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# Reconstructing Quantum Mechanics

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## Abstract

The reconstruction of quantum mechanics has often been connected with the interpretation of the quantum formalism, and has recently been so with the fuller consideration of the relation of information to quantum states. This has often involved the derivation of quantum theory specifically on the basis of informational principles, providing new perspectives on physical correlations and entanglement that can be used to encode information and to the view that information pertains directly to its foundations. By contrast to the traditional, interpretational approach toward engaging the foundations of quantum theory, which attempts directly to establish the meaning of the elements of the theory and often touches on metaphysical issues, the newer, more purely reconstructive approach may defer this task, focusing primarily on the mathematical derivation of the theoretical apparatus from simple principles or axioms. Most specifically, this sort of theory reconstruction is fundamentally the mathematical derivation of the elements of theory from explicitly presented principles involving a minimum of extra-mathematical content. It is, therefore, a means of carrying out of the mathematization of physics. Here, a representative series of specifically information-based reconstructions, both full and partial in extent, of quantum theory is reviewed, identifying its central aspects and trends.

# 1 Introduction

The reconstruction of quantum mechanics has historically been intertwined with the interpretation of the quantum formalism and, more recently, with the understanding of the relation of information to quantum states. This has naturally led to the idea of reconstructing quantum theory specifically on the basis of informational principles rather than physical principles which are understood as mechanical in nature.

The most basic physical principle of quantum mechanics is the superposition principle: Any superposition of physical states is also a physical state, the states lying in a Hilbert space or related mathematical structure. The traditional view has been that such superposition indirectly lends striking features to quantum information—that is, information encoded entirely via quantum systems—that are not found in its classical counterpart, that is, information encoded in classical mechanical states. The most striking of the features of the quantum state are those associated with entanglement, which arises in superpositions attributed to the composition of systems via another rule traditionally appearing in the formulation of quantum mechanics, namely, that which assigns the tensor-product space of the Hilbert spaces of the subsystems composing a larger system; as its consequence, quantum signal-state correlations are possible that are stronger than those between classical states—these correlations being often called “nonlocal” because they violate Bell-type inequalities (cf., e.g. [1]) and enable communication and information processing tasks to be accomplished that either cannot be done as efficiently or cannot be done at all using only classical mechanical signals, cf., e.g. [2].

That non-classical phenomena involving correlation can arise when quantum systems become entangled was evident to Albert Einstein [3] and to Erwin Schrödinger [4, 5], who named entanglement, early in the history of quantum mechanics even before the explorations of David Bohm [6], John S. Bell [7], and others for whom non-local correlations were a central pursuit and who conceived the first schemes for analyzing them. Moreover, it has become increasingly apparent since those analyses that entanglement is the rule rather than the exceptional, apparent absurdity which it initially seemed to be when it was first recognized as allowed by quantum theory [4, 5]. The consideration of information-theoretic questions has, in recent years, also provided a new perspective on entanglement, providing valuable insights into quantum-mechanical behavior by exploiting the tools of information theory, lending further credence to the view that information and the foundations of quantum mechanics are intimately related.

Two different, broad and interrelated approaches have been taken in attempts to come to terms with the significance of quantum information for what

has come to be considered quantum theory: the traditional, interpretational approach, which attempts directly to establish the meaning of the theory (cf. [8]) and often engages deep philosophical questions, and the reconstructive approach, which ostensibly defers these questions—that is, the establishment the meaning of the theory regarding the fundamental nature of the world—by proceeding with a mathematical derivation of as much of the theory as possible from simple principles. It is often hoped that the meaning of theory will arise in such derivations of traditionally accepted rules, and perhaps even a successor theory—cf. [9, 10, 11]. Fundamentally, theory reconstruction is the mathematical derivation of the elements of theory from formally presented principles with minimal extra-mathematical content; it is sometimes seen as the carrying out of the mathematization of physics as completely as possible.

Determining the relation of the physical world to information is crucial for both of these approaches to quantum foundations. At least three general positions have been delineated that might bear on this relation: (i) *Informational ontology*, (ii) *Digital ontology* and (iii) *Pancomputationalism* [12]. The Information-ontological position is that all things are reducible to information. The Digital ontological position is that the world is *discrete* in some sense, with a computable, deterministic temporal evolution; more precisely, it is the position that there are deterministic, discrete processes underlying all physical phenomena. Pancomputationalism is the very specific position that the universe is a computational system equivalent to a Turing machine of some sort. These three positions can also be combined and have been so in various ways: one recent approach embracing the informational ontology has explicitly also taken on board both the digital ontology together with a variant of pancomputationalism has argued that the universe as a whole is literally a quantum computer [13], a Turing machine thought of as a quantum cellular automaton [14]. The majority of informational reconstructions do not explicitly claim to take these positions but do implicitly invoke at least the digital ontology; fewer have also argued that quantum information theory not only illuminates physics but is its *essence*, thereby adopting an informational ontology; the fewest of the reconstructions adopt a sort of pancomputationalism.

That information can be considered physical is now often considered a valid assumption. However, that information might be more fundamental than matter has been met with suspicion, with energy sometimes put on an equal footing with it by some advocates of the view in concession, cf. e.g. [13]. An influential version of this idea involves the assumption that there is an ontological reduction of physical objects to information; this notion typically involves a form of “information immaterialism,” namely, that information is the most fundamental, basic entity or category of entities in the universe to which all others considered in the natural sciences might be so reduced. The

claim has been embraced by some of the most recent of informational reconstructions of quantum theory that are discussed in Sections 3 and 4, below.

## 2 Information and Physics

Before carrying out a review of a representative collection of attempts to reconstruct quantum theory based on principles related to information, it is valuable to briefly consider also the main positions regarding the relation of information and physics. The importance of reducing physics to information in some way was strongly advocated by John Wheeler, who presented it in the form of the *it from bit* thesis: “Every **it**, every particle, every field of force, even the spacetime continuum itself, derives its way of action and its very entirety, even if in some contexts indirectly, from the detector-elicited answers to yes or no questions, binary choices, **bits**. Otherwise stated, all things physical, all **its**. . . must in the end submit to an information-theoretic description” [15]. Similarly to the assumption of Zeno and his predecessors that all things have spatial extension, those claiming physics is reducible to information must assume all things must have what one might call *informational magnitude*. Indeed, for Wheeler quanta exist only because “what we call existence is an information-theoretic entity” [15].

Wheeler also set out a broader agenda, namely, to find “what, if anything, has to be added to distinguishability and complementarity to obtain *all* of standard quantum theory” [15]. This latter step represents a pioneering move in the direction of what have come to be known as “partial informational reconstructions” of quantum theory (cf. [16]) and has been echoed in various way since, as will be shown further below. The partial reconstructive approach—that of the reconstructing quantum theory beginning with simple fundamental principles that are known to capture *part*, though not all of the theory, so as to assist in the discovery of other, remaining simple principles allowing for complete reconstruction—is discussed in Section 4 below, after a review of attempts at full reconstructions of quantum mechanics in the following section. The example offered by Wheeler of a physical entity most plausibly informationally reducible is the black hole, which can be parameterized by its area. Wheeler’s deeper claim is that general relativity is reducible to quantum gravity as an approximation, and that space and time are “secondary ideas.” On that assumption, Wheeler’s position vis-à-vis matter may be more justified. For him, the “it from bit” thesis ultimately constituted the basis of a research program rather than an interpretation or reconstruction of quantum mechanics. Others have more taken up this picture in more stridently.

