1926

An analysis of several recent drill services in arithmetic or an investigation into the amount of drill given to the various combinations - according to their relative difficulty

Boston University

SCHOOL OF EDUCATION.

T H E S I S

An Analysis of Several Recent Drill Services in Arithmetic

or

An Investigation into the Amount of Drill Given to the Various Combinations - According to Their Relative Difficulty.

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There are one hundred primary or basic combinations each in addition, subtraction, and multiplication; and ninety in division. They are, in addition, the following direct combinations with their reverses:

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0
	1	2	3	4	5	5	5	5	5
	1	1	1	1	1	1	1	1	1
		2	3	4	5	6	7	8	9
		2	2	2	2	2	2	2	2
			3	4	5	6	7	8	9
			3	3	3	3	3	3	3
				4	5	6	7	8	9
				4	4	4	4	4	4
					5	6	7	8	9
					5	5	5	5	5
						6	7	8	9
						6	6	6	6
							7	8	9
							7	7	7
								8	9
								8	8
									9
									9

In subtraction they are these:

18-9	17-8	16-7	15-6	14-5
17-9	16-8	15-7	14-6	13-5
16-9	15-8	14-7	13-6	12-5
15-9	14-8	13-7	12-6	11-5
14-9	13-8	12-7	11-6	10-5
13-9	12-8	11-7	10-6	9-5
12-9	11-8	10-7	9-6	8-5
11-9	10-8	9-7	8-6	7-5
10-9	9-8	8-7	7-6	6-5
9-9	8-8	7-7	6-6	5-5
13-4	12-3	11-2	10-1	9-0
12-4	11-3	10-2	9-1	8-0
11-4	10-3	9-2	8-1	7-0
10-4	9-3	8-2	7-1	6-0
9-4	8-3	7-2	6-1	5-0
8-4	7-3	6-2	5-1	4-0
7-4	6-3	5-2	4-1	3-0
6-4	5-3	4-2	3-1	2-0
5-4	4-3	3-2	2-1	1-0
4-4	3-3	2-2	1-1	0-0

In multiplication, they are the following combinations with their reverses:

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0
	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1
		2	3	4	5	6	7	8	9
		2	2	2	2	2	2	2	2
			3	4	5	6	7	8	9
			3	3	3	3	3	3	3
				4	5	6	7	8	9
				4	4	4	4	4	4
					5	6	7	8	9
					5	5	5	5	5
						6	7	8	9
						6	6	6	6
							7	8	9
							7	7	7
								8	9
								8	8
									9
									9

And in division:

0÷1	1÷1	2÷1	3÷1	4÷1	5÷1	6÷1	7÷1	8÷1	9÷1
0÷2	2÷2	4÷2	6÷2	8÷2	10÷2	12÷2	14÷2	16÷2	18÷2
0÷3	3÷3	6÷3	9÷3	12÷3	15÷3	18÷3	21÷3	24÷3	27÷3
0÷4	4÷4	8÷4	12÷4	16÷4	20÷4	24÷4	28÷4	32÷4	36÷4
0÷5	5÷5	10÷5	15÷5	20÷5	25÷5	30÷5	35÷5	40÷5	45÷5
0÷6	6÷6	12÷6	18÷6	24÷6	30÷6	36÷6	42÷6	48÷6	54÷6
0÷7	7÷7	14÷7	21÷7	28÷7	35÷7	42÷7	49÷7	56÷7	63÷7
0÷8	8÷8	16÷8	24÷8	32÷8	40÷8	48÷8	56÷8	64÷8	72÷8
0÷9	9÷9	18÷9	27÷9	36÷9	45÷9	54÷9	63÷9	72÷9	81÷9

Note that there are no "zero " divisors, hence there are ninety and NOT one hundred basic combinations in division as in the other three operations.

But besides these simple combinations, there are yet others; all told there are 1680 combinations which may be classified in this wise (1):

Simple addition	100 comb's
Higher decade addition (up to and including all cases where 9 is to be carried)	755 "
Simple subtraction	100 "
The subtraction involved in short division	175 "
Simple multiplication	100 "
Short division	450 "
Total	1680 "

(1) W. J. Osburn; Corrective Arithmetic, Pg. 11.

It may be noted here that these combinations must be absolutely mastered by the child before he has passed many years in school; Osburn, before quoted, would say by the end of the fourth year. The learning of this large number of facts - and simply as facts - is a very great task, indeed, but there is no alternative. Of course the child must know that the symbol 5 represents 5 objects of some kind; he must know and from a varied experience that the putting together, say, of 3 apples and 2 apples gives 5 apples; or 4 blocks and 1 block make a total of 5 blocks, and so on.

But the combinations are not to be learned this way; instead, they are to be learned mechanically and as facts and incidentally let it be said, according to best thought today, not in table form. Tables are valuable in so far as they are used as means of remembering what has been learned; they are useless, perhaps even worse, as methods of learning.

In the past, we have gone entirely on the assumption that through the "transfer of training", learning for instance, that 6+5 is 11, the reverse combination 5+6 and the various corresponding decade combinations, as 26+5, 76+5, 96+5, etc, were all taken care of at the same stroke. This of course as we know now is not the case; each and every combination must be taken care of separately and learned independently.

Of the simple combinations in each of the four operations, given in full on pages 1,2, and 3 herein, much study has been given recently pertaining to their relative difficulty. An early study of this sort was made by H. V. Holloway at the University of Pennsylvania. A more recent investigation made by Frank L. Clapp of the University of Wisconsin is per-

haps more thorough and reliable. It will be described briefly in the pages that follow; in fact the results of this Clapp Study have been made the basis in determining the conclusions of the present thesis.

This study of Prof. Clapp (2) which was possibly even more comprehensive, so it is said, than the original investigation of Ayres in the field of spelling, tested more than 10,000 school children in various and widely separated sections of the United States. We cannot go into detail but briefly it may be said that among other things discovered were these;

1.) That certain combinations are much more difficult than others. For instance in addition, it is shown that the following combinations caused 43% of all mistakes:

5+5	6+5	7+5	8+5	9+5
5+6	6+6	7+6	8+6	9+6
5+7	6+7	7+7	8+7	9+7
5+8	6+8	7+8	8+8	9+8
5+9	6+9	7+9	8+9	9+9

and these are only twenty-five of the one hundred combinations in addition. In these tests it was shown that twenty times

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(2) The best description of the Clapp Study is found in : "The Number Combinations: Their Relative Difficulty and the Frequency of their Appearance in Text Books", by Frank L. Clapp. It is Bulletin #2 of the Bureau of Educatioal Research, University of Wisconsin, Madison, Wisconsin.

as many mistakes were made on some of the combinations as on others.

2.) Many more mistakes were made on certain combinations when they were presented in examples than when they were presented singly, while of others the reverse was true.

Based upon the results in these two regards, Clapp has prepared "Orders of Difficulty" of the combinations in each of the processes. The first Order of Difficulty that he gives us is termed Order A; the other Order B. In determining the former, pupils were tested on the combinations presented singly; while for the latter, the combinations were presented in examples. The A tests were timed and designed to measure the extent to which the associations represented by the various combinations are automatic in the minds of pupils. The B Tests involved no speed element and were designed to measure the difficulty of the combinations when pupils are free to determine the answers by any method they prefer. The results of the A Tests may be said to indicate the learning or teaching difficulty of the combinations while those of the B Tests represent their functional difficulty. Charts I, II, III, and IV on pages 9, 10, 11,& 12 of this work give in full both orders in all of the four processes.

The arrangement or line-up in the two Orders are quite different. One is surprised at noting so many combinations found in the first quartile of the one, appearing in the third or fourth quartile of the other. One very noticeable example is 2÷2. Presented singly, it is rated as the most difficult of all the ninety combinations in division. That is, it is found as the first combination of the first quartile of Order A where the form is in decreasing order of

difficulty. But this same combination, 2÷2, presented in examples, is one of the easiest; that is, it is found in Order B at nearly the end of the third quartile.

Not only is 2÷2 found in the first quartile (the most difficult) of Order A, but so also are:

9÷9	1÷1	8÷8	3÷3
6÷6	7÷7	5÷5	4÷4

That is to say, every digit divided by itself is among the twenty-five most difficult division combinations when presented singly. It seems that there is a marked tendency to respond "zero" to any number divided by itself. But these same combinations presented in examples do not give the same difficulty. None of them appear in either the first or second quartiles of the B Order.

In addition, the combination 2+9 in examples is one of the most difficult. It is found in Quartile I of Order B. But presented singly it is one of the less difficult and is found in Quartile IV of Order A. It is probable that if the child, having 2+9 presented singly, finds it bothering, he immediately and perhaps almost unconsciously reverses the order of the two addends thus having 9+2, which to many children would be easier. But in examples where one addend is most likely to be in decade form, as 32+9, reversing would not make the situation any easier.

It must be said, at least parenthetically at this point, that teachers often make the mistake of analyzing the work of pupils only when the combinations are presented singly and then giving attention to those combinations that cause

difficulty as indicated by such analysis. It is quite as important, if not more so, to analyze the work of pupils when the combinations are presented in examples and then give the necessary attention to those combinations that are giving difficulty as revealed by this analysis. Giving drill only to combinations which are proving troublesome singly will bring but little if any improvement in written work which of course makes up a large part of actual school and everyday arithmetic.

Chart I

Clapp Order of Difficulty

Order A — Addition — Order B

Learning Difficulty — Functional Difficulty

I	II	III	IV	I	II	III	IV
8+5	9+0	0+7	7+1	4+9	1+3	4+6	3+2
7+9	2+6	0+1	2+9	9+6	8+7	2+7	1+0
5+8	9+3	7+2	2+5	6+7	5+7	6+3	6+1
9+7	0+6	1+9	2+8	8+6	7+8	6+5	7+1
6+8	6+5	0+9	4+4	6+9	2+2	1+2	0+7
6+9	3+8	8+1	4+1	3+9	3+6	5+3	0+4
5+7	3+4	6+2	3+1	5+8	8+3	2+5	0+3
7+8	3+9	0+4	4+0	7+9	7+4	2+4	4+2
8+7	2+3	3+6	1+8	5+9	9+4	3+3	2+0
9+6	3+5	0+2	9+9	8+5	1+9	5+6	3+0
5+9	6+3	2+4	7+7	9+9	4+4	7+3	0+5
8+9	7+3	3+0	2+0	7+5	1+8	5+5	4+0
8+6	2+7	4+5	6+1	4+8	2+8	4+3	0+8
4+7	8+4	0+8	5+4	2+9	3+7	6+0	8+2
7+5	4+8	6+0	3+3	7+6	2+6	8+4	5+2
4+9	8+0	8+3	1+1	3+8	3+5	5+4	5+0
9+5	1+0	8+2	9+2	7+7	9+5	1+1	4+1
9+4	5+2	6+4	8+8	6+6	1+4	8+1	7+2
6+7	4+2	1+4	1+3	4+7	1+5	6+4	2+1
5+6	1+2	9+1	1+6	6+8	1+7	2+3	9+1
4+6	5+3	5+0	1+7	9+7	9+2	0+9	5+1
7+6	0+3	6+6	2+1	8+8	4+5	0+6	0+2
7+4	0+5	3+2	2+2	9+3	3+4	7+2	9+0
9+8	5+1	4+3	5+5	9+8	6+2	0+1	8+0
3+7	7+0	1+5	0+0	8+9	1+6	3+1	0+0

Chart II

Clapp Order of Difficulty

Order A — Subtraction — Order B

Learning Difficulty — Functional Difficulty

I	II	III	IV	I	II	III	IV
14-9	11-7	3-0	6-1	11-3	6-3	10-1	17-8
13-4	13-6	5-0	2-1	16-9	6-6	9-2	8-5
16-9	15-8	9-6	5-2	8-4	13-6	9-8	18-9
14-5	8-5	8-2	6-3	11-4	2-2	9-4	7-7
17-9	11-5	10-4	3-1	14-9	8-8	10-3	3-1
15-9	11-8	8-8	7-1	7-0	11-5	13-7	5-2
17-8	9-2	9-3	4-3	9-1	6-4	12-7	1-1
15-7	10-3	7-3	5-5	15-9	10-7	8-7	5-3
13-9	10-9	6-0	8-4	9-0	12-5	7-6	10-4
13-5	14-7	9-0	6-5	11-2	8-1	6-1	7-4
16-7	12-4	7-2	9-8	11-7	14-6	3-0	4-0
13-7	10-8	12-6	4-4	15-8	5-1	2-0	4-2
13-8	7-4	10-7	9-1	13-5	11-8	14-8	8-0
11-3	12-8	8-6	9-9	16-7	14-7	12-8	5-5
14-6	1-0	11-2	5-3	9-9	7-1	12-3	11-6
12-3	8-3	10-1	5-4	14-5	9-6	6-2	10-5
15-6	7-0	6-2	1-1	13-8	4-1	8-3	3-3
14-8	12-9	10-2	8-1	10-2	15-6	12-9	4-3
12-5	10-5	7-6	4-2	13-9	7-5	16-8	4-4
12-7	10-6	9-5	6-6	13-4	2-1	9-7	10-6
11-4	8-0	8-7	3-3	5-0	7-3	9-5	3-2
11-6	2-0	4-1	3-2	5-4	8-2	8-6	10-9
16-8	9-7	2-2	7-7	6-0	15-7	6-5	10-8
18-9	9-4	7-5	5-1	9-3	7-2	1-0	12-6
11-9	4-0	6-4	0-0	17-9	12-4	11-9	0-0

Chart III

Clapp Order of Difficulty

Order A — Multiplication — Order B

Learning Difficulty — Functional Difficulty

I	II	III	IV	I	II	III	IV
7x0	8x6	7x3	2x7	0x2	8x0	4x6	2x1
0x5	4x9	9x9	2x3	9x4	7x3	4x4	2x2
0x7	4x7	7x5	4x1	9x7	7x5	4x7	6x0
0x1	1x1	4x5	4x4	8x6	6x7	3x1	3x3
4x0	8x9	3x4	5x2	8x8	9x6	6x1	8x2
0x8	9x4	2x4	7x1	7x6	6x6	3x6	8x1
0x2	7x4	4x2	8x2	0x5	5x1	2x0	1x0
0x4	7x6	5x3	3x2	0x7	6x4	6x3	3x4
2x0	8x8	6x6	3x1	6x9	7x7	5x9	1x2
0x6	8x5	6x3	6x1	7x8	7x1	2x8	3x8
9x0	8x4	6x4	1x8	4x8	4x0	9x1	4x5
8x0	9x8	1x2	5x4	0x8	0x6	5x7	1x9
0x3	9x3	6x5	2x9	7x4	9x9	3x9	1x8
6x0	5x6	2x6	2x1	9x0	9x3	4x3	2x6
0x9	6x7	3x6	1x6	8x7	3x0	3x7	1x3
3x0	4x8	8x3	1x5	7x9	4x9	2x4	7x2
9x7	3x8	6x2	2x2	8x4	7x0	6x5	1x5
5x0	4x6	3x5	9x2	6x2	9x5	5x3	2x5
1x0	5x8	8x1	2x8	0x9	8x5	5x0	3x5
8x7	3x9	1x9	1x4	9x8	0x4	1x1	2x7
9x6	7x7	4x3	2x5	0x3	9x2	4x1	1x6
6x9	3x7	5x5	1x3	6x8	3x2	5x4	1x7
7x9	5x7	3x3	5x1	0x1	5x5	2x3	1x4
7x8	5x9	9x1	1x7	8x9	5x6	2x9	5x2
6x8	9x5	7x2	0x0	8x3	5x8	4x2	0x0

Chart IV

Clapp Order of Difficulty

Order A — Division — Order B

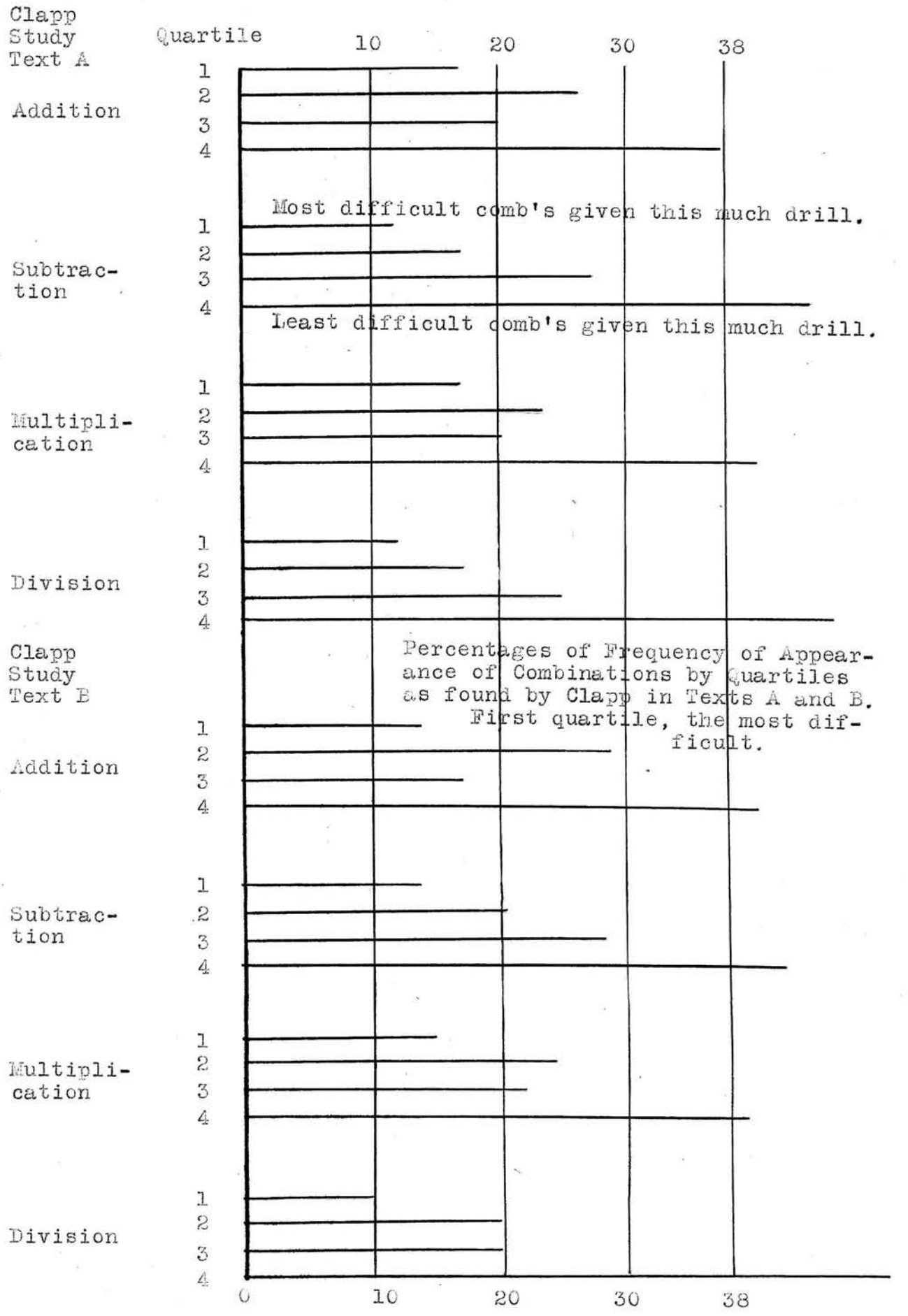
Learning Difficulty — Functional Difficulty

I	II	III	IV	I	II	III	IV
2÷2	0÷4	18÷2	40÷5	49÷7	42÷7	12÷3	9÷1
9÷9	0÷6	8÷4	21÷7	54÷6	54÷9	12÷6	4÷4
28÷4	42÷7	56÷7	21÷3	45÷9	30÷6	32÷8	0÷3
1÷1	36÷9	45÷9	30÷5	27÷3	28÷7	8÷4	3÷3
36÷4	28÷7	30÷6	12÷3	56÷7	24÷3	9÷3	5÷5
8÷8	63÷7	3÷1	15÷5	28÷4	27÷9	12÷2	0÷4
3÷3	27÷3	35÷7	12÷6	24÷4	64÷8	16÷8	9÷9
6÷6	24÷8	18÷3	36÷6	36÷6	81÷9	10÷5	0÷7
56÷8	48÷6	8÷1	16÷2	48÷6	36÷9	4÷2	0÷6
54÷6	24÷4	45÷5	14÷7	21÷7	18÷6	24÷6	0÷5
7÷7	2÷1	42÷6	20÷5	18÷2	63÷7	5÷1	6÷1
5÷5	49÷7	18÷9	8÷2	21÷3	42÷6	15÷5	0÷8
4÷4	40÷8	16÷8	12÷2	36÷4	56÷8	18÷9	1÷1
63÷9	72÷8	4÷1	15÷3	72÷9	16÷2	6÷6	0÷9
0÷5	18÷6	20÷4	14÷2	63÷9	24÷8	20÷5	8÷1
0÷9	24÷3	6÷1	10÷5	40÷8	72÷8	2÷2	2÷1
54÷9	72÷9	32÷8	6÷2	32÷4	16÷4	10÷2	0÷1
0÷2	9÷1	35÷5	81÷9	35÷7	20÷4	6÷3	8÷2
0÷7	24÷6	27÷9	9÷3	35÷5	30÷5	40÷5	0÷2
0÷8	0÷1	5÷1	10÷2	18÷3	14÷7	8÷8	4÷1
0÷3	6÷3	16÷4	25÷5	15÷3	14÷2	7÷7	7÷1
64÷8	32÷4	7÷1	4÷2	45÷5	25÷5	12÷4	3÷1
48÷8		12÷4		48÷8		6÷2	

Having the Orders of Difficulty set up (See Charts I, II, III, and IV on pages 9, 10, 11, and 12), Prof. Clapp next proceeded to analyze two widely used arithmetic texts to determine whether or not the typical text book in use up to the time had not given most drill to the less difficult combinations and least drill to the more difficult combinations. Chart V on the next page shows graphically his findings which were quite as surmised. For instance, in one text, that he designates as Text B - in addition - the percentages of frequency of appearance of the combinations by quartiles were as follows:

Quartile	I	14
	II	29
	III	17
	IV	40

This means that in this particular text the twenty-five most difficult combinations appeared only fourteen times to every forty times that the twenty-five least difficult combinations appeared. This is typical, as shown in this same chart, of the other three operations of this same text and also of the other book analyzed and referred to as Text A. In every case in all of the four operations of both texts, a great deal less drill is given to the more difficult combinations.


Clapp Study Text A
Quartile
10
20
30
38
1
2
Addition
3
4
Most difficult comb's given this much drill.
1
2
Subtrac-
tion
3
4
Least difficult comb's given this much drill.
1
2
Multipli-
cation
3
4
1
2
Division
3
4
Clapp Study Text B
Percentages of Frequency of Appearance of Combinations by Quartiles as found by Clapp in Texts A and B.
First quartile, the most difficult.
1
2
Addition
3
4
1
2
Subtrac-
tion
3
4
1
2
Multipli-
cation
3
4
1
2
Division
3
4
0
10
20
30
38

Having the findings of these analyses of typical texts widely used in this country, it has been thought desirable and worthwhile to analyze in the same manner several of the drill services in arithmetic recently offered by leading publishers. Such is the undertaking of the present study. Drill services by Hillegas, Osburn, and a series by Clapp, himself, are the objects; all of them bearing "copyrights" of either 1925 or 1926. These latest Services are coming to us with many claims made by their makers as meeting the demands of modern pedagogical and psychological research.
Are these texts really giving most practice and drill to the more difficult combinations, or do they fail in this respect as did so grossly those analyzed by Clapp? This is the question to which we seek an answer.

It is of course the contention of the present study that the work of Clapp, Holloway, and others makes it quite certain that there is a real intrinsic difference in difficulty in the combinations both as to learning or teaching, and functionally. And it is with the Clapp Order of Difficulty of the Combinations that we are making the comparisons which are so fundamental to our conclusions.

In analyzing a text every combination in the entire book must be "rounded-up", so to speak, and tabulated. In an example in multiplication, as will be shown in detail presently, not only are there the several multiplication combinations, but also numerous combinations in addition coming from the "carrying", and from the adding of the partial products - if the multiplier is of two or more places. One example in long division will give combinations in addition, subtraction,

and multiplication, as well as division. Thus they soon reach up into the thousands; in fact the present study has involved the tabulating, checking, plotting, etc. of a grand total of 57,730 combinations. Of these the Hillegas Book furnished 9,069, the three books of the Osburn Series 11,484, and the two books of the Clapp Series 37,177.

Further on, but as an illustration only, a complete set of the combinations in one of the operations and from one of the Services, will be included. See Chart VI.

Procedure of Analyzing Examples into Their Various Combinations.

In this example in addition the following combinations are obtained:

```
  2 7
  4 5
1 6 9
-----
2 4 1
```

9 + 5

1 4 + 7

2 + 6

8 + 4

1 2 + 2

1 + 1

This example in subtraction gives the combinations:

```
9 0 2 5
5 6 1 8
-------
3 4 0 7
```

1 5 - 8

1 - 1

1 0 - 6

8 - 5

We have the following combinations from this example in multiplication:

```
    3 7 0 9
        4 5
    -------
  1 8 5 4 5
1 4 8 3 6
---------
1 6 6 9 0 5
```

5 x 9
5 x 0
5 x 7
5 x 3
4 x 9
4 x 0
4 x 7
4 x 3

0 + 4
1 5 + 3
0 + 3
1 2 + 2
6 + 4

1 + 3
4 + 5
8 + 8
1 + 4
5 + 1

In analyzing this example in long division, we obtain combinations as follows:

```
      4 8 5
   -------
41 ) 1 9 8 9 6
     1 6 4
     -----
       3 4 9
       3 2 8
       -----
         2 1 6
         2 0 5
         -----
           1 1
```

1 9 ÷ 4; 4 x 1 and 4 x 4; 8 - 4 and 9 - 6

3 4 ÷ 4; 8 x 1 and 8 x 4; 8 - 8 and 4 - 2

2 0 ÷ 4; 5 x 1 and 5 x 4; 6 - 5 and 1 - 0

Note that subtraction combinations giving "zero" in long division are not included. This is following the Clapp procedure in his analyses.

As said before, in this study we are attempting to determine whether or not certain of the latest Drill Services in Arithmetic are giving more drill and practice to the most difficult combinations and less to the least difficult combinations. It has been shown already that the typical text book even up to the present has been notoriously deficient in this respect. The drill Services being considered in these pages are the following and in this order:

1. Osburn — Grade II
2. Osburn — Grade III
3. Osburn — Grade IV
4. Osburn — Grades II, III, and IV (combined)

Published by Houghton Mifflin Co.

5. Clapp — Grade IV
6. Clapp — Grade V
7. Clapp — Grades IV and V (combined)

Published by Silver Burdett & Co.

8. Hillegas (Horace Mann Supplementary Arithmetic)

Published by J. B. Lippincott Co.

In the results tabulated and graphed in Charts LXXX, LXXXI, LXXXII, and LXXXIII, are also given the results of the Clapp Analyses of the two texts before mentioned and known as Texts A and B. By means of these charts easy comparison may be made between the findings of the Clapp Study and those of the present one.

It would serve no purpose to include here the actual 57,000 and more combinations, or any large part of them, that go to make up the numerous examples of these several Services. However we are giving the combinations in addition of the Osburn Book for the third grade as an example. See Chart VI. An attempt will be made then to carry through the work in succesive steps by charts that follow that of # 6, in order to make clear to the reader the procedure followed in this study in its entirety.

