input-output linkages connecting industries. Together, idiosyncratic productivity shocks and aggregate liquidity shocks account for nearly three-quarters of aggregate volatility during this period.

Next, I evaluate the role of the credit network of industrial production in aggregate volatility. Recall from the quantitative analysis that the credit network amplifies shocks by transmitting them across industries. How much of the observed aggregate volatility in industrial production can be accounted for by the credit network amplifying the estimated shocks? The results are summarized in Table (4). Overall, the credit network accounts for nearly one-fifth of aggregate volatility. Put differently, in the absence of the credit linkage channel of propagation, aggregate volatility from 1997-2006 would be 17 percent lower. As was discussed in the theoretical analysis, the credit network primarily propagates liquidity shocks. Indeed, most of the effect
of the credit network is in amplifying the aggregate liquidity shock.

In summary, the main results of this analysis are that, when taking into account the credit linkages between industries,

1. Aggregate productivity shocks do not play an important role in aggregate fluctuations in industrial production

2. Aggregate volatility is driven primarily by idiosyncratic productivity shocks and aggregate liquidity shocks

3. The credit network of the economy plays an important role in amplifying fluctuations in aggregate output

How does this compare to the findings of studies? Foerster et al. (2011) show that, when accounting for the effects of input-output linkages in propagating shocks across industries, the role of aggregate productivity shocks in driving the business cycle is diminished; more of aggregate volatility in IP can be explained by industry-level productivity shocks. Nevertheless, they still find a quantitatively large role for aggregate productivity shocks. On the other hand, my analysis shows that when one takes into account the credit linkages between non-financial firms in the economy, the role of aggregate productivity shocks is minimal. On the contrary, aggregate liquidity shocks seem to play a vital role the business cycle. Indeed, the importance of shocks emanating from the financial sector to real economy as a whole is well-documented.

### Table 2.4: Contribution of Credit Network

<table>
<thead>
<tr>
<th>Contribution of Credit Network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Agg. Volatility</td>
</tr>
</tbody>
</table>
2.8.2 Great Recession

In this section, I perform an accounting exercise to evaluate how much of the peak-to-trough drop in aggregate industrial production during the Great Recession each type of shock can explain. To perform this accounting exercise, I do the following. I first restrict the sample to the time in which the peak-to-trough decline in aggregate IP occurred: 2007Q4: 2009Q2. For each quarter during this period, I use the estimated shocks to decompose the drop in aggregate IP into components arising from each type of shock. For each quarter, this yields a breakdown of the quarterly decline in aggregate IP across each shock. I then take a weighted sum of these breakdowns across quarters. I weight each quarterly breakdown by the fraction of the peak-to-trough decline in aggregate IP accounted for by each quarter. This yields a weighted average breakdown, describing, on average, how much of the total peak-to-trough decline in aggregate IP that occurred during the Great Recession can be accounted for by each type of shock.

I find that both aggregate and idiosyncratic productivity shocks were on average slightly positive during this period. As such, changes in productivity did not contribute to the decline in aggregate IP during the recession. On the contrary, the observed movements in aggregate IP can be accounted for by liquidity shocks. I find that 73 percent of the drop in aggregate IP is due to an adverse aggregate liquidity shock. This is natural given the financial crisis that occurred during the beginning of the recession.

Of the remaining 27 percent not explained by the aggregate liquidity shock, idiosyncratic liquidity shocks to the three most systemically important industries can account for a sizable fraction. Idiosyncratic shocks to the oil and coal products manufacturing, chemical products manufacturing, and auto manufacturing industries account for between one-third and all of the remaining decline in aggregate IP, despite
comprising only about 25 percent of aggregate IP. This suggests that idiosyncratic liquidity shocks to a few systemically important industries played a quantitatively significant role during the Great Recession.

Much of the previous literature has relied on aggregate productivity shocks to drive the business cycle. Yet by many accounts, this has been an unsatisfactory explanation due to the lack of direct evidence for such shocks. This paper, however, finds a minimal role for aggregate productivity shocks in the US business cycle, but a vital one for aggregate liquidity shocks. Hence, my results suggest that when one accounts for the effects of credit linkages between industries, aggregate liquidity shocks seem to displace aggregate productivity shocks as a prominent driver of the US business cycle. This finding is in line both with growing evidence on the importance of the financial sector for real activity, and with standard interpretations of the causes of the Great Recession. Thus, by explicitly accounting for the propagation generated by credit linkages, this paper captures the importance of financial shocks for aggregate fluctuations in the real economy.

2.9 Conclusion

In this paper, I show that inter-firm lending plays an important role in business cycle fluctuations. First, I introduced supplier credit into a network model of the economy. In this model, a shock to one firm’s liquid funds reduces its ability to make payments to its suppliers. The credit linkages between firms and their suppliers thus propagate the firm-level shock across the network, amplifying its aggregate effects. Thus, the endogenous response in cash-in-advance constraints to liquidity shocks is crucial for how the economy behaves in response to liquidity shock.

To evaluate the model quantitatively, I constructed a proxy of the credit linkages between US industries by combining firm-level balance sheet data and industry-level
input-output data. I then calibrated the model to assess the quantitative importance of this propagation mechanism. I found that, in response to an aggregate liquidity shock, the credit network of the US economy accounts for 22 percent of the fall in GDP. I also showed that US industries which are important suppliers of intermediate goods are also more vulnerable to nonpayment by their customers. This feature of the US economy exacerbates the aggregate impact of liquidity shocks.

Finally, I investigated which shocks drive the US business cycle when we account for the credit linkages between industries. To do so, I used a structural factor approach to estimate the contribution of productivity and liquidity shocks to observed aggregate fluctuations in US industrial production (IP) from 1997-2013, and find that these fluctuations were driven primarily by aggregate liquidity shocks and idiosyncratic productivity shocks. During the Great Recession, productivity shocks played a minimal role; rather, most of the drop in aggregate IP was driven by an aggregate liquidity shock. Thus, by explicitly accounting for the credit linkages between industries, this paper quantitatively and empirically captures the importance of financial shocks for US business cycle fluctuations.
Chapter 3

Cross-Border Hiring and Unemployment in a Global Economy

3.1 Introduction

A central issue in international trade is the relationship between international trade and unemployment. Both empirical and theoretical studies have produced conflicting evidence with regard to the response of unemployment to trade liberalization. Along these lines, a more comprehensive study of unemployment and international trade is needed to shed further light on their relationship.

In this paper, I develop a unique framework for examining the relationship between international trade and unemployment by allowing for cross-border hiring by firms, in which firms can hire labor both domestically and abroad. Foreign workers act as marketing agents who reduce the unit trade costs of exporting firms. Labor market frictions induce a response in the unemployment rate to trade liberalization via expected worker income and wages. This model incorporates important features of product and labor markets, and is novel in allowing for cross-border hiring. This framework is tractable and demonstrates that models in the literature which ignore cross-border hiring likely underestimate the upward force of trade liberalization on unemployment.

I incorporate Diamond-Mortensen-Pissarides labor market search and matching frictions into a framework with heterogeneous firms who can engage in cross-border
hiring – i.e. hire labor both domestically and abroad. Productivity is entirely firm-specific and directly observable by the firm. Labor market frictions induce multilateral bargaining between firms and their workers. In this framework, firms can export to the other country and can hire workers both domestically and abroad. There is a unit export cost which is a decreasing function of the measure of foreign workers that a firm has hired. While hiring foreign labor reduces an exporting firm’s trade costs, it does not directly increase the productive capacity of the firm, as hiring domestic labor does. The model is static, but the equilibria it produces are identical to the steady state equilibria that would prevail in a dynamic equivalent. Upon embedding the model in general equilibrium, I analyze how equilibrium unemployment changes when a country undergoes trade liberalization. Since my analysis relies on comparing static equilibria before and after liberalization, it is essentially a long-run analysis of the change in unemployment when a country experiences trade liberalization. Whether a firm produces, exports, or hires foreign labor in equilibrium is determined entirely by its productivity draw. When a country undergoes trade liberalization, more productive firms produce more and earn more revenue. My analysis shows, in accordance with Melitz (2003), that when trading countries experience liberalization, there is a reallocation of resources toward more productive firms in both countries. Hence, there is an increase in the minimum and average productivity of producing firms in equilibrium. The same is true for firms that export and hire foreign labor.

The main result of this paper is that there are two channels through which trade liberalization affects unemployment. The first is through the expected income of a worker in the country: trade liberalization pushes the unemployment rate down through a rise in expected worker income. The second result of this paper is that trade liberalization pushes the unemployment rate up via a rise in wages. The intuition for this is that liberalization causes less productive firms to exit, leaving only the
most productive firm to produce and engage in cross-border hiring in equilibrium. These more productive firms pay higher wages, which means that, in aggregate, firms want to hire fewer workers. Jobless workers who would have been hired by less productive firms before liberalization, are now left unemployed because of the smaller availability of jobs. The fact that the model produces separate and opposing effects on unemployment of expected worker income and wages is a direct result of the frictions in labor markets. This paper also outlines the conditions under which one effect dominates and thus yields a clear prediction about which effect should dominate, given a reasonable parameterization. Hence, this paper helps open the door for future research that could shed light on the relationship between trade and unemployment.

My analysis shows that cross-border hiring has significant implications for the magnitude of the response of unemployment to trade liberalization. Cross-border hiring enhances the quantitative effect of liberalization on unemployment. The fact that the unemployment rate in a country depends in part on the hiring behavior of foreign firms magnifies the wage effect on unemployment. Hence, the framework I outline in this paper shows that cross-border hiring enhances the sensitivity of the unemployment rate to liberalization. This represents the paper’s main contribution to the literature.

This paper is related to a literature examining the relationship between trade and unemployment which emphasizes the role of labor market frictions analyzing the macroeconomic effects of trade. Helpman, Itskoki, and Redding (2010) develop a model with search and matching frictions in the labor market and worker screening to analyze the effects of opening to trade on unemployment and the distribution of wages. They find that while opening to trade unambiguously raises wage inequality, the net effects on unemployment are unclear. Similar methods were used by Helpman and Itskohki (2009) to examine how the unemployment rate changes when a coun-
try opens to trade. They find that the response of unemployment depends on the parameterization of search and matching frictions in the country’s labor market, but that lower frictions do not, in general, imply that the unemployment rate falls when a country opens to trade. Cosar (2011) develops a dynamic model of labor market frictions and human capital accumulation to examine the dynamic response of labor markets to trade liberalization, finding that frictions and human capital accumulation can explain the slow transition of unemployment after liberalization. Felbermayr et al. (2008) incorporate Mortensen-Pissarides search frictions into the Melitz (2003) framework and find that trade liberalization unambiguously decreases a country’s unemployment rate for reasonable parameter values.

None of these papers allow for cross-border hiring by firms, and hence ignore a potentially significant determinant of a country’s unemployment rate. My main point of departure from these studies is to allow for cross-border hiring, thereby obtaining a more comprehensive and descriptive picture of the effects of trade liberalization on unemployment.

The rest of the paper is organized as follows. Section II outlines the model, sectoral equilibrium, and embeds the model in general equilibrium. Section III presents the results, specifically with regard to the effects of trade liberalization on the unemployment rate. Section IV concludes and Section V, the appendix, contains technical details.

