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# $\bar{B}-B$ Mixing Constrains Topcolor-Assisted Technicolor

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## Abstract

We argue that extended technicolor augmented with topcolor requires that all mixing between the third and the first two quark generations resides in the mixing matrix of left-handed down quarks. Then, the  $\bar{B}_d-B_d$  mixing that occurs in topcolor models constrains the coloron and  $Z'$  boson masses to be greater than about 5 TeV. This implies fine tuning of the topcolor couplings to better than 1%.

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The impressive agreement of the standard model’s predictions with experimental data does not lessen the need for new physics to explain the dynamics underlying electroweak and flavor symmetry breaking. This physics may manifest itself not only in high energy collider experiments but also in precision low energy measurements of meson decays and mixing. In turn, low energy measurements powerfully constrain flavor physics scenarios. A prime example is technicolor [1] with extended technicolor (ETC) [2,3], a natural, dynamical scheme for electroweak and flavor symmetry breaking. There,  $|\Delta S| = 2$  effects in the neutral kaon system require ETC gauge boson masses of  $10^2$ – $10^4$  TeV *and* walking technicolor to produce the correct first and second generation quark masses. With such large masses, ETC by itself cannot account for the top quark’s mass. Therefore, we take it to be augmented by topcolor, a system referred to as topcolor–assisted technicolor (TC2) [4,5,6].

All TC2 models assume that color  $SU(3)_C$  and weak hypercharge  $U(1)_Y$  arise from the breakdown of the topcolor groups  $SU(3)_1 \otimes SU(3)_2$  and  $U(1)_1 \otimes U(1)_2$  to their diagonal subgroups. Here  $SU(3)_1$  and  $U(1)_1$  are strongly–coupled,  $SU(3)_2$  and  $U(1)_2$  are weakly–coupled, with the color and weak hypercharge couplings given by  $g_C = g_1 g_2 / \sqrt{g_1^2 + g_2^2} \equiv g_1 g_2 / g_{V_8} \simeq g_2$  and  $g_Y = g'_1 g'_2 / \sqrt{g_1'^2 + g_2'^2} \equiv g'_1 g'_2 / g_{Z'} \simeq g'_2$ . Top and bottom quarks are  $SU(3)_1$  triplets. The broken topcolor interactions are mediated by a color octet of colorons,  $V_8$ , and a color singlet  $Z'$  boson, respectively. By virtue of the different  $U(1)_1$  couplings of  $t_R$  and  $b_R$ ,  $V_8$  and  $Z'$  exchange between third generation quarks generates a large contribution  $\hat{m}_t(1 \text{ TeV}) \simeq 160 \text{ GeV}$  to the top mass, but none to the bottom mass.

If topcolor is to provide a natural explanation of  $\hat{m}_t$ , the  $V_8$  and  $Z'$  masses ought to be  $\mathcal{O}(1 \text{ TeV})$ . In the Nambu–Jona-Lasinio (NJL) approximation—which we rely heavily upon here—the degree to which this naturalness criterion is met is quantified by the ratio [7]

$$\frac{\alpha(V_8) + \alpha(Z') - (\alpha^*(V_8) + \alpha^*(Z'))}{\alpha^*(V_8) + \alpha^*(Z')} = \frac{\alpha(V_8) r_{V_8} + \alpha(Z') r_{Z'}}{\alpha(V_8)(1 - r_{V_8}) + \alpha(Z')(1 - r_{Z'})}. \quad (1)$$

Here,

$$\begin{aligned} \alpha(V_8) &= \frac{4\alpha_{V_8} \cos^4 \theta_C}{3\pi}, & \alpha(Z') &= \frac{\alpha_{Z'} Y_{t_L} Y_{t_R} \cos^4 \theta_Y}{\pi}; \\ \tan \theta_C &= \frac{g_2}{g_1}, & \tan \theta_Y &= \frac{g'_2}{g'_1}, & r_i &= \frac{\hat{m}_t^2}{M_i^2} \ln \left( \frac{M_i^2}{\hat{m}_t^2} \right), \quad (i = V_8, Z'); \end{aligned} \quad (2)$$

and  $Y_{t_{L,R}}$  are the  $U(1)_1$  charges of  $t_{L,R}$ . The NJL condition on the critical couplings for top condensation is  $\alpha^*(V_8) + \alpha^*(Z') = 1$ . In this letter, we show that, for such large couplings, TC2 is tightly constrained by the magnitude of  $\bar{B}_d$ – $B_d$  mixing: it requires  $M_{V_8} \simeq M'_{Z'} \gtrsim 5 \text{ TeV}$  [8]. This implies that the topcolor coupling  $\alpha(V_8) + \alpha(Z')$  must be within less than 1% of its critical value, a tuning we regard as unnaturally fine.

