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THREE ESSAYS ON FIRM DYNAMICS AND MACROECONOMICS

by

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(Order No.)

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ABSTRACT

This dissertation examines three topics in macroeconomics. The first chapter studies the impact of severance payments on employment when firms can subcontract as a substitute for hiring workers. In countries with strict job security regulations firms use flexible staffing arrangements to buffer the regular workforce from economic fluctuations and avoid workers' firing costs. I set up a general equilibrium model in the tradition of Hopenhayn and Rogerson (1993) where firms can hire two types of workers: subcontractors that are totally flexible, and permanent workers that entail firing costs that increase with seniority in the job. Both types are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. I estimate the model using a simulated method of moments by fitting employment growth dynamics of Chilean manufacturing plants. I find that allowing firms to subcontract workers increases output, employment and productivity. This effect is stronger on output as subcontracted workers allow firms to respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity (TFP). When firms can subcontract, the negative effects of firing costs are less than previously estimated in the literature.

The second chapter analyzes the effects of capital adjustment costs on quantity dynamics and asset prices in a real business cycle model when the representative agent has Epstein-

Zin preferences. Capital adjustment costs make it costly for agents to smooth fluctuations in consumption through the production sector, inducing them to take more consumption risk. I show this model accounts for the main statistical features of macroeconomic aggregate quantities. At the same time, adjustment costs increase the equity risk premium, with the mean stock return and its standard deviation in the order of magnitude consistent with the data. The model also produces a stable risk-free rate, and comes close to matching its average return.

Finally, the third chapter (with Shuheng Lin) empirically examines the contribution of firm-level idiosyncratic shocks to aggregate fluctuations in the US, Germany, Canada, and the UK. We find shocks to large firms are of little relevance in the UK or Canada, but roughly explain one third of output fluctuations in the US and Germany. We argue the ability of the largest firms to transmit shocks is not universal, even when the firm size distribution is highly skewed as the theory suggests (Gabaix, 2011).

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List of Abbreviations

ENCLA	Encuesta Laboral
ENIA . .	Encuesta Nacional Industrial Anual
INE . . .	Instituto Nacional de Estadísticas

List of Symbols

* . . . pvalue<.1

** . pvalue<.05

*** pvalue<.01

Chapter 1

Firing Costs, Subcontracting and Employment Dynamics

1.1 Introduction

In many countries labor markets are constraint by strict legislations that protect workers against arbitrary actions, and give them higher job stability in the face of adverse economic conditions.[1] Even though these regulations are necessary in some cases, they also increase labor adjustment costs and impose heavy constraints to the firms that want to adjust their workforce in response to economic fluctuations. As a result, firms are increasingly turning to flexible staffing arrangements with less stringent rules, particularly with regard to firing costs. These contracts allow firms to buffer the stock of permanent workers, and overcome the potential costs associated with employment regulations during off-peak periods. As firms' use of contingent workers widespread, it is important to explicitly take into account this margin of adjustment for firms to properly evaluate the impact of firing costs. This paper provides a basic framework to perform such analysis.

A variety of evidence has pointed to a significant growth of flexible staffing arrangements, in particular, with regard to subcontracting. Subcontracting is a form of temporary employment in which a firm ('main firm') sublets to a third party ('subcontract firm') the performance of tasks or works, complete or partially, with its own dependent employees. When firms subcontract tasks to other firms, the employer of record for the worker performing the task changes, and the responsibility for all employment liabilities is trespassed to the subcontract firm. This way the firm avoids potential costs of dismissal and gains

[1] Legislation on employment protection usually regulates unfair dismissals, dismissals for economic reasons, mandatory severance payments, the use of fixed-term contracts, and minimum advance notice period in case of impending dismissal.

flexibility to terminate workers' contracts at will. But why then firms don't subcontract their entire workforce to circumvent firing costs? The hypothesis explored in this paper is that subcontractors' charges are higher than the firm's own production costs. Recognizing this, firms still hire permanent workers even when subcontracted workers do not entail firing costs.

Chile provides a particularly interesting setting to investigate the combine effects of firing costs and subcontracted workers. For many years, the country carried out asymmetric labor market reforms introducing a two-tier system: on the one hand, supporting job security provisions that greatly penalized employers for firing workers by imposing sizable tenure-dependent severance payments and, on the other, maintaining the market for subcontracted workers practically deregulated. This policy sustained a high employment protection legislation gap between both types of workers for years, triggering a widespread use of subcontracted workers, and a reallocation away from permanent workers.[2]

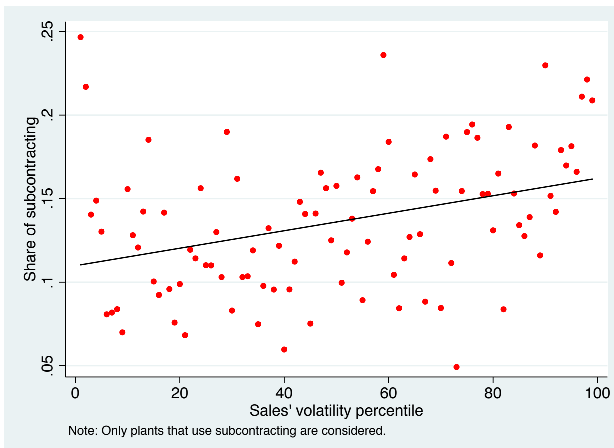
For many people, these arrangements are a politically viable way of achieving labor market flexibility when faced to important opposition from "insiders" (permanent workers). In this sense, they argue that keeping a proportion of low firing costs workers allows firms to regain flexibility to adjust their workforce, increasing their profits and the firms' value. In fact, we observe there is a positive relationship between the share of subcontracted workers and the establishments' sales volatility (see Figure 1.1). Establishments with more volatile sales, need to adjust labor more frequently and are more constraint by the costs of firing workers. These plants, therefore, employ subcontracted workers in a larger proportion.[3] For others, these developments have only lead to a dramatic reduction in

[2] A recent survey of employment conditions conducted by the Labor Directorate in Chile (ENCLA for its initials in Spanish) indicates subcontracting has widespread as a form of flexible employment in Chile: 25% of the firms use subcontracted for their main activities, while 38% declare to have subcontracted as least one service during 2011. The survey also shows that increasing number of workers is engaged in subcontracted employment relationships, and that the firm where and/or for whom they work no longer directly hires them: in 2011, 3 out 10 workers per firm were subcontracted, while 1 out of 10 was subcontracted for activities regarded as central to the business function.

[3] Micco and Pages (2006); Cingano et al. (2010) and Haltiwanger et al. (2014) find that employment

subcontracted workers' job stability, and an impoverishment in their working conditions. Indeed, permanent employment fluctuations are smoother and less frequent than fluctuations in subcontracted workers. As observed in Figure 1.2, a large proportion of plants that report mild or no changes in permanent employment per year coexists with many more plants adjusting subcontracted employment sharply.[4]

Figure 1.1: Establishments Sales' Volatility and Share of Subcontracting



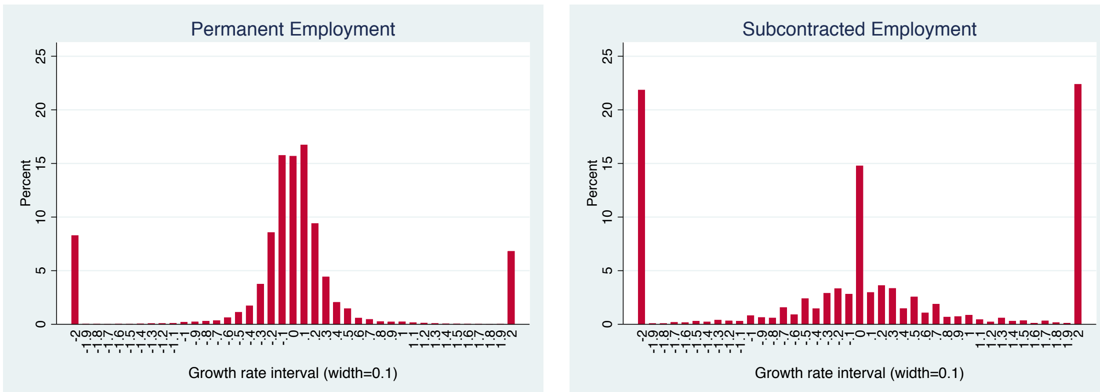
Notes: the figure shows the average share of subcontracted workers in an establishment by decile of sales volatility. Source: ENIA.

The simultaneous use of strong job security provisions for full-time workers with permanent contracts, and lax regulation on subcontracted workers is clearly contradictory, and raises important questions regarding the combined effect of both instruments on employment, and the desirability of such a policy from a normative perspective. In spite of these concerns, very little is known about the impact of severance payments on aggregate outcomes when firms circumvent the regulation using subcontracting as a substitute for

protection regulation is more binding in sectors exposed to higher volatility in demand/supply shocks or, similarly, with larger reallocation rates.

[4]For subcontracted employment, "exits" ($g_{it} = -2$) and "entries" ($g_{it} = +2$) do not necessarily correspond to plants that effectively entered or exit the market as is the case with permanent employment. These are also plants that start using subcontracted workers this period after not having employed any the previous period (entry), and plants that fired all their subcontracted workers after having employed some the previous period (exit). Further, these periods of sharp adjustment are usually followed by long periods of inaction.

Figure 1.2: Distribution of Employment Growth Rate



Notes: the figure represents the fraction of plants expanding (contracting) at different growth rate intervals (as measured in the horizontal axis). Growth rate is computed according to the standard Davis and Haltiwanger (1992) definitions: $g_{it} = (x_{it} - x_{it-1})/(0.5*(x_{it} + x_{it-1}))$, where x_{it} is the number of employees (subcontracted or permanent) in plant i at time t. The bars to the right of the origin correspond to job creation and to the left to job destruction. At the center, the proportion of plants for which employment remains unchanged, and exits (entries) correspond to the left (right) endpoint. Source: ENIA.

hiring workers. The aim of this paper is to contribute to this discussion from a theoretical and empirical perspective.

