2015-11-01

Mathematical story: a metaphor for mathematics curriculum

Leslie Dietiker. 2015. "Mathematical story: a metaphor for mathematics curriculum." EDUCATIONAL STUDIES IN MATHEMATICS, Volume 90, Issue 3, pp. 285 - 302 (18).
https://hdl.handle.net/2144/23743

Mathematical story: A metaphor for mathematics curriculum

Leslie Dietiker
Boston University

Abstract: This paper proposes a theoretical framework for interpreting mathematical content found in mathematics curriculum in order to offer teachers and other mathematics educators comprehensive conceptual tools with which to make curricular decisions. More specifically, it describes a metaphor of *mathematics curriculum as story* and defines and illustrates the mathematical story elements of mathematical characters, action, setting, and plot. Drawn from literary theory, this framework supports the interpretation of mathematics curriculum as art, able to stimulate the imagination and curiosity of students and teachers alike. In doing so, it is argued, this framework offers teachers and other curriculum designers a conceptual tool that can be used to improve the mathematics curriculum offered to students in terms of both logic and aesthetic.

Biographical note: Leslie Dietiker is an assistant professor of mathematics education at the School of Education, Boston University. Her interests include understanding and improving written and enacted mathematics curricula in order to inspire student curiosity and increase their desire to make sense of mathematics.

Mailing Address:
Leslie Dietiker
Boston University
Two Silber Way
Boston, MA 02215, USA

Email: dietiker@bu.edu. **Telephone:** 781-281-4443. **Fax:** 617-353-2909

Key words: mathematics curriculum, textbooks, narrative, aesthetic, mathematical story

1 Introduction

Mathematics teachers can greatly influence how mathematical content is manifested within the classroom (Brown, 2009; Tarr, Chávez, Reys, & Reys, 2006). Because of this, there is increasing evidence that one way to improve the mathematical quality of instruction, and thus influence student learning, is to support the curricular decisions of teachers such as selecting and sequencing tasks, deciding which numbers to use in a problem, and altering a mathematical activity so that it engages students in critical thinking (Choppin, 2009; Remillard, 1999). To address this need, recent studies have examined how teachers interact with mathematics materials, including how they plan with, adapt, or ignore the content contained within (Brown, 2009; Pepin, Gueudet, & Trouche, 2013a, 2013b; Sherin & Drake, 2009). In order to influence the curriculum use of teachers, researchers have called for an increased attention to how teachers make sense of the content of mathematics textbooks (Ball & Cohen, 1996; Breyfogle, Roth McDuffie, & Wohlhuter, 2010; Nicol & Crespo, 2006; Remillard & Bryans, 2004)(Ball & Cohen, 1996; Breyfogle, Roth McDuffie, & Wohlhuter, 2010; Nicol & Crespo, 2006; Remillard & Bryans, 2004). Specifically, these calls have pointed to the need for teachers to recognize and understand the temporality of curriculum beyond the top-level scope and sequence in tables of contents currently provided in textbooks (e.g., Ball & Cohen, 1996; Nicol & Crespo, 2006). As Ball and Cohen (1996) explain,

> Teachers' guides rarely help teachers to think about the temporal dimensions of curriculum construction. Teachers' guides could, for instance, contribute to teachers' thinking about content and activities appropriate in September as they begin to construct the classroom culture and environment. Teachers' guides could also help teachers to consider ways to relate units during the year... curriculum authors could discuss alternative representations of the ideas and connections among them (p. 7).

This paper supports this call by proposing a theoretical framework for interpreting sequences of mathematical content found in curriculum. The goal is to offer mathematics educators (which for the purposes of this paper includes mathematics teachers, curriculum developers, and researchers) a set of coordinated conceptual tools with which to make sense of written curriculum materials and to support curricular decisions. Specifically, literary theory is used to situate mathematics curriculum as a form of narrative for which the sequence of mathematical events are taken as a *mathematical story* (Dietiker, 2013). Examples of mathematical stories drawn from textbooks are used to explain how the metaphor *story* offers mathematics educators a way to perceive how mathematical content in textbooks emerges and changes within a sequence. This paper demonstrates how this conceptualization enables mathematics educators to recognize the changing content as a coherent integration of mathematical objects, procedures, and representations. In addition, this paper explains how this framework offers a way to attend to the aesthetic dimensions of mathematical sequences, enabling mathematics educators to recognize how a portion of textbook offers the potential for surprise or suspense (or not). The point of this paper is not to make claims about which mathematical stories are better than others; rather, this paper proposes a conceptual framework that enables such comparisons by mathematics educators.

The framing of curriculum proposed in this article, mathematics curriculum as *story*, uses a metaphor to situate mathematical content as a form of art. Although other art forms also resonate with mathematics curriculum (for example, architecture or woven art), metaphorically interpreting mathematics as a story takes advantage of their underlying pedagogic nature. Egan (1988) argues that stories are a “powerful form” that sparks our imagination and help us “make sense of the world” (p. 2). Stories are not solely a form of narrative; stories entertain and communicate messages (e.g., the moral of a story).

Though treating mathematics curriculum as a form of art may be uncommon, researchers have found it useful to consider the narrative qualities of mathematical texts and textbooks (see, for example, Andrà & Sinclair, 2012; Borasi & Brown, 1985; Dietiker, 2015; Gadanidis & Hoogland, 2003; Netz, 2005; Sinclair, 2005; Thomas, 2002). The *story* metaphor is appealing because theorists of literature have long focused on ways to characterize both the way events are sequenced and their overall effect. Thus, interpreting mathematics curriculum as a *story* draws attention to the sequence of content (i.e., how the story unfolds in the moment) and its aesthetic dimensions (i.e., surprising turns). In addition, the metaphor *story* also brings with it new descriptive language for what happens along the way (i.e., action), environment (i.e., where the story is set), objects of the sequence (i.e., its characters), as well as the interrelationships between parts of the sequence (i.e., foreshadowing).

The first section of this paper begins with a discussion of theory, including assumptions regarding reading, narrative, and metaphor, with which this curriculum framework was developed. Following this, the main section of this paper defines and illustrates core elements that have been used to understand and describe stories for thousands of years (e.g., Aristotle's *Poetics*), namely their characters, action, setting, and plot. The next section explains how a narrative interpretation of mathematics curriculum enables these elements to be recognized as a coherent integration of mathematical objects, procedures, and representations within a sequence, rather than being unrelated mathematical aspects of curriculum. The paper concludes with a discussion of the potential implications of this framework on teachers and mathematics curriculum.