A less radical view of the relation of information to quantum physics is that information reduces to physics, rather than the other way around, such as the following. “[T]he theory of information is not purely a mathematical concept, but...the properties of its basic units are dictated by the laws of physics” [17]. One fact supporting this view intuitively is that the physics of computation which some, including Neil Gershenfeld, have identified with information theory itself, provides “an explanation of how noise and energy limit the amount of information that can be represented in a physical system, which in turn provides insight into how to efficiently manipulate information in the system” ([18], p. 36). A strong reductionist version of the reduction of information to physics has been advocated by John Preskill: “Information, after all, is something that is encoded in the state of a physical system; a computation is something that can be carried out on an actual physically realizable device. So the study of information and computation should be linked to the study of the underlying physical processes. Certainly, from an engineering perspective, mastery of principles of physics and materials science is needed to develop state of the art computing hardware,” [19], p. 7. The idea is that information can and should be understood only as encoded in a physical system: “The moral we draw [from the major achievements in the physics of computation] is that ‘information is physical’ ” [19], p. 10.

The most influential version of this physicalist position is that of Rolf Landauer, presented in his paper “The physical nature of information” [20]. “Information is not a disembodied abstract entity; it is always tied to a physical representation. It is represented by engraving on a stone tablet, a spin, a charge, a hole in a punched card, a mark on paper, or some other equivalent. This ties the handling of information to all the possibilities and restrictions of our real physical world, its laws of physics and its storehouse of available parts. . . our assertion that information is physical amounts to an assertion that mathematics and computer science are a part of physics” [21]. Nonetheless, one should note that although physics *constrains* information in the technological context, the entirety of the behavior of information is not dictated by physics alone. This can be seen, for example, by realizing that, although the physical characteristics of a particular source and other elements of a physical communication system constrain an agent’s ability to transmit the information in question to a receiver, in the standard, Shannon conception precisely how much information is in fact communicated by a signal is ultimately dictated by the choices of the sending and receiving agents using the source and system, rather than the merely the physics of its signals, as was pointed out early by Warren Weaver [22].

The contrary, yet more revolutionary idea—that physics is strongly reducible to information—can also be questioned; consider the following two

issues, for example [8]. Recall that on this approach, physical objects must have information-theoretic characterizations that are also the most complete descriptions that can be given of them; it is not immediately clear that the intrinsic features of physical things correspond to informational magnitudes, although physical properties may in principle be simulated by quantum computational algorithms given unlimited resources for their simulation on an abstract machine capable of universal computation. A second, greater issue to be conquered is that any information-theoretic description is different from the existent it describes, by virtue of the fact that the latter belongs to the external world: Actual physical entities, unlike the virtual entities of simulations, cannot be immediately equated with their own descriptions or other information about their behavior, however complete: the unfolding of a mathematical simulation of physical behavior differs from the experience of physical behavior.

An attempt is generally made in reconstructions of this latter kind to minimize or avoid interpretation [11], with any attempted reduction being one following from the mathematical results obtained; some have gone so far as to adopt the position that such theories can be self-interpreting or need not describe an external world. For example, Radical Bayesians such as Chris Fuchs have advocated an approach they call “QBism” which attempts to obtain an “‘interpretation without interpretation’ for quantum mechanics” [23]. One QBist claim is that “quantum theory does *not* describe physical reality. What it does is provide an algorithm for computing *probabilities* for the macroscopic events (‘detector clicks’) that are the consequence of our experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists” [23]. Stronger informational reconstructions of quantum theory are sometimes also amenable to such operationalist renderings; more recent informational reconstructions of a pancomputationalist sort, discussed at the end of Section 4, involve cellular automata for which no material substratum is assumed.

Neither the thesis that physics is reducible to information nor the thesis that information is reducible to physics has been firmly established. What is clear is that because physics constrains information processing, similarly to the way neurochemistry constrains mental function, physics and information processing bear a close relationship that can be helpful in the investigation of both subjects. The informational reconstructive approach seeks to do this through the derivation of the theory from simple mathematical principles.

### 3 Informational Reconstructions

Because the informational reconstructive approach to quantum mechanics focuses less on meaning than on the mathematical derivability of its formalism, it has been suggested by some practitioners that the quantum measurement problem might be deprived of much of its significance by following the reconstructive path to the foundations of physics instead of the interpretational one [11]; this would also have the virtue of the increase of conceptual precision that the emphasis on the mathematical derivation of quantum theory from simple principles something called for by John Bell, among others—cf. [24]. These reconstructions can to some extent be viewed as information-centric successors to previous attempts to reconstruct quantum mechanics, including those of J. von Neumann [25], N. Zieler [26], V. S. Varadarajan [27, 28], C. Piron [29, 30], S. Kochen and Specker [31], Jauch [32], J. C. T. Pool [33, 34], E. Beltrametti and G. Casinelli [35], G. Ludwig [36], and I. Segal [37, 38]. Beyond its emphasis on informational notions, the information reconstructive approach places a stronger emphasis on simplicity than on logico-mathematical structure.

With respect to the emphasis on the role of information, nearly as important for the informational reductive approaches as Wheeler’s “It from bit” thesis has been the following concern of his student Richard Feynman regarding relationship between computation and space: “It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space?” [39]. Indeed, for example, two later workers engaged in the information reconstruction of quantum mechanics, Āeslav Bruckner and Anton Zeilinger, offered two intuitive postulates characterizing their position in the hope that they might “solve Feynman’s problem” by reconstructing quantum mechanics from “a natural limit when a system only represents one bit of information” ([40], p. 57). These postulates are the following.

(BZ1) The amount of information carried by any system is finite.

(BZ2) The amount of information carried is lesser the smaller the system in terms of the number of its parts, rather than its spatial extent.

Importantly, these investigators also suggest the use of a measure of information differing from the standard Shannon measure, namely, the sum of the individual probabilities of possible measurement outcomes weighted by those probabilities themselves  $I = \sum_i p_i p_i$ , which is motivated by “only features



known before an experiment is performed are the probabilities for various events to occur” [41]. Again, another important aim of their effort was to resolve the conceptual difficulties in the foundations of quantum mechanics by demonstrating the measurement problem (cf. [24]) to be a distractionary, false problem. However, as often found in other work of this kind, as is seen below in a number of examples, there remained an assumption of much of the quantum state space structure despite the determined attempt to avoid doing so.

Before proceeding, let us recall the basis of this mathematical structure. In standard quantum mechanics, the state of the individual physical system can be described by a vector  $v$  in a linear space, *complex Hilbert space*, usually written as a *ket*,  $|v\rangle$ ; the corresponding Hermitian adjoint being given by a *bra*,  $\langle v|$ . The inner product  $(v, w)$  of the two sorts of vector is written as the *braket*  $\langle v|w\rangle$ , a scalar; the row vector  $(\alpha_1^* \alpha_2^* \cdots)$  represents  $\langle v|$  and the column vector  $(\beta_1 \beta_2 \cdots)^T$  represents  $|w\rangle$ . Operators on the space acting from the left on a ket yield a ket and acting from the right on a bra yield a bra. A *ketbra*,  $|v\rangle\langle w|$ , is an operator that, when acting on a ket  $|u\rangle$ , yields  $|v\rangle\langle w|(|u\rangle) = \langle w|u\rangle|v\rangle = (w, u)|v\rangle$ . The statistical operator  $\rho$ , used describing statistical ensembles generally, can be written as a weighted, not necessarily unique, linear combination of projector ketbras  $P(|u_i\rangle) \equiv |u_i\rangle\langle u_i|$ , each corresponding to a pure statistical ensemble  $\rho$ , with weights  $p_i$  interpretable as probabilities. Recalling that the projector  $P(|v\rangle) = |v\rangle\langle v|$ , one sees that, for any  $|w\rangle$ ,  $P(|v\rangle)|w\rangle = (\langle v|w\rangle)|v\rangle$ ;  $P^2 = |v\rangle\langle v|v\rangle\langle v| = P$  because  $|v\rangle$  has norm 1, that is, projectors are idempotent.