There are two variants of TC2: The “standard” version [5], in which only the third generation quarks are  $SU(3)_1$  triplets, and the “flavor–universal” version [9] in which all quarks are  $SU(3)_1$  triplets. In standard TC2,  $V_8$  and  $Z'$  exchange gives rise to flavor–changing neutral currents (FCNC) that mediate  $|\Delta B| = 2$ . In flavor–universal TC2, only  $Z'$  exchange can generate such FCNC. Our results constrain both types of TC2 theory.

The coloron interaction at energies well below  $M_{V_8}$  is

$$\mathcal{H}_{V_8} = \frac{g_{V_8}^2}{2M_{V_8}^2} \sum_{A=1}^8 J^{A\mu} J_\mu^A, \quad (3)$$

where the coloron current written in terms of electroweak eigenstate (primed) fields is given by

$$J_\mu^A = \cos^2 \theta_C \sum_{i=t,b} \bar{q}'_i \gamma_\mu \frac{\lambda_A}{2} q'_i - \sin^2 \theta_C \sum_{i=u,d,c,s} \bar{q}'_i \gamma_\mu \frac{\lambda_A}{2} q'_i. \quad (4)$$

The dominant coloron interactions for  $|\Delta B| = 2$  come from  $\bar{b}'b'\bar{b}'b'$  terms in Eq. (3). When written in terms of mass eigenstate fields, they are (neglecting smaller terms proportional to  $\alpha_C$  alone):

$$\mathcal{H}_{V_8} = \frac{2\pi\alpha_C \cot^2 \theta_C}{M_{V_8}^2} \sum_{\lambda_1, \lambda_2=L,R} \left( D_{\lambda_1 bb}^* D_{\lambda_1 bd_i} D_{\lambda_2 bb}^* D_{\lambda_2 bd_i} \bar{b}_{\lambda_1} \gamma^\mu \frac{\lambda_A}{2} d_{i\lambda_1} \bar{b}_{\lambda_2} \gamma_\mu \frac{\lambda_A}{2} d_{i\lambda_2} + \text{h.c.} \right) \quad (5)$$

Here,  $d_i = d$  or  $s$ , and the unitary matrices  $D_{L,R}$  arise from vacuum alignment in the down–quark sector. We discuss them shortly.

In order that  $Z'$  exchange not induce large  $|\Delta S| = 2$  transitions, quarks of the two light (electroweak basis) generations must have the same  $U(1)_1$  charge. Then, the  $Z'$  interaction for  $|\Delta B| = 2$  is

$$\mathcal{H}_{Z'} = \frac{2\pi\alpha_Y \cot^2 \theta_Y}{M_{Z'}^2} \sum_{\lambda_1, \lambda_2=L,R} \left( D_{\lambda_1 bb}^* D_{\lambda_1 bd_i} D_{\lambda_2 bb}^* D_{\lambda_2 bd_i} \Delta Y_{\lambda_1} \Delta Y_{\lambda_2} \bar{b}_{\lambda_1} \gamma^\mu d_{i\lambda_1} \bar{b}_{\lambda_2} \gamma_\mu d_{i\lambda_2} + \text{h.c.} \right) \quad (6)$$

Here,  $\Delta Y_\lambda = Y_{b\lambda} - Y_{d\lambda}$  is the difference of  $U(1)_1$  charges.

Vacuum alignment in the technifermion sector leads to unitary matrices  $W = (W^U, W^D)$  which represent the mismatch between the directions of spontaneous and explicit breaking of technifermion chiral symmetries. A common feature of these matrices is that all their phases are rational multiples of  $\pi$ . That is, for  $N$  technifermion doublets, these phases may be integral multiples of  $\pi/N'$  for *various*  $N'$  from 1 to  $N$  [10]. Extended technicolor couples quarks to technifermions and generates the “primordial” quark mass matrices  $\mathcal{M}_{qij} = \Lambda_{iIJj}^{qT} W_{IJ}^T \Delta_T$  (except for the  $\hat{m}_t$  part of  $\mathcal{M}_{utt}$ ). Here  $(q, T) = (u, U)$  or  $(d, D)$ ;  $i, j = u, c, t$  or  $d, s, b$ ;  $I, J$  label the technifermion flavors. The  $\Lambda_{iIJj}^{qT}$  are *real* ETC couplings of order  $(100 - 1000 \text{ TeV})^{-2}$  and  $\Delta_T$  is the real technifermion condensate renormalized at

