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BOSTON UNIVERSITY
GRADUATE SCHOOL OF ARTS AND SCIENCES

Dissertation

ESSAYS ON INFLATION AND FINANCIAL INSTITUTIONS

by

ERIC SAMUEL HARDY

B.A., Oberlin College, 2011

Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

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Approved by

First Reader

Robert G. King, Ph.D.
Professor of Economics

Second Reader

Paul S. Willen, Ph.D.
Senior Economist and Policy Advisor
Federal Reserve Bank of Boston

Third Reader

Stephen J. Terry, Ph.D.
Assistant Professor of Economics

DEDICATION

This dissertation is dedicated to those who seek to understand.

ACKNOWLEDGMENTS

I would like to thank my advisor Robert G. King, without whose guidance this work would not have been possible.

ESSAYS ON INFLATION AND FINANCIAL INSTITUTIONS

ERIC SAMUEL HARDY

Boston University, Graduate School of Arts and Sciences, 2023

Major Professor: Robert G. King, Professor of Economics

ABSTRACT

This dissertation consists of three essays studying inflation and financial institutions. The first essay develops a neoclassical model to study how a welfare-maximizing government should conduct fiscal and monetary policy to finance exogenous government purchases and transfer payments. The government raises revenue through distortionary taxes, and finances its expenditures by issuing nominal one-period bonds and consols. I study a sequence of example economies to show that under flexible prices the government can support the complete markets real allocations of Lucas and Stokey (1983) with moderate inflation volatility and positive bond issuance. Policy that combines state-contingent expected inflation targets with consol issuance is most effective at reducing inflation associated with adverse government purchase shocks and adverse productivity shocks. However, increased consol issuance can increase inflation volatility when the market value of government debt varies little across states. Stylized facts using data from the example economies are comparable to their analogues in post-war U.S. data.

The second essay studies the relationship between shocks to fiscal conditions and inflation. Both unexpected increases to government purchases and unexpected increases to the real discount rate are associated with unexpected increases in inflation. The direction of these effects is consistent with the predictions of the

model developed in the first essay.

The third essay models an economy in which commercial banks produce financial instruments to facilitate transactions between households and firms, and in which a central bank conducts monetary policy. All modeled financial instruments would be familiar to a banker: banks create loans that are financed by bonds or deposits, and banks may choose to hold precautionary reserves of base money, or lend these reserves on the Federal Funds market. The single institutional constraint imposed on banks is that they must choose a quantity of reserves to hold before receiving an idiosyncratic deposit withdrawal shock at the end of each period. In equilibrium, monetary and banking policy are described by three interest rates: the Federal Funds rate, the rate on excess reserves, and the overdraft penalty rate. When the rate on excess reserves is below the Federal Funds rate, as was the case before the Global Financial Crisis, banks economize on reserves, holding roughly one to two cents in reserve for every dollar of deposits. When the rate on excess reserves is equal to the Federal Funds rate, the model implies that banks will optimally choose to hold one unit of reserves for every unit of deposits. This change in banks' optimal behavior can rationalize the increase in the size of the Federal Reserve's balance sheet beginning in the Global Financial Crisis.

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LIST OF SYMBOLS AND ABBREVIATIONS

P_t	Price Level
V_t	Market Value of Government Debt
$Q_{1,t}$	Price of a One-Period Bond
$Q_{\infty,t}$	Price of a Consol
$R_{1,t}$	One-Period Nominal Interest Rate
$r_{v,t}$	Real Return on Government Debt

CHAPTER 1

Optimal Fiscal and Monetary Policy with Flexible Prices and Long-Term Debt

1.1 INTRODUCTION

How should a welfare-maximizing government that raises revenue through distortionary taxation finance exogenous government purchases and transfer payments? Optimal policy will depend on both the type of exogenous shocks hitting the economy, and on the types of financial instruments available to the government. Lucas & Stokey (1983) characterize optimal policy for a Ramsey planner that finances its expenditures with state-contingent securities. The state-contingent returns on the portfolio of government securities allow the Ramsey planner to finance state-contingent expenses while smoothing distortionary taxes across states, thereby maximizing welfare.

I consider a government that finances its expenditures with nominal one-period bonds and consols. Government policy is described by a tax rate, a nominal one-period interest rate (equivalently, the government can set an inflation target), and a maturity structure for outstanding debt. I show that under flexible prices and abstracting from all monetary frictions, the government can conduct policy to support the complete markets real allocations of Lucas & Stokey (1983) with moderate inflation volatility, and non-negative debt issuance. Movements in consol prices in combination with state-contingent ex-post inflation allow the real returns on a government debt portfolio composed of a limited set of financial instruments to replicate the state-contingent real returns required to support complete markets real allocations.

Chari et al. (1991) study a flexible-price economy in which the government can

only issue one-period nominal bonds, and in which monetary frictions make the Friedman rule optimal. They find that when the government is always a net debtor, very volatile inflation policy is required to support the complete markets allocations. This is because changes in ex-post inflation are the only means by which policy can generate state-contingent real returns on a one-period nominal bond. Using their calibration, they report, “if the inflation rate were normally distributed, it would be higher than 20 percent or lower than -20 percent approximately a third of the time.”

Angeletos (2002) and Buera & Nicolini (2004) show that a government that finances itself with real bonds of different maturities can also construct portfolios that support the complete markets real allocations. The ex-post return on a multi-period bond is state contingent, and a carefully chosen bond portfolio can produce the appropriate state-contingent real returns. However, Buera & Nicolini (2004) find that debt positions required to replicate the Ramsey allocation are very high, “on the order of a few hundred times output for a simple four-state economy, and increasing in the number of states.”

I construct a dynamic program for a Ramsey planner that finances government purchases and transfer payments with nominal one-period bonds and consols, and draw parallels between its optimality conditions and the optimality conditions for the Ramsey planner in a complete markets economy. A Ramsey planner will smooth tax collection across states, leaving increases in fiscal needs to be financed with deficits. Debt management that supports the complete markets real allocations requires the market value of debt to decline in response to an increase in government purchases or transfer payments in order to transfer resources to the

government. When government debt is all issued as one-period nominal bonds, inflation is the only tool available to bring the real market value of government debt outstanding in line with the value required to support the complete markets real allocations.

Fluctuating consol prices present an additional means through which to adjust the real market value of debt outstanding. States with increased government purchases or transfer payments tend to also be states with higher marginal utility of consumption, and correspondingly, lower consol prices. When a portion of the debt is issued as consols, fluctuations in consol prices can be used, in addition to inflation, to adjust the real market value of debt across states. State-contingent expected inflation targets can be used to increase the volatility of consol prices. This allows changes in consol prices to account for a larger portion of the change in the real market value of debt from state to state.

In a War and Peace economy such as that studied by Barro (1989), the economy begins in peace, but must finance higher-than-usual purchases during consecutive war states before returning to peace. In this economy, a constant expected inflation target produces high inflation in the transition from peace to war. However, consol issuance in combination with increasing the expected inflation target during the war states has the effect of reducing the inflation from peace to war, while increasing inflation during the war. This policy smooths inflation in response to the spending shock over time, a mechanism emphasized by Cochrane (2022). Higher expected inflation in the war states is supported by higher one-period nominal interest rates during the war states. Investors know that the consol will trade at par upon the return to the peace state, and high nominal one-period interest rates in

the war states produce similarly high returns on the consol in the war states. In order for the consol to generate high returns in the war states and return to trading at par during peace, its market price in the first war state must be low. When government debt is all issued in consols and the expected inflation target during the war states is high, the decrease in the market price of government debt during the transition from peace to war can be so large as to require the price level to *fall* in the transition from peace to war.

Inflation volatility in the peace and war economy is monotonically decreasing in consol issuance, but it is not always the case that more consol issuance decreases inflation volatility. To show this, I introduce a three-state economy with fluctuating productivity and constant transfer payments. Inflation volatility in this economy behaves as it did in the war and peace economy: inflation volatility is monotonically decreasing in consol issuance, and is further reduced with the introduction of state-contingent expected inflation targets with higher expected inflation in the low-productivity state. Interestingly, average inflation must be greater than zero in this economy, since the lower expected inflation target in the high-productivity state requires a low nominal one-period interest rate that would be less than 1 in an economy with average inflation equal to zero.

To create an economy in which increased consol issuance increases inflation volatility, I introduce state-contingent transfer payments to the three-state economy with a constant inflation target. In this economy, transfer payments are higher in the high-productivity state, and vice-versa. The real market value of debt required to support the complete markets real allocations does not vary much across states. The state-to-state real returns on government debt do not vary much either. In this economy, nominal returns on consols are more volatile than the real returns

on the portfolio of government debt. When government debt is issued as consols, inflation is now required to offset the volatility of the consol returns.

Section 1.2 describes optimality conditions for a Ramsey planner in a complete markets economy that finances its expenditures with state-contingent contracts, and derives optimality conditions for a Ramsey planner that finances itself with nominal one-period bonds and consols. Equivalence of optimality conditions is then established. Section 1.3 describes a procedure to compute nominal bond prices, quantities, and ex-post inflation values that support the real allocations given an assumption about expected inflation policy, and describes the behavior of six example economies. Section 1.4 draws comparisons between outcomes in the example economies, and stylized facts from the postwar United States' historical experience. Model predictions for tax rates are discussed in Section 1.4.1. Taylor Rules and Phillips Curves are not structural ingredients for any of these economies. Nevertheless, Taylor Rule and Phillips Curve coefficients for these example economies can be estimated, and are reported in Section 1.4.2, alongside similar coefficients computed from simple Taylor Rules and Phillips Curves estimated on US data. Section 3.6 concludes.

1.1.1 Background

Economists since Ramsey (1927) have studied optimal policy for environments in which exogenous government expenditures are financed with distortionary taxes so as to maximize welfare by minimizing tax distortions. If a government has a need for additional funds to purchase labor and goods, to make transfer payments, or to offset a decline in its tax base, then how should it use taxes, unexpected

inflation, and changes in interest rates to impose the smallest possible cost on the economy?

Lucas & Stokey (1983) derive striking results for optimal taxation in a real point-in-time production economy with complete markets. In particular, a leading constant elasticity utility specification used in macroeconomics makes the labor income tax rate constant in an environment with state-contingent government purchases.¹ This result is in contrast to those of Barro (1979), who argued that tax rates should be smoothed, but that it was not feasible to do so with uncertain fiscal needs. Without explicitly adopting a Ramsey planner's point of view, Barro reasoned that changes in tax rates should be unpredictable and that the public debt should act as a buffer against transitory fiscal needs.²

1.1.1.1 *Complete Markets*

Chari et al. (1991) (CCK) recognized that in a monetary economy with a government that only issues one-period nominal debt, it is possible for the government to replicate the state-contingent real returns on government debt by varying inflation ex-post, and thereby replicate the optimal allocations from the complete markets economy of Lucas and Stokey. Additionally, cash-in-advance constraints produce optimal monetary policy that adheres to the Friedman rule, under which the gross nominal interest rate is equal to one in every period.³ However, optimal policy

¹Analysis of economies with capital found the optimal tax rate on capital to be zero. Chamley (1986) Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives, and Judd (1985) Redistributive Taxation in a Simple Perfect Foresight Model.

²Friedman (1957) Theory of the Consumption Function, and Hall (1978) Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence.

³This finding on the optimality of the Friedman rule is in contrast to the argument in Phelps (1973) that in economies in which governments must raise revenues with distorting taxes, it is optimal to tax all goods, including the liquidity services derived from holding money. It is also at odds with observed interest rate and inflation policy in countries around the world: implementation of the Friedman rule in an economy with positive real growth implies negative average rates

in such an economy involves highly volatile inflation, since inflation is the only mechanism through which ex-post real returns can vary across states. Furthermore, it is not possible to replicate arbitrary complete markets allocations in the CCK economy. Ljungqvist & Sargent (2018) point out that optimal policy under complete markets may require the government to be a net asset holder in one period, and a net debtor in the next period. Control of ex-post inflation alone cannot replicate this allocation.⁴

Angeletos (2002) and Buera & Nicolini (2004) consider governments that issue long-term real debt, and show that certain maturity structures can approximate a complete markets allocation. In general, if the economy can enter one of N states next period, the government can replicate the Arrow-Debreu allocations with at least N non-contingent debt instruments with different maturities. However, Buera & Nicolini (2004) find that debt positions required to replicate the Ramsey allocation are very high, on the order of a few hundred times output for a simple four-state economy, and increasing in the number of states.

1.1.1.2 *Incomplete Markets*

In order to match empirical facts about taxes, debt, and inflation, researchers have explored numerous departures from the complete markets setting of Lucas & Stokey (1983).

Aiyagari et al. (2002) (AMSS) restrict the government to issue one-period real debt. In doing so, they are able to recover a version of Barro's random walk tax-

of ex-post inflation.

⁴Optimal policy under complete markets in a "war and peace" economy with a government that begins with no initial debt produces debt policy in which the government becomes a net asset holder at the beginning of the war, and then finances wartime purchases through tax revenues as well as through spending down its assets.

smoothing outcome. However, in the absence of an upper bound on government assets, optimal policy may entail the government accumulating a “war chest” over time from which it finances all future expenditures, allowing it to set the tax rate equal to zero.

Marcet & Scott (2009) point out that under complete markets, the market value of debt has the same or less persistence as other variables in the economy, and that it declines in response to shocks that cause the deficit to increase. They contrast this with the behavior of debt and deficits in an incomplete markets economy in which agents can only issue one-period debt which pays a risk-free rate of return. Under this form of market incompleteness, debt shows more persistence than other variables, and it increases in response to shocks that cause a higher deficit. Marcet & Scott (2009) argue that that data for US government debt is more supportive of bond market incompleteness.

Optimal policy pursued by a Ramsey planner will depend on the types of frictions present in the economy. Chari et al. (1996) analyze flexible price monetary economies in which the only frictions are related to the use of money. In these economies the Friedman rule is optimal, and correspondingly, inflation is volatile. Siu (2004) and Schmitt-Grohé & Uribe (2004) study sticky-price models with one-period government debt, and find that optimal inflation is much less volatile. Even “for a miniscule degree of price stickiness (i.e., many times below available empirical estimates) the optimal volatility of inflation is near zero.” Under sticky prices, only some fraction of goods producers will be able to adjust prices in response to unexpected inflation, which introduces inefficiencies as households substitute consumption towards goods produced by firms with lower prices. For this rea-

son, the optimality of price level stability in the presence of sticky prices may rest heavily on the microfoundations of Calvo pricing or other staggered pricing arrangements. A similar result for price level stability is obtained by Khan et al. (2003), who study optimal monetary policy in an economy with sticky prices under a range of monopoly power, price stickiness, and monetary transactions cost distortions.

Lustig et al. (2008) and Leeper & Zhou (2021) study optimal policy in environments with sticky prices, and in which the government can issue long-term debt. Both find that the ability to issue long-term debt re-introduces a non-negligible role for inflation in financing fiscal shocks. Faraglia et al. (2013) also study an environment with sticky prices and a government that issues long-term nominal bonds, but find that while longer bonds lead to stronger and more persistent inflation, the role of inflation as an optimal response to expenditure shocks is still minor.

Faraglia et al. (2019) introduce incomplete markets by introducing small transactions costs on the repurchase and re-issuance of debt, which produce optimal debt management policy that more closely resembles the data: repurchase and re-issuance is negligible, short bond issuance is substantial and persistent, and short and long bonds positively covary. Bhandari et al. (2017) explore incomplete markets settings that result in an optimal target debt level that is negative but close to zero, in contrast to the surprising “war chest” result in Aiyagari et al. (2002).

1.2 REPLICATING COMPLETE MARKETS ALLOCATIONS

This section describes the welfare-maximizing complete markets real allocations and supporting behavior of tax rates and the real market value of government debt when the Ramsey planner can finance itself with Arrow-Debreu securities.

An example of a War and Peace economy similar to that studied in Barro (1989) builds intuition for outcomes under Ramsey optimal policy with state-contingent securities. Finally, this section describes the dynamic program for a Ramsey planner that finances itself with nominal one-period bonds and consols, and shows that optimality conditions will be identical to those under complete markets when the Ramsey planner can issue nominal securities.

1.2.1 Complete Markets Real Allocations

This section describes a method to solve for real allocations in a complete markets economy given exogenous states for productivity, government purchases and transfer payments, and a transition matrix governing the probability of transitioning from one state to another. The environment is similar to that in Ljungqvist & Sargent (2018), Chapter 16. The method allows for calculating a value for initial debt such that after an initial startup period, the debt-to-gdp ratio can be set to any feasible value.

1.2.1.1 The Household's Problem

The household maximizes lifetime utility as a function of consumption $c(s^t)$ and hours worked $n(s^t)$

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), n(s^t)) \quad (1.1)$$

subject to its flow budget constraint

$$(1 - \tau(s^t))w(s^t)n(s^t) + b(s^t; s^{t-1}) + z(s^t) - [c(s^t) + \sum_{s^{t+1}} \pi(s^{t+1}; s^t)q(s^{t+1}; s^t)b(s^{t+1}; s^t)] \geq 0 \quad (1.2)$$

where $\pi(s^{t+1}; s^t)$ is the probability of transitioning to state s_{t+1} conditional on being in state s^t . This allows bond prices to be written so that the household pays $\pi(s^{t+1}; s^t)q(s^{t+1}; s^t)$ in s^t for a unit of $b(s^{t+1}; s^t)$ in s_{t+1} . $(1 - \tau(s^t))w(s^t)n(s^t)$ represents after-tax labor income, and $z(s^t)$ represents transfer payments. This can be represented as an infinite-horizon budget constraint after repeated substitution, and used to form the household's Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t; s^0) u(c(s^t), n(s^t)) \\ & + \Lambda \left[\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t; s^0) q(s^t; s^0) [(1 - \tau(s^t))w(s^t)n(s^t) + z(s^t)] \right. \\ & \left. + b(s^0) - \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t; s^0) q(s^t; s^0) c(s^t) \right] \end{aligned} \quad (1.3)$$

where $b(s^0)$ is the initial quantity of government bonds owned by the household. First order conditions with respect to consumption and hours worked imply

$$\frac{q(s^{t+1}; s^0)}{q(s^t; s^0)} = \beta \frac{u_c(s^{t+1})}{u_c(s^t)} \quad (1.4)$$

$$\frac{u_n(s^t)}{u_c(s^t)} = -(1 - \tau(s^t))a(s^t) \quad (1.5)$$

with $q(s^{t+1}; s^t) = \frac{q(s^{t+1}; s^0)}{q(s^t; s^0)}$. These optimality conditions describe how state-contingent securities will be priced, and how the marginal rate of substitution between consumption and hours worked depends on the tax rate and the level of productivity. The Ramsey planner will take the household's optimality conditions as given when it formulates optimal policy.

1.2.1.2 The Planner's Problem

The Ramsey planner finances exogenous government purchases $g(s^t)$ and transfer payments $z(s^t)$ by raising revenue through a distortionary tax on labor income $\tau(s^t)$, and issuing state-contingent securities $b(s^{t+1}; s^t)$. The government's flow budget constraint is

$$f(s^t) - b(s^t; s^{t-1}) + \sum_{s^{t+1}} \pi(s^{t+1}; s^t) q(s^{t+1}; s^t) b(s^{t+1}; s^t) \geq 0 \quad (1.6)$$

where $f(s^t) = \tau(s^t)w(s^t)n(s^t) - g(s^t) - z(s^t)$ is the primary surplus. Iterating forward and using the household's optimality conditions to remove bond prices and tax rates produces the government's implementation constraint. The implementation constraint, along with a resource constraint in each period, are used to form the Ramsey Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c(s^t), n(s^t)) \\ & + \Phi \left[\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u_n(s^t)n(s^t) + u_c(s^t)[c(s^t) - z(s^t)]] - u_c(s^0)b(s^0) \right] \quad (1.7) \\ & + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \xi(s^t) [a(s^t)n(s^t) - g(s^t) - c(s^t)] \end{aligned}$$

where Φ is the constant multiplier on the implementation constraint, and $\xi(s^t)$ is the state-dependent multiplier on the resource constraint. Optimality conditions depend on $b(s^0)$ at $t = 0$, and do not depend on $b(s^0)$ otherwise.

$$(c(s^t)) : u_c(s^t) + \Phi \{ u_{cc}(s^t)[c(s^t) - z(s^t)] + u_c(s^t) \} - \xi(s^t) = 0 \quad (1.8)$$

$$(n(s^t)) : u_n(s^t) + \Phi \{ u_{nn}(s^t)n(s^t) + u_n(s^t) \} + \xi(s^t)a(s^t) = 0 \quad (1.9)$$

$$\begin{aligned} (c(s^0)) : u_c(s^0) + \Phi\{u_{cc}(s^0)[c(s^0) - z(s^0) - b(s^0)] + u_c(s^0)\} \\ - \xi(s^0) = 0 \end{aligned} \quad (1.10)$$

$$(n(s^0)) : u_n(s^0) + \Phi\{u_{nn}(s^0)n(s^0) + u_n(s^0)\} + \xi(s^0)a(s^0) = 0 \quad (1.11)$$

Combining household and Ramsey planner optimality conditions, the tax rates for $t \geq 1$ and $t = 0$ are

$$(1 - \tau(s^t)) = \frac{1 + \Phi\left\{\frac{u_{cc}(s^t)}{u_c(s^t)}[c(s^t) - z(s^t)] + 1\right\}}{1 + \Phi\left\{\frac{u_{nn}(s^t)}{u_n(s^t)}n(s^t) + 1\right\}} \quad (1.12)$$

$$(1 - \tau(s^0)) = \frac{1 + \Phi\left\{\frac{u_{cc}(s^0)}{u_c(s^0)}[c(s^0) - z(s^0) - b(s^0)] + 1\right\}}{1 + \Phi\left\{\frac{u_{nn}(s^0)}{u_n(s^0)}n(s^0) + 1\right\}} \quad (1.13)$$

To unify notation for s^0 and s^t , $b(s^0)$ can be treated as part of the transfer $z(s^0)$. In the special case of separable power utility in consumption and hours worked, and for an economy in which the government only finances government purchases, the tax rate will be the same in all states after $t = 0$.

Combining the government flow budget constraint from equation 1.6, and the household's optimality condition relating bond prices to marginal utilities of consumption produces

$$b(s^t; s^{t-1}) \leq f(s^t) + \sum_{s^{t+1}} \pi(s^{t+1}; s^t) \beta \frac{u_c(s^{t+1})}{u_c(s^t)} b(s^{t+1}; s^t). \quad (1.14)$$

The equation for the present value of surpluses is

$$F(s^t) = f(s^t) + \sum_{s^{t+1}} \pi(s^{t+1}; s^t) \beta \frac{u_c(s^{t+1})}{u_c(s^t)} F(s^{t+1}). \quad (1.15)$$

At optimality the implementation constraint will bind, producing the result that

the present value of surpluses is equal to the value of the outstanding debt in each state:

$$F(s^t) = b(s^t; s^{t-1}). \quad (1.16)$$

1.2.2 A War and Peace Economy

Suppose that the economy begins at peace (state 5), but with probability θ transitions into the first of four war states, in a war which lasts for four periods (states 1-4). The transition matrix is

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \theta & 0 & 0 & 0 & (1 - \theta) \end{bmatrix} \quad (1.17)$$

where $\theta = 0.05$ is the probability of transitioning to the war state from the peace state. Exogenous productivity $a(\varsigma)$, government purchases $g(\varsigma)$, and transfer payments $z(\varsigma)$ are defined in each state. In the War and Peace economy, productivity is constant at 1 in all states, and transfer payments are constant at zero. The resource constraint is

$$a(\varsigma)n(\varsigma) = g(\varsigma) + c(\varsigma) \quad (1.18)$$

In the peace state, government purchases are 0.1, and rise to 0.2 in each war state. The household has utility that is separable in consumption and hours worked

$$u(c(\varsigma), n(\varsigma)) = \frac{1}{1 - \sigma} c(\varsigma)^{(1 - \sigma)} - \frac{\chi}{1 + \eta} n(\varsigma)^{(1 + \eta)} \quad (1.19)$$

All outcomes are computed using the following parameters for the household's separable utility function

Parameter	Value	Description
β	0.95	Annual discount factor
σ	2	CRRA risk aversion, 1/IES
η	2	Frisch elasticity of labor supply
χ	3	scaling parameter for labor

All variables are calculated assuming the economy starts in the peace state with an initial debt-to-gdp ratio ($b0_an$) of 1.⁵ There is a positive relationship between $b(s^0)$, Φ , $\tau(\varsigma)$, and the mean of $F(\varsigma)$. Given a transition matrix M and values for exogenous variables $a(\varsigma)$, $g(\varsigma)$, and $z(\varsigma)$, it is always possible to find a $b(s^0)$ high enough so that $F(\varsigma) \geq 0$.

state	g	z	a	b0_an	PHI	n	c	un	uc
1	0.200	0.000	1.000	1.000	0.058	0.826	0.626	-2.047	2.552
2	0.200	0.000	1.000	1.000	0.058	0.826	0.626	-2.047	2.552
3	0.200	0.000	1.000	1.000	0.058	0.826	0.626	-2.047	2.552
4	0.200	0.000	1.000	1.000	0.058	0.826	0.626	-2.047	2.552
5	0.100	0.000	1.000	1.000	0.058	0.771	0.671	-1.782	2.223

state	tau	ft	Ft	q1	q10	qinf	er1	erinf
1	0.198	-0.036	0.412	0.950	0.535	0.885	1.053	1.053
2	0.198	-0.036	0.471	0.950	0.534	0.880	1.053	1.053
3	0.198	-0.036	0.535	0.950	0.534	0.875	1.053	1.053
4	0.198	-0.036	0.601	0.827	0.534	0.870	1.209	1.209
5	0.198	0.053	0.770	0.957	0.614	1.000	1.045	1.046

In the War and Peace economy, the Ramsey planner chooses a complete tax smoothing in the form of a constant tax rate due to the form of the utility function and the fact that all expenses are in the form of government purchases. Optimal

⁵Appendix A.2 describes the procedure for calculating the complete markets real allocations given an exogenous quantity of initial debt.

debt policy in the peace state requires that the government pay off its outstanding debt of 0.770 with its surplus of 0.053 and by issuing state-contingent securities with a market price equal to $(0.770 - 0.053)$. If the economy remains in the peace state, the portfolio of state-contingent securities should return the surplus to the household, and the government should maintain its outstanding debt level of 0.770. If the economy enters the first war state, this same portfolio of state-contingent securities should instead transfer wealth from the household to the government, reducing the market value of outstanding debt from 0.770 to 0.412. This policy is state-contingent, but not history contingent: quantities of debt do not vary depending on the number of periods spent in the peace state. As the economy transitions through the war states, successive deficits increase the market value of the debt until it has climbed back to its peace state value of 0.770.

1.2.3 The Ramsey Dynamic Program

This section develops a dynamic program that features lagged multipliers as state variables in the spirit of Marcet & Marimon (2019) for a government that finances state-contingent purchases and transfer payments in the face of fluctuating productivity by issuing nominal one-period bonds and consols. Beginning with two nominal assets, optimality conditions for the recursive optimization problem are analogous to the optimality conditions derived under complete markets.

This framework nests economies like those in Chari et al. (1991) (nominal one-period bonds) and Aiyagari et al. (2002) (real one-period bonds), and allows additional exploration of optimal policy with long-term debt. The inclusion of consols allows the study of dynamics similar to those in Angeletos (2002) and Buera & Nicolini (2004) in which the state-contingent returns on a portfolio of multi-period

real bonds can approximate the state-contingent returns under complete markets.

An economy with nominal bonds usually includes an institutional framework for circulation of money in the economy, and a way to treat seigniorage revenue. Other authors have collectively shown that the specific frictions chosen for consideration can influence optimal outcomes. For tractability, I will completely abstract from monetary institutions and associated frictions.

1.2.3.1 *Preferences, Technologies, and Shocks*

The household has utility that is separable in consumption and hours worked

$$u(c_t, n_t) = \frac{1}{1 - \sigma} c_t^{(1-\sigma)} - \frac{\chi}{1 + \eta} n_t^{(1+\eta)} \quad (1.20)$$

The resource constraint is

$$a_t n_t = g_t + c_t \quad (1.21)$$

Productivity a_t , government purchases g_t , and transfer payments z_t are again exogenous.