The above suggestion for reconstructing quantum mechanics on an informational basis was motivated by Zeilinger’s earlier Foundational Principle for quantum mechanics, which accords with the postulates BZ1 and BZ2.

(FP) “An elementary system carries 1 bit of information,” because “an elementary system represents the truth value of one proposition.”

He suggested that “this might also be interpreted as a definition of what is the most elementary system” and that it “underline[s] that notions such as that a system ‘represents’ the truth value of a proposition or that it ‘carries’ one bit of information only implies a statement concerning what can be said about possible measurement results.” ([42]). This picture can be seen as the result of applying the notion of Ludwig Wittgenstein that “the world is everything that is the case” and “the totality of facts, not of things” ([43], Propositions I and I.I) in the context of physics. To obtain an ‘elementary system,’ one is to decompose “a system which may be represented by numerous propositions into constituent systems”; “the limit” of this decomposition “is reached when an

individual system finally represents the truth value to one single proposition only.” Thus, he can be seen as espousing logical atomism in the specific form in which all facts are propositions represented by quantum state projectors.

Zeilinger focused on propositions associated with the two, directional spin components of a spin-1/2 particle, the mathematics of which applies to any two-state system. “The spin of the particle carries the answer to one question only, namely, the question, What is its spin along the  $z$ -axis? . . . Since this is the only information the spin carries, measurement along any other direction must necessarily contain an element of randomness. This kind of randomness must then be irreducible, that is, it cannot be reduced to hidden properties of the system, otherwise the system would carry more than a single bit of information.” However, because there exist well defined local hidden-variables models for the spin-1/2 system, such as that offered by Bell early in the investigation of the possibility of hidden variables models of quantum mechanics [44], only an analysis of measurement in a world described by such models can answer the question of whether the spin-1/2 system under a hidden-variables model could or could not be used to encode additional information, undermining the basis of the approach. The special role of entanglement must also be incorporated.

In a paper entitled, ‘Relational quantum mechanics,’ Carlo Rovelli proposed a somewhat similar but more explicitly logically oriented information reconstruction of quantum mechanics (later further developed by Alexei Grinbaum [11]) [45]; its two axiomatic principles can be formulated as

(R1) There exists a maximum amount of relevant information that can be extracted from a system.

(R2) It is always possible to obtain new information about the system.

to which are added assumptions of a purely logico-mathematical character, including that the set of dichotomic measurement results correspond to a complete, atomic orthocomplemented lattice [11]. Recall that a *lattice* is a poset for which there exists both a least upper bound and a greatest lower bound for every pair of elements. A *poset* (partially ordered set)  $P$  is a set  $S$  together with a binary (partial ordering) relation,  $\leq$ , with the following properties. 1) Reflexivity:  $a \leq a$ ; 2) Antisymmetry:  $a \leq b$  and  $b \leq a$  implies that  $a = b$ , for all  $a, b \in S$ ; and 3) Transitivity:  $a \leq b$  and  $b \leq c$  implies that  $a \leq c$ , for all  $a, b, c \in S$ . The least upper bound of two elements,  $a$  and  $b$ , under  $\leq$  is written  $a \vee b$ ; the greatest lower bound is written  $a \wedge b$ .

An *orthomodular poset* is a poset, with a unary operation  $^\perp$ , fulfilling four additional conditions: 1)  $0 \leq a \leq 1$  for all  $a \in P$ ,  $0$  being the zero element and  $1$  the unit; 2) For all  $a, b \in P$ ,  $(a^\perp)^\perp = a$ ,  $a \leq b \Rightarrow b^\perp \leq a^\perp$ ,

$a \vee a^\perp = 1$ ; 3) If  $a \leq b^\perp$  then  $a \vee b \in P$ ; and 4) If  $a \leq b$ , then there exists  $c \in P$  such that  $c \leq a^\perp$  and  $b = a \vee c$ . Condition 2 ensures that the operation  $^\perp : P \rightarrow P$ , corresponding to set-theoretic complementation, is an orthocomplementation; Condition 4 is the orthomodular law: Two elements  $a$  and  $b$  of an orthomodular poset are said to be *orthogonal* if  $a \leq b^\perp$ . A lattice contains both a zero element, 0, and an identity element, 1, if  $0 \leq a$  and  $a \leq 1$  for every one of its elements  $a$ . A lattice is a *complemented lattice* if there exists a complement,  $a^\perp$ , for every one of its elements,  $a$ —that is, if for every  $a$  there exists an element  $a^\perp$ , such that  $a \vee a^\perp = 1$  and also  $a \wedge a^\perp = 0$ . A lattice is a *distributive lattice* if for all triplets of elements  $a, b, c$ ,  $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$  and  $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$ . An *orthomodular lattice* is an orthomodular poset that is a lattice. 1)  $h(0) = 0$ ; 2)  $h(a^\perp) = h(a)^\perp$ ; (3)  $a \leq b$  if and only if  $h(a) \leq h(b)$ ; and 4)  $h(a \vee b) = h(a) \vee h(b)$  whenever  $a \perp b$ .

In Rovelli's reconstruction, the total amount of information regarding a quantum system will depend on the experiments done to it and the possibility that later measurements may render irrelevant information obtained in earlier ones; both measurement outcomes and information obtained via measurement thus depend on the *agent* making them. What makes information relevant is based on its relationship to information already known to possible agents in possession of it. Principle R1 and the formal definition of the information relevance of dichotomic questions to each other are together imply that they form an orthomodular lattice.

With the lattice of yes/no questions being isomorphic to the lattice of all closed subspaces of a Banach space constructed over a field, namely, the real or complex numbers or the quaternions, this Banach space is seen to be a Hilbert space; a *Banach space* is a normed space in which every Cauchy sequence converges. The observables of a quantum system form a real Banach (*i.e.* complete normed linear) space  $\mathcal{A}$  such that  $A \in \mathcal{A}$ . The powers of  $A$ ,  $A^n \in \mathcal{A}$  for  $n = 0, 1, \dots$ , are well-defined and such that the usual rules for operating with polynomials in a single variable hold. A *Banach \*-algebra* is an algebra of operators that form a Banach algebra with respect to the operator norm and has defined on it an *involution*  $A \rightarrow A^*$ , satisfying: 1)  $(A + B)^* = A^* + B^*$ ; 2)  $(cA)^* = c^*A^*$ ,  $\forall c \in \mathcal{C}$ ; 3)  $(AB)^* = A^*B^*$ ; and 4)  $(A)^{**} = A$ , for all  $A, B \in \mathcal{A}$ . In the context of traditional quantum mechanics, one identifies  $*$  with  $^\dagger$ , (Hermitian) adjoint. The set of linear operators on Hilbert space fulfills these conditions. It is argued in the Rovelli approach that it is R2 that endows the theory with its quantum character. Later reconstructions would come similarly to seek a principle or two specifically lending “quantumness” to their theoretical reconstructions.

