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Quadratic equations in one unknown

QUADRATIC EQUATIONS IN ONE UNKNOWN

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QUADRATIC EQUATIONS IN ONE UNKNOWN

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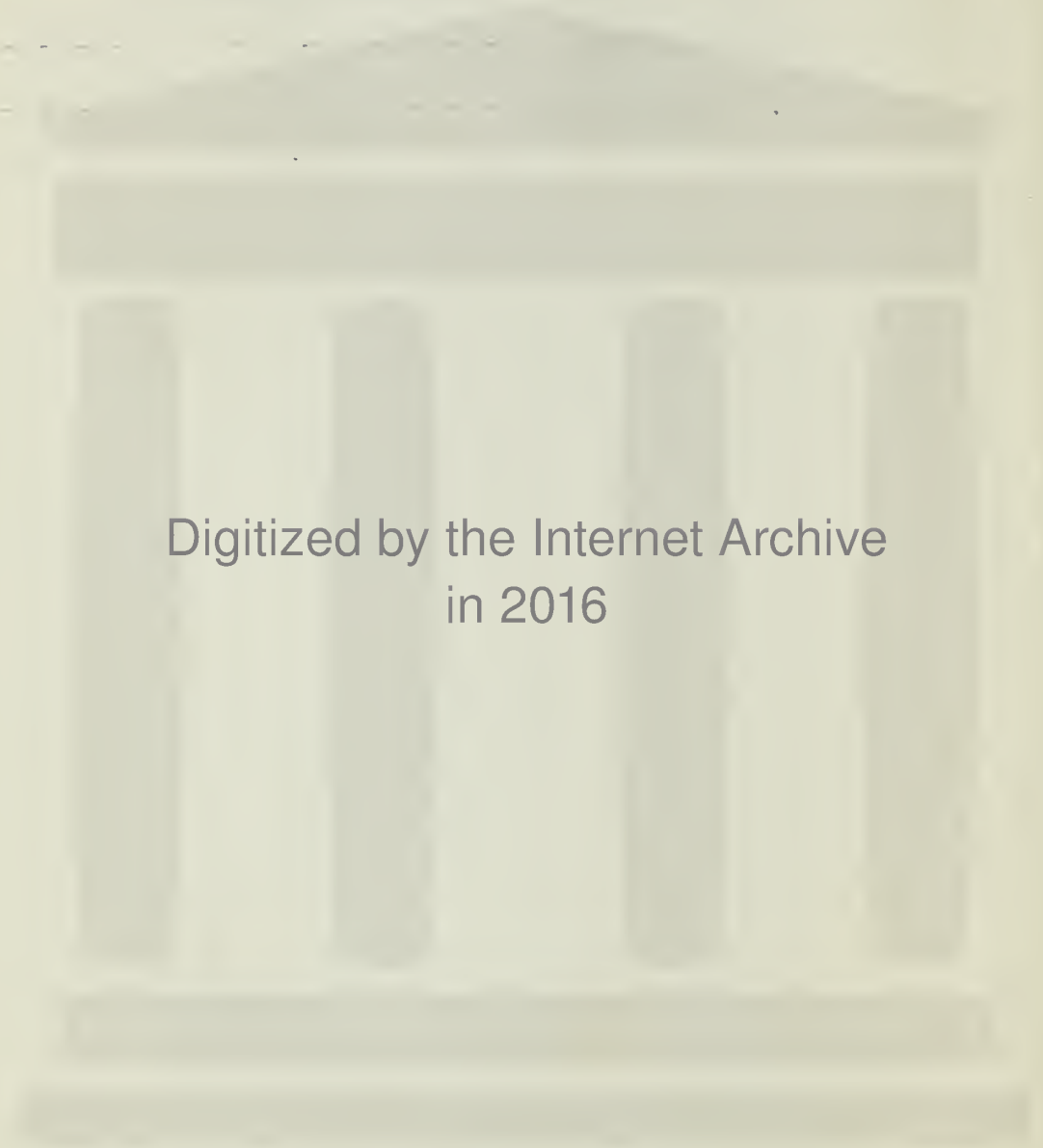

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CHAPTER I

THE PROBLEM AND ITS BACKGROUND

Statement of the Problem

The problem.-- The problem of this paper is to apply the basic principles and procedures set forth in Fundamentals of Secondary-School Teaching [1] and the course in the Unit Method in the Secondary School [2] to the unit organization and classroom presentation of the topic, quadratic equations in one unknown, in college-preparatory mathematics in a high school. The topic, quadratic equations in one unknown, was prepared for eleventh-grade students.

The Pupils to Be Taught

Class groups.-- The unit was prepared for two class groups of college-preparatory students enrolled in the eleventh-grade course in second-year algebra, which, at that time, was elective for both boys and girls in the college preparatory curriculum. Group A was composed of twenty-two students, eight boys and fourteen girls, all in the eleventh-grade. Group B contained twenty-one students, thirteen boys and eight girls, of whom two were in the twelth-grade and the remainder in the

[1] Roy O. Billett, Fundamentals of Secondary-School Teaching, Houghton Mifflin Company, Boston, Massachusetts, 1940, pp. xvi, 671.

[2] Roy O. Billett, "Unit Method in the Secondary School," Professor of Education, School of Education, Boston University Summer School 1948.

eleventh-grade. The classes met five periods a week for forty-three minute periods. Group A met during the second period in the day at 9:26 A.M. the first period of work, which followed the home-room period. Group B met at 10:25 A.M. which was the fourth period of work.

Statistics regarding the pupils.-- From the four-year record card of each pupil, filled in the office, it was possible to obtain the following information: chronological age, intelligence quotient based on the Otis Quick Scoring Test, and previous yearly averages in numerical scores of mathematics in grades nine and ten. From the guidance office, it was possible to secure the following information based on the test given by the Boston University Testing Bureau: reading comprehension, space-relations ability, problem-solving, and intelligence quotient. These data are recorded in Tables 1 to 3 on pages 3 to 6.

Chronological ages.-- At the time the pupils commenced working on the unit, the chronological ages varied from sixteen years, no months, to seventeen years, nine months, in Group A; and from fifteen years, ten months, to eighteen years, one month, in Group B.

Table 1. Chronological Ages of Pupils in Group A and Group B on March 1, 1949, When Starting the Unit on Quadratic Equations in One Unknown.

Ages in Years and Months	Group A Frequency	Group B Frequency
(1)	(2)	(3)
18-0 to 18- 5	0	2
17-6 to 17-11	1	1
17-0 to 17- 5	5	4
16-6 to 16-11	8	9
16-0 to 16- 5	8	4
15-0 to 15-11	0	1
Totals	22	21

The median ages were sixteen years, nine months, for Group A: and sixteen years, ten months, for Group B.

Intelligence quotients.-- According to the Otis Quick Scoring Test, the intelligence quotients for Group A ranged from 99 to 124; and according to the Boston University Battery Tests, Progressive Mathematics, Form A, ranged from 98 to 127, with a median of 114.05. 1/

1/ Boston University Battery Tests for the grade 11 pupils.
California Mental Maturity Test--Advanced Form.
Minnesota Test For Clerical Workers.
Minnesota Paper Form Board Test--Form A.
Cooperative Test of Reading Comprehension--Form C I T.
California Occupational Interest Inventory Test--Advanced Form A.
Progressive Mathematics Tests-Advanced Form A.

Table 2. Achievement Test Grades for Group A for the Unit on Quadratic Equations in One Unknown as Derived from the Boston University Battery Tests and Otis Quick Scoring Tests When the Pupils Were in Grades Nine and Eleven.

Range	Intelligence Quotients		Reading Comprehension	Space Relations	Problem Solving Ability
	Otis Test	Boston University Test			
(1)	(2)	(3)	(4)	(5)	(6)
120-129	4	4			
110-119	13	8			
100-109	4	9	1	1	1
90- 99	1	1	5	3	6
80- 89			6	2	3
70- 79			1	4	7
60- 69			1	2	0
50- 59			3	3	3
40- 49			2	1	0
30- 39			1	2	1
20- 29			1	3	1
10- 19			1	1	0
Totals	22	22	22	22	22

The intelligence quotients for Group B according to the Otis Quick Scoring Test ranged from 83 to 124 and according to the Boston University Battery Tests, Progressive Mathematics Form A, ranged from 95 to 125, with medians of 112.1 and 110.6 respectively.

Table 3. Achievement Tests Grades for Group B for the Unit on Quadratic Equations in One Unknown as Derived from the Boston University Battery Tests and Otis Quick Scoring Tests When the Pupils Were in Grades Nine and Eleven.

Range	Intelligence Quotients		Reading Comprehension	Space Relations	Problem Solving Ability
	Otis Test	Boston University Test			
(1)	(2)	(3)	(4)	(5)	(6)
120-129	7	2	2		
110-119	9	9	9		
100-109	1	6			
99- 99	2	2	2	3	6
80- 89	2	0	6	1	3
70- 79			2	2	6
60- 69			2	4	0
50- 59			2	2	2
40- 49			1	2	1
30- 39			1	1	0
20- 29			2	2	2
10- 19			1	2	0
Totals	21	19*	19	19	19

*The data for two of the pupils were not available.

The intelligence quotients of the lowest eight pupils indicated that they might expect to have trouble with the work to be covered in the unit for quadratic equations in one unknown.

Previous marks in mathematics.-- Previous marks in mathe-

matics for the pupils in Group A and Group B were studied to see if the marks could explain any difficulties which might occur in the study of the unit.

Table 4. Previous Marks in Mathematics for Pupils in Group A and Group B.

Percentage	Algebra 1	Plane Geometry	Algebra 1	Plane Geometry
95-100	1	1	1	0
90- 94	4	3	3	2
85- 89	7	4	6	3
80- 84	6	5	4	6
75- 79	2	2	1	4
70- 74	2	5	5	3
65- 69	0	0	0	1
60- 64	0	0	1	1
No Marks Available	0	2	0	1
Totals	22	22	21	21

The first-year algebra and plane-geometry marks for the previous two years of mathematics were given in numerical grades. Seventy per cent was the lowest passing mark except for that one boy in Group B. This boy's mark was sixty per cent. This students transfered from an out-of-state public high school after completing his eleventh-year of work. The lowest passing mark for this student was given as sixty per cent. Assuming that the general health and effort of the

students were good and the marking system fairly reliable, the ninth-year algebra marks and the tenth-year plane geometry marks indicated that there were five pupils in Group A and eight pupils in Group B who might be considered not mathematically inclined, and who might be expected to have trouble with the unit. As events disclosed, these pupils had difficulty with the work, especially the section covering solving quadratic equations in one unknown by the method known as completing the square. This is discussed further in Chapter II.

Achievement tests.-- The Bosotn University Battery Tests were given to the students when they were beginning the eleventh-grade. The Otis Quick Scoring Tests were given to the students when they were beginning the ninth-grade.

A study of the previous marks in mathematics indicated that there were four students in Group A who might be expected to have trouble with the work in the unit. The grades showed that in Group B there were six students who might be expected to have trouble with the work in the unit. There were fifteen pupils in Group A who could be expected to do the work in the unit with some individual help. In Group B there were eleven pupils who could be expected to do the work in the unit with additional help. These conditions had to be considered when planning the core activities and when choosing the reference books for the pupils' use.

Second Year Algebra [1] and Essentials of Algebra [2] were chosen with these pupils in mind. The remainder of the reference books, given in the pupils' reference list, were well fitted for the more able pupils of the two groups.

Home background of the pupils.-- Practically, without exception, the pupils had fine home backgrounds. The majority of the parents owned their own homes, most of which were single dwellings. Of the forty-three students represented, thirty-three lived in homes owned by their parents.

The parents were active in the parent-teacher associations in the nine elementary schools of the city. The interest of parents in their children's education continued into the high school. This was shown by the large percentage of attendance during parents' night which was held in the fall of the school year.

The parents of the pupils for whom the unit was planned were in various types of work in the city. The parents' occupations were as follows: jeweler, coalman, credit manager, factory superintendent, carpenter, accountant, insurance agent, photographer, toolmaker, lawyer, machinist, molder, purchasing agent, and dietician.

1/ Raleigh Schorling, Rolland Smith, John Clark, Second Year Algebra, World Book Co., Yonkers-on-Hudson. N.Y. 1942

2/ Walter W. Hart., Essentials of Algebra, D.C. Heath and Co., Boston, Mass.; 1947

Extracurricular and out-of-school activities.-- The members of both Group A and Group B showed a strong interest in athletics, and a few were varsity player. Several of the students belonged to the following school activities in which they took and active part: the glee club, the dramatic club, the school paper, the school annual, the student council, the band, the orchestra, the video club, the chorus, and the Junior Red Cross.

The many activities in which the pupils were participants outside of the school were The Young Men's Christian Association, the DeMolay, the Boy Scouts, the Girl Scouts, and the young peoples' religious organizations. Just about half of the students had part time employment after school, most of which was in the stores.

The students expressed interests in hobbies such as stamp collecting, knitting, music, photography, model plane building, sewing, and boat building. One student took an active part in working on automobiles at one of the local garages, while another student made a hobby of collecting phonograph records.

Colleges which the pupils were preparing to attend.-- Only seven of the total of forty-three students did not plan to attend college. The two groups were particularly interested in the following colleges and universities: Boston University, Northeastern, Bowdoin, Babson Institute, Mount Ida, Simmons, Wheaton, Brown, Catherine Gibbs, Harvard, University of Massacusetts, Cornell, Tufts, Wentworth Institute, Regis, General

Motor's Institute, United States Maritime Academy, Rhode Island School of Design, and Jackson.

The seven students not planning to attend a college or university had not made up their minds just what would be their chosen profession.

The School

Situation of the school.-- The school in which the unit was taught is the only high school in a small city of about twenty-five thousand inhabitants and situated about fifteen miles north of Providence, Rhode Island. The high school is a four-year school with approximately seven hundred and fifty pupils and thirty-five teachers. The main building was erected in 1912 and has a large wing that was added seventeen years later. The light yellow brick structure faces a lawn surrounded on one side by sloping grounds planted with shrubs. The other schools in the city consist of nine elementary schools, a jewelery trade school, and a vocational school.