3.2 Model

Environment

The initial setup of the model is essentially the same as in Melitz (2003). As such, it incorporates Melitz (2003) of heterogeneous firms I consider a world with two countries, Home (H) and Foreign (F). Variables in country F are denoted by an asterisk. In
each country, there is a continuum of households (workers) and a continuum of firms, each firm producing a different variety. The set of different goods in $H$ is denoted $J$, and $L$ denotes the measure of workers who populate $H$. As in Melitz, each agent has preferences given by a CES utility function over a continuum of goods, indexed by $j$, where the utility function is given below by $\Omega$.

$$\Omega = \left[ \int_{j \in J} p(j)^\rho \, dj \right]^{1/\rho}$$

(3.1)

It is assumed that the goods are substitutes, so that $\rho \in (0, 1)$, and the elasticity of substitution between any two goods is $\sigma = 1/(1 - \rho) > 1$. The well-known result of Dixit and Stiglitz (1977) is that we can model consumer behavior by treating the set of varieties as an aggregate good $Q \equiv \Omega$, with an associated aggregate price given by

$$P = \left[ \int_{j \in J} p(j)^{1-\sigma} \, dj \right]^{1/(1-\sigma)}$$

(3.2)

From now on, all quantities are measured in terms of the aggregate consumption good $Q$. Denoting aggregate expenditure on all varieties within the sector by $R$ and the price of variety $j$ by $p(j)$, the demand for variety $j$ can then be expressed as $q(j) = A^{\frac{1}{1-\rho}} p(j)^{\frac{1}{1-\rho}}$, where $A = R^{1-\rho} P^\rho$ is interpreted as a demand shifter. Defining equilibrium firm revenue as $r(j) = p(j) q(j)$, it follows this specification of sectoral demand that

$$r(j) = A q(j)^\rho$$

(3.3)

Given (1)-(3), the aggregate optimal consumption and expenditure decisions, respectively, in the economy for each variety $j$ are:
Here, total expenditure on varieties within the sector (aggregate revenue) is $R = PQ = \int_{j \in J} r(j)^{1-\sigma} dj$. Each firm faces a residual demand curve with constant elasticity $\sigma$, and therefore chooses the same profit-maximizing markup equal to $\frac{\sigma}{\sigma - 1} = \frac{1}{\rho}$. As in Melitz, I model higher productivity as producing a symmetric variety at a lower marginal cost. The pricing rule is therefore

$$p(\varphi) = \frac{w}{\rho \varphi} = \frac{1}{\rho \varphi} = MC \ast \text{markup}$$

where $w$ is a common wage rate normalized to 1. Plugging (5) into (4) yields

$$r(\varphi) = R(P \rho \varphi)^{\sigma - 1}$$

Production Technology

I now specify the production technology in more detail. I assume that each country has a number of sectors of the economy, in which the production technology may differ. When I later embed the model in general equilibrium, I will impose the assumption that both countries one sector. The production technology is as follows. Output $y$ of each variety depends on the firm’s productivity $\varphi$, and the measure of domestic workers hired by a firm in country $i \in \{H,F\}$ is given by $l_i^d$.

$$y_i^j(\varphi) = \varphi l_i^d(\varphi)$$

This production technology is essentially the same as in Melitz (2003), but differs from that of Helpman, Itskhoki, and Redding (2010) in that all productivity is entirely firm-specific.

There is a competitive fringe of potential firms who can enter the differentiated
sector by paying entry cost \( f_e > 0 \), measured in terms of the aggregate consumption good \( Q \). Once a firm has paid this cost, it draws its productivity \( \varphi \) from a Pareto distribution \( G(\varphi) = 1 - (\varphi_{\text{min}}/\varphi)^z \), where \( z \geq 1 \) and \( \varphi_{\text{min}} > 0 \). A firm can enter at most one sector – i.e. any firm draws a productivity level at most one time. Based on its productivity draw, the firm decides whether to produce (domestically) or exit. The firm then decides whether to exit, produce only domestically, produce for both the domestic and export markets, or produce for both markets and hire foreign labor. A firm is said to be producing domestically if it employs a domestic worker, and producing abroad if it employs a foreign worker. A firm in a given country is either idle; producing only domestically; producing domestically and exporting; or producing domestically, exporting, and producing abroad (i.e. producing using additional foreign labor). These decisions are described by productivity thresholds which will be described below.

It will be shown that there is a cutoff productivity \( \varphi_d \) such that a firm decides to produce if and only if it draws a productivity level \( \varphi \geq \varphi_d \). If it decides to produce, it pays a fixed domestic production cost \( f_d \). It then decides the measure domestic workers to hire \( l_d \) at the unit hiring cost \( b_d \). The workers and the firm then engage in Nash bargaining to split the surplus that results from production. I normalize the value of remaining unemployed to zero. Therefore, the decision of a contacted worker whether or not to accept the job is trivial, since the share of firm revenue received by the worker as a wage is non-zero (if it were not, then the firm would not have contacted the worker to begin with). For this reason, the job acceptance decisions by workers are ignored in this paper. The firm then decides whether or not to export to (sell output to consumers in) the other country. If it decides to export, it pays a fixed export cost \( f_x \) (in addition to \( f_d \)) and an export cost \( \tau(l_f) > 1 \) per unit of the good it exports. More precisely, the unit export cost \( \tau \) of an exporting firm in country i is
decreasing in the measure of foreign workers that the firm has hired $l_f$ (giving firms incentive to hire labor from abroad), and will be specified later. As in Melitz (2003), firms that export in equilibrium are larger and more productive than those that do not. It will be shown that there is a cutoff productivity $\varphi_x$ such that a firm exports in equilibrium if and only if its productivity draw $\varphi$ satisfies $\varphi \geq \varphi_x$ will export in equilibrium. An exporting firm then decides whether or not to hire foreign workers. While these workers do not contribute to production, they reduce the export costs. As such, firms who do not export have no incentive to hire foreign workers. If the firm decides to hire foreign workers, it pays a fixed foreign hiring cost $f_f$ (in addition to $f_d$ and $f_x$) and a unit hiring cost $b_f$. It will be show that there is a cutoff productivity $\varphi_f$ such that a firm hires foreign labor in equilibrium if and only if its productivity draw $\varphi$ satisfies $\varphi \geq \varphi_f$.

I now specify the unit export cost in more detail. Recall that an exporting firm must export $\tau > 1$ units of the good for 1 unit of the good to arrive in the foreign country. This unit export cost is a function of the measure $l_f$ of foreign workers hired by the exporting firm, the elasticity of substitution between varieties $\rho_{1/2}$, and a parameter $\eta > 1$.

$$\tau(l_f) = \left(\frac{\eta}{1 + l_f}\right)^{1/\rho-1}, \quad \tau' < 0, \quad \tau'' > 0 \quad (3.8)$$

The parameter $\eta$ determines the unit export cost of firms which do not hire foreign labor, and hence is assumed to be greater than one. The restriction $\rho > 1/2$ ensures that a firm’s unit export cost $\tau$ is decreasing in the measure of foreign workers $l_f$ that it hires. It is important to note that by the specification of production and the unit export cost, a firm cannot directly increase its production capacity by hiring foreign workers, but rather can only reduce its export costs. The more a firm hires workers from the other country, the less its unit export cost. However, the strict convexity of
the unit export cost implies that there is no amount of foreign labor that a firm can hire to eliminate all of its unit export cost. Therefore, by hiring foreign workers, the firm is essentially undergoing an improvement in technology. In this sense, foreign workers act as agents who help market the firm’s variety abroad, and thus facilitate a reduction in the unit export cost without directly contributing to production. This specification of unit export costs allows for analytical tractability. More precisely, the exponent in the expression for $\tau(l_f)$ allows for a closed-form representation of the firm’s optimal measure of foreign workers $l_f$ to hire, which will be derived later.

Because consumers have a love of variety and because of the fixed export cost, no firm will ever choose to export and not produce domestically. Similarly, because the only function of hiring foreign labor is to reduce export costs, no firm will choose to hire foreign labor if it is not exporting. In short, all firms which hire foreign labor both produce domestically and export, and all firms which export also produce domestically.

**Labor Markets**

In each sector, there are two labor markets – a domestic and foreign labor market – in which workers search for employment from domestic and foreign firms, respectively. Each market exhibits frictional search and matching, which is modeled using the standard Diamond-Mortensen-Pissarides approach. A worker searches for employment in exactly one sector, and can choose to search for a foreign job or a domestic job in this sector. A firm in a particular sector can choose how many workers to contact domestically and abroad. In any period, a worker in a given country is either unemployed, employed by a domestic firm, or employed by a foreign firm. From a worker’s perspective, the only difference between a foreign job and a domestic job is the wage they pay. As such, this model ignores issues of worker migration by assuming a worker does not need to relocate if hired by a foreign firm.
Once a worker is hired by a firm, the worker and firm engage in Nash bargaining, in the manner described in Stole and Zwiebel (1996), with equal weights over the division of revenue from production. Nash bargaining between the firm and its workers occurs such that the firm’s share of revenue is given exogenously by $\gamma \epsilon (0, 1)$, while the worker’s share is $1 - \gamma$. The outside option for hired workers is normalized to zero, so that workers get zero utility from being unemployed.

I let $b_d$ and $b_f$ denote the unit search cost for matching with domestic and foreign workers, respectively. These search costs will be determined endogenously in sectoral equilibrium. If a firm pays a search cost of $b_d n$ units of the aggregate consumption index, it can randomly match with a measure of $n$ domestic workers. A firm’s search cost in the foreign labor market, however, is assumed to be increasing in firm revenue, and is given by $b_f r(\varphi) n$.

While this specification of search costs improves the tractability of the model, it is easily justifiable: firms with higher revenue are usually more productive, and therefore prefer to hire workers of a higher skillset. But hiring more productive workers often entails some search and screening costs which increase as the desired worker productivity increases. It is therefore reasonable to assume that the higher a firm’s revenue, the higher its productivity, and so the higher its marginal search cost for skilled workers. While workers are homogeneous in this model, this representation of the search cost as an increasing function of firm revenue is thus a reduced form representation of worker screening in Helpman, Itskhoki, and Redding (2010), that firms screen the workers they contact and screen their productivities at a cost. The unit search costs $(b_d, b_f)$ are determined endogenously by the tightness of the labor market, and are explained in further detail below.

Let $N_d, N_f$ denote the measure of workers sampled in the sector by domestic and foreign firms, respectively, and let $U_d^i$ and $U_f^i$ denote the measure of workers searching
for employment in the domestic and foreign labor markets of the sector, respectively. Also, let the total measure of workers in the sector searching for employment be given by \( U^i = U^i_d + U^i_f \). For a country \( i \), market tightness is defined in the standard way, where \( k \in \{H, F\} \) and \( k \neq i \):

\[
x^i_d = \frac{N^i_d}{U^i_d}, \quad x^i_f = \frac{N^i_f}{U^i_f}
\]

(3.9)

\[
x^i = \frac{N^i_d + N^i_f}{U^i}
\]

Here, \( x^i_d \) and \( x^i_f \) denote the market tightness in the domestic and foreign labor markets, respectively, in country \( i \in \{H, F\} \), and also represent the probability that a worker searching for employment in the domestic or foreign labor market is hired. The unit search costs are specified as follows for country \( i \) using the standard Diamond-Mortensen-Pissarides approach, in which unit search cost for the domestic and foreign labor markets (\( b_d \) and \( b_f \)) are increasing in the market tightness of the domestic and foreign labor market (\( x_d \) and \( x_f \)), respectively.

\[
b^i_d = \alpha^d_0 (x^i_d)^{\alpha^d_1}, \quad b^i_f = \alpha^f_0 (x^i_f)^{\alpha^f_1}, \quad \alpha^d_0, \alpha^f_0 > 1, \quad \alpha^d_1, \alpha^f_1 > 0
\]

(3.10)

The restrictions on the alpha parameters ensure that search costs are increasing in market tightness to reflect that when there is little supply for labor relative to demand, it is costlier for firms to search for workers.

