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BOSTON UNIVERSITY

WHEELOCK COLLEGE OF EDUCATION & HUMAN DEVELOPMENT

Dissertation

INQUIRIES INTO LIBERATORY MATHEMATICS PEDAGOGY: CONVERSATIONS WITH CRITICAL EDUCATORS AND SCHOLARSHIP

by

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Submitted in partial fulfillment of the

requirements for the degree of

Doctor of Philosophy

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Leslie Dietiker, Ph.D. Associate Professor of Mathematics Education Urging all of us to open our minds and hearts so that we can know beyond the boundaries of what is acceptable, so that we can think and rethink, so that we can create new visions. I celebrate teaching that enables transgressions—a movement against and beyond boundaries. It is that movement which makes education the practice of freedom.

—bell hooks, *Teaching to Transgress*

DEDICATION

I dedicate this dissertation to educators who "see their action as a humble, yet essential, contribution to an extensive, collaborative, and enduring project of social change," who embrace "the responsibility we have to make sure that our actions contribute to the larger human goal of freedom for all" (Beauboeuf-Lafontant, 2002, pp. 83–84).

Thank you "for the bravery of your freedom in spaces of clear limitation. In spaces of muddy reality" (Gumbs, 2020, p. 40).

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This dissertation comes from many people. I extend gratitude to the participants who joined with me in research—for the generous spirit, time, and insight you offered through our collaborations; the relationships we'll continue to develop; and the "wading through" (Givens, 2021a) you do every day, struggling to bring a different world into being. By welcoming me into your practice, you helped me stay connected to the heartwork of teaching and learning. I hope these pages express the depths of what I've learned from you.

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vi

My work as an educator and educational researcher is inspired by my own teachers, from elementary through graduate school, who created classroom learning communities that practiced freedom: Louisa Cruz-Acosta, Liza Hernandez, Ali McKersie, Emily Moore, Eric Grossman, Sebastian Stoenescu, Lisa Smulyan, Frank Grossman, and Jennifer Lindsay. I studied your practice while a student in your classes and continue to channel your passion, creativity, and wisdom.

Support from my family sustains all that I do. Mom, Dad, Evan, and Andi—thank you for the unwavering love and encouragement that build me up in and beyond academics.

I close with a different kind of acknowledgement: of my reservations at having committed to a dissertation for the past two years. I'm grateful for all that participants, advisors, and peers have contributed to this piece of work; without them, it would not be. Still, the doctoral dissertation as a model of inquiry expects individuals to author consequential work largely on their own—a premise that I question, especially within fields that tout commitments to social justice and change. I regret that this project siphoned my intellect, energy, and creativity from more deeply collaborative endeavors, that I accommodated its individualist demands in many ways. Kimmerer (2013) reminds us that in flourishing "there are no soloists" (p. 15). Going forward, may the ideas and relationships seeded through this project grow toward inquiry that is fundamentally shared, honoring that we need each other when pursuing "big questions" (Lena, participating educator in study 1; Kate, participating educator in study 3).

vii

INQUIRIES INTO LIBERATORY MATHEMATICS PEDAGOGY: CONVERSATIONS WITH CRITICAL EDUCATORS AND SCHOLARSHIP ADA OKUN

Boston University Wheelock College of Education & Human Development, 2023

Major Professor: Beth Warren, Ed.D., Sylvia Earl Professor; Professor of Language and Literacy Education

ABSTRACT

This dissertation is a multi-part inquiry into the question, what could liberatory mathematics teaching and learning be? It works from an understanding of liberation as plural, collective, sociohistorically situated, radically imaginative, and practicable in the here and now (Combahee River Collective, in Taylor, 2017; Escobar, 2015; G. Gutiérrez, 1973/1988; Kelley, 2002; Walcott, 2021). Rather than pursue final or totalizing answers, the dissertation engages a question that holds infinite multiplicities (Martin et al., 2019). It includes three studies, two of which foreground the perspectives of K–12 educators; the third is an analysis of extant scholarship. All three center educational research and practice with explicit commitments to justice and liberation.

The first study profiles an elementary educator's reflections on teaching across school disciplines. Drawing on ethnographic data from a year-long collaboration, including observations, interviews, and video-based reflection conversations, the analysis explores a rift (Booker & Goldman, 2016) that this teacher experienced between liberatory pedagogy and school mathematics. The paper describes key facets of her liberatory praxis, which developed largely in the humanities, and considers challenges and possibilities of liberatory teaching and learning in mathematics. The second study is an integrative analysis (Torraco, 2016) of scholarship that takes critical perspectives on mathematics education. The paper brings extant literature from mathematics education and the learning sciences into conversation with ideas from Warren, Vossoughi, Rosebery, Bang, and Taylor's (2020) chapter, "Multiple Ways of Knowing: Re-imagining Disciplinary Learning." Warren and colleagues call educators and researchers across fields to pursue disciplinary learning that liberates from the EuroWestern normativity of academic disciplines and K–12 schooling. The paper highlights contributions, limitations, and future possibilities of critical mathematics education scholarship in light of this call.

The third study is an analysis of conversations with six mathematics educators spanning grade levels, roles, and institutional settings—who centered commitments to justice and liberation in their teaching. In one-on-one conversations, we discussed the educational histories, teaching experiences, and political values that shaped their praxis. The paper synthesizes themes across their accounts of liberatory mathematics teaching and learning, highlighting the prefigurative orientation these educators brought to pedagogy as they visioned and practiced freedom within systems that were far from liberatory (Bang & Vossoughi, 2016; Boggs, 1977; Givens, 2021a).

Across the studies, educators and researchers offer incisive critiques of mathematics education as a force for sociohistorical injustice and point toward its liberatory potential. Common themes from the three analyses include: a multi-scale political perspective on mathematics education; harm, healing, and social connection in mathematics; learning from life beyond school; and creative inspiration in mathematics.

ix

Themes unique to individual studies are: the microrelational work of liberatory pedagogy (Paper 1), turning a critical eye on mathematics as a discipline (Paper 2), and educators examining their own relationships with mathematics (Paper 3). Taken together, the studies suggest that mathematics pedagogy as the practice of freedom (Freire, 1973; hooks, 1994) can—and perhaps must—take multiple forms. These include critically navigating normative systems and seeking radical departures from them.

The dissertation concludes with directions for future inquiry in mathematics education and teacher professional learning, highlighting possibilities for critical collaborative study with educators.

TABLE OF CONTENTS

DEDICATIONv
ACKNOWLEDGMENTS vi
ABSTRACTviii
TABLE OF CONTENTS xi
LIST OF TABLESxvii
LIST OF FIGURES
CHAPTER ONE: PURPOSE AND OVERVIEW1
Introduction1
Overview of the Three Studies
Paper 1: A Teacher's Reflections on Liberatory Pedagogy Across Disciplines 5
Paper 2: Synthetic Analysis of Critical Scholarship on Mathematics Education9
Paper 3: Stories of Liberatory Pedagogy from Mathematics Educators
Interrelating the Three Papers17
Notes on Methodology19
Interanimating Studies
Theoretical Perspectives from Multiple Traditions
Caution in Looking Across Contexts
Reflective Conversations as a Methodological-Pedagogical Process
Humanizing Research Relations
Ongoing Methodological Reflection26
Additional Reflections on Researcher Positionality

Liberatory Possibilities	55
Situating This Paper	57
Methods	58
Origins of Partnership	58
Critically Reflexive Partnership	60
Context and Participants	62
Generating Data Sources	65
Data Analysis	71
A "Case" of Interest	74
Researcher Positionality	75
Findings	77
Liberatory Pedagogy	77
Reflections on Mathematics Teaching10	02
Possibilities for Liberatory Mathematics Pedagogy12	20
Discussion12	26
Systemic Layers of the Rift12	27
Towards Liberatory Mathematics Pedagogy12	28
Study Limitations	34
Conclusion	36
CHAPTER FOUR: TO CRITICALLY REIMAGINE MATHEMATICS LEARNING: A	4
SYNTHETIC LITERATURE ANALYSIS	38
Introduction13	38

Critically Reimagining Disciplinary Learning	140
Onto-Epistemic Heterogeneity	140
Entanglement in the Colonial Matrix of Power	141
Desettling and De-Linking	142
To Critically Reimagine Disciplinary Learning	143
Desettling Dominant Mathematics	144
Selecting Literature to Analyze	146
Critical Analysis of Power	146
Bring Seminal and Contemporary Works Into Conversation	147
Reflect Heterogeneity in Research	148
Analysis of Critical Literature on Mathematics Education	149
Examine Disciplinary Education's Entanglement in Empire	150
Attend to Intersections of Self- and World-Making in Disciplinary Learning	161
Critique and Refuse Disciplines' Onto-Epistemic Normativity	176
Explore the Multiplicity and Dialogicality in Disciplines	194
Re-Place Disciplinary Learning as People "Do Life"	209
Conclusion	229
Directions for Future Research	232
CHAPTER FIVE: "BREAKING MATH FREE": STORIES OF LIBERATORY	
PEDAGOGY FROM MATHEMATICS EDUCATORS	236
Introduction	236
Literature Review	238

"Liberatory" Perspectives on Mathematics Education	
Liberatory Education, Broadly	
Methods	
Participants	
Conversation Process	
Data Analysis	
Writing	
Researcher Positionality	
Findings	
Freedom From	
Freedom Toward: Describing Liberatory Mathematics Pedagogy	
The Work of Freeing	
Math Would Be: A Found Poem	
Discussion	
Limitations and Future Directions	
Conclusion	
CHAPTER SIX: PULLING THREADS	
Introduction	
Summary of the Three Studies	
Threads Across Studies	
Resonances	
Variations	

Directions for Future Inquiry	
Mathematics Education	
Teacher Learning	
Conclusion	
APPENDIX: GENERATING DATA SOURCES FOR STUDY 1	
BIBLIOGRAPHY	
CURRICULUM VITAE	

LIST OF TABLES

Table 1. Overview of the Three Studies	17
Table 2. Descriptive Information About Participants (Study 3).	
Table A1. Conversations with Lena	336
Table A2. Interviews with Children in Lena's Class	336
Table A3. Visits to Lena's Class	337

LIST OF FIGURES

Figure 1.	Themes of Lena's Liberatory Pedagogy	78
Figure 2.	Themes of Liberatory Mathematics Pedagogy	64

CHAPTER ONE: PURPOSE AND OVERVIEW

Introduction

Education as and for freedom is a notion with multiple meanings, histories, and agendas. It grows from various intellectual and educational traditions, each one of which also holds diversity and tension. These include but are not limited to Black liberatory education (Anderson, 1988; Du Bois, 1935/2017; Murrell, 1997; Payne & Strickland, 2008; Perlow et al., 2018; Shujaa, 1994; Woodson, 1933/2009); critical pedagogies (Duncan-Andrade & Morrell, 2008; Freire, 1970; Giroux, 1983, 1988; Shor & Freire, 1987); feminist pedagogies (Beauboeuf-Lafontant, 2005; Ellsworth, 1989; hooks, 1994; Weiler, 1991); decolonizing education (Bang, 2017; Cajete, 1994; R. Gutiérrez, 2017; Jacob et al., 2018; Richardson, 2011); and frameworks that build from these traditions, such as social justice, humanizing, and abolitionist teaching (e.g., Bartolomé, 1994; Brion-Meisels et al., 2010; Catone, 2014; Love, 2019; Muhammad, 2020; Salazar, 2013).

In common is a vision of education for self-determination, community thriving, and social transformation. Authors situate learning and schooling socioculturally and sociohistorically and call for education that not only provides access to opportunity within current societal systems but also works to change them. This vision is distinct from educational projects that further assimilation and reproduction of the status quo, as well as from notions of freedom that emphasize individual autonomy and opportunity apart from collective liberation. Further, liberatory¹ frameworks tend to unify education

¹ Educational theory and practice that centers the notion of freedom has been referred to as

of the mind with emotional, spiritual, and cultural development (Bartolomé, 2008; Beauboeuf-Lafontant, 2005; Cajete, 1994; Freire, 1998; Ginwright, 2016; Glanfield, 2016; hooks, 1994; Johnson et al., 2014; Ladson-Billings, 1995).

Liberatory perspectives have stronger roots in educational research in the humanities and social sciences than in scientific fields and have historically been treated as peripheral, if not counter to, mathematics in particular. The narrative of mathematics as a politically and culturally neutral domain has an entrenched history in both K–12 education and the professional discipline. Though critiques of the neutrality narrative trace back decades (Bishop, 1988; D'Ambrosio, 1985; Fasheh, 1982; Frankenstein, 1983; Joseph, 1987; Restivo, 1992), only recently have the sociocultural and sociopolitical dimensions of mathematics education become a more common subject of inquiry.

Over the past two decades, as educators and researchers increasingly attend to questions of power and (in)justice, the field of mathematics education has taken a "sociopolitical turn" (R. Gutiérrez, 2013; Martin, 2009a; National Council of Supervisors of Mathematics & TODOS, 2016). Sociopolitical perspectives regard mathematics education as fundamentally political, shaped by and also holding the potential to change power relations in society. Research and policy have focused attention on unequal access and achievement in school mathematics for historically marginalized groups (e.g., Berry et al., 2013; Moses & Cobb, 2001; National Council of Teachers of Mathematics, 2014a,

[&]quot;emancipatory education," "liberatory education," "education for liberation," and "education as the practice of freedom." I use "liberatory education" and "liberatory pedagogy," recognizing that in the U.S. context "emancipation" has historically connoted a legal status conferred by the state (Walcott, 2021). I use "freedom" and "liberation" synonymously. Later chapters discuss "education as the practice of freedom," which emphasizes the processual nature of freedom and the responsibility to manifest it in the present through daily practice (Freire, 1973; Givens, 2021a; hooks, 1994).

2014b; National Research Council, 2001). Much of this work centers normative measures of academic success, such as standardized test scores, course enrollment, graduation rates, and career pathways. Critical scholars underscore that, while success by these measures may be one avenue to empowerment, inclusion and attainment in mainstream mathematics can also cause harm, demanding that young people—particularly those with nondominant racial, linguistic, gender, and socioeconomic identities—assimilate to the cultural expectations of school mathematics (e.g., Buenrostro & Radinsky, 2019; Fasheh, 2015; R. Gutiérrez, 2008, 2019; Martin et al., 2019; Yeh & Rubel, 2020). Researchers have also pointed to the persistence of hierarchy and deficit-based framings in discourse about mathematics education, including within contexts focused on equity (Adiredja, 2019; Battey & Franke, 2015; Bullock, 2014; Louie, 2017, 2019).

Given the ways mathematics education sustains social inequalities, there is urgency for the field to not only illuminate manifestations of injustice but also pursue forms of teaching and learning that liberate from them. Referring to prior work on the sociopolitical turn, Gutiérrez (2022) proposes:

Uncovering our 'truths' in mathematics education (e.g., various ways that people have been marginalized, silenced, oppressed, and erased by structures, policies, and social interactions) is an important step in getting us to think more seriously about how we can get into right relations. That is what a sociopolitical turn catalyzed. But truth telling is an insufficient springboard for action because pointing out the problems we face does not guarantee that we know how to heal or bring back that which is erased or that which is not yet. For that, we must also

have our eyes on radical dreaming. (p. 386)

She writes that radical dreaming orients "toward ethical futures that are not yet here" (p. 381). This dissertation motions toward that not yet. Its three studies, in various ways, ask: what could liberatory mathematics teaching and learning be?

I articulate the question in the subjunctive mood ("could be") to signal that liberation invites new worlds and ways of being, not simply inclusion and incremental reform within existing systems. This distinction is crucial in mathematics education, where discourse around social change tends to emphasize the integration of marginalized groups into the spaces of those already in power (Martin, 2003, 2015; Sengupta-Irving & Vossoughi, 2019; Valero, 2008). Conversations about "equity" and "justice" often frame mathematical literacy as a form of power that allows for participation and mobility within existing educational, economic, and cultural systems (e.g., NCTM, 1989, 2014a, 2014b). While democratizing status-carrying skills is important, access paradigms can lose sight of questions such as, participation and achievement for whose purposes? They can overlook possibilities for more fundamental transformation of mathematics, schooling, and society (R. Gutiérrez, 2007b; Martin, 2019; Martin et al., 2019). To clarify, these are not mutually exclusive goals: as educators and authors in the dissertation's three studies suggest, participating critically within dominant systems can co-occur with efforts to explore beyond them (R. Gutiérrez, 2007a; Gutstein, 2006; Martin, 2019).

I elaborate on notions of liberation and liberatory education in Chapter Two, here noting that, since liberation is not a unitary concept, neither are possibilities for liberatory mathematics education. Collective liberation requires many simultaneous inquiries into

freedom—including, and perhaps especially, in places where it is assumed not to be pertinent or possible (e.g., school mathematics). This dissertation, itself multivocal, joins with extant scholarship that has considered meanings of liberation in mathematics education in various contexts and methodological traditions. While honoring multiplicity, I center critical perspectives—those which are sociopolitical (address power relations), structural (extend to institutions and systems), historicized (look across time and scales of activity), and intersectional (link multiple dimensions of identity). Criticality is discussed further in Chapter Two.

The rest of this chapter provides an overview of the dissertation's three papers, introducing the purpose, methodology, and key findings of each study and offering an initial statement on how the studies speak to one another. Then I share overarching thoughts on methodology across the studies. The chapter closes with an outline of the dissertation's remaining parts.

Overview of the Three Studies

This section provides an overview of the three studies. For each, I introduce the purpose, methodology, and key findings. Then I discuss how the three papers interrelate, a conversation that picks up in the dissertation's concluding chapter.

Paper 1: A Teacher's Reflections on Liberatory Pedagogy Across Disciplines

The first study is from a year-long collaboration with a 5th/6th grade teacher in which we reflected on her practice across school disciplines. Lena (pseudonym) used the term "liberatory" to describe her teaching, grounded in educational traditions that critique systemic injustice and position learners and educators as agents of social change

(Duncan-Andrade, 2009; Freire, 1970, 1987; Ginwright, 2016; hooks, 1994; Love, 2019; Swalwell, 2013). Paper 1 explores Lena's liberatory pedagogy, which developed across the humanities and extradisciplinary activity in her class, and the sense of disconnect she experienced from liberatory vision and practice in mathematics. The paper discusses key components of Lena's liberatory pedagogy, her perspectives on mathematics teaching, and tensions and possibilities in the relationship between them.

Methodology

In the paper's analysis, I borrow the notion of "rift" from Booker and Goldman (2016), who write of a separation between formal mathematics education and what is "valued and integral" to people's lives beyond school (p. 226). I examine the rift between Lena's perspectives on mathematics teaching and what was valued and integral to her liberatory pedagogy. The analysis asks: (1) What is the rift Lena narrated between her liberatory pedagogy and mathematics teaching when considering her work across school disciplines? How did she describe this rift in relation to: conceptions of mathematics, work with curriculum, relationships with children, and professional learning experiences? (2) What possibilities for liberatory mathematics teaching and learning surfaced through our conversations?

Lena taught at a small, independent, intentionally diverse K–6 school located in a large city in the northeastern United States. Children were grouped into mixed-age classes and looped with teachers for two years at a time. Lena taught the oldest group, equivalent to 5th/6th grade. At the time of the study, she had been in this role for five years and taught for seven. During the year of our collaboration (2020–2021), the school

convened in various settings to accommodate the COVID-19 pandemic, including an outdoor campus near the school's usual site, a public park, a local church, and the Zoom video conferencing platform. Due to pandemic precautions, my visits to Lena's class were limited to outdoor and Zoom settings.

The analysis for this paper draws on data sources from our larger collaboration, which included field notes, video and audio recordings, and transcripts from: conversations with Lena, observations of her class, and interviews with six of her students. The paper focuses on my conversations with Lena outside of class time, which included semi-structured interviews about her teaching philosophy and practice, videobased reflection conversations using recordings from my class visits, informal debriefs following each visit, unstructured conversations in informal spaces (e.g., walking outdoors), and member checking several months after the study period ended. Visits and conversations spanned subject areas and times of day in Lena's class. Our joint reflection often emphasized the microrelational aspects of teaching and learning (i.e., how Lena and children interacted and developed relationships with one another), which was of particular interest to Lena.

I used grounded thematic analysis to construct themes from 18 conversations with Lena (Charmaz, 2006; Glaser & Strauss, 1967; Shank, 2001, in Matthews, 2009). I also analyzed student interviews and class observations to triangulate the themes Lena raised. The paper refers to Lena's reflections on teaching as a "case," suggesting that an in-depth look at her reflections across school disciplines sheds light on the "particularity and complexity" of her pedagogy as well as questions of interest beyond her context (Stake,

1995, p. xi; Yin, 2012).

Key Findings

The paper presents themes in three sections: a description of Lena's liberatory pedagogy; her perspectives on mathematics teaching, emphasizing the rift with her liberatory vision; and "glimmers" of liberatory mathematics from our conversations. Lena described her liberatory pedagogy as politicized, joyful and creative, rooted in a sense of community and deep relationships, and honoring of her own personhood. Certain aspects of these themes carried over into mathematics in her class, specifically the ethic of knowing children as whole people and a critique of social injustice. Overall, however, Lena experienced a disconnect between liberatory vision and mathematics teaching. She spoke of entrenched hierarchy in the school discipline of mathematics, a divorce from curricular creativity, limited capacity to build class community, and less confidence in her professional training in mathematics. The third part of the findings shares moments from our conversations in which we considered mathematics learning as potentially liberatory: socially transformative, naturally emergent, embodied, creative, and community-oriented. The paper's discussion speculates about liberatory mathematics beyond Lena's context, inspired by the themes of her liberatory pedagogy.

Significance

Extant scholarship offers various perspectives on the relationships between mathematics education, (in)justice, and liberation. Many authors examine mathematics schooling as a force of exclusion and domination, directly at odds with liberation (e.g., Berry et al., 2013; Fasheh, 2012; Martin, 2019; Nasir & McKinney de Royston, 2013;

Skovsmose & Greer, 2012; Takeuchi, 2018; Yeh & Rubel, 2020). Others discuss efforts to challenge systems of injustice through mathematics teaching and learning, some in contexts beyond schools (e.g., Aguirre et al., 2012; Barajas-López & Bang, 2018; Frankenstein, 1983, 2013; R. Gutiérrez, 2018; Gutstein, 2003; Moses & Cobb, 2001). This work mostly focuses on mathematics in isolation from other disciplines. Additionally, accounts of teacher pedagogy tend to focus either on success stories or on challenges when educators are earlier in their development of sociopolitical perspectives on teaching. Paper 1 explores the perspective of an elementary educator who has a mature critical teaching praxis outside of mathematics. The paper examines her work across school disciplines to shed light on challenges and possibilities of liberatory pedagogy in mathematics.

Paper 2: Synthetic Analysis of Critical Scholarship on Mathematics Education

Paper 2 is a synthetic analysis of scholarship that centers critical perspectives on mathematics education. Torraco (2016) writes that synthetic, or integrative, literature reviews "tell a story" by analyzing extant scholarship and suggesting directions for future research in light of a conceptual framework that may be new to the literature under analysis (p. 419). While this paper presses on extant literature more than is typical for a literature review, the concept of "synthetic" or "integrative" aptly describes the analysis. I bring literature from mathematics education and the learning sciences into conversation with ideas from Warren, Vossoughi, Rosebery, Bang, and Taylor's (2020) chapter, "Multiple Ways of Knowing: Re-imagining Disciplinary Learning." Warren and colleagues call educators and researchers across fields to pursue disciplinary learning that

liberates from the EuroWestern normativity of academic disciplines and K–12 schooling. In the paper, I distill five principles of their call and highlight contributions, limitations, and future possibilities of critical mathematics education scholarship in light of these ideas.

Methodology

As Gutiérrez (2013) and others contend, researching critical questions "requires knowing multiple literatures outside the field of mathematics education and finding appropriate ways to draw upon them" (p. 57; also see Bullock, 2012; Larnell & Martin, 2021; Martin, 2009a). In this paper, I turn to a cross-disciplinary conceptual framework as a source of accountability and inspiration for those within the field of mathematics education. In their chapter, Warren and colleagues (2020) situate academic disciplines and K-12 schooling within the colonial matrix of power—a structure of cumulative, intersecting systems of oppression that organize people, traditions, and ideas in relations of domination and subordination (Mignolo, 2009; Quijano, 2000). In their call to liberate disciplinary learning from the colonial matrix, the authors invite researchers, educators, and youth to: (1) interrogate the colonial workings of disciplines, (2) attune to self- and relation-making in disciplinary learning, (3) critique and refuse settled forms of disciplinary knowledge and practice, (4) engage the multiplicity and dialogicality in disciplinary discourses, and (5) re-place disciplinary learning as people "do life" (Warren et al., 2020). This paper is organized into five sections, each guided by one of the principles above, which I present as calls to conversation for researchers and educators. My analysis asks: (1) In what ways does critical scholarship on mathematics education

interrogate the colonial matrix of power and open toward liberatory alternatives? and (2) In what ways could the literature further heed these calls?

I used a selection method that was a combination of "representative" and "pivotal," purposefully sampling works that illustrate or have seminally informed subsets of critical literature on mathematics education (Cooper, 1988). Three purposes guided my selection process: to foreground a critical analysis of power, bring seminal and contemporary works into conversation, and reflect theoretical and methodological diversity. The paper covers literature that addresses mathematics education's role in perpetuating and potentially disrupting sociohistorical systems of power; it does not include scholarship that analyzes teaching, learning, and schooling apart from their embeddedness in these larger systems. Across the paper, I bring seminal works into conversation with scholarship that has taken up and extended seminal authors' foundational ideas. In selecting contemporary literature, I included works that reflect the diversity of methods, contexts, and themes explored in critical scholarship on mathematics education, including from authors in the learning sciences and those who take international perspectives.

Key Findings

The paper discusses critiques, liberatory possibilities, and limitations in the literature under analysis. Authors offer critiques that trace colonial histories of mathematics as a discipline and school subject; analyze continuing sources of oppression in mathematics education, across scales of experience; and challenge narratives of a unitary, universal mathematics. They propose liberatory alternatives that leverage mathematics as a tool for critical social inquiry, enable individuals to construct selfdetermining identities as mathematics learners, and illuminate the heterogeneity of mathematical knowledge and practice across cultural communities. The paper also underscores where this literature stops short of liberatory possibilities. Limitations include a tendency to treat the broader discipline of mathematics as settled while focusing on transformation of school curriculum and pedagogy; a lack of attention, in classrooms and research analysis, to dialogicality among diverse forms of mathematics; and an underemphasis on youth as active participants in the critical interrogation and creation of mathematical knowledge.

Significance

With the sociopolitical turn of the past two decades, the field of mathematics education has increasingly foregrounded questions of power, justice, and liberation (R. Gutiérrez, 2013; Martin, 2009a; National Council of Supervisors of Mathematics & TODOS, 2016). While language of "equity" and "social justice" is prolific, the meanings of these terms vary across studies and places of practice. The principles of critical reimagining ground discourse on disciplinary learning in a historicized, global, and transdisciplinary perspective (Warren et al., 2020). Paper 2 seeks to clarify and expand critical conversations on mathematics teaching and learning by foregrounding this perspective. In addition to highlighting trends in extant scholarship that reflect the principles of critical reimagining, my analysis presses on the boundaries of this literature. The paper encourages readers to envision "a fundamentally new and different mathematics education" while honoring insurgent work taking place within the constraints of current systems (Martin, 2019, p. 469).

Paper 3: Stories of Liberatory Pedagogy from Mathematics Educators

Paper 3 is an interview-based study with six mathematics educators who centered commitments to justice and liberation in their teaching. In one-on-one conversations, participants shared the educational and professional histories and political visions that shaped their praxis. Their work spanned institutional settings, roles, age groups, and curricula, and their perspectives on liberatory mathematics teaching and learning varied. My analysis constructs a composite portrait, synthesizing themes across their accounts based on the following questions: (1) In what ways has mathematics teaching and learning been liberatory in these educators' experiences? As they have developed liberatory pedagogies, what freedoms, tensions, and questions have they encountered? (2) What are these educators' visions for what liberatory mathematics teaching and learning could be?

Methodology

This paper considers theories of liberation within and beyond the field of mathematics education. The analysis lifts notions of education as the practice of freedom (Freire, 1973; Givens, 2021a; hooks, 1994) to highlight the ways participating educators pursued liberatory possibilities while working within dominant systems. The participants in this study, who held between eight and 43 years of teaching experience, had taught learners from elementary through high school as well as in-service and pre-service teachers. They had worked in district public and independent schools, universities and

teacher residency programs, professional development and out-of-school youth organizations. They spoke to varied personal histories with school mathematics and a range of intellectual traditions that inspired their notions of liberatory education. Across this diversity, all participants voiced a clear analysis of sociohistorical forces of oppression in mathematics education and a sense of personal purpose in seeking liberatory alternatives.

I conducted one or two conversations with each participant. We followed a semistructured protocol, which included prompts to recollect past experiences with mathematics, reflect on opportunities and challenges of liberatory praxis, and imagine mathematics learning free from common institutional constraints. Our conversation process emphasized storytelling, and I offered my own reflections and recollections throughout. Data sources included field notes, video and audio recordings, and transcripts from the conversations. I analyzed transcripts using a variation of grounded thematic coding inspired by the reflexive "top down and bottom up" approach described by Erickson (2004). This involved a sustained, descriptive engagement with full transcripts before parsing the text to pull out themes.

The paper presents themes in three sections of qualitative analytic prose and a fourth section that includes a found poem constructed from participants' words. In search of "poetic methods" for qualitative analysis (Shotter, 1996), I listened for the beauty in educators' language and the poetic imagination (Kelley, 2002) in their stories of liberatory mathematics.

Key Findings

The first section of the paper's findings discusses the systems from which educators sought freedom: educators referred to pressures from mandated curriculum, the stratification of disciplinary content, accountability systems tied to standardized assessment, and hierarchical ways of labeling children—which they situated within broader sociohistorical systems of oppression. The next two sections address their experiences and visions of liberatory mathematics. Educators described venturing beyond boundaries of various kinds; centering learners' experiences and interests; supporting open-ended, collaborative inquiry; fostering deep social connection; critiquing power structures; and organizing physical space to reconfigure relations among teachers, learners, and disciplines. They emphasized that liberatory pedagogy involved simultaneous work within and beyond normative structures, the courage to invite others into liberatory practice, and critical reflection on their own experiences as mathematics learners. The paper's found poem, entitled "Mathematics Would Be," turns to the discipline more generally, opening up the who, where, and why of mathematics.

Significance

Prior research in mathematics education has conceptualized liberatory pedagogy in a variety of ways. Earlier work examines the epistemological assumptions of mathematics learning environments, expanding notions of how mathematical knowledge is produced and young people's roles in co-constructing its meaning (Gordon, 1978; Price & Ball, 1998). This work is not always clear on the sociopolitical contexts in which teaching and learning are embedded. Authors who situate teaching and learning

sociopolitically explore the diversity of mathematical practice around the world, the use of mathematics as a tool for critical social inquiry, and liberatory educational visions that reflect the needs and traditions of historically marginalized communities (Barajas-López & Bang, 2018; Davis, 2018; Frankenstein, 1983; Frankenstein & Powell, 1994; Glanfield, 2016; R. Gutiérrez, 2017; Gutstein, 2006; Martin, 2009a; Martin & McGee, 2009; Martin et al., 2019).

Martin and colleagues (2019) call for continued theorizations of liberatory mathematics education, "in infinite multiplicities" (p. 47; see also Martin, 2009a). They write about Black liberation, though the urge toward multiplicity, rooted in the particulars of human experience, is pertinent across contexts (McLaren & Lankshear, 1994). This paper contributes to that multivocality by sharing stories of liberatory teaching and learning from six mathematics educators variously situated within K–12 education in the United States. My analysis highlights the prefigurative orientation these educators brought to teaching as they visioned and practiced freedom within systems that were far from liberatory (Bang & Vossoughi, 2016; Boggs, 1977; Givens, 2021a).

Paper Title	Research Questions	Participants & Data Sources
 "But in Math": (1) What is the rift Lena narrated between he liberatory pedagogy and mathematics teachir when considering her work across school disciplines? How did she describe this rift in relation to: conceptions of mathematics, worl with curriculum, relations with children, and professional learning experiences? 	Lena, a 5th/6th-grade teacher; six children in her class	
	relation to: conceptions of mathematics, work with curriculum, relations with children, and	Field notes, transcripts, and video/audio recordings from class visits and
	(2) What possibilities for liberatory mathematics teaching and learning surfaced through our conversations?	interviews
To Critically Reimagine Mathematics Learning: A Synthetic Literature Analysis	(1) In what ways does critical scholarship on mathematics education interrogate the colonial matrix of power and open toward liberatory alternatives?	Critical research literature on mathematics education
	(2) In what ways could the literature further heed these calls?	
"Breaking Math Free": Stories of	(1) In what ways has mathematics teaching and learning been liberatory in these	Six mathematics educators
Liberatory Pedagogy from Mathematics Educators	educators' experiences? As they have developed liberatory pedagogies, what freedoms, tensions, and questions have they encountered?	Field notes, transcripts, and video/audio recordings from interviews
	(2) What are these educators' visions for what liberatory mathematics teaching and learning could be?	

Table 1. Overview of the Three Studies.

Interrelating the Three Papers

In various ways, the three studies speak to the question, What could liberatory

mathematics teaching and learning be? I begin with a story of one teacher and her

classroom in order to contextualize the dissertation's overarching inquiry in a local case

that raises critical questions about the relationship between liberatory pedagogy and

mathematics. The first paper ends by suggesting that the barriers and tensions Lena

experienced in mathematics were neither insurmountable nor simply peculiarities of her

individual situation. The synthetic literature analysis addresses the systemic layers (Booker & Goldman, 2016) of these tensions and discusses accounts of researchers, educators, and young people who have explored possibilities for liberatory mathematics. The third paper continues to consider liberatory possibilities, returning to a focus on K–12 teachers' pedagogy. Like Lena, the educators in this study expressed clear visions of what it means to teach toward justice and liberation, and they saw a specific place for that vision in mathematics.

Across the papers, educators and researchers offer incisive critique of mathematics education as a force for sociohistorical injustice and point toward its liberatory potential. Common themes from the three analyses include: a multi-scale political perspective on mathematics education; harm, healing, and social connection in mathematics; learning from life beyond school; and creative inspiration in mathematics. The studies also speak to one another's open questions. Papers 2 and 3 profile educators and scholars who share many of Lena's political commitments and have explored them in mathematics learning contexts. The empirical papers heed the literature synthesis' call for more stories of liberatory teaching and learning in action. They highlight the curricular decisions, microrelational work, culture building, and critical reflection required to enact liberatory values and disciplinary reimaginings in everyday practice. Papers 1 and 3 leave off with questions about the cross- and potentially trans-disciplinary nature of liberatory pedagogy, which resonate with the guiding conceptual framework of Paper 2 (Warren et al., 2020). In the next section, I discuss how the three studies relate methodologically.

Notes on Methodology

While each study had its own methodology, certain commitments and approaches cut across them. These include: letting the studies interanimate as they developed, drawing on theory from multiple intellectual traditions, analyzing across contexts, reflective dialogue with educators, a commitment to humanizing research relations (Bang & Vossoughi, 2016; Dance et al., 2010; Paris & Winn, 2014), and ongoing reflection on the tensions inherent in a researcher's role. I elaborate on each below and conclude with a statement on researcher positionality, though positionality is addressed throughout the section.

Interanimating Studies

The three studies did not develop linearly or in isolation. Through iterative processes of designing, analyzing, and writing, each study spoke to and shaped the course of the other two. I wrote the synthetic literature analysis (Study 2) first, based on a prior piece of work, grounding the dissertation's inquiry in principles and questions for the field of mathematics education. The collaboration with Lena, which began as I wrote Paper 2, animated these questions on a local scale that I got to witness personally. The question of "rift" (Booker & Goldman, 2016) emerged through my work with Lena and, in this way, Study 1 offered a kind of "problem statement" for the dissertation as a whole. Rather than deriving the dissertation's purpose solely from the literature analysis, as a conventional literature review or theoretical framework section might (Merriam & Tisdell, 2016), I articulated the what and why of the larger inquiry in part through Paper 1. Conversations with Lena informed what I asked of and saw in the literature in Study 2;

thus, the literature became a response to, not just a framing for, that empirical study.

I conceptualized the third study after analyzing data from the collaboration with Lena. The question of rift, while a necessary provocation, was not the dissertation's gravitational center: I wanted to ask what liberatory mathematics could be more centrally than the data sources from Study 1 supported. Study 3 was a chance to explore that question directly with mathematics educators. I held conversations with the six mathematics educators over several months, while also revising drafts of Papers 1 and 2. Returning to Paper 1 sharpened a sense of purpose as I crafted the protocol for, conducted, and analyzed transcripts from Study 3's conversations. Conversations about educators' experiences of freedom in mathematics also inspired a sense of hope as I added a section to Paper 1 speculating about liberatory mathematics in and beyond Lena's class.

Building the dissertation as a conversation highlighted that its inquiry is never finalized. Just as each study brought new life to the other two over successive iterations, every time I re-read the dissertation's chapters or share pieces of the work with others, new questions, connections, and possibilities emerge. I imagine that the interanimation will go on, and lead to further inquiry, as readers engage with these pages and I continue to collaborate with educators.

Theoretical Perspectives from Multiple Traditions

Throughout the dissertation, I turn to authors and intellectual traditions beyond the field of mathematics education and bring their critical perspectives to bear on questions of mathematics teaching, learning, schooling, and research. I touch on literature

20

from emancipatory pedagogies in the humanities (Freire, 1970; hooks, 1994), Black radical thought (Kelley, 2002; Givens, 2021a; Walcott, 2021), liberation theologies (Cone, 1970/2010; G. Gutiérrez, 1973/1988), decolonial theory (Maldonado-Torres, 2007; Mignolo, 2009; Quijano, 1992, 2000), and critical research methodologies (Bang & Vossoughi, 2016; Benjamin, 2016; Paris & Winn, 2014).

Educational researchers whose work unites liberatory politics and deep disciplinary learning call us to craft questions, tools, and methods that transgress disciplinary silos (Larnell & Martin, 2021; Vakil & Ayers, 2019; Warren et al., 2020). Bullock (2012) suggests that inter- and trans-disciplinary perspectives can expand methodological possibilities in mathematics education by exposing scholars to interpretive frameworks and methods of inquiry "that remain untapped in mathematics education research" (p. 34). Critical research in particular demands sociohistorical analysis and orientations to inquiry that are not common in—and sometimes regarded as counter to—the disciplines of mathematics and mathematics education (R. Gutiérrez, 2013; Larnell & Martin, 2021; Martin, 2009a). The call to transgress disciplinary boundaries in educational research runs across the dissertation's papers and resonates with the idea, proposed in the two empirical studies, of cross-disciplinary inquiry with teachers.

Caution in Looking Across Contexts

This dissertation inquires into liberatory mathematics in multiple contexts. It does not focus on a specific sociohistorical community's experiences of oppression and liberation, and only Paper 1 profiles one teaching and learning context in depth. This broad scope is reflected in each paper. The study in Paper 1 was situated in a racially, ethnically, and socioeconomically diverse classroom whose teacher was attuned to that diversity as she developed and discussed her liberatory praxis. Paper 2 discusses literature from multiple methodologies and learning contexts. Paper 3 shares the reflections of six educators who varied in their sociopolitical identities, educational experiences, and teaching settings.

In and beyond the dissertation, I am curious about the entailments of liberatory praxis within contexts that bring people together from different walks of life. This interest may stem from my own liberatory experiences in educational, faith, and political organizing communities that are intentionally diverse across race, ethnicity, language, social class, and age. In various ways, these experiences inspire the question, how do liberatory principles and practice develop when people commit to one another across difference?

I orient to such inquiry with caution, recognizing the pitfalls of undifferentiated conceptualizations of liberation (Weiler, 1991). Experiences of oppression and visions of liberation vary across communities differently situated with respect to sociohistorical power structures (Crenshaw, 1989; hooks, 1994; Taylor, 2017; Weiler, 1991). Looking across contexts could fall into the traps of political unclarity, homogenization, or erasure. While I have tried to highlight heterogeneity and particularity in my analyses, I recognize that the dissertation's broad analytic scope could be a limitation. Situating my own liberatory praxis, as a researcher and more generally, is an ongoing process and may require more political clarity than this current work expresses.

Reflective Conversations as a Methodological-Pedagogical Process

Reflective conversations with educators were a primary research method in the two empirical studies. Seidman (2006) writes that interviewing is rooted in an interest in and respect for other people's stories. I was curious to learn from teachers' stories as a window into the work of liberatory (mathematics) pedagogy. I also believe sharing stories is a way to humanize and liberate our relationships, in and beyond research. I prefer the term "conversations" to "interviews" because it better signals the mutuality and co-presence that characterized my interactions with educators (Brinkmann, 2013; Myers, 2011). In traditional research paradigms, interviewers take an ostensibly neutral stance, or a more reserved role, focusing on eliciting participants' views for the purposes of data collection. The conversations for this dissertation were dialogues, forms of joint inquiry in which both educators and I shared reflections, raised questions, and developed one another's thinking (Dennis, 2018; Pomerantz & Zemel, 2003; Vossoughi & Zavala, 2020).

Vossoughi and Zavala (2020) write of "interviews as pedagogical encounters," which hold the learning and development of both researcher and participant as a primary methodological purpose. In conversations with educators, I crafted questions and responses to "facilitate reflection aimed at new understandings," not simply to extract educators' already-formed ideas (Vossoughi & Zavala, 2020, p. 139; see also Byram, 1996; Pomerantz & Zemel, 2003). I raised probing questions, alternate framings, instances from class visits, or prompts to imagine differently, in an effort to complicate or expand perspectives that participants offered. I also expressed when my own ideas were changing through our dialogue. Conversations with Lena grew increasingly pedagogical as we built a reflective practice over the course of a school year, but I also documented pedagogical moments in the conversations with educators in Paper 3. In both studies, my invitations to imagine teaching and learning being otherwise (Benjamin, 2016) led to exchanges that felt opening and often reorienting.

Later in the dissertation I discuss the concept of praxis, entwined reflection and action on the world in order to transform it (Freire, 1970; G. Gutiérrez, 1973/1988). I see these research conversations with educators as a kind of joint praxis in which our reflection, imagining, and theorizing shaped possibilities for future action, in both teaching and research practice. Lena attested to the ways our conversations influenced choices she made or opportunities she saw in her work with children. Multiple educators in Paper 3 mentioned something they wanted to try or an idea they would continue to ponder in their work moving forward. Within the arc of my dissertation research, conversations with Lena inspired purpose and method for the conversations with mathematics educators months later. Finally, I expect that the experience of designing and engaging in these conversations will inform my future practice-based work with teachers.

Humanizing Research Relations

Paris and Winn (2014) characterize "humanizing approaches" to research as "those that involve the building of relationships of care and dignity....reciprocity and respect" (p. xvi). Other scholars add that such relationships require responsivity and answerability to participants, their needs, contexts, and full personhood (Bang et al., 2016; Dance et al., 2010; Dennis, 2018; Dillard, 2000). Describing the ethics of research in terms of relationships—and, specifically, relational qualities we might seek among family and friends—disrupts positivist notions that research should (and can) be objective, purely rational, and separate from the tenderness of life. It also amplifies the significance of each moment of interaction, from scheduling communications and brief exchanges during class visits, to the ways we hold space for and respond to one another's deepest reflections.

As I detail in Paper 1, Lena and I built a relationship of care, reciprocity, and responsivity over the course of our collaboration year. We spent several months getting to know one another, being in her class, and reflecting on teaching before envisioning a formal research study. Through ongoing dialogue, we worked to integrate our various questions, needs, and hopes, across the year and within any particular conversation. We also spent time together—on walks, over tea, on the phone—sharing about and supporting each other through our lives beyond work. Research relationships with the educators in Paper 3 were less extended, but within each interaction, participants and I tended to engage with a similar sense of mutuality and openness to one another. Educators in both studies reflected that our conversations felt giving to them—"healing," "life-affirming," a "gift"—and I would say the same for myself.

An essential piece of these research relationships has been knowing and interacting with participants beyond data collection activities. My connections with most educators in this dissertation preceded IRB applications, consent forms, and first interviews, sometimes tracing back multiple years. Moving forward, I expect to stay in touch with each educator; in some cases, we have already extended our connection beyond the dissertation research process. While taking care to maintain clarity about when research interactions begin and end (Tuck, in Bozalek & Kuby, 2021), developing relationships through broader work and living enhances those relationships and also humanizes the research process.

Ongoing Methodological Reflection

Methodological reflection is an ongoing part of my inquiry process. Through all phases of a study, I keep notes about my decisions, intentions behind them, challenges I encounter, sources of inspiration, and the ways I notice myself and the research process developing. These memos keep me accountable to the ethical commitments described above. They offer space to process the tensions that arise around my positionality as researcher and relationships with participants, including the power dynamics involved (Bang & Vossoughi, 2016; Erickson, 2006).

In the context of work with educators, I reflect often on the porous boundaries between the roles of researcher and colleague (Bang & Vossoughi, 2016). When in classrooms with children or in conversations with teachers about their practice, I tend toward engaging with participants as a colleague in their setting (e.g., as a learner in the same activity, or as a co-teacher or instructional coach). The distance, or sense of separation, that a researcher role entails feels unnatural, even troubling to me. At the same time, I recognize its value for making visible phenomena that can be hard to discern by those "closest to the action" (K. D. Gutiérrez & Vossoughi, 2010, p. 101). Emerson, Fretz, and Shaw (1995) suggest that it is possible to maintain a "socially close but experientially separate stance" as an ethnographic researcher, relating deeply with participants and at times engaging in their activity alongside them, while also being able to step back to document and analyze (p. 36). Erickson (2006) writes similarly of "sideby-side" work with teachers.

I am interested in and also struggle with the relationship between withness (Shotter, 2006) and witnessing (Ayala et al., 2020; Winn & Ubiles, 2011) in collaborative research—between being "centrally involved in unfolding activity" and bearing witness in order to analyze, mirror back, and tell stories from it (Bang et al., 2016, p. 33). Navigating the withness-witness relationship requires ongoing attention and care. Written memos are an important tool for me in this reflection. So is open communication with participants about our joint processes. For example, at the end of conversations with Lena, we often reflected on how the conversation's process felt to each of us, the ways I had engaged during recent class visits, and shifts we might try moving forward. While not always easy, these exchanges created a sense of withness in the very process of methodological reflection.

Additional Reflections on Researcher Positionality

Like the educators in this dissertation, I recognize that sociohistorical forces shape my work and living, across settings. My dominant social positions (as a white, Englishspeaking, cisgender, able-bodied person with generational financial wealth and U.S. citizenship status) intersect with colonizing histories of research and schooling in ways that demand critical attention. These positions grant undue power, enculturate harmful ways of being, and create blind spots, while also offering access to resources, spaces, and experiences that might be stewarded toward socially just ends. I believe that systems of oppression dehumanize us all and that, from our various histories, perspectives, and material locations, we might imagine our freedom as similarly entwined (Combahee River Collective, in Taylor, 2017; Freire, 1970; Hamer, 1971; Walcott, 2021). Through my work, I seek to reflect on my entanglement in these systems and to act in solidarity toward collective liberation, in all of its multiplicity.

From a young age, I learned that schools could be places that live out this commitment to collective liberation. The neighborhood elementary school I attended as a child communicated a clear political vision—of learning as a way to bring people together, from diverse walks of life, to dream, inquire into, and practice social change. My interest in liberatory mathematics is rooted in one classroom at the school. Mathematics learning in this class reflected what are now some of my deepest values: intellectual inquiry as a process of coming to know and care for one another, heterogeneity and collaboration as essential to creativity, and interdependence as a kind of flourishing.

I carried this orientation toward learning into my work as a teacher. Across disciplines and age groups, children and I explored questions, created work, and built relationships that reflected what I would now call "liberatory praxis," though I did not use the term then. I saw mathematics learning as a vital context for this praxis. Unlike in the humanities, however, I had fewer occasions as a teacher to articulate or collaborate with colleagues around a political vision for mathematics pedagogy. Moving into doctoral studies, I sought opportunities for research inquiry that would, in connection with practicing educators, pursue articulations of this vision.

This dissertation's emphasis on school-based teaching reveals my interest, and radical hope, in classroom communities as sites for social change. I believe insurgent work can and does happen in "liberatory pockets" within school systems, if we summon the courage to see, join, and incite it (Naya, participant in Paper 3). I am sensitive to the tendency of academic research to "lament 'lack of imagination' or 'proclivity to maintain the status quo' among teachers" in school-based roles (Philip et al., 2022, p. 72). Like Philip and colleagues (2022), I resist such characterizations and am wary of their origins in processes of educational research that are "far removed from the embodied and visceral experiences of teaching within the complex and layered constraints" of schools (p. 72). In partnering with educators and writing about their perspectives, I strive to balance appreciation for the complexities they face with critical attunement to the ways we all, across multiple professional and personal locations, participate in systems of oppression, sometimes unknowingly and counter to our deepest commitments.

By inquiring into liberatory pedagogy together, educators and I articulated how they worked to "incite the enfleshment" of radical social dreaming in current practice (Bang & Vossoughi, 2016, p. 178), the challenges they faced in doing so, and possibilities they had yet to actualize. Collaborating with educators also invited me to reflect on my own teaching experience and professional future. It illuminated questions I wish I had explored while classroom teaching and crystalized my commitment to forms of research inquiry that are embedded and ongoing in the work of school communities.

May this dissertation be one piece of a lifelong praxis of co-conspiring in (Love,

2019) liberatory teaching and learning. I orient toward this inquiry with urgency, humility, and hope, recognizing the long-haul, distributed work it demands while taking seriously my place in it in the here and now.

Dissertation Roadmap

To close, here is an outline of the remaining chapters. Chapter Two discusses key concepts that thread throughout the dissertation. These include: liberation, in and beyond educational contexts; criticality; pedagogy; and mathematics. The next three chapters are the stand-alone studies: Paper 1 an analysis of reflective conversations with Lena, Paper 2 an integrative literature analysis, and Paper 3 a synthesis of conversations with six mathematics educators. The dissertation concludes with a chapter that brings the three studies together, highlighting themes that developed through and across them and proposing directions for future inquiry.

CHAPTER TWO: GROUNDING CONCEPTS

Introduction

In this chapter, I discuss five terms that are central across the dissertation's chapters: liberation, education, pedagogy, mathematics, and criticality. Each holds multiple, sometimes contradictory meanings. My intention is not to fix definitions but to illuminate ideas and commitments that these words signal in my writing. Each of the subsequent three chapters also includes an explication of terms that are specific to that paper's analysis.

This is not a theoretical framework in the sense of an a priori set of ideas that a researcher applies systematically in order to "see" or "reveal" meaning in data (Anfara & Mertz, 2015). The concepts below developed in meaning as I wrote the three papers. My work with participants and engagement with extant literature shaped how I came to understand and articulate them. Further, as elaborated in the concluding chapter, participating educators in the empirical studies offered their own principles of liberatory (mathematics) pedagogy, which I consider theory in its own right (Ballenger & Rosebery, 2003; Cochran-Smith & Lytle, 2009; Philip et al., 2022). The purpose of this chapter is to share understandings of key terms that I use when narrating what I learned from these educators and from the scholarship in Chapter Four.

Liberation

Liberation can be conceptualized at different scales, in different contexts, and toward different ends. As definitions and experiences of oppression vary, so too do meanings of liberation. Further, liberation refers not only to what people seek freedom *from* but also the future possibilities they might orient *toward* (Ringer, 2005; Walcott, 2021). Rather than pinpointing a single thought tradition as a framework for the dissertation, I identify a set of meanings that liberation holds for me. This section presents an overarching statement, inspired by multiple authors and perspectives. Then it elaborates on three principles central to my understanding of liberation.

As across the dissertation, in this section I turn to authors within and beyond the field of education. Many write specifically about Black liberation. Walcott (2021) clarifies that Black freedom "is not one kind of freedom that sits alongside other kinds of freedom" but "a radical reordering phenomenon" that "inaugurates an entirely new human experience for everyone" (pp. 72, 5). If Black liberation would liberate all people, theorizations of it should make calls on us all. I seek to understand liberation in the interdependent terms Walcott and others describe, recognizing with humility my position as a white person learning from their work. When drawing connections between Black liberatory pedagogies and the teaching profiled in this dissertation, I do not presume a direct parallel between the historical contexts in which liberatory Black education has developed and the contexts in which participating educators worked (Anderson, 1988; Givens, 2021a; Johnson et al, 2014; Payne & Strickland, 2008). Rather, I lift inspiration from authors' descriptions of the values, sensibilities, and practical wisdom involved in teaching "within yet against" dominant systems of education (Givens, 2021a, p. 27).

Overarching Statement

Liberation frees from hierarchy, violence, and the hegemony of single stories (Adichie, 2009; Escobar, 2015; Walcott, 2021). It assures dignity and the capacity to

32

flourish; honors heterogeneity as fundamental; entwines self-determination and agency with belonging and interdependence; and nurtures reciprocal, sustainable relationships among all forms of life (Combahee River Collective, in Taylor, 2017; Espinoza et al., 2020; Evans & Vaandering, 2016; Gumbs, 2020; G. Gutiérrez, 1973/1988; Kimmerer, 2013; Rosebery et al., 2010). It is simultaneously intellectual, spiritual, material, and ecological, as these are inextricable from one another.

Liberation is distinct from notions of equity and justice because it prioritizes freedom from systems and cultures that oppress, not simply access or reparation within them. I understand equity as an assurance of resources, opportunity, and well-being within current structures, accounting for the differing positions and needs of various groups. Equitable distributions may also be called just, though justice brings particular attention to collective well-being and relational healing (Evans & Vaandering, 2016). Equity and justice can be necessary goals in liberatory movements, but, as discussed in the previous chapter, they do not in and of themselves constitute liberation. Liberation extends beyond inclusion and redress within existing systems, toward possibilities for something fundamentally different (Martin, 2003; Walcott, 2021). Below, I turn to authors who elaborate on this idea.

Plural and Collective

Liberation is plural and variegated; there is no single experience of it. The very essence of freedom necessitates a release from unitary visions of what that freedom could be. Feminist thinkers have critiqued theories of liberation that universalize human experience, calling instead for "differentiated" understandings of oppression and freedom based on social location (Weiler, 1991, p. 455; see also Crenshaw, 1989; hooks, 1994). Likewise, decolonial perspectives suggest that a liberated world would be one "in which many worlds fit—a pluriverse" of ways of flourishing² (Escobar, 2015, p. 14; see also Grosfoguel, 2013).

Pluriversal conceptions of liberation do not reduce to individualism. Our various liberations are interdependent and collectively transformative, as expressed in the idea that no one is free until all are free (Hamer, 1971; see also Combahee River Collective, in Taylor, 2017). Liberation requires a collective reorienting while holding space for heterogeneity and self-determination (Walcott, 2021). This understanding is distinct from EuroWestern notions of liberty that trace back to the Enlightenment, which portray freedom as the absence of social constraints on individual belief and behavior (Ringer, 2005). Individualist paradigms equate freedom with an abnegation of social responsibility, often masking continued relations of dominance and subjugation. A collective view recognizes the ways our lives are inherently interconnected, finding self-actualization and freedom through that interrelation.

Politically Clear and Imaginative

Theorizing liberation requires clarity about the sociopolitical structures that we seek freedom from, including a historicized understanding of those structures. As Kelley (2018) urges, we must "go to the root—the historical, political, social, cultural, ideological, material, economic root of oppression in order to understand its negation, the

² Inspired by the Zapatista notion of "a world where many worlds fit," Escobar (2015) uses

[&]quot;pluriverse" to refer to "the richness of the multiple worlds that make up socio-natural life" (p. 14).

prospect of our liberation" (p. 164; see also G. Gutiérrez, 1979). Political clarity in education demands the recognition that teaching, learning, and schooling are never neutral, that they always promote some configuration of power relations (McKinney de Royston & Sengupta-Irving, 2019). Politically clear pedagogy and scholarship devote explicit attention to the social hierarchies that education sustains and articulate a vision for justice and freedom in specific relation to those systems (Politics of Learning Writing Collective (PLWC), 2017).

At the same time, liberatory visions do not tether to oppressive structures as a fixed reality or primary referent, as this re-centering can limit movement for change to resistance and survival within current arrangements (Love, 2019; Martin et al., 2019; Walcott, 2021). Instead, authors call for "imagining a break" (Walcott, 2021), or "radical departure" (Martin et al., 2019), from the hierarchies of human society. Discussing histories of Black freedom dreaming in the United States, Kelley (2002) asks, "How do we produce a vision that enables us to see beyond our immediate ordeals?...Without new visions we don't know what to build, only what to knock down" (pp. x, xii). Similarly, Benjamin (2016) calls for methods of social science inquiry that "attempt to perceive otherwise," to "envisage a not-too-distant future" in which modes of social living are "reconfigured differently, defiantly, and hopefully" (p. 3). These authors make clear that the capacity to imagine otherwise is essential to liberatory social critique.