To address these issues, I set up a general equilibrium model in the tradition of Hopenhayn and Rogerson (1993) with heterogeneous firms and endogenous entry and exit, where firms can hire two types of workers: subcontractors that are totally flexible, and permanent workers that entail tenure dependent firing costs. Since increasing current employment determines firms' future firing costs, the existence of firing costs transforms the firms' problem into a non-trivial intertemporal one. Both types are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. Hence, firms can either hire full-time permanent workers and bear the potential adjustment costs in case of dismissal, or afford to pay a wage premium on subcontracted workers and benefit from the flexibility of terminating their contracts at zero cost. When subcontractors' charges are large enough and all permanent workers are subject to firing costs, the employment protection system studied in this paper reduces exactly to the separation tax regime analyzed by Hopenhayn and Rogerson (1993).

For the estimation, I set the model in partial equilibrium and use the Annual National Manufacturing Survey (ENIA for its initials in Spanish) conducted by the National Institute of Statistics of Chile (INE), which contains detailed information on subcontracted workers for more than 10,000 plants over the span of seven years. Since the model has no closed-form solution I use a simulated method of moments, and optimally choose the parameters to reproduce a set of moments that combine time-series employment dynamics, and cross-sectional industry characteristics. By studying permanent and subcontracted employment dynamics, I am able to measure the costs of adjusting permanent workers, and the wage premium on subcontracted workers firms are willing to pay to substitute for permanent workers. The importance of incorporating subcontracted workers becomes clear when comparing the different estimations performed. Finally, I embed my estimated model in a general equilibrium framework to quantify the costs of the regulation, and the potential benefits of removing it. Also, I measure the gains from subcontracting as a substitute for hiring workers when firms face strict job security regulations.

To anticipate my results, I find that severance payments in the manufacturing sector in Chile are equivalent to seven months' wages, and that workers get tenure after 4 years in the job. Further, firms are willing to pay a wage premium of 10 percent on subcontracted workers to substitute for hiring workers, and be able to buffer the regular workforce from economic fluctuations avoiding workers' firing costs. A naive researcher wanting to estimate firing costs in Chilean manufacturing plants without noticing that firms subcontract to substitute for hiring workers, would conclude that firing costs are substantially lower in the economy (i.e. between one and four months' wages), and that on average workers get tenure after approximately 3 year on the job.

The main finding of the paper is that allowing firms to subcontract workers in a heavily regulated environment increases output, employment and productivity. To overcome the potential costs associated with dismissing permanent workers, firms subcontract as a substitute for hiring workers to buffer the regular workforce from economic fluctuations. This way firms smooth out permanent employment fluctuations at the expense of an increase in subcontracted employment volatility. Provided subcontractors' charges are small relative to adjusting inside workers, subcontracting workers is an attractive alternative for the firms to cover peak demand or productivity shocks. When firms can subcontract they respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity (TFP). In this context, the negative effects of firing costs on aggregate outcomes are less than previously estimated in the literature. If the government decided to eliminate firing costs instead of allowing subcontracting to introduce flexibility to the labor market, the increase in productivity and output of this policy would be even stronger. However, such a policy would eliminate subcontracted workers, being permanent workers the big winners of the change.

Related literature This paper is related to the literature that evaluates the impact of job security provisions on labor markets performance, and productivity. Several models predict that employment protection raises the costs of workforce adjustments, distorting the efficient allocation of labor as firms retain unproductive workers, and divert from hiring workers whose productivity exceeds their market wage, ultimately affecting productivity growth.[5] Another line of research is more empirical, and looks at the impact of job security provisions on aggregate outcomes and employment dynamics. In line with the theoretical literature, there is much of a consensus regarding the impact of firing costs on job flows (both job creation and job destruction decrease) and productivity (also decrease), though the implications for employment are less clear.[6] When targeting em-

[5]See, among others, Bertola (1990), Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993), Bartelsman et al. (2004), Samaniego (2006), and Poschke (2009).

[6]For instance, for studies on the effects on job flows see Micco and Pages (2006), Cingano et al. (2010), and Haltiwanger et al. (2014); on productivity see Autor et al. (2007), Bassanini et al. (2009) and van

ployment protection on a specific group of workers or type of contract, several studies find that the legislation actually induces substitutions across groups or type of contracts.[7]

Few papers have studied the impact of firing costs on aggregate outcomes when firms can use flexible staffing arrangements to substitute away from permanent workers. The majority of the studies available have concentrated on the effect of temporary contracts within a partial equilibrium setting, and usually justify the use of temporary contracts exogenously; either imposing that all the new jobs are temporary, or modeling them as an exemption of the firing costs and forcing firms to open permanent positions.[8] In my paper, I set up a general equilibrium model and endogenously explain the choice between permanent and temporary workers. The approach is consistent with the labor regulation in Chile, in which subcontracted workers entail no costs of dismissal, but subcontracts' charges are higher than the firm's own production costs. In this setting, the choice of permanent and subcontracted workers can be easily understood as the trade-off between firms hiring full-time permanent workers and bearing the potential adjustment costs in case of dismissal, or paying a wage premium on subcontracted workers and gaining flexibility to terminate their contracts at zero cost. Subcontracted workers in this setting are substitutes to permanent workers.

The studies closet in spirit to mine are Alonso-Borrego et al. (2004), Veracierto (2007) and Alvarez and Veracierto (2012); the three studies perform a general equilibrium analysis of severance payments and temporary contracts with search frictions. While the first

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Schaik and van de Klundert (2013); and on the effects on employment see Lazear (1990),Heckman and Pagés (2000),Boeri et al. (2000),Di Tella and MacCulloch (2005) and Ljungqvist (2002) for a theoretical discussion on how the results on employment crucially depend on model assumptions. Several studies evaluate the effect of firing costs exploiting labor reforms as a source of exogenous variations. See, for example, Miles (2000),Autor et al. (2004), and Kugler and Pica (2008).

[7]See, for instance, Acemoglu and Angrist (2001), Fernández-Kranz and Rodriguez Planas (2011), Boeri (2011), Boeri and van Ours (2013), and Pierre and Scarpetta (2013).

[8]For models in partial equilibrium see, for instance, the labor demand model of Bentolila and Saint-Paul (1992), and Aguirregabiria and Alonso-Borrego (2014), the model of job creation and destruction of Cabrales and Hopenhayn (1997), and the matching model of Cahuc and Postel-Vinay (2002). While the relative consensus in this literature is that temporary contracts increase job turnover and employment volatility, the effects on aggregate employment remain in partial equilibrium and are less clear.

evaluates the quantitative effects of the labor regulation in the presence of contractual and reallocation frictions, the last two studies assume complete markets. Alvarez and Veracierto (2012) extends an island model with indirect search to study tenure dependent firing costs. In their framework they can analyze firing taxes as in Hopenhayn and Rogerson (1993) and temporary contracts as special cases. In a similar setting, Veracierto (2007) analyzes the short-term effects of introducing flexibility in the labor market which differ quite substantially from the long-run effects. All the three papers find that labor reforms that introduce temporary contracts increase allow firms to respond more aggressively to economic fluctuations, which enhances the allocation of labor and increases productivity. While they also produce an increase unemployment, the effects on welfare tend to be positive.

The remainder of this paper is organized as follows: Section 1.2 presents the institutional setting of the labor market in Chile, the data and some stylized facts. Section 1.3 describes the model economy, defines the equilibrium concept, and presents the calibration for the fixed parameters. Section 1.4 describes the simulated method of moments for the estimation, and discusses the selection of moments. Section 1.5 shows the estimation results for different specifications of the benchmark model. Section 1.6 presents the results for the policy experiments in the general equilibrium framework, followed by the conclusion in section 1.7. The Appendix outlines the solution algorithm.

1.2 Motivating Evidence

In this section, I first briefly explain the process of reform of the employment protection legislation (EPL) over the past decades, and describe the regulatory framework regarding full-time and subcontracted workers. Then, I present the data used in the analysis, and some stylized facts regarding the dynamics of permanent and subcontracted workers in

Chile.

1.2.1 Institutional background and the origins of a dual labor market

For the past three decades Chile has carried out asymmetric labor market reforms introducing a two-tier system; on the one hand, encouraging job security provisions that greatly penalized employers for firing full-time workers by imposing sizable tenure dependent severance payments and other restrictions to the firing process; on the other, maintaining the market for subcontracted workers practically deregulated.

The current institutional framework dates back to 1980 with the approval of the *Labor Code* by the military junta in the midst of an unprecedented liberalization process that had at its center the labor market. The aim of this new law was to increase labor market flexibility and eliminate labor market distortions, while still providing some minimum degree of job security to the workers.[9] In the early 1990s, with the reestablishment of a democratic government, the tables turned and the efforts to further the liberalization of the labor market unwound. Gradually, the employment protection legislation regarding full-time workers became more restrictive with major reforms occurring during the 1990s and 2000s. Subcontracted workers' lack of influence to lobby policy makers, and probably their still incipient use during this period resulted, instead, in the upholding of lax job security protection for almost three decades.