Chart VI.

Addition Combinations - from Osburn III.

(But one page of such included here.)

2+5	1+8	8+4	8+6	5+8	1+7	5+9	4+6	26+5	5+6
5 3	8 1	6 7	8 3	9 8	8 9	9 4	1 8	36 6	1 5
4 1	0 9	8 9	9 5	6 8	4 6	1 8	9 4	86 7	6 7
6 3	3 2	8 6	7 7	3 8	1 7	9 1	9 7	63 8	7 4
0 4	0 0	8 3	9 9	7 7	8 6	3 7	1 7	79 2	2 8
1 6	3 6	6 9	6 5	4 6	5 6	1 4	8 7	64 6	9 6
2 7	4 3	7 8	8 9	7 6	1 6	5 9	5 9	73 9	2 6
4 0	0 2	9 4	9 2	8 4	7 8	6 8	1 5	15 6	8 6
2 2	4 0	5 8	7 8	9 3	4 6	1 8	6 6	46 4	8 9
1 5	4 5	3 8	6 9	5 8	1 3	9 6	7 5	81 9	2 9
5 0	1 6	5 7	3 9	9 7	4 8	8 3	1 7	85 7	3 9
7 1	7 2	7 6	8 5	7 7	7 3	1 8	8 5	27 8	1 6
4 2	1 7	8 8	5 5	6 6	1 5	9 3	4 8	37 5	7 5
6 2	8 1	9 7	9 6	9 7	6 9	7 3	1 3	28 6	9 9
3 0	1 8	6 5	4 8	6 4	4 9	1 5	4 6	92 8	2 7
0 7	9 0	3 9	7 5	7 8	1 2	6 4	7 3	17 7	9 5
5 1	2 3	3 7	5 8	8 8	3 9	4 7	1 4	35 5	5 9
6 0	5 4	4 8	9 2	5 8	4 9	1 8	5 6	98 9	2 6
1 2	0 3	2 8	6 5	4 7	1 2	9 9	9 3	69 8	8 4
5 4	3 0	9 2	8 9	6 5	3 9	7 7	1 8	55 6	1 9
1 7	3 5	5 9	5 5	9 6	4 9	1 5	9 7	72 8	8 9
0 8	1 3	7 9	9 5	3 9	1 7	6 7	9 2	98 5	2 7
0 1	4 1	1 9	4 7	4 8	8 3	6 7	1 5	46 9	9 5
2 1	5 4	9 6	4 6	9 7	7 4	1 2	6 6	39 7	9 8
5 2	1 4	7 5	4 8	6 9	1 8	3 7	4 6	78 9	2 7
7 2	5 3	8 2	5 9	8 4	9 2	3 9	1 3	85 8	9 9
9 0	3 5	4 6	5 6	1 5	4 8	1 3	4 8	26 5	9 7
0 3	8 0	2 9	4 9	6 5	1 4	4 9	9 9	78 4	2 7
0 6	0 2	8 7	5 9	7 6	5 7	7 7	1 5	12 8	9 6
2 0	2 6	9 9	9 6	1 8	9 4	1 8	6 9	37 4	9 8
2 3	8 0	6 6	8 7	9 8	1 5	9 7	5 8	63 7	2 9
2 6	4 4	4 7	3 8	7 9	6 8	7 3	1 4	96 7	8 4
4 4	8 1	9 8	8 6	1 7	5 6	1 2	5 6	72 9	2 4
3 5	0 3	9 1	3 8	8 8	1 5	3 9	4 7	53 8	6 6
1 4	3 0	6 4	3 7	7 8	9 2	6 6	1 5	81 9	3 9
7 0	3 6	7 4	9 4	6 5	1 5	1 5	6 5	54 7	1 9
1 3	2 0	6 8	6 6	1 4	6 8	5 5	6 5	55 8	8 8
3 4	2 6	7 7	7 4	5 7	3 9	1 5	1 7	13 8	1 8
1 1	8 1	9 3	8 7	5 8	1 2	6 6	8 6	47 3	9 9
3 3	0 1	7 3	7 3	1 4	3 9	8 8	6 6	59 8	3 8
1 0	1 0	4 9	9 5	5 9	7 3	1 8	1 6	59 1	1 9
2 4	1 5	5 6	6 9	5 7	1 8	9 4	7 9	69 1	3 8
3 1	1 6	8 5	8 4	1 8	9 9	9 9	9 7	84 9	1 9
0 5	7 2	8 5	7 6	9 9	9 9	1 6	2 2	47 6	6 8
4 5	6 1	3 7	5 7	6 8	1 5	7 3	4 6	28 3	1 5
8 0	9 5	6 7	6 7	1 7	6 9	8 3	6 8	79 8	6 7
4 6	7 8	1 5	7 7	8 9	3 8	1 2	2 5	18 2	4 9
1 6	4 9	6 7	1 8	4 6	5 4	3 7	7 6	99 7	1 5

The combinations, a few of which were given on the preceding page are next tabulated as shown in Charts VII and VIII on the next two pages. From these tables, they are gathered together and recorded in another similar table. See Chart IX. To interpret this last and other charts of the same type throughout this work, note that the numbers above the diagonals are read from top to left as 1+9, a total of 38 times; 3+6 a total of 14 times. The numbers below the diagonals are read from left to top as , 9+1, a total of 16 times; 6+3, a total of 18 times. In every case the upper addend represents the number of primary or basic combinations, while the lower addend represents the number of decade or secondary combinations. Of course the sums are the total numbers of combinations involving the particular digits. The reader is referred to pages 1 to three inclusive for a complete summary of the primary combinations.

Only in addition has any distinction been made between primary and decade combinations. An interesting comparison might be made between the relative frequency of appearance of primary and decade combinations, but such falls outside the scope of this study.

The totals of the various combinations were next tabulated (Chart X) each in their proper quartile according to the Clapp Order B of Difficulty.

Order of Difficulty B, as we have already seen, has reference to the functional difficulty of the combinations. It has to do with the difficulty of combinations as they are met in examples as is the case in text book work, Hence comparisons are made with the Order B of Difficulty and NOT with Order

A, which has to do with combinations only when presented singly, and when the automatic response is what is sought.

Chart VII.

Number of Various Primary Combinations as Found in:

OSBURN III ADDITION.

Read: Left to top.

	0	1	2	3	4	5	6	7	8	9
9	4	5	10	11	14	9	11	18	10	16
8	7	7	4	10	13	10	13	13	13	16
7	3	5	6	11	11	13	12	14	12	12
6	6	9	4	5	7	14	12	11	13	19
5	3	4	5	4	5	8	11	14	12	13
4	4	5	6	6	6	5	13	14	13	13
3	7	3	3	3	4	8	3	17	12	16
2	10	7	8	10	9	14	11	12	16	16
1	9	7	18	17	20	36	27	24	32	21
0	6	12	15	12	9	10	7	5	10	7

Chart VIII.

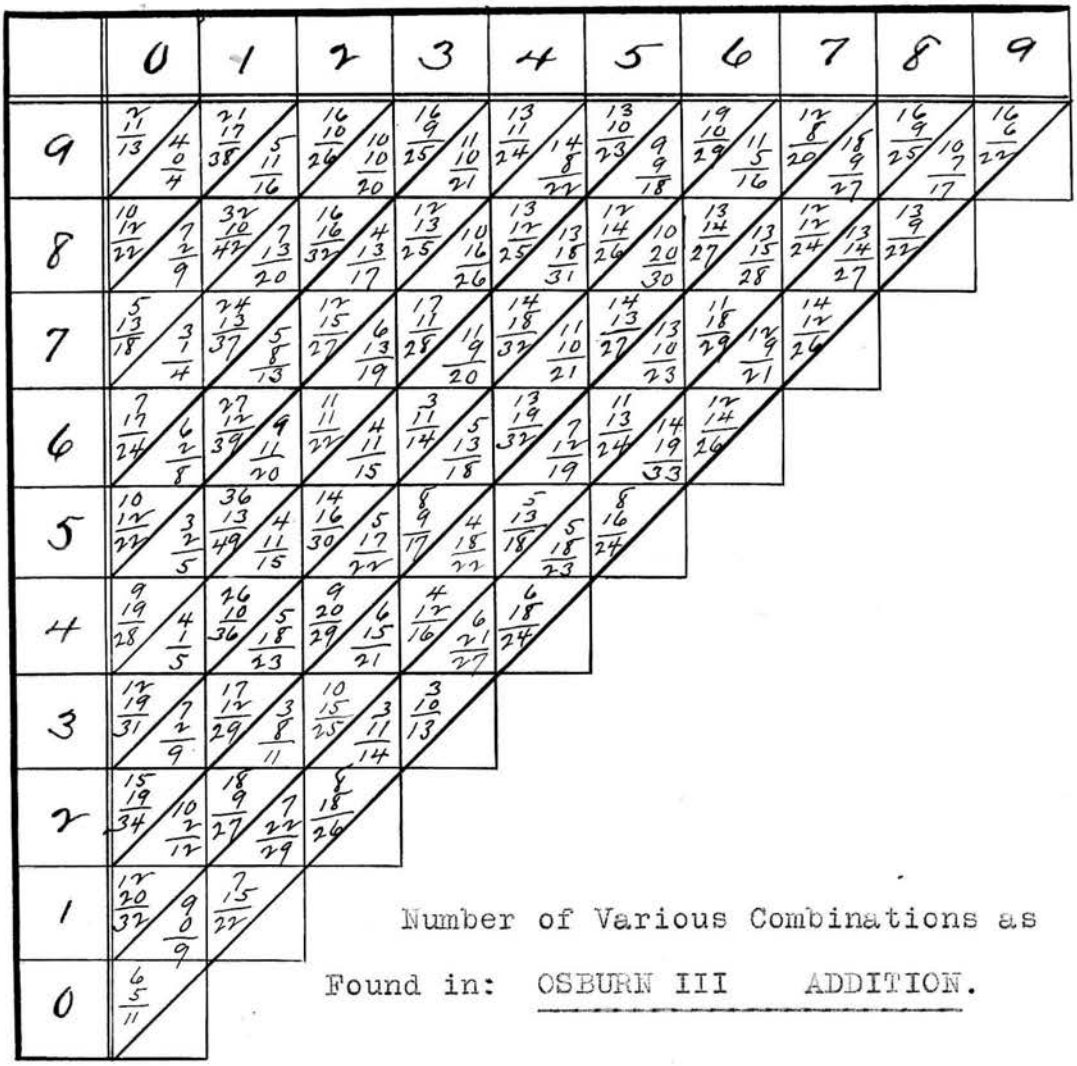
Number of Various Decade Combinations as Found in:

OSBURN III ADDITION.

Read: Left to top.

	0	1	2	3	4	5	6	7	8	9
9		11	10	10	8	9	5	9	7	6
8	2	13	13	16	18	20	15	14	9	9
7	1	8	13	9	10	10	9	12	12	8
6	2	11	11	13	12	19	14	18	14	10
5	2	11	17	18	18	16	13	13	14	10
4	1	18	15	21	18	13	19	18	12	11
3	2	8	11	10	12	9	11	11	13	9
2	2	22	18	15	20	16	11	15	16	10
1		15	9	12	10	13	12	13	10	17
0	5	20	19	19	19	12	17	13	12	11

Chart IX.



Number of Various Combinations as Found in: OSBURN III ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

Lower addends represent the number of decade combinations.

Upper addends represent the number of primary combinations.

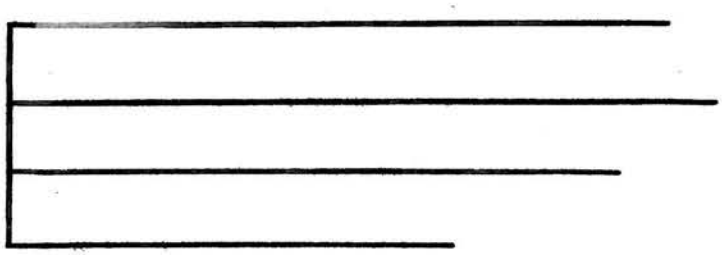
See page 21.

Chart X.

Osburn III Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	24	1+3	29	4+6	32	3+2	14
9+6	16	8+7	27	2+7	27	1+0	9
6+7	29	5+7	27	6+3	18	6+1	20
8+6	28	7+8	24	6+5	33	7+1	13
6+9	29	2+2	26	1+2	27	0+7	18
3+9	25	3+6	14	5+3	22	0+4	28
5+8	26	8+3	26	2+5	30	0+3	31
7+9	20	7+4	21	2+4	29	4+2	21
5+9	23	9+4	22	3+3	13	2+0	12
8+5	30	1+9	38	5+6	24	3+0	9
9+9	22	4+4	24	7+3	20	0+5	22
7+5	23	1+8	42	5+5	24	4+0	5
4+8	25	2+8	32	4+3	27	0+8	22
2+9	26	3+7	28	6+0	8	8+2	17
7+6	21	2+6	22	8+4	31	5+2	22
3+8	25	3+5	17	5+4	23	5+0	5
7+7	26	9+5	18	1+1	22	4+1	23
6+6	26	1+4	36	8+1	20	7+0	4
4+7	32	1+5	49	6+4	19	2+1	29
6+8	27	1+7	37	2+3	25	9+1	16
9+7	27	9+2	20	0+9	13	5+1	15
8+8	22	4+5	18	0+6	24	0+2	34
9+3	21	3+4	16	7+2	19	9+0	4
9+8	17	6+2	15	0+1	32	8+0	9
8+9	25	1+6	39	3+1	11	0+0	11
	615		667		573		413
	27%		29%		25%		18%

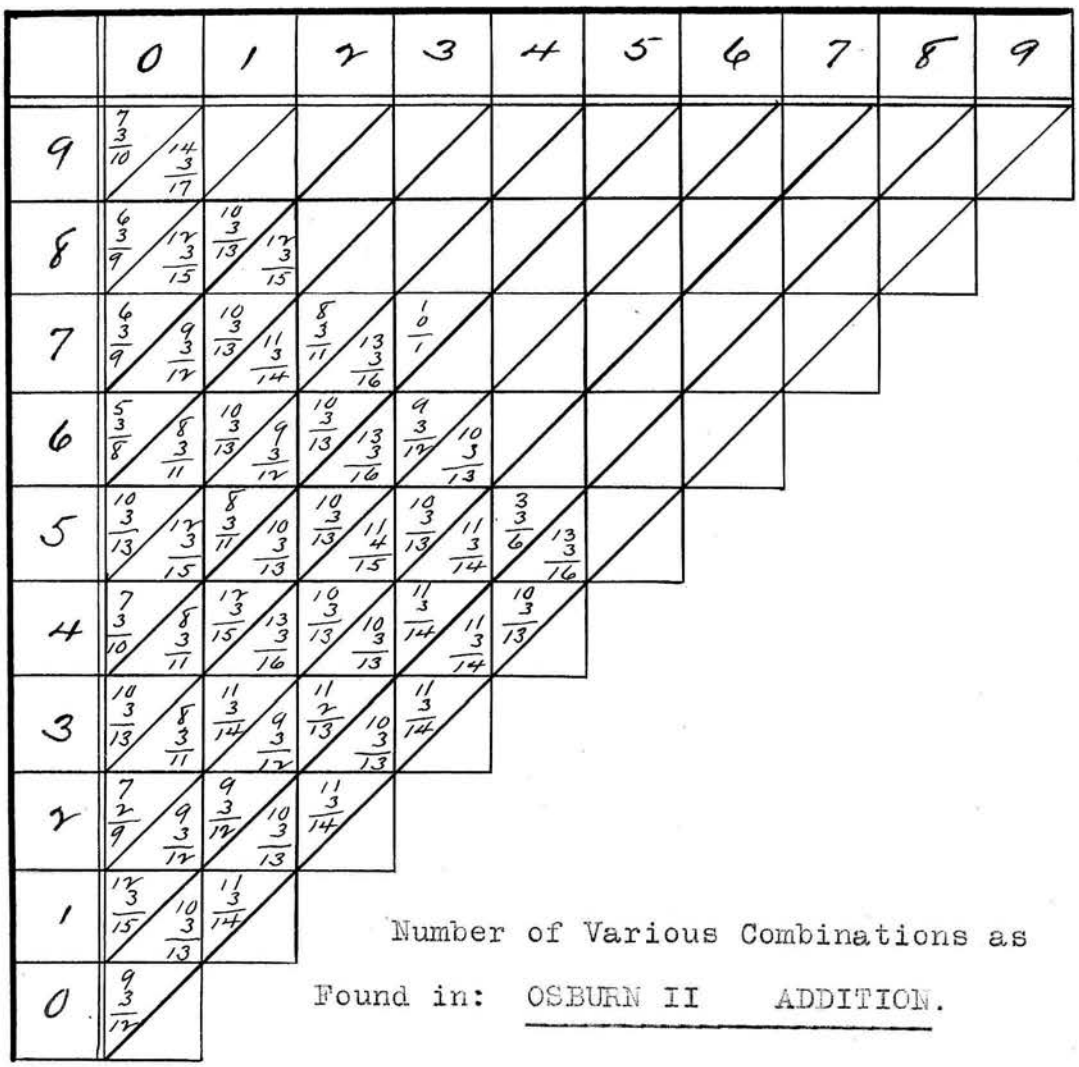


In Chart X, we note that the total number of combinations falling in Quartile I is 615. In the second, third, and fourth quartiles, they are respectively 667, 573, and 413. Reduced to percentages the combinations falling in Quartiles I to IV are respectively 27, 29, 25, and 18. Here we have results pretty much as they should be; especially would this be so, were the totals of Quartiles I and II interchanged. Compare this showing with those of the Clapp Study in Addition - either Text A or B - where it is readily seen that many less combinations fall in Quartile I (the most difficult) than any of the other three quartiles, and many more fallin Quartile IV than any of the other three. Said in other words, in these two texts, the easiest combinations are given much more drill; the most difficult combinations, much less. In the case of Osburn III, more drill and practice is allotted to the hardest combinations and less drill to the least difficult ones.

We have used Osburn III - Addition - as an example as said before, to illustrate the procedure followed herein. Commencing on page 28 each of the several Services are taken up one by one by means of tables and charts only, as just shown in the case of the one used for illustration. At the end of each section, results are shown graphically as well as numerically. All deductions and conclusions however are reserved until the final pages.

In the case of division, a somewhat different chart was necessary for the tabulating of combinations. The form as appearing in these pages seemed to meet the requirements best. It is as will be seen, quite similar to the one already made familiar and will not need further explanation.

Chart XI.



	0	1	2	3	4	5	6	7	8	9
9	7 3 10 / 14 3 17									
8	6 3 9 / 12 3 15	10 3 13 / 12 3 15								
7	6 3 9 / 9 3 12	10 3 13 / 11 3 14	8 3 11 / 13 3 16	1 0 1						
6	5 3 8 / 8 3 11	10 3 13 / 9 3 12	10 3 13 / 13 3 16	9 3 12 / 10 3 13						
5	10 3 13 / 12 3 15	8 3 11 / 10 3 13	10 3 13 / 11 4 15	10 3 13 / 11 3 14	3 3 6 / 13 3 16					
4	7 3 10 / 8 3 11	12 3 15 / 13 3 16	10 3 13 / 10 3 13	11 3 14 / 11 3 14	10 3 13					
3	10 3 13 / 8 3 11	11 3 14 / 9 3 12	11 2 13 / 10 3 13	11 3 14						
2	7 2 9 / 9 3 12	9 3 12 / 10 3 13	11 3 14							
1	12 3 15 / 10 3 13	11 3 14								
0	9 3 12									

Number of Various Combinations as

Found in: OSBURN II ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Upper addends represent the number of primary combinations.

Lower addends represent the number of decade combinations.

Chart XII.

Osburn II Addition.

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9		1+3	14	4+6		3+2	13
9+6		8+7		2+7	11	1+0	13
6+7		5+7		6+3	13	6+1	12
8+6		7+8		6+5		7+1	14
6+9		2+2	14	1+2	12	0+7	9
3+9		3+6	12	5+3	14	0+4	10
5+8		8+3		2+5	13	0+3	13
7+9		7+4		2+4	13	4+2	13
5+9		9+4		3+3	14	2+0	12
8+5		1+9		5+6		3+0	11
9+9		4+4	13	7+3		0+5	13
7+5		1+8	13	5+5		4+0	11
4+8		2+8		4+3	14	0+8	9
2+9		3+7	1	6+0	11	8+2	
7+6		2+6	13	8+4		5+2	15
3+8		3+5	13	5+4	16	5+0	15
7+7		9+5		1+1	14	4+1	16
6+6		1+4	15	8+1	15	7+0	12
4+7		1+5	11	6+4		2+1	13
6+8		1+7	13	2+3	13	9+1	
9+7		9+2		0+9	10	5+1	13
8+8		4+5	6	0+6	8	0+2	9
9+3		3+4	14	7+2	16	9+0	17
9+8		6+2	16	0+1	15	8+0	15
8+9		1+6	13	3+1	12	0+0	12
	00		181		234		290
	00%		26%		33%		41%

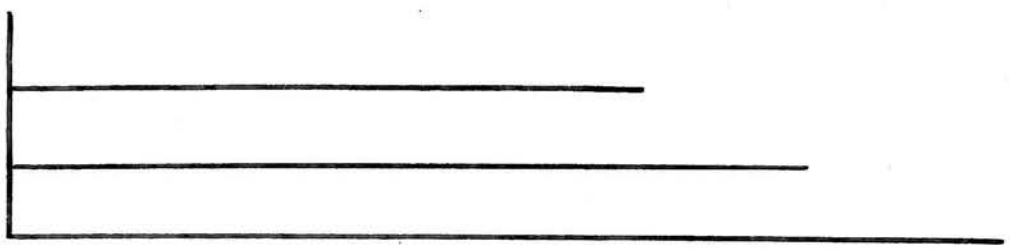


Chart XIII.

	0	1	2	3	4	5	6	7	8	9
9	8	5	7	5	6	8	6	10	8	7
8	6	6	6	7	6	6	4	7	5	
7	5	6	6	7	8	7	6	5		
6	5	5	8	7	7	6	5			
5	6	5	6	7	8	6				
4	6	5	6	6	6					
3	5	6	6	6						
2	6	6	6							
1	5	6								
0	5									

Number of Various Combinations as found in: OSBURN II SUBTRACTION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XIV.

Osburn II Subtraction.

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3		6-3	7	10-1		17-8	
16-9		6-6	5	9-2	7	8-5	6
8-4	6	13-6		9-8	8	18-9	
11-4		2-2	6	9-4	6	7-7	5
14-9		8-8	5	10-3		3-1	6
7-0	5	11-5		13-7		5-2	6
9-1	5	6-4	7	12-7		1-1	6
15-9		10-7		8-7	7	5-3	7
9-0	8	12-5		7-6	6	10-4	
11-2		8-1	6	6-1	5	7-4	8
11-7		14-6		3-0	6	4-0	6
15-8		5-1	5	2-0	6	4-2	6
13-5		11-8		14-8		8-0	6
16-7		14-7		12-8		5-5	6
9-9	7	7-1	6	12-3		11-6	
14-5		9-6	6	6-2	8	10-5	
13-8		4-1	5	8-3	7	3-3	6
10-2		15-6		12-9		4-3	6
13-9		7-5	7	16-8		4-4	6
13-4		2-1	6	9-7	10	10-6	
5-0	6	7-3	7	9-5	8	3-2	6
5-4	8	8-2	6	8-6	4	10-9	
6-0	5	15-7		6-5	6	10-8	
9-3	5	7-2	6	1-0	5	12-6	
17-9		12-4		11-9		0-0	5
	55		90		99		97
	16%		26%		29%		28%

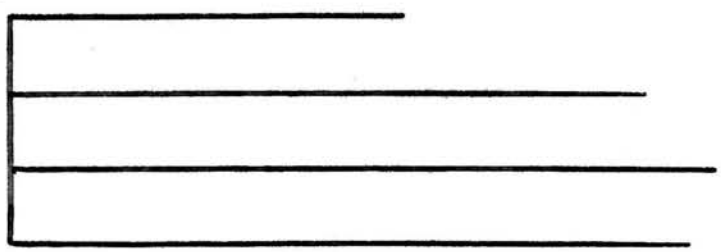
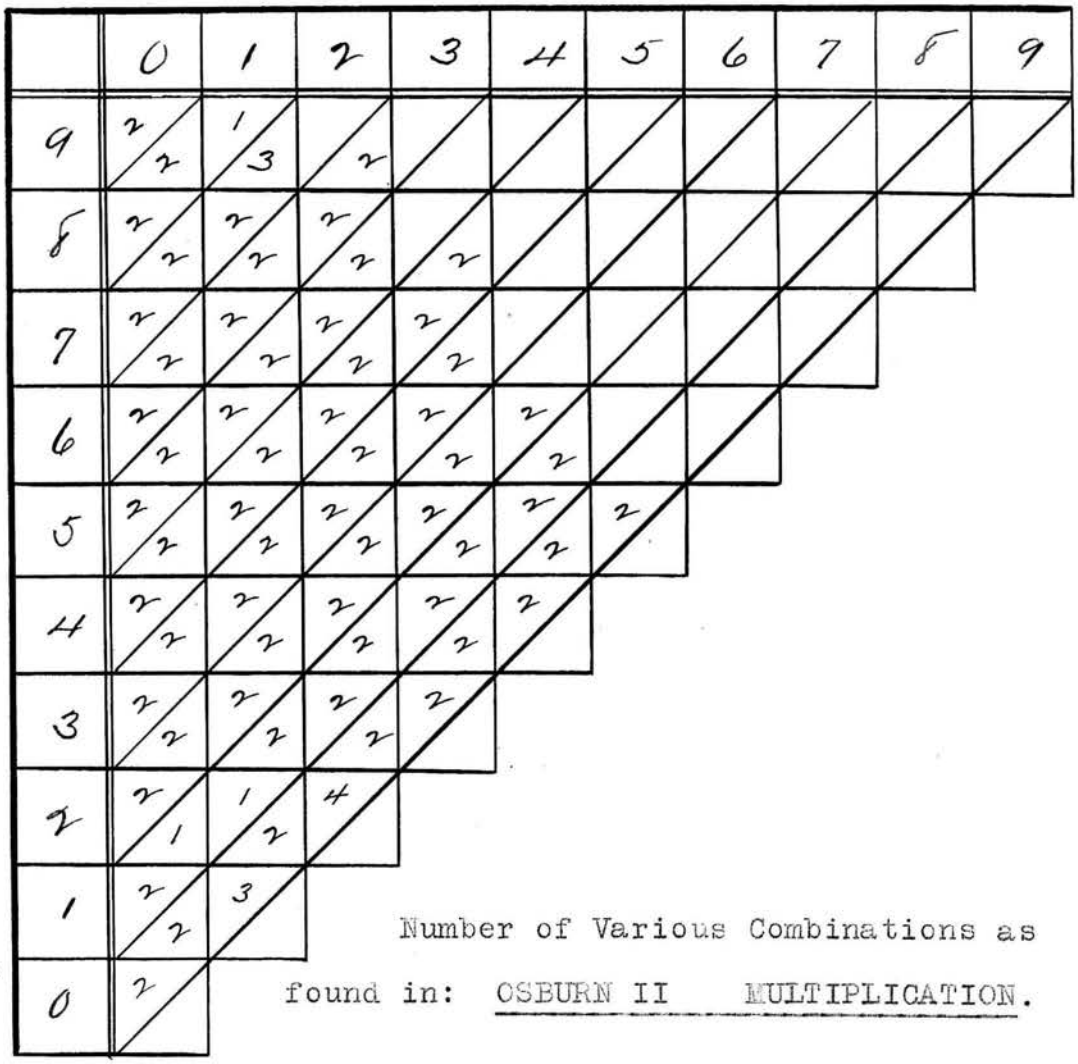


Chart XV.