Jeffrey Bub et al. [46, 47, 48] offered an informational reconstructive treatment of quantum theory involving a \*-algebra that seeks to derive

quantum theory, but from information-theoretic constraints rather than some propositional structure of binary alternatives. Recall that a Banach algebra with involution,  $\mathcal{A}$ , can be given a representation on a Hilbert space  $\mathcal{H}$ , by a linear map  $\pi : \mathcal{A} \rightarrow B(\mathcal{H})$ , into the bounded linear operators on  $\mathcal{H}$  such that: 1)  $\pi(AB) = \pi(A)\pi(B)$ ; and 2)  $\pi(A^*) = \pi(A)^*$ , for all  $A \in \mathcal{A}$ . Every norm-closed  $*$ -subalgebra of the algebra of all bounded operators on a Hilbert space, with the norm induced by the inner product is a Banach  $*$ -algebra; it also fulfills the relations  $\|A^*\| = \|A\|$  and  $\|A^*A\| = \|A\|^2$ , so that  $\|A^*A\| = \|A\|^2$ . It was argued by these workers that a quantum theory is best understood as a theory about the possibilities and impossibilities of *information transfer* under constraints, as opposed to a theory about the mechanics of physical entities. This is done by taking information itself to be physical and focusing on information rather than, for example, the motion of matter, the state of which is to be measured by other material systems.

In a terminological move that has become increasingly common, Bub considers “a quantum theory” to be one fitting a specific conception of what constitutes a theory that transcends standard non-relativistic quantum mechanics: A *quantum theory* is one in which observables and states have the algebraic structure of a  $C^*$ -algebra in which there is a non-commutative algebra of observables on individual, possible space-like separated systems. Bub’s is not an entirely new mathematical choice, however, in that in von Neumann’s standard formulation of quantum mechanics, the bounded operators have such a structure [25], which allows for state entanglement. Recall that a  $C^*$ -algebra is Banach algebra for which  $\|A^*A\| = \|A\|^2$ , for all its elements. Quantum mechanics can thus be viewed as a  $C^*$ -system where the  $C^*$ -algebra is that of the set of all *bounded* self-adjoint operators acting on finite-dimensional Hilbert space. Quantum states are in this formalism normalized positive linear function  $\rho$  on the  $C^*$ -algebra  $\mathcal{A}$ . Every state  $\rho$  assigns to each  $A \in \mathcal{A}$  a complex number  $\rho(A)$  in such a way that we have a  $\rho$  which is

- i)  $\rho(1) = 1$  , normalized,
- ii)  $\rho(A^*A) \geq 0$ , positive, and
- iii)  $\rho(aA + bB) = a\rho(A) + b\rho(B)$ , for every  $A, B \in \mathcal{A}$ ,

for all  $a, b \in \mathbb{C}$ . The algebra of observables can be taken to be the algebra  $\mathcal{B}(\mathcal{H})$  of all bounded observables acting on a complex Hilbert space  $\mathcal{H}$ . Every normalized vector  $\Psi \in \mathcal{H}$  defines a state  $\rho_\Psi(A) \equiv \langle \Psi | A | \Psi \rangle$ , for all  $A \in \mathcal{A}$ . Such a state is a vector state. Generally, every density operator  $\rho_D \in \mathcal{B}(\mathcal{H})$  defines a state  $\rho_D(A) \equiv \text{tr}(DA)$ , for all  $A \in \mathcal{A}$ . Such states are *normal states*. A normal state is pure if and only if  $D$  is a projection, *i.e.*  $D^2 = D$ ; it is therefore pure if and only if it is a vector state.

Again, what is significant in this reconstruction regarding the choice of algebra is the association of the characteristics of this algebra with information theoretic constraints. This reconstruction also contains specific structure to relate its results to *measurement*: In it, measurement devices correspond to the elements of the algebra of observables and yield specific states corresponding to well specified preparations with specified *average values* when subjected to statistical measurement. The state evolution is assumed to be described by non-trace-increasing completely positive maps, that is, trace-preserving or trace-reducing (such as occurs when subensembles are selected). A linear map  $L : \mathcal{L}(\mathcal{H}) \rightarrow \mathcal{L}(\mathcal{H})$ , where  $\mathcal{L}(\mathcal{H})$  is the space of bounded linear operators on  $\mathcal{H}$ , is said to be *positive* if  $L(\mathcal{O}) \geq \mathbb{O}$  for all  $\mathcal{O} \geq \mathbb{O}$ , that is, all  $\mathcal{O} \in B(\mathcal{H})$  for which  $\langle \psi | \mathcal{O} | \psi \rangle \geq 0$  for all  $|\psi\rangle \in \mathcal{H}$ , where a map  $L : \rho \mapsto L(\rho)$  is linear if  $L(\rho) = p_1 L(\rho_1) + p_2 L(\rho_2)$  for  $\rho = p_1 \rho_1 + p_2 \rho_2$ . The relation  $\geq$  is defined as follows. ( $A \geq B$  if  $A - B \geq \mathbb{O}$ , where  $\mathbb{O}$  is the zero operator. It is an ordering on the set of self-adjoint bounded operators.) A positive  $L$  is *completely positive* if, in addition, any  $\mathbb{I}_N \otimes L \in B(\mathbb{C}^N \otimes \mathcal{H})$  is positive, for all  $N \in \mathbb{N}$ . A map  $\mathcal{E} : \rho \rightarrow E(\rho)$  satisfies the three above conditions if and only if it can be written  $\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger$ , for some set  $\{K_i\}$  of not necessarily Hermitian linear operators for which  $\mathbb{I} - \sum_i K_i^\dagger K_i \geq \mathbb{O}$ .

An important result for this approach is the Clifton-Bub-Halvorson (CBH) theorem [46, 47], which concerns theories that satisfy the following requirements.

(CBH1) The observables of the theory are represented by the self-adjoint operators in a non-commutative  $C^*$ -algebra (with individual algebras commuting).

(CBH2) The states of the theory are represented by  $C^*$ -algebraic states (as positive normalized linear functionals on the algebra), and spacelike separated systems can be prepared in entangled states that allow remote state steering.

(CBH3) Dynamical changes are represented by completely positive linear maps.

The CBH theorem shows that quantum theories (those with observables and states of the kind specified just above) can be characterized in terms of just these three informational constraints.

The derivation of quantum theory of the reconstruction has the following general architecture [48]. It is argued that given appropriate constraints on the behavior of information, any mechanical theory that includes an account of the measuring instruments that reveal quantum phenomena must be empirically equivalent to a quantum theory, and assuming that the information-

theoretic constraints are in fact satisfied in our world, no mechanical theory of quantum phenomena that includes an account of measurement interactions can be acceptable. Dynamical changes must then be represented by completely positive linear maps, once one assumes that the world has built-in constraints on the acquisition, representation, and communication of information. This latter is the strongest of the assumptions made in this reconstruction.

The specific information-theoretic constraints involved in this argument are the following. *One* is the impossibility of superluminal information transfer between two physical systems by performing measurements on one of them. This condition means that when two agents perform non-selective local measurements, the measurements of one agent can have no influence on the statistics of outcomes obtained by the other, and vice-versa; selective measurements could otherwise alter the statistics of measurements performed at a distance simply because there is a change of the ensemble with regard to which statistics are taken. The subsystems possessed by the two agents are kinematically independent if every element of the  $C^*$ -algebra of one commutes pairwise with every element of the  $C^*$ -algebra of the other, that is, the algebras are mutually commuting.