The labor law regarding full-time workers mandates a minimum period of advance notice in case of impending dismissal, the causes considered as justified reasons for dismissals, and the compensation to hired workers in case of dismissal for unjustified reasons. Firms are required to notify workers in case of impending dismissal with at least one month in advance, and in case of termination for unjustified reasons they are entitled to one monthly

[9] Since their inception, job security provisions were intended to favor permanent workers over subcontracting, part-time, fixed-term, or any other kind of temporary contractual relationship.

wage per year of service with a maximum of eleven months. Until 1980 there was no upper limit to severance payments, and the new law of 1980 established an upper limit of five months; in 1990, this upper limit was raised again from five to eleven month wages. Since then there has been no change in the regulation regarding severance payments in Chile.[10]

Permanent workers have contracts of indefinite duration and cannot be fired without cause even if severance are paid. Causes for just or unjust dismissal were modified in several occasions during this period, in particular, in relation to economic and financial needs being just reasons for dismissal. The Labor Code of 1980 established that economic and financial needs, as well as serious misconduct such as criminal behavior or absenteeism were justified reasons for dismissals. In 1984, firms' economic or financial needs were excluded as justified causes for dismissal, restoring these workers' rights to severance payments. Further, the first democratic government in 1990 reclassified firms' economic and financial needs as just causes, but workers dismissed for these reasons were liable to severance pay. In case of dispute, severance would be paid with a 20 percent surcharge in the amount of the compensation if the firm failed to prove just cause. In 2001, the penalty for firms that fail to prove just cause on court was severely increased; it raised from an equivalent of 20 percent, to a range that goes from 30 to 100 percent surcharge in the amount of the severance.[11]

The extent of these reforms can be appreciated in Figure 1.3 by means of the "job security" index.[12] This index measures in monthly wages the expected cost of dismissing a full-time indefinite worker at the time of hiring, and it utilizes information on compulsory

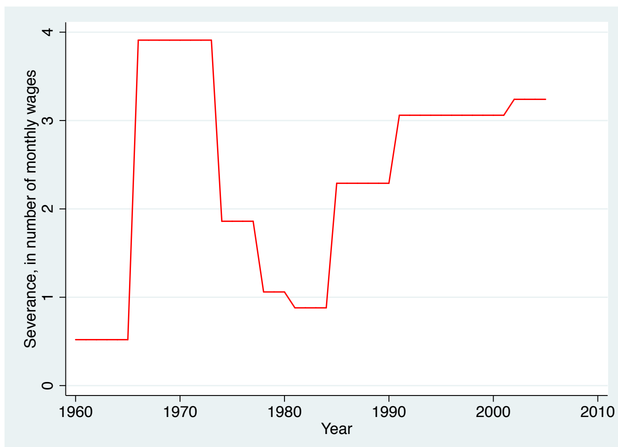
[10]According to Bentolila et al. (2012), severance pay for permanent workers dismissed for economic reasons in France are equivalent to 6 days of wages per year of service plus 4 days if seniority is higher than 10 years. In Spain, severance pay is equivalent to 20 days per year of service for workers dismissed for economic reasons. Unfair dismissals in Spain raise the severance pay to 45 days per year of service.

[11]See Edwards and Edwards (2000) for a complete description of the reforms to the labor market regulation from the early 1970s to the late 1990s. For a description on more recent reforms, see [complete]

[12]This index was constructed by Heckman and Pagés (2000) for 24 countries in OECD and Latin America, and subsequently updated for Chile from 1960-1996 by Montenegro and Pagés (2005) , and from 1996-2005 by Alvarez and Fuentes (2011).

advance notice periods, and compensations for dismissals. Given that the workers have the right to contest dismissals, the index also includes a measure of the likelihood that a firm's dismissal cause is considered unjust in court. After several years of low employment protection (late 1970s and beginning 1980s), job security more than doubled in the mid-1980s to continue trending up during the 1990s and 2000s. According to this index, firing costs in Chile are back to around 3 months' wages.

Figure 1.3: Index of Employment Protection in Chile: 1960-2005



Notes: the "job security" index measures in monthly wages the expected cost of dismissing a full-time indefinite worker at the time the worker is hired. The index was constructed by Heckman and Pagés (2000) for 24 countries in OECD and Latin America, and subsequently updated for Chile from 1960-1996 by Montenegro and Pagés (2005), and from 1996-2005 by Alvarez and Fuentes (2011).

In the meantime, regulation on subcontracted workers was kept isolated from the counter-reform process of the 1990s and 2000s. Towards the late-1970s the use of subcontracted workers was completely liberalized, extending their use to any activity inside the firm, and eliminating all the restrictions preventing firms from subcontracting activities regarded as central to the business function, and periodic machine maintenance (a key component of the production process at that time).[13] In addition, the requirement to provide the same working conditions, salaries and social benefits to permanent and subcontracted workers

[13]See Decree Law No. 2,200 of 1978, and Decree Law No. 2,759 of 1979.

was eliminated. The law also prohibited subcontracted workers to join a union in the user firm with the full-time workers under the rationale that their legal employer is the subcontract firm.[14]

It was not until October 2006 that subcontracted work was regulated in Chile for the first time in large detail (Law No. $20,123$). The subcontracting law (or "anti-subcontracting law" as has been known) changed the existing subsidiary responsibility of the user firm into a joint responsibility shared with the subcontractors. This means that both parties become jointly responsible for compliance with labor obligations, and in the case of injuries and fatalities to their workforce. The responsibility lies with the subcontractor, but the law also placed labor responsibilities on the user firm. On the other hand, the user firm has the right to require certifications of compliance with these obligations by the subcontractor, and may refuse to pay any amounts due in case of non-compliance.[15]

1.2.1.1 Enforcement of the Law

The extent to which the legal regulation is implemented and enforced plays a key role in terms of how it effectively applies to the firms. In this sense, the degree to which countries have the proper institutions to enforce the regulation determines the "true" rigidity of the labor laws. In Chile, the Labor Directorate (DT) has been historically the exclusive government body that enforces all labor, social security, and health and safety laws. This institution has the authority to enforce the labor law through different mechanism, among which the three most important are: policing, which means inspectors can visit workplaces at any time, with or without a preceding charge made by an employee

[14] The only protection that remained from the previous regulation was the subsidiary responsibility of the user firms in relation to their subcontractors' labor practices. This means the user firm is responsible for compliance with these obligations only after it is not possible to sue the subcontract firm. Law No. $16,757$ of 1968 regulated subcontracting before these modifications.

[15] A controversial issue was the fact the law did not limit the possibility of subcontracting tasks that are central to business function, and that it did not reinstate the requirement of providing the same working conditions, salaries and social benefits to permanent and subcontracted workers.

against the employer. They can also inspect physical premises or business records. If inspectors find that an employer has violated a labor law, they can fine the employer, order the suspension of working activities, or even close the workplace; administrative interpretation of the law (*dictamen*), which means they have the faculty to determine the meaning and scope of the labor legislation; and file a complaint in the labor courts in matters which it does not have authority (i.e. "unfair labor practices", violations of "fundamental rights" or "disloyal" actions against employees.)

From an international perspective, Chile has a rather strict enforcement of the regulation. Following the approach in Caballero et al. (2013), I proxy the level of enforcement of the labor regulation using Kaufmann et al. (2010) indicators of rule of law, government efficiency, and control of corruption. I assume countries with an effective government, stronger rule of law, and lower levels of corruption are more likely to have the ability to enforce the existing labor regulation. According to these measures, Chile ranks well above the average of Latin American countries, and about the mean of OECD countries in terms of the three different measures (See Table 1.1).

Table 1.1: Enforcement of the Labor Law: International Comparison

	Government efficiency	Rule of law	Control of corruption
	(rank percentile, 2012)		
Chile	87	88	91
Latin America	58	51	57
OECD	87	87	85
US	90	91	89
Best practice	100	100	100

Notes: the table shows Kaufmann et al. (2010) indicators of rule of law, government efficiency, and control of corruption. The data is reported in percentile rank, ranging from 0 (lowest rank) to 100 (highest rank).

1.2.2 Firm-level Data

The empirical analysis in the paper is performed using a panel of plant-level survey data from the Annual National Industrial Survey (hereinafter referred to by its Spanish acronym, "ENIA") collected by the National Institute of Statistics of Chile (INE). The survey encompasses all manufacturing establishments with at least 10 or more workers, and is updated annually incorporating all those plants that begin operating during the year plus the continuing plants, and excludes plants that stop operating or reduced their hiring below the survey's threshold. Each plant has a unique identification number which allows identification of entry and exit, and the computation of plant-level time-series.

The dataset is available for the period 1996 to 2011, but panel-data information for subcontracted work is only available from 2001 through 2007.[16] Plant-year observations are dropped if permanent employment is either zero or missing. I also excluded the tobacco industry and petroleum refineries from the analysis because they are organized as monopolies, operating with very few plants. This generates a sample of 10,906 plants and 69,938 observations with mean (median) employees of 72 (27). To ensure a reasonable sample size I run the estimation on the full panel, ignoring for now the specific industry to which the plants belong.

For each plant and year, the census collects detailed information on total number of employees, separated by the contractual relationship between the plant and the employee. Employers can hire workers under a permanent or full-time contract, or subcontract to a third party the performance of a certain task or work with their own independent employees. The survey also reports plants' use of subcontracted workers in 6 different occupations: engineer and drafting services, blue-collar production, production assistant (i.e.machine maintenance, storage and transportation services), accounting services,

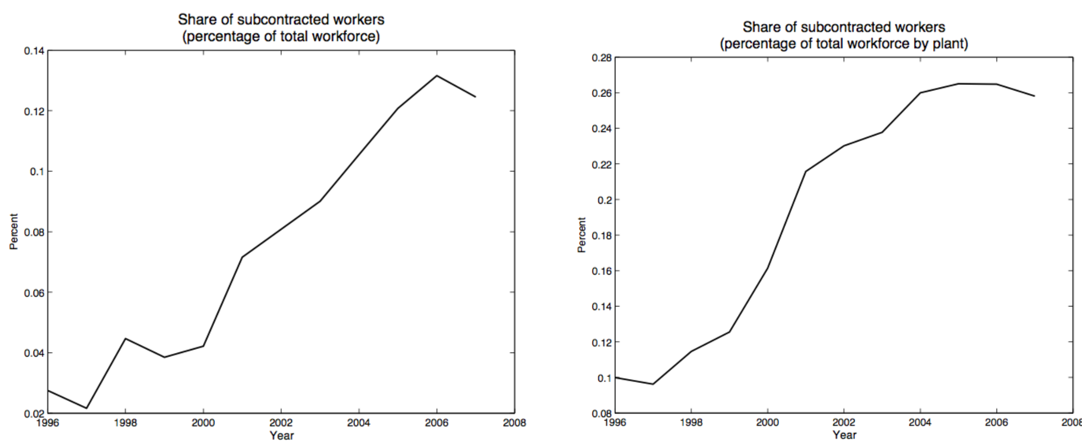
[16] Starting from 2008, the National Institute of Statistics ceased to release the plant unique identification number necessary to match the plants abandoning the panel-structure. Before 2001 the classification to register subcontracted worker was dramatically different so I also drop that information.

blue-collar non-production (i.e. janitorial and secretarial services), and salesperson on commission.

1.2.3 Stylized Facts

The effect of the employment protection gap between permanent and subcontracted workers can be appreciated by looking at Figure 1.4, where I present the evolution of subcontracted workers in Chile. Between 1996 and 2007, plants' use of subcontracted workers skyrocketed in Chile; in 2007, 12 percent of the plants use subcontracted workers (up from 3 percent in 1996), while among the plants that use subcontracted workers, around 3 out 10 workers per plant were subcontracted (up from 1 out of 10 in 1996).