the ETC breaking scale. If the  $\Lambda_{iI,Jj}^{qT}$  properly image  $W^T$ 's rational phases onto  $\mathcal{M}_q$ , there will be no strong CP violation, i.e.,  $\bar{\theta}_q = \arg \det(\mathcal{M}_u) + \arg \det(\mathcal{M}_d) \lesssim \mathcal{O}(10^{-10})$ . In this imaging, all elements of  $\mathcal{M}_u$  and  $\mathcal{M}_d$  have (generally different) rational phases that add to zero in the determinant. We assume this can happen in ETC models.

Vacuum alignment in the quark sector [11] is achieved by minimizing the quark energy  $E_q(U, D) = -\text{Tr}(\mathcal{M}_u U^\dagger + \mathcal{M}_d D^\dagger + \text{h.c.})$ . The up and down-quark alignment matrices  $U = U_L U_R^\dagger$  and  $D = D_L D_R^\dagger$  are  $3 \times 3$  diagonal blocks in the  $(SU(6)_L \otimes SU(6)_R)/SU(6)_V$  matrices.\* The ETC and TC2 interactions restrict the texture of the  $\mathcal{M}_u$  and  $\mathcal{M}_d$ . This, in turn, determines the form of  $U_{L,R}$  and  $D_{L,R}$  and, ultimately, of the TC2 amplitude for  $\bar{B}_d$ - $B_d$  mixing.

Flavor-changing neutral current limits imply that ETC contributions to  $\mathcal{M}_{u,d}$  are at most a few GeV, just enough to produce  $m_b(M_{ETC})$ . The TC2 interactions generate  $\hat{m}_t$ , but off-diagonal elements in the third row and column of  $\mathcal{M}_u$  come from ETC. They are expected to be no larger than the 0.01–1.0 GeV associated with  $m_u$  and  $m_c$ . Thus,  $\mathcal{M}_u$  is very nearly block-diagonal and, so,  $|U_{L,R tu_i}| \cong |U_{L,R u_i t}| \cong \delta_{tu_i}$ . Limits on  $\bar{B}_d$ - $B_d$  mixing induced by exchange of “bottom pions” [13,14], together with the need to generate appropriate intergenerational mixing in the Cabibbo–Kobayashi–Maskawa (CKM) matrix  $V = U_L^\dagger D_L$ , require that  $d_R, s_R \leftrightarrow b_L$  elements of  $\mathcal{M}_d$  are much smaller than the  $d_L, s_L \leftrightarrow b_R$  elements [6]. This makes  $D_R$  nearly  $2 \times 2$  times  $1 \times 1$  block-diagonal.

Since  $D_L$  contains almost all the mixing between  $b$  and  $d, s$  and, hence, between the third generation and the first two, the mixing pattern of  $V = U_L^\dagger D_L$  is essentially the same as that of  $D_L$ . To an excellent approximation,

$$\begin{aligned} |V_{td_i}| &= |U_{Ltt}^* D_{Lbd_i}| = |D_{Lbd_i}|; \\ V_{tb}^* V_{td_i} &= U_{Ltt} D_{Lbb}^* U_{Ltt}^* D_{Lbd_i} = D_{Lbb}^* D_{Lbd_i}. \end{aligned} \quad (7)$$

Our strongest limit on the  $V_8$  and  $Z'$  masses will arise from Eq. (7).

The dominant TC2 contribution to  $|\Delta B_d| = 2$  is given by the LL terms in  $\mathcal{H}_{V_8} + \mathcal{H}_{Z'}$ . The relevant observable is the  $B_L^0$ - $B_S^0$  mass difference. Since  $\Gamma_{12} \ll M_{12}$ , it is given by  $\Delta M_{B_d} = 2|M_{12}|$  [16]. Fierzing  $\mathcal{H}_{V_8}$  into a product of color-singlet currents and calculating the  $\bar{B}_d$ - $B_d$  matrix element in the usual vacuum-insertion approximation, the TC2 contribution to  $M_{12}$  is

$$2(M_{12})_{\text{TC2}} = \frac{4\pi}{3} \left[ \frac{\alpha_C \cot^2 \theta_C}{3M_{V_8}^2} + \frac{\alpha_Y \cot^2 \theta_Y (\Delta Y_L)^2}{M_{Z'}^2} \right] \eta_B M_{B_d} f_{B_d}^2 B_{B_d} (D_{Lbb}^* D_{Lbd})^2. \quad (8)$$

