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A course of study in arithmetic emphasizing the use of instructional aids for grade five, Ludlow, Massachusetts.

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Service Paper

A COURSE OF STUDY IN ARITHMETIC EMPHASIZING
THE USE OF INSTRUCTIONAL AIDS FOR GRADE FIVE,
LUDLOW, MASSACHUSETTS

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CHAPTER I

INTRODUCTION

Statement of the Problem:

The purpose of this project is to build a course of study for Grade 5 in Ludlow, Massachusetts, by elaborating on the textbook presently available in Ludlow through the extensive use of instructional aids and devices. By utilizing these materials it is intended that the course of study will provide for two main objectives: (1) meaningful arithmetic, and (2) socially significant arithmetic.

As to what is meant by "meaningful arithmetic," the writer considers Brownell's definition appropriate:

The 'meaning theory' conceives of arithmetic as a closely knit system of understandable ideas, principles, and processes. According to this theory, the test of learning is not mere mechanical facility in figuring. The true test is an intelligent grasp upon number relations and the ability to deal with arithmetical situations with proper comprehension of their mathematical as well as their practical significance.

By "socially significant" arithmetic we mean the value of arithmetic, its importance, its necessity in the modern social order. As Buckingham puts it, the idea of significance is


Another objective for this course of study, and a more specific one, is to facilitate the transitional learning through instructional aids from concrete objects and situations to the number or quantitative abstractions.

Before going any farther, it is necessary to define what we mean by instructional aids and materials. The definition given by Grossnickle, Junge, and Metzner seems adequate:

Instructional materials include anything which contributes to the learning process...any picture, book, real activity, model, or teaching aid which provides experiences to the learner for purposes of (a) introducing, enriching, classifying, or summarizing abstract arithmetic concepts, (b) developing desirable attitudes toward arithmetic, and (c) stimulating further interest and activity on the part of the learner in the subject.

Justification:

This project seems justified insofar as there is no course of study in arithmetic in Ludlow at present, and the program is confined, for the most part, to the textbook. It is hoped that this program might induce other teachers in the system to enrich their arithmetic programs in a like manner.

Scope:

The writer will use the existing textbook as a basis and elaborate on it by emphasizing the use of instructional aids

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and materials whenever feasible and at the same time adjust the program to local situations. The plan will embrace a school year's work, from September to June.
CHAPTER II

REVIEW OF RESEARCH

It is generally recognized that during the last thirty or forty years more fundamental improvements have been made in the teaching of the language arts than in the teaching of arithmetic. This may be so because in general the teaching of the language arts has been related more closely than the teaching of arithmetic to the children's own first-hand experiences. The improvements also may be due to the growing realization that language communication occurs only to the degree that language symbols have the same meaning for the listener that they have for the speaker, and the same meaning for the reader that they have for the writer.

If language teaching can be improved by relating language symbols to the child's concrete experiences, it seems probable that arithmetic teaching can be improved by relating arithmetic symbols to the child's concrete experiences, and if language teaching can be improved by emphasizing the meaning of language symbols, it seems probable that arithmetic teaching can be improved by emphasizing the meaning of arithmetic symbols. Do not arithmetic symbols, like language symbols represent things, ideas, and relationships? Are not arithmetic symbols, like language symbols, a means of communication?

So says Hickerson in the preface of his book, *Guiding Children's Arithmetic Experiences*; and it is upon this logical assumption that the writer bases this course of study. It will be noticed that Hickerson suggested two phases of arithmetic teaching, that of utilizing the children's own concrete experiences and emphasizing the meaning of arithmetic.

According to Brownell, these two phases have been

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neglected in the past, for he says:

The arithmetic programs of the past twenty-five years have been inadequate chiefly at two points. First, these programs have given children little chance to use ideas and skills already learned in solving their own personal problems. Second, these arithmetic programs have neglected the meanings and rational principles which make arithmetic a phase of mathematics.

Brownell mentions further that one of the great faults of elementary education is to classify arithmetic as a skill or a drill or a tool subject. When arithmetic is looked upon as such and taught accordingly, the results of such teaching indicate just what we have been getting in our schools for the past several decades; namely, arithmetical incompetence. When the tool conception of arithmetic is utilized, the teaching process tries to tell children what to do but not why they do it. Then by much drill they have to do it until they can demonstrate some degree of mastery.

Arithmetic, however, is not a tool or a drill subject, according to Brownell. Proficiency is necessary, of course, as most everyone will agree; but more than proficiency (speed and correctness in computation) is demanded by the conditions of life. We may get along with mechanical skills alone as long as those skills are used in situations that are familiar. But as we go from the familiar to the unfamiliar we must be able to

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1 Ibid.
2 Ibid.
think—and one does not think effectively with mechanical skills alone. Thinking is possible only to him who possesses rich meanings. We have been told for many years that skills can be used intelligently only when they have been acquired intelli-
gently; hence, the importance of meanings in arithmetic.

Thiele sums up the current trend in arithmetic when he says, "The keynote of the new arithmetic is that it should be meaningful rather than mechanical."

Through meanings we secure insights and note rela-
tionships which without meanings, we should not be likely to hit upon. The insights in turn enable us to foresee connections and to tie together various aspects of the learning task which without understanding, would have to be mastered separately, one at a time.

There are at least four good reasons why meanings should be taught in arithmetic and these are:

1. Arithmetic can function in intelligent living only when it is understood.
2. Meanings facilitate learning.
3. Meanings increase the chances of transfer.
4. Meaningful arithmetic is better retained and is more easily rehabilitated than is mechanically learned arithmetic.

If it is true then, that arithmetic must be made meaningful, and most authorities seem to think so, the question now arises—how are we to make arithmetic meaningful?


3 Ibid.
"The psychology of meaningful teaching suggests that books and courses of study should not contain rules and explanations, but instead, directions for activities with questions and suggestions to guide and aid children in generalized thinking."  

Dewey cautions us, however, when he states, "Meanings are not themselves tangible things." He said, "The children should put number into things."  

According to Thiele:  

One of the first problems of the middle-grade teacher is that of anchoring meanings to things...and if so it follows that number symbols and terms serve a definite purpose, namely, to designate objects as well as to indicate the ways in which objects are manipulated. If this principle is adhered to, symbolization and meaning must develop together. Children will neither deal with symbols which have no designation nor will they consider meanings which cannot be symbolized.  

Thiele also suggests that special emphasis be placed upon the need for concrete material in a meaningful program of  

1 C. L. Thiele, "From Concrete Experiences to the Higher Mental Processes of Arithmetic," Education. 61:480, April, 1941.  
5 Ibid.
arithmetic instruction. This is in keeping with the idea that children should first make number records of situations in which objects are manipulated before moving to described or abstract levels. In short, an effort should be made to indicate the nature of an arithmetic program which may be termed meaningful.

Meaningful arithmetic then does not mean the memorization of a few abstract statements; neither does it result from computing endless masses of abstract examples. Meaningful arithmetic comes only by engaging in a wide variety of appropriate activities, first with concrete objects by which the process or diagrammatic representations which reveal the function of the operation, and only finally by solving many verbal problems in which the need for the operation occurs in many language forms. No system of instruction, textbook, or course of study which fails to provide opportunity for the learning of these meanings can properly be said to produce meaningful learning regardless of the claims made for that program.

It has been stated earlier that another objective of an arithmetic program is that of social significance. Morton states: "The function of instruction in arithmetic is to teach the nature and use of the number system in the affairs of daily life and to help the learner utilize quantitative procedure effectively in the achievement of his purpose and those of the social order to which he is a part."

To make arithmetic truly significant for pupils, teachers find it necessary to study carefully the ways in which arithmetic functions in the home, school, and


community, and then devise methods of helping pupils to develop an awareness of the large variety of life situations in which arithmetic is used, and methods of helping pupils to deal successfully with the quantitative aspects of the situations discovered.\(^1\)

\(^2\) Sauble, when discussing textbooks themselves, says that they seek to emphasize the significance of numbers by including a great variety of problems to solve, problems of the type that arise in the everyday experiences of the children or their parents, or problems which the children may be called upon to solve in later life. The problems may be of two types: a series of problems all based upon a social situation, or a group of problems about different social situations, but all emphasizing the uses of some phase of the work, as fractions, decimals, or percentage. The success of textbook problems in giving significance to arithmetic depends upon the extent to which pupils recognize them as representative of real life situations.