This small city boasts of some seventy small industries, the majority of which are jewelry manufaturing concerns. There are several attractive residential sections, a park which includes a small zoo, and a very fine golf course. North of the city, some three to four miles, is located a small airport.

The classroom.-- The classroom, twenty-one feet by thirty-

three feet, is located on the second floor of the building and has a pleasant northeast exposure. The room overlooks a desirable residential area.

The furniture consisted of forty stationary desks. Three tables, three by five feet, were brought in for group work. The tables were placed in the front part of the room. One section of the built-in book case was reserved for the reference books, the card index file of optional related activities, and the manila folders holding the completed work and testing material. The students worked in groups and kept track of the members of each group. This was for a quick outdoor check in case of fire drill. The room had three doors, one in the front at the head of the stairway, and two on the south side opening into the corridor. The only other furniture in the room was the teacher's desk and chair. The use of the three tables was the best solution for the laboratory type of classroom that could be arranged under the existing conditions. The built in bookcase provided an excellent storage space for the reference books, tests, and illustrative materials.

The front wall of the classroom contained a fourteen foot blackboard. The south side of the classroom held an eighteen foot blackboard between the two side doors. The rear wall held a sixteen foot bulletin board. This bulletin board was very helpful during the work on the unit.

It was impossible to secure a filing cabinet so some small cardboard boxes were used. The boxes were just the right size

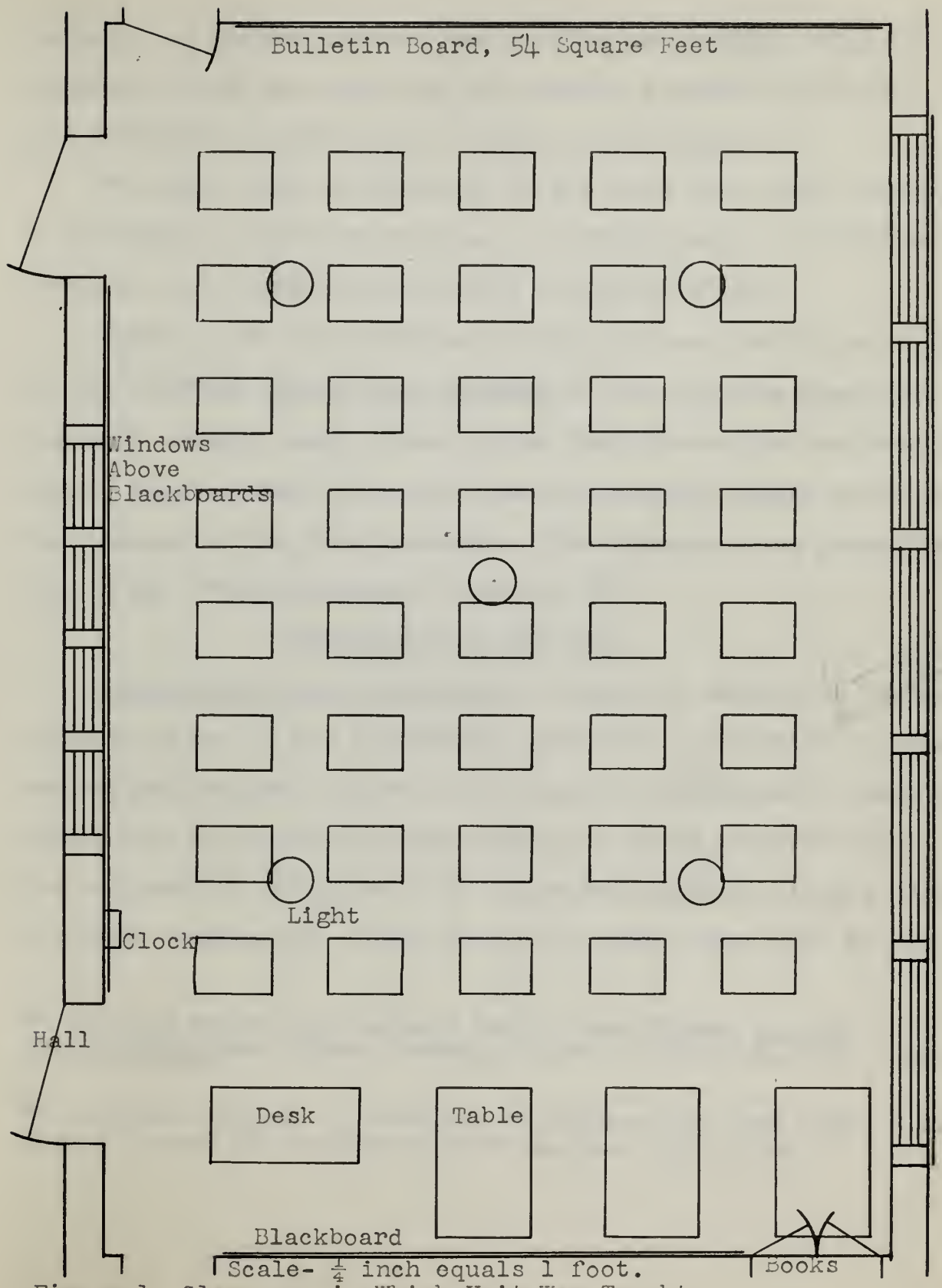


Figure 1. Classroom in Which Unit Was Taught.

to hold the manila folders used to file the pupils' work. A separate folder was kept for each pupil; and when any work was completed, it was filed in its correct place.

The only other furnishings in the room were those common to a mathematics classroom such as: a straight-edge, a blackboard compass, and a hanging blackboard for graphic work.

Books.-- At the beginning of the unit each pupil had a copy of the textbook Second Year Algebra [1] The bookcase contained one copy of each book listed in the "References for the Students' Use." Most of the reference books were sample copies given to the teacher by the book salesmen. The bookcase also contained copies of, "The Mathematics Teacher" [2]

Preparation of the Unit

Determination of objectives.-- Quadratic equations in one unknown is one of the fundamental sections of material for the second year algebra course of all college preparatory students. This topic is included in the course of study prescribed by the mathematics department for the eleventh-grade algebra class. A careful inspection of the course of study prescribed by the

[1] Raleigh Schorling, Rolland Smith, John Clark, Second Year Algebra, World Book Company, Yonkers-on-Hudson, N.Y. 1942

[2] National Council of Teachers of Mathematics, New York, N.Y. twenty issues of the Mathematics Teacher, 1947, 1948.

mathematics department showed that a detailed outline of the work to be covered must be made. This outline was worked out; and, from this outline of objectives, a general statement of the unit was written. This statement contained the main purpose to be accomplished. The delimitation was written in simple declarative sentences stating exactly what was to be taught. In addition there was a list of probable incidental and indirect learning products. The unit assignment of unit of experience sequence was then prepared. Incorporated in this were the introductory activities, the core-activities for which all pupils would be held responsible, the laboratory experiences, the optional related activities, predetermined instruction points, intervals for the "pooling of experiences" and a tentative time for the evaluation of the unit. For the pupils' use, a pupils' study and activity guide was prepared which contained the core activities with instructions, and detailed references to the textbook and reference books.

In the unit method of teaching, each pupil should have his own separate copy of the study guide and of the reference books. Copies of the study guide and reference books were prepared by the teacher, and run off on the duplicating machine in the high school office. Each pupil was given a copy of the study guide and a list of reference books the day the teaching of the unit started. The optional related activities were typed on three by five inch cards and placed in a box in the back

case.

Preparation of the examination.-- The examination consisted of four sections to be given during one class period. Section I contained forty true-false statements; Section II, fifteen completion items; Section III, ten matching word and sentence statements; and Section IV ten examples to be worked out.

The first section consisted of statements to test the understanding of concepts and the ability to apply these concepts. These statements were arranged to follow the items in the delimitation of the unit. The second section consisted of statements to test the understanding of the fundamentals of the work the unit covered. The third section consisted of matching words and statements to test the understanding of new mathematical words found in the study of quadratic equations in one unknown. The fourth section consisted of examples to be worked out to test the understanding of various methods of solving quadratic equations in one unknown.

The test was printed on the duplicating machine so that every pupil would have a copy, and it was given to all the students the day before the teaching of the unit was started and again when the student had completed the work on the unit. This was done in order to measure the pupil progress in the unit.

Summary of preparation.-- The teacher cut all the stencils and duplicated the study guides and tests. This work was com-

pleted during the mid-winter recess. All the materials were placed in the classroom to be ready when the work on the unit commenced. The built-in bookcase was cleaned out and all reference books, the card index of optional related activities, study guides, and testing material were placed on the shelves. The janitors were very cooperative in helping to get the tables and chairs to be used, and the office staff assisted running the duplicating machine. A log book was prepared to keep a daily record of the unit as it progressed. The log was also used to note omissions and corrections to be made on the unit. At the conclusion of the unit, the entries indicated those phases of the unit which should be improved or eliminated and those phases worthy of being retained.

Table 5. Pupils' Scores for Group A on Pre-Test on Quadratic Equations in One Unknown.

Pupil	Section I	Section II	Section III	Section IV	Totals
(1)	(2)	(3)	(4)	(5)	(6)
1	29	3	7	3	42
2	33	3	8	2	46
3	23	2	8	4	37
4	25	2	10	2	39
5	30	3	7	2	39
6	26	4	7	4	43
7	25	1	8	2	36
8	23	1	8	3	35
9	27	4	7	3	41
10	33	6	7	2	48
11	34	5	8	3	50
12	26	3	8	3	40
13	35	7	10	4	56
14	23	0	7	3	33
15	31	3	10	5	31
16	29	7	8	4	48
17	27	2	5	2	36
18	30	6	6	2	44
19	20	0	7	1	28
20	26	1	7	0	34
21	28	1	7	3	39
22	28	0	0	0	28

Table 6. Pupils' Scores for Group B on Pre-Test on Quadratic Equations in One Unknown.

Pupil	Section I	Section II	Section III	Section IV	Totals
(1)	(2)	(3)	(4)	(5)	(6)
23	23	1	7	3	34
24	23	3	3	2	31
25	25	2	0	0	27
26	27	0	4	0	31
27	27	2	8	0	37
28	25	1	4	1	31
29	26	6	5	0	37
30	21	0	5	2	28
31	26	2	0	2	30
32	29	2	5	3	39
33	32	6	7	4	49
34	20	5	5	4	34
35	12	3	4	0	19
36	18	1	7	3	29
37	22	1	7	3	33
38	22	2	3	2	31
39	29	4	8	4	45
40	25	1	6	2	34
41	28	4	7	0	39
42	19	2	4	5	30
43	19	0	4	0	23

Table 7. Pupils' Scores for Group A on Final Test on Quadratic Equations in One Unknown.

Pupil	Section I	Section II	Section III	Section IV	Totals
(1)	(2)	(3)	(4)	(5)	(6)
1	32	7	8	8	55
2	31	7	8	6	52
3	35	13	10	9	67
4	29	9	10	8	56
5	37	8	10	7	62
6	27	5	6	3	41
7	26	10	8	6	52
8	37	12	9	9	67
9	33	9	10	8	60
10	37	13	8	5	63
11	39	14	10	10	73
12	39	14	10	8	71
13	37	7	10	7	61
14	33	7	8	6	51
15	34	8	8	9	59
16	30	6	8	9	53
17	31	2	6	3	43
18	33	7	7	6	53
19	36	7	10	9	62
20	29	6	8	4	47
21	35	7	8	8	43
22	27	6	6	4	43

Table 8. Pupils' Scores for Group B on Final Test on Quadratic Equations in One Unknown.

Pupil	Section I	Section II	Section III	Section IV	Totals
(1)	(2)	(3)	(4)	(5)	(6)
23	30	3	7	7	47
24	30	7	8	6	51
25	27	4	0	6	37
26	27	6	7	4	44
27	35	13	10	5	63
28	32	13	8	8	60
29	33	6	10	6	55
30	33	12	10	7	62
31	31	12	10	7	60
32	38	14	10	9	71
33	35	9	8	10	62
34	32	14	10	7	63
35	26	6	6	3	41
36	31	12	10	5	58
37	32	6	8	5	51
38	39	14	8	10	71
39	37	13	8	5	63
40	36	13	8	8	65
41	37	15	8	7	67
42	31	7	8	9	55
43	31	8	3	0	42

CHAPTER II

UNIT ORGANIZATION OF THE TOPIC, QUADRATIC EQUATIONS IN ONE UNKNOWN

General Statement of the Unit

General Statement.-- A thorough understanding of the quadratic equation in one unknown, the methods of solution, and its use in problem solving, is necessary for the continued study of science and mathematics.

Delimitation of the Unit

1. The world needs men and women who have mathematical ability. The study of the parabolic path of comets in astronomy and the study of the parabolic curves of light and sound reflectors are examples of the many uses of the quadratic equation in one unknown.

2. The quadratic equation in one unknown is one of the most powerful tools we have in algebra. The quadratic equation is indispensable for figuring the curves of cables for suspension bridges, the parabolic curves of arches in construction, and problems of motion in physics.

3. The two types of quadratic equations in one unknown must be understood; namely,

   a. Incomplete quadratic equations.

      Type form, $ax^2 + c = 0$

   b. Complete quadratic equations.

      Type form, $ax^2 + bx + c = 0$

4. The study of the quadratic equation in one unknown involves the use of new words for the mathematical vocabulary, such as the discriminant, the abscessa, the ordinate, the axis of symmetry, and imaginary roots of an equation.

5. All quadratic equations in one unknown are of the standard type form, $ax^2 + bx + c = 0$.

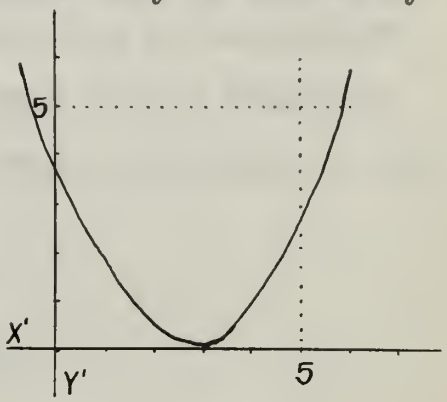
6. The quadratic equation is an equation of the second degree.