Radically Practical

While imaginative and future-oriented, liberatory praxis also works to enact transformative possibilities in the present. According to Freire (1970), praxis is the

interanimation of "reflection and action upon the world in order to transform it"—the means by which communities "emerge from" and "turn upon" the conditions that oppress them (p. 36). The notion of praxis is central to liberation theologies, which suggest that liberatory faith involves reflecting on "a future which is believed in and hoped for...with a view to action which transforms the present" (G. Gutiérrez, 1973/1988, p. 12; see also Cone, 1970/2010; G. Gutiérrez, 1979). Liberatory faith, education, and political organizing all commit to "concretely fashion[ing]" alternatives to oppressive political-economic structures in "the here and now" (G. Gutiérrez, 1979, p. 21).

Praxis requires a prefigurative orientation, modes of engaging in current work that "anticipate" the values, social relations, and experiences of a liberated world (Boggs, 1977, p. 103). Bang and Vossoughi (2016) describe the prefigurative nature of participatory design research in education, which "hold[s] space for radical critique and social dreaming...while inciting the enfleshment of these ideas in current practice" (p. 178). In his discussion of "fugitive pedagogy" in Black education in the United States, Givens (2021a) argues that education as and for freedom has been not only a philosophical ideal or hoped-for future but a practice that Black people "lived out and enacted," in large- and small-scale, overt and covert ways (p. 13). Givens (2021b) suggests that the work of freedom demands vigilant engagement with existing systems strategically "wad[ing] through" oppressive realities—as people subvert and create beyond them (p. 23; see also Grant et al., 2021; Harney & Moten, 2013). Each of these traditions frames liberatory praxis as a process of freedom-in-the-making, one that seeks to realize "those moments of the something more…inside of the dire conditions of our present" (Walcott, 2021, p. 2).

In this and the next part of the section, I discuss education *as* or *for* liberation. A stance of radical practicality also suggests that liberation is itself educational (Shor & Freire, 1987). While struggling to make freedom in the here and now, we encounter challenges and realize possibilities that invite us to refine our liberatory visions and methods for enacting them. As an iterative process of experience, reflection, and change, liberation might be seen as a form of learning.

Education

I understand education as a sustained, structured process of learning that can occur in a variety of contexts, including beyond formal institutions. Schools are a primary context for education, and much of this dissertation examines teaching and learning in school settings, but education is more expansive than formal schooling (Giroux, 1983; Shor & Freire, 1987).

Craft (1984) points to two different Latin roots of "education": *educare*, which means to "to train or mould," and *educere*, meaning "to lead out" (p. 9). Interpretations of the former often emphasize conformity and the uncritical reproduction of existing social structures (Craft, 1984), though "training" and "molding" can be subversive if practiced as a form of cultural uplift and survival by marginalized communities (e.g., Bang et al., 2015; Johnson et al., 2014). *Educere* signals possibilities for creation and new direction. While Craft (1984) discusses *educere* in individualistic terms—emphasizing learners' self-expression, creativity, and choice—the metaphor of "leading out" could also refer to sociopolitical transformation. The potential for education to lead to social change is central to philosophies of education for democracy and liberation (e.g., Brion-Meisels et al., 2010; Dewey, 1916; Du Bois, 1935/2017; Freire, 1973; Gutmann, 1987; Love, 2019).

This dissertation explores education as and for freedom, commonly referred to as "emancipatory education," "liberatory education," and "education for liberation." Liberatory education can take many forms, including but not limited to community organizing and institution building for educational self-determination, curricula that center cultural relevance and sociopolitical inquiry, and social relations of care and solidarity (Anderson, 1988; Beauboeuf-Lafontant, 2005; Freire, 1970; hooks, 1994; Ladson-Billings, 1995; Love, 2019; Murrell, 1997; Payne & Strickland, 2008; Perry, 2003). Across these, education is visioned and practiced as a process of change that frees people.

Pedagogy

I use "pedagogy" to refer to the commitments, relationships, and practices entailed in teaching. These may be attributed to a person or group institutionally assigned the role of teacher(s), or distributed across a community that designs its learning collectively. The empirical studies in this dissertation focus largely on pedagogy within formal K–12 settings, highlighting the ways educators organize classroom activity, make curricular decisions, interact and build relationships with learners, and reflect on their political and ethical commitments in the context of school-based education. It is important to note that pedagogy is also at work in out-of-school educational programs, professional learning settings, workplaces across industries, community organizations, and social movements (Barajas-López & Bang, 2018; Boreham & Morgan, 2004; Moses & Cobb, 2001; Rosebery et al., 2015; Shor & Freire, 1987). The participating educators in this dissertation, while often reflecting on their experiences in schools and school-adjacent programs, articulated pedagogies that could extend to learning beyond K–12 contexts.

An analytic focus on pedagogy is not meant to separate teaching and learning. I see the two as fundamentally entwined and would suggest that in liberatory contexts all participants engage as both teachers and learners (Freire, 1998; Glanfield, 2016). Still, teaching is a particular kind of activity, and teacher a particular position, within a learning community. While that activity may be distributed and positions can be fluid, inquiring into pedagogy directs attention to the distinctive, intentional work of teaching.

I view pedagogy as praxis and use the terms synonymously when discussing educators' work (e.g., "liberatory pedagogy" and "liberatory praxis"). Praxis is central to notions of liberatory pedagogy, which emphasize the reflexive relationship between action and reflection, practice and theory in education for social change (Freire, 1970; hooks, 1994; Perlow et al., 2018). This perspective moves away from an idea of pedagogy as the technical "science" or "methods" of teaching, somehow isolable from the philosophy that underlies and animates it (Bartolomé, 1994; Philip et al., 2018b). All practice, educational or otherwise, is shaped by values and theoretical perspectives, even when these are not conscious or self-determined on the part of practitioners. That said, in this dissertation, I use the term "pedagogy" when educators themselves theorize their practice, articulating the why, for-what, and for-whom of their teaching (Philip et al., 2018a). The two empirical studies profile educators who expressed "politically clear" pedagogies, attuned to the linkages between macro-level political forces and the everyday, microrelational moments of teaching and learning (Bartolomé, 1994; Freire, 1987; McKinney de Royston & Sengupta-Irving, 2019). These educators assumed responsibility for the political consequence of their work and were in ongoing reflection on the ways their classroom-level practice both reflected and shaped the broader sociopolitical world.

Mathematics

I regard mathematics as plural, including but not limited to the forms taught in schools and practiced by professional mathematicians. Dominant narratives promote "the myth of one and only one kind of mathematics," obscuring the diversity of mathematical knowledges and practices around the world and across time (Fasheh, 2012, p. 94). A plural view sees mathematics as cultural practice, developing through the heterogeneity of human social life, rather than as a universal body of knowledge that transcends human subjectivity and particularities of context (Bishop, 1988; R. Gutiérrez, 2017; Nasir, 2002; Triadafillidis, 1998).

Ethnomathematics is a scholarly tradition that explores the "radically different ways" of knowing and practicing mathematics across cultural communities, especially beyond institutions of formal education (D'Ambrosio, 1985, p. 44; see also Ascher, 1991, 2002; Bishop, 1988; Eglash, 1997). Scholarship in ethnomathematics also documents the multiplicity of cultural traditions that have contributed, often without credit, to dominant (i.e., Eurowestern) mathematics (Joseph, 1987, 1991/2011). This scholarship highlights that, not only is mathematics plural, but its many forms develop in dynamic relation to one another, through the hierarchies of power that structure all facets of human society.

Dominant Mathematics

I follow Gutiérrez (2002) in using the term "dominant mathematics" to refer to mathematical concepts, practices, discourses, pedagogies, and ways of knowing that assume a superior status—as *the* mathematics—in universities and K–12 schools. Scholars also refer to Western or Eurocentric mathematics (Bishop, 1990; Fasheh, 1990; R. Gutiérrez, 2017; Joseph, 1991/2011; Shelley, 1995/2005) and Mathematics with a capital 'M' (Bishop, 1988). All highlight that the form of mathematics that holds power in society, though often treated as universal, in fact reflects a culturally specific set of values and conceptualizations of knowledge, rooted in EuroWestern colonial thought (Fasheh, 1990; R. Gutiérrez, 2002, 2017; Shelley, 1995/2005). These values include the idealization of objectivity, generalization, rationalism, hierarchical classification, and written text (as elaborated in Chapter Four). Dominant mathematics includes various iterations of the discipline, all of which share, or at least leave unquestioned, EuroWestern assumptions about what it means to know mathematically.

I use the term "school mathematics" broadly, to refer to curriculum and pedagogy in use in formal schools. While instructional approaches vary—for example, "reform" and "traditional" mathematics hold different perspectives on the role of learners in constructing knowledge (Lampert, 1990; Trafton et al., 2001)—school-sanctioned approaches tend to reflect the EuroWestern assumptions of dominant mathematics. That said, each analysis in this dissertation holds open the possibility that mathematics

41

teaching and learning in schools might break from this normativity.

Treating dominant mathematics as culturally and politically situated is not a rejection of its value or legitimacy but rather a rejection of its normativity. Along with critical scholars, I question the notion that any single form mathematics is a "universal language" and critique the ways EuroWestern mathematics has been used to further colonial and imperial projects (Bishop, 1990; D'Ambrosio, 1985; Fasheh, 1982, 1990; Gutiérrez, 2019; Joseph, 1987; Vossoughi & Vakil, 2018). Viewing dominant mathematics as one among many "partial" forms of mathematics allows us to challenge its colonizing power, appreciate the particular ways of knowing it makes possible, and open to the plurality of mathematical knowledges beyond it (Gutiérrez, 2017, p. 5). These ideas are all elaborated later in the dissertation, most in depth in Chapter Four.

Criticality

Across its various meanings, "critical" signifies the questioning of taken-forgranted assumptions or an examination of what has not been explicitly stated. My use of the term does so specifically in reference to issues of power in society, which always underpin but often are not explicitly named in educational research and practice (PLWC, 2017). I describe perspectives as "critical" if they are sociopolitical (address power relations), structural (extend to institutions and systems), historicized (look across time and scales of activity), and intersectional (link multiple dimensions of identity) recognizing that a given piece of work may emphasize certain dimensions over others. This notion of criticality centers the politics and ethics of social life, highlighting unjust conditions and possibilities for resisting and transforming them (McKinney de Royston & Sengupta-Irving, 2019).

While critical work usually begins from analysis of oppressive systems, it can be reconstructive and hopeful (Duncan-Andrade, 2009; Luke, 2004). As discussed above, *liberatory* critical praxis is both politically clear and radically imaginative. It attends honestly to the harms of historical and current societal arrangements while simultaneously envisioning "differently, defiantly, and hopefully" (Benjamin, 2016, p. 3). In the context of mathematics education research, Skovsmose (2012) refers to a kind of critique that "searches for possibilities" and "radical changes" in mathematics teaching and learning (pp. 355, 359). Skovsmose and Borba (2004) write:

Doing critical research means not only to consider what is taking place but also to consider what could have taken place and what could be imagined as possible alternatives to what is taking place....Critical research points out that something could be different. (p. 211)

As explored across the dissertation, mathematics educators and educational scholars have gone about this radical inquiry in various ways.

While criticality involves analytic ways of knowing, it is not purely logical or even cognitive. Kelley (2002) refers to imaginative critical analysis as "poetic knowledge" (p. 9), suggesting that criticality can entail aesthetic and affective qualities. Ways of moving, feeling, and making may also be characterized as critical, though they are less often foregrounded in critical academic scholarship. In the papers that follow, I highlight where educational research and practice engage more-than-analytic modes of criticality, though this is an area of disciplinary education that demands further exploration.

This chapter has presented theoretical perspectives that ground my understanding of five key terms used throughout the dissertation: liberation, education, pedagogy, mathematics, and criticality. Given the wide range of meanings these words can hold, I have highlighted ideas and commitments that I associate with them. The concepts explored here weave through the following chapters, hopefully nuancing in meaning with each analysis. The next chapter is the first of the dissertation's three studies, focused on an elementary teacher's reflections on liberatory pedagogy across disciplines.

CHAPTER THREE: "BUT IN MATH...": AN ELEMENTARY TEACHER'S REFLECTIONS ON LIBERATORY PEDAGOGY ACROSS DISCIPLINES Introduction

Literature on justice-oriented mathematics teaching often takes inspiration from educational and intellectual traditions that cut across disciplinary boundaries, including Black liberatory education (e.g., Anderson, 1988; Dumas & ross, 2016; Murrell, 1997; Woodson, 1933/2009), critical pedagogies (Duncan-Andrade & Morrell, 2008; Freire, 1970; Giroux, 1983, 1988; Shor & Freire, 1987), culturally relevant and sustaining pedagogies (Gay, 2000; Ladson-Billings, 1995; Paris & Alim, 2014; Perry et al., 2003), funds of knowledge (Moll et al., 1992), and Indigenous epistemologies (e.g., Barnhardt & Kawagley, 2005; Cajete, 1994; Donald et al., 2012). These traditions center values and orientations that are less common in mathematics education. They emphasize attention to cultural identity and collective well-being, historicity, holistic ways of knowing, and visions for societal transformation, while the field of mathematics education has historically valued apoliticism, individualism, and narrowly rational views of cognition and meaning making (Ernest, 2012; R. Gutiérrez, 2019; McBride, 1994; Skovsmose & Greer, 2012; Triadafillidis, 1998). Cross-disciplinary perspectives thus have the potential to expand the ways we design for and practice mathematics teaching and learning.

Mathematics education literature that is cross-disciplinary in theoretical orientation (e.g., Frankenstein, 1983; González et al., 2001; Gutstein, 2006; Martin, 2019; Martin & McGee, 2009; Sterenberg et al., 2010) still tends to examine the practice of mathematics teaching and learning in isolation from other disciplines. That is, authors draw upon conceptual frameworks that transcend disciplines but collaborate with educators and youth, or consider their activity, only in mathematics. While curriculum and pedagogy have discipline-specific aspects, the underlying purposes and methods of teaching are not necessarily siloed by content area, especially in elementary classrooms.

This paper explores questions about teaching mathematics toward social justice and liberation in light of an elementary educator's practice across school disciplines. While this is not a case of interdisciplinary mathematics (i.e., mathematics curriculum and learning activity were not integrated with other school disciplines in the educator's class), the paper draws insight from the simultaneity of disciplines that characterized her work and our joint reflection. In doing so, the analysis raises questions and possibilities for mathematics teaching and learning that might not have surfaced in a study of mathematics on its own.

The paper shares reflections from Lena³, an upper elementary educator who approached teaching with striking political clarity (Bartolomé, 1994; Freire, 1987). Lena used multiple terms to refer to the political ethos of her teaching: social justice, antioppressive, abolitionist, and liberatory. "Liberatory," her preference, was an "allencompassing" orientation, describing relationships and experiences that free people from oppression and invite creative exploration of the world alongside others. This analysis examines Lena's articulation of her liberatory pedagogy, which developed across the humanities in her class⁴. It also considers a sense of alienation Lena experienced, in

³ All names are pseudonyms. Children in the study chose their pseudonyms.

⁴ In Lena's class, the humanities included study of literature, writing in various genres, art making, historical inquiry, and social action projects, as well as activity that might be considered extra-

mathematics, from the values and practices that guided her work with children⁵ at other times of day. Through grounded thematic analysis of conversations with Lena, supported by records from class observations and interviews with children, I bring Lena's perspectives on liberatory pedagogy and mathematics teaching alongside one another. Borrowing the metaphor of "rift" from Booker and Goldman (2016), the paper asks:

- What is the rift Lena narrated between her liberatory pedagogy and mathematics teaching when considering her work across school disciplines? How did she describe this rift in relation to: conceptions of mathematics, work with curriculum, relationships with children, and professional learning experiences?
- 2. What possibilities for liberatory mathematics teaching and learning surfaced through our conversations?

In examining these questions, I contend that the rift was not a matter of Lena lacking language or tools to translate her liberatory vision to mathematics. She sensed tensions that source to deep within the discipline of mathematics and systems of mathematics education. Through the analysis, I consider these systemic sources and also propose avenues for future inquiry into mathematics teaching and learning, inspired by Lena's liberatory pedagogy. The paper thus raises critical questions for the field of mathematics education, where notions of liberatory praxis are under-conceptualized

disciplinary, such as morning and afternoon circles, various "clubs," and one-on-one and small-group conversations with children about their social-emotional growth.

⁵ In this paper, I use the word "children" to refer to the young people in Lena's class because it emphasizes their humanity over their institutional position as "student." Lena often referred to them as "people," though she did sometimes use "student." While I mirror Lena's liberatory language when possible, the generic meaning of "people" seemed potentially confusing. I use "children" not in a diminutive sense, but rather with reverence for the wonder, courage, complex engagement, and hope that young people bring to learning and living.

(Martin, 2009a; Martin et al., 2019). While Lena and I did not, together, explore liberatory mathematics in her class, our inquiry suggests openings into, and principles that might guide, such exploration.

The paper begins by introducing three concepts that ground my analysis, followed by a review of literature that has addressed forms of rift between school mathematics and liberation. I then describe my methods for carrying out and analyzing conversations with Lena. The findings section, organized into three parts, illustrates the relationship between Lena's liberatory praxis and mathematics teaching: I introduce key facets of her liberatory pedagogy, examine her articulation of the rift with mathematics, and point toward possibilities for liberatory mathematics in our conversations. The discussion section considers these possibilities further, extrapolating from Lena's praxis to speculate about liberatory mathematics beyond her context. The discussion also draws connections to literature that addresses systemic dimensions of the rift and efforts by mathematics educators and educational researchers to overcome it. I conclude with implications for practice-based inquiry with mathematics teachers, inspired by the cross-disciplinary spirit of my collaboration with Lena.

Grounding Concepts

This section elaborates three concepts that are central to the paper's analysis: liberatory education, pedagogy as praxis, and the metaphor of rift.

Liberatory Education

Liberatory education—also referred to as "emancipatory education" or "education for liberation"—grows from multiple intellectual and educational traditions (e.g., Anderson, 1988; Beauboeuf-Lafontant, 2005; Brion-Meisels et al., 2010; Freire, 1970; R. Gutiérrez, 2017; hooks, 1994; Ladson-Billings, 1995; Love, 2019; Payne & Strickland, 2008; Shor & Freire, 1987; Woodson, 1933/2009). All emphasize the potential for education to free people from, rather than reproduce, unjust social systems. Liberatory education can take many forms, from the creation of community-run schools and programs, to curricula that center critical culturally-relevant inquiry, to relations of care and solidarity among teachers and learners. Across these, freedom is both a hoped-for ideal and a process of putting that ideal into practice, at institutional, curricular, and interpersonal scales. Authors thus refer to "education as the practice of freedom" (Freire, 1973; hooks, 1994).

In this paper, I do not build from a single tradition or refer to the liberation of a particular community. This is in part because Lena did not, though she attended carefully in her reflection and practice to individuals' varied experiences of oppression based on social location. Lena worked with a diverse group of children across racial, ethnic, gender, class, and dis/ability identities. While most literature on liberatory education devotes attention to historically marginalized communities, some scholarship has examined educational contexts in which people reflect on their privilege, power, and calls to solidarity within unjust systems (Jemal, 2017; Kokka, 2020; San Pedro, 2018; Swalwell, 2013). Lena's liberatory pedagogy integrated attention to both. I hope that, rather than universalizing experiences of liberatory praxis (hooks, 1994; Weiler, 1991), this analysis illuminates principles and questions pertinent to diverse learning communities such as Lena's class.

49

As discussed in Chapter Two, there are certain ideas that anchor this paper's broader understanding of liberation. Liberation is plural and collective: there are many possible experiences of freedom and these are interdependent—personally and collectively and across communities variously situated (Combahee River Collective, in Taylor, 2017; Hamer, 1971; Walcott, 2021). Additionally, visions of liberation are grounded in politically clear analysis of oppressive systems and cultures while also imagining radically beyond them (Benjamin, 2016; Kelley, 2002). Liberation is thus distinct from notions of equity and justice, which tend to prioritize access, integration, and reparation within existing systems. At the same time, liberation is radically practical, a process of enacting transformative possibilities in the here and now, amidst the real constraints of the present (Boggs, 1977; Freire, 1970; Givens, 2021a; G. Gutiérrez, 1973/1988).

In the paper's analysis, I elevate the meanings that Lena ascribed to "liberatory" as we inquired into her liberatory pedagogy. These meanings resonate with the ideas above and point to their significance for classroom teaching and learning.

Pedagogy as Praxis

I use "pedagogy" to refer to the commitments, relationships, designs, and practices entailed in teaching. This is not to separate teaching and learning; I see the two as fundamentally entwined and would suggest that in liberatory contexts all participants engage as both teachers and learners (Freire, 1998; Glanfield, 2016). Still, teaching is a particular kind of activity within a learning community, and Lena held the institutionally formalized role of "teacher" in her class. My analysis focuses on Lena's perspectives from this distinctive position.

Lena described her pedagogy as embedded within the constellation of relationships of her class community as well as broader relations of power in society. While teaching, and in her reflections, she attended carefully to the details of moment-tomoment interaction. She also held in view the ways micro- and meso-level relational patterns reflexively co-construct sociohistorical systems (Philip & Gupta, 2020). Throughout the paper, I try to engage this multi-scale vision of teaching.

I refer interchangeably to Lena's "liberatory pedagogy" and "liberatory praxis." The notion of praxis is central to theories of liberatory education that build from the work of Paolo Freire (1970), who described the interanimation of "action and reflection on the world in order to transform it" (p. 36). This perspective moves away from an idea of pedagogy as the technical "science" or "methods" of teaching, somehow isolable from the philosophy that underlies and animates it (Bartolomé, 1994; Philip et al., 2018b). Lena approached the work of teaching as an interplay between practice and reflection, and she engaged children in "action-reflection" in their humanities and extradisciplinary learning. She did not, however, theorize mathematics teaching or reflect with children on mathematics learning as she did in other areas of her work. For this reason, while I refer to Lena's "liberatory praxis" throughout the paper, I refrain from using "mathematics teaching praxis" or "mathematics pedagogy," other than speculatively or in reference to contexts beyond Lena's.

51

Rift

To explore discontinuities between Lena's liberatory praxis and mathematics teaching, I lift the term "rift" from Booker and Goldman (2016). These authors write of "a rift that repeatedly forms between experiences of formal mathematics education" and families' mathematical practices (p. 222). They discuss the exclusionary nature of school mathematics, which narrows our conceptualizations of mathematics and estranges mathematical activity in school from people's everyday living, particularly for historically marginalized communities. Booker and Goldman's use of the rift metaphor invokes images of a spatial separation, or a deep cleave or fissure in solid matter, though rift might also allude to a breach in social relations.

The rift I examine is not (directly) between home and school mathematics, but rather one within a teacher's work across disciplines. Later in the paper, I propose that the rift in Lena's teaching stemmed from the same exclusionary systems to which Booker and Goldman allude, related to a narrowing of what counts as mathematics and the purposes it furthers in the world. It is important to note that I layered the concept of rift onto Lena's reflections as part of the paper's analysis. While Lena did not contest this thematic framing when I shared it, we did not use the term "rift" in our conversations.

Framing the scope of the rift, Booker and Goldman suggest that "mathematical practice becomes a social, cultural, and historical problem in need of repair" (p. 223). They define repair as "a practice of caring for and restoring what is valued and integral to people's lives" (p. 226). As discussed later, I question the assumption that repair should be the ultimate goal, at least in the non-transformative sense of closing a gap or mending

cleaved pieces without critically examining those pieces and the consequences of (re)integrating them. I do, however, take up Booker and Goldman's notion of "caring for and restoring what is valued and integral to people's lives," asking what was valued and integral to Lena's liberatory pedagogy that might be cared for in contexts of mathematics learning.

Literature Review

As educational research devotes increasing attention to the political and cultural dimensions of mathematics teaching and learning (R. Gutiérrez, 2013; Larnell & Martin, 2021; Skovsmose & Greer, 2012), scholarship has explored multiple perspectives on the relationships between mathematics education, (in)justice, and liberation. While "rift" is not a widely used term in the literature, many authors examine mathematics schooling as a force of exclusion and domination, directly at odds with liberation⁶. Others highlight possibilities for challenging systems of injustice through mathematics teaching and learning, some in contexts beyond schools. Below, I review literature on both rift and liberatory possibility. I focus on work that takes a critical perspective, centering analysis of sociohistorical systems of power and the interconnectedness of individual and collective liberation.

⁶ Not all authors in this section use the terms "liberation" or "liberatory." Those who do signal a range of meanings with these words, spanning the various visions of anti-oppressive education outlined below (Davis, 2018; Frankenstein, 1983; Frankenstein & Powell, 1994; Gordon, 1978; Gutstein, 2006; Matthews, 2009; Martin, 2009a; Martin & McGee, 2009; Martin et al., 2019; Price & Ball, 1998).

Examination of Rift

Booker and Goldman's (2016) work is part of a larger body of research that examines the rift between school mathematics and the mathematical knowledge and practice extant in children's lives beyond school (Booker & Goldman, 2016; de Abreu, 1995; Fasheh, 2012; Ishimaru et al., 2015; Nasir & McKinney de Royston, 2013; Takeuchi, 2018; Taylor, 2009). Focusing on the experiences of nondominant communities—across age groups, countries, and cultures—authors highlight the exclusion that occurs when curriculum and pedagogy reflect a narrow, Eurowestern version of mathematics, ignoring or otherwise delegitimizing the diverse ways of practicing mathematics in children's communities outside of school. Authors acknowledge that the rift operates at epistemological and sociocultural levels, rooted in hierarchies around what counts as mathematical knowledge and practice in different contexts. Their analyses attend to local scales of experience as individuals negotiate identities, relationships, and moment-to-moment interactions across the rift (Buenrostro & Radinsky, 2019; Martin, 2000, 2006; Nasir, 2002; Sengupta-Irving & Vossoughi, 2019).

Scholarship has also illuminated the disconnect between school mathematics and liberated living at the scale of sociohistorical systems. Some authors examine the ways school mathematics ideologically and materially sustains oppressive political-economic projects, including racial capitalism, nationalism, militarism, globalization, and neocolonialism (Berry et al., 2013; Bishop, 1990; D'Ambrosio, 1985; Fasheh, 1982; Harouni, 2015; Larnell & Martin, 2021; Martin, 2013; Skovsmose, 2008; Vossoughi & Vakil, 2018). Others have traced connections between onto-epistemic norms of the discipline (e.g., universality, individualism, categorical rigidity, objectism) and the governing values of coloniality, white supremacy, and heteropatriarchy (Bishop, 1988; Ernest, 2012; R. Gutiérrez, 2019; McBride, 1994; Triadafillidis, 1998; Yeh & Rubel, 2020). In various ways, these works ask, "what is math education for?"—situating questions about mathematics education's purposes within a global political context that is far from liberatory (Greer & Mukhopadhyay, 2003; see also Martin, 2013; Pais, 2013; Pais et al., 2010; Skovsmose, 2008; Valero, 2008; Vossoughi & Vakil, 2018).

Liberatory Possibilities

Authors have considered possibilities for mathematics teaching and learning that counter forces of oppression, some in contexts beyond school. Literature in reform mathematics challenges assumptions of individualism, objectivity, and epistemic hierarchy by examining the mathematics classroom as an intellectual community in which authority for knowing is decentralized and knowledge is socially constructed (Ball, 1993; Boaler, 2002; Cobb & Yackel, 1996; Lampert, 1990). It is important to note that this literature can be politically unclear in its analysis, obscuring the sociopolitical situatedness of classroom practice by, for example, omitting discussion of children's and teachers' social identities or the broader purposes for reforming classroom epistemic cultures.

Critical and social justice mathematics, which builds upon work in critical literacy education (Freire, 1970; Murrell, 1997; Perry, 2003), engages learners directly in sociopolitical inquiry as they use mathematics to analyze issues of social injustice and

take action to counter those injustices (Frankenstein, 1983, 2013; Gutstein, 2003, 2006; Kokka, 2020; Skovsmose, 1994). Most cases in the literature focus on individual or small groups of teachers developing curricula for their classrooms. Seattle Public Schools' (2019) Math Ethnic Studies Framework is a rare example of a broader curricular framework.

Another strand of research explores approaches to culturally-responsive and culturally-sustaining mathematics pedagogy. This body of work is itself multifaceted. Some studies focus on modeling children's out-of-school contexts with school mathematics (Aguirre et al., 2012; Civil, 2007; González et al., 2001; Lipka et al., 2005; Moses & Cobb, 2001; Taylor, 2011; Turner et al., 2009). Others have designed and documented learning experiences that explore culturally diverse forms of mathematical practice (Barajas-López & Bang, 2018; Nasir, 2002; Trinick et al., 2015; Sterenberg et al., 2010). Literature has profiled educators' efforts to develop culturally relevant relationships and community in the mathematics classroom (Lipka et al., 2005; Matthews, 2009) and to partner with children's families in mathematics curriculum and research design (Booker & Goldman, 2016; Ishimaru et al., 2015; Lemons-Smith, 2009). Scholarship focused on teacher development has examined professional learning contexts that politicize educators' perspectives on mathematics instruction and support various aspects of the pedagogical work described above (Aguirre et al, 2012; Battey & Franke, 2015; Bell et al., 2021; Das & Adams, 2019; de Freitas & Zolkower, 2009; Gonzalez, 2009; R. Gutiérrez, 2012; Louie et al., 2021; Nicol et al., 2020; Rubel, 2017).

In addition to offering stories of possibility, extant scholarship highlights

challenges of justice-oriented mathematics teaching. Studies raise questions around how to connect authentically to young people's lives while maintaining mathematical rigor, consider diverse positionalities with respect to issues of (in)justice, avoid reductive paradigms of multiculturalism, respond to pressures from curriculum standards, navigate powered relationships between school and home communities, and develop teachers' understandings of structural oppression (Bartell, 2013; Bartell et al., 2021; Brantlinger, 2013; Ishimaru et al., 2015; Kokka, 2020; Leonard et al., 2010; Pais, 2011; Taylor, 2011).

Situating This Paper

The paper that follows builds on this literature in several ways. Most of the work reviewed above focuses on mathematics in isolation from other disciplines (for exceptions, see Barajas-López & Bang, 2018; Das & Adams, 2019). The current analysis examines an educator's pedagogy across school disciplines in order to shed light on mathematics teaching and learning. Second, while extant literature may venture beyond traditional approaches to school mathematics, analyses are not reliably guided by a *liberatory* social imaginary (Larnell & Martin, 2021). The sociopolitical vision driving instructional approaches may be underspecified, or justice-oriented curriculum and pedagogy may leave unquestioned exclusionary assumptions about what constitutes "mathematics" (Larnell et al., 2016). This paper examines pedagogical vision that is concerned with disrupting systems of oppression and seeking collective freedom. The analysis suggests that the discipline itself, in and out of schools, needs freeing in order for mathematics learning to be liberatory.

Finally, discussions of rift in the literature most often center the experiences of

children and families. Accounts of educators' experiences tend to focus on success stories without attending to the rift in its complexity, or they address challenges when educators are earlier in their development of sociopolitical perspectives on teaching. This study profiles an elementary educator who had a mature critical teaching praxis and spoke to deep tensions between her political vision and mathematics teaching. The analysis explores her articulation of these tensions and implications for the challenges and possibilities of liberatory pedagogy in mathematics.

Methods

This section shares context for my work with Lena, including the origins of our partnership, our orientation to collaboration, and the setting in which she taught. I also describe my methods for constructing and engaging with data and conclude with a statement on researcher positionality.

Origins of Partnership

Lena and I met the summer before the 2020–2021 school year. A common mentor put us in touch, knowing that Lena was seeking a thought partner to explore deep questions in her practice, that I wanted to work alongside teachers and children as part of my doctoral studies, and that the two of us shared commitments to liberatory education. We began with a conversation to share our professional and personal histories and the work we were each engaged in. Soon after, we met to imagine ways we might work together, specifically around Lena's classroom practice. Lena shared more about her teaching context and questions that were live as she moved into a new school year. I raised connections to my teaching experience and literature I was reading. I took notes on the conversation, highlighting questions that surfaced and reflecting them back to Lena.

A theme in our early conversations was relationship building in the classroom: the intimate ways Lena and children came to know each other, their politicized trust and care (Camangian & Cariaga, 2021; Rolón-Dow, 2005; Vakil et al., 2016; Watson, 2018), how these relations built moment-to-moment and over time, and their place within Lena's liberatory pedagogy. When I began visiting her class, I kept these ideas at the fore as I observed and later reflected with Lena. As Lena maintained, her relationships with children were "not housed in particular disciplines," so I joined the class for different periods of the day. In the early months of the school year, I visited every week or two and met with Lena after school hours to informally debrief⁷. Lena commented that these early conversations felt "pedagogical" to her, like "a fun teaching relationship."

When we decided to formalize a research study, I proposed what I heard emerging from our initial visits and reflections: through ethnographic documentation and continued practice-based reflection, we could inquire into the relational work of Lena's liberatory teaching. Our shared questions included: 1) What makes Lena's relationships with children? How do Lena and children come to know each other, moment-to-moment and over time? What sensibilities, practices, and ways of being are involved? 2) What is the role of this relationship building within Lena's liberatory pedagogy? 3) How do these relationships live and grow during different times of day (around different subject matter and activity)?

⁷ The first four months of class visits and reflection were not part of an IRB-approved study, so I do not draw directly on notes from these events in the paper's analysis.

When discussing an initial vision for the study, Lena and I articulated what each of us would "get" from such a collaboration. I would be a reflection partner for Lena, bringing an outside perspective while also developing an intimate view of her teaching. Lena later described my role as "another set of eyes and ears" who compassionately "pushed" her thinking about practice. Through our reflection, Lena would have a chance to deepen her understanding of the relationships that grounded and inspired her work; she might also consider new questions about these relationships, including in the context of mathematics teaching. In addition to the experience of co-developing a classroom-based research partnership, I would get to learn about a mature educator's liberatory praxis within and across elementary school disciplines.

Critically Reflexive Partnership

Lena and I strove for a partnership guided by values of reciprocity, care, responsivity, and answerability (Bang et al., 2016; Dance et al., 2010; Dennis, 2018; Dillard, 2000; Hermes, 1999). Paris and Winn (2014) conceptualize "humanizing approaches" to research "as those that involve the building of relationships of care and dignity....reciprocity and respect" (p. xvi). In the specific context of researcher-teacher collaborations, they characterize a classroom teacher's experience of care and dignity as "feeling valued by a 'worthy witness'" (Paris & Winn, 2014, p. xvi). Winn and Ubiles (2011) use the term "worthy witness" to describe a researcher who earns membership in a classroom community while generating ethnographic records that can serve as "a mirror and a window" for participants "to reflect and map new directions for practice" (p. 295). I joined Lena's class as a "worthy witness" and, over the course of our study, we coconstructed a reflection process that served as a mirror and a window for each of us.

Although this was not a case of practitioner inquiry in which the classroom teacher assumed a lead-researcher role (Cochran-Smith & Lytle, 1993, 2009; Erickson, 2006), Lena and I developed our methodology through ongoing dialogue and an explicit intention to "synthesize [our] objectives" (Winn & Ubiles, 2011, p. 301). At the same time, we maintained clarity about our different positionalities, purposes, and responsibilities within the collaboration. This allowed us to generate rich data, regard learning activity from multiple vantage points, and avoid blurring the role of teacher in the classroom (Erickson, 2006; K. D. Gutiérrez & Vossoughi, 2010; Winn & Ubiles, 2011). I carried out all ethnographic documentation, while Lena always planned for and led her class sessions. During reflection conversations, each of us raised questions, though I saw it as my role to facilitate consideration of those questions in a way that allowed us to "look anew" at Lena's practice together (K. D. Gutiérrez & Vossoughi, 2010, p. 101).

Through the foci and lenses of our "looking," we worked to balance Lena's curiosity, needs, and learning with my own. Many conversations followed the questions and concerns that felt most pressing to Lena, such as how to foster deep relationships in a virtual setting, emerging social conflicts among children, or the ways Lena's one-on-one counseling work intersected with broader group dynamics in the class. There were times when our conversations about mathematics seemed driven primarily by my interests, though Lena always engaged in them generously. The data analysis and writing for this

61

paper were less collaborative, as I constructed the analytic questions, themes, and written manuscript on my own. While the narrative is phenomenologically grounded in Lena's experience and integrates her perspective, it inevitably reflects my interests and priorities as the sole author.

In addition to discussing questions of teaching, Lena and I talked regularly about our "processes of partnering," including the power dynamics within our relationship (Bang & Vossoughi, 2016, p. 174). Early on, Lena and I named and told stories about the sociopolitical identities we carry in the world. Interviews and video-based conversations often ended with a reflection on how the conversation's process felt to each of us. Across our collaboration, we shared resources related to critical and decolonizing approaches to research. During a member checking conversation, we talked about the "off-balance" (researcher-skewed) nature of the data analysis and writing process: Lena commented on the ways this stage of research might feel "violating" to participants but also said that she felt "autonomy and power in our working relationship." As Erickson (2006) clarifies, side-by-side collaboration with educators "doesn't remove issues of power difference from the research process—but it makes them more visible within it," allowing for "greater candor" among collaborators about their ways of being and working together (p. 254; see also Fine, 1994).

Context and Participants

I will now detail the methods of our inquiry, beginning with descriptions of the school context and participants.

School Context

Lena taught at an independent K–6 school located in a large city in the northeastern United States. The school was small, with a total of four classes and 16 adult staff members. Classes were mixed-age after Kindergarten (i.e., 1st/2nd grade, 3rd/4th grade, and 5th/6th grade, though the school did not use "grades" nomenclature), so children looped with teachers for two years at a time. The school described itself as "intentionally diverse": it had a sliding-scale ("fair share") tuition policy and an ethnically and racially diverse student body. In Lena's class, children and their families identified as Asian, Black, Dominican, Eastern European, Haitian, Indian, Korean, Latinx, Mexican, Native American, Nigerian, Western European, White, and mixed-race. They spoke English, Haitian Creole, Korean, and Spanish. Most families lived in the neighborhood where the school was located, as did Lena and many staff members.

For the 2020–2021 academic year, classes gathered in various spaces to accommodate the COVID-19 pandemic. In the fall, the school rented part of an outdoor campus not far from its original site. During the winter months, classes transitioned to remote learning, with an outdoor gathering at a public park one afternoon per week. In the spring, the school rented a local church space, which allowed for a mix of indoor and outdoor activity; during this time, classes continued to meet remotely via Zoom two days each week.

Children in Lena's Class

Lena taught the oldest group in the school, which included eight "first years" (the equivalent of fifth graders in other schools) and eight "second years" (sixth graders). The

63

latter were in their second year with Lena. Most children had attended this school since Kindergarten, as had their siblings; three transferred from district public schools. Of the six children I interviewed, three were first years and three second years. Two were assigned male at birth and identified as boys; four were assigned female at birth and identified as girls at the time of the study, though one currently identifies as nonbinary. Three identified as white, two as Korean and/or Asian American, and one as Black and African American.

Lena

Lena identified as an Asian American woman, the daughter of Korean immigrants. She reflected on her social identities as experiences of "in-betweenness": a U.S. citizen with undocumented parents, neither Black nor white in a national context built around the Black/white racial binary, and living with access to financial and institutional resources as an adult that she did not have as a child. Art and crafting were core to her identity, and she described teaching as "the next closest thing to making art." Lena had been teaching for seven years, five at the school in this study. With a degree in Adolescent Prevention Practice, she also held a formal counseling role at the school. In this capacity, she met one-on-one with many children in her class, at scheduled times outside of academic periods, to support their social-emotional development. She facilitated, documented, and communicated with families about these sessions apart from her role as classroom teacher.

As educators' own schooling experiences shape their work as teachers, I will share briefly about Lena's relationship with school growing up. Lena attended district public elementary and middle schools, both urban and suburban; she was tracked into Gifted and Talented programs within two of these. She then went to a large, exam-based magnet high school specializing in science and technology. She attended a liberal arts college, where she completed a dual major in Peace and Justice Studies and Education and received her teaching license. Beginning at an early age, Lena loved reading, across genres and subject matter, in and out of school. As a child, she was drawn to historical fiction stories but disliked social studies classes in school that were geared toward coverage and memorization. She shared that her science education was not memorable in any particular way.

Lena's relationship with mathematics was complex. She was consistently placed in "accelerated" math classes in school, received clear messages that she was "very good at math," and, as an adult, saw herself as someone who understood the content she taught. Yet, Lena shared, "I hated it all the way through." She described her experiences of school math as socially isolating, focused on solitary workbook exercises and little to no meaningful interaction with her teachers. Her entwined mathematics and racial identities were part of this fraught relationship (Martin, 2006). She reflected, "math is so tied to negative stereotypes about Asian folks;" her efforts to trouble and free from those stereotypes involved resistance to school mathematics, even as she excelled in it by many school measures.

Generating Data Sources

During the 2020–2021 school year, I visited Lena's class outdoors and on Zoom; reflected with Lena following these visits, sometimes supported by video; conducted two

65

semi-structured interviews with Lena about her teaching experience and philosophy broadly; and interviewed six children in her class. Resulting data sources included video and audio recordings, field notes, transcripts, and initial memos. To transcribe interviews, I used various artificial intelligence services to prepare initial transcripts and then edited the text by listening back to the audio recording; I transcribed class visits myself. Next, I elaborate on each mode of data generation, focusing on conversations with Lena since they formed the basis for this paper's analysis. (See Appendix A for tables summarizing data generation activities.)

Class Visits

I visited the following periods in the class's daily schedule: Community Care (at the start of each day), Reflection (at the close of each day), Math, Social Studies, Literacy, Grad Projects (culminating research projects for sixth graders), Racial Justice Club, and AFAB (Assigned Female at Birth) Book Club. Sessions generally began with the whole class together, involved small break-out groups or independent work at various points, and closed with a return to the whole group. During visits, I moved between taking detailed field notes as an observer and more direct participation in class discussions and small-group activities. Immediately following each visit, I fleshed out field notes, including initial impressions and reflections (Emerson et al., 1995; Miles et al., 2020). I video- and audio- recorded remote class sessions via Zoom, usually turning off the recording when I joined small groups or conferred one-on-one with a child.

Conversations with Lena

Lena and I engaged in five types of conversations as part of our joint inquiry: semi-structured interviews about her teaching philosophy and practice; video-based reflection conversations using recordings from my class visits; informal debriefs following each visit; unstructured conversations in informal spaces (e.g., walking outdoors); and member checking several months after the study period had ended. I elaborate on each type of conversation below.

Especially as our collaboration went on, conversations with Lena took the form of "pedagogical encounters" in which both she and I shared our perspectives and worked towards mutual learning (Vossoughi & Zavala, 2020). The researcher's role in such conversations is distinct from a traditional interview. Rather than assuming an ostensibly neutral stance, focusing on eliciting Lena's views for the purposes of data collection, I crafted questions and in-the-moment responses to "facilitate reflection aimed at new understandings" and to generate questions we might continue to explore together (Vossoughi & Zavala, 2020, p. 139; see also Byram, 1996; Pomerantz & Zemel, 2003). For example, I pointed to moments from class visits that might complicate or expand Lena's view on a particular aspect of her teaching, or I shared my perspective on an interaction to offer an alternative framing for what was going on. Sometimes I asked Lena to imagine hypothetical scenarios and articulate what she might do or expect if she and children were free from certain constraints of their current context. Such an approach to research conversations holds learning and development, of both researcher and participant, as a primary purpose of the encounter (Goldman et al., 2022).

67

General Interviews

Lena participated in two semi-structured interviews, both early in the study. I asked about the theoretical commitments and frameworks that inspired her pedagogy, core routines and practices in her class, approaches to building relationships with children, and her goals and experiences teaching particular school disciplines. Of our conversations, these interviews were the most akin to a traditional interview. As Lena reflected after the first one, she felt like she was offering "a spiel," in contrast to our other encounters, which felt "like two teachers talking." Interviews were recorded via Zoom.

Video Reflection Conversations

At four points during our study, Lena and I reflected on a particular episode of her teaching by watching video recordings from a class visit. Two video reflections were based on a mathematics period, one on writing, and one on a gender-based affinity group discussion on masculinity. To prepare for these conversations, I selected one or more clips from the longer recording (usually totaling 10–15 minutes) that included moments of interest to discuss with Lena. These moments either raised a specific question I wanted to ask Lena or seemed like fertile examples for considering our overarching questions about liberatory relationship building.

The conversations began with a prompt for Lena to reflect on the purposes guiding her planning for this class session as well as initial impressions of how it unfolded. Then we watched the video clip together, pausing to discuss at stopping points I had planned and ones that struck either of us as we watched. Sometimes conversation was brief, and in other moments Lena and I engaged in a lengthy back-and-forth before

68

returning to the video. Video reflections were recorded via Zoom; the class visit playback video was embedded in the recording through the Zoom screen-share function.

Class Visit Debriefs

Lena and I also held debrief conversations following visits to her class, which did not involve video. These ranged in duration from twenty minutes to over an hour and typically occurred within a few days of the visit. Between the visit and debrief, I reviewed my observation field notes and prepared possible questions and reflections to raise with Lena. During the conversation, we prioritized anything that Lena wanted to discuss. With the exception of one, these conversations were not audio recorded, though I took detailed notes during and wrote reflections after them.

Visiting Conversations

To develop our relationship in a context that was not focused on work, Lena and I visited with one another by going for outdoor walks or sharing tea at one of our homes. These conversations meandered in focus, spanning topics related to teaching and research as well as our lives generally.

Member Checking

Several months after the school year, Lena and I held a member checking conversation in which I shared themes and conceptual framing from my developing analysis. This was guided by a document with both textual and graphic displays of ideas. At various points, I asked for feedback around a specific idea, but the general invitation was for Lena to share what resonated or shed light for her, anything that did not seem accurate to her experience or perspective, and questions that arose as she engaged with what I presented.

Interviews with Children

Interviews with children were important to our broader collaboration because they helped me understand the relational ethos of the class from children's perspectives, including the role they saw Lena playing within the community and their individual relationships with her. I explicitly framed for children that these interviews were an opportunity for them to teach me about their class and what mattered to them as its members. Sometimes I shared children's words (anonymously) with Lena during our reflections, since they often expressed appreciation or insight that children did not share directly with her. Reflecting children's words back to Lena was a way to affirm, honor, and sometimes see anew the work she and children were doing together. Lena commented at multiple points that my conversations with children were part of the "gift" she received through our partnership. While this paper focuses on Lena's perspective, I included children's voices in the data analysis and writing to illuminate and texture Lena's reflections, especially related to her liberatory pedagogy.

I interviewed six children in Lena's class, varying in age, gender and racial/ethnic identities, and habits of participation in class (e.g., more and less vocal during discussions). Interviews lasted between 20–30 minutes and took place on Zoom during the first block of the school day, while the rest of the class also gathered remotely. I invited children to describe what they and Lena do and how they feel during various parts of the school day. They selected at least three subject areas or class periods they wanted to discuss. I also invited children to reflect on a specific project or conversation related to

their class's learning about activism, justice, and oppression. These conversations, more so than my conversations with Lena, ran like traditional interviews, and they were recorded via Zoom.

Personal Reflexive Memos

I considered my own activity and relationships with participants as part of "the domain of 'the researched'" (Bang & Vossoughi, 2016, p. 174). To this end, I kept reflective memos about my participation and positionality as a researcher; relations with Lena and her students; and questions, tensions, and insights that grew through this work. I wrote memos after every class visit and conversation with participants as well as throughout the analysis and writing process.

Data Analysis

I used grounded thematic analysis to construct themes from my conversations with Lena (Charmaz, 2006; Glaser & Strauss, 1967; Shank, 2001, in Matthews, 2009). Interviews with children and class observations were not primary in this analysis, but I read through and coded these data sources to triangulate themes synthesized from conversations with Lena. Through the coding process, I sought to answer the paired questions: How did Lena describe and reflect on her liberatory pedagogy? How did she describe and reflect on her mathematics teaching?

Grounded Thematic Coding

My process involved four phases of coding and synthesizing. First I did open coding of transcripts and field notes from conversations with Lena (Miles et al., 2020). I highlighted excerpts ranging in length from one to several sentences and used a constant comparative method to develop codes inductively, creating in vivo code labels often. After a first pass through transcripts and notes, I consolidated and parsed categories by reading through all excerpts associated with each code, noting connections across and variations within them. I then worked with transcripts from class observations and children's interviews, creating some new codes but mostly adding excerpts to the existing codebook.

Finally, I engaged in a "resistant reading" of transcripts of Lena's conversations to note discrepancies and nuances in the themes I had constructed (Rogers, 2004). During this phase, I looked for moments that complicated existing codes or the developing underpinning of my analysis (i.e., that there was a rift between Lena's liberatory pedagogy and mathematics teaching). I also attuned to partially expressed ideas, conversation possibilities that we surfaced but did not pursue, and imaginative talk that lifted off from our lived experience. Here, I channeled Ellingson and Sotirin's (2020) notion of "data engagement," which they argue "is not merely about representing a given reality or experience 'grounded in the data'" but also calls for "animat[ing] new ways of thinking" with and from data sources (p. 822).

In each phase of the thematic analysis, I moved recursively between fine-grained nuances in the transcripts and broader trends and theoretical constructs (Erickson, 2004). For example, to set up for the first round of open coding, I organized the codebook into two overarching sections, *Liberatory Pedagogy* and *Mathematics Teaching* (recognizing that these were not necessarily mutually exclusive, i.e., that an excerpt could fall within both). Within *Mathematics Teaching*, I also created tentative subfolders to represent

various dimensions of teaching. I revised subfolder names as the codes within them developed, ending up with *school discipline, curriculum, relations with children,* and *professional learning*.

As Erickson (2004) contends, grounded thematic analysis is not theoryindependent. When Lena's ideas resonated with concepts from literature, I sometimes reflected those terms in my codes. She spoke to frameworks like restorative justice (Davis, 2019; Evans & Vaandering, 2016; Kaba, 2021), critical consciousness and unlearning oppression (Freire, 1970; Ginwright, 2016; Swalwell, 2013), engaged pedagogy (hooks, 1994), critical care (Antrop-González & De Jesús, 2006; Beauboeuf-Lafontant, 2002; Prieto & Villenas, 2012; Rolón-Dow, 2005; Thompson, 1998), and politicized trust and solidarity (Camangian & Cariaga, 2021; Vakil & McKinney de Royston, 2019). Related to mathematics education literature, Lena invoked notions of hierarchy (Bishop, 1990; Bullock & Meiners, 2019; D'Ambrosio, 1999; Louie, 2017; Martin, 2009b; Parks, 2010), binary logic (R. Gutiérrez, 2019; Fasheh, 2015; Triadafillidis, 1998; Yeh & Rubel, 2020), and authority for knowing (Ball, 1993; Lampert, 1990).

During each phase of coding, I kept initial analytic memos to track emerging themes and questions (Emerson et al., 1995). Following the coding process, I wrote lengthier memos guided by analytic questions derived from the study's research questions. At that point, I focused on articulating and fleshing out overarching themes to describe Lena's liberatory pedagogy, her perspectives on mathematics teaching, and the relationship between the two.

A "Case" of Interest

Throughout the paper, I use the term "case" to refer to the focus of this analysis and its significance. The larger project's methodology reflected aspects of but was not fully a case study. I collected multiple data sources to develop an in-depth understanding of one educator's pedagogy, and both this paper and our broader collaboration pursued descriptive questions (Yin, 2012). Yet the paper does not devote adequate attention to that range of data sources to be considered a case study analysis; it prioritizes one type of data, my conversations with Lena. Additionally, the process of our larger collaboration was more "interventive" and co-constructed than traditional paradigms of case study research, in which the researcher attempts "not to disturb the ordinary activity of the case" (Stake, 1995, p. 12). Through our joint reflection, Lena and I expected to shape, not only uncover or describe, her perspectives on teaching—and, potentially, her direct work with children.

Still, I find the notion of case useful for articulating the "boundedness" of what this paper analyzes and why it might be of "special interest" (Stake, 1995, p. xi). Stake (1995) suggests that, in contrast to a teacher herself, "teaching lacks the specificity, the boundedness, to be called a case" (p. 2). The focus of this paper, a teacher's reflections *on* teaching, may lie somewhere in between--perhaps not a discrete "entity," but bounded in the sense that only Lena could share her reflections (Yin, 2012). While corroborated by evidence from her classroom practice and the perspectives of her students, my analysis centers on the "particularity and complexity" of how Lena viewed her teaching (Stake, 1995, p. xi). As a researcher, I considered her reflection on practice worthy of study "itself" (Stake, 1995), as "unique" and "revelatory" as her actual teaching (Yin, 2012, p.6). In the paper's discussion, I suggest that "com[ing] to know well" Lena's reflections on teaching, specifically from a cross-disciplinary perspective, has the potential to inspire inquiry beyond her context as well (Stake, 1995, p. 8).

Researcher Positionality

I conclude this section with comments on researcher positionality, building from the broader positionality statement in Chapter One. The previous statement situated my work as a researcher, and my interest in liberatory mathematics pedagogy, within other educational and professional experiences and the broader sociopolitical world. Here I speak specifically to my role as a researcher partnering with a classroom teacher.

As a research partner, I consider myself involved in co-operative activity with participants (Goodwin, 2017). While collaborating with Lena, I saw class visits and conversations as opportunities for dialogue and joint learning, distinct from approaches to ethnographic research that position researchers as neutral observers who elicit information from participants as a form of extraction (Dennis, 2018; Smith, 2012; Vossoughi & Zavala, 2020). Throughout the process of data engagement, I was attuned to my presence in the data sources and to the ways interacting with them again was shaping my perspective. At the same time, a degree of distance from a local context allows researchers to "help make visible the practices, meanings, and contradictions that often become invisible to those closest to the action" (K. D. Gutiérrez & Vossoughi, 2010, p. 101). It took ongoing reflection to maintain a "socially close but experientially separate stance"—relating deeply with Lena and children, often engaging in activity alongside them, while also stepping back from that activity to document and analyze (Emerson et al., 1995, p. 36).

Across my work, including in this collaboration and my writing about it, I seek to honor the complexities and radical hope of teaching. I recognize the systems that constrain school-based educators (Philip et al., 2022) and believe insurgent work can happen in local places within those systems. Reflecting on his positionality as a researcher who writes intimately about others' teaching practice, Michie (2005) shares wisdom inspired by Sarah Lawrence-Lightfoot:

I made an effort to balance my "empathetic regard" for each teacher with "critical attention" and a "discerning gaze" (Lightfoot, 1983, p. 6). One thing that gave me pause in being critical, however, was that I knew from 9 years' experience just how hard teaching is, and that trying to teach with an eye on social justice makes it that much more of an uphill climb....My assumption from the outset was that these five teachers were doing good work in their classrooms, and that as Sara Lawrence-Lightfoot (1997) has noted, "The researcher who asks first 'what is good here?' is likely to absorb a very different reality than the one who is on a mission to discover the sources of failure" (p. 9). (p. 185)

Like Michie, I sought to weave empathetic regard and critical discernment as I worked with Lena and wrote about her teaching. Though the analysis that follows raises some difficult questions, it starts from and closes with, "what is good here?"

Findings

In this analysis, I explore the relationship between Lena's perspectives on liberatory pedagogy and mathematics teaching. I seek to understand the rift she experienced between them, including its various fault lines and where it was more and less clear-cut. I begin by presenting themes that illustrate her liberatory pedagogy, which derive primarily from our discussion of humanities teaching and work with children that was not explicitly disciplinary. Reflections on mathematics teaching sometimes resonated, though more often they informed these themes through contrast, as Lena expressed a sense of separation from the values and practices of her liberatory pedagogy. The next part of the analysis focuses on Lena's perspectives on mathematics teaching. It is organized into four subsections, representing different dimensions of teaching. In this part I consider connections to Lena's liberatory pedagogy by highlighting the ways she experienced mathematics teaching as peripheral and sometimes counter to her liberatory praxis. The section closes with possibilities for liberatory mathematics that surfaced in our conversations.

Liberatory Pedagogy

The themes below describe foundational components of Lena's liberatory pedagogy (see Figure 1 for a summary of themes). They are not intended to represent a comprehensive framework or finite description of her teaching but rather highlight recurrent threads from our conversations. I provide this description in order to give a sense of what was valued and integral to Lena's liberatory praxis (Booker & Goldman, 2016).

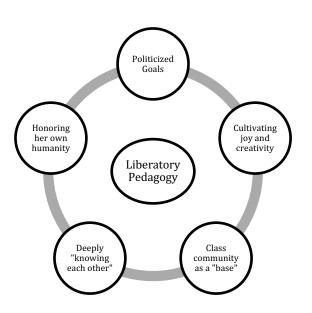


Figure 1. Themes of Lena's Liberatory Pedagogy.

