Evaluation of the Catherine Stern concrete materials as used in a sixth-grade remedial program

Larkin, John William Jr

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Boston University
Boston University
School of Education

Thesis

Evaluation of the Catherine Stern Concrete Materials
As Used in a Sixth-Grade Remedial Program

Submitted by

John William Larkin Jr.
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First Reader: Robert L. Burch, Assistant Professor of Education

Second Reader: Donald D. Durrell, Professor of Education
# TABLE OF CONTENTS

## CHAPTER

<table>
<thead>
<tr>
<th>LIST OF TABLES.</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>I  THE PROBLEM.</td>
<td>1</td>
</tr>
<tr>
<td>II REVIEW OF RESEARCH AND LITERATURE.</td>
<td>4</td>
</tr>
<tr>
<td>Remedial</td>
<td>8</td>
</tr>
<tr>
<td>Implications for this Study</td>
<td>12</td>
</tr>
<tr>
<td>The Place of the Concrete Illustration</td>
<td>13</td>
</tr>
<tr>
<td>Concrete</td>
<td>13</td>
</tr>
<tr>
<td>Concrete and Systematic</td>
<td>14</td>
</tr>
<tr>
<td>Concrete and Social</td>
<td>16</td>
</tr>
<tr>
<td>Social</td>
<td>18</td>
</tr>
<tr>
<td>Social and Concrete</td>
<td>19</td>
</tr>
<tr>
<td>Social and Systematic</td>
<td>21</td>
</tr>
<tr>
<td>Systematic</td>
<td>22</td>
</tr>
<tr>
<td>Systematic and Social</td>
<td>23</td>
</tr>
<tr>
<td>Systematic and Concrete</td>
<td>25</td>
</tr>
<tr>
<td>Implications for this Study</td>
<td>30</td>
</tr>
<tr>
<td>III PROCEDURES FOR THE STUDY</td>
<td>33</td>
</tr>
<tr>
<td>Construction and Description of the Materials.</td>
<td>33</td>
</tr>
<tr>
<td>Construction and Administration of the Concept Test.</td>
<td>34</td>
</tr>
<tr>
<td>Selection of the Pupils for the Remedial Group</td>
<td>46</td>
</tr>
<tr>
<td>Orientation of the Pupils to the Remedial Program.</td>
<td>49</td>
</tr>
<tr>
<td>Presentation of the Materials.</td>
<td>52</td>
</tr>
<tr>
<td>Post-Program Testing</td>
<td>58</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>IV ANALYSIS OF THE RESULTS.</td>
<td>62</td>
</tr>
<tr>
<td>The Concept Test as a Measuring Device</td>
<td>63</td>
</tr>
<tr>
<td>Selection of the Pupils for the Remedial Group</td>
<td>66</td>
</tr>
<tr>
<td>I. Q. Scores.</td>
<td>66</td>
</tr>
<tr>
<td>Computation, Reasoning, and Average Arithmetic.</td>
<td>70</td>
</tr>
<tr>
<td>Concept Test.</td>
<td>72</td>
</tr>
<tr>
<td>The Teacher's Evaluation.</td>
<td>73</td>
</tr>
<tr>
<td>Findings from the Orientation Periods.</td>
<td>75</td>
</tr>
<tr>
<td>Case Number One</td>
<td>76</td>
</tr>
<tr>
<td>Case Number Two</td>
<td>77</td>
</tr>
<tr>
<td>Case Number Three</td>
<td>79</td>
</tr>
<tr>
<td>Case Number Four.</td>
<td>80</td>
</tr>
<tr>
<td>Case Number Five.</td>
<td>81</td>
</tr>
<tr>
<td>Summary</td>
<td>82</td>
</tr>
<tr>
<td>Findings from the Remedial Sessions.</td>
<td>83</td>
</tr>
<tr>
<td>Evidence of Understanding</td>
<td>85</td>
</tr>
<tr>
<td>Evidence of Confusion</td>
<td>87</td>
</tr>
<tr>
<td>Analysis of the Results of the Post-Program Testing.</td>
<td>91</td>
</tr>
<tr>
<td>Individual Gain</td>
<td>92</td>
</tr>
<tr>
<td>The Group and the Class</td>
<td>97</td>
</tr>
<tr>
<td>V SUMMARY AND CONCLUSIONS.</td>
<td>101</td>
</tr>
<tr>
<td>Conclusions Related to Further Study of Concrete Materials</td>
<td>101</td>
</tr>
<tr>
<td>Conclusions Related to the Materials and the Remedial Work</td>
<td>102</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>Implications for Further Study.</td>
<td>103</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>105</td>
</tr>
<tr>
<td>APPENDICES:</td>
<td></td>
</tr>
<tr>
<td>A. Illustrations of the Materials.</td>
<td>108</td>
</tr>
<tr>
<td>B. Age, Scores, and Gain for All Pupils in the Class on All Tests.</td>
<td>113</td>
</tr>
<tr>
<td>C. The Concept Test.</td>
<td>115</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ADMINISTRATION TIME, CENTRAL TENDENCY, VARIABILITY, RELIABILITY, AND SOME CORRELATION COEFFICIENTS FOR THE TWO ADMINISTRATIONS OF THE CONCEPT TEST</td>
<td>64</td>
</tr>
<tr>
<td>2. MEASURES FOR THE THREE PUPILS WHO DID NOT MEET THE I. Q. CRITERION</td>
<td>67</td>
</tr>
<tr>
<td>3. COMPARISON OF THE FIVE REMEDIAL PUPILS WITH THE CLASS AVERAGE ON THE BASIS OF THE I. Q. CRITERION</td>
<td>69</td>
</tr>
<tr>
<td>4. SCORES FOR THE SEVEN PUPILS WHO MET THE COMPUTATION, REASONING, AND AVERAGE ARITHMETIC CRITERION, AND THE LOWEST QUARTILE SCORE FOR THE CLASS</td>
<td>71</td>
</tr>
<tr>
<td>5. SCORES FOR THE SEVEN PUPILS WHO MET THE CONCEPT CRITERION, AND THE LOWEST QUARTILE SCORE FOR THE CLASS</td>
<td>72</td>
</tr>
<tr>
<td>6. SUMMARY OF MEASURES FOR THE SEVEN PUPILS WHO MET THE THREE TEST CRITERIA</td>
<td>74</td>
</tr>
<tr>
<td>7. SCORES ON THE PRELIMINARY AND FINAL TESTS, SHOWING GAIN MADE, FOR EACH OF THE FIVE REMEDIAL PUPILS AND THE CLASS</td>
<td>93</td>
</tr>
<tr>
<td>8. MEAN SCORES AND GAIN FOR THE REMEDIAL GROUP AND THE CLASS</td>
<td>98</td>
</tr>
</tbody>
</table>
Chapter I

THE PROBLEM

The purpose of this study was to find whether the concrete materials designed by Dr. Catherine Stern for pre-school and primary grade arithmetic might not also serve as a successful approach to remedial work in arithmetic at the sixth-grade level. The materials have been designed to teach the four fundamental whole-number and fractional processes to the child from four to seven years of age. In the present investigation, these materials have been presented to five sixth grade pupils, in the manner that is prescribed by Dr. Stern in her recent book, *Children Discover Arithmetic.*¹ In other words, the teaching technique has remained unchanged while the learning situation has been altered. Five pupils were used because that number represented the optimum number that could be handled under the restrictions imposed by the supply of concrete materials.

It will become obvious that the method of solution chosen for the problem has severe limitations. The conclusions obtained will, of necessity, encompass a scope only so large as the remedial group. The efficacy of the concrete approach in general will remain in doubt. This would be serious were it not for the fact that justification is to be considered in terms of this work's being an orientation study. An orientation study,

in this case, is considered to be one that attempts to discover the problems which must be overcome before scientific research can be done in areas where concrete materials are used. A better understanding of this kind of justification will be made possible through the reading of Tobias Dantzig's words:

The history of mathematics...shows that the progress of mathematics has been most erratic, and that intuition has played a predominant role in it. Distant outposts were acquired before the intermediate territory had been explored. It was the function of intuition to create new forms; it was the acknowledged right of logic to accept or reject these forms, in whose birth it had no part. But the decisions of the judge were slow in coming, and in the meantime the children had to live, so while waiting for logic to sanctify their existence, they thrrove and multiplied.°

That mathematics, so rigorous as to be called "The Handmaiden of Science," should permit intuition may startle some. The quotation was chosen for that reason. The fears of some that research cannot be too strict must be allayed. It is true that procedures must be made as valid as possible. Logic is needed; systems must be built and maintained, but, guideposts are needed as much in the field of research as in the theory of number. This study is intended to be a guidepost. It is intended to indicate the research problems fostered in situations involving concrete presentation; not to solve those problems. An appreciation for this need of orientation in the area of research concerned with the use of concrete materials may be gained through the reading of Chapter II. The Review of Research and Literature, in which philosophy, as opposed to true research, is predominant. Strange as it may seem, the paucity of real research into

areas where concrete materials are employed justifies the existence of one more work which will not materially affect that paucity.

The Catherine Stern concrete materials were chosen for study for the reason that they satisfy the latest and most authoritative criteria for an arithmetic teaching technique. These standards include: 1) A mathematically systematic approach, 2) A concrete basis, 3) Meaningfulness, and, 4) Fulfillment of child development principles. All four points are admitted into the learning situation through the use of the Stern materials. The same cannot be said about any other known concrete approach. Most of them fail on point 1. Stern phrases the issue nicely when she says:

It has always been the job of the educator to put abstract number relations into a concrete form which is adapted to the child's interest and his mental capacities. But while we adjust our teaching to fit the inner nature of the child, we must do so without damaging the inner nature of mathematics.¹

Since the Stern materials have been designed to fulfill most of the criteria set up by the authorities in arithmetic, they should be evaluated as to their validity and use at higher educational levels than those for which they were built. At the same time, a good opportunity occurs to discover what research problems exist in experimental work which attempts to study the nature of learning that is based on the use of concrete materials.

¹ Stern, op. cit., p. 3.
Chapter II

REVIEW OF RESEARCH AND LITERATURE

In the course of the solution of this thesis, two main factors will have to be considered. The first factor is related to the nature of the study and has to do with remedial teaching. The second is related to the nature of the materials being studied and is concerned with the problems arising because of the use of the concrete approach.

One particular phase of the concrete approach is especially pertinent to this study. That is the "theory of measurement." This theory suggests, as its name implies, that number is used to measure quantity. Just as it is incorrect to add 3 feet to 18 inches without first transforming to a common unit, so is it incorrect to add 3 horses to 18 men without finding a common unit. In the latter case, some such unit as "animate object" would have to be used. Since the number system is always used to quantify homogeneous objects, any concrete representation of it will have to show this fact by being based on a standard unit. Some people have tried to represent the number system with non-standardized concrete objects, such as oranges, pebbles, buttons, and the like, but 3 oranges can stand for three oranges only; not for the number 3. If, however, as with the Catherine Stern materials, a standard unit of measurement is adopted and used, it is possible to say that a certain block represents the number 3 if it is exactly 3 times the length of the standard unit. Since the theory
of measurement is basic to every proper concretization of the number system, it has been assumed to be an aspect of the larger issue, the use of concrete materials, and will, therefore, receive due attention in the review of literature to take place.

Since the remedial and concrete approaches to arithmetic are so intimately related to the problem being undertaken, any previous research that has been done with them will be of importance to this work. One general observation may be made immediately, however. Almost no experimental research has been done involving the use of concrete materials in arithmetic. This is serious in view of the fact that all competent writers in the field have stressed the use of the concrete as a starting point for every teaching or learning situation.

The limitations imposed by the observation just made are important. Although much research having to do with the remedial teaching of arithmetic has been completed, the results obtained will not be directly significant to this study since the concrete approach has played no part. For that reason, the review of research that takes up remedial teaching in this chapter will be brief. Moreover, for the second factor—the concrete approach—the review of necessity will be confined to literature rather than research. This means that in place of a sound basis of scientific research, there will be a more theoretical foundation upon which to build the study.

The organization for the review of literature pertaining to the concrete approach has been couched in terms that might be called historical and philosophical. The attempt has been to indicate the emphasis which
has been placed on the use of concrete materials in the development of the many methodologies of teaching arithmetic.

To indicate where the emphasis must be placed in the writings of the various authors at least three main divisions are possible. They are: 1) Concrete, 2) Social, and 3) Systematic. Used in combination, nine separate categories emerge, with a possible tenth which is not yet applicable to any author. These ten categories are:

1. Concrete
2. Concrete and systematic
3. Concrete and Social
4. Social
5. Social and concrete
6. Social and systematic
7. Systematic
8. Systematic and social
9. Systematic and concrete
10. Concrete, systematic and social

In the interpretation of these nine categories, three points must be made: The first is that degree of emphasis is to be the criterion for placing an author in a category. For this reason, category 2) is not synonymous with category 9). Although both categories suggest that a synthesis of the two factors concrete and systematic has taken place, the synthesis is never complete. Therefore, an author shall be placed in category 1) when he emphasizes the concrete to a greater degree than the systematic, and he shall be placed in category 9) when the reverse is true.

The second point to be made cannot be stressed enough. These categories are arbitrary, non-rigid, and subjective. They are organizing generalizations whose function is to bring trends of methodology into relief. They must not be thought of as definitely placing an author in one pattern or another, nor must they be thought of as telling the whole story about the
content of his writings. Much overlapping of category boundaries takes place, and many points of importance are left out of this organizational scheme.

The last of the three points is that the three main divisions (concrete, social and systematic) are meant to be equally important aspects of larger philosophies. Such is not strictly the case. The aspect labeled "Social" is for the most part used as being synonymous with functionalism, which is close to being a full-pledged philosophy in its own right. Thus for example, one can believe in transfer of learning and use a concrete approach to learning, but one cannot readily believe in transfer of learning and functionalism simultaneously. Moreover, insofar as there is a time-lag between theory and practice, the historical development outlined below will not follow the actual practices that have gone on in the schools for the same period of time.

Whatever the difficulties may be in the form of organization described, it has proved to be of great value in analyzing the nature and proper place of the concrete approach to the teaching of arithmetic. Therefore, the review of literature will be based upon the following outline:

Remedial
   Implications for this study
The Place of the concrete
   Concrete
   Concrete and systematic
   Concrete and social
   Social
   Social and concrete
   Social and systematic
   Systematic
   Systematic and social
   Systematic and concrete
   Implications for this study
Remedial

Early remedial work in arithmetic was concerned with the diagnosis of computational difficulties and subsequent drill on the skill showing deficiency. For that matter, with few exceptions, the previous sentence characterizes present day remedial teaching, even under the influence of the enlightened "meaning" theory of arithmetic. It is not surprising, either, that less remedial work has been reported within the last ten years than the ten years previous. The remedial teaching of arithmetical concepts is much more difficult, less direct, and more variable than the remedial teaching of skills. Nor is it surprising that the greatest amount of research has been done in the skills area. There tends to be a greater correlation between the amount of research done in an area and the ease with which that area succumbs to experimental research than there is between amount of research done in an area and the importance of that area to the educational philosophy in favor at the time. The obvious, yet discouraging, suggestion is that research can be a sedative rather than a stimulant to rapid progress in the field of education.

In 1927, Otto reported a work which summarized the outcomes for a study he had done in remedial teaching. The standard remedial procedure was employed. There was diagnosis, twelve fifteen-minute periods of instruction, and subsequent testing. One hundred percent mastery was the aim. Flash cards and other drill techniques were employed to overcome specific computational difficulties. Five outcomes were reported and are here summarized.

1. Gain in computational skill was manifested.
2. Some permanence in learning was discovered.
3. Loss of initial gain resulted for some pupils.
4. A more favorable attitude toward arithmetic resulted.
5. The remedial procedure showed potentialities.

Except in a few cases, these outcomes, with some modifications and elaborations, are applicable to the studies preceding and succeeding Otto's research.

Two years later, Brownell,¹ in a preface to the presentation of four remedial case histories by Gabbert, Evans, Trousdale, and Whitson, suggested one major change in Otto's technique. Instead of working with deficient skills, new habits and methods of attack were to receive attention. The Buswell–John Diagnostic Test was the key to the discovery of faulty pupil techniques. Since this test involves a rational approach to arithmetic, the method of remediation was also necessarily rational. Significant gains were made in computational arithmetic, but the investigators noted that the results could probably have been improved through the use of drill near the end of the program to bring the new habits and methods of attack to the skills level.

In 1931, Cooke reported a study² in which he followed Brownell's method of treatment rather closely, except that he used many more diagnostic tests, fifteen in all, including the Buswell–John test. Certainly a case may be made for the belief that the testing movement played a large part in the popularization of diagnostic and remedial research. Cooke added to the list of outcomes already given above by observing that:

1. More achievement resulted with individual instruction alone than with individual instruction and classroom instruction together.
2. Lower I.Q. pupils needed more time for equal increments in achievement.
3. Little gain was made in areas where serious difficulties were encountered.
4. Improvement was possible, but grade norms could not always be achieved.

The importance of Cooke's work, however, lies in the fact that he published a follow-up study one year later. In this article he commented on the permanence and efficacy of remediation. He discovered that:

1. The order of permanence of elimination of process difficulties was division, subtraction, multiplication, and addition.
2. Seventy-one percent of the initial difficulties had been eliminated.
3. More new difficulties arose than were eliminated.

The conclusion drawn was that the program in remedial work must be a continuous one.

In 1942, Bemis and Trow did a follow-up study of remedial teaching in arithmetic which allowed a two-year time span to intervene. The initial learning situation was not discussed. Twelve pupils were paired for I.Q., age, and retardation. One group was given remedial work while the other was retained as a control group. The only conclusion drawn was that remedial instruction helps some but not others to achieve normal progress and to maintain it. They noticed, however, that the control group showed variations in progress and so suggested that better group measuring devices were necessary and that arithmetic progress is a function of a developmental factor.