2 Theoretical assumptions and framing

Before introducing and illustrating the mathematical story elements, I first articulate theory that grounds this work: reading theory and the framing of mathematics texts as narrative. It should be noted that the *story* metaphor can broadly be used to describe mathematics content in all curricular forms (i.e., as written, imagined, planned, and enacted). However, for the purposes of this paper, I have chosen to focus narrowly on the content organized within mathematics textbooks.

2.1 Assumptions about reading

It is assumed that texts do not "contain" meaning; instead, readers make meaning with a text through a transactional process (Rosenblatt, 1988, 1994). That is, neither the text nor the reader is a determinant factor in the production of meaning. All reading is interpretive and is done with a particular lens of the individual that is shaped by his or her knowledge, goals, and prior experiences (Rosenblatt, 1988). As Rosenblatt explains,

> The reader, we can say, interprets the text. (The reader acts on the text.) Or we can say, the text produces a response in the reader. (The text acts on the reader.) Each of these phrasings, because it implies a single line of action by one separate element on another separate element, distorts the actual reading process. The relation between reader and text is not linear. It is a situation, an event at a particular time and place in which each element conditions the other. (Rosenblatt, 1994, p. 16)

In short, a reading of textbooks is its interpretation by a reader and cannot be generalized to others. For example, it is important to note that the readings presented in this paper are the readings by a specific reader (i.e., myself). They do not (and cannot) reveal how another reader might interpret the same portion of text, particularly a student.

However, this does not detract from the goals of this paper as my purpose is not to propose particular readings of textbooks or to describe how students read textbooks. Rather, the goal is to propose a new metaphor of mathematics curriculum, generated from literary theory, that offers mathematics educators curricular insight beyond scope and sequence offered in a table of contents of a textbook.

However, this is not to say that a mathematics teacher cannot interpret a textbook for what it offers a novice. Although this is not the same as reading *as* a student, teachers can read a textbook for its potential meaning *for* students. Instead of employing expert knowledge developed outside of the mathematical story, reading for the potential mathematical story involves narrowly attending to only that which the mathematical story offers its readers. In this way, the readings described in this paper represent a knowledgeable reading limited to the content at hand based on a close reading of text. Doing so requires making assumptions of content knowledge that is reasonably expected of a reader at a given grade level. This type of reading reflects *potential* meaning because there is no claim that students will actually interpret the texts this way – only that the possibility exists when limited to the text at hand. Reading this way offers a teacher or curriculum designer an image of how the content might be withheld and/or revealed over time, allowing him or her to recognize the potential aesthetic consequences of revealing the punch line early in the lesson.

2.2 Mathematical story

Metaphors shape how we view, and thus interact with, the world. (Lakoff & Johnson, 1980). Thus, interpreting one thing in terms of another, the basis of metaphor, is one form of conceptual tool. As Lakoff and Johnson explain “Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature" (1980, p. 3). For example, perceiving time in terms of money, as in the "time is money" metaphor, explains why time is viewed as a limited resource, which can be "budgeted" or "wasted" (p. 8). Therefore, metaphors support "viewing as," where a selected metaphor draws attention to features and qualities of what Lakoff and Johnson refer to as the *target domain*. For the metaphor proposed in this paper, the target domain is purposefully-designed sequences of mathematical content (i.e., curriculum) and the metaphor *story* maps the concepts in the domain of literary stories to their counterparts in mathematics curriculum.

Thus, to define the metaphorical concept of mathematical story, it requires a definition of the concept of literary story. For this, I draw from the narratological framework of Bal (1986, 2009), who carefully articulates a set of enduring and well-regarded definitions of narrative and their elements of character, action, setting, and plot. Bal explains that narratives can be analyzed with respect to three interrelated layers of meaning (which she calls "text," "fabula," and "story"). These layers, which follow a Russian formalist tradition of separating the logical truths of the narrative ("fabula") from the unfolding revelations of the truths ("story"), help to explain how a story changes when the sequence of revelations changes such as how learning that Romeo and Juliet are doomed to die affects the interpretation of the story, which can occur at its beginning (as is found in Shakespeare's play) or at its end (as in the 1968 movie *Romeo and Juliet*). The *text* layer focuses on the role of the media for the narrative (e.g., live play vs. film).

In terms of mathematics curriculum, therefore, a *mathematical story* is the interpretation of the chronological sequence of mathematical changes ("events") in a mathematics textbook by a reader (Dietiker, 2013). Thus, the metaphorical interpretation of mathematical story is one of three narrative layers of mathematics curriculum. These layers recognize the roles of media ("mathematical text"), content and meanings developed throughout the narrative ("mathematical fabula"), and sequence and effects of the unfolding revelations ("mathematical story") of the mathematical sequences. I have provided a more rigorous analysis of the three metaphorical layers of mathematics curriculum in Dietiker (2013). This current paper extends this theoretical work by introducing and defining core elements of mathematical stories, specifically the framing of mathematical character, action, setting, and plot.

Despite the use of this metaphor in other work, I should note in the interest of clarity that the phrase “mathematical story” has been used in several different ways in other work. Perhaps the most common is the notion of a contextual word or “story problem.” Although story problems can be read as mathematical stories as described in this paper, the term “mathematical story” described here are not limited to contextual word problems. Also, rather than referring to narratives *about* mathematics (e.g., *Math Curse* by Scieszka and Smith), the term “mathematical story” in this current paper describes the way the mathematical content changes across a sequence as read; that is, they are stories *of* mathematics. Finally, the phrase “mathematical story” has also been used to refer to the mathematical histories of teachers (e.g., Drake, 2006), which again is not the intended meaning for the framework discussed here.

2.3 Mathematical sequences as a story

To highlight how this framing offers insight into the nature and quality of the mathematical content, consider the potential mathematical stories based on a set of tasks upon which a lesson on the roots of quadratics could be planned from a textbook of which I am an author (see Figure 1). That is, assuming that a lesson is being designed for a set of students who have explored rewriting quadratics and who have connected the roots of quadratics with the *x*-intercepts of their corresponding parabolas, how might the sequence of these three tasks differ? To benefit from the discussion that follows and references to these tasks throughout the rest of the paper, it is recommended that you first solve these tasks, assuming a beginning understanding of graphing, solving, and factoring quadratic functions (over rationals).

