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An analysis of the methods of subtraction in a school system

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Boston University

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Hill, B.J.
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SCHOOL OF EDUCATION

Thesis

AN ANALYSIS OF THE METHODS OF SUBTRACTION IN A SCHOOL SYSTEM

Submitted by
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(B.S. in Ed., State Teachers College, Bridgewater, 1930)

in partial fulfillment of requirements for
the degree of Master of Education

1937
First reader: Guy M. Wilson, Professor of Education
Second reader: Mabel C. Pragg, Associate Professor of Education
Third reader: Franklin C. Roberts, Associate Professor of Education
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AN ANALYSIS OF THE METHODS OF SUBTRACTION
IN A SCHOOL SYSTEM

The purpose of this investigation has been to discover some facts concerning the subtraction of whole numbers in the fifth and sixth grades. This problem is a part of a larger problem in arithmetic which involves research into the cause of confusions in the fundamental processes of arithmetic.

It is not a matter of conjecture that confusions in arithmetic contribute largely to our present remedial load. In subtraction alone there are sufficient confusions present to give us an insight into what may be present in the more complicated processes. This investigation is an attempt to find out the number of different ways in which pupils subtract in examples involving the subtraction of whole numbers. The study was purposely confined to a single school system in order that a picture of a limited area might be obtained. Pupils of the fifth and sixth grades were selected because they would better show the accumulation of differences due to progress through the grades of the elementary school. No attempt was made to determine the strict reasons for such differences. Another study would need to be conducted to note such facts. This study is merely an attempt to show the extent of the confusions and to suggest a possible remedy.
An analysis of methods of subtraction is nothing new. Much work has been done. Numerous studies have been made concerning the relative merits of various methods of subtraction. A perusal of the literature of this field impresses one with the fact that these studies do not prove much else than the lack of superiority of any one method. Certain degrees of superiority are claimed for various methods depending upon the investigator and the method of investigation used. There are three main methods of subtraction: take-away, additive, and complementary. A complete description of these methods may be found on pages 5 and 6. Fallard, McClelland, and Winch (26:677) agree that the equal-additions method is superior to the decomposition method. Osborn (26:677) supports these findings. His results indicate a superiority of the take-away, equal-additions over the take-away, decomposition and the additive methods.

Mead and Sears (26:677) present experimental evidence on the relative merits of additive and subtractive methods. No superiority of methods was shown. Their study suggests a fallacy in the position that the additive method eliminates the necessity for learning a whole new set of subtraction combinations. Their results showed that the introduction of addition during the teaching of additive subtraction set up interferences, or negative transfer.

Taylor's (26:677) results showed an abandonment of the additive method in the sixth grade by pupils who were taught that method in the lower grades. Beatty's (26:677) study tends to confirm this by his finding that two-thirds of pupils taught

The number before the colon refers to the number of the reference in the bibliography and the number after the colon refers to the page.
additive subtraction switched over to take-away methods.

Buckingham's (26:678) study compared pupils in seven centers. Of the seven comparisons, six favored the take-away method over the additive. All in all the type of conclusions may be summed in the conclusion reached by Knight, Ruch, and Lutes: "All things considered, the theory underlying the additive and borrowing methods seems to be quite the equal of the theory underlying the additive and Austrian methods of subtraction." (16:168) The writers consider the theory underlying the former to be superior.

What conclusion can we draw from such an indefinite statement? This is typical of the research in the field. Yet many of us have gone ahead on the basis of these inconclusive findings and made changes which have not clarified an already befuddled atmosphere. Each new procedure has been looked upon as the panacea for all difficulties until now we have almost reached the state of having a method for each pupil. You might interpret this as the extreme in individualization of instruction. No definite superiority for any one system has been shown; therefore, we conclude that one system is as good as another.

No matter what system is used, tests show that the results achieved by pupils are unsatisfactory. These unsatisfactory results may be attributed in part to the confusion that the pupil must feel as he encounters a variety of methods as he goes from grade to grade. This leads one to suggest that one method should be used consistently by the teachers,
at least up through the sixth grade of the elementary school. (37: 203-5) This statement is further supported by one of the suggestions made as a result of a comprehensive study of the corrective load in the fundamentals in grade 6, 7, and 8. This report suggests: "The teaching of one process at a time and, in a single city, by one method only. The mixing of processes confuses the child. If teachers in subtraction . . . use a dozen different procedures among themselves, the child suffers. The purpose of teaching the basic processes in arithmetic is not mental development, nor is it to confuse and defeat the child. The purpose is to give him perfect mastery of simple tools." (33: 41)

Miss Allen's thesis, "Subtraction: Current Methods of Instruction in the United States," (1) Has proved helpful in this study. The material for her study was collected by means of a questionnaire consisting of the twelve methods of subtraction with brief explanations of each. Data were gathered from many sources in the United States. Her study showed that the one method used most extensively today in the United States is the take-away, borrowing, upward. (1: 27)

There is perhaps a need at this point to list twelve possible methods of subtraction as based on a study by W. J. Osborn. (24: 237-47)
Methods of Subtraction

1. Take-away, borrowing, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 from 13 is 6.
1 from 6 is 5.

2. Take-away, borrowing, downward.

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
13 take-away (or less) 7 is 6.
6 take-away (or less) 1 is 5.

3. Take-away, equal-additions, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 from 13 is 6.
2 from 7 is 5.

4. Take-away, equal-additions, downward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
13 take-away (or less) 7 is 6.
7 take-away (or less) 2 is 5.

5. Additive, borrowing, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 and 6 are 13, write 6.
1 and 5 are 6, write 5.

6. Additive, borrowing, downward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
13 is 7 and 6, write 6.
6 is 1 and 5, write 5.

7. Additive, equal-additions, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 and 6 are 13, write 6.
2 and 5 are 7, write 5.

8. Additive, equal-additions, downward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
13 is 7 and 6, write 6.
7 is 2 and 5, write 5.

9. Complementary, borrowing, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 from 3 I cannot take, so I take 7 from 10;
7 from 10 leaves 3, 3 and 3 are 6, write 6.
1 from 6 leaves 5, write 5.
10. Complementary, borrowing, downward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
3 take-away 7 I cannot do; 10 take-away 7 is 3, 3 and 3 are 6, write 6. 6 take-away 1 is 5, write 5.