Politicized Goals

Lena's teaching goals were explicitly tied to the broader political world. In her reflections on practice and in conversation with children, she regularly named the systems of oppression that they were working to counter, including but not limited to: colonization, white supremacy, sexism, homophobia, and transphobia. Her class discussed how these systems and efforts to dismantle them spanned multiple levels of experience (which they referred to as the "four I's"⁸: internalized, interpersonal, institutional, and ideological). The following two goals—to develop critical consciousness and support internalized and relational healing—illustrate the multi-level nature of her politicized vision.

78

⁸ Throughout the findings section, I use quotation marks around words and statements that come directly from transcripts of conversations with participants. Unless otherwise noted, the speaker is the subject of the sentence or clause in which the quotation is embedded. If the subject is not a person, and no speaker is noted, Lena can be the assumed speaker. In the rare cases in which multiple sources are quoted in a single sentence, I use parentheses to denote attribution of each statement.

Developing Critical Consciousness

Lena referred to critical consciousness raising as "the fabric" of teaching and learning in her class. She drew on Freire's (1970) notion of critical consciousness, which entails critical analysis and action around issues of social injustice. By reflecting on unjust social, political, and economic conditions, learners come to understand the root causes of those conditions, their positions with respect to them, and their agency in creating change (Freire, 1970). Lena sought to support children in "analyzing power, understanding who has it, why they have it, how they have it, how they use it" and "the constructs and the stories that have been created throughout human history" to perpetuate relations of supremacy. This purpose extended to all children in her class, including those who held dominant positions in society. She regularly communicated the message that oppression "hurts the person who created or the person who is supposed to have power, too….White people, racism hurts you, too. Colonizers, colonization hurts us, too."

To nurture this critical awareness, Lena supported children to identify their responsibility and potential roles in efforts for social transformation. For example, after studying the history of disenfranchisement and voting rights in the United States, the class participated in a national postcard writing project during the 2020 presidential election and later in the year connected with a candidate for local political office around voter registration initiatives. The class read a novel about experiences of colonization among the Haudenosaunee peoples, wrote historical fiction stories about Cherokee Removal and Native American boarding schools, and studied the intersections of colonization and climate justice today. The class held regular conversations in response to current events—such as incidents of anti-Asian and anti-Black violence and state legislation limiting the rights of transgender youth—and shared about their personal experiences with racism, sexism, and homophobia. They convened as a Racial Justice Club to engage in local actions around some of these issues.

In children's words, this work inspired "a strong sense of justice" (Mariame), which included "feeling really, really mad about the issues" (Poppy) as well as a belief that "we can help and our voice matters" (Mariame). Carter acknowledged that the class's critical learning sometimes felt "awkward" and "not fun," giving the example of a conversation following Derek Chauvin's trial for the murder of George Floyd in April 2021. Still, Carter maintained, these conversations were "really good" to have "because they don't shed kids from reality."

Supporting Internalized and Relational Healing from Oppression

In addition to supporting children to reflect on and take action around injustices in society, Lena worked with her class to "heal" from the harm those injustices cause on internalized and interpersonal levels. At the close of an affinity group discussion with children who were assigned male at birth (AMAB), Lena pressed members of the group: "So we've been talking a lot today about society, or the world's stereotypes, but [in this part of the activity] I want you to think about you: you Elias, you Carter, like how do these questions relate to you?" She shared what personal connections might sound like, offering an example about an AMAB friend of hers, and later thanked participants "for reflecting and thinking about yourself."

Lena also encouraged children to consider connections between the systems of

oppression they learned about on a societal level and the power dynamics they reproduced through relationships with one another. She suggested that doing so deeply and sensitively could be challenging. During a book club meeting with children who were assigned female at birth (AFAB), Lena asked how the sexist and racist stereotypes the group was discussing "showed up" in their class. Participants mentioned AMAB peers' jokes about puberty and sexual assault and assumptions about feminine and masculine body types. Lena later reflected that she wished the group had more directly addressed the ways power dynamics manifested among AFAB members of the class, including during this very discussion. Poppy had spoken up when her classmate laughed in response to a comment from a peer. Lena later explained that the person laughing often articulated mature critical analysis of structural oppressions while continuing to exercise superiority within the class. Lena imagined raising the question, in a one-on-one context, "Don't you see how you're using your power?" but felt "frustrated" that she had not yet been "direct in that political way" with this child.

Lena drew upon her restorative justice training to facilitate one-on-one, small group, and whole-class processes for relational healing. I return to restorative justice practices below, here highlighting that Lena saw restorative circles as a way of "getting to what people's needs are," processing "internal fears" at the root of harmful social behaviors, and feeling compassion toward one another so that the group could liberate from the oppressive power dynamics they were reproducing. She posited that "interpersonal and internal shifts" among individuals were inextricable from the larger sociopolitical change the class discussed, each holding the power to shape the other. When reflecting on her politicized teaching goals, Lena often spoke of a tension between maintaining political clarity and avoiding "indoctrinat[ing] other people with my exact political beliefs." She said that she was cautious "all the time about sounding preachy." While she intended for class inquiries to unambiguously raise anti-oppressive themes—for example, the idea that white supremacy hurts everyone, or that the message of "one way is the right way" stems from systems of supremacy—she added, "I don't want to just say [these ideas] to them." In Lena's view, critical consciousness raising and healing from oppression were not about "pass[ing] this very specific perspective on to you," but rather inviting children to explore the world as open to critical interrogation and transformation, with an honesty about the structures of power set up to thwart such inquiry. Living out this distinction demanded ongoing reflection on her part.

Cultivating Joy and Creativity

Lena strove to cultivate an ethos of joy, creativity, and wonder in her class. Teaching in the humanities was a way to share her own fascination with the world and its stories. She spoke of curricular design as an art making process that gave her "great joy" and inspired creative experiences with children.

Sharing Love for Disciplinary Learning

Lena described her own relationship with the humanities as inspired. The activities she engaged in with her class—analyzing novels, writing personal narratives, studying connections between historical and current events—were ones that, "in my own human beingness, I love." Beginning as a young child, she found deep social and emotional connectedness through reading: "I always loved stories and learning about people's experiences and feeling the feelings of the people's experiences." She narrated a similar curiosity toward historical inquiry, hypothetically addressing children: "Isn't it so interesting? I just want you to know it and be fascinated by the world and by people like I am."

For Lena, teaching was a way to "revel" together with children in the world's stories, and she found opportunities for such reveling in the humanities. Lena believed that something inherent in the content—whether novels, pieces of art, or historical events—inspired children's engagement and thus "has done the work for me." For example, she found that books had the power to "bring everyone together" as "the characters come alive and become...a member of our class community." Speaking about social studies, she remarked, "It's like you show the world to the kids and then they just can't help but be...fascinated by it." At the same time, Lena alluded to the role of pedagogy in bringing this content alive:

I find that when I teach social studies, kids are very engaged....I'm sure if you look into anything with enough care and inquiry and questions, everything is fascinating in the world. But I just feel the ability to make that happen [in social studies].

Such care and inquiry also guided her work with children on their writing. Lena described a passion for "finding one thread of something to follow in a student's work and then really talking that through with them." She emphasized the importance of supporting children to find their stories and voice, and not letting her own enthusiasm for the art of writing lead her to project her visions onto their work. The children I spoke with discussed their humanities learning experiences with a similar sense of enthusiasm and purpose. Juju shared of her love for writing, which she said developed during her time in Lena's class: "I really like writing because I can kind of make a creative story....It's just such a big space you can write...like such a big world and everything. And it's really fun." During a class discussion about the purposes of reading and writing, she said that she reads to "imagine someone else's life" and, as Carter also put it, to consider the world from "multiple perspectives." Similarly, Cam and Poppy described social studies projects as intriguing encounters with other people's stories, which both expanded their view of the world and helped them better understand their place within it.

Curricular Design as Creative Process

Lena compared teaching to making art and discussed curriculum design as part of that creative process. When designing humanities curriculum, Lena both channeled her own creativity and looked to "creations" from other sources to anchor the class's work. She used the term "creations" broadly, to refer to pieces of art or literature as well as historical or scientific phenomena that inspired the class's inquiry. (As I elaborate later, Lena was unsure about what might constitute a mathematical "creation.") Each humanities unit centered around a novel that the class read together, with both small- and whole-group opportunities for discussion. These units, which Lena characterized as interdisciplinary and project-based, usually integrated an activism or community-based "field" component and culminated in a piece of creative work whose purpose tied into that "outside world" context. Lena thought carefully about how to craft an "arc" of experiences that would bring these elements together so that children sensed "the whole rich thing."

Interdisciplinarity was fundamental to this richness. Lena found that historical inquiry on its own could sometimes focus too heavily on "understanding that this timeline happened" and the "institutional" factors at play. During social studies, she sometimes felt "guilty, like I'm doing a lot of, 'These are the ways things are.'" Incorporating novels brought "human stories and characters" to the center, so that "suddenly kids are talking about, 'Oh, it must feel like this. Oh, what must it feel like to be...?"—which, she said, "helps get down to the different levels of the thing we're learning about." Community Care, which Lena considered part of the humanities, could similarly amplify the interpersonal, story-driven dimensions of social studies. Designing an integrated humanities curriculum treated various skill sets—related to historical inquiry, textual analysis, writing composition, and social-emotional development—as interconnected.

Lena saw curricular design as open to children's creative input. Children had a voice in shaping the culminating projects of each unit of study, both in their individual creative works and when the class collaborated on a larger project. Lena reflected:

I love the times when I'm like, "Okay...I have a vision for something we're gonna create—a mural, a book, a something. Should we do it in three parts, you guys?" And they're like, "Yeah, no, we should do it in four parts"....And then someone's like, "Somebody should do this," and somebody else is like, "Somebody should do this," and I'm like..."Okay, great, let's go. Do it. Try it." I love that...I love the energy, the creative energy, the designing energy.

Lena regarded children as fellow "creators" moment to moment during discussions as well. She especially enjoyed planning for conversations in which she brought questions and materials as provocations, "to spark different connections for different people," and then she and children explored the "diversity of ideas" and stories that came up.

Class Community as a "Base"

Lena described class community as a "base" that both grounded and grew through the class's learning. The class had routine practices that facilitated bonding, mutual accountability, and egalitarian relations. Lena also saw disciplinary learning in the humanities as a time for nurturing community connectedness. Building such a community required careful negotiation of her role as a leader.

Structures for Community Connection

Restorative justice circles were central to community building in Lena's class. They were held for a range of purposes: to get to know and trust one another, process a difficult event, facilitate conflict resolution, or welcome back a returning member of the class. As "holder and facilitator of the space," Lena set a purpose for each circle, posed prompts at various points, and supported children to listen and share. In Mariame's words, "Lena's kind of the one who's making sure we're really, who's helping us...speak from our hearts and from our mind." Within this guiding role, Lena looked to circles as an "equalizing force" that "puts the control out of my hands" and into a community process: When there's a process, I can be like, it's not me that's controlling you, and it's also not you who gets to say how this classroom works. It's like there's an equalizing force among us that...I think works because everybody feels heard....It's a process that helps us figure out how to move forward and how to be together without giving up on each other.

Circles fostered this sense of egalitarianism and shared ownership across contexts in the class. When focused on repairing harm, it took the form of mutual accountability. According to Graceyn, theirs was "a class where we believe in accountability": when someone was "harmed in our community," circles set an expectation and opened space for all members of the class to "hold ourselves accountable for...how we contributed to the problem." Circles also supported the community in moments that were not focused on conflict resolution, including during disciplinary learning. Graceyn continued:

We do a lot of stuff in circles, because it's a way that everyone feels valued, instead of, if you're all sitting at different places, you're sort of on different levels; but if you're all sitting right next to each other, it's like you're all in an equal position.

She underscored the physical and processual aspects of circles that helped create this egalitarian environment. Across interviews, children referred to the physical circle configuration as central to their experience in Lena's class.

Other class structures that fostered a spirit of connectedness included Community Care, affinity groups, and various clubs. During Community Care, the first period of each day, Lena and children engaged in group games, yoga, discussion of current events, sharing about life beyond school, or restorative circles. It was a space for them to connect with each other and the world in a range of ways—through play and embodied practice as well as serious contemplation and storytelling. The class also "bonded" (a term that came up repeatedly in interviews with children) through identity-based affinity groups; book clubs, sometimes organized as affinity group spaces; Racial Justice Club, which involved the whole class; and smaller "Lunch Bunch" groups focused on friendship building. In these groups, class members connected around "issues we care about" (Poppy), from matters of injustice in the broader world to questions of personal identity and adolescent social dynamics. According to Lena, an intention underlying these community processes was to "give us a taste of what it's like to be freed from all those…things that separate people," to come "to see each other and ourselves" as interconnected in mutually sustaining ways.

The class was able to maintain many structures for community building through transitions to new settings, but physical separation on Zoom posed challenges to the depth of social connection Lena and children wanted. Lena shared, "The energy of the room and the energy of activities is really important to me....I really thrive on creating connected-feeling energies...where like, 'dang, we are all really invested right now.'" She explained that creating this "certain mood" relied on being in close touch with children emotionally moment-to-moment, and that "I don't know how to read them online" with the same attunement. She also worried that children were less able to attend closely to one another and that, as a result, their virtual interactions felt "flat…like we're doing test-prep passages, emotionally speaking." Cam confirmed that "when we're in a whole group

on Zoom...I get a little tired, or it's just sort of hard to be attentive and participate a lot," whereas "if you're in a whole group in person, then it's more interactive." Carter also reflected that whole-group times on Zoom were "harder," specifically because "we don't have an actual circle." As an observer, I witnessed the class's connected-feeling energies alive on Zoom, though I understood Lena's and children's experience of them as sometimes muted, harder to access, or easily disrupted.

"Connected-Feeling Energies" Through Disciplinary Learning

The class's community base was a foundation for disciplinary activity in the humanities. Both Lena and children reflected that Community Care set a tone and relational infrastructure for the class's work during the rest of the day. Lena embedded circle processes into social studies and literacy periods, and the class's ethic of mutual care and accountability supported children as they collaborated on projects and engaged together in discussions. According to Lena, disciplinary learning was a context for "practicing" the community's valued ways of being together:

One way of...truly being kind to each other—and not just kind, but vulnerable and accountable to each other—is that we do work together....I love being with young people, and I love just caring about them, but if we all 16 of us were in a room and there was no academic content, I'm not really sure what we would be...living it through, practicing it through.

Lena also described disciplinary learning as developing the class's "connectedfeeling energies." Activism projects in social studies engendered a sense of common purpose and shared decision making. Lena reflected that the novels the class read together, and "the experience of sharing a book," were also "great unifiers." During reading time, all children worked through the same text, which contrasts with the typical school practice of segregating children by reading level. This shared text became an anchor for the class's discussions and creative projects throughout a humanities unit. Lena also designed book clubs as "unifying" experiences for particular subgroups of the larger class community. For example, she convened the AFAB Book Club in part to bridge divides between younger and older AFAB members of the class. In both small and whole-group contexts, Lena saw discussions in the humanities as a place "where community building happens," where children "connect with each other."

Negotiating Her Position of Authority

Lena thought carefully about her role as a leader of the class's "democratic community." She acknowledged that, "because I am the adult and the teacher, or the counselor in some cases, we're not equal pals....There's a power that I do think I have." While she rejected an interpretation of this power as, "it's me that's controlling you," Lena took seriously her responsibility to "provide structure" and "bounds," as well as inspiration and resources, to support children "to get to places that they couldn't get by themselves," both individually and collectively. Her guidance took the form of curriculum planning and circle facilitation as well as clear communication of "high standards" for their participation in class and ways of treating one another. Cam referred to Lena as "the organizer, or the supports" of their class, explaining:

I feel like "leader" isn't the right word because that's sort of like she has all the power and she's trying to get power from us, but she is....I guess she's a leader who doesn't try to take power away from us or give power to herself. And she just tries to make what we want possible when, like, we can't.

Lena and I discussed the tensions that arise when trying to distribute power in this way. For example, when working with children to address social conflicts, Lena often sensed an ethically "right choice" that she wanted a child or group of children to make, but she refrained from simply instructing them in what to do, recognizing that letting kids "learn to be" sometimes required them to make decisions without her direct guidance right away. As she planned for conflict resolution circles, she strove for: "not too controlled, but with bounds."

During class discussions more generally, Lena was wary of the "talking power" she held and tried to "reduce teacher voice" when possible, so as to encourage dialogue among children. Yet she did not disregard her guiding role. She articulated, "What feels really good is when the arrows aren't all pointing at me, they're pointing at each other...I put my body out there, my energy out in certain ways...and can step back more with my words." In other moments, she chimed in frequently and intentionally with her words. She saw her participation affirming children and opening space for them to share vulnerably and "expansively." Reflecting on her facilitation during a Kids of Color Affinity Group conversation, Lena shared that, when she "said the right in-between things"—encouraging a child to "say more," drawing connections among children's contributions, or sharing her own personal stories—she then noticed children "engag[ing] with each other." She reflected: I would sort of push, and I feel like the pushing made it so that then each kid would say something a little bit more expansive. And when students say interesting, expansive things, I don't think I have to do much to make the other students listen to what [they're] saying and be like, "Oh, wait, I have something to say about that."

While Lena made clear that children in conversation with one another was "where community building happens," she recognized that this did not preclude her active participation and that, in fact, she was in a particular position to keep the "arrows moving."

Deeply "Knowing Each Other"

Lena emphasized that at the core of class community was an ethic and practice of "knowing each other" in deep and transformative ways. Deep knowing involved the following qualities of relationship: authenticity, vulnerability, care, and trust⁹. These were fractal in nature, reflected in one-on-one relationships, within small groups, and across the class. Practicing these ways of being as a whole class supported individuals to carry them into moments with one another; simultaneously, each authentic, caring, vulnerable, and trusting moment between individuals wove into the larger community fabric.

Authenticity

To come to know one another, Lena called herself and children to "be real about who you are," to explore, express, and connect with others around what was "true for

⁹ Lena used these terms often when discussing the qualities of social relationships in her class. While the four are entwined, I separate them for the sake of clear elaboration.

you." Authenticity was a quality of individual expression as well as interactions and relationships with others. Lena made it a priority to foster "genuine, connected moments" across the day. These could take place in interactions around emotionally vulnerable content (e.g., relational tensions or personal fears) or the ordinary details of life (e.g., how a family pet was doing or who won a baseball game).

Individual counseling was one space for building authentic relationships with children. Lena commented, "The kids that I've had counseling with, I just know a lot about what's...real for them and what's live for them." Cam shared that when he talked with Lena during counseling sessions, "She knows what I want, or what I'm trying to get at, or who I am really well." Cam and others spoke about how their counseling experiences nurtured a relationship with Lena that carried into the rest of the day. Lena also discussed the ways her sensibilities as a counselor supported authenticity in her work with the class as a whole. These sensibilities included the ways she could "see" children and what they were "going through," modeled and expected honesty, and "lovingly" encouraged them toward self-awareness.

Vulnerability

Closely tied to authenticity, vulnerability was a quality in relationships that Lena reflected on often. She described it as a willingness to share feelings, stories, and "truth[s] that might be scary to say," in ways that brought class members closer together as they worked toward individual and collective growth. Lena modeled and encouraged vulnerability across the day, from one-on-one check-ins and restorative circles to disciplinary discussions and individuals' written work. Because the class practiced it so

regularly, children were able to lean into vulnerability in moments that called for it. Poppy reflected on the book club conversation in which she had asked classmates not to laugh at one another's contributions: "I wanted to say that because when you're being vulnerable in circle...it's harder to ask questions if you're embarrassed." She was comfortable holding her classmates publicly accountable in this way because they had participated in many restorative conversations before, "so we're already good at being vulnerable to each other."

Lena spoke about the transformative power of vulnerability across levels of experience. Vulnerability was involved in what Lena called "intra-processing moments," as children explored personal questions, aspirations, and traumas. Individuals sharing vulnerably in a group setting could then encourage others to open up, creating a culture of emotional risk-taking and solidarity. Further, Lena believed that personal experiences of vulnerability had the power to "shift" broader cultures and systems:

When one person or when a group of people speak out about their personal experiences with oppression and they heal some of the hurts that have happened in their lives because of that oppression, that is what makes the systems of oppression change....I want them to have this embodied experience that when you change an internal fear, or when you express an internal feeling, it literally can change this system, or this dynamic that you thought was never changeable.

Lena viewed the classroom, like other local settings, as a microcosm of society, where individuals (re)constructed broader social relations as they interacted with one another. She described vulnerability as a quality of personal interaction that could intervene on those systems by opening possibilities for mutual understanding, compassion, and transformation, where oppression would depend on alienation, fear, and emotional intransigence.

Care

Lena described care as a deeply felt concern for and desire to know others. She discussed care in terms of her relationships with individuals, as a communal ethic across the class, and as an orientation toward social life generally. Lena referred to her relationship with children as a "tender friendship" in which "I care very much" about the details of their interests, emotions, behaviors, and growth. She attuned to the moment-to-moment "cues" they sent during class and worked to develop an intricate understanding of each child over the course of their two years together. The children I spoke with experienced Lena's care intimately. They also recognized its political dimensions, as Juju expressed when reflecting on a class discussion about anti-Asian violence: "I'm Korean. It really felt important to me because that's part of my identity, and knowing that somebody's supporting me, like, really, really supporting....knowing that my teacher cares, it was something that really helped."

Expressing care for individual children, including "in public" before their classmates, was one way Lena cultivated an ethic of "communal care" across the class. She spoke of a "caring that [was] bigger than oneself," that enabled children to consider, listen to, and support one another compassionately, especially during moments of tension. Like vulnerability, care had politically transformative potential. Reflecting on the connection between liberation and care, Lena suggested that "we oppress and hate people when we don't know them"; knowing each other well "disrupts the ability to not really care about each other." To Lena, care was fundamental to disrupting a relational status quo in which people harm one another across their separation.

Trust

Engaging in authentic, vulnerable, and caring ways both depended on and further developed trusting relationships. Lena described trust as a kind of solidarity that supported individuals and groups through demanding learning experiences. She reflected most on her own trust-building with children, which she regarded as a particular strength in her relationships with them. According to Lena, children saw her as someone in whom they could confide because she was "concerned about their consent," non-judgemental, and "on their side." Trust enabled them to be honest, with Lena and in group spaces she facilitated, about the deepest parts of themselves. From this position of solidarity, Lena pushed children to grow, "holding them to high standards" and supporting them to work through challenges in and beyond academics.

While Lena's reflections focused on children trusting her and one another, children suggested that the trust was reciprocal. Juju shared that Lena "trusts us" by sharing about parts of her life and engaging in conversations about the world that other teachers might "hide." She described Lena "sitting close with us," both literally (e.g., hanging out at recess) and figuratively. Three other children used the phrase "with us," often with vocal emphasis on the "with," to describe the ways Lena was a trusted and trusting ally in their learning. For example, Cam said that Lena was "really with us" at the launch of their independent research projects and then gradually "let us go in a way" so that they could make self-directed choices. Later, describing Lena's role in the class's Racial Justice Club (RJC), Cam reflected: "She doesn't force us to do anything, or just she understands what we want for RJC, and she sort of helps that become true, and with all of our consent." The children I interviewed portrayed Lena, across contexts, as a reliable source of support who was responsive to their needs while believing in their agency and capacity to self-actualize.

Lena also talked about "trusting the space" and "the process," in reference to her role as a facilitator of group discussions. In discussions, Lena sought to create an environment that would support children to share vulnerably and connect with one another's ideas. She reflected that such facilitation required a trusting stance of "let's see where this goes," which was not always easy to sustain. When she felt "anxious" about barriers to communication (e.g., on Zoom) or pressured by an external demand on her work with children (e.g., state standards), she described herself more tightly "guiding each thing," pressured to "move on" through her planned material in ways that were not responsive to children's contributions and left "no empty spaces for us to feel something." Working to "open the space" demanded active facilitation of a different kind—still carefully guided, but motivated by a trust in what each person would bring to the conversation, in her own and children's capacity to "make connection" with and follow one another's ideas in the moment.

Knowing Each Other on Zoom

Across conversations, Lena reflected on the barriers to deep interpersonal knowing in a virtual setting. While not impossible, developing authentic care,

97

vulnerability, and trust was harder with limited opportunities for moment-to-moment connection. Whereas "during in-person school, there's so many moments of checking in...on how people are doing," those were less frequent on Zoom, as the class chose to take breaks away from their screens during transition periods and did not get to build the "rich before- and after-school life" together that they could in person. Lena shared:

It's mostly invisible to me. I'm not watching them play at recess, I'm not watching—usually, I'm watching every subtle detail of, [for example,] "I said it was lunchtime and these two usually gravitate, but today they didn't." Or, "I'm reading this book to them, and this topic comes up, and she has a sad face on." I'm just not getting any of the cues.

Lena was also concerned that children did not have the bonding opportunities with one another that in prior years had encouraged compassion across friend groups. Referring to exclusionary social dynamics among AFAB members of the class, Lena reiterated the connection between care, knowing each other, and anti-oppressive relations. She hypothesized that one child, who held social power in the class, may have been acting uncharacteristically harsh toward her classmates "because she has no idea what's going on for anybody else, so why would she care?" Lena was more hesitant to authentically "confront" this child's behavior, as she balanced caring for children's fragility amidst the stresses of the pandemic year: "there needs to be a container for that" kind of confrontation, and the online space was not always conducive to such "sensitive" work.

The challenges of Zoom interaction were not final: the class began and ended the

year together in person, half the class had a foundation with Lena from the prior year, and together they found ways to continue cultivating humanizing relations through virtual space. Still, Lena's reflections on these challenges suggested that liberatory ways of knowing each other developed in the interstices of classroom life, not always through discrete structures or practices that could be adapted to an online setting.

Honoring Her Own Humanity

As children intuited, Lena prioritized their needs, visions, and capacity for leadership in the class. She did not center herself as an authority, but nor did she sideline her own needs and values. Lena positioned herself as a fellow community member and human being alongside children.

Well-Being

Lena built a classroom that "felt giving to me," tending to her well-being as well as to children's. Reflecting on her intention to experience joy through the work she planned for the class, Lena expressed: "I'm interested in having fun, I'm interested in enjoying myself every day....I don't want to ask people to do something if it's not serving me either." She identified her own needs within the class' shared commitment to "taking care of ourselves"—noting that she sometimes decided to lead the class in yoga "because I literally need to move" or in meditation as "a way of taking care of my emotions"—and said she could gauge the effectiveness of these activities "when I feel better, too."

Lena was often transparent with children about her needs and emotions as a member of the class. For instance, on the afternoon following anti-Asian mass shootings in Atlanta in March 2021, Lena acknowledged the reason she had spent the morning out of class: "Thank you for giving me that time to do a little bit of community care and self care. I had a nice walk with [colleague]. And we talked about many things, including the news and how we were both reacting." Lena shared and honored her whole personhood— "my human beingness"—alongside children.

Values

As illustrated across the paper, Lena described her pedagogical decision making as values-driven. She used phrases like, "that comes from a value that I have" and "I know that I value that as a teacher." This was another way she recognized her humanity, as it made teaching "feel so personal"—authentic to who she was, honoring of her creative capacity, and supportive of her self-actualization. She reflected:

What will make me feel joy? What are the things I value? The decisions are made based on my skills and my desires. And that feels liberatory because it's like, I'm free as a human to bring myself into the classroom, and I'm not a machine made to fulfill certain standards or goals.

Feeling liberated to bring herself to teaching was vital to her efforts to build a liberatory environment for children.

The freedom to align practice with her values also raised questions, especially when Lena considered her membership within a schoolwide community of practice with other educators. Reflecting on where her pedagogical values "come from," Lena shared that when she considers her colleagues, she often thinks about how "different from me" many of them are. She continued, "Well, I guess it's good we're all so different. We all just teach in ways that...are true to us, and therefore each [class] at [this school] is pretty different. I just keep thinking, 'Is that good?'" Lena appreciated the pedagogical and curricular freedom that her school allowed teachers. She also wondered about the potential for more clearly articulated political and ethical values for teaching within her local professional community. This points toward questions, which Lena and I did not explore in depth, around deprivatizing notions of pedagogy—understanding it not simply as an individual teacher's vision and practice but as collectively developed and held. Such a view seems resonant with Lena's conceptualization of liberatory praxis as interweaving the personal and collective.

Summary

This section offers a view into Lena's liberatory pedagogy, discussing her goals for and ways of relating with children, the community ethos of her class, and her sense of identity as a teacher. Many of the ideas Lena and children invoked as they described her liberatory pedagogy resonated with scholarship on education for liberation: critical consciousness (Freire, 1970; Ginwright, 2016; Woodson, 1933/2009), restorative justice (Evans & Vaandering, 2016), relational histories and healing (Ginwright, 2016; Vossoughi et al, 2020), politicized trust and care (Antrop-González & De Jesús, 2006; Rolón-Dow, 2005; Vakil et al., 2016; Watson, 2018), the role of emotionality in learning (hooks, 1994; Lemke, 2013), and teachers' well-being and self-actualization (hooks, 1994). These were cross-cutting in many respects, spanning disciplines within the humanities, one-on-one and group contexts, and both structured and unplanned moments throughout the day. Mathematics teaching was largely to the side of this liberatory praxis, a trend I examine in the next section. In the paper's discussion section, I return to the literature connections mentioned above as inspiration to further conceptualize liberatory pedagogy in mathematics.

Reflections on Mathematics Teaching

Lena described a rift between her liberatory pedagogy and mathematics teaching as she spoke about her views of the discipline, interactions with curriculum, relations with children, and professional learning experiences. I elaborate on each aspect of pedagogy in a separate subsection below, though I recognize that they are interrelated. Since the rift came into sharpest relief when Lena reflected on mathematics teaching in comparison with her pedagogy in other areas, I highlight those points of contrast within each subsection.

Hierarchy and Instrumentalism in the School Discipline

When Lena referred to the discipline of mathematics, she mostly focused on its K-12 school forms. (Generally, our conversations about disciplinary learning had this focus, but in the humanities Lena was more likely to mention people reading, writing, and engaging in historical inquiry beyond school as well.) Lena characterized school mathematics as rigidly hierarchical, from micro to macro levels of experience, and its purposes as instrumental rather than aesthetic or ethical.

Hierarchy Across Levels of Experience

Lena described school mathematics as "constructed to be very hierarchical." She saw hierarchy operating through the logics of supremacy culture, institutional systems for evaluating and segregating children, classroom-level social dynamics, and internalized feelings of shame and competition. I touch on each of these across the larger section, focusing below on the ideological and internalized levels, which came up most in our conversations.

Logics of Supremacy Culture. Lena discussed the ties between supremacy ideology and hierarchical constructs in school mathematics, such as binary categorizations and authoritative notions of "right." These logics reinforce hierarchies of worth that position children and their ideas as "better than," "not good enough," in need of correction, or even "shameful." The binaries that came up in our conversations included: "good" or "bad" at math, "math person" or "not a math person," does or does not "know," right or wrong, advanced or behind, and abstract or concrete. Lena sourced binary thinking to sociohistorical systems (i.e., colonialism, white supremacy, heterosexism, and ableism) that perpetuate "this idea that something is right and civilized, and something else is not acceptable and not human." Even when binaries are not explicitly evaluative, they rely on the either-or logic that underpins exclusionary classification systems, and they often carry implicit value judgments (e.g., abstraction as more advanced than concrete reasoning).

The hegemony of predetermined "right ways" or answers was a particular source of tension for Lena as she taught mathematics. She was aware of her authority to act as an arbiter of right or valued mathematical thinking and that she sometimes leaned on "teacher-diagnostic" framings that reinscribed notions of "better," "right," and "good." During mathematics conversations, Lena tried to "stay very neutral" when responding to children's ideas, but she acknowledged that this did not always feel authentic. There were times when she was "looking for one answer" and, once a child "hit it on the head," she felt a sense of gratification followed by pressure to get the class to "converge" on that idea. Even when she facilitated conversations and activities that were open to "multiple ways of seeing", she often had "a certain thing" in mind for children to get:

You know those moments when you're like, oh yes, I secretly did have one thing I wanted everyone to say? I can't act like I had one thing I wanted everyone to say, because that's not the point of a constructivist exercise such as this¹⁰, but oh, Olive finally said the thing that I was like, "Hello, is anyone going to notice the equivalent ratios? Is that not the point of what we've been talking about for the past four weeks?" But, you know, working hard to not let that come out.

In these moments, signaling an openness to "different ways" seemed to Lena like a kind of feigned performance or "a trick," since she still adhered to a hierarchy of "important" ideas based on her curricular objectives and the discipline's notions of "sophisticated" mathematical understanding.

Lena noted that the tension between converging on a teacher-determined "key idea" and opening to a "divergence of ideas" was not restricted to math. Still, valuing multiple perspectives seemed integral to the humanities in ways it did not in mathematics. Even when Lena held a sense of a right or valued way to see or act—related, for example, to a historical question of injustice, an author's message, or a social conflict among

¹⁰ Here Lena reflected on a routine activity called "Which One Doesn't Belong?" She presented four quadrants—each containing a figure, symbol, or expression (in this case, a ratio of two numbers)— and invited children to notice relationships across them by identifying how each quadrant could be considered one that "doesn't belong" to a pattern that applied to the other three.

children in class—she maintained that her view "doesn't have to be the end all, be all." She reflected:

In literacy and social studies...if a student is like, "Is this right? Is this the answer?" my response is never..."Oh, yeah, that's right" or "Oh yeah, that's wrong." It's, "Well, what did the author say? What did the character say?"...Or in social studies, "Well from this perspective? From whose perspective? What makes you say that?"...It's not like I'm the knower of the right or wrong....we can analyze it together.

When the notion of rightness did come up in the humanities, it was in the sense of ethical principles (e.g., related to "how we treat each other") or a commitment to understand the "truth" of a particular source's perspective. In contrast, the authority for "right" in mathematics seemed arbitrary—disconnected from any moral value or particular position in the world—which lent "right ways" an almost unquestionable force.

Internalized Competition and Shame. Lena was concerned about the assumptions of competition and feelings of shame that children entered her class with, specifically around mathematics. She reflected on the ways prior school experiences, outof-school programs, messages from families, and "popular cultural narratives" about what it means to be "good at math" shaped children's mathematical identities in her class. For example, a few children were enrolled in out-of-school mathematics programs or worked through published curriculum materials with family members at home. Lena described their participation in mathematics as especially competitive and worried about the status hierarchies this reinforced across the class. While facilitating mathematics discussions, Lena felt "genuinely worried that everyone thinks...that these two [who attend a Russian Math program] know everything about math...and what everyone else has to say isn't as valuable." Lena was acutely aware of the shame children internalized because they did not "know" something that their classmates did or because they felt "left behind" over years in school mathematics. She recognized that this concern shaped her teaching moment to moment: "I feel like I think more about not traumatizing them than about their math thinking. I don't know, it really gets in my way."

Lena wished she could "reverse" the supremacy logics that children internalized through school mathematics, but she was not always confident in her ability to do so. She imagined:

If in math class someone could help students not feel that judgment, not feel that hierarchy—if someone could somehow help kids feel like, "What I'm doing is my own way of understanding something, and I don't have to match somebody else because they're not the right way to do it"...that would be an extremely interpersonal and internal shift in white supremacy culture....I just don't really feel like I do that very well.

Lena described her effort to attenuate competitive dynamics and internalized shame by communicating to children, "I don't approve of you more based on how right or wrong you are." Yet she did not see herself disrupting the foundational hierarchies that led to these dynamics. Referring to conversations she would have with individual children, Lena reflected:

At least I feel good that I'm talking to Adam about his feelings...[or helping Juju]

undo how she needs to perform to be accepted in the world. Maybe math is a time to do that, because math has that hierarchy. In other subjects, I feel like I can craft the time we have together to be outside the oppressions of how people think about learning or knowing or thinking. But in math...I guess I'm okay that that structure kind of exists because at least I get to help kids practice noticing it and then healing *some* part of it.

The qualifying language of "at least" and "some part of it" contrasted with the ways Lena spoke about healing from internalized oppression in other contexts. At other times, her work with children felt "subversive," with the power to "free" and "transform" on personal and systemic levels. In contrast, oppressive systems held a grip over mathematics that Lena did not see as within her capacity to fundamentally challenge.

Instrumental Purposes

Lena questioned the deeper purpose of learning and doing mathematics. Several times, when reflecting on a math lesson, she wondered, "Why are we doing this? What's the point?" This was a question that she invited children to consider regularly in the humanities: "I open a lot of things with, 'Let's talk about what we're doing and why we're doing it"—a practice she referred to as "placing" the discussion or activity. Lena recognized that she posed this question far less often in mathematics. During class visits, I noticed that she "placed" mathematics learning experiences by focusing on a discrete skill the class was practicing and locating its purpose in connection with a prior or upcoming lesson objective. For instance, to transition from the warm-up into the main portion of a lesson on solving algebraic equations, Lena framed for the class:

What you were just practicing there was writing expressions and thinking about how you can use parentheses and order of operations to solve expressions, right? And we talked yesterday and the day before about how order of operations is very important to remember as we're working on solving algebraic equations. So we're going to use [that skill]...but apply it to a place where there are variables.

At the start of a subsequent math period, she stated, "We've been talking for weeks now about expressions and equations, and you've all gotten very, very skilled and thoughtful about expressions and equations." In the humanities, Lena sometimes focused class activities on a particular skill (e.g., organizing sentences into a paragraph or identifying a type of figurative language), but these were always embedded within a larger creative, socially meaningful project.

In her reflections, Lena communicated that she felt "lost" or simply not compelled when she considered the broader significance of mathematical practice. She granted that the study of mathematics could be linked to social, political, and creative purposes. Beyond moving through grade-level standards to prepare for middle and high school, mathematics could be helpful in daily life and work, artistry, and even critical social analysis. She alluded to those who "use math for creativity" and derive aesthetic pleasure from it: "I know that there are people who can look at the numbers and look at the shapes and look at the patterns and be like, 'Oh, my gosh, there's a story here.'" Unlike in the humanities, however, Lena did not personally connect with that story, with the "substance of it." Further, the inspiration others find in mathematics may not constitute a story in Lena's sense of the term, which seemed to entail a clarity of social and ethical significance. She said that she could "think of what that creation and that story is for science, social studies, music, art, everything except math." As a result, she questioned her ability to encourage children to find a sense of meaning and fascination in mathematics.

Lena was also critical of instrumental framings of the discipline, suggesting that when mathematics is valued primarily as a "tool for" daily living, artistic creation, or sociopolitical analysis, the ways people relate with one another around it also feel instrumental. She reflected, "Having tools is helpful, [but] when I'm only teaching something that feels like a tool, it feels like I'm treating the students like workers, you know? As opposed to creators." Her critique extended to mathematical inquiry around sociopolitical themes:

You're trying to connect math to social justice, or you're just using the ability to find a percent of something as the tool to look at a question, and the question is a social science question, but math is the tool for it.

Lena questioned various narratives of the "use-value" of mathematics, or ways mathematics is seen as important because of its utility *for* other activities and purposes (Pais, 2013). Some might suggest that valuing mathematics for art making or sociopolitical inquiry is distinct from the utility most commonly assigned to mathematics, as a tool for economic mobility and reproduction of a capitalist society. Even when ascribed a creative or socially transformative use-value, however, mathematics as "tool for" was dissonant with what Lena experienced in other areas of her work with children: disciplinary activity as itself an experience of liberatory social living.

Uncreative Relationship with Curriculum

Lena described a less inspired relationship with curriculum in mathematics than she had in the humanities. When planning mathematics curriculum, she felt disconnected from creative vision and the themes that integrated the class's work in literacy and social studies. This disconnect stemmed in part from external pressures and constraints on her curricular choices that were specific to mathematics.

A Disciplinary Silo

Mirroring a schoolwide trend, mathematics curriculum in Lena's class was dissociated from her interdisciplinary humanities curriculum. (Science at the school, taught by a separate teacher, was sometimes treated as a disciplinary silo as well, but at other times it integrated with humanities studies.) The separation between mathematics and humanities was evident when I visited the class's end-of-year Learning Fair. Children presented at stations displaying writing, bead work, painting, collage, and interactive charts for visitors, all reflecting themes from the class's humanities inquiry across the year. There was one table devoted to mathematics, with tessellation designs that the class had explored with another teacher. Lena's mathematics work with children was not represented. Mathematics stood apart as just another "school subject" (Booker & Goldman, 2016)—seemingly disconnected from social justice themes, artistic creation, and what felt "real" and "important" to children.

Based on interviews, children clearly valued the integrated nature of their humanities learning. According to Graceyn, "You tie all of your subjects together around what you're learning about in social studies....I like when everything's tied together, because I think that gives you a more full learning of it." Cam appreciated the class' study integrating issues of climate change and (de)colonization, sharing that, before, he had "always thought about them as separate things." Unspoken in children's statements was that mathematics was not part of this "full learning."

External Pressures and Constraints

Lena mentioned school- and system-level constraints on curriculum that were less insistent, or did not exist, outside of mathematics. Math was the only subject area for which Lena's school purchased a curriculum¹¹. Though teachers followed the workbooks to varying degrees, and the school's principal was open to them creating beyond the provided materials, math was the one area in which teachers were not expected to craft their curriculum. Lena said she felt compelled to "get through" the curriculum's chapters so that children would not fall behind. Reinforcing this linear model, the school also segregated children by grade level for mathematics (i.e., dividing Lena's class into fifth and sixth grade groups), which was not usually the case at other times of day.

Beyond her particular school context, Lena felt pressures from mathematics state standards and standardized school entrance exams. The Independent School Entrance Examination (ISEE) included literacy components as well, but Lena felt a particular accountability to children and their families for the mathematics portion: "Parents will be like, 'My kid got a 70 percentile on the ISEE. Why did they get so many decimal problems wrong?'...And for some reason, no one's ever like, 'Why didn't you teach my kid how to read this specific ISEE test?'" Lena also invoked state curriculum standards in

¹¹ The school used the workbook series from *Math in Focus: Singapore Math* (Cavendish, 2012).

mathematics. In one of our earliest conversations, she mentioned standards frameworks in English Language Arts and Social Studies—which often "align with what I think are just inherently good and fun" to teach—but in subsequent conversations, curriculum standards only came up in relation to math. Lena suggested that this might be because she had less confidence in her ability to support all children to meet grade-level mathematics standards if she veered from an external curriculum. "Constrained" by these external pressures on her curriculum choices, Lena felt less free to imagine and create beyond their demands.

Disconnected from Creativity

Mathematics curriculum planning did not engage Lena's artistic sensibilities in the ways that planning in the humanities did. While her process was inventive in that she pulled from various resources, she saw those materials as a source of objectives, concepts, and activities "outside of me," disconnected from a sense of creative vision. Lena looked to the school's student workbook for a basic sequence of content and skills to follow. She also referred to open-source curricula online, specifically the "teacher narration" portion, to understand "the conceptual thinking we'll need to go through." She then pieced together problems and activities that connected to the concepts in focus. Lena said that she thought about "the choreography and rhythm" of instructional activities across the day and week, to ensure a balance of minilessons, partner tasks or games, and independent practice. Here, Lena referred to a kind of logistical ingenuity rather than creative relationship with the actual mathematical content or pedagogical goals.

Lena described her relationship with the content of mathematics curriculum as

missing "emotional connection" and creative inspiration:

Every time I plan math, I really am like, okay, well, these problems and these scenarios exist. I don't have to come up with them; they just exist in this workbook, in this curriculum, on this website, on Kahn Academy. [For example,] it exists that Mary had three dogs and Todd had four dogs, and you need to find a ratio of them....I feel extremely like I didn't put any creativity into what I'm doing....I feel like I'm just giving them stuff that someone else made, that I didn't have any emotional connection with.

When curriculum planning felt "dry" and devoid of personal connection, Lena then experienced the enacted curriculum similarly:

I don't have that much confidence in my process of how to help a group of students engage with that thing that exists outside of me....Did they get something from it? How do I...know that they felt something and that it meant something to them?

Lena expressed a disconnect from curricular vision and narrative in mathematics (Dietiker, 2015; Drake & Sherin, 2012), unsure of a "whole rich thing" that went deeper than a logical progression of concepts and skills. Whereas in the humanities, "I feel like there's meaning behind everything we're doing...[and see] how it connects to an arc we're going through....in math I don't feel that way when I'm planning." Using a sequenced workbook to determine mathematical goals for children and plans for their shared activity, Lena related to these as "objective[s] I'm apparently supposed to have as a teacher" rather than as personally meaningful visions for children and the world.

Supportive (Not Transformative) Relations with Children

Next, I explore an area of Lena's teaching in which the rift with mathematics was less gaping. Lena suggested that her relationships with children were not siloed by discipline, that authenticity, care, vulnerability, and trust cut across the content and contexts of their interactions. She commented that relationship building was "the only part of my math teaching that feels successful." Yet Lena did not describe social relations in mathematics as liberatory, in the sense of freeing from sources of harm and oppression. Further, Lena did not see mathematics as conducive to the "connected-feeling" community ethos that was central to her liberatory pedagogy.

One "Layer" of Knowing Children

Lena referred to multiple "layers" of relationship building, one of which involved close attunement to children's emotions, needs, and ways of thinking. This layer developed just as authentically in mathematics as at other times of day:

You build relationships with students by teaching them how to add fractions, right? In those moments of talking about adding fractions, Mariame gets frustrated, and I'm like, "Oh, what's frustrating?" And then they say what's frustrating them, and I'm like, it's not about the fraction anymore. It's about, how do I handle stress?...So even if the content isn't, write a poem about your greatest fear, you still build relationships.

Lena saw mathematics teaching as an opportunity to further understand and connect with children academically and emotionally. She also said that the "many hours of relationship building" that occurred throughout the day formed a "foundation" for each "teaching moment" in mathematics.

Lena found fewer occasions in mathematics for the second layer of relationship learning about and exploring with children the "content of their lives" beyond school. Lena explained:

When the content is about a student's perspective, or a student caring deeply about something, then there's like double the relationship building in that learning moment....When we're doing a project in Racial Justice Club, and Kya is there, and she's writing poetry about what's going on when her parents got divorced, and what happens for her when she goes to the DR [Dominican Republic] every summer but then comes back and everyone on her mom's side is white and she doesn't know how to deal with—when there's that layer of it, too, I'm still helping her through the basics of reading and writing and being in school, but then I'm also learning stuff that wouldn't come up in a fractions lesson, you know, like content about her life.

While Lena's efforts to know children as particular, complex people did not stop at disciplinary boundaries, there was a fundamental layer of knowing each other that did not seem to develop in mathematics, because school-based mathematical activity did not invite exploration of their multifaceted lives and identities.

Emotional Solidarity

Lena expressed compassionate solidarity with children during mathematics, allying with them through struggle and intervening in potentially exclusionary social dynamics. She reflected frequently on how she supported children for whom mathematics was "very hard," making space for them to name what was difficult and their feelings about it. She said that children "choose to tell me about" their "insecurities," that she "trust[s] they'll come to me to tell me when they're upset, and that when they do…that I can support them." Lena was similarly supportive in whole-group contexts, vigilant to competitive dynamics around "knowing" and "not knowing" and attuned to the feelings of inferiority and exclusion these could create. Reflecting on a moment from class in which Carter was not sure how to refer to square numbers in a problem, Lena shared:

I'm panicking inside because I'm like, oh no, Mia and Juju probably know what Carter's talking about....If I don't interrupt soon, one of them's probably gonna unmute herself and say [with a tone of ridicule], "You mean a square [number]?" And then somebody's gonna feel like, "Oh no, it's another math thing that I don't know." I just really have it in my mind that math could be a traumatizing time....It felt like my responsibility was to rephrase what Carter said so that it was valuable that Carter said that. And [I communicated], "you didn't need to know the word, that didn't really matter."

Lena worked to buffer against feelings of exclusion by framing all contributions to mathematical discussions as valuable and preventing children from expressing superiority over others. However, this was not the politically transformative care she spoke of at other times. Compassionate solidarity in mathematics was a source of affective support within, more than a freeing from, hierarchies and the internalized harm they could cause.

Obscured "Community Base"

When Lena reflected on social relationships in mathematics, she tended to focus on her interactions with individual children or on dynamics across the class that "separated" children from one another (e.g., competition, judgment). She did not reflect on mathematics teaching and learning as a collective experience that "created connectedfeeling energies" or a "community base" in the way that analyzing a novel or collaborating on an activist project did. The processes Lena turned to as "an equalizing force among us" at other times of day (e.g., restorative justice circles) did not seem to have a place or an equivalent in mathematics. Her caring, trusting relationships with children around mathematics also seemed disconnected from the community-wide sense of knowing each other that was so central to her liberatory pedagogy. When I raised this theme directly with Lena, she expressed that she was unsure about how to nurture class community in mathematics. Given Lena's insight and practice around community building with children at other times of day, this statement of not knowing highlighted an especially stark fault line of the rift with mathematics.

Limited Professional Learning in Mathematics

In this final part of the section, I consider the rift in Lena's professional learning experiences across disciplines. We did not discuss her professional learning history in depth, but it had implications for the other dimensions of rift elaborated above. Lena attributed a lot about her teaching to prior professional development, which included university-based teacher education, school-based coaching and mentorship, and training as a counselor and restorative justice facilitator. Few of these experiences focused on mathematics, a point she underscored multiple times. Lena recognized that when she did not feel "well-trained" she was more likely to work within normative expectations of mathematics teaching.

"I Haven't Been Taught"

When Lena expressed feeling "insecure" or "lost" in her mathematics teaching, she often commented that she hadn't been "taught" and needed to "learn more" in these areas. For example, in a conversation about creativity in mathematics, Lena alluded to patterns in the binary number system and said, "I get that I'm supposed to take some sort of meaning from that, but I haven't been taught yet how to do it." Lena also expressed that she was "not as confident" posing questions that facilitated sensemaking conversations, including all children "in the fold" during discussions while still furthering everyone's learning, or building class community around mathematics. Lena felt "undersupported, or just [as though] I don't know as much" and suggested that professional development sessions or instructional coaching might improve her confidence in these areas of teaching practice.

Outside of mathematics, Lena considered herself "well-trained" for the kinds of teaching she wanted to do. She referred often to the "frameworks" that informed her pedagogy, such as developmental models for building racial identity and critical consciousness, Ethnic Studies curriculum principles, and approaches for engaging with children in literary analysis. Lena did not invoke frameworks or principles for mathematics teaching and recognized that she neither had encountered nor actively "sought out" the professional learning opportunities that might introduce her to them.

Not Asking "Bigger Questions"

Lena noted that the questions she asked about mathematics teaching were different in nature than those she asked about her practice in other areas. She reflected that her professional inquiry got "stuck" in the concerns of dominant educational discourse, which kept her from "asking any bigger questions about math." While reflecting on a video segment in which she worked one-on-one with a child with a diagnosed learning disability, Lena shared:

When we talk about math, Ada, I feel like the things I bring up and spend time talking about...the things that stick with me are..."special education," or math disability, or learning disability, as opposed to...justice-oriented thinking in math. I feel like I can't even get there....In humanities, I'm like, "Okay, how do we talk about abolition?" But in math, I'm always just like, "Okay, how am I even supposed to be teaching the standards?"

She later continued:

My teaching condition in math is that I feel very limited, partially because of things I've described around my insecurities, or my feeling like I don't have the tools...and then also just by my fear of, well, parents will need—and they [children] have to take the ISEE [Independent School Entrance Exam].

Lena recognized the systemic forces—curriculum standards, entrance exams, and disability discourse—that limited her professional inquiry, specifically the freedom to ask politically courageous questions. Still, she did not dismiss possibilities for professional agency and growth. Later in this conversation, Lena suggested that teachers could be "bought into" institutional pressures, "even if we think we aren't." She acknowledged that when she felt less "well-trained" she was not as likely to imagine beyond constraints in the bold ways she did in the humanities. She sought "tools" and experiences that might open toward a more expansive praxis in mathematics.

Summary

This part of the section examined the rift between Lena's liberatory pedagogy and her experience of mathematics teaching. Lena spoke to hierarchy and instrumentality in the discipline, a divorce from creativity and thematic integration when working with curriculum, constraints on her ability to build class community and connect with children about "the content of their lives," and a need for professional learning experiences that would support her to reimagine her mathematics instruction. The depth of the rift was not uniform across these fault lines, as illustrated by Lena's caring relationships with children and attunement to power dynamics in the class during mathematics learning. Still, her reflections on social relationships in mathematics did not foreground the liberatory vision that guided other areas of her work. In the next section, I share moments from our conversations in which this vision came into clearer view around mathematics teaching and learning.

Possibilities for Liberatory Mathematics Pedagogy

This final part of the section builds from the preceding analysis to begin to consider possibilities for liberatory mathematics in Lena's class. I share "glimmers" of liberatory mathematics from my conversations with Lena, grounded in particular data excerpts. I call these moments "glimmers" because we did not explore them at length and they tended to be considerations of what could be, rather than reflections of Lena's dayto-day experience. This speculative analysis continues in the paper's discussion, where I suggest avenues for inquiry into liberatory mathematics inspired by the earlier description of Lena's liberatory pedagogy.

Glimmers of Liberatory Mathematics

During our conversations, Lena and I began to ask "bigger questions" about a freer, more humanizing mathematics. Sometimes these questions related to a particular class session that I visited; at other times, they departed from Lena's teaching practice and even school contexts generally. We considered mathematics as: playful, creative, and embodied; open to multiplicities of meaning and ways of seeing; embedded in the questions and purposes of people's lives beyond school; and germane to the ethics of communal living. As glimmers, these moments of talk raised possibilities without exploring them in depth. I will share three examples: in two, Lena imagined hypothetical scenarios with children, and the third was a reflection on my own experience.

Transformative Social Interactions Around Mathematics

In one exchange, Lena imagined a classroom-based dialogue that would disrupt cultures of supremacy and exclusion around mathematics learning. We were reflecting on an AMAB affinity group discussion with three members of her class. Though neither the group discussion nor our reflection conversation was focused on mathematics, Lena responded in this moment by envisioning children in the affinity group talking openly about norms of masculinity in mathematics classrooms.

In her reflection, Lena wondered whether the group discussion had been

meaningful to participants; she was not confident that it had opened a space for children to be vulnerable with one another and "shift a dynamic." I asked her to elaborate: "What would it be for these kids...the kind of shifting that you imagine?" She narrated a hypothetical dialogue among three boys about judgment and competition in mathematics class¹²:

Let's say we were together in a room, the four of us—me, Adam, Elias, and Carter—and we were talking about all these pressures to be better than other people, these pressures to never show what's hard for you as a boy, and Adam was finally like, "You guys, I've never really talked about this, but I feel like you guys are always watching me and judging me because you don't want to be my math partner, because you know that I'm not that fast at math, and I feel really worried about that." And he had that internal shift of turning that shame into something that he wanted to share and he wanted to have support for....I have this fantasy that that makes Elias be like, "Oh my gosh, sometimes I do do that, don't I? I am always telling—I am always being like, 'I'm done, first!' in math, and I never really realized that could be hurtful for you...." And then Carter could say something about how he's really nervous and he's always worried that someone's watching him and that he didn't know that Adam felt that way.

Lena imagined that she would respond by thanking Adam for sharing "something that's hard for you" and then suggesting, "'Maybe we could have a conversation as a math

¹² Lena chose mathematics as a backdrop for this hypothetical conversation. In the video segment we watched, Adam had mentioned being "strong at math" as an alternative type of strength to physical strength, though Lena and I were not, in this moment of reflection, discussing that point in the video.

group and change—I don't know, does everyone having the same lesson work for us?" She continued, "I could see it leading to some sort of, 'Let's try this different math structure for a while." Building on themes from prior reflection conversations, here Lena narrated children talking vulnerably with one another about dynamics of exclusion and competition and taking personal responsibility for collective healing. She imagined this restorative process prompting a change in the "structure" of their mathematics class and, on a larger scale, cultures of masculinity. This was a rare instance in which Lena associated mathematics-related classroom interactions with the power to disrupt oppressive systems.

Naturally Emerging, Embodied Mathematical Inquiry

The next example offers a glimmer of mathematical inquiry beyond classroom walls, as Lena imagined exploring mathematical questions through athletic play. Lena and I had just reflected on a video segment in which she worked one-on-one with Adam to review decimal place values. Lena commented that this session felt like "grasping for straws" as her language, symbolic representations, and visual models did not seem to resonate with Adam. She discussed the challenges of supporting mathematical sensemaking when children with learning disabilities are expected to meet certain grade level standards. Toward the end of the conversation, I asked, "If you could liberate yourself to imagine working with Adam around something that you consider math—and go to a really free place—what would you hope to do together?" After a long pause, Lena responded: Well, maybe I would want to pick something that he likes and cares about. He's really into cars these days. He's also into baseball and biking. And maybe we—yeah, if this is totally outside of the realm of what I have to do because of the constraints—then maybe we would—and it's, I don't know, a summer day, and for some reason, this is what we want to do—I don't know, maybe we could race each other on our bikes and then measure how long the distance was, and how fast it was for each person, and then see who was faster and how do we know? Maybe we'd try a bunch of times, and we'd see a pattern, like our rates increased when we were on a hill versus not....So, yeah, maybe I would like to explore a little mystery of the concrete world around Adam, and help him see some patterns. Because I feel often like he just doesn't see the patterns, or at least he doesn't know how to express that he sees the patterns. But when he's pumping his legs really fast to go up a hill and trying to meet the same rate, I feel like he would see the pattern.