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Fernald offers four reasons for failure in arithmetic: 1) Mental deficiency, 2) Reading disability, 3) Lack of adequate number concepts, and, 4) Blocking by emotional response and habits. She prescribes remedial techniques for each of the four areas. To the knowledge of this writer, Fernald is the only author who stresses the teaching of concepts in the remedial situation. Moreover, she has developed concrete materials to facilitate this teaching. It must be said, however, that her definition of the term concept is restricted to problem-type exercises. She uses concrete materials, such as beans, money, blocks, and fractions cut from pieces of paper, to bring out the mathematical relationships inherent in a problem. A further point of interest is that Fernald believes the use of concrete materials will aid in correcting faulty habits and responses which are based on emotional disturbances. Her experiments are of the case-history type and excellent results were reported.

McLatchy used a genetic-type study to show that pupils travel through the grades in groups based on their initial arithmetic ability in the early grades, rather than in groups based on their I.Q.'s. She believes that the phenomenon may be explained in terms of the pupil's pre-school lack of number readiness and the failure on the part of previous teachers to recognize basic disabilities. Her remedial technique utilizes markers based on tens which are supposed to impart understanding to the pupils. Like Fernald, her reports are of case histories.


The short historical sketch just presented shows that remedial teaching has traveled a path that started with one hundred percent mastery of skills through drill and has arrived at a point where concepts and understandings are stressed through the use of concrete materials.

Implications for this Study

In connection with this study, some of the points brought out are more important than others. It may be expected, however, that:

1. At least a more favorable attitude toward arithmetic will result with remedial teaching.
2. Where computational skill is stressed, permanence of learning will be less stable.
3. Where there are serious difficulties, little gain will result.
4. Improvement may take place, but grade norms may not be reached.
5. New difficulties will accompany the dispersal of old difficulties.
6. I.Q. scores will not necessarily be highly correlated to arithmetic achievement.
7. A developmental factor will be in operation.
8. Concrete materials should aid the learning of concepts.
9. The case history-type experiment will prove to be the most valid research procedure.
10. The remedial teaching of concepts will be more difficult than the remedial teaching of skills.
11. Faulty habits and responses based on emotional factors may be eliminated.

The following implications for this study could not be taken advantage of because of administration difficulties peculiar to the nature and locale of the study itself:

1. There is more achievement with individual instruction than both individual instruction and class room instruction.
2. Lower I.Q. pupils need more time for learning than higher I.Q. pupils.
3. The remedial program should be a continuous one.
4. Better group tests should be developed for the measurement of arithmetic achievement.
The Place of the Concrete Illustration

Concrete

Perhaps it was Rousseau (1712-1778) who was the early agitator for a more concrete approach to learning. At least, it is known that Pestalozzi (1746-1827) was influenced by his writings and that, as an educator, Pestalozzi stood behind a doctrine based on the importance of "sense impression." He defined sense impression simply as "nothing but the presence of the external object before the senses which rouses a consciousness of the impression made by it."\(^1\) His approach was almost completely through the use of concrete materials.

Even today, Pestalozzi's comments on the learning of arithmetic are well taken, some of them coinciding exactly with much of the modern thinking that is current in the field of arithmetic. The following remark, for example, compares favorably with the modern theory which asks that a meaningful concept be attached to the referent for which each number symbol stands.

"Number in itself, without a foundation of sense impression, is a delusive phantom of an idea which our imagination certainly holds in a dreary fashion, but which our reason cannot grasp firmly as a truth. The child must learn to know rightly the inner nature of every form in which the relations of number may appear, before he is in a position to comprehend one of these forms as the foundation of a clear consciousness of a few or many."\(^2\)

Pestalozzi attempted to teach the concept of "few or manyness" through the use of concrete objects intended to develop the senses. To that extent he belonged to a school of educators who believed in the theory of "facul-

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\(^2\) Ibid., p. 69.
ties." For much of the time after Dewey, the same concept, when it was taught, was developed through the functional problems a child might meet. By no means does this mean that concrete objects were not used. However, Stern believes that the meanings lying behind number symbols must be discovered through the use of objects which truly depict the system upon which the science of number is built.

Froebel (1787-1852) added little more than refinement to the sense-impression doctrine as set forth by Pestalozzi. He designed certain concrete objects, called "gifts," which were to develop the senses in a systematic way. It must be noted, however, that his method was not systematic with respect to the relationships that are a part of the mathematical design. It most certainly can be said, however, that he emphasized a concrete approach to learning.

Concrete and Systematic

The next phase in this historical development most nearly corresponds to the position taken by Catherine Stern. She has been classified under "systematic and concrete," since her greatest stress is placed on the importance of keeping the number system intact, although she has ingeniously devised to do so through the use of concrete materials.

The category now under discussion is best exemplified by the writings of Seguin (1812-1889) and Montessori (1870- ). The former was a physician who specialized in the treatment of mental diseases, concentrating much of his attention on idiots. He is mentioned here for the reason that Montessori utilized many of his methods and much of his equipment in developing her
"Montessori Method" for the teaching of both the mental defective and the normal child.

There is no doubt but what some of the inspiration for the Stern materials sprang from the Montessori materials. Chapter III will describe in greater detail the constitution of the Stern materials. For the purposes of this chapter, the following comparisons sufficiently indicate some of the areas common to both systems:

<table>
<thead>
<tr>
<th>Montessori Materials</th>
<th>Stern Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Different length rods for each number up to ten.</td>
<td>1. Different length, marked blocks for each number up to ten.</td>
</tr>
<tr>
<td>2. Two trays—one for number rods up to four, one for number rods up to ten.</td>
<td>2. Separate trays for each of the number blocks from one to ten.</td>
</tr>
<tr>
<td>3. Semi-concrete number patterns based on a five by two rectangle.</td>
<td>3. Pattern boards based on a five by two rectangle.</td>
</tr>
</tbody>
</table>

The important point to be noted is that an unusual amount of similarity between the two systems does exist. Only because Stern has insisted that the measurement aspect of number be emphasized has she managed to refrain from propounding what would otherwise be a repetition of the Montessori method. The measurement aspect of Stern's doctrine has, itself, forced her to advocate a mathematically systematic approach to the teaching of arithmetic. Montessori on the other hand, probably because of her physician's interest in the mentally defective, was more interested in the

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concrete approach. If her materials show some mathematical consistency, it came about more through expediency than through prime concern with a systematic approach.

Whatever the answer to the question of how much influence Montessori has had for Stern, it may be safely said that Stern is much closer, philosophically, to Rousseau than to Dewey. The meaning and importance to education of the cycle thus implied, certainly deserves more attention. Profit might also be gained from a study of those methods of teaching which have been based on an interest in mentally deficient children. Many of them have met with great success when used with normal children. What is the answer? Have teachers tended to be too abstract, too demanding, too obscure in the teaching of young children, so that the application of a method that must be rid of these faults shows important academic gains?

Concrete and Social

Dewey (1859– ) was the most important factor in swinging education away from the "faculty-sense impression" line that had started with Rousseau. It was he who started the trend toward functionalism, or what is here called the social aspect of learning. To some it may seem inconsistent to place Dewey in the same category with the concrete approach. It is not at all. That Dewey believed in the use of concrete materials is made plain by the following quotation from a book written by him in conjunction with McLellan:

"It is then almost equally absurd to attempt to teach numerical ideas and processes without things, as to teach them simply by things. Numerical ideas can be normally acquired, and numerical operations fully mastered only by arrangements of things—that is, by certain acts of mental construction, which are aided, of
course, by acts of physical construction; it is not the mere perception of the things which gives us the idea, but the employing of the things in a constructive way."

Many modern writers have blamed the proponents of the Rousseau doctrine for the decrease in the use of concrete materials. It is clear, however, from what has been said above, that those who suggested the development of faculties meant it to be done through a gradual transition from the concrete to the abstract faculty. Strangely enough, Dewey's writings have probably done as much to destroy the concrete basis for learning as have the writings of Rousseau, although it is equally clear that Dewey did not mean it to be so. An extreme interpretation of Dewey led to the situation which existed in the late 30's and early 40's wherein the so-called progressive schools relied so heavily on the child's interests that they found it impossible to introduce concrete materials not asked for by the child himself. The same sort of reasoning made it impossible to teach any subject systematically since a child's interests were not apt to develop in the same way that a particular subject should be organized.

Beyond the functional idea that has been over-stressed for Dewey by many of his adherents, there is also another idea present in the quotation given above. It has to do with arithmetic and is perhaps better said by Shouse. "The number idea cannot be anything but abstract. It must be thought into the concrete situation and is not inherent in it."2 What he really maintains is that the number system cannot be made concrete. He

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clarifies this assertion somewhat when he goes on to say:

"The problem in arithmetic comes to be seen, then, not primarily as the development of this understanding of abstract number through a succession of concrete experiences, but more largely as the development of ability to apply number known to successive concrete situations."¹

There is little solace here for Stern who claims that her materials, being mathematically consistent, bring out number understandings. The present writer, however, is of the opinion that Shouse is so obscure as to the relation of concrete experiences and numerical understandings as to make his remark inconsequential. It must not be forgotten, however, that much of the interpretation of Dewey's position is based on the premise that meaning must be "thought into" number by means of an active, functional process. At the very least, success with the Stern materials would mean a restatement of the salient features of Dewey's stand.

Social

Very little will be said about the social aspect of learning, since this paper is primarily interested in the use of concrete materials in the teaching of arithmetic. The social viewpoint does not reject concrete materials, but through lack of attention, the place of the materials has been subordinated.

The three authors who have been placed in the category called social are more than likely misrepresented. Others of their writings would no doubt reclassify their position.

Clapp believes that "our arithmetic in elementary schools...needs to be developed on lines that one might call historical, scientific, theoreti-

This is a tall order. Her elaboration on the subject, however, reveals that she would like the approach to arithmetic to be historical in order to create the interest and resultant understanding that has been obtained in other subjects through a similar approach.

Renfrow\(^2\) offers the same reasoning for his assertion that arithmetic should be taught historically. He would also like number to be given a cultural meaning. Conant\(^3\) too, takes the attitude that science, which presumably includes mathematics, should be taught historically. He advances the theory for different reasons and for higher levels of education, but he represents rather well the position taken by those who would like to expand on the historical, social, cultural, and functional aspects of the sciences, while deemphasizing the logical rigidity and the stereotyped methods of these same sciences. The accomplishment of this aim would tend to make social sciences out of what are now considered sciences-proper, or the "tools" of the social sciences. The reader is warned that there is much more to the social approach than can be said here, especially in the area that takes up the practical teaching of the content which makes up the historical or social curriculum.

Social and Concrete

In 1924, a book\(^4\) was published which was designed to stimulate interest


in the learning of arithmetic. It consisted almost entirely of games that could be played while learning number facts. There were also some suggestions for concrete objects which could be manipulated by the pupils, as well as a section on sense training. He was not explicit as to his educational philosophy, but there were indications that he differed from Dewey only in that he believed social activities, such as games, would create meaning, where Dewey wanted the activities to be both social and constructive.

Voorhees followed along with Dewey but gave stress to a point that had been largely overlooked in the many attempts to make learning functional. In reporting on the arithmetic program in a specific school, she said:

"Here we find that programs based on activities and experience necessarily follow two lines of development. The first, through actual use satisfies the demands that arithmetic shall meet an immediate and vital need in the child's life. The second, through imagination and applied use gives recognized opportunity for further discovery and expression."1

The "imagination and applied use" of which she speaks was the practice of introducing concrete materials which would interest the child yet carry him forward in arithmetic. One such object, for example, was a puzzle which when fitted together properly rendered a picture on one side and a correct number fact on the other. Although it seems necessary to find a way to allow Dewey's philosophy to permit progress, or, to put it another way, to permit children to expand their activities into important subject matter areas, it does not seem realistic, nor logically, sound to do so by the use of the kinds of subversive tactics employed by Voorhees.

Steiss and Baxter, in selecting number readiness as an important factor for very young children, manage to adhere to the social line, but to add the concrete approach by insisting that the readiness can come only through the manipulation of concrete objects. Their procedure is to have the children measure earth worms, count Eucalyptus buds, and the like.

Social and Systematic

Drummond, like Steiss and Baxter, picks number readiness as an all-important aspect of learning arithmetic. He, however, maintains that the readiness should come from social experiences; that the concrete is less valuable for developing number concepts and that, in fact, a little rote drill is not harmful. It will be seen from his thoughts on the nature of the number system, why his position is necessary.

"At the very beginning the child has to leave the world of concrete fascinating realities and concentrate on an abstraction, on a creation of the human intellect." 2

Since the number system is a creation of man's mind, and not a child's mind, at that, concretization of it is impossible, says Drummond. He thinks that all that can be done is to give the child a feeling for number, as he is capable of knowing it, through social experiences. At that point he introduces the number system as a system and continues to develop it as such with rote drill.

Thiele 3 does not deviate too far from Drummond's stand. He asks how


meaningful arithmetic can be taught now that it has been established. His own answer to the question is that meaning must be "put into number" via the McLellan-Dewey technique. The word meaning is used by Thiele to denote the relationships or concepts that are a part of the number system, and hence, can be thought of as implying a systematic approach to the teaching of arithmetic. Thiele comes very close to advocating that all three learning factors—concrete, social, and systematic—be included in the teaching of arithmetic. The mere recommendation that the McLellan-Dewey mixture of concrete materials and constructive social experiences be applied to the teaching of a systematic, or meaningful, arithmetic does not tell how it is to be done without destroying either the functional premise or the systematic premise.

**Systematic**

Judd, as far back as 1927, began to emphasize the importance of the number system as a set of abstract relationships united by unyielding, but arbitrarily founded rules. His is the typical systematic approach.

"The procedure of guiding the child to the complete understanding of number will be successful only when there is an intelligent analysis of the number system on the one hand and on the other hand an equally intelligent consideration of the child's modes of thinking and possibilities of development in the mastery of abstractions." ¹

Judd was also interested in the psychology of concrete learning, but after performing a group of interesting experiments devised to gain further knowledge about the nature of such learning, he apparently drew the conclusion that so little was known about the process as to limit a conviction in favor of the use of concrete materials. He had individuals do different

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tapping exercises and found great variation in individual capacities. His conclusion was not actually that some forms of the concrete were more difficult that others, but more that some applications of concrete approaches to learning were more difficult than other. Here is an instance of some real experimental work with the concrete. The limitations of the conclusion drawn are indicative of the immense amount of work that must be done in the area.

More recently, Wheat has phrased the issue of the systematic approach to arithmetic in quite concise terms.

"Arithmetic is a system of ideas. It is not a collection of objects. It is not a set of signs. It is not a series of physical activities. Arithmetic is a system of ideas...Arithmetic exists and grows only in the mind. Being a system, arithmetic must be taught as a system." 1

Even though a systematic program is not the full story to the problems of arithmetic methodology, it is a question which deserves the consideration it is now receiving. Somebody must solve the difficult synthesis of the systematic with the social and concrete facets of learning.

**Systematic and Social**

The area now to be discussed might easily be called the modern approach to the teaching of arithmetic. A great deal has been written by recent authorities, in consequence of which only a little will be said here. This category contains those who have attempted to combine the meaning theory with the functional theory, or, as it has been called here, the systematic with the social. Frequently, the concrete has been mentioned

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as being important; but the tone makes one feel that it has been mentioned for the sake of completeness rather than because of a strong conviction in its importance.

Buckingham is probably responsible for the introduction of the terms meaning and significance which have done so much to clarify the issue. He uses meaning to refer to the interrelationships within the number system, and significance to refer to the meaning which is gained through the constructive social use of quantitative thinking. The delimitation of the word meaning has been an important contribution to the field of arithmetic even though much more must be done before a concise definition will emerge.

Morton, for the National Council Committee on Arithmetic was quick to stand behind the synthesis of the two factors, meaning and significance.

"The committee stands for a kind of arithmetic in which both the mathematical and the social aims are clearly recognized as interdependent and mutually related."2

Bond amplified Morton's statement showing a little more of the two factors which were to be included as arithmetic objectives, but added not a thing more to what had already been said.

"There are then two very different but mutually helpful possible objectives for teachers of arithmetic. The one has for its aim the development by the pupil of a unified science of arithmetic... The other looks towards the uses of arithmetic in satisfying the quantitative needs of the social and economic life of our people... They are both needed. Indeed they are both necessary adjuncts of a power to use number to advantage."3

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Buswell, like Thiele, almost contrived to mention all three learning factors, concrete, social, and systematic, but, again, like Thiele, he did not produce a workable concrete program which would integrate well with the social and systematic factors.

"Arithmetic is based on a completely related and integrated number system. Its parts are interdependent. There is a sequence in its learnings proceeding from simple to complex; from immediate concretes to widely applicable abstractions. Some parts of arithmetic will not be related to the immediate needs of all children or even to their community. Some parts are essential to future needs."

Systematic and Concrete

The last stage of the historical development that has been presented considers those writers who have emphasized a systematic study of the number system, but who have also devised concrete materials to aid the teaching of it.

Spitzer, in an attempt to teach the tens idea, constructed a "ten block," with which he uses single blocks to represent the numbers from one through nine. His contention is that the use of bundles of rods or cards that can be broken down destroys the grouping-on-tens concept. He offers a warning about the use of concrete materials which seems to show with what struggle and difficulty the combination of the concrete and systematic with the social theory has been made.

"Mere use of concrete materials such as blocks, sticks, and marbles for early counting, adding, and subtracting exercises will not of themselves, however, promote understanding of some phases of our number system. In fact, unless certain precautions are taken, extensive use of objects in connection with counting may be a block in the way of the development of

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significant aspects of number."¹

Grosenickle, Metzner, and Wade have developed materials which are claimed to be organized around the number system. They say of their materials:

"They present experiences to pupils in an organized form. An organized subject matter such as arithmetic is demands organized experiences for its efficient study and understanding."²

From an observation of these materials as used by several elementary teachers, it seems valid to assert that either they are not completely organized in terms of the number system, or that the teachers do not know their number system. If the latter is true, it may mean that it truly takes more than a set of concrete materials to completely represent the continuity of arithmetical relationships.

Stern speaks up in her own behalf by repeating that the concrete approach is the only way for children to learn abstract concepts.