A. Find the roots of $y = x^2 - 6x + 3$ by factoring.

B. Justin says that $x^2 - 6x + 3$ can be rewritten as $(x-3)^2 - 6$. Do you agree? If so, how can you use the expression $(x-3)^2 - 6$ to find the roots of $y = x^2 - 6x + 3$?

C. Graph $y = x^2 - 6x + 3$ and estimate the *x*-intercepts, if any.

Fig. 1 Three tasks adapted from *Core Connections Algebra* (Dietiker, Kysh, Sallee, Hamada, & Hoey, 2012).

For this set of tasks, the order A-B-C prompts a reader to first factor a quadratic function (we will assume without any prior knowledge of this particular quadratic). Assuming that this reader has some basic understanding of factoring over rationals and may be surprised when it does not factor. Following this, Task B introduces a way of rewriting the function and prompts this reader to use this form of the expression to find solutions to the quadratic. The introduction of this strategy after a failed strategy suggests that it will likely be successful. When solutions are found, Justin's strategy is positioned as coming to the rescue, offering a successful strategy for solving for the roots of a quadratic when the factoring strategy did not. Finally, Task C prompts this reader to use a third strategy (graphing) to find its roots, which at in this sequence could be seen as redundant since this reader potentially has exact roots already. However, the graph could be an opportunity for checking the solutions to Task B.

In contrast, other sequences of these same three tasks offer important differences regarding the purpose, experience, and potential aesthetic for the reader. For example, in the reverse sequence C-B-A, a reader is able to recognize and approximate roots of the quadratic at the beginning by viewing the graph. Task B, then, introduces a strategy (rewriting the quadratic expression) that enables the reader to solve for exact roots. With this sequence, Task C then can lead to the expression of factors that take into account these exact roots, namely $(x-(3+\sqrt{6}))(x-(3-\sqrt{6}))$. In this mathematical story, the role of Task A has considerably changed (from being a failed strategy to being a strategy extended to include the factoring over irrational numbers).

Interestingly, the A-B-C and C-B-A sequences often emerge when I give these three tasks (unordered) to groups of mathematics teachers and curriculum designers to sequence for a lesson involving the roots of quadratics. The difference between the overall messages of these mathematical stories (which can be interpreted as the *morals* of the mathematical stories) is important; in A-B-C, the potential mathematical story is about how one strategy fails and another can succeed to find roots of a quadratic function in certain conditions while in C-B-A, it is instead about how quadratic functions that have x-intercepts when graphed can be rewritten as binomials that involve irrational roots. In their explanations for choosing these sequences, teachers and designers have referred to the familiarity of the progression or how the emergence of the content is incremental.

In contrast, when I ask, "is there a sequence that offers a sense of drama or suspense?", a new sequence A-C-B often emerges. This sequence, similar to A-B-C, starts by prompting a reader to solving the quadratic by factoring but then follows this with the graphing task. When this reader determines that the quadratic in Task A is unfactorable over rational numbers, he or she may assume that the graph will not intersect the *x*-axis (a reasonable assumption for a student in the United States who has a beginning understanding of factoring and solving for the roots of a quadratic). Therefore, this sequence offers a potential surprise when it is realized that the graph of $y = x^2 - 6x + 3$ indeed intersects the *x*-axis twice. This surprise can then lead a reader to wonder: *How can this be?* or *How else could I have known the parabola would intersect the x-axis?* Task B then offers this reader a way to answer these questions through a means other than factoring by solving an equivalent quadratic equation $(x-3)^2 - 6 = 0$. The A-C-B mathematical story is again about something different; its moral suggests that when the factoring strategy fails, the parabola may still have *x*-intercepts and that a rewriting strategy can identify the roots when factoring (over rationals) cannot.

The interpretations in this section help to demonstrate that when curricular parts (e.g., tasks definitions, statements, or other mathematical parts of curriculum) are sequenced and read for their temporal connections, a narrative structure can be perceived that can help a mathematics educator recognize how the content changes across the sequence and the potential aesthetic dimensions of the sequence. This framing, however, does not assume that all mathematics texts make sense as narrative. Some sequences make no sense when reading across the parts for a connected story, such as when each part of the story appears disconnected and unrelated from the others. Nor does it imply that those sequences that do offer narrative interpretations are interesting or compelling stories. Despite this, all mathematics texts can be read with a narrative lens by attending to how the parts alter and constrain the meaning of the others, if they do.

2.4 The grain size of mathematical stories

The preceding section involved potential mathematical stories at the lesson grain-size. That is, a lesson could be composed with the three tasks given in Figure 1. Beyond a small set of tasks, however, a mathematical story can be recognized across larger units of text, such as a chapter of a textbook or an entire course (or beyond). At each of these magnitudes, what constitutes a mathematical event differs, much like how describing important events in a particular episode of a serial novel will focus on much smaller details than descriptions across multiple volumes within a series (such as *Harry Potter*). A top-level interpretation of a mathematical story for an entire course describes in broader strokes how mathematical content changes across the sequence. Its parts represent "chunks" of curricular material that advances its mathematical story.

For example, consider the potential affordances and constraints in terms of how mathematical relationships are revealed for functions in an algebra course. At a top level, a common sequence found in algebra textbooks (including my own) starts with (A) a focus on the characteristics of linear functions in multiple representations (e.g., learning about slope and intercepts), followed by (B) the study of the qualities of quadratic functions (e.g., identifying intercepts, vertex, and concavity), followed by (C) the introduction of polynomials of higher order such as cubics, quartics, and so on. However, a different sequence (C-B-A) offers different insight: quadratics can instead be positioned as a degenerate case of cubics when $a = 0$. Likewise, linear functions are degenerate quadratics. While A-B-C highlights the effects of adding new terms to a polynomial, C-B-A instead draws attention to the interrelationships, positioning each in terms of the other.

Although a lot of information is left out at this top level, even these broad descriptions help show how these mathematical stories highlight different relationships between these sets of functions (which will be interpreted as families of characters in the next section). Even if the particular mathematical functions are the same across all of these stories, the way the content unfolds enables different aspects of these functions to be brought into focus and thus develops different meaning of the content. Fundamentally, for any sequence of "chunks" A-B-C at any grain size, it is how B builds from A and changes what we know about both A and B (and similarly how C then builds from B and changes what we know about A, B, and C) that engenders the mathematical story.

