Bargaining and informed agent's advice: theory and evidence

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Dissertation

BARGAINING AND INFORMED AGENT’S ADVICE:
THEORY AND EVIDENCE

by

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Sanghoon Kim
ABSTRACT

This dissertation investigates the market effect of informed agents’ advice to buyers and sellers. I compare two kinds of informed agents: one who maximizes an expected sales price, the price-maximizing agent, and one who maximizes the probability of trade, the trade-maximizing agent.

The first chapter presents a theoretical model and results. The model is based on a bilateral bargaining game with an informed agent, where the agent has more information than either the buyer or the seller. I show that if there exists a bargaining equilibrium in which a price-maximizing agent disseminates information truthfully, then there also exists a bargaining equilibrium in which a trade-maximizing agent disseminates information truthfully. However, the converse is not true. Thus, for some cases, a trade-maximizing agent is truthful, but a price-maximizing agent is not. In such cases, a trade-maximizing agent leads to a higher trade probability and a higher expected sales price than a price-maximizing agent.

The second chapter studies the model under specific assumptions. Here, both the buyer’s and seller’s good valuations are uniformly distributed, and the agent knows whether each of the buyer and seller has a high or low valuation. I analyze three cases:
an informed agent advises (1) a buyer only, (2) a seller only, and (3) both. Whereas a trade-maximizing agent is always truthful, a price-maximizing agent over-reports a seller’s valuation to increase the sales price.

The third chapter empirically tests the model prediction: a trade-maximizing agent leads to a higher trade probability than a price-maximizing agent. I do so using South Korean housing transaction data. I exploit the unique Korean real estate agents’ commission scheme to identify the two kinds of agents. I solve the missing listing data problem by applying cluster analysis, which is an unsupervised machine learning algorithm. I show the conditions under which the number of sales is the appropriate proxy for the trade probability even when listing data are missing. The results show that a trade-maximizing agent brings an approximately 0.2% greater number of sales in each ten-day period relative to a price-maximizing agent.
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<td>Avg</td>
<td>Average</td>
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<tr>
<td>CV</td>
<td>Coefficient of Variation</td>
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<td>FE</td>
<td>Fixed Effect</td>
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<td>FSBO</td>
<td>For-Sale-By-Owner</td>
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<td>MLS</td>
<td>Multiple Listing Service</td>
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<td>OLS</td>
<td>Ordinary Least Squares</td>
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Chapter 1

Bargaining and an Informed Agent

1.1 Introduction

This dissertation studies informed intermediary’s advice to buyers and sellers and the effect of that advice on a market where buyers and sellers trade goods under incomplete information.

When a buyer and seller bargain under incomplete information, the buyer aggressively bids a lower price, and the seller aggressively asks a higher price than their respective true valuations of the good. In other words, when not aware of the adversary’s valuation, a buyer understates and a seller overstates in order to maximize each one’s expected payoff. As a result, sometimes a trade is not closed even when the buyer’s true valuation is higher than the seller’s: it is ex-post inefficient. The above aspects are well-summarized in Chatterjee and Samuelson (1983).

If there exists someone who can reduce the information incompleteness between buyer and seller by advising them, then this person can reduce the ex-post inefficiency. In other words, if someone has more accurate information than either party regarding the adversary’s valuation, and reveals that information, then this person can make the trade more likely to occur.

An intermediary is a person who acts as a link between buyers and sellers. That is, an intermediary centralizes markets: either buying from sellers and then reselling to buyers, or matching buyers and sellers together. Through these activities, the inter-
mediary becomes more informed and experienced than his or her clients. Therefore, the intermediary can advise buyers and sellers before they interact in the market so that they are more likely to trade.

This advisory role is not significant in markets where an intermediary buys from sellers and resells to buyers. In this situation, the intermediary merely offers a bid price to sellers and an ask price to buyers rather than advising either of market conditions. In such markets, there are two bargaining processes: one between seller and intermediary and another between intermediary and buyer. As the intermediary is a direct party in both bargaining processes, he or she does not advise either buyer or seller.

However, in a market where an intermediary matches buyers and sellers together, only one bargaining process occurs, between buyer and seller. As the intermediary is not a direct party in this process, he or she can dispense advice to both buyer and seller by way of cheap talk. For example, buyers and sellers in the real estate market simply call or stop by a real estate agent’s office to obtain some information concerning the market. The real estate agent can tell them whether it is currently a buyer’s market or a seller’s market, even though the buyers and sellers pay nothing to the agent in return for that information.

Although an intermediary gives some advice to buyers and sellers for free, this is a strategic action: the intermediary is paid only if a mediated trade occurs. That is to say, the compensation scheme for the intermediary affects his or her strategic behavior not only in the stages of matching and closing deals but also in the advising stage. Many papers have focused on the matching and closing stages\(^1\); however, almost none focus on the advising stage. This dissertation studies how the dissemination of information by an intermediary during the advising stage is affected by

\(^1\)See the list of the papers in Section 1.1.1
different compensation schemes that reward the intermediary in return for matching and closing deals.

In this chapter, I build a theoretical model to analyze an intermediary’s strategic behavior during the advising stage. This model is based on the bilateral bargaining game in Chatterjee and Samuelson (1983) into which I introduce an informed agent. That is, there are three players in the model: a buyer, a seller, and an informed agent. The seller owns an indivisible good that the buyer wants to obtain. The buyer has an exact valuation in mind for the good but does not know the valuation held by the seller. Similarly, the seller conceives of an exact valuation for the good but is not aware of that held by the buyer. The informed agent receives finite signals about both valuations and then delivers a private message to each participant. After receiving the agent’s message, the buyer and seller submit their offers. If the buyer’s offer is greater than or equal to that of the seller, they trade the good at a final price equal to the average of their offers.

I compare two different kinds of informed agents: a price-maximizing agent and a trade-maximizing agent. The utility of the price-maximizing agent is increasing in the expected sales price, and that of the trade-maximizing agent is increasing in the probability of trade.

The main result is as follows: If there exists an equilibrium in which the price-maximizing agent truthfully disseminates information to both buyer and seller, then there exists an equilibrium in which the trade-maximizing agent also truthfully disseminates information to both parties. However, the converse of the above statement is not true. That is, there exists a case where the trade-maximizing agent truthfully disseminates information to the buyer and seller, but the price-maximizing agent does not. In other words, the trade-maximizing agent makes more detailed information available in the market, while the price-maximizing agent does not.
As a result, the trade-maximizing agent reduces the information incompleteness between buyer and seller and thus leads to a higher probability of trade and smaller ex-post inefficiency. Moreover, the trade-maximizing agent leads to a higher expected sales price than even the price-maximizing agent due to the higher probability of trade.

The rest of this chapter is organized as follows. In Section 1.1.1, I briefly review the related literature. In Section 1.2, I construct a theoretical model of a bargaining game with an informed agent. Section 1.3 and Section 1.4 analyze the model, and I conclude in Section 1.5. All the proofs of results in this chapter are in Appendix A.

1.1.1 Literature review

This chapter is concerned with bilateral trading and an intermediary liaising between a buyer and a seller. In this section, I briefly highlight some of the existing literature on these topics.

Myerson and Satterthwaite (1983) show that for two parties trading a good when they have private information regarding their valuations, there is no mechanism that satisfies all of the ex-ante individual rationality, incentive compatibility, weak balanced budget, and ex-post efficiency. Chatterjee and Samuelson (1983) add to this with their study of an equilibrium in a bilateral bargaining game. They show that relative to each party’s valuation, a buyer bids a lower price and a seller asks a higher price because each seeks to maximize the expected payoff. As a result, the bargaining rule in Chatterjee and Samuelson (1983) is also ex-post inefficient. However, the bargaining rule gives the highest expected total gains from trade, as noted in Myerson and Satterthwaite (1983). Leininger et al. (1989) also characterize the other equilibria of the bilateral bargaining game in their paper.

Farrell and Gibbons (1989) introduce into a bilateral bargaining game a cheap talk stage that occurs between buyer and seller before the bidding stage. When a

\footnote{Also known as a double auction.}
buyer and seller realize in the cheap talk stage that neither of them is keen to trade, they never continue to the bidding stage even though there could still be a positive probability of them making a trade. Thus, they reveal their types in the cheap talk stage as in Crawford and Sobel (1982).

Broadly, there are three types of intermediaries: a centralizer of a decentralized market, a quality guarantor, and an advisor. The first type either centralizes the market by buying from sellers and reselling to buyers or does so indirectly by matching buyers and sellers with one another (Rubinstein and Wolinsky, 1987; Biglaiser, 1993; Gehrig, 1993; Yavaş, 1994; Spulber, 1996a,b; Rust and Hall, 2003). The second type is particularly relevant in situations where a seller has private information or a hidden action of which the buyer is uninformed. In that case, the intermediary guarantees quality either by buying only a high-quality good and reselling it to the buyer or by certifying the good’s quality (Biglaiser and Friedman, 1994; Lizzeri, 1999; Biglaiser and Li, 2018).

This chapter is more concerned with scenarios in which the intermediary acts as an advisor; namely, the intermediary is informed while both buyer and seller are uninformed. The informed intermediary advises the buyer and seller by delivering the information that he or she has. In Suvorov and Tsybuleva (2010), uniform distributions are assumed for the buyer’s and seller’s respective valuations. They also assume the information structure of the intermediary to be the partition of the support of the valuation distribution. The partition structure has two subintervals over the support. They characterize the partition structure where the intermediary, who receives a fixed per-transaction fee, discloses information truthfully. I assume the distributions of the buyer’s and seller’s valuations and the information structure of the intermediary in a more general way. Moreover, I focus on the effects of different incentives for the intermediary.
1.2 Basic setting

There are three players: a buyer, a seller, and an informed agent. I assume that the seller owns an indivisible good that the buyer wants to obtain. The valuations of the buyer and the seller are denoted by $v^b$ and $v^s$, respectively. The buyer and seller are risk neutral, and each has an additively separable utility for the good and money. If they trade the good at a price of $p$, the buyer’s utility is $v^b - p$, and the seller’s utility is $p - v^s$. Their utilities are normalized to 0 if no trade occurs.

Moreover, I assume that $v^b$ and $v^s$ are independent random variables distributed over a non-negative interval of $[v, \bar{v}]$ with cumulative distribution functions of $F^b(\cdot)$ and $F^s(\cdot)$, respectively. That is, $F^b(v) = F^s(v) = 0$ and $F^b(\bar{v}) = F^s(\bar{v}) = 1$. Furthermore, both $F^b(\cdot)$ and $F^s(\cdot)$ are strictly increasing and twice differentiable on $[v, \bar{v}]$. The corresponding density functions of $F^b(\cdot)$ and $F^s(\cdot)$ are denoted as $f^b(\cdot)$ and $f^s(\cdot)$, respectively. Finally, I assume reflection-symmetric valuations of the buyer and seller as below.

Assumption 1 $F^s(v) = 1 - F^b(\bar{v} - v)$ for all $v \in [v, \bar{v}]$.

We can also interpret this model as there are continua of buyers and sellers over $[v, \bar{v}]$ with $F^b(\cdot)$ and $F^s(\cdot)$, respectively.
Note that given a specific price, the buyer’s utility is increasing in the buyer’s valuation, and the seller’s utility is decreasing in the seller’s valuation: a higher value buyer and a lower value seller are more keen to trade than a lower value buyer and a higher value seller, respectively. As we can see in Figure 1.1, the buyer’s and seller’s valuations being reflection symmetric means that \( f^b(\cdot) \) is the mirror image of \( f^s(\cdot) \), and vice versa: for a given keenness to trade, the respective likelihoods of the buyer and seller having that value are equal.

The informed agent has an information set of the partition of the interval \([v, \overline{v}]\) about the buyer’s valuation as \( V^b = \{ I^b_i = [v^b_i, v^b_{i+1}] | v = v^b_0 < v^b_1 < \cdots < v^b_n = \overline{v}, i = 0, 1, \ldots, n - 1 \} \). Similarly, the agent also has an information set of the partition of the interval \([v, \overline{v}]\) about the seller’s valuation as \( V^s = \{ I^s_j = [v^s_j, v^s_{j+1}] | v = v^s_0 < v^s_1 < \cdots < v^s_n = \overline{v}, j = 0, 1, \ldots, n - 1 \} \). That is, after the realization of \( v^b \), the agent receives a signal about the buyer’s valuation as one of the elements in \( V^b \). Similarly, after the realization of \( v^s \), the agent receives a signal about the seller’s valuation as one of the elements in \( V^s \). For instance, if an interval \( I^b_i = [v^b_i, v^b_{i+1}] \subset [v, \overline{v}] \) includes the realized \( v^b \) and an interval \( I^s_j = [v^s_j, v^s_{j+1}] \subset [v, \overline{v}] \) includes the realized \( v^s \) for some \( i, j \in \{0, 1, \cdots, n-1\} \), the agent knows that the realized \( v^b \) is within the interval of \( I^b_i \) and the realized \( v^s \) is within the interval of \( I^s_j \). The agent thus has finer information about the buyer’s valuation than the seller does because \( I^b_i \) is a proper subset of \([v, \overline{v}]\) for all \( i = 0, 1, \cdots, n - 1 \). Likewise, the agent has finer information about the seller’s valuation than the buyer does because \( I^s_j \) is a proper subset of \([v, \overline{v}]\) for all \( j = 0, 1, \cdots, n - 1 \).

Furthermore, I assume reflection-symmetric sets of \( V^b \) and \( V^s \) as below.

**Assumption 2** \( v^b_k - v = \overline{v} - v^s_{n-k} \) for all \( k = 0, 1, \cdots, n \).

As we can see in Figure 1.2, having reflection-symmetric sets of \( V^b \) and \( V^s \) means that the partition structure of the buyer’s valuation is the mirror image of that of the
seller’s valuation, and vice versa: the agent is equally informative to buyer and seller about the other’s keenness to trade. To illustrate, if the agent has an information partition of the buyer’s valuation that is finer on the lower value side, then the information partition possessed for the seller’s valuation is finer on the higher value side. That is to say, if the agent has finer information about a buyer who is less keen to trade than a buyer who is more keen to trade, then the agent also has finer information about a seller who is less keen to trade than a seller who is more keen to trade with the same magnitude as for the buyer.

Furthermore, I define two kinds of informed agents: a price-maximizing agent and a trade-maximizing agent. The utility of the price-maximizing agent is increasing in the expected sales price, and that of the trade-maximizing agent is increasing in the probability of trade.

Note that I assume the agent gets the utility from the expected sales price or the probability of trade rather than being paid by the buyer and seller based on either of those parameters. As I mentioned in Section 1.1, I study the informed agent’s strategic behavior during the advising stage where the buyer and seller receive some information from the agent for free. That is, it is cheap talk; the agent’s payoff does not directly depend on what is said to either buyer or seller. However, after the advising stage, the agent anticipates some payment from the buyer and seller based on the final sales price or the fact that they trade the good. This aspect gives an

Figure 1.2: Reflection-symmetric sets of $V^b$ and $V^s$
incentive for strategic behavior by the agent during the advising stage. Therefore, I assume that the agent maximizes either the expected sales price or the probability of trade even without monetary transfer from buyer or seller.

The timeline of the game is as follows. First, nature selects $v^b$ from the distribution of $F^b$ over $[\underline{v}, \overline{v}]$ and $v^s$ from the distribution of $F^s$ over $[\underline{v}, \overline{v}]$. Second, the buyer learns the valuation of the realized $v^b$, and the seller learns the valuation of the realized $v^s$. At the same time, the agent receives a signal about $v^b$ as the interval of $I^b_i$ such that the realized $v^b$ is within $I^b_i$ for some $i \in \{0, 1, \cdots, n-1\}$. Simultaneously, the agent also receives a signal about $v^s$ as the interval of $I^s_j$ such that the realized $v^s$ is within $I^s_j$ for some $j \in \{0, 1, \cdots, n-1\}$. Third, the informed agent delivers a private message to each buyer and seller concerning the valuation interval of the corresponding adversary. After receiving the message, the buyer and the seller simultaneously submit sealed offers of $b$ and $s$, respectively. If $b \geq s$, then they trade the good at a price of $\frac{b+s}{2}$. The above structure of the game is common knowledge.

### 1.3 Bargaining equilibrium

In this section, I analyze the buyer’s and seller’s equilibrium strategies for the bargaining game. Among the multiple potential equilibria, I only consider those strategies that give the most efficient outcome.\(^4\)

Consider the realized $v^b \in [\underline{v}^b, \overline{v}^b] \subset [\underline{v}, \overline{v}]$ and the realized $v^s \in [\underline{v}^s, \overline{v}^s] \subset [\underline{v}, \overline{v}]$. Assume that $[\underline{v}^b, \overline{v}^b]$, the support of the realized $v^b$, and $[\underline{v}^s, \overline{v}^s]$, the support of the realized $v^s$, are common knowledge.

If $\underline{v}^b \geq \overline{v}^s$, then the buyer and seller can trade the good with certainty as the buyer’s valuation is always greater than or equal to the seller’s valuation. Their equilibrium strategies are $b = s \in [\overline{v}^s, \underline{v}^b]$. Otherwise, they cannot trade the good with

\(^4\)For the detailed multiplicity of the equilibria, see Leininger et al. (1989)
certainty. In this case, I consider their equilibrium strategies as in Chatterjee and Samuelson (1983) such that the buyer’s offer strategy \( b(\cdot) \) and the seller’s offer strategy \( s(\cdot) \) are assumed to be bounded above and below and to be strictly increasing and differentiable except possibly at these offer bounds. The following lemma summarizes the equilibrium strategies of the buyer and seller.

**Lemma 1** Suppose it is common knowledge that \([v^b, \bar{v}^b] \subset [v, \bar{v}]\) includes the realized \( v^b \) and that \([v^s, \bar{v}^s] \subset [v, \bar{v}]\) includes the realized \( v^s \). Let \( \tilde{b}(\cdot) \) and \( \tilde{s}(\cdot) \) be the solutions to the simultaneous differential equations (1.1) and (1.2) for \( x, y \in [v, \bar{v}] \):

\[
\frac{1}{2} F^b(y) \tilde{s}'(y) + f^b(y) \tilde{s}(y) = \tilde{b}^{-1}(\tilde{s}(y))f^b(y) \tag{1.1}
\]

\[
\frac{1}{2} (1 - F^s(x))\tilde{b}'(x) - f^s(x)\tilde{b}(x) = -\tilde{s}^{-1}(\tilde{b}(x))f^s(x) \tag{1.2}
\]

(i) If \( v^b < \bar{v}^s \), then the buyer’s and seller’s equilibrium strategies are \( b(v^b) = \tilde{b}(v^b) \) and \( s(v^s) = \tilde{s}(v^s) \) with the following boundary conditions:

1) If \( \tilde{b}(\bar{v}^b) > \tilde{s}(\bar{v}^s) \), then \( b(v^b) = \tilde{s}(\bar{v}^s) \) for \( v^b > \tilde{b}^{-1}(\tilde{s}(\bar{v}^s)) \)

2) If \( \tilde{b}(\bar{v}^b) < \tilde{s}(\bar{v}^s) \), then \( s(v^s) = \bar{v} \) for \( v^s > \tilde{b}(\bar{v}^b) \)

3) If \( \tilde{b}(\bar{v}^b) > \tilde{s}(\bar{v}^s) \), then \( s(v^s) = \tilde{b}(v^b) \) for \( v^s < \tilde{s}^{-1}(\tilde{b}(v^b)) \)

4) If \( \tilde{b}(\bar{v}^b) < \tilde{s}(\bar{v}^s) \), then \( b(v^b) = \bar{v} \) for \( v^b < \tilde{s}(\bar{v}^s) \)

(ii) Otherwise, the buyer’s and seller’s equilibrium strategies are \( b(v^b) = s(v^s) \in [\bar{v}^s, \underline{v}^b] \) for all \( v^b \in [\underline{v}^b, \bar{v}^b] \) and \( v^s \in [\underline{v}^s, \bar{v}^s] \).

As we can see in (ii) of Lemma 1, if the buyer and seller can trade the good with certainty, there exist multiple equilibria. In each equilibrium, the buyer’s offer and the seller’s offer should be the same and within the interval of \([\bar{v}^s, \underline{v}^b]\), regardless of their respective valuations.