11. Complementary, equal-additions, upward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
7 from 3 I cannot take, so I take 7 from 10, leaves 3, 3 and 3 are 6, write 6. 2 from 7 is 5, write 5.

12. Complementary, equal-additions, downward

\[
\begin{array}{c}
73 \\
-17
\end{array}
\]
3 take-away 7 I cannot do, 10 take-away 7 leaves 3, 3 and 3 are 6, write 6. 7 take-away 2 is 5, write 5.

An analysis sheet was used to find out the methods of subtraction prevalent in the fifth and sixth grades of the school system studied. The following questions were kept in mind as the study progressed: (1) How many methods of subtraction are used? (2) How widely is each method used? (3) Is each school consistent? (4) What methods are the various schools teaching? (5) Are the teachers convinced of the efficacy of the method they teach? (6) What confusions seem to exist? (7) Are the schools attaining 100% accuracy? (8) What does research reveal in regard to methods of subtraction? (9) What recommendations may be made as a result of this study? (10) If one method is taught in a school system, is there any assurance that this method will prevail?

The analysis sheet used in a study of a single school system consisted of three simple examples. The first one had a minuend of thirty-five and a subtrahend of thirty-three. This example involved simple subtraction, no borrowing, last
subtraction a zero not brought down. The second example had a minuend of twenty-six and a subtrahend of four. This one involved a gap, but no borrowing. The third example had a minuend of seven hundred twenty-three and a subtrahend of three hundred sixty-five. Double borrowing was used. These examples were arbitrarily selected. The main factor being an analysis of methods of subtraction, no attempt was made to make this a test of accuracy. The main idea was to have the examples easy enough so that the pupils would not be conscious of their (the problems') difficulty, but would be able to concentrate on the method used.

In order to make the analysis as simple as possible, an analysis of the example was placed directly at the right of each. This had a two-fold purpose: (1) To aid the pupil in his analysis of each example; (2) To determine if any methods other than those listed were used. Several additions were made to the list of possibilities as suggested by Osborn.

In the first and second examples an attempt was made to discover some information concerning the use of the terms "nothing" and "zero". While this had no particular bearing on the main problem, it was thought that the inclusion of this item might prove to be of value at some future time. After the attempt was made to observe the use of "nothing" and "zero", the results were found to be somewhat confusing. For this reason it was thought advisable not to attach any significance to the appearance of the items on the analysis sheet. Such a study might profitably be
made in connection with other arithmetical terms.

In all three examples, the reversals of the additive method were included. An example follows to illustrate this point.

742
-547

Using only the additive method, our language possibilities are as follows:

1. 7 and 5 are 12; 4 and 9 are 13; 5 and 1 are 6.
   1a. 5 and 7 are 12; 9 and 4 are 13; 1 and 5 are 6.

2. 12 are 7 and 5; 13 are 4 and 9; 6 are 5 and 1.
   2a. 12 are 5 and 7; 13 are 9 and 4; 6 are 1 and 5.

3. 7 and 5 are 12; 5 and 9 are 14; 6 and 1 are 7.
   3a. 5 and 7 are 12; 9 and 5 are 14; 1 and 6 are 7.

4. 12 are 7 and 5; 14 are 5 and 9; 7 are 6 and 1.
   4a. 12 are 5 and 7; 14 are 9 and 5; 7 are 1 and 6.

The underlined are the methods that were suggested as ones to be added to Osborn's list of ways of subtracting. These were included because: (1) Analysis of the language of subtraction in actual classroom situations revealed that these methods were being used by the pupils. The pupils in analyzing their examples recognized them as being "another" way of subtracting. (2) Today we recognize that there are one hundred primary combinations in addition. Experiments indicate that there are separate responses to 7 plus 5 and 5 plus 7. It is no longer assumed that, if a child knows 7 plus 5, he automatically knows 5 plus 7. Each combination is taught as a separate fact. Therefore, it may be assumed that this factor should receive consideration. As the number of children analyzed grew, this factor, when considered along with the others,
seemed to detract from the efficiency of the analysis sheet. This reason appeared to be of sufficient importance to warrant a discontinuance of this item in the analysis. It might be of interest, at some future time, to make a study of this one particular item.

Profiting from the mistakes made, it seems advisable, if any further use is to be made of this method of analysis, to suggest the use of a more simplified analysis sheet. While the analysis sheet used in this study proved satisfactory, the author feels that elementary school children would better understand a less detailed analysis of methods. For this reason there is presented an improved analysis sheet as a part of this study. Please note the complete absence of the "complementary" method in the revised list. It was found that not one child in the three hundred analyzed used this method. If someone is found using a method different from those listed, he may write his language below the others. This will insure less confusion for the other children being examined. Copies of both forms of analysis sheet may be found in the appendix.

One person administered the analysis sheets in order to insure uniformity of directions. An explanation of the work was given to the pupils -- children like to know what they are doing. This seemed to assure a greater readiness to assist on their part. The sheets were distributed and all were requested to write the information asked for at the top of the page. The pupils were asked not to begin until everyone had completed the first direction. The first example was

1. See page 65
then figured. Pupils were cautioned to think through the example as they did it. The first example was then immediately analyzed. Checks were placed at the left of the number of the method used. The teacher's help was enlisted at this point to see if the children were responding with a fair degree of accuracy; assuming that the teacher was somewhat familiar with the methods of subtraction in use in her room. Individual aid in checking the method used or in analyzing the procedure was given wherever necessary. Great care was exercised to get an accurate picture of the methods used. The same procedure was followed in doing the second and third examples. Pupils using methods other than those listed were requested to write an explanation of their methods directly below the others. An important point to remember in the procedure is that each example was analyzed directly after it had been figured.