1.2.3.2 *The Household's Problem*

The household uses its after-tax labor income $(1 - \tau_t)w_t n_t$, transfer payments, and its bond portfolio to purchase the consumption good, nominal one-period bonds, and consols. The household maximizes lifetime utility subject to its budget constraint, so that its maximization problem is given by

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \quad (1.22)$$

$$\begin{aligned}
& + \sum_{t=0}^{\infty} \beta^t \lambda_t [(1 - \tau_t) w_t n_t + z_t + [b_{1,t} + (k + Q_{\infty,t}) b_{\infty,t}] \delta_t \\
& \quad - c_t - [Q_{1,t} b_{1,t+1} + Q_{\infty,t} b_{\infty,t+1}]]
\end{aligned} \tag{1.23}$$

where $\delta_{t,t+j} = \frac{P_t}{P_{t+j}}$ is the cumulative depreciation due to changes in the price level from t to $t + j$. Depreciation notation with a single time subscript indicates depreciation from the previous period. For example, $\delta_t \equiv \delta_{t-1,t}$. At optimality, a nominal one-period bond with a payout of 1 in period $t + 1$ is priced

$$Q_{1,t} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \delta_{t,t+1} \tag{1.24}$$

where $\lambda_t = u_c(c_t)$. A nominal consol with face value of 1 and coupon equal to a fraction k of its face value is priced

$$Q_{\infty,t} = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \delta_{t,t+1} (Q_{\infty,t+1} + k) \tag{1.25}$$

$$= k E_t \sum_{j=1}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \delta_{t,t+j} \tag{1.26}$$

Optimality conditions for consumption and labor imply

$$\frac{u_n(n_t)}{u_c(c_t)} = -(1 - \tau_t) a_t. \tag{1.27}$$

1.2.3.3 The Planner's Problem

The government budget constraint with nominal consols is

$$f_t + Q_{1,t} b_{1,t+1} + Q_{\infty,t} [b_{\infty,t+1} - b_{\infty,t} \delta_t] - [b_{1,t} + k b_{\infty,t}] \delta_t \geq 0 \tag{1.28}$$

Removing price terms, multiplying through by the marginal utility of consumption, the implementation constraint is

$$\begin{aligned} & \lambda_t f_t + k E_t \left[\sum_{j=1}^{\infty} \beta^j \lambda_{t+j} \delta_{t,t+j} \right] [b_{\infty,t+1} - b_{\infty,t} \delta_t] \\ & + E_t [\beta \lambda_{t+1} \delta_{t,t+1}] b_{1,t+1} - \lambda_t [b_{1,t} + k b_{\infty,t}] \delta_t \geq 0. \end{aligned} \quad (1.29)$$

Summing the discounted implementation constraints over time produces

$$\begin{aligned} \Psi_0 = & \sum_{t=0}^{\infty} (\theta \beta)^t \gamma_t \left\{ \lambda_t f_t + k E_t \left[\sum_{j=1}^{\infty} \beta^j \lambda_{t+j} \delta_{t,t+j} \right] [b_{\infty,t+1} - b_{\infty,t} \delta_{t-1,t}] \right. \\ & \left. + E_t \beta \lambda_{t+1} \delta_{t,t+1} b_{1,t+1} - \lambda_t [b_{1,t} + k b_{\infty,t}] \delta_{t-1,t} \right\} \geq 0 \end{aligned} \quad (1.30)$$

where the government discount rate $(\theta \beta)$ may be different from the household's discount rate β . Group terms at $t + j$ and define multipliers $\mu_t = [\mu_{1,t}, \mu_{\infty,t}]$ that begin at $\mu_{1,0} = 0$ and $\mu_{\infty,0} = 0$, and evolve according to

$$\mu_{1,t+1} = \theta^{-1} \gamma_t \quad (1.31)$$

$$\mu_{\infty,t+1} = \theta^{-1} \mu_{\infty,t} + \theta^{-1} (\mu_{1,t} - \gamma_t) k b_{\infty,t} \delta_{t-1,t} \quad (1.32)$$

to write the infinite-horizon implementation constraint as⁶

$$\Psi_0 = \sum_{t=0}^{\infty} (\theta\beta)^t \lambda_t \{ \gamma_t f_t + [(\mu_{1,t} - \gamma_t)[b_{1,t} + kb_{\infty,t}] + \mu_{\infty,t}] \delta_t \} \geq 0 \quad (1.35)$$

The Bellman-like functional equation is

$$\begin{aligned} W(s_t) = \min_{\gamma_t} \max_{c_t, n_t, b_t, \delta_t} \{ & u(c_t, n_t) \\ & + \gamma_t u_c(c_t) f_t \\ & + u_c(c_t) [(\mu_{1,t} - \gamma_t)[b_{1,t} + kb_{\infty,t}] + \mu_{\infty,t}] \delta_t \\ & + \theta\beta E_t W(s_{t+1}) | s_t \} \end{aligned} \quad (1.36)$$

$$\text{s.t.} \quad \mu_{1,t+1} = \theta^{-1} \gamma_t \quad (1.37)$$

$$\mu_{\infty,t+1} = \theta^{-1} \mu_{\infty,t} \delta_t + \theta^{-1} (\mu_{1,t} - \gamma_t) k b_{\infty,t} \delta_t \quad (1.38)$$

$$a_t n_t \geq g_t + c_t \quad (1.39)$$

$$u_c(c_t) f_t = u_n(n_t) n_t + u_c(c_t) [c_t - z_t] \quad (1.40)$$

⁶Grouping terms by λ_{t+j} involves

$$\begin{aligned} \Psi_t = & (\theta\beta)^t \gamma_t \lambda_t \{ f_t - [b_{1,t} + kb_{\infty,t}] \delta_{t-1,t} \} \\ & + (\theta\beta)^t \beta \gamma_{t+1} \lambda_{t+1} \{ f_{t+1} - [b_{1,t+1} + kb_{\infty,t+1}] \delta_{t,t+1} \} \\ & + (\theta\beta)^t \beta^2 \gamma_t \lambda_{t+2} \delta_{t,t+2} k [b_{\infty,t+1} - b_{\infty,t} \delta_{t-1,t}] \end{aligned} \quad (1.33)$$

$$\begin{aligned} & + (\theta\beta)^{t+1} \beta \gamma_{t+1} \lambda_{t+2} \delta_{t+1,t+2} [k [b_{\infty,t+2} - b_{\infty,t+1} \delta_{t,t+1}] + b_{1,t+2}] \\ & + (\theta\beta)^{t+2} \gamma_{t+2} \lambda_{t+2} \{ f_{t+2} - [b_{\infty,t+2} + kb_{\infty,t+2}] \delta_{t+1,t+2} \} \\ & + \dots \\ = & (\theta\beta)^t \gamma_t \lambda_t \{ f_t - [b_{1,t} + kb_{\infty,t}] \delta_{t-1,t} \} \\ & + (\theta\beta)^{t+1} \lambda_{t+1} \{ \gamma_{t+1} f_{t+1} + (\theta^{-1} \gamma_t - \gamma_{t+1}) [b_{1,t+1} + kb_{\infty,t+1}] \delta_{t,t+1} \\ & + \theta^{-1} (\theta^{-1} \gamma_{t-1} - \gamma_t) \delta_{t,t+1} k b_{\infty,t} \delta_{t-1,t} \} \\ & + (\theta\beta)^{t+2} \lambda_{t+2} \{ \gamma_{t+2} f_{t+2} + (\theta^{-1} \gamma_{t+1} - \gamma_{t+2}) [b_{1,t+2} \delta_{t+1,t+2} + kb_{\infty,t+2} \delta_{t+1,t+2}] \\ & + \theta^{-1} (\theta^{-1} \gamma_t - \gamma_{t+1}) \delta_{t,t+2} k b_{\infty,t+1} + \theta^{-2} (\theta^{-1} \gamma_{t-1} - \gamma_t) \delta_{t,t+2} k b_{\infty,t} \delta_{t-1,t} \} \\ & + \dots \end{aligned} \quad (1.34)$$

$$\xi'_{1,t}[b_{1,t+1} - \underline{b}_1] \geq 0; \quad \xi_{1,t}[b_{1,t} - \underline{b}_1] \geq 0 \quad (1.41)$$

$$\xi'_{\infty,t}[b_{\infty,t+1} - \underline{b}_\infty] \geq 0; \quad \xi_{\infty,t}[b_{\infty,t} - \underline{b}_\infty] \geq 0 \quad (1.42)$$

where equations 1.41 and 1.42 are lower bounds on the issuance of one-period bonds and consols, respectively. Optimality conditions are

$$(c_t) : u_c(c_t) + \gamma_t \{u_{cc}(c_t)[c_t - z_t] + u_c(c_t)\} \quad (1.43)$$

$$+ u_{cc}(c_t)[(\mu_{1,t} - \gamma_t)[b_{1,t} + kb_{\infty,t}] + \mu_{\infty,t}] \delta_t - \xi_t = 0$$

$$(n_t) : u_n(n_t) + \gamma_t \{u_{nn}(n_t)n_t + u_n(n_t)\} + X_{it}a_t = 0 \quad (1.44)$$

$$(\delta_t) : u_c(c_t)[(\mu_{1,t} - \gamma_t)[b_{1,t} + kb_{\infty,t}] + \mu_{\infty,t}] \quad (1.45)$$

$$+ [\mu_{\infty,t} + (\mu_{1,t} - \gamma_t)kb_{\infty,t}] \beta E_t W_{\mu_\infty}(s_{t+1}) = 0$$

$$(\gamma_t) : u_c(c_t)f_t - u_c(c_t)[b_{1,t} + kb_{\infty,t}] \delta_t \quad (1.46)$$

$$+ \beta E_t W_{\mu_1}(s_{t+1}) - kb_{\infty,t} \delta_t \beta E_t W_{\mu_\infty}(s_{t+1}) = 0$$

$$(b_{1,t+1}) : \theta \beta E_t W_{b_1}(s_{t+1}) | s_t + \xi'_{1,t} = 0 \quad (1.47)$$

$$(b_{\infty,t+1}) : \theta \beta E_t W_{b_\infty}(s_{t+1}) | s_t + \xi'_{\infty,t} = 0 \quad (1.48)$$

The Envelope Theorem yields

$$W_{b_1}(s_t) = u_c(c_t)(\mu_{1,t} - \gamma_t) \delta_t + \xi_{1,t} \quad (1.49)$$

$$W_{b_\infty}(s_t) = u_c(c_t)(\mu_{1,t} - \gamma_t) k \delta_t + (\mu_{1,t} - \gamma_t) k \delta_t \beta E_t W_{\mu_\infty}(s_{t+1}) + \xi_{\infty,t} \quad (1.50)$$

$$W_{\mu_1}(s_t) = u_c(c_t)[b_{1,t} + kb_{\infty,t}] \delta_t + kb_{\infty,t} \delta_t \beta E_t W_{\mu_\infty}(s_{t+1}) \quad (1.51)$$

$$W_{\mu_\infty}(s_t) = u_c(c_t) \delta_t + \delta_t \beta E_t W_{\mu_\infty}(s_{t+1}) \quad (1.52)$$

1.2.4 Interpretation of Equilibrium Conditions

Iterate equation 1.52 forward and

$$W_{\mu_\infty}(s_t) = u_c(c_t)\delta_t + \delta_t\beta E_t W_{\mu_\infty}(s_{t+1}) \quad (1.53)$$

$$= u_c(c_t)\delta_t + \delta_t \sum_{j=1}^{\infty} \beta^j u_c(c_{t+j})\delta_{t,t+j} \quad (1.54)$$

$$= u_c(c_t)\delta_t + \delta_t \frac{1}{k} u_c(c_t) Q_{\infty,t}, \quad (1.55)$$

so that $kW_{\mu_\infty}(s_t)$ is the depreciated cum-dividend value of a consol in marginal utility units

$$kW_{\mu_\infty}(s_t) = u_c(c_t)\delta_t[k + Q_{\infty,t}], \quad (1.56)$$

and will be positive so long as $\delta_t > 0$. A related implication is

$$k\beta E_t W_{\mu_\infty}(s_{t+1}) = u_c(c_t)Q_{\infty,t}. \quad (1.57)$$

Equations 1.46 and 1.51 yield

$$W_{\mu_1}(s_t) = u_c(c_t)f_t + \beta E_t W_{\mu_1}(s_{t+1}) \quad (1.58)$$

Iterating forward produces

$$W_{\mu_1}(s_t) = E_t \sum_{j=0}^{\infty} \beta^j u_c(c_{t+j})f_{t+j} \quad (1.59)$$

so that $W_{\mu_1}(s_t)$ is the present discounted value of real surpluses in marginal utility units. From equations 1.51 and 1.57

$$W_{\mu_1}(s_t) = u_c(c_t)[b_{1,t} + (k + Q_{\infty,t})b_{\infty,t}]\delta_t \quad (1.60)$$

so that $W_{\mu_1}(s_t)$ is also the real value of outstanding debt in marginal utility units. At optimality, the real market value of outstanding debt is equal to the expected value of future surpluses.

The optimality condition with respect to nominal depreciation, equation 1.45, in combination with equation 1.52 becomes

$$u_c(c_t)(\mu_{1,t} - \gamma_t)b_{1,t}\delta_t + [(\mu_{1,t} - \gamma_t)kb_{\infty,t} + \mu_{\infty,t}]W_{\mu_{\infty}}(s_t) = 0. \quad (1.61)$$

This optimality condition is satisfied when $\mu_{1,t} = \gamma_t$, and $\mu_{\infty,t} = 0$, because $W_{\mu_{\infty}}(s_t) > 0$. These conditions will be met when γ_t is equal to a constant γ in all periods. At optimality, the government will choose nominal depreciation so it faces the same multiplier on its implementation constraint in each period, as would be the case in a complete markets economy.⁷

Finally, the planner's and household's optimality conditions for consumption and labor produce an expression for the tax rate

$$(1 - \tau_t) = \frac{1 + \gamma_t \left\{ \frac{u_{cc}(c_t)}{u_c(c_t)} [c_t - z_t] + 1 \right\} + \frac{u_{cc}(c_t)}{u_c(c_t)} \zeta_t}{1 + \gamma_t \left\{ \frac{u_{nn}(n_t)}{u_n(n_t)} n_t + 1 \right\}} \quad (1.62)$$

where $\zeta_t = [(\mu_{1,t} - \gamma_t)[b_{1,t} + kb_{\infty,t}] + \mu_{\infty,t}]\delta_t$. When γ_t is equal to a constant γ in all

⁷Appendix A.1 shows that equation 1.61 does not hold under other conditions such as oscillatory multipliers.

periods and $\mu_{\infty,t} = 0$, $\zeta_t = 0$, and the expression for the tax rate reduces to

$$(1 - \tau_t) = \frac{1 + \gamma \left\{ \frac{u_{cc}(c_t)}{u_c(c_t)} [c_t - z_t] + 1 \right\}}{1 + \gamma \left\{ \frac{u_{nn}(n_t)}{u_n(n_t)} n_t + 1 \right\}}, \quad (1.63)$$

which is analogous to the expression for the tax rate under complete markets.

1.3 MONETARY POLICY THAT SUPPORTS COMPLETE MARKETS REAL ALLOCATIONS

This section describes a method to find nominal bond prices, bond quantities, and inflation policy given a (state-contingent) target for expected inflation that support the complete markets real allocations from Section 1.2.1. Bond pricing equations and the form of the government's budget constraint are from Section 1.2.3.

A variant of this method is first applied to the War and Peace economy to show that its allocations can be supported with real one-period bonds and consols, but that the government must be a net purchaser of one-period bonds during the peace state. When the government can issue nominal bonds, policy that supports real allocations with positive bond issuance often involves a one-time increase in inflation upon entry to the first war state.

We next study optimal policy in a three-state economy with fluctuating productivity and constant transfer payments, which is intended to capture features of the modern U.S. economy.

Drawing a connection between the complete markets real allocations and the Ramsey planner's problem with nominal one-period bonds and consols, the real

state-contingent return on government debt is

$$r_v(\zeta'; \zeta) = \frac{F(\zeta')}{F(\zeta) - f(\zeta)} = \frac{[\omega_1 + (k + Q_\infty(\zeta))\omega_\infty]}{Q_1(\zeta)\omega_1 + Q_\infty(\zeta)\omega_\infty} \delta(\zeta'; \zeta) \quad (1.64)$$

Nominal bonds are priced

$$Q_1(\zeta) = \frac{1}{u_c(\zeta)} \circ \beta(M \circ \delta)u_c(\zeta) \quad (1.65)$$

$$Q_\infty(\zeta) = \frac{k}{u_c(\zeta)} \circ (I - \beta(M \circ \delta))^{-1} \beta(M \circ \delta)u_c(\zeta) \quad (1.66)$$

The Markov environment allows consol prices to be written in terms of their coupon, expected future marginal utility of consumption, and δ , which denotes the full state-to-state matrix for nominal depreciation. Real returns on government debt also depend on its maturity structure, with the fraction of debt issued as one-period bonds and consols summing to 1, so that

$$1 = \omega_1 + \omega_\infty. \quad (1.67)$$

Given an expected inflation target in each state, denoted $\underline{\delta}(\zeta)$, and the exogenous maturity structure for debt, it is possible to iteratively find a unique value of δ that replicates the state-contingent real return on government debt, $r_v(\zeta'; \zeta)$. The full algorithm to compute δ is described in Section [A.2.2](#).

1.3.1 War and Peace Economy Supported with Real One-Period Bonds and Consols

One-period real bond prices and consol prices can be computed from the real allocations. k is chosen so that consols have a price of 1 in the peace state.

$$q_1(\varsigma) = \frac{1}{u_c(\varsigma)} \circ \beta M u_c(\varsigma) \quad (1.68)$$

$$q_\infty(\varsigma) = \frac{k}{u_c(\varsigma)} \circ (I - \beta M)^{-1} \beta M u_c(\varsigma) \quad (1.69)$$

where $M u_c(\varsigma)$ is understood to be matrix multiplication, and \circ denotes element-wise multiplication.

In order to support the real allocations, the government must issue debt with a market value of $F(5) - f(5) = 0.717$ in the peace state that returns the surplus of $f(5) = 0.053$ to the household in a transition to the peace state, but transfers resources to the government in a transition to the war state in the amount $F(5) - f(5) - F(1) = 0.305$ by reducing the market value of debt outstanding. This is a system of two equations and two unknowns:

$$F(5) = f(5) + q_{1,5} b_{1,5} + q_{\infty,5} b_{\infty,5} \quad (1.70)$$

$$F(5) = b_{1,5} + (k + q_{\infty,5}) b_{\infty,5} \quad (1.71)$$

In matrix notation,

$$\begin{bmatrix} F(5) - f(5) \\ F(5) \end{bmatrix} = \begin{bmatrix} q_{1,5} & q_{\infty,5} \\ 1 & (k + q_{\infty,5}) \end{bmatrix} \begin{bmatrix} b_{1,5} \\ b_{\infty,5} \end{bmatrix} \quad (1.72)$$

allows a solution for $[b_{1,5} \ b_{\infty,5}]' = [-2.514 \ 3.124]'$.

The bonds purchased in the peace state must also transfer 0.305 to the government by reducing the market value of debt to 0.412 in the war state.

$$F(1) = b_{1,5} + (k + q_{\infty,1})b_{\infty,5} = 0.412 \quad (1.73)$$

In the peace state, the government sells consols to the household and purchases one-period bonds from the household. In order for returns on this portfolio to replicate optimal policy with state-contingent securities, the portfolio must return the peace-state surplus to the household if the peace state continues, and generate high returns for the government if there is a transition to the first war state. The return on consols from peace to peace (1.046) is higher than the return on one-period bonds (1.045), so a policy of selling the high-return security and purchasing the low-return security will transfer resources away from the government. In the peace state, productivity is 1 and labor is 0.771, so that output is 0.771, while consumption is 0.671. The individual bond positions of $b_{1,5} = -2.514$ and $b_{\infty,5} = 3.124$ are each more than three times as large as output.

The first panel of the table shows nominal one-period bond and consol prices for this economy (Q1 and Qinf), which are identical to the real prices since there is no inflation in this economy. k is the coupon on the consol, chosen so that it trades at par in the peace state. b_1 and b_{inf} are the quantities of one-period bonds and consols issued by the government in each state. A negative value indicates that the government is a net purchaser of the asset. During each of the war states, expected returns on one-period bonds and consols are equal, so the government is indifferent between issuing one-period bonds and consols. In these cases, the government issues half of its debt in each type of security. R_{1t} and ER_{inf} are the nominal return on a one-period bond, and the nominal expected return on a

consol, respectively. Again, nominal returns are equal to real returns. In the peace state, there is a very small risk premium on consols over one-period bonds.

state	Q1	Qinf	k	b1	binf	R1t	ERinf
1	0.950	0.885	0.051	0.244	0.244	1.053	1.053
2	0.950	0.880	0.051	0.277	0.277	1.053	1.053
3	0.950	0.875	0.051	0.313	0.313	1.053	1.053
4	0.827	0.870	0.051	0.376	0.376	1.209	1.209
5	0.957	1.000	0.051	-2.514	3.124	1.045	1.046

1.3.2 Inflation Policy in the War and Peace Economy

1.3.2.1 Constant Expected Inflation Target

This section shows that increasing the face value of debt issued as consols decreases inflation in the transition from the peace state to the war state under a constant inflation target.

When the government only issues one-period nominal bonds, outcomes are very similar to those that would be obtained in the Chari et al. (1991) economies, except that monetary policy in their economies adheres to the Friedman rule, whereas monetary policy in this economy adheres to an inflation target.

Figures 1.1 and 1.2 plot “impulse responses” for a War and Peace economy that begins in the peace state in period 1, transitions again to the peace state in period 2, and then transitions to the first war state in period three, only returning again to the peace state by period 7, and remaining in the peace state in period 8.

The state transitions are as follows:

Period	1	2	3	4	5	6	7	8
State	5	5	1	2	3	4	5	5

Figure 1.1 shows that inflation from the peace state to the war state is 1.828 (an 82.8% increase in the price level) when all debt is issued as one-period nominal bonds, and falls to 1.637 (a 63.7% increase in the price level) when all debt is issued as consols. Optimal policy requires the government to reduce the real market value of its debt from the peace state to the war state. When all debt is issued as one-period nominal bonds, the only mechanism by which the real market value of debt can be reduced is inflation. In contrast, nominal consol prices decrease from 1 in the peace state to 0.887 in the war state. Therefore, when some portion of the debt is issued as consols, inflation is only required to further reduce the real market value of debt by an amount not already reflected in the decreased price of consols.

1.3.2.2 *State-Contingent Expected Inflation Targets*

Inflation in the peace-to-war transition can be reduced even further when expected inflation is higher during the war states, and some portion of the debt is issued as consols. This section sets expected gross inflation equal to 1.25, or 25% in each of the four war states, and varies consol issuance from zero to 100% of debt. Higher expected inflation in the war states coincides with higher nominal one-period interest rates in the war states. Since one-period bonds and consols get the same returns in the war states, higher nominal interest rates on one-period bonds require the consol price to drop precipitously in the transition from the peace state to the war state, so that it can produce high nominal returns in each war state, and return to trading at par in the peace state. This larger decrease in the consol price accounts for a larger portion of the required decrease in the market value of government debt from the peace state to the war state. In fact, expected gross inflation of 1.25 during the war states is high enough that the price level actually

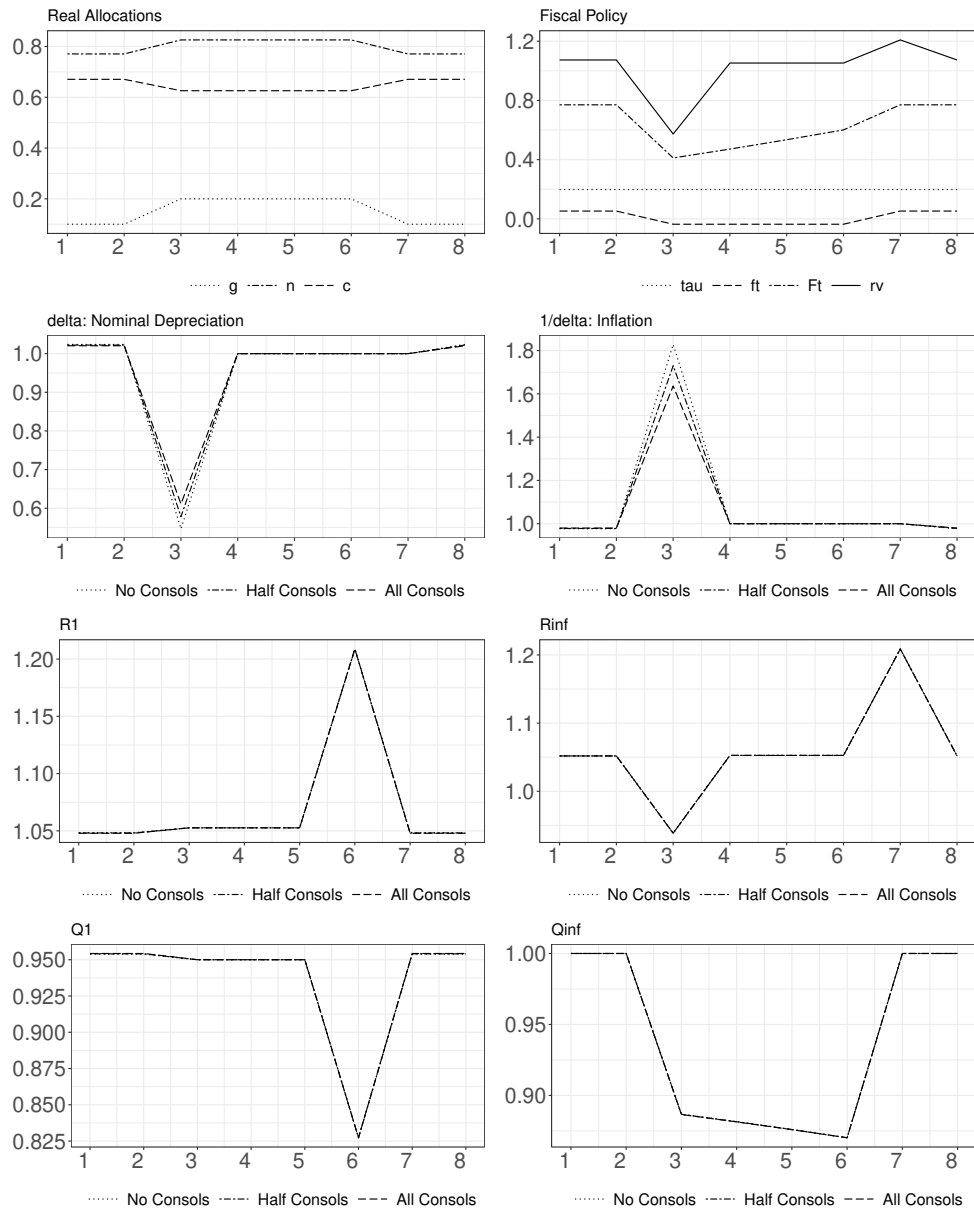


Figure 1.1: War and Peace Economy, Constant Expected Inflation Target

drops during the transition from peace to war when all debt is issued as consols. Figure 1.2 shows that with this state-contingent inflation target, gross inflation in the peace-to-war transition drops from 1.828 with no consol issuance, to 1.345 with half of debt issued as consols, to 0.924 when all debt is issued as consols.