Figure 1.4: Evolution of Subcontracted Work in Chile



Notes: the figures show the percentage of total workforce by year (on the left) and the share of subcontracted workers as a percentage of total workforce by plant (on the right). Source: ENIA.

It is interesting to note that even when this modality of employment initially emerged in routine and low skilled occupations such as janitorial and security services, now it is present in key value-adding functions, such as logistics and accounting services, and high-skilled production-related occupations such as engineer and drafting services. Between

2001 and 2007, plants subcontracted on average 25 workers out of every 100, where blue-collar production workers, and engineer and drafting services were the occupations that gathered the largest number of subcontracted workers (See Table 1.2). During this period, these were also the occupations that experienced the largest increase (see Figure 1.5). This echoes the fact that the form subcontracted work adopted in Chile is not the "specialized" one, where establishments subcontract skills they lack for their business. Instead, plants in Chile subcontract activities regarded as central to the business function, and subcontractors works in the premisses of the main firm, with machinery, inputs and raw materials also provided by the main firms.

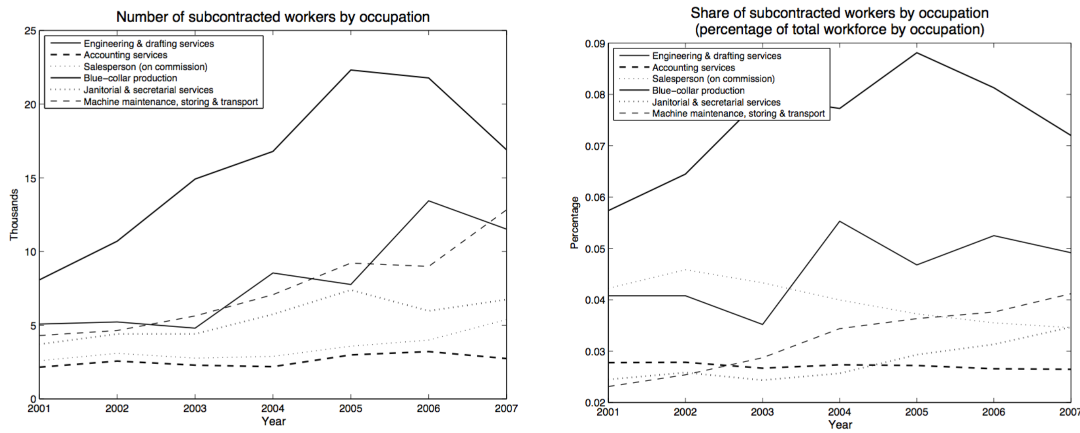
Table 1.2: Share of Subcontracted Workers by Occupation

	Average 2001-2007
Engineering & drafting services	4.6
Accounting services	2.7
Salesperson (on commission)	4.0
Janitorial & secretarial services	2.8
Blue-collar production	7.4
Machine maintenance, storing & transport	3.2
Total	24.7

Notes: the table reports the share of subcontracted workers by occupation as percentage over total workforce by plant. The figures are computed averaging across the N plants and then over the T periods.

In practice, when firms subcontract tasks or services to other firms, the employer of record for the worker performing the task changes, and the responsibility for all employment liabilities is trespassed to the subcontract firm. The client firm becomes the *de facto* employer of the worker, though the subcontract firm remains as its *de jure* employer (i.e. signs the labor contract and agrees on the wage to be paid). The subcontract firm then "lends" the worker to the client firm, which in turn charges a cost for the provided service. In this sense, the client firm gains flexibility to terminate workers' contracts at will, as it can proceed without indicating reasons, nor comply with the minimum period

Figure 1.5: Subcontracted Work by Occupation in Chile



Note: the figures show the total number of subcontracted workers by occupation (on the left) and the share of subcontracted workers as a percentage of the plant's workforce by occupation (on the right). Source: ENIA.

of advance notice or pay firing costs. One of the biggest advantages of these employment arrangements is that subcontracted workers are under the managerial authority of the client firm, but on the payroll of the firms that supplies them.

1.3 Description of the Model

In this section I introduce the model for the estimation which is an industry equilibrium model in the tradition of Hopenhayn and Rogerson (1993) with heterogenous firms and endogenous entry and exit, modified to include tenure dependent firing costs and two types of workers. For the estimation the model is set in partial equilibrium, and in Section 1.6, I embed it in a general equilibrium framework to perform the policy analysis.

I start by briefly motivating the elements in the theory. First, firms produce output using two types of workers: subcontractors that are totally flexible, and permanent workers that entail firing costs that increase with seniority in the job. Both types are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. Firms decide the division of labor input between permanent and subcontracted labor as the optimal response to shocks.

Second, there is ample evidence that employment adjustment at the plant level is characterized by periods of sharp adjustment followed by long periods of inactivity, and the case of Chile is not any different.[17] Thus, in the model I consider non-convex labor adjustment costs, in particular, piecewise linear adjustment costs, which can produce inaction and mimic these facts.

Third, I consider severance payments that increases with seniority in the job as the main characteristic of the employment protection regulation. In Chile, severance payments are equivalent to a month's wage per year of service with a maximum of eleven months. Instead of keeping track of the distribution of workers across tenure levels and increasing the dimension of the problem, I assume permanent workers randomly get tenure, and that only workers with tenure are entitled to severance payments.

Finally, there is a continuum of *ex ante* identical potential entrants, and selection occurs upon entry. Once firms enter the market they receive a random idiosyncratic productivity level, and they operate only if their first productivity draw is above the exit threshold. As the firm's productivity changes, it optimally chooses to grow, contract or exit the market. Since there are no aggregate shocks and the only source of uncertainty in the model is the firms' productivity, the distribution of firms over a size-productivity space is constant, and so all the aggregate variables.

[17] For evidence for the U.S., see Hamermesh (1989), and Caballero et al. (1997). For evidence for other countries, see Varejão and Portugal (2007), complete

1.3.1 Firms and Technology

There is an industry composed of a continuum of firms that produce an homogenous good. Firms behave competitively taking prices in the output and labor markets as given. Each firm operates a decreasing returns to scale, labor-only production function, using both permanent and subcontracted workers:

$$y_t = f(n_t, s_t, z_t) = z_t(n_t + s_t)^{\alpha} \tag{1.1}$$

where n_t are the workers with a permanent contract, s_t the workers with subcontracts, $\alpha \in (0,1)$, and z_t is the exogenous productivity that takes values in the finite set $Z \equiv \{\underline{z}, ..., \overline{z}\}$. The process for z_t follows a First Order Markov Process with transition matrix $\Pi(z, z')$ and is i.i.d. across firms. This implies there is no uncertainty at the aggregate level.[18]

The two types of workers are perfect substitutes in production, but they differ in their wages and firing costs:

i) Permanent workers are those with contracts of indefinite duration, and entail severance pay in case of dismissal. Permanent workers earn wage w. To avoid increasing the dimension of the problem and keeping track of the distribution of workers across tenure levels, I assume permanent workers have $(1-\lambda)$ probability of getting tenure, and only workers with tenure receive severance payments in case of dismissal. Workers with a permanent contract fired before tenure do not accrue severance pay.

   Thus, workers with a permanent contract evolve:

$$n_t = l_{t-1} + o_t \tag{1.2}$$

[18] These disturbances could also reflect shocks on the demand side, where firm produce differentiated goods and the distribution of consumer tastes across this differentiated goods is stochastic over time. See Hopenhayn and Rogerson (1993) for a more detailed description of this alternative structure.

where l_{t-1} is the number of permanent workers with tenure employed last period, and o_t is the number of workers hired or fired in t. The law of motion for permanent workers with tenure is:

$$l_t = \begin{cases} l_{t-1} + (1-\lambda)o_t, & \text{if } o_t > 0 \\ l_{t-1} + o_t, & \text{if } o_t \leq 0. \end{cases} \quad (1.3)$$

Since the optimal decision of current employment depends on the number of permanent workers last period, l_{t-1} is a state variable for the firm.

Firing costs on permanent workers with tenure take a form similar to the work of Hopenhayn and Rogerson (1993):

$$g(l_t, l_{t-1}) = \max\{0, \tau(l_{t-1} - l_t)\} \quad (1.4)$$

where τ is the fixed payment for every permanent worker laid-off. In principle, labor adjustment costs can consider the search, recruiting and training cost of hiring workers, but since the interest falls on the effect of severance payments I choose to ignore hiring costs for now. This specification for labor adjustment costs imply the marginal cost of changing employment is constant; hence, when the gains to changing the number of workers is small firms optimally choose not to adjust–marginal costs of adjustment do not go to zero as the size of the adjustment goes to zero, and there is no reason for the firms to smooth adjustment. In this setting, firms' labor adjustments are characterized by episodes of sharp adjustment followed by periods of optimal inactivity.

ii) Subcontracted workers are those with temporary contracts subject to no costs for laying them off. In turn, they are relatively more expensive than permanent workers as subcontractors' charges are higher than the firm's own production costs. Firms can employ subcontracted workers for occasional or seasonal purposes, or jobs for absent, as well as jobs for carrying out a specific task or service for a determined period of time related to the production process. The subcontract firm legally employs the worker (signs the contract and pays the wage w), which in turn works on the premises of the user firm who pays a *fee* per worker to the subcontract firm.[19] Hence, subcontracted workers earn $w_s = w(1+f)$, where f is the fee or wage premium on subcontracted labor. Provided the cost of subcontracting workers is small relative to the cost of adjusting in-house workers $[1-\tau(1-\lambda)]$, contracting out is an attractive alternative for the firms to cover peak demand or productivity shocks.[20]

The operative profits of an active plant are given by

$$py_t - wn_t - w_s s_t - pc_f - g(l_t, l_{t-1}) \tag{1.5}$$

The timing of the model for incumbents is as follows:

1. Enter period t with last period's shock z_{t-1} and permanent workers with tenure l_{t-1}

2. Decide whether to exit. If the firm exits, pays the adjustment costs $g(0, l_{t-1})$ for firing all workers from last period, and receives zero profits in all future periods avoiding to pay c_f.[21]

3. If the firm stays, it pays $p_t c_f$ and receives this period's shock, z_t

4. Firm chooses labor demand and the number of workers to hire under each type of

[19]Subcontracted workers may be restricted by law or mutual agreements between firms and unions, so that firms are obliged to hire a certain amount of employees on a permanent basis. For example, it could be assumed that the ratio between permanent and subcontracted workers can never fall below a minimum threshold $\bar{\psi}$. To remain faithful to the regulatory framework in Chile for the period under study, I assume no restrictions on subcontracted labor and no hiring cost.