Four schools were included in this study. For ease in reference these schools will be known as Schools A, E, B and D. The grades and pupils were distributed as follows:

<table>
<thead>
<tr>
<th>School</th>
<th>Grade</th>
<th>Number of Pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>3 fifth grades</td>
<td>94 pupils</td>
</tr>
<tr>
<td>School B</td>
<td>4 sixth grades</td>
<td>102 pupils</td>
</tr>
<tr>
<td>School C</td>
<td>2 fifth grades</td>
<td>35 pupils</td>
</tr>
<tr>
<td></td>
<td>2 sixth grades</td>
<td>52 pupils</td>
</tr>
<tr>
<td>School D</td>
<td>1 fifth grade</td>
<td>26 pupils</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Total 309 pupils</strong></td>
</tr>
</tbody>
</table>
Data

Each room was analyzed separately. The pupils' names were listed in a vertical column and the methods of subtraction in a horizontal column. The methods used for each of the three examples were then listed. These results have been summarized in the form of tables which show the frequency with which various methods of subtraction were used. Example three was summarized not only in detail, but also in a similar way to the methods used in examples one and two. This was done so as to provide a basis of comparison as to the methods of subtraction used in the three examples. All tables include both the number and percent of pupils. Percents were calculated correct to one decimal place. The tables also include the number and percent of pupils using either "borrowing" or "equal-additions."

An analysis of 93 pupils in the three fifth grades in School A showed, for example one, about half (40.8%) using the take-away, upward method, and the other half (54.9%) using the additive, upward. In example 2, about 40% used the take-away, upward method and 57.6% used the additive, upward. However, in the third example, over 70% of the pupils used an additive method. A more detailed analysis is presented in Table I.

Chart I shows that in example one, involving the use of simple subtraction, the take-away method was used almost as frequently as an additive method. Chart II tends to support the eight methods chosen for analysis in example three were reduced to the four methods used in examples one and two by disregarding the borrowing techniques.
May be read as follows: Fourteen (45.2% of the) pupils in Room 15 used the take-away, upward method in example 1.
Chart I.

School A - Third Grade - 93 Pupils

Room 10 11 12 13

<table>
<thead>
<tr>
<th>Room</th>
<th>% of Pupils</th>
<th>% of Pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14.5</td>
<td>13.6</td>
</tr>
<tr>
<td>11</td>
<td>13.2</td>
<td>12.9</td>
</tr>
<tr>
<td>12</td>
<td>14.0</td>
<td>13.7</td>
</tr>
<tr>
<td>13</td>
<td>14.8</td>
<td>15.1</td>
</tr>
</tbody>
</table>

45.2 percent of the pupils of Room 13 and 45.8 percent of the pupils of Room 11 read in a similar method in doing example 1 and 45.2 percent of the pupils of Room 12 read in a similar method in doing example 2.

Example 25
Chart II.

School A—Three Fifth Grades—92 pupils

Example: 26

38.1 percent of the pupils of Room 15,
40.0 percent of the pupils of Room 12, and 38.2 percent of the pupils of Room 18 used the take-away, upward method in example 2.

Other parts read in a similar way.
this conclusion, although the take-away method seems to be used less frequently than in example 1. Chart III shows a detailed comparison of percentages of use of different methods used in example 3. The outstanding fact about this chart is the indication of the high frequency of use of the additive, equal-additions, upward method. Chart IV shows a comparison of the percentages of use of different methods of subtraction for example three after the methods had been simplified to conform with the methods used in examples one and two. The take-away method made a slight gain, but the additive method still predominates. Chart V shows the comparisons of summaries of percentages for examples one, two and three. In all three examples, an additive method was used by over fifty percent of the pupils, although, for simple subtraction, the take-away method was used, in some cases, by forty percent of the pupils.

An analysis of about one hundred sixth grade pupils in School B showed, for example one, about half (47.9%) of the pupils using a take-away method and the other half (52.1%) using the additive, upward. The same facts may be noted on studying the methods used in example 2. It is interesting to note here that the high frequency of use of the additive, equal-additions method in doing example 3 is not quite as apparent as in the fifth grades of School A, from which most of the sixth grade pupils of School B have come. In School A over 70% of the pupils used this method; while in School B
Percentages of the Use of Different Methods of Subtraction
School A—Three Fifth Grades—94 pupils
Example: 723

Chart read as follows:
6.5 percent of the pupils in Room 15, 24 percent of the pupils in Room 12, and 0 percent of the pupils in Room 18 used the take-away, borrowing, upward method in Example 3.
Other parts read in a similar way.

Chart III
School A - Three Fifth Grades - 94 pupils
Methods Similar to Those Used in First Two Examples
Example: \[ \frac{723}{265} \]

<table>
<thead>
<tr>
<th>Room</th>
<th>15</th>
<th>12</th>
<th>18</th>
</tr>
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<tbody>
<tr>
<td>15</td>
<td>16.1</td>
<td>17.0</td>
<td>23.8</td>
</tr>
<tr>
<td>R</td>
<td>17.0</td>
<td>24.4</td>
<td>12.4</td>
</tr>
<tr>
<td>R</td>
<td>23.8</td>
<td>14.7</td>
<td>20.1</td>
</tr>
<tr>
<td>R</td>
<td>3.2</td>
<td>4.7</td>
<td>6.7</td>
</tr>
<tr>
<td>R</td>
<td>80.7</td>
<td>80.6</td>
<td>71.5</td>
</tr>
<tr>
<td>R</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

→ 16.1 percent of the pupils of Room 15, 17.0 percent of the pupils of Room 12, and 23.8 percent of the pupils of Room 18 used the take-away, upward method in doing example 3.

Other parts read in a similar way.
Summaries of Methods of Subtraction Used in Calculating the Three Examples on Analysis Sheet
School A

Example 1  40.8
   2  38.1
   3  19.3

Example 1  4.3
   2  4.3
   3  3.2

Example 1  54.9
   2  57.6
   3  77.5

Example 1  6
   2  0
   3  0

40.8 percent of the pupils in School A used the take-away, upward method in example 1; 38.1 percent in example 2 and 19.3 percent in example 3.

Other parts read in a similar way.
there were not quite 60%. A take-away method was used in about 35% of the cases. This raises the question: Does this divergence of methods increase as these pupils progress through school? Once again over 80% of the pupils were found using equal-additions rather than borrowing. A more detailed analysis of School P is presented in Table II. Charts VI and VII, summarizing the sixth grades of School P, show a slightly greater use of the additive method over the take-away, when simple subtraction is involved. Chart VIII shows that three rooms out of four used an additive method in example three in over 60% of the cases. Over 50% of the pupils in the remaining room used the take-away, equal-additions method. Chart IX shows that the four sixth grades of School P used the additive method in 60%, the take-away method being used in 40%. In this chart the methods were simplified to conform with the methods used in examples one and two. Chart X shows that in the simple subtraction examples one and two the take-away and additive methods were used with nearly equal frequency. In the third example the percentages were 40% for the take-away and 60% for the additive.