7. The quadratic equation in one unknown may be solved by four methods; namely,

a. The graphic method:

Example: $x^2 - 6x + 9 = 0$

x=	0	6	2	4	3
y=	9	9	1	1	0



b. The factoring method:

Example: $x^2 - 6x + 9 = 0$
$(x - 3)(x - 3) = 0$
$x = 3,\ x = 3$

c. The method of completing the square:

Example: $2x^2 + x - 6 = 0$
$x^2 + \frac{1}{2}x = 3$
$x^2 + \frac{1}{2}x + 1/16 = 3\ 1/16$
$(x + \frac{1}{4})^2 = 49/16$
$x = -2,\ x = 3/2$

d. Quadratic equation formula method:

Example: $x^2 - 6x + 9 = 0$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$x = \frac{6 \pm \sqrt{0}}{2}$$

$$x = 3$$
$$x = 3$$

8. The formula for solving all quadratic equations in one unknown is derived from the standard type form equation $ax^2 + bx + c = 0$.

Quadratic equation formula: $x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2a}$

9. All quadratic equations in one unknown have two roots.

10. The solutions of the quadratic equation in one unknown are the roots of the equation. The roots of the quadratic equation are combinations of the following, depending on the individual equation:

    a. Real or imaginary roots.

    b. Rational or irrational roots.

    c. Equal or unequal roots.

11. The solution of quadratic equations in one unknown may be determined by inspection of the character of the roots.

12. The discriminant and its use are important to the study of the quadratic equation in one unknown.

13. The quadratic equation in one unknown are the abscissa of the point where the parabola of the graph crosses the x-axis.

14. The quadratic equation in one unknown may be formed from the roots themselves.

15. The graphic solution of the quadratic equation in one

unknown is the parabola. The parabola is one of the types of a conic section.

16. One should be able to read intelligently and understand the parabolic graph of a quadratic equation in one unknown.

17. The effect of changing the constant term of a quadratic equation in one unknown is to raise or lower every point on the graph by a constant number of units.

18. The quadratic equation in one unknown is used to solve written problems dealing with an unknown power of the second degree; namely,

    a. Problems involving motion.

    b. Problems involving paths of projectiles.

    c. Problems involving paths of comets.

    d. Problems involving arches in construction.

    e. Problems involving reflectors of light and sound.

19. The second course in algebra, in many schools, is based on the quadratic equation. The quadratic equation in one unknown is the fundamental concept of the course in advanced algebra.

This unit will be followed by a unit on irrational and imaginary numbers.

List of Probable Indirect and Incidental Learning Products

The pupil will learn to appreciate the value of methematics for the continued study of mathematics and science.

The pupil will realize the advantages of working with unknown numbers.

The advantages of using more than one method for the solving of problems and examples is valuable.

The pupil will discover that methematics is not merely a collection of methods useful in the sciences, a vast unified system of reasoning which possesses many of the characteristics of a fine art.

The pupil will become acquainted with a logical structure of a methematical system and thus provide himself with a standard of exact reasoning which should help him to achieve a more critical attitude toward conclusions arrived at in other fields.

The pupil should develop and clarify understandings of algebra.

The pupil should see the necessity of understanding all fundamentals before advanced work may be accomplished.

The pupil should become more familiar with the terminology, notation, and symbolism of algebra, and to perfect operational facility.

Accuracy pays when working mathematical problems. A well thought out problem can be ruined by carelessness in compu-

tation.

This unit is necessary if we are to understand the following work in a second course in algebra.

The pupil will learn how many of the fundamental ideas of mathematics have their sources in physical experience.

The pupil will learn the practical application of algebraic ideas to physics, engineering, science, and art.

Persistent review and pratice, both in the skills of algebra and in their application, is necessary, otherwise they will deteriorate through disuse.

The pupil will learn the advantages of working together in groups.

The work should develop the student's ability to think out problems for himself, and learn by using his own initiative.

The work should develop, in the pupil, those abilities and qualities considered most desirable as a preparation for a better life.

The ability to read with percision and understanding and to form habits of reflective thinking are invaluable to the student.

The accuracy of work is important as it fosters and develops self-confidence.

The Unit Assignment

Tentative time.-- The tentative time for teaching the unit of Quadratic Equations in One Unknown is four weeks.

1. Introductory activity.-- An introductory talk was given by the teacher centering around the following topics:

a. The quadratic equation in one unknown has many uses in mathematics, as well as physics, engineering, astronomy, and art.

b. The parabola as a conic section.

c. The parabola as the graph of a quadratic equation in one unknown.

d. The difference between a linear equation and a quadratic equation in one unknown.

e. Applications of the parabola:

1. Parabolic paths of comets.

2. Parabolic curves of light and sound reflectors, and telescopes.

3. Parabolic curves used in art.

4. Parabolic curves used in construction.

5. Parabolic curves of projectiles.

f. The importance of the quadratic equation in one unknown for the further study of mathematics and science.

g. Wall chart showing a parabola worked out on a graph.

h. Blackboard demonstration showing the path of a projectile.

i. Pictures showing the use of the parabola in con-

struction.

2. The objective test.-- The objective test consists of questions covering all the topics included in the unit.

Core activities.-- The core activities are marked with an asterisk. Each item appears on the pupil's copy of the general study-and-activity guide.

*3. What goes up must come down. $S = 100t - 16t^2$ is an example of the equation necessary to solve a problem of motion. This formula enables us to find out how high an object propelled upward will go, how long it will take to go a certain height, and when it will reach the ground.

This equation cannot be solved as a linear equation can be solved; therefore, it is necessary to use other methods. This new equation is called a quadratic equation in one unknown.

Make a list of as many uses or examples of the quadratic equation in one unknown as you can, giving some illustrations of these examples that you have found in newspapers or magazines. (1:37; 2:249; 2:154; 2:155; 3:38; 3:150)

*4. There are two types of quadratic equations in one unknown that should be understood, the complete quadratic equation, and the incomplete quadratic equation. What type equation is $ax^2+bx+c=0$? What type equation is $x^2-4=0$? Can this equation be solved as a quadratic equation in one unknown? The solution of an incomplete quadratic equation has the

sign ±. Why is it necessary to use both the plus and the minus sign for the roots of these equations? How do quadratic equations differ from each other?

Work out five examples from one of the following references. (1:251 to 252; 2:158; 2:162)

*5. You will find new words for your mathematical vocabulary while working on this unit. Some of these words will be underlined in the study-and-activity guide. Look up the meaning of all the new words you find. Many of these words will be in the glossary which will be found on the reference table. Keep a list of all the new words and remember their meanings. Would it be possible to study and understand quadratic equations in one unknown without the use of new words?

*6. All quadratic equations in one unknown may be reduced to one standard type form $ax^2 + bx + c = 0$. Look through your previous work and select several quadratic equations in one unknown that are of this standard type form? Look through some of the algebra text books on the reference table and select several quadratic equations in one unknown that are not arranged in the order of the standard type form equation. List the original equation in one column; in the other column, list the equation after it has been rearranged in the order of the standard type form equation. Why is it advantageous to rearrange the equations in the order of the standard type form.

*7. The quadratic equation in one unknown is an equation of the second degree. What is an equation of the first degree

called? Give an example of an equation in the first degree. Give an example of an equation in the second degree. Why is it necessary for mathematicans to use equations of the quadratic type form? (1:39; 1:249; 2:111; 2:154; 3:38; 3:67)

*8. There are four methods which may be used to solve quadratic equations in one unknown.

a. The graphic method.-- The graphic method of solving quadratic equations in one unknown is not always accurate. The accuracy depends upon the exactness of the graph paper and the skill of the worker. What is the graphic form of a quadratic equation in one unknown? What is the path of a projectile such as a baseball, or a connon ball? What is the axis of symmetry? What is the y-intercept? What is the x-intercept? In what respect are all graphs of a quadratic equation in one unknown alike? How does a change in the value of the coefficient of x square affect the graph of a quadratic equation? Read one of the following references. (1:171; 1:232 to 234; 2:94; 3:152)

Work out one of the examples in each of the following references. (1:240 Exercise 2; 1:240 Exercise 3; 1:240 Exercise 4; 1:240 Exercise 5)

Optional: Work out any four examples in one of the following references. (2:159; 3:153)

b. The factoring method.-- The solution of quadratic equations in one unknown by the graphic method gives results correct to the nearest tenth only. It is necessary to use other

means of solving quadratic equations in one unknown which will yield more accurate results. One of these methods is by factoring. The solution by factoring is based upon the following principle: If the product of two factors is zero, one of the factors must be zero. Why? Is the factoring method of solving the quadratic equation in one unknown entirely satisfactory? Can any quadratic equation in one unknown be solved by the factoring method? Why must one side of the equation be equal to zero in order to factor the equation?

Work all the examples in one of the following references. (1:260 to 261; 2:160 to 161; 3:159)

c. Method of completing the square.-- The solution of the quadratic equation in one unknown is fairly simple provided the quadratic equation is factorable. Unfortunately this is not frequently the case in practical problems; and so, one must consider a new method, such as the method of completing the square. The method of completing the square is particularly important since it leads to the solution of a formula by means of which any quadratic equation in one unknown may be solved. What is the general type form of equation for all quadratic equations? What is a perfect trinomial square? Is it possible to add an expression or expressions to an equation without destroying the original value of the equation? Why must the term to be added to the trinomial equation always be a perfect square? How is the term to be added to the trinomial equation always obtained from the coefficient of x?

Work five examples from one of the following references.

(1:253 to 254; 2:162 to 163; 3:159 to 161)

d. The quadratic equation formula.-- The standard type form of a quadratic equation in one unknown is $ax^2 + bx + c = 0$. The solution of this standard type form equation will lead to a formula which may be used to find the roots of any quadratic equation in one unknown. Solve the standard type form equation by the method known as completing the square? What do the terms a, b, and c stand for in the standard type form equation? What is the quadratic equation formula? Why is this formula used? What was the quadratic equation formula derived from? When is it practical to use the quadratic equation formula?

Read one of the following references.

(1:257; 2:166; 3:162 to 164)

Work five examples from one of the following references.

(1:258; 1:259; 2:167; 2:168; 3:165)

*9. There is a standard type form formula that may be used to solve all quadratic equations in one unknown. You found this formula in your work previous to now. Look this formula up if you do not remember it. Study the deviation of this formula from the standard type form quadratic equation. What do the terms a, b, and c stand for in the standard type form equation? Can this formula be applied to a linear equation?

Read one of the following references.

(1:256 to 257; 2:166 to 167; 3:162 to 164)

*10. It is possible to summarize the facts about a quadratic

equation in one unknown by inspection of the equation. Construct a table for summarizing the facts about the quadratic equation in one unknown, their roots, and their graphic representation. How do you tell if the roots of a quadratic equation in one unknown are real or imaginary, rational or irrational, and equal or unequal? Does every equation of the second degree in one unknown have a solution? If so, how many roots does it have?

Read one of the following references.

(1:243; 1:255; 1:262; 2:188; 3:168 to 169)

*11. It is possible to determine the character of the roots of a quadratic equation in one unknown by inspection? This will save one the trouble of having to actually solve, mathematically, all the equations. By using the table prepared in Exercise 10, it is possible to determine the character of the roots by inspection. Are the roots of a quadratic equation in one unknown always integers? How do you check an equation to find out if the roots are correct? The character of the roots depends upon what term? What term expresses the sum of the roots?

Work out any ten of the examples from one of the following references. (1:262 to 263; 2:188 to 189; 3:170)

Work out any ten examples from one of the following references. (1:298 to 299; 2:190 to 191; 3:173)

*12. In the study of the quadratic equation in one unknown we come across the term, the discriminant. What is the discriminant? Of what use is the discriminant in the study of the

quadratic equation in one unknown? Look up the dictionary meaning of the word discriminant? Why is this term called the discriminant?

Work out any ten examples from one of the following references. (1:264; 2:189; 3:169; to 171)

*13. Up to the present time, you have solved quadratic equations in one unknown and found the roots. By reversing the process it is possible to write the equation which will have for its roots any given numbers. By taking the roots of a quadratic equation it is possible to write the equations. From your previous experience how would you form an equation if the roots were given to you? Write a statement, in your own words, that may be used as a result for forming an equation from the given roots.

Work out any ten examples from one of the following references. (1:261; 2:192; 3:173 to 175)

*14. How may one tell the solution of a quadratic equation in one unknown by looking at the graphic solution of the equation? What is the abscissa of a parabola?

Give the approximate solutions of the parabolas for the graphs in the following references. Do only one of the following reference groups. (1:233; 1:238; 1:239; 1:241) (2:155; 2:156; 2:159; 2:195) (3;151; 3:152; 3:155; 3:156; 3:157; 3:166; 3:176)

*15. Everyone has observed the curved path of a ball thrown obliquely upward. Knowledge of the muzzle velocity of the pro-

jectile and of the angle of elevation of the gun permits the determination of the parabolic path of the projectile. Arches in some of the best modern as well as ancient constructions are parabolic curves. What is a parabola? Construct a parabola on a sheet of graph paper and label all its parts. List as many examples of a parabola as you can think of in the various types of industry, science, and art. Do all parabolas eventually pass through a common point? Does the parabola turn upward or downward when the coefficient of x square is positive, negative? What is the turning point of a parabola? How does the curve of $y = ax^2$ change when "a" increases, is negative, approaches zero, becomes zero, becomes positive, becomes very large?

Read any two of the following references.

(1:234; 1:329; 2:159; 2:195; 3:151; 3:152; 3:153)

*16. Reading certain parts of newspapers, textbooks, and magazines, intelligently, one often needs to understand the graph of a quadratic equation in one unknown. The parabolic graph of a quadratic equation in one unknown pictures a gradual change. A well constructed graph is accurate, neat, well-spaced on the paper, and easily understood by the readers for whom it is intended, provided they have learned how to read such graphs. Prepare a graph showing the axis of symmetry, and the maximum and minimum values. Bring to class for discussion an example of a quadratic equation in one unknown, showing a gradual change. Examples may be found in

newspapers or business magazines.

Read two of the following references.

(1:234 to 235; 1:241; 2:159; 2:195; 3:175 to 179)

*17. Changing the constant term of a quadratic equation in one unknown produces a decided change in the behavoir of the equation. What is the constant term of a quadratic equation in one unknown? What is the effect on the position of the graph when the constant term is changed? Do all quadratic equations in one unknown have a constant term?