Number of Various Combinations as found in: OSBURN II MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XVI.

Osburn II Multiplication.

Comparison with Clapp Order of Difficulty -B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	2	8x0	2	4x6	2	2x1	2
9x4		7x3	2	4x4	2	2x2	4
9x7		7x5		4x7		6x0	2
8x6		6x7		3x1	2	3x3	2
8x8		9x6		6x1	2	8x2	2
7x6		6x6		3x6	2	8x1	2
0x5	2	5x1	2	2x0	1	1x0	2
0x7	2	6x4	2	6x3	2	3x4	2
6x9		7x7		5x9		1x2	1
7x8		7x1	2	2x8	2	3x8	
4x8		4x0	2	9x1	3	4x5	2
0x8	2	0x6	2	5x7		1x9	1
7x4		9x9		3x9		1x8	2
9x0	2	9x3		4x3	2	2x6	2
8x7		3x0	2	3x7	2	1x3	2
7x9		4x9		2x4	2	7x2	2
8x4		7x0	2	6x5		1x5	2
6x2	2	9x5		5x3	2	2x5	2
0x9	2	8x5		5x0	2	3x5	2
9x8		0x4	2	1x1	3	2x7	2
0x3	2	9x2	2	4x1	2	1x6	2
6x8		3x2	2	5x4	2	1x7	2
0x1	2	5x5	2	2x3	2	1x4	2
8x9		5x6		2x9		5x2	2
8x3	2	5x8		4x2	2	0x0	2
	20		26		39		48
	15%		20%		29%		36%

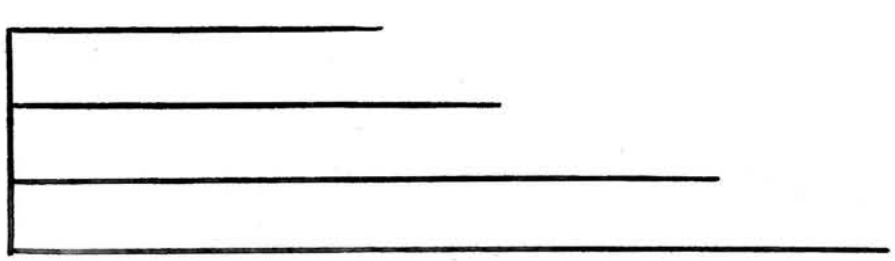


Chart XVII.

Number of Various Combinations as Found in:

OSBURN II DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors.

	0	1	2	3	4	5	6	7	8	9
9		9 3		9 5				49		9 5
8		8 3	5 4 1 3	18 4	8 5 28		18 6 48	28	8 4 48	18 4
7		7 3		27				7 4		27
6		6 3	6 5 16 3	6 7	16 3 36		6 7 36	56	16 4 56	36
5		5 3		15 3		7 5 3 4 15 25 35 45		35		45
4		4 3	4 4 14 3	24 3	4 5 24 3		24 4 54	14 4	24 4 64	54
3		3 4		3 5				63		63
2		2 3	2 5 12 4	12 3	12 4 32		12 4 42	42	32 72	72
1		1 3		21 3				21 3		81
0		0 2	0 5 10 4	0 5	0 5 20 2	0 5 10 20 30 40	0 6 30	0 3	0 4 40	0 9

Chart XVIII.

Osburn II Division

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
49÷7		42÷7		12÷3	3	9÷1	3
54÷6		54÷9		24÷6	4	4÷4	5
45÷9		30÷6		32÷8		0÷3	5
27÷3		28÷7		8÷4	5	3÷3	5
56÷7		24÷3	3	9÷3	5	5÷5	7
28÷4		27÷9		12÷2	4	0÷4	5
24÷4	3	64÷8		16÷8	4	9÷9	5
36÷6		81÷9		10÷5	4	0÷7	3
48÷6		36÷9		4÷2	4	0÷6	6
21÷7	3	18÷6	6	12÷6	4	0÷5	5
18÷2	3	63÷7		5÷1	3	6÷1	3
21÷3	3	42÷6		15÷5	3	0÷8	4
36÷4		56÷8		18÷9	4	1÷1	3
72÷9		16÷2	3	6÷6	7	0÷9	3
63÷9		24÷8	4	20÷5	5	8÷1	3
40÷8		72÷8		2÷2	5	2÷1	3
32÷4		16÷4	3	10÷2	4	0÷1	2
35÷7		20÷4	2	6÷3	7	8÷2	4
35÷5		30÷5		40÷5		0÷2	5
18÷3	4	14÷7	4	8÷8	4	4÷1	3
15÷3	3	14÷2	3	7÷7	4	7÷1	3
45÷5		25÷5	4	12÷4	4	3÷1	4
48÷8				6÷2	5		
	19		32		92		89
	8%		14%		40%		38%

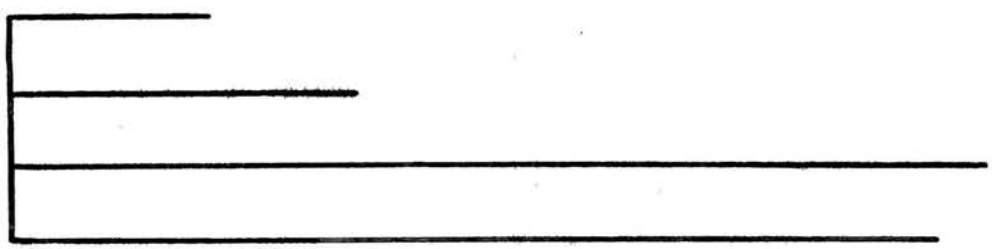


Chart XIX.

	0	1	2	3	4	5	6	7	8	9
9	12 / 11 / 13 — 4 / 0 / 4	21 / 17 / 38 — 5 / 11 / 16	16 / 10 / 26 — 10 / 10 / 20	16 / 9 / 25 — 11 / 10 / 21	13 / 11 / 24 — 14 / 8 / 22	13 / 10 / 23 — 9 / 9 / 18	19 / 10 / 29 — 11 / 5 / 16	12 / 8 / 20 — 18 / 9 / 27	16 / 9 / 25 — 10 / 7 / 17	16 / 6 / 22
8	10 / 12 / 22 — 7 / 2 / 9	32 / 10 / 42 — 7 / 13 / 20	16 / 16 / 32 — 4 / 13 / 17	12 / 13 / 25 — 10 / 16 / 26	13 / 12 / 25 — 13 / 18 / 31	12 / 14 / 26 — 10 / 20 / 30	13 / 14 / 27 — 13 / 15 / 28	12 / 12 / 24 — 13 / 14 / 27	13 / 9 / 22	
7	5 / 13 / 18 — 3 / 1 / 4	24 / 13 / 37 — 5 / 8 / 13	12 / 15 / 27 — 6 / 13 / 19	17 / 11 / 28 — 11 / 9 / 20	14 / 18 / 32 — 11 / 10 / 21	14 / 13 / 27 — 13 / 10 / 23	11 / 18 / 29 — 12 / 9 / 21	14 / 12 / 26		
6	7 / 17 / 24 — 6 / 2 / 8	27 / 12 / 39 — 9 / 11 / 20	11 / 11 / 22 — 4 / 11 / 15	3 / 11 / 14 — 5 / 13 / 18	13 / 19 / 32 — 7 / 12 / 19	11 / 13 / 24 — 14 / 19 / 33	12 / 14 / 26			
5	10 / 12 / 22 — 3 / 2 / 5	36 / 13 / 49 — 4 / 11 / 15	14 / 16 / 30 — 5 / 17 / 22	8 / 9 / 17 — 4 / 18 / 22	5 / 13 / 18 — 5 / 18 / 23	8 / 16 / 24				
4	9 / 19 / 28 — 4 / 1 / 5	26 / 10 / 36 — 5 / 18 / 23	9 / 20 / 29 — 6 / 15 / 21	4 / 12 / 16 — 6 / 21 / 27	6 / 18 / 24					
3	12 / 19 / 31 — 7 / 2 / 9	17 / 12 / 29 — 3 / 8 / 11	10 / 15 / 25 — 3 / 11 / 14	3 / 10 / 13						
2	15 / 19 / 34 — 10 / 2 / 12	18 / 9 / 27 — 7 / 22 / 29	8 / 18 / 26							
1	12 / 20 / 32 — 9 / 0 / 9	7 / 15 / 22								
0	6 / 5 / 11									

Number of Various Combinations as Found in: OSBURN III ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

Upper addends represent the number of primary combinations.

Lower addends represent the number of decade combinations.

See page 21.

Chart XX.

Osburn III Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	24	1+3	29	4+6	32	3+2	14
9+6	16	8+7	27	2+7	27	1+0	9
6+7	29	5+7	27	6+3	18	6+1	20
8+6	28	7+8	24	6+5	33	7+1	13
6+9	29	2+2	26	1+2	27	0+7	18
3+9	25	3+6	14	5+3	22	0+4	28
5+8	26	8+3	26	2+5	30	0+3	31
7+9	20	7+4	21	2+4	29	4+2	21
5+9	23	9+4	22	3+3	13	2+0	12
8+5	30	1+9	38	5+6	24	3+0	9
9+9	22	4+4	24	7+3	20	0+5	22
7+5	23	1+8	42	5+5	24	4+0	5
4+8	25	2+8	32	4+3	27	0+8	22
2+9	26	3+7	28	6+0	8	8+2	17
7+6	21	2+6	22	8+4	31	5+2	22
3+8	25	3+5	17	5+4	23	5+0	5
7+7	26	9+5	18	1+1	22	4+1	23
6+6	26	1+4	36	8+1	20	7+0	4
4+7	32	1+5	49	6+4	19	2+1	29
6+8	27	1+7	37	2+3	25	9+1	16
9+7	27	9+2	20	0+9	13	5+1	15
8+8	22	4+5	18	0+6	24	0+2	34
9+3	21	3+4	16	7+2	19	9+0	4
9+8	17	6+2	15	0+1	32	8+0	9
8+9	25	1+6	39	3+1	11	0+0	11
	615		667		573		413
	27%		29%		25%		18%

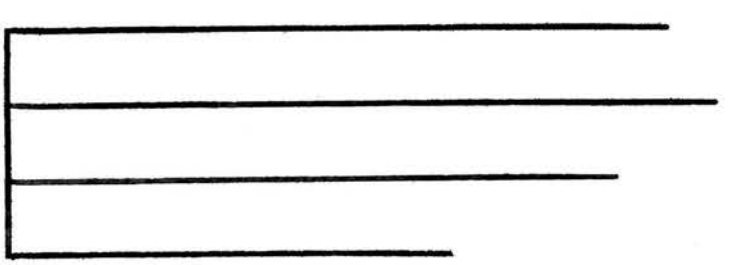


Chart XXI.

	0	1	2	3	4	5	6	7	8	9
9	11 / 7	11 / 8	6 / 11	11 / 12	11 / 12	12 / 10	5 / 15	15 / 20	7 / 17	10
8	13 / 3	16 / 6	13 / 9	15 / 9	11 / 11	12 / 15	8 / 14	6 / 7	20	
7	15 / 6	10 / 6	10 / 12	6 / 6	4 / 15	5 / 15	6 / 9	20		
6	15 / 6	15 / 6	13 / 15	12 / 9	8 / 11	8 / 19	27			
5	13 / 9	11 / 9	8 / 14	5 / 6	6 / 12	28				
4	13 / 12	9 / 6	6 / 20	8 / 8	53					
3	7 / 8	7 / 8	6 / 18	55						
2	7 / 11	4 / 11	49							
1	6 / 10	49								
0	5									

Number of Various Combinations as found in: OSBURN III SUBTRACTION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XXII.

Osburn III Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	7	6-3	9	10-1	6	17-8	6
16-9	5	6-6	27	9-2	11	8-5	15
8-4	11	13-6	12	9-8	17	18-9	7
11-4	9	2-2	49	9-4	12	7-7	20
14-9	11	8-8	20	10-3	7	3-1	8
7-0	6	11-5	11	13-7	6	5-2	14
9-1	8	6-4	11	12-7	10	1-1	49
15-9	12	10-7	15	8-7	7	5-3	6
9-0	7	12-5	8	7-6	9	10-4	13
11-2	4	8-1	6	6-1	6	7-4	15
11-7	10	14-6	8	3-0	8	4-0	12
15-8	12	5-1	9	2-0	11	4-2	20
13-5	5	11-8	16	14-8	11	6-0	3
16-7	6	14-7	4	12-8	13	5-5	28
9-9	10	7-1	6	12-3	6	11-6	15
14-5	6	9-6	15	6-2	15	10-5	13
13-8	15	4-1	6	8-3	9	3-3	55
10-2	7	15-6	8	12-9	6	4-3	8
13-9	11	7-5	15	16-8	8	4-4	53
13-4	8	2-1	11	9-7	20	10-6	15
5-0	9	7-3	6	9-5	10	3-2	18
5-4	12	8-2	9	8-6	14	10-9	11
6-0	6	15-7	5	6-5	19	10-8	13
9-3	12	7-2	12	1-0	10	12-6	13
17-9	15	12-4	6	11-9	11	6-0	5
	224		304		262		435
	18%		25%		21%		36%

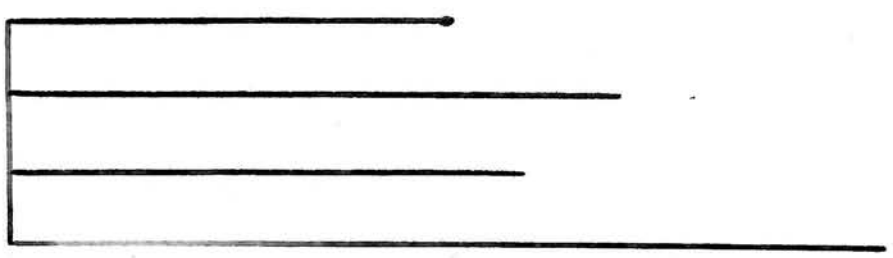


Chart XXIII

	0	1	2	3	4	5	6	7	8	9
9	2 / 11	10 / 8	8 / 10	10 / 15	13 / 17	19 / 20	20 / 21	20 / 17	21 / 21	18
8	2 / 15	7 / 5	10 / 5	12 / 8	19 / 16	16 / 16	22 / 21	24 / 16	15	
7	3 / 15	10 / 3	8 / 9	13 / 14	11 / 22	15 / 19	20 / 26	18		
6	2 / 20	7 / 9	9 / 9	7 / 10	15 / 9	18 / 14	16			
5	3 / 9	6 / 3	6 / 9	10 / 10	8 / 9	12				
4	2 / 10	4 / 3	5 / 6	9 / 6	5					
3	2 / 7	2 / 5	4 / 6	7						
2	2 / 6	6 / 3	3							
1	2 / 4	6								
0	2									

. Number of Various Combinations as found in: OSBURN III MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XXIV.

Osburn III Multiplication

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	2	8x0	15	4x6	15	2x1	3
9x4	17	7x3	14	4x4	5	2x2	3
9x7	17	7x5	19	4x7	11	6x0	20
8x6	21	6x7	20	3x1	5	3x3	7
8x8	15	9x6	21	6x1	9	8x2	5
7x6	26	6x6	16	3x6	7	8x1	5
0x5	3	5x1	3	2x0	6	1x0	4
0x7	3	6x4	9	6x3	10	3x4	9
6x9	20	7x7	18	5x9	19	1x2	6
7x8	24	7x1	3	2x8	10	3x8	12
4x8	19	4x0	10	9x1	8	4x5	8
0x8	2	0x6	2	5x7	15	1x9	10
7x4	22	9x9	18	3x9	10	1x8	7
9x0	11	9x3	15	4x3	6	2x6	9
8x7	16	3x0	7	3x7	13	1x3	2
7x9	30	4x9	13	2x4	5	7x2	9
8x4	16	7x0	3	6x5	14	1x5	6
6x2	9	9x5	20	5x3	10	2x5	6
0x9	2	8x5	16	5x0	9	3x5	10
9x8	31	0x4	2	1x1	6	2x7	8
0x3	2	9x2	10	4x1	5	1x6	7
6x8	22	3x2	6	5x4	9	1x7	10
0x1	2	5x5	12	2x3	4	1x4	4
8x9	21	5x6	18	2x9	8	5x2	9
8x3	8	5x8	16	4x2	6	0x0	2
	341		306		223		181
	32%		29%		21%		17%

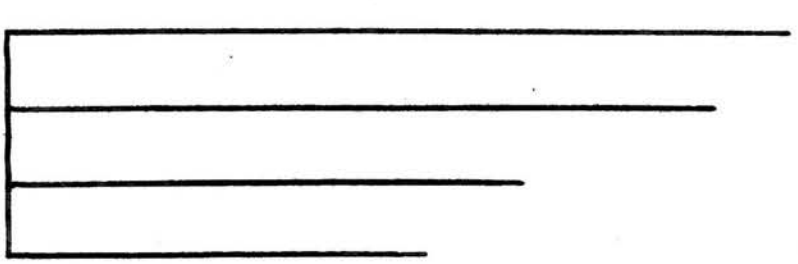


Chart XXV.

Number of Various Combinations as Found in:

OSBURN III DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors

	0	1	2	3	4	5	6	7	8	9
9	X	9 1		9 4				49 6		9 18
8	X	8 1	8 2 18 3	18 2	8 4 28 5		8 4	28 7	8 1 28 9	18 8
7	X	7 1		27 5				7 13		27 6
6	X	6 1	6 2 16 1	6 2	16 5 36 4		6 9 36 8	56 5	16 9 56 7	36 7
5	X	5 1		15 4		1		35 6		45 5
4	X	4 1	4 1 14 2	24 2	4 2 44 1	5 15 25 35 45	24 6 54 9	14 5	24 5 64 6	54 4
3	X	3 1		3 1				63 8		63 7
2	X	2 1 12 3	2 2	12 5 32 6	12 2		12 6 42 9	42 8	32 10 72 8	12 6
1	X	1 1		11 4				21 7		81 9
0	X	0 1 10 3	0 1	0 5 20 6	0 4 30 4	0 10 20 30 40	0 3 30 6	0 6	0 5 40 7	0 7

Chart XXVI.

Osburn III Division

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
49÷7	6	42÷7	8	12÷3	5	9÷1	1
54÷6	9	54÷9	4	24÷6	6	4÷4	2
45÷9	5	30÷6	6	32÷8	10	0÷3	5
27÷3	5	28÷7	7	8÷4	4	3÷3	1
56÷7	5	24÷3	2	9÷3	4	5÷5	4
28÷4	5	27÷9	6	12÷2	3	0÷4	4
24÷4	1	64÷8	6	16÷8	9	9÷9	12
36÷6	8	81÷9	9	10÷5	3	0÷7	6
48÷6	9	36÷9	7	4÷2	1	0÷6	3
48÷8	9	18÷6	4	12÷6	6	0÷5	4
21÷7	7	63÷7	8	5÷1	1	6÷1	1
18÷2	3	42÷6	9	15÷5	2	0÷8	5
21÷3	4	56÷8	7	18÷9	8	1÷1	1
36÷4	4	16÷2	1	6÷6	9	0÷9	7
72÷9	6	24÷8	5	20÷5	2	8÷1	2
63÷9	7	72÷8	8	2÷2	2	2÷1	2
40÷8	7	16÷4	5	10÷2	3	0÷1	1
32÷4	6	20÷4	4	6÷3	2	8÷2	2
35÷7	8	30÷5	3	40÷5	3	0÷2	1
35÷5	6	14÷7	5	8÷8	11	4÷1	2
18÷3	2	14÷2	2	7÷7	13	7÷1	1
15÷3	4	25÷5	3	12÷4	2	3÷1	1
45÷5	5			6÷2	2		
	131		119		111		68
	30%		28%		26%		16%

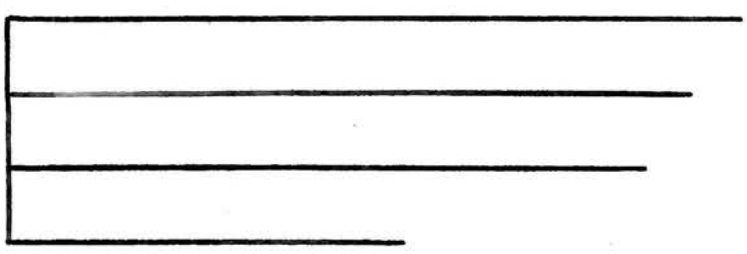


Chart XXVII.

	0	1	2	3	4	5	6	7	8	9
9	2/17/19 \ 2/9/11	11/25/36 \ 13/13/26	8/17/25 \ 10/18/28	8/21/29 \ 10/9/19	6/15/21 \ 8/10/18	9/14/23 \ 8/15/23	7/15/22 \ 12/18/30	5/10/15 \ 13/13/26	6/12/18 \ 5/17/22	7/10/17
8	2/19/21 \ 3/9/12	15/17/32 \ 8/21/29	7/12/19 \ 7/13/20	7/16/23 \ 3/15/18	6/16/22 \ 7/29/36	9/25/34 \ 12/14/26	3/15/18 \ 11/11/22	8/15/23 \ 4/13/17	4/18/22	
7	4/20/24 \ 4/3/7	11/21/32 \ 6/14/20	9/19/28 \ 6/16/22	7/18/25 \ 9/14/23	6/28/34 \ 8/17/25	6/12/18 \ 13/8/21	6/20/26 \ 6/12/18	8/14/22		
6	6/21/27 \ 3/4/7	16/14/30 \ 9/18/27	4/18/22 \ 4/17/21	2/21/23 \ 5/23/28	9/20/29 \ 13/17/30	5/23/28 \ 8/20/28	7/18/25			
5	10/27/37 \ 2/7/9	20/27/47 \ 7/22/29	8/27/35 \ 4/19/23	4/16/20 \ 1/26/27	5/19/24 \ 6/17/23	7/18/25				
4	10/27/37 \ 4/9/13	12/13/25 \ 7/23/30	5/28/33 \ 4/23/27	2/14/16 \ 6/21/27	5/21/26					
3	8/31/39 \ 2/10/12	10/22/32 \ 4/15/19	8/25/33 \ 2/16/18	5/13/18						
2	10/22/32 \ 7/9/16	13/17/30 \ 6/27/33	7/22/29							
1	5/24/29 \ 6/8/14	6/14/20								
0	8/5/13									

Number of Various Combinations as Found in: OSBURN IV ADDITION.

Below diagonals : Read left to top.

Above diagonals: Read top to left.

Upper addends represent the number of primary combinations.

Lower addends represent the number of decade combinations.

See page 21.

Chart XXVIII

Osburn IV Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	21	1+3	32	4+6	29	3+2	18
9+6	30	8+7	17	2+7	28	1+0	14
6+7	26	5+7	18	6+3	28	6+1	27
8+6	22	7+8	23	6+5	28	7+1	20
6+9	22	2+2	29	1+2	30	0+7	24
3+9	29	3+6	23	5+3	27	0+4	37
5+8	34	8+3	18	2+5	35	0+3	39
7+9	15	7+4	25	2+4	33	4+2	27
5+9	23	9+4	18	3+3	18	2+0	16
8+5	26	1+9	36	5+6	28	3+0	12
9+9	17	4+4	26	7+3	23	0+5	37
7+5	21	1+8	32	5+5	25	4+0	13
4+8	22	2+8	19	4+3	27	0+8	21
2+9	25	3+7	27	6+0	7	8+2	20
7+6	18	2+6	22	8+4	36	5+2	23
3+8	23	3+5	20	5+4	23	5+0	9
7+7	22	9+5	23	1+1	20	4+1	30
6+6	25	1+4	25	8+1	29	7+0	7
4+7	34	1+5	47	6+4	30	2+1	33
6+8	18	1+7	32	2+3	33	9+1	26
9+7	26	9+2	28	0+9	19	5+1	29
8+8	22	4+5	24	0+6	27	0+2	32
9+3	19	3+4	16	7+2	22	9+0	11
9+8	22	6+2	21	0+1	29	8+0	12
8+9	18	1+6	30	3+1	19	0+0	13
	580		631		653		550
	24%		26%		27%		23%

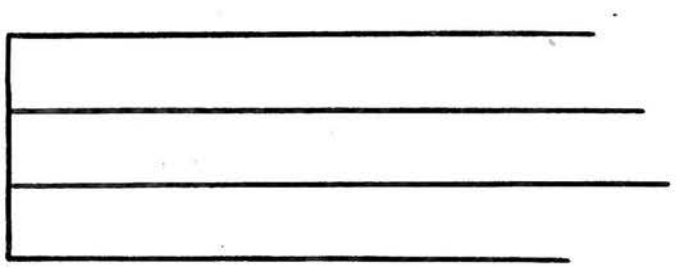


Chart XXIX.

	0	1	2	3	4	5	6	7	8	9
9	7 / 12	8 / 4	7 / 9	8 / 7	8 / 18	9 / 11	9 / 8	14 / 12	9 / 15	16 /
8	9 / 6	8 / 4	6 / 8	9 / 13	6 / 16	10 / 10	11 / 9	7 / 11	10 /	
7	6 / 9	11 / 4	8 / 9	8 / 5	7 / 12	11 / 14	6 / 7	8 /		
6	14 / 11	8 / 5	8 / 11	8 / 6	12 / 7	7 / 13	7 /			
5	6 / 8	8 / 3	9 / 11	8 / 6	6 / 14	11 /				
4	12 / 15	3 / 6	9 / 10	3 / 6	13 /					
3	5 / 12	9 / 7	8 / 10	15 /						
2	5 / 13	7 / 9	14 /							
1	3 / 13	13 /								
0	12 /									

Number of Various Combinations as Found in: OSBURN IV SUBTRACTION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XXX.

Osburn IV Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	9	6-3	6	10-1	3	17-8	7
16-9	9	6-6	7	9-2	9	8-5	10
8-4	16	13-6	8	9-8	15	18-9	9
11-4	3	2-2	14	9-4	18	7-7	8
14-9	8	8-8	10	10-3	5	3-1	7
7-0	9	11-5	8	13-7	8	5-2	11
9-1	4	6-4	7	12-7	8	1-1	13
15-9	9	10-7	6	8-7	11	5-3	6
9-0	12	12-5	9	7-6	7	10-4	12
11-2	7	8-1	4	6-1	5	7-4	12
11-7	11	14-6	12	3-0	12	4-0	15
15-8	10	5-1	3	2-0	13	4-2	10
13-5	8	11-8	8	14-8	6	8-0	6
16-7	6	14-7	7	12-8	6	5-5	11
9-9	16	7-1	4	12-3	8	11-6	8
14-5	6	9-6	8	6-2	11	10-5	6
13-8	9	4-1	6	8-3	13	3-3	15
10-2	5	15-6	7	12-9	7	4-3	6
13-9	8	7-5	14	16-8	11	4-4	13
13-4	3	2-1	9	9-7	12	10-6	14
5-0	8	7-3	5	9-5	11	3-2	10
5-4	14	8-2	8	8-6	9	10-9	7
6-0	11	15-7	11	6-5	13	10-8	9
9-3	7	7-2	9	1-0	13	12-6	8
17-9	14	12-4	9	11-9	8	0-0	12
	222		199		242		245
	24%		22%		27%		27%

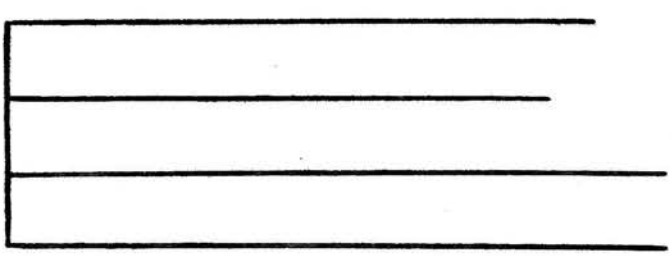


Chart XXXI.