Another constraint is the impossibility of perfectly broadcasting information possibly communicated using an unknown physical state, which for pure states amounts to requirement that states cannot be perfectly cloned: No-cloning (perfect copying) for general pure states is an immediate consequence in standard quantum mechanics of the superposition principle [50]. This constraint requires each of the two algebras to be non-commutative. A *third* constraint is the impossibility of communicating information so as to implement a quantum bit commitment protocol with unconditional security. The quantum bit commitment protocol is a primitive cryptographic protocol involving two agents, describable as follows. One agent supplies an encoded bit to a second agent as a warrant for commitment to a binary value, that is, 0 or 1. It should be not be possible for the second agent to infer that value at this initial stage, but the information provided, together with additional information supplied by the sender later, should allow one to infer that value *during the revelation stage* of the protocol. The receiver should also be certain that the protocol will not allow for ‘cheating’ by the sending of the initial information in a way that would allow the value to be changed after the initial, ‘commitment’ stage. The “no bit commitment” condition protects one against any *a priori* theoretical *preclusion* of entangled states violating local causality—for a particularly clear, detailed discussion of this constraint, see [51], Sect 8.3.

It is argued by Bub that the CBH theorem justifies taking the structure of quantum mechanical states and observables to be representable as a non-

commutative  $C^*$ -algebra. In particular, it is claimed, were the states and observables not of this kind, one of more of the constraints would not hold. Crucially, however, in order to go through, with quantum theories remaining physical theories at all, Bub's argument requires an additional and crucial background assumption, namely, "taking the notion of quantum information as a new physical primitive," clearly taking information to be itself physical in nature [49]. As Christopher Timpson has emphasized "By assumption, the world is such that the information-theoretic constraints are true, but this is too general and says too little: it is consistent with a wide range of ways of understanding the quantum formalism." [51], p. 177. This also shows that, in some cases, despite intentions to the contrary, reconstructions do not eliminate the role of interpretation, but often point out its importance in quantum physics.

A different informational reconstruction of quantum theory amenable to an operational treatment was subsequently given by Lucien Hardy, who identifies "five reasonable axioms" for quantum theory [52]. Going beyond the emphasis on probability of Zeilinger's approach, Hardy frames quantum mechanics as "a new type of probability theory" that follows from axioms "which might well have been posited without any particular access to empirical data," namely, the following.

- (H1) In the infinite limit of  $n$ , relative frequencies (measured by taking the proportion of times a particular outcome is observed) tend to the same value for any case where a given measurement is performed on an ensemble of  $n$  systems prepared by some given preparation.
- (H2)  $K$  is determined by a function of  $N$  where  $N = 1, 2, \dots$  and where, for each given  $N$ ,  $K$  takes the minimum value consistent with the axioms.
- (H3) A system whose state is constrained to belong to an  $M$ -dimensional subspace behaves like a system of dimension  $M$ .
- (H4) A composite system consisting of subsystems  $A$  and  $B$  satisfies  $N = N_A N_B$  and  $K = K_A K_B$ .
- (H5) There exists a continuous reversible transformation on a system between any two pure states of that system.

The approach is operationalist in that it is based on schematic "devices" (pictured as boxes) with the ability to prepare, transform and measure systems, which are describable by a state, characterized as a "mathematical object

which can be used to determine the probability for any measurement that could possibly be performed on the system when prepared by the associated preparation.” These probabilities are to depend on the preparation and *not* on the particular ensemble of systems used. The mathematical object of choice for him is a *minimum list of probabilities* from which the probabilities for all such measurements on any given system via the state they provide written taken as a column vector  $\mathbf{p} = (p_1, p_2 \cdots, p_k)^T$ , which “contains just sufficient information to determine the state and the state must contain sufficient information to determine this vector”; in short, the state and this vector are “interchangeable.”

Hardy considers a not necessarily unique set of  $K$  “fiducial measurements,” that can be used to determine the state. Any measurable probability, to be found as the proportion of the cases in which the event in questions occurs in an ensemble, is to be determinable by a function  $f(\mathbf{p})$  of the state and lie between 0 and 1, inclusive, in accordance with its method of determination by measurement. A preparing agent is taken to have the ability of randomly preparing a state  $\mathbf{p}_A$  with probability  $\lambda$  or in another state  $\mathbf{p}_B$  with probability  $1 - \lambda$  and to make a record of this choice. Another agent measuring a state so prepared will measure the probability  $\mathbf{p}_C = \lambda f(\mathbf{p}_A) + (1 - \lambda)f(\mathbf{p}_B)$ , checkable against the preparation record. Thus, for the fiducial measurements  $\mathbf{p}_C = \lambda \mathbf{p}_A + (1 - \lambda)\mathbf{p}_B$ . These two relationships imply that  $f(\lambda \mathbf{p}_A + (1 - \lambda)\mathbf{p}_B) = \lambda f(\mathbf{p}_A) + (1 - \lambda)f(\mathbf{p}_B)$  from which it follows that  $f$  is a linear function (as shown in his [52], Appendix 1), so that  $p_{\text{meas}} = \mathbf{r} \cdot \mathbf{p}$ , where  $\mathbf{r}$  is a vector associated with the measurement. From the fact that the  $k$ th fiducial measurement gives the  $k$ th component of  $\mathbf{p}_B$ , one obtains the  $k$  fiducial measurement vectors  $\mathbf{r}^i$  ( $i = 1, 2, \dots k$ ).

The transforming device transforms a prepared state  $\mathbf{p}$  to  $\mathbf{g}(\mathbf{p})$ , which again must be linear by virtue of the application of the same reasoning as is applied above to each component of  $\mathbf{g}$  so that transformation ‘devices’ act as  $\mathbf{p} \rightarrow Z\mathbf{p}$ , with  $Z$  being a  $K \times K$  matrix. The sets of prepared states, transformations, and measurements are named  $S, R$ , and  $\Gamma$ , respectively, and are shown to be convex. The existence of special states, null  $\mathbf{0}$  and the pure states—the extremal members of  $S$ , excepting  $\mathbf{0}$ —the ones that cannot be understood as non-trivial mixtures of others, is then pointed out. The identity measurement  $\mathbf{r}^I$  is given as the sum of all the probabilities of non-trivial measurement outcomes.

The first four of the axioms have relatively direct *physical* interpretations as well: Regarding H1 relating to probabilities, probability is a relative frequency; regarding H2, one takes the number of parameters needed to characterize a state to be connected to the number of states that can be distinguished in a single measurement; regarding H3 relating to subspaces, take systems



having the same information carrying capacity to have the same properties; regarding H4 relating to system composition, information carrying capacity is assumed to be multiplicative. The principle H5, which imposing continuity, is less clearly physically motivated but is commonly assumed physics; Hardy also views it as motivated by the operation of classical computers in performing finite calculations. The derivation of quantum mechanics from these principles proceeds in nine steps, these showing respectively that:

- (1)  $K = N^r$  with  $r = 1, 2, \dots$ ;
- (2) A choice of fiducial measurements is that in which the first  $N$  are some basis set of measurements and two additional measurements in each of the  $1/2N(N-1)$  two-dimensional subspaces, yielding  $N^2$  measurements;
- (3) The state can be represented as a vector of the type  $\mathbf{r}$ ;
- (4) Pure states must satisfy  $\mathbf{r}^T D \mathbf{r}$ ;
- (5)  $K = N$  is disallowed by Axiom 5 and that  $K = N^2$  by Axiom 2;
- (6) The case of  $N = 2$  is that of the qubit (Bloch sphere),  
i.e. the QM of spin-1/2 systems;
- (7) The trace formula and the conditions on the quantum states  $\rho$  and measurements  $\hat{A}$  follow;
- (8) The most general evolution consistent with the (H1-H5) is the Schrödinger evolution and the tensor product structure properly describes composite systems;
- (9) The most general evolution of the state after measurement (including but not only including von Neumann's projection) is that of quantum theory.