Read the following reference. (1:241 to 242)

*18. The quadratic equation in one unknown may be used to solve problems involving motion, paths of projectiles, paths of comets, arches in construction, reflectors of light and sound, problems of area in mathematics, problems in geometry, and problems involving cables for suspension bridges. Why is it advisable to use quadratic equations rather than linear equations in solving the above type of problems? Why is an equation often a more suitable way of stating the situation in a given problem than verbal language?

Work out one of the following reference groups.

(1:237 Examples 2 to 5; 1:268 Examples 4 to 6; 1:270 Examples 26 to 28) (2:157 Examples 19 to 22; 2:169 Examples 10 to 15; 2:170 Examples 5 to 10) (3:161 Examples 25; 3:162 Examples 26 to 27; 3:180 Examples 5 to 10)

*19. Prepare for the final examination, covering the unit, by working out the examples in the following reference.

(1:247 Examples 1 to 17)

*20. The objective test.-- The objective test consists of questions covering topics included in this unit.

General Study-and-Activity Guide

Pupils' copy.-- The pupils' copy of the general study-and-activity guide consisted of eight typewritten pages. Since this material was identical with the items marked with an asterisk in the unit assignment, it will not be repeated here.

Optional Related Activities

The following optional related activities were typewritten on three by five cards and placed in an indexed file for the use of the pupils.

1. Construct a cone out of some material such as cardboard and show the parabolic section made by passing a plane through the cone and parallel to the side of the cone.

2. Prepare, for a class demonstration, a drawing of the Newtonian telescope showing the parabolic reflector and the path of the light rays.

3. Prepare a collection of geometric designs showing the use of the parabola and present the material to the class in the form of a talk and discussion.

4. Prepare to show the class a collection of pictures of engineering constructions illustrating the use of the parabola. These pictures may be from ancient as well as modern constructions.

5. Construct a table showing how $ax^2 + bx + c = 0$ changes as x

changes in value. Such a table will show corresponding pairs of numbers.

6. Prepare a chart showing all the formulas, terms, and types of equations as well as the character of the roots, type of roots, types of solutions, et cetera that are involved in the study of quadratic equations in one unknown.

7. Prepare a chart for classroom demonstration showing the mechanical construction of a parabola.

References for the Pupils' Use

The books included in this reference list are arranged as in a bibliography with the addition of such notations as 1, 2, and 3. In the copy which was given to the pupils, only the notations, the name of the book, and the name of the author were given. The books with no notations were put at the end of the list when it was prepared for the pupils.

1. Barber, Henry C., A Second Course in Algebra, Houghton Mifflin Company, Boston; 1930.

2. Betz, William, Junior Mathematics For Today-Book Three, Ginn and Company, Boston; 1933.

3. Edgerton, Edward, and Carpenter, Perry, Second Course in Algebra, Allyn and Bacon, Boston; 1934.

4. Edgerton, Edward, and Carpenter, Perry, Advanced Algebra, Allyn and Bacon, Boston; 1941.

5. Freilich, Aaron, Schanholt, Henry, and McCormack, Joseph, Intermediate Algebra, Silver Burdett and Company, Boston; 1934.

6. Hart, Walter W., Essentials of Algebra, D. C. Heath and Copany, Boston; 1947.

7. Milne, William, and Downey, Walter, New Second Course in Algebra, American Book Company, Boston; 1935.

8. Orleans, Joseph, and Hart, Walter, Intermediate Algebra, D. C. Heath and Company, Boston; 1933.

9. Schorling, Raleigh, Smith, Rolland, and Clark, John, Second-Year Algebra, World Book Company, Yonkers-on-Hudson, New York; 1942.

References for the Teacher's Use

Billett, Roy O., Fundamentals of Secondary-School Teaching, Houghton Mifflin Company, Boston; 1940.

Butler, Charles, and Wren, F. Lynwood, The Teaching of Secondary Mathematics, McGraw-Hill Book Company, Inc., New York; 1941.

Cooley, Hollis, Gans, David, Kline, Norris, and Wohlert, Howard, Introduction to Mathematics, Houghton Mifflin Company, Boston; 1937.

Courant, Richard, and Robbins, Herbert, What is Mathematics? Oxford University Press, New York; 1946.

Griffin, Frank, An Introduction to Mathematical Analyses, Houghton Mifflin Company, Boston; 1936.

National Council of Teachers of Mathematics, The Mathematics Teacher, 525 West 120th St., New York, New York.
Numbers 1 to 7; 1947.
Numbers 1 to 7; 1948

Date:_____ Name:_____

Test

Quadratic Equations in One Unknown

Part - I	True and False Statements.	(40)	.____
-II	Completion Statements.	(15)	.____
III	Matching Statements.	(10)	.____
-IV	Multiple Choice Statements.	(10)	.____
		Total	.____

Test- Part I

Mark a plus (+) for the true statements and a minus (-) for the false statements in the parentheses to the left of the questions.

( ) 1. The quadratic equation in one unknown is useful for working problems in physics and engineering.

( ) 2. The equation $ax^2 + bx + c = 0$ is an example of a quadratic equation in one unknown.

( ) 3. The equation $2x^2 = 6x - 2$ is an example of an incomplete quadratic equation in one unknown.

( ) 4. The equation $(x - 5)^2 = 4$ is an example of a complete quadratic equation in one unknown.

( ) 5. All quadratic equations in one unknown can be solved by the factoring method.

( ) 6. All quadratic equations in one unknown can be solved by completing the square.

( ) 7. All quadratic equations in one unknown can be solved by the quadratic equation formula.

( ) 8. The graphic solution of the quadratic equation is the most accurate method of solution.

( ) 9. By changing the values of a, b, and c in the quadratic equation type form it is possible to obtain all possible quadratic equations.

( ) 10. A quadratic equation in one unknown may have only one root.

( ) 11. A quadratic equation in one unknown always has roots.

( ) 12. The roots of a quadratic equation in one unknown are always integers.

( ) 13. The roots of a quadratic equation in one unknown may be equal.

( ) 14. The roots of a quadratic equation in one unknown are always rational numbers.

( ) 15. The roots of a quadratic equation in one unknown may be complex numbers.

( ) 16. It is not necessary to actually solve a quadratic equation in one unknown to determine the character of the roots.

( ) 17. If the discriminant is a perfect square or zero, the roots of a quadratic equation in one unknown are rational.

( ) 18. If the discriminant of a quadratic equation in one unknown is zero, the roots are equal.

( ) 19. The discriminant is part of the quadratic equation formula.

( ) 20. The discriminant is used to determine the character of the roots of a quadratic equation in one unknown.

( ) 21. The discriminant of a quadratic equation is $b^2 + 4ac$.

( ) 22. The parabola is the graphic solution of a quadratic equation in one unknown.

( ) 23. The parabola of a quadratic equation in one unknown always crosses the x-axis.

( ) 24. The parabola has many practical applications to astronomy, physics, and engineering.

( ) 25. Quadratic equations in one unknown are useful for solving problems of motion in physics.

( ) 26. Quadratic equations in one unknown are of the first degree or of the second degree power.

( ) 27. A quadratic equation parabola always opens either up or down.

( ) 28. The parabolic curve is an example of a conic section.

( ) 29. It is possible to form a quadratic equation in one unknown from the roots that are given.

( ) 30. In the quadratic equation type form $ax^2 + bx + c = 0$, the term "c" may be zero.

( ) 31. All quadratic equations in one unknown may be reduced to the standard type form $ax^2 + bx + c = 0$.

( ) 32. One root of the quadratic equation in one unknown, $ax^2 + bx + c = 0$ is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

( ) 33. The quadratic equation formula was derived by factoring the standard type form equation $ax^2 + bx + c = 0$.

( ) 34. All second degree equations are quadratic equations.

( ) 35. A quadratic equation in one unknown may be of the first degree power.

( ) 36. Changing the constant term of a quadratic equation in one unknown raises or lowers every point on the graph.

( ) 37. A quadratic equation in one unknown may have only one root.

( ) 38. One root of a quadratic equation in one unknown may be zero.

( ) 39. The quadratic equation in one unknown is necessary for the study of a course in second year algebra.

( ) 40. The abscissa is the point where the graph of a quadratic equation in one unknown crosses the x-axis.

Test- Part II.

Fill in the blank spaces with only <u>one</u> word.

1. The graph of the equation $y = ax$ cuts the x-axis in ____ point(s).

2. The graph of the equation $y = ax^2 + bx + c$ is _____ a straight line.

3. If the discriminant is negative the roots are _____.

4. The _____ of the roots can be found from the term _____.

5. The square root of twenty-nine is an example of a _____ root.

6. If the discriminant is a perfect square the roots are _____.

7. The _____ is the graphic form of a quadratic equation in one unknown.

8. Four is an example of a _____ root.

9. The _____ of the roots can be found from the term -b/a.

10. A quadratic equation not containing a first degree term is called an _____ quadratic equation.

11. Minus the square root of five is an example of an

_____ root.

12. If the discriminant is equal to zero the roots are _____.

13. A straight line is the graphic solution of a _____ equation.

14. A quadratic equation containing both the first degree term and the second degree term is called a _____ quadratic equation.

15. In the quadratic equation $ax^2 + bx + c = 0$, the "c" term is called the _____ term.

Test- Part III.

Match the following statements with the number of the word that describes the statement best. Place the number of the word in the parentheses beside the statement, and to the left of the statement.

( ) 1. The distance measured along or parallel to the x-axis.

( ) 2. A number composed of the sum of a real number and an imaginary number.

( ) 3. A picture or drawing, every point of which exhibits a pair of corresponding values of two related quantities.

( ) 4. An indicated root of a number.

( ) 5. Any number that satisfies the equation.

( ) 6. An equation of the second degree.

( ) 7. A distance measured along or parallel to the y-axis.

( ) 8. A number that is expressed as an integer or as a fraction with integer terms.

( ) 9. A number that involves an indicated even root of a negative number.

( ) 10. An equation of the first degree.

1. Radical	6. Rational numbers
2. Quadratic equation	7. Abscissa
3. Ordinate	8. Root of an equation
4. Complex equation	9. Imaginary number
5. Linear equation	10. Graph

Test- Part IV.

The following statements all have possible answers. You may use the back of the other sheets for figuring. Place your answers in the space to the right of the equation.

1. What is the solution for the following equation?

$x^2 - 4 = 0$ Answer. _____

2. What should be added to the following expression to make it a trinomial square? $x^2 + 5x + (\ ) = 0$ Answer. _____

3. What are the roots of the following expression?

$x^2 - 6x - 16 = 0$ Answer. _____

4. What is the equation from which the roots 5 and -6 came from? Answer. _____

5. What three words describe the roots of the following equation best? $x^2 - 4x + 4 = 0$ Answer. _____

6. Determine "k" in the following equation so that the equation will have equal roots.

$x^2+3kx+9=0$ Answer. _____

7. Solve the following equation for the roots of x.

$x^2+6x=9$ Answer. _____

8. Solve the following equation for the roots of x.

$\frac{9}{x}=\frac{x}{4}$ Answer. _____

9. What must be added to the following equation to make it a trinimial square. $x^2+\frac{1}{2}x+(\ )=0$ Answer. _____

10. Solve the following equation for the roots of x.

$x^2+10ax=24a^2$ Answer. _____

Comments on the Key for the Examination on Quadratic Equations in One Unknown

Pupils' method of marking the test.-- The directions for the pupils to mark each item were printed at the beginning of each section of the examination.

The key was prepared so that it could be placed beside the pupils' answers for comparison with the correct answers. This method simplified correcting the examinations.

Other Methods of Evaluating the Pupils' Work

Teacher observation.-- Besides the use of an objective test there are other methods by which a student's work in the unit may be evaluated. The following questions will give a clue to how well the student grasped the subject material.

1. How does the pupil attack a problem? Does he use a systematic plan?

2. Does the pupil recall his previous work? Does the pupil understand what he is doing?

3. Is the pupil a help or a hindrance in the group work?

4. Can the pupil concentrate on new work that is being developed in the class, and does he offer constructive suggestions?

5. Does the pupil organize his work for use in future situations?

6. Can the pupil give good explanations of his work?

7. Can the pupil follow the work of others and detect errors and omissions in their work?

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5. Does the pupil organize his work for use in future situations?

6. Can the pupil give good explanations of his work?

7. Can the pupil follow the work of others and detect errors and omissions in their work?

8. Can the pupil think independently and correct his own mistakes?

9. Does the pupil choose the easy work or some of the more difficult work?

10. Are the pupil's questions intelligent or do they show a weak understanding of the situation?

The above questions may be answered by the teacher's observation of the pupil. The pupil's mark on the final examination may be affected by reading ability, illness, or some emotional factor, while his work over a period of time will disclose real ability in the subject.

The final evaluation of the pupil's progress in the unit was based upon his standing on the final examination and upon the teacher's judgment as to his classwork, group work, and individual work which has disclosed the qualifications indicated in the above questions.

the same test was given as a final examination. The testing results of the teaching of the unit are recorded in the tables found in Chapter II.

The pupil study-and-activity guide.-- The pupil's study-and-activity guide was given to all the students at the beginning of the introductory talk. The students were instructed to take care of the study-and-activity guides and make comments on the margins when they thought suggestions would be helpful. Many of these comments such as the following, for example, were entered in the teacher's log for future reference.

One of the students suggested that part of a period be turned over to an explanatory lecture, by the teacher, covering the use of graphs in quadratic equations in one unknown as well as the detailed construction of the graphs.

Another student suggested that the teacher review the methods of solution of quadratic equations in one unknown to each group when the group had completed the work on that section of the study-and-activity guide.

Another student suggested that the time for each group should be limited for the "pooling-of-experience" phase.

One of the boys suggested that more examples be added to the study-and-activity guide for the section of the unit covering the factoring method of solution and the section covering the character of the roots of quadratic equations

in one unknown.

The study-and-activity guides were duplicated on yellow manila paper and stapled together. The study-and-activity guide consisted of eight typewritten pages eight and one half by eleven inches.