**Firm’s Problem**

Since no firm will ever export and not serve the domestic market, an exporting firm allocates output to equate its marginal revenue in the domestic and export markets.

\[
r^i_d(\varphi) = r^i_x(\varphi)
\]
Using (3), and letting $y_d(\varphi)$ and $y_x(\varphi)$ denote the amount of an exporting firm’s output supplied to the domestic and export markets respectively, this condition can be written as the following. For simplicity of notation, I suppress country superscripts here, but denote the demand shifter of the ‘other’ country $A_k$.

$$\left(\frac{y_x(\varphi)}{y_d(\varphi)}\right)^{1-\rho} = \tau(l_f)^{-\rho}A_k$$

(3.11)

I define total output as the sum of output for the domestic and export markets, $y(\varphi) = y_d(\varphi) + y_x(\varphi)$. Then (11) implies that

$$y_d(\varphi) = \frac{y(\varphi)}{\Gamma(\varphi)}, \quad y_d(\varphi) = \frac{y(\varphi)\left[\Gamma(\varphi) - 1\right]}{\Gamma(\varphi)}$$

(3.12)

Here, $\Delta(\varphi) = 1 + I_x(\varphi)\tau(l_f)\frac{\rho}{A_k(\frac{\rho}{1-\rho})}$ captures the firm’s “market access”, or whether or not the firm chooses to serve both the domestic and export markets, where $I_x(\varphi)$ is an indicator function which takes the value of 1 if the firm is an exporter. Note that if the firm is not an exporter, $\Delta = 1$. I define total firm revenue by $r(\varphi) = r_d(\varphi) + r_x(\varphi)$ so that (3) and (10) imply

$$r(\varphi) = Ay_d(\varphi)^{\rho} + A_k^{\rho}\left[\frac{y_x(\varphi)}{\tau(l_f)}\right] = \Delta(\varphi)^{1-\rho}Ay(\varphi)^{\rho}$$

(3.13)

where $r_d(\varphi) = Ay_d(\varphi)^{\rho} = \frac{r(\varphi)}{\Delta(\varphi)}$, and $r_x(\varphi) = A_k^{\rho}\left[\frac{y_x(\varphi)}{\tau(l_f)}\right] = \frac{r(\varphi)[\Delta(\varphi)-1]}{\Delta(\varphi)}$. Suppressing the country superscripts, the firm’s problem is as follows. The firm chooses the measures of domestic and foreign workers to hire $l_d, l_f$ and whether or not to export, to maximize its profit. Using (13):

$$\pi(\varphi) = \max_{l_d, l_f} \left\{ \gamma\Delta(\varphi)^{1-\rho}Ay(\varphi, l_d)^{\rho} - f_d - I_x f_x - I_f f_f - b_d l_d - b_f r(\varphi)l_f \right\}$$
\[ s.t. \; I_x \in \{0, 1\}, \; I_f \in \{0, 1\} \]

where \( I_f \in \{0, 1\} \) takes the value of 1 if the firm has hired any foreign workers. The presence of a fixed production cost \( f_d \) implies that there is a zero-profit productivity cutoff \( \varphi_d \) such that a firm exits without producing if and only if its productivity \( \varphi \) satisfies \( \varphi < \varphi_d \). Similarly, the presence of a fixed export cost \( f_x \) implies that there is an export productivity cutoff \( \varphi_x \) such that a firm exports if and only if its productivity \( \varphi \) satisfies \( \varphi \geq \varphi_x \). Finally, the same reasoning implies there is a foreign hiring productivity cutoff \( \varphi_f \) such that a firm hires foreign labor if and only if \( \varphi \geq \varphi_f \). Each of these cutoff productivities will be defined and discussed later.

Defining \( \Delta_x = \Delta(\varphi) \) for all \( \varphi \geq \varphi_x \), the firm market access variable can then be written as:

\[
\Delta(\varphi) = \begin{cases} 
1 & \text{if } \varphi < \varphi_x \\
\Delta_x & \text{otherwise}
\end{cases}
\]  

where

\[
\Delta_x = 1 + \tau(l_f)^{\frac{1}{1-\rho}} \left( \frac{A^k}{A} \right)^{\frac{1}{1-\rho}} > 1
\]

The first order conditions of the firm’s problem are:

\[
l_d : \; b_d l_d(\varphi) = \gamma \rho r(\varphi)
\]

where \( b_d \) is the foreign labor cost.

\[
l_f : \; \tau'(l_f)^{\frac{1}{1-\rho}} + I_x(\varphi) \left( \frac{A^k}{A} \right)^{\frac{1}{1-\rho}} \tau'(l_f)^{\frac{1}{1-\rho}} \tau(l_f)^{\frac{\rho}{1-\rho}} = \frac{-\rho \gamma}{b_f r(\varphi)} r(\varphi) I_x(\varphi) \left( \frac{A^k}{A} \right)^{\frac{1}{1-\rho}}
\]

The first order condition for \( l_f \) is a quadratic equation. Therefore, this yields an
explicit solution for $l_f$ as a function of $\varphi$ – i.e. the policy function $l_f(\varphi)$.

The first order conditions (15) and (16) define the firm policy functions for $l_d$ and $l_f$. It can be seen that the measure of domestic workers a firm hires is proportional to the firm’s revenue. This is an unsurprising result, as larger, more productive firms tend to hire more workers both at home and abroad. Plugging the policy functions into the expression for revenue and profit yields explicit expressions of firm revenue and firm profit as functions of firm productivity $\varphi$.

$$r(\varphi) = \left[1 + I_x(\varphi)\eta^{\varphi/(\varphi-1)} \left(\frac{A^k}{A}\right)^{\frac{1}{1-\rho}} (1 + l_f(\varphi))^{\varphi/(\varphi-1)} \right] \left[A \left(\frac{\gamma \rho \varphi}{b_d}\right)\right]^{\frac{1}{1-\rho}} \tag{3.17}$$

$$\pi(\varphi) = [\gamma(1-\rho) - b_l l_f(\varphi)] r(\varphi) - f_d - I_x(\varphi) f_x \tag{3.18}$$

I assume that bargaining weights of both domestic and foreign workers are the same. Then a firm in country $i$ pays domestic and foreign workers wages $w_{id}$ and $w_{if}$ respectively, and the firm receives the residual $r^i - w_{id} l_d - w_{if} l_f$, which I call net firm revenue. For a firm in a given country, the wage of a domestic or foreign worker must equal the marginal net firm revenue of labor of this kind. This yields the following equations.

$$\frac{\partial}{\partial l_d} [r(l_d, l_f) - w_d(l_d, l_f) l_d - w_f(l_d, l_f) l_f] = w_d \tag{3.19}$$

$$\frac{\partial}{\partial l_f} [r(l_d, l_f) - w_d(l_d, l_f) l_d - w_f(l_d, l_f) l_f] = w_f$$

This system of differential equations in $(w_d, w_f)$, for each country, can be solved.
numerically, given the initial conditions $w_d(0,0) = w_f(0,0) = 0$, yielding the wages for each country.

**Labor Market Tightness and Search Costs**

I assume that workers are risk-neutral, as in the baseline case of Helpman, Itskhoki, and Redding (2010), and so the supply of workers searching for employment depends on their expected income outside the sector. Recall that a worker can choose to search for a foreign job or a domestic job in a given sector. Then a worker is indifferent between searching for a job inside or outside a sector if and only if the expected incomes of a domestic and foreign job in the sector are equal to the value of the worker’s outside opportunity $\omega$. Denoting $\omega_d^i$ and $\omega_f^i$ as the outside opportunities to a worker in country $i$ of having a domestic and foreign job respectively, these conditions are as follows.

\[
\omega_d^i = x_d^i w_d^i(\varphi) \quad (3.20)
\]

\[
\omega_f^i = x_f^i w_f^k(\varphi)
\]

\[
\omega^i = \omega_d^i = \omega_f^i \quad (3.21)
\]

where $k \in \{H,F\}$ and $k \neq i$. It is worth noting that the frictions in labor markets prevent wages across countries from equalizing. From (20), note that a worker has the same outside option regardless of the sector or type of job (domestic or foreign) that he has. The value of the worker’s outside option $\omega^i$ can therefore be interpreted as expected worker income for a worker in country $i$. Equations (20) and (21) show that domestic and foreign labor market tightness are inversely related to the domestic and foreign wages, respectively, and are proportional to expected worker income. This
is intuitive, since an increase in the wage should cause an increase in the supply of labor relative to the demand, causing a drop in labor market tightness. Similarly, an increase in expected worker income $\omega$ should cause an increase in demand for goods, causing firms to want to increase their production, create an increase in the demand for labor relative to the supply, and hence result in a rise in labor market tightness.

Equations (20) and (21) yield solutions for $(x_d, x_f)$ for given values of $(\omega_d, \omega_f)$ and $(w_d, w_f)$. Since $(w_d, w_f)$ is determined in (19), we thus have a solution for $(x_d, x_f)$ as a function of $(\omega_d, \omega_f)$. Using (10), this in turn yields solutions for $(b_d, b_f)$ in terms of $(\omega_d, \omega_f)$. In particular, these solutions are below.

\[ b_d = \alpha_0^d \left( \frac{\omega_d}{w_d} \right)^{\alpha^d_1}, \quad b_f = \alpha_0^f \left( \frac{\omega_f}{w_f} \right)^{\alpha^f_1} \quad (3.22) \]

Given the restrictions on the alpha parameters in (10), equations (22) imply that domestic and foreign search costs are increasing in expected worker income and decreasing in the domestic and foreign wage, respectively.

**Productivity Cutoffs**

I now derive the conditions which yield the productivity cutoff levels, mentioned above, that determine which firms produce domestically, export, and hire abroad based on firm productivity draws. The Zero Cutoff Profit (ZCP) condition requires that the profits of the firm with the lowest productivity in equilibrium must be zero. Letting $\varphi_d$ denote the lowest firm productivity in equilibrium, the ZCP condition is:

\[ \pi(\varphi_d) = 0 \]

Using (17) and (18), this condition is equivalent to

\[ \gamma(1-\rho) \left[ A \left( \frac{\gamma \rho \varphi_d}{b_d} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}} = f_d \quad (ZCP) \]
The ZCP condition pins down the cutoff productivity $\varphi_d$ as a function of parameters and the demand shifter $A$. When we embed the sectoral economy in general equilibrium, in turn solving for the demand shifter $A$, the ZCP will yield $\varphi_d$ as a function of only parameters. A similar condition, called the Exporting Cutoff (EC) condition, is applicable for the exporting cutoff $\varphi_x$. A firm with productivity $\varphi_x$ must be indifferent between producing only domestically and producing domestically and exporting. An argument akin to that for the ZCP condition applies to the EC condition: if the profit earned from exports for a firm with $\varphi_x$ were negative, the firm would not exit; if it were positive, there would be firms with $\varphi < \varphi_x$ which also export, contradicting the definition of $\varphi_x$. This EC condition is given below.

$$\gamma (1 - \rho) \left[ A \left( \frac{\gamma \rho \varphi_x b_d}{\rho} \right) \right]^{\frac{1}{1 - \rho}} [\Delta_x - 1] = f_x \quad (EC)$$

The EC condition pins down the cutoff productivity $\varphi_x$ as a function of the model parameters and the demand shifter $A$. Finally, a firm with productivity $\varphi_{for}$ should be indifferent between paying the fixed foreign hiring cost $f_{for}$ and hiring its desired measure of foreign workers and not. This, I call the Foreign Hiring Cutoff Condition and it pins down the cutoff productivity $\varphi_{for}$.

$$\gamma (1 - \rho) \left[ A \left( \frac{\gamma \rho \varphi_f b_d}{\rho} \right) \right]^{\frac{1}{1 - \rho}} [\Delta_x (l_f(\varphi_f)) - \Delta_x (l_f = 0)] = f_{for} \quad (FHC)$$

Because of the complexity of the expression for $l_f(\varphi_f)$, the FHC condition cannot be solved explicitly for $\varphi_f$. In any case, the above three conditions can be solved for the cutoff productivities to yield the following closed-form expressions for $\varphi_d$ and $\varphi_x$, and the implicit solution for $\varphi_f$.

$$\varphi_d = \frac{b_d}{\gamma \rho A^{1/\rho}} \left( \frac{f_d}{\gamma (1 - \rho)} \right)^{\frac{1 - \rho}{\rho}}$$

(3.23)
\[ \varphi_x = \frac{b_d \tau(l_f)}{\gamma \rho (A^k)^{1/\rho}} \left( \frac{f_x}{\gamma (1 - \rho)} \right)^{1 - \rho/\rho} \]

\[ \varphi_x = \frac{b_d}{\gamma \rho A^{1/\rho}} \left( \frac{f_x}{\gamma (1 - \rho) v} \right)^{1 - \rho/\rho} \]

where

\[ v = \left[ \frac{A^k}{A} \right]^{1/\rho} \eta^{\rho - 1} \frac{\rho}{(\rho - 1)} \left( 1 - [1 + l_f(\varphi_f)]^{1 - \rho/\rho} \right) \]

Using the above solutions for the cutoff productivities, it should first be noted that it was assumed that \( \eta > 1 \) and \( \rho > 1/2 \). Then from (23) it can be seen that a sufficient condition for \( \varphi_d < \varphi_x \) is that \( f_x \geq f_d \). In other words, it suffices that the fixed cost for domestic production is no smaller than the fixed cost required for exporting for \( \varphi_d < \varphi_x \). It is worth noting here that by the definition of \( \varphi_f \), \( l_f(\varphi) = 0 \) for all \( \varphi < \varphi_f \). Then it follows that \( l_f(\varphi_d) = l_f(\varphi_x) = 0 \).