Some of the activities that would give social significance to an arithmetic program are:

1. Dramatization of life situations and business procedures.
2. Interviewing experts to obtain information about topics being studied
3. Taking trips


\(^2\) Ibid., p. 182.
4. Preparation of posters, bulletin board displays, and other types of exhibits
5. Preparing and giving special reports
6. Constructing measuring instruments, of models, and of various devices
7. Making a class scrapbook or individual scrapbooks to hold clippings, illustrations, and other supplementary materials
8. Analyzing imaginary life situations having quantitative aspects about which first-hand information may be obtained
9. Participating in activities of the home, school, and community which have definite quantitative aspects.¹

When the social phase of arithmetic became recognized as important in education, it was greatly stressed in some quarters to the exclusion of the mathematical phase. In other words, the teachers stressed teaching pupils how to meet problems that they would encounter in their everyday lives and were not concerned about arithmetic as an orderly and systematic way of thinking. Now, however, the pendulum is swinging back and the authorities are recognizing both phases as equally important.²

Morton, speaking for the National Council Committee on Arithmetic, said: "The committee stands for a kind of arithmetic in which both the mathematical and the social aims are clearly recognized as interdependent and mutually related." Buckingham³

¹ Ibid.
follows the same point of view when he states: "A teacher should use a socially significant approach, but his teaching of a given unit is not complete until the goal of mathematically meaningful ideas has been reached...We must, therefore, do two things. We must teach arithmetic as a social study, and we must teach it as mathematics." Bond considers these two objectives as very different but mutually helpful.

Perhaps the most pertinent statement for our purposes is that of Brueckner when he writes:

Arithmetic is conceived as a closely knit system of understandable ideas, principles and processes, and an important test of arithmetical learning is an intelligent grasp of the number relations together with the ability to deal with arithmetical situations with proper comprehension of their mathematical significance. The desired outcomes, related to the mathematical and the social phases, can most likely be secured by making certain that the pupil has vital social experiences in which he is led to see the number involved in situations that are meaningful and significant to him, and if at the same time the teacher takes the steps needed to make certain that the mathematical meanings of the number elements involved is fully grasped by the learner...The aim of the teacher at all levels should be to develop in the pupils the ability and the disposition to view the affairs of life in orderly, systematic ways, and to use quantitative techniques, when feasible, to enable them to see the relationships more clearly and exactly.

Now that a dual purpose for our course of study has been

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established, it is necessary to determine how this can best be accomplished. The success of a meaningful and socially significant program in arithmetic depends, in a large degree, upon methods and materials of instruction which will suffice in all situations. The skillful teacher selects methods and materials in terms of the outcomes to be achieved and the needs and the interests of the children. If instruction in arithmetic is to insure a steady growth in understanding number relations, a wide variety of instructional materials must be used to enrich and to supplement the learner's experience.

It is now generally agreed that concrete materials must be accessible in the classroom. A great advocate of concrete materials, Catherine Stern, says that there is no other way for the child to acquire abstract concepts but to develop them by himself from first-hand experiences with concrete objects.

Sueltz suggests that schoolrooms have at least a minimum of mathematical equipment, such as a meter, standard measures, various containers, geometric forms, and similar things. He adds that this equipment should be handled as well as seen by the pupils.


Morton outlines the use of concrete materials and at the same time signifies the role of the teacher.

Certain very elementary phases of number experiences must be provided by the teacher. Because of their nature they cannot be provided by the textbook or the course of study. In the development of most topics, the first experiences should be concrete. That is they should be sensory in character; they should deal with objects which can be seen and handled. Later experiences may be semi-concrete; they may deal with pictures of objects and with diagrams. Obviously, the object stage cannot appear in the course of study or in the textbook. It must be supplied by the teacher. The course of study and the teacher's guide which accompanies the textbook should offer suggestions, but after that it is up to the teacher to do the job. The semi-concrete stage and the abstract stage can very well appear on the pages of the textbook. But the concrete stage must be supplied by the teacher.

The primary purpose of the use of such visual learning aids is to discover or demonstrate the meaning and nature of arithmetic processes and related facts. However, the use of such materials does not guarantee, and in some cases does not facilitate, the development of meaning. They are steps toward the development of abstract thinking, and, since they can be seen or manipulated, their use may do much to build up the child's confidence as well as his understanding.

The teacher needs to be careful when objective materials


are used for demonstration purposes. He must always make certain that the pupils go beyond the mere activity and try to sense the idea which is being demonstrated. The teacher can stimulate this type of thinking through his questions and by placing emphasis upon certain points.

In the same manner, pupils at work with their own objective materials need skillful teacher direction and guidance to help them to move from immature to more mature ways of thinking about numbers. It should be recognized that the manipulation of concrete objects represents only the first stage in the development of pupils' number ideas. In the second stage of progress pupils are able to "think" certain arrangements when the objects are present or imagined, and pupils have developed the ability to use the language of number. As pupils analyze, assemble, and compare groups of objects, the teacher needs to guide their thinking to the point that advancement will be made steadily toward a higher stage.

To summarize what has been written before, there seems to be general agreement now that an arithmetic program should stress the mathematical phase as well as the social phase. Instructional materials will aid in obtaining these objectives. They are not the whole story, however, and whether or not they

2 Ibid.
do the job depends upon the teacher's knowledge of their use and his application of them. As Ebert states:

Mathematics is not only computation and manipulation of symbols; it is also understanding of the principles of computation and of the meanings of the symbols. It deals with not only important concrete situations and experiences, so very necessary for meaningful learnings, but also with useful abstractions and generalizations which grow out of concrete experiences.

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CHAPTER III

OUTLINE OF CONTENT AND MATHEMATICAL UNDERSTANDINGS

INTRODUCTION

The following outline in this chapter is based upon the material in *Growth in Arithmetic*, Grade 5, by Clark, Moser, and Junge. The items are listed topically and would not necessarily be taught in the sequence in which they are listed. The purpose of this outline is to give the teacher a quick look at what material or phases of arithmetic should be taught in Grade 5 in Ludlow.

Immediately following the outline of content is a list of mathematical understandings which the writer feels should be reviewed or taught in fifth grade. Many of these understandings should have been taught before grade-five and should be reviewed for clearer understanding and possible arithmetical enrichment. Those understandings that should receive special emphasis in grade-five are starred (*). This list does not represent a complete list of understandings for grade-five. However, the writer feels the more basic and important ones are mentioned. In regard to the sequence of teaching these understandings, it should coincide with whatever phases of arithmetic these understandings are concerned.
ARITHMETIC OUTLINE FOR GRADE V

I. The Number System
   A. Review of work in the lower grades
   B. Understanding, reading and writing numbers up to one million
   C. Roman numerals to CC
   D. Meaning and terminology of common fractions
      1. Parts of wholes
      2. Parts of a group of wholes
      3. Meanings of terms
   E. Introduction to decimals
      1. Meaning of decimals
      2. Reading and writing decimals
      3. Uses of decimals

II. Addition of Whole Numbers
   A. Review and extension of concepts and skills of addition
      1. Meaning of carrying
      2. Use of key facts
      3. Adding by making tens (mental addition)
   B. Basic addition facts—review and complete diagnosis
   C. Estimating sums
      1. Rounding off numbers
III. Subtraction of Whole Numbers

A. Review and extension of concepts and skills of subtraction

1. Meaning of borrowing
2. Relationship to addition
3. Breaking the subtrahend into parts (mental subtraction)

B. Basic subtraction facts—review and complete diagnosis

C. Estimating differences

IV. Multiplication of Whole Numbers

A. Review and extension of concepts and skills of multiplication

1. Meaning of carrying
2. Relationship to addition
3. Meaning and placement of the second partial product

B. Basic multiplication facts—review and complete diagnosis

C. Reteaching of multiplying by two and three place numbers

D. Estimating products

1. Rounding of the multiplicand or multiplier

V. Division of Whole Numbers

A. Review and extension of concepts and skills of
division

1. Measurement meaning
2. Sharing or partition meaning
3. Different ways of writing division
4. Meaning of remainders

B. Basic facts—review and complete diagnosis

C. Estimating quotients
   1. Helping tables
   2. Hint system

D. Finding two and three-figure quotients by division
   1. Development of the meaningful process of dividing

E. Dividing with two-figure divisors
   1. Determining the number of figures in the quotient
   2. Understanding quotient figures
   3. Placing quotient figures
   4. Steps in division
   5. Zeros in the quotient
   6. Relationship to addition and subtraction