However, if the buyer and seller cannot trade the good with certainty, their offers vary according to each party’s valuation. To illustrate, let me classify buyer and seller into three types each: low-, medium-, and high-value buyer; and low-, medium-, and high-value seller. If \( v^b \) is less than \( \tilde{s}(\bar{v}^s) \), this is a low-value buyer; if \( v^b \) is higher than \( \tilde{b}^{-1}(\tilde{s}(\bar{v}^s)) \), this is a high-value buyer; and otherwise, this is a medium-value buyer.
Similarly, if $v^s$ is less than $\tilde{s}^{-1}(\tilde{b}(v^b))$, this is a low-value seller; if $v^s$ is higher than $\tilde{b}(v^b)$, this is a high-value seller; and otherwise, this is a medium-value seller.

As we can see in the first boundary condition in (i) of Lemma 1, a high-value buyer’s offer, according to the offer function $\tilde{b}(\cdot)$, is always higher than the seller’s offer: a high-value buyer trades the good with certainty. Thus, the offer function $\tilde{b}(\cdot)$ is suboptimal. If the buyer matches the highest-value seller’s offer, then the expected sales price is lower than in the case where the buyer uses the offer function $\tilde{b}(\cdot)$ while still trading the good with certainty. Therefore, a high-value buyer matches the highest-value seller’s offer.

In contrast, as shown in the fourth boundary condition in (i) of Lemma 1, a low-value buyer cannot trade at all because the seller’s offer is always higher than the buyer’s valuation of the good. Thus, the low-value buyer just submits $v$, which certainly makes the probability of trade zero.\(^5\)

Similarly but symmetrically, as we can see in the third boundary condition in (i) of Lemma 1, a low-value seller’s offer, according to the offer function $\tilde{s}(\cdot)$, is always lower than the buyer’s offer: a low-value seller trades the good with certainty. Thus, the offer function $\tilde{s}(\cdot)$ is suboptimal. If the seller matches the lowest-value buyer’s offer, then the expected sales price is higher than in the case where the seller uses the offer function $\tilde{s}(\cdot)$ while still trading the good with certainty. Therefore, a low-value seller matches the lowest-value buyer’s offer.

In contrast, as shown in the second boundary condition in (i) of Lemma 1, a high-value seller cannot trade at all because the buyer’s offer is always lower than the seller’s valuation of the good. Thus, the high-value seller submits $v$, which certainly makes the probability of trade zero.\(^6\)

\(^5\)In the equilibrium strategies described in Chatterjee and Samuelson (1983), a low-value buyer can submit any number less than $\tilde{s}(v^s)$ because all offers result in zero probability of trade. I only consider the equilibrium in which the low-value buyer submits $v$.

\(^6\)As with the footnote 5, I only consider the equilibrium in which the high-value seller submits $v$. 
A medium-value buyer uses the offer function $\tilde{b}(\cdot)$, which is optimal for the buyer given the seller’s strategy. Similarly, a medium-value seller uses the offer function $\tilde{s}(\cdot)$, which is optimal for the seller given the buyer’s strategy.

Note that a high-value buyer trades the good with certainty, a medium-value buyer trades with probability less than 1, and a low-value buyer does not trade at all. Similarly but symmetrically, a low-value seller trades the good with certainty, a medium-value seller trades with probability less than 1, and a high-value seller does not trade at all.

1.4 Information dissemination by the informed agent

As I assumed in Section 1.2, the informed agent receives a signal about the buyer’s valuation as one of the elements in $V_b^i = \{I^i_b = [v^b_i, v^b_{i+1}]|v = v^b_0 < v^b_1 < \cdots < v^b_i = \overline{v}, i = 0, 1, \ldots, n-1\}$ and a signal about the seller’s valuation as one of the elements in $V_s^j = \{I^j_s = [v^s_j, v^s_{j+1}]|v = v^s_0 < v^s_1 < \cdots < v^s_j = \overline{v}, j = 0, 1, \ldots, n-1\}$.\(^7\)

After the agent receives these signals, and before the buyer and seller submit their respective offers, the agent delivers a private message to each party. The message space to the seller about the buyer’s valuation is denoted by $M^b$, and the message space to the buyer about the seller’s valuation is denoted by $M^s$. Thus, the agent’s strategy for the buyer side is $m^s : V^s \rightarrow M^s$, and for the seller side is $m^b : V^b \rightarrow M^b$.

In the rest of this chapter, I consider a bargaining equilibrium with fully revealed information that the agent truthfully disseminates information to the buyer and seller, and in which the buyer and seller follow the equilibrium strategies described in Lemma 1.\(^8\) Then, without loss of generality, the message spaces can be assumed as $M^b = \ldots$

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\(^7\)If the realized $v^b = v^b_i$ for some $i \in \{1, 2, \ldots, n-1\}$, then the informed agent receives a signal of either $I^b_{i-1} = [v^b_{i-1}, v^b_i]$ or $I^b_i = [v^b_i, v^b_{i+1}]$ with a half probability each. The buyer observes what the agent receives. The same occurs on the seller’s side.

\(^8\)The bargaining equilibrium with fully revealed information is similar to the fully revealing equilibrium in Suvorov and Tsybuleva (2010)
\( \{m^b_i = [v^b_i, v^b_{i+1}] | i = 0, 1, \ldots n - 1 \} \), and \( M^s = \{m^s_j = [v^s_j, v^s_{j+1}] | j = 0, 1, \ldots n - 1 \} \).

In other words, the bargaining equilibrium with fully revealed information consists of the agent’s message \( m^s(\cdot) \) to the buyer, the agent’s message \( m^b(\cdot) \) to the seller, and the buyer’s and seller’s offer strategies such that: (1) for each \( j \in \{0, 1, \ldots, n - 1 \} \), \( m^s(I^s_j) = m^s_j \); (2) for each \( i \in \{0, 1, \ldots, n - 1 \} \), \( m^b(I^b_i) = m^b_i \); and (3) the buyer and seller follow the strategies described in Lemma 1.

I need the following lemma for the main result.

**Lemma 2** (i) Consider the two cases wherein \( v^s \in [v^s_j, v^s_{j+1}] \) and \( v^s \in [v^s_i, v^s_{i+1}] \) such that \( j \neq l \) for some \( j, l \in \{0, 1, \ldots, n - 1 \} \). Let \( \tilde{b}_j(\cdot) \) and \( \tilde{b}_l(\cdot) \) be the solutions of \( \tilde{b}(\cdot) \) in Lemma 1 corresponding to \( v^s \in [v^s_j, v^s_{j+1}] \) and \( v^s \in [v^s_i, v^s_{i+1}] \), respectively. Then, whenever \( j > l \), \( \tilde{b}_j(v^b) > \tilde{b}_l(v^b) \) for all \( v^b \).

(ii) Consider the two cases wherein \( v^b \in [v^b_i, v^b_{i+1}] \) and \( v^b \in [v^b_k, v^b_{k+1}] \) such that \( i \neq k \) for some \( i, k \in \{0, 1, \ldots, n - 1 \} \). Let \( \tilde{s}_i(\cdot) \) and \( \tilde{s}_k(\cdot) \) be the solutions of \( \tilde{s}(\cdot) \) in Lemma 1 corresponding to \( v^b \in [v^b_i, v^b_{i+1}] \) and \( v^b \in [v^b_k, v^b_{k+1}] \), respectively. Then, whenever \( i > k \), \( \tilde{s}_i(v^s) > \tilde{s}_k(v^s) \) for all \( v^s \).

Lemma 2 says that a medium-value buyer submits a higher offer when the seller has a higher value, and a medium-value seller submits a higher offer when the buyer has a higher value. Thus, if the buyer and seller always accept the agent’s message as true, the agent can make a party change the offer strategy by delivering false information about the adversary’s valuation. The following lemma shows how the agent can influence the buyer’s and seller’s strategies.

**Lemma 3** Assume that the true state is \( (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] \), and that \( v^b_i < v^s_{j+1} \) for some \( i, j \in \{0, 1, \ldots, n - 1 \} \). Suppose the buyer and seller always believe that the informed agent truthfully disseminates the information to the buyer and seller.

*Compared with the true message \( m^b_i \) to the seller,*

(i) A false message \( m^b_k \) for some \( 0 \leq k < i \) to the seller makes the seller more likely to submit \( \bar{v} \); if the seller submits an offer less than \( \bar{v} \), the seller submits a lower offer.
(ii) A false message $m^b_k$ for some $i < k \leq n$ to the seller makes the seller less likely to submit $\overline{v}$; if the seller submits an offer less than $\overline{v}$, the seller submits a higher offer.

Compared with the true message $m^b_j$ to the buyer,

(iii) A false message $m^b_l$ for some $j < l \leq n$ to the buyer makes the buyer more likely to submit $\overline{v}$; if the buyer submits an offer higher than $\overline{v}$, the buyer submits a higher offer.

(iv) A false message $m^b_l$ for some $0 \leq l < j$ to the buyer makes the buyer less likely to submit $\overline{v}$; if the buyer submits an offer higher than $\overline{v}$, the buyer submits a lower offer.

Suppose the agent lies to the seller that the buyer has a lower valuation than the true state and is believed by the seller. Then, on the one hand, the seller is more likely to submit $\overline{v}$ and so does not trade at all. This is due to a medium-value seller misconceiving that the buyer’s valuation is too low for the trade to occur. On the other hand, if the seller submits an offer less than $\overline{v}$, this offer is lower than in the case of the agent telling the truth, as a lower offer maximizes expected utility from the deceived seller’s point of view. These aspects are summarized in (i) of Lemma 3.

Suppose the agent lies to the seller that the buyer has a higher valuation than the true state and is believed by the seller. Then, on the one hand, the seller is less likely to submit $\overline{v}$. This is caused by a high-value seller misconceiving that the buyer’s valuation is high enough for the trade to occur. On the other hand, if the seller submits an offer less than $\overline{v}$, this offer is higher than in the case of the agent telling the truth, as a higher offer maximizes expected utility from the deceived seller’s point of view. These aspects are summarized in (ii) of Lemma 3.

Suppose the agent lies to the buyer that the seller has a higher valuation than the true state and is believed by the buyer. Then, on the one hand, the buyer is more likely to submit $\overline{v}$ and so does not trade at all. This is due to a medium-value buyer misconceiving that the seller’s valuation is too high for the trade to occur. On the
other hand, if the buyer submits an offer higher than \( v \), this offer is higher than in the case of the agent telling the truth, as a higher offer maximizes expected utility from the deceived buyer’s point of view. These aspects are summarized in (iii) of Lemma 3.

Suppose the agent lies to the buyer that the seller has a lower valuation than the true state and is believed by the buyer. Then, on the one hand, the buyer is less likely to submit \( v \). This is caused by a low-value buyer misconceiving that the seller’s valuation is low enough for the trade to occur. On the other hand, if the buyer submits an offer higher than \( v \), this offer is lower than in the case of the agent telling the truth, as a lower offer maximizes expected utility from the deceived buyer’s point of view. These aspects are summarized in (iv) of Lemma 3.

Now, I present the main result of this chapter.

**Theorem 1** *If there exists an equilibrium in which a price-maximizing agent truthfully disseminates information to both buyer and seller, then there also exists an equilibrium in which a trade-maximizing agent truthfully disseminates information to both buyer and seller.*

Theorem 1 says that for any given information partition structure of the informed agent and the distributions of \( v_b \) and \( v_s \) which satisfy the assumptions in Section 1.2, if there exists a bargaining equilibrium with fully revealed information when the informed agent is a price-maximizing agent, then that equilibrium also exists when the informed agent is a trade-maximizing agent.

However, the converse of Theorem 1 is not true. That is, for some information partition structures of the informed agent and some distributions of \( v_b \) and \( v_s \), there exists a bargaining equilibrium with fully revealed information when the informed agent is a trade-maximizing agent, but not when the informed agent is a price-maximizing agent.\(^9\)

\(^9\)One example is when both \( v_b \) and \( v_s \) are distributed uniformly over \([0, 1]\), and the informed
Note that the babbling equilibrium in which the informed agent does not disseminate any information at all and thus the buyer and seller complete the bargaining game without receiving finer information from the agent always exists. The following lemma indicates that if the bargaining equilibrium with fully revealed information exists, then the probability of trade is higher than for the babbling equilibrium case.

**Lemma 4** The probability of trade in the bargaining equilibrium with fully revealed information is higher than in the babbling equilibrium.

The above Lemma 4 says that if finer information from the informed agent is available in the bargaining game, then the probability of trade is higher. As we can see in Chatterjee and Samuelson (1983), the bargaining game fails in ex-post efficiency, which requires the buyer and seller to trade the good as long as the buyer’s valuation is greater than or equal to the seller’s valuation, because of the incomplete information between the buyer and seller. That incomplete information leads the buyer to aggressively submit a lower offer and the seller to aggressively submit a higher offer than each one’s true valuation of the good. Thus, if we can reduce the intensity of the information incompleteness between the buyer and seller, we can increase the probability of trade, which makes for smaller ex-post inefficiency. In my model, the informed agent can effect this because disseminating finer information causes the buyer and seller to submit less aggressive offers.

Moreover, as illustrated in the following lemma, if the bargaining equilibrium with fully revealed information exists, then its expected sales price is also higher than in the case of the babbling equilibrium.

**Lemma 5** The expected sales price in the bargaining equilibrium with fully revealed information is higher than in the babbling equilibrium.

agent’s information sets are $V^b = \{[0,4/7], [4/7, 1]\}$ and $V^s = \{[0,3/7], [3/7, 1]\}$. 
Note that the bargaining rule in which the buyer and seller trade the good at the average of their offers when the buyer’s offer is greater than or equal to the seller’s offer does not lean toward either buyer or seller. Moreover, the buyer’s valuation and the seller’s valuation are reflection symmetric by Assumption 1. Also, note that by Assumption 2, the informed agent is equally informative to both buyer and seller, as I mentioned in Section 1.2. Thus, as demonstrated in the proof of Lemma 5, the main reason for the bargaining equilibrium with fully revealed information having a higher expected sales price than the babbling equilibrium is that it has a higher probability of trade.

Now, I can compare trade-maximizing and price-maximizing agents in terms of the probability of trade and expected sales price. The below two corollaries hold under the assumption that the bargaining equilibrium with fully revealed information is selected over the babbling equilibrium if it exists.

**Corollary 2** *In the bargaining game with an informed agent, a trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent.*

**Corollary 3** *In the bargaining game with an informed agent, a trade-maximizing agent leads to a weakly higher expected sales price than a price-maximizing agent.*

Corollary 2 sounds trivial, but Corollary 3 does not. As we can see in Theorem 1, and as I mentioned after the theorem, there are some cases where a trade-maximizing agent makes finer information available in the market, but a price-maximizing agent does not. As a result, a trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent, which means weakly smaller ex-post inefficiency. Moreover, as I mentioned after Lemma 5, under the reflection-symmetric setting of the game, a higher probability of trade means a higher expected sales price. Therefore, a trade-maximizing agent leads to a weakly higher expected sales price than even a price-maximizing agent, due to having a weakly higher probability of trade.
1.5 Conclusion

In this chapter, I study a bargaining game with an informed agent who can disseminate finer information to a buyer and seller. If the agent truthfully disseminates this information to both the buyer and the seller, then the information incompleteness between them is reduced. As a result, both buyer and seller submit their offers in a less aggressive way. This has the effect of reducing ex-post inefficiency, which always exists in a bargaining game under incomplete information.

In this context, I compare two different kinds of informed agents, a price-maximizing agent and a trade-maximizing agent. I show that if a price-maximizing agent disseminates information truthfully to the buyer and seller in equilibrium, so does a trade-maximizing agent. That is, without any specific assumptions on the information structure of the informed agent and the distributions of the buyer’s and seller’s valuations of the good, we do not know whether there exists a bargaining equilibrium with fully revealed information, but if this equilibrium exists when the informed agent is a price-maximizing agent, then it also exists when the informed agent is a trade-maximizing agent.

Furthermore, the converse of the above statement is not true. That is, for some cases, a bargaining equilibrium with fully revealed information exists when the informed agent is a trade-maximizing agent, but it does not exist when the informed agent is a price-maximizing agent. As a result, a trade-maximizing agent leads to a higher probability of trade, which means smaller ex-post inefficiency, and also leads to a higher expected sales price than even a price-maximizing agent due to that higher probability. This is because finer information is available in the market when the agent is a trade-maximizing agent, but it is not available when the agent is a price-maximizing agent.
Chapter 2

Informed Agent’s Advice

2.1 Introduction

In this chapter, I study the model in the previous chapter under specific assumptions. Here, I assume that both buyer’s and seller’s valuations are uniformly distributed, and an informed agent knows whether each of the buyer and seller has a high or low valuation.

We can interpret the setting as follows. When each of the buyer and seller has a high (low) valuation, it is a seller’s (buyer’s) market. When a buyer has a high (low) valuation but a seller has a low (high) valuation, it is an expansion (a recession). The agent knows the current market condition as one of a seller’s market, a buyer’s market, an expansion, or a recession, and the agent can advise each buyer and seller about it.

Thus, there are four possible equilibria: (1) a babbling equilibrium in which an informed agent delivers no information at all, (2) a bargaining equilibrium in which an informed agent truthfully advises a buyer only, (3) a bargaining equilibrium in which an informed agent truthfully advises a seller only, and (4) a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller1. Note that the babbling equilibrium always exists.

I show that a bargaining equilibrium in which an informed agent truthfully advises

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1A bargaining equilibrium in which an informed agent truthfully advises both buyer and seller is the same as a bargaining equilibrium with fully revealed information in Chapter 1.
a seller only always exists regardless of whether the agent is a trade-maximizing agent or a price-maximizing agent. A bargaining equilibrium in which an informed agent truthfully advises a buyer only and a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller always exist when the agent is a trade-maximizing agent. However, when the agent is a price-maximizing agent, the above two equilibria do not always exist.

The intuition behind the above results is as follows. If an informed agent disseminates information about the market condition, the trade between a buyer and seller is more likely to occur. A trade-maximizing agent always disseminates the information because the agent’s utility is increasing in the probability of trade. However, a price-maximizing agent’s utility is increasing in the expected sales price. That is, a price-maximizing agent’s interest and a seller’s interest are perfectly aligned, but a price-maximizing agent’s interest conflicts with a buyer’s interest. Therefore, a price-maximizing agent is always truthful for a seller, but not always for a buyer.

In other words, a trade-maximizing agent makes more detailed information available in the market than a price-maximizing agent. As a result, a trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent. Moreover, a trade-maximizing agent leads to a weakly higher expected sales price than even a price-maximizing agent due to the weakly higher trade probability.

The rest of this chapter is organized as follows. Section 2.2 presents the model setting in this chapter. Section 2.3 analyzes the case where an informed agent advises a buyer only. The case that an informed agent advises a seller only is presented in Section 2.4, and Section 2.5 studies the case that an informed agent advises both buyer and seller. I conclude in Section 2.6. All the proofs of results in this chapter are in Appendix B.
2.2 Basic setting

The basic setting of the model and notations in this chapter are similar to those in the previous chapter. There are three players: a buyer, a seller, and an informed agent. I assume that the seller owns an indivisible good that the buyer wants to obtain. The valuations of the buyer and the seller are denoted by $v^b$ and $v^s$, respectively. The buyer and seller are risk neutral, and each has an additively separable utility for the good and money. If they trade the good at a price of $p$, the buyer’s utility is $v^b - p$, and the seller’s utility is $p - v^s$. Their utilities are normalized to 0 if no trade occurs.

Moreover, I assume that $v^b$ and $v^s$ are independent random variables distributed uniformly over $[0,1]$. That is, $F^b(v) = F^s(v) = v$ and $f^b(v) = f^s(v) = 1$ for all $v \in [0,1]$. Note that $F^b(v) = 1 - F^s(1-v)$ for all $v \in [0,1]$. Thus, it satisfies Assumption 1 in Chapter 1, reflection-symmetric valuations of buyer and seller.