Beyond the hypothetical example discussed here, there is evidence that changing the sequence of topics at the top level can alter the meaning of mathematical objects and their relationships change with sequence. For example, although most elementary textbooks in the United States start with the study of number and operations and then introduce measurement (A-B), the *Measure Up* curriculum series (Dougherty, 2003) starts with the qualitative comparison of measures instead and uses this to develop the concept of number and counting (B-A). Based on the work of Davydov and his colleagues, the curriculum designers explain that with this change in sequence, common tasks such as "Is $3 < 8$?" are read differently. Although most adult readers might read $3 < 8$ as a comparison between two abstract values and determine that the question presents an accurate comparison, the students in the *Measure Up* program interpret the question differently:

> At this stage, if asked whether 3 < 8 is a true statement, these children will respond that you have to know what the unit is. As one first grader commented, "If you have three really big units and 8 really small ones, 3 could be greater than 8. But if you're working on a number line, then you know that 3 is less than 8 because all the units are the same." Another first grader noted, when asked to describe what 5 = 5 meant, "It's probably true unless you have a big 5 and a little 5. Like 5 big units and 5 small units, then it isn't true" (Dougherty, 2003, pp. 19–20).

This example reveals the possible effect on mathematical meaning of a sequence ("What is the meaning of "3"?"). The children's responses provide evidence that in any curriculum, to predict the meaning of a statement or question from a student's point of view, it must be read in context of its broader mathematical story.

3 The elements of a mathematical story

Stories have descriptive elements that, at least since Aristotle, have been used by readers for descriptive analyses and comparisons of stories. In this section, I introduce a theoretical framing of the mathematical story elements characters, actions, settings, and plot. For each mathematical story element, its framing will start by introducing the construct through literary theory (predominantly relying on Bal (2009) to keep consistent within the framing of narrative layers). Then, the metaphorical mathematical construct is developed and illustrated with examples from textbooks. However, merely identifying the mathematical story constructs of a mathematical sequence, such as mathematical setting, is not useful unless this metaphorical lens brings with it new information. Therefore, beyond its framing, each construct is also examined in terms of what insight into mathematical curricular sequence it offers mathematics educators.

Due to space limitations, portions of mathematical stories from textbooks that highlight particular qualities of multiple elements (such as a useful example of both central mathematical characters and action) were selected and used for illustration. These generally unremarkable examples were intentionally selected to demonstrate the use of this framework with "everyday" textbooks and are not intended to demonstrate exceptional mathematics stories or their elements.

3.1 Mathematical character

Bal (2009) defines literary characters as "anthropomorphic figures with specifying features the narrator tells us about" (p. 112). Therefore, metaphorically, it is reasonable to expect that *mathematical characters* are "figures" brought into existence (objectified) through reference or inference in the text (e.g., naming, defining, or otherwise drawing attention to them) and which are given specifying features (a process that Bal refers to as "fleshing out"). Examples of mathematical characters of textbooks are mathematical objects such as a function ($y=x^2-6x+3$ in Figure 1) or a geometric object (such as a diagram of a 3-4-5 triangle). For example, for the sequence C-B-A for the tasks in Figure 1, what is known about the mathematical character $y=x^2-6x+3$ is extended by learning new names for the character, namely $(x-3)^2-6$ in Task B followed by $(x-(3+\sqrt{6}))(x-(3-\sqrt{6}))$ in Task A. In terms of a mathematical story, this temporal change of what is known about a mathematical character is *mathematical character development*.

What is objectified by a mathematical story is determined by its reader based on positioning of content within the text; in the case of an expression such as $3+5$, the numbers 3 and 5 may be treated as mathematical characters that are being added together or the expression itself may be positioned as a character. Whether or not to read "3+5" as a single character or as an action on two characters (the joining of two numbers) is determined by a reader and influenced by the sequence of tasks. For example, I interpret $x^2 - 6x + 3$ in Task A of Figure 1 as a single object rather than a collection of objects since the text does not draw my attention to any of the terms of the expression. The directions "Find the roots of $y = x^2 - 6x + 3$" could just as well been phrased "find the roots of *it*." This allows me to perceive the function as a single mathematical character, of which I then learn could have a different name (namely, $(x-3)^2 - 6$) in Task B. However, other expressions can be positioned differently so that the terms are each objectified. An example of how an expression such as $3+1$ could be positioned as two characters joined together will be offered in the next section.

3.1.1 Curricular insight offered by mathematical characters. The consideration of how mathematical characters are introduced and developed can offer new information about how textbooks temporally reveal the properties of characters. Similar to complex characters in literary stories, mathematical characters have characteristics (mathematical "properties") and often can be referenced by multiple names. After a mathematical character is formally introduced by a given definition, it is usually later further developed with additional characteristics.

The discussion of the different sequencings of the tasks in Figure 1 demonstrates how reading for mathematical character development offers a way to distinguish potential benefits of sequences. For example, what is potentially revealed to a reader about the function $x^2 - 6x + 3$ changes depending on whether Task A comes before or after Task B.

Reading sequences of textbooks for mathematical character development can potentially circumvent "expert blindness," which prevents teachers from recognizing the potential meanings of textbooks for students (Nathan & Petrosino, 2003). Rather than reading textbooks with a fully developed understanding of the complexity of mathematical objects, this framing enables a teacher to read how new properties are subtly revealed throughout the sequence. Specifically, a teacher might recognize how different tasks assume radically different characterizations of mathematical objects that may render them unrecognizable by students.

To illustrate how an analysis of mathematical character development offers potential insight into the content of mathematical textbooks, an elaborated example across multiple lessons of the Singapore's *My Pals are Here!* Grade 1 textbook (Kheong, Ramakrishnan, & Wah, 2012) is offered and will be referenced later in the paper. At its start, the counting numbers 1 through 10 are introduced with symbols and pictures of countable objects such as toy cars or candies, similar to the adaptation in Figure 2. With the instruction of "point with your finger and count" (p. 6) these mathematical characters are endowed with quantity and are initially related to each other through the sequence of counting 1, 2, 3, … 10.

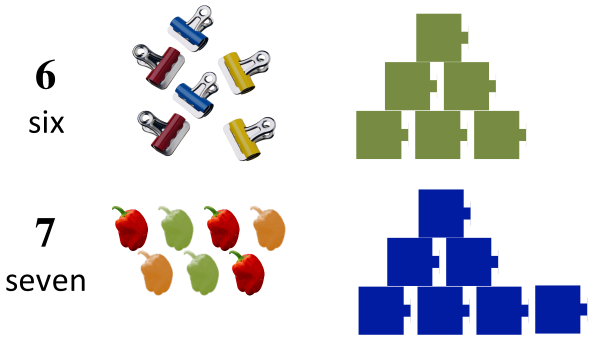

Fig. 2 Diagram introducing "6" and "7" adapted from *My Pals are Here!* (2012).