	0	1	2	3	4	5	6	7	8	9
9	3 / 10	17 / 11	7 / 6	16 / 19	17 / 17	18 / 15	29 / 16	18 / 20	22 / 21	15
8	3 / 24	13 / 10	8 / 8	13 / 10	11 / 17	11 / 21	26 / 19	24 / 11	20	
7	3 / 14	10 / 11	9 / 12	8 / 15	17 / 17	13 / 20	18 / 20	16		
6	3 / 8	9 / 13	11 / 9	11 / 10	18 / 18	14 / 19	25			
5	2 / 10	12 / 8	13 / 8	9 / 8	11 / 8	14				
4	2 / 12	13 / 5	11 / 9	12 / 11	10					
3	2 / 8	9 / 5	6 / 7	10						
2	2 / 6	6 / 7	8							
1	2 / 5	5								
0	4									

Number of Various Combinations as found in: OSBURN IV MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XXXII.

Osburn IV Multiplication

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	2	8x0	24	4x6	18	2x1	7
9x4	17	7x3	15	4x4	10	2x2	8
9x7	20	7x5	20	4x7	12	6x0	8
8x6	19	6x7	18	3x1	5	3x3	10
8x8	20	9x6	16	6x1	13	8x2	8
7x6	20	6x6	25	3x6	11	8x1	10
0x5	2	5x1	8	2x0	6	1x0	5
0x7	3	6x4	18	6x3	10	3x4	12
6x9	29	7x7	16	5x9	18	1x2	6
7x8	24	7x1	11	2x8	8	3x8	13
4x8	11	4x0	12	9x1	11	4x5	11
0x8	3	0x6	3	5x7	13	1x9	17
7x4	17	9x9	15	3x9	16	1x8	13
9x0	10	9x3	19	4x3	11	2x6	11
8x7	11	3x0	8	3x7	8	1x3	9
7x9	18	4x9	12	4x4	11	7x2	12
8x4	17	7x0	14	6x5	19	1x5	12
6x2	9	9x5	15	5x3	8	2x5	13
0x9	3	8x5	21	5x0	10	3x5	9
9x8	21	0x4	2	1x1	5	2x7	9
0x3	2	9x2	6	4x1	5	1x6	9
6x8	26	3x2	7	5x4	8	1x7	10
0x1	2	5x5	14	2x3	6	1x4	13
8x9	22	5x6	14	2x9	7	5x2	8
8x3	10	5x8	11	4x2	9	0x0	4
	338		344		258		247
	28%		29%		22%		21%

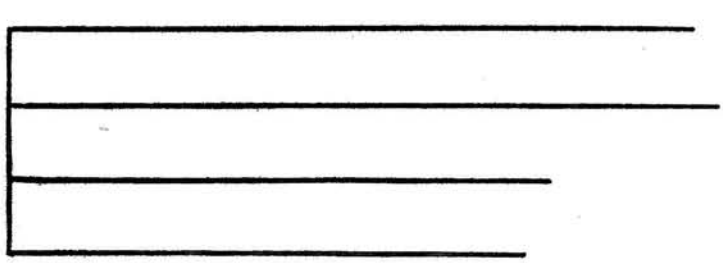


Chart XXXIII.

Number of Various Combinations as Found in:

OSBURN IV DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors.

	0	1	2	3	4	5	6	7	8	9
9	X	9 9		9 8				49 10		9 11
8	X	8 3	8 5 18 3	18 7	8 6 28 7		18 11 48 11	28 7	8 7 48 9	18 10
7	X	7 3		27 7				7 9		27 10
6	X	6 3	6 3 16 5	6 5	16 7 36 5		6 5 36 10	56 4	16 9 56 9	36 7
5	X	5 3		15 4		5 8 15 8 25 7 6 35 45 7		35 5		45 9
4	X	4 3	4 4 14 4	24 5	4 9 24 6		24 10 54 3	14 9	24 8 64 11	54 9
3	X	3 3		3 7				63 8		63 9
2	X	2 3	2 5 12 5	12 9	12 9 32 7		12 8 42 7	42 6	32 4 72 7	72 7
1	X	1 3		21 5				21 5		81 7
0	X	0 2	0 4 10 3	0 7	0 7 20 5	0 4 10 9 20 9 7 30 7 40 7	0 5 30 5	0 4	0 7 40 7	90 0 7

Chart XXXIV.

Osburn IV Division

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
49÷7	10	42÷7	6	12÷3	9	9÷1	9
54÷6	3	54÷9	9	24÷6	10	4÷4	9
45÷9	9	30÷6	5	32÷8	4	0÷3	7
27÷3	7	28÷7	7	8÷4	6	3÷3	7
56÷7	4	24÷3	5	9÷3	8	5÷5	8
28÷4	7	27÷9	10	12÷2	5	0÷4	7
24÷4	6	64÷8	11	16÷8	9	9÷9	11
36÷6	10	81÷9	7	10÷5	9	0÷7	4
48÷6	11	36÷9	7	4÷2	4	0÷6	5
48÷8	9	18÷6	11	12÷6	8	0÷5	4
21÷7	5	63÷7	8	5÷1	3	6÷1	3
18÷2	3	42÷6	7	15÷5	8	0÷8	7
21÷3	5	56÷8	8	18÷9	10	1÷1	4
36÷4	5	16÷2	5	6÷6	5	0÷9	7
72÷9	7	24÷8	8	20÷5	9	8÷1	3
63÷9	9	72÷8	7	2÷2	5	2÷1	3
40÷8	7	16÷4	7	10÷2	3	0÷1	2
32÷4	7	20÷4	5	6÷3	5	8÷2	5
35÷7	5	30÷5	7	40÷5	7	0÷2	4
35÷5	6	14÷7	9	8÷8	7	4÷1	3
18÷3	7	14÷2	4	7÷7	9	7÷1	3
15÷3	4	25÷5	7	12÷4	9	3÷1	3
45÷5	7			6÷2	3		
	153		160		155		118
	26%		27%		26%		20%

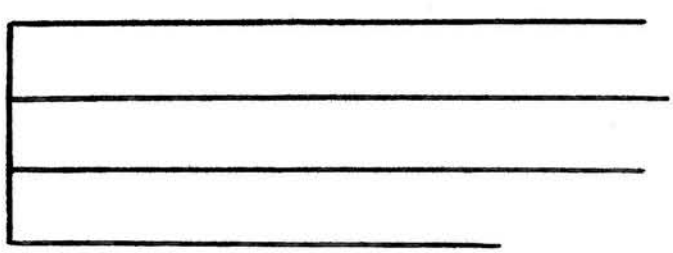


Chart XXXV.

Osburn II, III and IV Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	45	1+3	75	4+6	61	3+2	45
9+6	46	8+7	44	2+7	66	1+0	36
6+7	55	5+7	45	6+3	59	6+1	59
8+6	50	7+8	47	6+5	61	7+1	47
6+9	51	2+2	69	1+2	69	0+7	51
3+9	54	3+6	49	5+3	63	0+4	75
5+8	60	8+3	44	2+5	78	0+3	83
7+9	35	7+4	46	2+4	75	4+2	61
5+9	46	9+4	40	3+3	41	2+0	40
8+5	56	1+9	74	5+6	52	3+0	32
9+9	39	4+4	63	7+3	46	0+5	72
7+5	44	1+8	87	5+5	49	4+0	29
4+8	47	2+8	51	4+3	68	0+8	52
2+9	51	3+7	56	6+0	26	8+2	37
7+6	39	2+6	57	8+4	67	5+2	60
3+8	48	3+5	50	5+4	62	5+0	29
7+7	48	9+5	41	1+1	56	4+1	69
6+6	51	1+4	76	8+1	64	7+0	23
4+7	66	1+5	107	6+4	49	2+1	75
6+8	45	1+7	82	2+3	71	9+1	42
9+7	53	9+2	48	0+9	42	5+1	57
8+8	44	4+5	48	0+6	59	0+2	75
9+3	40	3+4	46	7+2	57	9+0	32
9+8	39	6+2	52	0+1	76	8+0	36
8+9	43	1+6	82	3+1	42	0+0	36
	1195		1479		1460		1253
	22%		27%		27%		23%

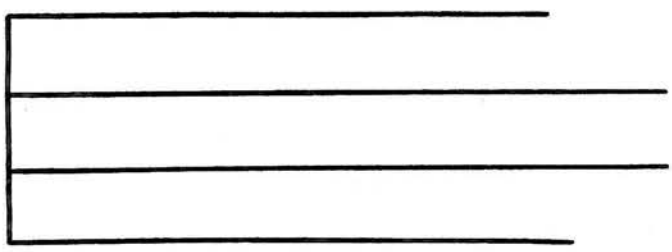


Chart XXXVI.

Osburn II, III and IV Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	16	6-3	22	10-1	9	17-8	13
16-9	14	6-6	39	9-2	27	8-5	31
8-4	33	13-6	20	9-8	40	18-9	16
11-4	12	2-2	69	9-4	36	7-7	33
14-9	19	8-8	35	10-3	12	3-1	21
7-0	20	11-5	19	13-7	14	5-2	31
9-1	17	6-4	25	12-7	18	1-1	68
15-9	21	10-7	21	8-7	25	5-3	19
9-0	27	12-5	17	7-6	22	10-4	25
11-2	11	8-1	16	6-1	16	7-4	35
11-7	21	14-6	20	3-0	26	4-0	33
15-8	22	5-1	17	2-0	30	4-2	36
13-5	13	11-8	24	14-8	17	8-0	15
16-7	12	14-7	11	12-8	19	5-5	45
9-9	33	7-1	16	12-3	14	11-6	23
14-5	12	9-6	29	6-2	34	10-5	19
13-8	24	4-1	17	8-3	29	3-3	76
10-2	12	15-6	15	12-9	13	4-3	20
13-9	19	7-5	36	16-8	19	4-4	72
13-4	11	2-1	26	9-7	42	10-6	29
5-0	23	7-3	18	9-5	29	3-2	34
5-4	34	8-2	23	8-6	27	10-9	18
6-0	22	15-7	16	6-5	38	10-8	22
9-3	24	7-2	27	1-0	28	12-6	21
17-9	29	12-4	15	11-9	19	0-0	22
	501		593		603		777
	20%		24%		24%		31%

Chart XXXVII.

Osburn II, III and IV Multiplication

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	6	8x0	41	4x6	35	2x1	12
9x4	34	7x3	31	4x4	17	2x2	15
9x7	37	7x5	39	4x7	23	6x0	30
8x6	40	6x7	38	3x1	12	3x3	19
8x8	35	9x6	37	6x1	24	8x2	15
7x6	46	6x6	41	3x6	20	8x1	17
0x5	7	5x1	13	2x0	13	1x0	11
0x7	8	6x4	29	6x3	22	3x4	23
6x9	49	7x7	34	5x9	37	1x2	13
7x8	48	7x1	16	2x8	20	3x8	25
4x8	30	4x0	24	9x1	22	4x5	21
0x8	7	0x6	7	5x7	28	1x9	28
7x4	39	9x9	33	3x9	26	1x8	22
9x0	23	9x3	37	4x3	19	2x6	22
8x7	27	3x0	17	3x7	23	1x3	13
7x9	38	4x9	25	2x4	18	7x2	23
8x4	33	7x0	19	6x5	33	1x5	20
6x2	20	9x5	35	5x3	20	2x5	21
0x9	7	8x5	37	5x0	21	3x5	21
9x8	42	0x4	6	1x1	14	2x7	19
0x3	6	9x2	18	4x1	10	1x6	18
6x8	48	3x2	15	5x4	19	1x7	22
0x1	6	5x5	28	2x3	12	1x4	19
8x9	43	5x6	32	2x9	15	5x2	19
8x3	20	5x8	27	4x2	17	0x0	8
	699		676		520		476
	29%		29%		22%		20%

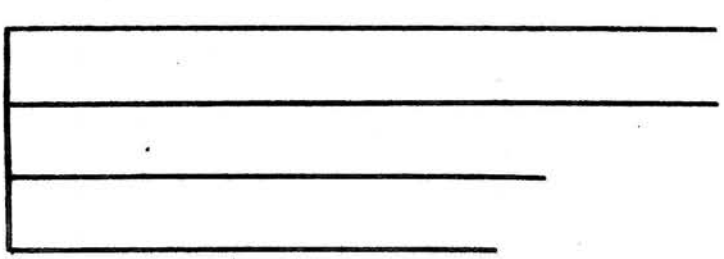


Chart XXXVIII.

Osburn II, III and IV Division

Comparison with Clapp Order of Difficulty - B (devreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
49÷7	16	42÷7	14	12÷3	17	9÷1	13
54÷6	12	54÷9	13	24÷6	20	4÷4	16
45÷9	14	30÷6	11	32÷8	14	0÷3	17
27÷3	12	28÷7	14	8÷4	15	3÷3	13
56÷7	9	24÷3	10	9÷3	17	5÷5	19
28÷4	12	27÷9	16	12÷2	12	0÷4	16
24÷4	10	64÷8	17	16÷8	22	9÷9	28
36÷6	18	81÷9	16	10÷5	16	0÷7	13
48÷6	20	36÷9	14	4÷2	9	0÷6	14
48÷8	18	18÷6	21	12÷6	18	0÷5	14
21÷7	15	63÷7	16	5÷1	7	6÷1	7
18÷2	12	42÷6	16	15÷5	13	0÷8	16
21÷3	12	56÷8	15	18÷9	22	1÷1	8
36÷4	9	16÷2	9	6÷6	21	0÷9	17
72÷9	13	24÷8	17	20÷5	16	8÷1	8
63÷9	16	72÷8	15	2÷2	12	2÷1	8
40÷8	15	16÷4	15	10÷2	10	0÷1	5
32÷4	13	20÷4	11	6÷3	14	8÷2	11
35÷7	13	30÷5	10	40÷5	10	0÷2	10
35÷5	12	14÷7	18	8÷8	22	4÷1	8
18÷3	13	14÷2	9	7÷7	26	7÷1	7
15÷3	11	25÷5	14	12÷4	15	3÷1	8
45÷5	12			6÷2	10		
	307		311		358		276
	24%		25%		29%		22%

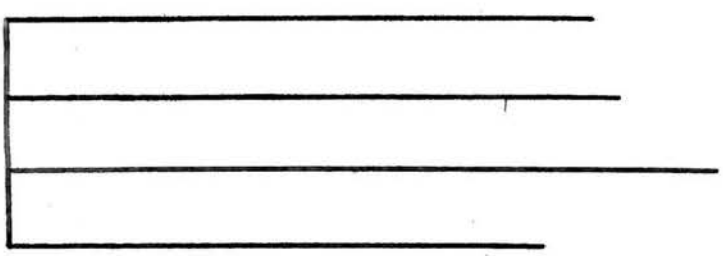
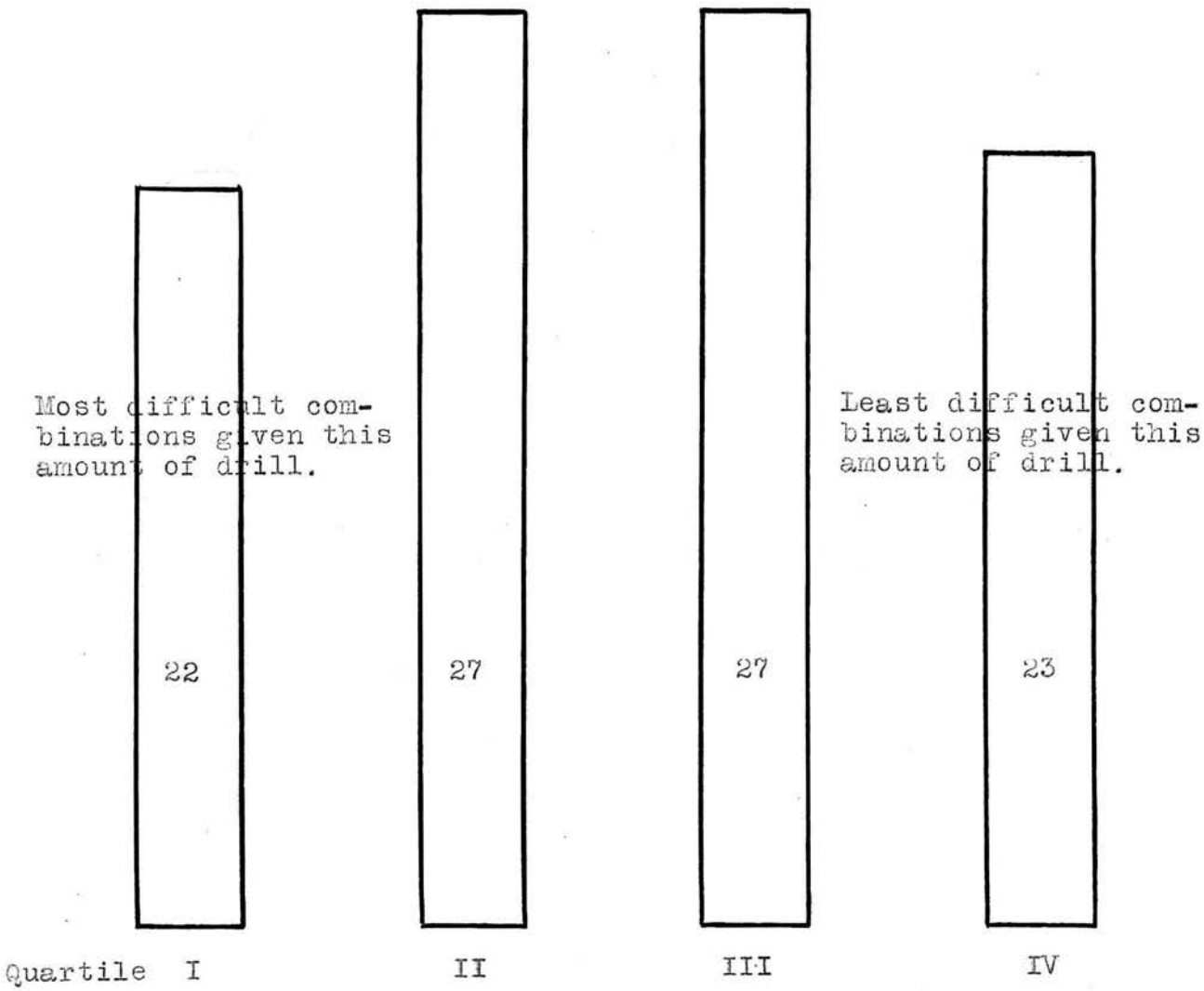


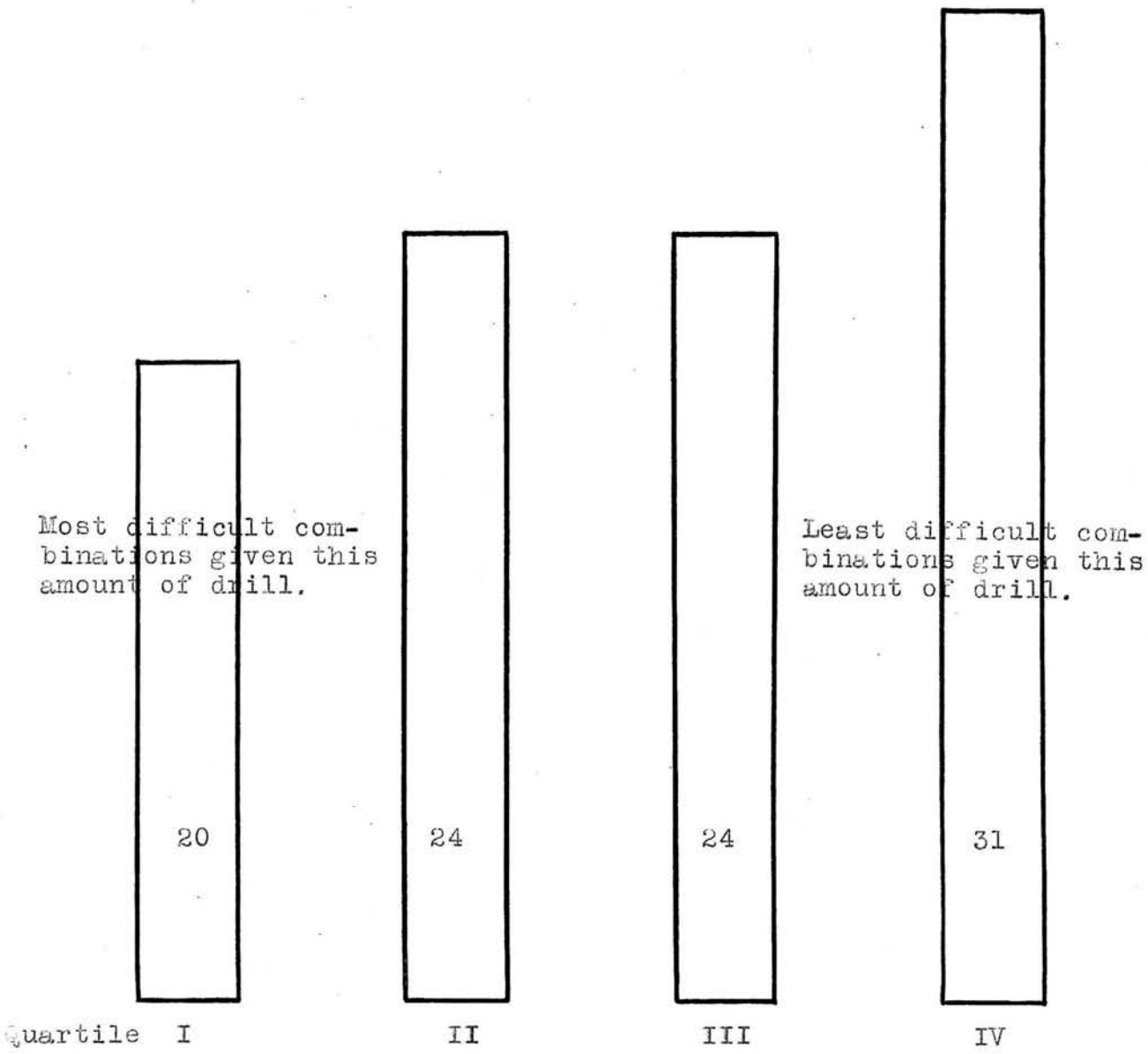
Chart XXXIX.



Osburn II, III and IV Addition.

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

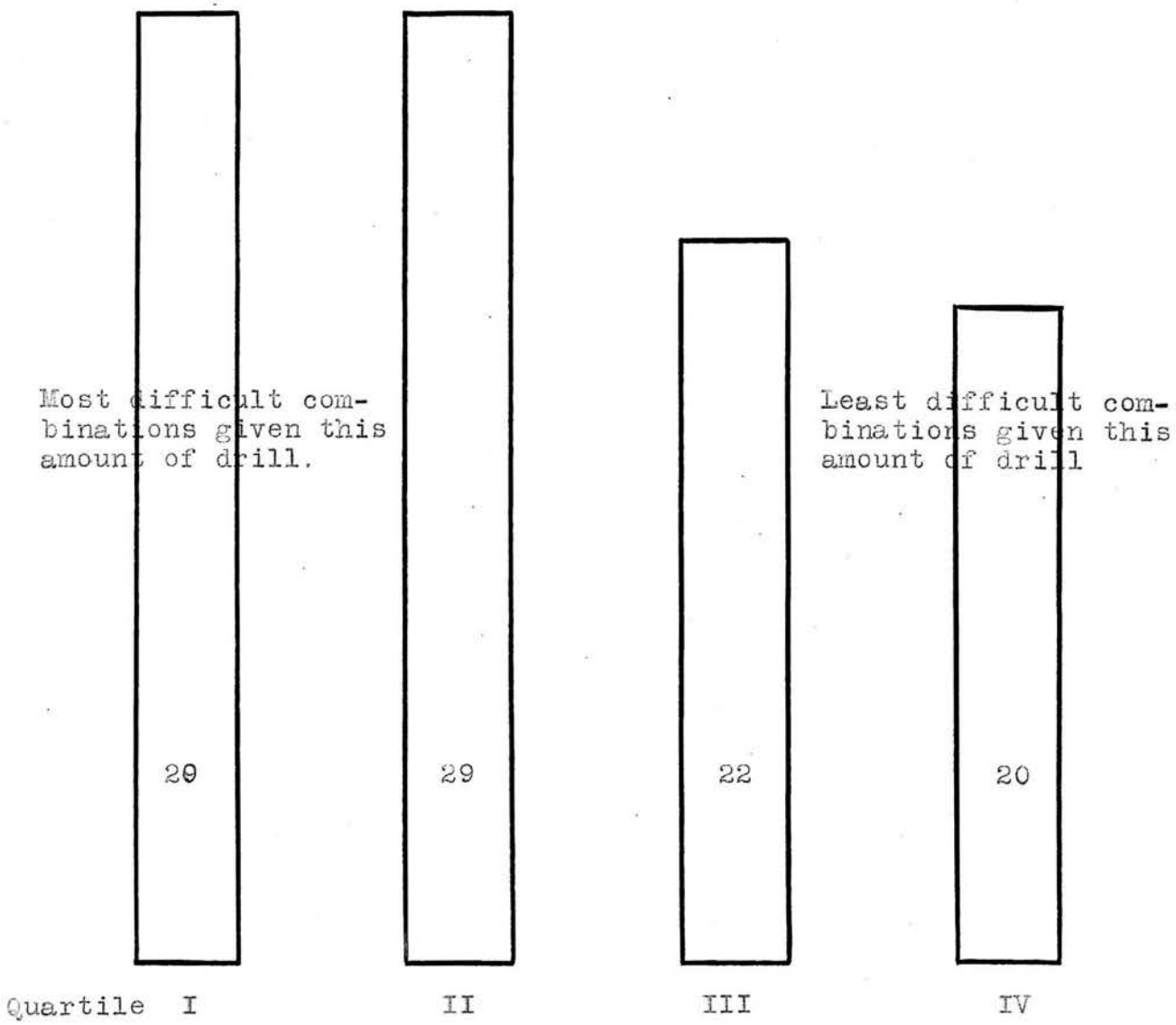
Chart XL.



Osburn II, III and IV Subtraction.