[20]The premium over subcontracted workers could also be justified on the basis of a compensation subcontracted workers demand to work on the firm considering their higher expected probability of losing the job.

[21]Fixed operating costs make the exit decision meaningful; plants exit to avoid paying the fixed cost instead of simply waiting for a better realization of z and bearing an output of zero.

contract.

The timing for a potential entrant:

1. Pay the one-time entry cost $p_t c_e$ and then draw a productivity level z_t from $\nu(z_0)$ (which is independent across firms)

2. Decide whether to stay in the industry. If the first productivity draw is above the exit threshold the firm stays and produces as in 4 above.

1.3.2 Static Subproblem of the Firm

For any plant with $z \in Z$ the optimal level of subcontracting solves the following static problem:

$$\begin{aligned} P(n, s, z) &= \max_{s} \{pz(n+s)^{\alpha} - n - w_s s - pc_f\} \\ st &: s \geq 0 \end{aligned} \tag{1.6}$$

Note that the wage rate for permanent employees has been normalized $w = 1$, hence does not appear explicitly in the expression.

The solution implies that the optimal subcontracted labor choice is:

$$s(n, z) = \begin{cases} \left(\frac{\alpha p z}{w_s}\right)^{\frac{1}{1-\alpha}} - n, & \text{if } \alpha p z n^{\alpha-1} > w_s \\ 0, & \text{if } \alpha p z n^{\alpha-1} < w_s \end{cases} \tag{1.7}$$

Then, evaluating the profit function $P(n, s, z)$ at the optimal subcontracted labor decision

$s(n,z)$, the operating profit of the plant $R(n,z)$ is:

$$R(n,z) \equiv P(n,s(n,z),z) = \begin{cases} \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha p z}{w_s^{\alpha}}\right)^{\frac{1}{1-\alpha}} + n(w_s-1) - pc_f, & \text{if } \alpha p z n^{\alpha-1} > w_s \\ pzn^{\alpha} - n - pc_f, & \text{if } \alpha p z n^{\alpha-1} < w_s. \end{cases} \tag{1.8}$$

1.3.3 Dynamic Optimization

Given that all uncertainty is idiosyncratic, I study a stationary equilibrium where $p_t = p$. In this equilibrium, firm undergo change over time, with some of them growing or contracting, even exiting the market and others starting up. Since there are no aggregate shocks, despite all these changes the distribution of firms over a size-productivity space is constant, and so all the aggregate variables.

1.3.3.1 Incumbent Firms

The dynamic programming problem of an incumbent plant that employed l_{t-1} permanent workers last period, decided to remain in the industry this current period, and received the new value for its shock z_t is described by the Bellman equation:

$$V(l_{t-1},z_t;p) = \max_{n}\left\{R(n_t,z_t;p) - g(l_t,l_{t-1}) + \beta \max[E_{z_{t+1}}V(l_t,z_{t+1};p), -g(0,l_t)]\right\}, \tag{1.9}$$

subject to equation (1.2) and (1.3), and labor adjustment costs as defined in equation (1.4).

$E_{z_{t+1}}$ denotes the expectation of z_{t+1} conditional on the current value of productivity z_t, and β is the discount factor. The value $V(l_{t-1},z_t;p)$ is the expected discounted stream of profits from operating a plant with productivity z_t and previous employment level l_{t-1}. Given that the firm does not receive any new information between the current decision point and the time of the exit decision at the beginning of next period, it chooses now whether to exit tomorrow. Conditional on this period's employment decision, the firm stays if the exit cost, $-g(0, l_t)$, is larger than the expected value of staying, $E_{z_{t+1}} V(l_t, z_{t+1}; p)$.

In this framework, there are two decisions of an incumbent firm: i) optimal composition of total employment $n_t = L(l_{t-1}, z_t; p)$, and $s_t = S(n_t, z_t; p)$, and ii) optimal exit decision next period $x_{t+1} = X(l_t, z_t; p) \in \{0, 1\}$ with convention that $X = 1$ corresponds to exit and $X = 0$ to stay.

1.3.3.2 Entry Decision

The decision whether to open a plant is also dynamic. It is profitable to open a new plant if:

$$V^e(p) = \int V(0, z; p) d\nu(z) \leq pc_e, \tag{1.10}$$

where the value of of operating a new plant with productivity z_t and no previous employment, $l_{t-1} = 0$, is:

$$V(0, z_t; p) = \max_n \left\{ R(n_t, z_t; p) + \beta \max[E_{z_{t+1}} V(l_t, z_{t+1}; p), -g(0, l_t)] \right\}. \tag{1.11}$$

subject to equation (1.3), (1.2) and labor adjustment costs as in equation (1.4).

That is, new plants are open as long as the discounted expected profits from operating a new plant are enough to cover the entry costs. In equilibrium with positive entry, the entry of new plants induces changes in the output price and the firm value until there are no gains from entering this industry, and the constraint is satisfied with equality.

1.3.4 Stationary Distribution

In this model the state of an individual firm is fully described by (z, l), and the state of the industry in turn is described by the distribution over the state variables for all firms. Let the incumbent firms at the beginning of the period be summarized by the measure $\mu(z, l)$ (after they have made their exit/stay decision and new realizations of z have arrived), and the mass of firms that enter be equal to M.

The law of motion for the distribution of firms is given by

$$\mu'(z,l) = \int_{z'} \int_{z} [1 - X(l, z; p)] F(z'/z) d\mu(z, l) + \int_{z'} M' d\nu(z) \tag{1.12}$$

A stationary equilibrium is such that this distribution reproduces itself, i.e. $\mu' = \mu$.

The equilibrium distribution of productivity and permanent employment is determined by the productivity of entrants, the stochastic process of productivity, the extent of selection, and the number of entrants. Once the distribution of the state variables has been determined it is possible to compute all aggregate variables.

Total supply in the industry is:

$$Q^s(\mu, M; p) = \int_{z^*} f(L(l, z; p), S(n, z; p), z) d\mu(z, l) + M \int_{z^*} f(L(0, z; p), S(n, z; p), z) d\nu(z). \tag{1.13}$$

Aggregate demand for this industry follows a standard representation: $Q^d = D(P)$

1.3.5 Definition of Equilibrium

A ***stationary industry equilibrium*** with positive entry and exit is a set of value functions and decision rules, a price p^*, a stationary distribution of firms μ^*, and a mass of entrants M^* such that:

1. Given prices, the value functions of the firms and the policy functions are consistent with firms optimization

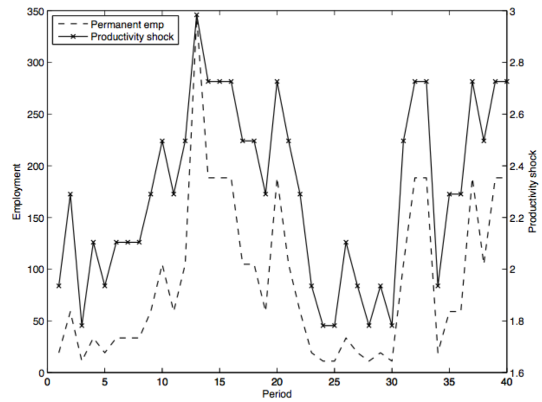
2. Markets clear: $p^* = D(Q^*)$ and $Q^* = Q^s(\mu^*, p^*, w^*)$

3. There is an invariant distribution over firms: $\mu^* = T(\mu^*, M^*; p^*)$

4. The free entry condition is satisfied: $V^e(p^*) = p^* c_e$

Before moving to the estimation of the model I discuss some properties of the policy function for labor implied by the model. Starting with the model without firing costs ($\tau = 0$), subcontracted workers are meaningless in this setting as they are more expensive than permanent workers, but provide no advantage in terms of firing costs. Hence, firms choose permanent workers so that their marginal product equates the wage: $l_t = (\alpha p z_t / w)^{1/(1-\alpha)}$. To illustrate the firm optimal behavior, Figure 1.6 simulates the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock. It is clear that when productivity this period is high, firms hire permanent workers, while if productivity is low they dismiss workers; current employment is determined entirely by the current value of the productivity shock.

Figure 1.6: Optimal Labor Decision: Model Without Firing Costs



Notes: the figure shows the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers do not entail firing costs. The parameters are given in Table 1.6 (Panel B, row 1, model with quick tenure) for $\tau = 0$.

When the government introduces a positive firing cost, and no subcontracting is allowed yet, current employment also depends on last period's employment. In this setting, the optimal employment decision for permanent workers with tenure $l_t(z_t, l_{t-1})$ follows:

$$\begin{aligned} l_t(z_t, l_{t-1}) &= l_{t-1} \text{ if } l_{t-1} \in \left[\underline{l}(z_t), \bar{l}(z_t)\right] \\ l_t(z_t, l_{t-1}) &= \underline{l}(z_t) \text{ if } l_{t-1} < \underline{l}(z_t) \\ l_t(z_t, l_{t-1}) &= \bar{l}(z_t) \text{ if } l_{t-1} > \bar{l}(z_t), \end{aligned} \tag{1.14}$$

where $\underline{l}(z_t)$ and $\bar{l}(z_t)$ are obtained from the first-order conditions of equation (1.9). Intuitively, $\underline{l}(z_t)$ is the largest amount of permanent workers a firm with productivity z_t wants to hire if it does not have to pay firing costs this period (i.e. is a firm that is expanding), and $\bar{l}(z_t)$ is the smallest amount of workers the firm hires if it has to pay firing costs this period (i.e. is a firm that is shrinking). For a firm with $l_{t-1} \in \left[\underline{l}(z_t), \bar{l}(z_t)\right]$, the gains from changing the number of workers is too small so they optimally choose not to adjust.