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\*Actually, quark vacuum alignment is based on first-order chiral perturbation theory, so it is inapplicable to the heavy quarks  $c, b, t$ . When  $\bar{\theta}_q = 0$ , Dashen's procedure is equivalent to making the mass matrices diagonal, real, and positive [12]. Thus, it correctly determines the quark unitary matrices  $U_{L,R}, D_{L,R}$  and the magnitude of strong and weak CP violation.

Here,  $\eta_B = 0.55 \pm 0.01$  is a QCD radiative correction factor for the LL product of color-singlet currents. We take  $f_{B_d}\sqrt{B_{B_d}} = (200 \pm 40)\text{MeV}$  [16], where  $f_{B_d}$  and  $B_{B_d}$  are, respectively, the  $B_d$ -meson decay constant and bag parameter. This TC2 contribution is to be added to the standard model one,

$$2(M_{12})_{\text{SM}} = \frac{G_F^2}{6\pi^2}\eta_B M_{B_d} f_{B_d}^2 M_W^2 S_0(x_t)(V_{tb}^* V_{td})^2, \quad (9)$$

where the top-quark loop function  $S_0(x_t) \cong 2.3$  for  $x_t = m_t^2(m_t)/M_W^2$  and  $m_t(m_t) = 167\text{GeV}$ .

To determine the  $\Delta M_{B_d}$ -restriction on TC2, we adopt the ETC-based analysis of quark mixing matrices made above. Then,  $D_{Lbb}^* D_{Lbd} = V_{tb}^* V_{td} \cong V_{td}$ , so that the standard model and TC2 contributions add coherently. Next, to simplify our discussion, we ignore for now the  $Z'$  contribution to  $\Delta M_{B_d}$  and  $\hat{m}_t$ . The NJL approximation then implies  $\cot^2 \theta_C = 3\pi/4\alpha_C(1\text{TeV}) \gtrsim 25$  for  $\alpha(V_8) \geq \alpha^*(V_8) = 1$  and  $\alpha_C(1\text{TeV}) \simeq 0.093$ . Using  $\Delta M_{B_d} = (3.11 \pm 0.11) \times 10^{-13}\text{GeV}$  [15], we plot in Fig. 1 the constraint in the  $(M_{V_8}, |V_{td}|)$  plane. The width of the allowed band comes mostly from the error in  $f_{B_d}\sqrt{B_{B_d}}$ .

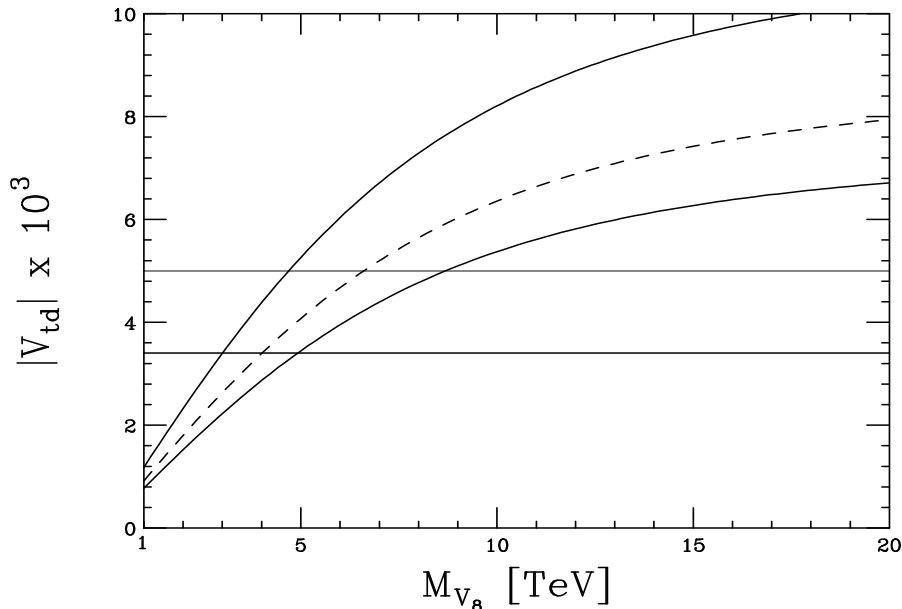


Figure 1: The allowed region in the  $(M_{V_8}, |V_{td}|)$  plane. The upper horizontal line comes from the lower limit on  $|V_{td}|$  derived by making use of  $\epsilon$  plus the CKM unitarity triangle. The lower line is the lowest value of  $|V_{td}|$  for which  $\text{Im}(V_{ts}^* V_{td})$  and, therefore,  $\epsilon'/\epsilon$  are not too small.