The similarity of Hardy's derivation in some respects to the constructive one offered by Julian Schwinger c. 1960 is noteworthy—cf. [54, 55, 56, 57]. One of the insights obtained from this reconstruction is that Axiom 5 is the one of these distinguishes the quantum probabilistic structure derived from a classical one. Hardy argues that this exercise helps point the way toward a modified version of quantum theory in which *quantum gravity* might be obtained, for a broader such framework he was later to offer, cf. [53]. This reconstruction has been criticized on the basis that Axiom H2 is somewhat unnatural with its issues being addressed in later work of Brukner and Borivoje Dakić, who also make connections with Zeilinger's Foundational Principle (FP, mentioned above), cf. [58, 59].

## 4 Partial Reconstruction

Some informational reconstructions, which have been called partially reconstructive, *intentionally do not* seek in themselves to fully reconstruct a form of standard quantum mechanics, but rather aim to identify *how much* of the theory can be traced back to simple principles, such as non-local correlation and causality, so as to assist in identifying what remains to find a full reconstruction and/or an alternative theory or theories similar yet not identical to quantum mechanics, under the broader umbrella of quantum theory [16]. Like attempted full reconstructions, these could eventually allow ontologies to be contemplated that might differ from the traditional material, wave/particle conceptions, and even more so. Here we consider in some detail primarily those of Popescu and Rohrlich [60] and D’Ariano et al. [61]. (Related “toy model” treatments will not be considered here—cf. [16].) Most of such work has focused on principles related to information and involve notions engaged by CBH and/or mathematical characteristics associated to them, which might involve state-spaces that are more general or slightly different from the one of traditional quantum mechanics but are still considered “quantum,” allowing for the imagining of successor quantum theory possibly differing from traditional quantum mechanics.

The most clear of these cases is found in the work of Sandu Popescu and Daniel Rohrlich that appeared shortly before the above-mentioned work of Rovelli, and which was inspired by earlier work of Abner Shimony, takes nonlocality (of correlation) as a fundamental characteristic of physical theory. Shimony asked the question of whether “*nonlocality plus no signalling plus something else simple and fundamental*” might help one get at what are the basic, most fundamental principles of quantum mechanics, echoing Wheeler’s earlier methodology mentioned above in Section 2 [62]: Popescu and Rohrlich call these two fundamental principles *relativistic causality* (in the form of no-signaling) and *nonlocality*. Those theories that involve nonlocal correlations and obey causality are identified without any dependence on quantum mechanics, per se ([60], p. 381).

Recall that Bell first helped distinguish quantum behavior from classical physical behavior by deriving an inequality that must be obeyed by local realistic physical systems, which are characteristic of classical physics [7]; John Clauser, Michael Horne, Abner Shimony, and Richard Holt (CHSH) reformulated it in a way more clearly amenable to experimental testing [63]. These early workers considered local physical models as local hidden-variables theories for situations similar to those earlier considered by Einstein, Podolsky, and Rosen [64], who had questioned the completeness of the quantum-mechanical formalism, having every complete physical state assign a definite probability to

a positive outcome in the measurement of a bivalent property of one subsystem when the hidden parameter describing it takes a given value independently of measurements performed on the other.

Specifically, CHSH modified Bell's original treatment so as to be applicable to any practical experimental arrangement that could be described as performing coincidence measurements of a bivalent property, for example, polarization of the directional component of spin for each spin-1/2 particle of a pair to obtain the inequality

$$|S| \leq 2, \quad (1)$$

$$\text{for } S \equiv E(\theta_1, \theta_2) + E(\theta'_1, \theta_2) + E(\theta_1, \theta'_2) - E(\theta'_1, \theta'_2), \quad (2)$$

where the  $E$ s are expectation values of the products of measurement outcomes given parameter values  $\theta_i$  and  $\theta'_i$  of the two different directions for the same side  $i$  of the two sides jointly constituting the joint-detection apparatus [63]. The correlation coefficients contributing to  $S$  in terms of experimental detection rates from which  $S$  is obtained are

$$E(\theta_i, \theta_j) = \frac{C(\theta_i, \theta_j) + C(\theta_i^\perp, \theta_j^\perp) - C(\theta_i, \theta_j^\perp) - C(\theta_i^\perp, \theta_j)}{C(\theta_i, \theta_j) + C(\theta_i^\perp, \theta_j^\perp) + C(\theta_i, \theta_j^\perp) + C(\theta_i^\perp, \theta_j)}, \quad (3)$$

where the  $C(\cdot, \cdot)$  are coincidence detection count rates,  $i$  is the index for particle 1,  $j$  the index for particle 2, and the parameter  $\theta^\perp$  is the direction perpendicular to  $\theta$  in the plane normal to particle propagation in this scheme.

In the context of quantum theory generally, the value of  $S$  has been taken as a important measure of how “quantum mechanical” the system observed is. Quantum mechanics provides a maximum violation of this inequality by a factor of  $\sqrt{2}$ , which is achievable when, for example, one prepares the quantum state  $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ , and measures with  $\theta_1 = \frac{\pi}{4}$ ,  $\theta'_1 = 0$ ,  $\theta_2 = \frac{\pi}{8}$ , and  $\theta'_2 = \frac{3\pi}{8}$ . For this set of parameters, one finds  $|S| = 2\sqrt{2}$ . Once  $|S|$  exceeds 2, the behavior of a system is no longer considered classical in nature but is instead considered quantum mechanical. Thus,  $|S|$  is an example of the sort of quantity useful for placing quantum mechanics in a broader context. It has been shown that such states allow communications tasks to be performed with improved efficiency [65].

By assuming the above two fundamental constraints on probabilities, nonlocality and relativistic causality, Popescu and Rohrlich found that indeterminacy and limitation on the classes of allowed measurements found in quantum mechanics appear and so can be associated with the allowance of “nonlocal action.” Measurements are conceived schematically in this approach via two laboratories, each conceived of via mathematical “boxes” allowing for a set of

evaluations via schematic test-like operations involving the related constraints: It is required that the the various test results assigned their various probabilities of obtaining are random and uniformly distributed while superluminal signaling between boxes is precluded, in that the probability of obtaining each possible results in one laboratory is independent of those tested in the other. The probability distributions obeying these requirements are then found to be more general than those assigned by quantum mechanics.

Popescu and Rohrlich first explicitly considered the possibility of broadening the context further, by considering theories exceeding the maximum quantum-mechanical value of  $S$  by introducing the “non-local box” schema [60]. They used the nonlocal box schema to find that the reasonable, that is, causally acceptable violation of the Bell inequality is not sufficient to characterize *all* nonlocal causal physical descriptions, referring to correlations stronger than those of quantum mechanics as “superquantum.” The structure of communication complexity theory under the assumption of the availability of both quantum [65] and super-quantum correlations as shared “resources” for communication was found to be particularly pertinent [67, 66].

To see this, consider the results of Bell–CHSH inequality reformulated in the following form, which is more convenient to these considerations than the original version. Any local realistic hidden-variables theory describing a joint system AB—with A to be measured in one laboratory and B in the other, distant one—must obey an inequality of the form

$$s = \sum_{x,y \in \mathbb{Z}_2} p(m_x^A + m_y^B \equiv x \cdot y) \leq 3, \quad (4)$$

where  $p(m_x^A + m_y^B \equiv x \cdot y)$  is the probability of obtaining measurement outcomes  $m_x^A, m_y^B \in \mathbb{Z}_2$  summing (mod 2) to the product of “measurement setting” parameters  $x$  and  $y$  in  $\mathbb{Z}_2$ , that is,  $\{0, 1\}$ . Quantum mechanics violates such an inequality by reaching the value  $s = 3.41 = 2 + \sqrt{2}$ . In fact, by sharing a Bell state, two agents, one in each laboratory, are able to approximate nonlocal boxes with probability  $\cos^2 \frac{\pi}{8} \approx 0.854$  [69].