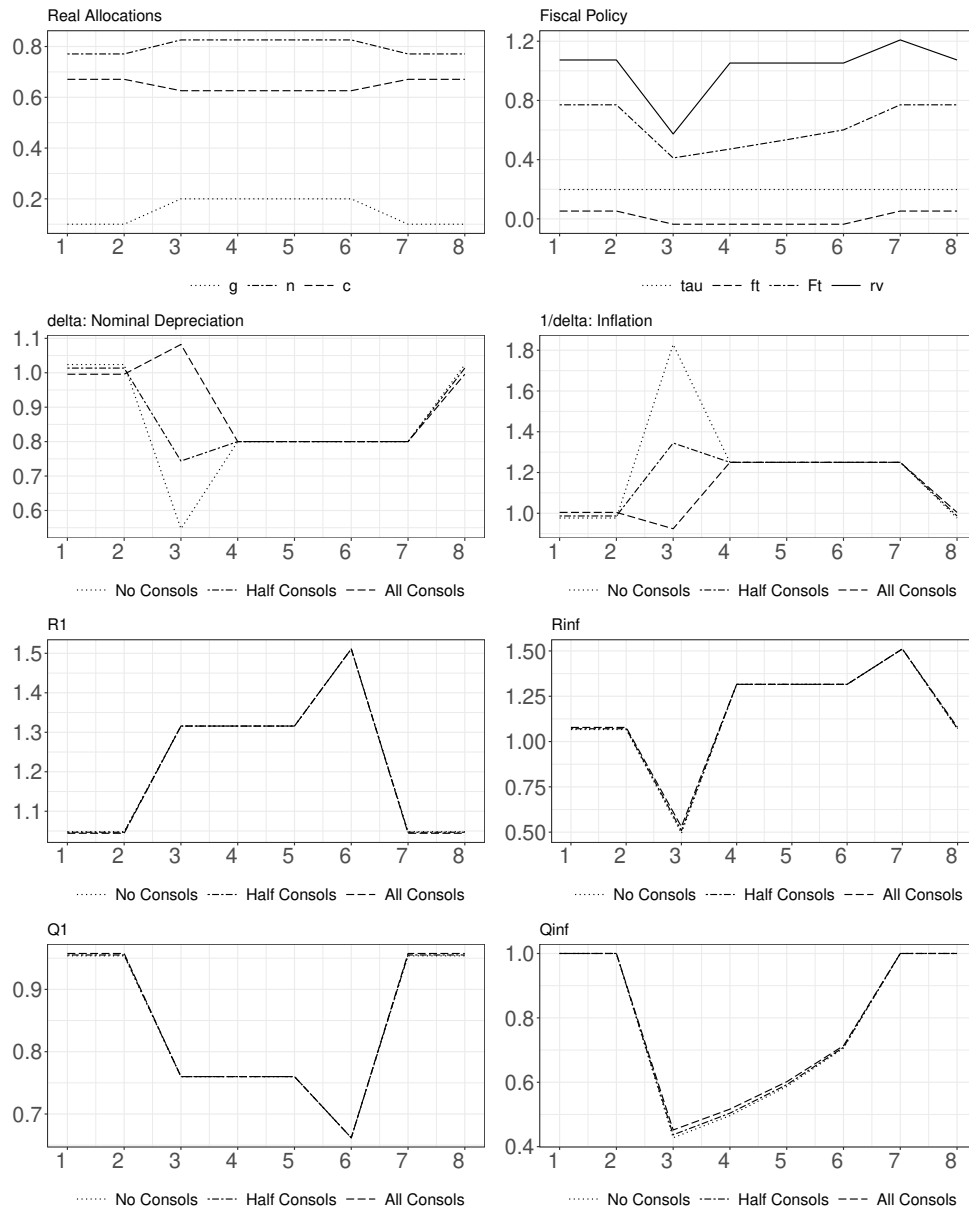


Figure 1.2: War and Peace Economy, State-Contingent Expected Inflation Target

1.3.3 Inflation Policy in a Fluctuating Productivity Economy

In the peace and war economy, inflation in the transition from peace to war is always decreasing in consol issuance. However, it will not always be the case that issuing a larger fraction of debt as consols will decrease inflation volatility. To produce an example of this, I introduce a three-state economy with fluctuating productivity and transfer payments, which is intended to approximate the modern US economy. As in the war and peace economy, inflation volatility is decreasing in consol issuance under a constant inflation target, and inflation volatility can be reduced even further with state-contingent inflation targets.

However, introducing state-contingent transfers that are higher in high-productivity states causes inflation volatility to increase when a larger portion of debt is issued as consols. This is because the real allocations that support state-contingent transfers require the market value of debt to vary less than consol prices across states. Therefore, as more debt is issued as consols, fluctuations in consol prices cause its market value to fluctuate more, and require inflation to offset the fluctuations in consol prices in order to generate market values of debt that do not vary much across states.

Finally, we return to an economy with constant transfers and a higher initial quantity of debt outstanding. In this economy, real returns on government debt vary less from state to state, and inflation is less volatile as well.

The real allocations for the three-state economy with constant transfer payments are

state	g	z	a	b0_an	PHI	n	c	un	uc
1	0.000	0.300	0.882	1.000	0.366	0.654	0.577	-1.283	3.005
2	0.000	0.300	1.000	1.000	0.366	0.629	0.629	-1.186	2.529
3	0.000	0.300	1.118	1.000	0.366	0.607	0.679	-1.107	2.169

state	tau	ft	Ft	FtWt	q1	q10	qinf	er1	erinf
1	0.516	-0.002	0.409	0.038	0.920	0.519	0.866	1.087	0.969
2	0.531	0.034	0.629	0.050	0.954	0.605	1.000	1.048	1.004
3	0.543	0.069	0.867	0.060	0.982	0.694	1.137	1.018	1.034

Figures 1.3, 1.4, and 1.5 plot “impulse responses” for a fluctuating productivity economy that begins in the medium-productivity state, and stays there for one period before transitioning to the high-productivity state, and from there cycling from the low-productivity state through the medium-productivity state and back to the high-productivity state before returning to the medium-productivity state. State transitions are

Period	1	2	3	4	5	6	7	8
State	2	2	3	1	2	3	2	2

1.3.3.1 Constant Expected Inflation Target

Figure 1.3 shows that inflation in the transition from the high-productivity state to the low-productivity state goes from 2.005 with no consol issuance to 1.576 with all consol issuance.

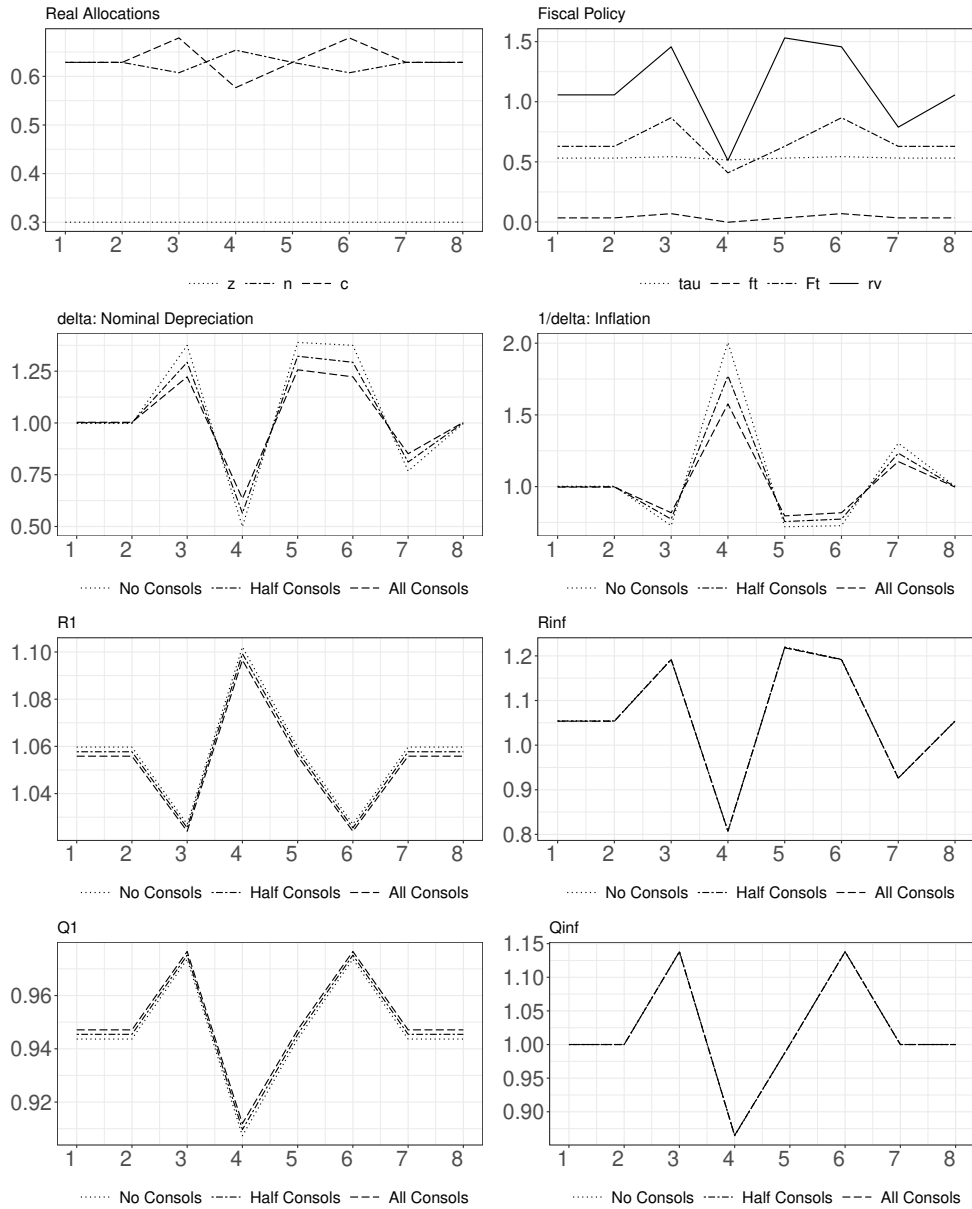


Figure 1.3: Fluctuating Productivity Economy, Constant Expected Inflation Target

1.3.3.2 *State-Contingent Expected Inflation Targets*

Gross inflation targets are 1.10 in the low-productivity state, 1.05 in the medium-productivity state, and 1.00 in the high-productivity state. It is not possible to set inflation targets at [1.05, 1.00, 0.95] because doing so would require values for $R_{1,t}$ of less than 1 in the high-productivity state. This hints at a rationalization for positive steady-state inflation: implementation of a state-contingent inflation target that reduces inflation volatility may require one-period nominal rates to be set lower in some periods than others, and positive steady-state inflation provides “room” for policymakers to reduce nominal rates when needed.

With only one-period bond issuance, inflation volatility is identical to that in the reference economy, but mean inflation is shifted up by 0.10 and 0.05 in the low- and medium-productivity states, respectively. As in the war and peace economy, only the combination of state-contingent inflation targets and consol issuance can substantially reduce inflation volatility. Figure 1.4 shows that inflation in the transition from the high- to low-productivity states is 2.005 with no consol issuance, drops to 1.554 when half of debt is issued as consols, and drops further to 1.235 when all debt is issued as consols.

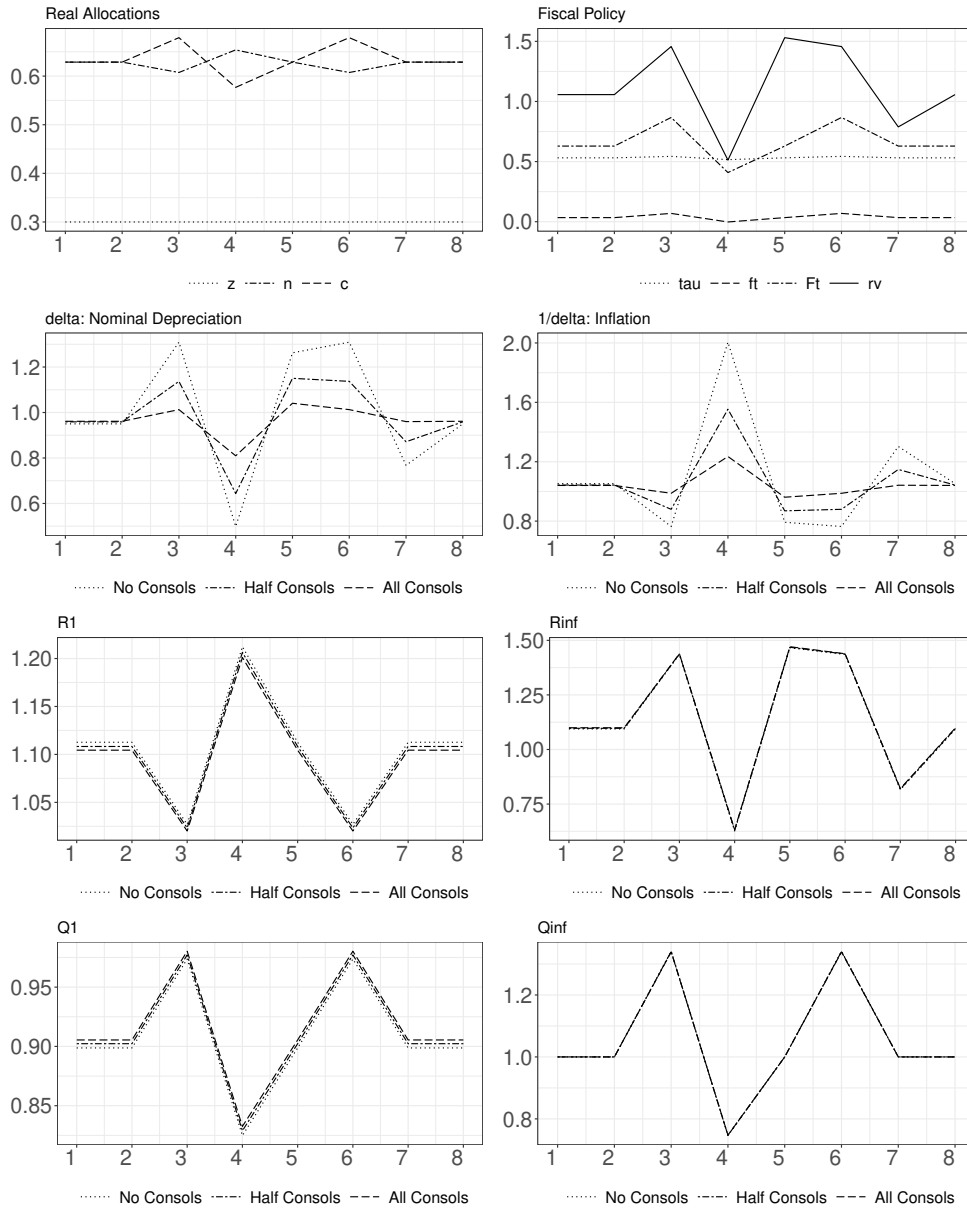


Figure 1.4: Fluctuating Productivity Economy, State-Contingent Expected Inflation Target

1.3.3.3 *State-Contingent Transfers*

Transfer payments are now set to $[0.250, 0.300, 0.350]'$ in the low- medium- and high-productivity states, respectively, rather than being set at 0.300 in all states. This change in policy produces a dramatic reduction in the variance of the market value of debt across states, and also requires the market value of debt to be *higher* in low-productivity, (high expected-productivity-growth) states. With constant transfers, debt was $[0.409, 0.629, 0.867]'$, but with state-contingent transfers, this becomes $[0.639, 0.632, 0.610]'$.

The expected gross inflation target is 1 in all states. Correspondingly, the nominal one-period interest rate is still lower in high-productivity (low expected-productivity-growth) states, so gross inflation is now less than one in the transition from a higher- to a lower-productivity state.

Real returns on the portfolio of government debt vary little across state-to-state transitions, from 1.004 to 1.094. When all debt is issued in one-period bonds, the appropriate real returns can be generated with inflation that moves the price level by less than 5% for any state-to-state transition.

In contrast, consol prices vary greatly across states, trading at more than 10% below and above par in the low- and high-productivity states, respectively. Figure 1.5 shows that when government debt is all issued as consols, inflation policy must counteract the volatility in consol prices in order to produce returns on the portfolio of government debt that vary little across states.

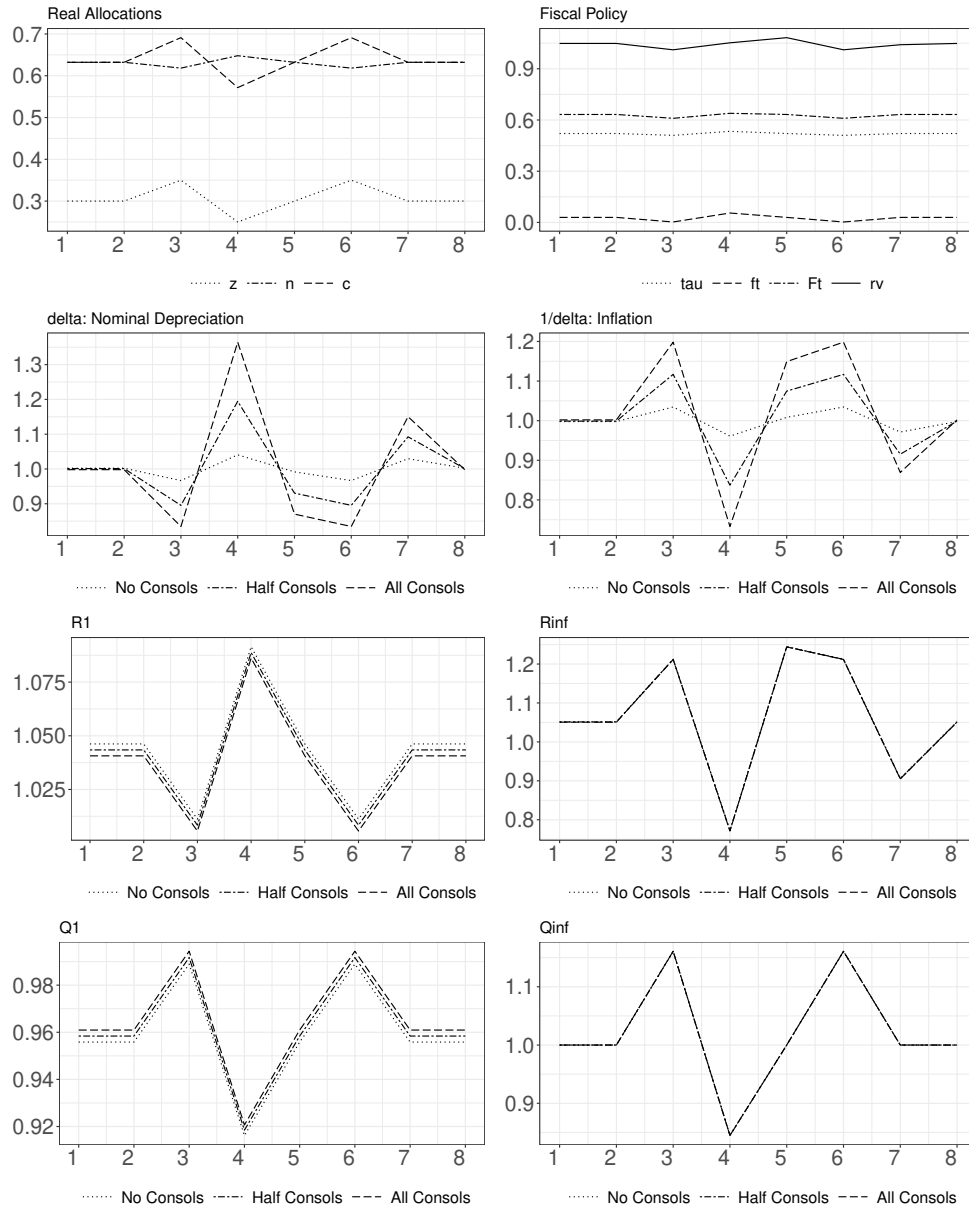


Figure 1.5: Fluctuating Productivity Economy, State-Contingent Transfer Payments

1.3.3.4 *Higher Initial Debt*

Everything is the same as in the fluctuating productivity economy with state-contingent expected inflation targets, except the initial debt-to-gdp ratio is now 2 instead of 1. Tax rates and primary surpluses are somewhat higher, while volatility in the real returns on government debt are lower. As a result, inflation volatility required to replicate the real returns is lower as well. As in previous economies, optimal policy produces a spike in inflation in the transition from the high-productivity state to the low-productivity state. When the debt-to-gdp ratio was 1 and all debt was issued as one-period bonds, inflation in this transition was 2: a 100% increase in the price level. Now that the debt-to-gdp ratio is 2, inflation in this transition is roughly 1.6, corresponding to a 60% increase in the price level. As consol issuance increases, inflation during this transition decreases. As in previous economies, when a large enough fraction of government debt is issued as consols, inflation will actually fall in the transition from the high-productivity state to the low-productivity state.

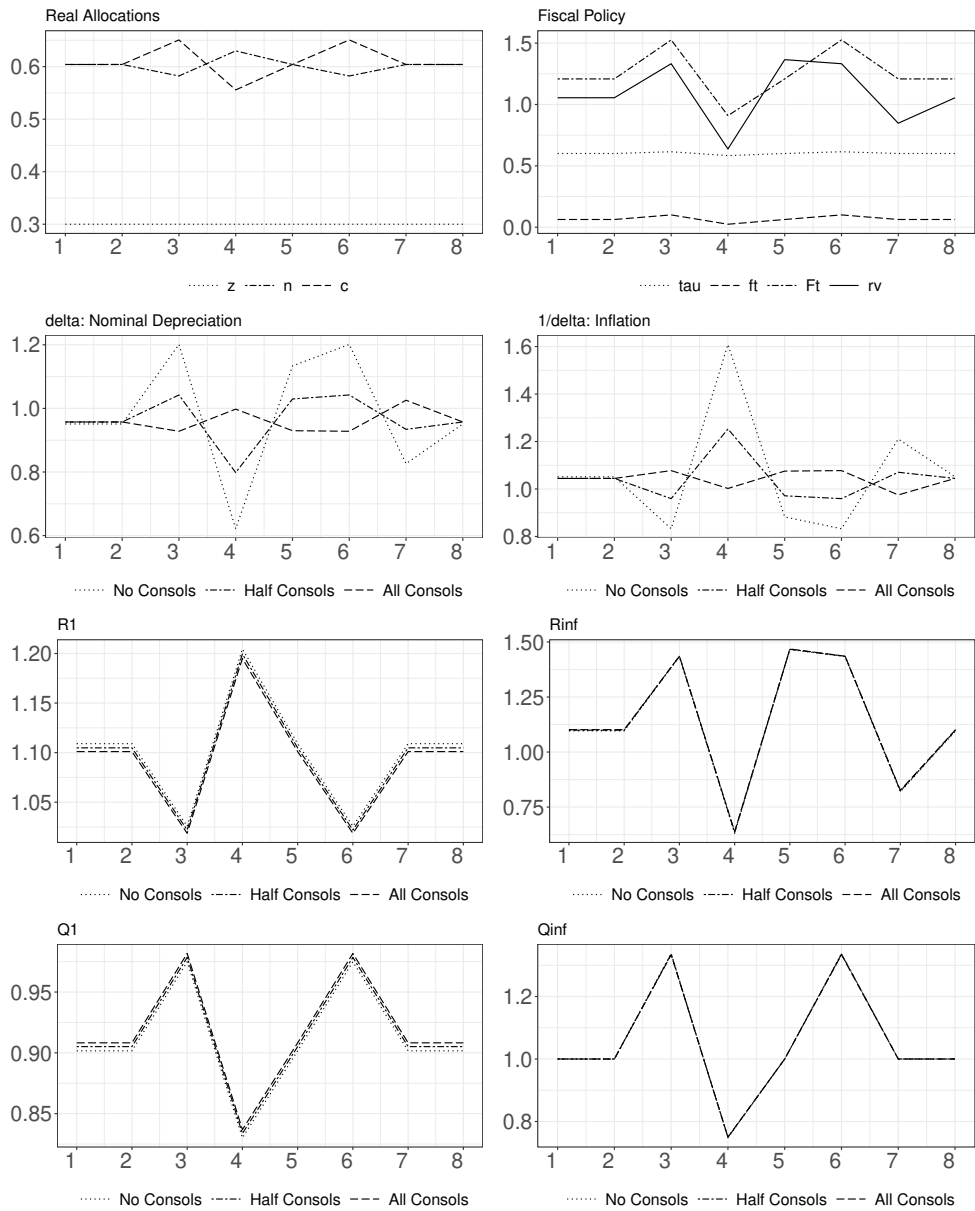


Figure 1.6: Fluctuating Productivity Economy, Higher Initial Debt

1.4 MODEL PREDICTIONS AND STYLIZED FACTS

1.4.1 Tax Receipts, Spending and Debt

In the model, the tax rate τ_t has three indistinguishable interpretations. It is the marginal tax rate, the average tax rate, and the fraction of output collected by the government as tax receipts. When thinking about τ_t in the context of a government that uses tax revenues in dollars to finance spending and interest payments, it may be most appropriately interpreted as the fraction of output collected as tax receipts.

The upper panel of Figure 1.7 plots the fraction of output collected as tax revenue since 1947. This series is trendless even before 1971, and becomes more procyclical by the 1990s, reaching a peak of just over 20% in 2000, and a trough just above 15% in 2009. Correspondingly, the three-state economy with constant transfer payments exhibits a tax rate that is somewhat higher in the high-productivity state than in the low-productivity state.

The middle panel of Figure 1.7 plots defense expenditures and total expenditures as fractions of output over the postwar period. Taking defense expenditures as a proxy for government purchases and non-defense expenditures as a proxy for transfer payments, this panel shows that transfer payments as a fraction of government expenditures has increased over the postwar period. This is consistent with the increased fluctuation in the fraction of output collected as tax receipts since 1971.

The lower panel of Figure 1.7 plots the ratio of the market value of debt to output over the postwar period.

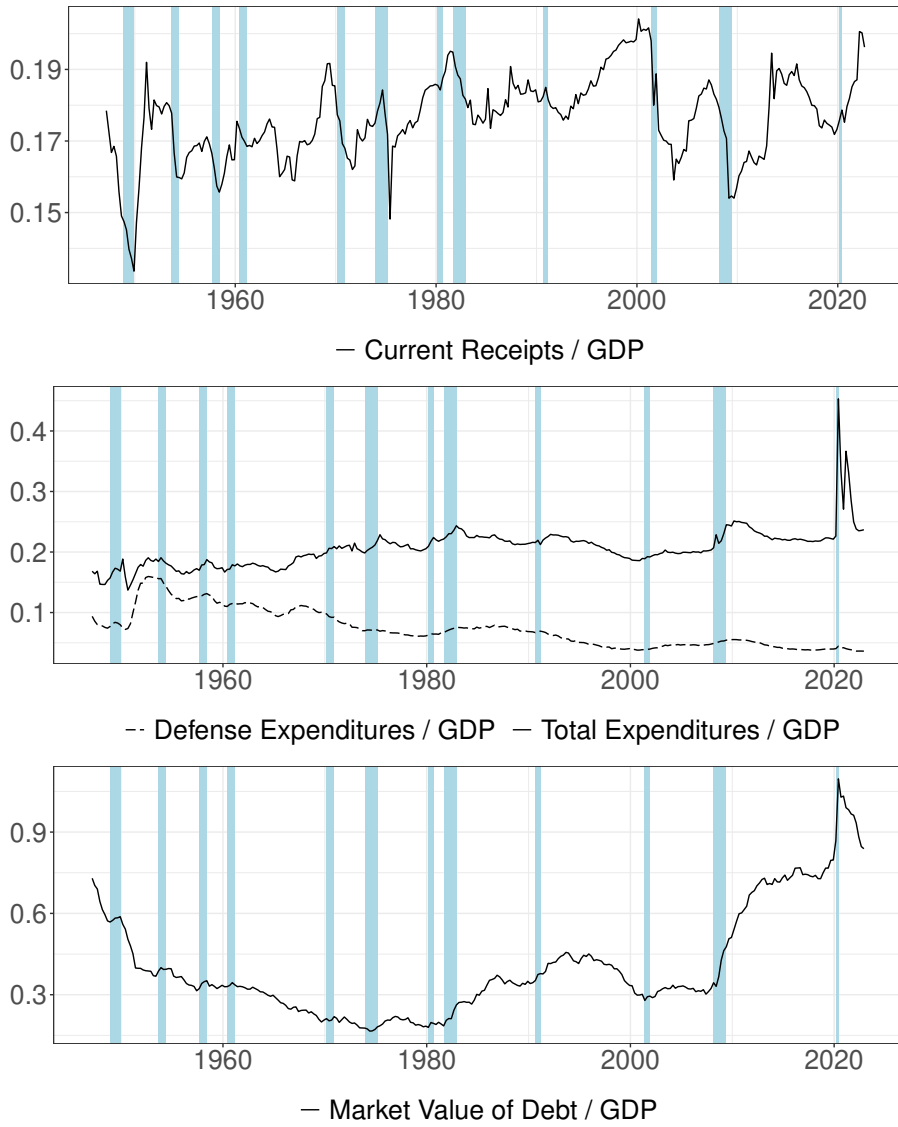


Figure 1.7: Tax Receipts, Spending and Debt

1.4.2 Taylor Rules and Phillips Curves

Neither a Taylor Rule nor a Phillips curve are included as a structural component of the model. Nevertheless, it is possible to estimate simple Taylor Rule and Phillips curve coefficients on both model-generated data and on postwar US data.

The Taylor Rule can be written

$$(R_t - R_t^*) = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t - y_t^*) \quad (1.74)$$

and coefficients can be estimated assuming that $(R_t - R_t^*)$ and $(\pi_t - \pi_t^*)$ represent the log deviation of R_t and π_t from a trend value. I will use the a two-sided HP filtered trend for R_t^* , and a constant 2% inflation target for π_t^* . While the 2% time-invariant inflation target is strict, the HP filtered trend for R_t^* allows for the “natural rate” to fluctuate. The expression $(y_t - y_t^*)$ represents the log deviation of output from its potential level. All variables are retrieved from FRED. R_t is the quarterly mean of the daily Federal Funds rate.