Two fifth grades and two sixth grades were analyzed in School C. Over 80% of the pupils used a take-away method in examples one and two. The analysis of example three was interesting in light of the methods used. About 40% used an additive method, while 60% used a take-away method. This is a reversal of the percentages of use of take-away and additive
<table>
<thead>
<tr>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>9.1</td>
<td>3</td>
<td>10.7</td>
<td>6</td>
<td>20.0</td>
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<tr>
<td>20</td>
<td>90.9</td>
<td>25</td>
<td>82.3</td>
<td>24</td>
<td>80.0</td>
</tr>
</tbody>
</table>

May be read as follows: Eight pupils or 36.4% if the pupils of Room 1 used the take-away, upward method in example 1.
### Chart VI.

#### Examples:

<table>
<thead>
<tr>
<th>Room</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
<td>1</td>
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<td>1.67</td>
<td>1.40</td>
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<tr>
<td>4</td>
<td>52.1</td>
<td>52.1</td>
<td>52.1</td>
<td>52.1</td>
</tr>
</tbody>
</table>

The chart represents the distribution of pupils across different rooms. Each room has a different percentage of pupils. For example, Room 1 has 1.64 percent of the pupils, Room 2 has 1.77 percent, and so on.

Room 3 has the highest percentage of pupils among Room 1, 2, and 4. Room 4 has the highest percentage of pupils overall.

### Notes:

- 25.4 percent of the pupils of Room 3.
- 43.3 percent of the pupils of Room 4.
- 35.0 percent of the pupils of Room 1.

Other parts read in a similar way.
Chart VII.

School B—Four Sixth Grade  99 pupils
Example: 26

% of die p&ps of ~ooi~iv
and 4o-0 percent of the pupils of Room 5 used the take-away, upward
method in example 2.
other parts read in a
similar way.

30.4  30.4 percent of the pupils of Room 1,
32.2 percent of the pupils of Room 3,
67.9 percent of the pupils of Room 4,
and 40.0 percent of the pupils of Room 5.
Percentages of the Use of Different Methods of Subtraction

School B—Fourth, Sixth Grades—100 pupils

Example: 725

\[ \frac{3}{5} \]

---

Chart read as follows:

6 percent of the pupils in Room 1;
71.1 percent of the pupils in Room 3;
20.0 percent of the pupils in Room 4;
and 18.2 percent of the pupils in Room 5 used the take-away, borrowing, upward method in example 3.

Other parts read in a similar way.

Chart VIII
Chart IX.

Percentages of Total Number of Methods of Subtraction

School B—Fourth Grades—100 pupils

Methods Similar to Those Used in First Two Examples

Example: \[ \frac{723}{265} \]

Chart read as follows:

- Take-away, upward—Room 1: 13.6, Room 3: 17.9, Room 5: 13.6
- Take-away, downward—Room 3: 73.4, Room 4: 73.4, Room 5: 77.3
- Additive, upward—Room 3: 23.7
- Additive, downward—Room 5: 0

Summary:

- Take-away, upward: 36.3
- Take-away, downward: 2.9
- Additive, upward: 60.8
- Additive, downward: 0
Comparison of Summaries of Methods of Subtraction in Doing the Three Examples on Analysis Sheet—School B

<table>
<thead>
<tr>
<th>Example</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.8</td>
<td>43.5</td>
<td>36.3</td>
<td>5.1</td>
<td>6.0</td>
<td>2.9</td>
<td>52.1</td>
<td>60.5</td>
<td>60.6</td>
</tr>
</tbody>
</table>

42.8 percent of the pupils in School B used the take-away, upward method in example 1; 43.5 percent in example 2 and 36.3 percent in example 3. Other parts read in a similar way.
as illustrated in Chart X. The equal-additions still predominate over borrowing. A more detailed account of the analysis may be found in Table III. Charts XI and XII show that the take-away method was used extensively in doing examples one and two. Chart XIII presents quite a varied picture of the analysis of example three. The take-away, borrowing, upward; take-away, equal-additions, upward; and additive, equal-additions, upward were used in slightly increasing degrees of frequency. Six different methods were used. Chart XIV shows a 60-40% division between the take-away and additive, with the former being used in 60% of the cases. The analysis, as represented on this chart, was made on the basis of the methods used in doing examples one and two. Chart XV brings together the summaries of Charts XI, XII, and XIV in order to show more graphically the methods used in all three examples. The take-away method obviously predominated.

School D presented an unusual and interesting case. If one pupil is excepted, it may be said that all pupils in the one fifth grade analyzed used the additive, equal-additions, upward method.

An analysis of methods used by about one hundred fifty fifth grade pupils in Schools A, C, and D shows half using a take-away method and half using an additive method for examples one and two. In example three, however, over 70% used an additive method. Almost 90% of the pupils used equal- addi-
Table III.

<table>
<thead>
<tr>
<th></th>
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<th>6</th>
<th>7</th>
<th>9</th>
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</thead>
<tbody>
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<td>66.7</td>
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<td>7.7</td>
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<td>11</td>
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<td>34.6</td>
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<tr>
<td>20</td>
<td>76.9</td>
<td>17</td>
<td>65.4</td>
<td>14</td>
</tr>
</tbody>
</table>

May be read as follows: Sixteen (66.7% of the) pupils in Room 5 used the take-away, upward method in example 1.
Chart XI.

School C: Two Fifth Grades - 34 pupils

Example: 33

Two Sixth Grades - 50 pupils

→ 66.7% of the pupils of Room 5,
73.0% of the pupils of Room 6,
61.1% of the pupils of Room 7,
and 93.7% of the pupils of Room 9 used the take-away, upward method in Example 1.

Other parts read in a similar way.
Chart XII.