This start represents an example of how mathematical characters can be introduced through implicit definition. Although it does not have many of the features of an explicit mathematical definition (with phrases such as, “Let 5 ...”) nor the goal (offering minimal but sufficient criteria to specify the object what the object is and what it is not), this table brings these numerical characters into being and characterizes them. Although the text is *explicitly* introducing mathematical characters, many of the characteristics described earlier are read *implicitly* through interaction with the diagrams (such as how the pictures show similar objects or comparing the ways different rows organize the grouped objects).

From a story perspective, these opening pages of this textbook do more than just offer initial characterizations of the counting numbers 1 to 10; they set up an expectation that the story from this point on will involve these characters, just as a story might start with “Once upon a time there were three little pigs ...”. Since this is so early in the mathematical story, there is not enough information to know how these numbers may be involved, only the expectation to see them again.

These numbers, however, are later fleshed out in a task that shows collections of different types of objects arranged in different configurations and directs the reader to “Count. What is the number?” (p. 9). This change is significant because all of the collections have seven objects. The teacher is also instructed to “ask pupils to name any 7 objects they can see around them” (p. 9). This activity extends the mathematical character “7” beyond a label of a quantity of a particular collection to refer to a general quantity for any collection of seven objects. Follow-up tasks similarly extend the other numerical characters.

The textbook then prompts readers to compare counting numbers, asking, “make a number train using (a) 4 [blocks of one color] and (b) 9 [blocks of another color]. Which number is greater?” (p. 16). This task does not reveal much about the properties of the characters of 4 and 9 (that is, 4 can be recognized as a quantity that is smaller than 9 before this by listening to the counting sequence 1, 2, 3, 4, 5, 6, 7, 8, and 9 as one touches and counts). However, it reveals that numbers can be useful for comparing quantities. This use is then found in the next lesson in prompts that ask questions like, “What is 1 more than 3?” (p. 19) with a photo of a number train built of 3 cubes of the same color and below it a photo of the same train of 3 cubes with another cube of different color at the end. This set of comparisons of “1 more” and “1 less” draws attention to the relationships between consecutive counting numbers such as 3 and 4. These relationships highlight a unitary and ordered quality of the counting numbers that, while present in the prior textbook portions, were not in focus and therefore were not yet realized.

As the mathematical stories discussed so far demonstrate, sometimes a mathematical story is less about the particular mathematical character (as is the case of the mathematical stories based on the tasks in Figure 1) and more about the relationships between the mathematical characters (as is the case for relating the numbers 1 through 10 in relation to each other). Just as in literary stories (Bal, 2009), mathematical characters can be “fleshed out” through the connection or comparison with others with similar or different qualities or dimensions. The mathematical characters can also be interpreted as central or marginal to a story. One way to recognize whether a mathematical character is central or not is to consider the impact of changing it to another mathematical object.

In terms of mathematics, Bal's reference to anthropomorphism may make recognizing mathematical objects as mathematical characters in textbooks challenging. Even in terms of literary narrative, Bal does not restrict the notion of character only to humans (there are many narratives without humans). Bal emphasizes that part of the way the story layer works is that a reader of a narrative interprets characters as "human-like" and assigns them human characteristics. Analogically, this means that it may be useful to think of mathematical characters as those objects in the mathematical story that can be related to by a reader through narrative modes of thought (a topic explored by Healy and Sinclair (2007)). That is, the human reader can be though of as the actor of the mathematical story, not only performing mathematical actions *on* mathematical characters (as is the case of adding 3+1) but also in terms of acting within the mathematical story (when acting *as* a mathematical character). More on mathematical actor is discussed in the next section.

3.2 Mathematical action

Another descriptive element of all literary stories is an act (or action), which is performed by an actor and results in some form of change (Bal, 2009). Since actions are needed to create a story (otherwise, nothing happens), actions create markers of change along the temporal experience of the story (called *events*). Thus, a *mathematical action* can be interpreted as a manipulation of a mathematical character, such as a number or shape, resulting in a mathematical change. Examples of mathematical actions include transformations (e.g., translating a function on a coordinate plane), decomposition or composition (e.g., breaking a trapezoidal region into a rectangular and two triangular regions), and operations (e.g., adding two numbers). In the case of Task B in Figure 1, rewriting $x^2 - 6x + 3$ as $(x-3)^2 - 6$ is a mathematical action on a mathematical character that changes what is mathematically known at that point (that this character has another form that allows it to be solved with existing solution strategies).

In mathematical stories, mathematical actions are performed by mathematical actors. Mathematical actors can include narrators, fictional characters, and human actors. For example, in Task B of Figure 1, a fictional character named "Justin" rewrote the quadratic function to advance the mathematical story. In contrast, the act of solving for the exact values of the roots in Task B was performed by a human actor (myself in this case, or a student in a classroom).

3.2.1 Curricular insight offered by mathematical action.

Reading for action is generally reading for transition, then, considering the questions *What is happening to the central characters?, What is being done?, What changes?* and *How does it change?* In the case of curriculum, some actions may be more integral to the mathematical story than others. For example, when learning about finding the area of a complex shape, the decomposition and rearrangement of regions into a new shape is important in changing the temporal story; prior to this act, the region might be been unmeasureable by a reader/actor. Other mathematical actions, however, may not have a major effect on moving the mathematical story forward, such as the repetitious practice of a procedure. This is not a critique of repetition or practice, but instead a recognition that the mathematical story may not advance with it. This statement is also not all-inclusive; there are ways that exercises that appear routine can advance what is known about the mathematical characters or actions.

Reading for mathematical action can enable a reader to recognize how mathematical actions change across a sequence with increasing sophistication and how a reader's physical actions can be mapped to mathematical actions. For example, the opening activity of Lesson 1-2 of *Everyday Mathematics* (The University of Chicago School Mathematics Project, 2007, Grade 3), introduces a number grid from –9 to 110 (see Figure 3 for a partial number grid) and poses a series of questions, such as:

[1] "What happens to the numbers in a row as you move from left to right?" (p. 24)

[2] "What happens to a number when you move to a number below it?" (p. 25)

[3] "What happens when you add 1 to a number that does not end in 9 or 0?" (p. 25)

[4] "What happens when you add 1 to a number that ends in a 0?" (p. 25).