Expression	Fred Variables
R_t	log(DFF)
π_t	log(GDPDEF/lag(GDPDEF,4))
$(y_t - y_t^*)$	log(GDP / NGDPPOT)

A Phillips curve relationship between inflation and the output gap can be written

$$(\pi_t - \pi_t^*) = \kappa(y_t - y_t^*) \quad (1.75)$$

Figure 1.8 plots the deviation of the Federal Funds rate from its trend, the deviation of inflation from its target, and the deviation of output from potential output.

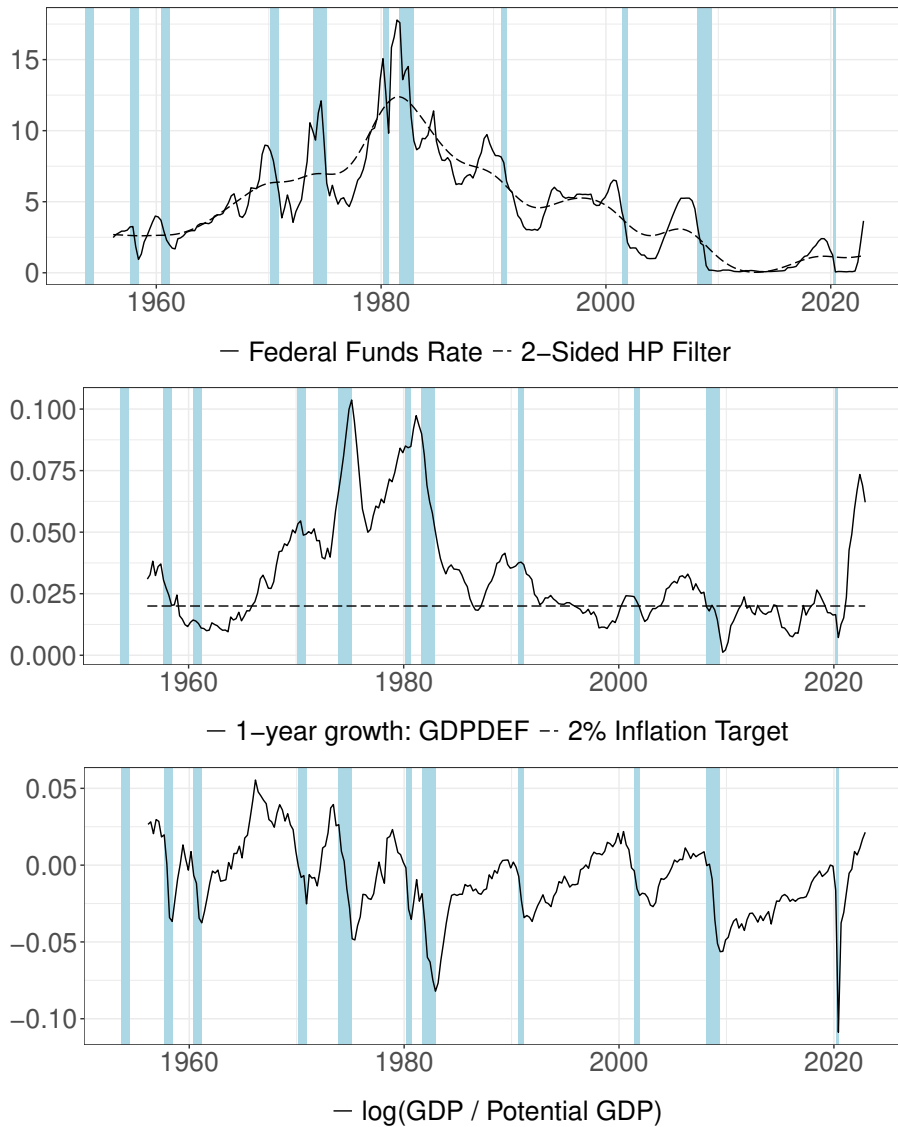


Figure 1.8: Data and Trends for Taylor Rules and Phillips Curves

Tables 1.1 and 1.2 display coefficients from these regressions using quarterly data covering two subsamples from 1956:Q1 to 1981:Q4, and from 1982:Q1 to 2022:Q4. In the estimated Taylor Rule, the coefficient on inflation is close to zero in the first subsample, and positive but less than one in the second subsample. The

coefficient on the output gap is positive in both subsamples, as predicted. The estimated Phillips curve shows a negative relationship between inflation and output in the first subsample, and a positive relationship in the second subsample.

	<i>Dependent variable:</i>	
	$(R_t - R_t^*)$	$(R_t - R_t^*)$
	(1)	(2)
$(\pi_t - \pi_t^*)$	0.008 (0.029)	0.381*** (0.100)
$(y_t - y_t^*)$	3.146*** (1.095)	15.186*** (2.010)
Sample Period 1956:Q1-1981:Q4 1982:Q1-2022:Q4		
Observations	104	164
R ²	0.075	0.308
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01	

Table 1.1: Quarterly Taylor Rule Regression

	<i>Dependent variable:</i>	
	$(\pi_t - \pi_t^*)$	$(\pi_t - \pi_t^*)$
	(1)	(2)
$(y_t - y_t^*)$	-1.095 (0.709)	5.727*** (1.002)
Sample Period 1956:Q1-1981:Q4 1982:Q1-2022:Q4		
Observations	104	164
R ²	0.023	0.167
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01	

Table 1.2: Quarterly Phillips Curve Regression

The subsequent tables display these same coefficients estimated using data generated by the War and Peace, and the Fluctuating Productivity economies. Data from these economies produce a similar mix of positive and negative coefficients.

econ_type	debt_inst	deltatarget	phi_pi	phi_y	kappa
War and Peace	No Consols	Constant	-0.076	0.750	2.503
War and Peace	Half Consols	Constant	-0.084	0.754	2.282
War and Peace	All Consols	Constant	-0.093	0.758	2.058

econ_type	debt_inst	deltatarget	phi_pi	phi_y	kappa
War and Peace	No Consols	State-Cont.	-0.128	4.399	4.787
War and Peace	Half Consols	State-Cont.	-0.661	6.148	3.538
War and Peace	All Consols	State-Cont.	0.148	3.559	1.935

econ_type	debt_inst	deltatarget	phi_pi	phi_y	kappa
Fluctuating Productivity	No Consols	Constant	0.059	-0.160	-4.614
Fluctuating Productivity	Half Consols	Constant	0.060	-0.197	-3.800
Fluctuating Productivity	All Consols	Constant	0.063	-0.230	-3.024

econ_type	debt_inst	deltatarget	phi_pi	phi_y	kappa
Fluctuating Productivity	No Consols	State-Cont.	0.048	-0.794	-4.617
Fluctuating Productivity	Half Consols	State-Cont.	-0.019	-1.062	-2.917
Fluctuating Productivity	All Consols	State-Cont.	-0.040	-1.054	-1.382

econ_type	debt_inst	deltatarget	phi_pi	phi_y	kappa
Fluctuating Productivity	No Consols	State-Cont.	0.116	-0.430	0.242
Fluctuating Productivity	Half Consols	State-Cont.	0.076	-0.483	1.052
Fluctuating Productivity	All Consols	State-Cont.	0.079	-0.550	1.834

1.5 CONCLUSION

Under flexible prices and abstracting from all sources of monetary frictions, a government that finances its expenses with only nominal one-period bonds and consols can conduct policy to support the complete markets real allocations from Lucas & Stokey (1983) with moderate inflation volatility and positive bond issuance.

Equilibrium outcomes are determined by government policy consisting of a tax rate, a nominal one-period interest rate, and a maturity structure for outstanding debt. An inflation-targeting central bank could use such a model when determining how its nominal interest rate policy should respond to evolving economic conditions.

CHAPTER 2

Empirical Shocks to Fiscal Conditions

2.1 INTRODUCTION

A complete markets economy such as that studied by Lucas & Stokey (1983) produces the surprising implication that the market value of government debt should decrease in response to an unexpected increase in government purchases. In contrast, incomplete markets models of government finance such as that studied by Aiyagari et al. (2002) imply that the market value of government debt should increase in response to the same unexpected increase in government purchases. I show that the real market value of debt increases in response to Ramey (2011) military spending shocks prior to 1971, consistent with Aiyagari et al. (2002), and decreases in response to the same shocks after 1971, consistent with Lucas & Stokey (1983). Taken together, these findings are consistent with the idea that the gold standard limited the inflation policy that the government could pursue.

I next turn to shocks to the real discount rate, measured by unexpected changes in the *cay* “consumption to assets and income” ratio as measured by Lettau & Ludvigson (2001). Under the gold standard, there is no obvious relationship between shocks to the real discount rate and the price level. However, the government debt flow identity implies that all else equal, an unexpected increase in the real discount rate should increase the price level in the absence of increases to nominal interest rates. Before 1971, *cay* is unrelated to the price level, but after 1971, unexpected increases in *cay* are associated with increases in the price level.

The theory provides no clear predictions on how the price level should respond to unexpected changes in interest rates on FOMC announcement dates. Neverthe-

less, responses to these shocks are reported as well. The “price puzzle” is only apparent after 1971. Controlling for unexpected changes in interest rates on FOMC announcement dates does not substantially alter the previous results for unexpected government purchases or changes in *cay*.

Tables report changes in variables from the beginning to the end of the quarter in which the shock was observed, along with Newey & West (1987) standard errors computed with four lags. Graphs plot the response to shocks over the subsequent four quarters using the local projections method described by Jordà (2005). Dotted lines are the 95% confidence intervals implied by the same Newey West robust standard errors with four lags.

2.1.1 Relating the Price Level to Fiscal Conditions

The government debt flow identity can be written

$$0 = P_{t+1}s_{t+1} - R_{t+1}^V V_t + V_{t+1} + (R_{t+1}^V - 1)M_t. \quad (2.1)$$

Variables are defined in Table 2.1.

V_t	Nominal Market Value of Government Debt
R_{t+1}^V (r_{t+1}^V)	Nominal (Real) Holding Period Return on Govt Debt
s_{t+1}	Real Primary Surpluses: (Tax Revenue - Non-Interest Spending)
P_{t+1}	Price Level
M_t (m_t)	Nominal (Real) Base Money

Table 2.1: Variables in the government debt flow identity

The fiscal authority finances surpluses $P_{t+1}s_{t+1}$ (deficits) and currently outstanding debt $R_{t+1}^V V_t$ by issuing new debt V_{t+1} and by receiving seigniorage revenue from the monetary authority $(R_{t+1}^V - 1)M_t$.

Equation 2.1 can be iterated forward to produce

$$V_t = P_t E_t \left[\sum_{j=1}^{\infty} \left(\prod_{k=1}^j \frac{1}{r_{t+k}^V} \right) \{s_{t+j} + (R_{t+j}^V - 1)m_{t+j-1}\} \right] \quad (2.2)$$

Equation 2.2 is the standard infinite-horizon representation of the government budget constraint, describing an equilibrium relationship between the nominal value of debt V_t , the price level P_t , and the present discounted value of real surpluses s_{t+j} and seigniorage revenue $(R_{t+j}^V - 1)m_{t+j-1}$.

For ease of notation, denote the expected present value of real surpluses as ψ_t , so that

$$V_t = P_t \psi_t. \quad (2.3)$$

Equivalently, the present value of real surpluses is

$$\frac{V_t}{P_t} = \psi_t \quad (2.4)$$

In order for currently outstanding government debt to maintain its (real) market value, investors must expect that the government will be able to raise enough real surplus revenue to pay back the principal and interest on the debt.

The relationship in equation 2.3 is implied by the government debt flow identity, and will always hold. Taking logs and denoting the log of each variable with a tilde, equation 2.3 can be written in first differences:

$$\Delta \tilde{V}_{t+1} = \Delta \tilde{P}_{t+1} + \Delta \tilde{\psi}_{t+1} \quad (2.5)$$

where $\Delta x_{t+1} \equiv (x_{t+1} - x_t)$.

The relationship in equation 2.5 holds ex-post, so it also holds ex-ante

$$E_t \Delta \tilde{V}_{t+1} = E_t \Delta \tilde{P}_{t+1} + E_t \Delta \tilde{\psi}_{t+1} \quad (2.6)$$

and can be written in innovations, the difference between the realized value of a variable and its expectation:

$$(\Delta \tilde{V}_{t+1} - E_t \Delta \tilde{V}_{t+1}) = (\Delta \tilde{P}_{t+1} - E_t \Delta \tilde{P}_{t+1}) + (\Delta \tilde{\psi}_{t+1} - E_t \Delta \tilde{\psi}_{t+1}) \quad (2.7)$$

2.1.2 Reduced Form Estimation Equation

Suppose there is a shock to fiscal conditions between period t and $t + 1$, denoted z_{t+1} . The identifying assumption is that the expected value of the shock at time t is zero: $E_t z_{t+1} = 0$. If the shock affects an outcome variable y_{t+1} , then z_{t+1} will help to explain innovations to the outcome variable, $(\Delta \tilde{y}_{t+1} - E_t \Delta \tilde{y}_{t+1})$, but will not affect any variables observed or forecasted at time t . Using first differences in the regression ensures that all variables are stationary.

$$\begin{aligned} \Delta \tilde{y}_{t+1} = & \beta_1 z_{t+1} \\ & + \beta_2 (S_t / C_t) + \beta_3 (V_t / C_t) \\ & + \beta_4 \ln(R_t^{3mo}) + \beta_5 \ln(R_t^{10yr}) + \beta_0 + \varepsilon_{t+1} \end{aligned} \quad (2.8)$$

where dependent variables are defined in Table 2.2,

and where independent variables are defined in Table 2.3

\tilde{P}_{t+1}	Core PCE Inflation
\tilde{V}_{t+1}	Growth in Nominal Market Value of Debt
\tilde{v}_{t+1}	Growth in Real Market Value of Debt

Table 2.2: Dependent variables for reduced-form estimation

z_{t+1}	Shock to Fiscal Conditions between t and $t + 1$
S_t/C_t	Imputed Surplus / PCE
V_t/C_t	Market Value of Debt / PCE
$\ln(R_t^{3mo})$	3-month Treasury Yield
$\ln(R_t^{10yr})$	10-year Treasury Yield

Table 2.3: Independent variables for reduced-form estimation

The estimation equations for local projections are identical, with cumulative changes in the dependent variable from t to $t + h$, where $h = 1, 2, 3, 4$, and is measured in quarters:

$$\begin{aligned} \Delta \tilde{y}_{t+h} = & \beta_1 z_{t+1} \\ & + \beta_2 (S_t/C_t) + \beta_3 (V_t/C_t) \\ & + \beta_4 \ln(R_t^{3mo}) + \beta_5 \ln(R_t^{10yr}) + \beta_0 + \varepsilon_{t,t+h} \end{aligned} \quad (2.9)$$

I use data on the value of government debt, the stock of base money, and monthly holding period returns on government debt from Hall et al. (2018).

The Core PCE Deflator (PCEPILFE) and the Monetary Base (BOGMBASE) series from FRED are used as data for the price level and monetary base, respectively.

2.2 SHOCKS TO FISCAL CONDITIONS

2.2.1 Ramey (2011) Military Spending Shocks

Ramey (2011) uses articles in Business Week to identify shocks to the present dis-

counted value of defense spending attributable to World War II, the Korean War, the Vietnam War, the Cold War, and the Wars in Afghanistan and Iraq. The series is available from 1939:Q1-2008:Q4. Figure 2.1 shows that these spending shocks are larger during the time of World War II and the Korean War, and much smaller for the remainder of the sample. Following the notation in Ramey (2011), this series is divided by the market value of debt outstanding at the end of the quarter preceeding the shock and denoted $pdvmil_{t+1}/V_t$.

A surprise increase in government purchases must be financed through some combination of increased taxes, issuance of debt, or an increase in the price level, which transfers resources to the government when the government is a net debtor by reducing the real value of its outstanding debt. Coefficient estimates on shocks may be attenuated because an unexpected increase in government purchases may be offset by a decrease in another category of spending, or because not all price adjustments have been measured by the end of the quarter in which the shock occurs.

Tables 2.4 and 2.5 show the growth in the price level, nominal debt, and real surpluses in response to a defense spending shock during the Ramey (2011) sample from 1939:Q1-2008:Q4 before and after the closing of the gold window on August 15th, 1971. The first column shows that a surprise increase in spending equal to 1% of the market value of government debt would raise the price level by 0.018% under the gold standard, but by 0.039%, nearly twice as much, afterward. More strikingly, the real present value of the debt does not change in response to the shocks under the gold standard, but *decreases* in response to government purchase shocks afterward, an empirical finding consistent with predictions of the complete markets framework in Lucas & Stokey (1983).

Figure 2.2 plots local projections for the same outcome variables as in Tables 2.4 and 2.5 over the four quarters following the shock to military purchases.

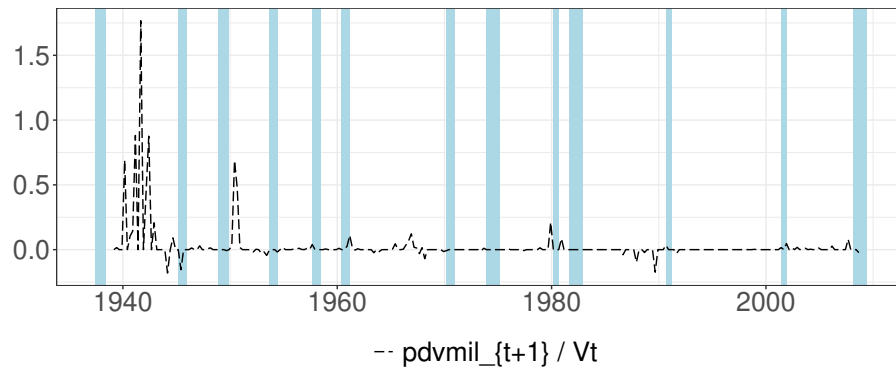


Figure 2.1: Ramey (2011) defense spending shocks are largest before 1971.

	<i>Dependent variable:</i>		
	$\Delta\tilde{P}_{t+1}$	$\Delta\tilde{V}_{t+1}$	$\Delta\tilde{\psi}_{t+1} \equiv \Delta\tilde{V}_{t+1} - \Delta\tilde{P}_{t+1}$
	(1)	(2)	(3)
$pdv\tilde{m}il_{t+1}/V_t$	0.018*** (0.003)	0.034** (0.014)	0.016 (0.016)
Observations	130	130	130
R ²	0.198	0.427	0.428

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.4: Ramey (2011) Defense Shocks 1939:Q1-1971:Q2

	<i>Dependent variable:</i>		
	$\Delta\tilde{P}_{t+1}$	$\Delta\tilde{V}_{t+1}$	$\Delta\tilde{\psi}_{t+1} \equiv \Delta\tilde{V}_{t+1} - \Delta\tilde{P}_{t+1}$
	(1)	(2)	(3)
$pdv\tilde{m}il_{t+1}/V_t$	0.039*** (0.013)	-0.130** (0.064)	-0.169** (0.072)
Observations	149	149	149
R ²	0.568	0.196	0.194

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.5: Ramey (2011) Defense Shocks 1971:Q3-2008:Q4

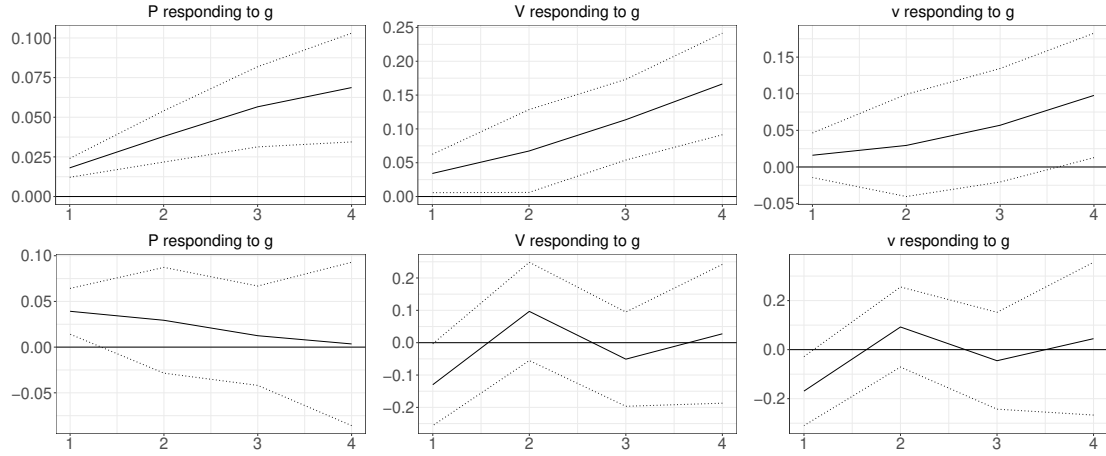


Figure 2.2: Response to g

2.2.2 Lettau Ludvigson (2001) *cay* Shocks

Cochrane (2022) emphasizes that decreases in real discount rates may be an important mechanism through which ψ_t increases during recessions, coinciding with an increase in the real market value of government debt. As a proxy for expected returns, I use the *cay* ratio developed by Lettau & Ludvigson (2001). Lettau and Ludvigson show that a higher consumption to total wealth ratio implies higher expected real returns, and forecasts higher realized returns. This variable is available from 1952:Q1-2019:Q2. Following Campbell & Mankiw (1989), Lettau & Ludvigson (2001) begin with the accumulation equation for aggregate wealth:

$$W_{t+1} = R_{t+1}^w (W_t - C_t) \quad (2.10)$$

After linearizing and iterating forward, the log consumption to wealth ratio is written as:

$$c_t - w_t = \sum_{j=1}^{\infty} \rho_w^j (r_{t+j}^w - \Delta c_{t+j}) \quad (2.11)$$

Lettau and Ludvigson argue that future changes in aggregate consumption will be unpredictable, so that any changes in the ratio of consumption to wealth will reflect changes in expected returns on wealth. They develop the variable cay , the ratio of consumption (c) to assets (a) and income (y), as a proxy for $c_t - w_t$.

When government debt and wealth are priced with a common SDF, consumption-based asset pricing implies a constant risk premium between wealth and government debt, and changes shocks to the SDF for wealth will also be shocks to the SDF for government debt.

To isolate the unexpected change in cay , I run the regression

$$cay_{t+1} = \beta_0 cay_t + cay_{t+1}^u \quad (2.12)$$

and treat the residual cay_{t+1}^u as the unexpected component of cay_{t+1} . The first lag cay_{t-1} in an AR(2) is insignificant so I conclude that this AR(1) specification is sufficient. Figure 2.3 displays cay and the residual cay_{t+1}^u .

Tables 2.6 and 2.7 show that an unexpected 1 percentage point increase in cay , which is associated with an increase in the real discount rate, leads to an 8% increase in the price level under the gold standard, which increases to a 13.6% increase in the price level afterward. In both periods, it is also associated with a decrease in the market value of government debt, consistent with its interpretation as an unexpected increase in real interest rates.

Figure 2.4 plots local projections for the same outcome variables as in Tables 2.6 and 2.7 over the four quarters following the shock to cay .

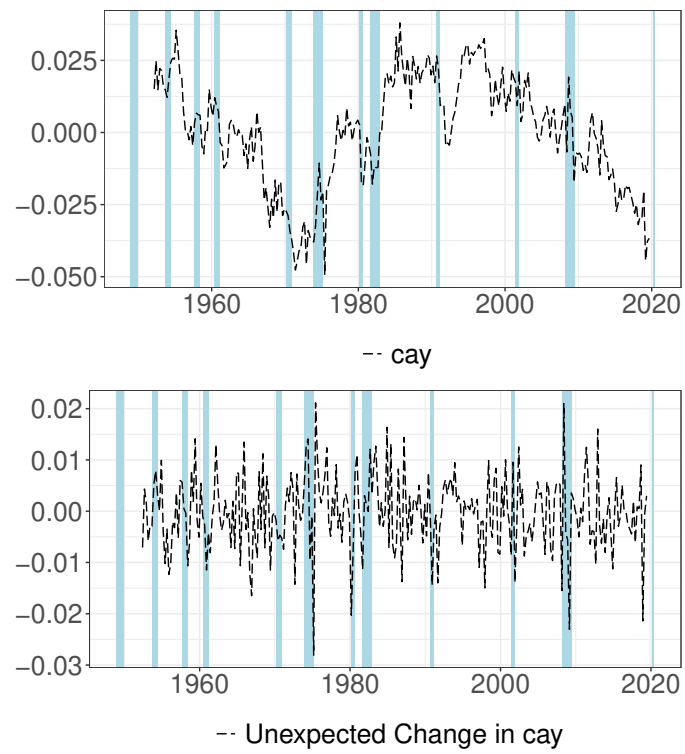


Figure 2.3: *cay* from Lettau and Ludvigson (2001), and its AR(1) residual cay_{t+1}^u .

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
cay_{t+1}^u	0.025 (0.033)	0.075 (0.335)	0.050 (0.348)
Observations	77	77	77
R ²	0.575	0.192	0.111

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.6: Lettau & Ludvigson (2001) Unexpected Changes in *cay* 1952:Q1-1971:Q2

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
cay_{t+1}^u	0.136*** (0.038)	-0.168 (0.246)	-0.303 (0.241)
Observations	192	192	192
R ²	0.537	0.180	0.156

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.7: Lettau & Ludvigson (2001) Unexpected Changes in *cay* 1971:Q3-2019:Q2

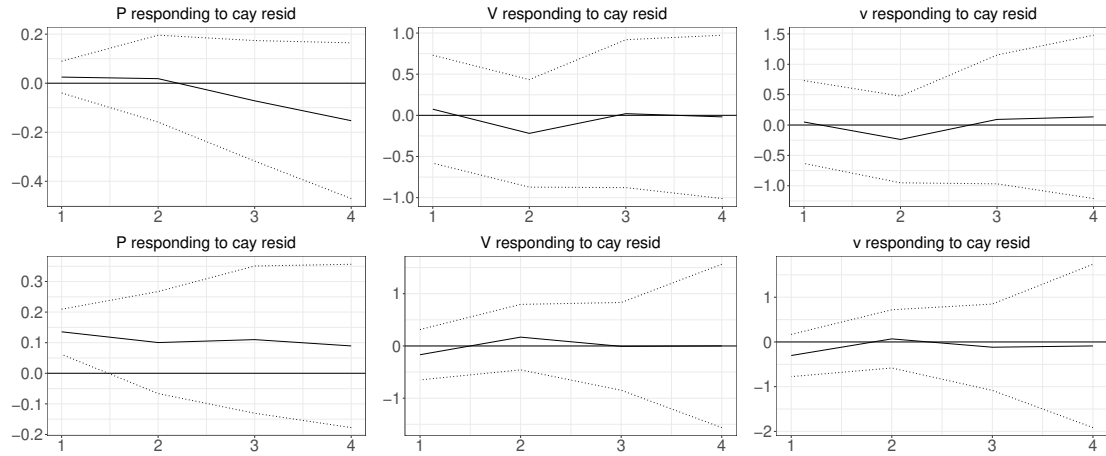


Figure 2.4: Response to cay

2.2.3 Shock to the Nominal Value of Government Debt

Finally, we consider unexpected changes in the market value of government debt on FOMC announcement dates as a shock to fiscal conditions. From the fiscal equation $V_t = P_t\psi_t$, if the market value of government debt unexpectedly increases, but expected future surpluses ψ_t remain constant, the price level is expected to increase.

Suppose that government debt is composed solely of 3-month bonds. With a pre-determined face value of debt outstanding G_t , an unexpected decrease in the Federal Funds rate $R_{1,t}$ will raise bond prices ($Q_{1,t}$), and raise the market value of debt V_t , since $V_t = Q_{1,t}G_t = \frac{1}{R_{1,t}}G_t$.

When government debt is composed of bonds of various maturities, it is important to consider the effects of monetary policy announcements. Inoue & Rossi (2018) emphasize that two FOMC announcements that have similar effects on the Federal Funds rate may nevertheless correspond to different movements in bond prices farther out on the yield curve. To capture the effects of an FOMC announce-

ment on the nominal value of government debt, I compute the change in the market value of debt from the day before to the day of an FOMC announcement using daily yield data for US Treasury securities of all maturities from Gürkaynak et al. (2006) and estimates of the maturity structure of government debt from Wharton.

Due to offsetting Fiscal and Fisher Effects, theory does not make an unambiguous prediction for the movement of the price level in response to an unexpected increase in the nominal market value of government debt. Tables 2.9 and 2.10 show that the price level does not respond to unexpected changes in the market value of debt that occur as the result of FOMC announcements. This is true even after controlling for the unexpected changes in the 3-month interest rate. The positive association between nominal interest rates and the price level has been referred to as the “price puzzle” since Sims (1992). Figure 2.5 illustrates the offsetting Fisher and Fiscal effects in response to a surprise increase in the short-term interest rate from 1.01 to 1.02 as described in Table 2.8.