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

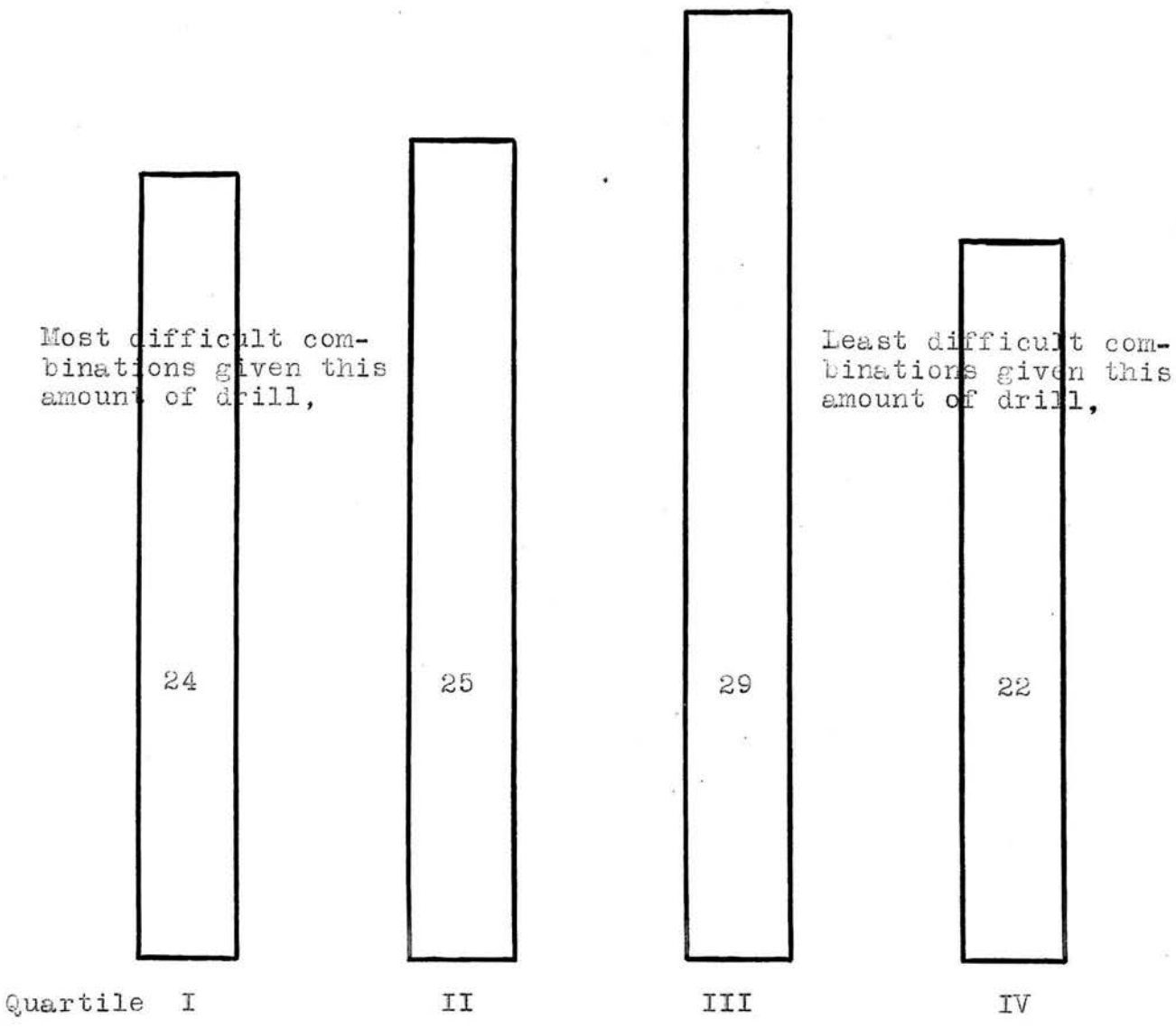
Chart XLI.



Osburn II, III and IV . Multiplication.

Comparisons of percentages of frequency of appearances of combinations with Clapp Order of Difficulty - B.

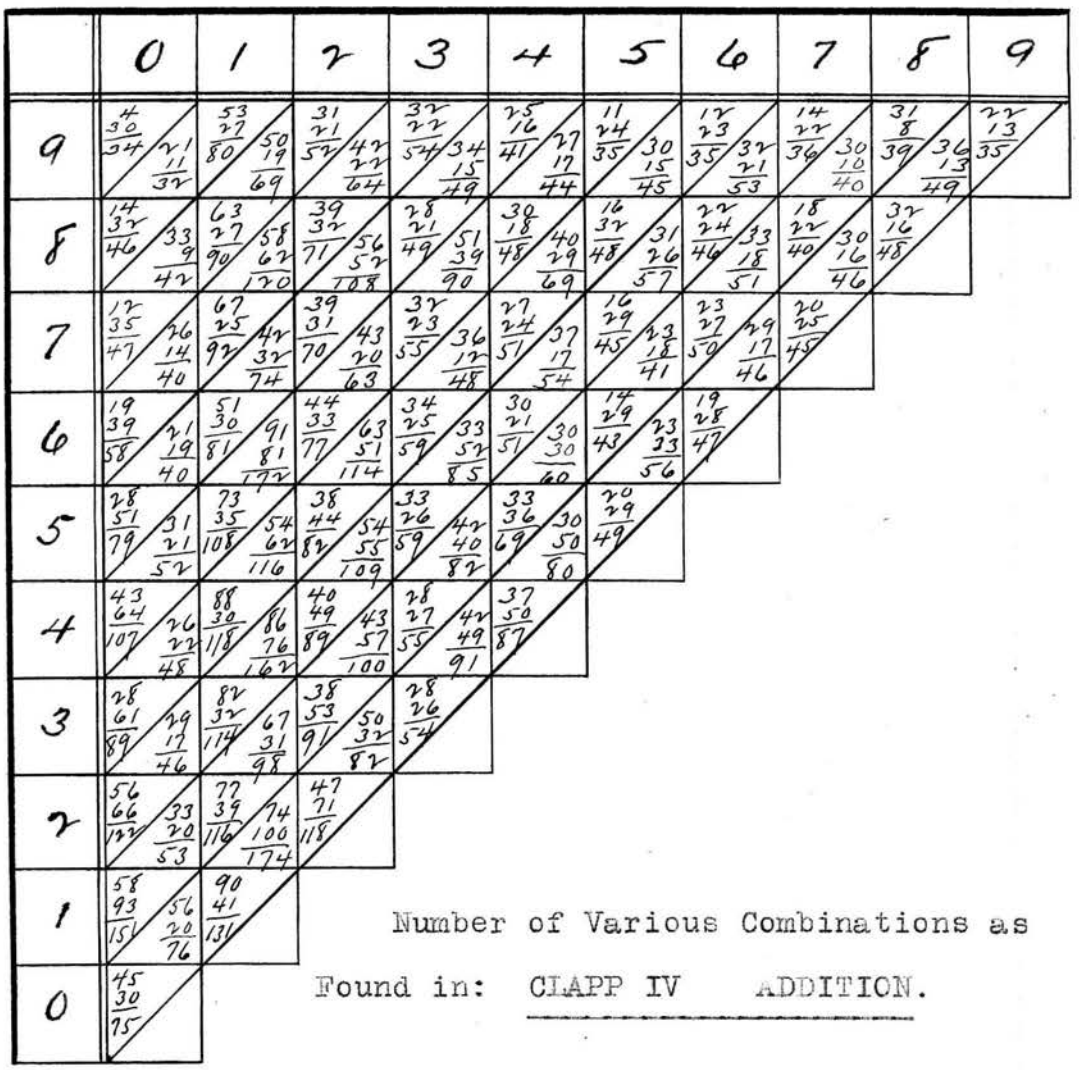
Chart XLII.



Osburn II, III and IV Division.

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

Chart XLIII.



Number of Various Combinations as Found in: CLAPP IV ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

Upper addends represent the number of primary combinations.

Lower addends represent the number of decade combinations.

See page 21.

Chart XLIV.

Clapp IV Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	41	1+3	114	4+6	51	3+2	82
9+6	53	8+7	46	2+7	70	1+0	76
6+7	50	5+7	45	6+3	85	6+1	172
8+6	51	7+8	40	6+5	56	7+1	74
6+9	35	2+2	118	1+2	116	0+7	47
3+9	54	3+6	59	5+3	82	0+4	107
5+8	48	8+3	90	2+5	82	0+3	89
7+9	36	7+4	54	2+4	89	4+2	100
5+9	35	9+4	44	3+3	54	2+0	53
8+5	57	1+9	80	5+6	43	3+0	46
9+9	35	4+4	87	7+3	48	0+5	79
7+5	41	1+8	90	5+5	49	4+0	48
4+8	48	2+8	71	4+3	91	0+8	46
2+9	52	3+7	55	6+0	40	8+2	108
7+6	46	2+6	77	8+4	69	5+2	109
3+8	49	3+5	59	5+4	80	5+0	52
7+7	45	9+5	45	1+1	131	4+1	162
6+6	47	1+4	118	8+1	120	7+0	40
4+7	51	1+5	108	6+4	60	2+1	174
6+8	46	1+7	92	2+3	91	9+1	69
9+7	40	9+2	64	0+9	34	5+1	116
8+8	48	4+5	69	0+6	58	0+2	122
9+3	49	3+4	55	7+2	63	9+0	32
9+8	40	6+2	114	0+1	151	8+0	42
8+9	39	1+6	81	3+1	98	0+0	75
	1136		1875		911		2120
	19%		31%		15%		35%

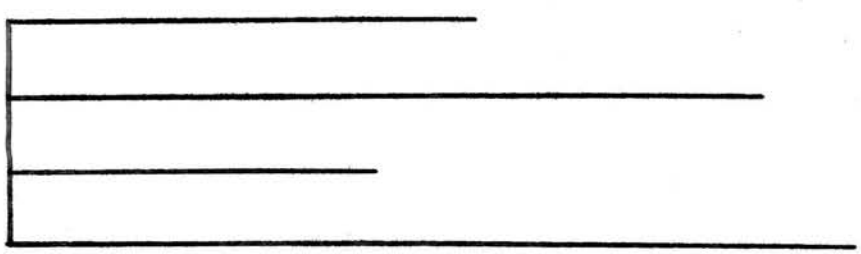


Chart XLV.

	0	1	2	3	4	5	6	7	8	9
9	37 / 40	40 / 29	39 / 39	34 / 34	34 / 42	37 / 47	31 / 46	31 / 48	33 / 70	27
8	61 / 52	39 / 29	50 / 42	36 / 36	53 / 48	39 / 48	41 / 71	44 / 46	42	
7	45 / 50	32 / 35	40 / 47	28 / 35	39 / 56	33 / 61	30 / 65	29		
6	51 / 47	43 / 33	44 / 53	44 / 38	48 / 59	33 / 51	34			
5	47 / 62	35 / 38	28 / 48	38 / 39	32 / 81	32				
4	48 / 68	45 / 35	32 / 66	31 / 55	38					
3	27 / 56	31 / 43	32 / 76	33						
2	46 / 77	27 / 54	40							
1	29 / 66	38								
0	64									

Number of Various Combinations as Found in: CLAPP IV SUBTRACTION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XLVI.

Clapp IV Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	31	6-3	38	10-1	29	17-8	44
16-9	31	6-6	34	9-2	39	8-5	48
8-4	48	13-6	44	9-8	70	18-9	33
11-4	45	2-2	40	9-4	42	7-7	29
14-9	34	8-8	42	10-3	27	3-1	43
7-0	50	11-5	35	13-7	28	5-2	48
9-1	29	6-4	59	12-7	40	1-1	38
15-9	37	10-7	45	8-7	46	5-3	39
9-0	40	12-5	28	7-6	65	10-4	48
11-2	27	8-1	29	6-1	33	7-4	56
11-7	32	14-6	48	3-0	56	4-0	68
15-8	39	5-1	38	2-0	77	4-2	66
13-5	38	11-8	39	14-8	53	8-0	52
16-7	30	14-7	39	12-8	50	5-5	32
9-9	27	7-1	35	12-3	32	11-6	43
14-5	32	9-6	46	6-2	53	10-5	47
13-8	36	4-1	35	8-3	36	3-3	33
10-2	46	15-6	33	12-9	39	4-3	55
13-9	34	7-5	61	16-8	41	4-4	38
13-4	31	2-1	54	9-7	48	10-6	51
5-0	62	7-3	35	9-5	47	3-2	76
5-4	31	8-2	42	8-6	71	10-9	37
6-0	47	15-7	33	6-5	51	10-8	61
9-3	34	7-2	47	1-0	66	12-6	44
17-9	31	12-4	32	11-9	40	0-0	64
	972		1011		1179		1193
	22%		23%		27%		27%

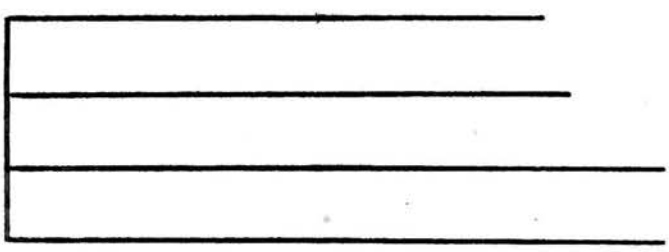


Chart XLVII.

	0	1	2	3	4	5	6	7	8	9
9	9 / 47	41 / 50	41 / 42	43 / 42	44 / 45	40 / 44	44 / 39	40 / 37	36 / 40	34 /
8	14 / 52	55 / 66	55 / 62	52 / 45	44 / 65	51 / 58	38 / 45	46 / 50	44 /	
7	18 / 48	55 / 73	57 / 73	49 / 57	49 / 62	46 / 50	42 / 54	44 /		
6	17 / 48	61 / 74	56 / 76	55 / 53	57 / 65	44 / 51	45 /			
5	25 / 52	64 / 80	59 / 76	55 / 69	50 / 66	50 /				
4	25 / 45	74 / 88	82 / 87	70 / 70	79 /					
3	27 / 59	63 / 84	64 / 72	67 /						
2	31 / 58	69 / 77	72 /							
1	33 / 49	77 /								
0	9 /									

Number of Various Combinations as Found in: CLAPP IV MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart XLVIII.

Clapp IV Multiplication.

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	31	8x0	52	4x6	57	2x1	77
9x4	45	7x3	57	4x4	79	2x2	72
9x7	37	7x5	50	4x7	49	6x0	48
8x6	45	6x7	42	3x1	84	3x3	67
8x8	44	9x6	39	6x1	74	8x2	62
7x6	54	6x6	45	3x6	55	8x1	66
0x5	25	5x1	80	2x0	58	1x0	49
0x7	18	6x4	65	6x3	53	3x4	70
6x9	44	7x7	44	5x9	40	1x2	69
7x8	46	7x1	73	2x8	55	3x8	52
4x8	44	4x0	45	9x1	50	4x5	50
0x8	14	0x6	17	5x7	46	1x9	41
7x4	62	9x9	34	3x9	43	1x8	55
9x0	47	9x3	42	4x3	70	2x6	56
8x7	50	3x0	59	3x7	49	1x3	63
7x9	40	4x9	44	2x4	32	7x2	73
8x4	65	7x0	48	6x5	51	1x5	64
6x2	76	9x5	44	5x3	69	2x5	59
0x9	9	8x5	58	5x0	52	3x5	55
9x8	40	0x4	25	1x1	77	2x7	57
0x3	27	9x2	42	4x1	88	1x6	61
6x8	38	3x2	72	5x4	66	1x7	55
0x1	33	5x5	50	2x3	64	1x4	74
8x9	36	5x6	44	2x9	41	5x2	76
8x3	45	5x8	51	4x2	87	0x0	9
	1015		1222		1539		1480
	19%		23%		29%		28%

Chart XLIX.

Number of Various Combinations as Found in:

CLAPP IV DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors.

	0	1	2	3	4	5	6	7	8	9
9	X	9 30		9 19				49 14		9 13
8	X	8 16	8 20 18 23	18 21	8 22 28 16		18 15 48 19	28 14	8 14 48 15	18 13
7	X	7 16		27 18				7 14		27 13
6	X	6 17	6 20 16 14	6 20	16 18 36 20		6 13 36 15	56 13	16 16 56 13	36 13
5	X	5 18		15 19		5 19 15 13 25 16 35 15 45 18		35 14		45 12
4	X	4 15	4 31 14 17	24 19	4 23 24 18		24 19 54 15	14 16	24 14 64 17	54 16
3	X	3 14		3 20				63 19		63 13
2	X	2 21	2 25 12 19	12 19	12 22 32 20		12 16 42 13	42 11	32 17 72 14	72 14
1	X	1 20		21 12				21 17		81 14
0	X	0 23	0 34 10 21	0 24	0 18 20 17	0 31 10 16 20 15 30 14 40 17	0 30	0 12	0 13 40 15	0 16

Chart L.

Clapp IV Division

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
49÷7	14	42÷7	11	12÷3	19	9÷1	30
54÷6	15	54÷9	16	24÷6	19	4÷4	23
45÷9	12	30÷6	12	32÷8	17	0÷3	24
27÷3	18	28÷7	14	8÷4	22	3÷3	20
56÷7	13	24÷3	19	9÷3	19	5÷5	19
28÷4	16	27÷9	13	12÷2	19	0÷4	18
24÷4	18	64÷8	17	16÷8	16	9÷9	13
36÷6	15	81÷9	14	10÷5	16	0÷7	23
48÷6	19	36÷9	13	4÷2	31	0÷6	13
48÷8	15	18÷6	15	12÷6	16	0÷5	15
21÷7	17	63÷7	19	5÷1	18	6÷1	17
18÷2	23	42÷6	13	15÷5	19	0÷8	13
21÷3	12	56÷8	13	18÷9	13	1÷1	20
36÷4	20	16÷2	14	6÷6	13	0÷9	16
72÷9	14	24÷8	14	20÷5	15	8÷1	16
63÷9	13	72÷8	14	2÷2	25	2÷1	21
40÷8	15	16÷4	18	10÷2	21	0÷1	23
32÷4	17	20÷4	17	6÷3	20	8÷2	20
35÷7	14	30÷5	14	40÷5	15	0÷2	34
35÷5	15	14÷7	16	8÷8	14	4÷1	15
18÷3	21	14÷2	17	7÷7	14	7÷1	16
15÷3	19	25÷5	16	12÷4	22	3÷1	14
45÷5	18			6÷2	20		
	373		329		423		423
	24%		21%		27%		27%

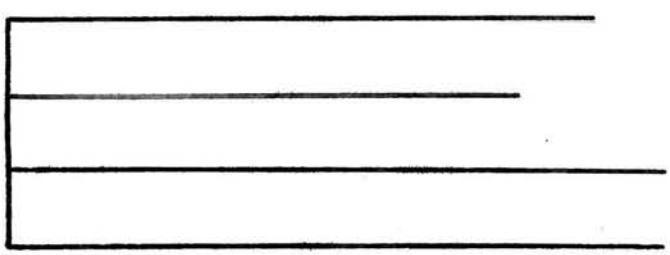
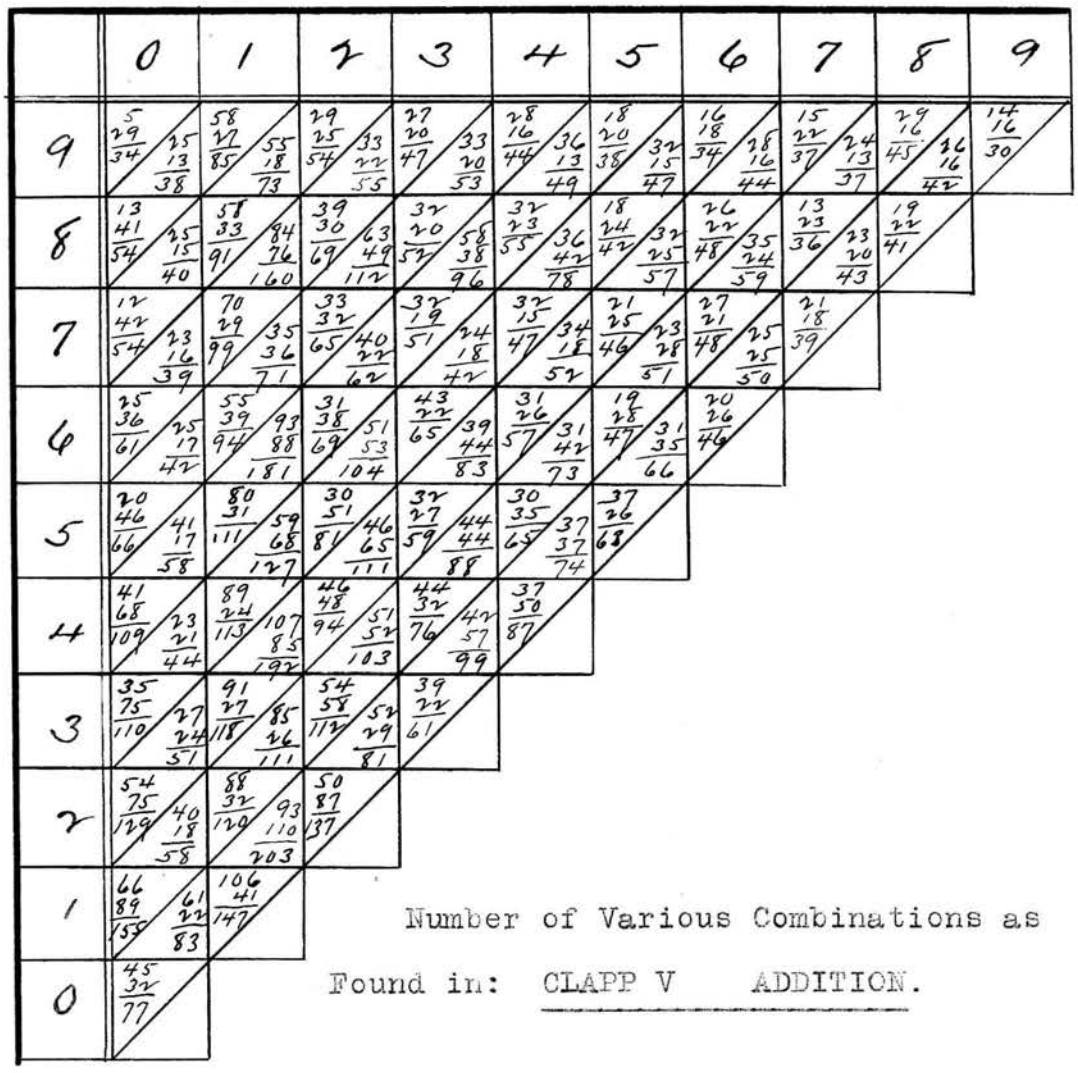


Chart LI.



Number of Various Combinations as

Found in: CLAPP V ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

Upper addends represent the number of primary combinations

Lower addends represent the number of decade combinations

See page 21.

Chart LII.

Clapp V Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	44	1+3	118	4+6	57	3+2	81
9+6	44	8+7	43	2+7	65	1+0	83
6+7	48	5+7	46	6+3	83	6+1	181
8+6	59	7+8	36	6+5	66	7+1	71
6+9	34	2+2	137	1+2	120	0+7	54
3+9	47	3+6	65	5+3	88	0+4	109
5+8	42	8+3	96	2+5	81	0+3	110
7+9	37	7+4	52	2+4	94	4+2	103
5+9	38	9+4	49	3+3	61	2+0	58
8+5	57	1+9	85	5+6	47	3+0	51
9+9	30	4+4	87	7+3	42	0+5	66
7+5	51	1+8	91	5+5	63	4+0	44
4+8	55	2+8	69	4+3	99	0+8	54
2+9	54	3+7	51	6+0	42	8+2	112
7+6	50	2+6	69	8+4	78	5+2	111
3+8	52	3+5	59	5+4	74	5+0	58
7+7	39	9+5	47	1+1	147	4+1	192
6+6	46	1+4	113	8+1	160	7+0	39
4+7	47	1+5	111	6+4	73	2+1	203
6+8	48	1+7	99	2+3	112	9+1	73
9+7	37	9+2	55	0+9	34	5+1	127
8+8	41	4+5	65	0+6	61	0+2	129
9+3	53	3+4	76	7+2	62	9+0	38
9+8	42	6+2	104	0+1	155	8+0	40
8+9	45	1+6	94	3+1	111	0+0	77
	1140		1917		2075		2264
	15%		26%		28%		31%

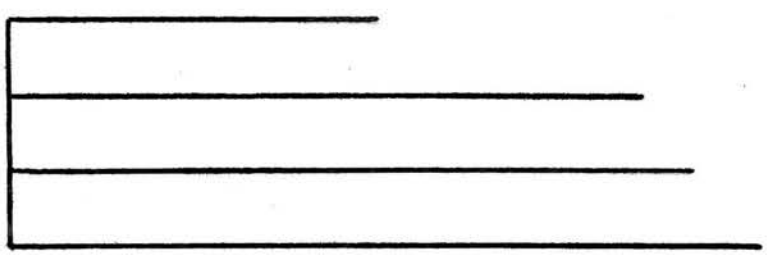


Chart LIII.

	0	1	2	3	4	5	6	7	8	9
9	43 / 40	38 / 32	42 / 42	34 / 33	35 / 47	39 / 40	32 / 53	29 / 43	36 / 66	32 /
8	73 / 47	46 / 29	51 / 51	35 / 39	47 / 42	51 / 55	39 / 70	45 / 58	37 /	
7	40 / 54	42 / 35	38 / 40	30 / 34	39 / 59	36 / 55	29 / 75	28 /		
6	49 / 51	49 / 35	44 / 56	41 / 48	42 / 63	33 / 60	36 /			
5	39 / 63	37 / 41	29 / 54	40 / 47	34 / 96	38 /				
4	55 / 82	44 / 44	33 / 80	33 / 57	37 /					
3	28 / 73	30 / 50	32 / 91	35 /						
2	41 / 90	33 / 65	42 /							
1	31 / 75	37 /								
0	69 /									

Number of Various Combinations as Found in: CLAPP V SUBTRACTION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart LIV.

Clapp V Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	30	6-3	48	10-1	31	17-8	45
16-9	32	6-6	36	9-2	42	8-5	55
8-4	42	13-6	41	9-8	66	18-9	36
11-4	44	2-2	42	9-4	47	7-7	28
14-9	35	8-8	37	10-3	28	3-1	50
7-0	54	11-5	37	13-7	30	5-2	54
9-1	32	6-4	63	12-7	38	1-1	37
15-9	39	10-7	40	8-7	58	5-3	47
9-0	40	12-5	29	7-6	75	10-4	55
11-2	33	8-1	29	6-1	35	7-4	59
11-7	42	14-6	42	3-0	73	4-0	82
15-8	51	5-1	41	2-0	90	4-2	80
13-5	40	11-8	46	14-8	47	8-0	47
16-7	29	14-7	39	12-8	51	5-5	38
9-9	32	7-1	35	12-3	32	11-6	49
14-5	34	9-6	53	6-2	56	10-5	39
13-8	35	4-1	44	8-3	39	3-3	35
10-2	41	15-6	33	12-9	42	4-3	57
13-9	34	7-5	55	16-8	39	4-4	37
13-4	33	2-1	65	9-7	43	10-6	49
5-0	63	7-3	34	9-5	40	3-2	91
5-4	96	8-2	51	8-6	70	10-9	43
6-0	61	15-7	36	6-5	60	10-8	73
9-3	53	7-2	40	1-0	75	12-6	44
17-9	29	12-4	33	11-9	38	0-0	69
	1024		1049		1245		1299
	22%		23%		27%		28%

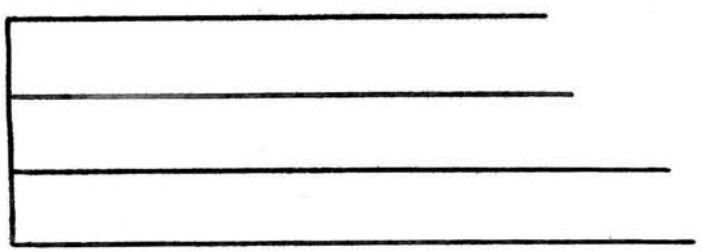


Chart LV.