The informed agent has an information set of the partition of the interval $[0,1]$ about the buyer’s valuation as $V^b = \{I^b_0 = [0,y], I^b_1 = [y,1]|0 \leq y \leq 1\}$. Similarly, the agent also has an information set of the partition of the interval $[0,1]$ about the seller’s valuation as $V^s = \{I^s_0 = [0,1-y], I^s_1 = [1-y,1]|0 \leq y \leq 1\}$. That is, after the realization of $v^b$, the agent knows whether the buyer has a high ($v^b \in [y,1]$) or low ($v^b \in [0,y]$) valuation. Similarly, after the realization of $v^s$, the agent knows whether the seller has a high ($v^s \in [1-y,1]$) or low ($v^s \in [0,1-y]$) valuation. Note that the value of $y$ denotes the agent’s information structure. Also, note that $y = 1 - (1-y)$ for all $y \in [0,1]$. Thus, it satisfies Assumption 2 in Chapter 1, reflection-symmetric sets of $V^b$ and $V^s$.

After the agent knows whether each of the buyer and seller has a high or low valuation, the agent delivers a private message to each party. The message space to
the seller about the buyer’s valuation is denoted by $M^b$, and the message space to the buyer about the seller’s valuation is denoted by $M^s$. Thus, the agent’s strategy for the buyer side is $m^s: V^s \rightarrow M^s$, and for the seller side is $m^b: V^b \rightarrow M^b$.

In the rest of this chapter, I consider equilibria in which the agent truthfully advises the buyer only, the seller only, and both the buyer and seller. Thus, without loss of generality, the message spaces can be assumed as $M^b = \{m^b_0 = [0, y], m^b_1 = [y, 1] | 0 \leq y \leq 1\}$ and $M^s = \{m^s_0 = [0, 1 - y], m^s_1 = [1 - y, 1] | 0 \leq y \leq 1\}$.

As in Chapter 1, I define two kinds of informed agents: a price-maximizing agent and a trade-maximizing agent. The utility of the price-maximizing agent is increasing in the expected sales price, and that of the trade-maximizing agent is increasing in the probability of trade.

The timeline of the game is as follows: First, nature selects $v^b$ and $v^s$ from the uniform distribution over $[0, 1]$. Second, the buyer learns the valuation of the realized $v^b$, and the seller learns the valuation of the realized $v^s$. At the same time, the agent receives a signal about $v^b$ as the interval of $I^b_i$ such that the realized $v^b$ is within $I^b_i$ for some $i \in \{0, 1\}$. Simultaneously, the agent also receives a signal about $v^s$ as the interval of $I^s_j$ such that the realized $v^s$ is within $I^s_j$ for some $j \in \{0, 1\}$. Third, the informed agent delivers a private message to each buyer and seller concerning the valuation interval of the corresponding adversary. After receiving the message, the buyer and the seller simultaneously submit sealed offers of $b$ and $s$, respectively. If $b \geq s$, then they trade the good at a price of $\frac{b+s}{2}$. The above structure of the game is common knowledge.

The following lemma is the counterpart of Lemma 1 in Chapter 1.

**Lemma 6** Consider intervals $[\underline{v}^b, \overline{v}^b] \subset [0, 1]$ and $[\underline{v}^s, \overline{v}^s] \subset [0, 1]$. Suppose it is common knowledge that $[\underline{v}^b, \overline{v}^b]$ includes the realized $v^b$ and that $[\underline{v}^s, \overline{v}^s]$ includes the realized $v^s$. Let

$$\tilde{b}(v^b) = \frac{2}{3} v^b + \frac{1}{12} \overline{v}^b + \frac{1}{4} v^s,$$

where
and
\[ \tilde{s}(v^s) = \frac{2}{3}v^s + \frac{1}{4}v^b + \frac{1}{12}v^s. \]  
(2.2)

(i) If \( v^b < \bar{v}^s \), then the buyer’s and seller’s equilibrium strategies are \( b(v^b) = \tilde{b}(v^b) \) and \( s(v^s) = \tilde{s}(v^s) \) with the following boundary conditions:

1) If \( \frac{4}{3}v^b - \frac{1}{3}v^s > v^s \), then \( b(v^b) = \frac{2}{3}v^s + \frac{1}{4}v^b + \frac{1}{12}v^s \) for \( v^b > v^s + \frac{1}{4}(v^b - v^s) \)

2) If \( \frac{4}{3}v^b - \frac{1}{3}v^s < v^s \), then \( s(v^s) = 1 \) for \( v^s > \frac{3}{4}v^b + \frac{1}{4}v^s \)

3) If \( \frac{4}{3}v^b - \frac{1}{3}v^s < v^s \), then \( s(v^s) = \frac{2}{3}v^b + \frac{1}{12}v^b + \frac{1}{12}v^s \) for \( v^s < v^b - \frac{1}{4}(v^b - v^s) \)

4) If \( \frac{4}{3}v^b - \frac{1}{3}v^s > v^s \), then \( b(v^b) = 0 \) for \( v^b < \frac{3}{4}v^s + \frac{1}{4}v^b \)

(ii) Otherwise, the buyer’s and seller’s equilibrium strategies are \( b(v^b) = s(v^s) \in [\bar{v}^s, \bar{v}^b] \) for all \( v^b \in [\bar{v}^b, \bar{v}^b] \) and \( v^s \in [\bar{v}^s, \bar{v}^s] \).

In the rest of the chapter, if \( \bar{v}^b \geq \bar{v}^s \), I consider the buyer’s and seller’s equilibrium strategies as \( b(v^b) = s(v^s) \) for all \( v^b \in [\bar{v}^b, \bar{v}^b] \) and \( v^s \in [\bar{v}^s, \bar{v}^s] \) among the multiple equilibria in (ii) of Lemma 6.

Note that the babbling equilibrium in which the informed agent delivers no information at all and thus the buyer and seller do the bargaining game as in Chatterjee and Samuelson (1983) always exists. Based on the above setting, there are three other candidate equilibria: (1) a bargaining equilibrium in which an informed agent truthfully advises a buyer only, (2) a bargaining equilibrium in which an informed agent truthfully advises a seller only, and (3) a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller. In the following sections, I consider those equilibria.

### 2.3 Advice for a buyer

In this section, I consider a bargaining equilibrium in which an informed agent truthfully advises a buyer only. That is, it consists of the agent’s message \( m^s(\cdot) \) to the buyer and the buyer’s and seller’s strategies such that: (1) for each \( j \in \{0, 1\} \), \( m^s(I^s_j) = m^s_j \), and (2) the buyer and seller follow the strategies described in Lemma
6. Note that the agent reveals information only to the buyer and babbles to the seller in this equilibrium.

The below two theorems say that the equilibrium exists when the agent is a trade-maximizing agent, but when the agent is a price-maximizing agent, the existence of the equilibrium depends on the value of $y$, the information structure of the agent.

**Theorem 4** Suppose an informed agent is a trade-maximizing agent and can only advise a buyer. Then, there exists an equilibrium in which the agent truthfully reveals information.

**Theorem 5** Suppose an informed agent is a price-maximizing agent and can only advise a buyer. Then, there exists an equilibrium in which the agent truthfully reveals information if and only if $y \in \left[0, \frac{3\sqrt{11}-17}{4} \approx 0.5523\right]$.

As we can see in Theorem 4, Theorem 5, and the proofs of those in Appendix B, when an informed agent advises a buyer only, a trade-maximizing agent always truthfully advises a buyer. However, for some values of $y$, a price-maximizing agent has an incentive to lie to a buyer that a seller has a high valuation ($v^s \in [1 - y, 1]$) in order to increase the buyer’s offer and thus to increase the sales price when the seller actually has a low valuation ($v^s \in [0, 1 - y]$). If that is the case, it goes back to the babbling equilibrium where the common knowledge is that $v^b$ and $v^s$ are distributed uniformly over $[0, 1]$. That is because a price-maximizing agent always says to a buyer that a seller has a high valuation regardless of the true state.

Note that in the babbling equilibrium, the probability of trade is $\frac{9}{32}$ and the expected sales price is $\frac{9}{64}$ as we can see in Chatterjee and Samuelson (1983). The below two lemmas say that the agent’s truthful advice for a buyer makes a weakly higher probability of trade as well as a weakly higher expected sales price relative to the babbling equilibrium.

**Lemma 7** In a bargaining equilibrium in which an informed agent truthfully advises a buyer only, the probability of trade is weakly higher than in the babbling equilibrium.
Lemma 8  In a bargaining equilibrium in which an informed agent truthfully advises a buyer only, the expected sales price is weakly higher than in the babbling equilibrium.

The agent’s finer information on a seller’s valuation can make the trade more likely to occur. Moreover, it also leads to a higher expected sales price due to the higher probability of trade. On the one hand, the finer information on a seller’s valuation is always available in the market when the agent is a trade-maximizing agent. On the other hand, when the agent is a price-maximizing agent, the finer information is not available in the market for all $y \in (\frac{3\sqrt{41}-17}{4}, 1]$ as we can see in Theorem 5. As a result, under the assumption that an equilibrium with finer information is selected over the babbling equilibrium if it exists, a trade-maximizing agent leads to a higher probability of trade and a higher expected sales price than a price-maximizing agent for all $y \in (\frac{3\sqrt{41}-17}{4}, 1)$.\(^4\) The following corollaries summarize the above aspects.

Corollary 6  A trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent.

Corollary 7  A trade-maximizing agent leads to a weakly higher expected sales price than a price-maximizing agent.

Corollary 6 sounds trivial, but Corollary 7 does not. As I mentioned above, because a price-maximizing agent always says to a buyer that a seller has a high valuation for all $y \in (\frac{3\sqrt{41}-17}{4}, 1]$, the agent’s message is not credible to the buyer in that case. Consequently, the agent’s finer information on a seller’s valuation is not available in the market, and thus it goes back to the babbling equilibrium. For this reason, a price-maximizing agent leads to a lower expected sales price than even a trade-maximizing agent due to a lower probability of trade.

\(^4\)Note that $y = 1$ means an uninformed agent. Thus, there is no difference between a trade-maximizing agent and a price-maximizing agent.
2.4 Advice for a seller

In this section, I consider a bargaining equilibrium in which an informed agent truthfully advises a seller only. That is, it consists of the agent’s message $m^b(\cdot)$ to the seller and the buyer’s and seller’s strategies such that: (1) for each $i \in \{0, 1\}$, $m^b(I^b_i) = m^b_i$, and (2) the buyer and seller follow the strategies described in Lemma 6. Note that the agent reveals information only to the seller and babbles to the buyer in this equilibrium.

The below two theorems say that the equilibrium exists regardless of whether an informed agent is a trade-maximizing agent or a price-maximizing agent.

**Theorem 8** Suppose an informed agent is a trade-maximizing agent and can only advise a seller. Then, there exists an equilibrium in which the agent truthfully reveals information.

**Theorem 9** Suppose an informed agent is a price-maximizing agent and can only advise a seller. Then, there exists an equilibrium in which the agent truthfully reveals information.

As we can see in Theorem 8 and Theorem 9, when an informed agent advises a seller only, both a trade-maximizing agent and a price-maximizing agent always truthfully advise a seller.

In contrast with Section 2.3, a bargaining equilibrium in which an informed agent truthfully advises a seller only always exists even when the agent is a price-maximizing agent. That is because both a price-maximizing agent and a seller prefer a higher expected sales price. In other words, a seller’s interest and a price-maximizing agent’s interest perfectly coincide, and thus a price-maximizing agent always truthfully advises a seller.
2.5 Advice for both buyer and seller

In this section, I consider a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller. That is, it consists of the agent’s message \( m^s(\cdot) \) to the buyer, the agent’s message \( m^b(\cdot) \) to the seller, and the buyer’s and seller’s strategies such that: (1) for each \( j \in \{0, 1\} \), \( m^s(I^s_j) = m^s_j \); (2) for each \( i \in \{0, 1\} \), \( m^b(I^b_i) = m^b_i \); and (3) the buyer and seller follow the strategies described in Lemma 6.

The below two theorems say that the equilibrium exists when the agent is a trade-maximizing agent, but when the agent is a price-maximizing agent, the existence of the equilibrium depends on the value of \( y \), the information structure of the agent.

**Theorem 10** Suppose an informed agent is a trade-maximizing agent and advises both buyer and seller. Then, there exists an equilibrium in which the agent truthfully disseminates information.

**Theorem 11** Suppose an informed agent is a price-maximizing agent and advises both buyer and seller. Then, there exists an equilibrium in which the agent truthfully disseminates information if and only if \( y \in \left[0, \frac{1}{28} \left\{ 10 - \frac{5 \times 22^{2/3}}{(27 + 7\sqrt{71})^{1/3}} + (22(27 + 7\sqrt{71}))^{1/3} \right\} \approx 0.4819 \right] \) or \( y \in \left[\frac{17 - \sqrt{193}}{6} \approx 0.5179 \right]. \)

**Corollary 12** If there exists an equilibrium in which a price-maximizing agent truthfully advises both buyer and seller, then there also exists an equilibrium in which a trade-maximizing agent truthfully advises both buyer and seller.

As we can see in the above Theorem 10 and Theorem 11, when an informed agent advises both sides of the buyer and seller, a trade-maximizing agent always truthfully advises them. However, a price-maximizing agent truthfully advises the buyer and seller only for some values of \( y \). Thus, we can check that Theorem 1 in Chapter 1 holds in the model of this chapter as we can see in Corollary 12.

Note that there are four possible equilibria in this chapter: (1) a babbling equilibrium in which an informed agent delivers no information at all, (2) a bargaining
equilibrium in which an informed agent truthfully advises a buyer only, (3) a bargain-
ing equilibrium in which an informed agent truthfully advises a seller only, and (4) a 
bargaining equilibrium in which an informed agent truthfully advises both buyer and 
seller. Also, note that the babbling equilibrium always exists.

The below two corollaries show the conditions under which each equilibrium exists 
when an informed agent is a trade-maximizing agent and a price-maximizing agent, 
respectively.

**Corollary 13** Suppose an informed agent is a trade-maximizing agent. Then, there 
extist all of the possible four equilibria.

**Corollary 14** Suppose an informed agent is a price-maximizing agent. Then, 
(i) there exist two equilibria, which are the babbling equilibrium and a bargaining 
equilibrium in which the agent truthfully advises a seller only, for \( y \in (\frac{3\sqrt{74} - 17}{4}, 1) \).

(ii) there exist three equilibria, but not a bargaining equilibrium in which the agent 
truthfully advises both buyer and seller, for \( y \in (\frac{1}{28}(10 - \frac{5 \times 22^{2/3}}{(27 + 7\sqrt{71})^{1/3}} + (22(27 + 
7\sqrt{71}))^{1/3}}, \frac{1}{2}) \) and \( y \in (\frac{17 - \sqrt{193}}{6}, \frac{3\sqrt{11} - 17}{4}] \).

(iii) there exist all of the possible four equilibria for \( y \in [0, \frac{1}{28}(10 - \frac{5 \times 22^{2/3}}{(27 + 7\sqrt{71})^{1/3}} + 
(22(27 + 7\sqrt{71}))^{1/3}] \) and \( y \in [\frac{1}{2}, \frac{17 - \sqrt{193}}{6}] \).

As we can see in Corollary 13, when an informed agent is a trade-maximizing 
agent, all of the possible four equilibria, which are the babbling equilibrium, a bar-
gaining equilibrium in which the agent truthfully advises a buyer only, a bargaining 
equilibrium in which the agent truthfully advises a seller only, and a bargaining equi-
librium in which the agent truthfully advises both buyer and seller, exist.

However, as we can see in Corollary 14, when an informed agent is a price-
maximizing agent, the existence of the equilibria depends on the value of \( y \). As 
we saw in the proofs of Theorem 5 and Theorem 11, for some values of \( y \), a price-
maximizing agent has an incentive to lie to a buyer that a seller has a high valuation 
in order to increase the buyer’s offer and thus to increase the sales price when the
seller actually has a low valuation. That is, in that case, a price-maximizing agent always says to a buyer that a seller has a high valuation regardless of the true state. Therefore, it makes the agent’s advice for the buyer incredible, and thus the truthful advice for the buyer is not an equilibrium in that case.

The following lemma compares the probability of trade in each equilibrium.

**Lemma 9** The probability of trade in a bargaining equilibrium in which an informed agent truthfully advises a buyer only

(i) is the same as in a bargaining equilibrium in which an informed agent truthfully advises a seller only.

(ii) is weakly higher than in the babbling equilibrium.

(iii) is weakly lower than in a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller.

The following lemma compares the expected sales price in each equilibrium.

**Lemma 10** (i) For \( y \in [0, \frac{2(8+3\sqrt{3})}{37} \approx 0.7133] \) \((y \in [\frac{2(8+3\sqrt{3})}{37}, 1])\), the expected sales price in a bargaining equilibrium in which an informed agent truthfully advises a buyer only is weakly higher (lower) than in a bargaining equilibrium in which an informed agent truthfully advises a seller only.

(ii) The expected sales price in the babbling equilibrium is weakly lower than all other equilibria.

(iii) The expected sales price in a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller is weakly higher than all other equilibria.

The agent’s finer information can make the trade more likely to occur. On the one hand, the finer information is always available in the market when the agent is a trade-maximizing agent. On the other hand, when the agent is a price-maximizing agent, the finer information is not fully available in the market for some values of \( y \). As a result, under the assumption that an equilibrium with finer information is selected if it exists, a trade-maximizing agent leads to a higher probability of trade and a higher expected sales price than a price-maximizing agent in that case. The following corollaries summarize the above aspects.
Corollary 15 A trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent.

Corollary 16 A trade-maximizing agent leads to a weakly higher expected sales price than a price-maximizing agent.

Again, Corollary 15 sounds trivial, but Corollary 16 does not. As I mentioned above, because a price-maximizing agent has an incentive to lie to a buyer that a seller has a high valuation regardless of the true state for some values of $y$, the agent’s message is not credible to the buyer in that case. Consequently, the agent’s finer information is not fully available in the market. For this reason, a price-maximizing agent leads to a lower expected sales price than even a trade-maximizing agent due to a lower probability of trade.

2.6 Conclusion

In this chapter, I analyze the model in the previous chapter under specific assumptions. We can check that all possible equilibria, which are the babbling equilibrium, a bargaining equilibrium in which an informed agent truthfully advises a buyer only, a bargaining equilibrium in which an informed agent truthfully advises a seller only, and a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller, always exist when the agent is a trade-maximizing agent. However, when the agent is a price-maximizing agent, the existence of the equilibria depends on the value of $y$, which is the agent’s information structure.

A trade-maximizing agent’s interest is to make the trade between buyer and seller more likely to occur and thus does not lean toward either side of a buyer or a seller. However, a price-maximizing agent wants to make the trade at a higher price and thus leans toward a seller’s interest. Therefore, a price-maximizing agent is always truthful for a seller but not always for a buyer, while a trade-maximizing agent is
always truthful for both buyer and seller.

As a result, a trade-maximizing agent leads to a weakly higher probability of trade than a price-maximizing agent. Moreover, a trade-maximizing agent also leads to a weakly higher expected sales price than even a price-maximizing agent due to the weakly higher probability of trade.
Chapter 3

Percentage Commission versus Fixed Commission: Evidence from the South Korean Real Estate Market

3.1 Introduction

In this chapter, I empirically test the implications of the models from previous chapters using South Korean housing transaction data from 2006 to 2017. That is, I test whether a trade-maximizing agent actually leads to a higher probability of trade, and therefore smaller ex-post inefficiency, than a price-maximizing agent.

In South Korea, real estate agents receive a percentage commission if they match and close a deal between a buyer and a seller. That is, the real estate agent receives some percentage of the sales price. However, there are five intervals of sales prices, and the commission rates in each interval are different. Some intervals feature maximum limits on the commission. Consequently, in some subintervals of sales prices, every real estate agent receives a fixed amount of money regardless of sales price.

If a real estate agent receives an unrestricted percentage commission, then that agent’s incentive is the same as for the price-maximizing agent; the agent’s expected payoff depends on the expected sales price. In contrast, if a real estate agent receives a fixed commission regardless of the sales price, then the agent’s incentive is the same as that of the trade-maximizing agent; the agent’s expected payoff depends solely on the probability of trade. Therefore, I can compare those two kinds of informed
agents by comparing the percentage commission intervals with the fixed commission intervals.

However, the original data source does not include listing activities. Although I observe the number of sales, I cannot directly measure the probability of trade due to the missing listing data. Therefore, I apply cluster analysis, which is an unsupervised machine learning algorithm, on top of the above identification strategy. This analysis groups together time periods that have the same average number of listings; by setting these as fixed effect groups in my regression model, I am able to use the number of sales as a proxy for the probability of trade. The results from the regression show that a trade-maximizing agent brings an approximately 0.2% greater number of sales in each ten-day period than a price-maximizing agent.