"It is certainly true that we have to make numbers concrete to make them accessible for the child. There is no other way for the child to acquire abstract concepts but to develop them by himself from first hand experience with concrete objects. However, not any concretization will do; number concepts can only emerge from a concrete representation that is true to the inner nature of numbers. Moreover, we must keep in mind that we are making numbers concrete to help the child recall number facts."³


² Foster E. Grosenickle, William Metzner and Francis A. Wade, Number as the Child Sees It, Point Pleasant, Bucks County, Pennsylvania, Clymer Sales Co., Inc., 1947, p. 3.

Because Stern makes the point that concretization only serves to help children recall number facts, the old problem that arose back with Dewey again comes into prominence. If concrete materials only help the child to remember what he already knows, where does the initial learning take place? Does it arise, as Dewey says, in a functional use of quantitative thinking? If so, the concrete materials act to give form and organization to what is already a part of the child's knowledge. Indeed, this is the interpretation that probably will prove to be most constructive, and which Stern herself would authorize. Her book\(^1\) is subtitled, "An Introduction to Structural Arithmetic," which itself implies the importance of the form that is given to number. Moreover, Stern acknowledges her indebtedness to Wertheimer, one of the most avid proponents for Gestalt psychology, which is, again, another indication of the fact that only organization is given to what the child already vaguely knows about number. For this writer, such a confession is strong proof of the need for giving children ample social opportunities to learn those vague notions which will later be crystallized into meaningful number concepts. A case may be made for the assumption that it is not the school's job to build these basic number notions. Some experimental work has pointed out that children enter school with a great deal of number knowledge. Perhaps the school should not attempt to do more than classify and organize whatever knowledge and feeling the child has gleaned by chance and inclination from his every-day experiences with number. Modern education, however, with its activity units and all, cannot afford to make such an assumption. For the Stern materials, then, strict limitations are met. They must be used as the second half of a learning situation, the first half

\(^{1}\) Children Discover Arithmetic, op. cit.
of which is the gaining of number notions, and the second half of which is the fixation of that already made a part of the pupil.

Limitations, though, are inevitable in any unified approach to arithmetic. Stern must receive more credit. Not only has she built an extensive set of concrete materials, with genius as to their creation, and meticulous care as to their mathematical consistency, but she has also attempted to put the materials to work, the better to test for their validity. The reader should bring to mind again the understanding that very little experimental work has been done with concrete materials. For that reason alone, Stern is to be further recommended for her work.

In general, the materials have been validated by the founding of a school in which the materials are the focal point for the teaching of arithmetic. In this way a genetic study is made possible. In her book Stern has presented some case-history type descriptions which she calls experiments. They are not adequate, but the scope of the book is not such as to include scientific validation of the materials. Stern has kept case history records for each pupil, and these, if published, would probably go far in establishing the efficacy of her approach. It is enlightening to observe that most comprehensive systems involving concrete materials have been put to work in schools specifically organized for that purpose. The Pestalozzi, Froebel, Montessori, and Stern schools are examples. It suggests that the genetic, experimental technique and the testing of concrete materials are closely related.

It would not be fair to leave this discussion without citing one of the known experiments that produces results contrary to what can be expected from the meaningful approach. McConnell did a study in which he prepared
two arithmetic learning books for the sixth grade, one based on meaningful arithmetic, and the other on traditional arithmetic. The traditional workbook, Method A, was given to one of two equated groups, and the workbook based on meaningful arithmetic, Method B, was given to the other. After time for instruction was allowed and testing had taken place, McConnell drew this conclusion:

"The extensive claims made by the configurationists for the superiority of the pedagogy of meaning and discovery over the pedagogy of mechanical repetition either are seriously exaggerated, or Method B sadly missed the main point of the Gestalt psychology of learning."¹

Based on his own method of measuring the results of learning, he then draws another rather startling conclusion:

"If the results of the meaningful method are so subtle that they cannot be measured in a reasonably forthright manner, the benefits of its use must be frankly left to the realm of subjective possibilities."²

The mb of McConnell's inability to discover an advantage for the meaningful, Gestalt approach to arithmetic over the traditional method is probably in his second quotation. The curious but close relation of the use of concrete materials to genetic studies has already been mentioned. Meaningful learning may need to extend over long periods of time and be measured in a new kind of learning unit. It is perhaps quite true that the traditional experimental techniques and measuring devices as conceived by McConnell are not useful when evaluating a meaningful approach to learning, but there are others that if not yet forthright, can be made to be so.


² Ibid., p. 144.
Moreover, it is more than possible that McConnell "sadly missed the point" of meaningful arithmetic. His Gestalt workbook was to be used with paper and pencil alone. The thesis here is that a concrete systematic approach is fundamental for the meaningful method.

Implications for this Study

The most significant discovery made from the review of research and literature just given has been that there is no research to speak of that deals with the use of concrete materials, and that what little there is uses a genetic study as an experimental vehicle. Two implications for this study are possible. The first is that, being handicapped by lack of research background, this study must become a pilot study whose function is to uncover experimental difficulties inherent in the evaluation of concrete materials, and, on that basis, to point the probable direction for future studies of its kind. The second is that a genetic study should be utilized in order to show whatever advantage systematic concrete materials really have over the traditional method of teaching arithmetic. The first implication has been incorporated as one of the purposes of this study, while the second could be applied only moderately. That is to say, only five pupils were used in this evaluation and that case-history type descriptions were used to indicate their progress, or lack of progress, based on the use of the Stern materials. The pupils could not be observed over a long period of time, which fact becomes a serious limitation to whatever conclusions are drawn.

Of lesser importance, but still significant, was the finding that no studies, and no arithmetic teaching methods, using a basic measurement system, were reported. This fact substantiates the statement that this paper must be a pilot study, not only for further work with concrete materials, but
also for further validation of teaching techniques based on the measurement aspect of arithmetic.

The review also implies, and McConnell states directly, that traditional group measuring methods seem inadequate for testing the results of the meaningful teaching of arithmetic. While this thesis has entailed the building of a concept test to measure the growth in arithmetical understandings made possible by the use of the Stern materials, absolutely no claim has been made that the test has solved the group measurement problem. It would be surprising if it had.

It has been observed that most concrete, systematic arithmetic teaching techniques have been studied genetically, and that children with no background in meaningful arithmetic show little or no progress when instructed with Gestalt figures and tested for growth with traditional measuring devices. Closely related to these two facts is the hypothesis that only children with history of meaningful arithmetic should be subjected to a remedial group such as the one being studied. It is not always easy, however, to find groups of pupils whose arithmetic background can be firmly established as bearing the stamp of a meaningful lineage. Moreover, the resources standing behind this work did not permit a wide search for such groups. Consequently, the group finally selected can be said to be of mixed arithmetic heritage—partly meaningful, and partly traditional. Limitations to the conclusions drawn from this study are, again, imposed.

If it can be agreed that the Stern materials function primarily to give form to number understandings already learned by the child in a vague way, as was suggested in the review of literature, then it seems fair to deduce that other things being equal, the older the child the more concepts he
will have which can be organized through the use of the materials. This assumes, of course, that concepts can be learned incidentally, even with traditional teaching. Following this line of reasoning, sixth-grade children have been used in the evaluation of the Stern materials, despite the fact that they were designed for younger children in the four to seven age bracket. Certainly, it does not seem necessary to dwell long on the hypothesis that the materials are not too difficult for sixth-grade pupils, whether below standard or not. If pre-school children can learn arithmetic with the materials, then sixth-grade children ought rightly to be expected to do the same. It will be seen later that the further expansion of the hypothesis that older children can more greatly benefit from the use of the materials has led to one of the conclusions for this study. That conclusion is that the materials may prove to be of most value in preparing teachers to teach meaningful arithmetic.

Only one more finding from the review of literature will be mentioned. It is in the form of a justification for the study. Certainly Stern's method of teaching arithmetic has enough theoretical and philosophical background to warrant its evaluation. The point is that the materials have not been constructed in a fanatical attempt to gain educational fame. They are a respectable contribution to the methodology of teaching arithmetic, and, as such, their proper place, use, and validity must be defined. This paper modestly hopes to be able to begin the work involved in such a definition.
Chapter III

PROCEDURES FOR THE STUDY

The procedure used in attacking the problem of evaluating the Catherine Stern materials may be thought of as involving seven general units of activity. One of these units—the analysis of the results of the experiment—will be considered in Chapter IV. A list of the remaining six which will be discussed in this chapter follows:

2. Construction of a concept test.
3. Selection of the pupils for the remedial group.
4. Orientation of the pupils to the remedial program.
5. Presentation of the materials.
6. Post-program testing and evaluation.

Construction and Description of the Materials

A letter inquiring about the availability of her materials was sent to Catherine Stern at the Castle School in New York. An excerpt from her reply, dated November 1, 1949, is reproduced forthwith.

"Arrangements have been made with Houghton Mifflin (2 Park Street, Boston, Mass.) for the issuing of all materials including workbooks. It is our present hope that the first set will be published shortly after the first of the new year."

As of the first of June of the following year, the blocks had not been made available through Houghton Mifflin, although some of the workbooks were being published. Indirect, oral communication with the publishing company made it obvious that construction of the materials would have to become part of the work of the thesis if the study were to come to a rapid
completion. Therefore, the materials were built according to plans and specifications that were pieced together from the text and illustrations of Stern's own book.

The Stern materials are based on the theory of measurement. The idea is that the concrete representation of the number 2 can be made to be twice as long as the representation for number 1. Of course, number 3 can be made three times as long as number 1, and so on, up to the number 10. It is block size in this case that makes concrete number symbolization possible. If a six block and a four block are laid end to end, it can be seen that the two together are as long as a single ten block. Moreover, a frame can be made that will just receive blocks of any size. If it is a frame for ten's blocks, all combinations of blocks that just fit the frame will be concrete representations of number facts that add up to ten. Counting is thus eliminated in the verification of number facts that have been demonstrated in concrete form.

It must be remembered, however, that where measurement takes place, an arbitrarily determined unit is implied. The pupils must become familiar with the unit of measurement before counting for verification can be eliminated. The chief aid to this familiarization is found in the use of color. The child comes to know that, in this case, the blue block is the longest block, and, consequently, stands for number ten. The same association is made between the remaining blocks and their characteristic color. In fact, color differentiation is not needed except to facilitate rapid recognition of the various blocks. Recognition by relative block size, or even, a printed number on each block, would not restrict the possible block manipulations. Failure to bring out some of the number system relationships is,
however, the main criticism of the latter arrangement. Since each number block is cut in such a way as to resemble separate single cubes, a ten’s frame filled with all the possible number facts that add up to ten looks like one hundred single cubes when color is not used. With color, it is quickly seen that there are really two sets of each of the number blocks from one to nine, and one number ten block. It is apparent, then, that color serves as an aid in identifying block size and in bringing out number relations.

What has been said so far indicates that a set of colored blocks related to each other by size is the core around which the complete set of materials is built (see picture, page 109). The remaining devices are simply various types of frames which will receive the blocks in different ways. The use of the frames permits the emphasizing of those specific number relationships which merit individual attention. Simply presenting the blocks to the child is much like giving him a set of number symbols from which it is hoped he will derive the many computational processes. The use of the blocks with the many different frames, along with guided activities, constitutes the arithmetic instruction that is usually done with number symbols alone. The guided activities used with the remedial group were those developed by Stern and will be discussed in the section headed “Presentation of the Materials.”

The blocks constructed for this thesis were fashioned from solid oak. These included one hundred fifty single cubes, fifteen each of the blocks representing the numbers from two through nine, and twenty-five ten’s blocks, or, altogether, two hundred ninety-five blocks. From one to ten, the blocks have the following colors: 1. Red, 2. Light blue, 3. Magenta, 4. Orange,
5. Gray, 6. Medium blue, 7. Yellow, 8. Dark green, 9. White, and
10. Dark blue. The single cubes measure approximately $3/10"$ on all sides, and the ten blocks therefore measure about $3/10" \times 3/10" \times 3"$. The intermediate blocks have been constructed according to the same scale. To avoid confusion in selecting and using the blocks, and for ease in transportation, a light weight box containing seven sliding trays was built to hold the complete set of blocks. It measures $3\frac{1}{2}" \times 2\frac{1}{2}" \times 1\frac{3}{2}"$, and when filled with the blocks weighs slightly over twenty-one pounds.

Stern provides for the teaching of fractions. Ideally, the standard unit of measurement upon which the materials are based, the single cube measuring $3/10$ of an inch on all sides, should be the unit for the fractions. The splitting of such a small cube, however, into the many fractions which should be taught would result in pieces too small to be of practical worth. It is necessary, therefore, to adopt a new basic unit for the fractions. In the case of the materials built for this thesis, the single whole unit has been made ten inches long, one inch wide, and $3/8$ inches thick (see picture, page 112). It represents the number one and is called, after Stern, a fraction plate. All the fractions from $\frac{1}{3}$ to $1/10$ have been constructed with this unit in mind. The plate which stands for the fraction $1/10$, therefore, measures $1" \times 1" \times 3/8"$, and ten of them will just equal the length of the single unit. There are, moreover, ten Fraction Frames (see picture, page 111), each built to receive one whole-number plate exactly. The bottom of each frame is segregated by lines into fractional parts, and each part printed with the corresponding fractional number, so that there is a frame for each fraction from $\frac{1}{3}$ to $1/10$. The frame for the whole unit has a scale marked on the top edge which runs from one to one hundred.
There are three whole plates and enough plates for each of the fractions from \( \frac{1}{2} \) to \( \frac{1}{10} \) to make up three whole units. Altogether then, there are one hundred sixty-five plates, each fashioned from ply wood. To prevent any possible association of color with number, a different color scheme has been used for the fraction plates than was used for the whole-numbers. Each plate and its respective Fraction Frame has been painted according to the following criteria: 1. White, 1/2. Dark blue, 1/3. Red, 1/4. Dark green, 1/5. Gray, 1/6. Yellow, 1/7. Light blue, 1/8. Orange, 1/9. Medium blue, and, 1/10. Magenta. In addition to its color, each plate is distinguished by the presence of the proper fraction number painted in black on one of its faces. A light weight carrying case has also been constructed for the plates. It measures 12" x 11" x 17" and contains nine partitions. Each recess thus formed holds three whole units, side by side, and on edge, and can, therefore, be used in teaching improper fractions. The box, with all the plates, weighs just under five pounds. The ten frames, also constructed of ply wood, weigh two and one half pounds.

There are four more devices yet to be considered. All of them are frames, or trays which have been designed to take the number blocks in different ways. All of them are constructed of ply wood. The Number Cases are used to familiarize the child with the counting, the addition facts, and the relationships of the numbers from one through ten. The Number Track is used in various ways to demonstrate all four computational processes. The Multiplication Machine is used for drill of the multiplication facts from one to ten. The Dual Board is used primarily to demonstrate the division process.
There are eleven Number Cases (see picture, page 112). Each is a square tray that will just accommodate in its length or width one of the number blocks from one to ten, except that there is an extra Unit Box. The inside measurements, for example, of the One Case and the Unit Box are respectively $8/10\text{"}$ square, and $3\text{"}$ square. The remaining boxes are built to the same scale. Each case is painted to correspond to the block that just fits into it, and they are built to pyramid, one inside the other. Their weight is three and one half pounds. There is also a Twenty Board (see picture, page 109) which will take two tens blocks in its length or width. It is used for illustration of the tens basis of the number system and the "Teens" number facts. It measures sixteen square inches on the inside, and has a horizontal dividing piece which cuts the box into two eight by sixteen inch rectangles. The box folds along the dividing piece to facilitate storage and is painted a medium-green, which is considered a basic color. Its weight is two pounds.

The Number Track (see picture, page 110) consists of ten trough-like pieces, open at both ends, which will just receive a single tens block. The inside measurement is, therefore, $8/10\text{"}$ wide, $8\text{"}$ long, and a little under $1\text{"}$ deep. There are nine grooves along the top which divide the pieces into ten equal spaces. On one side, directly beneath each groove is painted a number such that when the ten pieces are laid end to end the numbers from one to one hundred will appear consecutively along the side of the whole assembly. When assembled, the unit will measure one hundred sixty inches and will receive ten tens blocks laid end to end. Starting with the pieces that have the numbers from one to ten, and including each successive piece, the colors are the same as for the number blocks from one to ten. The
weight of the ten pieces that make up the complete Number Track is three pounds.

The Multiplication Machine (see picture, page 110) is nothing more than a tray with inside measurements equal to eleven single cubes in length and ten single cubes in width. A chart containing the Pythagorean Table, with the addition of a row of zeroes at the left side, covers the bottom of the tray. Eleven slides, the width of a cube and the length of the width of the machine cover the table. The numbers from one to ten were painted in black in equal intervals on the top of one of the sides. If, for example, six nine blocks are inserted in the machine, the drawing of the first free slide down to the bottom of the last block will reveal the number fifty-four. The machine is painted the basic medium-green. Its weight is one and one-half pounds.

The Dual Board (see picture, page 111) has two recesses. One is a tens block square, and the other, along side the first and separated by a thin partition, just receives a single tens block. The top edge of the larger recess is a strip that can be removed, leaving an open area. Painted on it in black, from right to left, are the numbers from ten to 100 by tens. In its place can be put any of the remaining nine strips which vary in size in such a way that their use will restrict the size of the recesses to rectangles which will receive any of the blocks from one to nine. Numbers are painted on each strip so that the first ten multiples of each number from one to nine appear on at least one strip, in ascending order and from right to left. Spaced across the bottom edge of the larger recess are the numbers from one to ten. At the top of the smaller recess, is an area the size of a single cube which is painted black. This is meant to indicate
that no more than nine single cubes should be placed in the smaller receptacle. Along the right hand side are painted, in black, the numbers from one to nine. The whole board, including the separate strips is painted the basic medium-green. Altogether, it weighs three and one-half pounds.