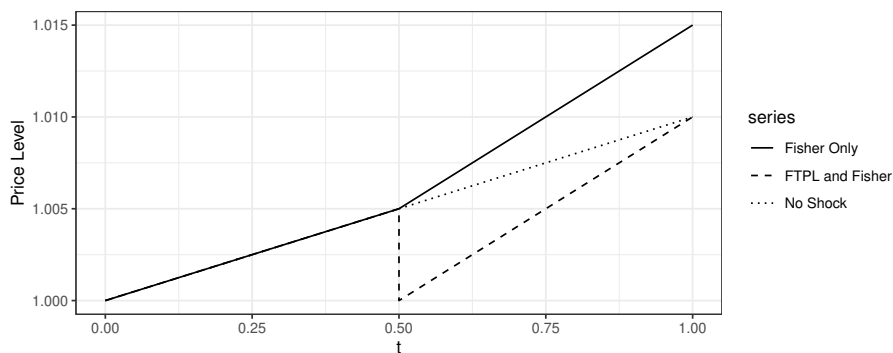


Figure 2.5: Offsetting Fisher and Fiscal Effects

Figure 2.6 plots the unexpected holding period returns on government debt (upper panel) and unexpected changes in the 3-month T-bill rate (lower panel) during FOMC windows. Tables 2.9 and 2.10 show that these shocks do not have a

t	R_t^F	V_t	P_t Fisher	P_t Fisher and Fiscal
1	1.01	.99	1.000	1.000
.5	1.02	.99	1.005	1.000
1	1.02	1	1.015	1.010

Table 2.8: Offsetting Fisher and Fiscal Effects: Numbers for Example

strong influence on the price level.

Figure 2.7 plots local projections for the same outcome variables following the shocks during FOMC windows.

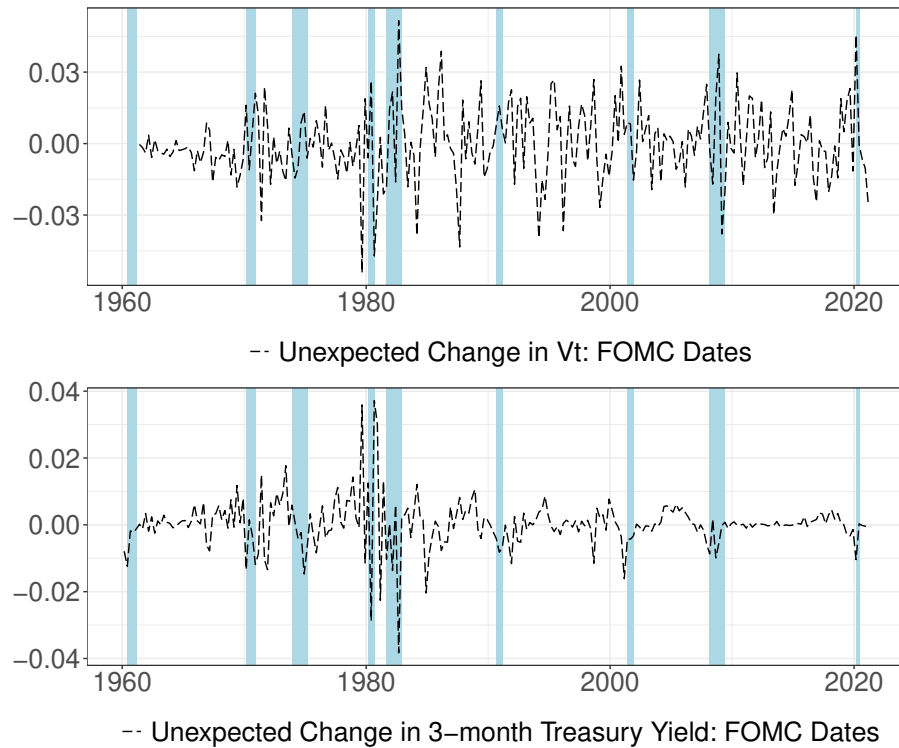


Figure 2.6: Shocks during FOMC windows.

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
ΔV_{t+1} FOMC	-0.022 (0.060)	2.531*** (0.335)	2.553*** (0.286)
ΔR_{t+1}^{3mo} FOMC	0.084 (0.117)	1.746*** (0.439)	1.662*** (0.432)
Observations	38	38	38
R ²	0.855	0.824	0.814

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.9: Market Value of Debt Shocks: 1962:Q1-1971:Q2

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
ΔV_{t+1} FOMC	-0.007 (0.020)	0.688*** (0.109)	0.695*** (0.111)
ΔR_{t+1}^{3mo} FOMC	0.095 (0.059)	-0.563*** (0.200)	-0.658*** (0.197)
Observations	198	198	198
R ²	0.522	0.474	0.457

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.10: Market Value of Debt Shocks: 1971:Q3-2019:Q2

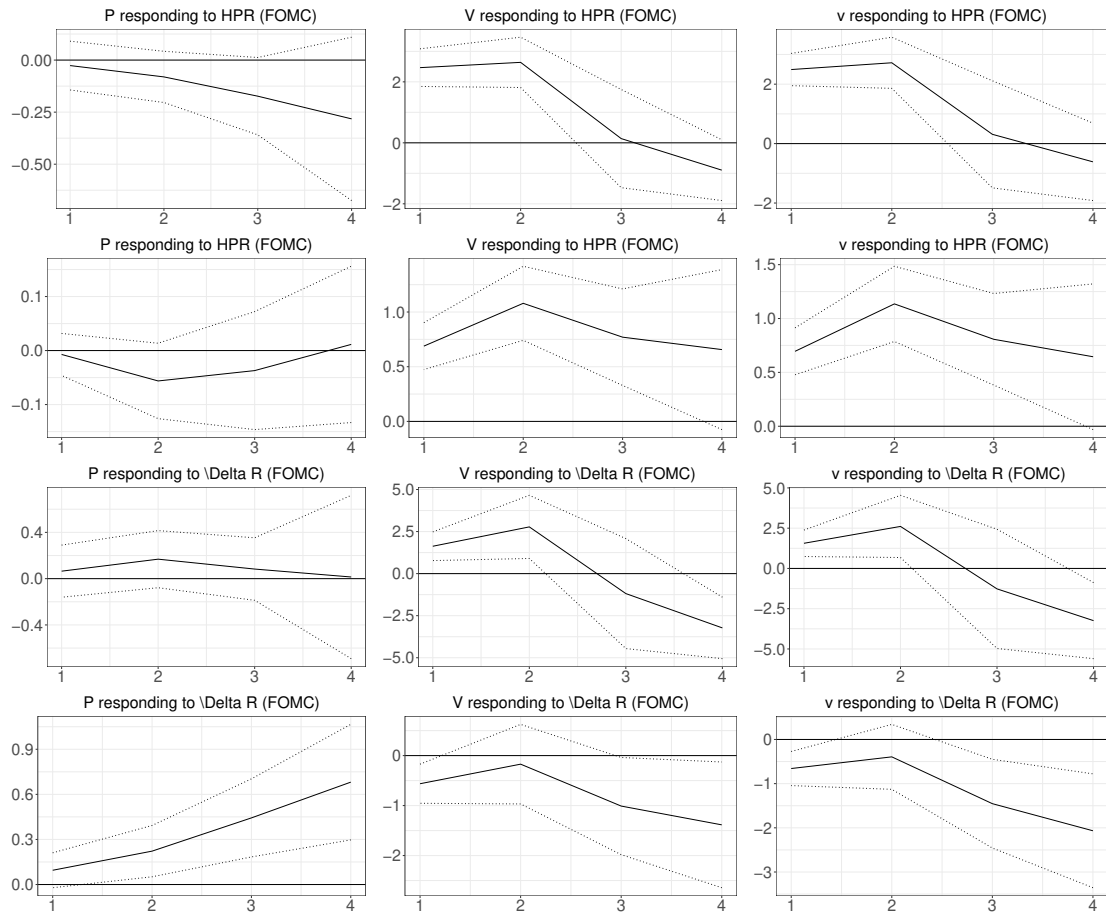


Figure 2.7: Response to FOMC

2.2.4 All Shocks

Tables 2.11 and 2.12 show that when the effects of all shocks are estimated in the same regression, their individual coefficients remain of similar magnitude.

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
pdv_{t+1}/V_t	-0.013*** (0.004)	0.024 (0.039)	0.037 (0.040)
cay_{t+1}^u	0.049** (0.020)	-0.220 (0.180)	-0.269 (0.167)
ΔV_{t+1} FOMC	-0.036 (0.063)	2.519*** (0.233)	2.555*** (0.206)
ΔR_{t+1}^{3mo} FOMC	0.020 (0.092)	1.807*** (0.473)	1.786*** (0.460)
Observations	41	41	41
R ²	0.874	0.808	0.799

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.11: All Shocks: 1961:Q1-1971:Q2

	<i>Dependent variable:</i>		
	$\Delta \tilde{P}_{t+1}$	$\Delta \tilde{V}_{t+1}$	$\Delta \tilde{\psi}_{t+1} \equiv \Delta \tilde{V}_{t+1} - \Delta \tilde{P}_{t+1}$
	(1)	(2)	(3)
pdv_{t+1}/V_t	0.028*** (0.008)	-0.147*** (0.028)	-0.175*** (0.029)
cay_{t+1}^u	0.121*** (0.036)	-0.343 (0.214)	-0.465** (0.226)
ΔV_{t+1} FOMC	-0.012 (0.027)	0.668*** (0.148)	0.681*** (0.164)
ΔR_{t+1}^{3mo} FOMC	0.075 (0.053)	-0.642*** (0.222)	-0.718*** (0.235)
Observations	149	149	149
R ²	0.661	0.583	0.583

Note: *p<0.1; **p<0.05; ***p<0.01

Table 2.12: All Shocks: 1971:Q3-2008:Q4

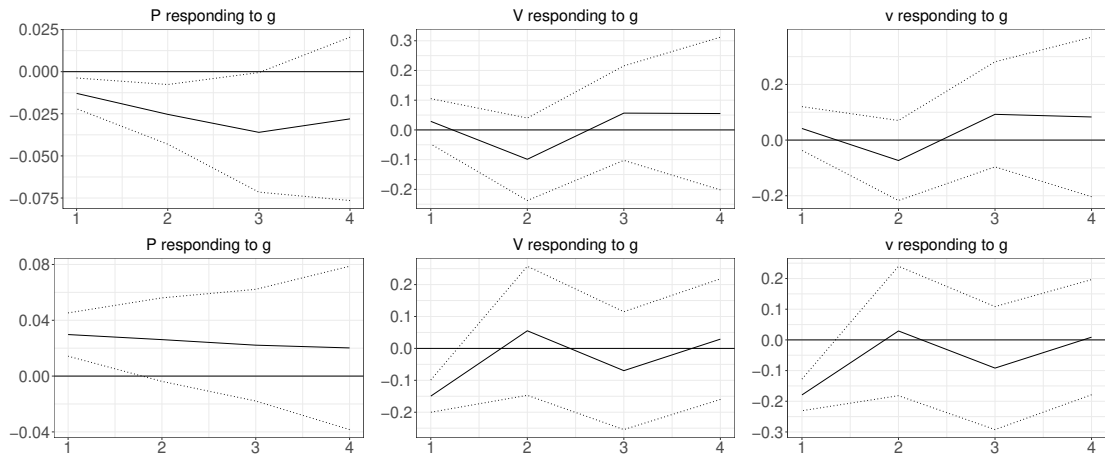


Figure 2.8: All Shocks: g

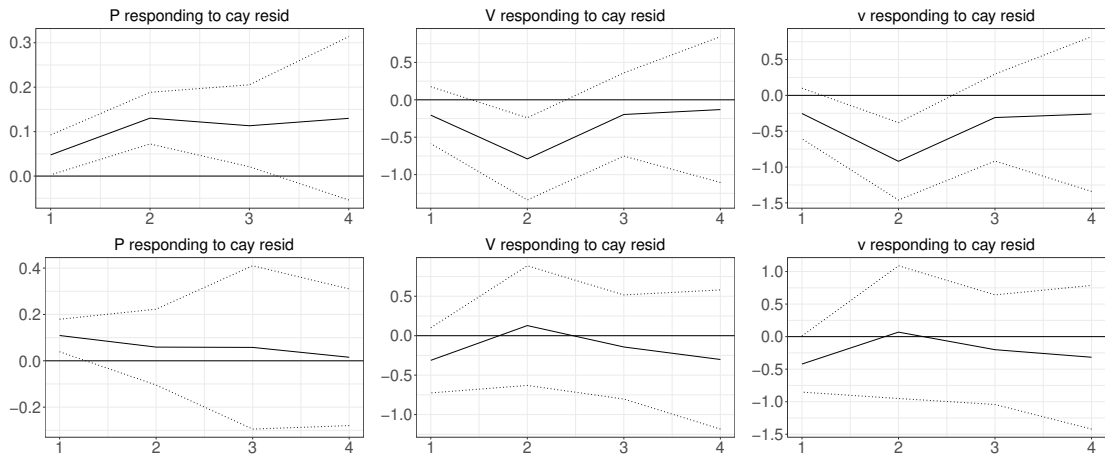


Figure 2.9: All Shocks: cay

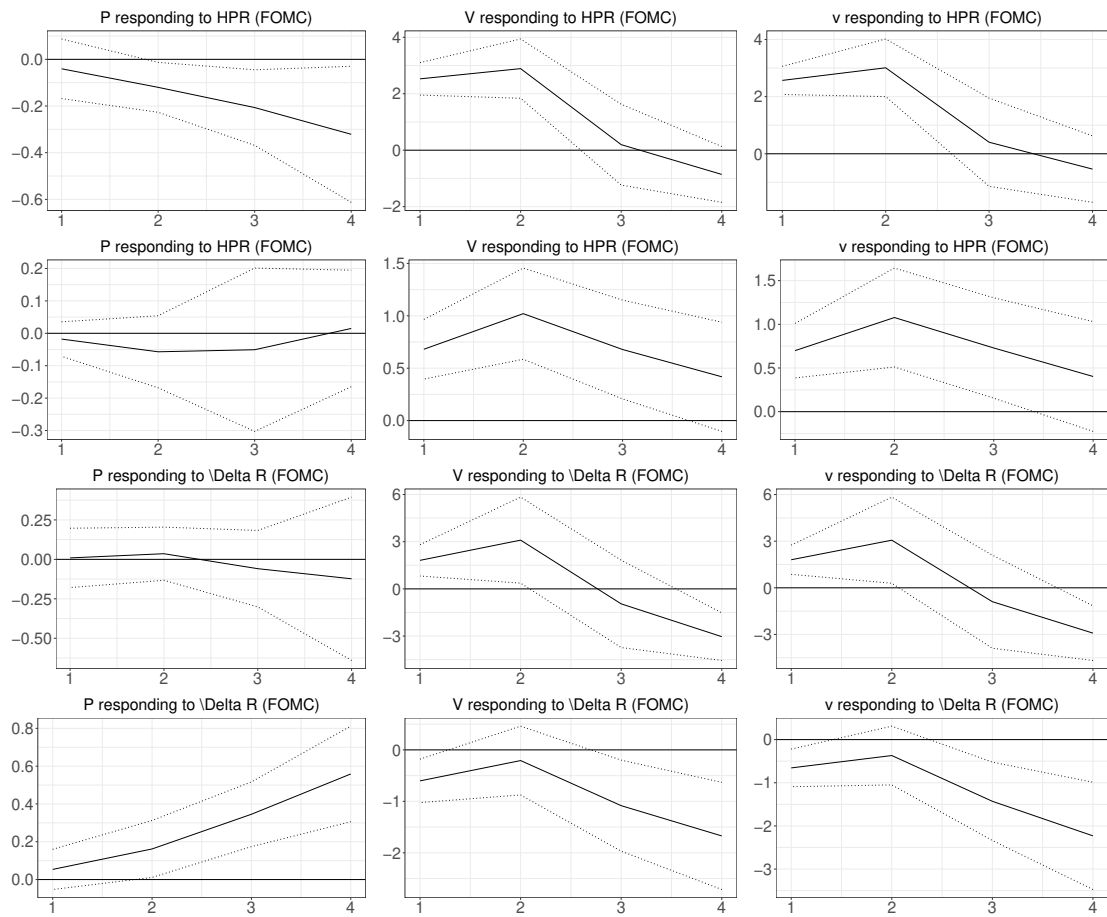


Figure 2.10: All Shocks: FOMC

2.3 CONCLUSION

The price level increases in response to unexpected increases in government purchases, and unexpected increases in the real discount rate, as predicted by the government debt flow identity when fiscal surpluses are constrained. After 1971, the real market value of debt decreases in response to the same shocks, a finding consistent with optimal fiscal policy in a complete markets environment.

CHAPTER 3

Financial Institutions in General Equilibrium

3.1 INTRODUCTION

I model an economy in which commercial banks produce financial instruments to facilitate transactions between households and firms, and in which a central bank conducts monetary policy. I take institutional details seriously, but allow for complete markets and frictionless exchange wherever possible. The equilibrium implied by this approach is nearly identical to the equilibrium of a model without an explicitly modeled financial sector. The explicit modeling of financial institutions illustrates the specific contributions of financial sector features to the emergent equilibrium outcomes.

This approach is in contrast to two broad approaches to modeling the financial system in macroeconomics. The first approach is to assume that banks are passive intermediaries between savers and borrowers, and to not explicitly model the financial system at all. The second approach is to model a financial sector rife with agency frictions, and constrain households so that they cannot finance firms directly. The households are then subjected to distortions caused by the financial sector. Such models can produce any number of inefficient macroeconomic outcomes, but their underlying assumptions about market segmentation and agency frictions are not obviously defensible.

I only allow for model components that map directly onto financial instruments that would be familiar to a banker: banks create loans that are financed by bonds or deposits, and banks may choose to hold precautionary reserves of base money, or lend these reserves on the Federal Funds market. Banks are not intermediaries of

the physical capital stock, they are not assumed to have deposit market power, and they need not be subject to regulatory capital or reserve requirements in order for an equilibrium to be reached. The single institutional constraint imposed on banks in this model is that they must choose the quantity of reserves to hold before they receive an idiosyncratic deposit withdrawal shock at the end of each period. This single imposed constraint is not new: it is consistent with partial equilibrium models of the Federal Funds market going back to Poole (1968). Further, it is consistent with narrative accounts of the daily sequence of events in the Federal Funds market (Furfine (2000)). This approach is also simpler than modeling search frictions in the interbank markets, as done by Ashcraft & Duffie (2007), Afonso et al. (2018), and Bianchi & Bigio (2022).

The model developed around this single institutional constraint produces many intuitive results. First, the interest rate that firms pay on loans is equal to the interest rate that households receive on bonds issued by the financial sector. Households are therefore indifferent between lending to firms directly through the bond market, or indirectly through banks. No market segmentation is required for bond, bank loan, and deposit markets to exist together in equilibrium.

Second, the interest rate banks pay on deposits is lower than the interest rate they pay on bonds, in the absence of deposit market power. This is because the bank faces two sources of expense when issuing deposits. The first is the standard interest expense, and the second is the possible overdraft penalty rate expense if the realized deposit withdrawal shock is large.

Third, risk-neutral banks choose finite leverage in the absence of regulatory reserve requirements or equity constraints. In many banking models, the deposit rate is lower than the loan rate, so a risk-neutral bank will choose to finance loans

with deposits until it hits a regulatory constraint. Not only is leverage in these models a corner solution, but these models imply that in the absence of regulatory constraints on leverage, risk-neutral firms would take on infinite leverage.

Fourth, in equilibrium, monetary and banking policy are described by three interest rates: the Federal Funds rate, the rate on excess reserves, and the overdraft penalty rate. Bank first order conditions imply that required reserves should be ignored when looking for the economically relevant reserve ratio of base money to deposits.¹ Banks held a ratio of unencumbered reserves to deposits of between 1-2% between 1959 and 2007.² Bank first order conditions also imply that when the federal funds rate is equal to the rate on excess reserves, as became true beginning in 2008, banks will hold one dollar of unencumbered reserves for each dollar of deposits. The empirical reserve ratio approached one beginning in 2008. Taking the price level and associated quantity of deposits as given, the behavior of the reserve ratio implies that banking policy that sets the rate on excess reserves below the Federal Funds rate will allow the Federal Reserve to reduce the size of its balance sheet.

The paper is organized as follows. Section 3.2 describes the economic environment, including the financial instruments and the timing of transactions during each period. Section 3.3 describes the objective functions and efficiency conditions for the bank, household, and firm, along with the behavior of the Central

¹Other authors, including Piazzesi & Schneider (2021) consider separate regimes of scarce and abundant reserves.

²In a paper asking “Does the money multiplier exist?”, Carpenter & Demiralp (2012) conclude that “there is no direct link between reserves and money - as defined by M2.” However, a trendless relationship between reserves and bank deposits can be found after making two data adjustments directly implied by the first order conditions of the model. First, we consider only demand deposits, the portion of M2 against which banks are most likely to hold reserves. Second, we subtract required reserves from total reserves in order to focus on reserves held by banks in excess of their reserve requirements.

Bank. Section 3.4 provides equilibrium conditions and plots impulse responses to productivity shocks and changes to the variance of withdrawal shocks under the assumption of uniformly distributed withdrawal shocks. Section 3.5 calculates an illustrative time-varying labor wedge that arises from firms' and households' use of financial instruments, and provides empirical support for the existence of a trendless reserve to deposit ratio that is implied by the model.

Appendix B.1 provides institutional details on the Federal Funds market and the federal reserve's daily operating procedures. Appendix B.2 illustrates the process of bank loan and deposit creation in the context of a transaction in which one person takes out a mortgage to purchase a house from another person.

3.2 THE ECONOMIC ENVIRONMENT

I describe a monetary economy including households, firms, commercial banks, and a central bank. Firms and households cannot contract directly with each other. Instead, goods and labor are paid for with deposits, some fraction of which must be held by the purchaser in advance of the purchase.

Firms take loans from the bank to pay for labor and capital, and produce real consumption goods with the CRS production function $y_t = A_t h_t^f$. Crucially, the firm must pay for a fraction θ of its labor inputs with currency before it can produce and earn revenue from sales of the consumption good. When $\theta > 0$, this restriction will require the firm to take out a loan from the bank in order to produce. Similarly, the household can only purchase the consumption good using deposits it already holds. When $\theta = 0$, the household will receive wages after paying for the consumption good, so the household will need to take out a loan in order to purchase goods. These constraints produce positive demand for deposits,

even when households have the option to simultaneously hold bonds.

Commercial banks use the monetary base to produce loans, bonds and deposits. At the end of each period, each bank faces an idiosyncratic deposit withdrawal shock, in which some fraction of deposits are either added to or withdrawn from each bank. These idiosyncratic withdrawal shocks sum to zero so that they do not influence aggregate demand for deposits. A deposit is a claim to one unit of the monetary base. If a bank has insufficient monetary base holdings to meet its withdrawal shock, it must pay an overdraft penalty rate to the clearing house or Central Bank. The presence of withdrawal shocks and the penalty for holding insufficient reserves creates a precautionary motive for banks to hold some amount of the monetary base in reserve for every unit of deposits they issue. In equilibrium, the monetary base will be entirely held by banks.

Commercial banks can operate under a commodity money standard, or in combination with a central bank. If commercial banks operate under a commodity standard, we assume that a clearing house sets a deposit overdraft penalty rate, but does not alter the supply of base money or provide a positive interest rate on excess reserves. If there is a central bank, the central bank will set three interest rates: the interbank rate, the overdraft penalty rate, and the interest rate on excess reserves. The central bank will set monetary policy which consists of an interbank rate which follows a Taylor rule. It will separately set banking policy, which consists of the overdraft penalty rate and the interest rate on excess reserves. Finally, the central bank will supply the quantity of base money required to support equilibrium by conducting Open Market Operations.

3.2.1 Financial Instruments

The financial sector uses base money to create one period nominal loans by issuing liabilities in the form of demand deposits and one-period nominal bonds. Loans cannot be repaid in the same period in which they are originated, and interest is paid on deposits and bonds held overnight. Firms do not purchase bonds³, and no agent may hold a negative amount of any financial instrument.

Base Money, denoted M_t , is the final settlement instrument for all transactions and credit contracts. Currency and bank reserves, the two forms of base money, are produced by the central bank. Currency may be held by households, firms, or banks. Bank reserves are only held by banks, and may be frictionlessly converted into currency at the bank's request. Two related assumptions simplify the movement of financial instruments. First, households and firms consider deposits and currency to be perfect substitutes for the purpose of facilitating transactions. This is plausible if there is no possibility of bank insolvency, and the costs of payment with debit card-like media are similar in size to the "shoe leather costs" of manually depositing and withdrawing currency. Second, households and firms choose to hold all their money in the form of bank deposits. Under these assumptions, all currency, as well as all bank reserves, will be held by banks. For example, a household may withdraw its deposits from the bank as currency to pay for goods from the firm, but the firm will immediately re-deposit the currency in the bank, so that no currency is *held* outside the bank.

Bank Loans, denoted L_t , are one-period credit contracts. Households and firms

³So long as the interest rate on loans is greater than or equal to the interest rate on bonds and deposits, as will be true in equilibrium, the firm should have no reason to purchase bonds.

may borrow base money from the financial sector at the bank's lending rate, R_t^L . When the bank originates a loan of quantity L_t , it simultaneously credits an equal amount of deposits D_t to the account of the borrower. To make a payment, the borrower transfers the newly created deposits in the form of currency, which the bank must supply to the borrower either from its reserves or its issuance of bonds (discussed below). Borrowers repay their bank loans in the following period with currency at interest in the amount $R_t^L L_t$.

Deposits, denoted D_t , are claims to currency with zero maturity. Deposits are used by the firm to purchase labor inputs, and by the household to purchase the consumption good from the firm. Households and firms may not directly exchange labor for goods; deposits must be used to make all purchases. Deposits held by the household are denoted D_t^h ; deposits held by the firm are denoted D_t^f .

If banks do not face default risk, so that households and firms may costlessly withdraw deposits in the form of currency to be used for payments, then these creditors will consider deposits and currency to be perfect substitutes for the purposes of facilitating transactions and storing wealth. If non-bank agents would like to hold more deposits, they must take out a loan, or spend fewer of their existing deposits.

Bonds, denoted B_t , are one-period credit contracts issued by the bank in which the household or firm lends currency to the bank. The bank returns deposits to the bondholders in the amount $R_t^B B_t$ at the beginning of the next period. In this model, bonds function like time deposits. There is no secondary market for bonds.

3.2.2 Timing of Transactions

Before each period begins, a level of productivity, A_t , and the support of the deposit withdrawal shocks, $X_{it} \sim U[-\mu_t D_{it}, \mu_t D_{it}]$, are realized. The central bank then sets its policy rate R_t^F , the overdraft penalty rate R_t^O , and the interest rate on excess reserves R_t^E , with knowledge of A_t and μ_t .⁴ Once A_t , μ_t , R_t^F , R_t^O , and REt (or equivalently M_t) are known, all information is known to all agents.

For expository reasons, I describe each period as a sequence of seven sub-periods. All information is known to all agents before any sub-period begins. The definition of sub-periods simply helps to specify the timing of transactions.

1. **Financial Sub-Period** Households and firms receive interest on currency and bonds held in the previous period, and repay loans from the previous period with interest. These pre-existing obligations, which sum to Z_t^h , Z_t^f and Z_t^b for the household firm and bank respectively, constitute their net financial wealth at the beginning of each period. Households and firms next allocate their initial financial wealth (possibly augmented by a bank loan) between quantities of deposits and bonds at the beginning of the period. Banks collect interest on loans and pay interest on bonds and deposits issued during the previous period. Households own the banks, so banks return any ex-post profit to the household, and the household pays any overdraft charges for banks with insufficient reserves to accommodate the previous period withdrawal shock. In the absence of a fiscal arm of the government, the central bank makes remittances of overdraft charges directly to the household. Finally, banks choose their balance sheets for the current period subject to loan

⁴If the economy instead has no central bank, and experiences exogenous shocks to the monetary base, M_t , that shock would be realized before each period begins as well. Details in sub-section ??.

bond and deposit demand from the firm and household, and the policy rate R_t^F , which is declared before the beginning of the Financial Sub-Period.

- (a) Firms must pay a fraction θ of wages in currency before they can produce and earn revenues. This requires firms to hold deposits $D_t^f \geq \theta W_t h_t^f$.
- (b) Households must purchase the consumption good at price $P_t c_t$ using deposits held during the financial Sub-Period, D_t^h , and their pre-production earnings from firms, $\theta W_t h_t^f$. This requires households to hold deposits $D_t^h \geq P_t c_t - \theta W_t h_t^f$.