The rest of the chapter is organized as follows. In Section 3.1.1, I briefly review the related literature. Section 3.2 presents the Korean real estate agents’ commission scheme and the data used in the empirical test. In Section 3.3, I introduce the cluster analysis. Section 3.4 presents regression models, and Section 3.5 shows results. I conclude in Section 3.6.

3.1.1 Literature review

This chapter is concerned with the real estate market. In this section, I briefly highlight some of the existing literature on this topic.

Zietz and Sirmans (2011) extensively review the literature of real estate brokerage research over the first decade of the 2000s, including efficiency, technology, performance, and agency relationships. I highlight some of this research here. Rutherford et al. (2005) and Levitt and Syverson (2008) compare sales of agent-owned properties with those of client-owned properties. Both papers find that agent-owned properties sell at higher premiums than client-owned properties. However, the latter paper finds that agent-owned properties stay on the market longer than client-owned properties,
while the former finds no significant difference in selling times. Huang and Rutherford (2007) similarly compare realtor listings with non-realtor listings. They find that non-realtor listings sell at lower prices, take more time to sell, and are less likely to sell than realtor listings. Miceli et al. (2007) look at the traditional compensation model for real estate agents; they argue that a percentage commission is socially unproductive and creates inefficiencies for buyers and sellers. Their work is consistent with the implication of my dissertation.

Out of the range of the literature review in Zietz and Sirmans (2011), Yinger (1981) points out the undesirable aspects of a percentage commission for real estate agents. Yavaş and Colwell (1999) indicate that the incentive of a real estate agent receiving a percentage commission differs from those of a buyer. Bernheim and Meer (2013) show that for sellers, the cost of using a real estate agent’s service exceeds its advantages. Some studies suggest alternative commission schemes for real estate agents (Miceli, 1991; Colwell et al., 1993, 1994). Hendel et al. (2009) compare two platforms, the Multiple Listing Service (MLS) and the For-Sale-By-Owner (FSBO). They find that the FSBO, with its strictly non-realtor listings, is less effective in terms of time to sell and the probability of a sale.

### 3.2 Background

The main result of the previous chapters is that a trade-maximizing agent is more likely to reduce the information incompleteness between buyer and seller than a price-maximizing agent is; this occurs through disseminating finer information to the buyer and seller. If that is the case, the trade-maximizing agent leads to a higher probability of trade, which means smaller ex-post inefficiency. In this chapter, I empirically test whether a trade-maximizing agent actually leads to a higher probability of trade than a price-maximizing agent.
For this test, as I mentioned in Section 3.1, I use data from the real estate market. To compare price-maximizing and trade-maximizing agents, I compare two different commission schemes for real estate agents: a percentage commission and a fixed commission.

Under a percentage commission, a real estate agent receives some percentage of the sales price after a buyer and seller trade a property. As the real estate agent’s payoff depends on the final sales price, his or her incentive is the same as that of a price-maximizing agent. In contrast, under a fixed commission, a real estate agent receives some fixed amount of money after a buyer and seller trade a property, regardless of the final sales price. As the real estate agent’s payoff depends solely on whether trade occurs, his or her incentive is the same as that of a trade-maximizing agent. Therefore, I can compare price-maximizing and trade-maximizing agents by comparing outcomes under percentage and fixed commissions.

However, it is generally difficult to directly compare these two different commission schemes using real-world real estate data, due to fixed commissions being uncommon in the real estate market. Moreover, even though some real estate agents receive fixed commissions, there exist other real estate agents who receive percentage commissions, and thus there are self-selection issues. Not only do real estate agents self-select their commission schemes, but buyers and sellers also self-select a real estate agent according to the agent’s commission scheme. Therefore, it is difficult to isolate pure differences between the commission schemes and therefore agent types.

Fortunately, the unique feature of the Korean commission scheme for real estate agents allows me to compare agents receiving percentage and fixed commissions directly. Thus, I can do so by using housing transaction data from South Korea. Moreover, as we can see in the following section, I do not need to worry about self-selection issues in the analysis.
3.2.1 Real estate agents’ commission scheme in South Korea

Table 3.1: Commission scheme for real estate agents in South Korea

<table>
<thead>
<tr>
<th>Sales Price*</th>
<th>Commission Rate</th>
<th>Maximum Limit*</th>
</tr>
</thead>
<tbody>
<tr>
<td>under W50</td>
<td>0.6%</td>
<td>W0.25</td>
</tr>
<tr>
<td>W50 – W200</td>
<td>0.5%</td>
<td>W0.80</td>
</tr>
<tr>
<td>W200 – W600</td>
<td>0.4%</td>
<td>—</td>
</tr>
<tr>
<td>W600 – W900</td>
<td>0.5%</td>
<td>—</td>
</tr>
<tr>
<td>over W900</td>
<td>0.9%</td>
<td>—</td>
</tr>
</tbody>
</table>

* In million W (Korean currency; approximately 1 US dollar equals 1,000 Korean Won)

Notes: This table shows commissions from one party. If a real estate agent works for both buyer and seller, the agent receives double the given commission.

Figure 3.1: Real estate agent commissions across sales price intervals

As we can see in Table 3.1, South Korean real estate agents basically receive percentage commissions. However, the percentage rates differ by sales price intervals, and these rates are defined by law. Moreover, some intervals feature maximum limits on commissions, which are defined by law as well. For example, there is a maximum limit of 250,000 Korean Won for commissions on sales with prices between 0 and 50 million Korean Won. This means that a real estate agent receives 0.6% of the sales price if the price is between 0 and 41.6 million Korean Won but a constant 250,000
Korean Won if the sales price is between 41.6 million and 50 million Korean Won. Therefore, for transactions with sales prices between 0 and 41.6 million Korean Won, a real estate agent’s incentive is the same as that of a price-maximizing agent, while for sales prices between 41.6 million and 50 million Korean Won, a real estate agent’s incentive is the same as that of a trade-maximizing agent.

Similarly, there is another maximum limit for commissions on sales prices within the interval of 50 million to 200 million Korean Won. Thus, for sales with prices between 50 million and 160 million Korean Won, a real estate agent receives a percentage commission of 0.5%, and for sales with prices between 160 million and 200 million Korean Won, the agent receives a fixed commission of 800,000 Korean Won.

In sum, for sales prices between 41.6 million and 50 million Korean Won and between 160 million and 200 million Korean Won, a real estate agent’s incentive is the same as that of a trade-maximizing agent. Across other price intervals, a real estate agent’s incentive is the same as that of a price-maximizing agent. Therefore, I can compare the probabilities of trade in those intervals in order to compare the probabilities of trade for a trade-maximizing agent and a price-maximizing agent.

Furthermore, the above commission scheme is defined by law and is identical for all real estate agents in South Korea. That is, real estate agents do not self-select between the two different commission schemes. Moreover, as we can see in Figure 3·2 and Figure 3·3, transactions are not bunching at the thresholds of 41.6, 50, 160, and 200 million Korean Won. This is due to sales prices being the results of bargaining between buyer and seller. That is, the price is not determined by the real estate agent. Instead, the real estate agent merely advises the buyer and seller before bargaining occurs, as I discuss in the first chapter. In sum, real estate agents cannot manipulate sales prices directly, and that is the reason why transactions are not bunching at the above thresholds even though the agent’s incentives are different on either side of
Therefore, the self-selection of the agent is not an issue in the identification strategy of this chapter. In addition, as we can see in Figure 3-1, the amount of the agent’s commission is continuous at the thresholds of 41.6, 50, 160, and 200 million Korean Won. Thus, the buyer and seller would pay the same commissions on either side of those thresholds and so have no incentive to manipulate sales prices.

However, transactions are evidently bunching at the left side of the thresholds of 600 and 900 million Korean Won, as shown in Figure 3-4; the values of agent commissions are discontinuous at these thresholds. Specifically, as we can see in Figure 3-1, the commission amount at a sales price of 600 million Korean Won jumps from 2.4 to 3 million Korean Won, and at a sales price of 900 million Korean Won, the commission amount jumps from 4.5 to 8.1 million Korean Won. Therefore, if the sales price is slightly above those thresholds, both buyer and seller want to manipulate it to be slightly lower so that they will pay less commission to the agent. Nevertheless, these bunching points remain within percentage commission intervals, and thus the
Notes: The first red vertical line indicates 160 and the second red vertical line indicates 200. The width of each bin is 10.

**Figure 3.3:** Distribution of sales by price between 100 and 500 (in million W)
Notes: The first red vertical line indicates 600 and the second red vertical line indicates 900. The width of each bin is 15.

**Figure 3-4:** Distribution of sales by price between 500 and 1,300 (in million W)
self-selection of buyers and sellers is also not an issue for the identification strategy of this chapter. In the following section, I describe the data that I use in the analysis.

3.2.2 Data

In South Korea, all real estate transactions should be declared to the Ministry of Land, Infrastructure, and Transport, which provides an open application programming interface for the transaction data, excluding private and personal information. Thus, I can access data on all South Korean real estate transactions from 2006 to 2017.

From this data, I specifically utilize housing transactions involving multi-unit apartment buildings for two reasons. First, the majority of the population lives in such buildings. Second, multi-unit apartment buildings in South Korea are usually large and contain hundreds of standardized units in each building, and thus a large number of transactions can be categorized according to a few types of units. This is a substantial advantage for dealing with the data, as unobserved unit characteristics can be controlled using a fixed effect model.

Within the above data source, I observe a total of 7,062,423 housing transactions involving multi-unit apartment buildings from 2006 to 2017. For each transaction, I have access to the sales price, sales date (in terms of ten-day periods encompassing the beginning, middle, or end of each month), unit type, floor of the unit, location of the building, and year of the building’s construction.

I drop 2,132 observations that are lacking or have incorrect information on the unit type or floor\(^1\) or for which the recorded sales year precedes the year in which the building was constructed. With the remaining 7,060,291 observations, I build unbalanced panel data by apartment unit type and time of sale. I then generate

\(^1\)For example, some observations have no information about their unit types or have nonpositive floor values.
the number of sales for each apartment unit type in each time period\(^2\). The built unbalanced panel data comprise a total of 4,585,859 observations spanning 126,396 apartment unit types\(^3\).

### 3.3 Cluster analysis

I use the number of sales in each time period as a proxy for the probability of trade, as the number of sales tends to be increasing in the probability of trade. However, this relationship is not always true. Even when the probability of trade is low, a large number of listings can still result in a large number of sales. Unfortunately, I cannot observe listing activities in this dataset; thus, I cannot directly use the number of sales as a proxy for the probability of trade. In the remaining part of this section, I discuss how to solve this issue.

The following theorem shows that the number of listings and the number of sales in a specific time period are Poisson distributed under some conditions.

**Theorem 17** Let \( n \) be the total number of apartment units, \( p \) be the probability of listing a unit, \( q \) be the probability of a sale, and \( r \) be the probability of the unit remaining listed after failing to sell. If \( n \to \infty \), \( \frac{p}{1-(1-p)(1-q)r} \to 0 \), and \( \frac{pq}{1-(1-p)(1-q)r} \to 0 \) as \( \frac{np}{1-(1-p)(1-q)r} \) and \( \frac{npq}{1-(1-p)(1-q)r} \) converge to finite limits, then the number of listings and the number of sales in each time period are Poisson distributed with parameters \( \frac{np}{1-(1-p)(1-q)r} \) and \( \frac{npq}{1-(1-p)(1-q)r} \), respectively.

**Proof.** See Appendix C. ■

According to Theorem 17, if each apartment unit type is represented by a large enough number of units, the probability of a new listing is both small enough and constant, and both the probability of a sale and the probability of remaining on the

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\(^2\)The time period is defined by a ten-day period.

\(^3\)Here, the apartment unit type is defined by each apartment unit type in each apartment complex. For example, different unit types in the same apartment complex and similar unit types in different apartment complexes are all defined as different apartment unit types.
list after failing to sell are constant, then the number of listings and the number of sales in a specific time period are Poisson distributed with some Poisson parameters. Specifically, these Poisson parameters are the average number of listings and the average number of sales in each time period. The parameters are constant unless any one of the probabilities of a new listing, a sale, or remaining on the list changes. In my panel data, the total number of units in each apartment unit type is sufficiently large, and the probability of a new listing in the housing market is sufficiently small. Therefore, the number of listings and the number of sales for a specific apartment unit type in each time period are Poisson distributed. Moreover, the two Poisson parameters, the average number of listings and the average number of sales, remain constant over any period of time that has constant probabilities of a new listing, a sale, and remaining on the list.

However, I do not directly observe the above probabilities and thus do not observe the lengths of those time periods with constant Poisson parameters. To determine each length of time that has the same Poisson parameter, I use cluster analysis, which is an unsupervised machine learning algorithm. More specifically, I apply a divisive (top-down) hierarchical clustering algorithm to the series of the number of sales for each apartment unit type. The algorithm is described below.

First of all, consider the likelihood ratio test of two different Poisson parameters. Let $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_m\}$ be two different series of random samples from $\text{Poisson}(\lambda_x)$ and $\text{Poisson}(\lambda_y)$, respectively.

\[
H_0 : \lambda_x = \lambda_y \; \text{vs} \; H_1 : \lambda_x \neq \lambda_y \tag{3.1}
\]
The log-likelihood ratio test statistic of the above test (3.1) is

\[
\log L = \left( \sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j \right) \log \left( \frac{1}{n + m} \left( \sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j \right) \right) - \left( \sum_{i=1}^{n} x_i \right) \log \left( \frac{1}{n} \sum_{i=1}^{n} x_i \right) - \left( \sum_{j=1}^{m} y_j \right) \log \left( \frac{1}{m} \sum_{j=1}^{m} y_j \right).
\]

The corresponding p-value is

\[
p = 0.5 \left\{ 1 - \chi^2_1 (-2\log L) \right\},
\]

where \( \chi^2_1(\cdot) \) is the cumulative distribution function of the chi-squared distribution with one degree of freedom.\(^4\)

For each apartment unit type, the entire series of the number of sales starts as a single cluster. For each point at which the cluster may be split into two, I calculate the log-likelihood ratio test statistic as in equation (3.2) and find the point for which that value is the lowest.\(^5\) If the corresponding p-value is less than a pre-set significance level,\(^6\) then I split the cluster into two clusters exactly at that point. I repeat these steps for each cluster until no further splitting points can be identified.

From this process, I obtain clusters of time periods within which the number of sales is Poisson distributed with constant Poisson parameters. Note that for each cluster, the number of listings is also Poisson distributed with a fixed Poisson parameter, as we can see in Theorem 17. In other words, for each apartment unit type, I can find time periods across which the average number of listings is constant.

Table 3.2 gives some descriptive statistics and the results of the cluster analysis. There are 126,396 apartment unit types in total, and from 2006 to 2017, the average

\(^4\) Gu et al. (2008)

\(^5\) Witten et al. (2011) propose using the log-likelihood ratio test statistic as a distance measure between data points in Poisson clustering, instead of the Euclidean distance.

\(^6\) I use 1% and 5% in the analysis.
Table 3.2: Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD**</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transactions data:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price (in million W)</td>
<td>218.54</td>
<td>195.42</td>
<td>1.55</td>
<td>8,200</td>
</tr>
<tr>
<td>Age</td>
<td>12.68</td>
<td>7.88</td>
<td>0</td>
<td>56</td>
</tr>
<tr>
<td>Area (m²)</td>
<td>74.31</td>
<td>26.40</td>
<td>9.26</td>
<td>424.32</td>
</tr>
<tr>
<td>Total number of observations</td>
<td>7,060,291</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Built unbalanced panel data:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of sales in each time period*</td>
<td>1.54</td>
<td>4.68</td>
<td>1</td>
<td>2351</td>
</tr>
<tr>
<td>Number of periods* for each apartment unit type</td>
<td>256.96</td>
<td>158.01</td>
<td>1</td>
<td>432</td>
</tr>
<tr>
<td>Total number of apartment unit types</td>
<td>126,396</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number of observations</td>
<td>4,585,859</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Clustering with 1% significance level:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of clusters for each apartment unit type</td>
<td>3.24</td>
<td>2.96</td>
<td>1</td>
<td>61</td>
</tr>
<tr>
<td>Number of periods* in each cluster</td>
<td>78.79</td>
<td>116.55</td>
<td>1</td>
<td>432</td>
</tr>
<tr>
<td>Total number of types × clusters</td>
<td>372,691</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Clustering with 5% significance level:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of clusters for each apartment unit type</td>
<td>8.38</td>
<td>8.15</td>
<td>1</td>
<td>101</td>
</tr>
<tr>
<td>Number of periods* in each cluster</td>
<td>29.18</td>
<td>62.42</td>
<td>1</td>
<td>431</td>
</tr>
<tr>
<td>Total number of types × clusters</td>
<td>800,631</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Ten-day period
** Standard period

number of ten-day periods between the first sale and the last sale of each apartment unit type is 256.96.

When clustering, I use 1% and 5% as the significance levels. Note that a lower significance level splits clusters less finely, and not splitting a cluster that actually contains two different series of observations with different Poisson parameters induces bias. In contrast, the choice of a higher significance level splits clusters more finely and thus reduces potential bias. However, if I split a cluster that actually contained only one series of observations with a constant Poisson parameter, then the analysis becomes less powerful. Therefore, I repeated the clustering using two different significance levels, 1% and 5%. As we can see in Section 3.5, both analyses give similar results.

When clustering at the 1% significance level, the number of clusters for each apartment unit type averages 3.24, and the average number of ten-day periods in
each cluster is 78.79. This results in a total of 372,691 clusters, that is, groups of time periods that each have different average numbers of listings.

When clustering at the 5% significance level, the number of clusters for each apartment unit type averages 8.38, and the average number of ten-day periods in each cluster is 29.18. This gives a total of 800,631 clusters, that is, groups of time periods that each have different average numbers of listings.

Figures 3·5 and 3·6 show the distributions of the number of clusters for each apartment unit type at significance levels of 1% and 5%, respectively. Figures 3·7 and 3·8 show the distributions of the number of ten-day periods in each cluster at significance levels of 1% and 5%, respectively. Clustering at the 5% significance level splits clusters more finely and thus produces a greater number of clusters for each apartment unit type and a smaller number of ten-day periods in each cluster.

**Figure 3·5:** Distribution of the number of clusters for each apartment unit type (Clustering with 1% significance level)
Figure 3.6: Distribution of the number of clusters for each apartment unit type (Clustering with 5% significance level)

Figure 3.7: Distribution of the number of ten-day periods in each cluster (Clustering with 1% significance level)
3.4 Regression model

After clustering time periods, I can use the number of sales in each time period as a proxy measure for the probability of trade by controlling it with the fixed effect of the cluster. More specifically, the regression model is

$$\log(y_{it}) = \alpha + \beta \cdot \text{FIX}_{it} + \gamma' X_{it} + \delta_{it} + \epsilon_{it}, \ t \in P^i_{\tau},$$

where $i \in \mathbb{N} = \{1, 2, \cdots, N\}$, $t \in T_i = \{1, 2, \cdots, T_i\}$; and $\mathbb{P}_i = \{P^i_1, P^i_2, \cdots, P^i_{K_i}\}$ is a partition of $T_i$ such that every $P^i_\tau \in \mathbb{P}_i$ is a set of consecutive integers, $\max P^i_\tau + 1 = \min P^i_{\tau+1}$ for all $\tau \in \{1, 2, \cdots, K_i - 1\}$, and $K_i \leq T_i$. Note that I find the partition $\mathbb{P}_i$ for each apartment unit type $i$ by using cluster analysis, which is described in Section 3.3.

$y_{it}$ is the number of sales of apartment unit type $i$ in period $t$. $\text{FIX}_{it}$ is the
proportion of sales for apartment unit type $i$ in period $t$ that fall within the fixed commission intervals\textsuperscript{7}. For example, if $y_{it} = 5$ and three out of five sales have prices that are within the fixed commission intervals, then $FIX_{it} = \frac{3}{5}$. Notably, if $FIX_{it}$ is close to one, a real estate agent’s incentive is close to that of a trade-maximizing agent; similarly, if $FIX_{it}$ is close to zero, a real estate agent’s incentive is close to that of a price-maximizing agent.