The complete set of materials as it was constructed for this study has been described. As a unit, the materials weigh about forty-two pounds and, when packed carefully, can be made to take up little more than two cubic feet of space. The construction of the materials was a meticulous and time-consuming task; so much so that it may be assumed that their purchase from a commercial source would call for a considerable amount of money.

The materials as just described do not represent the complete set as used by Catherine Stern. The following list gives those materials which were not used in this thesis.

- Counting Board
- Pattern Boards
- Street-Number Game
- Arithmetic Board
- Fretsw Nuterals
- Two-Place Holder
- Zig-Zag Board

These materials have been omitted because they were designed to introduce and develop number concepts and skills which may safely be assumed to be a part of any sixth-grade child's number repertoire. Examples of the types of accomplishment a sixth-grade pupil should have achieved are these:
1) Associating number symbols with number patterns, 2) Counting, 3) Writing numbers, 4) Writing common algorithms.

Also used by Stern is a small set of materials for extending the concretionization of numbers beyond one hundred and on into the thousands. Aside
from those materials which make up the systematic representation of the number idea, she also uses, in connection with the teaching of denominate numbers, some of the more common devices and objects, such as bottles or cans of different capacities, coins, calendars, etc. Moreover, there is a large group of semi-concrete materials that include such items as workbooks, graphs, charts, etc., which were also omitted as part of the equipment for this thesis.

Construction and Administration of the Concept Test

The Stern method involves a meaningful approach to arithmetic through a concrete medium. The word "meaningful" is used to describe a kind of teaching which concentrates on instructing the child in the arithmetical meaning of the number system. These meanings have often been called concepts or understandings.

Just as it is inconsistent to teach a child to swim and test him for learning by having him run a foot race, so is it inconsistent to teach meaningful arithmetic and test for learning with a traditional computational test. It may be said, however, that if the swimming lessons were intended to improve the physical fitness of the child, then he should be able to run a foot race better and faster. So may it also be said that if meaningful arithmetic lays claim to the giving of a firmer grasp of quantitative thinking, in general, then computational skills, as a part of quantitative thinking, should also be improved. The most obvious conclusion is that there is a place for both the traditional computational test and the concept test when the meaningful method is used, or to put it another way, the testing device should be consistent with the initial objectives for learning outcomes. However, since a short exposure to a teaching method cannot be
expected to bring about the broader educational objectives, it seems especially valid to maintain that a brief period of instruction, such as was received by the remedial group in this study, should be evaluated according to the more specific objectives. For this thesis, the implication is that meanings must be measured rather than computational skills, since the learning of meanings is the immediate goal, and the improvement in computational skills is part of the broader goal. This line of reasoning led to the conviction that a concept test should be used in evaluating the learning that resulted from the use of the Stern materials.

From the beginning, it was clear that the concept test would have to be constructed. There are a few measuring devices that purport to test for arithmetic understandings which are being prepared for publication at the present time, but none are yet available for use. There is a section in the Iowa Every-Pupil Test of Arithmetic Ability which is supposed to measure understandings, but it is much too incomplete for the purposes of this study.

It was distressing to find that almost no work had been done in defining and classifying concepts in any field, arithmetic or otherwise. This fact has made the building of the concept test more difficult than it might have been. On top of this must be added all the usual objections that go with a non-standardized test. Almost as a final blow, were the inadequacies of the completed test which are based on the fact that its construction received only minor attention in what can be called a minor study. Certainly the building of any test deserves no less time and work than can be given to it in a doctorate thesis. The reader is not expected
to think that the little space which this section offers to the problems of constructing a concept test is sufficient. Since this is considered a pilot study, however, there seemed to be some justification for the use of the concept test that finally emerged.

Because the concept test could not be thought of as a valid measuring device, it became necessary to use computational tests in addition to the concept test to measure the results of learning brought about by the use of the Stern materials. In a certain sense, the testing program has thus met with defeat. The instrument designed to measure the desired outcomes of the learning situation was not valid, while the valid instrument did not measure the right outcomes. This limitation to the study is serious and must be well marked.

The test items were drawn from a list of objectives for the meaningful program that was compiled from the literature on arithmetic. It might have been better to draw them from the learning experiences suggested by Stern, but the literature had the advantage of suggesting possible item forms. The objectives will not be discussed here, but the reader can get some idea of their kind and type by turning to the appendix and looking at some of the items in the sample test that will be found there. There are four parts to the test. Part I contains completion questions; Part II, multiple-choice; Part III, true-false, and, Part IV, derived multiple-choice. For the whole test, there are 28 items, but 64 answers are possible if the divisions of the main items are counted. There are four hectographed pages to the test, and the child marks his answer choice on the test itself. The usual precautions were taken to give concise but explicit directions to those taking the test.
The test was given to 32 pupils in a single sixth-grade classroom which usually comprised 35 members. The remedial group was to be drawn from this group. After approximately eleven weeks, the same test was again given to the same pupils. Absences reduced the number of pupils who had taken both tests to thirty. At both sittings, the pupils were allowed all the time they needed to complete the test.

The arithmetic mean, the median, the range, and the standard deviation for both test administrations were calculated. Also calculated was the average difficulty, in terms of percent of examples done right. This last figure is simply the arithmetic mean transformed into percent in order to show the difficulty of the test. Therefore, if the A.M. for a test of 64 examples was \( \frac{4}{4} \), the average difficulty of the test would be about 6 percent, meaning that only 6 percent of the examples were done correctly. Since the aim in constructing a test is to obtain about 50 percent average difficulty, this test would be considered too difficult. The difficulty of each test item can also be figured in much the same way, but it was not done for the concept test.

Not even a cursory item analysis was made but the reliability was estimated for the test as a whole. A set of random numbers was derived from two packs of playing cards and these numbers were used to split the test into two equal parts of 32 questions. Each paper in the first test administration was given a score for each of these parts. A reliability coefficient was then calculated and this figure corrected with the Spearman-Brown "chance-half" correction formula.
In order to check the second testing, and yet to avoid the time necessary for rescoring and correlating by a "chance-half" technique, the Kuder-Anderson formula for the estimation of the reliability coefficient was figured for both the initial and final tests. If the "chance-half" technique gives a high reliability coefficient on a first test administration, and the Kuder-Anderson estimation closely approximates this figure, then the estimation for the second test administration should approximate the one for the first sitting. If such is the case, it may be assumed that the use of the "chance-half" technique with the results of the second testing situation would yield about the same figure as it did for the first testing situation. If such is not the case, it may be assumed that a "chance-half" technique applied to the second sitting would, for some reason, yield a lower reliability coefficient.

The scores of the 30 pupils who took both administration of the concept test were also correlated. The results of this correlation should not reflect on the reliability of the test since eleven weeks had elapsed between the test and the retest situations. A lower correlation coefficient as compared with the reliability coefficient, should indicate differential learning over the intervening time-span, which, after all, was the aim for the remedial group.

Little could be done to establish validity figures. More out of curiosity than to prove validity, the scores for the first test administration were correlated with the Kuhlman-Anderson I.Q. scores and the Stanford Achievement scores in computation, reasoning and average arithmetic. The coefficients were based on the scores of 31 pupils, except with the figures for the I.Q.-concept correlation which was based on 32 pupils.
Selection of the Pupils for the Remedial Group

The pupils who received instruction in the remedial group were chosen from a single sixth-grade classroom in a city which is suburban to Boston and has a better than average economic rating. The particular school in which the study was made draws children from those families having the most adequate means in the city. It cannot be implied, however, that the children used in the remedial group were representative of this high-income status. Economic background was not considered an important factor in this work.

The school system of the city has a rather wide reputation for its competent staff, its modern public school methods, and its superior equipment. The school that entered into the study was equipped as well if not better than any in the city.

The teachers are permitted freedom in their choice of educational methods which resulted in a continuum of instructional techniques that extended from the so-called traditional to the so-called progressive. Since the school system used a 6-3-3 plan, the sixth grade was the highest level in the school, and, consequently, a sixth-grade pupil was apt to have received many types of instruction in his educational history. It can therefore be assumed that the pupils in the remedial group, all of whom had attended the school regularly, had had some traditional and some meaningful teaching in arithmetic sometime during their elementary schooling.

It was the practice in this school to start the systematic arithmetic program in the third grade, although the curriculum guide covered all six grades and also included the kindergarten. The arithmetic curriculum guide
had just been revised and rebuilt to give the program a meaningful orientation for the primary grades, but the sixth-grade children could not have received benefits from this act. It should not be assumed that the old guide contained a strictly traditional approach. It was ten years old but advocated some of the most modern practices and objectives for the teaching of arithmetic.

It has already been pointed out that the materials are cumbersome and difficult to construct. For that reason, no more than one set could be used. It was thought that the optimum number of children in the remedial group should, therefore, be five, for that number allowed close individual observation and close to maximum handling of the materials for each pupil.

Knowing that no more than five pupils would be in the remedial group, criteria were set up for their selection.

The first criterion was that the pupil must have an I.Q. not lower than 90. Cumulative records were available which gave the Kuhlman-Anderson I.Q. scores for group tests given in the third grade. In special cases the Stanford-Binet individual I.Q. test had been given, generally in the early primary grades, and these scores were also available. The pupil's age, and whether he was at grade for age, was not considered. It was decided, though, that the pupil's mental age should be no more than 6 months below the Kuhlman-Anderson norm for the sixth grade.

The second criterion was that the child's score on the Stanford Achievement test in computation and reasoning should be in the lower quartile of his class. This means, of course, that his average arithmetic score would also be in the lower quartile. These tests had been given in the Fall of the same year these pupils entered the sixth grade, and the results
had been entered into the cumulative records. The tests were administered by the classroom teachers as a part of the regular system-wide testing program. Form F of the Intermediate Battery was used, and the whole battery was given in two sittings over a period of two days. It was not to matter if the child's score fell in the lower quartile but still placed him above grade level. It was not likely that this would happen. Finally, only grade norms were to be used.

For the third criterion, it was decided that the pupil's score on the concept test, which has already been mentioned, should also fall in the lower quartile for his class. The test was given in February, one week prior to the beginning of the remedial class. It was administered by the writer who was practice teaching in the room at that time and had had four to five months familiarity with the children and the school.

The last criterion was subjective. It was that a list containing the pupils who fulfilled the first three criteria should be presented to the teacher and that he should choose, on the basis of his knowledge of the child through their homework, classwork and background, the five pupils he thought most needed remedial work and would most benefit from it.

The employment of the four criteria that have just been discussed should result in the selection of the five pupils whose arithmetic achievement could be improved. They should be pupils whose ability is not commensurate with their achievement—pupils who, therefore, are capable of learning more. Moreover, the teacher's subjective evaluation should aid in the selection of those pupils who are willing to learn.

It will be observed that no diagnostic test was given. The most important reason for this fact is that there is no test available that will aid
in diagnosing the presence or absence of understandings. The scope of this thesis could not possibly include the construction of such a test. It would have been possible to use a standard diagnostic test for discovering the areas of computational difficulty. Two obstacles to this move come to mind immediately. The first is that a knowledge of the computational deficiencies for each pupil in the remedial group might unduly influence the course of the remedial teaching of meanings, giving too much emphasis to arithmetic skills. The second, and largest, obstacle is that the materials were not designed for use in a diagnostic-remedial program. It seemed best to apply the materials and methods which Stern suggests for two or three years of pre-school and primary instruction to the five week remedial group, even though the resulting acceleration would produce drawbacks. Two factors were considered in this decision. The first was that a single concept is usually more fully understood when related to many computational processes. The second was that this study would rather evaluate the Stern methods then those that would have to be substituted if the materials were used to overcome diagnosed computational difficulties.

The results of the use of the criteria set forth in this section will be considered in Chapter IV.

Orientation of the Pupils to the Remedial Program

After the five pupils had been selected for the remedial program, two twenty-minute periods were devoted to orienting them to it.

One of the reasons for the introduction of an orientation period was that the radical nature of the instructional materials demanded explanation.

Another was that better rapport could be obtained between the instructor
and the pupils if they knew what activities they were to engage in, and if
the instructor knew a little more about the attitudes the pupils took toward
arithmetic.

Arrangements were made with the classroom teacher so that the five
pupils could go to a separate unused room during the early morning of two
successive days. The classroom schedule was organized in such a way that
the pupils were responsible for the completion of their arithmetic assign-
ment by the end of the day. The orientation periods took place during the
time usually set aside for instruction in, and class discussion of, arith-
metic. Therefore, the regular arithmetic assignments for the remedial
group were made about 20% shorter, but they were still required to do the
remaining classroom work. All five pupils were members of the lowest of
three arithmetic groups, each of which received different assignments every
day according to their ability in the subject.

During the first twenty-minute orientation period, the pupils were told
that they would be coming to the special room every morning for about a
month. The arrangement made for their regular arithmetic assignment was
then explained. They were told that they were chosen on the basis of their
scores on the tests which have been mentioned above, and with which they
were familiar. They had not seen the results of the achievement tests and
could not be shown the results on the concept test since it was to be used
again at the end of the program. They were also told that they were not
the worst pupils in the class in arithmetic, but that they had been chosen
because both the teacher and the instructor felt that they could do better.
The materials had been assembled on tables previous to the group's arrival so that they were open to the pupil's puzzled perusal. They were told that these were the materials that they would be using to help them learn arithmetic in a new way. They were allowed about five minutes to look at and handle the materials. During that time the instructor demonstrated how 25 and 9 could be added with the help of the Number Track. It was then decided who should get the materials out each day, who should put them away, and how they were to be handled.

During the time that had elapsed, it became apparent that the boys, especially, were worried about what would happen when their comrades found out that they were "playing with blocks." The derision that was bound to come could be seriously detrimental to any of the children's attitude toward the program and toward arithmetic in general. It was suggested that they decide, as a group, what they were going to tell their curious friends. The decision was startling for they decided that they would tell everybody who asked that they were "playing with blocks," but that it was fun and that they were going to learn a lot of arithmetic. With the close of the period, the pupils were told that on the next day they would have their voices recorded. This served to motivate them to want to come back.

During the next, and last, orientation period the materials were not brought out. Instead, a wire recorder had been set up. The plan was to use the recorder both to interest the child and to record the answers to an oral quiz that was to be given.

The pupils were more than agreeable to the suggestion that each one take a turn answering into the microphone, the four questions the instructor would ask. The four questions listed below were written on the board before
the recorder was turned on.

1. Do you like school?
2. Do you like your teacher?
3. Do you like arithmetic?
   a. What kind of arithmetic do you like best?
   b. What kind do you like least?
4. In what grade do you think you learned the most arithmetic?

The pupils were encouraged to give perfectly frank answers, which they seemed to do.

After all five pupils had had their turn at answering the questions, the group was told that the recorder would be running every day during the time they were to spend in the special room. It was demonstrated to the pupils that excessive noise did not record well, and that they should, therefore, be careful about interrupting one another in the future.

The close of the second orientation period ended that portion of the procedure which has been under discussion in this section.

Presentation of the Materials

The remedial group met every morning of the next 18 successive school days. The meetings varied from 15 to 30 minutes and averaged somewhere around 20 minutes' duration. Two pupils missed a day each due to absence. The same plans and arrangements that have already been explained for the orientation periods were applied to the remedial sessions.

In order to obtain the children's method of attacking the problems that confronted them, recordings were made of the whole of each meeting. The pupils were asked to "think out loud." They were to explain what manipulations they were making and why they made them. For the most part, the plan worked well. There were times, however, when the children worked individually and the recordings were then more noise than information. Moreover,
some of the children had more difficulty in rationalizing their actions than
others.

Although it could be said with some assurance that not all the pupils
had difficulty with all the fundamental processes, the plan was to go all
the way back to addition and work up to fractions. It was thought that
the concretization in a meaningful fashion of arithmetic process that had
already been covered would give a deeper insight into the number system
and thus strengthen whatever areas were weak because of mechanical learning.

There is reason to believe that a concrete meaningful approach to
arithmetic has serious disadvantages when used with pupils whose trouble
has started with an abstract computational approach to the subject. One of
the recurring difficulties was the pupil's inability to justify his own
faulty knowledge with the truths that were being demonstrated for him. It
was confusing. The pupil had not only to learn to take another viewpoint
of arithmetic and to learn the new facts that resulted, but he had also
to "unlearn" the viewpoints, and their corresponding faulty facts which
were already a part of him. If there is one thing harder than learning,
it is "unlearning." Although it was shown time and again that adding any
number larger than one to nine was best done with ten as a reference point,
and although the child saw this principle, understood it and could apply
it, when he came to doing his regular assignment he continued to count from
nine to the other addend to obtain the sum as he had always done. Added to
this was the fact that the child's own method of doing arithmetic was often
a mental block in the way of his understanding the new principle in the
first place.
For reasons such as those just put forth, the remedial program did not include work with counting or writing number symbols. Even though pupils probably had some misconceptions about counting, it would have been dangerous to try to remedy them. For them, counting was too easy; they knew all about it, and lost interest immediately if the subject was breached. Because the writer believes that remedial work in arithmetic should overcome the most basic misconceptions first, he must take the stand that a child in the sixth grade who has trouble based in the earliest phases of arithmetic is doomed to a certain amount of arithmetic incompetency more because he cannot be motivated to see his deficiency than because he cannot be taught the correct concepts.

The original plan was to describe the activities of each remedial meeting. Since then it has become clear that such a plan was too ambitious. The explanations that would be needed would cover too many pages and in the end would only be repetitious of the descriptions given by Stern. That space forbade a detailed discussion of the presentation is unfortunate for it is the real core of Stern's approach.

While the procedures prescribed by Stern in her book were followed closely, a few of the activities had to be omitted because their simplicity would be distasteful to sixth-grade pupils. It is possible, though, to give a general idea of the content of the program and that will be found in the following paragraphs.

It was possible to start with easy addition facts because the pupils found it interesting to fill in the Number Cases and then to write all the addition facts for each number that were thus found. They also found certain of the games and techniques used for teaching the bridging of the tens and
decades stimulating. So the reader who has Stern's book in hand and wants to know where instruction for the remedial group started must turn to Page 76. The activities for the group followed the order given by Stern starting on that page and extending to Page 259. A few of the simpler activities were omitted. Chapter XI, XII, XIII and XIX were also omitted, which means that problem solving, the structure of the two-place numbers, and denominate numbers were not given to the group. What is more, work with fractions was cut short because of time limitations. It will be noted that work with decimals and numbers over one hundred was not received by the group.