2. **Open Market Operations Sub-Period** The Central Bank conducts Open Market Operations by purchasing loans from banks in exchange for reserves with the objective of getting the realized interbank rate to trade near the target rate R_t^F . Open Market Operations, even if they are unexpected, will not affect household or firm decisions in the current period because all aggregate financial variables for the current period are chosen in the Financial Sub-Period.
3. **Production Sub-Period** Firms pay for a fraction θ of their total labor costs using deposits, so they must have chosen a large enough quantity of deposits during the Financial Sub-Period to pay for their labor inputs during the Production Sub-Period.
4. **Consumption Sub-Period** Households purchase goods from the firm using deposits chosen during the Financial Sub-Period, D_t^h , and wages earned during the Production Sub-Period, $\theta W_t h_t^f$.

5. **Post-Consumption Sub-Period** Finally, firms pay households the remaining portion of their wages, $(1 - \theta)W_t h_t^f$.
6. **Interbank Lending Sub-Period** Banks may borrow or lend reserves M_{it}^F on the interbank market. Reserves are in zero net supply during this sub-period ($\sum_i M_{it}^F = 0$), so the representative bank will not trade in this market. However, the existence of the interbank market allows for the computation of an interbank market-clearing interest rate, R_t^F .
7. **Withdrawal Shock Sub-Period** After trading on the interbank market to choose a final quantity of reserves $M_{it} + M_{it}^F$, each bank faces a deposit withdrawal shock, $X_{it} \sim U[-\mu_t D_{it}, \mu_t D_{it}]$ where $0 < \mu_t \leq 1$, and $X_{it} > 0$ denotes a net deposit inflow. If the withdrawal is smaller than the quantity of reserves held by bank i , $M_{it} + M_{it}^F + X_{it} \geq 0$, bank i earns the excess reserve rate R^E on its remaining quantity of reserves, $M_{it} + M_{it}^F + X_{it} > 0$. If the withdrawal is greater than the quantity of reserves held by bank i , bank i will be required to pay a penalty overdraft rate R^O for every dollar of its reserve shortage, $M_{it} + M_{it}^F + X_{it} < 0$. A reserve shortage also implies that bank i has not retained sufficient reserves to transfer the full amount of the deposit shock to other banks. The household, which owns bank i , will be required to make payments in the amount $M_{it} + M_{it}^F + X_{it}$ to banks $-i$ during the Financial Sub-Period in $t + 1$. All banks earn zero expected profit. With many banks, losses at some banks will be perfectly offset by positive profits at other banks.

The fraction θ will be important for determining the amount of deposits held by households and firms in the Financial Sub-Period, and determining who bears the inflation tax. When $\theta = 1$, the household optimally holds no deposits after

the Consumption Sub-Period, while the firm holds $D_t^f = P_t c_t$, the full amount of its revenues, as deposits overnight. When $\theta = 0$, firms pay their full labor costs *after* they produce and earn $P_t c_t$ in revenues. As a consequence, the household will need to pay for the consumption good before it receives its wages, and must take out a loan from the bank at the beginning of the period to cover the costs of purchasing the consumption good.

3.3 ACTORS

3.3.1 Banks (Exogenous M_t)

Banks use base money to create one-period nominal loans by issuing liabilities in the form of demand deposits and one-period nominal bonds.

When a bank creates a new loan, it simultaneously credits an equal amount of deposits to the account of the borrower. This is the only process through which inside money, D_t , can enter the economy in quantities greater than the monetary base. Inside money can be destroyed in two ways. First, borrowers can pay off loans using inside money. Second, any holder of inside money can use it to purchase bonds from the bank.⁵

The process of loan creation implies the following initial budget constraint for the bank

$$L_t + M_t - B_t - D_t = 0 \quad (3.1)$$

where L_t is the nominal quantity of loans issued, M_t is the base money held by the bank, B_t is the quantity of bonds purchased from the bank with inside money, and

⁵In principle, inside money can be destroyed in a third way: borrowers could redeem their bank notes for base money. However, in this model, there is no reason for households or firms to hold base money, and we assume all base money is held by the bank. Appendix B.2 provides a detailed example of loan and deposit creation.

D_t is the quantity of inside money (deposits).⁶

Table 3.1 displays the balance sheet implied by equation 3.1. The representative bank holds the monetary base and loans as assets. The fraction of loans below the dotted line are financed by bonds at zero profit. Returns from the fraction of loans above the dotted line are used to pay interest on deposits, again producing zero profit. The dotted line is presented for illustrative purposes only; there is no difference between loans above and below the dotted line. The fact that banks hold the monetary base as an asset against some fraction of their deposits creates a spread between the loan rate and the deposit rate.

Assets	Liabilities
M_t	D_t
L_t	

L_t	B_t

Table 3.1: Representative bank balance sheet

After banks have chosen their balance sheets, they may choose to borrow or lend a quantity of reserves M_{it}^F on the interbank market at the market-clearing interest rate R_t^F . Interbank borrowing does not alter the quantity of reserves in the system:

$$\sum_i M_{it}^F = 0 \quad \forall t. \quad (3.2)$$

Finally, banks are subject to deposit withdrawal shocks at the end of each period. Suppose there are many small banks, and each bank i receives the with-

⁶In this section, I assume that the monetary base M_t is determined exogenously before the beginning of each period. In Section 3.3.2.1 I discuss bank behavior in the presence of a central bank.

drawal shock X_{it} , with $X_{it} > 0$ indicating a net inflow of deposits, and $X_{it} < 0$ indicating a net withdrawal. Deposits in bank i are a claim to base money in bank i , and the bank is required to transfer the corresponding quantity of its base money in the event of a net withdrawal. We assume that deposit withdrawals from one bank are deposited into another bank, so that these withdrawal shocks do not change aggregate deposit demand:

$$\sum_i X_{it} = 0 \quad \forall t \quad (3.3)$$

If $M_{it} + X_{it} \geq 0$, the bank carries $R^E(M_{it} + X_{it})$ of reserves into the next period, where R^E is the return on excess reserves. If $M_{it} + X_{it} < 0$, the bank pays a penalty rate in base money for each unit of its reserve shortage: $R^O(M_{it} + X_{it})$. The return on reserves held in period t is:

$$R_t^M = \begin{cases} R_t^E & \text{if } M_{it} + X_{it} \geq 0 \\ R_t^O & \text{if } M_{it} + X_{it} < 0. \end{cases} \quad (3.4)$$

Maximization Problem. Banks maximize profits from lending, subject to their budget constraint:

$$\begin{aligned} \max_{L_{it}, B_{it}, D_{it}, M_{it}^F} E_t [\pi_{it}^b] &= R_t^L L_{it} - R_t^B B_{it} - R_t^D E_t [D_{it} + X_{it}] \\ &\quad - R_t^F M_{it}^F + E_t [R_t^M (\bar{M}_{it} + X_{it})] \end{aligned} \quad (3.5)$$

$$\text{s.t.} \quad 0 = L_{it} + M_{it} - B_{it} - D_{it} \quad (3.6)$$

$$\bar{M}_{it} = M_{it} + M_{it}^F \quad (3.7)$$

$$L_{it}, B_{it}, D_{it}, M_{it} \geq 0 \quad (3.8)$$

Variables with subscript it are for bank i in period t . Variables with a subscript t are summed across banks. For example, total loans $L_t = \sum_i L_{it}$. In general, total bank loans is the sum of loans to firms and households: $L_t = L_t^f + L_t^h$. The same is true for bonds: $B_t = B_t^f + B_t^h$. However, we assume that bonds are only purchased by households.

Efficiency Conditions. Solving for B_{it} using constraint 3.6 and taking derivatives of 3.5 with respect to loans L_{it} , deposits D_{it} , and the monetary base held by bank i , M_{it} , yields

$$\text{FOC } (L_{it}): R_t^L - R_t^B = 0 \quad (3.9)$$

$$(D_{it}): R_t^L - R_t^D + \frac{\partial}{\partial D_{it}} E_t[R_t^M (\bar{M}_{it} + X_{it})] = 0 \quad (3.10)$$

$$(M_{it}): -R_t^L + \frac{\partial}{\partial M_{it}} E_t[R_t^M (\bar{M}_{it} + X_{it})] = 0 \quad (3.11)$$

$$(M_{it}^F): -R_t^F + \frac{\partial}{\partial M_{it}^F} E_t[R_t^M (\bar{M}_{it} + X_{it})] = 0 \quad (3.12)$$

Equation 3.9 shows that the return on loans is equal to the return on bonds.⁷ Equation 3.10 shows that the spread between loans and deposits is equal to the derivative of the expected value of bank i 's current reserve holdings \bar{M}_{it} with respect to deposits. Holding reserves \bar{M}_{it} fixed, an increase in deposits increases the probability that the withdrawal shock will leave bank i holding a negative quantity of reserves, and require it to pay the penalty rate R_t^O . After describing the distri-

⁷ R_t^L is the net return on loans realized by the bank. In the presence of frictions like loan monitoring costs, which I do not consider in this document, loan interest rates faced by firms may be higher than the net return realized by the bank.

bution of withdrawal shocks, it will be the case that $\frac{\partial}{\partial D_{it}} E[R_t^M(\bar{M}_{it} + X_{it})] < 0$, and $R_t^L > R_t^D$. When banks issue a marginal bond, the only associated cost is repayment with interest in the next period. In contrast, when issuing a marginal deposit, banks incur interest costs and costs associated with larger withdrawal shocks. Equation 3.10 shows that the sum of interest costs and withdrawal costs on deposits are equal to interest costs on bonds in equilibrium.

Equation 3.11 shows that the equilibrium loan rate is equal to the marginal increase in returns from holding an additional unit of the monetary base. In section 3.3.2.1, which introduces a central bank to the model, the central bank will target the marginal value of reserves as its policy rate.

Finally, equation 3.12 shows that the equilibrium interbank market rate is also equal to the marginal increase in returns from holding an additional unit of the monetary base. The first order conditions imply that in equilibrium,

$$R_t^L = R_t^B = R_t^F. \quad (3.13)$$

Withdrawal shocks must be described in order to solve for the reserve to deposit ratio M_t/D_t . I first describe withdrawal shocks with a general distribution, and then provide a closed form solution for M_t/D_t as a function of policy interest rates under the assumption that withdrawal shocks follow a uniform distribution.

3.3.1.1 Generalized Withdrawal Shocks

Suppose that random cash inflows into bank i are

$$X_{it} = x_{it}\mu_t D_{it}, \quad (3.14)$$

where x_{it} is a random variable with an expected value of zero on $[d, u]$, with $d < 0$ and $u > 0$. Let the CDF of x be $F(x)$, and the PDF be $f(x)$. Define

$$\psi(x) = \int_d^x z f(z) dz \quad (3.15)$$

which is the conditional expectation of x_{it} given that $x_{it} < x$: $E_t[x_{it}|x_{it} \leq x]$. Note also that this function is zero for all $x < d$, and that

$$\int_x^u z f(z) dz = \underbrace{\int_d^u z f(z) dz}_0 - \int_d^x z f(z) dz = -\psi(x) \quad (3.16)$$

Suppose the bank initially holds deposits D_{it} and reserves M_{it} , with required reserves at the end of each period equal to $\phi_t^R [D_{it} + X_{it}]$. The bank has excess reserves if

$$M_{it} + X_{it} - \phi_t^R [D_{it} + X_{it}] > 0, \quad (3.17)$$

and has a reserve shortage if

$$M_{it} - X_{it} - \phi_t^R [D_{it} + X_{it}] < 0. \quad (3.18)$$

Suppose that the bank's chosen reserve ratio is $\phi_{it} = M_{it}/D_{it}$. Then, the two conditions become

$$[\phi_{it} - \phi_t^R + x_{it}\mu_t(1 - \phi_t^R)] D_{it} > 0 \quad (3.19)$$

and

$$[\phi_{it} - \phi_t^R + x_{it}\mu_t(1 - \phi_t^R)] D_{it} < 0. \quad (3.20)$$

Defining

$$\delta_t = -\frac{(\phi_{it} - \phi_t^R)}{\mu_t(1 - \phi_t^R)}, \quad (3.21)$$

the bank holds excess reserves when $x_{it} > \delta_t$, and has a reserve deficit when $x_{it} < \delta_t$. If the bank is paid R^R on required reserves, R_t^E on excess reserves, and pays an overdraft fee of R_t^O on reserve deficiencies (overdrafts), then for a given choice of ϕ_{it} , the return on reserves per unit of deposits can be written

$$\begin{aligned} E_t \left[\frac{R_t^M (M_{it} + X_{it})}{D_{it}} \right] &= R^R \phi_t^R \\ &+ R_t^E \left[\int_{\delta_t}^u (\phi_{it} - \phi_t^R + x\mu_t(1 - \phi_t^R)) f(x) dx \right] \\ &+ R_t^O \left[\int_d^{\delta_t} (\phi_{it} - \phi_t^R + x\mu_t(1 - \phi_t^R)) f(x) dx \right] \end{aligned} \quad (3.22)$$

The excess reserve and overdraft terms can be simplified to

$$R_t^E [(1 - F(\delta_t))(\phi_{it} - \phi_t^R) - \psi(\delta_t)\mu_t(1 - \phi_t^R)] \quad (3.23)$$

and

$$R_t^O [F(\delta_t)(\phi_{it} - \phi_t^R) + \psi(\delta_t)\mu_t(1 - \phi_t^R)], \quad (3.24)$$

respectively.

3.3.1.2 Example: Uniformly Distributed Withdrawal Shocks

Suppose that idiosyncratic withdrawal shocks X_{it} are uniformly distributed from $-\mu_t D_{it}$ to $\mu_t D_{it}$, where $0 < \mu_t \leq 1$:

$$X_{it} \sim U[-\mu_t D_{it}, \mu_t D_{it}]. \quad (3.25)$$

By the Law of Iterated Expectations, we can express the bank's expected return on reserves for given quantities of deposits D_{it} and reserves M_{it} :

$$\begin{aligned} E[R_t^M(M_{it} + X_{it})] &= R^E E[M_{it} + X_{it} | M_{it} + X_{it} \geq 0] P(M_{it} + X_{it} \geq 0) \\ &+ R^O E[M_{it} + X_{it} | M_{it} + X_{it} < 0] (1 - P(M_{it} + X_{it} \geq 0)) \end{aligned} \quad (3.26)$$

Under the assumption that withdrawal shocks are uniformly distributed, equation 3.26 becomes

$$E[R_t^M(M_{it} + X_{it})] = -\frac{1}{4} \left(\frac{M_{it}^2 + \mu_t^2 D_{it}^2}{\mu_t D_{it}} \right) (R^O - R^E) + \frac{1}{2} M_{it} (R^E + R^O). \quad (3.27)$$

Equation 3.27 can be differentiated with respect to deposits and base money to evaluate banks' optimality conditions.

R_t^L will be determined by the Household's Euler equation (3.52) with an exogenous monetary base, or by the Central Bank in the case of a Central Bank. Rearranging equation 3.11 yields an expression for the optimal *nominal* quantity of deposits issued by banks given the monetary base M_{it} , the loan rate R_t^L , overnight overdraft rate R^O , the return on excess reserves R^E , and the magnitude of the withdrawal shock, μ_t

$$D_{it} = \frac{1}{\mu_t} \left(\frac{R_t^O - R_t^E}{(R_t^O - R_t^L) - (R_t^L - R_t^E)} \right) M_{it} \quad (3.28)$$

Equation 3.28 describes the *nominal* quantity of deposits issued by bank i as a function of the monetary base held by bank i , the loan rate, and the policy overdraft rate R^O . R^O must be set to be larger than the loan rate R_t^L . This expression will allow the determination of an equilibrium price level as a function of the monetary base and interest rates. It is also worth noting that $D_{it} > \bar{M}_{it}$: this is a fractional

reserve banking system.

Figure 3.1 plots M_t/D_t as a function of R_t^L , the nominal return on loans. The intuition for this upward sloping deposit supply curve comes from equation 3.11: banks create deposits until the loan rate is equal to the marginal increase in expected returns from holding an extra unit of the monetary base. When the nominal return on loans R_t^L is higher, banks maintain a higher ratio of deposits to reserves to maximize profit. We define the endogenously determined ratio of reserves (monetary base) to deposits ϕ_t :

$$\phi_t \equiv \frac{M_t}{D_t} = \mu_t \frac{(R_t^O - R_t^L) - (R_t^L - R_t^E)}{R_t^O - R_t^E}. \quad (3.29)$$

By construction, $0 < \phi_t < 1$. To solve for the deposit rate as a function of the loan

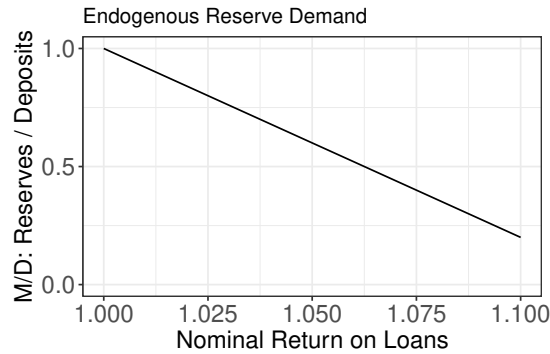


Figure 3.1: The ratio of reserves to deposits

rate and the magnitude of the withdrawal shock, combine equations 3.10 and 3.28:

$$R_t^D = R_t^L - \mu_t \frac{(R_t^O - R_t^L)(R_t^L - R_t^E)}{R_t^O - R_t^E} \quad (3.30)$$

Figure 3.2 plots the ratio of the deposit rate from equation 3.30 to the loan rate R_t^L . Equation 3.29 shows that banks issue more deposits for each unit of reserves as the nominal loan rate increases. This decrease in the reserve ratio increases the probability that banks pay the overdraft penalty rate as the result of a given deposit withdrawal shock, so banks offer a lower return on deposits relative to the return on bonds, which do not expose the bank to withdrawal shock risk.

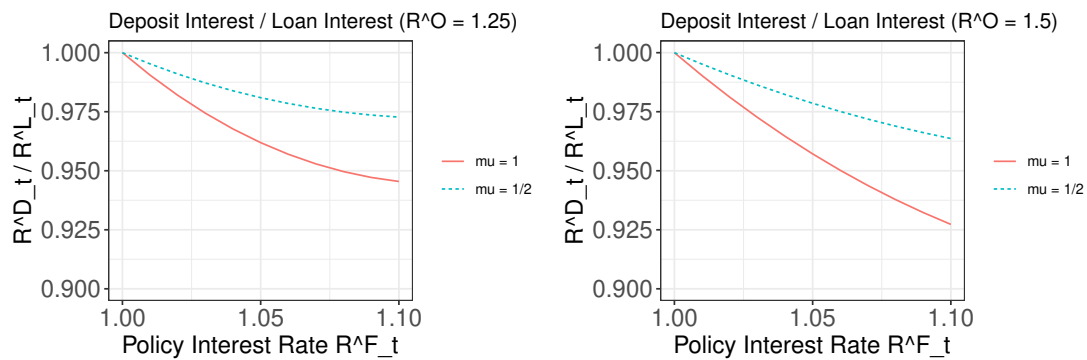


Figure 3.2: The ratio of deposit interest to loan interest

3.3.2 The Central Bank

The central bank sets an inflation target and conducts interest rate policy for R_t^F that follows a Taylor Rule. It also sets the overdraft penalty rate R_t^O , and the interest rate on excess reserves, R_t^E . The monetary policy rate R_t^F and the banking policy rates R_t^O , and R_t^E jointly determine the quantity of base money that the central bank must supply to the banking system through Open Market Operations (OMOs).

OMOs can be modeled in two ways. One approach is to assume that only the interest earned on loans held by the central bank is remitted to the household as

currency during the next financial sub-period. L_t^Ω , the quantity of loans purchased from banks during the OMO, is then $M_t^\Omega = M_t - M_{t-1}$.

A second approach is to assume that the central bank returns principal and interest on loans held in the previous period to the household. Using this approach, the central bank balance sheet returns to zero between periods, and $L_t^\Omega = M_t$.⁸

Following the second approach, Table 3.2 shows the central bank's balance sheet after conducting an OMO during period t . Table 3.3 shows the central bank's

Assets	Liabilities
L_t^Ω	M_t

Table 3.2: The central bank balance sheet

balance sheet after receiving interest on loans purchased in the previous period.

Assets	Liabilities
L_{t-1}^Ω	M_{t-1}
$(R_{t-1}^L - 1)L_{t-1}^\Omega$	Q_{t-1}^Ω

Table 3.3: The central bank balance sheet after receiving interest on its assets

3.3.2.1 Banks Under Central Banking

In Section 3.3.1, the monetary base M_t was exogenously given at the beginning of period t . In this section, central bank actions will determine the quantity of the monetary base by conducting Open Market Operations (OMOs). Now, M_t will be the quantity of reserves bankers expect to be present at the end of period t during

⁸In reality, only interest earned on Treasury securities held by the central bank is remitted to the government. This means that OMOs correspond to changes in the monetary base.

the Withdrawal Shock Sub-Period.

Maximization Problem

$$\begin{aligned} \max_{L_{it}, B_{it}, D_{it}, L_{it}^{\Omega}} E_t [\pi_{it}^b] = & R_t^L (L_{it} - L_{it}^{\Omega}) - R_t^B B_{it} - R_t^D E_t [D_{it} + X_{it}] \\ & - R_t^F M_{it}^F + E_t [R_t^M (M_{it} + X_{it})] \end{aligned} \quad (3.31)$$

$$\text{s.t.} \quad 0 = L_{it} + M_{i,t-1} - B_{it} - D_{it} \quad (3.32)$$

$$L_{it}^{\Omega} = M_{it}^{\Omega} \quad (3.33)$$

$$M_{it} = M_{i,t-1} + M_{it}^{\Omega} + M_{it}^F \quad (3.34)$$

$$L_{it}, B_{it}, D_{it}, M_{it} \geq 0 \quad (3.35)$$

Bank i chooses L_{it} , B_{it} , and D_{it} with respect to the expected quantity of base money that is consistent with the Central Bank's monetary policy.

The central bank chooses the aggregate quantity of reserves to inject into banks, M_t^{Ω} , and this implies a quantity of loans it must purchase, L_t^{Ω} . For the banking system as a whole:

$$M_t^{\Omega} = \sum_i M_{it}^{\Omega} \quad (3.36)$$

$$L_t^{\Omega} = \sum_i L_{it}^{\Omega} = M_t^{\Omega} \quad (3.37)$$

$$M_t = M_{t-1} + M_t^{\Omega} \quad (3.38)$$

Efficiency Conditions Solving for B_{it} using constraint 3.6 and taking derivatives of 3.5 with respect to loans L_{it} , deposits D_{it} , and the monetary base held by bank i ,

M_{it} , yields

$$\text{FOC } (L_{it}): R_t^L - R_t^B = 0 \quad (3.39)$$

$$(D_{it}): R_t^L - R_t^D + \frac{\partial}{\partial D_{it}} E[R_t^M (M_{it} + X_{it})] = 0 \quad (3.40)$$

$$(M_{it}^\Omega): -R_t^L + \frac{\partial}{\partial M_{it}^\Omega} E[R_t^M (M_{it} + X_{it})] = 0 \quad (3.41)$$

$$(M_{it}^F): -R_t^F + \frac{\partial}{\partial M_{it}^F} E[R_t^M (M_{it} + X_{it})] = 0 \quad (3.42)$$

Efficiency conditions under central banking have similar implications to those with an exogenous monetary base.

3.3.3 Households

Households are endowed with the initial monetary base M_t , and one unit of time each period which may be split between hours of labor at the firm, h_t^f , and hours of leisure, h_t^l . Households deposit their base money in banks in exchange for an initial quantity of demand deposits D_t^h .

Maximization Problem. Households maximize utility as a function of consumption c_t and leisure h_t^l subject to their nominal budget constraint, the constraint that consumption goods must be purchased with deposits, and their time constraint:

$$\max_{c_t, h_t^l} u(c, h^l) = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t^l) \quad (3.43)$$

$$\text{s.t. } D_t^h + B_t^h \leq R_{t-1}^D D_{t-1}^h + R_{t-1}^B B_{t-1}^h + Q_t^f + Q_{t-1}^\Omega \quad (3.44)$$

$$D_t^h \geq P_t c_t - \theta W_t h_t^f \quad (3.45)$$

$$D_{t-1}^h \geq (1 - \theta^h)W_{t-1}h_{t-1} \quad (3.46)$$

$$1 = h_t^f + h_t^l \quad (3.47)$$

At the beginning of each period, households receive interest payments on deposits and bonds held at the end of the previous period, $R_{t-1}^D D_{t-1}^h + R_{t-1}^B B_{t-1}^h$, dividends from the firm Q_t^f , and central bank remittances from open market operations in the previous period, Q_{t-1}^Ω . Constraint 3.44 shows that households divide their nominal wealth into deposits D_t^h in anticipation of purchasing goods from the firm, and purchases of bonds, B_t^h . After making this initial portfolio allocation, households will also receive wages paid in currency for hours of labor done at the firm, $W_t h_t^f$. Households receive the fraction θ of their nominal wages before production occurs, and the remaining fraction $(1 - \theta)$ after the firm receives its period revenue. Constraint 3.45 requires that households pay for all consumption with deposits, and shows that households choose D_t^h such that the sum of their initial deposits and pre-production wages is equal to their nominal consumption expenditures. Constraint 3.46 shows that deposits held by the household at the end of the previous period were at least equal to the fraction of wages paid post-production.

Efficiency Conditions. Combining constraints 3.44 - 3.47 and differentiating with respect to consumption, leisure and bond holdings yields

$$\text{FOC } (c_t): \beta^t u_c(c_t, h_t^l) - \Lambda_t P_t = 0 \quad (3.48)$$

$$(h_t^l): \beta^t u_{h^l}(c_t, h_t^l) - \beta^t \Lambda_t \theta^h W_t - \beta^{t+1} \Lambda_{t+1} R_t^D (1 - \theta^h) W_t = 0 \quad (3.49)$$

$$(B_t): -\beta^t \Lambda_t + \beta^{t+1} \Lambda_{t+1} R_t^B = 0 \quad (3.50)$$

where $u_c(\cdot)$ and $u_{h^l}(\cdot)$ denote the marginal utility of consumption and leisure, respectively. Rearranging yields the efficiency condition for labor and the Euler equation:

$$\frac{W_t}{P_t} = \frac{R_t^B}{\theta R_t^B + (1-\theta)R_t^D} \frac{u_{h^l}(c_t, h_t^l)}{u_c(c_t, h_t^l)} \quad (3.51)$$

$$1 = E_t \left[\beta \frac{u_c(c_{t+1}, h_{t+1}^l)}{u_c(c_t, h_t^l)} \frac{P_t}{P_{t+1}} \right] R_t^B. \quad (3.52)$$

Suppose the household's lifetime utility is

$$U(c, h^l) = \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma} c_t^{(1-\gamma)} - \frac{\chi}{1+\varphi} (1-h_t^l)^{(1+\varphi)} \right) \quad (3.53)$$

so that $u_c(c_t, h_t^l) = c_t^{-\gamma}$, and $u_{h^l}(c_t, h_t^l) = \chi(1-h_t^l)^\varphi = \chi h_t^{f\varphi}$. The efficiency condition for labor and Euler equation then become

$$\frac{W_t}{P_t} = \frac{R_t^B}{\theta R_t^B + (1-\theta)R_t^D} \frac{\chi h_t^{f\varphi}}{c_t^{-\gamma}} \quad (3.54)$$

$$1 = \beta R_t^B E_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right]. \quad (3.55)$$

3.3.4 Firms

Firms purchase labor from the household in order to produce output. Firms purchase labor with deposits, which they obtain from banks by taking out one-period loans of amount L_t^f at the lending rate R_t^L . Firms' nominal loans must be repaid with interest in the next period with revenues from sales of the consumption good.

Firms produce real output as a constant returns to scale function of labor

$$y_t = A_t F(h_t) = A_t h_t^f. \quad (3.56)$$

Maximization Problem

$$\max_{h_t^f} \pi_t^f = R_t^D \left[P_t y_t - (1 - \theta) W_t h_t^f \right] - R_t^L L_t^f \quad (3.57)$$

$$\text{s.t. } L_t^f \geq \theta W_t h_t^f \quad (3.58)$$

$$y_t = A_t h_t^f \quad (3.59)$$

The firm earns the deposit rate on any revenues it has not returned to labor in the post-consumption period ($P_t y_t - (1 - \theta) W_t h_t^f$). The firm then repays its loan with interest $R_t^L L_t^f$ with deposits that have also accrued interest $R_t^D [P_t y_t - (1 - \theta) W_t h_t^f]$. *Efficiency Conditions.* There is an interest rate wedge between the real wage and marginal product of labor:

$$\text{FOC } (h_t^f): \frac{W_t}{P_t} = \omega_t \frac{\partial}{\partial h_t^f} A_t F(h_t) = \omega_t A_t \quad (3.60)$$

where we define

$$\omega_t \equiv \frac{R_t^D}{\theta R_t^L + (1 - \theta) R_t^D} \quad (3.61)$$

for ease of notation.