$X_{it}$ is the vector of control variables for apartment unit type $i$ in period $t$. These variables are the building’s age\textsuperscript{8}, average price, coefficient of variation of price\textsuperscript{9}, average price gap\textsuperscript{10}, standard deviation of the price gap, average of the price gap squared, and floor controls\textsuperscript{11}. I include control variables regarding the price gap for the following reason: In a bargaining process, the buyer wants a lower sales price, while the seller wants a higher sales price. Thus, if a given sales price is far from the average market price, whether lower or higher, it is difficult for them to make a deal. Lastly, $\delta_{i\tau}$ is the fixed effect in cluster $\tau$ of apartment unit type $i$.

### 3.5 Results of the regression

Table 3.3 shows the results of the regression model from Section 3.4. For all columns, the dependent variable is the logarithm of the number of sales in each ten-day period, and the independent variable of interest is $FIX$. Column (i) is an OLS regression, column (ii) is the OLS regression with cluster-robust standard errors at the fixed

\textsuperscript{7}Between 41.6 million and 50 million Korean Won and between 160 million and 200 million Korean Won.

\textsuperscript{8}The sales year - the year of the building’s construction.

\textsuperscript{9}Standard Deviation(price)/Mean(price).

\textsuperscript{10}(price−Mean(price))/\sqrt{\text{Variance}(price)}$, the price gap is calculated within the level of fixed effect groups.

\textsuperscript{11}Floor controls are generated in the same way as $FIX_{it}$. That is, each control is the proportion of sales in each floor category. There are a total of 13 floor categories: (1) each floor from 2nd to 9th (8 categories), (2) 10th to 19th floors, (3) 20th to 29th floors, (4) 30th to 39th floors, (5) 40th to 49th floors, and (6) higher than 50th floor.
### Table 3.3: Regression results

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>log(Number of sales)</td>
<td>Variable of interest: <strong>FIX</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cluster (1%) FE</td>
<td>0.0023***</td>
<td>0.0023***</td>
<td>0.0023***</td>
<td>0.0023***</td>
<td>0.0023***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0005)</td>
<td>(0.0005)</td>
<td>(0.0006)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>372,691</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Cluster (5%) FE</td>
<td>0.0022***</td>
<td>0.0022***</td>
<td>0.0022***</td>
<td>0.0022***</td>
<td>0.0022***</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0005)</td>
<td>(0.0005)</td>
<td>(0.0006)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>800,631</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Fixed effect: No</td>
<td>0.0152***</td>
<td>0.0152***</td>
<td>0.0152***</td>
<td>0.0152***</td>
<td>0.0152***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0005)</td>
<td>(0.00016)</td>
<td>(0.0026)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>–</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Type FE</td>
<td>0.0016***</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0016</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0011)</td>
<td>(0.0017)</td>
<td>(0.0023)</td>
<td></td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>–</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Type × Year FE</td>
<td>0.0015*</td>
<td>0.0015*</td>
<td>0.0015*</td>
<td>0.0015</td>
<td>0.0015</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
<td>(0.0008)</td>
<td>(0.0009)</td>
<td>(0.0010)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>789,713</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Type × Quarter FE</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td>(0.0011)</td>
<td>(0.0009)</td>
<td>(0.0010)</td>
<td>(0.0010)</td>
<td>(0.0010)</td>
</tr>
<tr>
<td>Number of groups</td>
<td>–</td>
<td>1,899,779</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Cluster-robust SE I</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Cluster-robust SE II</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Cluster-robust SE III</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Cluster-robust SE IV</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>N</td>
<td>–</td>
<td>4,585,859</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The dependent variable is the logarithm of the number of sales. The independent variable of interest is **FIX**. FE and SE stand for fixed effect and standard error, respectively. All columns report the results of OLS regressions. Cluster-robust standard error I is clustered at the fixed effect group level. For the third panel, it is the robust standard error. Cluster-robust standard error II is clustered at the level of apartment unit type (126,396 groups). Cluster-robust standard error III is clustered at the town level (249 groups). Cluster-robust standard error IV is clustered at the city level (65 groups). Standard errors are in parentheses. *p ≤ 0.1, **p ≤ 0.05, and ***p ≤ 0.01.
effect group level\textsuperscript{12}, column (iii) is the OLS regression with cluster-robust standard errors at the level of apartment unit type (total 126,396 type groups), column (iv) is the OLS regression with cluster-robust standard errors at the town level (total 249 town groups), and column (v) is the OLS regression with cluster-robust standard errors at the city level (total 65 city groups).

The first and second panels contain the main results of the cluster fixed effect regression model, which is described in Section 3.4. The first panel is the result of clustering at the 1% significance level, while the second panel is the result of clustering at the 5% significance level. These results indicate that a fixed-commission real estate agent brings an approximately 0.22 or 0.23\%\textsuperscript{13} greater number of sales in each ten-day period than does a percentage-commission real estate agent. That is, a trade-maximizing agent leads to a higher probability of trade than a price-maximizing agent.

The results in this chapter are consistent with that from the previous chapters. If a price-maximizing real estate agent truthfully disseminates market information to buyers and sellers in equilibrium, reducing the intensity of information incompleteness between buyers and sellers, so does a trade-maximizing real estate agent. However, the converse of the above statement is not true. Thus, for some cases, a trade-maximizing agent truthfully disseminates market information, but a price-maximizing agent does not. As a result, the finer information possessed by a real estate agent is available to buyers and sellers when the agent receives a fixed commission but is not available when the agent receives a percentage commission. Therefore, the probability of trade is higher under a fixed-commission compensation scheme, which makes for smaller ex-post inefficiency.

I present the other four panels of Table 3.3, which contain the results from different

\textsuperscript{12}For the third panel, it is the OLS regression with robust standard errors.
\textsuperscript{13}e^{0.0022} \approx 1.0022, e^{0.0023} \approx 1.0023.
regression models when assuming I cannot perform cluster analysis, for the purpose of providing a benchmark. The third panel shows results from the regression without fixed effects. In this model, as I note in Section 3.3, the number of sales is not a good proxy measure for the probability of trade because, in some cases, we can observe a large number of sales given a large number of listings despite also having a small probability of trade. Moreover, it is likely that a higher probability of trade induces more listings and thus more sales. Therefore, when I do not control the above aspect, an upward bias results, as the obtained values reflect.

In the absence of cluster analysis, an alternative means of controlling the above aspect is to assume that the average number of listings for a given apartment unit type is the same during the same period of time, such as in the same sales year or in the same sales quarter. That is, I create the partition \( P_i \) of \( T_i \) in the regression model of equation (3.4) in Section 3.4 by myself rather than doing so with the clustering algorithm as described in Section 3.3. The fourth, fifth, and sixth panels give the results of those regression models.

The fourth panel is the apartment unit type fixed effect regression. In this model, the partition \( P_i \) has only one cell as \( P_i = \{T_i\} \) in the regression model of equation (3.4). There is a strict assumption that the average number of listings for a specific apartment unit type is constant regardless of the time period.

The fifth panel is the regression with fixed effects of the interactions between apartment unit types and years. In this model, there is a less but still strict assumption that the average number of listings for a specific apartment unit type is constant across a given sales year. In other words, in the regression model of equation (3.4), each cell of the partition \( P_i \) has time periods \( t \) that are in the same sales year. Thus, \( \tau \) indicates each year from 2006 to 2017.

The sixth panel is similar to the fifth panel; the only difference is the length of
the time period used. In this model, it is assumed that the average number of listings for a specific apartment unit type is constant within a given quarter of a given year. That is, in the regression model of equation (3.4), each cell of the partition $P_i$ has time periods $t$ that are in a single sales quarter of a given year. Thus, $\tau$ indicates each quarter from 2006 to 2017.

The coefficients of the fourth through sixth panels are not that much different from those in the first and second panels. However, most of them are insignificant, especially if I use cluster-robust standard errors. This is because the lengths of time periods having the same average numbers of listings are very different across apartment unit types. For some apartment unit types, the average number of listings changes in every short period of time, while for others the number is constant over several years. We can check this aspect in Figure 3·7 and Figure 3·8. If we simply assume that the average number of listings is constant within a given sales year or one sales quarter of a year for all apartment unit types, we may obtain an incorrect inference.

Another alternative method without using cluster analysis is to assume two different fixed effects, one for the apartment unit type and another for time, which is widely used in panel data regression analysis. In this case, the regression model is

$$
\log(y_{it}) = \alpha + \beta \cdot FIX_{it} + \gamma'X_{it} + \delta_i + \xi_{\tau} + \epsilon_{it},
$$

(3.5)

where $\delta_i$ is the apartment unit type fixed effect and $\xi_{\tau}$ is the time fixed effect. This regression model is appropriate if the time fixed effect $\xi_{\tau}$ is constant across all apartment unit types. Thus, if I only use transaction data from one region, which may have a constant time fixed effect across the observations, then this is an appropriate method. However, I am using transaction data from all over South Korea, and the time fixed effect can be different by region. Therefore, the regression model of
equation (3.5) is less powerful than that of equation (3.4) in Section 3.4.

The results of the regression model of equation (3.5) with various time fixed effects are given in Appendix C. In these models, cluster-robust standard errors are larger than those from the regression model of equation (3.4) in Section 3.4, which uses cluster analysis to determine the fixed effect groups. The full results from the regression model of equation (3.4) are also included in Appendix C.

3.6 Conclusion

I test the results in the previous chapters using housing transaction data from South Korea, exploiting the unique feature of the Korean commission scheme for real estate agents. This feature enables me to compare a price-maximizing agent and a trade-maximizing agent. The dataset does not include listing activities, so I use cluster analysis, which is an unsupervised machine learning algorithm, to group the number of sales observations so that the average number of listings is constant within each group. After this clustering, I can use the number of sales as a proxy measure for the probability of trade by controlling it with the group fixed effect.

The results show that a trade-maximizing agent brings an approximately 0.2% greater number of sales in each ten-day period than a price-maximizing agent. That is, a trade-maximizing agent leads to a higher probability of trade, which means smaller ex-post inefficiency. This is consistent with the result of the theoretical model of a bargaining game with an informed agent.

As we can see from both the theoretical model and the real-world data, it is likely that a price-maximizing agent does not disseminate finer information truthfully, while a trade-maximizing agent does make finer information available in the market. In such a case, a price-maximizing agent does not reduce the ex-post inefficiency in markets where a buyer and seller bilaterally bargain to trade a good under incomplete
information even though the agent could. Therefore, we need to make an informed agent’s incentive the same as that of a trade-maximizing agent, thereby reducing the ex-post inefficiency.
Appendix A

Appendix for Chapter 1

Proof of Lemma 1. (i) When the buyer’s valuation is $v^b$, the buyer’s problem is to maximize expected utility by submitting an offer $\tilde{B}$ given the seller’s strategy $s(\cdot)$:

$$\max_{\tilde{B}} \int_{v^b}^{s^{-1}(\tilde{B})} \left\{ v^b - \frac{\tilde{B} + s(v^s)}{2} \right\} \frac{f^s(v^s)}{F^s(v^s) - F^s(v^s)} dv^s. \quad (A.1)$$

The first order condition of the above buyer’s problem is

$$-\frac{1}{2} \left\{ F^s(s^{-1}(\tilde{B})) - F^s(v^s) \right\} + (v^b - \tilde{B}) f^s(s^{-1}(\tilde{B})) \frac{ds^{-1}(\tilde{B})}{d\tilde{B}} = 0, \quad (A.2)$$

which is identical to the first order condition of the buyer’s problem in Chatterjee and Samuelson (1983).

When the seller’s valuation is $v^s$, the seller’s problem is to maximize expected utility by submitting an offer $\tilde{S}$ given the buyer’s strategy $b(\cdot)$:

$$\max_{\tilde{S}} \int_{b^{-1}(\tilde{S})}^{v^b} \left\{ \frac{b(v^b) + \tilde{S}}{2} - v^s \right\} \frac{f^b(v^b)}{F^b(v^b) - F^b(v^b)} dv^b. \quad (A.3)$$

The first order condition of the above seller’s problem is

$$\frac{1}{2} \left\{ F^b(v^b) - F^b(b^{-1}(\tilde{S})) \right\} - (\tilde{S} - v^s) f^b(b^{-1}(\tilde{S})) \frac{db^{-1}(\tilde{S})}{d\tilde{S}} = 0, \quad (A.4)$$

which is identical to the first order condition of the seller’s problem in Chatterjee and Samuelson (1983).
For the rest of the proof, see the proofs of Theorem 2 and Theorem 3 in Chatterjee and Samuelson (1983).

(ii) Straightforward. ■

**Proof of Lemma 2.** (i) As we can see in the proof of Lemma 1, equation (A.2) is the first order condition of the buyer’s problem that maximizes the buyer’s expected utility by submitting an offer \( \bar{B} \) given the seller’s strategy \( s(\cdot) \) when the buyer’s valuation is \( v^b \). Note that the second order condition of the buyer’s problem is

\[
-\frac{3}{2} f^s(s^{-1}(\bar{B})) \frac{ds^{-1}(\bar{B})}{dB} + (v^b - \bar{B})\left[f'^s(s^{-1}(\bar{B}))\left\{\frac{ds^{-1}(\bar{B})}{dB}\right\}^2 + f^s(s^{-1}(\bar{B})) \frac{d^2s^{-1}(\bar{B})}{dB^2}\right] < 0.
\]

Let \( B \) be the left-hand side of equation (A.5). Then, from equation (A.2),

\[
\frac{dB}{dv^s} = -\frac{f^s(v^s)}{2B} > 0
\]

since \( f^s(v^s) > 0 \) for all \( v^s \in [\underline{v}, \bar{v}] \), and \( B < 0 \) by equation (A.5). That is, the buyer’s optimal offer \( \bar{B} \) is increasing in the seller’s lowest value. Therefore, \( \bar{b}_j(v^b) > \bar{b}_l(v^b) \) for all \( v^b \) since \( v^s_j > v^s_l \) whenever \( j > l \).

(ii) Similarly, as we can see in the proof of Lemma 1, equation (A.4) is the first order condition of the seller’s problem that maximizes the seller’s expected utility by submitting an offer \( \tilde{S} \) given the buyer’s strategy \( b(\cdot) \) when the seller’s valuation is \( v^s \). Note that the second order condition of the seller’s problem is

\[
-\frac{3}{2} f^b(b^{-1}(\tilde{S})) \frac{db^{-1}(\tilde{S})}{dS} - (\tilde{S} - v^s)\left[f'^b(b^{-1}(\tilde{S}))\left\{\frac{db^{-1}(\tilde{S})}{dS}\right\}^2 + f^b(b^{-1}(\tilde{S})) \frac{d^2b^{-1}(\tilde{S})}{dS^2}\right] < 0.
\]

Let \( S \) be the left-hand side of equation (A.7). Then, from equation (A.4),

\[
\frac{d\tilde{S}}{dv^b} = -\frac{f^b(\tilde{v}^b)}{2S} > 0
\]
since $f^b(\pi^b) > 0$ for all $\pi^b \in [v, \overline{v}]$, and $S < 0$ by equation (A.7). That is, the seller’s optimal offer $\tilde{S}$ is increasing in the buyer’s highest value. Therefore, $\tilde{s}_k(v^s) > \tilde{s}_i(v^s)$ for all $v^s$ since $v_{i+1}^b > v_{k+1}^b$ whenever $i > k$. ■

**Proof of Lemma 3.** Let $\tilde{b}_i(\cdot)$ be the solution of $\tilde{b}(\cdot)$ in Lemma 1 when the buyer receives a message $m^s_i$ from the agent and believes that information is truthfully disseminated to both buyer and seller. Similarly, let $\tilde{s}_k(\cdot)$ be the solution of $\tilde{s}$ in Lemma 1 when the seller receives a message $m^b_k$ from the agent and believes that information is truthfully disseminated to both buyer and seller.

(i) Note that $v^b_i < v^b_{k+1} \leq v^b_k < v^b_{i+1}$. Then, $\tilde{s}_k(v^s) < \tilde{s}_i(v^s)$ for all $v^s$ by (ii) in Lemma 2, and $\tilde{b}_j(v^b_k) < \tilde{b}_j(v^b_i)$ and $\tilde{b}_j(v^b_{k+1}) < \tilde{b}_j(v^b_{i+1})$ since $\tilde{b}_j(\cdot)$ is strictly increasing.

(ii) Note that $v^b_i < v^b_{i+1} \leq v^b_k < v^b_{j+1}$. Then, $\tilde{s}_k(v^s) > \tilde{s}_i(v^s)$ for all $v^s$ by (ii) in Lemma 2, and $\tilde{b}_j(v^b_k) > \tilde{b}_j(v^b_i)$ and $\tilde{b}_j(v^b_{k+1}) > \tilde{b}_j(v^b_{i+1})$ since $\tilde{b}_j(\cdot)$ is strictly increasing.

(iii) Note that $v^s_j < v^s_{j+1} \leq v^s_l < v^s_{l+1}$. Then, $\tilde{b}_j(v^b) > \tilde{b}_j(v^b)$ for all $v^b$ by (i) in Lemma 2, and $\tilde{s}_l(v^s_{l+1}) > \tilde{s}_l(v^s_{l+1})$ and $\tilde{s}_l(v^s_l) > \tilde{s}_l(v^s_{l+1})$ since $\tilde{s}_l(\cdot)$ is strictly increasing.

(iv) Note that $v^s_l < v^s_{l+1} \leq v^s_j < v^s_{j+1}$. Then, $\tilde{b}_l(v^b) < \tilde{b}_l(v^b)$ for all $v^b$ by (i) in Lemma 2, and $\tilde{s}_l(v^s_{l+1}) < \tilde{s}_l(v^s_{l+1})$ and $\tilde{s}_l(v^s_l) < \tilde{s}_l(v^s_{l+1})$ since $\tilde{s}_l(\cdot)$ is strictly increasing. ■

**Proof of Theorem 1.** Proof by contrapositive. Suppose there is a strategy profile in which an informed agent informs the buyer and seller of the true state, and the buyer and seller follow the strategies described in Lemma 1. I need to show that if a trade-maximizing agent has an incentive to deviate from the profile, then so does a price-maximizing agent.\(^1\)

Note that for a specific state $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$ such that $v^b_l < v^s_{l+1}$ for

\(^1\)Note that for any given states $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$ such that $v^b_i \geq v^s_{i+1}$ for some $i, j \in \{0, 1, \ldots, n - 1\}$, the probability of trade in the bargaining equilibrium with fully revealed information is equal to 1 by (ii) in Lemma 1. Therefore, the trade-maximizing agent has no incentive to deviate. Thus, I need to check the other states where $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$ such that $v^b_i < v^s_{j+1}$ for some $i, j \in \{0, 1, \ldots, n - 1\}$ in which the probability of trade is less than 1.
some $i, j \in \{0, 1, \cdots, n - 1\}$, the probability of trade is
\[
\int \int_{b(v^b) \geq s(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]} \frac{f^b(v^b)}{F^b(v^b_{i+1}) - F^b(v^b_i)} \frac{f^s(v^s)}{F^s(v^s_{j+1}) - F^s(v^s_j)} dv^b dv^s,
\]
(A.9)
and the expected sales price is
\[
\int \int_{b(v^b) \geq s(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]} \frac{b(v^b) + s(v^s)}{2} \frac{f^b(v^b)}{F^b(v^b_{i+1}) - F^b(v^b_i)} \frac{f^s(v^s)}{F^s(v^s_{j+1}) - F^s(v^s_j)} dv^b dv^s,
\]
(A.10)
where $b(\cdot)$ and $s(\cdot)$ are the buyer’s and seller’s offer strategies, respectively. The informed agent’s possible deviations are delivering a message $m^b_k$ for some $k \neq i$ to the seller, delivering a message $m^s_l$ for some $l \neq j$ to the buyer, or both; there are eight possible deviations in total.