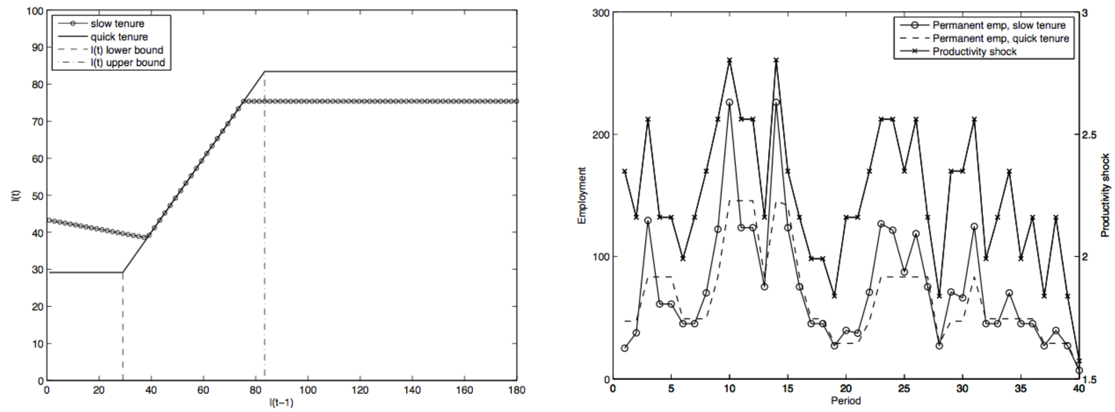
Figure 1.7 (left panel) illustrates this (s, S) type of rule for all workers with a permanent contract with quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$).[22] All firms with employment last period below "l(t) lower bound" hire workers up to this lower bound, while all firms with employment levels above the "l(t) upper bound" reduce their employment levels down to this upper bound. Note also that the band is narrower when $\lambda > 0$; this is, when the firm hires permanent workers knowing that with probability $(1-\lambda)$ they will actually get tenure.[23] The same figure, on the right, simulates the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock with quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$). Consistent with the policy function, firms hire permanent workers only if the productivity shock is large enough, and we observe periods of sharp adjustment followed by long periods of inactivity. When $\lambda > 0$, the fact that not all workers get tenure gives the firm some flexibility to adjust employment to changes in productivity more often. Employment becomes more volatile in this case, and firms can use resources more efficiently.

In the model economy with firing costs and subcontracted workers, firms use subcontracted workers to buffer the stock of permanent workers, and avoid their potential costs of dismissal during periods of lower productivity. When the firm receives a positive shock, it responds by increasing the number of subcontracted workers. Only if the shock is large enough, the firms increases their hiring of permanent workers. In the case of a negative productivity shock, the firms start by firing as many subcontracted workers as possible, and when it rans out of subcontracted workers, starts firing permanent workers and bearing their dismissal costs (see Figure 1.8, right panel). Consistent with this dynamic, Figure 1.8 (left panel) illustrates the policy function for a firm subject to firing costs (quick tenure) and with the possibility to subcontract. When firms can subcontract, the

[22] In the case when $\lambda = 0$, $n_t = l_t$ as there are no workers with permanent contracts that do not entail firing costs.

[23] The lower portion of the decision rule is downward slopping because smaller firms need to hire proportionally more permanent workers today to reach the "l(t) lower bound". Recall that when $\lambda > 0$, $n_t \neq l_t$.

Figure 1.7: Optimal Labor Decision: Model With Firing Costs and No Subcontracting



Notes: the figures illustrate the policy function for all workers with a permanent contract (on the left), and the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers entail firing costs (on the right). Two cases are plotted: quick tenure ($\lambda = 0$) and slow tenure ($\lambda > 0$). The parameters are given in Table 1.6 (Panel B, row 3, model with slow tenure).

"inaction band" narrows with respect to the case without subcontracting (compare the solid line labeled Total with the dashed line labeled Permanent no subcontracting) coming closer to reach the optimal level of employment without distortions. Hence, the extent to which resources are not allocated efficiently decreases. Also, the increase in employment up to the "lower bound", is attained by a combined increase of subcontracted and permanent workers. As explained before, firms begin subcontracting workers, and only if the productivity shock is large enough they increase their hiring of permanent workers.

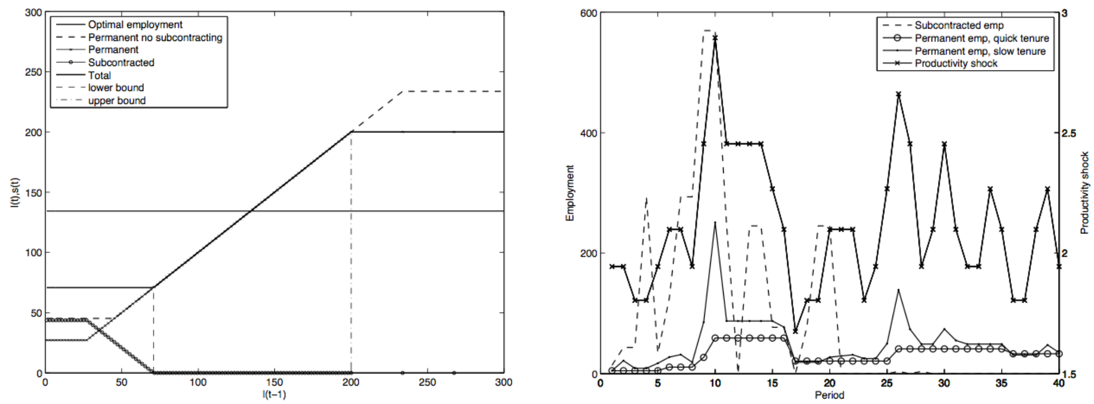
1.3.6 Solution Method

The model has no closed-form solution hence it is solved numerically. In appendix A.1 I present a detailed characterization of the computation method used to solve the model.

The model period is one year. I assume firm's idiosyncratic shocks follow an AR(1) process of the form:

$$\log z_t = \mu + \rho \log z_{t-1} + \varepsilon_t \tag{1.15}$$

Figure 1.8: Optimal Labor Decision: Model With Firing Costs and Subcontracting



Notes: the figures illustrate the policy function for all workers with a permanent contract and quick tenure (on the left), and the optimal labor decision of a single plant for 40 years for an arbitrary productivity shock when permanent workers entail firing costs and plants can subcontract (on the right). The parameters are given in Table 1.5 (Panel B, row 3, model with slow tenure).

where μ is a constant, ρ the persistence of the shocks, and ε_t is a random variable with standard normal distribution. I approximate the distribution of the idiosyncratic shocks using the quadrature-based method developed in Rouwenhorst (1995), which has been shown to be more reliable in approximating highly persistent processes, and choose the number of grid points $g_z = 30$. The initial distribution $\nu(z_0)$ is chosen to be the stationary distribution of the z process which matches well the size distribution of the firms age 0-1 years in the data.

Industry demand is given by a decreasing function. For simplicity, take the following iso-elastic functional form:

$$p = Q^{-\frac{1}{\eta}}, \tag{1.16}$$

where p is output price, Q is the industry output, and $\eta > 0$ is the price elasticity of demand elasticity.

To discretize the state space for permanent employment I assign a log-linear grid with size $g_n = 300$. Because permanent employment n is an endogenous variable, I have to be careful that the choice of the number of points in the grid does not affect the results. Sensitivity analysis indicates the choice was adequate.

1.4 Estimation Method

In this section I propose a simple technique for the estimation of the model based on simulation, and the selection of moments that summarize key features of the data.

1.4.1 Simulated Method of Moments

Since the model has no analytical closed form solution I use an estimation technique based on simulation to estimate the parameters of the model. Specifically, the estimation of the parameters is achieved by simulated method of moments (SMM) (McFadden, 1989; Pakes and Pollard, 1989; Duffie and Singleton, 1993), which minimizes the distance between key moments from actual data and model-generated moments.

The full set of parameters necessary to compute the model is the vector:

$$\theta = \{\beta, \alpha, c_f, c_e, \rho, \mu, \sigma_\varepsilon, \tau, f, \lambda, \eta\} \tag{1.17}$$

where β is the discount rate, α the curvature of the production function, c_f is the fixed operating costs, c_e is the entry cost, ρ, μ, and σ_ε are the parameters that define the idiosyncratic shock, τ is the fixed cost the firm must pay for each permanent job destroyed, f is the wage premium on subcontracted workers, λ is the probability that a permanent workers gets tenure, and η is the price elasticity of demand. From the full set of parameters, 7 are estimated, and the remaining 3 are predefined.

To perform the SMM estimation a set of statistics of interest Ψ^A is selected from the actual data for the model to match. For an arbitrary value of θ, the solution to the model is used to generate S simulated data sets of size (N, T), where N is the number of firms and T is the number of periods.[24] The simulated moments $\Psi^S(\theta)$ are computed on each data set and then averaged out to compute the minimizing criterion function: $\Gamma(\theta) = [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]'\ W\ [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]$. I use the same random draw for the productivity shock throughout each simulation.

The parameter estimate $\hat{\theta}$ is obtained by searching over the parameter space to minimize the (weighted) distance between the moments implied by the model and those computed from the data:

$$\hat{\theta} = \arg\min_{\theta\in\Theta}[\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)]'\ W\ [\Psi^A - \frac{1}{S}\sum_{s=1}^{S}\Psi^S(\theta)], \tag{1.18}$$

where W is a weighting matrix and Θ the estimated parameters space. $\hat{\theta}$ is consistent for any positive-definite weighting matrix (e.g. identity matrix) but the smallest asymptotic variance is obtained when the weighting matrix equals the inverse of the covariance matrix of the data moments, V. In this case, I use $W = \text{diag}(V^{-1})$ (diagonal elements equal to those of V and off-diagonal elements equal zero) because it has better small sample properties (see Altonji and Segal (1996)). V is calculated by bootstrap with replacement on the actual data.[25] To minimize the function I use Nelder-Mead simplex algorithm starting from $1,000$ different initial guesses to ensure the solution converges to the global minima.

To generate the standard errors of the parameter point estimates, I compute the numerical derivatives of the simulated moments with respect to the parameters and using the

[24] I set N=5,000 and T=200 which implies the number of firms in the simulation is approximately 10 times larger than in the data. I discard the first 50 periods of simulated data to start from the stationary distribution.