The non-local box of Popescu and Rohrlich causally providing nonlocal correlations is characterized by the condition that if the two-bit string  $xy$  is a member of the set  $\{00, 01, 10\}$  then  $p(m_x^A = 0, m_y^B = 0) = \frac{1}{2}$  and  $p(m_x^A = 1, m_y^B = 1) = \frac{1}{2}$ , and when  $xy$  takes the remaining possible value 11 then  $p(m_x^A = 0, m_y^B = 1) = \frac{1}{2}$  and  $p(m_x^A = 1, m_y^B = 0) = \frac{1}{2}$ , all other pertinent probabilities are zero. For this schema, on the left-hand side of the Inequality (4), one sees that the value 4 can be achieved. The “box” still obeys causality, because the outcomes on one side of the coincidence-counting apparatus still occur at random locally, as in the case of the fully correlated or anticorrelated

Bell states of spin, but with stronger correlations between joint measurement outcomes than in *any* quantum state.

The super-quantum correlations are, in the sense of the distributed computation they enable, too powerful. Consider the amount of information that must necessarily be communicated in order to obtain the value of a function  $f$  distributed between at least two parties; pertinent here are situations where  $f$  is a Boolean function from  $\mathbb{Z}_2^{\times 2}$  to  $\mathbb{Z}_2$ . Allowing *maximally* nonlocal such correlations to two spacelike separated agents allows them to perform *all* distributed computations with perfect accuracy given a *trivial* amount of communication, namely, one bit; that one bit is necessary for preserving causality [68]. Such a collapse of the classes of communication problems is remarkable given its fundamental to theoretical computer science [67] and suggests that nonlocal boxes themselves describe correlations that can't exist *on the grounds of information science* rather than traditional physics.

Giles Brassard and others later found an information-processing-centered constraint, one on computational complexity, appears to bring this sort of “physics” closer to that of quantum mechanics [69]. In particular, it was shown that any correlations “more nonlocal” than those of quantum mechanics result in trivial communication complexity: With them, all Boolean functions can be calculated by the two agents with the assistance of a single bit of information exchanged between the two laboratories. Nicholas Brunner and Paul Skrzypczyk [70] later showed the similar result that correlations could then also be achieved that are arbitrarily close to correlations allowed by the two-party Bell-type inequalities, that is, by classical theory, thereby circumscribing an informational principle identifying quantum theory: the preclusion of such trivial computational complexity.

Another line of informational partial-reconstruction work reconstructs quantum theory on the basis of informational principles as related to *quantum automata*. Mauro D’Ariano et al. begin with events involving qubits as its simples and implement a discrete characterization of the relationship between these events [71, 72, 73, 74]. In their approach to quantum theory, a set of six fundamental principles is identified, one of which picks out quantum theory specifically, echoing the CBH mode of reconstruction and others just mentioned. The most striking among the goals of this programme is the notion “pure information may underlie all of physics” with the inclusion not only that known quantum fields but also space and time, which are said to “emerge” in a specific way from quantum automata. With respect to ontology, it is claimed that “Looking at physics as pure information processing means to consider qubits as primitive entities. In simple words: qubits are not supported by matter, but matter is made of quantum information patterns. This is the It from Bit of Wheeler [15].” Space and time themselves are conceived explicitly

in these terms. Then, addressing the concern of Feynman noted previously, “quantum computational network is just the causal network from which the geometry of space–time should be derived.” In particular, it is argued that “the Lorentz transformations have been explicitly derived from a causal network with topological homogeneity, thus showing how relativity can be regarded as emergent from the quantum computation (a visual proof of time-dilation and space-contraction was given in Ref. [71]).”

Methodologically, this approach is operationalistic in that it is intended to reduce “the fundamental theoretical framework of physics to quantum theory only, and forcing the definition of each physical quantity to be given in operational terms [72, 73].” The reduction of quantum fields is to be accomplished by mimicking field theory by a quantum computation that involves the replacement of anticommuting Fermi fields with commuting locally interacting Dirac fields as well, bosons being precluded. The emphasis is on topological rather than metric notions of distance, in that “‘to be near’ for systems means just to be interacting, and the length of the graph links has no physical meaning. Space-time metric emerges from the pure topology by event counting” [74]. The precise sense in which the reconstruction considered by D’Ariano et al. is computational is that the behavior of the model involved is that of a dynamically homogeneous, unbounded *quantum circuit with a gate connection topology* embeddable in two (schematic, mathematical) dimensions, which are then taken to correspond to one spatial dimension in the mathematical sense and thought to be generalizable to higher dimensions [72]. The homogeneity incorporated is interpreted as “equivalent of the universality of the physical law, with the quantum circuit to be considered as a fundamental theory” [75]. A specific model of quantum circuit is used that incorporates structures naturally connected with the operations they perform: The basic “test” composed of a collection of events corresponding to test outcomes and an input “system” and an output “system” to which it is topologically connected.

The precise sense in which theory is said to be operationalist is the following. It is based (in the context of category theory) on “an operational structure” consisting of the triple  $\text{Op} = (\text{Transf}, \text{Outcomes}, \text{Tests})$ , where “Transf is a strict symmetric monoidal category, Outcomes is a collection of sets closed under Cartesian product, and Tests is a strict symmetric monoidal category, related to Transf and Outcomes... Intuitively, the operational structure describes 1. what can be done (connecting tests) 2. what can be observed (outcomes), and 3. what can happen (events)” [74].

The information-theoretical framework provides the fundamental ontology via this basic notion of *event* which general occurs probabilistically and relates to quantum systems, being considered as inputs and outputs to said systems. The notion of *quantum system* is articulated by D’Ariano as “an

immaterial support for quantum states, exactly in the same fashion as the bit in computer science is the abstract system having the two states 0 and 1. The analogous system of the bit in quantum theory is the qubit, having not only the two states 0 and 1, but also all their superpositions, corresponding to the possibility of having complementary properties which are absent in classical computer science. Therefore, we are left with states of qubits, namely pure quantum software: objects, matter, hardware, completely became vaporized” [74]. These are, nonetheless, said to be “just the usual physical systems” with which we are familiar.

A complete collection of such events in the sense of there being unit probability of an event occurring within unit probability within the collection constitutes a ‘test’ and is considered “physically a measurement instrument” [74]. From the perspective of information, “tests and events represent sub-routines, whereas the systems are registers on which information is read and written.” Tests appearing in the circuit schema can be composed to form compound events; likewise systems can be composed to form compound systems. Merging events corresponds to a “coarse graining,” with probability incorporated to provide joint probability to joint events: circuits forming closed networks—joint events (composite tests) with trivial input (preparation) and output (observation) systems—are described via joint probabilities. Composition of systems is associative, symbolically

$$A(BC) = (AB)C, \quad (5)$$

with the trivial system (in the sense that it is a system that cannot be used to communicate information) as identity [78].

$$AI = IA = A. \quad (6)$$

The theory describing these circuits presented in the form of six principles, discussed below, and attributed a dynamics carried out through discrete automaton gating steps which proceed at the rate

$$r = a/(\sqrt{dt_P}) \quad (7)$$

per step, each step of length  $a = \sqrt{dl_P}$ , where  $t_P$  is the Planck time and  $l_P$  is the Planck length [73]. The maximum propagation speed is taken to be  $r$  then identified as a universal constant, thus identified with the speed of light,  $c$ .