Here, ω_t acts like a tax wedge on real wages. When the spread between the loan rate and the deposit rate increases, the wedge increases. As described in the Bank's problem, $R_t^L > R_t^D$.

3.3.5 Markets

Households allocate one unit of time per period towards hours of work and leisure:

$$1 = h_t^f + h_t^l \quad (3.62)$$

All output is consumed each period:

$$c_t = y_t \quad (3.63)$$

Financial markets clear:

$$L_t + M_t - B_t - D_t^f - D_t^h = 0 \quad (3.64)$$

Households bid up the price of real output so that all deposits are exchanged for the consumption good during the consumption sub-period

$$P_t c_t = D_t \quad (3.65)$$

3.4 EQUILIBRIUM

These equations describe equilibrium under the assumption of uniformly distributed withdrawal shocks with support $X_{it} \sim [-\mu_t D_{it}, \mu_t D_{it}]$, where $0 < \mu_t \leq 1$.

$$y_t = A_t h_t^f \quad (3.66)$$

$$y_t = c_t \quad (3.67)$$

$$\frac{\chi h_t^{f\varphi}}{c_t^{-\gamma}} = \frac{R_t^D}{R_t^L} \frac{y_t}{h_t^f} \quad (3.68)$$

$$1 = R_t^B E_t \left[\beta \frac{c_{t+1}^{-\gamma} P_t}{c_t^{-\gamma} P_{t+1}} \right] \quad (3.69)$$

$$P_t c_t = \frac{1}{\mu_t} \left(\frac{R^O - R^E}{R^O + R^E - 2R_t^L} \right) M_t \quad (3.70)$$

$$R_t^D = R_t^L - \mu_t \frac{(R_t^L - R^E)(R^O - R_t^L)}{R^O - R^E} \quad (3.71)$$

$$D_t = \frac{1}{\phi_t} M_t = \underbrace{\frac{1}{\phi_t} \theta \omega_t M_t}_{D_t^f} + \underbrace{(1 - \theta \omega_t) \frac{1}{\phi_t} M_t}_{D_t^h} \quad (3.72)$$

$$L_t = D_t^f \quad (3.73)$$

$$B_t = M_{t-1} - D_t^h \quad (3.74)$$

$$\hat{R}_t^F = \gamma E_t \left[\log \left(\frac{c_{t+1}}{c_t} \right) \right] + \tau \log \left(\frac{P_t}{P_{t-1}} \right) + \hat{\nu}_t \quad (3.75)$$

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \varepsilon_{t+1}^a \quad (3.76)$$

$$\hat{\mu}_{t+1} = \rho_\mu \hat{\nu}_t + \varepsilon_{t+1}^\mu \quad (3.77)$$

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \varepsilon_{t+1}^\nu \quad (3.78)$$

A circumflex denotes the log deviation of a variable from its steady state value. Equation 3.75 is a slightly unusual monetary policy rule: it implies that the central bank sets the policy rate in response to all expected changes in consumption, not simply the steady-state change in consumption, or the “natural rate”. I chose this rule to give the central bank the maximum “benefit of the doubt” as it responds to real and financial shocks.

3.4.1 Evolution of Exogenous Variables

Technology evolves according to

$$\hat{a}_{t+1} = \rho_a \hat{a}_t + \varepsilon_{t+1}^a. \quad (3.79)$$

Banks are subject to deposit withdrawal shocks at the end of each period. There are many small banks, and each bank i receives the withdrawal shock X_{it} . For now, we assume that X_{it} is uniformly distributed from $-\mu_t D_{it}$ to $\mu_t D_{it}$, where D_{it} is the quantity of deposits in bank i , and $0 < \mu_t \leq 1$:

$$X_{it} \sim U[-\mu_t D_{it}, \mu_t D_{it}], \quad (3.80)$$

with $X_{it} > 0$ indicating a net withdrawal of deposits, and $X_{it} < 0$ indicating a net inflow. The magnitude of μ_t will influence the spread between the loan rate R_t^L and deposit rate R_t^D , and size of the “money multiplier”. When μ_t is not subject to shocks, $\mu_t = \bar{\mu} = 1 \forall t$. When μ_t is subject to shocks it will take on a mean of 0.50, and evolve according to

$$\hat{\mu}_{t+1} = \rho_\mu \hat{\mu}_t + \varepsilon_{t+1}^\mu. \quad (3.81)$$

Finally, shocks to the policy rate R_t^F evolve according to

$$\hat{\nu}_{t+1} = \rho_\nu \hat{\nu}_t + \varepsilon_{t+1}^\nu. \quad (3.82)$$

3.4.2 Calibration and Impulse Response Functions

In this subsection, I show how the economy without capital responds to productivity shocks, and to changes in the variance of idiosyncratic withdrawal shocks.⁹ Figure 3.3 shows that real variables h_t and c_t respond to a productivity shock even when γ , the parameter of CRRA utility, is set to 1. This is because the monetary policy response to the productivity shock changes the labor wedge. Figures 3.4 and 3.5 show that real variables h_t and c_t respond to financial shocks (to μ_t and

⁹For now, I continue to use the simplifying assumption of uniformly distributed withdrawal shocks. Calibrations with more general withdrawal shocks are coming soon.

Parameter	Value	Interpretation
β	0.95	Discount rate
γ	1.00	Coefficient of CRRA utility
φ	1.33	$1/\varphi$: Frisch elasticity of labor supply
χ	1.00	Coefficient on the disutility of labor
R^O	1.50	Overdraft penalty rate
R^E	1.00	Interest rate on excess reserves
τ	2.00	Central bank response to inflation
μ	1.00	Support of withdrawal shock
a	0.00	$\log(\text{Productivity})$
ρ_a	0.90	Persistence of AR(1) shock to a_t
ρ_μ	0.90	Persistence of AR(1) shock to μ_t
ρ_ν	0.90	Persistence of AR(1) shock to ν_t

Table 3.4: Parameters for calibration

R_t^F , respectively.

3.5 EMPIRICAL IMPLICATIONS

3.5.1 An Illustrative Labor Wedge

The FDIC provides annual aggregate data on FDIC-insured banks from 1934-present. FRED provides the prime loan rate, and the 3-month certificate of deposit (CD) rate. Figure 3.6 displays the three relevant interest rates: the prime loan rate R_t^L , the 3-month CD rate R_t^B , and the deposit rate R_t^D . The model hypothesizes an interest rate wedge of the form:

$$\frac{R_t^D}{\theta R_t^L + (1 - \theta) R_t^D} \frac{\theta R_t^B + (1 - \theta) R_t^D}{R_t^B} A_t = \frac{u_{h^l}(c_t, h_t^l)}{u_c(c_t, h_t^l)} \quad (3.83)$$

θ is the parameter governing the fraction of wages the firm must pay to the household before production can take place. When $\theta = 1$, the firm must take a loan to pay for its full wage bill before it earns revenues from selling output.

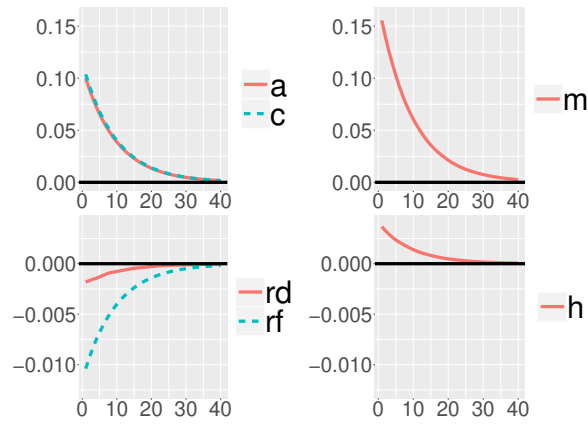


Figure 3.3: Impulse Responses for a shock to productivity

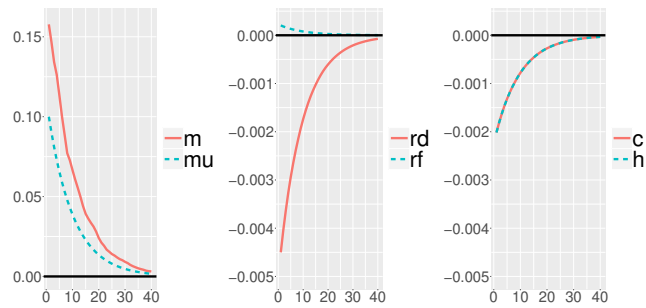


Figure 3.4: Impulse Responses for an increase in withdrawal shock variance

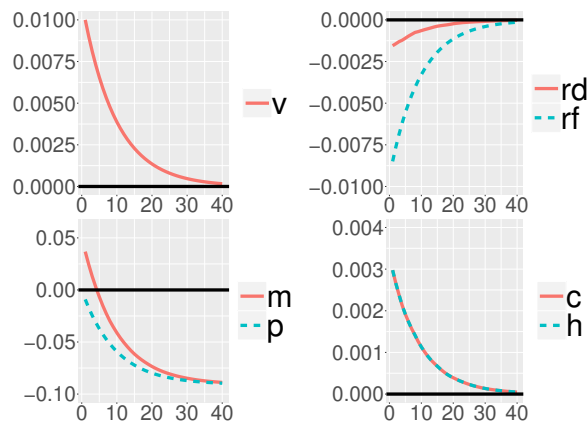


Figure 3.5: Impulse Responses for a shock to the policy rate

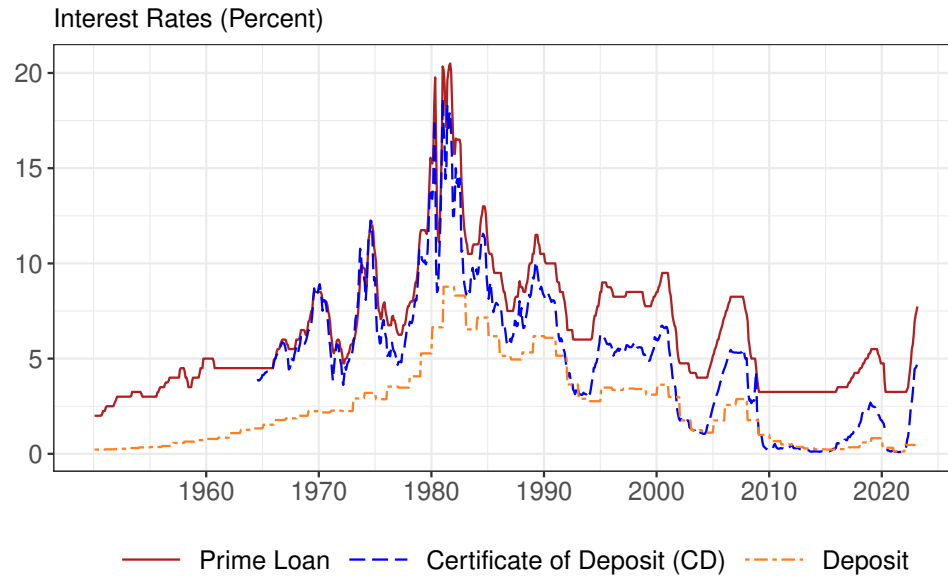


Figure 3.6: Historical interest rates

Figure 3.7 shows the implied interest rate wedge series for three values of θ . All wedges increase sharply during the 1980 recession. The size of the wedge appears pro-cyclical up to the mid-1980s, and then counter-cyclical beginning in the great moderation.

Not all firms are able to borrow at the prime rate. In particular, Adrian et al. (2013) document that credit spreads for bank and bond financing rise in downturns. For this reason, this illustrative labor wedge may be smaller in downturns than a labor wedge using data on firms' actual borrowing costs. However, the size of the wedge depends on the time-period over which it is considered. When considering monthly rather than annualized interest rates, the wedge remains vanishingly close to one over the whole sample period.

Not all firms are able to borrow at the prime rate. In particular, Adrian et al. (2013) document that credit spreads for bank and bond financing rise in down-

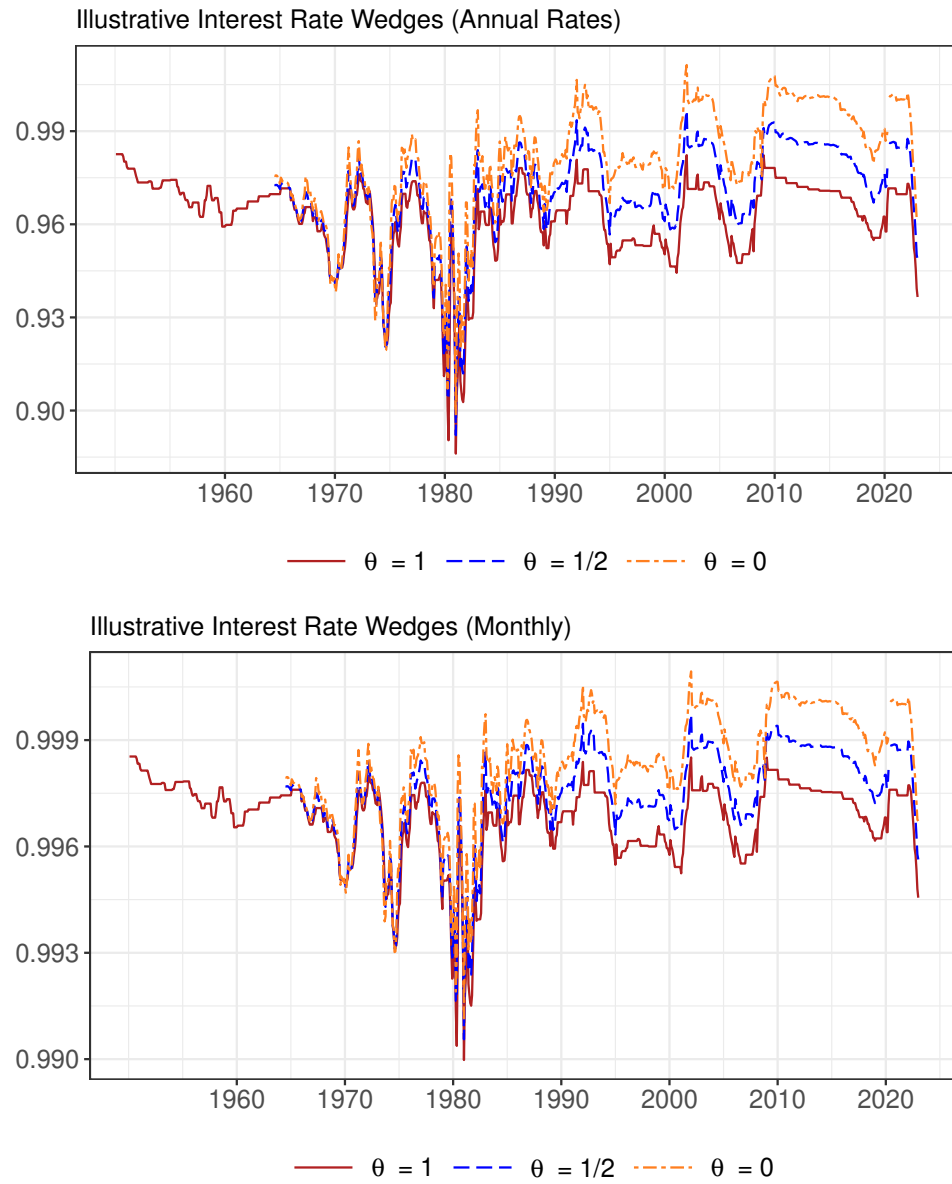


Figure 3.7: Interest rate wedges

turns. For this reason, this illustrative labor wedge may be much smaller in downturns than a labor wedge using data on firms' actual borrowing costs.

3.5.2 A Trendless Unencumbered Reserve Ratio from 1959-2007

The model predicts that banks choose the ratio of reserves to deposits in anticipation of deposit withdrawal shocks. One implication of this is that when policy rates R_t^O , R_t^E and R_t^F are trendless, the ratio of excess reserves to deposits should also be trendless. The model first order conditions suggest calculating the ratio of bank reserves held in excess of reserve requirements to the total quantity of deposits created by banks.

FRED provides monthly data on monetary aggregates and bank reserves beginning in 1959. A measure of Divisia M1, which includes Money Market Deposit Accounts (sweep accounts) which were introduced in 1994 is available on FRED until 2013, and available from the Center for Financial Stability website afterward.

I will consider the relevant measure of deposits to be the portion of M2 created by banks and against which they are likely to hold reserves. This is the sum of demand deposits and sweep accounts. Figure 3.8 displays the components of M2, with the relevant component labeled Demand Deposits. It is not possible to differentiate between reserves held against demand deposits and reserves held against savings deposits using aggregate data. However, as depositors can withdraw from a savings account at most six times per month, it is natural to assume that the majority of precautionary reserves are held against demand deposits which place no restrictions on the frequency of withdrawals. Figure 3.9 separates total banking system reserves into reserves required by statute, and two measures of economic

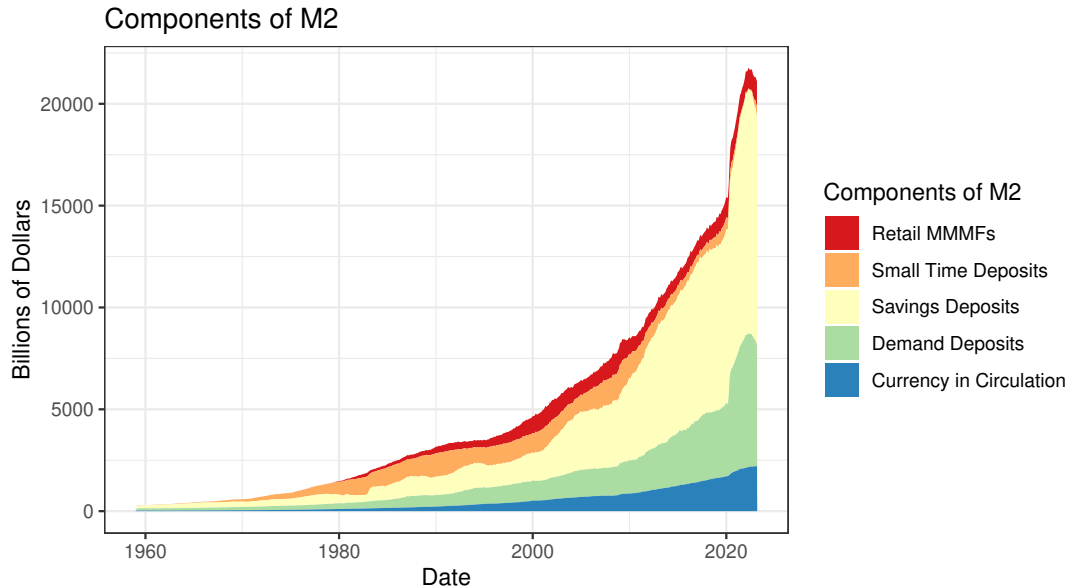


Figure 3.8: Components of M2

reserves: reserves held in excess of reserve requirements, excluding contractual clearing balances, and reserves held in excess of reserve requirements, including contractual clearing balances. I will describe reserves held in excess of reserve requirements as “unencumbered reserves”; the model predicts that this measure of reserves is relevant for calculating the reserve ratio.

Figure 3.10 shows the measures of required and total reserves divided by Total Deposits. These ratios decreased post-1980 due to decreases in reserve requirements from the Monetary Control Act of 1980, and banks’ use of software beginning in 1994 to “sweep” transactions deposits into savings deposits overnight, thereby reducing their quantities of reservable deposits.

The upper panel of Figure 3.11 shows that the ratio of economic reserves to total deposits was trendless before 2008. The model predicts that the reserve to deposit ratio should increase as the rate of interest on excess reserves increases,

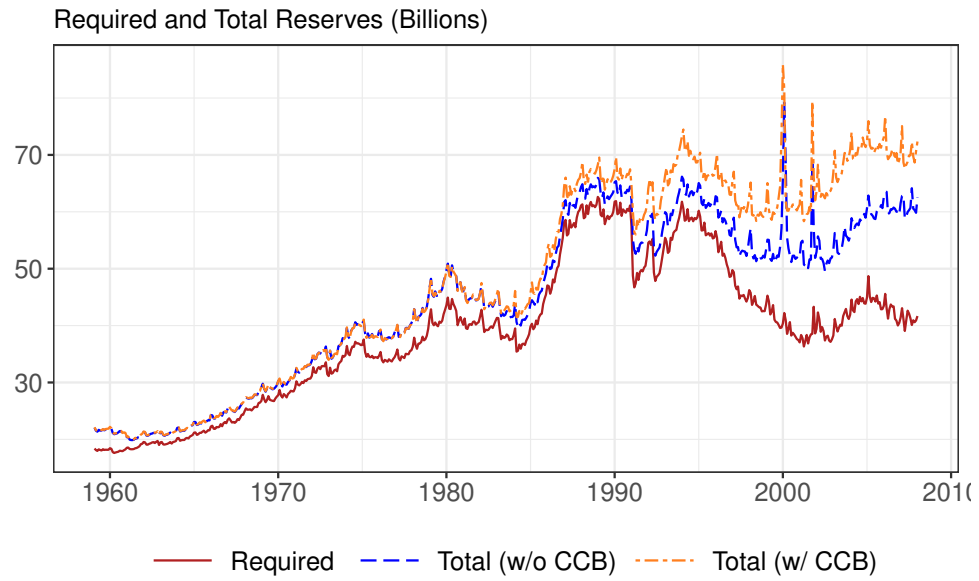


Figure 3.9: Required and unencumbered reserves

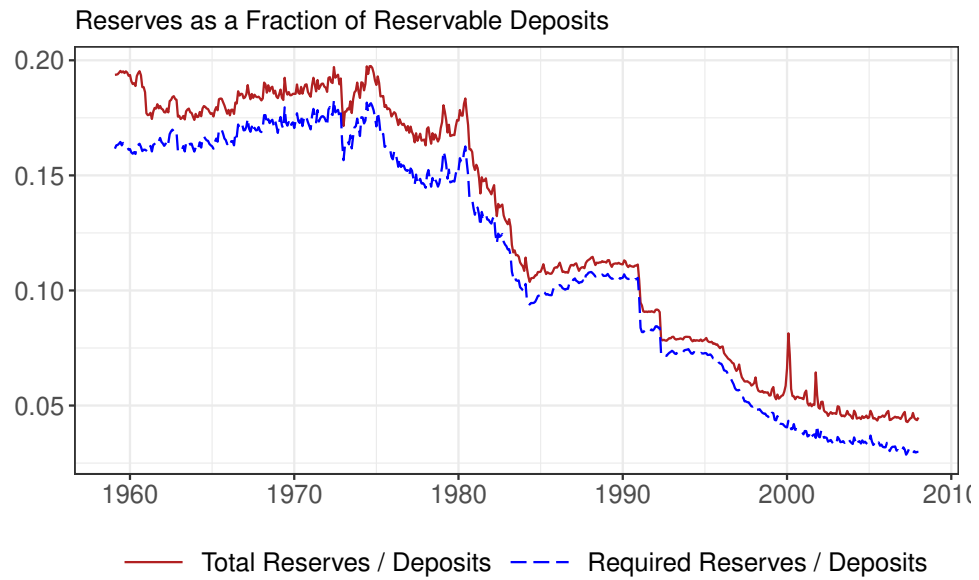


Figure 3.10: Total reserves and required reserves as a fraction of demand deposits

and as the federal funds rate decreases. Beginning in 2008, decreases in the Federal Funds rate and increases in the interest rate paid on excess reserves reduced the opportunity cost of holding reserves. This reduced opportunity cost of holding reserves is consistent with the dramatic increase in the reserve ratio beginning in 2008. The lower panel of Figure [3.11](#) shows that beginning in 2008, banks have held between \$0.50 to \$1 of reserves for every dollar of demand deposits.

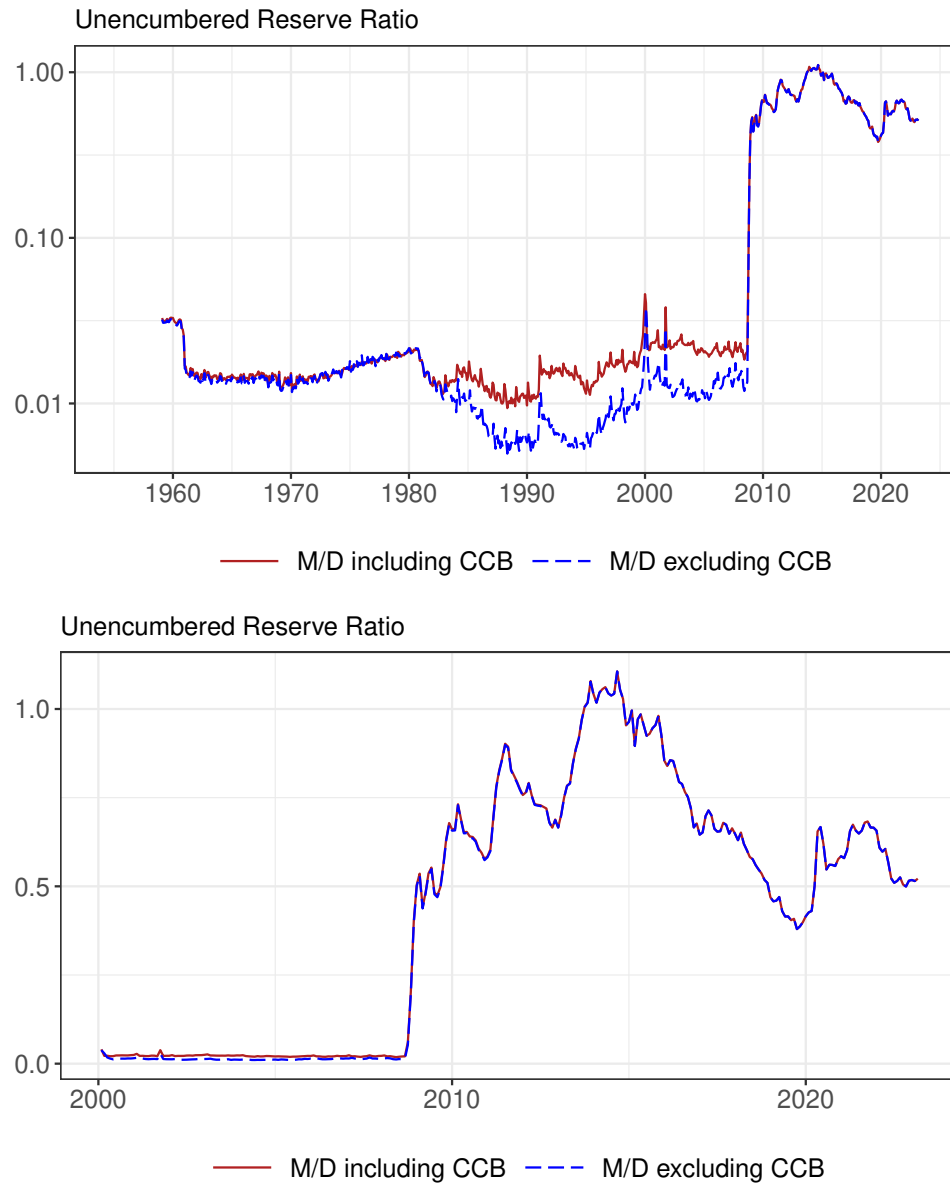


Figure 3.11: Unencumbered reserves as a fraction of demand deposits

3.6 CONCLUSION

In this paper, I explicitly model financial institutions in general equilibrium. Households and firms transact using financial instruments created by the financial sector. Banks in the financial sector issue loans, bonds, and deposits, and hold precautionary reserves of base money in anticipation of idiosyncratic deposit withdrawal shocks. Bank reserve management creates a spread between the loan rate R_t^L and the deposit rate, R_t^D . The loan rate is above the deposit rate because banks face two costs of deposit issuance. First, banks pay interest on deposits. Second, banks face expected overdraft costs from deposit withdrawal shocks. The spread between the loan and deposit rates creates a time-varying labor wedge which transmits financial shocks to real variables even under flexible prices. An illustrative labor wedge is shown to be pro-cyclical until the 1979-82 recession, and countercyclical afterward. However, using monthly period lengths and data for interest rates reveals the labor wedge to be small. As a result, equilibrium outcomes in the presence of a financial sector are nearly identical to those in an economy without an explicitly modeled financial sector.