However, the trade-maximizing agent never delivers a message $m^b_k$ for some $i < k \leq n$ to the seller, a message $m^s_l$ for some $0 \leq l < j$ to the buyer, or both since the messages always reduce the probability of trade. That is because, as we can see in (ii) and (iv) of Lemma 3, the above messages make the seller submit a higher offer, the buyer submit a lower offer, or both. Moreover, even though a message $m^b_k$ for some $i < k \leq n$ to the seller makes the seller less likely to submit $v^b_i$, and so the seller’s offer changes from $v^b_i$ to less than $v^b_i$, the seller’s submitted offer is still higher than the highest-value buyer’s offer. Similarly, even though a message $m^s_l$ for some $0 \leq l < j$ to the buyer makes the buyer less likely to submit $v^s_l$, and so the buyer’s offer changes from $v^s_l$ to higher than $v^s_l$, the buyer’s submitted offer is still lower than the lowest-value seller’s offer. Thus, the seller and buyer still have zero probability of trade. Therefore, the above false messages solely reduce the probability of trade, and so the trade-maximizing agent never delivers those messages.

Moreover, the trade-maximizing agent never delivers a message $m^b_k$ for some $0 \leq
$k < i$ to the seller and a message $m^s_l$ for some $j < l \leq n$ to the buyer at the same time. This is because either of the above false messages guarantees that the buyer and seller trade the good with certainty unless the buyer submits $\bar{v}$ or the seller submits $\bar{v}$. Thus, adding another false message can only reduce the probability of trade. For example, if the agent delivers a message $m^b_k$ for some $0 \leq k < i$ to the seller, then the seller misconceives that the buyer’s valuation is always less than or equal to the buyer’s true lowest value. Thus, if the seller submits an offer lower than $v$, the seller always submits a lower offer than does the buyer who submits an offer higher than $v$. Therefore, they trade the good with certainty unless the buyer submits $\bar{v}$, the seller always submits a lower offer than does the buyer who submits an offer higher than $v$. However, if the agent delivers a message $m^s_l$ for some $j < l \leq n$ to the buyer at the same time, then the buyer is more likely to submit $v$ by (iii) in Lemma 3. Therefore, doing so just reduces the probability of trade, and thus it is not a profitable deviation for the trade-maximizing agent.

Now, I have four deviations left to check.

i) A false message $m^b_k$ for some $0 \leq k < i$ to the seller:

If there exists a state $(v^b, v^s) \in [v^b_{i-1}, v^b_i] \times [v^s_{j+1}, v^s_{j+1}]$ for some $i, j \in \{0, 1, \cdots, n-1\}$ such that a false message $m^b_k$ for some $0 \leq k < i$ to the seller results in a higher probability of trade than would the true message $m^b_i$, then there exists a state $(v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i-1}]$ such that a false message $m^s_{n-k-1}$ to the buyer results in a higher probability of trade than would the true message $m^s_{n-i-1}$, according to Assumption 1 and Assumption 2. Note that $n-k-1 > n-i-1$. Thus, the false message $m^s_{n-k-1}$ to the buyer makes the buyer who submits an offer higher than $v$ submit a higher offer than would be submitted in the case of sending the true message $m^s_{n-i-1}$ in the state $(v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i}]$ by (iii) in Lemma 3. Therefore, the false message $m^s_{n-k-1}$ to the buyer results in a higher expected sales price than if sending the true message $m^s_{n-i-1}$ in the state $(v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i}]$. 


ii) A false message $m_i^s$ for some $j < l \leq n$ to the buyer:
Assume that there exists a state $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$ for some $i, j \in \{0, 1, \cdots, n - 1\}$ such that a false message $m_i^s$ for some $j < l \leq n$ to the buyer results in a higher probability of trade than would the true message $m_j^s$. Then, the false message $m_i^s$ to the buyer also results in a higher expected sales price than if sending the true message $m_j^s$ in the state $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$. That is because $l > j$, and thus the false message $m_i^s$ to the buyer makes the buyer who submits an offer higher than $v$ submit a higher offer than if sending the true message $m_j^s$ by (iii) in Lemma 3.

iii) A false message $m_k^b$ for some $i < k \leq n$ to the seller and a false message $m_i^s$ for some $j < l \leq n$ to the buyer:
Assume that there exists a state $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$ for some $i, j \in \{0, 1, \cdots, n - 1\}$ such that sending a false message $m_k^b$ for some $i < k \leq n$ to the seller and a false message $m_i^s$ for some $j < l \leq n$ to the buyer at the same time results in a higher probability of trade than would be the case if sending the true messages $m_k^b$ to the seller and $m_j^s$ to the buyer. Then, sending the false message $m_k^b$ to the seller and the false message $m_i^s$ to the buyer at the same time also results in a higher expected sales price than in the case of sending the true messages $m_k^b$ to the seller and $m_j^s$ to the buyer in the state $(v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]$. The reason is as follows. First, $k > i$, and thus the false message $m_k^b$ to the seller makes the seller who submits an offer lower than $v$ submit a higher offer than in the case of sending the true message $m_i^b$ by (ii) in Lemma 3. Second, $l > j$, and thus the false message $m_i^s$ to the buyer makes the buyer who submits an offer higher than $v$ submit a higher offer than in the case of sending the true message $m_j^s$ by (iii) in Lemma 3.

iv) A false message $m_k^b$ for some $0 \leq k < i$ to the seller and a false message $m_i^s$ for some $0 \leq l < j$ to the buyer:
If there exists a state \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\) for some \(i, j \in \{0, 1, \ldots, n - 1\}\) such that sending a false message \(m^b_k\) for some \(0 \leq k < i\) to the seller and a false message \(m^s_l\) for some \(0 \leq l < j\) to the buyer at the same time results in a higher probability of trade than would be the case if sending the true messages \(m^b_i\) to the seller and \(m^s_j\) to the buyer, then there exists a state \((v^b, v^s) \in [v^b_n - j - 1, v^b_{n-j}] \times [v^s_n - i - 1, v^s_{n-i}]\) such that sending a false message \(m^s_{n-k-1}\) to the buyer and a false message \(m^b_{n-l-1}\) to the seller at the same time results in a higher probability of trade than would be the case if sending the true messages \(m^s_{n-i-1}\) to the buyer and \(m^b_{n-j-1}\) to the seller, according to Assumption 1 and Assumption 2. Note that \(n - l - 1 > n - j - 1\). Thus, the false message \(m^b_{n-l-1}\) to the seller makes the seller who submits an offer lower than \(v\) submit a higher offer than in the case of sending the true message \(m^b_{n-j-1}\) in the state \((v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i}]\) by (ii) in Lemma 3. Note that \(n - k - 1 > n - i - 1\). Thus, the false message \(m^s_{n-k-1}\) to the buyer makes the buyer who submits an offer higher than \(v\) submit a higher offer than in the case of sending the true message \(m^s_{n-i-1}\) in the state \((v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i}]\) by (iii) in Lemma 3. Therefore, sending the false message \(m^s_{n-k-1}\) to the buyer and the false message \(m^b_{n-l-1}\) to the seller at the same time results in a higher expected sales price than in the case of sending the true messages \(m^s_{n-i-1}\) to the buyer and \(m^b_{n-j-1}\) to the seller in the state \((v^b, v^s) \in [v^b_{n-j-1}, v^b_{n-j}] \times [v^s_{n-i-1}, v^s_{n-i}]\).

**Proof of Lemma 4.** Let \(b_j(\cdot)\) and \(s_i(\cdot)\) be the buyer’s and seller’s offer strategies in a state \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\), described in Lemma 1, in the bargaining equilibrium with fully revealed information. Let \(b(\cdot)\) and \(s(\cdot)\) be the buyer’s and seller’s offer strategies, described in Lemma 1, in the babbling equilibrium. Then, I
need to show that
\[
\sum_{i=0}^{n} \sum_{j=0}^{n} \int_{b_j(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]} f^b(v^b) f^s(v^s) dv^b dv^s > \int_{b(v^b) \geq s(v^s), (v^b, v^s) \in \mathbb{R}^2} f^b(v^b) f^s(v^s) dv^b dv^s
\]

(A.11)

for any information partition structure such that \( \underline{v} = v^b_0 < v^b_1 < \cdots < v^b_n = \overline{v} \) and \( \underline{v} = v^s_0 < v^s_1 < \cdots < v^s_n = \overline{v} \).

Let \( \tilde{b}(\cdot) \) and \( \tilde{s}(\cdot) \) be the solutions of \( b(\cdot) \) and \( s(\cdot) \) in Lemma 1 for the case of the babbling equilibrium. Then, note that \( b(v^b) = \tilde{b}(v^b) \) and \( s(v^s) = \tilde{s}(v^s) \) for the buyer and seller who submit an offer higher than \( \underline{v} \) and lower than \( \overline{v} \), respectively, by the boundary conditions in (i) of Lemma 1 since \( \tilde{b}(\underline{v}) < \tilde{s}(\underline{v}) \) and \( \tilde{b}(\overline{v}) < \tilde{s}(\overline{v}) \).

Let \( \tilde{b}_j(\cdot) \) and \( \tilde{s}_i(\cdot) \) be the solutions of \( b(\cdot) \) and \( s(\cdot) \) in Lemma 1 in a state \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\) for the case of the bargaining equilibrium with fully revealed information.

i) For any states \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\) such that \( v^b_i < v^b_{i+1} \) and \( i, j \in \{0, 1, \cdots, n-1\} \), \( \tilde{b}_j(v^b) \geq \tilde{b}(v^b) \) for all \( v^b \in [v^b_i, v^b_{i+1}] \) by (i) in Lemma 2, and \( \tilde{s}_i(v^s) \leq \tilde{s}(v^s) \) for all \( v^s \in [v^s_j, v^s_{j+1}] \) by (ii) in Lemma 2, both with strict inequality if \( i \neq n - 1 \) or \( j \neq 0 \).

Note that in the bargaining equilibrium with fully revealed information, the buyer who matches the highest-value seller’s offer trades the good with certainty, and the seller who matches the lowest-value buyer’s offer also trades the good with certainty.

Moreover, the buyer and seller who submit \( \underline{v} \) and \( \overline{v} \), respectively, in the bargaining equilibrium with fully revealed information cannot trade the good in that case nor in the babbling equilibrium. This is because the buyer’s offer according to the offer function \( \tilde{b}_j(\cdot) \) is lower than \( \tilde{s}_i(v^s_j) \), and \( \tilde{s}_i(v^s_j) \leq \tilde{s}(v^s_j) \), and because the seller’s offer according to the offer function \( \tilde{s}_i(\cdot) \) is higher than \( \tilde{b}_j(v^b_{i+1}) \), and \( \tilde{b}_j(v^b_{i+1}) \geq \tilde{b}(v^b_{i+1}) \).
Therefore,
\[
\int \int b(v^b) \int \int b(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] \frac{f^b(v^b)}{F^b(v^b)} \frac{f^s(v^s)}{F^s(v^s)} dv^b dv^s
\]
for any states \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\) such that \(v^b_i < v^s_{j+1}\) and \(i, j \in \{0, 1, \cdots, n-1\}\).

ii) For any states \((v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}]\) such that \(v^b_i \geq v^s_{j+1}\) and \(i, j \in \{0, 1, \cdots, n-1\}\),
\[
\int \int b(v^b) \int \int b(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] \frac{f^b(v^b)}{F^b(v^b)} \frac{f^s(v^s)}{F^s(v^s)} dv^b dv^s = 1
\]
by (ii) in Lemma 1.

By i) and ii), the probability of trade in the bargaining equilibrium with fully revealed information is higher than in the babbling equilibrium.

**Proof of Lemma 5.** Note that the expected sales price in the babbling equilibrium is
\[
\int \int b(v^b) \int \int b(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b, v^s] \frac{b(v^b) + s(v^s)}{2} f^b(v^b) f^s(v^s) dv^b dv^s
\]
(A.14)
by the bargaining rule and Assumption 1. Note that the expected sales price in the
bargaining equilibrium with fully revealed information is

\[ \sum_{i=0}^{n} \sum_{j=0}^{n} \int \int b_j(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] \frac{b_j(v^b) + s_i(v^s)}{2} f^b(v^b) f^s(v^s) dv^b dv^s \]

\[= \frac{v + \overline{v}}{2} \sum_{i=0}^{n} \sum_{j=0}^{n} \int \int b_j(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] f^b(v^b) f^s(v^s) dv^b dv^s \]

by the bargaining rule, Assumption 1, and Assumption 2. Thus, the expected sales price in the bargaining equilibrium with fully revealed information is higher than that in the babbling equilibrium since

\[ \sum_{i=0}^{n} \sum_{j=0}^{n} \int \int b_j(v^b) \geq s_i(v^s), (v^b, v^s) \in [v^b_i, v^b_{i+1}] \times [v^s_j, v^s_{j+1}] f^b(v^b) f^s(v^s) dv^b dv^s \]

\[> \int \int b(v^b) \geq s(v^s), (v^b, v^s) \in [\underline{v}, \overline{v}]^2 f^b(v^b) f^s(v^s) dv^b dv^s \]

by Lemma 4.

**Proof of Corollary 2.** Straightforward from Theorem 1 and Lemma 4.

**Proof of Corollary 3.** Straightforward from Theorem 1 and Lemma 5.
Appendix B

Appendix for Chapter 2

**Proof of Lemma 6.** Note that equation (A.2) becomes

\[-\frac{1}{2}\left\{ \tilde{s}^{-1}(\tilde{B}) - v^s \right\} + (v^b - \tilde{B}) \frac{d\tilde{s}^{-1}(\tilde{B})}{dB} = 0, \tag{B.1}\]

and equation (A.4) becomes

\[\frac{1}{2}\left\{ \tilde{v}^b - \tilde{b}^{-1}(\tilde{S}) \right\} - (\tilde{S} - v^s) \frac{d\tilde{b}^{-1}(\tilde{S})}{dS} = 0 \tag{B.2}\]

with the assumption that \(v^b\) and \(v^s\) are independent random variables distributed uniformly over [0, 1]. \(\tilde{b}(\cdot)\) in equation (2.1) and \(\tilde{s}(\cdot)\) in equation (2.2) solve the system of equations (B.1) and (B.2) such that \((\tilde{B}, \tilde{s}) = (\tilde{b}, \tilde{S}) = (\tilde{b}, \tilde{S})\) for each \(v^b\) and \(v^s\). The rest of the proof is straightforward from Lemma 1. \(\blacksquare\)

**Proof of Theorem 4 and 5.** When the true state is \((v^b, v^s) \in [0, 1] \times [1 - y, 1]\), if the agent tells the true state only to the buyer, then common knowledge is that \(v^b\) is uniformly distributed over [0, 1], and \(v^s\) is uniformly distributed over \([1 - y, 1]\). Thus, by Lemma 6, the buyer’s and seller’s strategies are

\[b_1(v^b) = \begin{cases} 
0 & \text{if } v^b \in [0, \frac{3}{4}(1 - y) + \frac{1}{4}] \\
\frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1 - y) & \text{if } v^b \in \left[\frac{3}{4}(1 - y) + \frac{1}{4}, 1\right] 
\end{cases} \tag{B.3}\]

and

\[s(v^s) = \begin{cases} 
\frac{5}{3}v^s + \frac{1}{12} + \frac{1}{4}(1 - y) & \text{if } v^s \in [1 - y, \frac{3}{4} + \frac{1}{4}(1 - y)] \\
1 & \text{if } v^s \in \left(\frac{3}{4} + \frac{1}{4}(1 - y), 1\right]
\end{cases} \tag{B.4}\]
respectively. Then, the probability of trade in the equilibrium is

$$\int_{1-y}^{\frac{3}{4} + \frac{1}{4}(1-y)} \int_{\frac{1}{4}y}^{\frac{1}{4}y} \frac{1}{y} dv^b dv^s = \frac{9y}{32}, \quad (B.5)$$

and the expected sales price is

$$\int_{1-y}^{\frac{3}{4} + \frac{1}{4}(1-y)} \int_{\frac{1}{4}y}^{\frac{1}{4}y} \left\{\frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1-y) + \frac{2}{3}v^s + \frac{1}{4} + \frac{1}{12}(1-y)\right\} \frac{1}{y} dv^b dv^s = \frac{9(2-y)y}{64}. \quad (B.6)$$

If the agent delivers a false message $m_0^b$ to the buyer and the buyer believes the agent's message, then the buyer's strategy is

$$b_0(v^b) = \begin{cases} 0 & \text{if } v^b \in [0, \frac{1}{4}) \\ \frac{2}{3}v^b + \frac{1}{12} & \text{if } v^b \in [\frac{1}{4}, 1] \end{cases} \quad (B.7)$$

for $y \in [0, \frac{1}{4}]$ or

$$b_0(v^b) = \begin{cases} 0 & \text{if } v^b \in [0, \frac{1}{4}) \\ \frac{2}{3}v^b + \frac{1}{12} & \text{if } v^b \in [\frac{1}{4}, (1-y) + \frac{1}{4}] \\ \frac{2}{3}(1-y) + \frac{1}{4} & \text{if } v^b \in [(1-y) + \frac{1}{4}, 1] \end{cases} \quad (B.8)$$

for $y \in [\frac{1}{4}, 1]$ by Lemma 6. Note that for $y \in [0, \frac{1}{4}]$, $b_0(1) = \frac{2}{3} + \frac{1}{12} < s(1-y) = \frac{2}{3}(1-y) + \frac{1}{4} + \frac{1}{12}(1-y)$; and for $y \in [\frac{1}{4}, 1]$, $b_0(1) = \frac{2}{3}(1-y) + \frac{1}{4} < s(1-y) = \frac{2}{3}(1-y) + \frac{1}{4} + \frac{1}{12}(1-y)$. That is, if the agent lies to the buyer, the buyer and seller do not trade the good at all because the buyer’s highest offer is always lower than the seller’s lowest offer. Therefore, neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium.

When the true state is $(v^b, v^s) \in [0, 1] \times [0, 1-y]$, if the agent tells the true state only to the buyer, then common knowledge is that $v^b$ is uniformly distributed over $[0, 1]$, and $v^s$ is uniformly distributed over $[0, 1-y]$. Thus, for $y \in [\frac{1}{4}, 1]$, the buyer's
and seller’s strategies are

\begin{equation}
\begin{cases}
0 & \text{if } v^b \in [0, \frac{1}{4}) \\
\frac{2}{3} v^b + \frac{1}{12} & \text{if } v^b \in \left[\frac{1}{4}, (1 - y) + \frac{1}{4} \right] \\
\frac{3}{4}(1 - y) + \frac{1}{4} & \text{if } v^b \in ((1 - y) + \frac{1}{4}, 1] 
\end{cases}
\end{equation}

(B.9)

and

\begin{equation}
\begin{align*}
s(v^s) &= \frac{2}{3} v^s + \frac{1}{4} \quad \text{for all } v^s \in [0, 1 - y],
\end{align*}
\end{equation}

(B.10)

respectively, by Lemma 6. Then, the probability of trade in the equilibrium is

\begin{equation}
\int_0^{1-y} \int_{(1-y)+\frac{1}{4}}^1 \frac{1}{1-y} dv^b dv^s + \int_0^{1-y} \int_{v^s+\frac{1}{4}}^{(1-y)+\frac{1}{4}} \frac{1}{1-y} dv^b dv^s = \frac{1}{4} + \frac{1}{2} y,
\end{equation}

(B.11)

and the expected sales price is

\begin{equation}
\int_0^{1-y} \int_{(1-y)+\frac{1}{4}}^1 \frac{1}{2} \left(\frac{2}{3}(1 - y) + \frac{1}{4} + \frac{2}{3} v^s + \frac{1}{4}\right) \frac{1}{1-y} dv^b dv^s \\
+ \int_0^{1-y} \int_{v^s+\frac{1}{4}}^{(1-y)+\frac{1}{4}} \frac{1}{2} \left(\frac{2}{3} v^b + \frac{1}{12} + \frac{2}{3} v^s + \frac{1}{4}\right) \frac{1}{1-y} dv^b dv^s = \frac{5 + 20 y - 16 y^2}{48}.
\end{equation}

(B.12)

If the agent delivers a false message \( m^i_s \) to the buyer and the buyer believes the agent’s message, then the buyer’s strategy is

\begin{equation}
\begin{cases}
0 & \text{if } v^b \in [0, \frac{3}{4}(1 - y) + \frac{1}{4}) \\
\frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4}(1 - y) & \text{if } v^b \in \left[\frac{3}{4}(1 - y) + \frac{1}{4}, 1] 
\end{cases}
\end{equation}

(B.13)

by Lemma 6. Then, the probability of trade is

\begin{equation}
\int_0^{1-y} \int_{\frac{3}{4}(1-y)+\frac{1}{4}}^1 \frac{1}{1-y} dv^b dv^s = \frac{3}{4} y,
\end{equation}

(B.14)
and the expected sales price is

\[
\int_0^{1-y} \int_0^1 \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} (1-y) + \frac{2}{3} v^s + \frac{1}{4} \right) \frac{1}{1-y} dv^b dv^s = \frac{(19-10y)y}{32}. \tag{B.15}
\]

Note that \( \frac{1}{4} + \frac{1}{2} y \geq \frac{3}{4} y \) for all \( y \in [\frac{1}{4}, 1] \). That is, the trade-maximizing agent has no incentive to deviate from the equilibrium.