There is an additional Free Entry (FE) condition that the expected profit of an entering firm must be zero, using the assumption that there is free firm entry. It is worth noting that free entry here does not mean costless entry, as there are entry costs; rather, it means ‘unrestricted’ entry in that there exists an endless pool of potential firms who, if profitable, can pay the entry cost and enter the economy at any period. Freely entering and exiting firms will drive expected firm profit to zero:

\[ \int_{\varphi_d}^{\infty} \pi(\varphi) \, d\varphi = f_e \quad (FE) \]

After solving for general equilibrium, I will show that the FE and ZCP conditions define two relationships between average firm profit and the domestic productivity cutoff \( \varphi_d \) and use them to solve for their unique sectoral equilibrium values.
Additional Conditions Determining Sectoral Equilibrium

Recall that, in equilibrium, the outside opportunities for a worker in a country must be the same if the worker is searching for a job in the domestic or the foreign labor market. In addition, we have the following market clearing conditions for the domestic and foreign labor markets in a given country: the total expected worker income for all workers searching for employment in the domestic labor market must equal the total wages actually paid to domestic workers by all firms in the sector. The analogous market clearing condition for the foreign labor market is that for a country \( i \), the total expected worker income for all workers searching for employment in the foreign labor market must equal the total wages actually paid to foreign workers by foreign firms (in the other country) in the sector. I let \( M^i, M^i_x, \) and \( M^i_f \) denote the measure of firms in country \( i \) who produce domestically, export, and engage in cross-border hiring, respectively. Note that by definition, \( M^i \geq M^i_x \geq M^i_f \). (Recall that all firms that produce in equilibrium produce domestically, and all firms that hire foreign workers are exporters). Then these two conditions are expressed mathematically below. For a country \( i \), where \( i, k \in \{H, F\} \) and \( k \neq i \),

\[
\omega^i_d U^i_d = M^i \int_{\varphi^i_d} w^i_d(\varphi) l^i_d(\varphi) \, dG(\varphi) \quad (3.24)
\]

\[
\omega^i_f U^i_f = M^k_f \int_{\varphi^k_f} w^k_f(\varphi) l^k_f(\varphi) \, dG(\varphi)
\]

The intuition behind these conditions is as follows. Take the version of (24) for the domestic labor market. If the left-hand side were greater than the right-hand side, then more unemployed workers would search for employment outside the domestic labor market, reducing \( U^i_d \) and the left-hand side. On the other hand, if the left-hand side were less than the right-hand side, then less unemployed workers would search
for employment outside the domestic labor market, raising $U_d$ and the left-hand side. Therefore, the equality must hold in equilibrium.

There is also a market clearing condition for product markets: the sum of domestic and foreign revenues that supply varieties to the domestic market must equal the total domestic expenditure on differentiated varieties. Then for $i, k \epsilon \{H, F\}$ and $k \neq i$,

$$R^i = M^i \int_{\varphi_d^i} r_d^i(\varphi) l_d^i(\varphi) \, dG(\varphi) + M^k \int_{\varphi_x^k} r_x^k(\varphi) \, dG(\varphi)$$

(3.25)

I now impose the simplifying assumption that the fixed foreign hiring cost $f_f$ is zero. While this assumption improves tractability, it is inconsequential for the central results of the model. All exporting firms will then hire some measure of foreign workers, and so the cutoff productivity for foreign hiring $\varphi_f$ will fall to equalize with that of exporting $\varphi_x$ so that $\varphi_f = \varphi_x$. Then the mass of exporting firms is identical to that of firms who engage in cross-border hiring, so that for country $i$, $M_x^i = M_f^i$.

(26) Note also that this assumption implies that

$$\int_{\varphi_d^i}^{\infty} r_d^i(\varphi) \, dG(\varphi) = \int_{\varphi_d^i}^{\infty} w_d^i(\varphi) l_d^i(\varphi) \, dG(\varphi)$$

(3.26)

$$\int_{\varphi_x^i}^{\infty} r_x^i(\varphi) \, dG(\varphi) = \int_{\varphi_x^i}^{\infty} w_f^i(\varphi) l_f^i(\varphi) \, dG(\varphi)$$

(3.27)

Then we have a system of ten unknowns ($U_d^i, U_f^i, M_x^i, M_f^i, M^i$ for all $i \epsilon \{H, F\}$) and ten equations (24-27), yielding solutions to these variables in terms of firm revenue, wages, policy functions, and aggregate expenditure. All of these have been solved for already, with the exception of expected worker income in each labor market. This will be pinned down when I embed the model in general equilibrium in the next section.

I also define $U^i$ to be the total measure of workers in country $i$ seeking employment.
\[ U^i \equiv U^i_d + U^i_f \]  

(3.28)

**General Equilibrium**

In this section, I embed the model in general equilibrium, determining expected worker income \( \omega \), aggregate income \( \Theta \), and prices. Here, I impose the assumption that the economy has only a single sector. However, as demonstrated by Helpman, Itskhoki, and Redding (2010), the analysis generalizes in a straightforward manner to the case in which there are multiple sectors in the economy, when each sector contains a continuum of horizontally-differentiated varieties. It will be shown that under this framework, expected worker income will change as a result of a liberalization of trade. Consequently, the results of this section have important implications for how the unemployment rate responds to trade liberalization.

All fixed costs are denominated in terms of the aggregate consumption index \( Q \). An implicit assumption here is that these fixed costs require use of output of each differentiated variety in the exact same way that consumers demand each variety. I choose the aggregate consumption index in \( H \) as the numeraire so that \( P = 1 \). Then the equations for the demand shifter in \( F \) and total expenditure in \( F \) yields the price index in \( F \). Since \( P = 1 \), it follows that

\[ A = R^{1-\rho} \]  

(3.29)

and

\[ R = Q \]  

(3.30)

So we have
\[ A = Q^{1-\rho} \] (3.31)

From the ZCP condition and equations (22) and (25), we then have the following equation in \( Q \) and \( \omega \).

\[
Q = \frac{f_d}{\gamma(1-\rho)} \left( \frac{\gamma\rho\varphi_d}{\alpha^d_0 \left[ \frac{\omega}{w_d} \right]^{\alpha^d_1}} \right)^{\frac{\rho}{\rho-1}} \gamma^{\rho-1} \frac{1}{\rho(\alpha^d_1+1)-1} \] (3.32)

Also, there is an additional market clearing condition relating the aggregate consumption index \( Q \) and expected worker income \( \omega \) which states that aggregate expenditure must equal aggregate income, where I have replaced \( R \) with \( Q \):

\[
\gamma Q = U \omega \] (3.33)

Then (32) and (33) determine \( Q \) and \( \omega \).

\[
Q = \frac{U}{\gamma} \left[ \frac{f_d}{\gamma(1-\rho)} \left( \frac{\gamma\rho\varphi_d}{\alpha^d_0 \left[ \frac{\omega}{w_d} \right]^{\alpha^d_1}} \right)^{\frac{\rho}{\rho-1}} \right] \gamma^{\rho-1} \frac{1}{\rho(\alpha^d_1+1)-1} \] (3.34)

Since \( \varphi_d \) is given by (26), \( w_d \) is determined in (20), and \( U \) is given by (29), this determines \( \omega \) and \( Q \) as functions of the model parameters. Since \( 0 < \rho < 1 \) and \( \alpha^d_1 > 0 \), expected worker income \( \omega \) is positively related to the domestic wage \( w_d \) and the domestic cutoff productivity \( \varphi_d \). The intuition for this is straightforward: a higher domestic cutoff productivity means that the minimum and average productivity of producing firms is higher, and hence the wages that workers earn should be higher, which means that workers earn more income on average. Aggregate income is then the expected worker income given above multiplied by the supply of labor in the economy.
3.2.1 Results

Effects of Trade Liberalization on Productivity Cutoff Levels

In this section, I utilize the results of Melitz (2003) in order to determine the qualitative response of the equilibrium values of the cutoff productivity levels respond to trade liberalization. The standard prediction of Melitz (2003), under the assumptions of firm heterogeneity and fixed entry and export costs, is that when a country experiences trade liberalization, there is a reallocation of resources to more productive firms. Product markets become more competitive, forcing less productive and smaller firms to exit, leaving larger more productive firms to produce more. As a result, the average productivity level of producing firms increases. Therefore, less productive firms are forced to exit and the minimal productivity level $\varphi_d$ of firms producing in equilibrium rises. The assumptions central to these results hold in this framework, so that $\varphi_d$ increases after liberalization in this model as well. The mathematical reasoning has been relegated to the appendix.

Given that $\varphi_d$ increases unambiguously when an economy experiences liberalization, I now determine how the other productivity cutoff levels, $\varphi_x$ and $\varphi_f$, are affected by liberalization. To do this, I express $\varphi_d$ as a function of $\varphi_x$, and $\varphi_x$ as a function of $\varphi_f$. Taking the ratio of both sides of the ZCP and the EC conditions yields the following.

\[
\frac{\gamma(1 - \rho) r_d(\varphi_d)}{\gamma(1 - \rho) r_x(\varphi_x)} = \frac{f_d}{f_d + f_x} \quad (3.35)
\]

Using the production function, the first order conditions of the firm’s problem, and equations (9) and (16), this yields

\[
\varphi_d(\varphi_x) = \varphi_x \left( \frac{f_d B}{f_d + f_x} \right)^{\frac{\nu - \rho}{\nu}} \quad (3.36)
\]
Here,

\[ B = \frac{A^k}{A} \left( \frac{[\gamma(\varphi_x) - 1]}{\tau(l_f(\varphi_x))} \right)^\rho \]

Thus, I have expressed \( \varphi_d \) as a function of \( \varphi_x \). From (36) we can see that there is a linear relationship between \( \varphi_d \) and \( \varphi_x \). Then \( \varphi_x \) also increases (by the same proportion as \( \varphi_d \)) when the economy opens to trade. I now perform the same operations to produce \( \varphi_x \) as a function of \( \varphi_f \). Taking the ratio of both sides of the EC and FHC conditions imply the following relationship.

\[ \frac{\gamma(1 - \rho)\tau_x(\varphi_x)}{\gamma(1 - \rho)\tau_x(\varphi_f)} = \frac{f_x}{f_f} \]

Using the first order conditions and equations (10) and (15) yields the following.

\[ \varphi_x(\varphi_f) = \left( \frac{f_x}{f_f} \right)^{1-\rho} \left( \frac{\tau(\varphi_x)}{\Delta(\varphi_x) - 1} \right)^{1-\rho} \frac{\varphi_f}{\tau(l_f(\varphi_f))} \]

(3.37)

Thus, I have expressed \( \varphi_x \) in terms of \( \varphi_f \). (It is worth noting that in the expressions for \( \tau(\varphi_x) \) and \( \Delta(\varphi_x) \), \( \varphi_x \) does not appear). However, from (37) it is not immediately clear how \( \varphi_x \) depends on \( \varphi_f \). To determine how \( \varphi_f \) must change after liberalization, I take the derivative of \( \varphi_x(\varphi_f) \) with respect to \( \varphi_f \).

\[
\frac{d\varphi_x(\varphi_f)}{d\varphi_f} = \left( \frac{f_x}{f_f} \right)^{1-\rho} \left( \frac{\tau(\varphi_x)}{\Delta(\varphi_x) - 1} \right)^{1-\rho} \left( \varphi_f \eta^{1-2\rho} \frac{1}{1 - 2\rho} \left[ 1 + l_f(\varphi_f) \right]^{\frac{2+\rho}{2-\rho}} \frac{dl_f(\varphi_f)}{d\varphi_f} + \frac{1}{\tau(l_f)} \right)
\]

Since it was assumed that \( \eta > 1 \) and \( \rho > 1/2 \), the only term in the above expression whose sign is not immediately clear is \( \frac{dl_f(\varphi_f)}{d\varphi_f} \). The intuition for why \( \frac{dl_f(\varphi_f)}{d\varphi_f} < 0 \) at \( \varphi = \varphi_f \) is as follows. A higher \( \varphi_f \) means that for foreign hiring to be profitable, a higher firm productivity level is required, so less firms hire abroad in equilibrium. Then for a firm with \( \varphi \geq \varphi_f \), it is still profitable to hire workers from abroad and now the foreign labor market is less tight, making it easier for them to hire foreign workers. Therefore,
the measure of foreign workers hired by a firm with $\varphi > \varphi_f$ would not decrease if $\varphi_f$ increased. Therefore, I conclude that $\frac{d \varphi_f(\varphi_f)}{d \varphi_f} \geq 0$. Since $\eta > 1$ and $\rho > 1/2$, all other terms are also positive, implying that $\frac{d \varphi_x(\varphi_f)}{d \varphi_f} > 0$. It was shown that $\varphi_x$ increases after liberalization, and therefore $\varphi_f$ does as well. Thus, trade liberalization causes all productivity cutoff levels to.