The best general terms for describing the work that the group covered are addition, subtraction, multiplication, division, and fractions. The list below gives the chapters from which the activities were taken and the titles of the sections that served as guides for the presentation of the materials to the children. Some idea of the kind of work that children did can be gained from these sub-titles.

Chapter VII  WRITING NUMBERS AND NUMBER STORIES

Recording of Number Stories

Chapter VIII  INTRODUCTION OF SUBTRACTION

Subtraction in the Number Cases
The Process of Comparison
Finding the Difference
Adding and Subtracting with the Number Track

Chapter IX  MASTERY OF ADDITION

Zero Facts in Addition
Story of the Climbing 1
Story of the Climbing 2
Analysis of 10

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Chapter X  MASTERY OF SUBTRACTION

Zero facts in Subtraction
A Remainder of 1
A Remainder of 2
Subtracting from 10

Chapter XIV  ADDITION AND SUBTRACTION IN THE RANGE FROM 1 - 100

Adding and Subtracting Full Tens
The Story of 20 in the 20-Case
Addition and Subtraction in the Number Track
Adding and Subtracting 10
Adding and Subtracting Several Full Tens
The Completion Game

Chapter XV  STRUCTURAL TECHNIQUES TO MASTER BRIDGING

Bridging From 9 in the 20-Case
The Doubles and their Neighbors
The Analysis of 11 and 12
A Remainder of 9
A remainder of 8
The Doubles in Subtraction
Subtracting from 15, 14, 13, and so on.
Bridging from 9 or 8 in any Decade
Adding 9 in any Decade
Subtracting 9
Finding "Distances" in the Number Track

Chapter XVI  CARRYING AND BORROWING

Experiments in Positional Notation
Simple Addition and Subtraction with the Dual Board
The Completion Game in the Dual Board
Carrying with the Dual Board
Borrowing with the Dual Board
Column Addition with Blocks

Chapter XVII  MULTIPLICATION

First Experiments in Multiplication
The Tables of 10 and 5 in the Dual Board
The 9-Table in the Dual Board
The 2-Table Based on Doubling
The 3-Table in the Number Track
The 4-Table Arranged in Patterns
The 6-Table, the 7-Table, and the 8-Table in the Number Track
Mastery of the 7-Table and the 8-Table
The Multiplication Machine
Chapter XVIII  DIVISION

Even Division
Division with Remainders
Division with the Dual Board
Common Multiples and Common Measures
An Experiment in Partition
Dividing into Fractional Parts
Detective Work in Partition

Chapter XX  FRACTIONS

Experiments with Halves
Experiments with Thirds
Experiments with Fractions from Fourths to Tenths
Adding and Subtracting Like Fractions
Multiplying a Fraction by a Whole Number
Proper and Improper Fractions and Mixed Numbers
A Fraction of a Fraction
Equivalent Fractions

The long list of subject matter areas given in the list suggests that not much attention could be given to each one. That is so. Work with concrete materials should be a slow deliberate process. There is not much doubt but what the children received too much, too quickly. It should be done differently another time. Perhaps the biggest lack was that little provision was made for the transition stage between the concrete and the abstract. This should be especially harmful for children who are already familiar with a few abstract notions. With the presentation what it was, transfer played too large a role. Actually the children had a great many concrete experiences related to many arithmetic concepts. Whether or not it was possible for the child to relate the experiences and the concepts to his everyday quantitative thinking is another question.

The materials were also presented to adults. One group of people taking a course in the teaching of primary arithmetic and another group in an arithmetic seminar had the materials demonstrated and explained to them.
Three or four acquaintances of the writer were also subjugated to a discourse on the materials' merits. These exposures were never more than an hour long, and there was little chance for experimentation and manipulation. This is mentioned here because many of the adults professed to have learned something about arithmetic that they had not previously known. They seemed to learn quickly and easily and were highly stimulated. Work should be done to see if the more mature mind, or the high I.Q. child, might not benefit most through the use of the materials.

One final word: it was not thought wise to allow the remedial pupils to neglect their regular arithmetic lessons entirely. This produces a research problem since it cannot be said that their level of achievement at the end of the program was solely a result of the remedial work. To that extent there are limitations to the findings in the next chapter which pertain to achievement as a result of the use of the materials.

Post-Program Testing

At the end of the 13 remedial sessions, every pupil in the classroom from which the group had been selected was subjected to the two arithmetic sections of the Stanford Achievement Tests, Intermediate Battery, Form EM. These deal with arithmetic computation and reasoning. The average of these two sections gives a third score which is known as the "average arithmetic score." Like Form F of the same battery, which was used in selecting the remedial group, Form EM was administered by the regular classroom teacher as part of a system-wide testing program. The whole battery was given in two sittings covering a period of two days. The tests were machine scored.

About six and one-half weeks after the last remedial group meeting, the same concept test which had been used in selecting the group was again
administered to the whole class. The test was given by the writer who was no longer connected with the class but who made a special trip to the classroom for the testing occasion.

Altogether, then, there were 5 test scores used in this study. Six of these scores were from the standardized Stanford Achievement Test, 3 of which were obtained from Form F which was given 3 to 4 months prior to the inauguration of the remedial group, and 3 of which were obtained from Form EM, which was given after the close of the group's work. Two of the eight scores were not standardized. These were the scores obtained from the concept test. One score was obtained directly before the first remedial session, and another from the same test was obtained about six and one-half weeks after the remedial work had been completed. All tests were given to the whole class from which the remedial group was selected.

These arithmetic scores were used to compare the remedial group with the rest of the class before and after remedial instruction, as well as to compare the pre-program scores of the five pupils in the group with their post-program scores. Some indication of the effect of the use of the Stern materials could be found in this way.

Unfortunately, no one test was given directly before and after the remedial program. The standardized scores covered a period of about six months, a month of which included the remedial work. The concept test scores covered a period of about two and one-half months, a month of which, again, included the remedial work. For that reason, the effect of the remedial program cannot be considered as the only factor in the change in scores.
It was not possible to obtain comparable scores including a time span exactly as long as the remedial program. The remedial program could not be made to fit with the system-wide testing plan, nor could the reverse be managed. The concept test could not have been given directly before and after the remedial program because this would have placed the two testings with the same test too close together. However, a measure of retention was needed, so the concept test was given directly before and then not again until 6½ weeks after the program. It did not seem wise to give the same concept test three times since the effect of learning the test would nullify the comparability of the test scores.

Since the same concept test was given two times, it became necessary to discover how much memory of the first test administration affected the scores of the second administration. Three pupils from the remedial group were taken aside one at a time and, with the test before them, asked how much of the test they remembered. Even though the three pupils were taken from the remedial group, they were considered representative of the whole class. They were taken from the remedial group because the scores for this group were considered slightly more vital than for the whole class.

Since the test was of a unique form and type for the class, it was thought that memory would play a large part. Such did not prove to be the case. Memory was apparently a surprisingly small variable. Although the class was told that they had already taken the concept test once before, many could not remember it even in this general way. As to the three pupils who were questioned, two of them remembered the general form of the first question and the others did not remember it at all. The technique for answering the last question was remembered by one of the three, but he did
not remember the specific questions. The remaining two had no recollection of the last question whatsoever. The middle section of the test was recognized only in a hazy way, even with prompting. The true-false section seemed to escape notice more than any other single section.

After observing the class take the test and after questioning the 3 pupils from the remedial group, the conclusion was drawn that, in general, even if there was some slight memory of the form of the entire test, or of a single section, each specific question was attacked as if it had never been seen before. Perhaps two observations are pertinent. The first is that the pupils who were tested for test-memory had already taken the test twice by that time and that questioning may have prompted more memory than actually existed. The second is that if the better students had been questioned, more memory might have resulted. This assumption is based on the hypothesis that the better student gives more thought to each question and, therefore, sees its relationships and structure more thoroughly than the poorer student.

It has been noted elsewhere that there is reason to believe that computational tests are not adequate for measuring the results of a meaningful arithmetic program. The same can be said about the adequacy of a paper and pencil test, concept or otherwise, for measuring the results of an arithmetic program which has utilized concrete materials almost entirely. It seems fair to maintain that where concrete materials have been used, especially over a short period of time, the only valid test of learning is one which uses the materials themselves in the evaluative process. At the very least, more study of the problem is indicated.
Chapter IV

ANALYSIS OF THE RESULTS

The results of this thesis were derived from five of the six procedural units presented in Chapter III. The chapter now being presented is organized around these five types of results. The first section is a discussion of the concept test. It gives a brief account of the adequacy of the test as a measuring device. The second section considers the selection of the pupils for the remedial group. It is concerned primarily with the results of the pre-program testing and evaluation procedures. The third section presents the findings that emerged from the two orientation periods. It deals, therefore, with the results of the oral quiz given to the five remedial pupils before the start of the work with the concrete materials. The fourth section gives the findings which were associated with the presentation of the materials. This section offers an interpretation of the recordings which were made during each remedial session. The fifth and last section contains an analysis of the results of the post-program testing. Subject to the limitations of the study, this section attempts to relate the arithmetic progress of the remedial group to the use of the Catherine Stern materials.

The reader is asked to remember that one of the purposes of this thesis is to discover the research problems connected with the experimental study of learning based on the use of concrete materials. With this in mind it can be seen that certain of the findings from the review of literature, and the construction of the materials, are valid conclusions to the study. An
analysis of these findings is not presented in this chapter. Whenever conclusions of this type exist they have been given in the sections of the previous chapter which discuss these problems. A summary of all conclusions, however, will be given in Chapter V.

The Concept Test as a Measuring Device

Table 1 gives the figures that bear on the adequacy of the concept test as a measuring device. These figures are based on a first sitting of 32 pupils and a second sitting of 30 pupils. The median time and the range of time for administering both tests are given. The arithmetic means and median scores are given to express the central tendency of both testings. The range of scores and standard deviation for both administrations are given to express the variability of the test. The reliability of the test is indicated by the "chance-half" correlation coefficient for the first sitting. This figure was corrected by using the Spearman-Brown correction formula and the corrected coefficient is also given. The Kuder-Anderson "footrule" estimation of the reliability is also given for both sittings. The validity of the test could not be established, but as an indication of the relation of the test to other standardized tests, four correlation coefficients were calculated. These are also given in the table. They were based on the scores of 31 pupils, except with the figures for the I.Q.-concept coefficient which was based on 32 pupils.
Table 1

Administration Time, Central Tendency, Variability, Reliability, and Some Correlation Coefficients for the Two Administrations of the Concept Test

<table>
<thead>
<tr>
<th></th>
<th>First Administration</th>
<th>Second Administration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Administration Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Median Time</td>
<td>25&quot;</td>
<td>23&quot;</td>
</tr>
<tr>
<td>Range in Time</td>
<td>15&quot;-40&quot;</td>
<td>17&quot;-37&quot;</td>
</tr>
<tr>
<td>Central Tendency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.M.</td>
<td>38.18</td>
<td>43.74</td>
</tr>
<tr>
<td>Median</td>
<td>41.5</td>
<td>43.5</td>
</tr>
<tr>
<td>Variability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Range</td>
<td>8-60</td>
<td>18-62</td>
</tr>
<tr>
<td>S.D.</td>
<td>12.95</td>
<td>10.36</td>
</tr>
<tr>
<td>Reliability</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;Chance-half&quot;</td>
<td>.925</td>
<td>.912</td>
</tr>
<tr>
<td>Corrected</td>
<td>.961</td>
<td></td>
</tr>
<tr>
<td>&quot;Footrule&quot;</td>
<td>.922</td>
<td>.912</td>
</tr>
<tr>
<td>Correlation Coefficients</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concept-Reasoning</td>
<td>.71</td>
<td></td>
</tr>
<tr>
<td>Concept-Ave. Arith.</td>
<td>.71</td>
<td></td>
</tr>
<tr>
<td>Concept-Computation</td>
<td>.59</td>
<td></td>
</tr>
<tr>
<td>Concept-I.Q.</td>
<td>.55</td>
<td></td>
</tr>
</tbody>
</table>

The median time for the first sitting was 25 minutes and for the second, 23 minutes. The range of time for the first sitting was from 15 minutes to 40 minutes; while for the second the range was from 17 minutes to 37 minutes.

The arithmetic mean for the first testing was 38.18, while the median was 41.5. The variability was expressed in terms of the range which yielded scores running from 8 to 60, and the standard deviation, which was calculated to be 12.95. For the second testing the arithmetic mean was 43.74, and the median 43.5. The range of scores was from 18 to 62 and the standard devia-
tion was 10.36. On the basis of the mean, the average difficulty, in terms of percent of examples done right, was, in rounded numbers, 60% for the first test exposure and 69% for the second.

The figures just given suggest that some learning took place over the eleven-week period. The mean raised 5.56 units and the median time to take the test was reduced by two minutes. Apparently the variability was decreased chiefly by the better performances of those that did poorly during the first testing situation since the lower limits of the range of scores rose from 5 to 16. The test was not too easy nor too hard for any pupil since the best score was 62 out of 64, and the poorest score was 3 out of 64.

The "chance-half" reliability coefficient was .925 which became .961 when the Spearman-Brown correction formula was applied. These coefficients suggest that the test is highly reliable. The Kuder-Anderson "footrule" formula gave .922 and .912 as estimations of the reliability coefficient for the first and second sittings respectively. Since these figures are close to one another and to the "chance-half" coefficient, it may be assumed that the second administration of the test was as reliable as the first and that, therefore, no extraneous factors caused high reliability figures when this was not a true property of the test.

The coefficient that resulted when the scores for the 30 pupils that took both tests were correlated was .797. Since this figure is lower than the reliability coefficient, it may be said that differential learning took place over the eleven-week time-span between the two administrations of the test.
The least amount of correlation, .55, was found between the results of the I.Q. and concept tests. This is encouraging since it can be said that the concept test is not altogether a repetition of the I.Q. test. Correlating the concept test with the reasoning and average arithmetic scores gave the same coefficient, .71, which was the highest found. If the concept test is truly valid it can be said that a knowledge of arithmetical concepts helps the pupil to some extent in achieving high average arithmetic and reasoning scores. The relatively low coefficient, .59, that resulted when the concept test was correlated with the computational test suggests, first, that facility with concepts is more closely related to arithmetical reasoning than computational skills, and, secondly, that computational skills can be taught without meanings but with results.

The reader is cautioned that the figures that have been given in this section have not been accompanied by estimations of significance, probable error, or coefficients of alienation, and that, furthermore, the statistics are based on a small, selected population.

Selection of the Pupils for the Remedial Group

The criteria used in selecting the five pupils for the remedial group were enumerated in Chapter III. The results of the employment of each criterion will be discussed in this chapter.

I.Q. Scores

The first criterion, that the pupils' I.Q. be not less than 90, was established in order to permit reasonable assurance that the child was capable of doing sixth-grade arithmetic. It developed that this safeguard was not altogether necessary. Only three pupils out of the whole class of 35 had Kuhlman-Anderson I.Q.'s below 90.
Table 2 gives the code number, chronological age, Kuhlman-Anderson I.Q., Stanford-Binet I.Q.'s for pupils 2, 3 and 35, extrapolated mental ages, and the grade equivalent on the whole Stanford Achievement battery for these three pupils. An extrapolated mental age score is considered to be one which is computed from the pupil's I.Q. on some other chronological age than the one used in deriving the I.Q. at the time of the I.Q. test. The mental ages were figured on the pupil's April 1, 1950, chronological ages and on the Binet I.Q., except for pupil 2 for whom no Binet I.Q. was available. The M.A. for pupil 2 was figured on his Kuhlman-Anderson I.Q.

Table 2

Measures for the three Pupils Who Did Not Meet the I.Q. Criterion

<table>
<thead>
<tr>
<th>Pupil No.</th>
<th>Chron. Age</th>
<th>Intelligent Quotient K-A</th>
<th>Mental Age</th>
<th>Grade Equiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>13-3</td>
<td>87</td>
<td>---</td>
<td>11-5</td>
</tr>
<tr>
<td>3</td>
<td>12-10</td>
<td>87</td>
<td>103</td>
<td>13-2</td>
</tr>
<tr>
<td>35</td>
<td>13-2</td>
<td>87</td>
<td>86</td>
<td>11-3</td>
</tr>
</tbody>
</table>

On the basis of the information given in Table 2, pupil number 35 was made ineligible for the remedial group. Neither quotient for the pupil met the criterion. It is true that her M.A., because of excess age for grade, met the second I.Q. qualification, that the M.A. be not more than six months below grade level, but her very low achievement score for all areas, a grade equivalent of 3.6, made it clear that she needed more help than could be given in the proposed remedial program.
Since the Stanford-Binet I.Q. was to take precedent over the K.A. pupil number 3 was considered to have met the criterion. An I.Q. of 103 and a mental age of 13-2 can be thought of as adequate for sixth-grade work. The fact that her grade equivalent for all subjects of the Stanford Achievement Battery was only 5.2 indicated that she was not progressing as well as she could be expected to.

Pupil number 2 was admitted into the remedial group even though he did not meet the I.Q. criterion. There seemed to be good reason for the admission. Although pupil 2 was three points below the I.Q. standard of 90, he was doing fairly well in his school subjects if his grade equivalent of 5.9 was to be believed. Because of this fact, it was thought by both the teacher and the writer that his Kuhlman-Anderson score was not wholly reliable. It was possible, of course, that his being the oldest child in the class contributed to a false feeling as to what his I.Q. should be. Certainly his mental age, even based on the low I.Q. was sufficient for his grade. At any rate, in a not too rigorous way, pupil number 2 was entered into qualification for the remedial group.

It can be seen from the brief discussion of the three pupils who did not meet the original I.Q. criterion that the decision as to their eligibility for the remedial group was based on rather flexible considerations. This is, perhaps, as it should be when working with scores which are as subject to variations as are I.Q.'s and mental ages. Only one of the three was definitely made ineligible according to the I.Q. criterion.