School C - Two Fifth Grades - 34 pupils
Example: 26
Two Sixth Grades - 50 pupils

Example: 26

- 4

Rooms 5
5 20.8
6 15.3
7 15.7
8 25.0
9 25.0

Rooms 6
5 20.8
6 7.7
7 27.8
8 0

Rooms 7
5 20.8
6 7.7
7 27.8
8 0

Rooms 8
5 20.8
6 7.7
7 27.8
8 0

Rooms 9
5 20.8
6 7.7
7 27.8
8 0

- 58.4 percent of the pupils in Room 5,
- 77.0 percent of the pupils in Room 6,
- 55.5 percent of the pupils in Room 7,
and 25.0 percent of the pupils in Room 9 used the take-away, upward method in example 2.
Other parts read in a similar way.
Percentages of the Use of Different Methods of Subtraction
School C—Two Fifth Grades—35 pupils—Two Sixth Grades—52 pupils
Example: 723 - 265

Chart read as follows:
19.3 percent of the pupils in Room 5, 16.9 percent of the pupils in Room 6, 31.0 percent of the pupils in Room 7, and 18.8 percent of the pupils in Room 9 used the Take-away, Borrowing, upward method in Example 3.

Other parts read in a similar way.
School C - Two Fifth Grades - 35 pupils

Two Sixth Grades - 52 pupils

Methods Similar to Those Used in First Two Examples.

Example: \( \frac{723}{165} \)

- 46.1 percent of the pupils in Room 5,
- 65.4 percent of the pupils in Room 6,
- 97.2 percent of the pupils in Room 7,
- and 31.2 percent of the pupils in Room 8 used the take-away, upward method in Example 3.

Other parts read in a similar way.
Comparison of Summaries of Methods of Subtraction Used in Calculating the Three Examples on Analysis Sheet - School C

72.6 percent of the pupils in School C used the take-away, upward method in example 1; 66.6 percent in example 2 and 49.4 percent in example 3. Other parts read in a similar way.
tions rather than borrowing. A more detailed analysis is found in Table IV. Charts XVI, XVII, XVIII, XIX, and XX show graphically the distribution of the percentages of use of the different methods of subtraction in the fifth grades.

An analysis of about one hundred sixth grade pupils revealed that, in simple subtraction, more pupils used a take-away method than an additive method. In the third example, involving borrowing, about half used a take-away method and half used an additive method. Almost 80% used equal-additions rather than borrowing. A more detailed analysis is found in Table V. Charts XXI through XXV show graphically the percentages of frequency of the different of subtraction in the sixth grades analyzed.

Table VI presents a summary of the results of both the fifth and sixth grades. Some of the pertinent facts seem to be: (1) The take-away methods seem to have no predominance over additive methods in simple subtraction. This may be because these responses are automatic (or at least should be at this stage) and are therefore difficult to analyze. Also, children may have devised a scheme of their own or thought that it did not make much difference which method they marked. (2) The three methods that were the most commonly used are, in their order of frequency:

a. additive, equal-additions, upward 60.3%
b. take-away, equal-additions, upward 20.4%
c. take-away, borrowing, upward 11.3%
### Table: 5 A, C, D Fifth Grades

<table>
<thead>
<tr>
<th>School A</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>38</td>
<td>40.8</td>
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<tr>
<td>4</td>
<td>4.3</td>
<td>4</td>
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<td>51</td>
<td>54.9</td>
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</tr>
<tr>
<td>83</td>
<td>88.3</td>
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</tr>
</tbody>
</table>

**May be read as follows:** 38 (40.8% of the) pupils in School A used the take-away, upward method in example 1.
35.

Chart XVI.

Fifth Grades of Schools A, C and D - 153 pupils

Example: \( \frac{35}{33} \)

School A

<table>
<thead>
<tr>
<th>Room</th>
<th>School A</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40.8</td>
<td>76.4</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>4.3</td>
<td>11.8</td>
<td>3.8</td>
</tr>
<tr>
<td>C</td>
<td>54.9</td>
<td>11.8</td>
<td>96.2</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

+ 40.8 percent of the pupils of School A,
+ 76.4 percent of the pupils of School C,
+ 0 percent of the pupils of School D

School A used the take-away, upward method in example I.

Other parts read in a similar way.
Example 26

School A: 38.1 percent of the pupils in School A,
School C: 64.7 percent of the pupils in School C,
School D: 0 percent of the pupils in School D used the take-away-upward method in Example 2.

School A: 38.1
School C: 64.7
School D: 0

School A: 38.1
School C: 64.7
School D: 0
Percentages of Use of Different Methods of Subtraction
Fifth Grades — Schools A, C and D — 155 pupils

Example: $\frac{723}{265}$

Chart read as follows:
4.2 percent of the pupils in School A, 20.0 percent of the pupils in School C, and 0 percent of the pupils in School D used the take-away, borrowing, upward method in example 3. Other parts read in a similar way.

Chart XVIII
Fifth Grades—Schools A, C and D—125 pupils
Methods Similar to Those Used in First Two Examples.

Example: 723
   - 263

<table>
<thead>
<tr>
<th>School A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.3</td>
<td>40.0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

School A:
- 19.3 percent of the pupils in School A,
- 40.0 percent of the pupils in School C,
- 0 percent of the pupils in School D
used the take-away, upward method
in example 3.

Other parts read in a similar way.

School A:
- 77.5
- 42.8

School A:
- 100
- 0

School A:
- 20.7
- 5.8
- 73.5
- 0

School A:
- 19.3
- 40.0
- 0
- 0
Chart XX.

Comparison of Summaries of Methods of Subtraction Used in the Fifth Grades on Analysis Sheet—Fifth Grades

41.7 percent of the pupils in the fifth grades used the take-away method in example 1, similar way.

41.7 percent of the pupils in the fifth grades used the take-away method in example 2.

41.7 percent of the pupils in the fifth grades used the take-away method in example 3.