**Effects of Trade Liberalization on Wages and Unemployment**

I now analyze how wages respond to trade liberalization. As was described above in the general equilibrium section of this paper, both the domestic and foreign wages, $w_d$ and $w_f$, unambiguously increase in response to an increase in the cutoff productivities, irrespective of firm productivity. The reason for this is that higher cutoff productivities imply only the most productive firms survive in equilibrium, and workers employed by these firms are compensated more. Hence, the increase in productivity levels that occurs with trade liberalization (described in the previous section) causes the wages of both domestic and foreign workers to unambiguously increase. It is worth noting that the increase in wages affected by trade liberalization does not necessarily result in an increase in expected worker income $\omega$ because of frictions in the labor market: from (20) we can see that the positive effect on expected worker income of the increase in wages, may be counteracted by a decrease in labor market tightness.

For $i, k \in \{H, F\}$ and $k \neq i$, the sectoral unemployment rate in the domestic and foreign labor markets and the total sectoral unemployment rate in country $i$, denoted $u^i_d$, $u^i_f$, and $u^i$ respectively, are given by

$$u^i_d \equiv 1 - x^i_d = 1 - \frac{\omega^i}{w^i_d}, \quad u^i_f \equiv 1 - x^i_f = 1 - \frac{\omega^i}{w^i_f}$$

(3.38)

$$u^i \equiv 1 - x^i = \frac{U^i_d u^i_d}{U^i_d + \frac{U^i_f}{U^i_f} u^i_f}$$
Recall that $x_d^i$ and $x_f^i$ denote the market tightness of the domestic and foreign labor markets in country $i$, and were defined in (9). Equations (42) show that the total unemployment rate in country $i$ is a weighted average of the unemployment rates in the domestic and foreign labor markets in that country, where the weights are simply the proportion of all unemployed workers looking for employment in each labor market.

I now analyze the effects on the sectoral unemployment rate of liberalization under the framework I have outlined above. Using the results from sectoral equilibrium, it will be seen that, like in Helpman, Itskohoki, and Redding (2010), there are two channels through which liberalization affects change in the sectoral unemployment, which I call the expected income and wage effects. The latter of these channels is substantially different in Helpman, Itskohoki, and Redding (2010); in fact, it is not really a wage effect at all.

The first channel through which the openness of a country affects its sectoral unemployment rate is through the worker’s expected income $\omega$. Recall from the general equilibrium analysis that expected worker income is increasing in both the domestic wage $w_d$ and the domestic cutoff productivity level $\varphi_d$. Also recall that when a country experiences liberalization, both $\varphi_d$ and $w_d$ increase. Hence, liberalization also increases expected worker income $\omega$. From (42) it can then be seen this increase in $\omega$, ceteris paribus, reduces the sectoral unemployment rate $u$. This is the first channel through which liberalization affects the sectoral unemployment rate, and is present in the analysis of Helpman, Itskohoki, and Redding (2010).

The second channel – which I call the wage effect on unemployment – through which liberalization affects the unemployment rate of a country differs from that of Helpman, Itskohoki, and Redding (2010), and represents the contribution of this paper. As was shown above, liberalization causes an increase in the average productivity
of surviving firms, and hence, the cutoff productivities \((\phi_d\text{ in particular})\) increase. From (18), we can see that both wages are a function of the productivity cutoffs, i.e. \(w_d = w_d(\phi_d, \phi_f)\) and \(w_f = w_f(\phi_d, \phi_f)\). The increase in both \(\phi_d\) and \(\phi_f\) that results from liberalization causes the wages to increase. The reason for this is that higher cutoff productivities mean that the average productivities of firms who produce and export in equilibrium are higher. This higher productivity implies that workers will be compensated more, hence the higher wages. From (19) and (33), this then implies that the tightness in both the domestic and foreign labor markets decreases, and the sectoral unemployment rate increases in both the domestic and foreign labor markets.

The increase in \(w_d\) and \(w_f\) causes the sectoral unemployment rate in the domestic labor market \(u_d\) and that of the foreign labor market \(u_f\) to increase, respectively. As a result, the total sectoral unemployment rate \(u\) increases. Thus, this second effect of liberalization on the sectoral unemployment rate is realized entirely through the effect of opening to trade on wages.

The reason that the expected income and wage effects can be separated in this model is a direct result of labor market frictions. If there were no frictions, an increase in wages that results from liberalization would cause expected worker income to increase as well— the two variables would move in the same direction. However, the presence of labor market frictions implies that these variables need not be perfectly correlated: indeed, expected worker income depends on labor market tightness as much as it does the wage.

An important result from this analysis is that the literature may understate the positive wage effect of trade on unemployment. In particular, the wage effect on unemployment is exacerbated by the increase in the wage of foreign workers \(w_f\). The drop in market tightness of the foreign labor market induced by trade liberalization strengthens the positive effect on the unemployment rate. Jobless workers who would
have been hired by less productive firms before liberalization, are now left unemployed because of the smaller availability of jobs. The wage effects on the unemployment rate under this framework imply that the change in the unemployment rate after liberalization is even greater when the model allows for foreign hiring. Hence, models which ignore foreign hiring of labor underestimate the effects of liberalization on unemployment.

In Helpman, Itskhoki, and Redding (2010), there are two sources of unemployment. A worker is unemployed either because he is not matched with a firm or because the match-specific productivity is below the threshold. The latter source of unemployment comes from the fact that there are complementarities between firm and worker productivity. As a result, firms have incentive to screen workers (after contacting them but before hiring them) to retrieve information about what their contribution to total firm productivity would be. This yields a threshold match-specific productivity below which the firm does not hire the worker. Workers who are not hired remain unemployed; hence this second source of unemployment originates from the hiring rate. In summary, more intensive screening that results from an opening to trade reduces the hiring rate, thereby increasing the unemployment rate.

However, in this model, productivity is entirely firm-specific, and so the second source of unemployment that exists in Helpman, Itskhoki, and Redding (2010) described above is absent this model. Although unemployment is modeled more parsimoniously, this allows me to build a tractable model of cross-border hiring, yielding a more comprehensive picture of unemployment. The lack of match-specific productivity means there is no incentive for firms to screen workers that they contact. Indeed, workers are homogeneous. Therefore, any worker contacted by a firm is hired: the hiring rate is one for all firms. Accordingly, a worker is unemployed only because he is not matched with a firm. If a firm’s productivity is below the domestic production
threshold $\varphi_d$, it immediately exits the market and does not produce. Of course, firms that do not produce do not hire labor, and so firm exit that occurs due to a low productivity draw naturally affects market tightness: less workers are hired, increasing the sectoral unemployment rate. Thus, this framework yields two channels through trade liberalization affects the sectoral unemployment rate, each of which has an opposing effect. Therefore, the overall effect of liberalization on the unemployment rate is ambiguous.

While this ambiguity is consistent with the lack of consensus regarding the relationship between unemployment and trade (see Davidson and Matusz (2009)), a reasonable parameterization would yield a strong prediction of the qualitative response of the overall unemployment rate to liberalization. If the proportional increase in the domestic and foreign wage is greater than that of expected worker income, then the net effect on the unemployment rate of the domestic and foreign labor markets, respectively, is positive. The magnitude of these changes ultimately determines the overall effect on the country’s total unemployment rate. Clearly, the parameterization of the model is central to determining the qualitative response of unemployment to liberalization which, in principle, could be determined by performing counterfactuals, given a reasonable parameterization. Hence, this paper produces a framework which can yield a precise and comprehensive prediction of the response of the unemployment rate to trade liberalization.

### 3.3 Conclusion

In this paper, I develop a novel framework for examining the relationship between international trade and unemployment, which incorporates cross-border hiring by firms. I show that there are two channels through which trade liberalization affects unemployment. The first is through the expected income of a worker in the country:
trade liberalization pushes the unemployment rate down through a rise in expected worker income. The second result of this paper is that trade liberalization pushes the unemployment rate up via a rise in wages. Jobless workers who would have been hired by less productive firms before liberalization, are now left unemployed because of the smaller availability of jobs.

By allowing for cross-border hiring, I obtain a more comprehensive and descriptive picture of the effects of trade liberalization on unemployment. This framework is tractable and demonstrates that cross-border hiring has significant implications for the magnitude of the response of unemployment to trade liberalization. Cross-border hiring enhances the quantitative effect of liberalization on unemployment. The fact that the unemployment rate in a country depends in part on the hiring behavior of foreign firms magnifies the wage effect on unemployment. Hence, the framework I outline in this paper shows that cross-border hiring enhances the sensitivity of the unemployment rate to liberalization. Thus, this paper demonstrates that models in the literature which ignore cross-border hiring likely underestimate the upward force of trade liberalization on unemployment.

This paper also outlines conditions on the model parameters under which the qualitative response of the unemployment rate to liberalization will be positive. Hence, this paper lends itself to future research by laying the foundation for future calibration exercises and empirical surveys to make definitive conclusions about the model predictions, and the relationship between international trade and unemployment.
Appendix

A1. Agency Problem

Each firm has some ability to enforce debt repayment. Firm $i$ can pledge at most $\theta_{i,i-1}$ fraction of its end-of-period revenue repay the trade credit $\tau_{i-1}$ from its supplier. If the firm repudiates the trade credit contract, it loses $\theta_i p_i x_i$, and keeps the remainder for itself. To protect itself from this possibility, the supplier $i - 1$ takes care that the loan size does not incentivize its customer to repudiate the contract. Thus, there is an incentive constraint on the trade credit from $i - 1$ to $i$, which states that the payoff to $i$ of repudiating the contract does not exceed the payoff of collecting all of its receivables and paying the trade debt.

$$(1 - \theta_i) p_i x_i \leq p_i x_i - \tau_{i-1}$$

Rearranging terms, I call this firm $i$’s borrowing constraint.

$$\tau_{i-1} \leq \theta_i p_i x_i \tag{3.39}$$

Since trade credit must satisfy this constraint, firms never have enough incentive to default on their trade credit contracts in equilibrium.

A2. Borrowing and Lending Decisions

(The following results hold for an infinite horizon setting). Let $\delta$ denote the rate at which the firm discounts end-of-period payouts, and let $r^R$, $r^P$, and $r^B$ respectively denote the interest rates on accounts receivables, accounts payables, and bank debt. Let $d_{i0}$ and $d_{i1}$ respectively denote the payout to the firm at the beginning and end of the period. It follows that
\[ d_{i0} = p_i x_i - \tau_i - (p_{i-1} x_{i-1} - \tau_{i-1}) - w n_i + b_i \]

\[ d_{i1} = \tau_i (1 + r^R) - \tau_{i-1} (1 + r^P) - b_i (1 + r^B) \]

At the beginning of the period, the firm must make a net cash payment of \( p_{i-1} x_{i-1} - \tau_{i-1} \) to its supplier, and \( w n_i \) to the household. In addition, it receives a cash-in-advance payment of \( p_i x_i - \tau_i \) from its customer along with a cash loan \( b_i \) from the bank. At the end of the period, firm \( i \) must repay its loans with interest, and receives repayment for its trade credit with interest.