[25] To preserve the original time-series structure of the data to conduct inference I resample firm's complete time-series.

standard SMM formula compute the asymptotic variance:[26]

$$SE(\hat{\theta}) = \left[(J'WJ)^{-1}\right]^{1/2}, \tag{1.19}$$

where $J = E(\partial\Psi^S(\theta)/\partial\theta)$ of dimension p (#moments) $\times$ q (#parameters). Given the underlying discontinuities of the value functions, I follow the methodology in Bloom (2009) to compute the numerical derivatives. I calculate the numerical derivative as $f'(x) = \frac{f(x+\varepsilon)-f(x)}{\varepsilon}$ for an ε of $\pm 5\%$, $\pm 2.5\%$, and $\pm 1\%$ of the midpoint of the parameter space. Then, I simply compute the median value of these derivatives.

1.4.2 Predefined Paramaters

The predefined parameters are shown in Table 1.3. Parameter β is set to be equal to 0.965, which is equivalent to annual real interest rate over the period of study of 3.62%. Because the curvature of the production function is difficult to identify in the data, I also set its value a priori. α not only captures the labor share in the total revenue, but also decreasing returns to scale and the elasticity of demand of firms' output. If capital is flexible, the elasticity of demand is infinite, and there is constant return to scale, then α should equal one. Relaxing any of these assumptions leads to an $\alpha < 1$ (See Roys and Gourio (2013)). I choose α equal to 0.85 so that for $\eta = 4$ the labor share is consistent with previous estimations for Chile.[27] The value of c_e is chosen so that the free-entry condition (1.10) holds under $p = 1$, and the wage rate of permanent workers is normalized to 1.

[26] See Gouriéroux and Monfort (1997).

[27] Estimations for the labor share parameter in Chile range from $0.53 - 0.6$. These estimates are somehow lower than those for the US economy because of a larger participation of natural resources in the GDP, and a low stock of human capital.

Table 1.3: Predefined Parameters in the Model

Parameter	Description	Value
β	Discount rate	0.965
α	Curvature production function	0.85
η	Price elasticity of industry demand	4

1.4.3 Selection of Moments

The choice of moments is guided by their "informativeness" regarding the underlying structural parameters to be estimated. In particular, the exact choice of moments is directed by a combination of cross-sectional industry characteristics and time-series employment dynamics. Heuristically, a moment is informative about a certain parameter if that moment varies when the parameter varies. Table 1.4 shows the elasticities of model moments with respect to the model parameters.

To pin down the fixed operating costs parameter I attempt to match the exit rate, the average firm size, and the firm size and employment distribution. An increase in fixed operating costs c_f increases the minimum level of productivity needed for incumbents firms to survive. This, in turn, intensifies market selection, and decreases entry barriers, resulting in a distribution of surviving firms with a larger proportion of high productivity establishments (see column (1) in Table 1.4). These same moments are also informative about the mean μ, persistence ρ and volatility σ_ε of the productivity process. An increase in μ or the volatility σ_ε, increase the exit rate, and decrease the average mean size of firms shifting the size distribution towards more small firms. Instead, the persistence parameter ρ increase the average size of firms and decreases the exit rate, shifting the size distribution towards more large plants (see columns (3), (4) and (5) in Table 1.4).

To study employment dynamics I use a modified definition of employment growth following Davis and Haltiwanger (1992): $g_{it} = (x_{it} - x_{it-1})/(0.5 * (x_{it} + x_{it-1}))$, where x_{it} is the number of employees (subcontracted or permanent) in plant i at time t. This growth measure is symmetric about zero, and lies in the close interval $[-2, 2]$ with deaths (births) corresponding to the left (right) endpoints. The conventional growth rate measure (change in employment divided by lagged employment) does not allow for an integrated treatment of "exits" and "entries". However, a significant fraction of the adjustments in subcontracted employment corresponds to these cases so this information cannot be ignored; this is, plants that hire subcontracted workers this period after not having employed them the previous period ("entry"), and plants that cease to subcontract today after having hired subcontracted workers the previous period ("exit"), even when they still remain in operation. For consistency, growth in both types of employment is computed using this measure.

A key feature of the employment data is that permanent employment fluctuations are smoother and less frequent than fluctuations in subcontracted workers. It is transparent that the distribution of permanent employment growth rates is more peaked and with heavier tails, implying that there is a higher proportion of extreme events (even when sharp adjustments are still rare). Instead, the distribution of subcontracted employment growth rates indicates more smooth and persistent adjustment. Further, the permanent employment growth distribution has a considerable amount of mass around 0 (see Figure 1.2 in Section 1.1). I select moments that describe these features of the distribution of both permanent and subcontracted growth rates; this is, volatility and kurtosis of the distribution, and inaction rate of permanent employment.

To pin down λ, τ and f I attempt to match the volatility and kurtosis of permanent and subcontracted employment growth, and the inaction rate of permanent employment growth. When τ increases, firms use more subcontracted workers as they rely more on these workers to buffer permanent employment from economic fluctuations. As a consequence, the volatility of permanent employment decreases, the inaction rate of employment growth increases, and the kurtosis increases (see column (7) in Table 1.4). In turn, when λ decreases (the probability of getting tenure for permanent workers increase), firms have to rely more on permanent workers increasing (decreasing) the volatility (kurtosis) of permanent workers growth rate (see column (2) in Table 1.4). Similarly, when the premium on subcontracted work f increases the volatility of subcontracted workers increases as firms use subcontract workers more infrequently (see column (6) in Table 1.4). The variance of permanent employment growth rate is informative about the mean, persistence and volatility of the productivity process.

Lastly, to complete the selection of moments I choose to match the proportion of subcontracted workers over the firm workforce as this is informative about the fixed lay-off cost τ (i.e. higher firing costs more subcontracting by the firms), the premium over subcontracted workers f (i.e. higher the premium less subcontracting), and λ (i.e. an decrease in the probability of getting tenure, decreases the adjustment costs of permanent employment, and the advantage of using subcontracted workers). Note also that the share of subcontracting is informative about the persistence (i.e. more persistent the risk decreases and firms use less subcontracted workers), and the volatility of the productivity process (i.e. an increase in the volatility increases the risk and firms rely more on subcontracted workers).

1.5 Empirical Results

In this section I present the estimates from the simulated method of moment. In Table 1.5, the column labeled Data reports the actual moments from ENIA, and next to it the associated standard errors. These show that permanent employment fluctuations are smoother and less frequent than fluctuations in subcontracted workers (the volatility of employment growth rate is more than two times for subcontracted work than for perma-

Table 1.4: Sensitivity of Model Moments to Parameters

Moments	(1) c_f	(2) λ	(3) ρ	(4) μ	(5) σ_ε	(6) f	(7) τ
Average firm size	1.094	0.005	1.963	-0.201	-0.388	-0.207	-0.045
Exit rate	1.502	0.005	-7.903	0.736	1.550	0.003	-0.006
Fraction of plants in each bin:							
10-19 emp.	-1.225	-0.004	-2.240	0.073	0.253	-0.160	0.017
20-99 emp.	0.903	0.004	2.165	-0.055	-0.229	0.131	-0.006
100-499 emp.	1.219	-0.001	1.113	-0.029	-0.140	0.002	-0.004
500+ emp.	1.331	-0.001	0.933	-0.084	-0.166	-0.005	-0.001
Share of employment in each bin:							
10-19 emp.	-1.944	-0.033	-1.125	0.077	0.110	0.085	0.041
20-99 emp.	-0.127	0.029	1.276	-0.037	-0.218	0.216	-0.106
100-499 emp.	0.151	-0.008	-0.329	0.086	0.136	-0.064	0.032
500+ emp.	0.296	-0.008	-0.149	-0.001	0.051	-0.072	0.056
Volatility g_l	0.838	0.419	-4.293	0.378	0.842	0.056	-0.054
Volatility g_s	-0.456	0.117	-0.808	0.038	0.335	0.349	-0.330
Kurtosis g_l	-0.952	-0.241	5.196	-0.476	-1.095	-0.200	0.224
Kurtosis g_s	0.182	-0.057	1.617	-0.296	-0.553	-0.148	0.140
Share of subcontracting	0.406	-0.120	-4.916	0.304	0.796	-0.336	0.365
Inaction rate g_l	-0.302	0.000	1.648	-0.348	-0.464	-0.086	0.155

Notes: this table presents elasticities of model moments with respect to the model parameters. To calculate the elasticities the numerical derivatives of the model moments with respect to the parameters are multiplied by the ratio of the baseline parameters to the baseline moments. The numerical derivative is the median value of the numerical derivatives $f'(x) = (f(x+\varepsilon) - f(x))/\varepsilon$ for an ε of $\pm5\%$, $\pm2.5\%$, and $\pm1\%$ of the midpoint of the parameter space.

nent work). Similarly, the higher kurtosis of the distribution of permanent employment growth rates indicates there is a higher proportion of extreme events, alongside long periods of no adjustments (the share of plants not changing permanent employment in a year is around 18%). Instead, the lower kurtosis of the distribution of subcontracted employment growth rates indicates more smooth and persistent adjustments.[28]

The column labeled Slow Tenure in Table 1.5 presents the moments from the full model ('benchmark model') as presented in Section 1.3 evaluated at the estimated parameters.

[28]Even when it seems that the distribution of subcontracted employment growth rates would have the most kurtosis (it appears to have all of its mass in its tails as seen in Figure 1.2 in Section 1.1), being its variance a lot larger in fact it only has few mass in its tails. Instead, even when the distribution of permanent employment growth rates seems to have fewer mass in its tails, its kurtosis is larger because those events are much farther away from the mean.

The model fits the data quite well with the exception of the kurtosis of both permanent and subcontracted employment distribution, and the inaction rate for g_l. The fact that the model cannot match these facts suggests the need to incorporate some restriction on the degree of substitutability between both types of labor, or some fixed cost to the use of subcontracted workers.[29] Given that both types of labor are perfect substitutes in production, firms rely more on subcontracted workers, and adjustments of permanent employment are neither as frequent not as sharp as in the data. The fact that the volatility of subcontracted employment growth given by g_s fits well the data is also related to the fact that the model fits relatively high firing costs. In terms of fitting industry characteristics such as firm and employment distribution, the yearly exit rate and the average firm size the model performs well.