F. Dividing a number by a larger number

VI. Fractions and Mixed Numbers

A. Review work of previous grades
   1. Meaning of fractions
      a. Collection idea
      b. Fractional-scale idea
c. Denominator
d. Numerator

B. Comparing fractions

C. Kinds of fractions
   1. Common
   2. Improper

D. Adding and subtracting fractions to whole or mixed numbers
   1. Finding common denominators

E. Subtracting fractions with and without borrowing

F. Multiplying whole numbers by fractions and mixed numbers (easy examples)

G. Estimating and mental computation

VII. Decimals and Percentage

A. Introduction to meaning, reading, writing, and uses of tenths and hundredths
   1. Relationship to common fractions

B. Review of United States money

C. Introduction to adding and subtracting tenths and hundredths

VIII. Measurement

A. Review and extension of measures of length, liquid, weight, and time

B. Area of rectangular figures
1. Meaning and understanding of sq. in., sq. ft., sq. yd.

C. Perimeter of rectangular figures
D. Dry measure

IX. Business Usage
A. Review of work of previous grades
B. School savings accounts
C. Keeping a record of money earned
D. Sharing party expenses

X. Graphs, Tables, and Charts
A. Reading and making simple bar graphs
B. Reading and making tabular reports

XI. Geometric Concepts, Scale Drawings
A. Review work of previous grades
B. Reading maps drawn to scale; reading and making simple scale drawings
C. Finding area in a rectangle in sq. in., sq. ft., sq. yd.
D. Uses of simple geometric figures to build meanings of fractions and mixed numbers

XII. Problem Solving
A. Pupil generalization concerning thought patterns underlying problems

1. Discovering and formulating basic relationships
B. Helpful problem-solving techniques

1. Estimating
2. Mental arithmetic
3. Choosing among alternate solutions
5. Relating problems that seem difficult to easier problems of the same type.

C. Three-step problems

ARITHMETICAL UNDERSTANDINGS FOR GRADE FIVE

Our Number System

1. In a two digit number the number on the left represents the number of tens in the group.

*2. The ratio of the bases from left to right is ten to one; the ratio of the bases from right to left is one to ten.

3. Zero is used as a place holder to indicate that there is no frequency to record in a place in a number.

4. The absolute value of a digit in a number is its positional value.

5. The relative value of a digit depends on the amount of transformation (change). The relative value of 6 in 69 is 60 ones.
Addition of Whole Numbers

1. The order of the addends does not affect the sum, i.e., interchanging the addends in a series does not change the sum.

2. When zero is added to a number the value of the number is not changed.

3. Only those numbers can be added together which represent objects that have a common characteristic(s).

4. Only digits in like places can be added.

5. The sum of any column of figures cannot exceed the product of the largest figure in the column times the number of figures in the column.

6. In addition we begin at the right with the one's column, so that if there are ten or more ones they can be changed into an equivalent number of tens and added to the ten's column.

Subtraction of Whole Numbers

1. Only digits in like places can be subtracted.

2. In subtraction, when the figure in the one's place in the top number is smaller than the figure in the bottom number, it is necessary to "borrow" a ten from the ten's place in the top number, transform (change) into 10 ones and then add them to the figure in the unit's place in the top number.
3. In subtraction the minuend (top number) is equal to the sum of the subtrahend (bottom number) and the remainder.

4. "Borrowing" is a method of changing a ten into 10 ones; a hundred into 10 tens, a thousand into 10 hundreds.

*5. Carrying and borrowing are reverse processes.

Multiplication of Whole Numbers

*1. Multiplication is a quick way of adding several equal numbers.

*2. Interchanging the numbers in a multiplication fact (or example) does not change the answer.

3. Multiplying by two is the same as doubling the number.

*4. When multiplying by the figure in the ten's place, it is necessary to leave a space in the one's place of the second partial product, or fill it in with a zero.

5. In multiplying, when the product of two digits in the one's place is ten or more it is necessary to transform (change) the number into the greatest number of tens and add the number of tens to the ten's product.

*6. When multiplying by a two digit number, the second partial product has a larger value than the first partial product.

7. In a multiplication table, the product increases by the value of the table being developed.
8. The answer to a multiplication example can be found by adding.

9. When multiplying whole numbers, the answer is always larger than either number.

  *10. When the digits are identical in a two digit multiplier (e.g., 33), the second partial product has the value ten times that of the first partial product.

11. Carrying is a method of changing ten or more units into an equivalent number of tens and adding this to the product of the ten's place.

12. When multiplying by a number with two or more digits the answer is the sum of the partial products.

13. Any partial product is smaller than the answer (product), unless multiplying by an even ten, etc.

  *14. Placing a zero on the right of a number (annexing) increases the number ten times.

**Division of Whole Numbers**

  *1. Division is a quick way of finding how many times one number is contained in another.

  *2. Division is a method for finding how many groups like the divisor there are in the dividend.

  *3. The quotient tells how many times the divisor can be subtracted from the dividend.

  *4. Multiplication and division are related processes.
*5. The remainder in division is less than the divisor and indicates that there is not enough left over to make another group the size of the divisor.

*6. The dividend is equal to the product of the divisor and the quotient.

*7. Multiplying both the divisor and the dividend by the same number does not change the value of the quotient.

*8. Whether or not the remainder in a division problem is to be expressed as a fractional part of the divisor depends upon the nature of the problem.

*9. The remainder always refers to the same thing as the dividend.

*10. If different division examples have the same dividend, the larger the divisor the smaller the quotient, or the larger the quotient, the smaller the divisor.

*11. If different division examples have the same divisor, the example which has the largest quotient has the largest dividend, and in a similar manner, the example which has the smallest quotient has the smallest dividend.

**Common Fractions**

1. When an object or group of objects is divided into two equal parts, each part is one-half.

*2. Both numerator and denominator of a fraction may be multiplied by the same number without changing the value of
the fraction.

*3. In a group of fractions having the same numerator the fraction that has the largest denominator has the smallest value or size.

*4. In a group of fractions having the same numerator the fraction that has the smallest denominator has the largest value or size.

*5. A common fraction may represent an indicated division in which the numerator is a dividend and the denominator is a divisor.

*6. A fraction may represent the relationship between one or more equal parts of a group.

*7. A fraction may represent the relationship between one or more equal parts of an object.

*8. A fraction may represent the ratio between two amounts by comparing two quantities such as two and four.

*9. In a group of fractions having the same denominator the fraction that has the largest numerator has the largest value or size.

*10. In a group of fractions having the same denominator the fraction that has the smallest numerator has the smallest value or size.

*11. A common fraction has a value less than one.

*12. An improper fraction has a value equal to one or
greater than one.

*13. When a fraction is reduced, its size or value remains the same but its terms are smaller.

*14. Fractions must be changed to a common size (or value, or denominator) before they can be added or subtracted.

*15. Both numerator and denominator of a fraction may be divided by the same number without changing the value of the fraction.

*16. Adding the same number to both numerator and denominator of a fraction will change the value of a fraction.

*17. Subtracting the same number from both numerator and the denominator of a fraction will change the value of a fraction.

*18. The numerator of a proper fraction is less than the denominator.

*19. A mixed number contains a whole number and a fraction.

*20. A mixed number is greater than one whole.

*21. A mixed number is greater than a proper fraction.

*22. A proper fraction is always less than an improper fraction.

*23. Reduction of fractions means that fewer parts are found, but the parts are larger.

*24. When we change a fraction to higher terms we produce
more parts but the parts formed are smaller.

Decimal Fractions

*1. A decimal fraction can be written as a common fraction with a denominator of ten or a power of ten, hundred, thousand, ten thousand, etc.

*2. In a mixed decimal with identical digits (e.g., 6.6) the digits in the tenth's place has a value one-tenth that of the digit in the one's place.