	0	1	2	3	4	5	6	7	8	9
9	13 / 48	47 / 60	43 / 52	46 / 37	47 / 49	39 / 47	44 / 39	41 / 40	40 / 35	42
8	14 / 64	56 / 83	61 / 73	50 / 51	54 / 70	46 / 61	46 / 52	43 / 52	52	
7	11 / 55	66 / 79	61 / 75	55 / 59	53 / 64	58 / 50	48 / 55	48		
6	16 / 55	65 / 84	83 / 81	62 / 62	58 / 69	57 / 53	53			
5	20 / 63	78 / 102	79 / 83	67 / 73	55 / 78	63				
4	24 / 67	95 / 111	87 / 106	77 / 92	89					
3	17 / 58	87 / 107	86 / 90	83						
2	20 / 95	96 / 127	117							
1	28 / 63	98								
0	11									

Number of Various Combinations as Found in: CLAPP V MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart LVI

Clapp V Multiplication

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	20	8x0	64	4x6	58	2x1	127
9x4	49	7x3	59	4x4	89	2x2	117
9x7	40	7x5	50	4x7	53	6x0	55
8x6	52	6x7	48	3x1	107	3x3	83
8x8	52	9x6	39	6x1	84	8x2	73
7x6	55	6x6	53	3x6	62	8x1	83
0x5	20	5x1	102	2x0	95	1x0	63
0x7	11	6x4	69	6x3	62	3x4	77
6x9	44	7x7	48	5x9	39	1x2	96
7x8	43	7x1	79	2x8	61	3x8	50
4x8	54	4x0	67	9x1	60	4x5	55
0x8	14	0x6	16	5x7	58	1x9	47
7x4	64	9x9	42	3x9	46	1x8	56
9x0	48	9x3	37	4x3	92	2x6	83
8x7	52	3x0	58	3x7	55	1x3	87
7x9	41	4x9	47	2x4	87	7x2	75
8x4	70	7x0	55	6x5	53	1x5	78
6x2	81	9x5	47	5x3	73	2x5	79
0x9	13	8x5	61	5x0	63	3x5	67
9x8	35	0x4	24	1x1	98	2x7	81
0x3	17	9x2	52	4x1	111	1x6	65
6x8	46	3x2	90	5x4	78	1x7	66
0x1	28	5x5	63	2x3	86	1x4	95
8x9	40	5x6	57	2x9	43	5x2	83
8x3	51	5x8	46	4x2	106	0x0	11
	1040		1373		1819		1832
	17%		23%		30%		30%

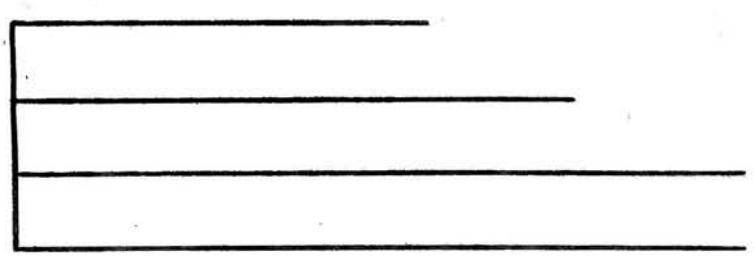


Chart LVII.

Number of Various Combinations as Found in:

CLAPP V DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors.

	0	1	2	3	4	5	6	7	8	9
9		9 11		9 26				49 14		9 18
8		8 11	8 43 18 11	18 21	8 30 28 15		18 18 48 15	28 13	8 25 48 14	18 17
7		7 20		27 6				7 15		27 20
6		6 21	6 38 16 16	6 33	16 30 36 15		6 26 36 17	56 17	16 24 56 15	36 17
5		5 20		15 22		5 33 15 19 25 19 35 19 45 11		35 18		45 14
4		4 23	4 44 14 22	24 19	4 38 24 24		24 19 54 13	14 14	24 18 64 19	54 16
3		3 21		3 37				63 12		63 14
2		2 24	2 43 12 21	12 32	12 33 32 22		12 22 42 16	42 15	32 18 72 15	72 14
1		1 26		21 24				21 19		81 14
0		0 32	0 31 10 43	0 29	0 19 20 29	0 17 10 34 20 18 30 24 40 18	0 20 30 14	0 14	0 20 40 18	0 17

Chart LVIII.

Clapp V Division

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq.
49÷7	14	42÷7	15	12÷3	32	9÷1	14
54÷6	13	54÷9	16	24÷6	19	4÷4	38
45÷9	14	30÷6	14	32÷8	18	0÷3	29
27÷3	6	28÷7	13	8÷4	30	3÷3	37
56÷7	17	24÷3	19	9÷3	26	5÷5	30
28÷4	15	27÷9	20	12÷2	21	0÷4	19
24÷4	24	64÷8	19	16÷8	24	9÷9	18
36÷6	17	81÷9	14	10÷5	34	0÷7	14
48÷6	15	36÷9	17	4÷2	44	0÷6	20
48÷8	14	18÷6	18	12÷6	22	0÷5	17
21÷7	19	63÷7	12	5÷1	20	6÷1	21
18÷2	11	42÷6	16	15÷5	19	0÷8	20
21÷3	24	56÷8	15	18÷9	17	1÷1	26
36÷4	15	16÷2	16	6÷6	26	0÷9	17
72÷9	14	24÷8	18	20÷5	18	8÷1	11
63÷9	14	72÷8	15	2÷2	43	2÷1	24
40÷8	18	16÷4	30	10÷2	31	0÷1	31
32÷4	22	20÷4	29	6÷3	33	8÷2	43
35÷7	18	30÷5	21	40÷5	18	0÷2	31
35÷5	19	14÷7	14	8÷8	25	4÷1	23
18÷3	20	14÷2	22	7÷7	16	7÷1	20
15÷3	22	25÷5	19	12÷4	33	3÷1	21
45÷5	11			6÷2	38		
	376		392		607		524
	20%		21%		32%		27%

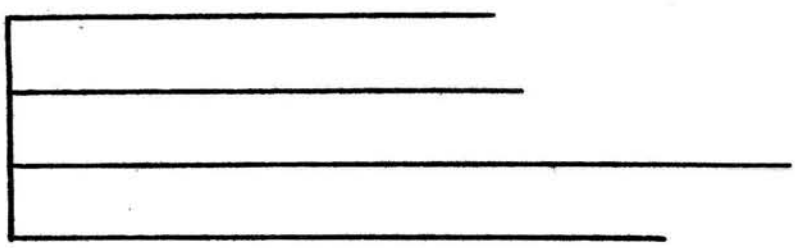


Chart LIX

Clapp IV and V Addition

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	85	1+3	232	4+6	108	3+2	163
9+6	97	8+7	89	2+7	135	1+0	159
6+7	98	5+7	92	6+3	168	6+1	353
8+6	110	7+8	76	6+5	122	7+1	145
6+9	69	2+2	255	1+2	236	0+7	101
3+9	101	3+6	124	5+3	170	0+4	216
5+8	90	8+3	186	2+5	183	0+3	199
7+9	73	7+4	106	2+4	163	4+2	203
5+9	73	9+4	93	3+3	115	2+0	111
8+5	114	1+9	165	5+6	90	3+0	97
9+9	65	4+4	174	7+3	90	0+5	145
7+5	92	1+8	181	5+5	112	4+0	92
4+8	103	2+8	140	4+3	190	0+8	100
2+9	106	3+7	106	6+0	82	8+2	220
7+6	96	2+6	146	8+4	147	5+2	220
3+8	101	3+5	118	5+4	154	5+0	110
7+7	84	9+5	92	1+1	278	4+1	354
6+6	93	1+4	231	8+1	280	7+0	79
4+7	98	1+5	219	6+4	133	2+1	377
6+8	94	1+7	191	2+3	203	9+1	142
9+7	77	9+2	119	0+9	68	5+1	243
8+8	89	4+5	134	0+6	119	0+2	251
9+3	102	3+4	131	7+2	125	9+0	70
9+8	82	6+2	218	0+1	306	8+0	82
8+9	84	1+6	175	3+1	209	0+0	152
	2276		3792		2986		4384
	17%		28%		22%		33%

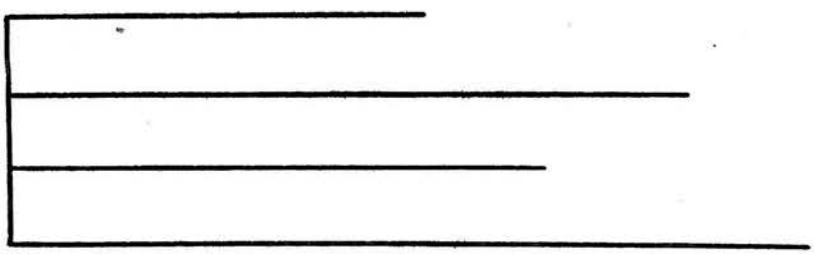


Chart LX.

Clapp IV and V Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	61	6-3	86	10-1	60	17-8	89
16-9	63	6-6	70	9-2	81	8-5	103
8-4	90	13-6	85	9-8	136	18-9	69
11-4	89	2-2	82	9-4	89	7-7	57
14-9	69	8-8	79	10-3	55	3-1	93
7-0	104	11-5	72	13-7	58	5-2	102
9-1	61	6-4	122	12-7	78	1-1	75
15-9	76	10-7	85	8-7	104	5-3	86
9-0	80	12-5	57	7-6	140	10-4	103
11-2	60	8-1	58	6-1	68	7-4	115
11-7	74	14-6	90	3-0	129	4-0	150
15-8	90	5-1	79	2-0	167	4-2	146
13-5	78	11-8	85	14-8	100	8-0	99
16-7	59	14-7	78	12-8	101	5-5	70
9-9	59	7-1	70	12-3	64	11-6	92
14-5	66	9-6	99	6-2	109	10-5	86
13-8	71	4-1	79	8-3	75	3-3	68
10-2	87	15-6	66	12-9	81	4-3	112
13-9	68	7-5	116	16-8	80	4-4	75
13-4	64	2-1	119	9-7	91	10-6	100
5-0	125	7-3	69	9-5	87	3-2	167
5-4	177	8-2	93	8-6	141	10-9	80
6-0	98	15-7	69	6-5	111	10-8	134
9-3	67	7-2	87	1-0	141	12-6	88
17-9	60	12-4	65	11-9	78	0-0	133
	1996		2060		2424		2492
	22%		23%		27%		28%

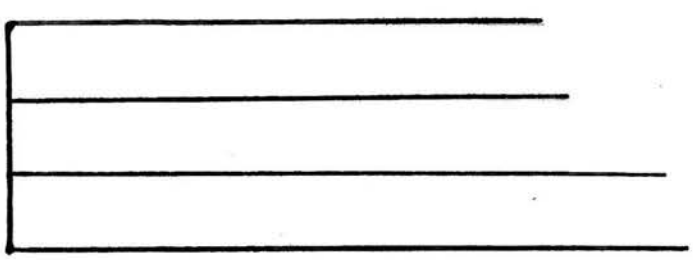


Chart LXI.

Clapp IV and V Multiplication.

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
0x2	51	8x0	116	4x6	115	2x1	204
9x4	94	7x3	116	4x4	168	2x2	189
9x7	77	7x5	100	4x7	102	6x0	103
8x6	97	6x7	90	3x1	191	3x3	150
8x8	96	9x6	78	6x1	158	8x2	135
7x6	109	6x6	98	3x6	117	8x1	149
0x5	45	5x1	182	2x0	153	1x0	112
0x7	29	6x4	134	6x3	115	3x4	147
6x9	88	7x7	92	5x9	79	1x2	165
7x8	89	7x1	152	2x8	116	3x8	102
4x8	98	4x0	112	9x1	110	4x5	105
0x8	28	0x6	33	5x7	104	1x9	88
7x4	126	9x9	76	3x9	89	1x8	111
9x0	95	9x3	79	4x3	162	2x6	139
8x7	102	3x0	117	3x7	104	1x3	150
7x9	81	4x9	91	2x4	169	7x2	148
8x4	135	7x0	103	6x5	104	1x5	142
6x2	157	9x5	91	5x3	142	2x5	138
0x9	22	8x5	119	5x0	115	3x5	122
9x8	75	0x4	49	1x1	175	2x7	118
0x3	44	9x2	94	4x1	199	1x6	126
6x8	84	3x2	162	5x4	144	1x7	121
0x1	61	5x5	113	2x3	150	1x4	169
8x9	76	5x6	101	2x9	84	5x2	159
8x3	96	5x8	97	4x2	193	0x0	20
	2055		2595		3358		3312
	18%		23%		30%		29%

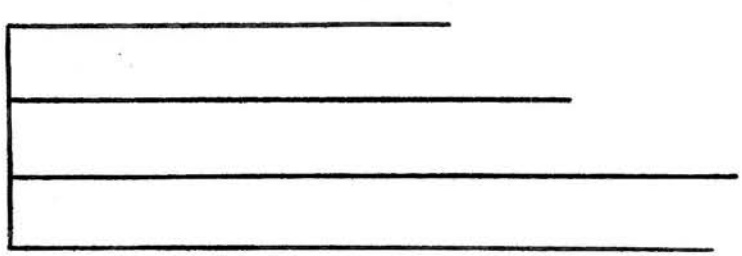


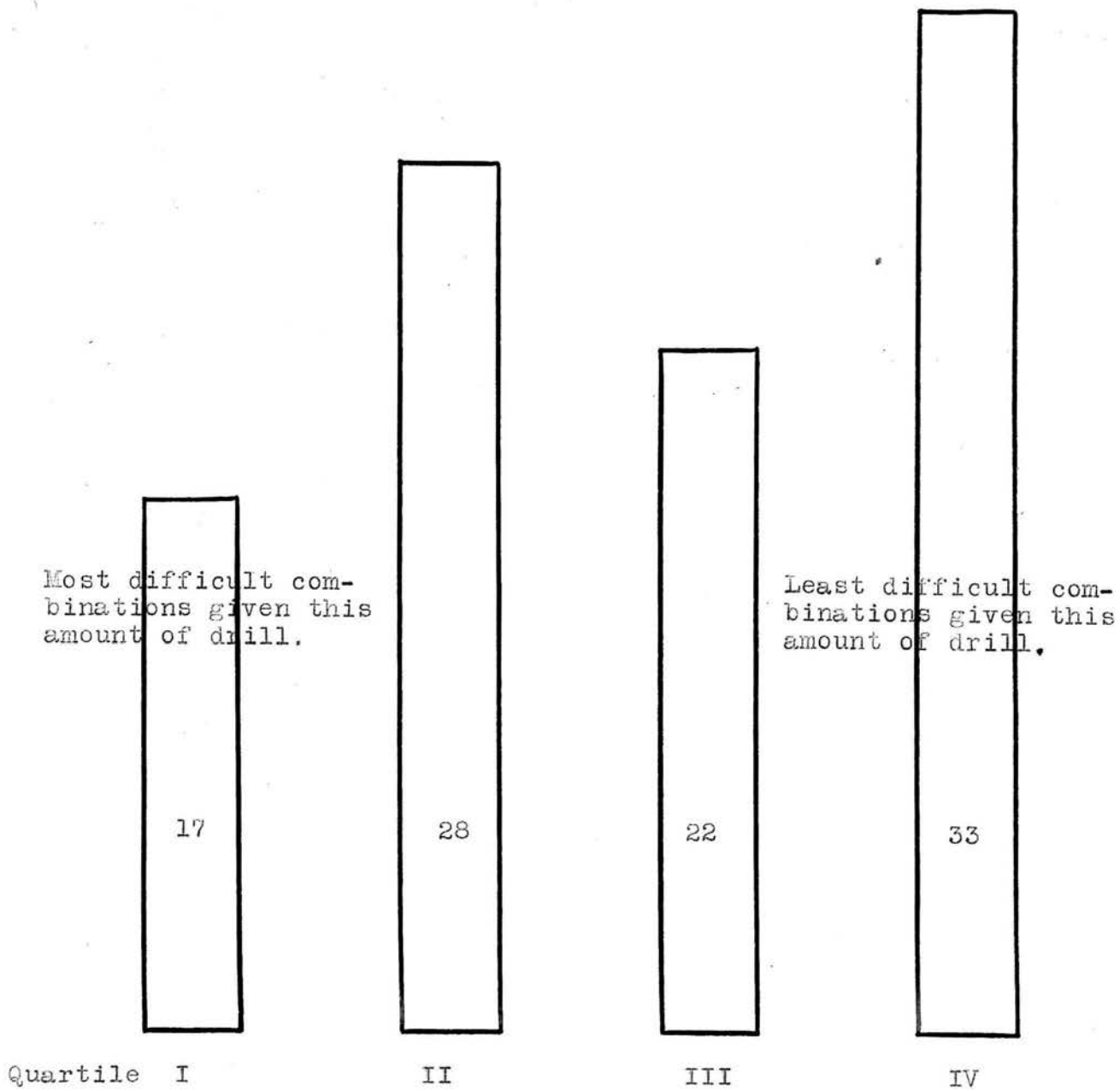
Chart LXII.

Clapp IV and V Division

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq.
49÷7	28	42÷7	26	12÷3	41	9÷1	44
54÷6	28	54÷9	32	24÷6	25	4÷4	61
45÷9	26	30÷6	26	32÷8	35	0÷3	53
27÷3	24	28÷7	27	8÷4	52	3÷3	57
56÷7	30	24÷3	38	9÷3	45	5÷5	49
28÷4	31	27÷9	33	12÷2	40	0÷4	37
24÷4	42	64÷8	36	16÷8	40	9÷9	31
36÷6	32	81÷9	28	10÷5	50	0÷7	37
48÷6	34	36÷9	30	4÷2	75	0÷6	33
48÷8	29	18÷6	33	12÷6	38	0÷5	32
21÷7	36	63÷7	31	5÷1	38	6÷1	38
18÷2	34	42÷6	29	15÷5	38	0÷8	33
21÷3	36	56÷8	28	18÷9	30	1÷1	46
36÷4	35	16÷2	30	6÷6	39	0÷9	33
72÷9	28	24÷8	32	20÷5	33	8÷1	27
63÷9	27	72÷8	29	2÷2	68	2÷1	45
40÷8	33	16÷4	48	10÷2	52	0÷1	54
32÷4	39	20÷4	46	6÷3	53	8÷2	63
35÷7	32	30÷5	35	40÷5	33	0÷2	65
35÷5	34	14÷7	30	8÷8	39	4÷1	38
18÷3	41	14÷2	39	7÷7	30	7÷1	36
15÷3	41	25÷5	36	12÷4	55	3÷1	35
45÷5	29			6÷2	58		
	749		721		1030		947
	22%		21%		30%		27%

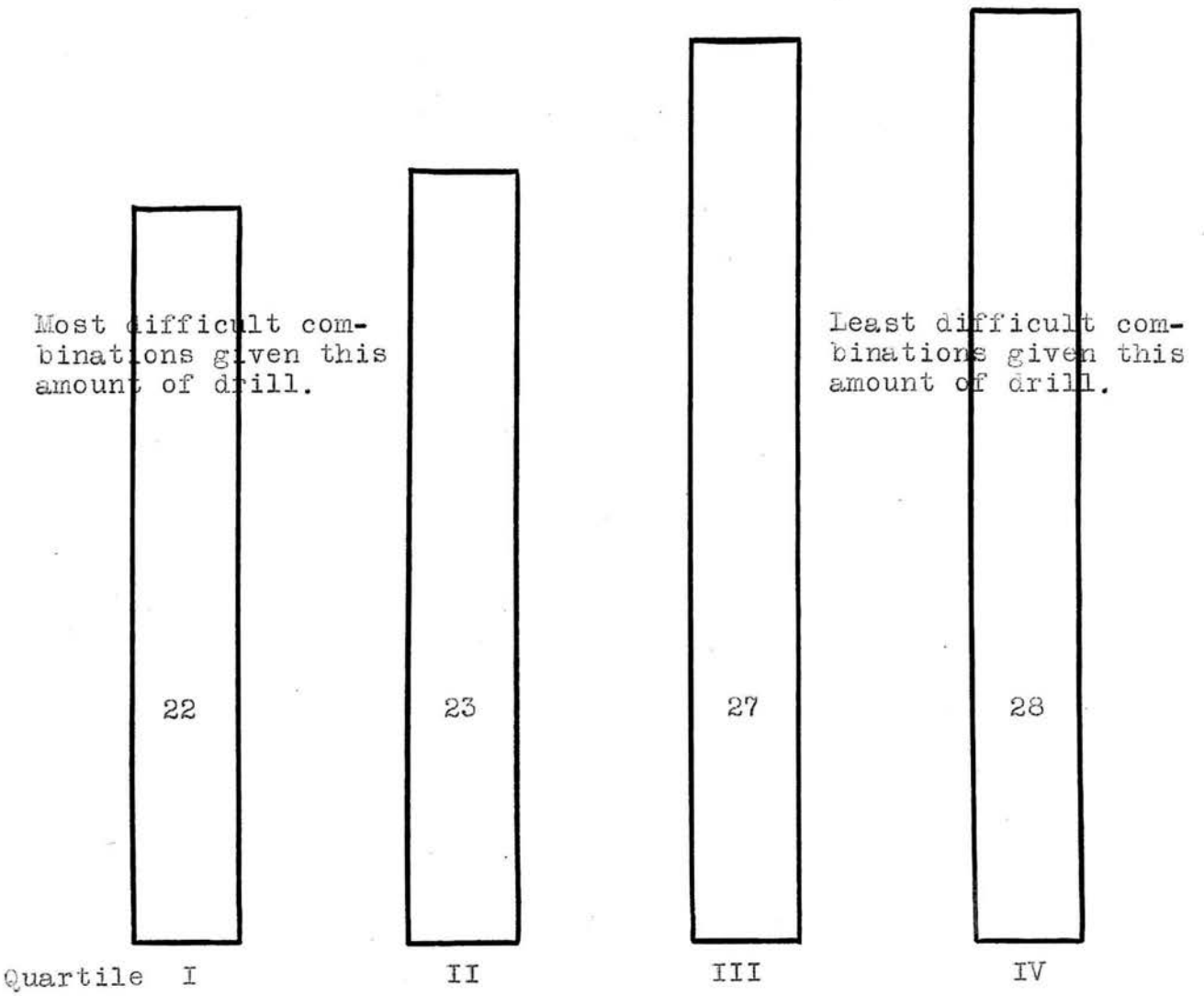
Chart LXIII.



Clapp IV and V Addition.

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

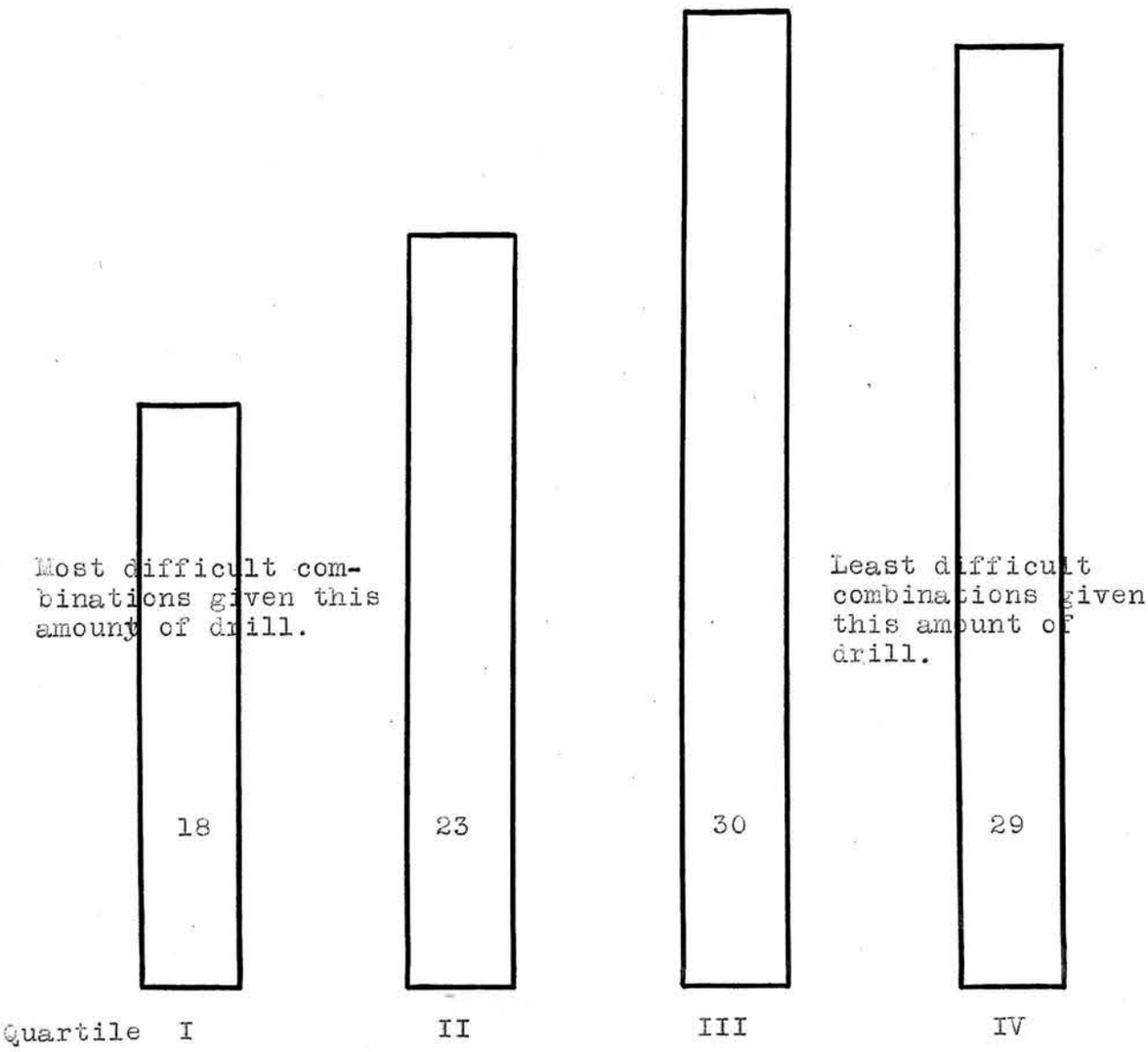
Chart LXIV.



Clapp IV and V Subtraction

Comparisons of Percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

Chart LXV.