The firm’s objective is to choose how much to borrow from its supplier and the bank, and how much to lend to its customer to maximize its discounted payout. Because the firm is a price-taker, it does not internalize the effect that its borrowing and lending decisions have on the demand it faces from its customer, nor does it internalize its supplier’s liquidity needs. Therefore, the problem of the firm is to choose \( \tau_{i-1}, \tau_i, \) and \( b_i \) taking prices and quantities of output as given, to maximize its discounted payouts subject to its borrowing constraints.

\[
\max_{\tau_{i-1}, \tau_i, b_i} d_{i0} + \delta d_{i1} \\
\text{s.t. } \tau_{i-1} \leq \theta_i p_i x_i \\
\quad b_i \leq B_i p_i x_i + \alpha \tau_i
\]

The firm’s first order conditions are

\[
\delta (1 + r^R) + \alpha \lambda^B = 1
\]
\[ \delta (1 + r^P) + \lambda^T = 1 \]

\[ \delta (1 + r^B) + \lambda^B = 1 \]

where \( \lambda^T \) and \( \lambda^B \) denote the shadow values of trade and bank credit, respectively.

Suppose that the interest rates for accounts payables and receivables are equal, so that \( r^R = r^P \). Then it follows that

\[ \lambda^T = \alpha \lambda^B \]

Because \( \alpha \geq 0 \), we have that \( \lambda^T > 0 \) if and only if \( \lambda^B > 0 \). In words, firm \( i \)'s trade credit borrowing constraint is binding if and only if its bank credit constraint is binding.

Now suppose that \( \frac{1}{\delta} > 1 + r^B \). (A sufficient condition for this is that \( r^B = 0 \) and \( \delta < 1 \).) This implies that \( \lambda^B > 0 \). Intuitively, if the interest rate on bank debt is sufficiently low, then the firm will always want to borrow more from the bank, and the bank borrowing constraint always binds. In equilibrium, all firms are maxing out their bank credit, and borrowing the maximum from their suppliers and lending the maximum to their customers. Under these conditions, each firm will want to borrow the maximum from its supplier and lend the maximum to its customer because the interest paid on trade debt is same as that received on trade credit, and the firm can relax its bank borrowing constraint because trade credit is collateralizable.

In reality, firms often collateralize their accounts receivable to borrow from financial intermediaries. Burkart and Ellingsen (2004) also find that this assumption is critical to explain why liquidity-constrained firms often grant delayed payment terms to their customers. Omiccioli (2005) examines empirically the prevalence of collat-
eralizing trade credit using data on Italian firms. In the remainder of the paper, I assume that \( \delta < 1 \) and \( r^R = r^P = r^B = 0 \) for simplicity. These conditions are sufficient that all firm borrowing constraints bind in equilibrium.

Nevertheless, binding borrowing constraints are not critical for the results. The qualitative results go through for sufficiently large liquidity shocks, even when borrowing constraints are not binding in equilibrium. As for the quantitative results, I assess how sensitive they are to these assumptions by checking their robustness to varying the value of \( \alpha \). The results are summarized in the text.

**A3. Simple Model Solution**

Solved in closed-form recursively, starting with the final firm in the chain, firm M.

**Firm M**

Recall that firm M collects none of its sales from the household up front (does not give the household any trade credit, \( \tau_M = 0 \)). Then its problem is to choose its input purchases, loan from the bank, and the trade credit loan from M-1, to maximize its profits, subject to its cash-in-advance, supplier borrowing, and bank borrowing constraints.

\[
\max_{n_M, x_{M-1}, b_M, \tau_{M-1}} \quad p_M x_M - wn_M - p_{M-1} x_{M-1}
\]

\[
s.t. \quad wn_M + p_{M-1} x_{M-1} \leq b_M + \tau_{M-1} + p_M x_M - \tau_M
\]

\[
b_M \leq B_M p_M x_M + \alpha \tau_M
\]

\[
\tau_{M-1} p_{M-1} x_{M-1} \leq \theta_{M,M-1} p_M x_M
\]
Recall that the firm does not collect any cash-in-advance from the household, so that its trade credit $\tau_M = 0$. Also recall that its borrowing constraints () and () bind in equilibrium, so that the problem can be rewritten

$$\max_{n_M, x_{M-1}, b_{M-1}, \tau_{M-1}} p_M x_M - wn_M - p_{M-1} x_{M-1}$$

s.t. $wn_M + p_{M-1} x_{M-1} \leq \chi_M p_M x_M$

where

$$\chi_M = \theta_{M,M-1} + B_M$$

Notice that $\chi_M$ is given by exogenous parameters.

If firm $M$ is unconstrained in equilibrium, then the optimality conditions equate the marginal cost of each type of input with the marginal revenue.

$$w = \eta_M \frac{p_M x_M}{n_M}$$ (3.40)

$$p_{M-1} = (1 - \eta_M) \frac{p_M x_M}{x_{M-1}}$$ (3.41)

Firm M's expenditure in inputs is then

$$wn_M + p_{M-1} x_{M-1} = (\eta_M + (1 - \eta_M)) p_M x_M$$ (3.42)

Then firm 3 is then unconstrained in equilibrium if and only if its expenditure at its unconstrained optimum is less than its liquidity at this optimum.

$$p_M x_M < \chi_M p_M x_M$$ (3.43)
i.e.

\[ \chi_M > 1 \]

If firm M is constrained in equilibrium, then its binding cash-in-advance pins down its level of output. The only choice left to make is how much labor to hire \( n_M \) versus how much intermediate goods \( x_{M-1} \) to purchase, given its level of output \( x_M \). Because \( \chi_M \) is independent of M’s choice of \( n_M \) and \( x_{M-1} \), the problem of maximizing profits subject to the binding cash-in-advance is equivalent to minimizing its expenditure \( n_M + x_{M-1} \) subject to producing \( x_M \). Thus, it solves the following cost-minimization problem.

\[
\min_{n_M, x_{M-1}} wn_M + p_{M-1} x_{M-1}
\]

s.t. \( x_M = z_M n_M^{\eta_M} x_{M-1}^{(1-\eta_M)} \)

Then firm M’s optimality condition equates the ratio of expenditure on each input with the ratio of each input’s share in production.

\[
\frac{wn_M}{p_{M-1} x_{M-1}} = \frac{\eta_M}{(1 - \eta_M)} \quad (3.44)
\]

Using this, we can rewrite M’s binding cash-in-advance as

\[
wn_M \left(1 + \frac{(1 - \eta_M)}{\eta_M}\right) = \chi_M p_M x_M \quad (3.45)
\]

Rearranging yields

\[
w = \eta_M \chi_M \frac{p_M x_M}{n_M}
\]
Combining \( () \) with its analog \( () \) in the unconstrained case, we can see that

- if \( \chi_M > r_M \) (i.e. if firm \( i \) is unconstrained in equilibrium)
  
  \[ w = \eta_M \frac{p_M x_M}{n_M} \]

- otherwise
  
  \[ w = \eta_M \chi_M \frac{p_M x_M}{n_M} \]

These two cases imply that we can write

\[ w = \phi_M \eta_M \frac{p_M x_M}{n_M} \]

where

\[ \phi_M \equiv \min\{1, \chi_M\} \]

\( \phi_M \) represents the distortion in firm \( M \)'s optimal labor usage due to its cash-in-advance. Financial frictions introduce wedge between firm’s marginal benefit and cost of production. The wedge between these two objects is increasing in the tightness \( \chi_M \) of \( M \)'s constraint, and decreasing in the returns-to-scale of firm \( M \)'s production function.

**Firm M-1**

Given firm \( M \)'s solution, we can proceed to firm M-1’s problem.

\[
\max_{n_{M-1}, x_{M-2}, \tau_{M-2}} p_{M-1} x_{M-1} - wn_{M-1} - p_{M-2} x_{M-2}
\]

s.t. \( wn_{M-1} + p_{M-2} x_{M-2} \leq \chi_{M-1} p_{M-1} x_{M-1} \)

where
\[ x_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - (1 - \alpha) \frac{\tau_M}{p_{M-1}x_{M-1}} \]

The binding borrowing constraint implies

\[ x_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - (1 - \alpha) \frac{\theta_{M,M-1} p_M x_M}{p_{M-1}x_{M-1}} \]

And () and () imply that \( \frac{p_M x_M}{p_{M-1}x_{M-1}} = \frac{1}{\phi_M \omega_{M,M-1}(1 - \eta M)} \). Therefore,

\[ x_{M-1} = \theta_{M-1,M-2} + B_{M-1} + 1 - \alpha \frac{\theta_{M,M-1}}{\phi_M (1 - \eta M)} \]

Since \( \phi_M \) is given by (), this is a closed-form expression for \( x_{M-1} \). Note that, since \( \phi_M \) depends on \( x_{M} \), \( x_{M-1} \) is an increasing function of \( x_{M} \); this interdependence of cash-in-advances comes from the trade credit relationship between M and M-1.

Given \( x_{M-1} \), the solution to firm M-1’s problem takes the same form as that of firm M. (Note that \( x_{M-1} \) does not depend directly on M-1’s choice of \( n_{M-1} \) versus \( x_{M-2} \). Therefore, when constrained in equilibrium, M-1 will solve the analogous cost-minimization problem as M to maximize profits.) The cash-in-advance places a wedge \( \phi_{M-1} \) between the marginal benefit of hiring labor and the marginal cost

\[ w = \phi_{M-1} \eta_{M-1} \frac{p_{M-1} x_{M-1}}{n_{M-1}} \]

Given the above expressions for \( x_{M-1} \) and \( x_{M} \), the the wedge \( \phi_{M-1} = \min 1, \) \( x_{M-1} \) is a closed-form expression.

**Equilibrium:** Each other firm’s problem is symmetric. Continuing recursively, I obtain the closed-form solution for each firm. To summarize, I have, for each firm \( i \)

\[ w = \phi_{i} \eta_{i} \frac{p_{i} x_{i}}{n_{i}} \]

where
\[ \phi_i = \min\{1, \chi_i\} \quad \text{and} \quad \chi_i = B_i + \theta_i + 1 - \theta_{i+1} \frac{1}{\phi_{i+1}\omega_{i+1}(1 - \eta_i)} \]

Market clearing conditions are given by

\[ C = Y \equiv x_M, \quad N = \sum_{i=1}^{M} n_i \]

Given these expressions, the task is to write each \( n_i \) as a function of aggregate output \( x_M \), starting with firm M-1. From the firm optimality conditions, we have the following three expressions:

\[ wn_{M-1} = \phi_{M-1}\eta_{M-1}p_{M-1}x_{M-1}, \quad wn_M = \phi_M\eta_MP_Mx_M, \quad p_{M-1}x_{M-1} = wn_M \frac{(1 - \eta_M)}{\eta_M} \]

Combining these yields \( n_{M-1} \) as a function of \( x_M \).

\[ wn_{M-1} = \phi_M\phi_{M-1}\eta_{M-1}\omega_{M,M-1}(1 - \eta_M)p_Mx_M \]

Continuing recursively, we can write \( n_i \) as a function of \( x_M \), for each \( i \) (LEFT OFF HERE)

\[ wn_i = p_Mx_M \left( \prod_{j=i}^{M} \phi_j \right) \left( \prod_{j=i}^{M-1} \omega_{j+1,j}(1 - \eta_j) \right) \eta_i \]

The household’s preferences and optimality conditions imply

\[ w = \frac{V'(N)}{U'(x_M)} = x_M \]

Let good M be the numeraire. Combining (5) with (4) yields a closed-form expression for each firm’s labor use.
Recall that the production functions imply that aggregate output can be written

Then (\ref{eq:aggregate_output}) and (\ref{eq:aggregate_output_2}) yield a closed-form expression for aggregate output.