*3. In a decimal fraction with the same digits (.44), the digit in the tenth's place has a value ten times that of the digit in the hundredth's place.

*4. The sum of two or more decimals can be found by changing them to common fractions and adding.

*5. The difference between two decimals can be found by first changing them to common fractions having the same denominator and subtracting.

*6. It is easier to add decimals if they are listed with the decimal points in one column, the tenths in another column, the hundredths in another column, etc.

*7. To multiply a decimal by ten move the decimal point one place to the right.
CHAPTER IV

INSTRUCTIONAL UNITS

INTRODUCTION

In order to give the teacher an idea of how mathematical understanding and social significance can be added to a program in arithmetic, two outlines of instructional units are included in this chapter. The first is a subject matter unit concerned with readiness for teaching division with two-digit divisors. This unit features the use of visual aids and correlates division with multiplication and subtraction. Its primary purpose is to review the meaning of division and to get children to utilize what they already know about arithmetic to solve division problems.

Social significance can be added to an arithmetic program through the use of experience units. The unit on traffic problems in Springfield, Massachusetts, is an effort to illustrate this. Much more could have been done with the social-studies side of this unit, but, since this paper deals with arithmetic, the writer considered arithmetic objectives as much as possible. It would be very proper for the teaching

of a subject matter unit, as indicated earlier, to be included in the teaching of this experience unit. Together, with appropriate use of manipulative materials and devices, these units would give any program the mathematical meaning and social significance desired.

INTRODUCTION TO DIVISION WITH TWO-Figure DIVISORS

Overview

This unit is an approach to the teaching of division to help overcome many of the learning difficulties associated with two-figure divisors. There are two important features to this unit.

1. The children spend most of their time in the introductory lessons solving problems. Correct quotients are found by reference to tables and other devices so that the pupils attention is focused on the total situation. Step procedures are not introduced until the process is understood.

2. The steps in the division operation gradually develop out of the learning activities of the children. Instead of memorizing the complicated division formula of estimate, multiply, compare, subtract, etc., the pupils discover it.

Objectives

1. To allow children to discover the complicated division process for themselves
2. To develop a further understanding of the meaning of division
3. To give children experiences at solving a partition-type division problem as well as the measurement type
4. To give children experience in using charts and tables

Understandings
1. We can find quotient figures by making a multiplication table of the divisor.
2. We can find quotient figures by subtracting.
3. The division algorism will tell us if we have the correct quotient figure.
4. Estimation is a way of eliminating the work of making a helping table.
5. The division algorism is a shortened version of repeated subtraction.

Outline of Content
1. Dividing by tens (two-figure divisor)
   A. Problems within the experiences of the children are utilized in accordance with the following chart:
PET STORE TICKETS 10¢ EACH

<table>
<thead>
<tr>
<th>Tickets sold so far</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Collected</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
<td>90%</td>
</tr>
</tbody>
</table>

B. Question: What does each of these divisions tell about the chart?

\[
\begin{array}{cccccccccc}
2 & 3 & 4 & 5 & 6 & 9 \\
\frac{2}{10} & \frac{3}{10} & \frac{4}{10} & \frac{5}{10} & \frac{6}{10} & \frac{9}{10}
\end{array}
\]

C. Arrange counters in piles of ten.

\[
40 \text{ counters} = \begin{array}{c}
\includegraphics[width=0.5\textwidth]{counter_piles.png}
\end{array}
\]

\[
40 \div 10 = 4
\]

\[
10 \overline{\div 40}
\]
II. Using a price-list to show the interrelation between multiplication and division

<table>
<thead>
<tr>
<th>FRESH VEGETABLES--PRICE LIST</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetables</td>
</tr>
<tr>
<td>Spinach</td>
</tr>
<tr>
<td>Lima Beans</td>
</tr>
<tr>
<td>Cauliflower</td>
</tr>
</tbody>
</table>

A. Questions:

1. How many pounds of lima-beans can you get for $.93?

2. What would 3 lbs. of cauliflower cost?

B. Making a multiplication table from the above chart

1x22= 22
2x22= 44
3x22= 66
4x22= 88
5x22= 110
C. Further understanding: There is a division fact for every multiplication fact.

\[
\begin{array}{c}
22 \\
\times 2 \\
\hline
44 \\
22 \overline{\div 2}
\end{array}
\]

III. Using a multiplication table to make number discoveries

\[
\begin{align*}
1 \times 21 &= 21 \\
2 \times 21 &= 42 \\
3 \times 21 &= 63 \\
4 \times 21 &= 84 \\
5 \times 21 &= 105 \\
6 \times 21 &= 126
\end{align*}
\]

A. Statement: Each page of a notebook holds 21 words.

B. Questions:

1. How many words on six pages?
2. Seven 21's equal?
3. Which fact tells us how many pages of 21 words it will take to write 84 words?

\[
\begin{array}{c}
21 \overline{\div 3}
\end{array}
\]

IV. Finding part of a number (partition type division)

\[
\begin{align*}
20 \times 1 &= 20 \\
20 \times 2 &= 40 \\
20 \times 3 &= 60 \\
20 \times 4 &= 80 \\
20 \times 5 &= 100 \\
20 \times 6 &= 120 \\
20 \times 7 &= 140 \\
20 \times 8 &= 160 \\
20 \times 9 &= 180
\end{align*}
\]
A. Problem: The children in the class have 80 cans of fruit. They have 20 baskets. How many cans will they put in each basket?

1. \( \frac{1}{2} \) of 80 = \( 20 \) \( 80 \)
2. \( 20 \times \ ? = 80 \)

V. Finding quotient figures by (a) subtracting, (b) making helping tables, (c) division algorithms

\[
\begin{array}{ccc}
98 & \ 1 \times 32 = 32 \\
-32 & \ 2 \times 32 = 64 \\
66 & \ 3 \times 32 = 96 \\
-32 & \ 4 \times 32 = 128 \\
34 & \ 5 \times 32 = 160 \\
-32 & \ 6 \times 32 = 192 \\
2 & \ 7 \times 32 = 224 \\
\end{array}
\]

A. Questions:

1. Which division below is correct?
2. How can you tell?

\[
\begin{array}{ccc}
6 \ r \ 6 & 32 \) 198 \\
32 & 192 \\
6 & 224 \\
\end{array}
\]

VI. Using the Hint System to find the quotient figure

A. Problem: \( 23 ) 69 \)

B. Statement: Divide the first figure of the dividend by the first figure of the divisor.

C. Statement: \( 6 \div 2 = 3 \) so there are probably three 23's in 69.
I. Overview

The tremendous increase in automobile ownership and the subsequent increase in traffic problems have created a situation vital to the people in the vicinity of Springfield, Massachusetts. Since many of the parents of Ludlow children work in and around Springfield, they are aware of the prevailing hazardous traffic conditions. Ludlow children should understand the importance of the new highway in regard to traffic problems as well as an improvement in transportation facilities. They have the opportunity of watching it being built. Emphasis in this unit will be placed upon the mathematical skills and understandings involved in the building and operation of this highway.

II. Objectives

A. Social Understandings

To develop the understanding that:

1. Roads produce nothing, but are as important as farms and mines.
2. Roads make it possible to move materials, objects and people from one place to another.
3. Roads are marks of civilization and progress.
4. There are many kinds of roads.
5. Roads have improved greatly in the past twenty
years.

6. Modern roads are more durable, more comfortable, and safer.

7. Large cities have difficult street-traffic problems.

8. The cost of building and maintaining a highway should be shared by all the people.

9. Good highways save time.

10. Many people travel long distances, daily, to work.

11. Many different kinds of machines are used to build modern highways.

12. Good highways are important to our nation's defense.

13. Good highways add beauty to our country.

B. Mathematical Understandings

To develop the understanding that:

1. We can also express thousands and a million with our number system.

2. Multiplication is a quick way of adding several equal numbers.

3. Interchanging the numbers in a multiplication fact (or example) does not change the answer.

4. When multiplying by the figure in the ten's
place, it is necessary to leave a space in the one's place of the second partial product, or fill it in with the zero.

5. When multiplying by a two digit number, the second partial product has a larger value than the first partial product.

6. When the digits are identical in a two digit multiplier (e.g., 33), the second partial product has the value ten times that of the first partial product.