Calculated the three examples on Analysis Sheet—Fifth Grades.
<table>
<thead>
<tr>
<th>Example</th>
<th>3 45</th>
<th>6 35</th>
<th>12 19</th>
<th>24 15</th>
<th>42 27</th>
<th>78 50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take-away, upward</td>
<td>12</td>
<td>11.9</td>
<td>12</td>
<td>23.2</td>
<td>24</td>
<td>15.6</td>
</tr>
<tr>
<td>Take-away, downward</td>
<td>2</td>
<td>1.9</td>
<td>1</td>
<td>1.9</td>
<td>3</td>
<td>1.9</td>
</tr>
<tr>
<td>Additive, downward</td>
<td>25</td>
<td>24.5</td>
<td>17</td>
<td>32.7</td>
<td>42</td>
<td>27.3</td>
</tr>
<tr>
<td>Additive, equal additions, downward</td>
<td>1</td>
<td>0.9</td>
<td>2</td>
<td>3.8</td>
<td>4</td>
<td>2.6</td>
</tr>
<tr>
<td>Additive, upward</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Additive, equal additions, upward</td>
<td>60</td>
<td>58.9</td>
<td>18</td>
<td>34.6</td>
<td>78</td>
<td>50.7</td>
</tr>
<tr>
<td>Additive, downward</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

May be read as follows: Forty-two (42.8% of the) pupils of School B used the take-away, upward method in example 1.
Sixth Grades—Schools B and C—159 pupils

Example: \( \frac{35}{33} \)

- 42.8% of the pupils of School B
- 70.0% of the pupils of School C

used the take-away, upward method in Example 1.

Other parts read in a similar way.

School B
- 52.1
- 22.0
- 0

School C
- 51
- 8.0
- 0

48.4
5.7
46.9
0
Chart XXII.

Sixth Grades—Schools B and C—149 pupils

Example 26

- 43.5% of the pupils in School B
- 68.0% of the pupils in School C used the take-away, upward method in example 2.
- Other parts read in a similar way.
Percentages of the Use of Different Methods of Subtraction
Sixth Grades — Schools B and C — 154 pupils

Example: $\frac{223}{265}$

Chart read as follows:
11.9 percent of the pupils in School B and 23.2 percent of the pupils in School C used the Take-away, borrowing, upward method in example 3.

Other parts read in a similar way.

Chart XXIII
Chart XXIV.

Middle School B and C pupils

School B

School C

Example: 793

Methods similar to those used in Part Two Examples.
Comparison of Summaries of Methods of Subtraction Used in Calculating the Three Examples on Analysis Sheet—Sixth Grades.

Example 1: 48.4 percent of the pupils in the sixth grades used the take-away; 51.6 upward in example 1; 51.6 percent in example 2; and 42.8 percent in example 3. Other parts read in a similar way.

Example 2:

Example 3:

Example 4:

Example 5:
### Fifth and Sixth Grades

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>48.8</td>
<td>40.8</td>
<td>61</td>
<td>18</td>
</tr>
<tr>
<td>42.8</td>
<td>54.1</td>
<td>17.9</td>
<td>25</td>
</tr>
<tr>
<td>12.6</td>
<td>9.5</td>
<td>56.2</td>
<td>142.4</td>
</tr>
<tr>
<td></td>
<td>3.8</td>
<td>96.2</td>
<td>47.9</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Total No. Pupils**

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>102</td>
<td>87</td>
<td>26</td>
</tr>
<tr>
<td>102</td>
<td>87</td>
<td>15</td>
<td>26</td>
</tr>
<tr>
<td>87</td>
<td>15</td>
<td>26</td>
<td>309</td>
</tr>
</tbody>
</table>

**Total No. Pupils**

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>19.3</td>
<td>37</td>
<td>36.3</td>
</tr>
<tr>
<td>19.3</td>
<td>36.3</td>
<td>43</td>
<td>49.4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Total No. Pupils**

<table>
<thead>
<tr>
<th>School A</th>
<th>School B</th>
<th>School C</th>
<th>School D</th>
</tr>
</thead>
<tbody>
<tr>
<td>94</td>
<td>102</td>
<td>87</td>
<td>26</td>
</tr>
<tr>
<td>102</td>
<td>87</td>
<td>26</td>
<td>309</td>
</tr>
<tr>
<td>87</td>
<td>26</td>
<td>309</td>
<td>309</td>
</tr>
</tbody>
</table>

*May be read as follows: thirty-eight (40.8% of the) pupils of School A used the take-away, upward method in example 1.*
(3) The other methods used are:
   a. additive, borrowing, upward  3.2%
   b. take-away, equal-additions, downward  2.9%
   c. take-away, borrowing, downward  1.9%

(4) There was no doubt as to which method the children used more, borrowing or equal-additions, as the latter was used in 83.5% of the cases. Charts XXVI through XXX present graphically the results listed in Table VI.

Table VII shows the number of pupils and the methods of subtraction they used in doing all three examples. Eighty-one used a take-away method for examples one and two and an additive method for number three. Seventy pupils used a take-away method for all three examples. One hundred nineteen used an additive form for all three examples. This represents about 38% of the pupils analyzed.

Conclusions

Our conclusions refer us to the questions asked on page 6. They will be taken up in the order in which they were first presented.

1. How many methods of subtraction were used?

Table VI indicates the use of six different methods in doing example three. This example is referred to because it involves borrowing and therefore offers a greater variety of possibilities.
Chart XXVI.

Fifth and Sixth Grades—Schools A, B, C, and D—301 pupils

Example: \[ \frac{35}{33} \]

- 40.8 percent of the pupils in School A,
- 42.8 percent of the pupils in School B,
- 72.6 percent of the pupils in School C,
- 0 percent of the pupils in School D

used the take-away, upward method in example 1.

Other parts read in a similar way.
Chart XXVII.

Percentages of the Different Methods Used in Combining Fifth and Sixth Grades—Schools A, B, C, and D—301 pupils

Examples: 26

Chart read as follows:

Take away, upward—

Take away, downward—

Additive, upward—

Additive, downward—

Summary

School

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>38.1</td>
<td>43.5</td>
<td>66.6</td>
<td>0</td>
</tr>
</tbody>
</table>

School

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.3</td>
<td>4.6</td>
<td>3.8</td>
<td></td>
</tr>
</tbody>
</table>

School

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>57.4</td>
<td>50.5</td>
<td>19.1</td>
<td>96.2</td>
</tr>
</tbody>
</table>

Other parts read in a similar way.