The second day of the teaching of the unit.-- When the teacher entered the classroom, from a three-minute corridor duty, the pupils were in their seats waiting for the class to begin. The teacher explained that they were to work on their study-and-activity guides in groups or individually as they wished. As many groups could sit at the tables in the front of the room as accommodations permitted. It was just a matter of a few minutes for the groups to form. It was strange for the students to be working on their own initiative. The teacher was busy every minute of the period going from group to group. Most of the groups picked out the first item of the study-and-activity guide as their work for this second day of teaching the unit.

The groups were somewhat noisy the first day but nothing was said about this. The noise was no more than the normal attitude of youngsters working together. The second class was more noisy than the first but not enough to disturb the classes next door.

The third day of the teaching of the unit.-- The pupils went to work as soon as they entered the classroom and did

not even wait for the bell to ring. The room was a little more noisy than the normal hum-drum of a classroom but not enough to cause any trouble.

The fourth day of the teaching of the unit.-- The work on the study-and-activity guide was progressing satisfactorily. By now the students were turning in completed work to the teacher. When a section of the study-and-activity guide was completed it was given to the teacher. This material was corrected and filed away in the folder provided for completed work. The students were not all working on the same items now, but were selecting various sections of the study-and-activity guide which were of particular interest to them.

The fifth day of the teaching of the unit.-- This was the first experience the pupils had with "the pooling-of-experience phase" of the work. At first, the students were a little hesitant about explaining their work to the entire class. The students worked in groups with the leader demonstrating the work while the rest of the group offered suggestions and helped to answer the many questions asked of them.

The majority of the pupils took an active part in this "pooling-of-experience phase". Once the discussion was started, the students were anxious to express their own opinions. The pupils seemed quite proud of their accomplishments.

The "pooling-of-experience phase" was announced two days

in advance of the class period in which it was to take place. The students were asked to turn in a brief outline of the subject their topic would cover. The topics were posted on the bulletin board the day before the "pooling-of-experience phase" was to take place. The topics were listed in the order they would be discussed.

With six to seven alert and interested groups all working at once there was a bustle and stir, but the concentration of each group was so intense that one group's attention was not distracted by the other groups. The teacher did not have a breathing spell all period. As soon as the teacher finished with one group another group was waiting. The groups did not sit idly waiting but continued working on other sections of the study-and-activity guide until the teacher found time to get to them.

Highlights of the second week.-- The second week of the teaching of the unit was not as noisy as the first week. The students realized what was expected of them and went about their work in earnest, The slower pupils realized that this method gave them much more opportunity for help and took advantage of this fact.

The third and fourth week of the unit.-- The third and fourth weeks progressed satisfactorily. The lack of necessary equipment such as more text books, more reference books, additional tables for the groups, and a file cabinet was evident but did not hinder the progress of the unit.

Several of the groups would work at the blackboards with the individual students taking turns working out the examples. This method very often attracted other groups to join them, and they would all work together.

At the conclusion of the fourth week, one period was taken for a review of the work on the unit. The last day of the teaching of the unit was given to the final examination. The final examination consisted of five typewritten papers stapled together.

Pupil assistance.-- The pupils were allowed to form their own groups. The girls grouped together as well as the boys. Two of the more promising students worked by themselves but did consult the groups occasionally for information or to offer their suggestions. One of the pupils in each group was chosen by his fellow students to act as a leader. The pupils did find advantages in the group method of work. They discovered that they were able to help each other very much. When the teacher worked with a group, he acted as one of the members when offering assistance. The decided contrast to their previous attitude toward the subject was shown in their own accomplishments, in the way they kept members of their group working, and the way they became interested in the programs and bulletin board displays.

Optional related activities.-- The optional related activities were not as popular with the students as was at

first expected. This was probably due to the fact they had enough work in the study-and-activity guide and a probable poor selection of items from which to choose.

The end of the class work.-- The fourth week of work on the unit would end the tentative four-week time alloted for the unit. On Friday the teacher sounded out the class on the day for the final examination. Many of the students wanted more time to prepare for the final examination. It was pointed out to them that much of this work could be done as home study. The following Monday was devoted to a review of questions and answers. This would be the last "pooling-of-experience" period. The final examination was given the following day.

The final examination.-- To evaluate the unit on Quadratic Equations in One Unknown a separate study was made of each section of the final examination before the study of the total scores was made. The results for the study were obtained from Tables 5, 6, 7, and 8 of Chapter I. The method used to find the arithmetic mean and the standard deviation for the forty three pupils on the final examination is on the following pages of this chapter.

The following method 1/ was used to obtain the arithmetic

1/Henry A. Greene, Albert N. Jorgensen, and J. Raymond Gerbrich, Measurement and Evaluation in the Secondary School, Longmans, Green and Company, New York; 1943, p 518-553.

mean and the standard deviation for the table recording the results of the final examination.

The Arithmetic Mean.

1. Select an assumed mean, the mid-point of the class interval near the middle of the frequency distribution.

2. Mark the deviations from the assumed mean in the deviation column.(d)

3. Fill in the frequency deviation column. (fd)

4. Find the algebraic sum of the values in the frequency deviation column. (fd)

5. Divide the algebraic sum of the frequency deviation by the number of frequencies.($\lesssim$ fd/N) This is the correction.(c)

6. Multiply the algebraic sum of the frequency deviation by the class interval.($\lesssim$ fd/N $\times$ c.i.)

7. Algebraically add item six to the assumed mean. This will give the arithmetic mean.(A. M.)

The Standard Deviation.

1. Select an assumed mean the mid-point of a class interval near the middle of the frequency distribution.

2. Mark the deviation from the assumed mean.(d)

3. Fill in the frequency deviation column.(fd)

4. Find the algebraic sum of the values in the fd column. ($\lesssim$ fd)

5. Divide $\lesssim$ fd by the number of frequencies.(N) $\lesssim$ fd/N gives the correction. (c)

6. Square the correction.

7. Fill in the frequency deviation squared column.(fd^2).

8. Find the sum of column fd^2. This is Σfd^2.

9. Divide Σfd^2 by the number of frequencies.($\Sigma fd^2/N$)

10. Subtract the correction squared from $\Sigma fd^2/N$.

11. Take the square root of item Number 10.

12. Multiply item Number 11 by the class interval.(c.i.)

13. The results give the standard deviation.

Table 9 shows the arithmetic mean to be 32.7 and the standard deviation to be 3.6.

Table 9 Arithmetic Mean and Standard Deviation Scores of Forty-Three Pupils on Section I of the Final Examination in a Grouped Frequency Distribution.

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
39	3	7	21	147
38	1	6	6	36
37	6	5	30	150
36	2	4	8	32
35	4	3	12	36
34	1	2	2	4
33	5	1	5	4
32	4	0	0	0
31	6	-1	-6	6
30	3	-2	-6	12
29	2	-3	-6	18

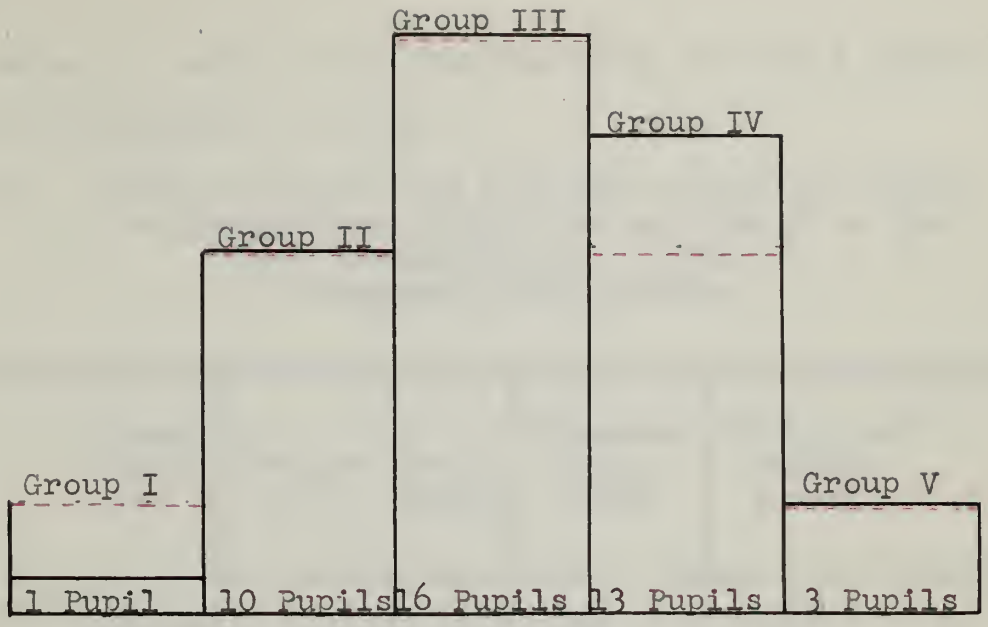
Table 9. (concluded)

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
28	1	-4	-4	16
27	4	-5	-20	100
26	1	-6	-6	36
Totals	43		36	598

High	39	Median	32.7
Low	26	Standard Deviation	3.6
Range	13	Arithmetic Mean	32.8

These results were used for the relative growth scale and for the histogram (Figure 2) which was drawn to show the actual distribution of pupil growth.

The normal distribution is represented by red dotted lines (Figure 2). The results of Section I of the final examination on Quadratic Equations in One Unknown as shown by the actual distribution of pupil growth in Figure 2 showed fewer pupils in Group V than the teacher expected and more pupils in Group II than the teacher expected.



Actual distribution of pupil growth with the normal distribution in red.

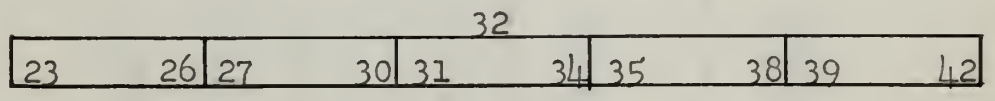


Figure 2. Relative growth 1/ scale.-- Section I of the examination on Quadratic Equations in One Unknown.

None of the scores of the pupils for Section I of the final examination fell below the lowest point on the relative growth scale.

A distinct advantage of the unit method in algebra is that a pupil must do thorough work during the entire time that the unit is being taught. The student can not rely on the "pooling-of-experience" phase to obtain the information necessary to carry him successfully through the final exami-

1/ Roy O. Billett, Fundamentals of Secondary-School Teaching, Houghton Mifflin Company, Boston; 1940, p. 633-634.

nation.

Table 10. shows the arithmetic mean to be 9.1 and the standard deviation to be 3.4.

Table 10 Arithmetic Mean and Standard Deviation Scores of Forty-Three Pupils on Section II of the Final Examination in a Grouped Frequency Distribution.

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
15	1	7	7	49
14	4	6	24	144
13	6	5	30	150
12	5	4	20	80
11	0	3	0	0
10	1	2	2	4
9	3	1	3	3
8	3	0	0	0
7	9	-1	-9	9
6	7	-2	-14	28
5	1	-3	-3	9
4	1	-4	-4	16
3	1	-5	-5	25
2	1	-6	-6	36
Totals	43		45	553

High	15	Median	9.3
Low	2	Standard Deviation	3.4
Range	13	Arithmetic Mean	9.1

These results were used for the relative growth scale and for the histogram (Figure 3) which was drawn to show the actual distribution of pupil growth. The normal distribution is represented by the red dotted lines (Figure 3). The results of the final examination on Quadratic Equations in One Unknown as shown by the actual distribution of pupil growth in Figure 3 showed more pupils in Group I, Group II, and Group IV than the teacher expected. Group IV contained seven students more than the normal distribution would show. This section of the examination covered terminology of the quadratic equation. The students that were weak on this particular point had to rush to complete their work on the unit. They had left the section on terminology until last.

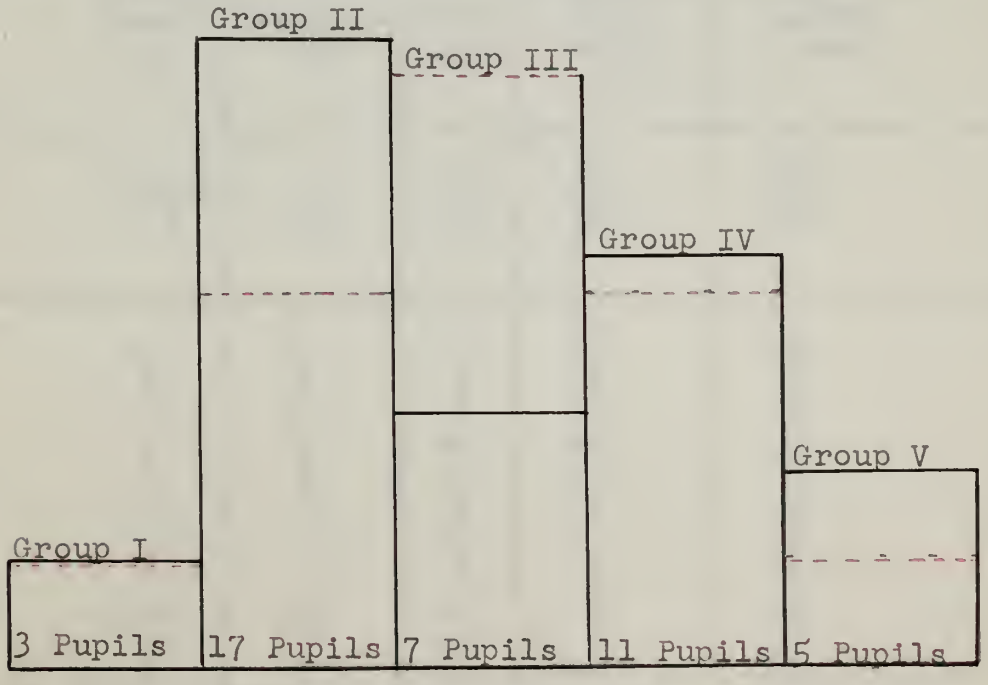


Figure 3. Actual distribution of pupil growth with the normal distribution in red.