7. Placing a zero on the right of a number (annexing) increases the number ten times.

8. Division is a quick way of finding how many times one number is contained in another.

9. Division is a method for finding how many groups like the divisor there are in the dividend.

10. The quotient tells how many times the divisor can be subtracted from the dividend.

11. Multiplication and division are related processes.

12. The remainder in division is less than the divisor and indicates that there is not enough left over to make another group the size of the divisor.

13. The dividend is equal to the product of the divisor and the quotient.

14. Multiplying both the divisor and the dividend
by the same number does not change the value of the quotient.

15. Whether or not the remainder in a division problem is to be expressed as a fractional part of the divisor depends upon the nature of the problem.

16. The remainder always refers to the same thing as the dividend.

17. If different division examples have the same dividend, the larger the divisor the smaller the quotient, or the larger the quotient, the smaller the divisor.

18. If different division examples have the same divisor, the example which has the largest quotient has the largest dividend, and in a similar manner, the example which has the smallest quotient has the smallest dividend.

19. Man has devised units of measure which enables him to deal with the quantitative aspects of the environment in a precise way.

20. We can depict information through visual aids.

III. Skills and Abilities

A. Social Skills and Abilities

To develop the ability to:

1. Ask intelligent questions.
2. Read with better understanding.
3. Use reference material with ease.
4. Organize work
5. Write reports clearly, forcibly and correctly
6. Make oral reports
7. Write legibly
8. Recall past learnings and apply them to the present

B. Mathematical skills and abilities

To develop the ability to:

1. Read and write large numbers up to one million
2. Subtract a small number from a large one
3. Add whole numbers
4. Add and subtract dollars and cents
5. Multiply a number by a two or three-place multiplier.
6. Divide by one or two-place numbers
7. Use our units of measure
8. Find area
9. Find perimeter
10. Find average
11. Read line, bar and picture graphs
12. Draw diagrams to scale
13. Draw maps
14. Solve one-and two-step problems
15. Estimate the answer of an example or problem

C. Attitudes and Appreciations
To develop the attitude of:

1. Pleasure toward arithmetic
2. Cooperation
3. Understanding
4. Determination to take an active part in developing our community and nation and our public utilities
5. A feeling of responsibility of sharing the cost of transportation facilities and also taking care of them
6. Respect of individual personality
7. Research and inquiry
8. Willingness to do one's share
9. Unselfishness
10. Appreciation toward technological progress

D. Habits

To develop the habit of:

1. Listening well
2. Thinking constructively
3. Organizing our work systematically
4. Working well together
5. Obeying rules of good citizenship
6. Individual participation
7. Observing accurately
8. Following directions carefully
9. Practicing good posture
10. Memorizing necessary facts

11. Making associations

12. Working independently

E. Facts

To review or acquire the fact that:

1. Periods (ones, thousands, a million) are separated by commas.

2. The numbers you add in an addition example are called addends.

3. The answer in an addition example is called the sum.

4. The bottom number in a subtraction example is called the subtrahend.

5. The top number in a subtraction example is called the minuend.

6. The answer in a subtraction example is called the difference or remainder.

7. The top number in a multiplication example is called the multiplicand.

8. The number by which you multiply in a multiplication example is called the multiplier.

9. The answer in a multiplication example is called the product.

10. The number to be divided is called the dividend.
11. The number by which you divide is called the divisor.

12. The answer in division is called the quotient.

13. The amount left over is the remainder.

14. The linear measure table consists of:
   - 12 inches equal 1 foot (ft.)
   - 3 feet equal 1 yard (yd.)
   - 16 feet equal 1 rod (rd.)
   - 320 rods equal 1 mile (mi.)
   - 5280 feet equal 1 mile

15. The table of weight consists of:
   - 16 ounces (oz.) equal 1 pound (lb.)
   - 100 pounds equal 1 hundredweight (cwt.)
   - 2000 pounds equal 1 ton (T)

16. The table of square measure consists of:
   - 144 square inches (sq. in.) equal 1 square foot (sq. ft.)
   - 9 square feet equal 1 square yard (sq. yd.)

17. The time table consists of:
   - 60 seconds equal 1 minute (min.)
   - 60 minutes equal 1 hour (hr.)
   - 24 hours equal 1 day (da.)
   - 7 days equal 1 week (wk.)
   - 30 days equal 1 month (mo.)
   - 12 months equal 1 year (yr.)
   - 365 days equal 1 year
   - 366 days equal 1 leap year
   - 10 years equal 1 decade
   - 100 years equal 1 century

IV. Possible Approaches

A. To initiate the unit:

1. Arrange pictures and posters on a bulletin board.
board showing modern highways and city traffic jams.

2. Discuss with the children a motor ride through the city of Springfield, Massachusetts, on a pleasant summer's Sunday afternoon.

3. Have the children discuss with their parents the problem of going to Springfield to shop or work.

4. Visit the site of the new highway to see the work that is being done.

V. Planning Period

By guiding discussion with the children the teacher will bring out the need for the new highway. The children's own experiences with motoring through Springfield will aid here. The teacher should lead the children to see some of the problems concerned with building a new highway and, whenever possible, point out the use and need of arithmetic in solving some of these problems.

Under teacher guidance the pupils should plan on what they want to know about the new highway, and how they can go about finding their answers. Committees should be chosen according to the children's interests and capabilities to cover the different phases of work.

VI. Learning Experiences

A. Consult the local chief of police concerning traffic problems.
B. Visit the site of the new highway to watch the building operations.

C. Visit the local highway department to view highway equipment and discuss highway building and maintenance with the superintendent.

D. Construct a mural showing highway construction.

E. Draw a map showing how the highway passes through Ludlow and by-passes Springfield center.

F. Construct a miniature modern highway.

G. Collect pictures of highway construction and modern traffic.

H. Gather items from newspapers and magazines concerning state, national, and local highway construction.

I. Write to U. S. Roads Commission and Department of Public Works.

J. Write to the Massachusetts Department of Public Works.

K. Write to the Springfield Department of Public Works.

L. Show filmstrips and films on highway transportation.

M. Draw line and bar graphs showing possible daily fuel consumption of some of the mechanized equipment used on the new highway.

N. Gather data regarding the work on the new highway (e.g., number of men working, hours worked, fuel consumption of vehicles, miles traveled, cost of materials and equipment,
salaries of employees). These data are to be used for developing quantitative understanding and for providing meaningful practice in:

1. Reading and writing large numbers.
2. Subtracting small numbers from large numbers.
3. Adding numbers.
4. Multiplying numbers by a two-and three-place multiplier.
5. Dividing with a one-or two-place divisor.
6. Working with our units of measure.
7. Reading line, bar, circle, and picture graphs and maps.
8. Solving problems.
10. Arrange an exhibit of unit projects.

VII. Evaluation Suggestions

A. Written test of mathematical understandings

1. In which number below does the 5 stand for half a million?

   5,347,921   2,507,278   6,450,000

2. (a) Extend this table far enough to show the number of 32's in 195.

   3x32: 96
   4x32: 128
   5x32: 160

   (b) Now copy and complete this division. 32[195]
3. What would be the remainder if five 30's were subtracted from 158?

4. Which example shows how to find how many quarts there are in 8 gallons? \(8 \div 2\) \(8 \times 4\) \(8 + 4\)

5. In this scale drawing Line(a) represents $40.
   a. ________  b. ________
   How much money does Line (b) represent? $60  $50  $80

6. \(24 \div 12 + 24 = ? \times 12\)

7. Which two numbers do you multiply to check this example? \(406 \div 58 = 7\)

8. In the example below, which number would go under the 4 in 204? 6 4 5 0

   \[
   \begin{array}{c}
   34 \\
   \times 16 \\
   \hline
   204 \\
   34 \\
   \hline
   344 \\
   \end{array}
   \]

9. Which example (a, b, or c) shows the same thing as these subtractions?
   \(78 \div 24 = \frac{54}{6}\)
   \(-24 \div 30 = \frac{-24}{6}\)

   a. \(\frac{24}{6} \div 6\)  b. \(\frac{24}{6} \div 6\)  c. \(\frac{24}{6} \div 6\)
10. $15 \times 53 = 785$, so how much would you add to 785 to find $16 \times 53$?