**A4. Production Influence Vector**

\[
\bar{v} = \begin{bmatrix}
v_1 & v_2 & v_3 & \cdots & v_M \\
0 & v_1 & v_2 & \cdots & v_M \\
0 & 0 & v_1 & \cdots & v_M \\
\vdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & v_1 \\
\end{bmatrix} 1_{Mx1}
\]

\(v_i = \bar{\eta}_i\) captures downstream propagation (supply effects). But misses upstream demand effects. Total effect is sum \(\sum_{j=1}^{i} v_i\)

\[
v' = \left[ \eta_i \prod_{k=2}^{M} (1 - \eta_k) \omega_{k,k-1} \cdots \eta_j \prod_{k=j+1}^{M} (1 - \eta_k) \omega_{k,k-1} \cdots \eta_M \right] = \]

**A5. Proof of Proposition 1**

**Proof:** From the definition of \(\chi_i\) (4) and the unnumbered equation after (6), we have

\[
\phi_i = \min \left\{ 1, \frac{1}{r_i} \left( B_i + \theta_{i,i-1} - \theta_{i+1,i} \frac{1}{\phi_{i+1}\omega_{i+1,i}(1 - \eta_{i+1})} \right) \right\}
\]

Here, \(r_i = 1\) denotes firm \(i\)'s returns-to-scale. It follows that

\[
\frac{d \phi_{i-1}}{d B_i} = \begin{cases} 
\frac{1}{r_i \phi_{i-1}\omega_{i-1,i}(1 - \eta_{i-1})} > 0 & \text{if } \phi_{i-1} < 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
\frac{d \phi_j}{d B_i} = 0 \forall j > i \text{ and } \frac{d \phi_j}{d B_i} = \frac{1}{r_i} > 0 \text{ for } j = i
\]
Putting these cases together, we can write \( \frac{d \log \phi_j}{d B_i} \) for any \( j \).

\[
\frac{d \log \phi_j}{d B_i} = \begin{cases} 
\frac{1}{r_i} > 0 & \text{if } j = i \\
\frac{1}{\phi_j \mu_j} \frac{\theta_{kj}}{\phi_k \omega_{kj}(1-\eta_k)} \frac{d \phi_k}{d B_i} \geq 0 & \forall k \text{ if } j < i \\
0 & \text{otherwise}
\end{cases}
\]

It follows that \( \frac{d \log \phi_i}{d B_i} \geq 0 \) and \( \frac{d}{d \theta_{ij}} \left( \frac{d \log \phi_i}{d B_i} \right) \geq 0 \).

**A6. Solution Procedure in General Model**

Claim: solution procedure takes same form in general model as in stylized.

Firm \( i \)'s problem is to maximize profits subject to its cash-in-advance.

\[
\max_{n_i, \{x_{is}\}_{s \in I}} p_i x_i - w n_i - \sum_{s=1}^{M} p_s x_{is}
\]

\[
w n_i + \sum_{s=1}^{M} p_s x_{is} \leq \chi_i p_i x_i
\]

where \( \chi_i \) denotes the tightness of \( i \)'s cash-in-advance.

\[
\chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i}
\]

If firm \( i \) is unconstrained in equilibrium, \( i \). Consider the case when \( i \) is constrained in equilibrium. For profit maximization to be equivalent to minimizing its expenditure subject to producing \( x_i \), we must have that \( \chi_i \) is independent of \( i \)'s choice of \( n_i \) and \( x_{is} \) for each \( s \) (or that firm \( i \) does not internalize these effects). First, suppose that \( \chi_i \) is independent of this choice. I will later verify that this indeed the case.

Firm \( i \)'s solution takes the same form as in the simple version of the model. The equilibrium system of \( M^2 + 5M + 2 \) nonlinear equations (for every \( i \) and \( j \)
\[ x_i = z_i^\eta_i n_i^{\eta_i} \left( \prod_{j=1}^{m} x_{ij}^{\omega_{ij}} \right)^{1-\eta_i} \]

\[ \phi_i = \min \left\{ 1, \frac{1}{r_i} \left( B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \right) \right\} \]

\[ \sum_{i=1}^{M} c_i^{\beta_i} = N^{1+\epsilon} \]

\[ n_i = \phi_i \eta_i \frac{p_i}{w} x_i \quad \quad x_{ij} = \phi_i (1 - \eta_i) \omega_{ij} \frac{p_i}{p_j} x_i \]

\[ \frac{p_i c_i}{p_j c_j} = \frac{\beta_i}{\beta_j} \quad p_1 = 1 \]

\[ N = \sum_{i=1}^{M} n_i \quad \quad x_i = c_i + \sum_{c=1}^{M} x_{ci} \]

\[ M^2 + 5M + 2 \text{ unknowns} \]

\[ \{ \{n_i, c_i, x_i, \{x_{ij}\}_{j \in I}, \phi_i, p_i\}_{i \in I}, N, w\} \]

I now verify that \( \chi_i \) is independent of \( i \)'s choice of \( n_i \) and \( x_{is} \) for all \( s \). Note that

\[ \frac{p_c x_c}{p_i x_i} = \frac{p_c x_c}{p_i x_{ci}} \frac{p_i x_{ci}}{p_i x_i} = \frac{\theta_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \nu_{ci} \]

where the second equality follows from firm \( c \)'s optimality condition for intermediate good \( i \), and from the definition of \( \nu_{ci} \). The term \( \frac{1}{\phi_c (1 - \eta_c) \omega_{ci}} \) represents the inverse of firm \( c \)'s demand for good \( i \), and is independent of \( i \)'s choice of \( n_i \) versus \( x_{is} \). The term \( \nu_{ci} \) represents firm \( c \)'s share of \( i \)'s total output, and is determined by each customer \( c \)'s optimal behavior. Thus, firm \( i \)'s choice of intermediates vs labor doesn't (directly) affect \( \chi_i \). This verifies the conjecture that, when constrained, profit maximization is
equivalent to expenditure minimization.

A7. Log-Linearized System

Stars are points around which the system is approximated. Calibrated equilibrium values.

For all $i$ and $j$

In order: firm $i$’s optimality condition for input $j$, firm $i$’s optimality condition for labor, definition of wedge $\phi_i$, household optimality condition for consumption of each good, market clearing for good $i$, production function for firm $i$, household budget constraint, labor market clearing condition, household optimality for labor versus aggregate consumption.

\[
\tilde{p}_j + \tilde{x}_{ij} = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \quad \tilde{w} + \tilde{n}_i = \tilde{\phi}_i + \tilde{p}_i + \tilde{x}_i \\
\tilde{\phi}_i \begin{cases} 
\tilde{c}_i & \text{if } \phi_i < 1 \\
0 & \text{otherwise}
\end{cases}
\]

\[
\tilde{\phi}_i^c = \frac{B_i}{\bar{r}_i \bar{\phi}_i} \hat{B}_i + \frac{\alpha}{\bar{r}_i \bar{\phi}_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \phi_c - \frac{\alpha}{\bar{r}_i \bar{\phi}_i} \sum_{c=1}^{M} \frac{\theta_{ci} \nu_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \nu_{ci}
\]

\[
\tilde{\phi}_i + \tilde{c}_i = \tilde{p}_j + \tilde{\phi}_j \quad \tilde{x}_i = \left( \frac{p_i c_i^*}{p_i x_i^*} \right) \tilde{c}_i + \sum_{c} \left( \frac{p_i x_{ci}^*}{p_i x_i^*} \right) \tilde{x}_{ci} \quad \bar{x}_i = \tilde{z}_i + \eta_i \bar{n}_i + (1 - \eta_i) \sum_{s} \omega_{is} \bar{x}_{is}
\]

\[
\tilde{w} = \sum_{i} \beta_i (\tilde{c}_i + \tilde{p}_i) \quad \sum_{i} \left( \frac{n_i^*}{N} \right) \tilde{n}_i = 0 \quad (1 + \epsilon) \bar{N} = \sum_{i} \beta_i \tilde{c}_i
\]

A8. Counterfactual

Recall the definition of $\phi_i$
Replace $\frac{p_c x_c}{p_i x_i}$ with firm c’s optimality conditions for good i yields

$$\tilde{\phi}_i = \min \left\{ 1, \frac{1}{r_i} \left( B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \frac{\theta_{ci}}{\phi_c (1 - \eta_c) \omega_{ci}} \right) \right\}$$

Log-linearizing $\tilde{\phi}_i$ yields

$$\tilde{\phi}_i = \begin{cases} \left( \frac{B^*_i}{r_i \tilde{\phi}_i} \right) \tilde{B}_i + \sum_{s=1}^{M} \left( \frac{\theta_{cs}}{\omega_{ci}(1 - \eta_c) \omega_{ci}} \right) \tilde{\phi}_c & \text{if } \phi^*_i < 1 \\ 0 & \text{otherwise} \end{cases}$$

Thus, in the full model wedges respond endogenously to direct liquidity shocks $B_i$ and to changes in its customers’ wedges $\phi_c$ through the credit linkage channel. This second term captures the propagation due to the credit linkages between firms. In performing my counterfactual, I compute the response in GDP to the aggregate liquidity shock $B_{\text{tilde}}=.01$ for all i, and then do the same by after imposing

$$\tilde{\phi}_i = \begin{cases} \left( \frac{B^*_i}{r_i \tilde{\phi}_i} \right) \tilde{B}_i & \text{if } \phi^*_i < 1 \\ 0 & \text{otherwise} \end{cases}$$

This latter exercise gives me the model’s response without propagation via the credit network. Then the marginal contribution to the change in GDP of including the credit linkages is given by the difference in ...

A9. Effect of Credit Linkages in General Model

Effect of Credit Linkages in General Model. In the model the trade credit parameters $\theta_{cs}$ show up only in the wedges $\phi_i$. Therefore, to see effect of credit network in propagating liquidity and productivity shocks, it suffices to show how $\phi_i$ responds to shocks to other industries. Recall
\[ \phi_i = \min \left\{ 1, \frac{\chi_i}{r_i} \right\} \]

where

\[ \chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_i} \]

\[ = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{p_c x_c}{p_i x_{ci}} \frac{x_{ci}}{x_i} \]

Let \( \nu_{ci} \equiv \frac{x_{ci}}{x_i} \) represent the share of \( c \) in \( i \)'s total revenue. Substituting \( c \)'s optimality condition for good \( i \) in for \( \frac{p_c x_c}{p_i x_{ci}} \) yields

\[ \chi_i = B_i + \sum_{s=1}^{M} \theta_{is} + 1 - \alpha \sum_{c=1}^{M} \theta_{ci} \frac{\phi_c (1 - \eta_c) \omega_{ci}}{\phi_{ci} (1 - \eta_c) \omega_{ci}} \nu_{ci} \]

The response in \( \phi_i \) to some shock can be summarized by the log-linearized expression for \( \phi_i \).

\[ \tilde{\phi}_i = \begin{cases} 
\tilde{\phi}_i^c & \text{if } \phi_i < 1 \\
0 & \text{otherwise} 
\end{cases} \]

where

\[ \tilde{\phi}_i^c = \frac{B_i}{r_i \phi_i} \tilde{B}_i + \alpha \frac{\theta_{ci} \nu_{ci}}{r_i \phi_i} \frac{\phi_c (1 - \eta_c) \omega_{ci}}{\phi_i (1 - \eta_c) \omega_{ci}} \tilde{\phi}_c - \alpha \frac{\theta_{ci} \nu_{ci}}{r_i \phi_i} \frac{\phi_c (1 - \eta_c) \omega_{ci}}{\phi_i (1 - \eta_c) \omega_{ci}} \tilde{\nu}_{ci} \]

and

\[ \tilde{\nu}_{ci} = \tilde{x}_{ci} - \tilde{x}_i \]

This expression says that industry \( i \)'s wedge can change either from direct liquidity shock to \( i \) (given by \( \tilde{B}_i \)), changes in the wedges of customers (given by \( \tilde{\phi}_i \)) through
credit linkages \( \theta_{ci} \), or changes in the composition of industry \( i \)'s sales (given by \( \tilde{\nu}_{ci} \) for all customers \( c \)), also through credit linkages.