In this scene, mathematics was embedded in embodied, place-based inquiry. Mathematical questions emerged naturally in the context of activity that inspired joy and connected people to "the concrete world" around them. Lena imagined that bicycle racing would allow Adam to *feel* the mathematics of rate—not through language and symbols, but in his body's relationship to land and space. She also underscored that they would engage in this activity "because this is what we want to do," apart from a predetermined need to solve a mathematics problem. Though set in a place and time beyond school, this vision suggests qualities of mathematical encounters that could extend to school-based learning: rooted in activities children "care about," engaging embodied modes of sensemaking, and encouraging playful, exploratory relationships among adults and children.

A few conversation turns later, Lena commented, "I really like that question....I do feel constrained, so it is nice to hear a question that's like, you don't have to be." We acknowledged that institutional and systemic constraints were real and attended to them across our reflections. Occasional invitations to imagine, however, freed us from feeling so "stuck" in them that we were not asking bigger questions, including about what liberatory mathematics could be.

In Creative Community

The following excerpt describes feelings of social connectedness and creative inspiration within a mathematics classroom. During a conversation in which Lena said she felt disconnected from sources of creativity in mathematics, I shared about my own elementary school experience. I reflected on a class in which mathematical inquiry sometimes carried on for multiple hours at a time, driven by a shared sense that we were creating something consequential together. Small groups of children spread out around the classroom and hallway outside, exploring various questions and patterns related to a common topic. We would then convene for a whole-class conversation about what we had discovered and the new questions our inquiries raised. I reflected:

I really believed that...the reason we were together and excited to be communicating and close to one another was because there was something that we were participating in creating. There are dominant narratives of the discipline—that structure, beauty, meaning are out there in some objective world that we are separate from and can come to discern, or measure, or control—but there was something different about how I related to it as an elementary student. We were making it.

My recollection invoked the creative, "connected-feeling energies" Lena described in her humanities teaching. I told of an experience of mathematics learning that brought people together for inspired exploration, in the ways books and writing did in Lena's class. In response, Lena reiterated that mathematics, in her experience, felt governed by "right and wrong answers," though she acknowledged, "I know we've talked about how it doesn't have to be that way." Months later, in a conversation about building community around mathematics, Lena recalled, "I think of that teacher of yours, and I always think, how'd they do it?" We agreed this might be an area for future inquiry together.

Summary

This final part of the analysis began to imagine beyond the rift. In each exchange above, Lena and I envisioned mathematics learning being otherwise (Benjamin, 2016), gesturing beyond Lena's actual practice to physical settings and relational contexts that freed from some of the assumptions that constrained her mathematics teaching. Though brief, these glimmers suggested that mathematics might not be irreconcilable with liberatory praxis, a proposition I explore in the discussion that follows.

Discussion

The previous analysis profiled an elementary teacher's perspectives on practice across disciplines, illustrating the liberatory pedagogy that guided her work and the tensions she experienced between this liberatory vision and mathematics teaching. The section that follows extends analysis of the rift and possibilities for repair beyond Lena's context. I first draw connections to scholarship in mathematics education that addresses systemic "layers" of the rift Lena experienced (Booker & Goldman, 2016, p. 232). I then consider possibilities that Lena's praxis raises for liberatory pedagogy in mathematics.

Systemic Layers of the Rift

In their analysis of a rift between experiences of formal mathematics education and families' mathematical practices, Booker and Goldman (2016) suggest that this divide reached beyond the local context of their project. They point to "layers of systemic experience that render math a school subject more so than a human practice" and frame this as "a social, cultural, and historical problem" (pp. 232, 233). Authors who examine the sociohistorical and ideological dimensions of mathematics education highlight its collusion with the very systems of oppression that Lena strove to liberate from through her teaching—among them, colonialism, racial capitalism, white supremacy, and heteropatriarchy (e.g., Greer & Mukhopadhyay, 2003; Martin, 2013; Pais, 2013; Pais et al., 2010; Skovsmose, 2008; Valero, 2008; Vossoughi & Vakil, 2018; Yeh & Rubel, 2020). Their critiques speak to many of Lena's concerns about school mathematics, including its binary and universal logics, hierarchical orderings, capitalist framings, and seeming disconnection from creativity and ethics.

Perhaps the rift in Lena's teaching reflected not simply a lacuna in her practice but an intuitive political response to the "place mathematics occupies" in the world and the pressures she is under to participate in this hegemonic "structural arrangement" as an educator (Pais, 2013, p. 17). Skovsmose (2008) writes that educators often receive the message that they "must serve as an ambassador of mathematics" (p. 16). It is understandable that Lena would feel ethical conflict as an emissary of a mathematics that sustains the political, economic, and cultural status quo (p. 16). She expressed a desire to resist the oppressive workings of school mathematics, similar to the ways she approached humanities teaching "subversively," but did not see clear possibilities for how to do so. Liberatory mathematics pedagogy seemed to demand broader reimagining of the purposes and values of school mathematics.

Towards Liberatory Mathematics Pedagogy

This part of the section brings the spirit of Lena's broader pedagogy to mathematics teaching and learning. Though Lena and I did not, together, explore liberatory mathematics, our conversations presented openings for such exploration. I propose avenues for future inquiry inspired by Lena's praxis, toward conceptualizing liberatory mathematics pedagogy beyond her specific context. I draw connections to extant literature on mathematics education as well as scholarship on liberatory education more broadly.

This part takes up Booker and Goldman's (2016) call to "car[e] for and restor[e] what is valued and integral to people's lives" (p. 226). Having examined what was valued and integral to Lena's liberatory pedagogy, I consider how that might be cared for in contexts of mathematics teaching and learning. This is not a "restoration" in the sense of simply reinstating or rehabilitating something that once existed. It is also not a question of direct translation from humanities to mathematics. I contend that the disconnect Lena

felt was not a matter of lacking the language or tools to translate her liberatory praxis to mathematics. As discussed above, she sensed tensions that source to deep within the discipline of mathematics and systems of mathematics education. She intuited the insufficiencies of simply importing aspects of liberatory practice into mathematics classrooms and calling this "liberatory mathematics pedagogy," without addressing deeper shifts and openings needed to liberate mathematics learning in school. Below, I consider what a deeper reimagining might entail, inspired by key facets of Lena's pedagogy.

Politicized Goals

Lena's pedagogical goals were explicitly politicized. These included developing critical consciousness and supporting internalized and relational healing from oppression. Scholarship on critical and social justice mathematics, which builds upon work in critical literacy education (Freire, 1970; Murrell, 1997; Perry, 2003), examines curriculum and pedagogy that engage learners in sociopolitical inquiry with mathematics (Frankenstein, 1983, 2013; Gutstein, 2006; Kokka, 2020; Skovsmose, 1994). Projects that integrate mathematics and social justice themes have the potential to develop young people's critical consciousness around issues of (in)justice in the world (Gutstein, 2003). As Lena cautioned, however, curricular integration tends to treat mathematics as a predetermined set of concepts and tools to be applied instrumentally *to* questions of social change. In addition to exploring sociopolitical issues through quantitative lenses, liberatory curricula would address the politics of mathematical knowledge (Frankenstein, 2013), investigating diverse histories of mathematics around the world, power structures that

validate certain forms of mathematics and nullify others, the ways mathematics has been used to further imperial and colonial projects, and the political narratives people craft with mathematics. Existing scholarship explores these sociohistorical foundations of the discipline but rarely in contexts that invite youth into such inquiry (Ascher, 1991; Bishop, 1990; D'Ambrosio, 1985; Das & Adams, 2019; Eglash, 1997; Joseph, 1991/2011; Martin, 2013).

Lena saw sociopolitical inquiry as tied to internalized and relational healing among children in her class. Extending Ginwright's (2016) notion of relational healing to mathematics classrooms, Kokka (2019, 2022) has examined social justice mathematics pedagogy that invites young people to process their emotions as they critically examine problem contexts. This emotional reflection, however, focuses on the societal issues that problem contexts address (e.g., immigration, labor policy), not necessarily young people's experiences with mathematics and mathematics learning. Drawing on restorative justice principles (Evans & Vaandering, 2016), future work could explore how youth and educators reflect on their relationships, identities, and experiences of harm and freedom in contexts of mathematics learning. For example, what notions of self-worth and assumptions of hierarchy have people internalized through mathematics teaching and learning (Fasheh, 2012; Louie, 2019)? When has mathematics felt dignity-affirming and socially connected (Espinoza et al., 2020; Evans & Vaandering, 2016; Sengupta-Irving & Vossoughi, 2019)? How do these experiences reflect or disrupt systems of oppression in society at large? In addition to reflection, restorative pedagogies would design for mathematical activity that nurtures politically transformative relationships, discussed

further below.

Cultivating Joy and Creativity

Lena made it a priority to cultivate joy and creativity in her class. Centering joy is important to philosophies of liberatory education (hooks, 1994; Love, 2019; Muhammad, 2020). Some discussions of mathematics as a discipline speak to the joy, wonder, and aesthetic appreciation that professional mathematicians experience (Burton, 1999, 1995/2005; Su, 2017), but school mathematics is not usually designed or felt as this kind of encounter, particularly for people from historically marginalized communities (R. Gutiérrez, 2018). Further, the world of professional mathematics has not reflected cultural heterogeneity in a way that would open it to diverse interpretations of joy and creativity (R. Gutiérrez, 2017; Riling, 2020).

Inquiry into liberatory mathematics might ask where and how people find joy and creative expression through mathematics, across diverse contexts, some of which may not conventionally be considered mathematical (Ascher, 2002; Sengupta-Irving & Vossoughi, 2019). Young people could explore the mathematics naturally embedded in activities of daily living, such as the bicycle riding Lena imagined with Adam or cultural practices important to their communities (Barajas-López & Bang, 2018; González et al., 2001; Nasir, 2002; Sterenberg et al., 2010). Educators and youth might investigate mathematics as a form of creation or storytelling, similar to the ways Lena's class approached historical and literary inquiry (Dietiker, 2015; Dietiker & Richman, 2021; R. Gutiérrez, 2018). More conventional types of classroom activity can also be sites for joy and creativity, as children co-construct mathematical ideas with one another (Ball, 1993;

Fosnot & Dolk, 2001; Lampert, 1990). In these contexts, it is important to examine the cultural expectations that shape what kinds of participation qualify as creative and mathematically valuable (de Freitas, 2012; J. F. Gutiérrez & Scott, 2019; Riling, 2020). There is also a need for professional learning experiences that enable educators to connect with their own joy, creativity, and wonder in mathematics, which they can then channel in work with young people.

Class Community as a "Base"

Lena's class developed a sense of community that both supported and grew through disciplinary learning in the humanities. While literature on mathematics classrooms has addressed forms of intellectual community that nurture collaboration, shared ownership, and distributed authority for knowing (Ball, 1993; Cobb & Yackel, 1996; Featherstone et al., 2011; Lampert, 1990), further work is needed to explore community building in mathematics that carries the political clarity of Lena's pedagogy. Such work would examine issues of identity and power in the social relations that develop around mathematics learning. In addition to designing activities that center interdependence and distributed authority, educators might facilitate direct conversations with young people to process power dynamics and possibilities for relational healing in their mathematics learning. Crucially, liberatory community building demands an approach to facilitation that positions teachers not apart from and above children, orchestrating their activity, but rather "with" them (as children in Lena's class often expressed) in critical solidarity (Camangian & Cariaga, 2021) and relationally responsive understanding (Shotter, 2006a, 2006b). This means that teachers, too, would need to

critically reflect on their roles in classroom power dynamics and their personal relationships with mathematics.

Deeply "Knowing Each Other"

The foundation for liberatory community in Lena's class was an ethic of "knowing each other," characterized by authenticity, vulnerability, care, and trust. The microrelational work of liberatory teaching and learning is underexamined in literature on mathematics education. Future research could extend studies of critical care and trust (Antrop-González & De Jesús, 2006; Rolón-Dow, 2005; Vakil et al., 2016; Watson, 2018) to mathematics learning contexts, tracing the ways these relations develop both moment-to-moment and over time (Vossoughi et al, 2020). Such work might highlight the role of emotionality in relationship building—as Lena described, the "connectedfeeling energies" that develop when there are "spaces to feel" and express "emotional truths" during disciplinary learning (see also hooks, 1994; Kokka, 2019, 2022; Lemke, 2013; Toliver, 1993). This would open up a more sociopolitically situated perspective on affective experience than has been common in mathematics education literature (e.g., Burton, 1999; Cobb et al., 1989; Gordon, 1978).

Honoring Her Own Humanity

Lena recognized her own "human beingness"—her needs, interests, and values alongside children's. Her commitment to self-actualization through teaching (hooks, 1994) often surfaced tensions in the context of mathematics. When educators have a "gut negative reaction to math" (Lena), formed through personal histories with and political analyses of school mathematics, what professional learning experiences might support them to both honor these perspectives and open toward new relationships with mathematics? How could the discipline, in and beyond schools, also be framed as open to transformation in this process? Critical scholarship on mathematics teacher learning has emphasized the need to politicize educators' perspectives on mathematics instruction (Aguirre et al, 2012; Battey & Franke, 2015; Bell et al., 2021; Das & Adams, 2019; de Freitas & Zolkower, 2009; Gonzalez, 2009; R. Gutiérrez, 2012; Louie et al., 2021; Nicol et al., 2020). Further work is needed to design and study professional learning contexts that would inspire teachers like Lena, who have a sophisticated analysis of educational injustice in mathematics, to envision and experiment with liberatory alternatives.

I return to implications for future inquiry, specifically around teacher development, in the concluding section. First, I address limitations of this paper's analysis.

Study Limitations

The COVID-19 pandemic shaped my work with Lena in multiple ways, limiting the forms of research collaboration that were possible, impacting methods of data generation, and influencing the topics we discussed and areas Lena was willing to open up in her practice. During the period reported in this paper, Lena and I did not get to experiment "anew" in her class (K. D. Gutiérrez & Vossoughi, 2010). A major barrier to classroom-based co-design was the shifting nature of "classroom" during this school year (across outdoor, Zoom, and limited-capacity indoor settings), which constrained my participation in Lena's class and possibilities for collaborative work alongside children. Through design-based inquiry in the classroom, we might have been able to explore, in practice, the questions this paper raises about liberatory mathematics pedagogy. A deeper focus on children's experiences of mathematics would have added to my analysis of both the rift and possibilities for liberating from it.

Turning to ethnographic documentation and reflection conversations as primary modes of research activity, we still faced barriers to data generation. During the middle of the study period, the school moved to an indoor site several days a week. I was not able to visit at all indoors; during physically-distanced outdoor visits at this location, audio recording was difficult, so I relied fully on fieldnotes. Across the semester, many class visits were virtual, as Lena and children gathered on Zoom. Although I was able to participate in their activities virtually, and video recording was actually easier, the context of the class's interaction was not what Lena and children were used to or desired. Conversations with Lena regularly surfaced challenges that were particular to teaching in a virtual setting, and we processed these together. We were also intentional to expand our reflective purview beyond this particular school year, to honor aspects of Lena's pedagogical vision and practice that flourished during non-pandemic times.

I close the section with a note about analytic validity, pertaining specifically to the relationship between this paper's analysis and the larger collaboration with Lena. I have attempted, through the narrative, to stay close to Lena's language and meanings for her experiences (Emerson et al., 1995; Erickson, 1979). That said, the paper's guiding questions are my own, leading to an analysis that disproportionately reflects my learning from our partnership, and only a slice of that. Lena expressed that our reflections on mathematics teaching were formative for her, and she engaged generously in such

conversations, but critical examination of mathematics education was not of primary interest to her or integral to our joint inquiry.

Conclusion

Examining what was valued and integral to Lena's liberatory pedagogy, and asking how that might be "cared for" in contexts of mathematics learning (Booker & Goldman, 2016), sheds light on a need to further conceptualize liberatory pedagogy in mathematics (Martin, 2009; Martin et al., 2019). This speaks to growing calls within the field to radically reimagine mathematics teaching and learning (Bullock, 2012; Larnell & Martin, 2021; Gholson, 2019; R. Gutiérrez, 2017; Martin et al., 2019). While this paper does not tell a story of pedagogical redesign, it suggests possibilities for future practice-based inquiry with teachers. Reflective conversations of the kind Lena and I engaged in might be seen as a "lift off" from practice, creating openings to see and rearticulate teaching anew (K. D. Gutiérrez & Vossoughi, 2010; Philip, 2011). Accompanied by opportunities to plan and try out new experiences with young people, such conversations could support educators to explore in practice what it might mean to teach and learn mathematics for liberation.

Lena's case underscores the value of cross-disciplinary approaches to critical pedagogical inquiry, particularly in elementary settings in which teachers work with children across content areas. Multidisciplinarity characterizes most elementary classrooms, but professional learning experiences often silo rather than engage the potential conversation among disciplines, especially across humanities and STEM domains. It is hard to imagine investigating liberatory mathematics pedagogy with Lena irrespective of the commitments that drove her work with children in other areas. Our collaboration also generated questions and possibilities that may not have emerged if mathematics teaching was not situated alongside her critical praxis in the humanities. As explored across the paper, Lena's pedagogy beyond mathematics—grounded in traditions of liberatory education in the humanities and social sciences—offers lenses, practices, and sensibilities that are not commonly, or as deeply, considered in the field of mathematics education. These include explicitly politicized pedagogical vision, creative artistic inquiry, community building based on restorative justice principles, an understanding of academic development that centers social relationships and emotional well-being, and an orientation toward pedagogy that attends to educators' values and growth.

This study suggests that an understanding of pedagogy as more than technical methods (Bartolomé, 1994), as politically and ethically grounded, demands that we view teaching praxis beyond the boundaries of academic disciplines. Inquiring into liberatory mathematics pedagogy invites us to ask what constitutes liberated learning and living generally and, then, how teachers, children, and researchers might engage those perspectives and ways of being in contexts involving mathematics. This paper begins to consider such questions; subsequent chapters explore them further, in contexts beyond Lena's class.

CHAPTER FOUR: TO CRITICALLY REIMAGINE MATHEMATICS LEARNING: A SYNTHETIC LITERATURE ANALYSIS Introduction

Over the past two decades, research in mathematics education has taken a "sociopolitical turn," increasingly foregrounding questions of power and (in)justice (R. Gutiérrez, 2013). This work regards mathematics teaching, learning, schooling, and research as fundamentally political—shaped by and also holding the potential to transform social hierarchies in society. The sociopolitical turn in mathematics education coincides with a trend in research in the learning sciences that underscores the political and ethical dimensions of learning and educational research (McKinney de Royston & Sengupta-Irving, 2019; Philip et al., 2018; PLWC, 2017; Warren et al., 2020). This scholarship examines how learning environments and the study of them can reflect, sustain, and reconfigure relations of power, with consequences for the well-being of individuals and societies (PLWC, 2017).

This paper considers literature—from mathematics education and the learning sciences—that interrogates mathematics education's entanglement in systems of oppression and pursues transformative possibilities. While language of "equity" and "social justice" has proliferated in research on mathematics education, not all work addresses these concepts from a historicized, structural perspective (Apple, 1992; Bullock, 2014; Martin, 2003, 2015). This paper centers scholarship that situates questions of equity and justice within an analysis of power that I refer to as "critical." Critical scholars assume that school mathematics and the broader discipline are complicit in—not

set apart from or simply a lever for rectifying—sociohistorical oppression. Their work ventures beyond questions of access and inclusion within current structures of mathematics education, to possibilities for changing and liberating from them.

To ground the analysis, I turn to ideas from Warren, Vossoughi, Rosebery, Bang, and Taylor's (2020) chapter, "Multiple Ways of Knowing: Re-imagining Disciplinary Learning." These authors examine the ways disciplinary education, across fields, asserts the authority of EuroWestern¹³ systems of thought, nullifying the world's epistemic and cultural heterogeneity (Warren et al., 2020). They call researchers, educators, and young people to refuse EuroWestern normativity by designing for learning that both critiques and frees from its assumptions (Warren et al., 2020). I refer to the ideas in Warren and colleagues' (2020) chapter, collectively, as "critical reimagining."

This paper brings together scholarship that, from various angles and methodological approaches, reflects a commitment to critically reimagining mathematics education. Through the synthesis, I highlight the literature's contributions and point to its limitations or silences, in light of key principles of critical reimagining. I begin the paper by elaborating on those principles and describing the criteria used to select literature for analysis. Each section of the analysis opens with a framing passage from Warren and colleagues' chapter, which I interpret as a call to conversation for educators and researchers. I discuss critical scholarship on mathematics education that echoes, builds

¹³ Scholars use a variety of terms to describe the hegemony of Northern- and Western-Europeandescended people, cultures, ideas, and political-economic systems. These include Eurocentric, Western, EuroWestern, Euro-American, Western colonial, Modern Western, Western mainstream, Whitestream, and more. I use "EuroWestern" to encompass multiple, interrelated, and ongoing histories of domination through European imperialism and colonialism, including those which developed in the Americas.

upon, or begins to heed that call; and I suggest where the literature might extend further. The paper concludes with questions to inspire continued inquiry into liberatory possibilities for mathematics teaching and learning.

Critically Reimagining Disciplinary Learning

This section introduces the theoretical concepts that ground my analysis of the literature. In addition, I present the paper's guiding questions, which stem from this conceptual framework. The section closes with a few clarifying points for bringing these ideas to bear on mathematics education.

Onto-Epistemic Heterogeneity

Critical reimagining is rooted in appreciation for the world's heterogeneity—the diversity of values, visions, ways of knowing, cultural practices, and forms of life that exist and that might be a foundation for learning (Rosebery et al., 2010; Warren et al., 2020). Escobar (2015), inspired by the Zapatista notion of "a world where many worlds fit," uses the term "pluriverse" to refer to "the richness of the multiple worlds that make up socio-natural life" (p. 14). Pluriversality challenges a "one-world world" perspective, which presumes a single, dominant reality that subordinates, subsumes, or acts as an unquestioned reference point for all other perspectives (Law, 2011, in Escobar, 2015, p. 14). Pluriversality orients toward "a multiplicity of mutually-entangled and co-constituting but distinct worlds" and maintains that these worlds can "flourish in mutually enhancing ways" (Escobar, 2015, pp. 19, 15).

Onto-epistemic heterogeneity refers specifically to the multiplicity of knowledges and ways of making meaning in the pluriverse (Grosfoguel, 2013; Santos, 2007). An onto-epistemic perspective presumes that "knowing and being are inextricably tied," that embedded in systems of knowledge production are assumptions about what it means to know (epistemology) as well as claims about who and what count as being (ontology) (Warren et al., 2020, p. 278). That is, conceptualizations of knowledge and knowing carry implications about whose histories, lives, and worldviews matter (Bang, 2017; Barad, 2003; Maldonado-Torres, 2007; Santos, 2007). Critical reimagining takes this ethical consequentiality seriously, attuning to the ways learning environments and their constitutive theories of learning may restrict or unleash onto-epistemic heterogeneity.

Entanglement in the Colonial Matrix of Power

Informed by decolonial theory, critical reimagining situates academic disciplines and systems of schooling within a "colonial matrix of power" (Mignolo, 2009; Quijano, 2000). The colonial matrix is a system of cumulative, intersecting power hierarchies that classify peoples, geographic regions, and cultural traditions—often according to binary categories (e.g., white/black, civilized/primitive, first-world/third-world)—in order to rationalize and sustain relations of domination and subordination (Grosfoguel, 2013; Maldonado-Torres, 2007; Mignolo, 2009). It is rooted in racialized notions of who counts as "being," with origins in the conquest of the Americas and systems of capitalist exploitation (Maldonado-Torres, 2007; Quijano, 1992). The colonial matrix of power operates through geopolitical, economic, cultural, and epistemological systems. Coloniality is a term used to refer to the totalizing logic that underpins these various systems: distinct from the political-economic structures of colonialism, coloniality also governs ways of being, knowing, and imagining (Maldonado-Torres, 2007; Ouijano, 1992).

Critical reimagining draws particular attention to coloniality's grip on academic disciplines (Warren et al., 2020). Disciplinary canon building functions as a mechanism of empire by centering EuroWestern histories, works, and epistemologies—a privileging that not only silences a multiplicity of voices but also treats EuroWestern perspectives as universal (Morrison, 1989). Mignolo (2009) refers to the assumption of universality as "zero point epistemology" because it frames knowledge making as an objective quest for truth, dislocated from particular vantage points and political relations in the world (see also Castro-Gómez, 2005). As a result, dominant disciplinary practices and conceptions of knowledge operate as a settled, "neutral baseline," exerting a totalizing force while remaining unnamed and therefore exempt from critique (Harris, 1993, p. 1715; Warren et al., 2020). This supremacist structuring eclipses the world's onto-epistemic heterogeneity, through the same imperial logics that drive geopolitical conquest (Grosfoguel, 2013; Santos, 2007; Warren et al., 2020).

Desettling and De-Linking

Critical reimagining calls researchers, educators, and young people to "desettle" normative expectations of disciplines in and beyond schools¹⁴ (Bang et al., 2012). Desettling involves questioning dominant assumptions about disciplinary knowledge and practice and exploring expansive alternatives, both resurgent and newly created (Warren et al., 2020). It is integral to de-linking (*desprenderse*), a term Quijano (1992) coined for

¹⁴ Bang and colleagues (2012) note that "desettling expectations" is informed by notions of decolonization (Smith, 1999/2012) and by Harris's (1993) critical race analysis of "settled expectations" (see footnote in Bang et al., 2012, p. 304).

the process of liberating from the colonial matrix of power. De-linking is distinct from reform: rather than tweaking mechanisms or expanding access to power within current structures, it orients from and toward "a different place," radically shifting the ways we understand and engage with the world (Mignolo, 2011, p. 45). In particular, de-linking opens to the world's onto-epistemic heterogeneity. In educational settings, this requires desettling governing notions of disciplines, knowledge, children, and pedagogy in order to envision and sustain "multiple values, purposes and arcs of human learning" (Warren et al., 2020, p. 278).

To Critically Reimagine Disciplinary Learning

For analytic purposes, I have delineated five parts of Warren and colleagues' call to critically reimagine disciplinary learning—not to suggest that they are discrete, but to clarify the multifaceted nature of this work. Critical reimagining invites researchers, educators, and young people to: (1) examine disciplinary education's entanglement in empire, (2) attend to intersections of self- and world-making in disciplinary learning, (3) critique and refuse disciplines' onto-epistemic normativity, (4) explore the multiplicity and dialogicality in disciplines, and (5) re-place disciplinary learning as people "do life" (Warren, et al., 2020). In this paper, I explore how critical research on mathematics education speaks to these calls, asking: In what ways does critical scholarship on mathematics education interrogate the colonial matrix of power and open toward liberatory alternatives? In what ways could the literature further heed these calls?

Desettling Dominant Mathematics

Before turning to the analysis of literature, I clarify three of my assumptions about desettling and de-linking in mathematics education. The first has to do with defining "settled" forms of mathematics. I understand settled mathematics, or "dominant mathematics," as mathematical ideas, practices, and pedagogical approaches that reflect "a Western (colonial) frame of reference," while assuming the guise and authority of a zero point epistemology (R. Gutiérrez, 2002, p. 151; see also Bishop, 1988; D'Ambrosio, 1985; Fasheh, 1990; Joseph, 1987; Shelley, 1995/2005). Shelley (1995/2005) writes that "the logic which permeates western Mathematics, having been the basis of accepted rationality in western thinking for centuries...is now a dominant culture in the world, and, as with any dominant culture, it lays claim to universality"-when in fact it, like all forms of knowledge, is "value-laden" (p. 261). Dominant mathematics encompasses a range of discourses and pedagogies found in U.S. schools, including those within both "traditional" and "reform" camps (Trafton et al., 2001). Though school mathematics varies across time and place, seemingly disparate approaches can ultimately share, or at least leave unquestioned, many of the basic values and assumptions of Western colonial thought, which I elaborate later in the paper.

A second clarification is that desettling is not an unequivocal rejection of dominant mathematics itself, but rather a rejection of its normativity. As scholars across this paper suggest, it is possible to develop and leverage skills in dominant mathematics without treating them as *the* (singular and superior) form of mathematics (Barajas-López & Bang, 2018; Frankenstein, 2013; Mutegi, 2011). Through critical educational experiences, learners can build fluency with this code of power (Delpit, 1988; Shor & Freire, 1987) while also examining its sociohistorical foundations, the ends toward which it has been used, and the diverse forms of mathematics that exist in the world (Bishop, 1988; D'Ambrosio, 1985; Davis, 2018; Skovsmose, 1994).

Finally, desettling and de-linking require moving beyond equity efforts centered around access, representation, and inclusion, which aim to diversify participation in dominant forms of the discipline without questioning the terms of that participation (Rogers & Kaiser, 1995/2005; Sengupta-Irving & Vossoughi, 2019; Vossoughi et al., 2016; Warren et al., 2020). Efforts to "empower" young people through school mathematics—even when framed as democratizing access (NCTM, 1989, 2014a, 2014b), claiming civil rights (Moses & Cobb, 2001), or reconfiguring status hierarchies in classrooms (Featherstone et al., 2011)—rarely examine how school mathematics has come to hold this status-conferring power or the exclusions upon which its authority depends. As Sengupta-Irving and Vossoughi (2019) state in the context of STEM education broadly, "the diversification of who wields power displaces the questioning of power structures themselves" (p. 481). Universalizing access to dominant mathematics, if done uncritically and without attention to epistemological heterogeneity, becomes a form of assimilation into a single version of mathematics (Bullock, 2019; Martin, 2003, 2015, 2019; Rogers & Kaiser, 1995/2005; Sengupta-Irving & Vossoughi, 2019).

Rather than tinkering or expanding participation within an exclusionary status quo, this paper calls for inquiry into radical alternatives, or what Rogers and Kaiser (1995/2005) might call "a vision of another type of mathematics" (p. 9). Throughout the paper, I argue that educators and educational researchers have a responsibility to confront mathematics education's ongoing entanglement in the colonial matrix of power particularly the consequences for communities whose cultural identities and ways of knowing have been discounted by school—and to pursue "expansive and insurgent ways of learning, being, and acting" in and with mathematics (Warren et al., 2020, p. 278).

Selecting Literature to Analyze

Next, I discuss criteria for including literature in the analysis. This is not an exhaustive literature review: I used a selection method that was a combination of "representative" and "pivotal," purposefully sampling works that illustrate or have seminally informed subsets of critical scholarship on mathematics education (Cooper, 1988). Three purposes guided my selection process: to foreground a critical analysis of power, bring seminal and contemporary works into conversation, and reflect theoretical and methodological diversity.

Critical Analysis of Power

I use the term "critical" to describe research and practice that conceptualizes power as sociopolitical (i.e., addressing power relations in society), structural (i.e., extending to institutions and systems), historicized (i.e., looking across time and scales of activity), and intersectional (i.e., linking multiple dimensions of identity). Critical works may emphasize certain of these dimensions over others.

In line with this definition, the paper covers literature that addresses mathematics education's role in perpetuating and potentially disrupting historically structured power hierarchies. It does not include scholarship that omits an analysis of power or that focuses on notions of "mathematical power" or "mathematics for all" (NCTM, 1989) without examining the sociohistorical roots and consequences of these ideas (for critiques in this vein, see Bullock, 2012, 2019; Martin, 2003; Yolcu & Popkewitz, 2018). When this work highlights sociopolitical concerns, it tends to dwell in the politics of access, representation, and achievement, focusing on the inclusion of students from historically marginalized communities within dominant mathematics and the economic opportunities such access affords (R. Gutiérrez, 2007a, 2007b; Sengupta-Irving & Vossoughi, 2019). I devote attention to scholarship that interrogates the ways mainstream educational opportunity structures actually collude to perpetuate social inequality and injustice (Martin, 2003, 2013). Whenever possible, I include work that explores possibilities for mathematics learning that liberate from, rather than simply extend opportunities within, existing systems of power.

Bring Seminal and Contemporary Works Into Conversation

The paper profiles seminal works in various areas of critical mathematics education research. These include: ethnomathematics and the cultural pluralism of mathematics (Ascher, 1991, 2002; Bishop, 1988; D'Ambrosio, 1985; Joseph 1991/2011), cultural identity in mathematics learning (de Abreu, 1995; González et al., 2001; Nasir, 2002), the racialization of mathematics education and research (Martin, 2000, 2006), critical mathematical literacy (Frankenstein, 1983; Skovsmose, 1994), teaching mathematics for social justice (Gutstein, 2003, 2006; Gutstein & Peterson, 2005), and rehumanizing mathematics (R. Gutiérrez, 2012, 2018). Across the paper, I bring seminal works into conversation with scholarship that has taken up, extended, and critically addressed these authors' foundational ideas. In selecting contemporary literature, I included works that are representative of the methods and themes explored in the broader literature on critical mathematics education. Over the past half decade, this scholarship has increasingly included work by authors in the learning sciences (e.g., Barajas-López & Bang, 2018; Booker & Goldman, 2016; Calabrese Barton & Tan, 2019; Sengupta-Irving & Vossoughi, 2019; Takeuchi, 2018; Vakil, 2020).

Reflect Heterogeneity in Research

A third purpose guiding my selection of literature was to reflect heterogeneity in critical research on mathematics education, as critically imaginative work demands conversation among diverse perspectives. The paper draws on literature that is international in scope, though it does skew toward works published in the United States. I incorporate both theoretical and empirical scholarship, whose projects include: proposing conceptual critiques and frameworks, tracing histories of mathematics education, documenting mathematics teaching and learning in practice, and narrating personal experiences with mathematics. This literature varies along a number of methodological dimensions as well, employing historiographic, ethnographic, narrative, and design-based methods; grounding in both school and out-of-school contexts; attending to various scales of educational activity; and foregrounding different systems of oppression in their analysis. Works from the learning sciences contribute to this methodological diversity, expanding the conceptual and empirical terrain typically traversed within the field of mathematics education—by, for example, investigating learning apart from the institution

of schooling, using design-based research methods, historicizing political analysis of teaching and learning, and questioning assumed disciplinary boundaries.

Finally, I sought heterogeneity across authors' social positionalities. Researchers' identities and experiences inform the lenses, values, tools, and contexts reflected in their work. While the field of mathematics education continues to be a white institutional space (Martin, 2009b), and the broader discipline heteronormative and male-dominated (Gholson, 2016; Rands, 2009; Yeh & Rubel, 2020), a more diverse group of voices has shaped critical scholarship on mathematics education. I turn primarily to literature written by Black, Indigenous, queer, and women scholars who design and narrate liberatory possibilities for mathematics learning.

Analysis of Critical Literature on Mathematics Education

Each section of this synthesis is organized around one of the five component calls of critical reimagining. Without suggesting a linear progression from one call to the next, I have sequenced sections so that their emphasis moves from critical interrogation to expansive reimagining. The first section discusses scholarship that critiques mathematics education's entanglement in colonial histories and systems, and the final two explore the diversity of possible contexts for mathematics learning. Across sections, I foreground theoretical and empirical efforts to liberate mathematics education from the workings of the colonial matrix of power. These include: critical mathematical literacy (Frankenstein, 1983, 2013; Skovsmose, 1994); teaching mathematics for social justice (Gutstein, 2003, 2006; Gutstein & Peterson, 2005); embodied modes of mathematical sensemaking (Barajas-López & Bang, 2018; Ma, 2017); reclaiming and rehumanizing mathematics identities (R. Gutiérrez, 2018; Martin, 2006; Nasir, 2002); culturally responsive and embedded mathematics pedagogies (Aguirre et al., 2012; Barajas-López & Bang, 2018; González et al., 2001; Lipka et al., 2005; Taylor, 2011); and epistemologies of mathematics rooted in Indigenous ways of knowing (Glanfield, 2016; R. Gutiérrez, 2017; Sterenberg et al., 2010).

Each section begins by interpreting the call and "sounding" it in the context of mathematics education. I then synthesize literature that speaks to the call, highlighting the ways authors have critically interrogated the colonial matrix of power and opened toward liberatory alternatives. To conclude each section, I address the bounds of the literature, noting where it remains tied to dominant paradigms or could further pursue the liberatory call.

Examine Disciplinary Education's Entanglement in Empire

[Morrison] shines a searing light on canon building as a process of exclusion, erasure, and onto-epistemic violence that nullifies presences-assumed-not-to-exist (Morrison, 1989) in the form of white imaginings of African Americans (Gates, 1984) or settler imaginings of Native people (Smith, 2012; Vizenor, 2000). Morrison draws our attention to the necessity of analyzing 'the workings as well as the work' (p. 162) in order to understand the processes and motives—racial, colonial, patriarchal—that have driven canon building as empire building in the West and in disciplinary learning and teaching in U.S. schools. (Warren et al., 2020, p. 277)

Critical reimagining calls for interrogation of the links between academic disciplines and empire. Warren and colleagues open their chapter with a line from Toni Morrison: "Canon building is empire building" (Morrison, 1989, p. 132). Examining the white literary imagination that governs canonical American literature, Morrison elucidates canon building "as a process of exclusion, erasure, and onto-epistemic violence that nullifies presences-assumed-not-to-exist" (Warren et al., 2020, p. 277). Like geopolitical empire, academic disciplines are constructed to center historically powerful actors—their interests, perspectives, and ways of knowing—and marginalize or efface those deemed inferior. Erasure may occur by way of overt exclusion or through "a veneer of liberal inclusion" as scholarship, policies, and pedagogies profess to open toward historically marginalized traditions only to enclose them within dominant systems of thought and practice (Warren et al., 2020, p. 278; see also Richardson, 2011). Following Morrison, critical reimagining invites us to interrogate and expose the imperial "workings" of those systems, through historicized inquiry into the social, economic, and cultural forces that forge academic disciplines (Morrison, 1989, p. 162).

Sounding the Call in Mathematics

Understanding canon building as empire building within mathematics requires asking whom and what have been excluded in order to establish the current bounds of the discipline. Critical scholars have examined the sociohistorical forces that narrow whose communities, knowledges, and cultural traditions are reflected in canonical mathematics. Their critical readings explore the academic discipline's co-development with imperialism, colonialism, racial capitalism, antiblackness, nationalism, militarism, and intersecting systems of oppression—highlighting the ways such systems regulate what counts as knowledge, who produces and has access to it, and the purposes for which it is used. While this paper emphasizes the workings of K–12 school mathematics, the current section also addresses constructions of the discipline within the realms of universities and colonial government.

I foreground historical analyses in this part in order to frame mathematics education's collusion with empire as a centuries-long political project. Authors address the entwined processes of appropriation and erasure involved in canon building, charting how colonizers established particular forms of mathematics as authoritative while appropriating and distorting the mathematical knowledge and practices of colonized communities (D'Ambrosio, 1985; Joseph, 1987). Other scholarship focuses on the ongoing collusion between mathematics education and contemporary political-economic systems of oppression. Across this literature, authors underscore the work systems of education do to hierarchize knowledge traditions and communities of people.

History of Entanglement in the Colonial Matrix

For decades, critical scholars have examined the "symbiotic relationship" between the development of EuroWestern mathematics and the geopolitical and cultural processes of colonization (D'Ambrosio, 1985, p. 47). Some authors take a global perspective, while others offer histories of particular countries, regions, or time periods.

International Perspectives. D'Ambrosio (1999) referred to the control of knowledge as an "instrument of hierarchical power" (p. 144). His work underscores the entwined processes of expropriation and erasure that have contributed to the development of university-based mathematics, a system designed to formally exclude groups of people from intellectual, political, and economic power (D'Ambrosio, 1985). D'Ambrosio (1985) clarifies that the formalization of academic mathematics did not simply eradicate the "radically different ways" cultural communities around the world practice

152

mathematics (p. 44). Rather, canon building involved selective appropriation as imperial systems of knowledge adopted and assimilated colonized people's mathematical advancements and ways of knowing without honoring their origins. Scholars point to the contributions of mathematicians in Egypt, Mesopotamia, China, India, and Indigenous communities in the Americas, whose roles "as transmitters and creators of knowledge" are often ignored in dominant histories of the discipline (Joseph, 1987, p. 19; see also Anderson, 1990; Bishop, 1990; Joseph, 1991/2011). As Joseph (1987) explains, exclusionary histories of mathematics—many written in the late 19th and early 20th centuries, at the pinnacle of European imperialism in Africa and Asia—have been wielded as an ideological rationale for geopolitical subjugation.

Bishop (1990) examines EuroWestern mathematics as a product and tool of imperialism. He discusses the ways European colonizers, through commerce and government administration, imposed their systems of measurement, number, and currency, supplanting or assimilating Indigenous communities' mathematics. Bishop (1990) emphasizes that embedded in these systems were broader "languages" and "values" for understanding and structuring human society, such as the "language of hierarchy," an "obsession with naming and classification," the privileging of deductive reasoning, and a drive to objectify and master the physical environment (p. 57). Like D'Ambrosio, Bishop (1990) situates the "cultural invasion" of EuroWestern mathematics within economic and political systems of colonial rule (p. 53).

Harouni (2015) focuses on how the rise of industrial capitalism shaped the development of EuroWestern mathematics, drawing links between school mathematics

153

and early capitalist systems of labor. He offers a historiography of mathematics curricular content in western European schools, tracing the insurgence of "commercial-administrative" (or "consumer") mathematics in the late 17th and early 18th centuries and the accompanying marginalization of "artisanal mathematics." As the western European bourgeoisie class sent children to school in increasing numbers, consumer mathematics came to dominate the curriculum, emphasizing calculation with money, questions focused on a single "final answer," and algorithms for efficient computation (Harouni, 2015, p. 62). According to Harouni (2015), consumer mathematics was premised on "simple interactions" that turn people, materials, and labor "into abstract quantities" according to their exchange value within a system of economic production (pp. 61–62).

Artisanal mathematics reflected a different "cluster of values" (Bishop, 1990, p. 56). According to Harouni (2015), artisanal mathematics used systems of measurement derived from the particular characteristics of craftspeople's materials, instruments, and interactions—privileging the relativity and improvisation demanded by complex creative labor. Rooted in relations of apprenticeship between experienced and novice crafters, learning artisanal mathematics contrasted with consumer mathematics, which reduced the pedagogical relationship to a transmissionist exchange of zero point knowledge via textbook exercises and standardized explanations (Harouni, 2015). Harouni (2015) explains that the rise of consumer mathematics relegated artisanal forms of mathematics to an inferior status in schools and society, illustrating the ways schooling can contribute to the hierarchical stratification of knowledges, practices, and people (Warren et al., 2020).

Fasheh (1990, 2012, 2015) explores epistemological hierarchy on a personal level, through scholarship that interweaves his experiences as an educator, mathematician, and community organizer. Across his work, Fasheh critiques the mathematics education he received in British colonial schools in Palestine, highlighting its impact on his family relationships and ethnic identity. He discusses a chasm between the mathematics he learned in school and his "illiterate" mother's mathematics, which was integral to her craftwork as a seamstress yet "invisible to eyes trained by formal education" (Fasheh, 1990, p. 22). Schooling rendered Fasheh's mother's complex geometrical knowledge not only inferior but illegible as a form of mathematics: "Why is my kind of mathematics considered knowledge while hers is not?...The mathematics I studied and taught suppressed and won over my mother's...by devaluing, ignoring, and belittling her mathematics" (Fasheh, 2012, p. 94). Fasheh connects this family history to his professional experiences leading the adoption of "foreign" curricula in West Bank schools, which he saw as part of a broader project of "military, political, and economic power and control" over Palestinian people¹⁵ (Fasheh, 1990, p. 22). I return to Fasheh's work later in the paper but for now underscore the ways his accounts augment the sociohistorical analyses put forth by D'Ambrosio, Bishop, and Harouni. Together, these authors illuminate that disciplinary canon building is a project of political and cultural hegemony with repercussions on individual, community, and (inter)national scales.

¹⁵ Here Fasheh (1982) refers to the "New Math" curriculum materials developed for Arab states through an initiative of the United Nations Educational, Scientific and Cultural Organization (UNESCO). Fasheh helped lead the early implementation of this curriculum in West Bank high schools in the early 1970's (p. 2).

U.S.- Focused Perspectives. Critical scholarship on the history of mathematics education in the United States emphasizes its entanglement in capitalist and militarist national projects. Berry and colleagues (2013) trace the co-evolution of U.S. mathematics education and developments in global economic and military competition over the past century. They write that during the new industrialism of the early 20th century, mathematics curriculum emphasized basic arithmetic and drill-and-practice pedagogy, geared towards "increasing productivity in various industries such as agriculture, mechanics, carpentry, [and] factory systems" (p. 25). This focus shifted during the early decades of the Cold War, when the launch of Sputnik prompted the federal government to invest defense funds in the development of new mathematics and science curricula. The "New Math"—which replaced drill-and-practice approaches with conceptual understanding, discovery learning, and the theoretical topics explored in university mathematics—was intended to foster young people's innovative capacities in service of the nation's economic, military, and technological competitiveness (Berry et al., 2013; Schoenfeld, 2004).

Concerns about the United States' military-industrial preeminence continued to steer mathematics curricular reform in the following decades (Berry et al., 2013; Vossoughi & Vakil, 2018). The publication of *A Nation At Risk* (National Commission on Excellence in Education, 1983), which tied education outcomes to anxieties about national security and economic competitiveness, led to the development of the first national standards for school mathematics (NCTM, 1989). The standards were published following a National Educational Summit in which U.S. political leaders announced their goal to "make U.S. students first in the world in mathematics and science by the year 2000" (Berry et al., 2013, p. 38). Berry and colleagues (2013) note that subsequent national standards, as well as the ensuing "Math Wars" over the instructional shifts they drove, maintained an underlying narrative focused on asserting U.S. global power, often neglecting race- and class-based educational injustice within the United States (see also Martin, 2015). Similarly, Vossoughi and Vakil (2018) argue that STEM education reform that "begins with and organizes learning around the needs and political agendas of the [U.S.] state" has not only overlooked, but often directly countered, the needs and visions of the nation's historically marginalized communities (p. 133). The authors below elaborate on this point in relation to contemporary contexts.

Ongoing Collusion in Oppressive Systems

Next I spotlight scholarship, mostly U.S.-based, that addresses mathematics education's ongoing entanglement in the colonial matrix's intersecting systems of oppression. Martin (2013) discusses relationships between "the mathematics education enterprise" and larger "racial projects" in the United States, examining mathematics education as a "white institutional space" that is dominated by white researchers, educators, and policymakers and guided by their logics and interests (p. 323). He critiques the "Mathematics for All" rhetoric that drove the standards movement of the 1990s and persists in policy discourse today, arguing that this rhetoric is authored by and towards white people, employing "equity" language without addressing the systemic change needed to address the ongoing disenfranchisement and dehumanization of Black, brown, and poor communities in mathematics education (Martin, 2015, p. 19). Across his work, Martin (2006, 2007, 2009a, 2009b, 2019) specifically illustrates the ways antiblackness operates in mathematics education, through deficit discourses about Black children's mathematical ability; a white-dominated teaching force that is insensitive to the social realities, visions, and agency of Black communities; tracking policies that "create naturalized relationships between Blackness and remediation" (Martin et al., 2019, p. 44); and curricula that perpetuate the myth of a culturally neutral, colorblind mathematics.

Authors who build on Martin's work address the collusion between mathematics education and neoliberal racial projects. Barajas-López and Larnell (2019) underscore the market-driven discourse of national mathematics standards, which promotes "mathematical literacy" and "college-and-career readiness" in ways that reduce the purposes of learning mathematics to advancing opportunities for mobility within the United States' capitalist economic structure (see also Apple, 1992; Martin, 2003). These authors also question iterations of the economic access argument that emphasize equitable teaching practices (Bartell et al., 2017) or democratic citizenship (Moses & Cobb, 2001), suggesting that these perspectives do not challenge the hegemonic "values [and] norms" that underlie national standards (Barajas-López & Larnell, 2019).

Bullock and Meiners (2019) link the stratifying logics of mathematics tracking systems with the organizing principles of the U.S. prison-industrial complex. They critique the ways gatekeeper courses and categories of mathematical ability (e.g., "advanced," "remedial," "regular") sort young people according to racialized hierarchies that pass as a "logical and necessary" academic order (Bullock & Meiners, 2019, p. 340). As Vossoughi and Vakil (2018) clarify, initiatives that expand access to particular strata within such hierarchies—such as "advanced" tracks, "college-level" courses, or high-status STEM careers—may diversify participation within oppressive systems but do not fundamentally upend the relations and ideologies on which they are based.

Skovsmose (2008) interrogates the role of mathematics education in "cofabricating" global capitalist structures, though his analysis suggests possibilities for transformation, not simply reproduction, of those structures (p. 7). Like the authors above, Skovsmose (2008) regards mathematics "as a technology of power" that can function to stratify groups of people and regulate their status as capable, worthy members of society (p. 7; see also Kirchgasler, 2017; Yolcu & Popkewitz, 2018). He delineates various social positions people assume with respect to the construction and use of mathematical knowledge within a neoliberal society and argues that mathematics education often prepares learners to fulfill these roles uncritically. For example, "constructors" learn to create new knowledge and tools for society, and "operators" carry out complex procedures with that mathematical technology, but neither group is educated to critique "how the system is operating" or the sociopolitical ends furthered by their work (Skovsmose, 2008, p. 12). While underscoring the reproductive function of education, Skovsmose (2008) also considers how mathematics education might foster "critical literacy" from various vantage points in this social structure-encouraging learners to question the seemingly natural order of society through their study of mathematics. I return to critical notions of mathematical literacy in a later section.

To Further Heed the Call

The authors in this section, writing from a range of subfields and perspectives, argue that mathematics education is rooted in and sustains oppressive political and economic systems. Collectively, their work interrogates mathematics education as an "agent" within the colonial matrix of power—nationally and globally, historically and into the present day (Bullock & Meiners, 2019, p. 339). In so doing, the authors challenge the narrative of mathematics as a politically neutral, universally empowering, inherently beneficent domain of knowledge and practice (D'Ambrosio, 1999; Valero, 2008).

This scholarship offers primarily theoretical and systems-level analysis. To further illuminate mathematics education's entanglement in the colonial matrix, this work might be paired with accounts that document the lived particulars of canon building as empire building. That is, what are the routine ways in which mathematics curricula, institutional policies, and classroom interactions (re)construct macro-level patterns of control, exclusion, and erasure? Works profiled later in the paper touch on this question, though they do not always situate their analysis historically or geopolitically, as do the works in this section. Further research might bring these two scales of analysis together in more intimate ways (Gholson, 2016; Gholson & Martin, 2019). In doing so, it is important to maintain a critical eye on university and professional mathematics as well, not allowing a focus on the particulars of K–12 contexts to leave the larger discipline "exempt from reproach or historicity" (Warren et al., 2020, p. 278). As the authors in this section demonstrate, critical readings of mathematics education require attention to the workings of power within the academic discipline, in addition to the schooling apparatus built around it.

In the introduction to their edited volume, *Opening the Cage: Critique and Politics of Mathematics Education*, Skovsmose and Greer (2012) note that, within research that critiques "the imperial/colonial enterprise" in mathematics education, there is an absence of analyses on resistance to that enterprise (p. 5). Most authors profiled in this section leave open, or underspecified, the question of how mathematics scholars, educators, and learners might refuse and transform the systems they critique. As Warren and colleagues (2020) caution, "remaining solely in the mode of critique invites enclosure such that powered ways of knowing continue to set the terms" (p. 278). The following sections foreground scholarship that begins to resist and reimagine mathematics education toward transformative ends.

Attend to Intersections of Self- and World-Making in Disciplinary Learning

Morrison serves as a guide in imagining radically different horizons of possibility for disciplinary knowing and learning as the making and sharing of worlds woven with the making and sharing of selves (Morrison, 1993; Smith, 2012). (Warren et al., 2020, p. 278)

Critical reimagining addresses the ways larger histories and structures play out and might be shaped through intimate scales of experience. While sociohistorically situated and sociogenetically consequential, disciplinary learning is also deeply personal, bound up with learners' sense of self and relations with others. Learning environments communicate assumptions about the ways of being that are valued through intellectual activity. Patterns of exclusion and effacement—and possibilities for resistance and transformation—occur not only at macro levels, but also through the identities, relationships, and local communities individuals form as they engage in disciplinary learning. Critical reimagining calls us to attend to the interwoven nature of these scales of activity.

Sounding the Call in Mathematics

There is extensive literature on the ways people negotiate their identities and relationships within mathematics learning contexts (e.g., Boaler & Greeno, 2000; Gholson & Martin, 2019; Larnell, 2019; Martin, 2006; Nasir, 2002; Philip & Gupta, 2020; Stinson, 2013; Takeuchi, 2018; Turner et al., 2013; Wood, 2013). This work recognizes that experiences of mathematics "are always also experiences of the self" and that learning environments carry implications for who individuals believe they are or might become in relation to one another (Sengupta-Irving & Vossoughi, 2019, p. 497). In various ways, authors explore how conceptions of the self, moment-to-moment interaction, and sustained relationships develop through and around mathematical activity.

Not all mathematics education research on identity and social relations is explicitly political in orientation (Gholson & Wilkes, 2017). Seminal scholarship in this area—on learners' dispositions toward mathematical activity (Boaler & Greeno, 2000; Gresalfi & Cobb, 2006), sociomathematical classroom norms (Cobb & Yackel, 1996), and relations among teachers, students, and disciplinary content (Cohen & Ball, 1999; Lampert, 2001; Wood, 2013)—often omits analysis of the sociopolitical systems in which these are embedded. The critical scholarship included below emphasizes the ways local experiences of learning mathematics "reflect, refract, [and] rewrite" broader discourses and power structures (Sengupta-Irving & Vossoughi, 2019, p. 479).

The section begins with a politicized definition of disciplinary identity (Martin, 2006; Vakil, 2020). I then discuss scholarship that addresses the intersection of mathematics and sociopolitical identities, beginning with works that highlight patterns of exclusion in mathematics learning contexts. Next I consider studies that examine the ways youth navigate, resist, and reconfigure discourses about personhood and belonging in mathematics. The section concludes with a discussion of rehumanizing mathematics (Fasheh, 2015; R. Gutiérrez, 2018).

Politicizing Disciplinary Identity

Critical researchers attend to disciplinary identity as interwoven with cultural and political identities—"dynamically connected to the particular constellations of relationships, values, goals, and worldviews of [learners'] communities" and shaped by the power relations among these (Ishimaru et al., 2015, p. 7). Martin (2000, 2006) writes about the racialization of mathematics identity and activity. He defines mathematics identity as "the dispositions and deeply held beliefs that individuals develop, within their overall self-concept, about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (Martin, 2006, p. 206). Racialized meanings ascribed to mathematics—pertaining to how mathematical knowledge and competence are defined, the ways of communicating or acting that are deemed mathematical, or the purposes mathematics can serve—influence this sense of self and can in turn "facilitate, legitimize, or inhibit meaningful participation in mathematics" (Martin, 2006, p. 206). Martin underscores that, like racial identity,

mathematics identity is a "negotiated self," encompassing a person's self-understanding and performances as well as the ways other people and systems construct that individual in relation to mathematics (Martin, 2006, p. 206; see also Gholson & Martin, 2019). Though not the explicit focus of Martin's work, other facets of political identity (e.g., gender, sexuality, language, dis/ability status) can be understood similarly, as coconstructed with mathematics identity.

In his work on disciplinary identity in STEM learning settings, Vakil (2020) examines the "ethical meanings" that regulate "the kind of person one has to be, or become, in order to participate" in disciplinary communities of practice (p. 93). Such meanings—related to the nature of knowledge, who can create it, and the purposes of disciplinary practice—are communicated at local (e.g., classroom), institutional (e.g., school), and cultural-historical scales. As youth define themselves and their purposes in relation to mathematics, they navigate these terms of participation in various ways accommodating, contesting, and at times reconfiguring assumptions about what it means to do and learn mathematics (Philip & Gupta, 2020; Stinson, 2013; Vakil, 2020). As the works below illustrate, the negotiation of mathematics identity can be dehumanizing and exclusionary or dignifying and enabling, depending on context.

Exclusion

Critical identity-based research examines how the expectations of school mathematics demand that youth from historically marginalized communities dismiss or assimilate parts of themselves in order to identify with mathematics and be considered legitimate participants in mathematics classrooms (Calabrese Barton & Tan, 2019; R.

Gutiérrez, 2012, 2018; Ishimaru et al., 2015; Larnell, 2019; Martin, 2006, 2012; Stinson, 2013; Yolcu & Popkewitz, 2018; Yeh & Rubel, 2020). Yolcu and Popkewitz (2018) describe this pressure as a form of "subjectification" in which the "regulatory and corrective practices of school mathematics" define who and what counts as "normal," "rational," mathematically able, and, ultimately, worthy (p. 252). Authors interrogate the regulatory norms of dominant mathematics along various dimensions of identity, including gender, race, language, and culture.