B. Written test on mathematical skills

1. $9.75 + 47.63 + 82.44$
2. Take $73.42$ from $91$.
4. $974536$
5. $3872356$

6. The salaries of five fathers who work on the new highway are as follows: $50, 65, 75, 80, 90$.
   a. What is the average salary?
   b. How many are below average?

7. A truck that travels 240 miles in eight hours must travel how many miles an hour on the average?

8. Jack's father's car can travel 22 miles on a gallon of gasoline. How many gallons would it need to go 660 miles?

9. A bulldozer working on the highway uses 2 quarts of oil a day. How many gallons of oil would it use in 30 days?

10. (a) A highway crew covered a section of the highway with asphalt that was 40 ft. in length and 20 ft. in width. How much land did they cover (surface)? (b) To keep people off from the new surface they encircled the area with
rope. How much rope did they need? (c) For every 100 sq. ft., 1 ton of asphalt was used. How many tons of asphalt were used? How many pounds was that?

Bibliography (children)


Bibliography (teacher)


Materials

Road maps
Rulers
Yardstick
Hundred-board
Egg boxes
Square yard wall chart
Cardboard
Newspapers
Flour and Water
Oak tag to make squares and rectangles
Poster paper
Mural paper
Pint and quart measures
Graph paper
Calendar
CHAPTER V

SUGGESTIONS FOR USING SELECTED TEACHING DEVICES

INTRODUCTION

Listed below are some of the selected teaching devices and visual aids which the writer feels are necessary in teaching fifth-grade arithmetic. This is not, by any manner of means, a complete listing of materials that can be used in teaching fifth-grade arithmetic since these aids are almost limitless, depending upon the ingenuity of the teacher and the availability of the items.

On the following pages there are illustrations of how some of these selected devices can be used in the classroom to teach certain skills and understandings in fifth-grade arithmetic. No effort was made to include all the skills and understandings to be taught in grade five but enough has been illustrated to show what materials can be used in the different phases of arithmetic and the procedure for using them.

SELECTED TEACHING AIDS

Tickets or sticks to show groupings and regroupings of ones, tens, and hundreds

A hundreds' board
Counting discs or bottle caps
United States money (pennies, dimes, and dollars)
Place value charts
An abacus
Fraction charts
Coherograph (or flannel board) with cut-outs of fractional parts of circles and other figures
Rulers and tape measures
Wall chart to show units of square measure
Egg boxes (some with two rows of six and some with three rows of four)
A mileage indicator
Posters made with letter cut-outs

SUGGESTIONS FOR USING TEACHING AIDS

Understanding to be taught: The ratio of the bases from left to right is ten to one; the ratio from right to left is one to ten. That is why our system is called a decimal system.

Teaching aids: Place value pockets with cardboard slips, some of which are tied together in groups of ten, some in groups of 10 tens.
Steps in manipulation:

1. Place 1 one in the one's place.
2. Tie 10 ones together and place them in the ten's place.
3. Tie 10 tens together and place them in the hundred's place.

The first three steps will show the one to ten ratio.

Reverse the procedure to show the ten to one ratio.

Understanding to be taught: The relative value of a digit depends on the amount of transformation (change). The relative value of 6 in 60 is 60 ones.

Teaching aids: Place value pockets with cardboard slips

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

66 equals 6 groups of ten or 60 plus 6 ones.
Steps in manipulation:

1. Take the six groups from the ten's place and untie them.
2. Allow the children to count the 60 ones.

Understanding to be taught: (a) A smaller letter to the left of a larger letter is subtracted in Roman numerals. (b) A smaller letter to the right of a larger letter is added.

Teaching aids: Posters showing the following letters—I, V, X, L, C.

Steps in procedure:

1. Let children cut out letters and make posters.
2. Write a number on the board, such as 61.
3. Let children take their posters and form the Roman numeral LXI (61) in front of the class.
4. Call attention to the position of the smaller letters and their value.
5. Repeat process with the number 41.
Understanding to be taught: The order of combining groups has no effect on the sum.

Teaching aids: Flannel board with fractional parts of discs

Steps in manipulation:
1. Place the fractional parts on the flannel board in the order indicated in figure A, and let the children count the total number of eighths.
2. Place the fractional parts on the flannel board as indicated in figure B, and let the children count the
number of eighths.

3. Point to the fractional parts in column A as the children count the eighths in the following manner: one-eighth plus two-eighths equal three-eighths, three-eighths plus three-eighths equal six-eighths.

4. Let children add the eighths in column B in a similar manner.

Understanding taught: Carrying in addition is a method of changing a number greater than nine, (90, 900, 9,000) into an equivalent number of tens (100s, 1,000s).

Teaching aids: Place value pockets with strips in units and groups of ten, pennies and dimes, and an abacus.

\[
\begin{array}{c|c}
\text{Tens} & \text{Ones} \\
\hline
\text{Tens} & \text{Ones} \\
9 & \mathbf{11} \\
\hline
\end{array}
\]

Steps in manipulation:

1. Place nine units in the one's place.

2. Add another unit to the nine and then tie them all into one group of ten units.
3. Place the ten in the ten's place.
4. Place ten pennies on a desk.
5. Allow the children to count them.
6. Substitute one dime for the ten pennies.
7. Ellicit the similarity between the two procedures.
8. Demonstrate the transformation on the abacus.

9 ones plus 1 more one equal 1 ten and no ones.

Understanding to be taught: When the sum of the one's column is ten or more the sum is transformed (changed) into the greatest number of tens and added to the digits in the ten's column. When the sum of the ten's column is ten or more the sum is transformed into the greatest number of tens and added to the digits in the hundred's column.

Teaching aid: Place value pockets

Problem: \[ \begin{array}{c}
246 \\
175 \\
\hline 421
\end{array} \]
Steps in manipulation:

1. Place two groups of one hundred tickets in the hundred's pocket, four groups of ten tickets in the ten's pocket, and six single tickets in the one's pocket. This equals the top addend (246).

2. Place one group of one hundred tickets in the hundred's pocket, seven groups of ten in the ten's pocket, and five single tickets in the one's pocket. This equals the lower addend (175).

3. Have the children add the tickets in the one's pocket—six plus five equals eleven.

4. Take ten single tickets and tie them into a group of ten and place it in the ten's pocket. We now have one single ticket left in the one's pocket and an extra group of ten carried to the ten's pocket.
12 tens equal 1 hundred plus 2 tens

5. Have the children now count the number of groups in the ten's pocket—five plus seven equal twelve.

6. Take ten groups of ten and tie them into one group of one hundred and place it into the hundred's pocket. Now there are 2 tens left in the ten's pocket and one single ticket left in the one's pocket.

7. Have the children add the groups of one hundred in the hundred's pocket—three plus one equals four.

Understanding to be taught: In subtraction, when the figure in the one's place in the top number is smaller than the figure in the bottom number, it is necessary to "borrow" a ten from the ten's place in the top number. Then change it into 10 ones and add them to the figure in the units place in the top number.

Teaching aids: Place value pockets, dimes and pennies, and algorithms
Problem: 42
\[
\begin{array}{c}
\text{Step 1:} \\
\text{Step 2:} \\
\text{Step 3:} \\
\text{Step 4:} \\
\text{Step 5:}
\end{array}
\]

Steps in manipulation:

1. Place tickets in the place value pockets as indicated in figure A.

2. Ellicit the fact that when subtracting 18 from 42 we cannot subtract 8 ones from 2 ones. We must, therefore, "borrow" 1 ten from the ten's column and change it into 10 ones.

3. Take 1 ten from the ten's place. Untie the group of tickets and place the 10 ones in the one's place as indicated in figure B.