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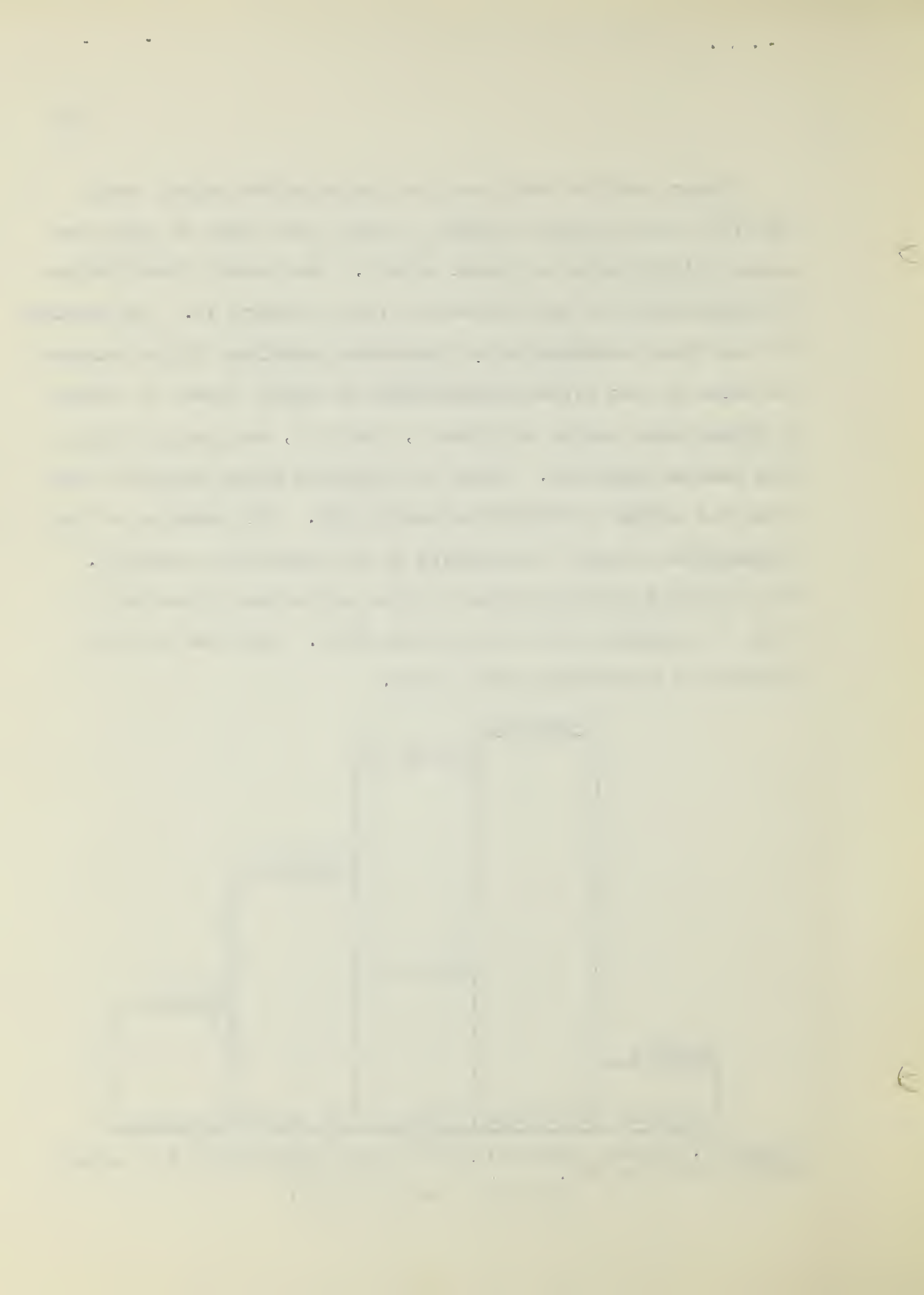

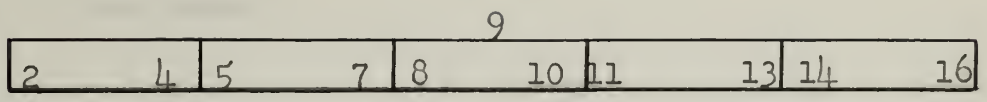

Figure 3.(Concluded) Relative growth scale.-- Section III of the examination on Quadratic Equations in One Unknown.

Table 11 showes the arithmetic mean to be 7.6 and the standard deviation to be 1.8. These results were used for the relative growth scale and for the histogram (Figure 4) which was drawn to show the actual distribution of pupil growth.

Table 11 Arithmetic Mean and Standard Deviation Scores of Forty-Three Pupils on Section III of the Final Examination in a Grouped Frequency Distribution.

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
10	15	2	30	60
9	1	1	1	1
8	18	0	0	0
7	4	-1	-4	4
6	4	-2	-8	6
5	0	-3	0	0
4	0	-4	0	0
3	0	-5	0	0
2	0	-6	0	0
1	0	-7	0	0
0	1	-8	-8	64
Totals	43		11	145

Table 11 (Concluded)

High	10	Median	7.6
Low	0	Standard Deviation	1.8
Range	10	Arithmetic Mean	8.3

The normal distribution is represented by red dotted lines (Figure 4). The results of Section III of the final examination on Quadratic Equations in One Unknown as shown by the actual distribution of pupil growth in Figure 4 showed that there were more pupils in Group II and Group III than the teacher expected.

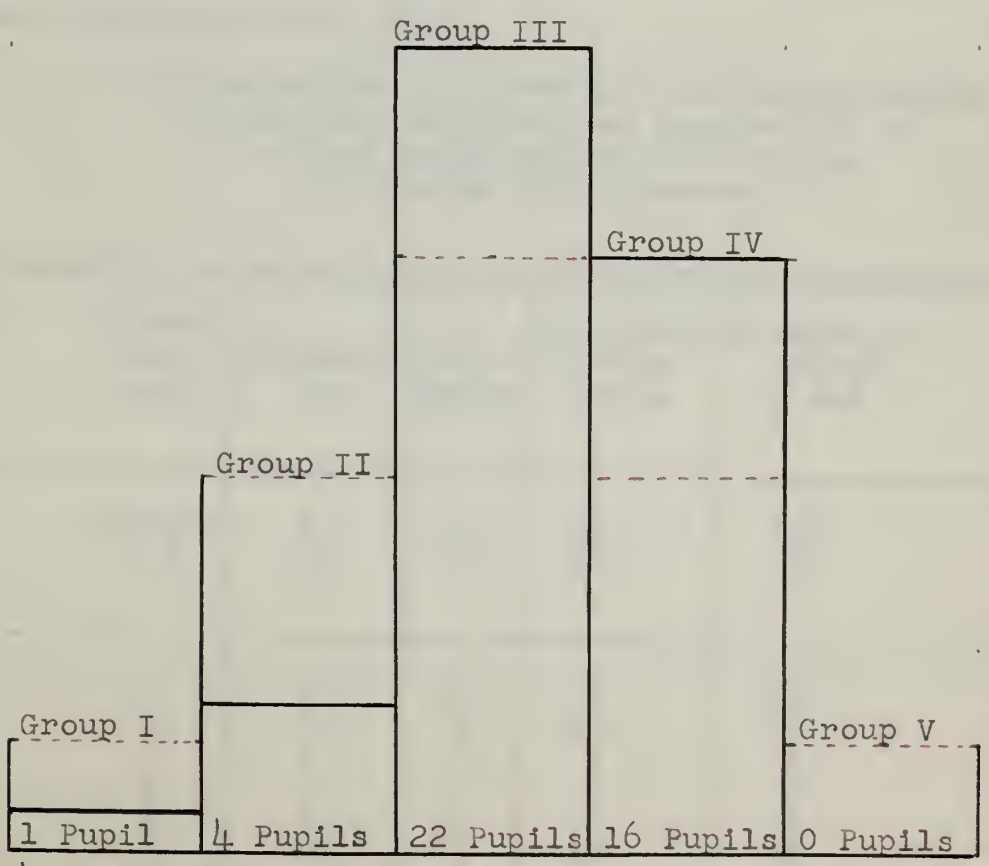

Figure 4. Actual distribution of pupil growth with the normal distribution in red.

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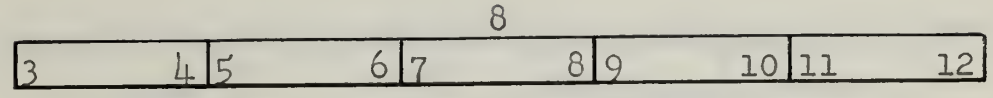


Figure 4.(Concluded) Relative growth scale.-- Section III of the examination on Quadratic Equations in One Unknown.

The probable reason for twenty-two of the pupils to fall into Group III was the small number of test items. Another reason was probably the nature of the limited number of test items. The majority of the pupils gave correct responses to the majority of the test items.

Table 12. shows the arithmetic mean to be 6.7 and the standard deviation to be 2.2.

Table 12 Arithmetic Mean and Standard Deviation Scores of Forty-Three Pupils on Section IV of the Final Examination in a Grouped Frequency Distribution.

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
10	3	3	9	27
9	7	2	14	28
8	7	1	7	7
7	7	0	0	0
6	7	-1	-7	7
5	5	-2	-10	20
4	3	-3	-9	27
3	3	-4	-12	48

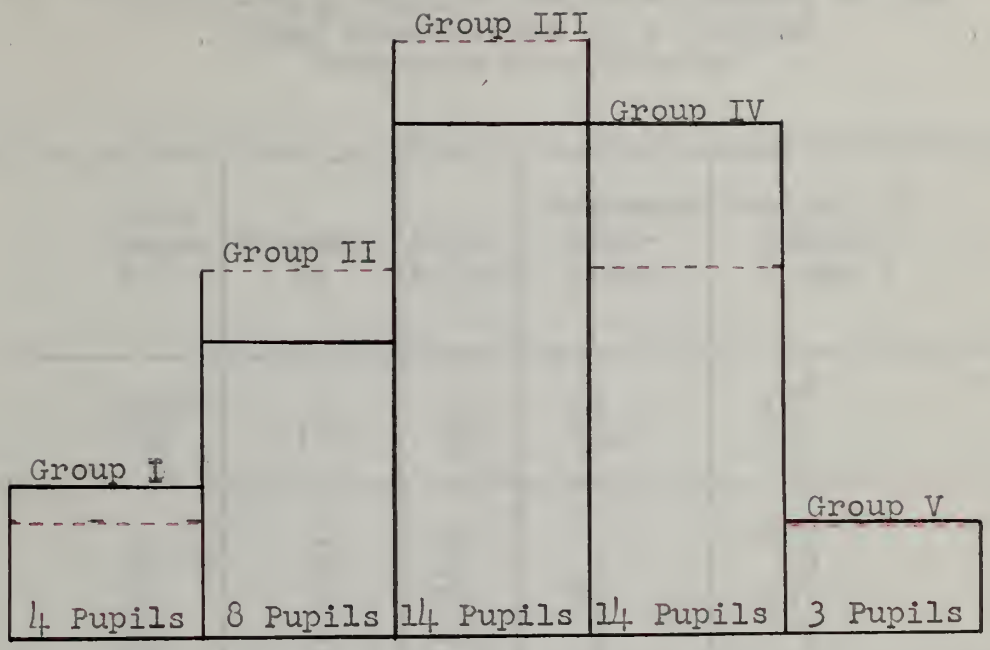
Table 12. (Concluded).

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
2	0	-5	0	0
1	0	-6	0	0
0	1	-7	-7	49
Totals	43		-15	213

High	10	Median	6.6
Low	0	Standard Deviation	2.2
Range	10	Arithmetic Mean	6.7

These results were used for the relative growth scale and for the histogram (Figure 5) which was drawn to show the actual distribution of pupil growth.

The normal distribution is represented by the red dotted lines (Figure 5). The results of Section IV of the final examination on Quadratic Equations in One Unknown, as shown by the actual distribution of pupil growth in Figure 5 indicated that there were more pupils in Group II than was expected.



Actual distribution of pupil growth with the normal distribution in red.

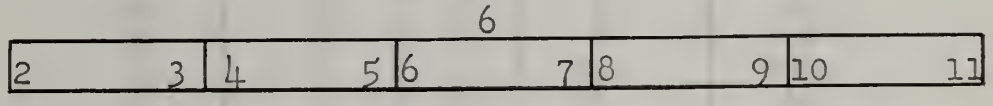


Figure 5. Relative growth scale.-- Section IV of the final examination on Quadratic Equations in One Unknown.

Section IV of the final examination covered work in the actual solution of examples. There were only ten items. The probable reason that fourteen pupils fell into Group II was the nature of the items as well as to the limited number of items. A limited number of test items will not give a satisfactory relative growth scale.

Table 13 shows the arithmetic mean to be 56.6 and the standard deviation to be 9.2.