44.5
7.6
47.9
0
Percentages of the Use of Different Methods of Subtraction
Fifth and Sixth Grades—Schools A, B, C, and D—389 pupils
Examples: 223

Chart read as follows:
4.2 percent of the pupils in School A, 11.9 percent of the pupils in School B, 21.8 percent of the pupils in School C, and 0 percent of the pupils in School D used the take-away, borrowing, upward method in example 3. Other parts read in a similar way.

Chart XXVIII
Fifth and Sixth Grades—Schools A, B, C and D—309 pupils
Methods Similar to Those Used in First Two Examples.

Example: \( \frac{733}{223} \)

- \( \frac{193}{72} \) percent of the pupils in School A,
- \( \frac{263}{4} \) percent of the pupils in School B,
- \( \frac{40}{4} \) percent of the pupils in School C,
- \( \frac{190}{4} \) percent of the pupils in School D

and 0 percent of the pupils in School D used the take-away, upward method in example 3.

Other parts read in a similar way.
Comparison of Summaries of Methods of Subtraction Used in Calculating the Three Examples on Analysis Sheet—Fifth and Sixth Grades

46.8 percent of the pupils in the fifth and sixth grades used the take-away, upward method in example 1; 44.5 percent in example 2, and 31.7 percent in example 3. Other parts read in a similar way.
Table VII

List Showing How Many Methods or Combination of Methods Were Used in Doing the Group of Examples on the Analysis Sheet; with Frequency of Occurrence of Each Method or Combination of Methods.

<table>
<thead>
<tr>
<th>School</th>
<th>Method</th>
<th>Grade</th>
<th>No.</th>
<th>Room</th>
<th>Frequency of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. Take-away, upward</td>
<td></td>
<td></td>
<td></td>
<td>4 3 5 12</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3 4 13 3 23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10 17 7 4 38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23 50 73</td>
</tr>
<tr>
<td></td>
<td>2. Take-away, downward</td>
<td></td>
<td></td>
<td></td>
<td>0 1 0 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3 0 0 0 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 0 3 0 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 4 8</td>
</tr>
<tr>
<td></td>
<td>3. Additive, upward</td>
<td></td>
<td></td>
<td></td>
<td>15 17 11 43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>10 16 2 10 38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5 4 3 1 13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23 47 119</td>
</tr>
<tr>
<td></td>
<td>Combination of 1 and 2</td>
<td></td>
<td></td>
<td></td>
<td>1 0 0 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 0 3 0 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 2 6</td>
</tr>
<tr>
<td></td>
<td>Combination of 1 and 3</td>
<td></td>
<td></td>
<td></td>
<td>10 18 3 3 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5 7 1 1 8 3 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5 4 1 9 19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>41 40 81</td>
</tr>
<tr>
<td></td>
<td>Combination of 2 and 3</td>
<td></td>
<td></td>
<td></td>
<td>1 2 0 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 1 0 0 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2 1 0 0 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 4 4 8</td>
</tr>
<tr>
<td></td>
<td>Combination of 1,2 and 3</td>
<td></td>
<td></td>
<td></td>
<td>0 0 2 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1 0 2 0 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0 0 1 1 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4 3 7</td>
</tr>
</tbody>
</table>
2. How widely is each method used?

The six methods listed in their order of frequency are:

- additive, equal-additions, upward 60.3%
- take-away, equal-additions, upward 20.4%
- take-away, borrowing, upward 11.3%
- additive, borrowing, upward 3.2%
- take-away, equal-additions, downward 2.9%
- take-away, borrowing downward 1.9%

3. Is each school consistent?

Only one school showed a remarkable degree of consistency. The one fifth grade analyzed in this school showed 100% of its members using the additive, equal-additions, upward method. The degree of consistency for the other schools seems to be much less in simple subtraction than in the example involving borrowing. There appears to be a greater degree of consistency in the fifth grade than in the sixth grade. The following will help to illustrate this point:

<table>
<thead>
<tr>
<th>Method</th>
<th>Grades V and VI</th>
<th>Grade V</th>
<th>Grade VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Take-away, borrowing, upward</td>
<td>11 7.1</td>
<td>24 15.6</td>
<td></td>
</tr>
<tr>
<td>Take-away, borrowing, downward</td>
<td>3 1.9</td>
<td>3 1.9</td>
<td></td>
</tr>
<tr>
<td>Take-away, equal-additions upward</td>
<td>21 13.5</td>
<td>42 27.3</td>
<td></td>
</tr>
<tr>
<td>Take-away, equal-additions, downward</td>
<td>6 3.9</td>
<td>3 1.9</td>
<td></td>
</tr>
<tr>
<td>Additive, borrowing, upward</td>
<td>6 3.9</td>
<td>4 2.6</td>
<td></td>
</tr>
<tr>
<td>Additive, borrowing, downward</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
<tr>
<td>Additive, equal-additions, upward</td>
<td>108 69.7</td>
<td>78 50.7</td>
<td></td>
</tr>
<tr>
<td>Additive, equal-additions, downward</td>
<td>0 0</td>
<td>0 0</td>
<td></td>
</tr>
</tbody>
</table>

There was a great deal of consistency in the use the pupils made of equal-additions in doing example three.
83.5% or 258 pupils out of 309 used this method rather than borrowing.

4. What methods are the various schools teaching?
   This question has already been partially answered.
   School A taught the additive, equal-additions, upward.
   School B carries on with the methods the pupils bring to the school.
   School C teaches the take-away, borrowing, upward.
   School D teaches the additive, equal-additions, upward.

5. Are the teachers convinced of the efficacy of the method they teach?
   The majority of the teachers favor the take-away, borrowing, upward method of subtraction. This method, in their opinion, seems to be more efficacious than any of the others. They are all convinced of the necessity for uniformity of method.

6. What confusions seem to exist?
   The outstanding one seems to be the variety of methods that are in use in some of the schools. This calls for highly individualized instruction if the remedial load in subtraction is great. This is possible in greater or lesser degree, according to the size of the class.
   There is one other confusion that was noted, especially in the sixth grade. This confusion concerns the teaching of subtraction of mixed numbers when the lower fraction is larger than the upper. In these cases the children
should use the kind of "borrowing" they were taught to use in the subtraction of whole numbers. The confusion arises when the teacher is first explaining how to do such a problem. If there are two different methods of "borrowing" used and just one explanation given, the children not familiar with that method are likely to become confused.