Yeh and Rubel (2020) draw attention to the norms of masculinity that often govern mathematics settings, including emotional detachment, assertiveness, competition, and hierarchy. Yeh and Rubel contend that these ways of being exclude women, queer, and gender-noncomforming people from feeling seen and competent in mathematics, unless they choose to "perform and participate in a masculine space" (p. 233; see also Rands, 2009; Rogers & Kaiser, 1995/2005). Specifically, these are norms of white masculinity. Gholson (2016) addresses "the ways in which Black girls and women are obfuscated, erased, and discounted" through the intersection of gendered and racialized standards for success in mathematics (p. 294). She suggests that performing masculinity does not necessarily translate into mathematical achievement for Black girls and women, whose assertiveness may be dismissed as disruptive behavior. Black boys and men, too, are often constructed as defiant when navigating racialized norms of masculinity in schools (Ferguson, 2000), and scholarship has examined the particular tensions Black males face when working to build positive, racially affirming mathematics identities (Berry, 2008; Nasir, 2002; Stinson, 2013; Terry, 2011). I return to this literature

165

in the next section on resistance and reclamation, as many accounts of Black males offer counterstories to the "White male math myth" (Stinson, 2013, p. 71). Here, I underscore Gholson's (2016) call for mathematics education research to attend further to the experiences of Black girls and women, as the majority of gender-specific scholarship on Black learners focuses on boys and men (for exceptions see Gholson & Martin, 2014, 2019; Pringle et al., 2012).

Gholson and Martin (2019) explore the intersection of racialized and gendered exclusion in mathematics learning through the "everyday and every moment" relational dynamics between a Black girl and her white teacher (p. 394). The authors discuss the symbolic and epistemological violence this young person experiences, and the relational labor required to "perform," within a mathematics classroom in which every move and utterance "creates the opportunity to be wrong and vulnerable" in the eyes of her teacher (Gholson & Martin, 2019, p. 401). Focusing on spoken and embodied interaction, Gholson and Martin (2019) analyze the teacher's repeated "mathematical microcorrections" as "small refutations and invalidations that can shape the performance of Black girlhood—its expressiveness and the certainty of self" (p. 401). In this microgenetic analysis, Gholson and Martin (2019) illuminate broader sociohistorical narratives about Black girls, in and beyond mathematics, that influence the possible selves and storylines available to them moment to moment in local contexts (see also Nasir et al., 2012).

Another strand of identity-based literature examines children's language identities in mathematics classrooms, often focusing on the positioning of multilingual learners during mathematics discussions (Enyedy et al., 2008; LópezLeiva et al., 2013; Setati, 2005; Turner et al., 2013). While this literature includes accounts of learning spaces that honor multiple language resources and multilingual learners' mathematical contributions, all studies underscore that English functions as the language of epistemic authority and therefore a "dominant symbolic resource" in U.S. mathematics classrooms¹⁶ (Setati, 2005, p. 448). The hegemony of English regulates not only how young people are expected to express themselves, but also whose ideas are rendered mathematically valuable. In a study of whole-group discussions in a high school algebra classroom, Envedy and colleagues (2008) found that when participants contributed ideas in Spanish, the teacher often revoiced and elaborated on the contributions in English, authorizing English as the primary resource for understanding and communicating publicly about mathematics. The authors suggest that, in some cases, translating into English may even have denied original speakers access to the validated versions of their ideas. Building on this work in an elementary context, Turner and colleagues (2013) found that teachers needed to do explicit discursive work to position learners of English as "competent problem solvers" and their ideas as "mathematically worthy of consideration" (p. 212).

Identity-based scholarship focused on language is part of a broader literature on the exclusionary divide between school mathematics and nondominant communities' cultural repertoires (Booker & Goldman, 2016; de Abreu, 1995; Fasheh, 2012; R.

¹⁶ While the other studies cited here are U.S.-based, Setati's (2005) study took place in a primary classroom in South Africa. Setati notes that "although the language-in-education policy of South Africa, which recognizes 11 official languages, is intended to address the over-valuing of English and the undervaluing of African languages, in practice English still dominates" (p. 448). Her framing of the institutional and societal power of English resonates with many U.S. bilingual education contexts: "although English is the home language of a minority, it is a dominant symbolic resource" (p. 448).

Gutiérrez, 2019; Ishimaru et al., 2015; Nasir & McKinney de Royston, 2013; Takeuchi, 2018). I return to this literature in a later section, here highlighting the alienation and dehumanization learners experience when their ways of knowing, being, and communicating are delegitimized by school mathematics (Ishimaru et al., 2015).

Resistance and Reclamation

Critical scholarship has also explored possibilities for agency and resistance as young people negotiate their sociopolitical identities in the context of mathematics learning. Building on Martin's work, Stinson (2013) discusses the "robust mathematics identities" of four young Black men who, through interviews and autobiographical writing as adults, narrated their experiences navigating society's "White male math myth discourse" (p. 71). Using a poststructural lens, Stinson (2013) examines the ways participants "exerted their agency by developing opposing discourses" to dominant narratives about Black males in mathematics (p. 88). Some learners refused to "act White" as they pursued high grades and identified as "successful" in their mathematics courses, often selectively deciding which classes were worthy of this effort. Others expressed general disdain or unconcern toward school mathematics while making use of its tools for their own purposes outside of school (e.g., to support a church community's finances). Stinson (2013) emphasizes that the young men's disciplinary identities were "multiplicitous and fragmented," at times accommodating aspects of the discourses they contested (p. 76). For instance, participants' reflections tended to uphold individualistic notions of achievement and the narrative of mathematics as a race- and culture-free domain.

Buenrostro and Radinsky (2019) also use narrative analysis to examine how a young adult stories his identity as a mathematics (and science) learner in and out of school. Buenrostro and Radinsky (2019) discuss the ways Calvin, a Latinx man, "appropriated" ideas from school about the value of learning mathematics and "made [them] his own, employing them as tools for understanding the world and himself" outside of institutional learning settings (p. 391). Calvin's accounts of school mathematics centered "the agency of his teachers and the curriculum," including in classrooms inspired by reform and critical pedagogies (p. 369). Yet Buenrostro and Radinsky (2019) underscore that Calvin identified elements of those classroom experiences that he found "important" and chose to incorporate into his self-directed learning experiences outside of school. For instance, when reflecting on a high school Mathematics for Social Justice course, Calvin did not frame its value in terms that educators commonly use (e.g., mathematics as a tool for sociopolitical critique and empowerment) but rather by "translating" ideas from the course to contexts in which he explored "his desired, scientific pursuits," such as examining planetary orbits (p. 399).

Sengupta-Irving and Vossoughi (2019) share about two girls of color who "refuse a loss of self" by narrating personally meaningful purposes for participating in mathematics and science (p. 498). One child found meaning in science as she engaged in household repair work with her parents. The second, Amina, resisted patriarchal norms in her low-track high school algebra class and articulated purposes for learning mathematics that extended beyond school- and market-driven narratives for academic achievement. Sengupta-Irving and Vossoughi (2019) describe how Amina worked to "remake" and "refract" expectations around rule compliance, competition, and hierarchical rankings by building relationships with female classmates based in compassion, mutual protection, and "affirmed dignity" (p. 489). She valued doing well in algebra because it was a way to honor her mother's academic pursuits and would eventually help her be a good mother for her own children. Similar to the young men in the studies above (Buenrostro & Radinsky, 2019; Stinson, 2013), Amina's identification with mathematics was rooted in relationships and aspirations that did not conform to dominant expectations about young people's participation in school mathematics.

Nasir (2002) examines the identities African American youth construct as they engage with mathematics in and out of school (see also Nasir & Hand, 2008; Nasir & McKinney de Royston, 2013). She views the development of mathematical knowledge and identity as "a socially distributed, interpersonal process" intimately tied to participation in communities of practice (Nasir, 2002, p. 240). In her seminal study, Nasir (2002) highlights young people's sense of "ownership and agency" around mathematics as they played basketball and dominoes within racially and culturally affirming communities outside of school (p. 220). In a subsequent study, Nasir and McKinney de Royston (2013) note discontinuities between the mathematical goals, practices, and identities African American youth developed while playing basketball versus in school. While young people "felt a strong sense of identity and competence" solving problems during basketball games, they expressed lower self-confidence and a sense of disconnect from their mathematical knowledge as they engaged with similar concepts in "school worksheet" tasks (Nasir & McKinney de Royston, 2013, p. 275). Nasir and McKinney de Royston (2013) underscore that the basketball court, domino table, and school classrooms are characterized by distinct social relationships, forms of belonging, and ways of drawing on mathematical knowledge—all of which influence the mathematical identities that young people are able to "access and perform" within these contexts (p. 270).

In a discussion of Nasir's work, Gholson and Wilkes (2017) underscore the opportunities for Black youth to "author themselves into" mathematics as they "claim or reclaim practices, tools, and meanings" within contexts that affirmed their humanity and racialized belonging (p. 232). Authors throughout this section highlight various ways young people of color reclaim, refract, and reimagine mathematical identities by pursuing self-determined purposes and in some cases carving out their own spaces for engaging in mathematics (Buenrostro & Radinsky, 2019; Sengupta-Irving & Vossoughi, 2019). Their analyses speak to questions Larnell (2019) poses in his critique of "productive disposition," a concept commonly invoked in discussions of STEM disciplinary identity. Larnell (2019) asks, "For whose purposes should learners form an attachment with mathematics? For what purposes should learners habitually see mathematics as useful or worthwhile?" (p. 135). Such questions challenge the assumption that learners should be encouraged to develop deep connection with the discipline without interrogating the ends that might be furthered through this affiliation. Larnell (2019) contends that learners should develop a "personal relationship" with mathematics "on their own terms" rather than to conform to STEM education discourses that focus narrowly on advancement within oppressive schooling and economic systems (p. 135).

(Re)Humanization

Sengupta-Irving and Vossoughi (2019) suggest that accounts of resistance and reclamation can illuminate "the ingenuity and humanity that goes unseen" in STEM education spaces and the narratives about them (p. 495). The concept of (re)humanization threads across critical scholarship on identity and relationships in mathematics learning. Gutiérrez's (2018) vision of rehumanizing mathematics holds that every individual "should be able to feel whole as a person" and in mutually affirming relationship with others through mathematics. Gutiérrez (2018, 2020) suggests that a rehumanizing mathematics education positions all young people as creators of mathematical knowledge; embraces heterogeneous forms of meaning making; reflects the histories and cultures of people across the world; and nurtures dignified, reciprocal relations among people and their more-than-human relatives.

Gutiérrez's vision resonates with work in mathematics education that centers the notion of healing. In a study of a middle school mathematics class that integrated social justice tasks (Gutstein, 2006) and trauma-informed care, Kokka (2019) proposes radical healing (Ginwright, 2016) as a purpose of justice-oriented mathematics pedagogy. While using mathematics to analyze issues of injustice, young people in this class also responded to written and conversation prompts to process their personal experiences related to the issues under study (e.g., immigration, housing insecurity, or minimum wage). Kokka suggests that such reflection created opportunities for individual healing and restorative relationship building within the mathematics classroom, a setting that typically does not make space for narrative reflection or deep emotional response.

In his "story of healing," Fasheh (2015) reflects on the emotional, relational, and epistemic harm that members of colonized communities often experience through school mathematics. Through an interweaving of autobiographical and sociological narrative, Fasheh (2015) works to "reclai[m] my sense of being" by envisioning a mathematics education that would counter that dehumanization. Like Gutiérrez, he calls for mathematics learning rooted in "reciprocal relationships" and the principle of *yuhsen*, which in Arabic refers to "the worth of a person" (Fasheh, 2015, p. 49). The notion of *yuhsen* echoes Sengupta-Irving and Vossoughi's (2019) call to "disentangle [people's] worth from the normative discourses of STEM" by (re)storying mathematics learning as integral to family life, friendship, and community flourishing (p. 497). In some cases, restorying may involve engaging with knowledge and practice that veer significantly from dominant mathematics (see below for a discussion of multiplicity in the discipline). For others, like Amina (Sengupta-Irving & Vossoughi, 2019) and the young men in Stinson's (2013) study, (re)claiming a sense of worth entails redefining the why, for whom, and with whom of otherwise conventional mathematical activity-centering relationships that dignify and purposes that lift up the thriving of communities historically disempowered by mathematics and school.

To Further Heed the Call

The authors in this section address mathematics education's entanglement in the colonial matrix at the intimate scale of learners' identities and relationships. Their scholarship helps to link "the suprapolitics within the field" to "everyday and every moment" experiences of mathematics learning (Gholson & Martin, 2019, p. 394). Some

works examine the exclusionary norms of educational spaces, which require youth to invisibilize, assimilate, or reconfigure parts of themselves in order to be recognized as people who are capable of mathematics. Authors emphasize that such norms stem from broader systems of oppression, including heteropatriarchy, antiblackness, and the supremacy of English. Other studies highlight that, within learning environments shaped by macro-level structures and ideologies, "people are always repurposing tools, reimagining themselves, [and] renegotiating relationships" (Philip & Gupta, 2020, p. 212).

A limitation of some of the works discussed here is that they "leav[e] mathematics as a subject matter intact and unchallenged," rather than regarding the discipline as also in-the-making as youth develop their identities and relationships through mathematical activity (Gholson & Wilkes, 2017, p. 246; R. Gutiérrez, 2013). Many authors address the construction of self and social relations *around* mathematics without naming that young people's efforts to resist, self-author, and rehumanize might contribute to (re)creating mathematics itself (R. Gutiérrez, 2018). Philip and Gupta (2020) confirm this trend in their review of STEM-focused scholarship that approaches microgenetic and micro-interactional analysis from a critical perspective. In their review, the studies that highlighted the "co-constructed nature" of disciplines and learners' identities all focused on science classrooms, leaving mathematics unaddressed (Philip & Gupta, 2020, p. 200). Delinking requires reconceptualizing and remaking not just the people who participate in mathematics, but the discipline itself—including the activities, tools, and language that count as mathematical; the contexts and purposes people choose for engaging in mathematics; and the onto-epistemic values guiding this engagement (Rogers & Kaiser, 1995/2005). This call is explored further in the following sections, and attuning to possibilities for disciplinary expansion at local scales could be a focus for future research.

Another area in need of continued research is the politicization of mathematics identity for youth who occupy positions of power in society (Kokka, 2020; Skovsmose, 2016). Most critical scholarship on self-making in mathematics centers young people from historically marginalized communities, for good reason. Yet the field may perpetuate the naturalization of whiteness, masculinity, ableism, and other forms of hegemony if it does not address how these oppressions (re)produce through the experiences of people traditionally empowered in mathematics (Abu El-Haj, 2006; Fine, 1997/2004). Critical analyses could examine the ways young people construct, view, and potentially dismantle their privilege in mathematics learning environments, as well as the distinct forms of rehumanization involved in unlearning oppressive ways of being. To be clear, this is not a call to center dominant identities in critical research or detract attention from the experiences and narratives of those most harmed through mathematics education. Rather, I argue for a comprehensive analysis of the colonial matrix's microlevel workings, including a sharpened focus on the normalized relationship between "productive" mathematics and dominant social identities.

I conclude with a caution raised across the literature in this section—not a limitation of this scholarship but rather a call to vigilance about the reductive ways it can be interpreted. Many authors question discourses of inclusion that emphasize broadened participation within school mathematics without critiquing the regulatory nature and often dehumanizing consequences of such participation. Yolcu and Popkewitz (2018) refer to this phenomenon as "inclusion with exclusions" because it frames a binary choice between alienation from or assimilation within an exclusionary norm (p. 252; see also Rogers & Kaiser, 1995/2005). Cultivating disciplinary identities should not "subjectify" learners into a particular version of "mathematical"—restricted to certain ways of knowing, communicating, and acting—but rather open possibilities for learners to *be(come)* themselves through, and in the process expand, mathematics (Yolcu & Popkewitz, 2018).

Calabrese Barton and Tan (2019), referring to youth historically marginalized within STEM, theorize "rightful presence" as "legitimate membership" in a disciplinary learning community "because of who one is (not who one should be)" (p. 3). This distinction resonates with Martin's (2019) notion of "refusal *in*" dominant systems, which may entail a demand for recognition and access but not "for the sole purpose of being accepted into anti-Black and white supremacist spaces" (p. 471). In addition to demands for rightful presence within existing systems, Martin (2019) calls for "refusal *of*" them— a concept I return to in the next section.

Critique and Refuse Disciplines' Onto-Epistemic Normativity

Apprentice students into habits of critiquing and refusing the treatment of knowledge as 'settled,' and cultivate a disposition towards concepts and practices as always reflective of culturally and politically situated ways of knowing....Shift from seeing Western knowledge-making as natural, normalized, and totalizing to analyzing it as one, dominant and dominating, way of being and knowing the world. (Warren et al., 2020, pp. 280, 289)

Critical reimagining draws attention to normative conceptualizations of knowledge and knowing. Across disciplines, Western colonial logics govern the ways of knowing that are considered disciplinary, intellectual, and even human (Maldonado-Torres, 2007; Warren et al., 2020; Wynter, 2003). These logics idealize notions of neutrality and universality, divorce rationality from embodied and affective modes of sense making, privilege textual modes of communication, and construe the world in terms of subject-object relations (Bang, 2017; Barad, 2003; Grosfoguel, 2013; Law, 2004; Maldonado-Torres, 2007; Mignolo, 2003, 2009; Quijano, 1992, 2000; Santos, 2007; Shotter, 2006, 2015). Critical reimagining denounces the onto-epistemic violence perpetuated when these governing assumptions go unquestioned. It encourages researchers, educators, and young people to refuse their totalizing force by examining dominant ways of knowing as sociohistorically situated and open to contestation (Warren et al., 2020).

Sounding the Call in Mathematics

Challenging EuroWestern normativity in mathematics education requires denaturalizing the "grand narrative" of a politically and culturally neutral mathematics (Skovsmose & Greer, 2012, p. 381). Standards frameworks and curricula for school mathematics are often viewed as "neutral assemblage[s] of knowledge" distilled from a larger disciplinary core; in this view, the purpose of pedagogy is to reveal the essential structure of that body of knowledge to learners (Apple, 1993, p. 222; Popkewitz, 2004). Scholars who highlight the politically contentious nature of mathematics curriculum and pedagogy do not necessarily recognize that the competing approaches they consider still reflect EuroWestern systems of thought (e.g., Schmidt et al., 2005; Schoenfeld, 2004). Further, the broader discipline from which school subject matter "derives" (Schmidt et al., 2005) is rarely scrutinized as politically and culturally situated. Critical research in mathematics education as well as the philosophy of mathematics has worked to expose and disrupt this zero point authority of dominant mathematics.

The authors in this section explore possibilities for critical inquiry in and with mathematics. Theoretical critiques illuminate the onto-epistemic hierarchies that undergird mathematics as a discipline across professional and K–12 settings. Some empirical analyses examine the ways these hierarchies are embedded in school mathematics curriculum. Others discuss efforts to develop critical mathematical literacy with educators and young people. Distinct from notions of mathematical literacy that focus narrowly on the mastery of skills and concepts valued in state standards and STEM industries, critical mathematical literacy interrogates the very constitution of that knowledge and how it is used (Frankenstein, 2013; Larnell et al., 2016).

The works below vary in the extent to which they refuse the norms of dominant mathematics. Many suggest forms of "refusal in" existing systems of mathematics education (Martin, 2019)—for example, critically examining the onto-epistemic assumptions of academic mathematics or reconsidering the purposes toward which it might be leveraged. Other authors address "refusal of" dominant mathematics by exploring knowledges, practices, and worldviews that have been excluded from normative definitions of "mathematics" and "mathematical." This section introduces two cases of learning designs that exemplify "refusal of" dominant mathematics. While the rest of the literature dwells within the realm of critique, subsequent parts of the paper will build from this critical foundation to consider a more fundamental reimagining of mathematical activity.

Denaturalizing Onto-Epistemic Norms

Through histories of imperialism and settler colonialism, a certain "cluster of values" has come to shape what counts as mathematics in schools and society, rendering alternative systems not only inferior but often non-mathematical (Bishop, 1990, p. 56; see also Barnhardt & Kawagley, 1995; R. Gutiérrez, 2017). Scholars have underscored differences between norms of inquiry and conceptions of knowledge in the professional discipline as compared to K–12 classrooms (Burton, 1999a; Lampert, 1990; Peck et al., 2021; Popkewitz, 2004; Watson, 2008). While school mathematics can be reductive in particular ways, the authors below point to an onto-epistemic normativity that cuts across academic settings, transcending disciplines and the discipline-schooling divide.

Philosophical Critique. Philosophical and theoretical scholarship has shined a light on the onto-epistemic hierarchies, or governing values, that underpin dominant mathematics. These values include: universalism, rationalism, objectism, rigid classification, and the privileging of written text (Bishop, 1988; Davis & Anderson, 1979; Ernest, 2008, 2012; R. Gutiérrez, 2019; J. F. Gutiérrez & Scott, 2019; Otte, 1983; Shelley, 1995/2005; Thomas, 1987; Triadafillidis, 1998). Below, I elaborate on each by synthesizing the arguments of various authors.

Universality. The notion of universality holds that mathematical knowledge and practice transcend human subjectivity and particularities of context (Ernest, 2008;

Shelley, 1995/2005; Thomas, 1987; Triadafillidis, 1998). Mathematicians are concerned with generalization as a tool of objectivity—using axioms, algorithms, and theorems to verify the truth of knowledge claims which they then assert as "necessary" conclusions (Shelley, 1995/2005, p. 255). The logic of generalization and objectivity is hierarchical in that it tends to discount sources of understanding such as personal experience, traditional wisdom, or trial and error (Bishop, 1988; Davis & Anderson, 1979; R. Gutiérrez, 2019; Triadafillidis, 1998). Even from perspectives that reject absolute truth and view knowledge as continuously evolving within local communities of practice, mathematical meaning making is framed as a process of proposing and certifying provable conjectures, as tentative as those proofs may be (Lakatos, 1976; Lampert, 1990).

Rationalism. Rationalism is "at the heart of [Western] mathematics," a domain commonly associated with cerebral reasoning, abstract logical argument, and dispassionate communication (Bishop, 1988, p. 62). Dominant conceptions of mathematical rigor render embodied, affective, intuitive, empirical, and spiritual modes of sense making as less sophisticated or simply non-mathematical (Burton, 1999b; de Freitas, 2012; Gordon, 1978; Gónzalez et al., 2001; R. Gutiérrez, 2019; J. F. Gutiérrez & Scott, 2019; Triadafillidis, 1998; Yeh & Rubel, 2020). Like universality, rationalism obscures the ways subjective experience and sociocultural context are integral to the ways people interpret and shape the world with mathematics (Burton, 1999b; de Freitas, 2008; Ernest, 2012; Nelson-Barber & Estrin, 1995; Gordon, 1978). When conflated with notions of "intelligence" or even "human," rationality sustains the hierarchies of being that are used to justify systems of oppression (Maldonado-Torres, 2007; Quijano, 1992). *Objectism*. Mathematical language and practice construct "a world of things" for humans to quantify, classify, model, and operate on (Bishop, 1988, p. 65; Ernest, 2008; Thomas, 1987; Triadafillidis, 1998). According to Bishop (1988), objectism renders the material environment, living beings, and ideas as objects available for human subjects to master. It does so by atomizing and decontextualizing discrete entities which might otherwise be understood within a network of dynamic relations (Barad, 2003; Bishop, 1988; Mellone et al., 2021). On a linguistic level, we objectify mathematical processes by expressing them as nouns, a discursive phenomenon known as nominalization (Lunney Borden, 2011; Morgan, 1996). By setting humans apart from the "thingified" (Barad, 2003) world they reason about and manipulate, objectism can enable relations of supremacy and violence, such as (dis)possession, domination, and exploitation (R. Gutiérrez, 2019; Shelley, 1995/2005).

Rigid Classification. Classification is integral to mathematics, across cultural contexts (Bishop, 1990). Dominant mathematics privileges rigid forms of categorization, presuming that entities can be sorted into clearly delineated groups with fixed properties and that fluidity and multiplicity signal a lack of precision or coherence (de Freitas & Zolkower, 2009; R. Gutiérrez, 2019, 2020; Triadafillidis, 1998; Yeh & Rubel, 2020). Mathematical classification is often structured by binary logic, which establishes mutually exclusive, opposing categories, sometimes in hierarchical relation to one another—such as true/false, positive/negative, and abstract/concrete (Fasheh, 2015; R. Gutiérrez, 2019; Yeh & Rubel, 2020; Walkerdine, 1990). While not all forms of classification are harmful, those characterized by rigidity and hierarchy tend to underpin

systems of oppression as groups in power establish exclusionary boundaries that "com[e] to have the force of the natural order" (Bernstein, 1996, cited in Veel, 1999, p. 207).

Primacy of Written Text. Like other academic disciplines in the EuroWestern tradition, dominant mathematics elevates written forms of communication, assuming that meaning grows in sophistication as it becomes increasingly textual and abstract (Law, 2004; Matusov & Hayes, 2000). While experiences of doing mathematics can be multi-modal, its "public face" relies heavily on "symbol-rich written text," in the form of worded explanations, formal proofs, labeled diagrams, graphs and tables, or computation displays (Ernest, 2008, p. 5; Otte, 1983). To be deemed convincing or sophisticated, written representations tend to emphasize abstract symbolism and follow expectations of EuroWestern expository literate discourse, such as topic-centeredness, explicitness, conciseness, and linearity (Davis & Anderson, 1979; McBride, 1989; Otte, 1983). In mathematical contexts that center oral reasoning, communication may still reflect these norms of written argumentation (de Freitas, 2012).

The authors above do not suggest that generalization, abstract logic, reasoning about objects, classification, or written text are inherently oppressive. Rather, ontoepistemic hierarchies position these particular ways of knowing mathematics as *the* way, delegitimizing and often effacing the actual heterogeneity of mathematical thought and practice.

Critical Curricular Inquiry. Empirical studies have examined the ways written mathematics curricula reinforce onto-epistemic hierarchies through the messages they communicate about what it means to know and do mathematics (Love & Pimm, 1996;

Morgan, 1996; Otte, 1983). Some studies analyze the micro-linguistic and semiotic features of texts, while others focus on narrative structure and the sequencing of content. Building on prior work by Love and Pimm (1996) and Morgan (1996), Herbel-Eisenmann and Wagner (2007) present a framework for analyzing the ways words and accompanying images in textbooks position people and mathematics in relation to one another. For example, they found that, by omitting first person pronouns, textbooks reinscribe dominant conceptions of mathematical activity as abstracted from the particular perspectives and choices of human agents. This finding is a microlinguistic example of the norm of universality.

Other authors analyze the sociopolitical messages reflected in the content of curricular tasks (de Freitas, 2008; Dowling, 1996; McBride, 1989, 1994; Yeh & Otis, 2019). McBride (1994) critiques the ways textbook word problems privilege rationalism by framing mathematics as a "mere neutral tool" with which "decisions about complex social problems get made on a very narrow, rational basis" (pp. 39–40). For example, she analyzes a task that asks readers to generate linear models to estimate the volume of timber in black cherry trees based on measurements from a sample of logged forest trees. She notes that readers are not invited to consider the (likely extractive) purposes for calculating timber volume or whether cutting down a sample of trees was ethically justified. McBride (1994) argues that such tasks decontextualize mathematical reasoning from the politics and ethics of social life, sending the message that quantitative information is sufficient for making sound mathematical judgments about the world (McBride, 1994; see also de Freitas, 2008).

De Freitas and Zolkower (2009) discuss the need for teachers to critically analyze the "regulatory norms" in curricular texts. The authors suggest that teachers should learn to "decode" the language and visual representations in texts for "how they constitute the difference between the mathematical and the non-mathematical" (De Freitas & Zolkower, 2009, p. 194). For instance, images of material reality are commonly juxtaposed with symbolic notation in ways that assume the (universal) lexicon of mathematical notation unproblematically maps on to "particular truth claims" from lived experience¹⁷ (p. 194). De Freitas and Zolkower also highlight the rigid classification structures in school mathematics tasks that expect people to sort entities into mutually exclusive categories in order to essentialize their qualities. The authors call for professional learning experiences that develop teachers' "critical understanding of the symbolic domination work they often unknowingly perform" when they treat such norms as the natural and only way to engage in mathematics (p. 189; see also Herbel-Eisenmann & Wagner, 2007; Mellone et al., 2021).

Refusing Dominant Norms

Recent work in the learning sciences has documented mathematics learning

¹⁷ For an example illustrating this assumption (which I constructed), suppose an early numeracy task asks children to count or calculate the total number of individual flowers displayed in a set. A graphic of six flowers, arranged in two rows of three, is notated with cardinal counting numbers (i.e., 1, 2, 3...) below each individual flower in the image or with a statement to the side of the image representing an operation to find the total (e.g., $3 + 3 = _$ or 2 groups of 3 equals __). Such symbolic mapping would assume that flowers (or any life form) can and should be treated as atomized objects rather than parts of an ecological whole. In the latter conceptualization of plant life, it might not "make sense" to count out discrete flowers; the image just described might be seen as "one whole" patch of flowers, or might be missing key information because none of the interdependent living beings in the flowers' home were portrayed. This is not to say that 3 + 3 = 6 is invalid, but it is not a universally meaningful representation of the context.

experiences that begin to refuse EuroWestern onto-epistemic norms by exploring alternative ways of practicing mathematics. I return to culturally expansive alternatives later in the paper; here, I highlight the "generative refusal" in these examples (R. Gutiérrez, 2019).

Ma (2017) recounts an experience for middle and high school youth built around walking scale geometry, an approach to interactive spatial reasoning that regards "wholebody interactions" as integral resources for mathematical problem-solving and communication. In an outdoor setting that included tools such as ropes and flags, young people represented "conventional mathematical forms" (e.g., graphs and geometric objects) and communicated mathematically with one another in ways that engaged their whole bodies in motion (p. 143). Walking scale geometry liberates from the constraints of logocentric, pencil-and-paper tasks, which tend to divorce embodied and cognitive forms of meaning making (de Freitas, 2012). It invites youth to develop "new" geometric tools, "problematizing implicit mathematical relations hidden in the taken-for-granted representational infrastructure" of school mathematics (Ma, 2017, p. 144).

Barajas-López and Bang (2018) tell of a STEAM summer program in which Indigenous youth explored mathematics as they engaged in outdoor walking, clay making, and storytelling. The program's pedagogies refused onto-epistemic hierarchies of EuroWestern mathematics in various ways. Across activities, educators and youth oriented to "relationships with animate materials that have life courses to fulfill" rather than objectifying and seeking mastery over nature (p. 16). Additionally, clay making "as an embodied form of mathematics," and walking as a way to explore mathematical knowledge in nature, disrupted the EuroWestern divide between cognitive and embodied modes of understanding (p. 13). Barajas-López and Bang (2018) underscore that this learning experience "directly refused" hegemonic conceptions of knowing, and the cultural erasure such hegemony enacts, not simply through critique of dominant systems but by recentering "Indigenous ways of knowing, being, and making" (p. 13).

Critical Mathematical Literacy

The preceding examples illustrate refusals of the regulatory norms of dominant mathematics but do not explore possibilities for engaging youth in analysis of those norms and the systems of power in which they are embedded. While there is less literature on youth participating in onto-epistemic critique, scholarship on critical mathematical (and STEM) literacy calls for pedagogies that engage young people in sociopolitical analysis with mathematics, in some cases mobilizing it toward socially transformative ends (Frankenstein, 1983; Gutstein, 2003, 2006; Mutegi, 2011; Skovsmose, 1994; Tan et al., 2012; Terry, 2011).

Frankenstein (2013), whose work extends Freire's (1970) critical pedagogy to mathematics, outlines two interrelated pieces of critical mathematical literacy: "the mathematics of political knowledge" and "the politics of mathematical knowledge." The former addresses how mathematics can be leveraged to examine and shape political structures of society. It involves building fluency with dominant mathematical lexicon, tools, and practices; and learning to critically analyze the ways humans mobilize this discourse to represent and act in the world. The politics of mathematical knowledge recognizes that mathematical tools and methods are themselves products of political struggle, both reflecting and (re)producing power relations (Frankenstein, 2013; Skovsmose & Greer, 2012). This strand of critical literacy takes interest in the social construction of the discipline and works to elevate "the contributions of all the world's peoples to the development of mathematical knowledge" (Frankenstein, 2013, p. 39). Extant literature includes more accounts of teaching and learning that center the mathematics of political knowledge than the politics of mathematical knowledge, though I provide examples of each below.

Mathematics of Political Knowledge. In an example of the mathematics of political knowledge, Terry (2011) recounts a participatory action research project in which youth used mathematics to contest racist narratives related to incarceration and educational attainment. Terry draws on Critical Race Theory's notion of counterstory as an account or portrayal that intervenes on a dominant narrative by exposing its contradictions and telling an alternate story (Solórzano & Yosso, 2002). In an out-ofschool learning space, high-school-aged Black youth examined documents published by universities, state prisons, and a city police department, focusing on reported trends in homicide and incarceration rates and the representation of Black males in state institutions of higher education. The group analyzed how quantitative information had been represented and interpreted to construct the reports' deficit narratives of Black men and how that data might be reconstructed to tell a counterstory. According to Terry (2011), through the project youth "re-experience[d] mathematics" as "a critical cultural activity" and "a tool to engage and act upon the world around them," with a critical focus on issues that directly impacted their communities (pp. 23, 41).

Terry's study echoes literature on teaching mathematics for social justice (Gutstein, 2003, 2006; Gutstein & Peterson, 2005), in which learners use the tools of school mathematics to critically analyze and take action around social injustices. This literature, building on the work of Frankenstein (1983) and similarly inspired by Freirean critical pedagogy, describes learning experiences organized around the mathematics of political knowledge—or, as Gutstein (2006) puts it, "reading and writing the world with mathematics." In his high school classes, Gutstein supports young people to develop fluency with dominant mathematics as they practice critically "reading" their sociopolitical world with those mathematical tools.

In a year-long study of his 12th-grade mathematics class, Gutstein (2016) reflects on a curricular unit in which youth applied college-preparatory mathematics content to study issues of gentrification, migration, and deportation (p. 454). A "central political goal" of this curriculum was for the African American and Latinx youth in his class "to understand the social and economic forces that drove displacement differentially and similarly across their communities" (p. 484). The class used regression models and differential equations to explore questions related to income inequality, the housing market, immigration trends, and the economic policies that displace migrant farmers from their land. As a culminating project, youth created a presentation to share their analyses with family and community members. Like the youth in Terry's project, Gutstein's students employed dominant mathematics to critique the workings of structural inequality in their lives and develop narratives that might challenge those structures. While not a critique of dominant mathematics itself, teaching mathematics for social justice expands the purposes toward which this particular set of disciplinary tools might be used.

Politics of Mathematical Knowledge. Gutiérrez (2019) shares an example of curricular reform that addresses the politics of mathematics knowledge. The Revisioning Reclaiming Reconciling School Mathematics project (RRRSM), led by a group of Indigenous and non-Indigenous mathematics educators in Canada's Saskatchewan province, engages youth in critically examining "Western school mathematics" through a "culture based" lens (R. Gutiérrez, 2019). The project seeks to "move beyond an acultural presentation of school mathematics" by opening it to critical study (R. Gutiérrez, 2019, section 3, para. 3). The curriculum addresses the history of dominant mathematics, its "everyday" uses at both local and societal levels, and traditional and contemporary forms of mathematics among Indigenous communities in the region. Although the RRRSM initiative treats school mathematics as culturally and politically situated, Gutiérrez questions its emphasis on constructing "analogues" between Indigenous and EuroWestern forms of mathematics. She argues that "to simply overlay Indigenous knowings onto Eurocentric ones" reifies the authority of dominant mathematics and can "strip" Indigenous practices of their cultural meanings—which runs counter to the transformative aims of critical literacy (R. Gutiérrez, 2019, section 3, para. 5; see also Eglash et al., 2006; Stavrou & Miller, 2017). I return to this point later in the paper.

In a case that weaves together Frankenstein's two strands of critical mathematical literacy, Das and Adams (2019) describe a professional inquiry community of STEM teachers investigating "critical numeracy." Critical numeracy regards numbers as a form

189

of discourse, "constructed, languaged, and communicated" through sociohistorical power relations (Das & Adams, 2019, p. 295). Educators explored number relationships and operations in the context of a simulation activity about a fictive system of capitalist production. Participants reflected on their "unfolding relations" as they interacted around labor roles, wages, and profit, eventually weaving number and arithmetic into that social analysis (p. 301). For example, they analyzed the unequal distribution of profits that had motivated workers and unemployed people in the simulation to organize for greater control of wealth. Guiding questions for the activity included, "What relevance or use does number have to understand the world around us?" and, "How are particular historical conditions reified and transformed through the asserting of number?" (Das & Adams, 2019, p. 302). Similar to Terry's (2011) and Gutstein's (2016) projects, this inquiry "repositioned the use and purpose of employing numbers" toward a critical analysis of unjust social relations, treating number as politically mobilizable rather than as a neutral and static representation of reality (Das & Adams, 2019, p. 301).

Das and Adams (2019) focus their account of the activity on the mathematics of political knowledge. Although they allude to the politics of mathematical knowledge referring to the multiplicity of "number systems, classifications, logic systems and categorizations [that] have been created within human relations" (p. 297)—this thread of critical numeracy did not come up in their investigation with teachers. The group of educators seemed to stick with the tools of canonical numeracy rather than inquiring into the historical development of that system or alternative ways of thinking and communicating mathematically within the social context of the simulation. In this sense, their case resembled accounts of teaching mathematics for social justice more than an exploration of the cultural and political foundations of mathematics.

Another example that brings together the two strands of critical mathematical literacy, with a greater emphasis on the politics of mathematical knowledge, is People's Mathematics, part of the broader People's Education movement in apartheid-era South Africa (Julie, 1993; Vithal, 2003). Considered a "subversive" alternative to the stateimposed, assessment-driven curriculum at the time (Vithal, 2003), the People's Mathematics program "open[ed] up the question of what counts as mathematical knowledge" and who produces it (Julie, 1993, p. 36). Julie (1993), whose study focuses on the implementation of People's Mathematics in a pre-service teacher education course, outlines the program's goals. One was to apply mathematics to the study of sociopolitical problems, with an emphasis on "political conscientisation" (Julie, 1993, p. 35). Another was to "demystify mathematics" as sociohistorically constructed, by encouraging the study of the social contexts in which mathematical ideas have been developed and put to use (Julie, 1993, p. 36). Demystification involved critique of the ways mathematics and mathematics education have reproduced societal inequalities as well as exploration of mathematical diversity across cultures, including in learners' daily lives. Though no longer in practice in South African schools (Vithal, 2003), People's Mathematics represents a rare example of a school-based effort to explore the politics of mathematical knowledge.

To Further Heed the Call

The literature in this section challenges the narrative of a universal, value-free mathematics. Some authors interrogate the onto-epistemic norms of dominant mathematics, in certain cases documenting efforts to refuse those norms with educators and young people. Others reconceptualize the purposes toward which dominant mathematics might be practiced, inviting youth to use the tools of conventional school mathematics to critically analyze the political world. A less-researched strand of critical mathematical literacy inquires into the sociohistorical construction of the discipline.

I will highlight two ways in which the above research might expand its critique and refusal of dominant mathematics. First, critical refusal calls for more than the instrumental use of dominant mathematics *for* social critique. Pedagogies that center the mathematics of political knowledge will not, on their own, desettle canonical notions of the discipline and can actually reassert their totalizing force. If not paired with critical analysis of the politics of mathematical knowledge, these pedagogies may leave intact the assumption that conventional school mathematics *is* mathematics in its totality. A limitation of many cases of teaching mathematics for social justice is that, in using dominant mathematics as a tool to study sociopolitical contexts, educators and youth regard mathematics as a relatively settled body of knowledge that maps onto other (potentially transformable) aspects of social reality (e.g., Gutstein, 2006, 2016; Terry, 2011). Missing from their inquiry is a view of school mathematics as itself open to critical problem posing and transformation. Problem posing about the discipline would investigate the histories of dominant mathematical tools, concepts, and practices;

192

consider the ways of knowing and living they value and exclude; and explore expansive definitions of mathematics.

Second, most literature that addresses the politics of mathematical knowledge does not explore possibilities for engaging youth in critical epistemological and historical analysis (for an exception see Vithal, 2003). Many onto-epistemic critiques present theoretical analyses of the discipline. Empirical works focus on written curriculum, discuss critical inquiry with educators, or suggest the potential for critical epistemological analysis with youth but do not elaborate on what this would entail¹⁸. For example, de Freitas and Zolkower (2009) propose that teachers might invite students to examine the regulatory discursive norms of school mathematics but do not elaborate on the idea. Das and Adams (2019) leave unaddressed how their critical numeracy work with educators did or might translate into teachers' classroom practice. Accounts of critical refusal with youth underscore the importance of "onto-epistemic navigation" between multiple ways of knowing but do not discuss whether young people explicitly consider the processes and purposes of the onto-epistemic refusal in which they participate (Barajas-López & Bang, 2018, p. 15; Ma, 2017). Further research might design for and analyze learning that apprentices youth into such critical reflection (Warren et al., 2020).

¹⁸ Seattle Public Schools (2019) developed a K-12 Math Ethnic Studies Framework which incorporates both strands of critical mathematical literacy, including attention to the intellectual history of the discipline. In its articulation of the core theme of "Power and Oppression," Seattle's framework names that EuroWestern mathematics is often seen "as the only legitimate expression of mathematical identity and intelligence." Corresponding "learning targets" state that students will be able to: "analyze the ways in which ancient mathematical knowledge has been appropriated by Western culture," identify how the development of mathematics has been erased from learning in school," and "identify how math has been and continues to be used to oppress and marginalize people and communities of color." Future research might examine the implementation of this or similar frameworks.

Explore the Multiplicity and Dialogicality in Disciplines

Teachers and elders focused attention on the compressed, often intentionally suppressed, dialogical reverberations in the deep historical strata of disciplinary discourses. Refusing settled disciplinary lenses and engaging emergent tensions in these ways seeds possibilities for insurgent readings of disciplinary discourses....We see young people rupturing matrices of power by expressing their desire for onto-epistemic heterogeneity, or by wrestling with complex, sometimes contradictory, meanings of heteroglossic texts within processes of socio-political education. Not having yet been disciplined into dominant Western ideology, or actively resisting it as a form of enclosure to their learning and becoming, they readily attune to disciplines as moving fields where a multiplicity of words, points of view, tones, and values demand attention. (Warren et al., 2020, pp. 287, 289)

Critical reimagining attunes to the multiplicity and dialogicality in disciplines.

Multiplicity refers to the heterogeneity of ideas, practices, tools, values, and purposes that have contributed to disciplines over time—including those that have been discounted within dominant narratives (Grosfoguel, 2013; Santos, 2007; Warren et al., 2020). Critical reimagining assumes that disciplines are "moving fields," shaped as multiple systems of knowledge come into contact, clash, and interanimate around asymmetrical relations of power (Warren et al., 2020, p. 289; see also Pratt, 1991). Dialogicality "bring[s] to life present, distant, and absent voices" that participate in this interaction—illuminating that multivocality and contestation are constitutive, even generative, aspects of disciplines (Warren et al., 2020, p. 286). Multiplicity and dialogicality are "routinely invisibilized in schools," where disciplines tend to be treated as "static, known, and finalized domains" (Warren et al., 2020, p. 280). By contrast, learning environments that cultivate these sensibilities invite people to engage with disciplines as continually in-the-making, exploring their multivocal histories and participating in their ongoing evolution (Warren et al., 2020).

Sounding the Call in Mathematics

Scholarship that highlights multiplicity and dialogicality destabilizes "the myth of one and only one kind of mathematics," instead regarding mathematics as a "plurality of knowledges" (Fasheh, 2012, p. 94; see also Bishop, 1988; R. Gutiérrez, 2017). Dominant discourses of mathematics and mathematics education obscure the variety of knowledge systems and cultural traditions that have contributed to mathematics, within and beyond academic settings (D'Ambrosio, 1985; Joseph, 1987, 1991/2011). The scholars in this section demystify those discourses by inquiring into mathematical heterogeneity and the processes of its erasure. Rather than dismissing dominant mathematics, authors position it as a particular and partial set of ideas, tools, and practices—in relationship with but not intrinsically superior to those that historically have been devalued as "informal," less "rigorous," or simply not a recognized form of mathematics (D'Ambrosio, 1985; R. Gutiérrez, 2017; Khan, 2011).

The literature in this section avoids simplified interpretations of multiplicity and dialogism commonly found in equity-oriented work in mathematics education. Multiplicity is often presented as "multiple pathways to the same destination" or multiple points of access to an already-established field of exploration (Warren et al., 2020, p. 283). In classrooms, this can manifest as an openness to multiple strategies, representations, or entry points to a problem while pedagogy drives toward predetermined, unitary conclusions (Popkewitz, 2004). Multiplicity may also be conflated with liberal multicultural notions of diversity. For example, curricula may celebrate mathematicians from historically marginalized backgrounds, or present story problems that integrate contexts and characters from young people's lives, without substantively engaging a diversity of ways of knowing mathematics (Pais, 2011; Rands, 2009). Similarly, dialogicality is often reduced to classroom discussion in which participants share and respond to different mathematical ideas. Less emphasized are the epistemological assumptions that regulate these conversations, the cultural and historical "voices" animated through participants' contributions, and multivocality within the larger discipline (Warren et al., 2020).

The authors in this section consider more expansive notions of multiplicity and, to an extent, dialogicality. Their work illuminates the onto-epistemic heterogeneity that often gets flattened or invisibilized in schools and the wider discipline, highlighting the variety of cultural traditions, purposes, and meaning making practices that constitute mathematics across contexts. After introducing scholarship that frames the concept of "many mathematics" in sociohistorical terms, I turn to studies that document children's experiences navigating home and school learning contexts, with particular emphasis on the powered relationships between the forms of mathematics they encounter. As discussed below, research on the multiplicity of mathematics, including work that centers young people's experiences, does not always address possibilities for explicitly examining that plurality with youth. The section includes a few classroom-based cases of curricula that bring together varied mathematical traditions, though it concludes with a call for deeper attention to dialogicality in this work.

"Many Mathematics"

Critical scholarship calls attention to the "many mathematics around the world," disrupting Eurocentric histories of the discipline by working to illuminate both pattern and variation in mathematics across cultural contexts (R. Gutiérrez, in Strong & Das, 2018). Ethnomathematics is an area of research that inquires into the "radically different ways" of practicing mathematics across cultural groups and time periods (D'Ambrosio, 1985, p. 44; see also Ascher, 1991, 2002; Eglash, 1997). According to Gutiérrez (2002), ethnomathematics honors "the diversity of mathematical knowledge that is created, transmitted, diffused, and institutionalized within different cultures, especially outside of formal education" (p. 158). She and others clarify that, while ethnomathematics shifts attention beyond traditional academic settings, dominant mathematics is itself culturally situated and should be regarded as a form of ethnomathematics (R. Gutiérrez, 2017; Khan, 2011; Pais, 2011).

Joseph (1987) counters the narrative that mathematics was discovered and developed primarily by white European civilizations, and he critiques European scholarship's representations of non-European mathematical traditions. His work highlights that Egyptian, Babylonian, Arab Islamic, and East and South Asian societies were not merely "borrowers" and "custodians" of ancient Greek mathematics; they both influenced and created beyond it (p. 19). In a historiography that spans centuries and continents, Joseph (1991/2011) details the mathematical innovations of these societies, exploring points of convergence, influence, and diversity across traditions. Identifying Joseph's work as part of a subfield of research on "non-Western mathematics," Eglash and colleagues (2006) note that this scholarship tends to give precedence to "the empire civilizations," whose economic and administrative structures tended to require kinds of mathematics that could "easily translate" to dominant EuroWestern forms (p. 348). Research in ethnomathematics is distinct in that it focuses on the culturally embedded mathematical practices of "small-scale (indigenous, traditional) societies," which may not have direct EuroWestern analogues (Eglash, 1997, p. 81).

Scholarship on the plurality of mathematics examines common mathematical activities that take various forms across time, space, and culture. Bishop (1988) explains that practices such as counting, measuring, locating, and classifying are common across cultural groups but that there is a range of ways people engage in each. For example, tools for quantifying may include beads, body parts, knotted string, and wood carvings (Bishop, 1988). Saxe's work among the Oksapmin people in Papua New Guinea (Saxe, 1982; Saxe & Esmonde, 2005) offers an example of a counting system whose number cycle and representational infrastructure—built around 27 parts of the human body—are wholly different from the base-ten system of dominant mathematics. Bishop (1988) points out that the spatial understandings underlying Euclidean geometry—which emphasize atomistic, object-oriented notions of points, planes, and figures—are but one way to conceptualize space; many Indigenous communities understand shape and space as dynamic processes rather than finite entities that can be objectified and subdivided (Sterenberg et al., 2010; Trinick et al., 2015). Ascher (2002) calls attention to diverse methods for recording mathematical ideas, extending beyond written marks on paper, to include: seed arrays, lines in dust, strings of plant material tied together, incised pieces of wood, and inscribed stone. In her work, Ascher (2002) also elaborates various cultural approaches to divination logic, mapping, marking time, and relating and categorizing objects.

In addition to furthering historical and sociological research on the diversity of mathematics, some authors advocate teaching about the sociohistorical foundations of the discipline (Bishop, 1988; Joseph 1987). Later in the paper I address classroom-based efforts to explore culturally diverse forms of mathematics. Here I highlight Anderson's (1990) work with college students to examine the historical roots of that diversity. Anderson (1990) describes courses he taught that opened to study the mathematics that "ordinary people" have created within contexts usually ignored in dominant histories of the discipline. For example, his classes discussed mathematical innovations developed by women farmers in early African communities; pre-Hellenic geometry in China, India, and Egypt; and research centers across Asia, Africa, and the Middle East that contributed to advancements in astronomy and algebra. Anderson's (1990) curriculum also emphasized the "intimacy" of mathematical knowledge and oppressive political systemshighlighting, for instance, the ways mathematical innovation supported shipbuilding for the Transatlantic slave trade and continues to underpin the development of military weapons systems (p. 356). According to Anderson (1990), integrating critical historical inquiry into traditional mathematics courses influenced students' perspectives on and relationships with mathematics, often inspiring their sense of efficacy, purpose, and willingness to enroll in subsequent courses¹⁹.

¹⁹ Anderson (1990) reports that students in his classes across two universities, particularly people of

Multiplicity as Powered and Evolving

Scholarship on mathematical heterogeneity varies in the extent to which it acknowledges the evolving nature of mathematics and the power relationships among its various forms. Some authors caution against essentialist perspectives that treat cultural communities and their mathematical knowledge as static and in a fixed hierarchy (Eglash et al., 2006; R. Gutiérrez, 2017). Gutiérrez (2017) contends that all forms of mathematics are "in motion," continually developing in dynamic relation to one another. Drawing on work by Santos (2007), she proposes the notion of an "ecology of [mathematical] knowledges" in which various knowledge systems co-exist and evolve interdependently (R. Gutiérrez, 2017, p. 5). Gutiérrez (2017) emphasizes that each member of this ecology, including dominant mathematics, is "partial"-relevant to and "legitimate" in particular contexts, rather than universally meaningful (p. 5). This perspective resonates with calls from Indigenous educators to "embrac[e] a wholistic and interconnected view of mathematical knowledge" and to look to a variety of local knowledges as resources for mathematics teaching and learning (Sterenberg et al., 2010, p. 9). The next part of this section includes examples that address such calls.

Sociohistorical structures of power both shape and are influenced by the interaction that takes place within ecologies of mathematical knowledge (D'Ambrosio, 1999; R. Gutiérrez, 2017). Analyses of mathematical diversity that recognize its dynamic

color, had "a more positive, self-assured attitude about themselves successfully doing mathematics"; 85% passed his algebra course as compared, for example, to a roughly 50% passing rate in traditional algebra classes at one of the universities; and, of those who passed, a majority pursued at least one mathematics course that they had previously planned to avoid (p. 358).

nature do not always address its powered dimensions. For instance, in their account of Oksapmin mathematical practices, Saxe and Esmonde (2005) allude to a history of colonization—explaining that through trade, plantation labor, and missionary schools, Australian colonizers introduced EuroWestern styles of counting and arithmetic among the Oksapmin people—but the authors emphasize the "remarkable heterogeneity" of colonial and Indigenous quantifying systems without acknowledging the power differential between them (p. 187). In a very different context, Gutstein (2016) discusses the "fluidly interconnected knowledge" involved in teaching mathematics for social justice, which "dialectically" interweaves "classical" or dominant mathematical knowledge, young people's "community knowledge," and the "critical knowledge" required for analyzing injustice (p. 458). Gutstein (2016) writes that "these types of knowledge mutually interact, evolve, overlap" but does not address the societal power that classical mathematics holds over nondominant community knowledges or the ways interweaving them has the potential to disrupt that hierarchy (p. 469).

The studies below devote attention to the power relations among interacting systems of knowledge. The first two highlight divisions created by these hierarchies, while subsequent ones investigate possibilities for bridging across and perhaps transforming those divisions.

Division and Delegitimation. De Abreu (de Abreu, 1995; de Abreu & Cline, 2007) examines the unequal "social valorization" of school and family mathematical practices in a Brazilian farming community. She describes "sugarcane farming mathematics" as a body of knowledge and skills that farmers learn independent of formal education, with "specific features" that distinguish it from the EuroWestern-inspired mathematics taught in many Brazilian schools (de Abreu, 1995, p. 124). These features include units of measurement derived from farming materials and tools, nonstandard formulas and procedures, and a mix of oral and written calculation. Additionally, children in her study learned farming mathematics through apprenticeship—an "interchange of knowledge" with family elders—whereas mathematics in school was organized around a teacher-centered, transmissionist model of learning (de Abreu, 1995, p. 138). De Abreu (1995) found that children devalued their families' ways of knowing, denying that these qualified as "proper mathematics, even as they continued to employ alternative mathematical practices in their daily work and living. De Abreu suggests that participants' views reflected social class hierarchies in Brazilian society, which positioned farming as a lower-status occupation.

In an ethnographic study with Filipina immigrants and their children living in an urban area of Japan, Takeuchi (2018) found a similar "discontinuity" between home and school learning, marked by notions of (il)legitimate mathematical knowledge. Both mothers and children in her study tended to "undervalue" the mathematics parents had learned in their communities of origin in the Philippines (Takeuchi, 2018, p. 13). Takeuchi (2018) focuses on a finger-based method for multiplication, which mothers chose not to openly teach at home because they believed Japanese schools would consider it illegitimate. For children, navigating this divide involved "hiding or not appropriating what was taught at home" and, in some cases, reluctance to learn their

mothers' method because it did not conform to the disembodied ways of practicing mathematics privileged at school (Takeuchi, 2018, p. 10). Like de Abreu, Takeuchi situates this epistemological violence within a broader sociopolitical context, in which urban schooling conferred economic status and the intersection of sexism and xenophobia discounted immigrant mothers' funds of knowledge.

Possibilities for Transformation. Literature that addresses power sometimes portrays hierarchies as immutable and the divide between various forms of mathematics unbridgeable. A growing body of work by Indigenous scholars and educators considers spaces of contact between EuroWestern mathematics and various Indigenous knowledge systems, highlighting possibilities for reorganizing relations between them (Barajas-López & Bang, 2018; Glanfield, 2016; Lipka et al., 2005; Nelson-Barber & Estrin, 1995; Parra et al., 2016; Ruef et al., 2020; Stavrou & Miller, 2017; Sterenberg et al., 2010; Trinick et al., 2015; Wiseman et al., 2017).

Lipka and colleagues (2005) analyze the implementation of Math in a Cultural Context, a "culturally based mathematics curriculum" co-designed by Yup'ik elders and mathematics educators in Alaska. Acknowledging that all curriculum and pedagogy are culturally based, the authors specify that their curriculum modules integrated disciplinary content knowledge, pedagogical approaches, and contextual knowledge informed by both EuroWestern and Yup'ik epistemologies. Lipka and colleagues' (2005) case study of two teachers implementing the curriculum highlights shifts in the "social organization of the classroom," including in teacher-student relationships, participation structures, norms of inquiry, and the nature of classroom talk about mathematics (Lipka et al., 2005, p. 371). For example, the authors found that the modules opened space for children to coconstruct ideas with one another without as much teacher mediation. In one classroom, they noted "culturally congruent ways" of communicating about mathematics, including nonverbal modalities and "familial" forms of talk such as joking (pp. 379–380). Lipka and colleagues (2005) refer to these classrooms as "third spaces," or "place[s] in which historically silenced knowledge of Indigenous peoples such as the Yup'ik is privileged alongside traditional academic discourses....creat[ing] possibilities for social and epistemological change" (pp. 369–370). I return to the notion of epistemological third spaces in this section's conclusion.