[5] "What happens when you add 10 to a number?" (p. 25).

– 9	– 8	– 7	– 6	– 5	– 4	– 3	– 2	– 1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Fig. 3 A partial number grid adapted from Grade 3 *Everyday Mathematics* (The University of Chicago School Mathematics Project, 2007)

The physical action prompted in [1] of moving to the right draws attention to the relationships across a row and to help a reader recognize that the numbers increase. Prompt [2] increases the complexity by challenging a reader to describe the result of shifting down a row. By suggesting no particular starting number, this prompt draws attention on the mathematical action "adding 10," allowing it to be generated through repeated shifts of adding 1 or by recognizing the change in the tens unit. Prompt [3] offers the possibility of connecting the mathematical action of adding 1 and the physical action from [1]. Prompt [4] increases the complexity since "adding 1" is no longer just moving one square to the right. Note that while a student may generate "add 10" for question [2], this mathematical action has not necessarily been performed, only recognized. Thus, prompt [5] then opens the possibility that a reader will connect the action of moving down a row with adding 10.

In addition to increasing in complexity to advance the mathematical story, mathematical action can result in the creation of a new mathematical character, offering a surprise for a reader. For example, following the start described in *My Pals are Here!* (Kheong et al., 2012), the text prompts the teacher to "hold 10 unit cubes in your hand. Ask pupils to count the number of cubes aloud. Remove 1 cube and get pupils to count the remaining cubes. Repeat this until there are no cubes left" (p. 6). Thus, for students, this action continues until, quite surprisingly, there are no cubes left. This sudden result opens the question "what do we call the quantity when there is nothing to be counted?" since zero as a quantity had not yet been named or otherwise referred to in the mathematical story.

3.3 Mathematical setting

With the mathematical characters and mathematical actions on these characters defined, it is reasonable to turn the attention to where and how these mathematical characters and action are manifested in the story. In terms of literature, a *setting* describes the space (or spaces) in which the literary characters are located. The choice of setting in a narrative both enables and constrains what is possible in a story and provides clues for a reader about what types of actions to expect (Bal, 2009).

Metaphorically, then, a *mathematical setting* is the manifestation of the content; that is, the mathematical representation. Mathematical representations create “spaces” in which the mathematical characters emerge and are acted upon. Perhaps the tasks in Figure 1 offer the most startling way in which mathematical setting is a critical component of a mathematical story. When the function $y = x^2 - 6x + 3$ in Task A is in symbolic form, the mathematical action available at that point (i.e., factoring) does not enable the roots to be found. However, when the mathematical setting is changed to a coordinate plane in Task C, the roots are evident (as x-intercepts) and able to be approximated. Changing the mathematical setting this way enables the story to advance so that a complicated relationship can be recognized. Just like literary stories, the mathematical setting thus enables certain characteristics of mathematical characters to become noticeable and thus potentially understood.

3.3.1 Curricular insight offered by mathematical setting. A study of mathematical settings in mathematical stories can help a mathematics educator recognize how a mathematical setting might offer either new challenges or new insights for a reader. For example, the number grid of Lesson 1-2 of *Everyday Mathematics* structures a numerical space and foregrounds certain relationships and properties of mathematical characters (i.e. place value) and mathematical actions (i.e., adding 10). If this lesson instead relied on a number line, the number 10 would likely not emerge as a central character as shifting 10 units to the right is not distinguishable from shifts of 9 or 11 units.

Mathematical settings are not restricted to those found on paper. For example, in Unit 2 of *My Pals are Here!* (Kheong et al., 2012), the mathematical characters (numbers) are found on a balance scale: a fulcrum with equally spaced numbers 1, 2, 3, etc. extending in both directions. In this mathematical setting, weights are used to show equivalence relationships. For example, hanging a weight at "3" and "4" to the left of the fulcrum balances with a weight at "7" on the right of the fulcrum, indicating that $3+4=7$. Within this mathematical setting, this mathematical action for addition is very different than in a different setting (for example, the joining of 3 linking cubes with 4 linking cubes). Hanging weights to create a balance draws attention to spatial locations and enables a potential *inequality* of quantities, two mathematical properties absent from the mathematical action of joining. This is not a suggestion that the practice of joining is problematic; rather, it is a demonstration of how the analysis of mathematical setting and its affording and constraining effect on the mathematical actions and characters of the mathematical story enables the critique of curriculum.

The notion of context and space used here includes all forms of representations in which a mathematical action occurs and mathematical characters are found. Certainly story problems construct a setting in which the mathematical characters are set and acted upon. In fact, in terms of a mathematical story, the most distinguishing story element of story problems may be its mathematical setting. For example, in the task "José had $76 in his bank account. He withdrew $29. How much money was in his bank account then?" (The University of Chicago School Mathematics Project, 2007, p. 125), a fictional bank is constructed in which the mathematical character $76 is acted upon. This imaginary space is a setting that similarly limits the potential mathematical actions based on the prior experiences of reader/actors with bank accounts.

When the mathematical setting is not explicit, a reader is left to construct his or her own setting which in part can explain some puzzling student responses. For example, in the *Everyday Math* lesson described earlier, a student may claim that 1 cannot be added to 20 because "adding one" has been connected to moving to the right in the number grid. Lockhart (2009) offers a more advanced example. A question such as "what is the sum of the angles of a triangle?" would evoke to most middle school mathematics teachers (and presumably students) the response "180°." However, Lockhart draws attention to the importance of the context:

> Mathematical structures, useful or not, are invented and developed within a problem context, and derive their meaning from that context. Sometimes we want one plus one to equal zero (as in so-called 'mod 2' arithmetic) and on the surface of a sphere the angles of a triangle add up to more than 180 degrees. There are no 'facts' per se; everything is relative and relational. *It is the story that matters, not just the ending*. (Lockhart, 2009, p. 17, italics added)

As Lockhart suggests, the setting of the mathematical story (whether it be two or three-dimensional, for example) influences its mathematical meaning.