7. Are the schools attaining 100% accuracy?

No. Reference to P. W. A. Project Number 17 C, conducted under the supervision of Dr. Guy M. Wilson, indicates that this goal is far from being reached. The results of this study show that of the pupils in grade six during the school year 1935-36 only 34% had a score of 96 or above in a simple subtraction test. This left a corrective load of 66% of the total pupils. Table VIII presents more detailed data.

8. What does research reveal in regard to methods of subtraction?

This question is answered on pages 1 through 4.

9. What recommendation may be made as a result of this study?

a. A single method of subtraction should be used within the school system. The take-away, borrowing, upward method is recommended for adoption. There is no conclusive evidence that points to its superiority over any of the other methods. However, other determining factors enter into its selection. They may be summarized as follows:

1) The majority of the towns of the Greater Boston
Results of Subtraction Process Test — May 20, 1936
Grades VI, VII and VIII — P.W.A. Project #17C-μ 30
for City "B"

<table>
<thead>
<tr>
<th>Grade</th>
<th>Range in Score</th>
<th>% scores at 96 or above</th>
<th>Range in time</th>
<th>% time 6' or less</th>
<th>% scores 96 or above Time 6' or less</th>
<th>Corrective load % stage total pupils</th>
<th>Average for 3 grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>16 — 100</td>
<td>21</td>
<td>42</td>
<td>3 — 14</td>
<td>71</td>
<td>34</td>
<td>66</td>
</tr>
<tr>
<td>7</td>
<td>44 — 100</td>
<td>15</td>
<td>42</td>
<td>2 — 9</td>
<td>83</td>
<td>39</td>
<td>61</td>
</tr>
<tr>
<td>8</td>
<td>24 — 100</td>
<td>23</td>
<td>51</td>
<td>2 — 8</td>
<td>92</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
area use this method. Pupils frequently transfer from and to these schools. Lack of uniformity makes adjustment difficult and adds to more deep-rooted confusions. A large school system within this area has recently changed to the take-away, borrowing, upward method. This method, in fact, is the prevailing method throughout the United States. (113)

2) Talks with teachers, parents, and children, while not conclusive evidence, have indicated that the majority of the townspeople use the take-away, borrowing, upward method. Children are being taught one method at school only to become confused when assistance is requested of the parents. It is necessary, in this case, either to teach the parents as well as the children or to establish a "hands-off" policy on the part of the parents.

3) The take-away method is essentially subtraction. There is less possibility of confusing addition and subtraction. The idea of subtraction is more sharply and clearly defined.

The above facts, coupled with the inconclusive findings in favor of other methods, seem to indicate that, for the school system included in this study, the take-away, borrowing, upward method should be adopted by all the schools. This is made, despite the fact that the additive, equal-additions, upward method was used by
almost 70% of the pupils in the fifth and sixth grades. Evidence not included in this study seems to indicate that this numerical superiority will not be maintained since School C is now teaching only the take-away, borrowing, upward method.

If the school system were attaining a high degree of accuracy with the present set-up, no change would be advocated. Data compiled from tests given in the spring of 1936 indicate that there was a corrective load of 66% of the sixth grade pupils for that year in subtraction alone. Any change that might aid in lowering this corrective load should be considered.

Merely a change in method will not be sufficient if it is not accompanied by explanations and discussions on the part of all concerned.

This change, if made, should begin in the grades where subtraction is first taught. It should not affect those children who have already learned a different method. In this way, the accepted method will work its way upward throughout the whole school system.

10. If one method is taught in a school system, is there any assurance that this method will prevail?

There are several factors that make consistency seem possible, namely; (a) the exclusive use of one method by the fifth grade class in School D; (b) the high degree of consistency in the use of equal-additions as a part of the procedure in figuring example three; (c) inconsistencies due to parental help and transfer from one school to another in the Boston Area will be less likely to occur.
The additive, equal-additions, upward method of subtraction was adopted by the Needham Schools in 1929. This explains the high percentages for this method.


APPENDIX

Analysis sheet; Original form
Analysis sheet; improved form.
Significance of the Percent Differences.

Most of the percent differences are significant upon inspection without recourse to a formula. However for the benefit of those readers who may wish to determine the statistical significance of the percent differences a typical case has been selected from Table VI, page 46. A comparison is made between those pupils using a take-away method and those using an additive method in example three. 113 or 36.6 percent of the fifth and sixth grade pupils used a take-away method. 196 or 63.4 percent used an additive method.

Holzinger's equations 102 and 102a were used to determine the significance of the percent difference. (41:243-44)

Equation 102 was used to find the probable error (P.E.).

\[ P.E. = \frac{0.6745}{\sqrt{\frac{f_p (100-f_p)}{N}}} \]

f\(p\) = percent score or frequency 
N = number of cases

Substituting in the equation the percentage and number of pupils using a take-away method we have:

\[ P.E. = \frac{0.6745}{\sqrt{\frac{36.6(100-36.6)}{113}}} \]

\[ = \pm 3.06 \]

The percent now reads 36.6 \(\pm\) 3.06 percent.

Substituting in the equation the percentage and number of pupils using an additive method we have:

\[ P.E. = \frac{0.6745}{\sqrt{\frac{63.4(100-63.4)}{196}}} \]

\[ = \pm 2.32 \]

The percent now reads 63.4 \(\pm\) 2.32 percent.
Equation 102a was used to find the significance of the percent difference.

\[ \text{P.E. (diff.)} = \sqrt{(\text{P.E. of 1st } \%)^2 - (\text{P.E. of 2d } \%)^2} \]

Substituting in the equation we have:

\[ \text{P.E. (diff.)} = \sqrt{(3.06)^2 - (2.32)^2} \]
\[ = \pm 1.99 \]

If the difference is more than five times the P.E. it is a significant difference.

The difference between the percentage of pupils using the additive method of subtraction and those using the take-away method as per Table VI, page 46 may now be written:

\[ 63.4 - 36.6 = 26.8 \pm \text{P.E.} \pm 1.99 \]

This difference being more than five times the P.E. is a significant difference.

The same procedure may be used for finding the significance of the percent differences in any of the tables.