4. Show the children that we now have 3 tens and 12 ones.

5. Take 8 ones from the one's place and 1 ten from the ten's place to illustrate the subtraction as shown in
figure C.

\[
\begin{array}{c}
\text{4 dimes} + \text{2 pennies} = \\
\text{3 dimes} + \text{12 pennies} \\
\text{1 dime} + \text{8 pennies}
\end{array}
\]

Steps in manipulation:

1. Illicit the fact that the four dimes and two pennies equal the minuend (42).

2. Since we cannot subtract 8 pennies from 2 pennies, take one of the dimes and change it into ten pennies.

3. Show that we now have three dimes and twelve pennies.

4. Take eight pennies from twelve pennies and one dime from three dimes, leaving two dimes and four pennies (24).

\[
\begin{array}{c}
42 \\
-18 \\
\hline
24
\end{array}
\]

42:4 dimes plus 2 pennies

18:1 dime plus 8 pennies
We cannot take eight pennies from two pennies so we take one dime and change it to ten pennies.

\[
4 \text{ dimes} - 1 \text{ dime} = 3 \text{ dimes} + 10 \text{ pennies} + 2 = 12 \text{ pennies} \\
-1 \text{ dime} \\
\frac{2 \text{ dimes}}{10 \text{ pennies}} \\
\frac{-3 \text{ pennies}}{4 \text{ pennies}}
\]

42
-18
24

We can extend the understanding of "borrowing" in subtraction to a higher level by utilizing an example with numbers in the hundreds (625-179). When using the place value pockets to illustrate this process, the same method of "borrowing" is used in the hundred's column as was previously shown for the ten's column. When using United States money to illustrate this process, one-dollar bills are used in place of hundreds. The children should know that there are ten dimes in one dollar.

Understanding to be taught: In multiplication, when the product of two digits in the one's place is ten or more, it is necessary to transform the number into the greatest number of tens and add the number of tens to the ten's product.

Teaching aids: Place value pockets

Problem: \(2 \times 36 = ?\) or \(36 \times \frac{2}{??}\)
Steps in manipulation:

1. Place tickets in the place value pockets, as indicated in figure A, to show 2x36.

2. Point out the 12 ones in the one's place.

3. Take 10 ones from the one's place and tie them into 1 ten.

4. Place the 1 ten in the ten's place. Thus:

\[
\begin{align*}
\text{36} & \\
\times \ 2 & \\
\hline \\
\text{72} &
\end{align*}
\]

Understanding to be taught: Division may mean how many times one number is as large as another number, or how many groups of one size there are in a larger group.

Teaching aids: Pennies or other counting discs

Problem: How many three-cent stamps can you buy for 15¢?
Steps in manipulation:

1. Place fifteen pennies on a desk.

2. Have a child divide the pennies into groups of three to show how many groups of a certain size there are in another group.

\[
\begin{array}{c}
\text{16} \\
\text{16} \\
\text{16} \\
\text{16} \\
\text{16} \\
\end{array}
\]

3 \( \div \) 15

Understanding to be taught: Division may mean the size of the equal parts into which a number is divided.

Teaching aids: Thirty pennies or counters

Problem: Five girls went to share thirty cents equally.

How many pennies should each receive?

<table>
<thead>
<tr>
<th>Anne</th>
<th>Jane</th>
<th>Mary</th>
<th>Rita</th>
<th>Patricia</th>
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Steps in procedure:

1. Have five girls stand in the front of the class.
2. Hand to the girls, one penny at a time, the thirty pennies so that each has an equal number of pennies.
3. Let each girl tell the class how many pennies she has.

\[
\frac{1}{5} \text{ of } 30 \neq 6 \neq 30 \neq 5 \neq 6 \neq 6
\]

Understanding to be taught: The quotient tells how many times the divisor can be subtracted from the dividend.

Teaching aids: Fifteen pennies or other counters

\[\begin{array}{c}
\text{15} \\
- \text{5} \\
\text{10} \\
- \text{5} \\
\text{5} \\
- \text{2.5} \\
\text{0}
\end{array}\]

Steps in manipulation:

1. Place fifteen pennies or counters on a desk.
2. Write the subtraction algorism on the board.
3. As each five is subtracted, take five counters from the desk to show that when 3 fives are taken all fifteen pennies are gone.

\[
5 \frac{3}{15}
\]
Understanding to be taught: The remainder in division is less than the divisor and indicates that there is not enough left over to make another group the size of the divisor.

Teaching aid: Place value pockets

Problem: \(3 \div 14\)

Steps in manipulation:

1. Place 1 ten and 4 ones in the place value pockets.

2. Transform the ten into 10 ones and place in the one's place as indicated in figures A and B.

3. Divide the 14 ones into four groups of threes with two left over as indicated in figure C.

4. Illicit the fact that if we had one more left over we could make another group. Since we don't, we say we have a remainder of two.
Understanding to be taught: A fraction equals one or more equal parts of a unit or object.

Teaching aids: Flannel board with fractional parts of discs, one pint bottle, and one quart bottle

Steps in manipulation:

1. Place discs divided into equal parts on the flannel board as indicated.

2. Place 1 one-half over the other to show that they are the same size.

3. Have the children count the number of halves in the whole disc.

4. Repeat with the discs divided into fourths, eighths, and other fractional parts.

5. Fill a pint bottle with water and dump it into a quart bottle to show that one pint is one-half a quart.

6. Fill the pint bottle again with water and dump it into the quart bottle to show that there are two pints in a quart.
Understanding to be taught: A fraction may mean that a group of wholes has been divided into equal parts.

Teaching aids: Counting discs

Problem: Three boys found six marbles. What part of the six marbles will each boy get?

Steps in manipulation:

1. Have three boys stand in front of the class.
2. Divide the six counters evenly among the three boys.
3. Illicit the fact that each boy has two counters or one-third of the group of six.

Understanding to be taught: A fraction may mean that a number is divided by a larger number.

Teaching aids: Flannel board with cut-out discs.

Problem: How can we divide two slices of watermelon between three boys evenly?
Steps in manipulation:

1. Place two whole discs that are divided into thirds on the flannel board.
   2. Have three boys come to the front of the class.
   3. Divide the six-thirds evenly among the three boys so that each boy has two-thirds of a whole.

Understanding to taught: The denominator of a fraction tells us the number of equal parts an object is divided into, or the size of the parts.

Understanding to be taught: The numerator of a fraction tells the number of equal parts of an object we are talking about.
Teaching aids: Flannel board with discs divided into fourths. Three of the fourths should be one color, while one of the fourths should be another color.

Steps in manipulation:

1. Place the fourths of the disc on the flannel board as indicated above.

2. Illicit the fact that the shaded fourths is the part of the pie that was eaten.

3. Ask the class, "What part of the pie was eaten?"

4. Illicit the fact that in the fraction the number above the line is the numerator and it tells us the number of equal parts of the pie that were eaten.

5. Illicit the fact that the number below the line is the denominator. It tells us that the pie was divided into four equal parts. When a whole is divided into four equal parts, each part is called a fourth.

Understanding to be taught: The denominator tells how large the parts are.

Teaching aids: Flannel board with fractional parts of discs.
Steps in manipulation:

1. Place on the flannel board the discs that are divided into fractional parts as indicated above.

2. Have the children count the number of parts in each disc and compare them with the corresponding denominators.

3. Take one-half and place it over one-third to show the difference in size.

4. Take one-fourth and place it over one-sixth to show the difference in size.

5. Illicit the fact that the more equal parts an object is divided into, the smaller parts.

Understanding to be taught: The numerator tells how many equal parts we are talking about.

Teaching aids: Flannel board with fractional parts of discs, and rectangular figures.
Steps in manipulation:

1. Place the figures on the flannel board as indicated above.

2. Call attention to the number of equal parts in each figure.

3. Have children go to the flannel board and take off fractional parts as you dictate—one $\frac{1}{4}$, three $\frac{3}{4}$s.

Understanding to be taught: The numerator of a proper fraction is less than the denominator. Therefore, a proper fraction is always less than one whole.

Teaching aids: Flannel board with fractional parts of discs.
Steps in manipulation:

1. Place the whole discs and fractional parts of discs as indicated.

2. Tell the children that in the fraction \( \frac{1}{2} \), the denominator 2 shows that the object was divided into two equal parts. Point to figure A.

3. Tell the children that the numerator 1 indicates just one of those equal parts. Point to figure B.

4. Place the \( \frac{1}{2} \) fractional part over the whole disc to show that one-half is less than one whole.

5. Repeat with figures C and D.

Understanding to be taught: Both the numerator and denominator of a fraction may be divided by the same number without changing the value of the fraction.