Note that \( \frac{5+20y-16y^2}{48} \geq \frac{(19-10y)y}{32} \) if and only if \( y \in [\frac{1}{4}, \frac{3\sqrt{41} - 17}{4}] \), and \( \frac{5+20y-16y^2}{48} < \frac{(19-10y)y}{32} \) if and only if \( y \in (\frac{3\sqrt{41} - 17}{4}, 1] \). That is, the price-maximizing agent has no incentive to deviate from the equilibrium if and only if \( y \in [\frac{1}{4}, \frac{3\sqrt{41} - 17}{4}] \).

For \( y \in [0, \frac{1}{4}] \), the buyer’s and seller’s strategies are

\[
b_0(v^b) = \begin{cases} 0 & \text{if } v^b \in [0, \frac{1}{4}) \\ \frac{2}{3} v^b + \frac{1}{12} & \text{if } v^b \in [\frac{1}{4}, 1] \end{cases} \tag{B.16}
\]

and

\[
s(v^s) = \begin{cases} \frac{2}{3} v^s + \frac{1}{4} & \text{if } v^s \in [0, \frac{3}{4}] \\ 1 & \text{if } v^s \in (\frac{3}{4}, 1-y] \end{cases} \tag{B.17}
\]

respectively, by Lemma 6. Then, the probability of trade in the equilibrium is

\[
\int_0^{\frac{3}{4}} \int_0^1 \frac{1}{1-y} dv^b dv^s = \frac{9}{32(1-y)}, \tag{B.18}
\]

and the expected sales price is

\[
\int_0^{\frac{3}{4}} \int_0^1 \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{2}{3} v^s + \frac{1}{4} \right) \frac{1}{1-y} dv^b dv^s = \frac{9}{64(1-y)}. \tag{B.19}
\]

If the agent delivers a false message \( m_1^s \) to the buyer and the buyer believes the agent’s message, then the buyer’s strategy is

\[
b_1(v^b) = \begin{cases} 0 & \text{if } v^b \in [0, \frac{3}{4}(1-y) + \frac{1}{4}) \\ \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} (1-y) & \text{if } v^b \in [\frac{3}{4}(1-y) + \frac{1}{4}, 1] \end{cases} \tag{B.20}
\]
by Lemma 6. Then, the probability of trade is

\[
\int_0^{\frac{3}{4}} \int_0^{\frac{1}{y}+\frac{1}{4}} \frac{1}{1-y} dv^b dv^s = \frac{9y}{16(1-y)}, \tag{B.21}
\]

and the expected sales price is

\[
\int_0^{\frac{3}{4}} \int_0^{\frac{1}{y}+\frac{1}{4}} \frac{1}{2} \left\{ \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} \right\} v^s + \frac{1}{4} \frac{1}{1-y} dv^b dv^s = \frac{9(3-y)y}{64(1-y)}. \tag{B.22}
\]

Note that \(\frac{9y}{32(1-y)} > \frac{9y}{64(1-y)}\) and \(\frac{9(3-y)y}{64(1-y)}\) for all \(y \in [0, \frac{1}{4}]\). That is, neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium. ■

**Proof of Lemma 7 and 8.** When the true state is \((v^b, v^s) \in [0, 1] \times [0, 1 - y]\), in a bargaining equilibrium in which an informed agent truthfully advises a buyer only, the probability of trade is \(\frac{9y}{32(1-y)}\) by equation (B.18), and the expected sales price is \(\frac{9y}{64(1-y)}\) by equation (B.19), for \(y \in [0, \frac{1}{4}]\). For \(y \in [\frac{1}{4}, 1]\), the probability of trade is \(\frac{1}{4} + \frac{1}{2} y\) by equation (B.11), and the expected sales price is \(\frac{5+20y-16y^2}{48}\) by equation (B.12).

When the true state is \((v^b, v^s) \in [0, 1] \times [1 - y, 1]\), in a bargaining equilibrium in which an informed agent truthfully advises a buyer only, the probability of trade is \(\frac{9y}{32}\) by equation (B.5), and the expected sales price is \(\frac{9(2-y)y}{64}\) by equation (B.6).

Thus, for \(y \in [0, \frac{1}{4}]\), the probability of trade is

\[
(1-y) \frac{9}{32(1-y)} + y \frac{9y}{32} = \frac{9(1+y^2)}{32}, \tag{B.23}
\]

and the expected sales price is

\[
(1-y) \frac{9}{64(1-y)} + y \frac{9(2-y)y}{64} = \frac{9(1+2y^2-y^3)}{64}. \tag{B.24}
\]
Note that in Chatterjee and Samuelson’s (1983) babbling equilibrium, the probability of trade is \( \frac{9}{32} \) and the expected sales price is \( \frac{9}{64} \). \( \frac{9(1+y^2)}{32} \geq \frac{9}{32} \) and \( \frac{9(1+2y^2-y^3)}{64} \geq \frac{9}{64} \) for all \( y \in [0, \frac{1}{4}] \).

For \( y \in [\frac{1}{4}, 1] \), the probability of trade is

\[
(1 - y)(\frac{1}{4} + \frac{1}{2}y) + y\frac{9y}{32} = \frac{8 + 8y - 7y^2}{32}, 
\]
(B.25)

and the expected sales price is

\[
(1 - y)^5 + 20y - 16y^2 + y\frac{9(2 - y)y}{64} = \frac{20 + 60y - 90y^2 + 37y^3}{192}. 
\]
(B.26)

\( \frac{8+8y-7y^2}{32} \geq \frac{9}{32} \) and \( \frac{20+60y-90y^2+37y^3}{192} \geq \frac{9}{64} \) for all \( y \in [\frac{1}{4}, 1] \).

**Proof of Corollary 6.** Straightforward from Theorem 4, Theorem 5, and Lemma 7.

**Proof of Corollary 7.** Straightforward from Theorem 4, Theorem 5, and Lemma 8.

**Proof of Theorem 8 and 9.** When the true state is \( (v^b, v^s) \in [0, y] \times [0, 1] \), if the agent tells the true state only to the seller, then common knowledge is that \( v^b \) is uniformly distributed over \([0, y]\), and \( v^s \) is uniformly distributed over \([0, 1]\). Thus, by Lemma 6, the buyer’s and seller’s strategies are

\[
b(v^b) = \begin{cases} 0 & \text{if } v^b \in [0, \frac{1}{4}y) \\ \frac{2}{3}v^b + \frac{1}{12}y & \text{if } v^b \in [\frac{1}{4}y, y] \end{cases} 
\]
(B.27)

and

\[
s_0(v^s) = \begin{cases} \frac{2}{3}v^s + \frac{1}{4}y & \text{if } v^s \in [0, \frac{3}{4}y] \\ 1 & \text{if } v^s \in (\frac{3}{4}y, 1] \end{cases} 
\]
(B.28)
respectively. Then, the probability of trade in the equilibrium is

\[
\int_0^{\frac{3}{4}y} \int_{v^s+\frac{1}{4}y}^{y} \frac{1}{y} dv^b dv^s = \frac{9y}{32}, \tag{B.29}
\]

and the expected sales price is

\[
\int_0^{\frac{3}{4}y} \int_{v^s+\frac{1}{4}y}^{y} \left(\frac{2}{3}v^b + \frac{1}{12}y + \frac{2}{3}v^s + \frac{1}{4}y\right) \frac{1}{y} dv^b dv^s = \frac{9y^2}{64}. \tag{B.30}
\]

If the agent delivers a false message \(m^b_1\) to the seller and the seller believes the agent’s message, then the seller’s strategy is

\[
s_1(v^s) = \begin{cases} 
\frac{2}{3}v^s + \frac{1}{4} & \text{if } v^s \in [0, \frac{3}{4}] \\
1 & \text{if } v^s \in (\frac{3}{4}, 1]
\end{cases} \tag{B.31}
\]

for \(y \in [0, \frac{1}{4}]\) or

\[
s_1(v^s) = \begin{cases} 
\frac{2}{3}y + \frac{1}{12} & \text{if } v^s \in [0, y - \frac{1}{4}] \\
\frac{2}{3}v^s + \frac{1}{3} & \text{if } v^s \in [y - \frac{1}{4}, \frac{3}{4}] \\
1 & \text{if } v^s \in (\frac{3}{4}, 1]
\end{cases} \tag{B.32}
\]

for \(y \in [\frac{1}{4}, 1]\) by Lemma 6. Note that for \(y \in [0, \frac{1}{4}]\), \(s_1(0) = \frac{1}{4} > b(y) = \frac{2}{3}y + \frac{1}{12}y\); and for \(y \in [\frac{1}{4}, 1]\), \(s_1(0) = \frac{2}{3}y + \frac{1}{12} > b(y) = \frac{2}{3}y + \frac{1}{12}y\). That is, if the agent lies to the seller, the buyer and seller do not trade the good at all because the buyer’s highest offer is always lower than the seller’s lowest offer. Therefore, neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium.

When the true state is \((v^b, v^s) \in [y, 1] \times [0, 1]\), if the agent tells the true state only to the seller, then common knowledge is that \(v^b\) is uniformly distributed over \([y, 1]\), and \(v^s\) is uniformly distributed over \([0, 1]\). Thus, for \(y \in [\frac{1}{4}, 1]\), the buyer’s and seller’s strategies are

\[
b(v^b) = \frac{2}{3}v^b + \frac{1}{12} \text{ for all } v^b \in [y, 1] \tag{B.33}
\]
and

\[ s_1(v^s) = \begin{cases} 
\frac{2}{3}y + \frac{1}{12} & \text{if } v^s \in [0, y - \frac{1}{4}) \\
\frac{2}{3}v^s + \frac{1}{7} & \text{if } v^s \in [y - \frac{1}{4}, \frac{3}{4}) \\
1 & \text{if } v^b \in (\frac{3}{4}, 1]
\end{cases} \quad \text{(B.34)} \]

respectively, by Lemma 6. Then, the probability of trade in the equilibrium is

\[ \int \int \frac{1}{1-y} dv^b dv^s + \int \int \frac{1}{1-y} dv^b dv^s = \frac{1}{4} + \frac{1}{2}y, \quad \text{(B.35)} \]

and the expected sales price is

\[
\frac{3}{4}y \int_0^1 \int_0^1 \frac{1}{1-y} dv^b dv^s + \frac{3}{4}y \int_{y-\frac{1}{4}}^{v^s+\frac{1}{4}} \int_{v^b}^{1} \frac{1}{1-y} dv^b dv^s = \frac{5y(1+2y)}{32}. \quad \text{(B.39)}
\]

If the agent delivers a false message \( m^b_0 \) to the seller and the seller believes the agent’s message, then the seller’s strategy is

\[ s_0(v^s) = \begin{cases} 
\frac{2}{3}v^s + \frac{1}{7}y & \text{if } v^s \in [0, \frac{3}{4}y) \\
1 & \text{if } v^b \in (\frac{3}{4}y, 1] \quad \text{(B.37)}
\end{cases} \]

by Lemma 6. Then, the probability of trade is

\[ \int \int \frac{1}{1-y} dv^b dv^s = \frac{3}{4} y, \quad \text{(B.38)} \]

and the expected sales price is

\[ \frac{3}{4}y \int_0^1 \int_0^1 \frac{1}{2} (\frac{2}{3}v^b + \frac{1}{12} + \frac{2}{3}v^s + \frac{1}{4}y) \frac{1}{1-y} dv^b dv^s = \frac{5y(1+2y)}{32}. \quad \text{(B.39)}
\]

Note that \( \frac{1}{4} + \frac{1}{2}y \geq \frac{3}{4}y \) and \( \frac{16y^2+4y+7}{48} > \frac{5y(1+2y)}{32} \) for all \( y \in [\frac{1}{4}, 1] \). That is, neither the
trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium.

For \( y \in [0, \frac{1}{4}] \), the buyer’s and seller’s strategies are

\[
b(v^b) = \begin{cases} 
0 & \text{if } v^b \in [0, \frac{1}{4}) \\
\frac{2}{3} v^b + \frac{1}{12} & \text{if } v^b \in [\frac{1}{4}, 1] 
\end{cases} \tag{B.40}
\]

and

\[
s_1(v^s) = \begin{cases} 
\frac{2}{3} v^s + \frac{1}{4} & \text{if } v^s \in [0, \frac{3}{4}] \\
1 & \text{if } v^s \in (\frac{3}{4}, 1] 
\end{cases} \tag{B.41}
\]

respectively, by Lemma 6. Then, the probability of trade in the equilibrium is

\[
\int_{\frac{y}{2}}^{\frac{1}{2}} \int_{0}^{1} \frac{1}{1 - y} dv^b dv^s = \frac{9}{32(1 - y)}, \tag{B.42}
\]

and the expected sales price is

\[
\int_{\frac{y}{2}}^{\frac{1}{2}} \int_{0}^{1} \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{2}{3} v^s + \frac{1}{4} \right) \frac{1}{1 - y} dv^b dv^s = \frac{9}{64(1 - y)}. \tag{B.43}
\]

If the agent delivers a false message \( m^b_0 \) to the seller and the seller believes the agent’s message, then the seller’s strategy is

\[
s_0(v^s) = \begin{cases} 
\frac{2}{3} v^s + \frac{1}{4} y & \text{if } v^s \in [0, \frac{3}{4} y] \\
1 & \text{if } v^b \in (\frac{3}{4} y, 1] 
\end{cases} \tag{B.44}
\]

by Lemma 6. Then, the probability of trade is

\[
\int_{0}^{\frac{3}{4} y} \int_{y}^{1} \frac{1}{1 - y} dv^b dv^s = \frac{3y}{4}, \tag{B.45}
\]
and the expected sales price is

\[
\int_0^{\frac{3}{2}y} \int_y^1 \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{2}{3} v^s + \frac{1}{4} y \right) \frac{1}{1 - y} dv^b dv^s = \frac{5y(1 + 2y)}{32}.
\]

(B.46)

Note that \( \frac{9}{32(1-y)} > \frac{3y}{4} \) and \( \frac{9}{64(1-y)} > \frac{5y(1+2y)}{32} \) for all \( y \in [0, \frac{1}{4}] \). That is, neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium.

\[\Box\]

**Proof of Theorem 10 and 11.** When the true state is \((v^b, v^s) \in [y, 1) \times [0, 1 - y]\), if the agent tells the true state to the buyer and seller, then common knowledge is that \( v^b \) is uniformly distributed over \([y, 1]\), and \( v^s \) is uniformly distributed over \([0, 1 - y]\).

For \( y \in [\frac{1}{2}, 1] \), the buyer’s and seller’s strategies are \( b_0(v^b) = s_1(v^s) = \frac{1}{2} \) for all \( v^b \in [y, 1] \) and \( v^s \in [0, 1 - y] \). Thus, the buyer and seller trade the good with certainty, and the expected sales price is \( \frac{1}{2} \). Note that the trade-maximizing agent has no incentive to deviate from the equilibrium since the probability of trade equals to 1.

Note that the agent’s possible deviations are delivering a false message \( m^b_0 \) to the seller or a false message \( m^s_1 \) to the buyer or both. Because of the reflection-symmetric nature of the setting, the false message \( m^b_0 \) to the seller and the false message \( m^s_1 \) to the buyer has the same effect on the change in the probability of trade. Moreover, if the agent delivers the false message \( m^b_0 \) to the seller and the false message \( m^s_1 \) to the buyer at the same time, as we saw in the proof of Theorem 1, it has a lower probability of trade than a one-sided false message of \( m^b_0 \) to the seller or \( m^s_1 \) to the buyer.

Also, note that the false message \( m^b_0 \) to the seller makes the seller submit a lower offer when the seller submits lower than 1 by (i) in Lemma 3, and the false message \( m^s_1 \) to the buyer makes the buyer submit a higher offer when the buyer submits higher
than 0 by (iii) in Lemma 3. Therefore, delivering the false message \( m^*_1 \) to the buyer is the most profitable deviation of the price-maximizing agent among the possible deviations. Thus, I only need to check the false message \( m^*_1 \) to the buyer.

If the agent delivers the false message \( m^*_1 \) to the buyer and the buyer believes the agent’s message, then the buyer’s strategy is

\[
b_1(v^b) = \begin{cases} 0 & \text{if } v^b \in [y, \frac{3}{4}(1 - y) + \frac{1}{4}) \\ \frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1 - y) & \text{if } v^b \in [\frac{3}{4}(1 - y) + \frac{1}{4}, 1] \end{cases}
\]

(B.47)

for \( y \in [\frac{1}{2}, \frac{4}{7}] \) or for \( y \in [\frac{4}{7}, 1] \),

\[
b_1(v^b) = \frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1 - y) \quad \text{for all } v^b \in [y, 1] \quad \text{(B.48)}
\]

by Lemma 6. Thus, the expected sales price is

\[
\int_0^{1-y} \int_{\frac{3}{4}(1-y)+\frac{1}{2}}^{1-y} \frac{1}{\frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1 - y) + \frac{1}{2}} \frac{1}{(1 - y)^2} dv^b dv^s = \frac{3y(3-y)}{16(1-y)} \quad \text{for } y \in [\frac{1}{2}, \frac{4}{7}]\]

(B.49)

for \( y \in [\frac{4}{7}, 1] \). Note that \( \frac{14+y}{24} > \frac{1}{2} \) for all \( y \in [\frac{4}{7}, 1] \), \( \frac{3y(3-y)}{16(1-y)} \leq \frac{1}{2} \) if and only if \( y \in [\frac{1}{2}, \frac{17-\sqrt{193}}{6}] \), and \( \frac{3y(3-y)}{16(1-y)} > \frac{1}{2} \) if and only if \( y \in (\frac{17-\sqrt{193}}{6}, \frac{4}{7}] \). That is, the price-maximizing agent has no incentive to deviate from the equilibrium if and only if \( y \in [\frac{1}{2}, \frac{17-\sqrt{193}}{6}] \).