Trinick and colleagues (2015) tell of efforts by a Māori community in New Zealand to "reintroduce traditional ethnomathematical practices" in primary school classrooms, following decades under a state-mandated curriculum that reflected a "Western mathematics perspective" (p. 417). The project focused specifically on Māori views of space, location, and direction, which orient around actions and signs of the sun, wind, plants, and geographical landforms (p. 420). Trinick and colleagues (2015) explain that Māori spatial frameworks are different from those privileged in most New Zealand schools, which abstract from embodied, land-based sources of knowing to Cartesian coordinate systems and representations. Children in the study interviewed community elders to learn about the spatial orientation frameworks used in their traditional tribal areas; they then created a scaled map of their area from memory, referencing sources of direction that came up in interviews (e.g., wind patterns). Trinick and colleagues (2015) underscore that the purpose of this learning design was not to replace EuroWestern mathematics but to explore "how mathematical ideas are constructed and applied, depending on the needs and circumstances of a particular community" (p. 429). These classrooms valorized and sustained children's connection to Māori mathematical knowledge—which, along with the Māori language, had been excluded from most schools for over a century and were in danger of falling out of use (Trinick et al., 2015).

Several studies, often connected to broader Indigenous language and cultural revitalization projects, examine mathematical heterogeneity and interanimation at the level of language. Ruef and colleagues (2020) share about a mathematics curriculum developed in the Yakama language of Ichishkíin that was designed for implementation in an Ichishkiin immersion classroom in the Pacific Northwest. A goal of the project was to "bridge Indigenous language and worldviews with western mathematics" through teaching and learning about fractions (Ruef et al., 2020, p. 317). The authors analyze language as a key mediator in the integration of Yakama cultural contexts and fractions concepts found in the Common Core State Standards for Mathematics (National Governors Association, 2010). Their curriculum development process began by asking a tribal elder to "recount ways in which fractions were evident in Yakama lived experiences"—for example, in the measurements involved in tipi building and price negotiation among seafood traders (Ruef et al., 2020, p. 322). This elder's stories also surfaced experiences of oppression related to land partitioning and blood quantum policies. The curriculum authors then inquired into Ichishkíin words that described the mathematical concepts embedded in these cultural contexts; if no word existed, they worked to create new words that reflected Yakama values and worldviews (Ruef et al.,

2020).

Sterenberg and colleagues (2010) note the tensions that can arise in such curricular integration efforts, specifically around language. Within a collection of reflections by different authors, Lunney Borden shares her experiences teaching and collaborating with a Mi'kmaw community in Nova Scotia that was working to align culturally-responsive pedagogies with a provincially approved curriculum. Lunney Borden highlights tensions between state-sanctioned school mathematics and "Mi'kmaw ways of reasoning about things seen as mathematical" (Sterenberg et al., 2010, p. 10). For example, the Mi'kmaw language embeds "a sense of motion...that is not apparent in school-based mathematics": words for shape and space act as verbs, reflecting the dynamic and processual nature of mathematical ideas, in contrast to dominant mathematics' tendency to turn actions and processes into nouns (Sterenberg et al., 2010; p. 12; see also Lunney Borden, 2011). Lunney Borden (2011) worked with classroom teachers to draw on the verb-based discourse of Mi'kmaq as they facilitated mathematical explorations with children, describing geometric figures and their characteristics using language that communicated ongoing process (e.g., "coming to a point," "sitting still," "look at you with its six faces," and using hand motions to track the behavior of part of a figure).

These authors complicate the notion that mathematical concepts can simply be "translated" between languages (Parra et al., 2016). Since language reflects cultural values and worldviews, terms and the concepts they index do not always have clear-cut equivalents across traditions. Further, as Parra and colleagues (2016) contend, translation

206

in multilingual educational settings often occurs unidirectionally (i.e., *from* children's community languages *to* the language of school mathematics), reifying the authority of school mathematics and undermining linguistic and cultural heterogeneity. They suggest that honoring cultural multiplicity requires a dialogical sensibility—a multidirectional, ongoing interanimation process, creating networks of mathematical ideas, practices, and representations that "expand the original fields of knowledge of each culture" (Parra et al., 2016, p. 78).

Toward Dialogicality

Dialogicality opens avenues for examining interconnection and contestation among the many mathematics young people encounter, positioning the knowledge of diverse cultural communities in dynamic, potentially transformative relationship with school mathematics. In the examples above, analyses gesture toward the critical potential of dialogicality but do not fully explore it. In the concluding section that follows, I discuss this as an area for further inquiry.

To Further Heed the Call

The literature in this section illuminates the plurality of mathematical knowledge and practice, challenging notions of a single, fixed mathematics while also highlighting continuities across cultures and contexts. Some authors examine mathematical heterogeneity on a global-cultural scale, while others focus on local sites of contact. Several studies profile curriculum projects that open classrooms to multiple ways of knowing, in an effort to decolonize school mathematics and imagine culturally sustaining mathematics pedagogies (Ruef et al., 2020; Stavrou & Miller, 2017; Sterenberg et al., 2010). These authors point to possibilities for "epistemological change" when classrooms expand the language, practices, and perspectives that undergird mathematical inquiry (Lipka et al., 2005, p. 370; see also Rogers & Kaiser, 1995/2005).

Scholarship that foregrounds multiplicity does not reliably attend to the dialogic potential in encounters with multiple forms of mathematics. Further work is needed to explore the ways diverse mathematical traditions resonate, clash, and shape each other as they come into contact in educational spaces (Pratt, 1991)—and how engaging with this dialogicality can deepen disciplinary learning. Below, I propose two foci for continued inquiry: inviting youth to explicitly examine this interanimation process and positioning them as active participants in the creation of mathematical knowledge and practice (R. Gutiérrez, 2018; Warren et al., 2020).

Exploring multiple disciplinary discourses "alongside" one another (Lipka et al., 2005) is not inherently counter-hegemonic and may actually reify epistemological hierarchies if power relations are not also the subject of critical inquiry in classrooms. Lipka and colleagues (2005) describe the classrooms in their study as "third spaces" (p. 369). According to Gutiérrez and colleagues (1995), third spaces are sites of cross-cultural contact, and potentially social change, in which people interrogate, contest, and imagine beyond "dominant scripts." In addition to drawing on diverse local knowledges as resources for mathematics pedagogy (Sterenberg et al., 2010), mathematics classrooms as third spaces would invite youth to examine the relationships among these knowledges, devoting explicit attention to the histories of domination and resilience that formed them, the cultural ways of knowing they reflect, and their dynamic interrelation with one

another. Further, critically dialogic pedagogies would position school mathematics as itself multivocal and subject to transformation through cross-cultural contact. Rather than either rejecting dominant mathematics or upholding its hegemony, dialogically-oriented classrooms enable a more complex engagement with it (Lipka et al., 2005; Trinick et al., 2015).

Future research might also design for and analyze learning spaces that recognize young people's creative agency in the (re)making of mathematics. In addition to considering the multiple traditions that have contributed to mathematics historically, youth would also come to see themselves as participants in the construction, and potential expansion, of an ever-evolving ecology of mathematical knowledges (R. Gutiérrez, 2017; Rogers & Kaiser, 1995/2005; Warren et al., 2020). In the learning contexts documented throughout this section—both dehumanizing and culturally affirming—what might have been possible had young people been invited to critically study the relationship between their families' and communities' mathematical practices and those typically privileged in school? Rather than assuming that these "belong to different worlds," some more legitimately mathematical than others, learners might come to see all mathematical knowledges as socially constructed, interdependent, and open to ongoing creation (Fasheh, 2012, p. 93).

Re-Place Disciplinary Learning as People "Do Life"

Horizontality highlights the boundless variety of places, cultural contexts, and practices through which people learn. In other words, learning is infinitely deeper and broader than school....By documenting the depth and breadth of learning as ongoing in the lives of youth, research across disciplines has exposed the hegemonic function of categories such as everyday/non-academic/concrete and disciplinary/academic/abstract, i.e., how they organize learning along a vertical axis that stratifies people, knowledges, and practices. (Warren et al., 2020, pp. 283–284)

Critical reimagining "re-places" disciplinary learning within the heterogeneous contexts, practices, and purposes of young people's lives—an idea Warren and colleagues (2020) refer to as "horizontality" (p. 283). Horizontality is distinct from curricular approaches that extract topics and situations from youth's out-of-school living to integrate them into otherwise conventional school activity; it fundamentally reconceptualizes where, how, why disciplinary learning happens. Critical reimagining questions distinctions between "disciplinary" (or "academic") and "everyday" knowledge and experience. This binary de-legitimates cultural and intellectual repertoires that do not reflect the EuroWestern perspectives privileged in formal academic settings, rendering them less sophisticated than and therefore subject to reformatting or displacement by what youth learn in school. Horizontality de-links disciplinary learning from the confines of school-based activity, opening toward the knowledges and practices that people cultivate as they "do life" (Warren et al., 2020, p. 283).

Sounding the Call in Mathematics

Re-placing mathematics learning orients to the boundless variety of contexts, practices, and purposes through and toward which people engage with mathematics as they go about their daily lives. Horizontality assumes that mathematical ingenuity and activity "are already present in diverse forms in all communities" and that learning mathematics is meaningful insofar as it sustains communities' self-determined purposes (Vossoughi et al., 2016, p. 218). Maier (1991) proposes the term "folk mathematics" to

refer to the mathematical inquiry that naturally arises in "the world outside school, the world most important to most people" (p. 63). While the term "folk mathematics" can reify a sense of hierarchy between academic or professional mathematics and "everyday" people's "world outside" (Warren et al., 2005), Maier honors the "wide and probably infinite variety" of mathematical problems, purposes, and methods involved in people's lives, which are often estranged from the mathematics youth encounter in school. Scholarship that centers horizontality both critically interrogates and ventures beyond that divide.

It is important to distinguish horizontality from related notions that diminish or devalue out-of-school living. First, re-placing is not simply a matter of "mathematizing" life by mapping the symbols, procedures, and concepts of school mathematics onto "real world" experiences. This asymmetrical overlaying of one onto the other suggests that out-of-school contexts are only mathematically legible when (re)formatted by school mathematics—a notion that reinscribes the "demathematisation" of many young people's lives and the colonizing power of school methods (Skovsmose, 2012, p. 350). Horizontality is also distinct from instrumentalist pedagogies that integrate discrete bits of young people's complex worlds as a lead into or backdrop for school-designed tasks (Pais, 2011). Such approaches overlook the "how and why" of mathematics done outside of school—the diverse reasons and methods for mathematical practice, not simply the topics and settings involved (Maier, 1991, p. 64). Third, horizontality should not be equated with reductive notions of "applied" mathematics, rooted in a hierarchy that positions "pure" mathematics as a more scholarly subfield because of its emphasis on theoretical concepts and structures independent of their contextualized use (D'Ambrosio, 1985; Frankenstein & Powell, 1994; Harouni, 2015; Joseph, 1987; Walkerdine, 1990). Instead, re-placing entails a "critical widening" of what it means to be mathematical and where mathematics happens (Sengupta-Irving & Vossoughi, 2019).

The scholarship in this section inquires into the diverse mathematics extant in people's naturalistic settings (Martin, 2019). Some studies document the mathematics embedded in daily social and professional activities. Others examine the relationship between culturally embedded mathematical activity and school mathematics, considering possibilities for grounding curriculum and pedagogy authentically in contexts beyond school. Across this literature, authors underscore the care and political clarity required to re-place mathematics learning in ways that avoid flattening cultural heterogeneity. While the literature below models such care, I note where certain framings may inadvertently reify the authority of dominant mathematics.

In Situ Mathematical Activity

Through both ethnographic and participatory design methods, researchers have investigated *in situ* mathematics learning, often focusing on cultural practices in communities whose ways of knowing are invisibilized in schools. Lave (1988, 2011) examines the work of Vai and Gola tailors in Liberia, exploring the ways mathematical problems emerged and were interpreted within the "social fabric" of a tailor shop (Lave, 2011, p. 119). Tailors developed problem-solving strategies "on the spot," employing "specialized" metrics and calculations that responded to moment-to-moment shifts in quantities and the social demands of their interactions with customers (Lave, 2011, p. 126). Like Lave, Fasheh (1990, 2015) emphasizes the mathematical flexibility involved in his mother's seamstressing. He describes "my mother's mathematics" as "linked to immediate and concrete needs" of her work, juxtaposing her context-sensitive practice with the drive toward abstract generalization in his schooling experience (Fasheh, 1990, p. 22). Fasheh (2015) writes:

My mother made thousands of dresses that fit numerous women, each with her own geometry! Whereas I had to deal with problems and theorems about identical geometric figures, she made thousands of dresses, no two of which were identical....Whereas my knowledge claims to be universal, hers formed a harmonious universe rooted in life. (p. 40)

In an ethnographic study exploring the mathematical practices in families' households, González and colleagues (2001) mark a similar contrast and add that "academically validated school knowledge of math seemed to obscure" the "reservoirs of knowledge" involved in seamstressing work. The authors describe their experience learning from a household member to design and sew a dress. Drawing explicit connections to Fasheh's mother's story, they note the seamstress's "flexibility with mathematical knowledge" as she took measurements and pieced together fabric (González et al., 2001, p. 124). While it was initially difficult for researchers to "see" the complex mathematics she called upon—just as Fasheh, as a child, did not recognize his mother's mathematical knowledge—the authors share that "the socially mediated nature of mathematical meaning making came into focus" as they immersed in the "authentic setting" and developed familiarity with the particular goals and practices of the textile

work (González et al., 2001, p. 127). Fasheh's (2015) use of the terms "harmonious" and "rooted" speak to the meaning of horizontality in these stories: mathematics and life are not inherently separate domains to be bridged or applied to one another. School-based practices, which often seem divorced from the modes and purposes of daily living, are not the totality of mathematics; when the mathematics in focus is that which is integral to people's social worlds, mathematical learning is natural to life.

Scholarship has also profiled *in situ* mathematics learning among young people outside of school. I will elaborate on two studies introduced earlier in the paper. In a study of the mathematics African American youth engaged as they played dominoes and basketball, Nasir (2002) examines the intersecting mathematical and "practice-linked" goals that "arise in the context of activity, when mathematical operations become a normal and required part of the cultural practice" (p. 225). Her analysis highlights that young people's mathematical problem solving evolved in relation to their social goals, as they developed experience at the game and connectedness with a community of players. For example, while playing dominoes, younger children were concerned primarily with "maintaining the flow of play and being able to complete one's turn" without assistance, which led them to focus on matching number representations between their hands and the tiles on the table (Nasir, 2002, p. 224). Older youth, concerned with scoring and blocking other players' moves, calculated potential scores by carrying out strings of addition and subtraction problems based on multiple hypothetical trajectories of play. Nasir (2002) highlights that the mathematical learning involved in dominoes (and basketball) was tied to the authentic purposes and sense-making practices young people developed as they

participated in these socially meaningful activities.

Barajas-López and Bang (2018) discuss the mathematics learning embedded in the activities of a STEAM camp for Indigenous youth. The program was designed "to engage Indigenous youth in understanding the changing lands and waters in the Pacific Northwest through direct engagement with phenomena in the world" (Barajas-López & Bang, 2018, p. 15). While on nature walks, youth inquired into the mathematics of the natural world by observing "symmetry, shapes, angles, tessellations, lines, and curves in plant relatives" native to their area (Barajas-López & Bang, 2018, p. 17). In their clay work, youth drew from the patterns they noticed on these walks to design their pieces. Clay making involved approximating proportions of water and dry clay and using mathematical modeling to translate between two-dimensional sketches and the threedimensional pieces. Barajas-López and Bang (2018) illustrate that "mathematical knowledge is not separate or divided from a cultural activity" but rather embedded in and constructed through it (p. 20). Engaging in practices of walking and clay making provided occasions for, and animated, mathematical learning while sustaining youth's cultural traditions (Barajas-López & Bang, 2018; R. Gutiérrez, 2019).

Scholarship that re-places mathematics learning intervenes on the myth of a universal mathematics, generalizable and meaningful apart from the particularities of context. Authors describe forms of mathematics that emerge through the "complex relationship between people and materials," as practitioners engage with the mathematical problems, tools, and processes entailed in their work (Harouni, 2015, p. 63). Lave (2011) describes this view of mathematics as "the doing of quantitative relations" (p. 140). The examples above—tailors and seamstresses assessing dimensions for clothing repair, Brazilian farmers measuring sugarcane stalks, youth envisioning domino play sequences or estimating amounts of clay to mix with water—all demonstrate a kind of quantitative responsivity to the materials and needs of people's contextualized activity. In these cases, mathematics is not an authoritative body of knowledge and practice originating external to and then transposed onto a social context; rather, it is constructed through the very doing of social life, in all of its heterogeneity.

In drawing parallels across this literature, I do not want to gloss over important distinctions between the social practices profiled in each study. For instance, while sugarcane farming and seamstressing are part of a global system of economic production that in many ways devalues the makers' labor, clay making in the Native STEAM camp was "a vehicle to contribute to and manifest the continuity of Indigenous practices and people" (Barajas-López & Bang, 2018, p. 16). Still, threading across cases of *in situ* mathematical activity is the notion that mathematical learning is "ongoing" in people's lives, deriving from the authentic purposes and practices of their everyday activities (Warren et al., 2020, p. 284).

Implications for School-Based Mathematics

I now turn to scholarship that considers the challenges and possibilities of exploring horizontality in school contexts. Booker and Goldman (2016) frame the relationship between in- and out-of-school forms of mathematics not as an inherent divide but rather as a sociohistorically constructed "rift" in need of "repair." The authors in this section approach possibilities for repair from various perspectives, some more critically imaginative than others. This literature demonstrates that, while opening classroom learning to the richness of young people's lives can be humanizing, it must be conceptualized and enacted with caution in order to avoid enclosure by the dominant systems of schooling.

Modeling Community Contexts with Mathematics. Many authors discuss efforts to develop school mathematics curriculum in response to the contexts and issues in young people's lives. They investigate opportunities for students "to deal with the mathematics in their own environments....encouraged to formulate, attempt to solve, and communicate their discoveries about mathematical questions arising in their classrooms, their playgrounds, their homes," and other community settings (Maier, 1991, p. 66).

Building from her work with the BRIDGE Project (González et al., 2001), Civil (2007) reflects on curriculum designs that drew on families' mathematical funds of knowledge, primarily in working-class Latinx communities. She describes the process of developing and implementing curriculum modules with teachers—and in some cases family members—that engaged young people in gardening, construction design, building furniture, and figuring purchasing costs. Civil (2007) writes that the most "successful" modules mirrored key aspects of children's out-of-school environments, including: apprenticeship learning structures, a degree of control over tasks and strategies on the part of the people carrying them out, and mathematics that was embedded without being "the center of attention" (p. 115). These qualities are resonant with the mathematical activity of tailors and basketball players that Lave (1988, 2011) and Nasir (2002) explore in their work, as discussed earlier in the paper. Civil (2007) also highlights tensions

around what counts as mathematics and how to maintain the cultural authenticity of tasks, which I address later in the section.

Aguirre and colleagues (Aguirre et al., 2012; Turner et al., 2019) design and analyze "mathematical modeling" experiences that are rooted in young people's cultural and community contexts. The researchers define modeling as a process of "connect[ing] core mathematical ideas to real world experiences through analysis, representation, interpretation, and revision" (Turner et al., n.d., Overview section, para. 1). As part of a professional development project with elementary teachers, they designed classroom tasks that required complex, socially meaningful decision making, grounded in situations that reflected the life experiences of children in participating teachers' school communities. Task contexts range from gardening and birthday gift planning to making pupusas and upcycling jump ropes; some invite young people to engage in these activities in class, while others form the backdrop for an imagined problem situation. Aguirre and colleagues explore the ways children leverage their life experiences as they interpret, model, and solve problems, as well as the professional learning required for teachers to support this work (Aguirre et al., 2012, 2019; Turner et al., 2019).

In the Algebra Project curriculum, which Moses and Cobb (2001) refer to as an "experiential" and "culturally-based" approach to middle school algebra, mathematical modeling and discourse grow out of explorations of young people's local environments (p. 120). Moses developed a multi-step process that revolves around a community field trip, or "physical event," which youth work to represent in increasingly abstract ways. After creating pictures of aspects of the experience that interest them, youth discuss and write about the event using "people talk," or "the language they use to negotiate their daily lives" (Moses & Cobb, 2001, p. 121). These stories are then "encoded" with "feature talk," or "regimented language" that distills features of the experience to be symbolized with mathematical notation (p. 121). Finally, youth construct and discuss symbolic representations of their ideas, building shared meaning and often redesigning mathematical symbols as part of a collaborative process with peers. Moses and Cobb (2001) narrate a bidirectional relationship between experience and symbolic representation as young people "form abstract conceptualizations out of their reflection, and then...apply the abstraction back on their experience" (pp. 119–120). In this process, children are positioned as "creators" of their own mathematical notation who make decisions about what aspects of experience are of value to represent (p. 122)—a sense of authorship over mathematical discourse that is not always evident in accounts of modeling with mathematics.

Social justice mathematics is another curricular approach focused on the mathematical modeling of community contexts (Gutstein, 2003, 2006; Gutstein & Peterson, 2005). Since this work was discussed in prior sections, I will briefly highlight its connection to the notion of modeling. Gutstein (2016) describes social justice mathematics as an "interweaving" of "reading the world and reading the (mathematical) word" (p. 456). It involves "movement between real-world contexts with mathematics, decontextualized mathematics, and real-world contexts without mathematics" (p. 476). As with other examples in this section, mathematical exploration is grounded in "themes" germane to young people's lives, whether or not youth initially see these as

mathematical. Through classroom-based modeling, they engage with mathematics as both a language for explaining sociopolitical issues and a tool for intervening in them. Unlike the Algebra Project's curriculum, teaching mathematics for social justice tends to frame the language of mathematics as unitary: while young people create narratives and actions in the world with mathematics, they are not necessarily co-constructing the mathematical word itself. Mathematics is treated as an *a priori* set of symbols, tools, and lenses that can explain phenomena in youth's lives.

Co-Design with Families. Researchers guided by participatory design methods have centered family partnerships in an effort to honor the "familial disciplinary knowledge" and repertoires of practice in communities historically marginalized by school mathematics (Ishimaru et al., 2015, p. 5). Ishimaru and colleagues (2015) call for educators and researchers to engage young people's families not simply as "sources of data and knowledge" but also as co-designers whose pedagogical insight might shape school curriculum (p. 4; see also Civil, 2007). According to these authors, co-design can be a learning experience for all involved: while families have opportunities to see their funds of knowledge in dialogue with school mathematics, classroom educators expand their "understandings of mathematics with respect to students' cultural practices" (Ishimaru et al., 2015, p. 12).

Booker and Goldman (2016) reflect on a collaboration among researchers, teachers, and families, many from immigrant communities of color, that focused on "making people's everyday math visible" (p. 224). Through immersive observations in families' everyday spaces and a series of co-designed workshops, the design team worked to repair the "rift" between home and school practices that "render[s] math a school subject more so than a human practice" (Booker & Goldman, 2016, p. 232). Researchers expanded their sense of where and how mathematics happens by accompanying families through their daily rounds—observing and discussing activities such as setting budgets, shopping for a prom dress, attending a baseball game, optimizing bus routes, and keeping inventory and books for a home business. Booker and Goldman (2016) suggest that, as families identified the problems, tools, and methods embedded in their daily routines, they began to "reclaim" their ways of knowing as mathematical. Through workshops on nutrition, cooking, and gardening, some family members also took on leadership roles designing and facilitating learning experiences for other adults. After the project's initial workshop "felt a lot like school"—mirroring the activities and relational structures typical in classrooms—the team worked to decenter teachers as primary facilitators, positioning them as "fellow learners" and heeding parents'

Lemons-Smith (2009), building on funds of knowledge work (Moll & González, 2004), shares about a project called Mathematics Beyond the School Walls, in which she collaborated with a group of elementary teachers to develop classroom tasks based on families' documentation of their children's out-of-school mathematics experiences. Though family members did not co-design the ultimate curriculum, the project turned to them as ethnographers of the mathematics in young people's lives. Families constructed artifacts of children's experiences by either taking photographs or cutting out printed pictures of objects that they believed reflected children's interactions with mathematics in home and community settings. They were invited to narrate each artifact's meaning in relation to children's mathematical learning. Lemons-Smith (2009) and collaborating teachers then developed standards-aligned tasks that drew upon "the mathematical content embedded in the artifacts" (p. 132). Though more tethered to the demands of dominant school mathematics than other examples in this section, the Beyond School Walls project illustrates another approach to partnering with families in the work of replacing mathematics learning.

Challenges of Curricular Integration. Critical literature also discusses challenges involved in efforts to connect in-school mathematics learning with young people's broader life contexts. One is that curricular integration can inadvertently reify the rift educators intend to bridge and uphold the authority of school mathematics. Dominant narratives about school mathematics are so powerful that youth and their families may not consider their home and community knowledge bases as "real" or complex mathematics (Civil, 2007; see also Chapman, 2022; González et al., 2001). Additionally, educators and researchers sometimes view culturally responsive curriculum as "at the expense of mathematics"—because they cannot identify "rich" mathematics in the focal activities or because inquiry ends up foregrounding other school disciplines (Civil, 2007, p. 107). For instance, in a professional development program focused on leveraging young people's "everyday experiences" in classroom mathematics, Taylor (2011) found that teachers made creative connections to children's out-of-school contexts but had difficulty thinking deeply "about the *mathematics* that children used in that context" (p. 279).

Scholarship has highlighted this challenge particularly around social justice mathematics. In a study of secondary mathematics teachers working to "incorporate social justice goals" into their lessons, Bartell (2013) found that participants "seemed to divide these two foci," concentrating the "mathematics goals" at the beginning of their lessons and attending to "social justice goals" at the end (p. 159). Turner and colleagues (2009) discuss a similar pattern among elementary teachers who saw the ideas within social-justice-themed mathematics investigations as "not necessarily connected to mathematics" and therefore in competition with state-mandated content objectives (p. 150). These analyses underscore that efforts to "connect the dynamism outside the classroom to life inside the classroom" (Gutstein, 2016, p. 493) can reify separation between the two if mathematics is treated as a school content area distinct from—to be integrated with, rather than integral to—youth's lived experiences.

Scholarship has also highlighted challenges related to the authenticity of culturally responsive mathematics curricula. Civil (2007) warns that "the pedagogical transformation of mathematical funds of knowledge for classroom implementation" can lead to superficial applications that do not honor the essence of families' experiences and cultural practices (p. 105; see also Nelson-Barber & Estrin, 1995). Multiple authors discuss this pitfall—or, evasion of horizontality's demands—in the context of word problem tasks that "insert" young people's lived experiences "not in their totality, but as a curiosity...a 'starter' to the real mathematics" (Pais, 2011, p. 222; see also Boaler, 1993; Maier, 1991). As Dowling (1996) argues, approaches to curricular integration can serve a "regulating" function as the school curriculum "casts a mythologising gaze onto these

activities" while "imposing its own structure on them" (p. 407).

To address these challenges, researchers have developed tools to assess the authenticity of culturally-responsive mathematics curricula. Taylor (2011) proposes a two-part framework for evaluating the extent to which classroom activities resemble young people's everyday practices (are "personally authentic") and the extent to which mathematics is naturally embedded within those activities (p. 283). Along the second dimension, he distinguishes between applying school mathematics to young people's out-of-school contexts and engaging the mathematics that is organically part of their lives. The framework includes four resulting pedagogical approaches:

- Mathematics in Practice: drawing on the mathematical knowledge and practice embedded in young people's activities;
- Prescribed Authentic Engagement: designing classroom-based "real-world activities" that children would not engage in outside of school but that genuinely require mathematical problem solving;
- Mathematizing Activities: "adding a mathematical lens" to out-of-school contexts; and
- Vicarious Mathematics Engagement: inviting children to observe others engage in "real-life practices" and then use this experience "as a basis for solving mathematical problems" (Taylor, 2011, pp. 282–283).

Taylor (2011) notes that the first of these, "mathematics in practice," is "both highly personally authentic and highly connected mathematically," whereas each of the other approaches tends to achieve one goal without the other (p. 284).

Aguirre and Zavala (2013) developed a tool for analyzing classroom lessons according to eight aspects of "culturally responsive mathematics teaching," including funds of knowledge and social justice connections. The tool's guiding questions include, "How does my lesson help students connect mathematics with relevant/authentic situations in their lives?" and "How does my lesson support students' use of mathematics to understand, critique, and change an important equity or social justice issue in their lives?" (p. 169). Rubric descriptors attune educators to the difference between "intentional, sustained connections" to family activities, student interests, and local community issues, on the one hand, and task contexts that "stereotypically" or superficially link to community funds of knowledge, on the other (Aguirre & Zavala, 2013, pp. 170, 175). Though oriented to young people's lives beyond school, Aguirre and Zavala's (2013) tool and associated professional development sessions stick to fairly conventional forms of classroom activity.

Some scholars question whether it is possible to authentically re-place learning within the confines of a school setting (R. Gutiérrez, 2019; Mutegi, 2011; Pais, 2011). Maier (1991) suggests that, while there "need not be a chasm," there "may be something inherent in schools, in the constraints and demands placed upon them, that will prevent school maths and folk maths from ever being the same" (p. 66). Gutiérrez (2019) questions whether bridging is necessarily desirable, given the tendency for school structures to "subsume" Indigenous ways of knowing and self-determined purposes for learning. Her vision of "living mathematx" does "not separate mathematics from a way of life" and, she contends, may only be able to thrive in "our own protected learning

spaces...that operate outside of school systems" (R. Gutiérrez, 2019, section 2, para. 2; section 3, para. 6). Gutiérrez (2019) clarifies that, while "seek[ing] to create something different of our own," communities historically oppressed by mathematics education should also demand "more humane" experiences for children in schools (section 3, para. 10–12). Her stance resonates with Martin's (2019) call for refusal *in* and *of* oppressive systems: these authors recognize that humanization and opportunity within existing schooling structures, while important, should not stand in for more liberated visions of mathematics learning.

To Further Heed the Call

The literature in this section "count[s] numerous and very different activities as being mathematical," illuminating the diversity of mathematics extant in people's daily lives (Skovsmose, 2012, p. 349). By embracing horizontality, this work seeks to disrupt the hierarchies that stratify people, knowledges, and practices across settings (Warren et al., 2020). Some authors document (and, in certain cases, cultivate) the mathematics embedded in communities of practice outside of schools, in contexts ranging from recreational activities and trade work to home routines and cultural traditions. Others examine the rift between in- and out-of-school mathematics, centering the experiences of communities whose ways of knowing are often demathematised or ignored in schools (Booker & Goldman, 2016; Skovsmose, 2012). This scholarship explores possibilities for bridging the rift as well as the challenges entailed in such work.

Re-placing mathematics learning as people do life requires vigilance and imagination, given the persistence of dominant cultural frames and the assimilative pressures of schooling, which can undermine efforts to critically widen disciplinary learning (Sengupta-Irving & Vossoughi, 2019). As discussed by authors in this section, efforts at curricular inclusion risk either flattening the cultural heterogeneity they seek to embrace or "pushing [school] mathematics into the culture" of historically marginalized communities (R. Gutiérrez, 2019, section 1, para. 27). A common pitfall of school-based attempts at horizontality is that they fail to situate learning in forms of mathematical practice that are naturally embedded in young people's lives (Taylor, 2011). Presenting a written plan for conserving household water use (Turner et al., 2019) or running regression analyses to examine neighborhood gentrification trends (Gutstein, 2016), while connected to issues that youth may find meaningful, do not necessarily reflect the practices involved in young people's daily living. Applying dominant mathematics *to* life activities does not, on its own, constitute a re-placing of mathematics learning.

A related caution is that efforts to "mathematize" everyday cultural activity can take the form of a hierarchically ordered translation process by which dominant mathematics recodes, and possibly overwrites, alternative ways of knowing and communicating mathematically. For instance, while Moses and Cobb (2001) describe the Algebra Project's approach as a bidirectional interpretive process, they also give voice to a linear-sounding purpose: to "bridge the transition from real-life to mathematical language and operations....mov[ing] toward their standard expression in school mathematics" (p. 120). Civil's (2007) question, "Where is the math?" is potentially expansive but can also invite a reductive mapping of mathematical heterogeneity to school mathematics frameworks. In these ways, metaphors such as "bridging," "interweaving," and "aligning" may reinforce the binary they intend to disrupt by denoting certain out-of-school contexts as "nonmathematical" or "without mathematics," to be mathematized by the authoritative "mathematical word" of school (Gutstein, 2016, pp. 476, 479). In other cases, efforts to translate among various forms of mathematics may overlook the inevitably problematic nature of cultural translation—that there is not always a direct mapping of practices and concepts across knowledge systems (Eglash et al., 2006; R. Gutiérrez, 2019). Horizontality demands that we authentically pluralize our notion of mathematics, not simply seek, apply, or translate to a certain version of mathematics in a variety of places.

The analysis in this section raises the question of whether re-placing mathematics learning is the work of school-based educators and curriculum. As Richardson (2011) argues, cultural responsivity can turn into a form of enclosure when it is pre-structured by the normative purposes of schooling, valorizing diverse experiences and epistemologies insofar as they fit within the terms of the official curriculum. Educators and researchers working within school systems are faced with state standards and assessments, college-and-career-readiness discourses, and developmental frameworks that organize learning around vertical progressions and siloed content domains. Further, when a majority of U.S. schoolteachers reflect the dominant culture (i.e., white, European-American, and middle-class), possibilities for culturally expansive curricula and pedagogy are limited, if not inherently problematic, because of the lack of collective diversity in educators' "wells" of cultural knowledge (Murrell, 2007, p. 36; see also Martin, 2007). This is true even when educators bring a critical political analysis and liberatory intentions to their

teaching (Murrell, 2007). Re-placing learning demands that educators, particularly those who hold dominant identities and work within mainstream institutions, recognize the limitations of their positions and work in solidarity with nondominant cultural communities and educational spaces beyond school (Love, 2019).

Literature in this and prior sections discusses attempts to diversify and rehumanize mathematics learning in schools, recognizing while also accommodating institutional constraints to various degrees. As mathematics education research continues to design and document such attempts, it might concurrently explore more liberatory contexts, opening to mathematics learning as "infinitely deeper and broader than school" (Warren et al., 2020, p. 283).

Conclusion

Inspired by Warren and colleagues' (2020) call to critically reimagine disciplinary learning, this paper pursues "insurgent readings of disciplinary discourses" in mathematics education research (p. 287). Critical reimagining elucidates the ways academic disciplines and their school instantiations operate as a form of coloniality, asserting the superiority of EuroWestern ways of knowing and negating or assimilating pluralistic conceptions of knowledge. Further, it invites us to explore possibilities for disciplinary learning that de-link from these assumptions (Mignolo, 2009). Guided by critical reimagining, I analyze scholarship that critiques mathematics education's entanglement in the colonial matrix of power and considers liberatory possibilities for mathematics learning.

The paper is organized around the following calls to researchers and educators:

(1) examine disciplinary education's entanglement in empire, (2) attend to intersections of self- and world-making in disciplinary learning, (3) critique and refuse disciplines' onto-epistemic normativity, (4) explore the multiplicity and dialogicality in disciplines, and (5) re-place disciplinary learning as people "do life" (Warren et al., 2020). I analyze literature that speaks to each call in relation to mathematics education. This scholarship spans contexts and methodologies, including philosophical critiques of the discipline, historiographies of mathematics education, ethnographic accounts of mathematics learning in and out of school settings, studies of mathematics pedagogy and teacher professional learning, design-based research with youth and their families, and personal narratives reflecting on experiences with mathematics.

I have synthesized this literature by pulling on its insurgent threads—identifying the critiques it puts forth, liberatory alternatives it proposes, and ways it might delve deeper into critical reimagining. Authors of the work analyzed engage in critique by:

- tracing the sociohistorical construction of mathematics as a discipline and school subject, underscoring the conquest, exclusion, and erasure integral to this history;
- highlighting the interconnectedness of intrapersonal, micro-interactional, and systemic manifestations of oppression in mathematics education;
- denaturalizing the colonial values that govern the discipline in and out of school;
- disrupting narratives of a unitary, universal, and fixed mathematics; and
- questioning paradigms of diversity and inclusion that focus only on broadening participation in dominant forms of mathematics or that treat heterogeneity in reductive ways.

This scholarship also offers possibilities for reimagining mathematics teaching and learning, by:

- describing how youth resist and reconfigure dominant narratives as they negotiate their political and cultural identities as mathematics learners;
- leveraging dominant mathematics as a tool for critical social inquiry with educators and youth;
- encouraging educators and young people to critically analyze the governing norms and sociopolitical consequences of dominant mathematics;
- illuminating the heterogeneity and dynamism in mathematical knowledges and practices across cultural communities; and
- designing for mathematics learning that engages cultural repertoires typically excluded from schools.

Throughout the paper, I have also named where literature in critical mathematics education stops short of or restricts liberatory possibilities. For instance, many works that critique school policy around mathematics, or that seek to integrate school activity with youth's broader lives, treat the broader discipline as settled rather than similarly open to critique and transformation. Scholarship that recognizes multiplicity in the discipline may not attune to the dialogical, powered relations among diverse forms of mathematics. Relatedly, research and pedagogy that regard mathematics as plural do not always position youth and educators as contributing to that plurality, whether through historicized onto-epistemic inquiry or the creation of new forms of mathematics.

Directions for Future Research

Despite the justice-oriented intentions of individual educators, curricula, and programs, dominant systems are flexible and durable, with wide-ranging capacities to accommodate attempted disruptions (Gholson, 2019; Martin, 2019). In the preceding analysis, I have pressed on the boundaries of critical mathematics education literature, not to diminish the contributions of this scholarship or the complexity of its undertakings. Rather, I write from a sense of urgency to envision "a fundamentally new and different mathematics education," beyond the enclosures of inclusion, integration, and empowerment paradigms that adhere to the expectations of existing systems (Martin, 2019, p. 469; see also Rogers & Kaiser, 1995/2005). Building from this paper's analysis, I propose the following lines of inquiry for future research:

- Which, if any, aspects of mathematics education's entanglement in the colonial matrix of power are specific to the institution of K–12 schooling, and which extend to the professional discipline? How do disruption and transformation within one arena intersect with change in the other?
- How might research methodology more closely integrate micro, meso, and macro scales of analysis to illuminate how entanglement in and liberation from the colonial matrix occur through everyday experiences of mathematics learning (Gholson, 2016; Gholson & Martin, 2019)?
- What are the particular, and possibly cross-cutting, elements of liberatory mathematics for various groups that have been historically oppressed through mathematics (e.g., Black, Indigenous, Latinx, women and nonbinary, queer,

dis/abled, and poor people)? What does liberation entail for youth whose social identities grant them access, recognition, and power in and through mathematics (e.g., people who are white, of European descent, male, heterosexual, able-bodied, and/or with financial wealth)?

- How might mathematics curriculum and pedagogy cultivate epistemic disobedience as a central purpose (Mignolo, 2009)? What types of learning experiences would support educators and youth to question the normativity inherent in mathematics tasks, tools, language, and classroom cultures; interrogate the social history of the discipline; and view mathematics as open to radical expansion through this inquiry?
- How, and in what settings, can researchers, educators and young people explore the many mathematics across the world and in their lives, while resisting forces that threaten to enclose epistemic heterogeneity within the dominant culture's terms (Richardson, 2011)? To what extent is re-placing mathematics learning the work of school-based educators and curriculum, specifically in settings where white, European-American, monolingual educators do not share access to the "wells" of cultural knowledge of their students (Murrell, 1997)?

Finally, the field needs more accounts of practice that explore the above questions through stories of lived experience. What does liberatory mathematics look, sound, and feel like, for educators and for young people? Where does pedagogical design encounter tensions between liberatory propositions and the mess of practice, and how might research honor this complexity without losing sight of liberatory vision (Law, 2004)? There is a need for nuanced, contextualized description of radical alternatives in-themaking, as partial, indefinite, and problematic as these efforts may be.

To de-link from the colonial matrix of power, educational research methodologies must themselves free from the constraints of EuroWestern normativity. It is beyond the scope of this paper to explore what liberated methodologies might entail (for examples focused on mathematics education see, e.g., Donald et al., 2011; Larnell & Martin, 2021; Skovsmose & Borba, 2004; Stavrou & Miller, 2017; Stinson & Bullock, 2015). Here I highlight calls to disrupt disciplinary silos in academic research. As Vakil and Ayers (2019) propose, constructing "newly imagined worlds of STEM where liberatory politics and deep disciplinary learning co-exist and co-develop will require new kinds of interand transdisciplinary methodological design and inquiry" (p. 455). Increasingly, scholars have advocated for work on mathematics teaching, learning, and schooling that transcends disciplinary boundaries (Bullock, 2012; R. Gutiérrez, 2013; Larnell & Martin, 2021; Martin, 2009a). In this paper, I have brought together literature from mathematics education and the learning sciences—fields which are themselves diverse—to expand the methodological terrain and intellectual histories we consider when inquiring into liberatory mathematics learning. Further research could explore the intersections and divergences of these and other epistemic traditions, in an effort to reflect, methodologically, the commitments to heterogeneity and sociohistorical perspectives on knowledge that ground critical studies of mathematics education. I imagine that such work might expand the focus of inquiry so that studies of mathematics learning are approached as inquiries into learning and living generally.

234

Vakil and Ayers's (2019) proposition for "newly imagined worlds" of educational inquiry echoes Smith's (1999/2012) contention that liberatory political projects must "touch on, appeal to, make space for, and release forces that are creative and imaginative" (p. 203). I draw inspiration from educators and researchers who have begun this bold work and, through the present chapter's synthesis of literature, hope to join their movement to release critically imaginative forces in and beyond mathematics education—perhaps "opening towards another thing" (Mignolo, 2011, p. 50).

The next chapter is a study based on conversations with mathematics educators who are reimagining mathematics teaching and learning in various ways and contexts. In addition to inviting recollections from experience, the conversations appealed to educators' imaginations as they articulated their liberatory mathematics pedagogies.

CHAPTER FIVE: "BREAKING MATH FREE": STORIES OF LIBERATORY

PEDAGOGY FROM MATHEMATICS EDUCATORS

In the spirit of prefigurative politics—a set of traditions rooted in social movements that seek to *enact* the type of future they are struggling to bring into being... (Bang & Vossoughi, 2016, p. 178)

How do we produce a vision that enables us to see beyond our immediate ordeals?...To imagine something different, to realize that things need not always be this way. (Kelley, 2002, pp. x, 9)

To think, 'What is the alternative?' can be really freeing. (Kate, mathematics teacher and teacher educator)

Introduction

Educational research has devoted significant attention, empirically and

theoretically, to the ways mathematics education reproduces oppression, from system-

level policies to moment-to-moment interaction in classrooms (e.g., Barajas-López &

Larnell, 2019; Gholson & Martin, 2019; Martin, 2013; Skovsmose & Greer, 2012;

Vossoughi & Vakil, 2018; Yeh & Rubel, 2020). This literature offers important

counternarratives to the myths of mathematics as a politically neutral discipline and

mathematics education as inherently empowering (D'Ambrosio, 1999; Martin, 2003,

2019; Valero, 2008). Yet, if the politicized stories we tell of mathematics education are

concerned primarily with the harms of, or potential for inclusion within, current

conditions, we may lose sight of the "something different" (Kelley, 2002)-alternative

possibilities for mathematics teaching and learning-that the authors above would call us

to consider.

This paper, while informed by critiques of educational injustice, orients toward the alternatives that educators strive to bring into being (Bang & Vossoughi, 2016). It considers liberatory possibilities for mathematics teaching and learning, grounded in the experiences of six mathematics educators who seek justice and liberation through their work. In one-on-one conversations, we discussed prior experiences as teachers and learners of mathematics, across roles, settings, and social identities; visions for mathematics learning that had not yet come to be; and the commitments that inspired their pedagogies. Through thematic analysis of the conversations, I explore the following questions:

- 1. In what ways has mathematics teaching and learning been liberatory in these educators' experiences?
 - a. As they have developed liberatory pedagogies, what freedoms and possibilities have they encountered?
 - b. What tensions and questions have they encountered?
- 2. What are these educators' visions for what liberatory mathematics teaching and learning could be?

This analysis foregrounds multiplicity and multivocality. Educators underscored that what needs liberating is manyfold, including disciplinary content and practice, school curriculum and pedagogy, physical learning environments, social activity and relationships around mathematics, mathematics identities, and broader societal purposes for mathematics. Bringing together the perspectives of a diverse group of educators, I assumed that experiences of oppression vary and that no individual practitioner or learning context can apprehend liberation for all. At the same time, clear themes wove across participants' reflections, suggesting that liberatory mathematics pedagogies hold certain commitments and potentialities in common.

The paper begins by reviewing literature on mathematics education that describes its orientation as "liberatory." I then offer a conceptual grounding for my use of the term, drawing on scholarship from multiple fields. The methods section shares details on the six participants, our conversation process, and my approach to analyzing and writing about the data. I present findings according to three overarching themes: the oppressive systems educators sought freedom from, their visions of liberatory mathematics pedagogy, and the work entailed in moving between these. Following this thematic analysis, I offer a found poem that brings the six educators' words into direct conversation, amplifying the poetic expression and vision in their talk (Kelley, 2002; Shotter, 1996). The paper's discussion ties participants' reflections on mathematics pedagogy to theories of liberation beyond the field mathematics education. It underscores that liberatory praxis is delicate and demanding work, a continual dance between visions of a "not yet" and attempts at freedom in the here and now.

Literature Review

This section reviews mathematics education literature that uses the term "liberatory." I highlight various intellectual traditions, scales of analysis, and degrees of political specificity in this literature. I then turn to scholarship on (educational) liberation more broadly, to offer conceptual framing for the paper's discussion of liberatory pedagogy.

"Liberatory" Perspectives on Mathematics Education

Literature in mathematics education that uses the term "liberatory" explores alternatives to dominant conceptions of mathematics teaching and learning. Some earlier work examines the epistemological underpinnings of learning environments, challenging notions of mathematics as a dispassionate and value-free pursuit of universal truth (Ernest, 1991; Gordon, 1978). Authors seek to liberate perspectives on how knowledge is produced and, specifically, young people's roles in co-creating and verifying mathematical knowledge (Price & Ball, 1998). Much of this work focuses on the experiences of individuals and the intellectual culture in classrooms, with less clarity on the sociopolitical contexts in which teaching and learning are embedded. For example, Gordon (1978) argues that "liberation requires the creation of personal meaning" and space for "emotion, belief, and commitment" in mathematics learning, yet he leaves unspecified the ways "personal aspects of the mathematics experience" are situated within broader social contexts that require liberating (pp. 252, 260). In an analysis of elementary school and pre-service teacher education classrooms, Price and Ball (1998) index "larger issues of race, class, and gender relations of power," though they do not fully explicate connections between those sociohistorical systems and the "culture[s] of sense-making" and "mathematical discourse" they examine at the classroom level (p. 257).

Authors who situate teaching and learning sociopolitically conceptualize mathematics education for liberation across individual, community, and societal scales. Frankenstein and Powell (1994) discuss scholarship in ethnomathematics that "redefine[s] conventional notions of mathematical knowledge," exploring the ways people across diverse communities create mathematics as a "cultural product" (Frankenstein & Powell, 1994, pp. 74, 85). Building from ethnomathematics as an area of anthropological research (Ascher, 1991; D'Ambrosio, 1985), Frankenstein and Powell (1994) raise the question of how "teaching and learning ethnomathematics can play a role in the economic and political action needed to create a liberatory society" (p. 92).

Frankenstein (1983) proposes "critical mathematical literacy" as a type of "knowledge for liberation" that might be developed through formal education. Building on Freire's (1970) ideas about liberatory literacy pedagogy, Frankenstein (1983, 2013) examines classrooms in which learners pose and analyze sociopolitical problems through a mathematical lens. Gutstein (2003, 2006, 2016) builds on Frankenstein's work in his accounts of "teaching mathematics for social justice," narrating how middle and high school youth in his classes use mathematics to investigate issues of injustice in their communities. Gutstein (2006) acknowledges a range of educational traditions that inspire his teaching and "embrace liberation as fundamental" (p. 23). These include Frierean critical pedagogy (Freire, 1970, 1998), culturally relevant pedagogy (Ladson-Billings, 1995), and histories of African American liberatory education (e.g., Anderson, 1988; Du Bois, 1935/2017; Perry, 2003; Woodson, 1933/1990).

Scholarship rooted in the African American liberatory tradition has worked to conceptualize liberatory mathematics education for Black youth specifically (Davis, 2018; Martin, 2009a, 2019; Martin & McGee, 2009; Martin et al., 2019). Authors highlight the multiplicity of their visions while distinguishing them from mainstream

240

notions of educational equity, which center white interests, logics, and experiences (Davis, 2018; Martin, 2009a, 2015, 2019). Davis (2018) argues that liberatory mathematics education would be "responsive to the distinct historical and contemporary needs of the collective Black community," developing knowledge of Black people's contributions to the field of mathematics and young people's skills as "agents of change" (pp. 70, 75). Similarly, Martin and colleagues (2019) envision a Black Liberatory Mathematics Education that "stress[es] collectivity and Black humanity," connecting mathematics learning with historical inquiry into structural racism and possibilities for its transformation (p. 48). In a case study of classroom teachers of Black children, Matthews (2009) examines cultural relevance, in curriculum and teacher-student relationships, as core to liberatory mathematics teaching. Together, these authors emphasize that "there is no prescription" (Martin & McGee, 2009, p. 212) or "generalizable truth" (Matthews, 2009, p. 84) when it comes to conceptualizing liberatory mathematics education, for Black children or more broadly. There are multiple ways to refuse oppressive systems; what is key is that communities who have experienced oppression lead in visioning and practicing the alternatives (Martin, 2009a; Martin et al., 2019).

I conclude this part of the review by acknowledging scholarship on mathematics education by and about Indigenous communities (Barajas-López & Bang, 2018; Glanfield, 2016; R. Gutiérrez, 2017; Stavrou & Miller, 2017; Sterenberg et al., 2010; Trinick et al., 2015). These authors do not necessarily use terminology of "liberation" or "freedom," more often framing their perspectives as "decolonizing," given the particular histories of Indigenous colonization that inform their work. Still, like literature cited earlier, they speak to epistemological and relational expansions that break with dominant approaches to teaching and learning mathematics. Studies examine pedagogies rooted in Indigenous knowledge systems, offering accounts of mathematical exploration through clay work, canoe building, map making, and storytelling (Barajas-López & Bang, 2018; Sterenberg et al., 2010; Trinick et al., 2015). They discuss ways of knowing mathematics guided by principles of reciprocity and multiplicity and grounded in historicized understandings of cultural relevance (R. Gutiérrez, 2017; Stavrou & Miller, 2017).

Across the literature in this section, authors consider curriculum, pedagogy, and epistemologies that might free mathematics teaching and learning from the logics of an oppressive status quo.

Liberatory Education, Broadly

This part of the section acknowledges ideas and traditions that form my understanding of liberatory education, which both developed through and shaped my conversations with educators. It begins with a statement about liberation generally and then elaborates on the notion of education as the practice of freedom (Freire, 1973; hooks, 1994), which figures centrally in the paper's discussion. I turn to authors both within and beyond the field of education. Many write specifically about Black freedom, clarifying that Black liberation would lead to liberation for all people, "a freedom that inaugurates an entirely new human experience for everyone" (Walcott, 2021, p. 5; see also Combahee River Collective, in Taylor, 2017; Hamer, 1971). When drawing connections to Black liberatory pedagogies, I do not presume a direct parallel between the historical contexts in which liberatory Black education has developed and the contexts in which participating educators worked (Anderson, 1988; Givens, 2021a; Johnson et al, 2014; Payne & Strickland, 2008). Rather, I lift inspiration from authors' descriptions of the values, sensibilities, and practical wisdom involved in teaching "within yet against" dominant systems of education (Givens, 2021a, p. 27).

Liberation

As discussed in Chapter Two, three principles anchor my understanding of liberation. Liberation is plural and collective: there are many possible experiences of freedom and these are interdependent—personally and collectively and across communities variously situated (Combahee River Collective, in Taylor, 2017; Hamer, 1971; Walcott, 2021). Additionally, visions of liberation are grounded in politically clear analysis of oppressive systems and cultures while also imagining radically beyond them (Benjamin, 2016; Kelley, 2002). Liberation is thus distinct from notions of equity and justice, which tend to prioritize access, integration, and reparation within existing systems. At the same time, liberation is radically practical, a process of enacting transformative possibilities in the here and now, amidst the real constraints of the present (Boggs, 1977; Freire, 1970; Givens, 2021a; G. Gutiérrez, 1973/1988).

Education as the Practice of Freedom

Educational traditions that embrace these principles—commonly referred to as "emancipatory education," "liberatory education," or "education for liberation" highlight the potential for education to free people from, rather than reproduce, unjust social systems (e.g., Anderson, 1988; Beauboeuf-Lafontant, 2005; Brion-Meisels et al., 2010; Freire, 1970; R. Gutiérrez, 2017; hooks, 1994; Ladson-Billings, 1995; Love, 2019; Payne & Strickland, 2008; Shor & Freire, 1987; Woodson, 1933/2009). Liberatory education can take many forms, from the creation of community-run schools and programs, to curricula that center critical culturally-relevant inquiry, to relations of care and solidarity among teachers and learners. Across these, freedom is both a hoped-for ideal and an experience of putting that ideal into practice—at institutional, curricular, and interpersonal scales—in the here and now (Givens, 2021a). Authors thus refer to "education as the practice of freedom" (Freire, 1973; hooks, 1994).

Education as the practice of freedom entails a prefigurative orientation, "anticipating" through ongoing work the values, social relations, and experiences of a liberated world (Boggs, 1977, p. 103). Bang and Vossoughi (2016) describe the prefigurative nature of participatory design research in education, which "hold[s] space for radical critique and social dreaming...while inciting the enfleshment of these ideas in current practice" (p. 178). In an examination of Black education in the United States from slavery through Jim Crow, Givens illustrates the ways Black people have "lived out and enacted" education as and for freedom, in large- and small-scale, overt and covert ways, while "wad[ing] through" the oppressive realities of U.S. schools and society (Givens, 2021a, p. 13; Givens, 2021b, p. 23). These authors suggest that liberatory praxis demands vigilant, creative, often subversive engagement with existing systems as people dream beyond them.

As Martin and colleagues (2019) propose, we need to continue theorizing liberatory education in mathematics specifically, and to do so "in infinite multiplicities," rooted in the particulars of human experience across diverse contexts (p. 47). This paper, itself multivocal, contributes a small piece to that multiplicity (though not focusing on education for Black children, as Martin and colleagues and Givens do). It shares the perspectives of six educators with politically clear pedagogies and the audacity to imagine and enact, in diverse ways, mathematics teaching and learning being otherwise (Benjamin, 2016; Kelley, 2002). While liberatory pedagogies are not limited to spaces of formal education, these educators' experiences largely speak to the practice of freedom within K–12 schools.

Methods

This section introduces the six educators, including their teaching experience, educational histories, and sources of inspiration for liberatory pedagogy. I then describe the design and process of our conversations and my methods for analyzing data sources. The section closes with a statement on researcher positionality.

Participants

In gathering a group of educators, my goal was not to generalize from their stories. I did not presume to bring together perspectives that would either be representative of a particular population or capture a broad range of political views on mathematics education. Erickson (2020) suggests that qualitative researchers select sites and participants not as a sample of a larger population, but as "a particular place in which certain things of research interest are happening...a 'telling case'" (p. 5). I believe that the six educators in this study have something to "tell" about liberatory mathematics teaching. I took interest in their stories in large part because of the political vision they brought to their work. In diverse ways and contexts, they were all committed to disrupting systems of oppression through mathematics teaching; believed liberation could be practiced on a daily basis; and inquired into their praxis ongoingly, in order to share it with other educators.

I purposefully brought together a diverse group of educators. Participants varied in their work contexts, roles, years of teaching experience, personal histories with mathematics education, and social identities (see Table 2). At the time of the study, participants had been educators for between eight and 43 years, mostly in large cities in the northeastern United States. They had taught learners from elementary grades through high school, and four also worked as teacher educators with in-service or pre-service teachers. They had worked in district public schools and independent schools, universities and teacher residency programs, professional development and out-of-school youth organizations. Individually and collectively, they had engaged with a range of instructional approaches and curricula. All but one attended public schools growing up, and they held degrees from small liberal arts colleges and large universities. All held Masters degrees, two held Ph.D.'s, and three were in the process of completing their doctorates. Two participants self-identified as male and four as cis-women or female. One participant identified as Asian American, one as Asian/South Asian, two as Black/African American, and two as white.