Two sorts of automaton are defined here, the Weyl automaton and the Dirac automaton, the latter obtained by coupling pairs of the former sort and called as such because in the limit as  $\hbar$  is much less than unity a Dirac-form equation is obtained. Given the discrete nature of the circuit operation,

D’Ariano and coworkers are motivated to define a local Hamiltonian matrix in terms of the discrete time-derivative of the field and another Hamiltonian which generates a unitary evolution interpolating the discrete time determined by the automaton steps to a continuous time  $t$ . One sees that D’Ariano’s reconstruction incorporates the digital ontology and information ontology, as well as pancomputationalism. This model is part of a broader programme of grounding quantum field theory on causality (its means of incorporating the Information ontology) and two principles relating more directly to physical law (its means of ostensibly reducing physics to information), namely,

(D1) the Deutsch–Church–Turing principle, and

(D2) the topological homogeneity of interactions

in conjunction with the principles of quantum theory [74]. Causality, considered more specifically below, here means specifically “‘no signaling from the future’”. In simple words it says: in a cascade of measurements on the same system, the outcome probability of a measurement does not depend on the choice of the measurement performed at the output. The principle also implies no-signaling without interaction—shortly ‘no-signaling’, and also commonly known as ‘Einstein causality’ ” [74]. Principles D1 and D2 are to entail a quantum cellular automaton based extension of quantum field theory in that localized states and measurements are incorporated, something that standard quantum field theory has not managed adequately to do.

For D1, D’Ariano rephrases the Deutsch–Church–Turing principle as “Every physical process describable in finite terms must be perfectly simulated by a quantum computer made with a finite number of qubits and a finite number of gates” and says that this means that every finite experimental protocol is perfectly simulated by a finite quantum algorithm and implies two things, that the density of (quantum) information is finite (i.e. “the dimension of the Hilbert space for any bounded portion of reality is finite”) and that “interactions” are local (i.e. “the number of quantum systems connected to each gate is finite”) [74]. Recall that Turing machines were conceived of by Alan Turing as a model of human “computation” [76]; later, Alonzo Church conjectured that any computation can be carried out by some Turing machine. This conjecture is known as Church’s thesis and today it is generally accepted as true.

Principle D2 is the proposition that “the quantum algorithm describing a physical law is a periodic quantum network,” capturing that idea that “in the informational paradigm the physical law is represented by a finite set of connected quantum gates, corresponding to a finite protocol, theoretically



specular of a finite quantum algorithm” [74]. D’Ariano and coworkers view this as requiring the locality of “interactions,” which is needed in order to define a physical law as a finite protocol under the local control of the experimenter. The universality of physical law is understood to correspond to a homogeneity requirement. Thus, physical law is to be understood algorithmically and represented by a quantum unitary cellular automaton with the Planck distance and the Planck time being the fundamental intervals of this discrete “reality.” The notions of space and time are thus understood in a context where the vacuum is identified as any state that is locally invariant under the automaton evolution; localized states are the states that differ from it for finite numbers of systems. The evolution is to be evaluated in the future causal cone of these systems, an evaluation which does not require boundary conditions [74].

The grounding of quantum field theory in such automata is ultimately to be accomplished via six fundamental principles, considered axioms, the first being causality as defined above, with the following theoretical functions. “In addition to causality [(i)], there are five other informational principles that are needed for deriving quantum theory [72]: (ii) local tomography, (iii) perfect distinguishability, (iv) atomicity of composition, (v) ideal compressibility, and (vi) purification. All six principles apart from vi hold for both classical and quantum information: only the purification one singles out quantum theory” [74]. Principle ii is the requirement that joint states of multiple systems be discriminable from one another by measurements on the individual systems involved. Principle iii relates probability and logic. Principle iv is the requirement that two transformations suffice to identify their composition. Principle v allows for sub-systems. Principle vi allows any form of irreversibility or state mixing to correspond to discarding part of the environment of the system in question.

The predictive element of the theory is developed by adding “a probabilistic structure on top of the operational structure,” to form an ‘operational-probabilistic theory’ (OPT) to provide what is considered an extension of probability theory, in which events are topologically connected in circuits. This is an impressive mathematical feature, demonstrating the value of the value of the quantum–digital heuristic. A primary task for informational reconstructive approaches to quantum theory has long been to provide, at a minimum, an informatic motivation for the mathematical structures assumed in quantum mechanics, such as complex Hilbert state space, Hermitian operators representing system properties, as seen in the previous examples discussed above. In this case, the representability of every state of a system as a Hermitian operator on a complex Hilbert space of said dimension is achieved by showing the identity of the upper bounds on the probability of conclusive teleportation of an unknown state in terms of the maximal number of perfectly discriminable

states and the dimension of the vector space spanned by system states.

Holism in quantum mechanics is generally understood as exemplified by the existence of entangled states, entangled states being just those that are *inseparable*—that is, those that generally cannot be prepared by local operations. In an OPT, the two are tantamount: Entangled states have the feature that the marginal probabilities pertaining to subsystems are mixed, with maximal knowledge of their composite without maximal knowledge of the parts, and the converse—that is, that such maximal knowledge of the composite without that of the parts implies its being entangled. Principle ii implies that any two distinct states of a composite system have different joint probability distributions for some local measurement, allowing them to be distinguished by local measurements on the subsystems alone via the corresponding correlations and that the composite system dimension is the product of the subsystem state dimensions: The full information of the entangled state is that contained in these correlations.

The quantum OPT is accordingly said to be “holistic, but not too much.” This statement given a precise meaning via the notion of “degrees of holism,” [61], Sect. 6.5. This is made possible by noting the possibility of principles different from local discriminability, such as principles requiring also the consideration in addition of joint measurements on various sets of subsystems to account for observed behavior—for example, in so-called “real quantum theory” in which the Hilbert space of states is real rather than complex; cf., e.g., [77]. From the perspective of this approach, Principle vi is that of greatest significance because it serves to distinguish quantum OPT from classical OPT. The basic idea is that every mixed state can be prepared by discarding part of a larger pure composite system state to which it belongs. The principle can also be seen as capturing a law of conservation of information in that every irreversible process can be carried out uniquely via a reversible interaction of the system undergoing it with an environment in a pure state before interaction begins.

## 5 Conclusion

Quantum theory reconstruction as a foundational activity distinct from the interpretation of quantum mechanics, or as an “interpretation without ‘interpretation’, ” has been pursued in the last twenty years often specifically on the basis of informational principles, rather than mechanical principles or pure logic as was previously more often attempted. Here, a representative set of informational reconstructive attempts was reviewed and a number of additional prominent themes were noted, namely, an attempt to isolate simple principles

from which the theory could be derived, in some cases beginning with an incomplete set with an aim toward identifying others or bringing in information notions not traditionally considered in the study of the foundations of the theory.

The most ambitious of these attempts at reconstructing quantum mechanics and quantum field theory is the recent modeling of quantum theory in the general sense as the operation of an information processor describable as a quantum automaton, which can be viewed as a weak quantum realization of the so-called Fredkin–Zuse thesis, that “the universe is being deterministically computed on some sort of giant but discrete computer” [79, 80] (see [14] for a discussion of strong such pursuit of this thesis). The reconstructive approach generally has also been of increasing interest as a way not only of avoiding the quantum measurement problem but also as a way of moving forward toward the goal of integrating quantum theory and gravitational theory.

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