Similarly, for \( y \in [\frac{1}{4}, \frac{1}{2}] \), the buyer’s and seller’s strategies are

\[
b_0(v^b) = \begin{cases} \frac{2}{3}v^b + \frac{1}{12} & \text{if } v^b \in [y, (1 - y) + \frac{1}{4}] \\ \frac{3}{3}(1 - y) + \frac{1}{4} & \text{if } v^b \in [(1 - y) + \frac{1}{4}, 1] \end{cases}
\]

(B.51)
and
\[ s_1(v^s) = \begin{cases} \frac{2}{3}y + \frac{1}{12} & \text{if } v^s \in [0, y - \frac{1}{4}] \\ \frac{3}{4}v^s & \text{if } v^s \in [y - \frac{1}{4}, 1 - y] \end{cases}, \]
respectively, by Lemma 6. Then, the probability of trade in equilibrium is
\[
\int_0^{y-\frac{1}{4}} \int_y^{1-y} \frac{1}{(1-y)^2} dv^b dv^s + \int_{y-\frac{1}{4}}^1 \int_y^{1-y} \frac{1}{(1-y)^2} dv^b dv^s 
+ \int_{y-\frac{1}{4}}^1 \int_{v^s+\frac{1}{4}}^{1-y} \frac{1}{(1-y)^2} dv^b dv^s + \int_{y-\frac{1}{4}}^{1-y} \int_{(1-y)+\frac{1}{4}}^{1} \frac{1}{(1-y)^2} dv^b dv^s = \frac{7+16y-32y^2}{32(1-y)^2},
\]
and the expected sales price is
\[
\int_0^{y-\frac{1}{4}} \int_y^{1-y} \{2 \frac{2}{3}v^b + \frac{1}{12} + \frac{2}{3}y + \frac{1}{12}\} \frac{1}{(1-y)^2} dv^b dv^s 
+ \int_{y-\frac{1}{4}}^1 \int_y^{1-y} \{\frac{2}{3}(1-y) + \frac{1}{4} + \frac{2}{3}y + \frac{1}{4}\} \frac{1}{(1-y)^2} dv^b dv^s 
+ \int_{y-\frac{1}{4}}^{1-y} \int_{v^s+\frac{1}{4}}^{1-y} \{\frac{2}{3}v^b + \frac{1}{12} + \frac{2}{3}v^s + \frac{1}{4}\} \frac{1}{(1-y)^2} dv^b dv^s 
+ \int_{y-\frac{1}{4}}^{1-y} \int_{(1-y)+\frac{1}{4}}^{1-y} \{\frac{2}{3}(1-y) + \frac{1}{4} + \frac{2}{3}v^s + \frac{1}{4}\} \frac{1}{(1-y)^2} dv^b dv^s = \frac{7+16y-32y^2}{64(1-y)^2}.
\]
If the agent delivers a false message \(m_1^s\) to the buyer and the buyer believes the agent’s message, then the buyer’s strategy is
\[ b_1(v^b) = \begin{cases} 0 & \text{if } v^b \in [y, \frac{3}{4}(1-y) + \frac{1}{4}] \\ \frac{2}{3}v^b + \frac{1}{12} + \frac{1}{4}(1-y) & \text{if } v^b \in [\frac{3}{4}(1-y) + \frac{1}{4}, 1] \end{cases}, \]
by Lemma 6. Then, the probability of trade is
\[
\int_{0}^{1-y} \int_{\frac{1}{4}(1-y)+\frac{1}{4}}^{1} \frac{1}{(1-y)^2} dv^b dv^s = \frac{3y}{4(1-y)},
\]
and the expected sales price is

\[
\int_{0}^{y-\frac{1}{4}} \int_{\frac{3}{4}(1-y^2)}^{1} \left( \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} (1 - y) + \frac{2}{3} y + \frac{1}{12} \right) \right) \frac{1}{(1-y)^2} dv^b dv^s 
\]

and

\[
\frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} (1 - y) + \frac{2}{3} y + \frac{1}{12} \right) \frac{1}{(1-y)^2} dv^b dv^s = \frac{y(77 - 124y + 56y^2)}{128(1-y)^2}. 
\]

Note that \( \frac{7+16y-32y^2}{32(1-y)^2} > \frac{3y}{4(1-y)} \) for all \( y \in \left[ \frac{1}{4}, \frac{1}{2} \right] \). That is, the trade-maximizing agent has no incentive to deviate from the equilibrium.

Note that \( \frac{7+16y-32y^2}{64(1-y)^2} \geq \frac{y(77 - 124y + 56y^2)}{128(1-y)^2} \) if and only if \( y \in \left[ \frac{1}{4}, \frac{1}{28} \left( \frac{10 - \frac{5x222/3}{(27+7\sqrt{71})^{1/3}} + (22(27 + 7\sqrt{71}))^{1/3}}{2} \right) \right] \), and \( \frac{7+16y-32y^2}{64(1-y)^2} < \frac{y(77 - 124y + 56y^2)}{128(1-y)^2} \) if and only if \( y \in \left( \frac{1}{28} \left( \frac{10 - \frac{5x222/3}{(27+7\sqrt{71})^{1/3}} + (22(27 + 7\sqrt{71}))^{1/3}}{2} \right), \frac{1}{2} \right] \). That is, the price-maximizing agent has no incentive to deviate from the equilibrium if and only if \( y \in \left[ \frac{1}{4}, \frac{1}{28} \left( \frac{10 - \frac{5x222/3}{(27+7\sqrt{71})^{1/3}} + (22(27 + 7\sqrt{71}))^{1/3}}{2} \right) \right] \).

For \( y \in [0, \frac{1}{4}] \), the buyer’s and seller’s strategies are

\[
b_0(v^b) = \begin{cases} 
0 & \text{if } v^b \in [y, \frac{1}{4}) \\
\frac{2}{3} v^b + \frac{1}{12} & \text{if } v^b \in [\frac{1}{4}, 1] 
\end{cases} 
\]

and

\[
s_1(v^s) = \begin{cases} 
\frac{2}{3} v^s + \frac{1}{4} & \text{if } v^s \in [0, \frac{3}{4}] \\
1 & \text{if } v^s \in (\frac{3}{4}, 1 - y]
\end{cases} 
\]

respectively, by Lemma 6. Then, the probability of trade in the equilibrium is

\[
\int_{0}^{\frac{3}{4}} \int_{v^s + \frac{1}{4}}^{1} \frac{1}{(1-y)^2} dv^b dv^s = \frac{9}{32(1-y)^2}. 
\]
and the expected sales price is
\[
\int_0^{\frac{3}{4}} \int_{v^b + \frac{1}{4}}^1 \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{2}{3} v^s + \frac{1}{4} \right) \frac{1}{(1 - y)^2} dv^b dv^s = \frac{9}{64(1 - y)^2}. \tag{B.61}
\]

If the agent delivers a false message \( m_s^* \) to the buyer and the buyer believes the agent’s message, then the buyer’s strategy is
\[
b_1(v^b) = \begin{cases} 
0 & \text{if } v^b \in [y, \frac{3}{4}(1 - y) + \frac{1}{3}] \\
\frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} & \text{if } v^b \in [\frac{3}{4}(1 - y) + \frac{1}{4}, 1]
\end{cases}
\tag{B.62}
\]
by Lemma 6. Then, the probability of trade is
\[
\int_0^{\frac{3}{4}} \int_{\frac{3}{4}(1 - y) + \frac{1}{4}}^1 \frac{1}{(1 - y)^2} dv^b dv^s = \frac{9y}{16(1 - y)^2}, \tag{B.63}
\]
and the expected sales price is
\[
\int_0^{\frac{3}{4}} \int_{\frac{3}{4}(1 - y) + \frac{1}{4}}^1 \frac{1}{2} \left( \frac{2}{3} v^b + \frac{1}{12} + \frac{1}{4} (1 - y) + \frac{2}{3} v^s + \frac{1}{4} \right) \frac{1}{(1 - y)^2} dv^b dv^s = \frac{9(3 - y)y}{64(1 - y)^2}. \tag{B.64}
\]

Note that \( \frac{9}{32(1 - y)^2} > \frac{9y}{16(1 - y)^2} \) and \( \frac{9}{64(1 - y)^2} > \frac{9(3 - y)y}{64(1 - y)^2} \) for all \( y \in [0, \frac{1}{4}] \). That is, neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate from the equilibrium.

Similarly, it is easy to check the following. When the true state is \( (v^b, v^s) \in [0, y] \times [0, 1 - y] \), the trade-maximizing agent has no incentive to deviate from the equilibrium, and the price-maximizing agent has no incentive to deviate from the equilibrium if and only if \( y \in [0, \frac{25 + 3\sqrt{41}}{64} \approx 0.6908] \). When the true state is \( (v^b, v^s) \in [y, 1] \times [1 - y, 1] \) or \( (v^b, v^s) \in [0, y] \times [1 - y, 1] \), neither the trade-maximizing agent nor the price-maximizing agent has an incentive to deviate.
**Proof of Corollary 12.** Straightforward from Theorem 10 and Theorem 11. ■

**Proof of Corollary 13.** Straightforward from Theorem 4, Theorem 8, and Theorem 10. ■

**Proof of Corollary 14.** Straightforward from Theorem 5, Theorem 9, and Theorem 11. ■

**Proof of Lemma 9.** It is straightforward to calculate the probability of trade in each equilibrium. The probability of trade in the babbling equilibrium is \( \frac{9}{32} \). In both bargaining equilibria in which an informed agent truthfully advises a buyer only and an informed agent truthfully advises a seller only, the probability of trade is \( \frac{9(1+y^2)}{32} \) for \( y \in [0, \frac{1}{4}] \) and \( \frac{8+8y-7y^2}{32} \) for \( y \in [\frac{1}{4}, 1] \). The probability of trade in a bargaining equilibrium in which an informed agent truthfully advises both buyer and seller is \( \frac{9(1+2y^2)}{32} \) for \( y \in [0, \frac{1}{4}] \), \( \frac{7+16y-14y^2}{32} \) for \( y \in [\frac{1}{4}, \frac{1}{2}] \), \( \frac{41-100y+86y^2}{32} \) for \( y \in [\frac{1}{2}, \frac{4}{7}] \), and \( \frac{3(3+4y-4y^2)}{32} \) for \( y \in [\frac{4}{7}, 1] \).

\[
\begin{align*}
\frac{9(1+y^2)}{32} &\geq \frac{9}{32} \text{ for all } y \in [0, \frac{1}{4}], \text{ and } \frac{8+8y-7y^2}{32} \geq \frac{9}{32} \text{ for all } y \in [\frac{1}{4}, 1]. \\
\frac{9(1+2y^2)}{32} &\geq \frac{9(1+y^2)}{32} \text{ for all } y \in [0, \frac{1}{4}], \text{ and } \frac{7+16y-14y^2}{32} > \frac{8+8y-7y^2}{32} \text{ for all } y \in [\frac{1}{4}, \frac{1}{2}], \\
\frac{41-100y+86y^2}{32} &> \frac{8+8y-7y^2}{32} \text{ for all } y \in [\frac{1}{2}, \frac{4}{7}], \text{ and } \frac{3(3+4y-4y^2)}{32} \geq \frac{8+8y-7y^2}{32} \text{ for all } y \in [\frac{4}{7}, 1].
\end{align*}
\]

■

**Proof of Lemma 10.** It is straightforward to calculate the expected sales price in each equilibrium. The expected sales price in the babbling equilibrium is \( \frac{9}{64} \). The expected sales price in a bargaining equilibrium in which an informed agent truthfully advises a buyer only is \( \frac{9(1+2y^2-3y^3)}{64} \) for \( y \in [0, \frac{1}{4}] \) and \( \frac{20+60y-90y^2+37y^3}{192} \) for \( y \in [\frac{1}{4}, 1] \). The expected sales price in a bargaining equilibrium in which an informed agent truthfully advises a seller only is \( \frac{9(1+y^3)}{64} \) for \( y \in [0, \frac{1}{4}] \) and \( \frac{28-12y+48y^2-37y^3}{192} \) for \( y \in [\frac{1}{4}, 1] \). The expected sales price in a bargaining equilibrium in which an informed
agent truthfully advises both buyer and seller is $\frac{9(1+2y^2)}{64}$ for $y \in [0, \frac{1}{4}]$, $\frac{7+16y-14y^2}{64}$ for $y \in \left[\frac{1}{4}, \frac{1}{2}\right]$, $\frac{41-100y+86y^2}{64}$ for $y \in \left[\frac{1}{2}, \frac{4}{7}\right]$, and $\frac{3(3+4y-4y^2)}{64}$ for $y \in \left[\frac{4}{7}, 1\right]$. 

\[
\frac{9(1+2y^2-3y^3)}{64} \geq \frac{9(1+y^3)}{64} \geq \frac{9}{64} \quad \text{for all } y \in \left[0, \frac{1}{4}\right], \quad \frac{20+60y-90y^2+37y^3}{192} \geq \frac{28-12y+48y^2-37y^3}{192} \geq \frac{9}{64} \quad \text{for all } y \in \left[\frac{1}{4}, \frac{2(8+3\sqrt{3})}{37}\right], \text{ and } \frac{28-12y+48y^2-37y^3}{192} \geq \frac{20+60y-90y^2+37y^3}{192} \geq \frac{9}{64} \quad \text{for all } y \in \left[\frac{1}{2}, \frac{4}{7}\right].
\]

\[
\frac{9(1+2y^2)}{64} \geq \frac{9(1+2y^2-3y^3)}{64} \quad \text{for all } y \in \left[0, \frac{1}{4}\right], \quad \frac{7+16y-14y^2}{64} > \frac{20+60y-90y^2+37y^3}{192} \quad \text{for all } y \in \left[\frac{1}{4}, \frac{1}{2}\right], \quad \frac{41-100y+86y^2}{64} > \frac{20+60y-90y^2+37y^3}{192} \quad \text{for all } y \in \left[\frac{1}{2}, \frac{4}{7}\right], \text{ and } \frac{3(3+4y-4y^2)}{64} \geq \frac{28-12y+48y^2-37y^3}{192} \quad \text{for all } y \in \left[\frac{4}{7}, \frac{2(8+3\sqrt{3})}{37}\right].\]

**Proof of Corollary 15.** Straightforward from Corollary 13, Corollary 14, and Lemma 9. ■

**Proof of Corollary 16.** Straightforward from Corollary 13, Corollary 14, and Lemma 10. ■
Appendix C

Appendix for Chapter 3

Proof of Theorem 17. Let $Y_i$ be the random variable of the number of new listings in period $i$, $X_i$ be the random variable of the number of listings that are unsold in period $i$ and remain on the list for period $i+1$, $Z_i$ be the random variable of the total number of listings in period $i$, and $W_i$ be the random variable of the number of sales in period $i$.

Then,

$$Z_0 = Y_0 \sim Binomial(n, p), W_0 \sim Binomial(n, pq),$$

and

$$X_0 \sim Binomial(n, p(1-q)r).$$

Conditional on $X_0 = x_0$,

$$Y_1 \sim Binomial(n-x_0, p).$$

That is, unconditionally,

$$Y_1 \sim Binomial(n, p\{1 - p(1-q)r\}).$$

Thus,

$$Z_1 = Y_1 + X_0 \sim Binomial(n, p\{1 + (1-p)(1-q)r\}),$$

$$W_1 \sim Binomial(n, p\{1 + (1-p)(1-q)r\}q),$$
and

\[ X_1 \sim \text{Binomial}(n, p \{1 + (1 - p)(1 - q)r\}(1 - q)r). \]

Conditional on \( X_1 = x_1 \),

\[ Y_2 \sim \text{Binomial}(n - x_1, p). \]

That is, unconditionally,

\[ Y_2 \sim \text{Binomial}(n, p \{1 + (1 - p)(1 - q)r\}(1 - q)r\}). \]

Thus,

\[ Z_2 = Y_2 + X_1 \sim \text{Binomial}(n, p \{1 + (1 - p)(1 - q)r + (1 - p)^2(1 - q)^2r^2\}), \]

and

\[ W_2 \sim \text{Binomial}(n, p \{1 + (1 - p)(1 - q)r + (1 - p)^2(1 - q)^2r^2\}q). \]

Similarly,

\[ Z_k \sim \text{Binomial}(n, p \{\sum_{i=0}^{k} (1 - p)^i(1 - q)^i}r^i\}), \]

and

\[ W_k \sim \text{Binomial}(n, p \{\sum_{i=0}^{k} (1 - p)^i(1 - q)^i}r^i\}q). \]

As \( k \to \infty \),

\[ Z \sim \text{Binomial}(n, \frac{p}{1 - (1 - p)(1 - q)r}), \text{ and } W \sim \text{Binomial}(n, \frac{pq}{1 - (1 - p)(1 - q)r}). \]

By the Poisson Limit Theorem,

\[ Z \sim \text{Poisson}(\frac{np}{1 - (1 - p)(1 - q)r}), \text{ and } W \sim \text{Poisson}(\frac{npq}{1 - (1 - p)(1 - q)r}). \]
Results of the regression models with two-way fixed effects of apartment unit type and time

Table C.1: Results of the regression models with two-way fixed effects of apartment unit type and time

<table>
<thead>
<tr>
<th></th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>log(Number of sales),</td>
<td><strong>Variable of interest:</strong></td>
<td>FIX</td>
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<td></td>
</tr>
<tr>
<td>Type &amp; year FEs</td>
<td>0.0024***</td>
<td>0.0024***</td>
<td>0.0024**</td>
<td>0.0024</td>
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<tr>
<td></td>
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<td>(0.0009)</td>
<td>(0.0011)</td>
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<td>(0.0019)</td>
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<td>Number of groups</td>
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<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
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<td>Type &amp; year &amp; quarter FEs</td>
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<td>0.0028***</td>
<td>0.0028***</td>
<td>0.0028**</td>
<td>0.0028</td>
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<td>(0.0007)</td>
<td>(0.0010)</td>
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<td>(0.0019)</td>
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<tr>
<td>Number of groups</td>
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<td>126,396</td>
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<td>65</td>
</tr>
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<td>Type &amp; year × quarter FEs</td>
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<td>0.0028***</td>
<td>0.0028***</td>
<td>0.0028**</td>
<td>0.0028</td>
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<tr>
<td></td>
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<td>(0.0007)</td>
<td>(0.0010)</td>
<td>(0.0015)</td>
<td>(0.0019)</td>
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<tr>
<td>Number of groups</td>
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<td>1,899,779</td>
<td>126,396</td>
<td>249</td>
<td>65</td>
</tr>
<tr>
<td>Type &amp; year &amp; month FEs</td>
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<td>0.0037***</td>
<td>0.0037***</td>
<td>0.0037***</td>
<td>0.0037***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0006)</td>
<td>(0.0009)</td>
<td>(0.0014)</td>
<td>(0.0017)</td>
</tr>
<tr>
<td>Number of groups</td>
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<td>126,396</td>
<td>249</td>
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<tr>
<td>Type &amp; year × month FEs</td>
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<td>0.0037***</td>
<td>0.0037***</td>
<td>0.0037***</td>
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<td>(0.0006)</td>
<td>(0.0009)</td>
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<td>Number of groups</td>
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<td>126,396</td>
<td>249</td>
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<td>No</td>
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Notes: The dependent variable is the logarithm of the number of sales. The independent variable of interest is FIX. FEs and SE stand for fixed effects and standard error, respectively. All columns report the results of OLS regressions. Cluster-robust standard error I is clustered at the level of the interacted group between two-way fixed effects of apartment unit type and time in each panel. Cluster-robust standard error II is clustered at the level of apartment unit type (126,396 groups). Cluster-robust standard error III is clustered at the town level (249 groups). Cluster-robust standard error IV is clustered at the city level (65 groups). Standard errors are in parentheses. *p ≤ 0.1, **p ≤ 0.05, and ***p ≤ 0.01.
Full results from the regression model with 1% clustering

**Table C.2:** Full results from the regression model with 1% clustering

<table>
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<td>0.0023***</td>
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<td>(0.0005)</td>
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<td>(0.0006)</td>
<td>(0.0005)</td>
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<td>0.0002***</td>
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<td>1.79e-06***</td>
<td>1.79e-06***</td>
<td>1.79e-06***</td>
<td>1.79e-06***</td>
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<td></td>
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<td>(6.40e-08)</td>
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<td>-0.0258***</td>
<td>-0.0258***</td>
<td>-0.0258***</td>
<td>-0.0258***</td>
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<td>0.0282***</td>
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<td>-0.0001**</td>
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|                  | Cluster-robust SE I | No | Yes | No | No |
| Cluster-robust SE II | No | No | Yes | No | No |
| Cluster-robust SE III | No | No | No | Yes | No |
| Cluster-robust SE IV | No | No | No | No | Yes |

| N               | 4,585,859 |

Notes: The dependent variable is the logarithm of the number of sales. The independent variable of interest is *FIX*. Avg, CV, SD, and SE stand for average, the coefficient of variation, standard deviation, and standard error, respectively. All columns report the results of OLS regressions. Cluster-robust standard error I is clustered at the fixed effect group level (372,691 groups). Cluster-robust standard error II is clustered at the level of apartment unit type (126,396 groups). Cluster-robust standard error III is clustered at the town level (249 groups). Cluster-robust standard error IV is clustered at the city level (65 groups). Standard errors are in parentheses. *p ≤ 0.1, **p ≤ 0.05, and ***p ≤ 0.01.
Full results from the regression model with 5% clustering

Table C.3: Full results from the regression model with 5% clustering

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Notes: The dependent variable is the logarithm of the number of sales. The independent variable of interest is FIX. Avg, CV, SD, and SE stand for average, the coefficient of variation, standard deviation, and standard error, respectively. All columns report the results of OLS regressions. Cluster-robust standard error I is clustered at the fixed effect group level (800,631 groups). Cluster-robust standard error II is clustered at the level of apartment unit type (126,396 groups). Cluster-robust standard error III is clustered at the town level (249 groups). Cluster-robust standard error IV is clustered at the city level (65 groups). Standard errors are in parentheses. *p ≤ 0.1, **p ≤ 0.05, and ***p ≤ 0.01.
References