To set a bound on  $M_{V_8}$ , we must find the lowest possible value of  $|V_{td}|$ . If ETC contributions to the kaon CP-violating parameter  $\epsilon$  are negligible, this lowest  $|V_{td}|$  can be read off the  $(\bar{\rho}, \bar{\eta})$  plane fits using  $\epsilon$  and  $|V_{ub}/V_{cb}|$ . (Data on  $\Delta M_{B_{d,s}}$  are not used as these

quantities are affected by TC2.) The most recent fit [17] gives  $\bar{\rho} < 0.40$  and  $\bar{\eta} > 0.20$  at the 95% C.L. This yields

$$|V_{td}| = \lambda|V_{cb}|\sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} > 5 \times 10^{-3}, \quad (10)$$

where we used the  $2\sigma$  lower limit value  $|V_{cb}| = 0.036$ . This bound is displayed as the upper horizontal line in Fig. 1. Its intersection with the left edge of the band gives the conservative lower limit for  $M_{V_8}$ ,

$$M_{V_8} \geq 4.8 \text{ TeV} \quad \Rightarrow \quad \frac{\alpha(V_8) - \alpha^*(V_8)}{\alpha^*(V_8)} \leq 0.0075, \quad (11)$$

The tuning of  $\alpha(V_8)$  is unnaturally fine.

If ETC contributes significantly to  $\epsilon$ , we must remove its constraint from the standard model  $(\bar{\rho}, \bar{\eta})$  plane. The remaining limitation on  $|V_{td}|$ , other than unitarity of  $V$ , comes from  $\epsilon'/\epsilon$ . We expect that  $\epsilon'$  is unaffected by ETC [18]. In the standard-model Wolfenstein parameterization [15], it is given by

$$\frac{\epsilon'}{\epsilon} \equiv \text{Im}(V_{ts}^* V_{td})S = A^2 \lambda^5 \bar{\eta} S. \quad (12)$$

Here,  $A = |V_{cb}|/\lambda^2 = 0.83$  is obtained using  $|V_{cb}| = 0.04$ . Also,  $S = P^{(1/2)} - P^{(3/2)}$  where  $P^{(\Delta I)}$  contains the hadronic matrix elements in the  $|\Delta I| = 1/2$  and  $3/2$  amplitudes [19]. In Eq. (12),  $\epsilon$  is taken from experiment, so that potential ETC contributions to it are not an issue. From Ref. [19], hadronic matrix element calculations imply  $S < 21.6$ . The world average is  $\epsilon'/\epsilon = (19.3 \pm 2.4) \times 10^{-4}$  [20]. The experimental situation is still somewhat unsettled; we conservatively assume the lower limit  $\epsilon'/\epsilon > 12 \times 10^{-4}$ . This gives  $\bar{\eta} > 0.156$ . Using this, and the 95% C.L. upper limit  $|V_{ub}/V_{cb}| < 0.14$ , we find  $\bar{\rho} < 0.60$  and

$$|V_{td}| > 3.4 \times 10^{-3}. \quad (13)$$

Intersecting this value with the allowed band in Fig. 1 yields

$$M_{V_8} \geq 3.1 \text{ TeV} \quad \Rightarrow \quad \frac{\alpha(V_8) - \alpha^*(V_8)}{\alpha^*(V_8)} \leq 0.016 \quad (14)$$

Although somewhat a matter of taste, we regard this level of fine tuning at the edge of acceptability for a dynamical theory with naturalness as a design goal.