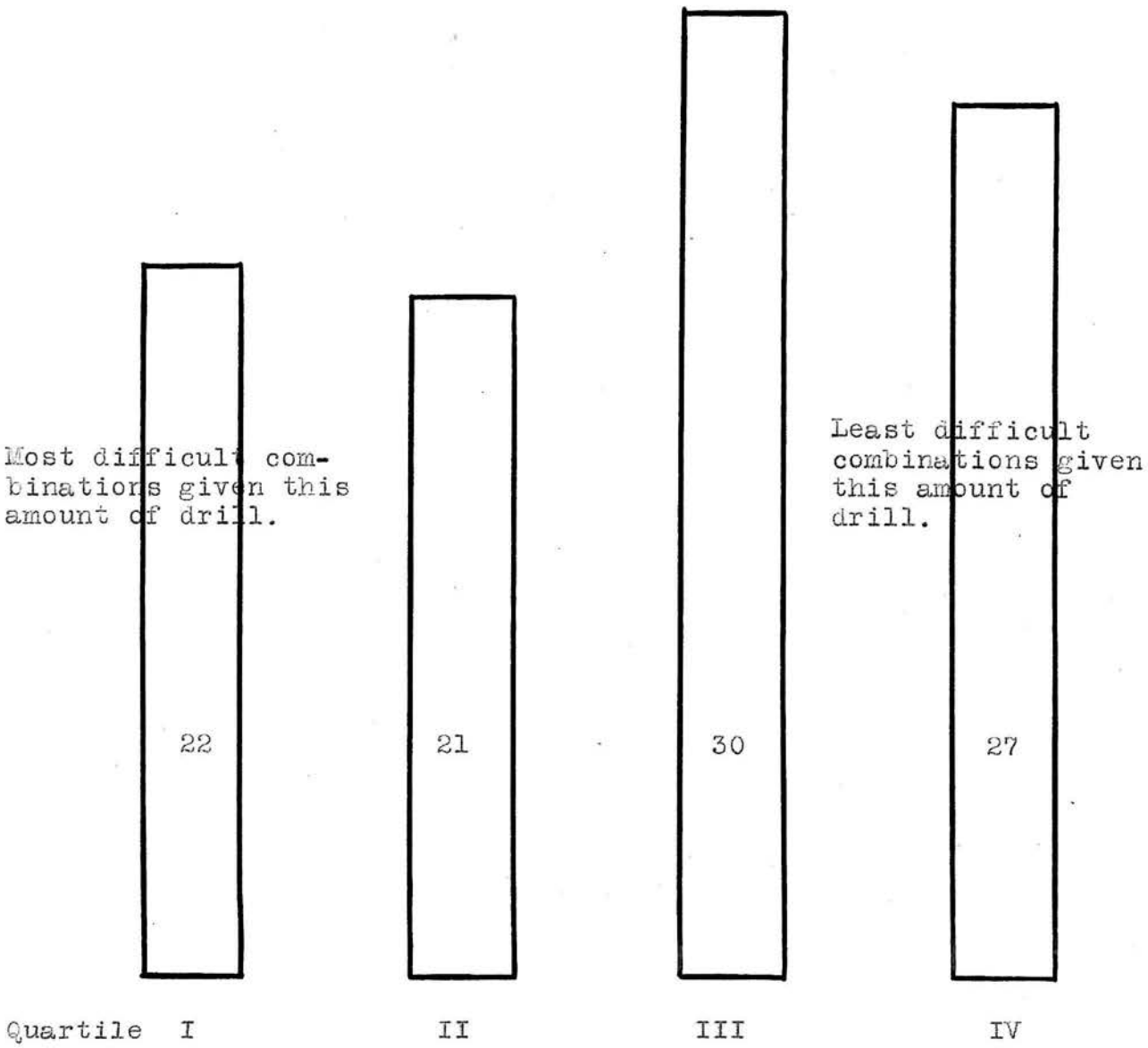


Clapp IV and V, Multiplication

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty -B.

Chart LXVI

Chart LXVI



Clapp IV and V Division.

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

Chart LXVII.

	0	1	2	3	4	5	6	7	8	9
9	2 / 10 / 12 — 17 / 2 / 19	24 / 6 / 30 — 14 / 7 / 21	18 / 10 / 28 — 16 / 5 / 21	11 / 7 / 18 — 8 / 7 / 15	13 / 5 / 18 — 9 / 8 / 17	7 / 5 / 12 — 14 / 4 / 18	13 / 4 / 17 — 9 / 8 / 17	10 / 2 / 12 — 14 / 4 / 18	17 / 3 / 20 — 11 / 3 / 14	
8	4 / 8 / 12 — 9 / 1 / 10	23 / 9 / 32 — 28 / 22 / 50	19 / 10 / 29 — 19 / 35 / 54	9 / 12 / 21 — 18 / 25 / 43	7 / 10 / 17 — 16 / 19 / 35	9 / 6 / 15 — 12 / 16 / 28	15 / 10 / 25 — 15 / 11 / 26	13 / 8 / 21 — 19 / 9 / 28	15 / 6 / 21	
7	7 / 15 / 22 — 6 / 4 / 10	21 / 11 / 32 — 20 / 7 / 27	14 / 9 / 23 — 20 / 7 / 27	7 / 9 / 16 — 12 / 8 / 20	8 / 11 / 19 — 15 / 6 / 21	9 / 9 / 18 — 18 / 6 / 24	11 / 10 / 21 — 15 / 6 / 21	14 / 9 / 23		
6	6 / 9 / 15 — 15 / 4 / 19	28 / 9 / 37 — 33 / 22 / 55	15 / 14 / 29 — 26 / 19 / 45	11 / 8 / 19 — 20 / 15 / 35	5 / 19 / 24 — 10 / 28 / 38	13 / 13 / 26 — 14 / 14 / 28	13 / 17 / 30			
5	4 / 17 / 21 — 4 / 4 / 8	18 / 11 / 29 — 17 / 14 / 31	16 / 18 / 34 — 20 / 28 / 48	13 / 7 / 20 — 19 / 13 / 32	13 / 12 / 25 — 14 / 20 / 34	12 / 10 / 22				
4	3 / 26 / 29 — 6 / 5 / 11	22 / 7 / 29 — 23 / 21 / 44	15 / 19 / 34 — 16 / 20 / 36	13 / 11 / 24 — 21 / 22 / 43	17 / 24 / 41					
3	5 / 24 / 29 — 8 / 4 / 12	24 / 9 / 33 — 17 / 3 / 20	17 / 10 / 27 — 22 / 7 / 29	17 / 10 / 27						
2	5 / 17 / 22 — 9 / 5 / 14	26 / 11 / 37 — 27 / 37 / 64	18 / 24 / 42							
1	5 / 18 / 23 — 13 / 6 / 19	26 / 9 / 35								
0	4 / 8 / 12									

Number of Various Combinations as Found in: HILLEGAS ADDITION.

Below diagonals: Read left to top.

Above diagonals: Read topto left.

Upper addends represent the number of primary combinations.

Lower addends represent the number of decade combinations.

See page 21.

Chart LXVIII.

Hillegas Addition

Comparison with Clapp Order of Difficulty - B.(decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
4+9	18	1+3	33	4+6	24	3+2	29
9+6	17	8+7	28	2+7	23	1+0	19
6+7	21	5+7	18	6+3	35	6+1	55
8+6	26	7+8	21	6+5	28	7+1	22
6+9	17	2+2	42	1+2	37	0+7	22
3+9	18	3+6	19	5+3	32	0+4	29
5+8	15	8+3	43	2+5	34	0+3	29
7+9	12	7+4	21	2+4	34	4+2	36
5+9	12	9+4	17	3+3	27	2+0	14
8+5	28	1+9	30	5+6	26	3+0	12
9+9	15	4+4	41	7+3	20	0+5	21
7+5	24	1+8	32	5+5	32	4+0	11
4+8	17	2+8	29	4+3	43	0+8	12
2+9	28	3+7	16	6+0	19	8+2	54
7+6	21	2+6	29	8+4	35	5+2	48
3+8	21	3+5	20	5+4	34	5+0	8
7+7	23	9+5	18	1+1	35	4+1	44
6+6	30	1+4	29	8+1	50	7+0	10
4+7	19	1+5	29	6+4	38	2+1	64
6+8	25	1+7	32	2+3	27	9+1	21
9+7	18	9+2	21	0+9	12	5+1	31
8+8	21	4+5	25	0+6	15	0+2	22
9+3	15	3+4	24	7+2	27	9+0	19
9+8	14	6+2	45	0+1	23	8+0	10
8+9	20	1+6	37	3+1	20	0+0	12
	495		699		720		654
	19%		27%		28%		25%

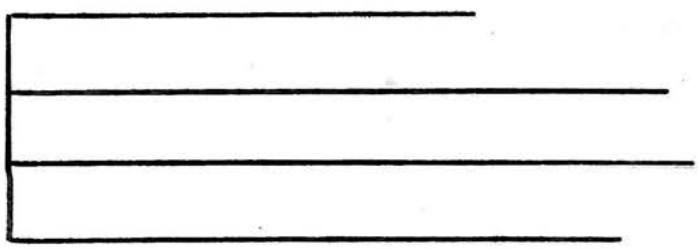


Chart LXX.

Hillegas Subtraction

Comparison with Clapp Order of Difficulty - B (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq
11-3	23	6-3	35	10-1	26	17-8	15
16-9	14	6-6	30	9-2	27	8-5	18
8-4	40	13-6	10	9-8	17	18-9	19
11-4	13	2-2	33	9-4	17	7-7	18
14-9	14	8-8	14	10-3	30	3-1	18
7-0	6	11-5	20	13-7	11	5-2	45
9-1	26	6-4	40	12-7	12	1-1	31
15-9	11	10-7	14	8-7	23	5-3	26
9-0	10	12-5	27	7-6	15	10-4	26
11-2	33	8-1	42	6-1	57	7-4	30
11-7	23	14-6	25	3-0	7	4-0	9
15-8	12	5-1	30	2-0	7	4-2	41
13-5	10	11-8	18	14-8	9	8-0	9
16-7	14	14-7	18	12-8	8	5-5	21
9-9	13	7-1	19	12-3	21	11-6	27
14-5	19	9-6	19	6-2	47	10-5	18
13-8	14	4-1	41	8-3	47	3-3	21
10-2	23	15-6	19	12-9	12	4-3	39
13-9	12	7-5	26	16-8	14	4-4	35
13-4	15	2-1	54	9-7	18	10-6	13
5-0	6	7-3	24	9-5	19	3-2	26
5-4	31	8-2	56	8-6	24	10-9	10
6-0	12	15-7	12	6-5	27	10-8	7
9-3	22	7-2	33	1-0	11	12-6	14
17-9	14	12-4	24	11-9	15	6-0	7
	430		673		521		543
	20%		31%		24%		25%

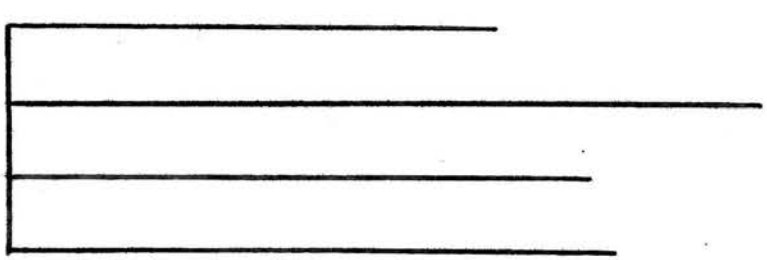


Chart LXXI.

	0	1	2	3	4	5	6	7	8	9
9	13 / 7	21 / 25	34 / 28	38 / 32	38 / 33	32 / 32	42 / 32	44 / 34	35 / 37	39
8	11 / 7	26 / 25	33 / 28	36 / 28	38 / 26	29 / 33	45 / 39	38 / 38	38
7	10 / 6	28 / 30	29 / 25	38 / 28	41 / 33	29 / 34	30 / 36	45
6	11 / 5	22 / 31	36 / 23	34 / 31	34 / 32	29 / 29	34
5	11 / 6	21 / 27	25 / 21	32 / 22	33 / 27	26
4	12 / 24	22 / 37	37 / 46	37 / 25	30
3	9 / 25	20 / 40	43 / 38	45
2	5 / 21	16 / 28	37
1	4 / 4	14
0	4

Number of Various Combinations as Found in: HILLEGAS MULTIPLICATION.

Below diagonals: Read left to top.

Above diagonals: Read top to left.

See page 21.

Chart LXXII.

Hillegas Multiplication

Comparison with Clapp Order of Difficulty - B. (decreasing order)

quart 1	Freq	Quart 2	Freq	Quart 3	Freq	uart 4	Freq
0x2	5	8x0	7	4x6	34	2x1	28
9x4	33	7x3	28	4x4	30	2x2	37
9x7	34	7x5	34	4x7	41	6x0	5
8x6	39	6x7	30	3x1	40	3x3	45
8x8	38	9x6	32	6x1	31	8x2	28
7x6	36	6x6	34	3x6	34	8x1	25
0x5	11	5x1	27	2x0	21	1x0	4
0x7	10	6x4	32	6x3	31	3x4	37
6x9	42	7x7	45	5x9	32	1x2	16
7x8	38	7x1	30	2x8	33	3x8	36
4x8	38	4x0	24	9x1	25	4x5	33
0x8	11	0x6	11	5x7	29	1x9	21
7x4	33	9x9	39	3x9	38	1x8	26
9x0	7	9x3	32	4x3	25	2x6	36
8x7	38	3x0	25	3x7	38	1x3	20
7x9	44	4x9	38	2x4	37	7x2	25
8x4	26	7x0	6	6x5	29	1x5	21
6x2	23	9x5	32	5x3	22	2x5	25
0x9	13	8x5	33	5x0	6	3x5	32
9x8	37	0x4	12	1x1	14	2x7	29
0x3	9	9x2	28	4x1	37	1x6	22
6x8	45	3x2	38	5x4	27	1x7	28
0x1	4	5x5	26	2x3	43	1x4	22
8x9	35	5x6	29	2x9	34	5x2	21
8x3	28	5x8	29	4x2	46	0x0	4
	677		701		777		626
	24%		25%		28%		23%

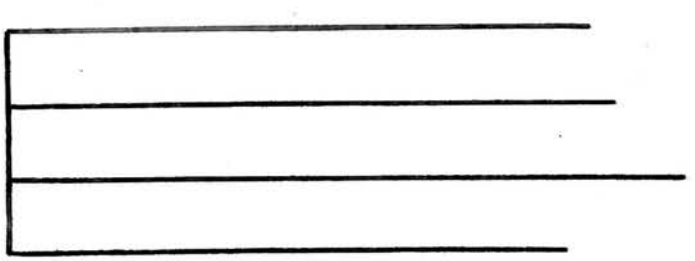


Chart LXXIII.

Number of Various Combinations as Found in:

HILLEGAS DIVISION.

Red figures in body denote dividends.
Top row of digits are divisors.

	0	1	2	3	4	5	6	7	8	9
9		4 44		9 37				44 19		9 23
8		8 44	8 29 18 13	18 13	8 36 45 14		18 14 45 16	28 17	8 34 45 19	18 15
7		7 7		27 14				7 27		27 18
6		6 9	6 34 18 17	6 37	18 11 36 17		6 30 36 15	56 18	18 19 36 16	36 17
5		5 9		15 9		5 25 15 10 25 15 35 19 45 14		35 14		45 19
4		4 10	4 27 14 13	24 11	4 40 24 13		24 16 54 16	14 15	24 13 64 19	54 22
3		3 11		3 34				63 13		63 23
2		2 9	2 24 14 17	14 10 32	14 15 32 19		14 14 42 20	42 17	32 18 42 17	42 17
1		1 8		41 15				91 19		81 18
0		0 3	0 14 10 17	0 14	0 15 40 14	0 14 10 9 20 12 30 17 40 14	0 23 30 15	0 44	0 24 40 16	0 29

Chart LXXIV.

Hillegas Division

Comparison with Clapp Order of Difficulty - B. (decreasing order)

Quart 1	Freq	Quart 2	Freq	Quart 3	Freq	Quart 4	Freq.
49÷7	19	42÷7	17	12÷3	10	9÷1	4
54÷6	16	54÷9	22	24÷6	16	4÷4	40
45÷9	19	30÷6	15	32÷8	18	0÷3	14
27÷3	16	28÷7	17	8÷4	36	3÷3	34
56÷7	18	24÷3	11	9÷3	32	5÷5	25
28÷4	14	27÷9	18	12÷2	12	0÷4	15
24÷4	13	64÷8	19	16÷8	19	9÷9	23
16÷6	15	81÷9	18	10÷5	9	0÷7	24
48÷6	16	36÷9	17	4÷2	27	0÷6	23
48÷8	19	18÷6	14	12÷6	14	0÷5	14
21÷7	19	63÷7	13	5÷1	9	6÷1	9
18÷2	13	42÷6	20	15÷5	16	0÷8	22
21÷3	15	56÷8	16	18÷9	15	1÷1	8
36÷4	17	16÷2	12	6÷6	30	0÷9	29
72÷9	17	24÷8	13	20÷5	12	8÷1	4
63÷9	23	72÷8	17	2÷2	24	2÷1	9
40÷8	16	16÷4	11	10÷2	10	0÷1	3
32÷4	19	20÷4	14	6÷3	37	8÷2	29
35÷7	14	30÷5	12	40÷5	14	0÷2	12
35÷5	19	14÷7	15	8÷8	34	4÷1	10
18÷3	13	14÷2	13	7÷7	29	7÷1	7
15÷3	9	25÷5	15	12÷4	15	3÷1	11
45÷5	14			6÷2	34		
	373		339		472		369
	24%		22%		30%		24%

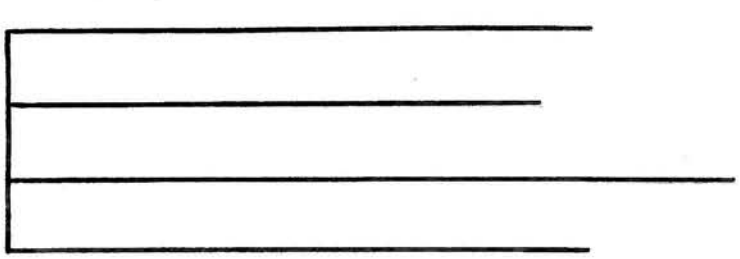
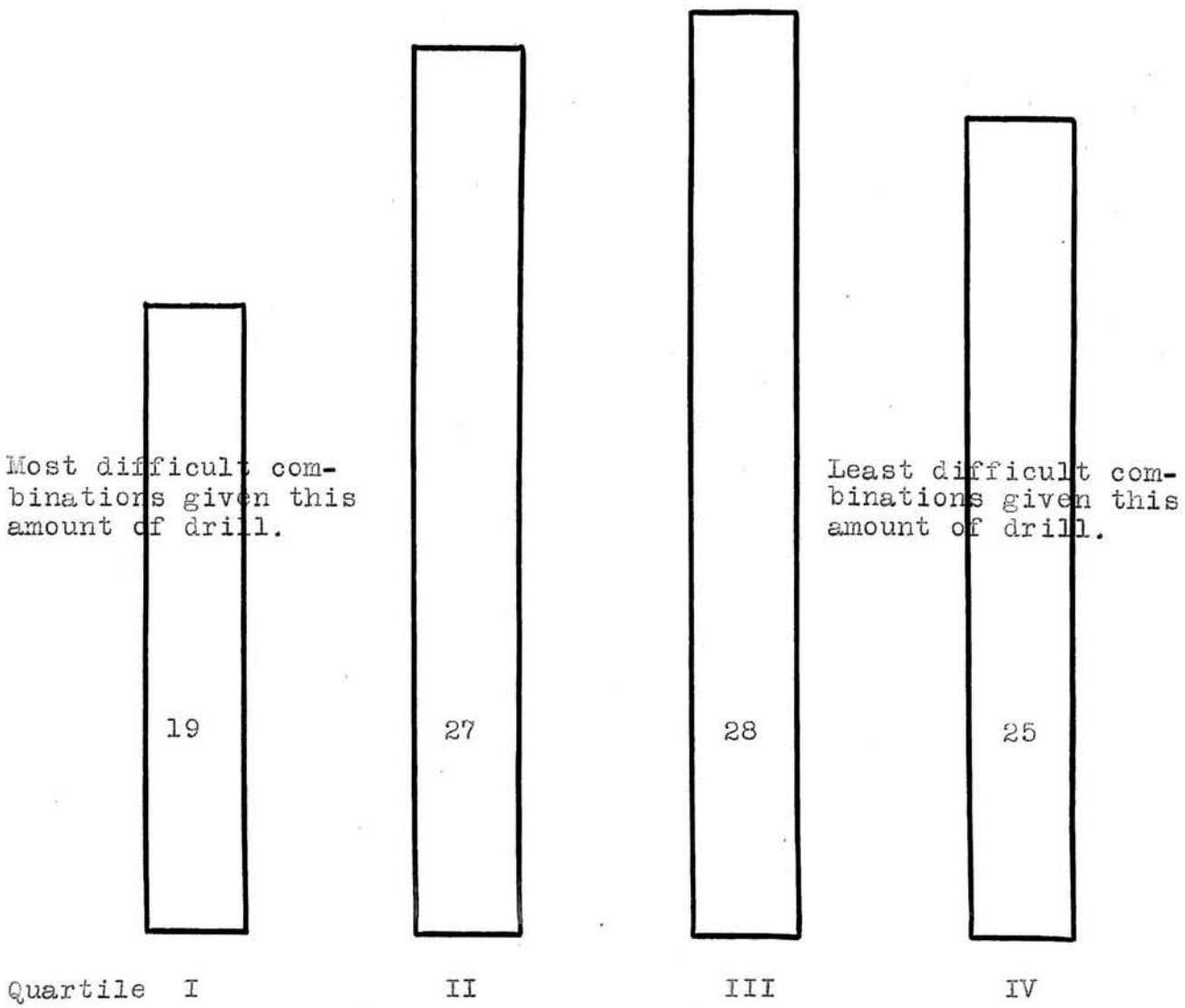


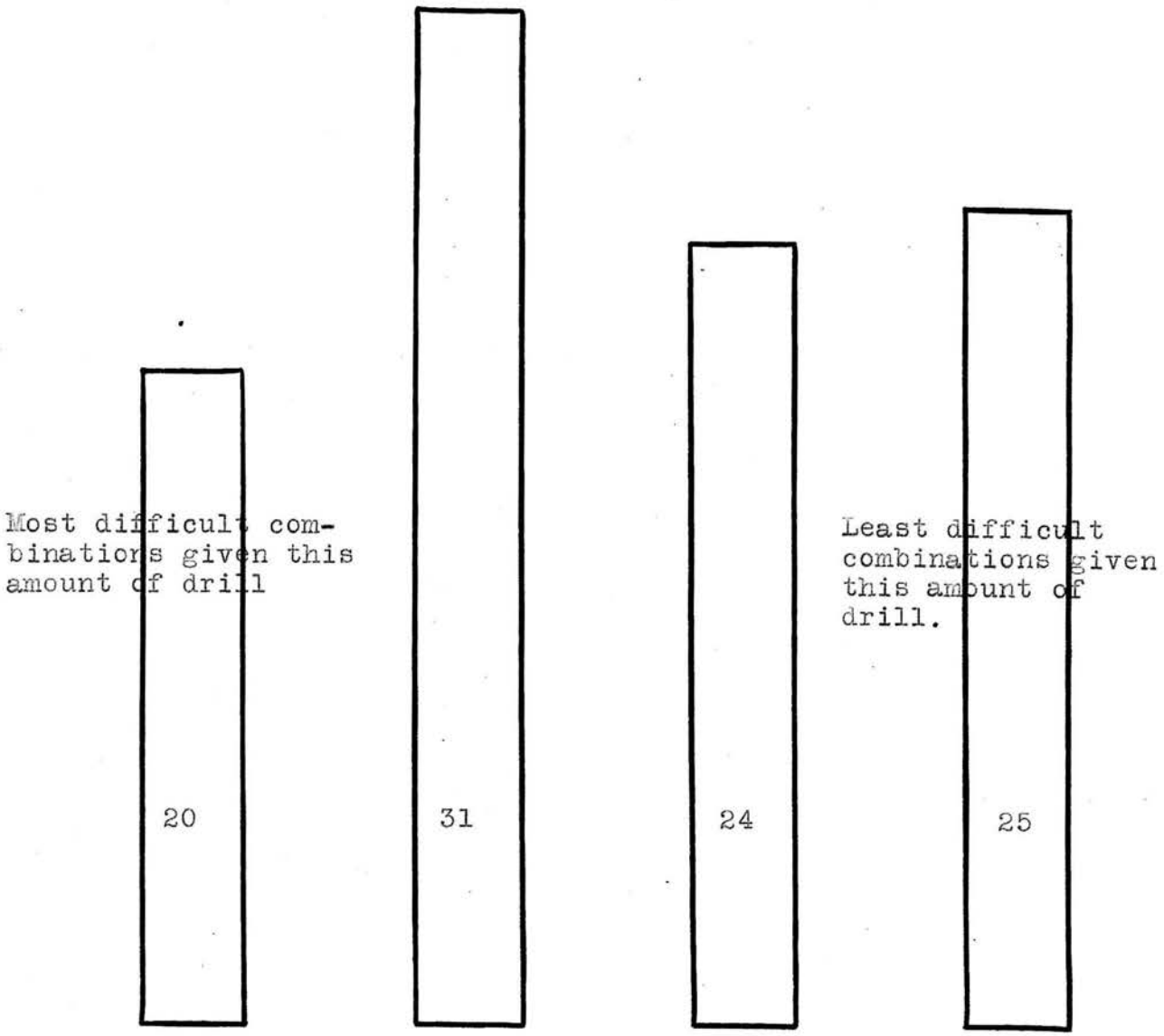
Chart LXXV.



Hillegas Addition.