3.4 Mathematical plot

In addition to placing characters and actions in settings, a literary story can temporally affect a reader as it unfolds in a manner referred to as its *plot*. The concept of plot is closely connected to story and in some discussions of narrative these words are used interchangeably. Indeed, plot is fundamentally linked to a sequence of events (through actions), since changing a sequence can have a strong effect on a reader. As Bal explains,

> Playing with sequential ordering is not just a literary convention; it is also a means of drawing attention to certain things, to emphasize, to bring about aesthetic or psychological effects, to show various interpretations of an event, to indicate the subtle difference between expectation and realization, and much else besides. (2009, p. 81)

Bal's (2009) reference to expectation and realization emphasizes the tension between what is guessed at (i.e., is anticipated) by a reader and what emerges as truth (i.e., is realized) at different moments along the sequence. As a reader reads a story, he or she asks questions about what is to come, extends patterns noticed throughout the story to make predictions, and looks for implicit and explicit interrelationships (or lack thereof) between the parts of the story. Thus, plot is defined as the tension between what Bal refers to as realization and expectation as the reader constructs the story structure; that is, what is known or believed by the reader and what is desired to be learned by the reader throughout the reading of the story. Consider, for example, how the description of the wizard at the beginning of *The Wizard of Oz* allows a reader to realize (i.e., come to know) that the wizard is able to do magic and therefore anticipate that the wizard will be able to help Dorothy go home by the end of the story. When Dorothy and her friends arrive in Oz and learn (i.e., realize) that the wizard is a fraud, this realization violates this expectation and allows for a reader's surprise (and therefore, a reader may feel that the plot has "turned"). Thus, the interaction of a reader's expectation and realization throughout a story explains how a sequence can evoke an aesthetic response because delaying realization offers the possibility of intrigue and wonder, while enabling expectation allows both surprise if an expectation is violated and relief and satisfaction when an anticipated result (such as the confirmation of "who done it") is met.

Given this framing, a *mathematical plot* describes the aesthetic response of a reader as he or she experiences a mathematical story, perceives its structure (and thus, look for order, find patterns, sense rhythm, etc.), and anticipates what is ahead (by wondering, imagining, asking questions). As a tension between the pursuit of mathematical ideas through inquiry and the revelation of information, it is the potential temporal dynamics of the story, that which encourages (or discourages) a reader to continue to advance through the mathematical story. This is not to say that all readers will find the same mathematical story aesthetically pleasurable, only that mathematical sequences can generate mathematical impulses that motivate a reader to read on.

3.4.1 Curricular insight offered by mathematical plot. By reading for the mathematical plot, the potential way in which a mathematical story provokes and maintains curiosity and wonder (or not) is foregrounded. If a mathematical story can be described as answering questions as soon as they are raised, then the lesson offers little for the reader to look forward to (except being asked more questions!). However, mathematical questions that are opened and sustained throughout a mathematical story can offer a reader suspense and wonder and motivate him or her to continue with the story.

The multiple sequencings of the tasks in Figure 1 offer insight into how mathematical plot can support the critique of curriculum by a teacher. When examined for how the information about the quadratic unfolds, the tensions available in each sequence can be distinguished. In the A-C-B sequence of tasks, an expectation set up from the inability to factor creates a tension once the graph reveals a contradiction. This sudden change, or "plot twist," is an invitation for a reader to generate mathematical inquiry, enabling him or her to enter Task B with a genuine interest in its outcome. When this strategy enables the roots to be identified, it shows that this strategy works in cases when factoring does not. With this mathematical story, the teacher can take advantage of the surprise in order to motivate the need for a new strategy. That is not to say that in the A-B-C sequence, a reader would not necessarily raise questions (e.g., why did the factoring strategy fail?), but since what follows does not offer support to answer this question, the inquiry is not supported. Using this conceptualization of plot, a teacher can recognize the irrelevance of Task C and use the potential mathematical question raised in Task B to generate a new ending of the sequence (such as focusing on why the traditional factoring strategy does not reveal irrational roots or focusing on how irrational roots can be used to create factors of the quadratic).

4 Coordinating the mathematical story elements to make sense of curriculum

In this paper, I began with an argument that a new metaphor of mathematics curriculum is needed in order to support the recognition of interrelationships between and across curricular parts. Although mathematical objects, processes, and representations have long been studied independently for their important roles in mathematical work, it is the *coordination* of all of these aspects of mathematics together in a structural conceptual metaphor (Lakoff & Johnson, 1980) that enables one to recognize how the parts work together to make (or not make) a coherent and aesthetic whole. For example, the mathematical objects of curriculum can certainly be recognized without the story framework, but within the framework, their temporal changes, interactions, interdependence, as well as the influence of setting and action on recognizable characteristics can be recognized and evaluated. Likewise, this framework draws attention to shifts in representations and the changing complexity of procedures.

I should note that while mathematical character, action, and setting directly correspond to a single mathematical phenomenon, the conceptualization of a mathematical plot has no single corresponding mathematical correlate. In contrast, mathematical plot interrelates both logic (making sense of what is happening in a story) and aesthetic (a reader's response to the story, such as surprise), mathematical qualities that are usually considered separately and thought to be independent. In perceiving a plot of a mathematical story, a reader can attend to both its logic and aesthetic as co-dependent forces. That is, the logical sense a reader constructs and seeks while reading a mathematical sequence alters his or her aesthetic experience, and vice versa.

5 Implications for mathematics curriculum

As curriculum (broadly) affects nearly every aspect of schooling (from planning to enacting to assessing), the metaphor *curriculum as story* could support a renaissance in mathematics curriculum in which the design of curriculum takes into account both logic and aesthetic. The notion of *mathematical plot* offers a new vision of what mathematics curriculum can be: an enticing mathematical story that provokes curiosity and a reader's desire for more. This metaphorical lens supports new questions for teachers to consider during curricular planning, such as what types of mathematical stories am I offering my students? How might changing the mathematical characters impact the story? What role(s) does the action play? And how can this sequence build excitement or suspense for what is to come?

Attending to the temporality of character might help a teacher predict how students (i.e., non-experts who are typically limited to the mathematical story enacted in the class and as represented in a textbook) could interpret the mathematics text. This framework could also enable a teacher to recognize opportunities for tension within the story between what is temporally known and can allow a teacher to take advantage of opportunities during the lesson to capitalize on aesthetic opportunities. As a way of reading curriculum, it may prevent expert blindspot and potentially reveals to teachers unintended discontinuities in the mathematical story and supports their remedy.

Just as a literary story can appeal to some readers and not to others, this framework assumes that mathematical stories will be interesting to some students and not others. However, as is evidenced by the enthusiasm shown for literature such as *Harry Potter*, there are some stories that have wider appeal than others. With the recognition of different potential genres of mathematical stories (such as the action-heavy addition story or the character study of $x^2 - 6x + 3$) this naturally provokes questions such as what types of mathematical stories attract the attention of struggling students? It also enables the consideration of new questions about mathematics curriculum, such as how do some mathematical sequences generate mathematical curiosity, provoke interest, or stimulate imagination and others not? Are particular types of mathematical stories more memorable for students, and do these lead to greater retention? If the mathematical plots that have wide student appeal are carefully analyzed, what is learned can be shared with teachers and other curriculum designers to improve mathematics curricula for the benefit of those with the most to gain: our students.