To estimate the effect of the  $Z'$  on this naturalness criterion, we suppose that  $V_8$  and  $Z'$  exchange contribute equally to generating  $\hat{m}_t$ , i.e., that  $\alpha^*(V_8) = \alpha^*(Z') = \frac{1}{2}$  in the NJL approximation. We also assume  $(\Delta Y_L)^2 \simeq Y_{tL} Y_{tR}$ . Then,  $\cot^2 \theta_C \geq \cot^2 \theta_C^* = 3\pi/8\alpha_C(1 \text{ TeV}) = 12.5$  and

$$\frac{\alpha_C \cot^2 \theta_C}{3M_{V_8}^2} + \frac{\alpha_Y \cot^2 \theta_Y (\Delta Y_L)^2}{M_{Z'}^2} \gtrsim \frac{\pi}{8} \left( \frac{1}{M_{V_8}^2} + \frac{4}{M_{Z'}^2} \right). \quad (15)$$

Equating the right-hand side to its maximum value  $\pi/(4(4.8 \text{ TeV})^2)$  obtained when we neglected  $Z'$  gives

$$M_{Z'} = 2 \left[ \frac{2}{(4.8 \text{ TeV})^2} - \frac{1}{M_{V_8}^2} \right]^{-\frac{1}{2}} > 6.8 \text{ TeV} \quad \Rightarrow \quad \frac{\alpha(Z') - \alpha^*(Z')}{\alpha^*(Z')} < 0.0042 \quad (16)$$

when  $M_{V_8} \rightarrow \infty$ . For  $M_{V_8} = 4.8 \text{ TeV}$ , we have  $M_{Z'} = 9.6 \text{ TeV}$  and

$$\frac{\alpha(V_8) r_{V_8} + \alpha(Z') r_{Z'}}{\alpha(V_8)(1 - r_{V_8}) + \alpha(Z')(1 - r_{Z'})} \simeq 0.0049. \quad (17)$$

In short, adding the  $Z'$  contribution to  $\Delta M_{B_d}$  does not make TC2 more natural.

It is possible to avoid this naturalness problem if, contrary to ETC expectations, all mixing between the third generation and the two light ones is contained in the up-sector matrices  $U_{L,R}$ . Even this possibility is constrained by  $V_8$  and  $Z'$ -exchange contributions to the neutron electric dipole moment,  $d_n$ . They affect only the up-quark moment. The  $V_8$  contribution suffices; it is

$$(d_u)_{\text{TC2}} = \frac{2e}{3} \frac{\alpha_C \cot^2 \theta_C}{4\pi} \frac{4}{3} \frac{m_t(m_t)}{M_{V_8}^2} \text{Im}(U_{Ltu}^* U_{Ltt} U_{Rtt}^* U_{Rtu}). \quad (18)$$

The limit [15]  $d_n < 0.63 \times 10^{-25} e\text{-cm}$  yields <sup>†</sup>

$$M_{V_8} \gtrsim 1 \text{ TeV} \sqrt{\frac{|\text{Im}(U_{Ltu}^* U_{Ltt} U_{Rtt}^* U_{Rtu})|}{10^{-7}}}. \quad (19)$$

Since  $U_{Ltu} = \sum_{d_i} D_{Lbd_i} V_{ud_i}^* \cong D_{Lbb} V_{ub}^*$  implies  $|U_{Ltu}| \gtrsim 2 \times 10^{-3}$ , a natural  $V_8$  mass of a 1–2 TeV suggests  $|U_{Rtu}| \lesssim 5 \times 10^{-5}$ . Thus, the analog of the Kominiis constraint [13,14] applies to  $U_R$ .

In summary, generation of light quark masses via extended technicolor strongly suggests that all quark mixing occurs in the left-handed down sector. Then, the TC2 mechanism for  $m_t$  requires, *in the NJL approximation*,  $V_8$  and  $Z'$  masses exceeding 5 TeV. This, in turn, needs fine-tuning of the topcolor couplings to within less than 1% of their critical values. We do not know how this difficulty will be resolved.

*Note added in proof:* The contribution to  $Z^0 \rightarrow b\bar{b}$  from top-pions, the pseudoGoldstone bosons arising from top condensation, also significantly constrains TC2 [21]. This contribution can be made consistent with experiment by increasing the top-pion decay constant and/or its mass. Increasing either requires raising the TC2 scale embodied in  $M_{V_8}$  and  $M_{Z'}$ . In the NJL approximation, we estimate that this implies a fine-tuning of about 1%, comparable to what we found above from  $\Delta M_{B_d}$ .

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<sup>†</sup> $\text{Im}(U_{Ltu}^* U_{Ltt} U_{Rtt}^* U_{Rtu}) = 0$  in ETC, but we cannot appeal to that here.

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