Table 3 gives the same information as Table 2, but for the five pupils who were finally chosen for the remedial group. There is the exception that the grade equivalents for the whole Stanford Achievement Battery are
not given. These scores were not considered to bear on the I.Q. criterion except in those cases already given where the criterion was not met directly. Table 3 also gives the arithmetic means for the ages and I.Q.'s of the 30 pupils who took both administrations of the reasoning, computation, and concept test in common.

Table 3

Comparison of the Five Remedial Pupils with the Class Average on the Basis of the I.Q. Criterion

<table>
<thead>
<tr>
<th>Pupil No.</th>
<th>Chron. Age</th>
<th>Intelligent Quotient</th>
<th>Mental Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11-3</td>
<td>124</td>
<td>11-5</td>
</tr>
<tr>
<td>2</td>
<td>13-3</td>
<td>87</td>
<td>11-5</td>
</tr>
<tr>
<td>3</td>
<td>12-10</td>
<td>87</td>
<td>13-2</td>
</tr>
<tr>
<td>4</td>
<td>13-1</td>
<td>92</td>
<td>13-3</td>
</tr>
<tr>
<td>5</td>
<td>11-6</td>
<td>111</td>
<td>13-0</td>
</tr>
</tbody>
</table>

| Class Ave. | 11-9 | 111 | 13-0 |

It can be seen from the information in Table 3 that pupils 1, 4, and 5 also met the I.Q. criterion. It can also be seen that the mental ages for all five pupils were commensurate with their grade level. The lowest M.A. was that of pupil 2, whose case has been discussed above. Pupil 5 showed an M.A. of 11-6, just one month short of the average C.A. for the whole class of 30 pupils. This score is based on the Stanford-Binet I.Q. even though the Kuhlman-Anderson I.Q. was higher. The remaining three pupils had mental ages above the average C.A., 11-9, for the whole class of 30 pupils.
Pupil number 1 had an I.Q. of 124, 13 points above the 111 average for the class. He placed 10th in his class on the basis of the complete Stanford Achievement Battery with a grade equivalent of 7.8. It will be seen later that his arithmetic scores were below average, which could only mean that he was doing exceptionally well in all other subjects. It seemed plausible that his difficulty was of an instructional nature since he gave every evidence of being capable of learning arithmetic. Pupils number 4 and 5 showed Stanford-Binet I.Q.'s of 102 and 103 respectively, slightly above average but below the Kuhlman-Anderson class average of 111.

With the exception of pupil 2, all five members of the remedial class showed an I. Q. above 100 on either one or the other of the two intelligent tests. They fulfilled the I.Q. criterion and should, therefore, be pupils who could be expected to achieve normal progress in arithmetic. Such was not the case as will be seen in the next division of this section.

**Computation, Reasoning, and Average Arithmetic**

The second criterion set up was that each pupil in the remedial group should have made scores in the computational and reasoning sections of the Stanford Achievement Test which were in the lowest quartile of the class scores. This would also place their average arithmetic scores, a composite of the two just mentioned, in the lowest quartile. The second criterion was established because it was necessary to select pupils for the remedial group who were not making satisfactory progress in arithmetic.

Table 4 gives the computational, reasoning, and average arithmetic scores of the five members of the remedial group and two others. It also gives the upper limits of the lowest quartile for the scores of the 30 pupils who took all the tests given in conjunction with this thesis.
Scores for the Seven Pupils who met the Computation, Reasoning, and Average Arithmetic Criterion, and the Lowest Quartile Score for the Class

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.6</td>
<td>5.2</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>5.5</td>
<td>5.1</td>
<td>5.4</td>
</tr>
<tr>
<td>3</td>
<td>5.6</td>
<td>5.0</td>
<td>5.4</td>
</tr>
<tr>
<td>4</td>
<td>5.5</td>
<td>5.2</td>
<td>5.4</td>
</tr>
<tr>
<td>5</td>
<td>5.6</td>
<td>5.1</td>
<td>5.4</td>
</tr>
<tr>
<td>25</td>
<td>6.2</td>
<td>5.5</td>
<td>5.9</td>
</tr>
<tr>
<td>27</td>
<td>5.9</td>
<td>5.5</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Q₁ for the Class: 5.9 5.5 5.8

The upper limits of the lowest quartile of scores for the 30 pupils in the class was 5.9 on the computation test, 5.5 on the reasoning test, and 5.8 on the average arithmetic scores. It is immediately clear that all members of the remedial group, pupils 1 - 5, met the second criterion by falling below these scores.

Of the remaining two pupils, pupil number 27 just met the upper limits of the lowest quartile on all three scores. Her record read, Computation - 5.9, Reasoning - 5.5, and Average Arithmetic - 5.8. She was included in the list of pupils qualified for the remedial group. The other pupil, number 25, met the upper limits of the lowest quartile in reasoning, but achieved a score of 6.2 on the computation test, thus disqualifying her for the remedial group. Her average arithmetic score was 5.9 just .1 of a point above the criterion level. In order to give the teacher some choice of pupils in
making his subjective choice, this pupil was also included in the list handed to him.

**Concept Test**

The third criterion was that the five pupils in the remedial group should receive low scores on the non-standardized concept test. These scores were to fall in the lowest quartile of the scores for the 30 pupils in the class. The use of the third criterion was intended to give more weight to the need for work with arithmetic understandings. The use of the computational and reasoning scores alone, was too apt to work in such a way as to select those pupils with computational deficiencies.

Table 5 gives the scores on the concept test for the five pupils chosen for the remedial group plus two other qualifying pupils.

**Table 5**

Scores for the Seven Pupils Who Met the Concept Test Criterion, and the Upper Limits of the Lowest Quartile Score for the Class

<table>
<thead>
<tr>
<th>Pupil No.</th>
<th>Concept Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>25</td>
<td>19</td>
</tr>
<tr>
<td>27</td>
<td>36</td>
</tr>
<tr>
<td>Q₁ for the class</td>
<td>23</td>
</tr>
</tbody>
</table>
The upper limit of the lowest quartile, 26, is also given. It is apparent that all five pupils in the remedial group, pupils 1 – 5, met the criterion. Pupil 1 was only one score unit below the criterion level, while pupil 5 just met it. Pupils 25 and 27, included in the list according to considerations of the previous division of this section, achieved scores of 19 and 36 respectively. Pupil 25 met the third criterion, but pupil 27 did not. She barely fell below the upper limit of the second quartile. Again, however, she was included in the list handed to the teacher in order to give him some choice in his selection. One other pupil, number 24, fell into the lowest quartile of the concept test scores with a mark of 17. Since the other scores, in computation, reasoning and average arithmetic were all above the criterion level, this pupil was not included in the list of those pupils qualified for the remedial group.

The Teacher's Evaluation

The fourth and last criterion was that the teacher should select the five pupils he thought would benefit most from the remedial program. The list from which he was to choose these pupils contained the names of the pupils eligible for the work according to the first 3 criterion. The fourth criterion was supposed to aid in selecting those pupils who would be willing to learn more arithmetic. His choice, therefore, was intended to help reduce the motivational problem.

Table 6 gives a composite of the information given in Tables 3 through 5. Table 6 is thus a replica of the list given to the teacher, except that the table uses code numbers in place of the pupil's name.
Table 6
Summary of Measures for the Seven Pupils Who Met the Three Test Criteria

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11-3</td>
<td>12½</td>
<td>14-5</td>
<td>5.6</td>
<td>5.2</td>
<td>5.5</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>13-3</td>
<td>87</td>
<td>11-5</td>
<td>5.5</td>
<td>5.1</td>
<td>5.4</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>12-10</td>
<td>97</td>
<td>13-2</td>
<td>5.6</td>
<td>5.0</td>
<td>5.4</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>13-1</td>
<td>92</td>
<td>13-3</td>
<td>5.4</td>
<td>5.2</td>
<td>5.4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>11-6</td>
<td>111</td>
<td>11-8</td>
<td>5.6</td>
<td>5.1</td>
<td>5.4</td>
<td>28</td>
</tr>
<tr>
<td>25</td>
<td>11-½</td>
<td>102</td>
<td>11-6</td>
<td>5.2</td>
<td>5.5</td>
<td>5.9</td>
<td>19</td>
</tr>
<tr>
<td>27</td>
<td>11-8</td>
<td>121</td>
<td>14-1</td>
<td>5.9</td>
<td>5.5</td>
<td>5.8</td>
<td>36</td>
</tr>
</tbody>
</table>

Q₄ for the class: 5.9 5.5 5.8 28

This table, plus what has been said previously in this section, makes it clear that the originally established criteria for selecting the remedial group pupils were too rigid. Only 4 pupils out of the class of 30 met all 4 criteria fully. Those were pupils 1, 3, 4, and 5. Pupil 2 failed to meet the I.Q. criterion; pupil 25 failed to meet the computational test, and, hence, the average arithmetic, criterion. Pupil 27 failed to meet the concept test criterion. Obviously, the teacher could not select the necessary five pupils from a list of 4 pupils. Therefore, pupils 2, 25, and 27 were in the list handed to him.

The teacher selected the first five pupils on the list for the remedial group. Hence, only one pupil, number 2, did not meet the criterion. His case has been discussed in the division of this chapter labeled "I.Q."

Of the five pupils finally chosen for the remedial group, none were up to grade level norms, which at the time of the test would have been 6.2.
The same cannot be said about the concept test because it was not standard-
ized. With the exception of pupil 2, all I.Q.'s were above 100. That the
pupils could be expected to do better was a justifiable statement. In making
his selection of pupils, the teacher took the stand that the five he had
chosen were capable of doing better and, he believed, would cooperate in
the experiment.

Findings from the Orientation Periods

The orientation periods were made a part of this study in order to
improve pupil-instructor rapport. This section offers the results of the
short oral quiz which was given to the remedial group during the second
orientation period. Each of the five members of the remedial group receives
individual attention in this section. Insofar as more information than just
that obtained from the oral quiz is given, these brief sketches may be
thought of as being of the case-history type. The reports given are highly
subjective. Fused with the data gathered from the oral quiz are certain
remarks and bits of knowledge about each pupil, which the writer gathered
during his six or more months' work with these pupils.

Other than the wire recordings made of the oral quiz, no anecdotal
record, check list, nor any other kind of device was put to work in the
interests of a more objective report. As pointed out in the review of
literature, there is reason to believe that the case-history type of ex-
perimental research should be improved upon and adapted to studies which
employ concrete materials. The case history, even in brief form, tends to
control, by bringing into relief, the variables that cannot be handled with
objective tests alone.
Case Number One

Pupil number 1 was a small boy for his age. He was generally active, even wild, but seemed to have no serious emotional troubles. He had two older brothers, both of whom had done exceptionally well in school and both of whom had recently received college honors. It sometimes seemed that pupil 1 attempted to obtain recognition in the family by building for himself the personality of a humorist. Although he had done very well in all subjects but arithmetic, this role tended to differentiate him from his brothers who, perhaps, had done a better all-around scholastic job. Since he carried his role into the classroom as the acknowledged wit and mimic of the sixth grade, he sometimes became a discipline problem. One other facet of his personality affected the discipline aspect. He was intensely interested in literature and read avidly all books that interested him, to the exclusion of all outside stimuli. He had several times to be admonished for not leaving his reading to tend to other school subjects. Frequently, he did not seem to hear these admonitions. His interest in literature and his persistence in maintaining his status as class humorist left little time for serious study of arithmetic, the one subject in which he did not excel. It is possible that he could have been interested in arithmetic through the historical literature on the subject, but at the time he was more fascinated by pure adventure.

Pupil 1 was frequently difficult to handle in the remedial periods. He wished more often than not to derive humorous remarks from the materials than to work with them directly. Once in a while he became engrossed in his work with the materials and manipulated them expertly. He professed to really want to learn more about arithmetic, for he was confident that he
had only missed some little trick to the whole thing somewhere along the instructional line. More than once he asked if he might have the special help in arithmetic that had been promised to him.

He claimed to like school very much, but thought it would be much better if everyone could study what he wanted to study. He said he sometimes felt like staying home so that he might read. He said he liked his teacher but wished the teacher would not keep telling him to do his work. He said he didn't like arithmetic very much because it, especially the problems, was too confusing. If he had to do arithmetic he would rather do exercises for he sometimes "caught on" to these. Question 4, "Where did you learn the most arithmetic," was not a good question. In general, the pupils did not seem to think of learning in this way. Pupil 1 thought he might have learned most in the fourth grade but wasn't sure. His teacher that year handled his problems nicely, and so it may be guessed that the pupils thought of amount of learning in terms of their most contented school year.

Case Number Two

Pupil 2 was a boy who stuttered. His family background was not known by the writer. He had been retarded and was consequently the oldest child in the class. He was fairly well accepted by the class for his interests seemed to have more in common with the sixth grade, or even a lower grade, than with what would have been his grade had he progressed normally. He was not athletically inclined and so, perhaps, got along better with the girls than with the boys. He sang well and liked music. He was the type that led the morning songs, but he frequently became disturbed if the songs were not completed properly.
He had general difficulty in all subjects. He read very poorly. He
did all his work conscientiously, but not effectively. In fact, pupil 2
was more conscientious about proper behavior than any one in the class, but
also more often reprimanded than any one else. He became excited over
simple happenings and then lost himself, becoming talkative and wild.
Pupil number 2 was either extremely naive, or lacked discernment. Pro-
bably the latter, if his work with the Stern materials was indicative of his
make-up. He had great difficulty in understanding the abstractions of which
the materials were a concrete example. He repeated constantly that the
materials were fun but made no sense. He liked to manipulate the materials
and occasionally gave evidence of discovering a particular understanding
in genuine fashion and spirit. In his everyday arithmetic he accomplished
most if he was given individual help and a set of rules to follow for each
specific computational process, but he forgot quickly and had little success
with heterogeneous exercises.

He had difficulty in deciding whether he liked school or not, but
decided that he liked some of it, but not all. He admitted that he liked
his teacher and was quick to add that he knew he was at fault for the fre-
quent redresses he received. He thought, however, that at times he was
right. He did not like arithmetic in any form; in fact, outside of music,
there was hardly a subject that he would say he enjoyed. There were times,
very few in number, when he liked arithmetic, but always when he understood
it and always when the work involved computational problems. He didn't see
any sense to problem solving and rarely ever got them right. Like the others,
he was not sure where he learned the most arithmetic but thought it might
have been in the fifth grade.
Case Number Three

Pupil number 3 was the third boy in the group. He was the tallest person in the class and was athletically oriented. He and his special friend more or less dominated the sport activities for the class, in and out of school. As a result he held a place of esteem in the class social structure, regardless of his generally poor academic work. He was a sincere, courteous boy, but often awkward and shy in the presence of adults. He always completed his school work to the best of his ability, knowing full well that his accomplishment would be of a low order. He thrived on praise and encouragement, but in a quiet way that made him return to longer and harder work than might ordinarily have been the case. If his first paper on a new subject in arithmetic was a good one, he could be expected to produce commendable results for as long as a week because of the initial impetus, but if he became entangled in difficult work which he could not do, he quickly lost interest and his work suffered. He wanted to do better in all his subjects and was grateful for whatever extra help he received. He knew, too, when he was and wasn’t doing a good job.

This pupil caused class trouble by not being able to resist invitations from his colleagues to play forbidden classroom tag and the like. He was often spotted as unruly simply because he was less adept at covering his activities than the others, but he responded quickly to a word of caution, at least for the moment. His desire to maintain his social position in the class, a position fostered by his physical prowess, caused him agonized worry when he discovered he was to work with "blocks." He pleaded to be omitted from the remedial class, but when it was pointed out to him that his comrades had secretly pleaded to be allowed to enter the class, he viewed
their taunts with a new light and began to worry less about the stigma. To the end, however, he tried to disassociate himself from the remedial group and to deemphasize his connection with it. Actually, he seemed to enjoy the opportunity to learn more arithmetic, and was more serious about his work with the materials than any of the other four members.

When asked, he said he liked school, but wished they could "play more ball." He liked his teacher, who, incidentally, had charge of the after-school recreation program, and thought he was fair in all his dealings. He thought he liked arithmetic pretty much, certainly better than English. He liked number problems better than problem solving since the latter often proved to be "tricky." He thought he was learning a lot of arithmetic in his present grade, although he wasn't sure but what he had learned a lot in the fifth grade, too.

Case Number Four

Pupil number ½ was a girl. She had been retarded and was over age for her grade. In no sense could she be called conscientious about her school work. It was always carelessly and haphazardly done. Her aim in attacking any assignment was to complete it with as much dispatch as possible. What activities she then turned to was not known since she seemed to have no special interests. For the most part, she was quiet, but she could be numbered amongst those five or six pupils who always annoyed class proceedings. She did it though, in a quiet way, by being out of her seat or whispering at the most inopportune moment, when at another time it wouldn't have mattered. In that sense she seemed to be thoroughly self-contained, going her own way completely and letting class activities disturb her only momentarily.
Maybe it was her independent nature that made her seem so phlegmatic. In the remedial work she only joined in the activities when asked to do so. She was neither bored nor stimulated by the materials. She was indifferent. The remedial class seemed to be for her only another school task which she could meet and be done with without too much effort. For that reason she was an easy person to have in the group, but it was obvious that she was not making the most of the situation.

She thought school was "O.K." if one had to do something. She liked her teacher but thought he should work more with art than science, although it was hard to see why this request should be made since she was not especially interested in art work. She liked arithmetic about as well as any subject. She liked examples but sometimes got tired of doing the same thing over and over again. She thought problem solving was "terrible." She wasn't sure where she learned the most arithmetic. The writer always felt that he did not understand pupil 4 and that a more thorough study of her case should have been made.