Participant	Age	Racial/Ethnic and Gender Identities (self-identified)	Years of Experience as an Educator	Roles as an Educator
Brent	35	White, male	15	Grades 7–12 mathematics teacher at an independent K–12 girls school; formerly a university-based mathematics education researcher and teacher educator
Cliff	29	Black / African American, male	12	Director of STEM Programs at an out-of- school youth mentoring program; formerly taught middle schoolers and mentored high school youth teachers; doctoral student in Mathematics and Science Education
Kate	49	White, ciswoman	22	Adjunct Assistant Professor at a university; independent consultant with multiple schools and districts; course designer and facilitator at a mathematics and science professional development organization; formerly co- director at a PreK–8 research and professional development center for mathematics education
Lydia	67	Black / African American, female	43	Math and Equity Consultant with an urban public school district and professional and youth development programs; formerly a 1st, 5th, and 6th grade teacher in public and independent schools and a teacher educator in schools and teacher preparation programs
Naya	33	Asian / South Asian, ciswoman	9	School improvement coach for a large urban district; formerly a high school mathematics teacher and Restorative Justice Coordinator
Ту	31	Asian American, ciswoman	9	Adjunct Lecturer in Education at a university; doctoral student in Mathematics and Science Education; formerly a teacher of grades 9–12 mathematics in public schools

Table 2. Descriptive Information About Participants (Study 3).

Note. All names are pseudonyms, with the exception of Cliff, who chose to use his actual nickname.

Participants traced their notions of liberatory mathematics education to a variety of intellectual traditions and personal experiences. They referenced scholarship in and beyond mathematics education, on histories of systemic racism and Black freedom struggle, women's studies, ethnic studies, critical pedagogy, abolitionist teaching, ethnomathematics, and rehumanizing mathematics (Alexander, 2010; Delpit, 1988; Freire, 1970; R. Gutiérrez, 2018; hooks, 1994; Joseph, 2011; Love, 2019; Moses & Cobb, 2001; Muhammad, 2020; Rogers & Kaiser, 1995/2005; Yeh et al., 2021). Participants also spoke about individuals and communities in their lives that had shaped their understandings of liberatory education, including family members, mentors, educational and professional communities, and public intellectuals on social media.

Educators indicated that meanings of "liberatory" were still developing for them, especially with respect to reflecting on their own practice. For some, the term was a "new way of naming" what they had been practicing: one participant reflected, "I haven't thought of this term 'liberatory' as a lens for my classroom, but I guess the stuff I did could apply toward it." Another shared that it had recently "emerged in my life as some kind of purpose" to explore intersections of mathematics and her longer-term work around anti-racist and restorative justice pedagogies. Still another "ha[d] been trying for years" to explore how classroom mathematics might be freeing and was still working to "figure out how to get to the 'liberatory stage'" (referencing McIntosh, 1983, in Rogers & Kaiser, 1995/2005).

Conversation Process

I met with educators individually for between 50–100 minutes, with the average conversation lasting just over an hour. In most cases, we met only once, though one conversation needed to be split into two parts, due to an emergency interruption during the first meeting. In another case, we met for a 45-minute follow-up so that I could ask a participant to elaborate on two facets of her work that came up in our first conversation. I also emailed a follow-up clarifying question to two other participants. All conversations took place over Zoom and were recorded through the video conferencing platform.

To facilitate the conversations, I used a semi-structured protocol which included an opening framing on my view of interviews as forms of mutual learning; prompts for participants to recollect past experiences with mathematics, as both learners and teachers; prompts to reflect on the opportunities and challenges of liberatory praxis; an invitation to imagine mathematics learning free from common institutional constraints of K–12 schools; and a question about the texts, authors, and experiences that shaped participants' perspectives on liberatory mathematics teaching. The order of questions varied based on the flow of each conversation. For instance, in the first few meetings, I noticed a marked shift in participants' talk when they responded to the prompt to imagine. The pace and tone of participants' voices changed; the flow of their responses sounded more exploratory; and they described people, settings, and activities that had not come up in their previous responses. In subsequent conversations, I used this question to intentionally create openings, at times when I sensed participants' responses were dwelling in limitations and challenges of their current situations.

Since a central purpose of these conversations was to explore participants' meanings of liberatory mathematics teaching (Emerson et al., 1995; Erickson, 1979), I did not offer my own definitions at the outset, other than to say that I saw teaching and learning as fundamentally entwined and used "liberation" and "freedom" interchangeably. To surface educators' meanings of freedom through the details of personal story (Seidman, 2013), I began conversations with the prompt to "recall a time when you engaged in or witnessed math teaching and learning and felt something you might call 'free." If participants asked about the word "free," I encouraged them to consider any ideas that resonated for them, but we did not discuss terminology or conceptual frameworks initially. I later shared that I considered "liberation" and "freedom" more expansive than terms commonly used in discourse about mathematics education (e.g., social justice, equity-oriented, humanizing) but that participants could speak to these alternatives as well. Sometimes we had extended discussion about the uses and limitations of various terms.

Mutual Learning and Benefit

I stated upfront to educators that I hoped our conversations would be an experience of joint inquiry in which we explored ideas together without needing to land at coherent or final conclusions. I shared my own thoughts, recollections, and uncertainties and encouraged participants to raise questions. At some points, I followed up on participants' responses in ways that seemed to lead them to new reflections. I also named how their ideas were shaping my own. While these conversations were mutual in many respects, participants and I assumed distinct roles: I asked most of the questions, shepherding us through the conversation, and participants' experiences and reflections were at the center of our talk.

During and after our meetings, educators expressed that the interview process had been generative for them, emotionally and intellectually. They described the conversations as "fun," "exciting," "deep," and "healing." One referred to the process as "an experience, not just an interview," and another shared, "It's kind of like a gift to get to talk about this." Participants also signaled when our discussion was developing their thinking, commenting, "I want to keep thinking about this," "I'm thinking about that thing you posed in the beginning...," or "I hadn't thought of that before....I will have to come back to that." Such reflections challenge traditional notions of interviews: rather than a process of information extraction or sharing of already-formed perspectives, the interview "becomes a potentially meaningful learning experience for those who are having the conversation" (Vossoughi & Zavala, 2020, p. 4).

Our mutual exchange extended beyond the initial meeting. Educators and I emailed academic articles and pieces of our own work that we had mentioned during our conversations. In one case I shared an assignment I had created for an undergraduate course on mathematics teaching methods, to follow up on a participant's comment about this type of task. Another educator said, "I'm just excited at the possibilities of where this conversation can go," expressing that she had been seeking intellectual companionship around questions of liberatory mathematics teaching and looked forward to finding ways to collaborate. As I reflected in a memo early in the process, "These conversations are themselves forces going out into the world, which might ripple into the thoughts and work we do after our 'interview' is over."

Data Analysis

To analyze the conversations, I used a variation of grounded thematic coding inspired by the reflexive "top down and bottom up" approach described by Erickson (2004). Whereas conventional grounded theory attempts "to identify parts first and then work up analytically from there," Erickson (2004) suggests a process of "pars[ing] analytically from whole to part and then down again and again, successively identifying subsequent next levels and their constituents" (p. 491). This involved six stages of analysis. First, I transcribed all conversation recordings, completing two from scratch and repairing the others from rough transcripts generated through the Zoom platform. While creating transcripts, I highlighted any passages that stood out to me, to track for later analysis. I kept short memos, jotting key phrases and ideas from the passages of interest as well as patterns and variations across transcripts if they occurred to me.

Second, I read through the full transcripts and wrote "observer's fieldnotes" about what I heard "going on" in each, including within my own contributions (Erickson, 2004, p. 490). I modified Erickson's suggestion to write these in narrative form, instead combining descriptive sentences, brief jottings or headings, and direct quotations from the transcripts. Notes for all six transcripts were kept in the same document, as separate entries. Third, I read through the document of descriptive notes, adding brief margin comments that responded to Erickson's (2004) prompt: "What are the different kinds of things going on in the…talk here[?]" (p. 491). My purpose was to identify "locally relevant distinctions" and "qualities" in the understandings participants were sharing (Erickson, 2020, p. 4). At this stage of analysis, Erickson (2004) recommends looking for "shifts in activity" and "lines of contrast" (p. 491). I found it helpful to consider pattern and variation (Rogoff, 2003), identifying themes in the talk and then multiplicity within particular themes.

In a fourth phase, I synthesized margin comments from all six transcripts in a separate analytic memo. I grouped and parsed themes, generating what, in some analytic approaches, would be called a system of codes. Fifth, I returned to the full transcripts, bringing the collection of themes to each conversational turn and coding excerpts,

252

between one and several sentences in length, in NVivo. During this process, I added, consolidated, and parsed some themes for better "fit" with the transcripts (Erickson, 2004). After going through all transcripts, I did another round of consolidating and parsing. I wrote analytic memos throughout the stages just described, including on the refined set of themes.

With this analytic process, I tried to complicate approaches to qualitative coding that Viruru and Rios (2021) characterize as "colonial artistry" (p. 1152; see also Tuck & Yang, 2014). By such methods, researchers seek to make participants' experiences and perspectives "knowable" by "breaking down data into intelligible-sized pieces" and then managing those pieces through a process of "thematic extraction" (Viruru & Rios, 2021, pp. 1146, 1150). While my process did involve breaking down participants' responses, Erickson's (2004) suggestion to work from "whole to part" required a sustained, descriptive engagement with the full transcripts before pulling out themes. This felt like a more holistic way to relate with participants' words than in conventional approaches to open coding. Still, I ultimately parsed, extracted, and reconstituted parts of their talk to write this report (Viruru & Rios, 2021).

Writing

In writing up the analysis, I constructed a composite description of liberatory mathematics pedagogy, weaving together educators' reflections from across the six conversations. Rather than approaching the report as a comparative case study—which might present a fuller story of each participant's teaching in order to draw out similarities and differences across individual practitioners—I synthesized the ideas from this series of conversations into a more integrative, though still multifaceted, narrative.

The first three parts of the findings section are organized thematically and written as qualitative analytic prose. The last part presents a found poem that I constructed from lines of talk, both participants' and my own, across the conversations. The poem's sections loosely map onto a set of thematic categories that I developed during transcript analysis, though through the crafting process themes blended and transcript excerpts drifted across themes. My first draft of the poem included longer blocks of text from each participant and a more explicitly thematized structure. I then experimented with a sparser and freer style, interweaving multiple speakers' language more thoroughly while working to preserve each person's meaning.

This part of the findings section pursues what Shotter (1996) might call "poetic methods," an approach to research inquiry that releases from analysis "about" participants and their words, moving instead to "writing and talking 'with' them" (p. 38). Such methods attune to the "poetic forms of talk" in live communication and records of interaction, seeing these "as providing, not information, but different possible relational openings," both for the ways researchers and participants interact with one another and in the content that they communicate about (Shotter, 1996, p. 38). Writing this section of the paper required poetic attunement as I engaged with data sources. I listened for the poetic language in each person's talk and worked with words on the page to amplify and create from that poetry.

Listening for the beauty in people's "ordinary ways of uniquely expressing themselves" is an orientation to social interaction and meaning making that, "instead of

254

helping us 'find' or 'discover' something already existing"—as positivist approaches to educational research and practice often do—might "help us grasp something new, as yet unseen" (Shotter, 1996, pp. 38, 46). This sensibility is not restricted to contexts with a designated focus on language arts: any space in which people express themselves and search for meaning can be alive with the poetic, including mathematics classrooms and professional conversations about mathematics teaching.

Researcher Positionality

I conclude this section with comments on researcher positionality, building from the broader positionality statement in Chapter One. The previous statement situated my work as a researcher and educator sociopolitically. Here I return to the educational and professional experiences that formed my interest in liberatory mathematics pedagogy. I also comment on my relational histories with the six participants in this paper.

My experiences as a learner and teacher bring me to this inquiry. The elementary school I attended as a child communicated a clear political vision—of learning as a way to bring people together, from diverse walks of life, to dream about, inquire into, and in small ways practice social change. My interest in liberatory mathematics specifically stems from one classroom at the school. Mathematics learning in this community reflected what are now some of my deepest values: intellectual inquiry as a process of coming to know and care for one another, heterogeneity and collaboration as core to intellectual creativity, and interdependence as fundamental to flourishing. I have carried this orientation toward mathematics learning into my work as a teacher and educational researcher. Unlike in the humanities, however, I had fewer occasions as a classroom

teacher to articulate or collaborate with colleagues around a political vision for mathematics pedagogy. I approached this paper's inquiry as an opportunity to commune with educators (G. V. Larnell, personal communication, September 20, 2021) who are inspired by similar questions, exploring with them how to give voice to the liberatory values underlying our mathematics teaching.

I knew all participants, to varying degrees, before asking them to participate in this study. Kate facilitated a professional learning community that I joined in my early years as a classroom teacher, and I later shadowed a week-long professional development workshop that she led. Lydia collaborated with research faculty at the university where I am a doctoral student, and we were in a few research project meetings together. Naya facilitated a webinar on restorative justice and mathematics teaching that I attended the summer before designing this study. I first heard Brent share about his teaching on a conference panel, and we later connected for a phone conversation about our shared interests. As fellow doctoral students, Cliff, Ty, and I engage in ongoing conversation about the politics of mathematics teaching, learning, and research.

My prior relationships with participants formed an important foundation for this paper's inquiry. The contexts of our shared work, trajectories of previous conversations, and ways of relating we had established with one another set us up for the conversations I analyze below. Whether developed over years or one or two recent encounters, these relationships brought a sense of mutual respect, trust, and curiosity to our conversations from the start. While listening to recordings and analyzing transcripts, I noticed a depth of inquiry and a willingness to think boldly together that might have been hard to achieve in a one-time conversation with someone I did not know.

My relational histories with participants also reveal broader commitments at the core of my work as an educator and educational researcher. Inspired in part by professional development facilitated by Kate and her colleagues, I approach mathematics teaching as co-inquiry with children and believe educators need professional learning experiences that mirror this view of learning. I first encountered Naya and Brent at professional events focused, respectively, on restorative justice and equitable design in classroom mathematics. Lydia and I met through a research project investigating race, culture, and power in disciplinary learning. Cliff, Ty, and I have chosen to connect outside of the required work for our doctoral program because we recognized our shared interests in the politics of mathematics education. This is not to say that participants and I were uniform in our views, but we shared certain lenses, orientations, and questions that allowed us to explore deeply together.

Findings

The following analysis builds from the stories and visions educators shared as we discussed liberatory mathematics teaching. I first address the systems educators sought freedom from, including normative approaches to school mathematics and the broader sociohistorical systems in which these are embedded. The section begins here in order to underscore the work it took to imagine and practice beyond those systems. The remainder of the section focuses on liberatory possibilities. I share themes from participants' reflections on their experience as learners and teachers, and their dreams of what could be, in and beyond school classrooms. The section concludes with a nod to the broader

reimagining of mathematics that underpinned educators' reflections. This part takes the form of a found poem that I constructed with participants' words after listening and analyzing across the six conversations.

Freedom From

As they discussed liberatory mathematics teaching and learning, educators referenced dominant systems, practices, and norms of mathematics education that they sought freedom from. Without centering these or assuming they were inevitable, educators were honest about the pressures and barriers they encountered as they strove to do insurgent work.

Entangled in Sociohistorical Systems

All educators situated their reflections sociopolitically, naming the broader societal systems in which their practice, and schooling generally, were embedded. They spoke to multiple systems of oppression tied to colonialism: imperialism, patriarchy, extractive capitalism, white supremacy and intersecting forms of racism.

Naya referred repeatedly to the triad of "colonialism-imperialism-capitalism" and "how much damage that's done to our understanding of math." She discussed the erasure of cultural traditions, knowledge systems, and identities through histories of schooling that valorize and impose a narrow, EuroWestern model of mathematics and who can practice it. She and Kate emphasized testing and accountability systems as mechanisms of "capitalist schooling" today.

Cliff drew linkages between education and systems that have historically oppressed Black people in the United States, including enslavement, related systems of labor exploitation, and mass incarceration. Lydia spoke to racism at the scale of classroom interaction and participation structures, which "do harm" to Black and brown children as they negotiate identity and belonging within mathematics learning environments.

Brent, who worked at an all-girls school, discussed sexism in terms of inequitable access and achievement (e.g., underrepresentation of girls and women at prestigious mathematics competitions) as well as gendered disciplinary norms (e.g., competition and individualism). Both Ty and Kate drew upon intersectional feminist perspectives as a lens for understanding injustice in mathematics education.

Describing Normative Mathematics Teaching

Informed by this sociohistorical perspective, participants described the ways structural oppressions shaped their experiences of mathematics teaching day to day. The pressures of accountability systems was a prominent theme, especially for educators who worked in or with public schools. Kate, who traveled across classrooms and districts in her role as a consultant, discussed "living within a testing paradigm," in which the physical set-up of learning environments "resembled testing factories" and teachers hesitated to break from prescribed curricula out of fear that their instruction, and student learning, would not "count" according to the expectations of standardized assessments. She and Lydia spoke to the rampant hierarchy discourse in schools (Louie, 2019), which labels children and their mathematical ability using language of "top"/"bottom," "high"/"low," and "advanced"/"remedial" and narrows the kinds of data schools consider to understand how students are engaging with mathematics. Educators portrayed classroom activity that was "beholden" to this testing paradigm as lifeless—as Naya described, "so boring and rudimentary that it has no connection to [young people's] real lives...it literally is just steps on a page."

Participants spoke to the stratification and narrowing of mathematics content, as reinforced by testing systems but also linked to broader structures of schooling and disciplines. All educators who had taught high school expressed frustration with rigid curriculum sequences that partitioned mathematics into siloed content strands (e.g., algebra, geometry, calculus, and statistics) and organized instruction around the need to cover prerequisite knowledge for subsequent courses. Across grade bands, they discussed pressures to drive toward a predetermined "end result" (Brent), or "hit your target" (Kate) as defined by standardized curricula, which, according to Brent, "limits severely the topics that you're able to broach meaningfully." Naya also brought up the siloing of school disciplines from each other:

It's part of the Western schooling system...[that] you learn English separate from history, separate from math, separate from art....you can be a mathematician in [one] period, and then you can be a historian in the next period...We perpetuate the idea that these things are separate.

According to educators, separation could also be a quality of social relations in mathematics classrooms. Kate described the traditional set-up of classrooms, designed as "a box" with "all the desks in little rows facing the teacher, facing the material," which can prevent learners from "acknowledging other students in the room." Cliff also discussed how a front-facing physical arrangement forecloses possibilities for collaboration and cross-pollination of ideas. Others alluded to norms of "working alone, under the pressure of the clock" (Brent) and the competition and "public shaming" (Lydia) that individually-structured, timed activities often encourage. Lydia shared about her experience as a learner in college-level math courses whose fast-paced, board-facing culture made her feel excluded, as though:

I'm the only one sitting here who doesn't know this stuff, the only one sitting here who has all these questions, and everybody else—I mean, the professor would just write on the board, all this stuff, write it so quickly. There was no way I was making sense of it.

In the context of her work with elementary teachers, she noted that even classroom discussions, which may appear more collaborative, can be organized around participation structures that center individual performance and competition, discouraging children from engaging deeply with one another.

Sentiments Around Normative Mathematics Teaching

Educators expressed feeling uninspired, discouraged, and ethically challenged by the kind of mathematics teaching described above. Kate reflected that classrooms caught in an "endless cycle" of producing and evaluating "felt oppressive" and "dreadful" to her, not "giv[ing] anybody any life or any motivation" and not "at all creative." Brent also used the word "oppressive" to refer to instructional activity in which "the problems come pre-made, and then we solve them, and then we move on to the next questions. The entire design is so antithetical to...my beliefs about mathematics." He described the broader "algebra-geometry-algebra sandwich" of high school course sequences as not "engaging" or "fun." Lydia expressed ethical unease about the ways she had participated in instructional practices that "do harm to kids." She recalled that she "didn't feel good about what was happening" when she grouped students based on testing data or tightly prescribed classroom learning activity. Approaching the work of teaching in this way was to be a "person that I just absolutely can't stand."

Educators reflected on negative experiences of mathematics from positions as learners as well. Building on the comment excerpted earlier, Lydia spoke to the exclusion she felt transitioning to college mathematics, after prior experiences identifying positively with school mathematics: "I thought I loved math, I loved math, it was for me, I could do it—until I got to college and the messages were real clear: 'It's not for you, you can't do it." Naya said she "felt horrible at math" and uninspired throughout her mathematics education: "I have very limited memories of math class from my K-12education....I don't even remember my teachers' faces or names. So that is telling of how I felt in math class, probably: not very inspired." Naya also shared stories of family members who internalized oppression through school mathematics. She described an art practice of her mother's, which, though "very mathematical," her mother abandoned upon immigrating to the United States, believing that "her craft wasn't [the] math" that "was gonna make money." Wrapped into this story was the gendered hierarchy of intelligence in her mother's family, based on whether children had attended British colonial or local Indian schools. Naya also mentioned the emotional traumas of family members who had been "forced" to study towards a career in mathematics or were pathologized for not "achieving in math classes," which she attributed to the model

minority myth. Lydia's and Naya's identities as women of color, and Naya's explicit mention of legacies of colonization, amplify the structural dimensions of harm in their stories.

Summary

To open this section, I illustrated how educators situated their visions of liberatory mathematics pedagogy within a critical sociopolitical analysis of teaching and learning in schools. While educators were clear about the systems that they sought freedom from, our conversations did not dwell there. The next sections focus on what educators worked to free *toward* through their mathematics teaching.

Freedom Toward: Describing Liberatory Mathematics Pedagogy

This section pulls threads across educators' reflections on liberatory mathematics teaching and learning (see Figure 2 for a summary of themes). In some cases, educators offered "glimpses" of liberatory possibilities (Benjamin, 2016; Walcott, 2021)—an imagined scenario or a "fleeting moment" from their experience (Kate). At other points, they shared about something more core to their practice, a "liberatory pocket" that reliably offered space for practicing freedom while "living within the structures" (Naya). I present the first part of the section as a "break" (Walcott, 2021), shifting attention from the barriers educators confronted as they worked within an oppressive status quo, toward what they saw as possible. Then the next part addresses how educators oriented to the work of freeing, including the limitations and tensions involved in enacting their liberatory visions.

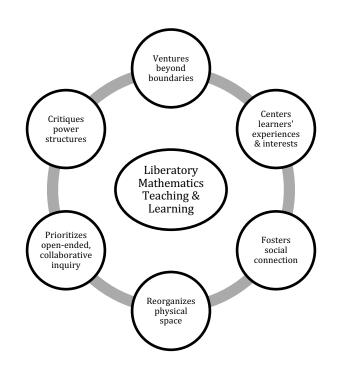


Figure 2. Themes of Liberatory Mathematics Pedagogy.

Beyond Boundaries

Educators described mathematics learning that transcended expected boundaries related to physical space, forms of activity, and communities of engagement. All participants described at least one learning environment outside of "math class," even as some remained within school walls. Ty reflected that the "biggest importance" across her examples of freeing experiences was that "they all didn't happen in math class; they were structures provided ancillary to traditional school." These involved after-school math teams, studying mathematics at home with her father, and out-of-school programs that "brin[g]...young people to the outside world" to "explore." Brent shared about a summer research group in college as well as two activities from his high school classes: a "game" that explores calculus power series by walking city streets and a problem-posing activity

264

in which students create memes of mathematics problems to share on social media. Imagining mathematics learning free from constraints, Lydia commented that "the brickand-mortar structure of school itself" factored into her consideration: "so much of it wouldn't be happening inside the classroom."

The most radical departures (Martin et al., 2019) from the classroom were stories Cliff and Naya told about educational contexts they had witnessed or heard about from others. Naya described a boarding school in the mountains of Ladakh, India (which she had visited) that is designed for young people who have failed state exams. Most activity at the school takes place outdoors, and "all of the learning that they do is intrinsically part of their lives." Mathematics is embedded in shared projects like building houses that will withstand local weather patterns, maintaining an irrigation system, and gardening. Cliff shared a vision of a public space for mathematical inquiry, passed down by Bob Moses, founding leader of the Algebra Project²⁰ and an important mentor of his. Cliff described this space as similar in ethos to a public park, with a physical layout like the Apple Store:

You walk in, you got tables everywhere, it's open space....Every day [that] you come in, there's a problem. And everybody, anybody from the community comes in: you know, the people—think about the guys who play chess in the park, they

²⁰ The Algebra Project is a national network of mathematics educators, administrators, and community members who seek to develop young people's mathematical literacy toward the goal of full participation in a highly technical, global society. Established by civil rights activist and mathematics educator Bob Moses, the Algebra Project draws on the organizing tradition of Ella J. Baker and the Mississippi Theater of the U.S. Civil Rights Movement in its approach to mathematics pedagogy and educational advocacy (The Algebra Project, Inc., n.d.; Moses & Cobb, 2001). Lydia was also connected with the Algebra Project, beginning in its early days as a local curriculum development project; like Cliff, she mentioned Bob Moses's influence on her as an educator (discussed later in the paper).

would come to this place to try to answer the question. And maybe you have this problem that has different levels, maybe there's some for kids, some for little kids, big kids, adults....But every day there's a problem. You come in, you pick up the paper, and then you sit somewhere, you collaborate with somebody, you bring them in, and you talk about what the problem's asking. And this is a place to jam, to hang out. Maybe there's some coffee, and it's literally a space to dialogue about, "Let's make sense of what this problem is asking about."...People make it in their routine to come to this spot to see what the problem of the day is. And they just sit down for a cool 20 minutes to an hour, or whatever, and they go about their day.

While Naya told of a formally organized educational setting and Cliff a public recreational space, both described social and spatial contexts that gave rise to mathematical activity not typical of school mathematics classrooms. Neither spoke specifically to the mathematics content with which people engaged in these spaces, but they suggested it was organically developed, free-flowing, embodied, place-based, and collective in nature.

Another commonality between the stories from Naya and Cliff, and echoed by other participants, was that mathematics connected learners across age groups and communities. At the Ladakhi school, young people work alongside adults and also travel regularly between the school and local villages to teach others what they have learned to do. Cliff described problem invitations "for little kids, big kids, adults" that would bring together people from different walks of life. In some respects, this multi-age context mirrored the Young People's Project, the youth mentoring program that Cliff had been involved with as a learner, youth teacher, and organizational leader²¹ (which I return to later in the paper). Ty shared that her favorite part of after-school math teams was that "all of the different [grade] levels were in one room practicing together....standing at whiteboards, solving problems together...learning from older students." Brent described his summer research group as creating knowledge that "gets pushed out to wider mathematics communities" and his classroom meme-creating activity "engag[ing] people, even outside of our own school community, with mathematics."

In some conversations, educators alluded to mathematics learning that crossed the boundaries of the traditional school discipline, though this idea was less elaborated in their responses. Kate associated pedagogical freedom with "moments within math class where I see people being wildly creative and thinking well outside of mathematics to engage learners." Lydia similarly reflected that liberating mathematics conversations with children "made me feel like I felt when we would talk about other things that weren't necessarily called math." Brent, Ty, and Naya all mentioned crafts that are inherently mathematical but rarely practiced as mathematical activity in schools (e.g., sewing, knitting, sculpture, kite design, and Kolam). These comments suggested that, in addition to the where and who of mathematics learning, liberatory pedagogy transgressed boundaries around *what* counts as mathematical.

²¹ The Young People's Project, which grew out of the Algebra Project, trains and employs highschool-aged youth to teach mathematics to young people in elementary and middle school.

Centering Learners

Educators discussed how they centered learners' life experiences, interests, and questions in their teaching—while facilitating mathematics conversations and in curriculum design across a unit or school year. Kate and Lydia, reflecting on their work with children, emphasized making space for children's questions and ideas to lead the trajectory of class discussions. Kate shared, "I see a space where the learners' questions are really vital and the teacher's role is to highlight them, connect them, elevate them, and help us to pursue them, to follow them." She described students' ideas as, "almost like this physical thing that we pass back and forth, and I sometimes hold it for kids and highlight something important inside of it, but ultimately it's not really mine."

Similarly, Lydia underscored that when she honors that "it's their conversation that matters, not what I have to tell them or my interpretation," she is better able to see all that children bring to discussions. She gave an example from her sixth-grade classroom:

I just remember...a conversation with kids about equivalence, and it's hard to explain, like this kind of floating feeling, of looking at it from above, like watching what was happening and realizing that they had a lot to say about what they considered to be equivalent. And mind you, it was in really familiar contexts; they had come up with the contexts themselves for their, they were called makedo stories²². And so, kids who weren't participating when we were sitting around

268

²² "Make-do stories" are part of the initial stage of a curricular process that Lydia co-developed with leaders of the Algebra Project. The first step of the process, referred to as the Physical Event, includes storytelling about life experiences related to a particular concept. In this case, children shared experiences related to the concept of equivalence. According to Lydia, "Everybody had some example of a time when they did not have something they needed, but used something else instead....Their

the table doing fractions, had a lot to say about equivalence in their make-do stories....I can remember just thinking that there's a lot that they are thinking

about, and that they have experience with, that they're bringing to the table. In this case, the group's inquiry was grounded in stories from children's lived experiences, which inspired them to "have a lot to say" about the mathematics in question. Importantly, the mathematics of equivalence did not precede these stories as an abstract concept with fixed, coherent meaning that the class was simply working to illustrate or prove. Children's lived examples gave rise to the meaning of equivalence; their experience, and how they narrated it, was itself mathematical content.

Educators also shaped their longer-term instructional design based on students' experiences and interests. Kate shared about a practice called empathy interviews, which she adapted from work she had done around design thinking. These interviews supported both Kate and the teachers she collaborated with to come to know young people "more broadly," to build a sense of how their classroom-based experiences "fit into the larger part of who they are." In an empathy interview:

I'm trying to understand this person's experience in the world, but also in this space that we're gonna share together. So I ask a lot of questions about life, and how you get to school, and your family's life, and the transition between a school building and a home place....I ask a lot about the relationships and all the feelings that are attached, and I try to elicit little tiny stories that I think are symbolic of

understanding of equivalence was grounded in [the notion of], 'not the exact thing but could still get the job done'" (Lydia, personal communication, March 4, 2022).

this person's experience.

She emphasized that, even when part of mathematics-focused instructional design, empathy interviews:

Don't just ask, "What's it like for you in math class?" For me, that's way too narrow....You want to get a sense of who the child is in the world....what stories do they want to tell you. And build from wherever they are.

Kate invited teachers to consider young people who they were "curious about," did not know as well, or might be "making assumptions about." She clarified that building from young people's stories was not "a gimmick" that invited students to share a part of themselves only to "pivot to this really artificial task that has nothing to do with that question, and nothing to do with your lives." As Lydia put it, "kids could see through that," when it "wasn't really an authentic part of them that was coming to the table." Instead, Kate saw empathy interviewing as an "orientation [that] shifts us to kids," offering an alternative to models of curriculum design that encourage teachers to "know exactly where they're going well before they've met the kids." While Kate did not speak to how empathy interviews inform mathematics experiences specifically, the practice assumes that disciplinary learning should take shape around the particular lives, desires, and questions of the individuals engaging in it.

Several educators shared examples of efforts to center young people's interests in mathematics curriculum. Cliff recalled a task he designed around the computer coding program Scratch, for an afterschool class he co-taught as part of the Young People's Project. After observing a computer science class in which many young people chose to play the online game Go Math rather than engaging with their teacher's planned activity, Cliff and a colleague devised a coding activity inspired by Go Math. After facilitating the activity with the class, Cliff reflected:

That was freeing, I think, for the students but also for myself and my peers, to see that, with just subtle changes like that—and caring about what students bring into the class, caring what students want to do, want to talk about, want to engage with—can free up, one, their learning experiences, but also the relationships that they have with "teachers."

As a teacher in this instance, Cliff honored young people's responses to instruction as "feedback" to "build with." He regarded their desires and interests as pedagogical resources that, when tapped, could "free up" learning for all involved. Grounded in the community organizing philosophy of the Young People's Project, Cliff's pedagogy prioritized "learning about things that young people need" and using "the perspectives of what's happened on the ground to make this space a place that serves, and that is a platform for, young people."

There were several other examples of educators developing curriculum to reflect what students "want to engage with." In one activity, Brent leveraged social media platforms as sites for engaging around mathematics. He invited students to create "math memes" based on mathematics problems they had worked on together and to track the conversation these memes sparked within their online social networks. Offering an example of a student-created meme that had received nearly 2000 reactions on Facebook, with people from different places "discussing how they've coded up solutions or proving solutions using modular arithmetic," Brent reflected:

All these people getting involved and connecting around this math question—and it's like a silly meme! I think giving students the opportunity to create memes— I've been thinking about, how can students not be just consumers of culture, but also creators of it? For me, this has been a hope and example of something liberatory. Not just writing a proof, not just writing a problem set write-up, not just "here's my answer, here's the justification," but, how do I turn this into something that is gonna engage people?...When they see that they can make things that are interesting, I think they enjoy it.

In another example, Brent's classes explored "social justice math trails," generating mathematical questions related to societal issues they cared about (e.g., food waste or the gender pay gap) and pursuing answers by analyzing relevant data they found through online research²³. Naya told of curriculum units with a similar purpose, in which "there's a problem that's emerging amongst the community that [students] are in"—she offered the example of paying college loans—"and you're using the math to help make sense of that problem....you're allowing the students to come up with their own opinion using the

²³ Social justice math trails were inspired by Toliver's (1993) Math Trail activity, in which students research the history of their neighborhood and design a course for their peers to walk through the community, with stops along the way to visit sites of interest and ask mathematical questions about aspects of that place. The class creates a book compiling the written mathematics problems and photographs or illustrations to document the journey. Shoaf, Pollak, and Schneider (2004) have also developed this idea: "A mathematics trail is a walk to discover mathematics. A math trail can be almost anywhere—a neighborhood, a business district or shopping mall, a park, a zoo, a library, even a government building. The math trail map or guide points to places where walkers formulate, discuss, and solve interesting mathematical problems" (p. 6). Brent's adaptation of math trails involved problem posing beyond a shared physical environment (i.e., exploring mathematics problems through online research).

math." In the case of math memes, the platform for communicating about mathematical problem solving was grounded in youth culture. In the latter two examples, the mathematics problems themselves developed from generative themes in young people's lives (Gutstein, 2016), and mathematical reasoning involved forming and expressing opinions about the sociopolitical world.

Open-Ended Collaborative Inquiry

Educators described mathematical learning that was open-ended and collaborative, an experience of joint inquiry whose end goal was not finite or predetermined. Brent reflected on what felt "freeing" about summer undergraduate research in which he, a professor, and a small group of peers investigated a conjecture in non-Archimedean dynamics. He underscored that members of the group did not know where their inquiry would lead:

When we asked him [professor] for guidelines around, "What are we supposed to show here, or what are we trying to prove?" he said, "This is unexplored territory."...He was not sure what we were actually trying to show....I think there was something freeing about not having, already in mind, the goal....I was like, "Okay, we're just gonna think about this and then write down ideas for a few weeks and see what happens."

Brent noted that this openness contrasted with his previous experiences of school mathematics which were structured around a predetermined "end result that you're trying to get to." As a teacher, he tried to foster a culture of inquiry that broke from this norm: " I don't want them [students] to just think, 'Okay, the problems come pre-made, and then we solve them, and then we move on to the next questions.' I want them to create their own problems." Brent acknowledged that when his students generated mathematical questions, he, like his college professor, genuinely did not know the answer and so assumed an inquiry stance along with them. Lydia also shared that open-ended inquiry allowed her to "be a learner" alongside children. Describing mathematics discussions, she recalled:

I didn't enter it with, "I know what's going to happen here," or, "I know what they're going to say," or, "I have something that I'm waiting to hear them say." Entering a conversation with kids in that way is very freeing....Being a learner in it means that I didn't have to be like the expert, right? And it would be okay to say, "Hm, don't know about that one. Let's put that one up, right, and let's figure that one out tomorrow."

Like Brent, Lydia suggested that it could be pedagogically freeing to release from the expectation that teachers need to know exactly where mathematical inquiry will lead.

Considering young people's responses to open-ended mathematical activity, Naya recalled a moment in which inquiry brought mathematical concepts "alive for kids" in new ways. She shared about a lesson that she designed as a student-teacher to explore the concept of dimensionality with high schoolers. The class sat in a circle, in pairs, each with a box containing various objects. Naya offered "really open-ended questions," inviting young people to discuss how they might describe the dimensions of the objects. After extended conversation in pairs and as a whole group, one student said, "You're blowing my mind! I'm gonna have to be thinking about this for the rest of the day." Naya

reflected to me, "That is the reaction that you want. Not having the answer at the end of class is the feeling of freedom, right?" Being asked to (re)consider the mathematical qualities of everyday objects in an open-ended way radically expanded this student's thinking about the lesson's focal content and, potentially, her assumptions about mathematics class. The "feeling of freedom" Naya named could be the student's or Naya's, once again suggesting that moving away from a narrow pursuit of predetermined answers liberates classroom mathematics for both learners and teachers.

Across educators' accounts, open-ended activity was also collaborative, a process of co-constructing knowledge that fostered interdependence and mutual support. Kate described classrooms in which people "create math for each other" and assume shared ownership over the process of its creation. During class discussions, she said:

I'm trying to offer to kids that, if this work is ours, and not just mine and not just yours, then....ultimately nobody owns these ideas. You can, once it's out in the classroom space, we can all nuance them or build on them.

Cliff described co-constructing knowledge in an afterschool computer science class, through an activity called Finish My Story. He and another youth teacher invited learners to, in two minutes, begin developing a game on Scratch, after which they rotated to a nearby computer to build on the code a peer had started. The rotation continued for several rounds, with young people "adding on to something that wasn't yours and trying to make it new." Cliff noted that, through Finish My Story, the class culture grew more collaborative as learners turned to one another for help or new ideas. He also spoke to the open-endedness of this activity, which was designed to honor heterogeneity of ideas: "how people were thinking about their problem, every game was different. We wanted them to know that when you're doing problems like this...you can reach a destination differently than the person next to you" while "building with" their ideas. Cliff later used the word "jamming" to describe the collaborative problem solving in his imagined learning space open to the public; the metaphor of people jamming together captured the spirit of participants' classroom-based stories as well.

Social Connection

As they supported collaborative activity, the environments educators described nurtured a kind of social connectedness that is not common in mathematics classrooms. Reflecting on his experiences learning and teaching with the Young People's Project, Cliff spoke of "a very unique and rare community" of elders, mentors, and mentees whom he saw as simultaneously "scholars and intellectuals," fellow community organizers, and a kind of family. Growing up in this "immersive community," whose mission centered the empowerment of Black youth, "poured into" Cliff a love for mathematics that was entwined with a sense of racialized belonging and justice-oriented political vision.

Multiple educators described community building as foundational to their teaching practice. Both Ty and Naya included restorative circles in their high school mathematics classes as a way to build deep relationships with and among students. Ty's classes gathered in circle every week or two, as "a tool for us to get to know each other, to trust each other as full individual humans, not just students of mathematics." In circles, young people shared about their daily routines, hobbies, preferences, emotions, visions for the future, and responses to current events. Circles sometimes addressed mathematics directly (e.g., discussing data related to electoral politics or the demographics of honors course enrollment at their school), but Ty saw them as a relational structure of their class that transcended disciplinary content.

Kate's work with teachers often focused on "building a math community" and fostering "a deep sense of belonging within the math classroom." When Kate joined another teacher's class as a co-facilitator, her priority was to "relate to kids," often opening sessions by inviting them to share about something they "care about," to "tell a few stories" that welcome young people in their full humanity. As a visiting teacher educator, she was not a long-term member of the class community, but she saw herself giving teachers "an image of the possible...what it could feel like" to "know a little bit more about each other" in the math classroom. She then worked with teachers to consider how they might develop that culture of connection, trust, and belonging. Similar to Ty, Kate described a process of community building that extended beyond the specifics of mathematics content. At the same time, she saw this relational work as crucial to young people "feeling whole" in the context of mathematics specifically. As Lydia put it, the ways teachers and children engaged with one another's mathematical ideas had the power to communicate who, as people, "mattered," and whether mathematics could be "for you" (Lydia).

A clear thread in these conversations was that the social relationships people formed around mathematics were essential to mathematical activity feeling liberatory. Whether Ty practicing mathematics with her father and competition teammates, Cliff playing Flagway and Finish My Story with youth in the Young People's Project, Naya solving mathematics problems with her adult friends, or Kate and Lydia building classroom communities with children and teachers, educators recalled experiences of mathematics teaching and learning that were meaningful in large part because of the human relationships being nurtured.

(Re)Organizing Physical Space

The work of building a community of belonging involved the intentional organization of physical space. Earlier in the section, I shared participants' stories of learning settings beyond school classrooms. Here, I discuss their reflections on the ways classroom spaces could be set up, and often reconfigured, to support the sense of community they sought to build. Multiple educators mentioned the configuration of a circle. In addition to facilitating social connectedness, circle formations reflected an orientation toward knowledge as collectively developed and held. In Naya's account of her lesson on dimensionality, rearranging desks into a circle contributed to the collaborative, open-ended spirit of their intellectual inquiry. Ty said that standing at whiteboards around the perimeter of the room was a "really important" part of her experience on math teams because it created an "exciting" culture of "solving problems together" and looking to one another's in-process work for inspiration. Upon taking the lead in the computer science class with middle schoolers, Cliff and his colleague decided to "rearrange the room" so that computers were set up in a circle rather than front-facing rows. As a result, they noticed that "collaboration was more frequent" because young people could "easily go to someone else's computer and say, 'Oh, how'd you do that

thing?'...it was an easy way to move around." He added that teachers were also better able to move from learner to learner within the circle configuration.

Kate shared a story, from her perspective as a learner, about the ways physical setting, movement of bodies, and movement of ideas were interrelated. She described a mathematics class during one of her first weeks in high school:

I walked into the room, and I remember the room looking like...many other rooms: just rows of desks facing the forefront, with chalkboards in the front and a teacher's desk, and chalkboards on the side, and a row of windows. And in the middle of the lesson, my very animated high school math teacher decided that we weren't seeing and making sense of the geometry that he wanted us to see. And so he just kind of broke into this frenzy...he got us out of our desks...he pushed all the desks out of the way, and he started drawing polygons all over the floor....We were standing, not sitting, and we were in this beautiful circular shape, which wasn't associated with math, because it was...expansive, and it felt like we were actually looking at each other. Right, so the whole way in which we were interacting with each other students in the room. I got the possibility that we don't have to organize the space that way, and something happens when we orient towards each other and not towards the teacher and the "material."

Kate narrated a radical reconfiguration of people, furniture, and mathematical representations in physical space—a shift that presented new possibilities for "seeing and making sense" of geometric ideas and for seeing and being with one another. This

279

"glimmer" of the possible made its way into Kate's work as a teacher educator. She typically began collaborations with teachers by "reshap[ing]" the classroom meeting space into a circle: "We look at each other, we look to each other, there's no clear evidence of a front of a classroom....I want people to feel like, 'I'm in this special place where I can see and hear everyone."" Kate, along with other educators, underscored that liberatory learning is embodied and that reorganizing physical space can shift the relationships among learners, teachers, and disciplinary content.

Critiquing Power Structures

A final theme, which traces through preceding parts of this section, is that educators saw the mathematics classroom as a place to develop young people's critical consciousness (Freire, 1970). One way was through mathematical problem posing around issues of societal injustice, as mentioned in the earlier examples from Brent's and Naya's classrooms. In Ty's class, young people also examined their own powered social interactions around mathematics. As a weekly routine, students worked in small groups on a mathematics problem, exchanged ideas across groups, and then discussed strategies and questions as a whole class. Rather than facilitate the discussion portion, Ty observed and took detailed notes about students' participation. She later reflected back the dynamics she had noticed, including who stood up to demonstrate at the board, who posed questions or asked for help, and whose contributions were taken up by other members of the class. She asked individuals to consider the roles they had assumed during the discussion and why particular class members might have participated in the ways they did. Ty saw this protocol as a way to engage students in the "content we had to actually learn" while also centering conversations about "positionality and power in the math classroom."

In yet another kind of critical analysis, Brent invited young people to interrogate epistemological norms that govern mathematical activity (e.g., individualism, competition, speed, a focus on single answers). He gave an example about preparing for the American Mathematics Competition (AMC), which determines admission to the International Mathematics Olympiad, a global mathematics championship for high school youth. According to Brent, participation in the Olympiad is predominantly, and some years entirely, white and male. In addition to discussing inequitable access to participation in this prestigious mathematical event, Brent and his students (all young women, and most of them white) questioned the exclusionary expectations of the admission exam's content and process. He described naming these expectations with his classes and then working to "shift" them:

Rather than adhering to the traditional format of the AMC—which is that people take them [exams] individually, they're timed, and the goal is to answer a multiple choice question and move on—we have sort of open-ended time periods for people to collaborate in answering the problems and then sometimes modifying the problems or asking follow-ups....[We consider,] "What other problems can we ask? How did the test designers create this?" I think, for me, that's an instantiation of naming the gate....And the way of challenging it is by saying, "Let's basically shift everything. Oh, the actual AMC's on your own? Let's collaborate. The actual one's multiple choice? Let's allow it to be openended. Oh, the actual AMC says move on to the next question? Let's pose our own problems as follow-ups."...I find that naming the components that feel oppressive and then flipping them, where individual becomes collaborative, etc., is—well, I don't know if it's liberatory, but I hope so.

Importantly, this shifting work did not take place as a curriculum planning process apart from young people; Brent and his classes explicitly discussed the norms they were challenging. They also examined critical texts on issues of gender and race in mathematics competitions and had conversations about students' own "views on what mathematics is or what mathematicians do." While examples earlier in this section address issues of power in society *using* mathematics, or classroom social dynamics *around* mathematics, Brent's class confronted power dynamics within and upheld by the discipline itself.

Summary

This part of the analysis presented themes threading across the six educators' accounts of liberatory mathematics teaching and learning. They described liberatory pedagogy as venturing beyond expected boundaries of various kinds; centering learners' experiences and interests; supporting open-ended, collaborative inquiry; fostering deep social connection; reorganizing physical space to disrupt normative relations among teachers, learners, and disciplines; and critiquing power structures.

In closing this section, I comment briefly on teaching mathematics for social justice (Gutstein, 2003, 2006), a prominent theme in mathematics education literature, one that often comes to stand in for justice-centered, liberatory pedagogy. Four of the six

educators I spoke with mentioned mathematical inquiry that explores sociopolitical problems, and the three who had taught high school mathematics offered examples from their classrooms. As mentioned earlier, they tried "social justice math trails," problem investigations that emerged from concerns in young people's lives, and circles responding to quantitative data on issues of societal injustice. Yet within the larger analysis, social justice mathematics was a relatively minor theme. For these educators, it was not the cornerstone but rather one possibility among many for what liberatory mathematics could be.

The Work of Freeing

Educators' reflections on liberatory mathematics were at times speculative visions of what could be that had not yet or fully materialized. They also shared stories from their lived experience which—though emergent, imperfect, and sometimes ephemeral—communicated liberatory possibility in the here and now. As educators recounted efforts to live out these possibilities, they described various relationships between liberatory pedagogy and the normativity of school mathematics. These ranged from critical navigation of dominant systems to radical departures from them. Multiple educators talked about the work of freeing as a process of engaging with invitations to imagine otherwise. They suggested that this work was energizing and hopeful and also involved persistent tensions. For all participants, it entailed examining their own conceptions of and relationships with mathematics, and seeing these as open to change. In this section of the paper, I devote less attention to examples from classroom practice, since many are addressed in prior sections, and focus instead on the ways teachers framed the work they and others were doing in those moments.

Relationships Between Liberatory and Normative Pedagogies

Educators discussed the relationship between liberatory practice and normative systems in various ways. They described supporting others to critically navigate dominant codes and spaces, finding "liberatory pockets" within existing systems (Naya), and seeking radical departures from them. Often, participants spoke to multiple orientations at once, suggesting that they were not mutually exclusive and all had a role to play in liberatory mathematics pedagogy.

Critically Navigating Powered Systems

One way educators oriented to normative systems of mathematics education was to support access to their powered "currencies" (Cliff) while critiquing, and to an extent disrupting, their exclusivity. Ty, discussing Delpit's (1988) notion of "codes of power" in literacy education, posited, "If you teach people the unwritten rules of society, then they are free to go break them....That license to be creative once you know what the rigid rules are can be freeing." Educators shared various ways that they and others had exposed the unwritten rules of school mathematics.

In a conversation about mathematics as a "gatekeeper" (Stinson, 2004), Brent mentioned the duality between "tearing down the gates" and "helping students get through the gates," particularly for groups that historically have been denied access. In the earlier example about the American Mathematics Competition, Brent created opportunities for his students to build fluency with the mathematics content of AMC tasks, a currency that held power beyond this particular exam, while also reconfiguring the ways young people interacted with that content and each other. In this way, he reflected,

They are getting at some of the math that is involved in that type of gatekeeping—which is, access to some of the post-secondary math experiences, and the camps that train people and prepare them to take graduate-level math courses as undergrads...but at the same time, we're not adhering to the traditionalist structure.

Brent described this as a kind of "repurposing" of dominant mathematics.

Like Brent, Kate initiated conversations with teachers and young people about unspoken assumptions in school mathematics. For example:

Sometimes I'll just ask the question, "Who decided we have to learn integers in seventh grade?" I just ask the question, "Who?" And no one can really say who, and yet everybody knows that's what we do. And so sometimes I feel like you nudge towards more liberating practices by just tapping on things, by just uncovering things....We start to wonder, "What's that about? Whose test is this? Who wanted us to sit in silence thinking about these questions for hours and hours on end?"

Kate made clear that critical empowerment went beyond the notion of, "the math I'm gonna teach you is so useful...Some day you'll have to do this, when you have a checking account." Rather than "defending math" in this manner, she raised questions about taken-for-granted ways of organizing mathematics in school—without ignoring the high-stakes pressure teachers and youth were under to move through that system.

Cliff shared about early experiences building fluency and confidence with mathematics within a learning community that framed mathematics literacy as an issue of racial justice. According to Cliff, the program built a "floor...under me, set me up on a path where, if I didn't have that floor, then I would have done the lower track of math and probably wouldn't have...excelled in college, even if I ever made it to college." Cliff described access to the institutional power mathematics affords as well as "the richness of [the] discipline" that was "being poured into me." Importantly, the Young People's Project also equipped Cliff and his peers to "know how to navigate mathematics" with a critical perspective and a sense of community support. Youth in the program "had a community around them...talk[ing] about, 'You're gonna get this math and you're gonna learn how to navigate the system with your brown skin."" In another context, having mathematics "poured into" him might have been oppressive, but Cliff spoke of disciplinary access supported within a community of racialized belonging and empowerment.

Finding "Liberatory Pockets"

Another orientation toward dominant systems was to find spaces for flexibility and change within them. Naya referred to these as "liberatory pockets." Discussing her work as a district-level coach assigned to schools with the express goal to "improve their math scores"—specifically among students of color and students identified as having "special education needs"—Naya shared:

When we're working within these very restrictive environments, it's like, where are the little pockets of liberation?...We have this vision, as if we were outside of

286

these structures, but living within the structures...the work becomes, where can I

foster moments of liberation within these very oppressive, racist structures? She gave examples such as making space for young people to speak to each other and even smile in mathematics classrooms. She narrated the conversations she typically has with collaborating teachers:

Some of the first conversations are like, "What if we took this problem that you were going to walk them through, and we just gave it to them on a piece of paper...we created some groups, and we gave it to them? We gave them some prompts to talk about the problem, and we gave them a big chart paper and asked them to just draw out what they thought about the problem? Or, what if we took this problem from the curriculum...and we gave them the tools, and they started building it themselves, and then we told them to discuss observations that they had? Let's see what happens; let's just see what happens."

In this "pocket," Naya did not suggest abandoning the curriculum task the teacher had to work with but rather reconfiguring the social relationships and norms of inquiry that would engage students with the task. Naya reflected that opening these possibilities sometimes required working with teachers to shift their own relationships with the mathematics they were teaching. She mentioned teachers who "themselves are so traumatized by the math [that] they're not giving themselves any space to deeply understand it at a conceptual level." Naya tried to create that space by engaging in mathematics with teachers, exploring conceptual questions, connections to teachers' lives beyond school, and the diversity of ways of thinking about a problem. In a later part of the section, I return to teachers' shifting relationships with mathematics; for now, I highlight this as a "pocket" Naya identified in her role as teacher educator.

Kate also spoke about finding "openings" for liberatory work as a teacher educator. She often invited collaborating teachers "to name the things in their world, as teachers, as math teachers, that just are not working...[the] structures that get in the way of work that teachers want to be doing" and then to "problematize it." She gave the example of not having enough time in the school day to experiment beyond the mandated curriculum while still "hitting the targets" teachers were accountable to through student testing and teacher evaluation systems. Kate acknowledged, "[For] some of these I can find openings, and others I can't. Like I can't magically make more time. I can help people use their time differently, prioritize what's important to them." By spending time alongside teachers in their classrooms and "get[ting] the kids' perspective on what's happening in this space," Kate, like Naya, then worked with teachers to re-envision an instructional set-up that attended to external mandates (e.g., regarding content coverage) while shifting activity structures and ways of interacting in the classroom.

Naya shared another example from her experience as a classroom mathematics teacher at a school that was exempt from state exit exams. In many ways, this context was less "restricted" and one in which teachers felt freedom to "explore." Still, Naya clarified, "even though we weren't testing, there was this looming feeling that they wouldn't do well in college unless we followed a curriculum exactly or [if] we didn't teach math in a certain way." At this school, Naya found a group of teachers of color who supported one another to interrogate that feeling—including its ties to "our own internalized colonization around math"—and to explore freer ways of teaching. As an informal inquiry community, these colleagues shifted to "meet[ing] the needs of the students in front of them as their impetus." Naya explained:

Learning from the kids, what is it they want from a math class, and then gauging, are my students enjoying themselves in my class? When they tell me they're enjoying a lesson, what was in that lesson, what was in that unit? When they felt inspired, what were we learning? And through that type of exploration I think I learned a lot.

In this case, Naya and her colleagues created a liberatory pocket in their own professional learning, which in turn helped liberate their instructional practice with students.

Toward Radical Departures

At other times, educators framed their efforts as more fundamental departures from the expectations of dominant systems. Kate reflected:

There have been moments, for sure, in my work co-teaching with teachers...[where] I've tried to offer permission to teachers not to teach the standard curriculum, not to teach the standards at all in some places. And to...take big risks in terms of knowing what's right for kids and being able to defend that....When I returned to those classrooms and I saw teachers...teaching things that were actually interesting to children, listening to kids' questions, or, even better, following kids' questions as things worthy of study—I felt more free, and I know for a fact some of those teachers felt more free....This notion that the standards, the curriculum, you know, we're beholden to it, we're at some level oppressed by it-getting teachers to think, "What is the alternative?" can be really

freeing. Also overwhelming, but it can be really freeing, just to imagine. As in her earlier example about "tapping on things" through critical questions, Kate saw herself in a position to "uncover" alternatives where teachers initially saw little room for change. Here she took that possibility further, inviting teachers to replace the standard curriculum with inquiry that was driven primarily by children's interests and questions. Lydia also described implementing a curriculum that did not "look anything like what the math textbook look[ed] like," grounded instead "in the experiences of kids and their families and communities":

It was just so different from any experience *I'd* ever had as a learner or as a teacher....When you're listening to these conversations that kids are having, and you're seeing the interactions and the willingness...that's when you realize just how powerful that is, and how different it was from, "Here's this book, here are these set of problems, this is how you do it, and now go do them." It just turned everything on its head, in a beautiful way.

This point in Lydia's career predated the era of accountability that she, Kate, and Naya described working within as teacher educators. Still, normative expectations at the time were to teach mathematics prescriptively according to a textbook, and Lydia noted the "dissonance" she felt when encouraged to try something very different.

Another example of departure was Naya's story about the boarding school in the mountains of Ladakh. As she explained, "The whole teaching philosophy of the place is just totally radical and revolutionary....[the founder's] big interest is breaking free from

the institutionalization of math." Naya acknowledged that the school's leaders had to reckon with pressures young people faced to eventually prepare for college, but they oriented to those demands by maintaining that young people could gain formative experiences with disciplinary content and the English language through nontraditional learning experiences. Naya compared the Ladahki school to Freedom Schools in the United States, noting that the former was unique for its blend of radical political vision and "a math focus."

Inspiration and Fear in "Let's See"

In many of the examples above, educators described invitations to expand ideas about mathematics teaching and learning. Sometimes teachers invited one another into expansive practice, as in the professional inquiry community at Naya's former school. Often, the invitations were from teacher educators to collaborating classroom teachers. Referring to her practice of raising critical questions with teachers, Kate noted:

In my role as a staff developer, I'm kind of the perfect person to do that. I'm a little bit of an insider-outsider. I'm one of them, in that I'm part of their community, and I know their kids, and I know who they are....But I'm to some degree an outsider, too, and so I can just kind of wonder aloud.

Lydia said similarly:

I'm the coach, and I can actually talk to the teacher about..."Just because the curriculum says this is what you should do, doesn't always mean that's a good thing to do for kids....We don't have to do that. If we really want to do right by kids, if we really want to nurture their growth and their identity, what's a different

way we could, what's something [else] we could do?"