Table 13 Arithmetic Mean and Standard Deviation Scores of Forty-Three Pupils for Total Scores of the Final Examination in a Grouped Frequency Distribution

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
73	1	15	15	225
72	0	14	0	0
71	3	13	39	507
70	0	12	0	0
69	0	11	0	0
68	0	10	0	0
67	3	9	27	243
66	0	8	0	0
65	1	7	7	49
64	0	6	0	0
63	4	5	20	100
62	4	4	16	64
61	1	3	3	9
60	3	2	6	12
59	1	1	1	1
58	2	0	0	0
57	0	-1	0	0
56	1	-2	-2	4
55	3	-3	-9	27
54	1	-4	-4	16
53	2	-5	-10	50
52	2	-6	-12	72
51	2	-7	-14	98
50	0	-8	0	0
49	0	-9	0	0
48	0	-10	0	0
47	1	-11	-11	121
46	0	-12	0	0
45	0	-13	0	0
44	2	-14	-28	392
43	2	-15	-30	450
42	1	-16	-16	256

Table 13 (Concluded)

Class Interval	Frequency	Deviation	Frequency Deviation	Product of Column 4 and 5
Limits (1)	f (2)	d (3)	fd (4)	fd^2 (5)
41	2	-17	-34	573
40	0	-18	0	0
39	0	-19	0	0
38	0	-20	0	0
37	1	-21	-21	441
Totals	43		-57	3715

High	73	Median	56.6
Low	37	Standard Deviation	9.2
Range	66	Arithmetic Mean	56.7

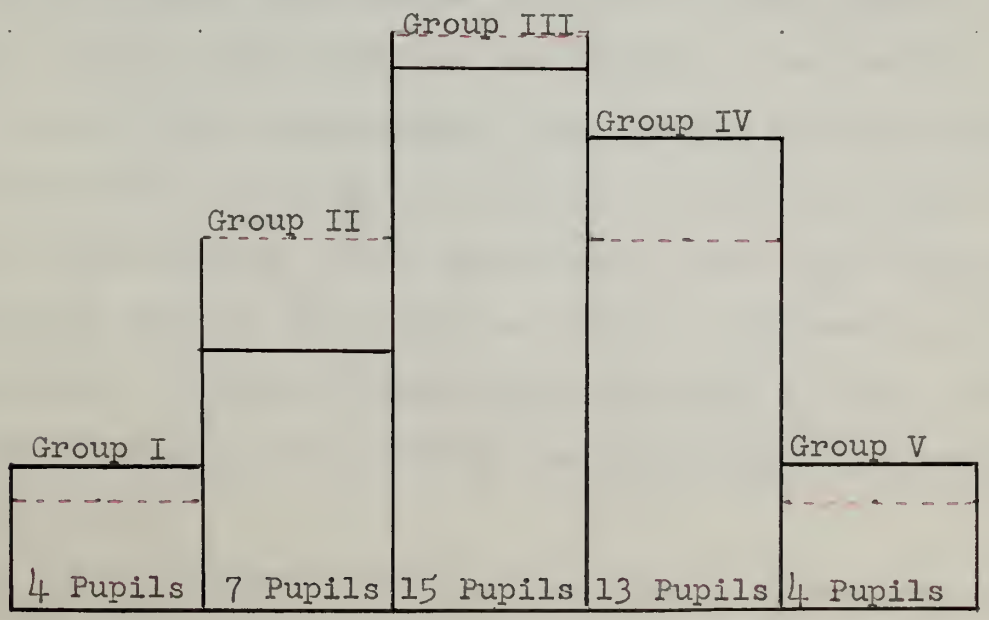


Figure 6. Actual distribution of pupil growth with the normal distribution in red.

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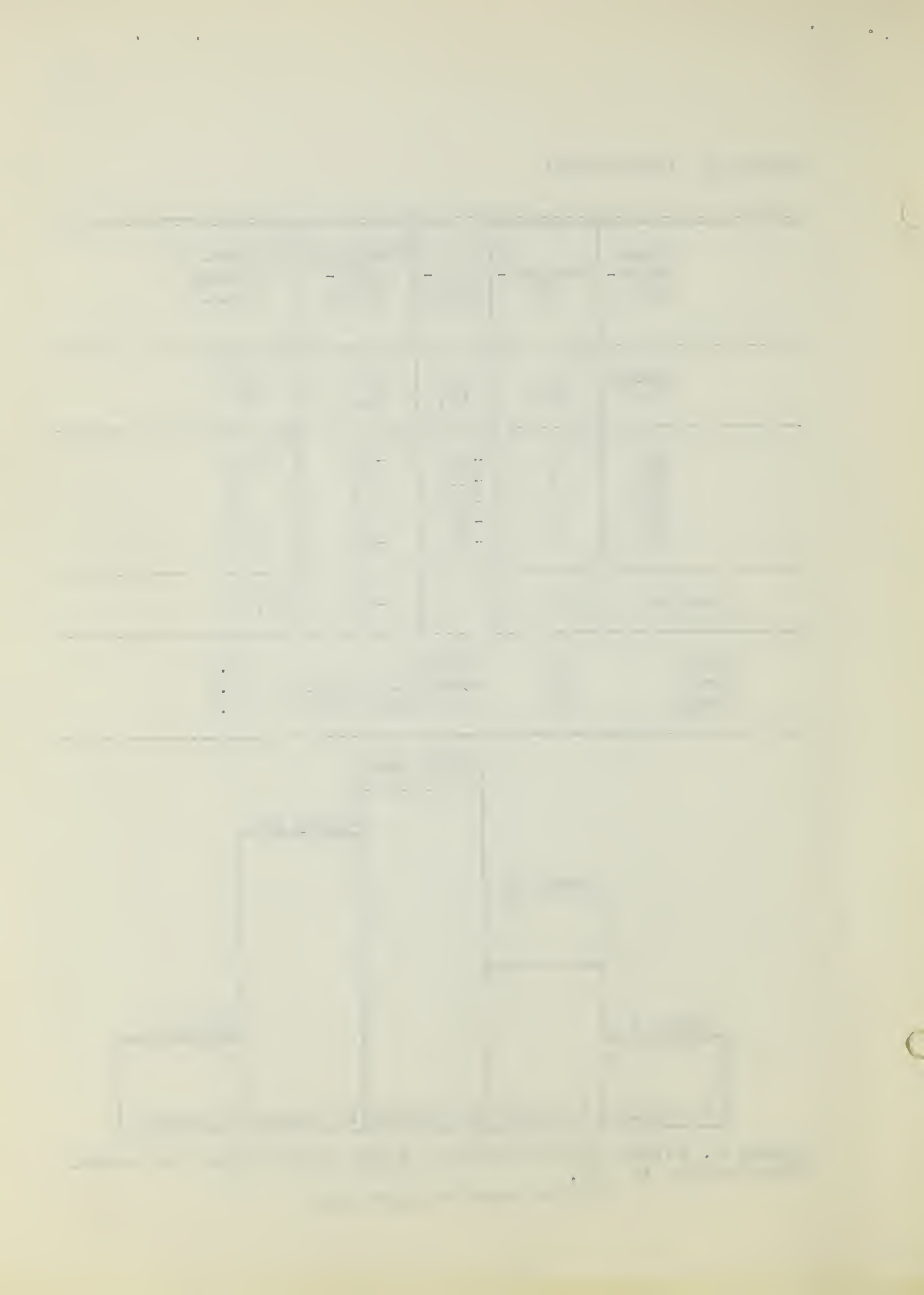

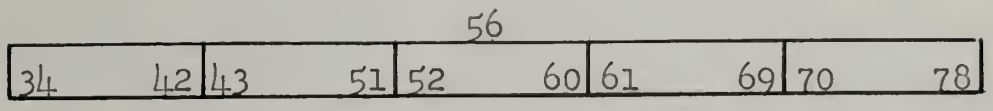

Figure 6. (Concluded) Relative growth scale.-- Total scores of the examination on Quadratic Equations in One Unknown.

These results were used for the relative growth scale and for the histogram (Figure 6) which was drawn to show the actual distribution of pupil growth. The normal distribution is represented by the red dotted lines (Figure 6). The results of the total scores of the final examination on Quadratic Equations in One Unknown, as shown by the actual distribution of pupil growth in Figure 6, showed that there were more pupils in Group II than was expected. None of the pupils' scores fell outside the lowest point in the relative growth scale.

The few pupils the teacher expected to have trouble with the unit did more work than was anticipated. The results showed on the final examination. One student did not complete the work on the unit, and as a result, he did not do well on the final examination. This particular student had transfered from another school. His previous marks in mathematics were just passing. He had an intelligence quotient of 95. The teachers, as well as his parents, had to prod him constantly to do his work.

Two of the students who had been failing in algebra did very well with the work on the unit. Their interest was

aroused in such a way that they will probably pass the course in June. These two students are now ardent enthusiasts of the unit method of teaching. They realized the advantages of group work and more individual help.

It was evident throughout the unit that the class spirit was different. The teacher-pupil relationship had been pleasant. This relationship was one of pleasant cooperation, as the pupils accepted the teacher as one of the group. They did not hesitate to consult the teacher when they ran into difficulties.

Pupil reaction.-- The class just mentioned was the composite of the individual reactions to the unit. The teacher could sense the pupils' appreciation of the responsibilities which had been delegated to them. They went about their work in a natural and business like manner.

There was more of a demand on the individual pupil in the unit method of teaching.

During the "pooling-of-experience" phase the pupils had to be alert in order to answer the questions of their classmates. The group leaders were selected according to mutual recognition of their ability.

It was quite notable how the pupils took to long-range planning and to face difficulties when they appeared. Some of the work was divided among the group members to be done at home in order to complete all the study guide assignments.

Most of the pupils did more work on an exercise than was required, especially on an item that happened to be more difficult than the rest of the work.

Pupil opinion of the unit.-- The pupils were asked to answer four questions at the conclusion of the work on the unit. (1) Did you prefer the unit method of teaching to the traditional method of teaching? (2) Did you work harder on the unit than with the traditional method of teaching? (3) Was the unit method more interesting than the traditional method of teaching? (4) Do you think you learned more by the unit method of teaching? Thirty-two of the total forty-three pupils stated they would prefer the unit method of instruction. Twenty-eight of the students admitted that they worked harded than with the traditional method of teaching. Thirty-eight of the students definitely said the unit method of instruction was more interesting, and thirty of the pupils felt they learned more with the unit method of instruction.

Many of the pupils liked the idea of being able to work at their own speed, and they realized the advantages of group work. They felt they were given more individual help and thus were able to accomplish more work. The pupils' favorable comments, for the unit method were very encouraging.

Concluding comments.-- The writer is conscious of the shortcomings of the first experience with the unit method

of teaching and has pointed out some of the ways in which this unit may be improved. These comments are found at the beginning of Chapter III.

The general impression is that of a classroom humming with activity of boys and girls who were eager to start work on their own initiative. There is satisfaction in knowing the progress made in the concepts and abilities listed in the delimitation of the unit. This indicates that the unit on quadratic equations in one unknown was worth while; and that, not only is it desirable to improve this unit, but that others in the field of algebra should be written.

Table 14 Scores for Tests on Quadratic Equations in One Unknown Group A, Twenty-Two Students.

	Pre-Test					Final-Test					
	Section					Section					
Pupil	I	II	III	IV	Total	I	II	III	IV	Total	Gain
1	29	3	7	3	42	32	7	8	8	55	13
2	33	3	8	2	46	31	7	8	6	52	6
3	23	2	8	4	37	35	13	10	9	67	30
4	25	2	10	2	39	29	9	10	8	56	17
5	30	3	7	4	44	37	8	10	7	62	18
6	26	4	7	1	38	27	5	6	3	41	3
7	25	1	8	2	36	28	10	8	6	52	16
8	23	1	8	3	35	37	12	9	9	67	32
9	27	4	7	3	41	33	9	10	8	60	19
10	33	6	7	2	48	37	13	8	5	63	15
11	34	5	8	3	50	38	13	10	9	70	20
12	26	3	8	3	40	37	13	10	8	68	28
13	35	7	10	4	56	37	7	10	7	61	5
14	23	0	7	3	33	33	7	8	6	54	21

Table 14 (Concluded)

Pre-Test						Final-Test					
Pupil	I	II	III	IV	Total	I	II	III	IV	Total	Gain
15	31	3	10	5	49	34	8	8	9	59	10
16	29	7	8	4	48	30	6	8	9	53	5
17	27	2	5	2	36	31	2	6	3	43	7
18	30	6	6	2	44	33	7	7	6	53	9
19	20	0	7	1	28	36	7	10	9	62	34
20	26	1	7	0	34	29	6	8	4	47	13
21	28	1	7	3	39	35	7	8	8	58	19
22	28	0	0	0	28	27	6	6	4	57	15
High	35	7	10	5	50	28	13	10	9	70	34
Low	23	0	0	0	28	27	2	6	3	41	3
Range	12	7	10	5	22	11	11	4	6	29	31
Median	23	3	7	2.5	41	33	9	9	7	57	16

Table 15 Scores for Test on Quadratic Equations in One Unknown, Group B, Twenty-One Students.

Pre-Test						Final-Test					
	Section						Section				
Pupil	I	II	III	IV	Total	I	II	III	IV	Total	Gain
23	23	1	7	3	34	30	3	7	7	47	14
24	23	3	3	2	31	30	7	8	6	51	20
25	25	2	0	0	27	27	4	0	6	37	10
26	27	0	4	0	31	27	6	7	4	44	13
27	27	2	8	0	37	35	13	10	5	63	26
28	25	1	4	1	31	32	12	8	8	60	29
29	26	6	5	0	37	33	6	10	6	55	18
30	21	0	5	2	28	33	12	10	7	62	34

Table 15 (Concluded)

Pre-Test						Final-Test					
	Section					Section					
Pupil	I	II	III	IV	Total	I	II	III	IV	Total	Gain
31	26	2	0	2	30	31	12	10	7	60	30
32	29	2	5	3	39	35	14	10	9	68	29
33	32	6	7	4	49	35	9	8	10	62	13
34	20	5	5	4	34	32	14	10	7	63	19
35	12	3	4	0	19	26	6	6	3	41	22
36	18	1	7	3	29	31	12	10	5	58	29
37	22	1	7	3	33	32	6	8	5	51	18
38	22	2	5	2	31	37	13	8	10	68	37
39	29	4	8	4	45	37	13	8	5	63	18
40	25	1	6	2	34	36	13	8	8	65	31
41	28	4	7	0	39	37	15	8	7	67	28
42	19	2	4	5	30	31	7	8	9	55	25
43	19	0	4	0	23	31	8	3	0	42	19
High	32	6	8	5	49	37	15	10	10	68	37
Low	12	0	0	0	10	26	3	0	0	37	10
Range	20	6	8	5	39	11	12	10	10	31	27
Median	24	2	6	2	33	33	9	8	6	57	23

Comments on the Final Examination for Quadratic Equations in One Unknown.

Test items.-- The analysis of the results of the final examination on quadratic equations in one unknown indicated that there was a weakness in the following test items.

Section I.-- Item Number 1 received all correct responses.

Item Number 10 received one incorrect re-

sponse.

Item Number 13 received all correct responses.

Item Number 14 received one incorrect response.

Item Number 22 received all correct responses.

Item Number 24 received all correct responses.

Item Number 29 received two incorrect responses.

Item Number 31 received two incorrect responses.

Item Number 39 received two incorrect responses.

The above questions will be studied for validity and changed if found necessary for the teaching of the unit next year.

Section II of the final examination on quadratic equations in one unknown consisted of ten questions. The response to each question was varied enough so that none of the questions will have to be changed.

Section III of the final examination on quadratic equations in one unknown consisted of ten items. Several of the students admitted they guessed the answers.

Section III.-- Item Number 1 received three incorrect responses.