Monetary policy in this model is described by three interest rates: an interbank market target rate, a rate on excess reserves, and an overdraft penalty rate. Banks will choose finite nominal balance sheets so long as the overdraft rate is above the target interbank rate. Model first order conditions suggest calculating the “unencumbered reserve” to deposit ratio as the ratio of reserves held by banks in excess of required reserves to the total quantity of deposits issued by banks. This reserve to deposit ratio is trendless before the 2008 financial crisis, which is consistent with the model’s hypothesis that banks face i.i.d. deposit withdrawal shocks. Furthermore, bank first order conditions imply that when the interbank rate is equal to the

rate on excess reserves, banks will hold one dollar of unencumbered reserves for every dollar of deposits. This model prediction is consistent with banks' aggregate reserve holding behavior after 2008.

APPENDIX A

**Appendices for Optimal Fiscal and Monetary Policy
with Flexible Prices and Long-Term Debt**

A.1 EVOLUTION OF MULTIPLIERS*A.1.0.1 Evolution of Multipliers with One-Period Nominal Bonds*

First, consider the implications of this constraint in the one-period-bond economy by setting $b_{\infty,t} = 0$, and $\mu_{\infty,t} = 0$:

$$u_c(c_t)(\mu_{1,t} - \gamma_t)b_{1,t}\delta_t = 0 \tag{A.1}$$

Suppose we do not consider infinite inflation, which would result in $\delta_t = 0$. $u_c(c_t)$ will always be positive, so when $b_{1,t} \neq 0$, it must be that $\mu_{1,t} - \gamma_t = 0$. Now suppose the government must be a net debtor, so that $b_{1,t} - \underline{b}_1 \geq 0$, with $\underline{b}_1 = 0$. In periods when the constraint binds, so that there is no government debt outstanding, there may be a difference between γ_t and $\mu_{1,t}$.

When $\gamma_t \neq \mu_{1,t}$, it seems likely that $\gamma_t < \mu_{1,t}$. To see this, suppose that instead, $\gamma_t > \mu_{1,t}$, but $b_{1,t} = \underline{b}_1$. If the government could reduce the tightness of its constraint in the current period by issuing debt, it would do so: debt issuance would simultaneously reduce γ_t and move $b_{1,t}$ away from its lower bound \underline{b}_1 . On the other hand, it would not be optimal to issue more debt if $\gamma_t < \mu_{1,t}$, indicating that the government budget constraint binds less tightly in the current period than it has in the past. If this is the case, it suggests that ex-post, the government could have chosen a policy that would have resulted in lower γ_{t-1} , and equivalently, lower $\mu_{1,t}$, with the tradeoff that policy in the current period would involve $b_{1,t} > \underline{b}_1$.

This leads me to conjecture that γ_t will be constant across states in economies with inflation, $\gamma = \gamma_t \forall t$, just as Φ was constant across states in the complete markets economy. I conjecture that the way in which the non-negativity constraint on debt will change the real allocations is by leading to a higher constant value of γ than the value of Φ that would be attained in the equivalent economy with complete markets: $\gamma > \Phi$.

A.1.0.2 Evolution of Multipliers with Nominal Consols

We next consider the optimality condition for inflation for a government that only issues nominal consols:

$$[(\mu_{1,t} - \gamma_t)kb_{\infty,t} + \mu_{\infty,t}]W_{\mu_{\infty}}(s_t) = 0, \quad (\text{A.2})$$

and recall the evolution equation for $\mu_{\infty,t}$:

$$\mu_{\infty,t+1} = \theta^{-1}\delta_t[\mu_{\infty,t} + (\mu_{1,t} - \gamma_t)kb_{\infty,t}] \quad (\text{A.3})$$

It will always be the case that $W_{\mu_{\infty}}(s_t) > 0$. When γ_t has been constant in previous periods, $\mu_{\infty,t}$ will be zero, and the same analysis as in the one-period bond case applies. This means that either $b_{\infty,t} > \underline{b}_{\infty}$ or $(\mu_{1,t} - \gamma_t) = 0$, so that even when the constraint binds, $\mu_{\infty,t+1} = 0$ when the constraint is set to $\underline{b}_{\infty} = 0$. Because of this, when the government can only issue nominal consols, $\mu_{\infty,t}$ will always be equal to zero, and there will be a constant $\gamma > \Phi$ following the same logic as in the one-period-bond economy.

From equation A.2, we know that

$$(\mu_{1,t} - \gamma_t)kb_{\infty,t} = -\mu_{\infty,t} \quad (\text{A.4})$$

If for some reason $\mu_{\infty,t} > 0$, $(\mu_{1,t} - \gamma_t) < 0$, and vice-versa: if $\mu_{\infty,t} < 0$, $(\mu_{1,t} - \gamma_t) > 0$. From the evolution equation for $\mu_{\infty,t}$, $\mu_{\infty,t+1} = 0$, returning the economy to the $\mu_{\infty,t} = 0$ case described above.

A.1.0.3 Evolution of Multipliers with

Nominal One-Period Bonds and Consols

Again consider the FOC with respect to nominal depreciation, equation 1.45, in combination with 1.52 becomes

$$u_c(c_t)(\mu_{1,t} - \gamma_t)\delta_t b_{1,t} + [(\mu_{1,t} - \gamma_t)kb_{\infty,t} + \mu_{\infty,t}]W_{\mu_{\infty}}(s_t) = 0 \quad (\text{A.5})$$

and the evolution equation for $\mu_{\infty,t}$:

$$\mu_{\infty,t+1} = \theta^{-1}\delta_t[\mu_{\infty,t} + (\mu_{1,t} - \gamma_t)kb_{\infty,t}] \quad (\text{A.6})$$

Suppose that $\mu_{\infty,t} = 0$, and that $\underline{b}_1 = 0$ and $\underline{b}_{\infty} = 0$, so that $b_{1,t} \geq 0$ and $b_{\infty,t} \geq 0$. If either $b_{1,t} > 0$ or $b_{\infty,t} > 0$, $(\mu_{1,t} - \gamma_t) = 0$, and $\mu_{\infty,t+1} = 0$. On the other hand, if $(\mu_{1,t} - \gamma_t) \neq 0$, both $b_{1,t} = 0$, and $b_{\infty,t} = 0$, so that $\mu_{\infty,t+1} = 0$. In other words, in an economy in which $b_{1,t} \geq 0$ and $b_{\infty,t} \geq 0$, when $\mu_{\infty,t} = 0$, all future values of $\mu_{\infty,t}$ will remain equal to zero. Again there will be a constant $\gamma > \Phi$, following the same logic as in the one-period-bond economy, and the nominal consol economy.

Finally, suppose that $\mu_{\infty,t} \neq 0$, and again require that $b_{1,t} \geq 0$ and $b_{\infty,t} \geq 0$. It

cannot simultaneously be the case that $(\mu_{1,t} - \gamma_t) = 0$, because equation 1.45 would not hold. When $\mu_{\infty,t} > 0$, $(\mu_{1,t} - \gamma_t) < 0$. When $b_{1,t} = 0$, this is equivalent to the consols-only economy, and $\mu_{\infty,t+1} = 0$. When $b_{1,t} > 0$, rearrange to produce

$$\frac{k}{k + Q_{\infty,t}}(\mu_{1,t} - \gamma_t)b_{1,t} + (\mu_{1,t} - \gamma_t)kb_{\infty,t} + \mu_{\infty,t} = 0 \quad (\text{A.7})$$

so that if $b_{\infty,t} = 0$, $\mu_{\infty,t+1} = \theta^{-1}\delta_t\mu_{\infty,t}$, and $\mu_{\infty,t+1}$ evolves with nominal depreciation. With both one-period bonds and consols outstanding, denote the face value of the debt as $b_t = b_{1,t} + b_{\infty,t}$, and use the evolution equation for $\mu_{\infty,t+1}$,

$$\mu_{\infty,t+1} = \theta\delta_t^{-1}[(\mu_{1,t} - \gamma_t)k(b_t - b_{1,t}) + \mu_{\infty,t}] \quad (\text{A.8})$$

$$= \theta\delta_t^{-1}\mu_{\infty,t} + \theta\delta_t^{-1}(\mu_{1,t} - \gamma_t)k(b_t - b_{1,t}) \quad (\text{A.9})$$

so that when $b_t > b_{1,t} > 0$,

$$\theta\delta_t^{-1}\mu_{\infty,t} > \mu_{\infty,t+1} > 0, \quad (\text{A.10})$$

indicating that if $\mu_{\infty,t+1}$ is ever positive, it will evolve towards zero without oscillation so long as inflation (δ_t^{-1}) is moderate.

A.2 NUMERICAL IMPLEMENTATION

A.2.1 Complete Markets Numerical Implementation

Estimating $n^*(s^t)$ Given Φ

Take values for a, g, z and Φ as given. n^* is between g/a (the value of n for which

$c = an - g = 0$) and 1. Use binary search to find n^* that satisfies the planner's FOCs

$$0 = \frac{u_n + \Phi\{u_{nn}n + u_n\}}{u_c + \Phi\{u_{cc}[c - z] + u_c\}} + a \quad (\text{A.11})$$

Use initial guess $\hat{n} = (ub - lb)/2 + lb = (1 - g/a)/2 + g/a$. If the RHS of equation A.11 is positive, \hat{n} is too low, and the new lower bound is $lb = \hat{n}$. If the RHS of equation A.11 is negative, \hat{n} is too high, and the new lower bound is $ub = \hat{n}$. Repeat until the guess for n^* makes the RHS of A.11 close to zero.

Estimating Φ

Begin with the Ramsey Planner's FOCs for consumption and labor

$$u_c(s^t) + \Phi\{u_{cc}(s^t)[c(s^t) - z(s^t)] + u_c(s^t)\} - \lambda(s^t) = 0 \quad (\text{A.12})$$

$$u_n(s^t) + \Phi\{u_{nn}(s^t)n(s^t) + u_n(s^t)\} + \lambda(s^t)a(s^t) = 0 \quad (\text{A.13})$$

Multiply the equations by $[c(s^t) - z(s^t)]$ and $n(s^t)$, respectively, sum, and rearrange terms:

$$\begin{aligned} & (u_c(s^t) + \Phi\{u_{cc}(s^t)[c(s^t) - z(s^t)] + u_c(s^t)\} - \lambda(s^t)) [c(s^t) - z(s^t)] + \\ & (u_n(s^t) + \Phi\{u_{nn}(s^t)n(s^t) + u_n(s^t)\} + \lambda(s^t)a(s^t)) n(s^t) = 0 \end{aligned} \quad (\text{A.14})$$

$$\begin{aligned} & (1 + \Phi)\{u_c(s^t)[c(s^t) - z(s^t)] + u_n(s^t)n(s^t)\} \\ & + \Phi\{u_{cc}(s^t)[c(s^t) - z(s^t)]^2 + u_{nn}(s^t)n(s^t)^2\} \\ & - \lambda(s^t)[[c(s^t) - z(s^t)] - a(s^t)n(s^t)] = 0 \end{aligned} \quad (\text{A.15})$$

Multiply by $\beta^t \pi(s^t)$ and sum over states:

$$\begin{aligned}
& (1 + \Phi) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u_c(s^t) [c(s^t) - z(s^t)] + u_n(s^t) n(s^t)] \\
& + \Phi \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u_{cc}(s^t) [c(s^t) - z(s^t)]^2 + u_{nn}(s^t) n(s^t)^2] \\
& - \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \lambda(s^t) [[c(s^t) - z(s^t)] - a(s^t) n(s^t)] = 0
\end{aligned} \tag{A.16}$$

The first line is the implementaiton constraint, which binds and is equal to zero.

Putting the resource constraint into the last line produces

$$\begin{aligned}
& \Phi \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) [u_{cc}(s^t) [c(s^t) - z(s^t)]^2 + u_{nn}(s^t) n(s^t)^2] \\
& + \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \lambda(s^t) [g(s^t) + z(s^t)] = 0
\end{aligned} \tag{A.17}$$

The $u_{cc}(s^t)$ and $u_{nn}(s^t)$ terms will be negative, with all other terms positive, so a positive Φ will solve this equation.

We will make guesses for Φ until the two lines of equation A.17 sum to zero. When the LHS of equation A.17 is positive, raise Φ . When the LHS of equation A.17 is negative, lower Φ . Given a guess for Φ , and the associated guesses for $n(s^t)$ and $c(s^t)$, $\lambda(s^t)$ can be calculated from the Ramsey Planner's FOC for consumption:

$$\lambda(s^t) = [1 + \Phi \{ \frac{u_{cc}(s^t)}{u_c(s^t)} [c(s^t) - z(s^t)] + 1 \}] u_c(s^t) \tag{A.18}$$

Suppose that $b(s^0) = 0$. Given a transition matrix M , and column vectors $g, z, c, n, u_c, u_{cc}, u_n, u_{nn}$, and λ , the LHS of equation A.17 can be calculated

$$\Phi[I - \beta M]^{-1}[u_{cc}(c - z)^2 + u_{nn}n^2] + [I - \beta M]^{-1}\lambda[g + z] \quad (\text{A.19})$$

Guesses for Φ also progress according to a binary search process. When $b(s^0) \geq 0$, we include it in the optimality conditions for s^0 so that

$$\begin{aligned} 0 = & \Phi[u_{cc}(c - z - b_0) + u_{nn}n^2] + \lambda_0[g + z + b_0] \\ & + \beta[\Phi[I - \beta M]^{-1}[u_{cc}(c - z)^2 + u_{nn}n^2] + [I - \beta M]^{-1}\lambda[g + z]] \end{aligned} \quad (\text{A.20})$$

This allows $t = 0$ to be treated as a “startup period”, with the transition matrix M governing state transitions beginning in $t = 1$.

A Value for Φ Determines Endogenous Real Variables

The planner’s FOCs and the resource constraint imply n_t^* , c_t^* , and marginal utilities of labor and consumption. Marginal utilities of labor and consumption determine the tax rate. The tax rate, labor supply, and exogenous state variables (a, g, z) determine the primary surplus.

Using the transition matrix to compute expectations, expected marginal utilities are

$$u_{c,t+j} = M^j u_c \quad (\text{A.21})$$

so that j -period bond prices are

$$q_{j,t} = \beta^j \frac{1}{u_c} \circ (M^j u_c) \quad (\text{A.22})$$

The PDV of surpluses is

$$F_t = f_t + \beta E_t \left[\frac{u_{c,t+1}}{u_{c,t}} F_{t+1} \right] \quad (\text{A.23})$$

$$F = f + \frac{1}{u_c} \circ \beta M (u_c \circ F) \quad (\text{A.24})$$

$$= \frac{1}{u_c} \circ [I - \beta M]^{-1} (u_c \circ f) \quad (\text{A.25})$$

where \circ denotes element-wise multiplication.

A.2.2 Iteration to find Nominal Bond Prices and Inflation Policy

Nominal bonds are priced

$$Q_1(\varsigma) = \frac{1}{u_c(\varsigma)} \circ \beta (M \circ \delta) u_c(\varsigma) \quad (\text{A.26})$$

$$Q_\infty(\varsigma) = \frac{k}{u_c(\varsigma)} \circ (I - \beta (M \circ \delta))^{-1} \beta (M \circ \delta) u_c(\varsigma) \quad (\text{A.27})$$

where δ denotes the full matrix of state-to-state price level depreciation terms, and \circ denotes element-wise multiplication. To find nominal bond prices, bond quantities, and inflation values that support the complete markets real allocations for this economy, follow the following iterative procedure:

Before beginning the iteration, assume that inflation is equal to its target $\underline{\delta}$ between all states i and j : $\delta(j; i) = \underline{\delta}$. When $\underline{\delta} = 1$, this would be equivalent to the assumption of a real economy.

1. Compute bond prices given the real allocations and assumption about inflation ($\underline{\delta}$)

2. For each row i in the transition matrix, focus on the diagonal transition from i to j if the diagonal element is non-zero, and the left-most non-zero element j in row i otherwise. Given previously-computed quantities $F(i)$, $f(i)$, and $F(j)$, and current guesses for $Q_1(i)$, $Q_\infty(i)$, $Q_\infty(j)$, and $\delta(j; i)$, compute bond quantities that must hold for the economy to transition from state i to state j :

$$\begin{bmatrix} F(i) - f(i) \\ F(j) \end{bmatrix} = \begin{bmatrix} Q_1(i) & Q_\infty(i) \\ 1\delta(j; i) & (k + Q_\infty(j))\delta(j; i) \end{bmatrix} \begin{bmatrix} b_1(i) \\ b_\infty(i) \end{bmatrix} \quad (\text{A.28})$$

Bond quantities $b_1(i)$ and $b_\infty(i)$ are chosen in state i , and are state variables in state j .

- (a) If the matrix of bond prices is singular, expected returns on one-period debt and consols are equal. Guess that half of government debt is issued in each type of bond.
- (b) If all $F(i) \geq 0$ and we have imposed the restriction that $b \geq 0$ where $b(i) = [b_1(i), b_\infty(i)]$, and one of the bond quantity guesses is less than zero, the other bond quantity must be positive. To satisfy the $b \geq 0$ constraint, set the guess for the negative bond quantity to zero, and guess that all debt in state i is issued in the other bond.
3. Given the bond quantities found in state i , find values for inflation that are required for the economy to transition from state i to state j . In particular, find $\delta(j; i)$ such that the real value of the debt in state j is equal to the present value of future surpluses $F(j)$:

$$F(j) = [b_1(i) + (k + Q_\infty(j))b_\infty(i)]\delta(j; i) \quad (\text{A.29})$$

4. Values for $\delta(j; i)$ combine to produce a new guess for δ .
5. Re-normalize delta so that expected inflation from one state to the next is always equal to its target: $\underline{\delta} = E_i[\delta(j; i)]$.
 - (a) If the previous step produces a nominal one-period bond price greater than 1 in state i , divide each $\delta(j; i)$ guess by the one-period bond price to increase expected inflation in that state. To maintain expected inflation equal to its target over time, expected inflation must be slightly decreased in other states until $\underline{\delta} = E[\pi(i)\delta(j; i)]$, where $\pi(i)$ is the long-run probability that the economy is in state i , so that $E[\cdot]$ is an unconditional expectation, and re-calculate bond prices. This will guarantee that the re-calculated nominal one-period bond price is exactly equal to 1 in state i .
6. Continue iterating until bond prices converge from one iteration to the next.

APPENDIX B

Appendices for Financial Institutions in General Equilibrium

B.1 INSTITUTIONAL BACKGROUND

As described in Furfine (2000), “Each business day, banks send and receive nearly \$2 trillion of large-value payments as a result of their own business activity and the activity of their customers. Both inflows and outflows of payments over the Federal Reserve’s large-value wire transfer system, Fedwire, are reflected immediately as debits or credits to a bank’s reserve account. Banks active in the payment system typically send and receive payments whose value is around 30 times greater than the bank’s overnight reserve balance, with the most active banks typically having a ratio of payments to balances of nearly 200.”

The sequence of daily events in the market for bank reserves is below:

1. Before the day begins, banks have received deposits and chosen how much to invest and how much to hold as reserves.
2. The Fed enters the market to conduct OMOs at 9:30 AM, before the vast majority of payments and withdrawals have taken place
3. Banks face withdrawals and receive deposits over the course of the day
 - (a) Banks are charged interest at an annualized rate of 27bp (Furfine (2000)) for daylight overdrafts, in which their Fed accounts are temporarily overdrawn during the business day. This rate is low enough that banks are generally willing to realize daylight overdrafts.¹

¹Aggregate data on daylight overdrafts are available at https://www.federalreserve.gov/paymentsystems/psr_data.htm.

4. In the afternoon, banks trade on the federal funds market (Furfine (1999) finds that the weighted mean transaction is delivered at 4:12 PM, but may have been arranged hours before it was recorded.)
5. Banks continue to face withdrawals until 6:30 when their branches close and Fedwire² ceases operation for the day. (As of December 8, 1997, Fedwire operated from 12:30 AM to 6:30 PM.)
 - (a) A bank in “sound financial condition” may submit collateral to its regional Fed in exchange for a short term loan from the Discount Window.
 - (b) Banks are charged an overnight overdraft fee of the Federal Funds rate + 400bp (Marquis (2002)) on any reserve shortage remaining at the close of Fedwire. This penalty rate is not the only cost of an overnight overdraft: more intensive regulatory supervision or legal proceedings may ensue. (Furfine (2000) and other authors proposing models of bank reserve management consider the avoidance of overnight overdrafts to be an important motivator of bank behavior.)

Figure B.1 shows the target Federal Funds rate set by the Federal Open Market Committee (FOMC), and the Effective Federal Funds rate, which is determined by banks’ reserve management decisions. To keep the Effective Federal Funds rate trading near its target, the New York Fed’s Trading Desk (the Desk) conducts Open Market Operations (OMOs). The Desk makes a forecast of reserve demand at the target federal funds rate, and then buys or sells a quantity of reserves with the objective that reserves will trade at rates near the target rate.

²Fedwire is a real-time gross interbank settlement system that enables participants to initiate funds transfers that are immediate, final, and irrevocable once processed. Institutions that hold an account with a Federal Reserve Bank are eligible to participate in Fedwire services. https://www.federalreserve.gov/paymentsystems/fedfunds_about.htm

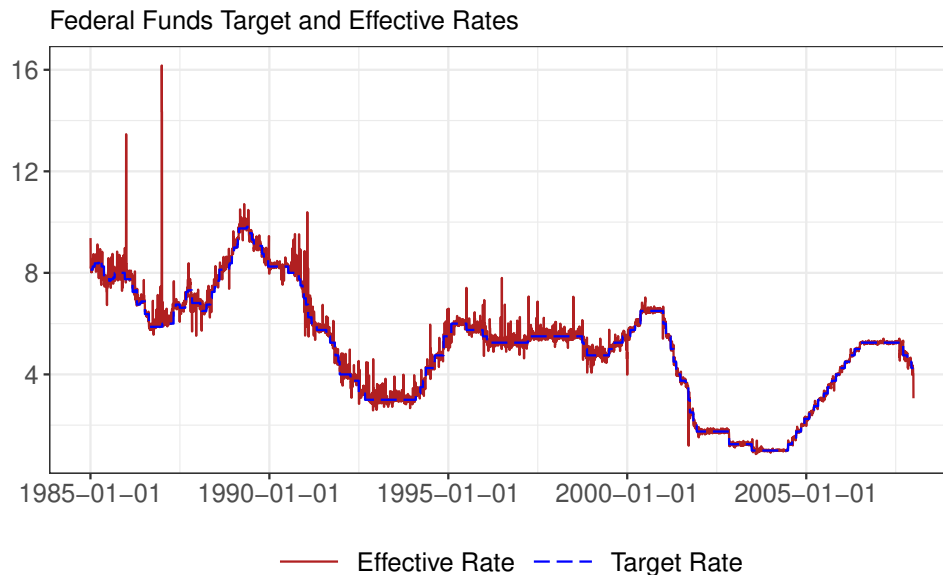


Figure B.1: Target and effective Federal Funds rates

The fact that federal funds transaction volumes are decided before the end of the business day, combined with banks' desire to avoid costly overnight overdrafts implies a demand for excess reserves. Figure B.2 shows that banks have been holding a substantial volume of excess reserves since reserve accounting changes in the early 1990s.

B.1.1 Borrowing from the Federal Reserve

If a bank is unable to secure a loan on the interbank market, it may choose to borrow directly from the Federal Reserve. Banks in "sound financial condition" may submit collateral to their regional Federal Reserve branch in exchange for a short-term loan from the Discount Window. The Discount Window offers two types of credit. Before January 9, 2003, these were Adjustment credit, provided to banks in sound financial condition, and extended credit, provided to weak institutions in

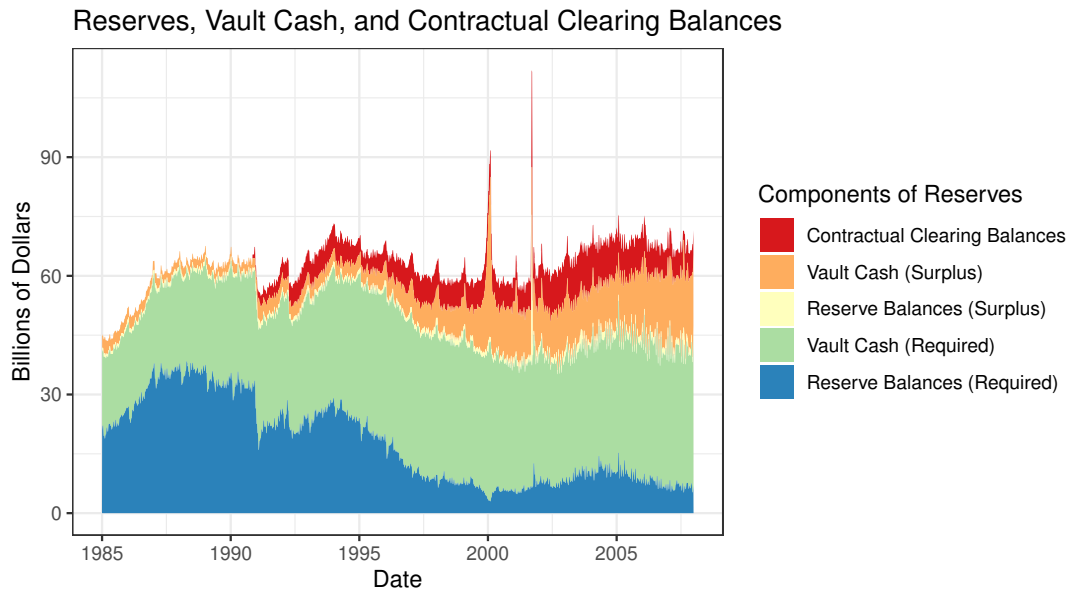


Figure B.2: Categories of reserves

close consultation with the institution's primary regulator as a means to facilitate an orderly resolution of funding problems (Klee (2011)). Beginning on January 9, 2003, the Fed offered primary credit at 100 basis points above the federal funds target rate to healthy banks. Primary credit was provided on a no questions asked basis and with no restrictions on use, including arbitraging the federal funds market. Secondary credit was provided at 150 basis points above the federal funds target rate to banks not eligible for primary credit, and involved additional administrative overhead.

Banks with a reserve shortage that do not borrow reserves from other banks or the discount window are required to pay an overnight overdraft penalty rate to the Fed. As of July 2012, the first three overdrafts in a rolling 12-month window require penalty rate payments of the primary credit rate plus 4 percentage points (annual rate). After the third overdraft, the penalty fee increases by one percentage

point per overdraft.³

B.2 THE PROCESS OF LOAN AND DEPOSIT CREATION

	Bank1		HH1		HH2		Bank2	
	Asset	Liab	Asset	Liab	Asset	Liab	Asset	Liab
(1)	M	D			House			
(2)	M +L	D +D	+D	+L	House			
(3)	M-M L	D D	+House D	L	House +M			
(4)	M-M L	D	+House	L	+D M		+M	+D
(5)	M-M+M L	D +B	House	L	D +B		M	D
(6)	M L	D B	House	L	B			

Table B.1: The process of loan and deposit creation

1. HH1 would like to purchase the house from HH2. Bank 1 has some pre-existing quantities of base money and deposits.
2. HH1 gets a mortgage from Bank 1. Bank 1 creates an asset (L) and a liability (D) for itself, and simultaneously creates an asset (D) and a liability (L) for HH1.
3. HH1 uses deposits to withdraw base money from Bank 1, and uses the base money to pay HH2 in exchange for the house.
4. HH2 deposits the base money into Bank 2

³https://www.federalreserve.gov/paymentsystems/oo_policy.htm

- (a) Models of inside money creation that assume all loans are financed by deposits stop here. This creates a tight relationship in the model between the quantity of deposits and the quantity of loans. In Step 4, aggregate deposit demand appears greater than it was in step 1. However, it may be that HH2 does not want to hold deposits, and would rather hold an asset like a bond that pays a higher rate of interest. Similarly, Bank 1 may be below its desired reserve to deposit ratio, and would like to increase its reserve holdings.
5. HH2 withdraws base money from Bank 2 and uses it to purchase a bond from Bank 1. This transaction leaves HH1's balance sheet unchanged.
 6. HH1 started with no assets, but now has a house as an asset, and a bank loan (mortgage) as a liability. HH2 started with a house as an asset, and now owns the bond which is financing the loan to HH1. Bank 1 has effectively financed an expansion of its balance sheet with a bond, but deposits were created and then destroyed in the process.

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CURRICULUM VITAE