As coaches and staff developers, Kate, Lydia, and Naya modeled and encouraged teachers to assume a stance of, "let's see" (Naya), opening space to consider alternatives where teaching felt prescribed, dehumanizing, or simply unexamined. They reflected that such invitations could inspire excitement and feelings of freedom as well as skepticism and fear.

Excitement and Freedom

Participants shared about the excitement and freedom that they and other educators experienced when exploring alternatives in their mathematics teaching. Lydia described a sense of elation-"happiness" and a "floating feeling"-as she witnessed children engaging in a discussion of equivalence inspired by a new curriculum that centered their experiences. She also said that it could be "freeing" for teachers she worked with to, with the support of a coach, recognize patterns of harm they unintentionally perpetuated in their classrooms and realize "a different way" was possible. By reflecting on classroom video and modeling instruction so that teachers could step back to observe, Lydia invited teachers to see their classrooms from new perspectives. For example, she pressed teachers to reconsider practices like cold calling on students and using timed tests, which "can do harm" to children's cognitive development and "to their sense of who they are." As teachers realized, "We don't have to do that," that there were "things you can do to disrupt the harm that we see," Lydia would notice them "get so excited." She said it could be "such a high" for teachers to witness their students interacting and sharing ideas more freely and to consider the

pedagogy that made that possible.

Skepticism and Fear

As Kate reflected, newness in one's practice could be "thrilling," and it could also inspire fear and doubt. Multiple participants acknowledged the power that systems exert over teachers and schools, discouraging attempts to explore creatively. They had worked with teachers who seemed drawn to the predictability, security, and even gratification that came from following the expectations of curriculum pacing guides, standardized tests, and settled patterns of interaction in classrooms. Teachers sometimes "pushed back" on invitations to experiment or see anew, out of fear that these efforts would not be "rewarded" or "count" according to the metrics of accountability systems. Kate quoted teachers saying things like, "Systems and structures don't give us the latitude to do the things that you're describing or you're modeling," and, "I don't get rewarded in the system for building a community in the way that I get rewarded for...test scores....Is this still a math class? Is it legitimate? Will it count?"

Lydia narrated her initial skepticism, early in her career, when first introduced to the Algebra Project's curriculum:

It just looked like, "What is this? What is this? It doesn't look anything like what the math textbook looks like."...That feeling, it was just such dissonance, like, how can this be the math that we're going to do, when there aren't any equations, and there aren't any rules for kids to remember?

It was through collaboration with mentors, and a community organized around justicedriven mathematics reform, that Lydia came to see and trust in the new curriculum's "power." Across my conversations with educators, we discussed the weight of the unknown and how intimidating it can feel to open up teaching practice, especially in contexts that lack the professional support necessary to sustain such inquiry. Even when teachers are willing to "go off script" and encounter "images of the possible," the freedom they experience can be "fleeting" if it is not held and affirmed collectively (Kate).

Changing Relationships with Mathematics

Each educator I spoke with touched on the idea that liberating mathematics teaching and learning involved changing people's relationships with mathematics. Participants described shifts they witnessed in others as well as evolution in their personal relationships with mathematics.

Supporting and Witnessing Changing Relationships

Educators spoke about the ways their pedagogy strove to, and often did, change people's "orientation to mathematics" (Kate). They framed the work of shifting as breathing new life into what felt lifeless, healing trauma, and expanding who and what was considered mathematical. Naya described "reinvigorating" mathematics for individuals who had previously found it dull, scary, or exclusionary. In the classroom, she made space for young people to "play" with mathematical ideas and materials that they had "a lot of fear around." For example, when she allowed students to explore with fractions bars, not tied to written tasks or specific problems to answer, she noticed their anxiety "unravel" and a sense of "excitement" build around mathematical ideas. Naya also talked about enlivening mathematics for teachers, who in their own education had experienced it as "procedural":

Sometimes I have to sit with math teachers and say, "Let's just do this problem together and see what happens. Let's just see. And I want you to talk about, why are we doing what we're doing? What are the concepts behind this? What are the themes that we're finding in these different topics? Can we make connections?

Where would this exist in real life for us? And, how did we all do it differently?" Naya found that such questions engaged teachers as learners, creating opportunities for them to rediscover mathematics as interesting and personally meaningful. Kate shared that she often began professional development sessions by setting the intention that she hoped teachers would "experience math as joyful" through their work together acknowledging, "I know some of you don't like teaching math....[but] I think I can show you how to kind of change that orientation." She and others made clear that changing orientations was not about conforming individuals to an oppressive version of mathematics but rather shifting the social activity, intellectual norms, and conceptions of mathematics in learning spaces—a point I elaborate below.

Addressing trauma from prior educational experiences was crucial to shifting orientations to mathematics, and educators framed "healing" in both personal and sociopolitical terms. Ty, referencing work by Gutiérrez (2020), invoked the idea of "restoring who mathematics is by and for." Restoration, to Ty, entailed "understanding that different cultures across the world are doing mathematics all the time and have been for a long time before Europeans staked a claim and said that math has to be this way and that." It also encouraged young people to "re-write the oppressive experiences they've had in math classes" by recognizing that "they can do math" and "tell their own stories" in math. Naya echoed:

There's so much trauma that happens in schooling...in a particular way in math classrooms. That is a huge part of what it means to be liberatory in a math classroom: restoring people's and my own relationship to what it means to be

doing math, and breaking down this idea of what math can be or can't be.

In addition to stories with young people and colleagues, Naya shared about doing mathematics with friends who had felt alienated from or shamed by mathematics in school. Whether collaborating to figure out a restaurant bill or supporting friends and their children with mathematics homework, Naya worked to facilitate a shared experience that nurtured social belonging, excitement, and inquiry around mathematics. She described these experiences as "healing" because they opened her friends to "something that they had a wall up against," within a social context that felt "bonded" and joyful. Like Ty, Naya underscored the sociopolitical dimensions of math trauma, naming the ways racial and gender identity shaped who experienced "math anxiety" and in what contexts.

Educators made clear that healing relationships with mathematics required expanding who and what was considered mathematical. One of Lydia's goals in her work with teachers and children was to "dismantl[e] some of the myths about who does math." She suggested that this could be freeing for people across social positions:

I think it's liberating for those from a privileged group to actually see and hear the thinking and voices of folks who don't look and sound like them, [who] they may not have thought of as math doers. And for those who may be in a group that have not been thought of as math doers, it's liberating for them to be in a place that acknowledges that what they're doing is mathematical also, and valued.

Educators clarified that developing identities as "math doers" was not about conforming to the normative expectations of school mathematics but instead identifying with a more expansive, dignifying version of mathematics. Brent was wary of the aphorism, "Everyone is a math person" because it can carry a homogenizing force, "collapsing multiple identities into a single umbrella term." He added, "There's a lot of people for whom they feel, deeply in their hearts [that] 'I'm a math person'—they feel this is false." Encouraging young people to fit into a high-status, institutionally sanctioned identity could be traumatizing in its own way. Brent preferred the notions of "participating in" and "creating" mathematics, dynamic processes that positioned mathematics itself as open to change. Similarly, Ty suggested that "helping people realize that what they are doing is mathematical" should be entwined with "redefin[ing] what it means to do mathematics."

Reflecting on Personal Relationships with Mathematics

In addition to supporting others' evolving relationships with mathematics, educators reflected on their own mathematical histories and identities, suggesting that this self-reflection was part of the work of liberatory teaching. Participants recalled past experiences as learners, some empowering and others dehumanizing, that later informed their teaching practice. Ty and Kate traced connections between formative moments in high school—on the math team and in one teacher's geometry class—and their approaches to organizing learning activity as classroom teachers. Cliff continued to work as an educator and organizational leader at the youth organization that had been such an affirming mathematical community for him as an early adolescent. Brent raised the question, "what sort of educational experiences [do teachers] need, to put them in a position to facilitate liberatory experiences?" He reflected that, while his own mathematics education was "somewhat traditionalist," it developed a "robust background" that supported him to recognize and facilitate inquiry around the mathematics in students' "organic" questions.

Naya and Lydia shared about more fraught relationships with school mathematics. Although neither explicitly mentioned their identities as women of color in these moments, their broader reflections pointed to the racialization of experiences like theirs. As mentioned earlier, Naya experienced school mathematics as dull and socially disconnected. This required that she later work to rediscover mathematics as an adult during her first year teaching (a story I return to later in the section). While Lydia identified as a confident mathematics student through high school, she experienced a traumatic "break" in college:

In math classes up until I went to [university], I felt like I had a voice, I could ask questions...and I felt smart. But as soon as I got to [university] and started going to those math classes, I really thought, "I'm the only one sitting here who doesn't know this stuff, the only one sitting here who has all these questions....There was no way I was making sense of it...The message I got was, "This is what math is. Whatever you were doing before, don't know what that stuff was, but this is math, and it's not for you." So, leaving that math world was really upsetting....I thought

I loved math, it was for me, I could do it, I could do it—until I got to college and

it was like, the messages were real clear: "It's not for you, you can't do it."

Lydia later found a professional community as a mathematics educator that helped her heal from this exclusionary experience and draw insights from it into her own teaching. Realizing "that there were kids, who were sitting in my math classes, who were having that same feeling that I had when I was a student at [university]" reinforced Lydia's commitment to making sure each child knows they matter in the mathematics classroom. She carried this attunement into her coaching work, often choosing to focus conversations on, "Let's look at who has voice; let's look at who doesn't," as a way into "creat[ing] equitable learning communities" with teachers and children.

Several participants shared about transformative experiences that "open[ed] up" (Naya) their perspectives on mathematics. Cliff regarded his early experiences with the Young People's Project as pivotal in shaping his positive relationship to mathematics, saying that "I really wasn't too serious about understanding and doing well in math until I joined." Through the program, Cliff discovered that mathematics could be "in me," a source of strength that sparked his curiosity, connected him to a racially affirming intellectual community, and set him on a trajectory of academic success. Through her work as a teacher with the Algebra Project, Lydia rediscovered that mathematics could be "for me." She recalled the particular influence of Bob Moses, the Algebra Project's founding leader, who encouraged her to "be a learner" as she explored new ways of teaching mathematics: It was like, "You can still be a learner, too, Lydia. I give you permission to still be in this." Right, to not have to be that person that I just absolutely can't stand, who really clearly was like, "Well, this is the way it is, this is what we do, and you need to learn how to do this thing." Just, having had that experience [in college] that was so off-putting and discouraging and really turned me away from thinking that math could be this beautiful thing, [and then] working with Bob and really starting with what kids could bring to the table from their experiences—like I said, it was giving me permission to be a learner in it also....Which is absolutely not what was happening when I left math.

Invoking the experience in college that had turned her away from mathematics and her identity as a mathematics learner, Lydia reflected that working with Moses allowed her to reencounter mathematics as affirming, exploratory, and collaborative. She underscored that "permission to be a learner in it" was key to this transformation, suggesting that liberatory pedagogy is possible when educators are positioned as ongoing learners of mathematics and mathematics teaching.

Naya also shared about re-engaging as a learner of mathematics. The transformation she described was largely self-initiated, though it involved turning to others as models of a different relationship with the discipline. Naya received her early professional training through a teaching fellows program that persuaded her to become a mathematics teacher, despite her ambivalence after an "uninspiring" K–12 education. She made a commitment in her first few years in the classroom to "teach myself the curriculum again," more expansively:

I was looking in a lot of different places for learning, which I don't think a lot of math teachers, maybe, do; they don't reteach themselves. But you know, I fostered a love and understanding that I'm still learning....I listened to people...in different types of math fields, talk about math and their wonderment around different math concepts, and I soaked it up. Because I'm like, "Oh, that is not how I was taught calculus."...You don't learn the beauty of what infinity means, or you don't learn about pattern and repetition....When you leave traditional education and then you're just trying to learn math on your own, you realize that math is super interesting....Talking to people who are inspired, who are confident and very inspired by math, and already feel liberated in doing math, I think helped a lot....Those types of experiences open up your mind.

Naya offered several examples of encounters with individuals who shared their "fascination" with mathematics. One was a conversation with an engineer who reformed coastal lines in response to beach erosion: "I was like, 'What? That is so fascinating,' how he was talking about his application and understanding of geometry...in this very specific context that also has environmental repercussions that could be tied into it." Naya also mentioned a co-teacher who recommended "philosophical" mathematics texts to her and shared her passion about the mathematics of astrology. Recognizing that these examples were of white people, Naya then turned to the art that her mother used to practice, which she learned about on a visit to her grandparents' house in India:

I found all these pieces of this type of art that they do in India. They take planks of wood and delicately hammer in nails in a very intricate design, and then they take string and they intricately design very beautiful images....I also found paintings that she had done—examples of my mother when she was young, and the type of liberation that she felt with this art that was very mathematical....Yeah, it's funny that I don't go there when I think of inspired math.

Reclaiming her mother's art making as "inspired math" was an opening in Naya's perspective that seemed to take place during our conversation. In addition to connecting with people working in applied mathematics fields and educators passionate about the discipline, encountering her mother's art became one of "those types of experiences" that expand what mathematics could be.

Summary

This section discussed how educators framed the work of freeing mathematics teaching and learning. They described various relationships between liberatory and normative pedagogies, the excitement and resistance they encountered when inviting other educators into liberatory practice, and their own experiences as mathematics learners. Across stories, educators communicated that liberatory praxis was collective, entwining the visions and growth of learners and educators alike—and, in many cases, collapsing the distinction between these roles. Participants also made clear that the work of freeing was ongoing, not a landing place or final accomplishment. As Cliff warned, "going against the grain" demanded vigilance and continued interrogation of teaching practice, given systemic pressures to "go back into what the traditional space does." While recognizing that freedom could be "fleeting," "not quite there yet," or at the scale

of a "small pocket," these educators made tangible and persistent efforts to bring it into being.

Math Would Be: A Found Poem

Underlying educators' reflections on teaching and learning was a broader reimagining of mathematics. Educators spoke of mathematics as naturally emergent in people's lives, dynamic and plural, playful and exciting, aesthetically inspiring, and radically inclusive. They discussed people practicing mathematics as a form of generational knowledge sharing and toward heterogeneous, self-determined purposes. Our conversations also considered the value, and potential enclosures, of labeling people, practices, and ways of living "mathematical."

These ideas weave throughout the analysis above. In this section, rather than returning to specific examples from educators' reflections, I offer a poem—in some sense, an imagined conversation—composed of language participants used to express their visions of liberatory mathematics. I also include fragments from my own contributions to the conversations. By closing the paper's analysis in this way, I hope to illuminate "opportunities of a poetic kind" that lived in our conversations, which may "help us grasp something new, as yet unseen"—in this data, in mathematics, and in possibilities for mathematics teaching and learning (Shotter, 1996, pp. 38, 46).

All text in the poem is directly quoted from conversation transcripts, and each color marks a different speaker. Often, consecutive lines in a single color signal one person's contiguous speech, though occasionally I bridged related phrases from a turn of talk by cutting out words in between, in order to heighten the sonic quality of the line while preserving what I interpreted to be the speaker's meaning.

Math Would Be

I. What math would be if we hadn't been colonized corporations coming in the criminal justice system forced to do math for the purpose of money We would be weaving crocheting braiding building making kites and going out to fly I may have been able to live near my grandmother learn from her ways of knowing math To say "I care about this" an inherent necessity to it (not just to answer someone else's problem) II. "A mathematician is someone who..." I found paintings she had done my mother when she was young

Those people around still you gotta listen I always thought my mom was brilliant A lotta wisdom Soak it up I soaked it up The liberation she felt with this art that was very mathematical

III. How free can you be if you're told math is this fixed thing the "universal language" already discovered one path to some place already in mind "Forty-five minutes later all thirty-two diverse humans are going to have the same epiphany about variables" Really?

Math is done by people ever-evolving People being wildly creative moved inside by how many different ways Seek the multiplicity of those languages Hear more voices ways of seeing Infinite possibilities here

IV.

What if there was a space not real yet hopefully I see a space You come in There's a problem Little kids, big kids, adults connecting around this math question everybody knows something about Problems emerge naturally Just walk over say "Hey I have this question" The beginning of more conversations

A place to jam

play around feel so fun deeply serious moments but it's loose asking really big questions for anyone

who wants to come in

I could imagine This world we're moving toward that we might all feel free We would be floating breaking free from the path so many cool questions you can ask freed up from the closed nature of the textbook Breaking math free we might be hopefully What if we Free up! Let's just see

Discussion

This paper explores meanings of liberation in the context of mathematics education, based on conversations with six educators in which they recollected and speculated about mathematics teaching and learning. Across our conversations, we considered liberation to be multifaceted, concerning the what, where, how, why, by and for whom of mathematics learning. Educators reflected on the systems they sought freedom from, which spanned curriculum and assessment frameworks, patterns of classroom interaction, and global political histories. Their pedagogies also ventured beyond the constraints of existing systems, disrupting expectations about the physical, social, and epistemological organization of mathematics learning. They described learning contexts that cut across typically segregated spaces and communities, centered young people's interests and experiences, inspired open-ended joint inquiry, fostered deep social connection, reorganized physical space, and attended explicitly to issues of power.

When discussing their visions of liberatory mathematics, educators reimagined the social relationships, norms of inquiry, and spatial arrangements of learning and

306

teaching. Our conversations delved less deeply into the mathematics itself—the disciplinary ideas and practices that people engaged with in these spaces. Across their accounts, educators did gesture toward a different kind of mathematics than is common in school and societal narratives of the discipline: they suggested mathematics could be place-based and context-dependent, embodied, improvisational, continually evolving, and rooted in (not simply applied to) questions about life. Still, there is more to explore about the nature of the discipline in these and other potential stories of liberatory mathematics. As discussed in Chapter Four, the broader field of mathematics education is in need of work that opens the discipline itself to reimagining.

My conversations with educators also illuminated the complex work involved in pursuing liberatory possibilities. Educators discussed the multiple ways they responded to normative pressures, supporting young people and fellow teachers to critically navigate dominant systems while also making space to explore more radical alternatives. This orientation to teaching inspired hope and excitement as well as skepticism and fear. It demanded an openness to learning and change on the part of young people, teachers, and teacher educators alike. Participants were also honest about the sources of resistance they faced, the vigilance and steadfastness required, and the precarious nature of hope as they "waded through" realities that were far from liberatory (Givens, 2021b).

What came through unequivocally in our conversations was the "poetic knowledge" these educators held—their willingness and capacity to imagine otherwise within mathematics teaching contexts (Benjamin, 2016; Kelley, 2002). Educators communicated that liberation was simultaneously a "not real, yet" future (Cliff) and an ongoing process of actualizing new possibilities, albeit imperfectly and sometimes fleetingly, in the present (Bang & Vossoughi, 2016). While acknowledging that their conceptions of liberatory pedagogy were neither fully developed nor absolute, they believed there were alternatives to conventional approaches to teaching and learning mathematics. They assumed that they had a role to play enacting those alternatives and extending "let's see" invitations for their colleagues to do the same (Naya). Bang and Vossoughi (2016) caution against discourses of practicality that "narrow possible courses of action" according to notions of "what is practicable given, at minimum, current disciplinary and political-economic structures" (p. 178). The educators I spoke with modeled the imaginative practicality that Bang, Vossoughi and others encourage, orienting toward teaching as a practice of freedom-in-the-making.

Limitations and Future Directions

Before concluding, I acknowledge this study's limitations and future directions for research. As a composite of six educators' perspectives, my analysis does not offer a thorough portrait of any one person's pedagogy. Each participant offered in-depth reflections, including particular examples from their practice, but the conversations were not designed to explore their approach to teaching comprehensively. A complementary study might focus on one or two participants, pursuing an up-close, in-depth understanding of how each enacted and theorized liberatory mathematics teaching in their specific settings (Yin, 2012). A case study approach would inquire into pedagogy in context, triangulating interview data with records of classroom practice and perhaps the perspectives of youth and teachers with whom these educators collaborated.

Though diverse along many dimensions, the study's participant group could have been more varied in a few ways. First, all participating educators had taught-and, largely, attended schools-in large urban centers. They did not speak specifically to geographic location as a major factor in their teaching, but it is important to note the absence of perspectives on teaching in rural or other non-metropolitan areas. Second, while educators had taught a range of grade levels, their experience skewed toward older age groups. Only Kate and Lydia had worked extensively with children and teachers in the elementary grades, and both had spent more time in upper elementary and middle school classrooms. In our conversation, Lydia stated that she was unsure how certain aspects of her practice would extend to settings with younger children. Third, this paper would offer a fuller vision of liberatory pedagogy, its scope of possibilities and questions, if it included more stories from informal educational settings. Cliff's experiences were based in an out-of-school youth program, though one largely organized around school mathematics; other participants had worked primarily in K-12 schools. Further research is needed to explore liberatory mathematics beyond the boundaries of formal educational institutions. These are all areas of the broader field that deserve continued study.

Finally, a note on disciplinarity. Many of the ideas that developed during my conversations with educators were not necessarily unique to mathematics. The sensibilities, principles, and practices participants discussed are pertinent to—and more commonly found in—other disciplinary learning contexts (e.g., in the humanities). By profiling educators' perspectives on mathematics teaching, I do not intend to fix their pedagogy within disciplinary boundaries but rather to highlight that liberatory praxis *does*

have a place in mathematics classrooms—a notion that is still developing in the field and in need of further exploration. A closer analysis of the cross- and potentially transdisciplinary nature of liberatory pedagogy deserves its own study.

Conclusion

While harnessing the power of imaginative thought, this study offers more than speculative glimpses into a future liberatory mathematics (Benjamin, 2016). It pulls themes from six educators' accounts of practice, some spanning decades in the teaching profession. The stories educators shared are themselves "enfleshments" of liberatory praxis, instances of practitioners striving daily to bring a not-yet into being amidst the myriad counterforces of K–12 education systems (Bang & Vossoughi, 2016, p. 178). Educators illuminated the demanding work entailed in designing for, trying out, and learning from the possibilities of liberatory mathematics. I hope their reflections might inspire others with the courage to say "let's see" in our own spaces of practice.

CHAPTER SIX: PULLING THREADS

Introduction

This dissertation searches for articulations of liberatory mathematics pedagogy. Through conversations with educators and critical scholarship, I explore purposes, possibilities, and challenges of pursuing liberation in and through mathematics education. While parts of the analysis address educational systems broadly, the two empirical papers center the perspectives of K–12 educators to understand how they conceptualize liberation in their praxis.

This chapter begins by summarizing the dissertation's three studies and the key ideas developed through each. The main part of the chapter synthesizes themes from across the papers in light of the overarching question, What could liberatory mathematics teaching and learning be? I consider how each analysis resonates with and extends ideas in the other two. The chapter concludes by suggesting implications and avenues for future inquiry, with an emphasis on mathematics education while acknowledging that questions of liberatory education, teacher pedagogy, and professional learning naturally invite inquiry across fields.

Summary of the Three Studies

The first paper examines a 5th/6th grade teacher's reflections on practice across school disciplines. Lena approached teaching with political clarity, rooted in traditions of liberatory education that critique systemic injustice and position learners and educators as agents of social change (Duncan-Andrade, 2009; Freire, 1970, 1987; Ginwright, 2016; hooks, 1994; Love, 2019; Swalwell, 2013). Lena used the term "liberatory" to describe

her praxis though expressed a disconnect from this vision while teaching mathematics. Through grounded thematic analysis of conversations with Lena, supported by records from class observations and interviews with children, I consider Lena's liberatory pedagogy and mathematics teaching alongside each other.

Lena's pedagogy beyond mathematics was politicized, with attention to critical consciousness and restorative justice; joyful and creative, from curriculum design to moment-to-moment interaction with children; grounded in a deep sense of class community and an ethic of "knowing each other"; and honoring of her own values and well-being. Certain aspects of these themes carried over into mathematics in her class, specifically the ethic of knowing children as whole people and articulating a critique of oppression in education. However, Lena did not describe these as transformative in the ways she did in other areas of her work with children. She spoke of entrenched hierarchy in mathematics as a school discipline, a divorce from creativity and thematic integration when working with curriculum, inhibited capacity to build class community, and less confidence in her professional training around mathematics. The first paper inquires into that rift (Booker & Goldman, 2016) and proposes possibilities for liberatory mathematics in and beyond Lena's class, inspired by the themes of her liberatory pedagogy in the humanities. The paper proposes cross-disciplinary, practice-based inquiry with teachers as a way to critically reimagine mathematics learning.

The second paper is a synthetic analysis of scholarship that interrogates mathematics education's entanglement in systems of oppression and explores liberatory alternatives across a range of contexts and methodological approaches. The analysis is guided by ideas from Warren, Vossoughi, Rosebery, Bang, and Taylor's (2020) chapter, "Multiple Ways of Knowing: Re-imagining Disciplinary Learning," which calls for learning that liberates from the EuroWestern normativity of academic disciplines and K– 12 schooling. The paper is organized around the following invitations to educators and researchers: (1) examine disciplinary education's entanglement in empire, (2) attend to intersections of self- and world-making in disciplinary learning, (3) critique and refuse disciplines' onto-epistemic normativity, (4) explore the multiplicity and dialogicality in disciplines, and (5) re-place disciplinary learning as people "do life" (Warren et al., 2020). I highlight the contributions of extant literature on mathematics education and point to its limitations or silences, in light of these five calls.

The paper discusses critiques, liberatory possibilities, and limitations in the literature under analysis. Authors critique colonial histories of mathematics as a discipline and school subject; continuing sources of oppression in mathematics education, across scales of experience; and narratives of a unitary, universal mathematics. Liberatory alternatives include exploring mathematics as a tool for critical social inquiry, reconfiguring dominant narratives as people negotiate identities as mathematics learners, and illuminating the heterogeneity of mathematical knowledge and practice across cultural communities. The paper also underscores where this literature stops short of liberatory possibilities. One limitation in the literature is a tendency to treat the broader discipline of mathematics as settled while focusing on transformation of educational practice. Relatedly, there is a lack of attention, in classrooms and research analysis, to dialogicality among diverse forms of mathematics—to the ways multiple systems of

mathematical knowledge contest and shape one another and how critically examining this interanimation, with youth, can deepen disciplinary learning.

The third paper returns to the scale of classroom teaching, bringing together the perspectives of six mathematics educators who centered liberatory commitments in their practice. In one-on-one conversations, participants and I discussed the educational histories, teaching experiences, and political visions that shaped their praxis in mathematics. Like Lena from Paper 1 and the scholars in Paper 2, these educators expressed clear critique of the ways mathematics education perpetuates sociohistorical injustice. Through their roles as classroom teachers, teacher educators, and instructional leaders, they worked, along with colleagues and young people, to disrupt those patterns of injustice and enact liberatory alternatives.

Educators described mathematics pedagogy that ventured beyond physical, social, and epistemological boundaries; centered learners' experiences and interests; supported open-ended, collaborative inquiry; fostered deep social connection; critiqued power structures; and organized physical space to reconfigure relations among teachers, learners, and disciplines. They emphasized that liberatory pedagogy involved simultaneous work within and beyond normative structures, feelings of hope and fear as they invited others into liberatory practice, critical reflection on their own experiences as mathematics learners, and a broader reenvisioning of mathematics. The paper's analysis highlights the prefigurative orientation these educators brought to teaching as they strove daily to practice freedom within systems that were far from liberatory (Bang & Vossoughi, 2016; Givens, 2021a; hooks, 1994).

Threads Across Studies

What could liberatory mathematics teaching and learning be? The synthesis below highlights what I have learned about this question through conversations with critical educators and scholarship. It assumes that researchers and teachers alike can contribute to theorizations of liberatory mathematics pedagogy. Rather than elevating research literature as the source of conceptual lenses for making sense of practitioners' work, I pull threads across published scholarship and educators' reflections on their daily practice. In doing so, I contend that theory is made every day in spaces of educational practice and that teachers can theorize their work in ways that illuminate principles for the broader field (Ballenger & Rosebery, 2003; Cochran-Smith & Lytle, 2009; Philip et al., 2022). To clarify, my focus on educators' accounts does not presume that individual teachers or classrooms can, on their own, liberate (Louie & Zhan, 2022). I seek to honor the radical practicality of participants' pedagogies while holding in view the broader systems in which their teaching is situated (Bang & Vossoughi, 2016; Givens, 2021a).

When drawing connections across the papers, I considered how each analysis resonated with and helped illuminate the other two, as well as any ideas it surfaced that the others did not. I moved among the following questions:

• How do Lena's liberatory pedagogy and the corresponding tensions she felt with mathematics teaching resonate with the calls to critically reimagine disciplinary learning in Paper 2 and with the reflections on liberatory mathematics from educators in Paper 3?

- How does the composite vision of liberatory mathematics in Paper 3 resonate with, and possibly extend, Lena's liberatory pedagogy, and how does it speak to her concerns about mathematics teaching? How do educators' reflections speak to the calls of critical reimagining and the liberatory possibilities presented in critical literature on mathematics education?
- Do the perspectives offered by educators in Papers 1 and 3 speak specifically to aspects of critical reimagining that are underexplored in extant literature on mathematics education (i.e., to the areas for future inquiry identified in Paper 2's sections entitled, "To Further Heed the Call")?
- Does any paper raise ideas or questions that are unaddressed in the other two, suggesting possible areas for future inquiry?

I do not attempt to fully answer these questions. This section begins a conversation that I hope will be picked up in future work and expanded to include more voices.

Resonances

The following themes weave across the dissertation's studies: a multi-scale political perspective on mathematics education; harm, healing, and social connection in mathematics; learning from life beyond school; and creative inspiration in mathematics. Below, I summarize how the three papers speak to each theme.

A Multi-Scale Political View of Mathematics Education

All papers consider mathematics teaching, learning, and schooling from a critical perspective. This was a methodological intention, as I connected with participants and turned to scholars who centered questions of oppression, justice, and freedom in their

work. Specifically, analyses underscore the multi-scale nature of power in education and society, discussing how it operates simultaneously at macro, meso, and micro scales. With her class and in reflections on teaching, Lena often invoked the "four I's" of oppression and liberation (internalized, interpersonal, institutional, and ideological). Similarly, the educators in Paper 3 spoke to intersections among moment-to-moment interaction, school system policies, and histories of colonialism and racial capitalism. Following the calls of critical reimagining, Paper 2 synthesizes scholarship that includes historiographies and global political critiques as well as autobiographical reflections and microgenetic analyses. That paper also points to the need for further work linking "the suprapolitics" of mathematics education with "everyday and every moment" experiences of teaching and learning (Gholson & Martin, 2019, p. 394).

To develop this political clarity, participants and scholars turned to intellectual traditions beyond the field of mathematics education. The authors included in Paper 2 referenced, among other traditions, decolonial theory, Critical Race Theory, Black liberatory fantasy, Indigenous knowledges, and queer theory. The educators I interviewed drew inspiration from Black studies, women's studies, ethnic studies, critical pedagogy, abolitionist teaching, ethnomathematics, and rehumanizing mathematics (Alexander, 2010; Delpit, 1988; Freire, 1970; Ginwright, 2016; R. Gutiérrez, 2018; hooks, 1994; Joseph, 2011; Love, 2019; Moses & Cobb, 2001; Muhammad, 2020; Rogers & Kaiser, 1995/2005; Yeh et al., 2021). Collectively, they made clear that questions of liberatory mathematics education demand a wide-ranging analytic perspective.

Harm, Healing, and Social Connection Through Mathematics

Healing from oppression is another common theme across papers. Educators who invoked the idea situated personal and relational healing within an analysis of the systemic issues that hinder well-being. They discussed the emotional, epistemological, and ontological harm school mathematics can cause, to individuals and communities, and the specifically racialized nature of this harm. Lena, Naya, and Ty, all with backgrounds in restorative justice, saw possibilities for healing through circle processes, critical conversations, and opportunities for play and inquiry in classrooms. Kate and Lydia described how mathematics classrooms can disrupt patterns of harm by valuing belonging, mutuality, and multiple perspectives rather than shame, competition, and single right answers. Although critical reimagining does not center the notion of healing, some scholars in Paper 2 address it in their analyses of restorative relationships and self making around mathematics (Fasheh, 2015; R. Gutiérrez, 2018; Kokka, 2019; Sengupta-Irving & Vossoughi, 2019). On this theme, the second two papers extend a key strand of Lena's liberatory pedagogy: while making clear that healing transcends disciplinary boundaries, they suggest that it is possible, potentially transformative, and especially urgent in mathematics.

Across papers, stories of healing are rooted in social connection with others around mathematics. Lena reflected on the deep "knowing" and emotional solidarity she developed with individual children across disciplines, including in mathematics when she sensed children felt shame or exclusion. Educators in Paper 3 recounted bonding with friends over mathematics problems in daily life, belonging to a multigenerational community of STEM intellectuals that resembled a form of kin, and building classroom environments that honored each member's full personhood. Scholars who write about humanizing relationships in mathematics examine youth's communities of racialized belonging, relations of care and dignity among family and peers, and individual worth tied to community purposes for mathematical activity (Fasheh, 2015; R. Gutiérrez, 2018; Nasir, 2002; Sengupta-Irving & Vossoughi, 2019). In this literature, few stories of rehumanization are situated in school classrooms, signaling an area for further inquiry to which the educators in Papers 1 and 3 might contribute.

Life Beyond School

Considering both in- and out-of-school contexts, the three papers underscore the importance of mathematics learning that connects authentically to people's lives beyond school. In Lena's classroom, disciplinary learning in the humanities involved exploring stories and engaging in community projects that brought the broader world into the classroom in rich ways. Connecting with "the content of [children's] lives" was core to Lena's pedagogy, though she did not see authentic openings for it in mathematics. Educators in Paper 3 spoke to possible openings, through social justice math trails (Brent), "make-do stories" about mathematical concepts (Lydia), empathy interviews (Kate), and curriculum as a "platform" for young people's interests (Cliff).

Critical reimagining, specifically the principle of horizontality, turns attention to the diversity of mathematics extant in people's daily lives and calls for re-placing disciplinary learning "as youth do life" (Warren et al., 2020, p. 283). The scholarship in Paper 2 explores *in situ* mathematical practice in workplace, home, and recreational contexts beyond school as well as culturally embedded forms of mathematics that educators integrate into curriculum (Aguirre et al., 2012; Barajas-López & Bang, 2018; González et al., 2001; Lave, 2011; Lipka et al., 2005; Nasir, 2002; Taylor, 2011). Focusing on the cultural repertoires of historically marginalized communities, authors warn of tendencies to flatten or assimilate cultural heterogeneity within the confines of school mathematics (Aguirre & Zavala, 2013; Civil, 2007; Pais, 2011; Warren et al., 2020). Educators in Paper 3 also cautioned against inauthenticity as they discussed curriculum design that builds from students' experiences. As Paper 2 concludes, educational research should continue to engage the question of whether re-placing mathematics learning can truly be the work of school-based curriculum and pedagogy.

Creative Inspiration in Mathematics

Threading across the papers is a vision of mathematics learning as joyful and creative. This spirit was fundamental to Lena's liberatory pedagogy: it came through in her ways of engaging with children, personal relationship with humanities disciplines, and orientation to curriculum building as a kind of "art making." The analysis in Paper 1 asks how Lena and others might find a similar sense of creative inspiration in mathematics. Educators in Paper 3 shared stories of joy in mathematics class during group games, class discussions, and math team practice. They spoke most clearly of creative inspiration when they speculated about mathematics learning beyond their immediate teaching contexts—in various kinds of crafts, the work of environmental engineers, and imagined spaces for community inquiry around mathematics.

The papers also surface a common tension around this theme. Recognizing

mathematics in creative domains that are not often considered mathematical can expand our conception of the discipline—who practices it, where, and toward what purposes. If not approached with sensitivity, it can also lead to a reductive mapping of dominant mathematics onto artistry. Lena was wary of instrumentalist notions of mathematics "for" art making. Scholars in Paper 2 caution against curricular efforts that appropriate culturally embedded forms of creative practice as a backdrop for conventional school mathematics, often losing touch with the authentic purposes and cultural meanings in those practices (Barajas-López & Bang, 2018; Civil, 2007; González et al., 2001; Taylor, 2011). All three studies suggest a need for further research on educational designs that replace mathematics learning in authentic creative activity (Warren et al., 2020).

Variations

In this part, I discuss a theme from each paper that is less prominent in the other two. Then I share the analyses' various takes on a core dilemma of liberatory teaching: how to engage with the normative structures of K–12 school systems.

Paper 1 Highlight: The Microrelational Work of Liberatory Teaching

Paper 1 highlights the microrelational work involved in Lena's teaching, the ways liberatory praxis involved something beyond "more radical subject matter" (hooks, 1994, p. 148). Through moment-to-moment interaction and relationship building across the year, Lena developed deep mutual trust, care, and commitment with children. She attended to the smallest details of interaction—while teaching and in her reflections articulating how these were embedded within the social fabric of her class community and broader relations of power in society. This insight extended to mathematics teaching and learning, though she was less confident in its transformative potential in mathematics contexts.

The theme of microrelational dynamics came up but was not central in the other two studies. In Paper 3, Kate and Lydia described teachers' efforts to create cultures of belonging, trust, and "mattering" in mathematics classrooms, specifically while facilitating class discussions. However, moment-to-moment interaction was not emphasized in all conversations, and I would say that my work with Lena attuned me to it in Kate's and Lydia's reflections. The microrelational scale of liberatory pedagogy emerged in Paper 2 as a topic for further research, particularly in the context of classroom teaching. Critical analyses of social interaction around mathematics tend to focus on oppressive contexts, out-of-school settings, or relationships among young people apart from their teachers (e.g., Barajas-López & Bang, 2018; Gholson & Martin, 2014, 2019; Nasir, 2002; Sengupta-Irving & Vossoughi, 2019; for an exception see van Es et al., 2022). Lena's attention to the microgenetic aspects of liberatory classroom pedagogy, and the questions our conversations raised about its transformative power in mathematics, point to an area for continued inquiry.

Paper 2 Highlight: Turning a Critical Eye on the Discipline

The synthetic literature analysis calls for turning a critical eye on mathematics as a discipline—on the politics of mathematical knowledge, not only the social interaction, pedagogies, and policies that develop around it. Extant literature on the politics of mathematical knowledge is largely theoretical or from fields of research that do not specifically explore educational practice (e.g., Ascher, 1991; D'Ambrosio, 1985; Ernest, 2012; Triadafillidis, 1998). Practice-based scholarship in this area engages undergraduates and pre- and in-service teachers in epistemic critique, but less often youth (e.g., Anderson, 1990; Das & Adams, 2019; de Freitas & Zolkower, 2009; McBride, 1994). Paper 2 points to the need for learning designs that invite young people to examine the sociohistorical foundations of mathematics, its multivocality, and the powered relations among diverse forms of mathematics.

While Papers 1 and 3 address disciplinary learning that furthers political knowledge, educators reflected less on the politics of disciplinary knowledge (Frankenstein, 2013). Lena's class read, wrote, and made art to learn about (de)colonization and voting rights; Naya, Ty, and Brent explored the mathematics of student loans, gerrymandering, and the gender pay gap with their students. Less common in participants' classrooms, as across K-12 education, was critical problem posing about disciplines themselves, including their histories, governing norms, and ongoing evolution. When we spoke, Brent was just beginning conversations with his classes that interrogated definitions of "mathematics" and "mathematician." He and other educators, including Lena, spoke about mathematics as human cultural practice, its diversity of forms, and the discipline's onto-epistemic normativity, but these were not the subject of inquiry with young people. As Paper 2 suggests, future research might focus on this kind of critical disciplinary learning, positioning youth and educators as active participants in the critique and creation of mathematical knowledge (R. Gutiérrez, 2018; Warren et al., 2020).

Paper 3 Highlight: Teachers Examining Their Relationships with Mathematics

A theme unique to Paper 3 is that educators' histories and identities as learners of mathematics shape their teaching. Further, reflecting on personal relationships with mathematics can offer a sense of clarity as teachers develop their mathematics pedagogy. All educators in Paper 3 had, at some point in their lives, experienced mathematics learning as liberatory, though this was not necessarily in school and, for some, occurred later in life. Several participants described formative moments as mathematics learners that were decidedly not liberatory—points in their educational trajectories at which they had uninspired or fraught relationships with mathematics. What was critical in their stories was that they viewed these relationships and their disciplinary identities as open to change, often through the support of professional community. Believing that they could "still be a learner in it" (Lydia) enabled educators to experiment with and trust in liberatory mathematics with children.

This theme speaks to Lena's case. Lena suggested that her personal history of school mathematics, tied with broader questions of racial identity and social belonging, shaped her "gut negative reaction to math" as a teacher. While she had deep insight into these linkages, she expressed a need for professional experiences that would open toward alternatives, for her own relationship with mathematics and her classroom pedagogy. I have imagined the learning that would be possible, for all involved, in a professional collaboration among Lena and the teacher educators in Paper 3. The broader field of mathematics education could benefit from professional learning designs that invite teachers to reflect critically on their personal relationships with mathematics, a

suggestion elaborated below.

Various Perspectives: Responding to Normative Structures of Schools

The three studies offer various responses to the question of how educators with liberatory commitments might engage with the normative demands of K–12 schools. These demands include mandated curriculum, accountability systems tied to standardized assessment, and vertical trajectories of disciplinary content and school attainment. The possibilities put forth across the papers were not mutually exclusive, and, as each analysis suggests, liberatory praxis can encompass multiple possibilities simultaneously.

The educators in Paper 3 described critically navigating dominant systems, finding liberatory pockets within them, and seeking radical departures. Some participants emphasized one orientation, but all communicated that they saw a place for each approach. At an independent school that valued teacher autonomy, Lena enjoyed more professional freedom than educators who worked in district settings. Still, in mathematics she felt similarly pressured by prescribed curriculum sequences and middle school entrance exams, often not finding liberatory pockets or possibilities for departure from them. In her pedagogy beyond mathematics, Lena supported critical navigation through humanities projects that integrated dominant literacy and communication skills with critical consciousness raising, creative expression, and community engagement. Her pedagogy departed from normative schooling most radically in the community ethos of her class and the relationships she built with children, as discussed above.

Paper 2 emphasizes radical departure. While I analyze literature that discusses critical navigation (e.g., Aguirre et al., 2012; Frankenstein, 1983; Gutstein, 2006; Lipka

325

et al., 2005) and liberatory pockets (e.g., Buenrostro & Radinsky, 2019; Lunney Borden, 2011; Ma, 2017), the paper is guided by calls to delink from the governing assumptions of EuroWestern schooling (Mignolo, 2009; Quijano, 1992; Warren et al., 2020). Some scholars question whether radical departure is possible within K–12 schools; educators in Paper 3 raised a similar question as they described liberatory contexts beyond classroom walls. As I suggest in Paper 2, educators and researchers might continue to make space for liberatory praxis within K–12 settings—critically analyzing, diversifying, and rehumanizing mathematics teaching and learning—as the field concurrently explores possibilities that extend "infinitely deeper and broader than school" (Warren et al., 2020, p. 283).

Directions for Future Inquiry

The previous section proposed a number of avenues for future research. These include inquiry into the microrelational aspects of liberatory mathematics pedagogy, the possibilities and limitations of horizontality in school-based settings, mathematics learning embedded in authentic creative activity, learning spaces that engage youth in epistemic critique and the creation of new mathematics, and professional learning that supports teachers to critically reflect on their mathematics identities. Below, I highlight additional implications for work in mathematics education and teacher learning, from across the dissertation's papers.

Mathematics Education

This dissertation touches on but does not explore in depth the ways participants' local contexts (e.g., institutional setting, professional culture, student demographics, and

teacher positionality) shape their vision and practice of liberatory pedagogy. On one hand, this allows for an analysis that cuts across experience and may speak to contexts, such as Lena's class, that are diverse along multiple dimensions of identity. On the other, it may signal a lack of political specificity or attention to the situated nature of oppression and liberation (Crenshaw, 1989; hooks, 1994; Taylor, 2017; Weiler, 1991). Research should continue to pursue stories of liberatory mathematics that highlight the varied forms liberatory pedagogy can take across contexts. Martin and colleagues (2019) have begun to theorize Black Liberatory Mathematics Education; Gutiérrez (2017, 2019) puts forth mathematx, reflecting Indigenous principles and ways of knowing; and scholarship on culturally relevant and embedded mathematics pedagogy emphasizes the particular knowledge systems, languages, cultural practices, and racial identities of the communities in which mathematics is situated (e.g., Barajas-López & Bang, 2018; González et al., 2001; Lunney Borden, 2011; Matthews, 2009; Nasir, 2002). As Paper 2 suggests, further research could examine the demands of liberatory pedagogy with learners who hold dominant social identities and have historically been empowered through school mathematics (Kokka, 2020; Skovsmose, 2016).

This dissertation would also offer a fuller vision of liberatory pedagogy if it included windows into mathematics teaching and learning with very young children and in informal educational settings. Lena worked with the oldest group of children at her elementary school; the educators in Paper 3 mostly had experiences at upper elementary, middle, and high school levels; and the majority of empirical studies analyzed in Paper 2 focused on learning settings with older youth. Few studies examine questions of justice and liberation in mathematics learning with young children. In what ways do the liberatory pedagogies discussed in this dissertation extend to early childhood contexts, and what are the unique challenges and possibilities with younger age groups? Research should also continue to explore liberatory mathematics teaching and learning in the diversity of contexts outside of schools. Prior work has examined various out-of-school programs and settings, some of which tether to the expectations of school mathematics more than others (e.g., Barajas-López & Bang, 2018; Gresalfi & Chapman, 2017; LópezLeiva et al., 2013; Nasir, 2002; Terry, 2011). Further work is needed to shed light on the mathematics learning that takes place beyond the reach of school systems.

In any context, situated studies of liberatory mathematics would triangulate interview data with records of teaching and learning in action. While the methods in Paper 1 included observations from Lena's class, the analysis focused on Lena's reflections, and neither empirical study involved co-design with educators in their classrooms. Understanding the opportunities and challenges of liberatory pedagogy requires studying it in practice. Participatory and community-based design research (Bang et al., 2016; Bang & Vossoughi, 2016), critical action research (Fine, 2018; Mendoza et al., 2018), and social design experiments (K. D. Gutiérrez & Jurow, 2016; K. D. Gutiérrez & Vossoughi, 2010) are all approaches to co-design that could build on the critical conversations with educators in this dissertation. Further, these approaches center the voices and visions of young people and their communities, which too often are left out of research on liberatory mathematics education.

A broader question that emerges across the three papers is: what needs liberating,

328

and where should liberatory efforts focus? Educators and scholars attend variously to mathematics content (e.g., concepts, methods, representations), mathematical practice and ways of knowing, the questions and purposes to which mathematical knowledge is applied, and the social relations people develop around mathematics. Since domination and exclusion manifest in each of these, our collective liberatory praxis should address them all. As discussed in Paper 2, critical scholarship tends to focus on the applications of mathematical content and practice. This dissertation's studies make clear that using dominant mathematics as a tool for critical and culturally relevant inquiry is one piece, but not the totality, of liberatory pedagogy. In Paper 1, Lena expressed reservations about treating mathematics as a "tool for" sociopolitical analysis or everyday tasks rather than as a freeing activity in itself—that is, an experience of interacting with others, ideas, and the material world that, moment to moment, creates liberatory relations. Additionally, more work could focus on the actual ideas and practices that people engage with in contexts of liberatory mathematics.

Finally, Papers 1 and 2 raise questions about the metaphors we use to conceptualize liberatory mathematics pedagogy. Often, discourse on social justice and culturally relevant mathematics emphasizes "integrating," "braiding," or "bridging" (dominant school) mathematics with themes from young people's lives, without problematizing what constitutes "mathematics" (Larnell et al., 2016). As with the notions of rift and repair (Booker & Goldman, 2016), we should be wary of metaphors that assume solutions lie in integrating or narrowing the gap between school mathematics and liberated living. Integration and repair do not necessarily address underlying power structures that need to change. Metaphors that suggest something more transformative include opening the cage (Skovsmose & Greer, 2012), leading out (Craft, 1984), taking flight (Gholson, 2019), and breaking free (de Freitas, 2012; see also participants in Paper 3). Since metaphors shape the ways we engage with the world (Lakoff & Johnson, 1980), taking care with the terms we use is an important part of liberatory praxis.

Teacher Learning

This dissertation also raises questions related to teacher learning, in and beyond mathematics education. How did the educators in Papers 1 and 3 develop the political analysis, ethical commitments, and instructional practices involved in their liberatory pedagogy? What forms of professional development cultivate and sustain these, especially within educational systems that pressure teachers to reproduce the status quo? Below, I suggest possibilities for teacher inquiry.

Reflective Conversations on Teaching Practice

Reflective conversations like those in Studies 1 and 3 can be a context for educators to ask "big questions" about their practice (Lena, Study 1) and envision alternatives to normative ways of teaching. Conversations might be one-on-one with colleagues or teacher educators, or bring together groups of educators across classrooms and schools. They could involve storytelling (e.g., Sengupta-Irving et al., 2013), descriptive study of student work (e.g., Himley & Carini, 2000), video-based reflection (e.g., McDuffie et al., 2014; van Es et al., 2022; van Es & Sherin, 2008), or debrief of classroom visits. To maintain political clarity, it is important that reflective conversations situate questions about teaching practice within structures of power in society (Bartell et al., 2021; Calabrese Barton et al., 2020; Louie et al., 2021; Rosebery et al., 2015). This includes attention to the onto-epistemic norms that govern learning interactions and tasks (de Freitas & Zolkower, 2009; J. F. Gutiérrez & Scott, 2019; Lemke, 1990)—a subject less often addressed with teachers, even in critical work on mathematics education. Finally, educators could reflect on their personal relationships with mathematics, examining how their sociopolitical and disciplinary identities have developed in relation to one another, in and out of school, and how these shape their work with young people (Martin, 2006).

Shared Study

There are various forms of collaborative inquiry that could augment mathematics educators' reflection on instructional practice. These include study of the discipline's social history, diverse traditions of mathematical practice, and critical and creative works not specific to mathematics. Critical study of the discipline would explore its historical foundations, including patterns of cultural expropriation and erasure of the many forms of mathematics around the world (D'Ambrosio, 1985; Joseph, 1991/2011; Restivo, 1992). This "social studies" of the discipline would also demystify dominant assumptions about what it means to know in mathematics (e.g., objectivity, rationalism, certainty), tying these to broader cultures of supremacy in society and examining how they manifest in everyday curriculum and pedagogy (J. F. Gutiérrez & Scott, 2019; McBride, 1989; Yeh & Rubel, 2020).

As the educators in Study 3 suggested, engaging as ongoing learners of mathematics—in contexts that support joint inquiry and expansive notions of what counts

as mathematics—can develop educators' disciplinary identities in ways that support liberatory work with young people. One purpose of doing mathematics together would be to allow educators to experience mathematics more expansively than is common in schools: as conceptually rich rather than procedural, collaborative rather than individualistic and competitive, and open to multiplicities of meaning rather than driven by singular answers. Educators could also engage with mathematical ideas and practices from a range of cultural traditions, considering connections to their lives and the ways such exploration might enrich mathematics curriculum (e.g., Ascher, 1991, 2002; Bishop, 1988; Joseph, 1991/2011; Sterenberg et al., 2010).

Finally, educators could examine critical and creative works from a range of disciplines (e.g., essays, scholarly and news articles, poetry, fiction, visual arts, music, dance) that raise themes of identity, oppression, and freedom. They could engage in collective processes of observation and response to discuss the meanings these pieces evoke. While such study might draw explicit connections to mathematics pedagogy, its main purpose would be to deepen educators' political-ethical perspectives and sensibilities more broadly.

Further Considerations

To close this section, I offer three considerations related to directions for professional learning. First, each form of study suggested above would ideally be linked to opportunities to design curriculum or try out pedagogical approaches inspired by the critical perspectives educators are developing. Since the demands of school accountability systems so often constrain expansive design, instructional leaders need to make intentional space for it and take caution not to flatten teachers' learning into a set of reductive practices or lesson plan templates. Supporting teachers to assume a stance of "let's see" and seek out "liberatory pockets" in their practice (Naya, Study 3) does not require a total abnegation of school system mandates, but it does necessitate a sense of freedom to ask, "What could be an alternative?" (Kate, Study 3) as well as ongoing vigilance against the pressures to "go back into what the traditional space does" (Cliff, Study 3).

Second, as in any learning setting, critical professional development spaces need to consider the diversity of experience and areas for growth that educators bring. Some may be new to questions about the sociopolitical dimensions of teaching and learning. Others, like Lena, may have a mature political analysis but not perceive liberatory possibilities within mathematics specifically. Still others may have developed particular approaches to liberatory mathematics teaching and be curious to explore further possibilities.

Third, Paper 1 highlights the potential of cross-disciplinary approaches to critical pedagogical inquiry, particularly in elementary settings in which teachers work with children across content areas. In my conversations with educators, many of the principles and practices participants discussed were not unique to, and often had origins outside of, mathematics. Identifying cross-cutting elements of liberatory pedagogy and exploring them in various disciplinary contexts would broaden possibilities the field considers for mathematics teaching. Such inquiry could also be generative for teams of teachers coming together across departmentalized subject areas. I am cautious not to advise

simply cross-pollinating methods and perspectives across settled boundaries; further work is needed to understand the transdisciplinary nature of liberatory pedagogy, questioning our assumed parameters for disciplinary learning, teaching, and knowledge making (Gaztambide-Fernández, 2014; Takeuchi, 2020; Warren et al., 2020).

Conclusion

In Chapter Two, I described liberation as freeing from hierarchy, violence, and the hegemony of single stories; assuring dignity and the capacity to flourish; honoring heterogeneity as fundamental; entwining self-determination and agency with belonging and interdependence; and nurturing reciprocal, sustainable relationships among all forms of life (Adichie, 2009; Combahee River Collective, in Taylor, 2017; Escobar, 2015; Espinoza et al., 2020; Evans & Vaandering, 2016; Gumbs, 2020; G. Gutiérrez, 1973/1988; Kimmerer, 2013; Rosebery et al., 2010; Walcott, 2021). The educators and scholars in Chapters Three through Five discuss pedagogies that strive toward and in various ways enact this vision. The dissertation as a whole raises questions about how mathematics itself might be liberatory—in addition to, and perhaps as a result of, the ways we organize mathematics teaching and learning.

Asking about liberation in contexts where it is less often considered (e.g., doing mathematics) makes space for the practice of freedom in unexpected places. It can also clarify, nuance, and extend our notions of liberation. Considering why an educator with deep liberatory commitments did not see a place for them in mathematics, and imagining her mathematics classroom as one that could be liberatory, invites us to (re)examine the entailments of liberatory praxis. A decolonial analysis of mathematics education

scholarship might contribute to our theories of delinking (Mignolo, 2009; Quijano, 1992) in education and more broadly. Reflecting on mathematics teaching as the practice of freedom, with educators who centered liberatory vision but did not always use this language to describe their work, could broaden our view of where freedom work is happening.

At the close of our interview, Naya shared that she was "excited at the possibilities of where this conversation can go." I hope that the conversations in this dissertation inspire continued inquiry into liberatory education, in and beyond mathematics. I am excited about where such inquiry can go—for the seven participating educators, practicing teachers generally, and those of us learning with and from them.

APPENDIX: GENERATING DATA SOURCES FOR STUDY 1

Conversation Type	Number	Duration (min.)	Focus
General interviews	2	65–75	Frameworks for teaching, class routines and practices, hopes for children, relationship building, goals and experiences specific to disciplinary learning (in mathematics, reading, writing, and social studies)
Video reflection conversations	4	50-80	Math, whole class Math, 1–1 with a child Writing, half group with second–years Affinity group with three male first-years
Class visit debriefs (without video)	5	20–75	Events of most recent class visit (usually same day or within 1–2 days after)
Visiting conversations	6	30–70	Range of topics, professional and not
Member checking after school year	1	90	Themes and conceptual framing from my analysis; partnership process

Table A1. Conversations with Lena.

Participant	Age Group	Racial/Ethnic Identity and Sex Assigned at Birth (self-identified)	Duration of interview (min.)
Cam	Second-year	Korean and Asian American, male	28
Carter	First-year	White, male	22
Graceyn	Second-year	White, female	22
Juju	First-year	Korean, female	27
Mariame	First-year	White, female	22
Рорру	Second-year	Black and African American, female	23

Table A2. Interviews with Children in Lena's Class.

Period of the School Day	Number of Visits	
Mathematics	3	
Literacy (Reading, Writing)	3	
Social Studies	1	
Rap Around / Affinity Groups	2	
Grad Projects	2	
Clubs	2	
Community Care	3	
Reflection (end of day)	3	
Learning Fair (one-time event)	1	

Table A3. Visits to Lena's Class.

Note. I often visited more than one period on the same day. Most periods lasted between 45–60 minutes; Community Care was 30 minutes and Reflection 15 minutes. *Note*. Social Studies sometimes extended into Literacy, Community Care, and Reflection.

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