Item Number 3 received three incorrect responses.

Item Number 6 received one incorrect response.

Item Number 7 received three incorrect responses.

Item Number 10 received one incorrect response.

The results of testing Section III of the final examination on quadratic equations in one unknown showed a weakness in the above mentioned questions. They will be studied for validity and revised for the teaching of the unit another year.

Section IV of the final examination on quadratic equations in one unknown consisted of ten questions. The questions were in the form of algebraic examples that had to be worked out mathematically.

Section IV.-- Item Number 1 received one incorrect response.

Item Number 3 received four incorrect responses.

Item Number 4 received three incorrect responses.

The results of testing Section IV of the final examination on quadratic equations in one unknown showed a weakness in the above mentioned questions. These questions will be studied for validity and changed if it seems advisable for the teaching of the unit in another year.

The final examination was prepared for a forty-five minute period. The day the final examination was given to the students the periods had been shortened to thirty-five minutes. There were only six of the students that did not complete the final examination in this thirty-five minute period.

The final examination will be lenthened before the unit is taught the following year. The logical conclusion will be to revise the questions mentioned above and add more questions to Sections II, III, and IV.

APPENDIX

Table 16, Raw Score Data for Pre-Test and Final Examination of the Unit Quadratic Equations In One Unknown, for Group A.

	Pre-Test Section				Final Examination				Raw Score Gain			
Pupil	I	II	III	IV	I	II	III	IV	I	II	III	IV
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	29	3	7	3	42	32	7	8	8	55	3	4	1	5	13
2	33	3	8	2	46	31	7	8	6	52	2	4	0	4	6
3	23	2	8	4	37	35	13	10	9	67	12	11	2	5	30
4	25	2	10	2	39	29	9	10	8	56	4	7	0	6	17
5	30	3	7	4	44	37	8	10	7	62	7	5	3	3	18
6	26	4	7	1	38	27	5	6	3	41	1	1	-1	2	3
7	25	1	8	2	36	28	10	8	6	52	3	9	0	4	16
8	23	1	8	3	35	37	12	9	9	67	14	11	1	6	32
9	27	4	7	3	41	33	9	10	8	60	5	5	3	5	19
10	33	6	7	2	48	37	13	8	5	63	4	7	1	3	15
11	34	5	8	3	50	39	14	10	10	73	5	9	2	7	23
12	26	3	8	3	40	39	14	10	8	71	13	11	2	5	31
13	35	7	10	4	56	37	7	10	7	61	2	0	0	3	5
14	23	0	7	3	33	33	7	8	6	54	10	7	1	3	21
15	31	3	10	5	49	34	8	8	9	59	3	5	-2	4	10
16	29	7	8	4	48	30	6	8	9	53	1	-1	0	5	5
17	27	2	5	2	36	31	2	6	3	43	4	0	1	1	7
18	30	6	6	2	44	33	7	7	6	53	3	1	1	4	9
19	20	0	7	1	28	36	7	10	9	62	16	7	3	8	34
20	26	1	7	0	34	29	6	8	4	47	3	5	1	4	13
21	28	1	7	3	39	35	7	8	8	58	7	6	1	5	19
22	28	0	0	0	28	27	6	6	4	43	-1	6	6	4	15
High	35	7	10	5	50	39	14	10	10	73	16	11	6	8	34
Low	23	0	0	0	28	27	2	6	3	41	1	-1	-2	1	3
Range	12	7	10	5	22	12	12	4	7	32	15	12	8	7	31
Median		3	7	3	41	33	9	9	7	57	-	-	-	-	16

Table 17. Raw Score Data for Pre-Test and Final Examination of the Unit Quadratic Equations In One Unknown, for Group B.

	Pre-Test Section				Final Examination				Raw Score Gain			
Pupil	I	II	III	IV	I	II	III	IV	I	II	III	IV
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
1	23	1	7	3	34	30	3	7	7	47	7	2	0	4	14
2	23	3	3	2	31	30	7	8	6	51	7	4	5	4	20
3	25	2	0	0	27	27	4	0	6	37	2	2	0	6	10
4	27	0	4	0	31	27	6	7	4	44	0	6	3	4	13
5	27	2	8	0	37	35	13	10	5	63	8	11	2	5	26
6	25	1	4	1	31	32	12	8	8	60	7	11	4	7	29
7	26	6	5	0	37	33	6	10	6	55	7	0	5	6	18
8	21	0	5	2	28	33	12	10	7	62	12	12	5	5	34
9	26	2	0	2	30	31	12	10	7	60	5	10	10	5	30
10	29	2	5	3	39	38	14	10	9	71	9	12	5	6	32
11	32	6	7	4	49	35	9	8	10	62	3	3	1	6	13
12	20	5	5	4	34	32	14	10	7	63	12	9	5	3	19
13	12	3	4	0	19	26	6	6	3	41	14	3	2	3	22
14	18	1	7	3	29	31	12	10	5	58	13	11	3	2	29
15	22	1	7	3	33	32	6	8	5	51	10	5	1	2	18
16	22	2	5	2	31	39	14	8	10	71	17	12	3	8	40
17	29	4	8	4	45	37	13	8	5	63	8	9	0	1	18
18	25	1	6	2	34	36	13	8	8	65	11	12	2	6	31
19	28	4	7	0	39	37	15	8	7	67	9	11	1	7	28
20	19	2	4	5	30	31	7	8	9	55	12	5	4	4	25
21	19	0	4	0	23	31	8	3	0	42	12	8	-1	0	19
High	32	6	8	5	49	39	15	10	10	71	17	12	10	8	40
Low	12	0	0	0	19	26	3	0	0	37	0	0	-1	0	10
Range	20	6	8	5	30	13	12	10	10	34	17	12	11	8	30
Median		2	6	2	32	32	9	8	6	56	-	-	-	-	23

Table 18. Response to Test Questions, Final Examination, Section I

Item	Group A Response		Group B Response		Group A and B Response	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	22	0	21	0	43	0
2	11	11	11	10	22	21
3	17	5	15	6	32	11
4	18	4	17	4	35	8
5	20	2	21	0	41	2
6	8	14	5	16	13	30
7	18	4	21	0	39	4
8	21	1	19	2	40	3
9	18	4	12	9	30	13
10	21	1	21	0	42	1
11	20	2	19	2	39	4
12	16	6	16	5	32	11
13	22	0	21	0	43	0
14	22	0	20	1	42	1
15	19	3	16	5	35	8
16	21	1	18	3	39	4
17	18	4	21	0	39	4
18	19	3	19	2	38	5
19	19	3	20	1	39	4
20	20	2	19	2	39	4
21	16	6	12	9	28	15
22	22	0	21	0	43	0
23	15	7	21	0	36	7
24	22	0	21	0	43	0
25	22	0	18	3	40	3
26	16	6	11	10	27	16
27	7	15	4	17	11	32
28	18	4	20	1	38	5
29	22	0	19	2	41	2
30	14	8	15	6	29	14
31	21	1	20	1	41	2
32	21	1	15	6	36	7
33	11	11	6	15	17	26
34	18	4	15	6	33	10
35	20	2	18	3	38	5

Table 18. (Concluded)

Item	Group A Response		Group B Response		Group A and B Response	
	Correct	Incorrect	Core rect	Incorrect	Correct	Incorrect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
36	20	2	19	2	39	4
37	20	2	17	4	37	6
38	19	3	20	1	39	4
39	21	1	20	1	41	2
40	11	11	13	8	24	19

Table 19. Response to Test Questions, Final Examination, Section II

Item	Group A Response		Group B Response		Group A and B Response	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	18	4	15	6	33	10
2	19	3	13	8	32	11
3	9	13	13	8	22	21
4	17	5	17	4	34	9
5	9	13	12	9	21	22
6	12	10	14	7	26	17
7	21	1	18	3	39	4
8	1	21	3	18	4	39
9	19	3	18	3	37	6
10	20	2	13	8	33	10
11	13	9	14	7	27	16
12	16	6	14	7	30	13
13	21	1	17	4	38	5
14	18	4	15	6	33	10
15	14	8	14	7	28	15

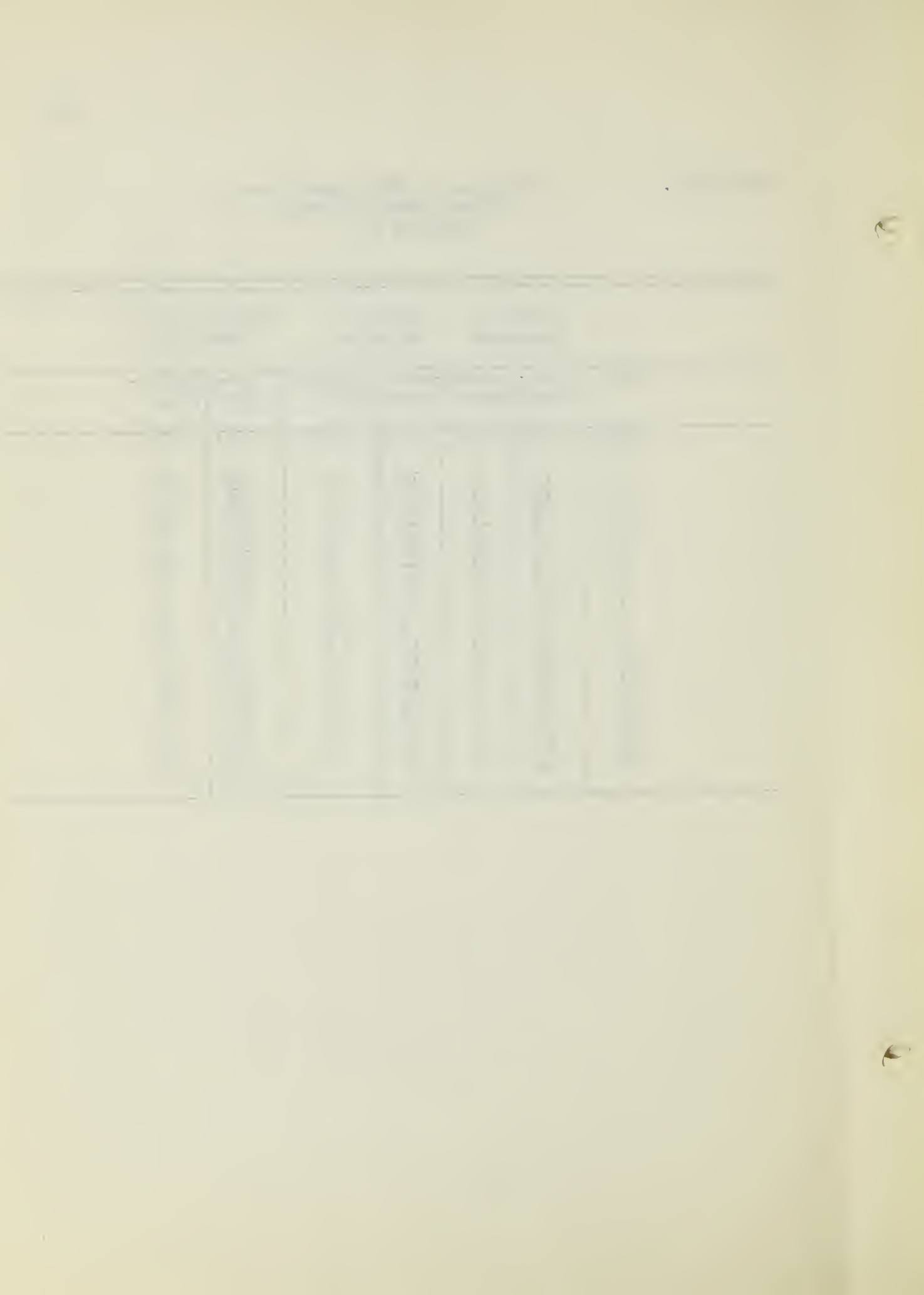

Table 20. Response to Test Questions, Final Examination, Section III

	Group A Response		Group B Response		Group A and B Response	
Item	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	22	0	18	3	40	3
2	13	9	14	7	27	16
3	21	1	19	2	40	3
4	20	2	11	10	31	12
5	18	4	20	1	38	5
6	22	0	20	1	42	1
7	22	0	18	3	40	3
8	11	11	8	13	19	24
9	15	7	17	4	32	11
10	22	0	20	1	42	1

Table 21. Response to Test Questions, Final Examination, Section IV

	Group A Response		Group B Response		Group A and B Response	
Item	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	22	0	20	1	42	1
2	8	14	5	16	13	30
3	19	3	20	1	39	4
4	21	1	19	2	40	3
5	12	10	13	8	25	18
6	17	5	15	6	32	11
7	7	15	6	15	13	30
8	17	5	17	4	34	9
9	15	7	11	10	26	17
10	14	8	9	12	23	20

Key to Test on Quadratic Equations in One Unknown

Part I				Part II		Part III		Part IV	
1.	(+)	21.	(-)	1.	Two	1.	7	1.	± 2
2.	(+)	22.	(+)	2.	Never	2.	4	2.	$25/4$
3.	(-)	23.	(-)	3.	Imaginary	3.	10	3.	$8; -2$
4.	(+)	24.	(+)	4.	Product	4.	1	4.	$x^2+x-30=0$
5.	(-)	25.	(+)	5.	Irrational	5.	8	5.	Equal, Real, Rational.
6.	(+)	26.	(-)	6.	Rational	6.	2	6.	± 2
7.	(+)	27.	(-)	7.	Parabola	7.	3	7.	$\pm 3; \pm 3/\overline{2}$
8.	(-)	28.	(+)	8.	Real	8.	6	8.	± 6
9.	(+)	29.	(+)	9.	Sum	9.	9	9.	$1/16$
10.	(-)	30.	(+)	10.	Incomplete	10.	5	10.	$-12a; 2a$
11.	(+)	31.	(+)	11.	Imaginary				
12.	(-)	32.	(+)	12.	Equal				
13.	(+)	33.	(-)	13.	Linear				
14.	(-)	34.	(+)	14.	Complete				
15.	(+)	35.	(-)	15.	Constant				
16.	(+)	36.	(+)						
17.	(+)	37.	(-)						
18.	(+)	38.	(+)						
19.	(+)	39.	(+)						
20.	(+)	40.	(+)						

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