Case Number Five

Pupil number 5 was one of the two girls in the remedial group. The most salient fact in her case was that she was not socially adjusted. She knew it and it bothered her. She came from a large family, and there were at least three or four from the family sprinkled throughout the school and several more, who had been through the school, at higher educational levels. The family was not well off financially and at times it reflected in the girl's appearance. This may have been the reason she was not accepted in the social life of the class. She was not aggressive, nor overly reticent. Although she did not do well in her school work, one always felt that it
was because she was not happily adjusted. In conversing with her, it became clear that she could not understand why she was not chosen for tag and the rest of it, nor why she wasn't liked, nor what she should do about it. It never became a personal affair, however. When she received individual help, she seemed to understand what was asked of her, but when left alone to do tests or complete projects, she rarely ever fulfilled these tasks. She was apt to ask for more help than she needed, perhaps for the social encounter it offered.

In the remedial program she followed the same pattern. She seemed to understand all that went on, but to gain nothing because of it. She was always too anxious to help to get the materials out or put them away. She seemed to like the idea of the program, but was not particularly interested in the work that went on there. She frequently asked if the remedial group could not be made longer.

She admitted to not liking school very much, but said she didn't mind the subjects. She liked her teacher very much. As to arithmetic, she liked it "about medium," liked exercises best, and thought the problems were hard. She didn't know where she learned the most arithmetic. She did not speak freely during the oral quiz and had to be drawn out skillfully if more than one-word answers were to be received.

Summary

It is not possible to summarize case histories in the strictest sense. It can be said, however, that all five members of the remedial group had a healthy and normal reaction to school and to their teacher. Although these pupils represented those with very low arithmetic achievement, they did not confess to disliking the subject as heartily as might have been expected.
All, without exception, found computational arithmetic the most pleasurable, if not the easiest, and all, vouched for the difficulty of problem solving. The question which asked where each child learned the most arithmetic yielded little information. During the oral quiz, the children seemed to strike a level of frankness which they enjoyed.

It was apparent from the type of case history consideration just given to these pupils, that there was more reason for their lack of arithmetic achievement than just instructional difficulty. Frequently the need for a kind of social and emotional adjustment seemed to carry more weight than the need to make the subject clear and concise.

Findings from the Remedial Sessions

The preceding section gave brief subjective descriptions of each pupil in the remedial group. Included in each case history was a passage which described the child's general behavioral reaction to the whole remedial program. The section now being presented is reserved for a consideration of the group's reaction to the materials. Whereas parts of the previous section attempted to answer the question, "Did the children like the idea of the program?" this section attempts to answer the question, "Did the children understand the materials?"

The latter question would be difficult to answer under any circumstances. The recordings that were made during every remedial session, however, yielded information that was useful in answering the question. The verbalizations made by the children as they worked with the materials were analyzed and interpreted with the question in mind. Of course, the results of this type of experimental technique can be no more valid than the criteria used by the analyst in his interpretations. Since no attempt
was made to objectify the criteria, the findings in this section must be thought of as being subjective.

It is permissible to ask why it is important to find out whether or not the children understand the materials. Perhaps the best answer that can be given is that the whole theory of Stern's method is based on the assumption that children can "discover arithmetic" by manipulating materials which bring out the structure of the number system. If they do not understand the materials, they cannot discover, and, hence, learn arithmetic. Since the purpose of this thesis is to evaluate the Stern materials as used in a sixth-grade remedial program, it is important to know whether or not these children understand the materials.

Two good reasons why the remedial group might not have understood the materials have already been given elsewhere in the paper. The first is that each pupil's faulty arithmetic concepts might have served as a block to his learning of the proper concepts. The second was that the program presented too much content in too little time. The first difficulty would not occur with young children just being introduced to arithmetic. It is not possible to say that the second difficulty could be avoided.

In listening to the recordings made during the remedial sessions, two broad generalizations materialized. The first was that every member of the group at one time or another seemed to discover at least one mathematical concept with true spirit. The second was that every pupil in the group also revealed himself to be truly confused about many of the manipulations he made.
Evidence of Understanding

The first generalization was encouraging. Moreover, the observation of the manifestations of the discoveries made gave the instructor a real feeling of satisfaction. One boy had been introduced to the Unit Box and had filled it. His reaction indicated understanding and enjoyment. Later he was introduced to the 9-Case and after working with it for a while, shouted, "Oh, I see, everything adds up to nine. No matter what you put in, it makes nine!" Immediately he went back to the Unit Box and applied his discovery to that box. He had no difficulty with the remaining Number Cases and seemed to be greatly pleased with himself as he continued his work.

One of the girls had built the 20-stair in the Twenty Case. When asked to point to the number 17, she pointed to the seven block in the upper half of the Case. The instructor asked if she had pointed to the whole number representation. She seemed uncertain, but after looking at the case for a moment, she said, "Oh, you have to put ten with it. All these numbers up here...(pointing to upper half of Case)...go with these tens down here." When asked to assemble blocks to make the number 27, she did so correctly. She had made a discovery that might eventually lead her to a better understanding of the tens basis of the number system.

On one occasion, the group was playing a game together. Starting with any number in the 90's on the Number Track, each pupil was supposed to move his marker nine units down the track according to the instructions shown on a die that each was casting in turn. With the excitement of the contest, the game was progressing rapidly. One boy was having difficulty. Since he could not subtract fast enough, he had resorted to counting nine units in a
downward direction in order to find his new position. As a result he was hopelessly retarded and afraid of losing the game because of missing a turn. In compassion, another boy breathlessly took the time to explain that the marker had only to be moved down 10 units and then up 1 unit. The second boy had discovered a new way to subtract. After verification to make sure the system did not deprive him full progress, the first boy learned the principle, and entered into the game with new confidence and much noise.

Another of the boys was intrigued by the Multiplication Machine. He tried a great many block combinations to make sure the correct products were given to multiplication facts which he already knew. He expressed surprise that the numbers could be placed in such a way that the right answer was always in the proper position. To understand this better, he took all the slides out of the machine and studied the table printed on the bottom of it. Although he did not discover all the relationships existing in the Pythagorean Table, he did notice that each vertical series of numbers had its counterpart in a horizontal row of numbers. When asked why this had to be, he said he didn't know. Later in the period he returned to the instructor. He had used two \( \frac{1}{2} \) blocks in the empty machine to show that the answer 2 had to be present twice, once when the blocks were placed horizontally and once when placed vertically. He offered this as an answer to the instructor's first question. When asked why some numbers were not repeated twice, like 25, he properly showed that five 5 blocks were "just as long one way as the other." This boy had proved to himself that \( \frac{1}{2} \times 2 \) had to equal \( 2 \times \frac{1}{2} \). Moreover, he had what was probably his first introduction to the idea of squares. With more experiences of this kind, the boy might be led to the commutative law. His discovery was a good beginning for such a generalization.
tion.

One of the girls was working with fractions. She had all the fraction materials to herself and had laid all the frames out, one above the other. She had discovered the concretization of a fact she already knew, that 1/2 equalled 2/4. When asked what other fractions just equalled 1/2, she gave the answer 5/10. This, too, must have been a part of her knowledge. After placing the five 1/10 plates in the frame and comparing the results with what she had done, she suddenly exclaimed, "They're all the same size!" Quickly and easily she placed the three 1/6 plates and the four 1/5 plates in their place. This girl was beginning to see the importance of measurement in number work. In time, the word "denominator" might come to mean more to her than a long, hard-to-spell word.

There were, of course, other examples that tended to show the presence and spirit of discovery. No doubt, too, there were some that never got verbalized. The little that has been said, however, is sufficient to show that the children were able to learn from the use of the materials.

Evidence of Confusion

The second generalization made, which, in essence, was that the children did not always understand the materials, was discouraging. Even out of context, the above samples of discovery display the limitations of the derived learning. Even though the boy in the first example knew that everything in the 9-Case added up to nine, did he realize that no other whole numbers could be made to add up to nine? It was doubtful. There was nothing in the nature of the materials to suggest it to him. Perhaps, to him, the 9-Case was a "magic box," forced, like a magic square, to add up to nine horizon-
tally and vertically. Similar remarks could be made about the other examples of discovery.

Of course, the trouble is that the child must be made thoroughly aware of the fact that the materials are mathematically consistent. With this awareness, in the case cited above, the boy is in a better position to understand that no other whole numbers can be made to add up to nine. Then, too, the child must be led to generalize wherever possible. If 9 can be subtracted from a number by subtracting 10 and adding 1, so can 8 be subtracted from a number by subtracting 10 and adding 2. Again, the materials do not suggest that this kind of generalization is important. Even so, these are not serious problems, and it is possible that with proper use of the materials these difficulties would not have to exist.

There were, however, more emphatic examples of the children's inability to understand the materials. Although one girl had, with guidance, successfully filled the Unit Box with all the combinations of 10, when asked to do the same for the 9-Case, she supplied herself with a great many blocks and filled it as if she were doing a jig-saw puzzle. "Look," she said, "I filled every hole." Yes, she had, but some blocks ran vertically, and some did not. With zest and enthusiasm she had built some sort of number abstraction which even the instructor could not easily rationalize. The importance of guidance when working with the materials is, thus, illustrated. The manipulations that are possible with the materials are unlimited, but some of them do not make sense. The child has to be led to make the proper moves and to draw the proper conclusion.

One boy was asked to demonstrate for the group how 5 could be subtracted from 43 on the Number Track. He placed three single cubes at the
beginning of the Track, and then 4 tens. Then he said, "I can't take 5 away from this 10, so I make it into two 5's and take one of them away. The answer... (looking at the numerals alongside the Track)... is 35." He had obtained the right answer with the wrong procedure. It was almost as difficult to make him see why he was not altogether right as it would have been to relate his faulty procedure to borrowing in the abstract subtraction algorithm.

Another boy was asked to use the Dual Board to find out how many 8's there were in 59. He could not even begin since he did not know what strip to place in the larger compartment of the board. After he had been led to select the strip with the multiples of 8, he was still confused. He tried to build the number 59 by using five 10 blocks laid crosswise, and 9 single cubes placed in the open section above the 10 blocks. With much questioning from the instructor and many suggestions from his companions, he was finally able to complete the project.

The Dual Board, especially, caused difficulty. A girl was using it to add 23 and 18. She placed two 10 blocks in the larger compartment, and three single cubes in the smaller compartment. Then she placed another ten block in the larger compartment, and seven of the remaining eight single cubes in the smaller compartment. "Seven and 3 make 10," she said, "and I have one left over, so I put it here." (Placing it in the larger compartment with the three 10's) "The answer is 31." Of course, it was easy enough to rectify the mistake by pointing out that the 10 single cubes had to be made into a 10 block, and the one remaining cube placed in the unit column, giving an answer of 41. But the child had made the mistake and was, consequently, not too sure that the materials were as good as they should be.
One of the boys was trying to add 1/2 plus 1/3. He simply put a 1/2 fraction plate and a 1/3 fraction plate in the frame for thirds. He was disturbed that no answer emerged. He knew that 2/3 could not be the answer and finally settled for "bigger than 2/3" as the result of his fractional addition. After the boy had been persuaded to find the frame that would give him an exact answer, and after he had been made to see why the 1/6 frame had to be used, his confusion subsided, and he had less difficulty with the rest of his work in fractions. It would have been interesting to see what his reaction would have been if the instructor had not been present to guide his thinking.

There were other examples of confusion and doubt, but these that have been given will suffice to help prove the point that the children did not always understand the materials.

There may be advantages, however, to the child's confusion over the materials, if it is interpreted in a special sense. If 100% mastery of number concepts is the goal of the remedial program, the instructor needs no further test of the lack of this mastery than the observation that the child cannot handle a new process with the materials. If, in the case cited above, the boy does not build the number 4/3 in the Number Track as it should be built, he obviously needs more work with the structure of the higher decade numbers. He should not be allowed to subtract 5 from 4/3 until he has mastered the make-up of the numbers with which he is to work. Since the materials are apparently mathematically consistent, they should be introduced to the child slowly and carefully. The program cannot skip along touching the high spots. It must be systematically organized.
Furthermore, if the pupil has a meager store of mathematical concepts, he will need to learn more than he could possibly learn in twenty 20-minute remedial sessions. Therefore, if the viewpoint is taken that remedial work should cause arithmetic progress, whether it be accompanied by comprehension or not, the Stern methods are, perhaps, not as useful as other procedures. If the viewpoint is taken that remedial work should establish thorough understandings and mastery, regardless of the time consumed, the materials may prove to be an excellent remedial technique.

This section has given a few of the more important findings from the wire recordings made during the remedial sessions. They have been given in the form of interpretations of the remarks made by the pupils while working with the Stern materials and, as such, have been subjective. Subject to the limitations noted, these interpretations were made in order to discover the adequacy of the materials for sixth-grade remedial work.

Analysis of the Results of the Post-Program Testing

This section, the last of this chapter, will attempt to analyze the gains in arithmetic made by the remedial group. The gains of each pupil will be discussed and compared to the average gain of the class as a whole. Also, the average gain of the group will be compared with the average gains of the class.

It would be well to keep several points in mind. First, none of the tests given to the group or to the class exactly paralleled the time-span for the remedial group. Second, the remedial group participated in their regular arithmetic class during the whole of the remedial program. Both of these points should cause caution when attributing the gains made by the
remedial group to the use of the Stern materials alone. Third, the remedial group, and the class, were small, selected groups. Therefore, the statistics in this section cannot be considered completely valid. The fourth, and last point is a corollary of the third point. Since the group was small and selected, no probable errors or levels of significance could be given.

At the very off-set, these limitations were so severe as almost to nullify any conclusions that might be drawn. It was, therefore, necessary to make an assumption which would allow conclusions of some sort to emerge. If pupils who are in the lowest quartile of their class make progress which nets them more gain than the class average gain, it may be assumed that at least some of the gain in achievement has been affected by the remedial program. In a more general form, the assumption is that pupils can be expected to maintain a steady rate of learning. If the rate of learning increases, some extraneous factor, such as a remedial program, may well be the cause. The review of remedial research suggested that remedial pupils may improve, but grade norms may not necessarily be reached. The assumption is a generalization of this suggestion.

**Individual Gain**

Table 7 gives the scores and gains made by each of the five remedial pupils on the pre- and post-program Stanford Achievement Tests in arithmetic computation, reasoning, and average arithmetic, and on the non-standardized concept test. Also given are the arithmetic means and gains for the whole class of 30 on the same four scores. Better comparisons could have been made if the 5 pupils in the remedial group had been compared with the remaining 25 members of the class. However, since the 5 remedial group members were chosen on the basis of criteria which acted to place their scores in
the lowest quartile of 30 pupils, the gains made by each group member were also compared to that same class of 30. If the remedial group members' gain equals the average class gain in any arithmetic score, it can be said that at that relative rate of gain through the elementary grade, their scores could not have fallen in the lowest quartile of the class scores.

With so many variables out of control, the scores and averages were not considered significant to more than 2 decimal places, and were rounded to the nearest tenth.

Table 7

Scores on the Preliminary and Final Tests, Showing Gain Made, for Each of the Five Remedial Pupils and the Class

<table>
<thead>
<tr>
<th></th>
<th>Pupil No. 1</th>
<th>Pupil No. 2</th>
<th>Pupil No. 3</th>
<th>Pupil No. 4</th>
<th>Pupil No. 5</th>
<th>Class Ave.</th>
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<tbody>
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<td><strong>Computation</strong></td>
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<td>6.1</td>
<td>5.9</td>
<td>5.9</td>
<td>7.1</td>
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<td>4.4</td>
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<td>4.4</td>
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<tr>
<td>Preliminary</td>
<td>5.2</td>
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<td>5.0</td>
<td>5.2</td>
<td>5.1</td>
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<tr>
<td>Final</td>
<td>7.6</td>
<td>6.2</td>
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<tr>
<td>Gain</td>
<td>2.4</td>
<td>1.1</td>
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<td>7.7</td>
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<tr>
<td>Gain</td>
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<tr>
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<td>33</td>
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<tr>
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<td>25</td>
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<td>18</td>
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<tr>
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<td>5</td>
<td>23</td>
<td>10</td>
<td>2</td>
<td>5.9</td>
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The figures in Table 7 show that pupil 1 bettered the average class
gain on all scores but computation. Since he gained but .2 of a year in
computation, while the class gained .4, most of his average arithmetic in-
crement was a reflection of his exceptional gain in arithmetic reasoning.
Although his reasoning norm was .4 of a grade behind his computational norm
at the start of the program, he gained 2.4 years in reasoning, placing this
score more than .6 of a year beyond his grade level, and giving him a
better post-program score in reasoning than in computation. A gain of 2.4
years in any subject may be considered excellent achievement. The whole
class made a normal year’s progress in reasoning. In average arithmetic,
pupil 1 increased his achievement by 1.1 years, while the class gained .7
of a year. On the concept test, he gained almost twice as much as the
class average gain; his increment was 11, and for the class it was 5.8.

The class surpassed their grade norms on all tests. Pupil 1 surpassed
the grade norm on only one test—the post-program arithmetic reasoning test.
His post-program average arithmetic grade placement, however, fell only
about .2 of a year behind his grade level. He still needed help in compu-
tation. His computation score after the program was at least a year below
his grade level. On the concept test, pupil 1 narrowed the gap between
his score and the class average from 11 points on the pre-program test to
5.6 points on the post-program test.

Pupil 1 made the best showing in reasoning and concepts. His I.Q. of
124 may have been responsible for this. If so, it could be said that the
materials would be used to best advantage with the higher I.Q. pupils.

Pupil 2 bettered the class gain on only one test, the arithmetic
reasoning test. However, on all other scores he lacked no more than .1 of
a year or a point of equalling the average class gain. On the computation
test his increment was .3 of a year, while for the class it was .4 of a
year; on the reasoning test his was 1.1 years, while the class’s was 1.0
years; on the average arithmetic score, his was .6 of a year, and the class
made .7 of a year; on the concept test his gain was 5, while the class
gained 5.9. His gain of more than a year in reasoning was encouraging,
especially considering his low I.Q. scores.