Comparison of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

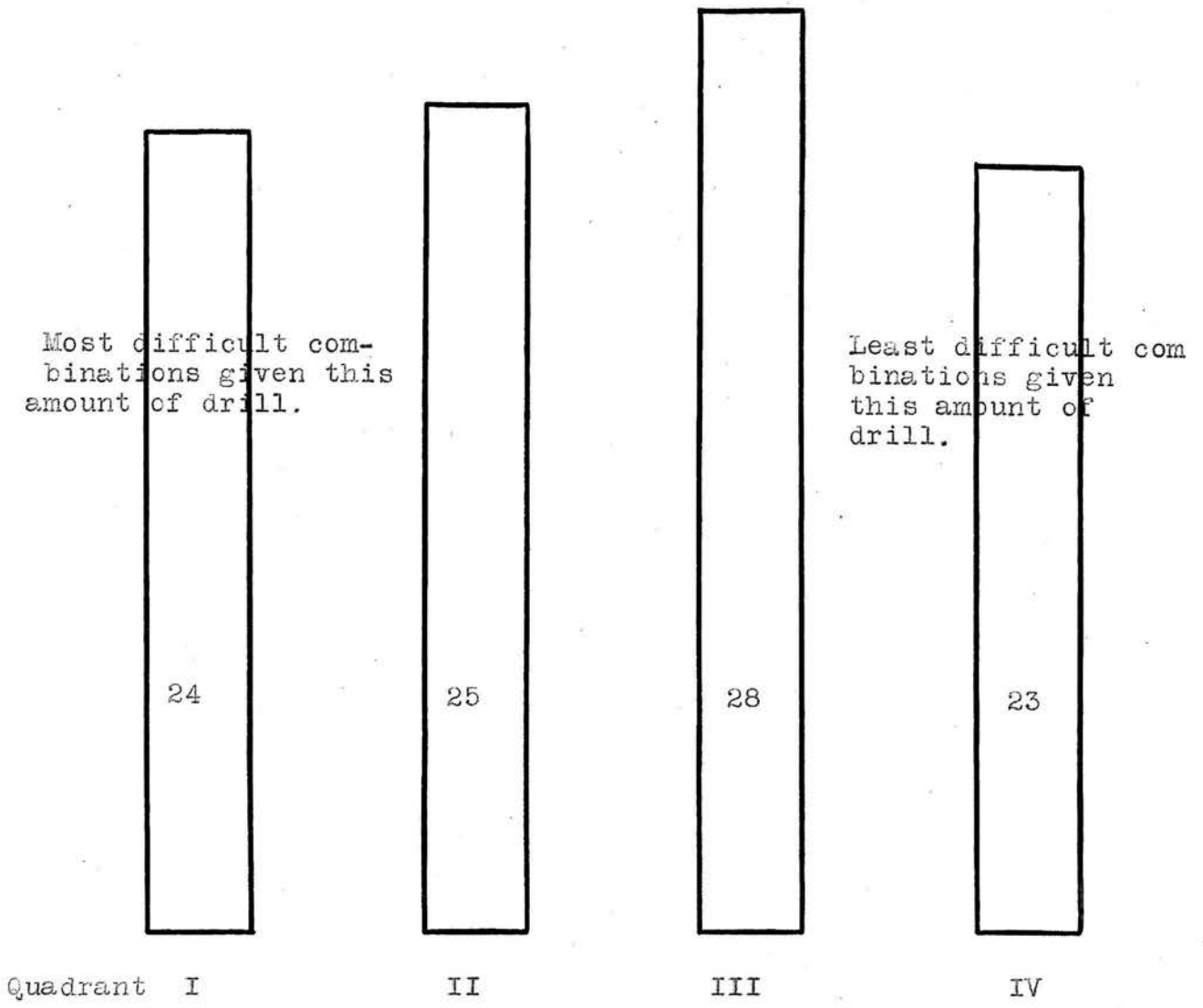
Chart LXXVII



Hillegas Subtraction

Comparisons of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

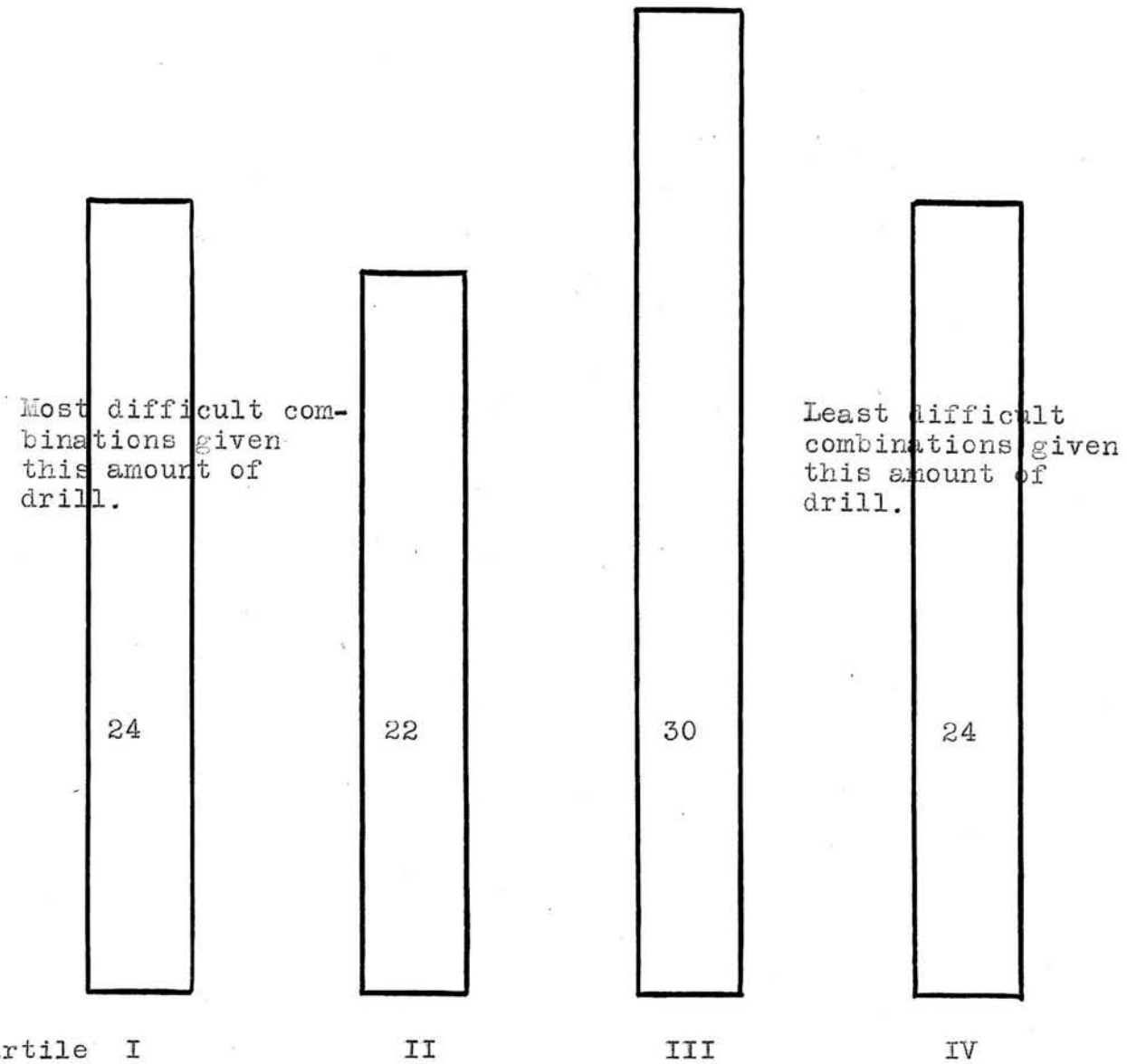
Chart LXXVII.



Hillegas Multiplication.

Comparison of percentages of frequency of appearance of combinations with Clapp Order of Difficulty - B.

Chart LXXVIII



Hillegas - Division.

Comparison of frequency of appearance of combinations with Clapp Order of Difficulty -B.

Chart LXXIX.

Percentages of Frequency of Appearance of the Combinations in Drill Services by Quartiles

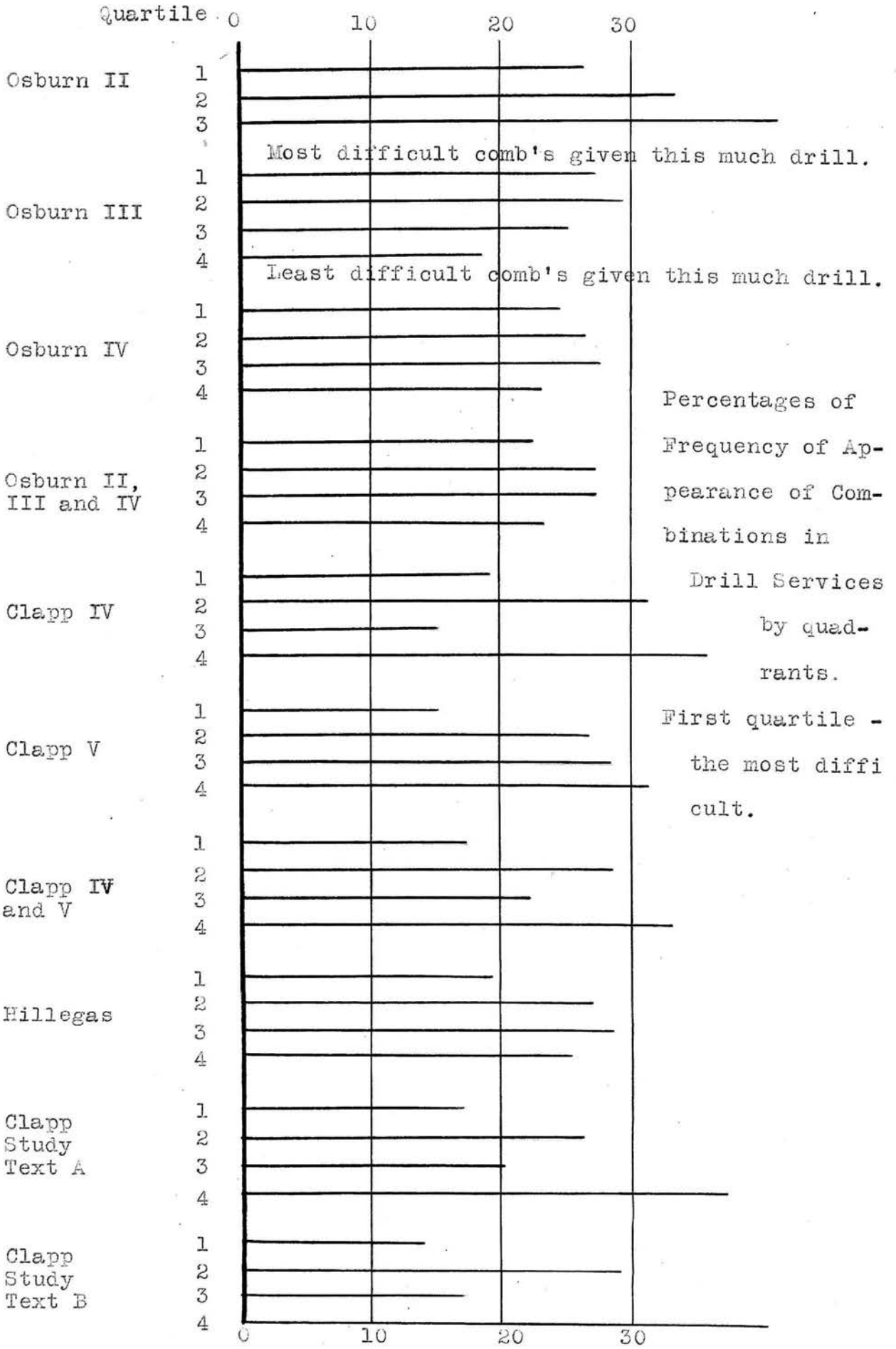
(Quartiles arranged in decreasing order of difficulty)

For these same results graphically, see charts on the next four pages.

		2 Osb	3 Osb	4 Osb	2,3 4 Osb	4 Clapp	5 Clapp	4,5 Clapp	Hilleg	Text A Clapp Study	Text B Clapp Study
Add.											
Quad	1	00	27	24	22	19	15	17	19	17	14
	2	26	29	26	27	31	26	28	27	26	29
	3	33	25	27	27	15	28	22	28	20	17
	4	41	18	23	23	35	31	33	25	37	40
Subt.											
Quad	1	16	18	24	20	22	22	22	20	12	14
	2	26	25	22	24	23	23	23	31	17	16
	3	29	21	27	24	27	27	27	24	27	28
	4	28	36	27	31	27	28	28	25	44	42
Mult.											
Quad	1	15	32	28	29	19	17	18	24	17	15
	2	20	29	29	29	23	23	23	25	23	24
	3	29	21	22	22	29	30	30	28	20	22
	4	36	17	21	20	28	30	29	23	40	39
Div.											
Quad	1	8	30	26	24	24	20	22	24	12	10
	2	14	28	27	25	21	21	21	22	17	20
	3	40	26	26	29	27	32	30	30	25	20
	4	38	16	20	22	27	27	27	24	46	50

Chart LXXXI

Addition



Percentages of Frequency of Appearance of Combinations in Drill Services by quadrants.

First quartile - the most difficult.

Chart LXXXII

Subtraction

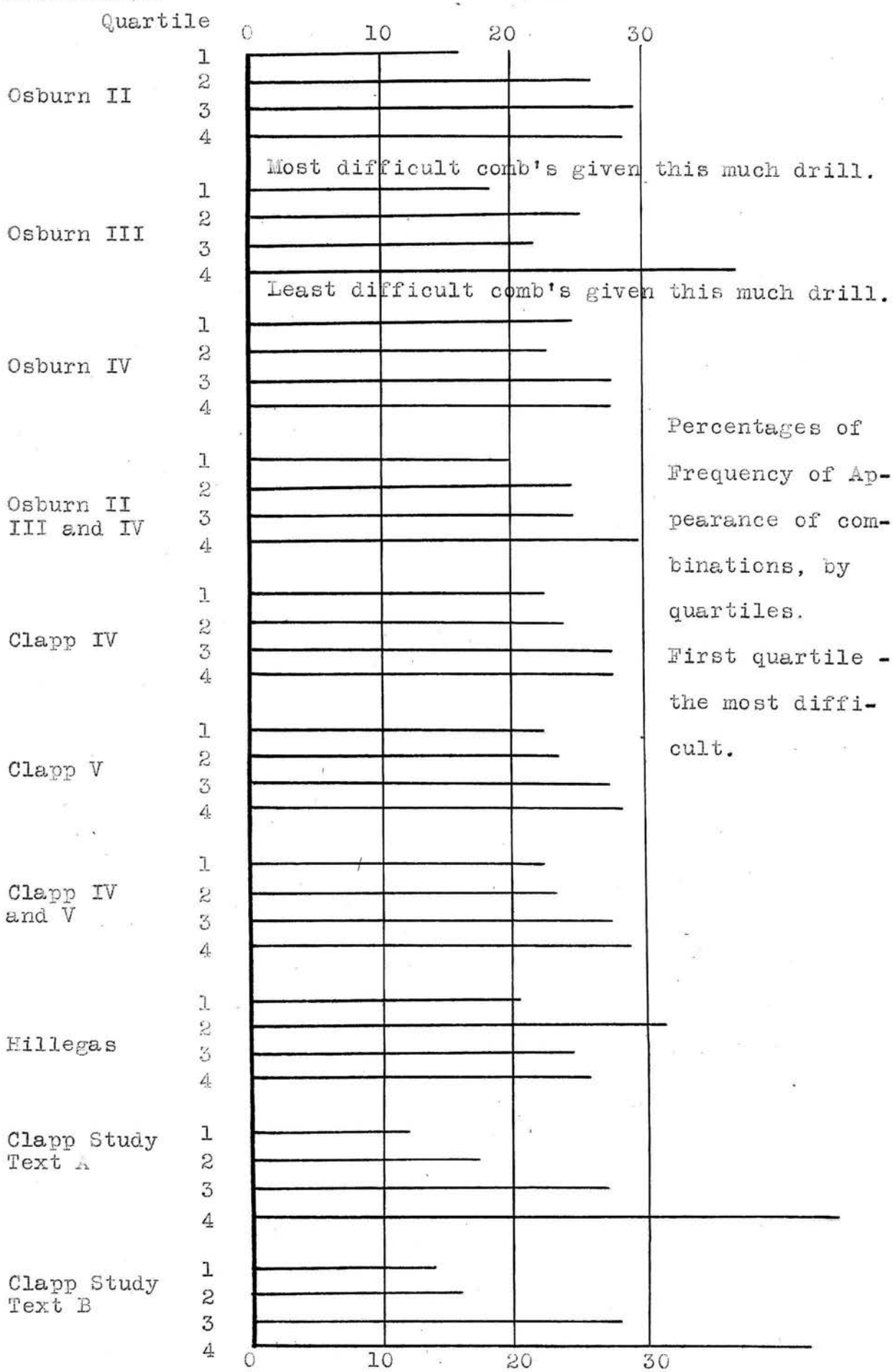


Chart LXXXIII

Multiplication

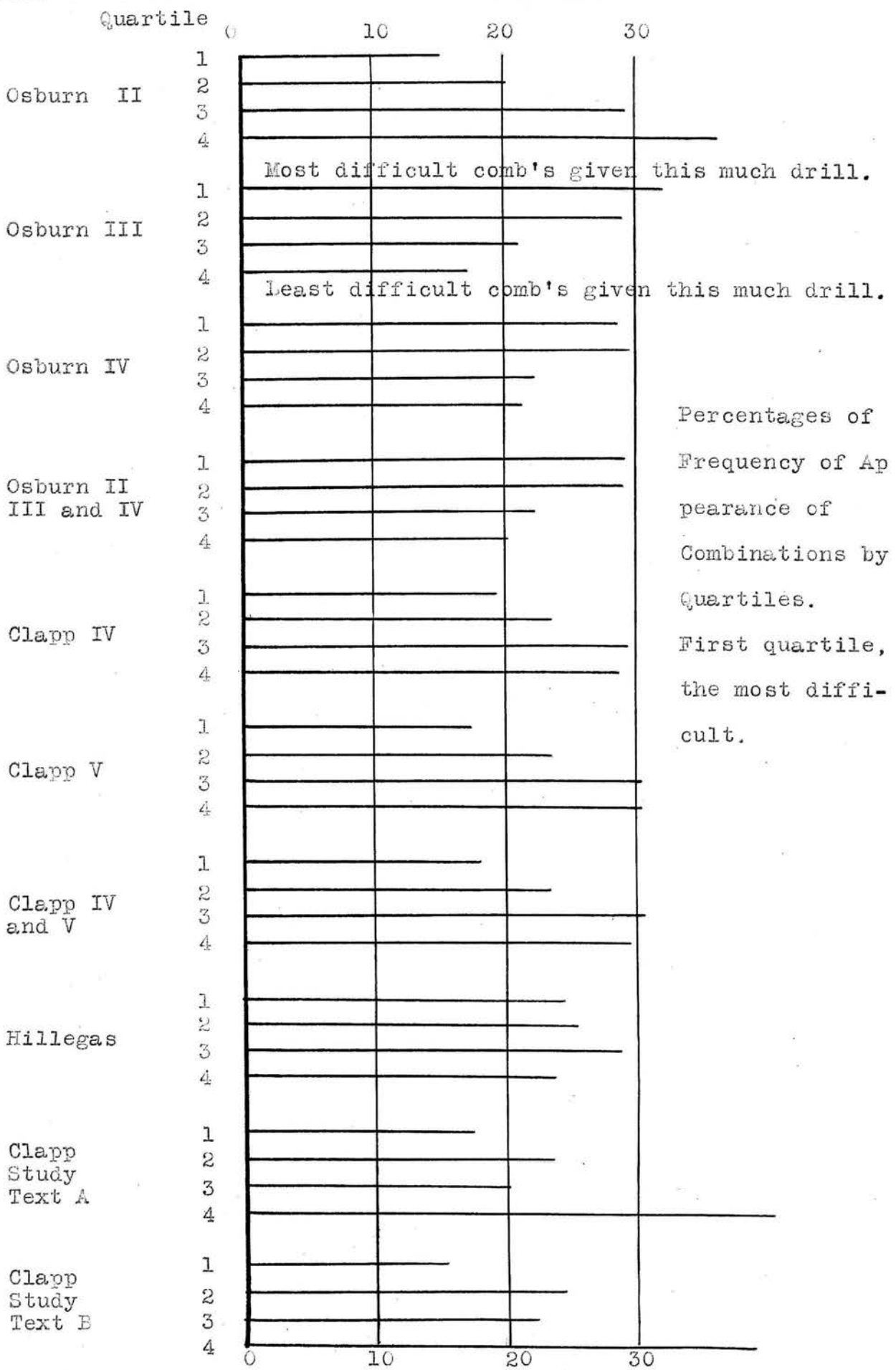
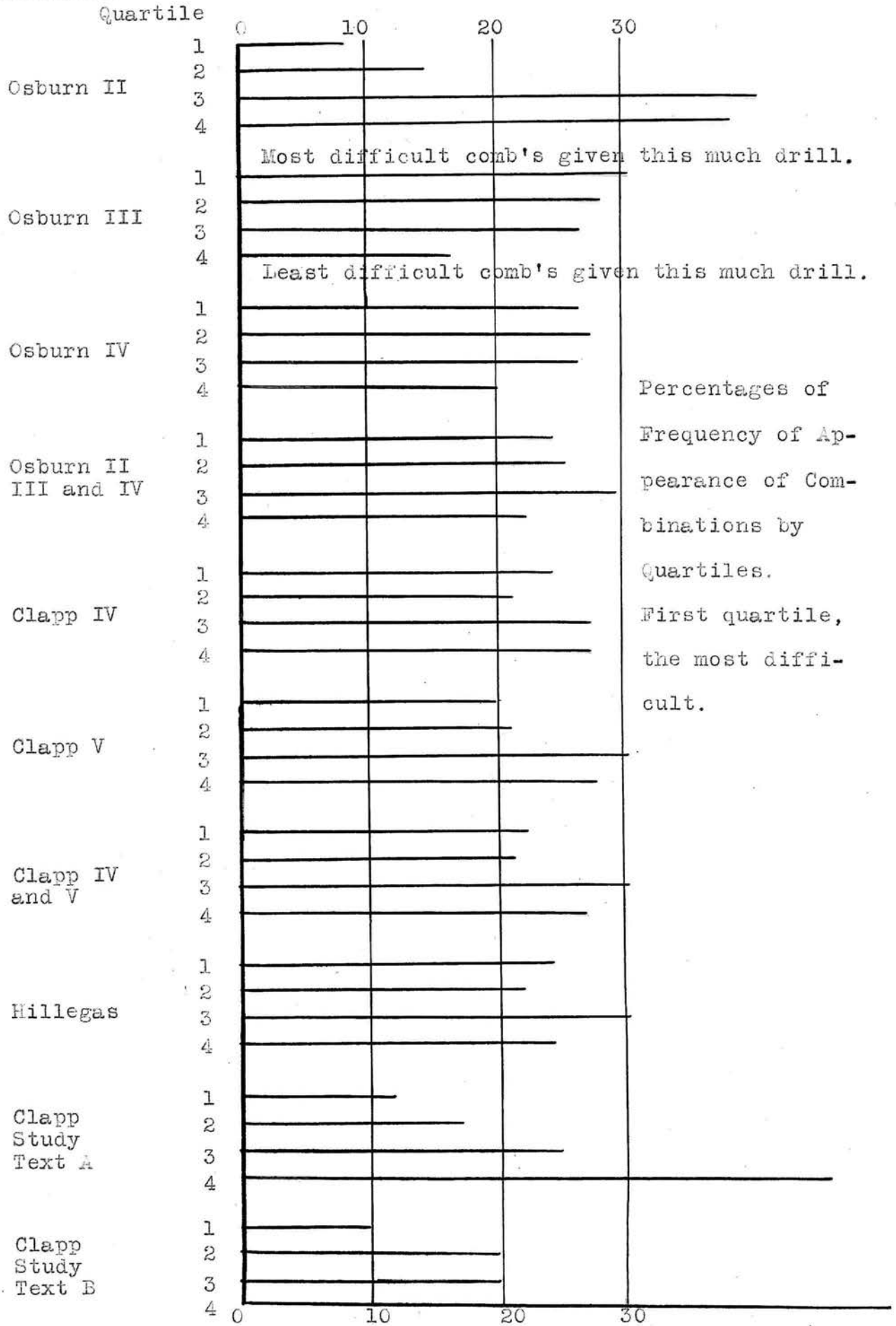


Chart LXXXIV

Division



If my reader has followed through carefully the numerous tables, etc, as found in Charts XI to LXXXIII; even if he has hastened overthem and contented himself with but a cursory review of the final graphs, there is but one conclusion at which he can arrive. One must be impressed with the fact that in every case the Services analyzed make a much better showing than did the texts studied by Clapp. This is not saying that certain of these Services do not make a better showing than others; indeed, that such is so is very evident at first glance. But that there may be unseen influencing factors entering in to disturb what would have been otherwise a gradual decrease in the length of the graphical bars of the four quartiles in certain instances is perhaps the after thought on a moment's reflection. Whether or not such is the case we shall see presently.

But if what has been said is true, that the texts we have analyzed show a considerable improvement over Texts A and B analyzed by Clapp; as a second conclusion quite equal in weight with the first, it will not be denied that there is still room for considerable improvement in more than one instance.

Turning back to the four charts of graphical results - Charts LXXX, LXXXI, LXXXII, and LXXXIII, let us see what they have to tell us in detail. The same is told numerically by Chart LXXIX. These graphs, as has been explained represent the percentages of frequency of appearance of the combinations of the particular service. The first of each of the four bars represents the percentage of combinations falling in the first quartile which it will be remembered in our scheme is the quar-

tile of greatest difficulty. In the same way, the last of each of the four bars represents in percentages the number of combinations falling in the fourth, or quartile of easiest combinations.

Commencing with Osburn II and following through the addition, subtraction, multiplication, and division charts we are struck by the fact that in every case, according to the bars, the most difficult combinations appear less frequently and the least difficult combinations appear much more frequently; especially marked is this, for instance, in addition, where of the most difficult combinations, there is not even one appearance. But is this not as it should be for Grade II? Most certainly it is, that isif any of this formal drill is to be introduced as early as the second grade; but we doubt very much the feasibility of this entire procedure so early, and the Fourth Year Book of the Department of Superintendence just published sustains us in this opinion.

Quite the reverse condition is found in the results of the two succeeding books of this series. Referring to the fourth set of bars of each of these four charts,, we have before us the results of the three books of the series summarized. In multiplication, at least, conditions are quite ideal; while in the other three operations, if not quite as satisfactory are still many times better than was the case with Texts A and B with which comparisons should be made constantly.

Taking the Clapp Series next, separately and collectively, we note results not quite as satisfactory as were those of Osburn. In the case of multiplication, for instance, in the Osburn Series - combined - there is a gradual decrease

in the length of the bars from the first to the fourth as we should desire. But in the Clapp Series - combined - there is a gradual increase, rather than decrease, in the length of these four bars in multiplication. Even the casual observer will recognize perhaps yet other discrepancies.
But as hinted before, perhaps there is some other factor entering in here, which we may discover later, and which does not enter into the Osburn Series.

Now turning to the Hillegas Service, it is apparent that there is not the wide divergence in the length of bars, from what would be the median bar, whether it be the upper or lower ones. This indicates a somewhat more staple number of combinations, on the whole at least, allotted to each of the four quartiles of relative difficulty of the combinations according to the scheme of Clapp which we repeat again is so fundamental to this study. Here as elsewhere, we should have expected a somewhat heavier allotment of combinations to the first two quartiles. This would have been indicated in somewhat longer bars of the first quartiles with a corresponding decrease in length of the lines of the third and fourth quartiles.

But it bears repetition that in the case of everyone of the several books analyzed, the results are much more gratifying than were the conditions found existing in the texts studied by Clapp. That one or other of our Services apparently meets our standards better than do the others, is but saying again what has been pointed out already in another way.

Here the reader is referred to a statement on page 16 which at the time probably appeared innocent enough.
But it has considerable bearing on the matter at hand.

There it was said that the present study has involved a handling of a grand total of 57, 730 combinations, and of which the Hillegas Book furnished 9,069, the Osburn Series 11,484, while the Clapp Series furnished 37,177 combinations.
We see that Clapp was working with more than three times as many combinations as was Osburn, and with more than four times as many as Hillegas. Osburn makes a rather good showing according to our standards with his 11,000 combinations.
To do as well handling 37,000 combinations, the task of Clapp would have been tremendously greater. Using almost 2500 fewer combinations than did Osburn, Hillegas certainly makes the poorest showing by far of any of the three - again be it understood, according to the standards we have set up herein.

Moreover and very worthy of note, in both books of Clapp (Grades IV and V) analyzed herein, he has introduced no less than thirty-nine pages of concrete or statement problems which would no doubt to no small extent increase his task in allotting to the more difficult combinations the greater recognition. And even more, he has introduced six pages of beginning work in fractions in each of the four operations, which despite almost anything he could do would tend to throw the alignment off. Neither Hillegas nor Osburn offer any work at all in either concrete problems or work in fractions.

On the other hand, Osburn has given over entirely one of his books to the very beginning work in the four operations thus necessitating for the time being a much heavier use of the easier combinations. But as we have seen, taking his series as a whole, the results have not been appreciably affected even by this.

As was said at the outset, the tendency in text books in arithmetic of the past, and even quite up to the present, has been to give much more drill and practice to the easiest combinations and much less drill to the more difficult combinations. It was as stated at that time our task to determine which, if any, of the more modern Services in Drill in the Fundamentals was most nearly approaching the ideal condition. Everything considered, it is our judgment based on the evidence as presented that the Osburn Series most nearly conforms to this standard, with of course the reservations as suggested. The Hillegas Book, especially as we recall the comparatively fewer combinations used, approaches this ideal less nearly of any.