Teaching aids: Flannel board with fractional parts of discs.

\[
\begin{align*}
A & & B \\
\frac{1}{4} & & \frac{1}{4} \quad \frac{1}{4} \\
\frac{1}{4} & & \frac{1}{4} \quad \frac{1}{4}
\end{align*}
\]

\[
\frac{6}{8} \div \frac{1}{2} = \frac{3}{4}
\]
Steps in manipulation:

1. Place a disc divided into fourths on the flannel board.

2. Take two-eighths and place them over one of the fourths to show that two-eighths equal one-fourth as indicated in figure A.

3. Take six-eighths and place them over three-fourths to show that six-eighths equal three-fourths.

4. Write the division algorism on the blackboard as indicated in figure B.

5. Illicit the fact that as the parts get larger there are fewer in number.

Understanding to be taught: Both the numerator and denominator can be multiplied by the same number without changing the value of the fraction.

Teaching aids: Flannel board with fractional parts of discs.

\[
\frac{3}{4} \times 2 \times \frac{3}{4} = \frac{6}{8}
\]
Steps in manipulation:

1. Represent three-fourths with fractional parts.
2. Show that three-fourths equal six-eighths after both the numerator and denominator has been multiplied by two.
3. Place fractional parts (three-fourths over six-eighths) to show equality of size.
4. Illustrate the fact that as the parts get smaller in size there are more in number.

Skill to be taught: Adding like fractions: e.g., \( \frac{1}{4} + \frac{3}{4} \)

Teaching aids: Flannel board and fractional parts of discs.
Steps in manipulation:

1. Represent each addend with fractional parts 
   \( \frac{1}{4} \) and \( \frac{3}{4} \).

2. Combine parts.

---------------------

Skill to be taught: Adding mixed numbers: e.g., \( 2 \frac{1}{4} + 1 \frac{1}{4} \)

Teaching aids: Flannel board with fractional parts of discs

\[ \begin{align*}
\text{\( \frac{1}{4} \)} & = \frac{1}{4} \\
\text{\( \frac{3}{4} \)} & = \frac{1}{4} \\
\text{\( \frac{5}{4} \)} & = 1 \frac{1}{4} \\
\text{\( \frac{7}{4} \)} & = 1 \frac{3}{4} \\
\text{\( \frac{10}{4} \)} & = 2 \frac{1}{2}
\end{align*} \]
Steps in manipulation:

1. Represent each addend with fractional parts and wholes \(2\frac{1}{4} \text{ and } 1\frac{1}{4}\).

2. First combine the parts \(\frac{1}{4} \text{ and } \frac{1}{4}\); then combine the wholes.

3. Change the \(\frac{2}{4}\) to \(\frac{1}{2}\).

-------------------------

Skill to be taught: Adding unlike but related fractions:

\[ \frac{1}{4} + \frac{1}{4} \]

Teaching aids: Flannel board with fractional parts of discs

\[ \frac{1}{2} + \frac{1}{2} = \frac{2}{4} \]

\[ \frac{1}{4} : \frac{1}{4} \]

Steps in manipulation:

1. Represent addends with fractional parts \((\frac{1}{4} \text{ and } \frac{1}{4})\).

2. Change \(\frac{1}{2}\) to \(\frac{2}{4}\).

3. Combine fractional parts (fourths).
Skill to be taught: Adding unlike but related fractions when the sum is more than one whole and demands reduction: e.g.,

Teaching aids: Flannel board and fractional parts of discs.

\[ \frac{1}{2} + \frac{3}{5} = \frac{2}{6} + \frac{5}{6} \]

Steps in manipulation:

1. Represent addends with fractional parts.
2. Change the one-half to three-sixths.
3. Combine the fractional parts (sixths).
4. Change six-sixths to one whole and two-sixths to one-third.
Skill to be taught: Subtracting a fraction from a whole number: e.g. $4 - \frac{1}{4}$

Teaching aids: Flannel board with whole and fractional parts of discs.

Steps in manipulation:

1. Represent the minuend with four whole discs.
2. Change one of the wholes to four-fourths.
3. Remove one-fourth from the minuend to show the difference.
Skill to be taught: Subtracting unlike but related fractions: e.g., $\frac{3}{4} - \frac{1}{2}$

Teaching aids: Flannel board with fractional parts of discs.

Steps in manipulation:

1. Represent minuend and subtrahend with fractional parts (three-fourths and one-half).
2. Change the one-half to two-fourths.
3. Remove two fourths from both subtrahend and minuend to show the difference.
Skill to be taught: Subtracting unlike and unrelated fractions: \( \frac{3}{2} - \frac{1}{4} \)

Teaching aids: Flannel board with fractional parts of discs

\[
\begin{align*}
\text{\(1\frac{3}{2} - \frac{1}{4}\)} &= \frac{8}{6} - \frac{3}{6} \\
&= \frac{5}{6}
\end{align*}
\]

Steps in manipulation:

1. Represent minuend and subtrahend with fractional parts and wholes (one and one-third and one-half).
2. Change one and one-third to eight-sixths.
3. Change one-half to three-sixths.
4. Remove three-sixths from both the subtrahend and the minuend to show the difference.
Skill to be taught: Multiplying a fraction by a whole number: \( 3 \times \frac{3}{4} \)

Teaching aids: Measuring cups and water; fractional parts of discs with a flannel board

\[
3 \times \frac{3}{4} = 2 \frac{1}{4}
\]

Steps in manipulation:
1. Fill three measuring cups three-fourths full of water.
2. Pour this water into three other measuring cups—filling two full and another one-fourth full.
Steps in manipulation:
1. Represent the multiplicand with fractional parts.
2. Place the three-fourths representation three times on the board to illustrate the multiplier.
3. Combine the fractional parts to show the product.

Skill to be taught: Multiplying a whole number by a fraction: e.g., $\frac{3}{4} \times 3$

Teaching aids: Flannel board with fractional parts of discs and whole discs

$$\frac{3}{4} \times 3 = ?$$
Steps in manipulation:

1. Place three whole discs on the flannel board.

2. In order to find three-fourths of three we must first change the three wholes to fourths, as in figure B.

3. Divide the fourths into four equal groups. We now have four equal groups of three-fourths.

4. Take three of these equal groups of three-fourths (which is one-fourth of twelve fourths) and place them together on the flannel board as in figure D, showing nine-fourths.

5. Change the nine-fourths into two wholes and one-fourth.

Understanding to be taught: A decimal fraction is another way of writing a common fraction. Both indicate an
equal part of a whole.

Teaching aids: Hundred board and a bicycle mileage indicator

Procedure:

The children should count the squares in the hundred board. One square should then be marked so that the children can see it is one equal part of the whole board. Since there are one-hundred squares in the whole board, the square marked would be \( \frac{1}{100} \) or .01 of the whole board.

Let the children count the number of rows of squares
on the board. Mark off one row. Since there are ten rows of squares on the board, one row would be \( \frac{1}{10} \) of the board, which may be written also as .1.

The mileage indicator can be used to help the pupils understand the mathematical meaning of a decimal fraction.

The indicator should be set so that the dial registers zero in each place of the trip dial. The pupil should turn the stem on the back of the instrument. This stem controls the rotation of the numbers which appear on the dial. He will find that when the dial on the extreme right makes a full rotation the dial next on the left makes one-tenth of a turn. Thus, after the first rotation of the dial on the right, the number shown will be 10. A different color differentiates the number in the tenth's place from the figures to the left of this place. The number indicated is 1.0, which means one whole and no tenths. The number shown in the dial above is 1.7. This amount may be expressed with common fractions as \( \frac{17}{10} \).
Understanding to be taught: To find the number of square inches in a rectangle we multiply the number rows one inch wide by the number of square inches in a row.

Teaching aids: Wall chart, checker board, egg boxes, and a hundred board

Procedure:

1. Have the children measure the square inches in the center block.
2. Have the children count the number of rows of square inches and the number of square inches in each row.
3. Multiply the two.
4. Have the children count by ones all the square inch blocks in the center block to see if their answers
coincide.

After practicing with the aforementioned wall chart, a checkerboard, egg boxes, and hundred board may be used to illustrate the rule for finding area.
BIBLIOGRAPHY


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