Pupil 2 did not make grade norms on any test, but, then, he would have
had to gain about 1.5 years on all scores to do so. Although his grade
level at the time of the tests was about 6.8, his post-program score for
computation was 5.8, for reasoning, 6.2, and for average arithmetic, 6.0.
Like pupil 1, pupil 2 brought his post-program reasoning score above his
post-program computation score. At the start of the program, the situation
was reversed. At the end of the program, pupil 2 was still about 17 points
behind the class average on the concept test.

Pupil 3 bettered the class average gain on all scores. In computation
he gained .5 of a year and the class gained .4 of a year; so he bettered
the class gain by but .1 of a year. In arithmetic reasoning he gained 1.6
years, or .6 of a year more than the class. In average arithmetic he
gained one year, or .3 of a year more than the class. On the concept test,
pupil 3’s increment was 4 times as large as the average class increment.
It would be difficult to say that pupil 3 did not benefit from the program.

Even so, pupil 3 did not achieve grade level on any test. In computation
he made a score of 6.1, .7 points below grade level. On the reasoning
test he achieved at the 6.6 level, only .2 of a year below grade level. His
average arithmetic score was 6.4, .4 points below grade level. Like pupils
1 and 2, pupil 3's post-program reasoning score surpassed his computation score while before the program such was not the case. Where his pre-program concept test score was 26 points below the class average, it fell only about 3 points below after the test.

Pupil 3 made a better showing on all scores than any other remedial group pupil. As pointed out in his case history, he seemed to attack the program with the most seriousness and determination, and this may be associated with his success.

Pupil number 4 did not better the class average gain on any score except the concept test score. In computation she just equalled the class gain with an increment of \( \frac{1}{4} \) of a year. In reasoning, however, she gained only \( \frac{1}{6} \) of a year, or \( \frac{1}{4} \) of a year less than the class gained. Her average arithmetic gain was \( \frac{1}{5} \) of a year, \( \frac{1}{2} \) less than for the class. Yet on the concept test she gained 10 points, slightly less than twice as much as the class gained. Pupil number 4 probably made her normal progress, and it, therefore, seems that she was not greatly affected by the remedial program.

Pupil 4 did not make grade level on any score. Even after the program she was just about a full year below grade norm on all the standardized tests. Where her reasoning score was \( \frac{1}{3} \) of a year behind her computation score before the program, it was only \( \frac{1}{1} \) below after the program, but this difference cannot be highly significant. Also, after the program her concept test score was still more than 25 points below the class average, even with her better than average gain on this test.

Pupil 4 did not respond well to the program. She was not stimulated by it. Moreover, the instructor did not understand the pupil, nor know what had to be done to interest her. This may have attributed to her poor showing.
Pupil number 5 did not better the class average gain on any score. On the computation test she fell .1 of a year behind the class gain of .4 of a year. In reasoning she fell .3 of a year below the class gain of 1 year, with a gain of .7 of a year. Her average arithmetic gain was .5 of a year, .2 of a year below that for the class. The class gain on the concept test, 5.9 was almost 3 times better than the gain made by pupil number 5. Like pupil 4, this pupil probably made her normal progress and was, therefore, apparently not much affected by the remedial program.

Pupil 5 did not achieve grade level on any score. Again, like pupil 4, she was almost a year below grade norm on all standardized scores. Whereas her pre-program reasoning score was one half of a year below her computation test score, her post-program reasoning score was only .1 of a year below her computation score. This, too, cannot be considered highly significant. Her post-program concept test score was almost 13 points below the class average, while before the program it was only 10 points below the class average.

As pointed out in her case history, pupil 5 was bothered by her lack of social adjustment. The writer was convinced that if this problem could have been overcome, pupil 5 could have made gains which were commensurate with her Kuhlman-Anderson I.Q. score of 111.

The Group and the Class

Table 8 shows the arithmetic means for the scores and gains made by both the remedial group and the thirty members of the class in computation, reasoning, average arithmetic, and on the concept test. It is not always wise to take averages for such a small group, but this procedure has been
followed in order to compare the remedial group as a whole to the class as a whole.

Table 8
Mean Scores and Gain for the Remedial Group and the Class

<table>
<thead>
<tr>
<th></th>
<th>Class Gain</th>
<th>Remedial Group Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>6.7</td>
<td>5.6</td>
</tr>
<tr>
<td>Final</td>
<td>7.1</td>
<td>5.9</td>
</tr>
<tr>
<td>Gain</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>7.3</td>
<td>5.1</td>
</tr>
<tr>
<td>Final</td>
<td>8.3</td>
<td>6.4</td>
</tr>
<tr>
<td>Gain</td>
<td>1.0</td>
<td>1.3</td>
</tr>
<tr>
<td>Ave. Arith.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>6.8</td>
<td>5.4</td>
</tr>
<tr>
<td>Final</td>
<td>7.5</td>
<td>6.1</td>
</tr>
<tr>
<td>Gain</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Concept</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preliminary</td>
<td>38</td>
<td>19.2</td>
</tr>
<tr>
<td>Final</td>
<td>43.6</td>
<td>29.4</td>
</tr>
<tr>
<td>Gain</td>
<td>5.9</td>
<td>10.2</td>
</tr>
</tbody>
</table>

As a group the remedial pupils did not gain as much on the computation test as the whole class did. The group gained .3 of a year, and the class gained .4 of a year. Neither of these figures was large. The remedial group was up to grade level neither before nor after the program, being about a full grade level below norm at the close of the program. The class, however, was beyond its grade level norm at both testings. Even despite the small gain made during the second computation test, the class was .3 of a year above its grade level.

The remedial group made a better than satisfactory gain on the arithmetic reasoning test. It gained 1.3 years while the class gained just one
year. The group still fell short of its proper grade level by about $\frac{1}{4}$ of a year. On the other hand, the class was a full year and a half above its grade level with an average of 8.3, even though it was above grade level on the first test. Both the class and the group made the best showing in arithmetic reasoning. Perhaps, as suggested in the review of remedial research, a developmental factor was at work, the effect of which was to cause large increments in arithmetic reasoning for all sixth-grade pupils. If this were so, the remedial group could be expected to make larger gains since their scores were lower at the start of the program, and the Stern materials would have to receive less credit for the remedial group gain.

Both the group and the class made the same increment on the average arithmetic scores, .7 of a year. The class, however, was above grade level by about .7 of a year, and the group was below grade level by about the same amount.

The group bettered the gain made by the class on the concept test by about 5 units. Still, they were about $\frac{1}{4}$ units below the class average. If concepts are the backbone of meaningful arithmetic, the remedial group was still weak in that area, but gaining at a greater rate than the class, possibly because of the use of the Stern materials.

The general pattern was that pupils $\frac{1}{4}$ and 5, the two girls, achieved what might be considered their normal rate without the benefit of the program. Pupil 2 made good gain in arithmetic reasoning, and pupils 1 and 3 did well in arithmetic reasoning and on the concept test. The group as a whole made the largest gains in arithmetic reasoning and conceptual arithmetic. All five members of the remedial group gained at a greater rate
on the reasoning tests than on the computational tests. This should mean
that the remedial materials were effective to some degree since their
meaningful basis would tend to affect these scores more than the others.

The matter of retention cannot be adequately considered. The concept
test was given six and one-half weeks after the program ended, but it was
not standardized. It can be said, however, that since the group made almost
twice as large a gain as the class did on this test, neither the remedial
work nor the lapse of time seriously affected the pupils' scores. In fact,
better measuring devices and more rigid procedures may show that with time
the concepts are transferred to computational and reasoning arithmetic,
causing better scores in these areas than might have been achieved imme-
diately after meaningful learning.

The close of this section ends the chapter that dealt with the analysis
of the results of the experiment. The next chapter, which is the last,
will summarize the more general conclusions of this study, and give the
implications for further study which have been suggested throughout this
work.
Chapter V

SUMMARY AND CONCLUSIONS

Only the more general conclusions and implication for further study will be given in this chapter. The assumptions and limitations which affect these conclusions have been directly stated, or implied, throughout all the paper. They are as much a part of the conclusions as the conclusions themselves. It is therefore, recommended that the summary given in this chapter be used as a guide to the location of the conclusions in their proper context.

Since there were two purposes to this study, to evaluate the Stern materials in a sixth-grade remedial program, and to discover some of the research problems connected with the experimental study of concrete materials, there will be two types of conclusions. These have been given in separate sections. The implications for further study have been grouped together and appear in a single section.

Conclusion Related to Further Study of Concrete Materials

1. The most general statement that can be made about future studies of learning that employ concrete materials is that they should be genetic studies. Some of the reasons for this statement are these:

   a. Concrete materials are too costly and cumbersome to put into the hands of a large sample of pupils.

   b. Work with concrete materials demands constant supervision if mere play is to be avoided.
c. As a result of the first two points, the usual large-sample statistical techniques are not feasible, and case histories must therefore be used.

d. Work with good concrete materials is not like drill work. It is more meaningful. It teaches deep-seated concepts which cannot be learned adequately over short periods of time.

e. Concrete materials are normally designed to supplement developmental phases of the child's growth. The genetic approach takes full advantage of such growth.

2. If pupils are to be subjected to short exposures of concrete materials, the more mature mind or more intelligent child should be used for study.

3. Pupils should be carefully prepared for their introduction to concrete materials. Some of them have a psychological aversion to what they consider "immature plaything."

4. Great care should be taken to carry the concrete work through the transitional stage to the abstract arithmetic algorithm.

5. Since an educational history of mechanical learning frequently produces blocks to meaningful learning, the subjects for experimentation should be chosen with care. Very young children should make good subjects for the systematic presentation of concrete materials.

Conclusions Related to the Materials and the Remedial Work

1. The Stern Materials

   a. If purchased, the materials would be expensive; if constructed, much labor would be entailed.

   b. Storage facilities should be provided for the materials.

   c. When used over a short period of time, some discovery, but more confusion, will result.
d. The materials are not of a too juvenile nature for sixth-grade pupils.

e. The materials can be made to substitute as a testing device.

f. Sixth-grade pupils will frequently be blocked in their attempt to learn with the materials because of their previously learned arithmetic knowledge.

g. If the development of concepts and not immediate skills is the goal, the materials are useful in a sixth-grade remedial program.

2. The remedial group

a. The group had normal reactions to their teacher, school and arithmetic.

b. Psychological maladjustment as well as inability to learn arithmetic were operative.

c. Two pupils were not significantly affected by the use of the materials in the remedial group.

d. The remedial work did not affect computational achievement.

e. As a whole, the group made significant gains in conceptual and reasoning arithmetic.

Implications for Further Study

1. More experimental study should be made of the use of concrete materials in the learning process. The following aspects, especially, deserve attention:


b. Materials that use a measurement approach.

c. The place of concrete materials in demonstrating and re-teaching arithmetic concepts to adults.

d. The relation of concrete materials to developmental phases of mental and physical growth.
2. A philosophical study should be made in order to synthesize the use of concrete materials with the "functional" and "systematic" schools of thought.

3. Work should be done to develop measuring devices that are in accord with the outcomes of meaningful and concrete learning. Group concept tests and diagnostic concept tests are needed.

4. A well-controlled, genetic study of the Catherine Stern materials used with children above the primary grade should be made.
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Appendix A

ILLUSTRATIONS OF THE MATERIALS
1. The Unit Box, showing all combinations of ten.

2. The 20-Stair in the 20-Case.
3. Two sections from the Number Track, showing 6 subtracted from 14.

4. The Multiplication Machine, showing 8 times 7.
5. The Dual Board, showing that there are six 8's in 51, with 3 left over.

6. Four of the Fraction Frames, showing 1/3 equals .33 equals 3/9 equals 2/6.
7. The Number Cases nested.

8. The Dual Board with its strips.

9. The carrying case for the blocks.

10. The carrying case for the fraction plates.
Appendix B

AGE, SCORES AND GAIN FOR THE THIRTY PUPILS IN THE CLASS ON ALL TESTS
Table 8  
Age, Scores and Gain for the Thirty Pupils in the Class on all Tests

<table>
<thead>
<tr>
<th>Pupil</th>
<th>Age</th>
<th>I.Q.</th>
<th>Ave. Arith.</th>
<th>Computation</th>
<th>Reason</th>
<th>Concept</th>
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<td></td>
<td></td>
<td>K-A S-E 1st 2nd</td>
<td>△ 1st 2nd</td>
<td>△ 1st 2nd</td>
<td>△ 1st 2nd</td>
<td>△ 1st 2nd</td>
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<td>-.8 7.2 7.8</td>
<td>.6 50 55</td>
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<td>.5 7.6 7.0</td>
<td>-.6 9.0 10.9</td>
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<td>.7 9.3 8.5</td>
<td>-.8 7.4 9.5</td>
<td>2.1 41 49</td>
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<td>11-6</td>
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<td>-.0 6.4 7.0</td>
<td>-.8 7.4 6.8</td>
<td>-.6 41 57</td>
</tr>
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<td>11-9</td>
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<td>.6 8.5 9.8</td>
<td>1.3 38 47</td>
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<td>-.9 7.0 7.2</td>
<td>-.2 5.9 6.6</td>
<td>.7 17 32</td>
</tr>
<tr>
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<td>11-4</td>
<td>102</td>
<td>5.9 6.4</td>
<td>-.5 6.2 7.2</td>
<td>1.0 5.5 5.8</td>
<td>.3 19 31</td>
</tr>
<tr>
<td>26</td>
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<td>127</td>
<td>6.8 6.8</td>
<td>0.0 6.2 6.8</td>
<td>.6 7.4 6.6</td>
<td>-.8 38 42</td>
</tr>
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<td>27</td>
<td>11-8</td>
<td>121</td>
<td>5.8 5.8</td>
<td>0.0 5.9 6.5</td>
<td>-.3 5.5 5.8</td>
<td>.3 36 40</td>
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<td>28</td>
<td>11-8</td>
<td>123</td>
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</tbody>
</table>

Ave. 11-9 111 6.8 7.5 .7 6.7 7.1 .4 7.3 8.3 1.0 38 436 5.9
Appendix C

THE CONCEPT TEST
1. Read the problems and follow the directions carefully.
In problems 2 through 9, write the answer in the blank
space given for each problem.

1. Blacken lines b, c, d, and e to make them as long in
comparison with line a as you are told to make them.
Look at the line in the example. It has been made
1/2 as long as line a.

Example

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make b 5/6 as long as a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make c 3/6 as long as a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make d 2/3 as long as a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Make e 6/6 as long as a</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Write the largest 5 place number you can. __________
Without using 0, write the smallest 5 place number
you can. __________

3. Using the figures in the number 82,513, write the
largest number you can. __________
Use the same figures to make the smallest number
you can. __________

4. 682 equals 500 plus________
682 equals 602 plus________
682 equals 600 plus 80 plus________
682 equals 400 plus 200 plus________
682 equals 500 plus 200 less________
5. Write the 2 numbers that come next when counting by:

<table>
<thead>
<tr>
<th>Place</th>
<th>Digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>10's</td>
<td>36</td>
</tr>
<tr>
<td>100's</td>
<td>736</td>
</tr>
<tr>
<td>1000's</td>
<td>58336</td>
</tr>
<tr>
<td>10,000's</td>
<td>58336</td>
</tr>
</tbody>
</table>

6. Write the number that has 5 tens, 90 hundreds and 3 ones.

7. Write the following numbers in order of size, from smallest to largest:

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
</tr>
<tr>
<td>.5</td>
</tr>
<tr>
<td>26</td>
</tr>
<tr>
<td>1/3</td>
</tr>
<tr>
<td>39</td>
</tr>
<tr>
<td>57</td>
</tr>
<tr>
<td>834</td>
</tr>
</tbody>
</table>

Write the following numbers in order of size, from largest to smallest:

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>096</td>
</tr>
<tr>
<td>.96</td>
</tr>
<tr>
<td>111</td>
</tr>
<tr>
<td>420</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>.071</td>
</tr>
<tr>
<td>93</td>
</tr>
</tbody>
</table>

8. Express the idea "3 out of 6"
   as a fraction
   as a decimal

9. Counting by 3's, how many numbers will there be from 3 through 99?
II. Read the directions very carefully and then put a circle around the correct answer.

1. Circle the number with the largest figure in the 10's place.
   a. 907,  b. 8,  c. 42,  d. 363

2. Which number is about 5 times as large as 190?
   a. 500,  b. 1000,  c. 9000,  d. 4500

3. Which is the shortest?
   a. 1/4 yd.,  b. 1/6 yd.,  c. 1/5 yd.

4. When the decimal point is removed from .93, that number is:
   b. Divided by 100.
   c. Multiplied by 100.
   d. Divided by 10.
   e. Unchanged.

III. Some of the following statements are true and some of them are false. If you think the statement is true, put a circle around the T. If you think it is false, put a circle around the F.

T F  Example. 19 is greater than 91.

T F  1. 0 times a number equals that number.

T F  2. When numbers other than 0 are added, the answer is greater than any of the numbers added.

T F  3. When whole numbers are divided by whole numbers, the answer is always greater than the number divided.
T  F  4. If a number is multiplied by 1, that number is increased.
T  F  5. If 0 is subtracted from a number, that number is decreased.
T  F  6. If a fraction is multiplied by a number greater than 1, that fraction is increased.
T  F  7. If the same number is added to both parts of a fraction, the value of that fraction is unchanged.
T  F  8. The fractions 1/2 and 1/4 can be added without first changing them.
T  F  91 The larger the measure you use, the more times you use it in finding the amount of water in a barrel.
T  F  10. Six place numbers are always larger than five place numbers.

IV. In the following problems, you do not need to do the examples to find the right answer. In each of the problems 1 through 5, put an X under the example which will give the largest answer, and a 0 under the example which will give the smallest answer. Look at the example before you start.

Remember:
X for the largest answer  0 for the smallest answer

Example:  5 plus 3   5 plus 9   5 plus 7   5 plus 2

1.  1903   1903   1903   1903
   X 42   X 101   X 61   X 99

2.  2975713   1.0175713   1375713   975713

3.  8275   8275   8275   8275.00
   -413   -99   -12   -107

4.  1/2 X 3/2   1/2 X 1/9   1/2 X 8/4

5.  1/3 plus 2/3   1/3 plus 3/2   1/3 plus 1