**Liquidity Shock to \( j \)** Consider first a liquidity shock to industry \( j \), given by \( \tilde{B}_j < 0 \). How does this affect \( \phi_i \), and how does this effect depend on \( i \)'s credit linkages with \( j \)? From (), we can see that there are two effects. First, the shock reduces \( \phi_j \), so that \( \tilde{\phi}_j < 0 \). This pushes \( \phi_i \) down. Second, because \( i \) has \( M \) customers, \( x_{ji} \) falls by more than \( x_i \) falls. Therefore, \( j \)'s share of \( i \)'s output \( \nu_{ji} \) falls, and \( \tilde{\nu}_{ji} < 0 \). This pushes \( \phi_i \) up. The stronger is \( j \)'s downstream credit linkage \( \theta_{ji} \) with \( i \), the stronger are both of these effects.

But there is a more indirect way by which \( \phi_i \) changes in response to \( \tilde{B}_j < 0 \). The initial fall in \( \phi_i \) is transmitted to each of \( i \)'s customers \( c \) as a supply shock, causing all \( c \) to cut back on output. Then the fall in \( p_c x_c \) causes \( \phi_c \) to fall, as the amount of credit \( c \) is giving per unit of its revenue is lower. Since all industries are interconnected, industry \( c \) is also industry \( i \)'s customer. As a result, the fall in \( \phi_c \) causes \( \phi_i \) to fall via the credit linkage from \( i \) to \( c \). This fall in \( \phi_i \) effect is increasing in \( i \)'s downstream linkage with \( c \theta_{ci} \). Thus, the greater \( \theta_{ci} \) for all \( c \), i.e. the larger \( i \)'s credit out-degree, the more that \( \phi_i \) will respond to the shock to \( j \), and the larger will be the aggregate impact.

**Productivity Shock to \( j \)** Now consider an adverse productivity shock to industry \( j \), given by \( \tilde{z}_j < 0 \). This shock affects neither \( \phi_j \) nor \( \phi_i \) directly. However, it has an indirect affect on \( \phi_i \) through the composition of \( i \)'s sales \( \nu_{ji} \). In particular \( j \)'s share of \( i \)'s total output \( \nu_{ji} \) falls, and so \( \tilde{\nu}_{ji} < 0 \). This reduces the amount of trade credit per dollar of revenue that \( i \) is giving its customers, and so \( i \)'s wedge increases: \( \tilde{\phi}_i > 0 \). This effect is increasing in \( i \)'s downstream credit linkage with \( j \), \( \theta_{ji} \). Therefore, stronger credit linkages mitigate the impact of the productivity shock. This effect is
not present in the stylized model, because \( \nu_{ji} = 1 \) for \( j = i + 1 \) and 0 for all other \( j \); there is no change in the composition of \( i \)'s sales. Nevertheless, this mitigation effect is quantitatively small, as discussed in the quantitative analysis.

**A10. Identification of Productivity vs. Liquidity Shocks**

Recall the production functions, optimality conditions for labor use, and definition of the wedges. First, the employment and output of an industry are linked by the industry production function \( x_{it} = z_{it} n_{it} \left( \prod_{s=1}^{M} x_{ist}^{\omega_{is}} \right)^{1-\eta_i} \). Therefore, a change in the TFP of industry \( i \) is given by

\[
\tilde{z}_{it} = \tilde{x}_{it} - \eta_i \tilde{n}_{it} - (1 - \eta_i) \sum_{s=1}^{M} \omega_{it} \tilde{x}_{ist}
\]

The constant returns-to-scale of industry \( i \)'s production function implies that if an observed change in industry \( i \)'s output \( \tilde{x}_{it} \) from period \( t - 1 \) to \( t \) exceeds that of \( \eta_i n_{it} \left( \prod_{s=1}^{M} x_{ist}^{\omega_{is}} \right)^{1-\eta_i} \), then there must have been an increase in \( i \)'s TFP such that \( \tilde{z}_{it} > 0 \).

Industry \( i \)'s optimality condition for labor equates the ratio of its wage bill to revenue with labor's marginal product, times the wedge, i.e. \( \frac{w_{it} x_{it}}{p_{it} x_{i}} = \eta_i \phi_i \). In log-changes from period \( t - 1 \) to \( t \), this can be written as

\[
\tilde{w}_t + \tilde{n}_{it} - \tilde{p}_t - \tilde{x}_{it} = \tilde{\phi}_{it}
\]

This says that an observed change in industry \( i \)'s ratio of labor expenditure to revenue from time \( t - 1 \) to \( t \), must have come from a change in the firm's wedge \( \tilde{\phi}_{it} \) from \( t - 1 \) to \( t \).

Finally, recall the definition of industry \( i \)'s wedge.
\[ \phi_i = \min \left\{ 1, \frac{1}{r_i} \left( B_i + \sum_{s=1}^{M} \theta_{is} + 1 - (1 - \alpha) \sum_{c=1}^{M} \frac{\theta_{ci}}{\phi_c(1 - \eta_c) \omega_{ci}} \nu_{ci} \right) \right\} \]

This implies that a change in industry \( i \)'s wedges must be driven by changes in liquidity, either directly shock to \( B_i \), or through credit linkages via \( \phi_c \). In this way, the model attributes a change in the ratio of industry \( i \)'s wage bill to revenue to a liquidity shock. In a later section, I discuss the extent to which the model’s predicted liquidity shocks are correlated with some industry-level measures of credit spreads, an indication of changes in liquidity conditions computed from an independent dataset.

Because the model can track how a liquidity shock or productivity shock to one industry spills over to other industries via their credit and input-output linkages, the model can back out exactly how much of a change in an industry’s output and employment is coming from spillover effects versus a direct shock, and can identify the industry which was shocked. In this manner, for any combination of \( 2M \) observations \( \tilde{x}_{it} \) and \( \tilde{n}_{it} \), the model exactly identifies the sequence of liquidity and productivity shocks \( \tilde{B}_{it} \) and \( \tilde{z}_{it} \) faced by each industry between periods \( t - 1 \) and \( t \).

**A11. Aggregate Volatility**

Recall that the growth in industry output can be written as a function of the industry liquidity and productivity shocks. Recall that \( X_t \) is a vector of the percentage change \( \tilde{x}_{it} \) in each industry’s output at time \( t \).

\[ X_t = G_X B_t + H_X z_t \]

And the shocks \( B_t \) and \( z_t \), in turn, are composed of an aggregate and idiosyncratic components.

\[ B_t = \Lambda_B F_t^B + u_t \quad F_t^B = \gamma_B F_{t-1}^B + \iota_t^B \]
\[ z_t = \Lambda z + v_t \quad F_t^z = \gamma z_{t-1} + t_t \]

Then letting \( \Sigma_{XX} \) denote the variance-covariance matrix of \( X_t \) (and similarly for the other variables), we have

\[ \Sigma_{XX} = G_X \Sigma_{BB} G_X' + H_X \Sigma_{zz} H_X' \]

\[ \Sigma_{BB} = \Lambda_B \Sigma_{FF} \Lambda_B' + \Sigma_{uu} \]

\[ \Sigma_{zz} = \Lambda_z \Sigma_{FF} \Lambda_z' + \Sigma_{vv} \]

where \( \Sigma_{uu} \) and \( \Sigma_{vv} \) are diagonal matrices.

Aggregate manufacturing output at time \( t \) is defined as \( \Sigma_i x_{it} \). Let \( \bar{s}_t \) denote the vector of industry shares of aggregate output at time \( t \). Then the growth of aggregate output at time \( t \) is given by

\[ \bar{s}_t X_t \]

Suppose that industry shares don’t fluctuate much over time, so that \( \bar{s}_t \approx \bar{s} \) for all \( t \). Then growth in aggregate output at time \( t \) can be approximated by \( \bar{s} X_t \). Then the variance of aggregate output, i.e. aggregate volatility in the economy, is approximately given by

\[ \sigma^2 \equiv \bar{s}' \Sigma_{XX} \bar{s} = \bar{s}' G_X \Sigma_{BB} G_X' \bar{s} + \bar{s}' H_X \Sigma_{zz} H_X' \bar{s} \]

Then the contribution of aggregate liquidity shocks to aggregate volatility is given by
\[
\frac{s'G_X \left( \Lambda_B \sum_{FF} A'_B \right) G'_X \bar{s}}{\sigma^2}
\]

And the aggregate volatility generated by the credit network in propagating aggregate liquidity shocks is then given by

where \( G_{NoTC} \) maps \( B_t \) into \( X_t \) when the credit linkage channel is shut-off. Similar expressions can be derived for the contribution to aggregate volatility of idiosyncratic liquidity shocks, and aggregate and idiosyncratic liquidity shocks.

**A14. Construction of Proxy for Inter-Industry Credit Flows**

For each firm in the sample, I want a measure of its cost of goods sold (COGS) financed with accounts payable (AP) in each year \( t \), which I call its payables financing (\( PayFin \)) at time \( t \). Since a firm may repay its accounts payable irregularly, simply taking the ratio \( \frac{AP_t}{COGS_t} \) may in part reflect a spuriously high or low repayment of its accounts payable in that year. Therefore, I take a take a moving average of AP to smooth it over time. Thus, I compute firm \( f \)'s payables financing at time \( t \) as

\[
PayFin_{f,t} = \frac{5(\text{AP}_{f,t-1} + \text{AP}_{f,t})}{\text{COGS}_{f,t}}
\]

I do this only for years in which there is data for both AP and COGS for each firm. I obtain a firm-level measure of payables financing by taking the median of \( PayFin_{f,t} \) across time, to minimize effect of outliers and get a representative firm-level estimate of the average COGS financed with trade credit. Then to get an industry-level measure of payables financing, I take the median of \( PayFin_f \) across all firms \( f \) in each three-digit level NAICS industry. In this way, I obtain a measure of payables financing for each of my industries.

Raddatz (2010) uses this industry-level measure of PayFin to construct \( q_{ij} \). However, since he only uses AP data, he must impose that \( q_{ij} = q_{ik} \) for all \( j, k \). In other
words, he assumes that each industry finances the same fraction of purchases with trade credit, across all of its suppliers. This is a fairly strong assumption that he is forced to make due to the paucity of data on trade credit. However, I improve on this proxy by making use of additional data on accounts receivables to obtain a more precise and industry-pair-specific measure of $q_{ij}$.

In particular, I construct an industry-level measure of the fraction of total sales made on credit to customers, which I call the industry’s receivables lending ($\text{RecLend}$), using each firm’s accounts receivable (AR) and sales each year.

$$\text{RecLend}_{f,t} = 0.5 \left( \frac{AR_{f,t-1} + AR_{f,t}}{Sales_{f,t}} \right)$$

I then aggregate across time and across firms in each industry to obtain an industry-level measure of receivables lending.

The measure $\text{PayFin}_i$ tells me how much trade credit each industry $i$ receives from all of its suppliers collectively; it does not tell me how this breaks down across each of its suppliers. Similarly, $\text{RecLend}_i$ tells me how much trade credit each industry $i$ gives to all of its customers collectively; it does not tell me how this breaks down across each of its customers. Therefore, to construct $q_{ij}$ the fraction of industry $j$’s sales to industry $i$ made on trade credit, I take a weighted average of $\text{PayFin}_i$ and $\text{RecLend}_j$. In the next section, I consider two weighting schemes and compare their aggregate accuracy. My baseline proxy uses weights given by each industry’s total sales.

$$\hat{q}_{ij} = k_{ij} \text{PayFin}_i + k_{ji} \text{RecLend}_j, \quad k_{ij} \equiv \frac{p_i x_i}{p_i x_i + p_j x_j}$$

Therefore, a larger industry will carry more weight in determining the trade credit flows to and from it. Alternative weighting schemes, such as equal weights to both customer and supplier, do not significantly alter the results. Given my proxy $\hat{q}_{ij}$,
inter-industry trade credit flows are then proxied as

$$\hat{\tau}_{ij} = \hat{q}_{ij} p_j x_{ij}$$

In the Appendix, I discuss the conditions under which the weighting scheme $k_{ij}$ described above is optimal for constructing the proxy of trade credit flows. Intuitively, this weighting scheme minimizes the mean squared errors in the observed accounts payables of each industry, when I impose the restriction that any given industry’s lending or borrowing not vary greatly across suppliers or customers.
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