Acknowledgements: I owe a deep gratitude to Drs. Glenda Lappan and Nathalie Sinclair for their support and advice throughout the development of this framework. In addition, the strategic recommendations of Aaron Brakoniecki, Kate Johnson, and Marianna Levin, made this paper even better.

6 References

Andrà, C., & Sinclair, N. (2012). Mathematics lessons as stories. Presented at the 18th MAVI (Mathematical Views) Conference, Helsinki, Finland. Retrieved from http://blogs.helsinki.fi/mavi-2012/files/2012/09/2_AndraSinclair_def-on-line.doc

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is – or might be – the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher*, *25*, 6–8, 14.

Bal, M. (1986). Tell-tale theories. *Poetics Today*, *7*(3), 555–564. http://doi.org/10.2307/1772511

Bal, M. (2009). *Narratology: Introduction to the theory of narrative*. (C. Van Boheemen, Trans.) (3rd ed.). Toronto, CA: University of Toronto Press.

Borasi, R., & Brown, S. I. (1985). A "novel" approach to texts. *For the Learning of Mathematics*, *5*(1), 21–23. http://doi.org/10.2307/40247872

Breyfogle, M. L., Roth McDuffie, A., & Wohlhuter, K. A. (2010). Developing curricular reasoning for grades pre-K-12 mathematics instruction. In B. Reys, R. E. Reys, & R. Rubenstein (Eds.), *Mathematics curriculum : Issues, trends, and future directions* (pp. 307–320). Reston VA: National Council of Teachers of Mathematics.

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). New York: Routledge.

Choppin, J. M. (2009). Curriculum-context knowledge: Teacher learning From successive enactments of a standards-based mathematics curriculum. *Curriculum Inquiry*, *39*(2), 287–320.

Dietiker, L. (2013). Mathematics texts as narrative: Rethinking curriculum. *For the Learning of Mathematics*, *33*(3), 14–19.

Dietiker, L. (2015). What mathematics education can learn from art: The assumptions, values, and vision of mathematics education. *Journal of Education*, *195*(1), 1–10.

Dietiker, L., Kysh, J., Sallee, T., Hamada, L., & Hoey, B. (2012). *Core connections: Algebra*. Sacramento, CA: CPM Educational Program.

Dougherty, B. J. (2003). Voyaging from theory to practice in learning: A view from Hawaii. In *International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17–32). Honolulu, HI: International Group for the Psychology of Mathematics Education. Retrieved from http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED500874

Drake, C. (2006). Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education*, *9*(6), 579–608.

Egan, K. (1988). *Teaching as story telling: An alternative approach to teaching and the curriculum*. Chicago, IL: Routledge.

Gadanidis, G., & Hoogland, C. (2003). The aesthetic in mathematics as story. *Canadian Journal of Science, Mathematics and Technology Education*, *3*(4), 487–498.

Healy, L., & Sinclair, N. (2007). If this is our mathematics, what are our stories? *International Journal of Computers for Mathematical Learning*, *12*, 3–21.

Kheong, F. H., Ramakrishnan, C., & Wah, B. L. P. (2012). *My pals are here! Maths* (2nd ed., Vol. 1A). Singapore: Marshall Cavendish.

Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. University of Chicago Press.

Lockhart, P. (2009). *A mathematician's lament*. New York: Bellevue Literary Press.

Nathan, M. J., & Petrosino, A. (2003). Expert blind spot among preservice teachers. *American Educational Research Journal*, *40*(4), 905–928.

Netz, R. (2005). The aesthetics of mathematics: A study. In P. Mancosu, K. F. Jørgensen, & S. A. Pedersen (Eds.), *Visualization, explanation and reasoning styles in mathematics* (pp. 251–293). Netherlands: Springer.

Nicol, C. C., & Crespo, S. M. (2006). Learning to teach with mathematics textbooks: How preservice teachers interpret and use curriculum materials. *Educational Studies in Mathematics*, *62*(3), 331–355.

Pepin, B., Gueudet, G., & Trouche, L. (2013a). Investigating textbooks as crucial interfaces between culture, policy and teacher curricular practice: two contrasted case studies in France and Norway. *ZDM*, *45*(5), 685–698. http://doi.org/10.1007/s11858-013-0526-2

Pepin, B., Gueudet, G., & Trouche, L. (2013b). Re-sourcing teachers' work and interactions: a collective perspective on resources, their use and transformation. *ZDM*, *45*(7), 929–943. http://doi.org/10.1007/s11858-013-0534-2

Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry*, *29*(3), 315–342. http://doi.org/10.1111/0362-6784.00130

Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education*, *35*(5), 352–388.

Rosenblatt, L. M. (1988). *Writing and reading: The transactional theory* (Technical Report No. 416). University of Illinois at Urbana-Champaign: Reading Research and Education Center.

Rosenblatt, L. M. (1994). *The reader, the text, the poem: The transactional theory of the literary work*. Carbondale and Edwardsville: Southern Illinois University Press.

Sherin, M. G., & Drake, C. (2009). Curriculum strategy framework: investigating patterns in teachers' use of a reform-based elementary mathematics curriculum. *Journal of Curriculum Studies*, *41*(4), 467–500. http://doi.org/10.1080/00220270802696115

Sinclair, N. (2005). Chorus, colour, and contrariness in school mathematics. *THEN: Journal*, *1*(1). Retrieved from http://thenjournal.org/feature/80/

Tarr, J. E., Chávez, Ó., Reys, R. E., & Reys, B. J. (2006). From the written to the enacted curricula: The intermediary role of middle school mathematics teachers in shaping students' opportunity to learn. *School Science and Mathematics*, *106*(4), 191–201. http://doi.org/10.1111/j.1949-8594.2006.tb18075.x

The University of Chicago School Mathematics Project. (2007). *Everyday mathematics* (3rd ed.). Chicago, IL: Wright Group/McGraw-Hill.

Thomas, R. S. D. (2002). Mathematics and narrative. *The Mathematical Intelligencer*, *24*(3), 43–46. http://doi.org/10.1007/BF03024731