In Table 1.5 I also display the results for the model restricted to $\lambda = 0$; this is, to the case permanent workers gets tenure immediately after they are hired. As shown by the increase in the criterion function (from 1,342.5 to 5,265.9), in comparison to the full model the fit is worst. The reduction in fit is due both to the worst fit of firms and employment dynamics, suggesting that ignoring the tenure-dependency of firing costs is problematic. Given the cost of subcontracted workers, and the proportion in which the plants use subcontracted workers, for the model to fit the low inaction rate for permanent workers it requires a rather low τ. In the benchmark model, much of the flexibility in employment adjustment is coming from the fact that only a fraction of workers get tenure, and not only from subcontracting. The low level of firing costs, in turn, produces an excessive volatility of g_l, and an even lower kurtosis of the distribution of permanent employment growth rates.

[29] A natural extension of the model would be to assume firms have a CES production functions such that: $y = z(an^{\gamma} + (1-a)s^{\gamma})^{\frac{\alpha}{\gamma}}$, where γ is the degree of substitutability of the two types of labor, $\alpha < 1$ returns to scale parameters, a share parameter, and z is firm's productivity. Similarly, we could assume plants need some level of sophistication or installed capacity to subdivide tasks and be able to subcontract.

Table 1.5: Simulated Moments Estimations for the Full Model

Panel A: Moments

Moments	Data	S.E.	Simulated Moments Slow Tenure ($\lambda > 0$)	Simulated Moments Quick Tenure ($\lambda = 0$)
Average firm size	71.95	1.8782	71.53	64.79
Exit rate	0.091	0.0012	0.098	0.135
Fraction of plants in each bin:				
10-19 employees	0.386	0.0049	0.398	0.457
20-99 employees	0.447	0.0049	0.436	0.407
100-499 employees	0.145	0.0038	0.148	0.121
More than 500 employees	0.022	0.0016	0.018	0.016
Share of employment in each bin:				
10-19 employees	0.064	0.0021	0.062	0.084
20-99 employees	0.260	0.0081	0.264	0.296
100-499 employees	0.417	0.0118	0.398	0.371
More than 500 employees	0.260	0.0173	0.275	0.249
Volatility g_l	0.688	0.0160	0.781	0.818
Volatility g_s	2.161	0.0618	2.118	2.519
Kurtosis g_l	5.144	0.0606	3.141	2.689
Kurtosis g_s	1.973	0.0273	1.645	1.704
Inaction rate g_l	0.181	0.0026	0.231	0.175
Share of subcontracting	0.247	0.0053	0.253	0.278
Criterion, $\Gamma(\theta)$			1,342.52	5,265.9

Panel B: Parameter Estimates

	c_f	λ	ρ	μ	σ_ε	f	τ
Quick tenure	4.807	-	0.903	0.023	0.139	0.095	0.160
($\lambda = 0$)	(0.0353)	-	(0.0197)	(0.0047)	(0.0198)	(0.0027)	(0.0421)
Slow tenure	6.384	0.758	0.913	0.029	0.129	0.101	0.593
	(0.0403)	(0.0284)	(0.0113)	(0.0025)	(0.0121)	(0.0025)	(0.0284)

Notes: Panel A reports the targeted moments and their corresponding standard errors, and the simulated moments evaluated at the estimated parameters. The bottom table reports the parameters' point estimates and their standard errors in parenthesis.

Panel B of Table 1.5 contains the point estimates of the parameters for both models with the associated standard errors. In the benchmark model with slow tenure, estimated firing costs are equivalent to seven months' wages, and on average workers get tenure after 4 years in the job.[30] In terms of the wage premium on subcontracted workers, the model estimates are consistent with the data for manufacturing plants in ENIA. On average, subcontracted workers earned 8 percent more than permanent workers in the period 2001-2007.[31] Finally, shocks to productivity are estimated to be 14 percent per year, the mean growth rate of productivity 2.3% and the persistence of idiosyncratic shocks 0.903. As mentioned, for the model with quick tenure to fit well the relative flexibility of permanent employment as observed in the data moments, it requires firing costs that are substantially lower (only two months' wages). Consistent with the estimations for the benchmark model, the wage premium on subcontracted workers remains around 10 percent, and the rest of the parameters summarizing firm dynamics are also relatively stable.

For interpretation, Table 1.6 presents estimations for two additional restricted models. First, a model without subcontracting, and a positive probability of not getting tenure in the column labeled Slow tenure. We see the fit of the model is slightly worse in comparison to the benchmark case in spite of the reduction in the number of moments to fit. In terms of employment dynamics, the model also has problem fitting the volatility and the kurtosis of g_l when the inaction rate is too low as observed in the data. In the column labeled Quick tenure I present the estimates of a model that also restricts subcontracting, but assumes all workers get tenure. Note that this specification of the model is the same model as in Hopenhayn and Rogerson (1993). For this model to fit such a low inaction rate for permanent employment growth rate is even more problematic. Panel B in Table 1.6 displays the point estimates of the parameters for both models with the associated

[30] There is no obvious translation between these parameters estimates and the "job security index" presented in Section 1.2, but the 3-4 month wages suggested by this index do not seem too far of considering that the estimation might be capturing other costs associated to dismissing workers.

[31] The wage of subcontracted (permanent) workers is computed as the total wage paid by the establishment to all subcontracted (permanent) workers divided by the number of subcontracted (permanent) workers employed by the establishment in that same period. The results are robust to the inclusion of bonuses on permanent workers' wages. The widespread perception that subcontracted jobs pay substantially less than permanent ones is largely contaminated by the decline in relative wages of low-skilled workers, and low-skilled jobs are still subcontracted in a larger proportion than permanent ones.

standard errors. In the model with quick tenure, estimated firing costs are equivalent to one month' wages, while in the model with slow tenure they increase to four months' wages, as workers get tenure on average after 3 years on the job.

In conclusion, a naive economist that estimates firing costs from these data moments ignoring firms subcontract to substitute permanent workers would arrive to the conclusion that firing costs are rather low in this industry. However, the results from the benchmark model show they are rather high, and the flexibility observed in the data comes from subcontracted workers being used as an adjustment margin for firms to accommodate economic shocks.

1.6 Policy Implications

In this section, I extend the partial-equilibrium model to a general equilibrium framework, and using the parameters' estimates I carry out several policy analysis. I use the estimations for the four models to analyze the implementation of two alternative labor market reforms: first, the elimination of subcontracted workers and, second, the reduction of firing costs to zero when suitable. This experiment is relevant in light of the debate that pits workers' demands to limit the use of subcontracting as a way to improve their working conditions, with those of the business community that advocate a reduction in firing costs. Finally, it is important to clarify that the model is not appropriate for welfare analysis as it only considers a frictionless economy in which firing costs have no potential benefits, but to distort the job reallocation process. The equilibrium allocation without government intervention is Pareto optimal, hence there is no space for improvement coming from firing costs.[32]

[32] See, for example, Alvarez and Veracierto (2001) and Alonso-Borrego et al. (2004) who analyze the impact of firing costs in an economy with imperfect insurance markets and search frictions.

Table 1.6: Simulated Moments Estimations for the Model Without Subcontracting

Panel A: Moments

Moments	Data	S.E.	Simulated Moments Slow tenure ($\lambda > 0$)	Quick tenure ($\lambda = 0$)
Average firm size	66.76	1.7310	67.97	78.71
Exit rate	0.091	0.0012	0.100	0.113
Fraction of plants in each bin:				
10-19 employees	0.402	0.0049	0.418	0.321
20-99 employees	0.440	0.0049	0.434	0.482
100-499 employees	0.139	0.0038	0.130	0.173
More than 500 employees	0.019	0.0015	0.018	0.024
Share of employment in each bin:				
10-19 employees	0.071	0.0023	0.076	0.057
20-99 employees	0.272	0.0084	0.283	0.275
100-499 employees	0.423	0.0121	0.368	0.398
More than 500 employees	0.234	0.0177	0.274	0.270
Volatility g_l	0.688	0.0160	0.833	0.806
Kurtosis g_l	5.144	0.0606	3.035	2.834
Inaction rate g_l	0.181	0.0026	0.153	0.244
Criterion, $\Gamma(\theta)$			1,524.4	2,937.9

Panel B: Parameter Estimates

	c_f	λ	ρ	μ	σ_ε	f	τ
Quick tenure	7.756	-	0.871	0.048	0.144	-	0.133
($\lambda = 0$)	(0.0263)	-	(0.0092)	(0.0032)	(0.0068)	-	(0.0048)
Slow tenure	5.654	0.684	0.915	0.016	0.133	-	0.285
	(0.0546)	(0.0234)	(0.0283)	(0.0017)	(0.0247)	-	(0.0268)

Notes: Panel A reports the targeted moments and their corresponding standard errors, and the simulated moments evaluated at the estimated parameters. The bottom table reports the parameters' point estimates and their standard errors in parenthesis.

1.6.1 General Equilibrium Model

The economy is populated by a continuum of identical two member households: workers that supply labor under a permanent contract and subcontracted workers. Each household has preferences defined over consumption and labor supply given by:

$$\sum_{t=1}^{\infty} \beta_t [\log(c_t) - B \frac{n_t^{1+\phi}}{1+\phi}], \tag{1.20}$$

where $c_t > 0$ is total consumption, and n_t is labor effort. Parameters B and ϕ represent preferences for leisure, and the inverse of the Frisch elasticity of labor supply, respectively. Households take all of the income from all of the workers, and allocate it to the individuals within the household. Also, they allocate total hours worked independent of which workers performs the effort.

The output price is normalized to one, and the households supplies labor to the market at the wage $w = -u_n/u_c = Bcn^{\phi}$. As before, both members of the household are perfect substitutes in production, but permanent workers are relatively less expensive as subcontractors' charges are higher than the firm's own production costs. Subcontract firms incur in a real cost for "creating" subcontracted workers, and the premium they charge to the main firm per worker is just enough to cover the real cost c so that their profits are zero: $\pi(s_t) = (w_t^s - c)s_t = 0$. The cost for firms to subcontract a worker is $w_t^s = w_t(1 + f)$. I consider a stationary equilibrium, so all prices and aggregates in the economy are constant, and household maximization implies the interest rate satisfies $1/(1 + r) = \beta$.

An individual firm that employed l_{t-1} permanent workers last period and draws a productivity shock z_t this period has expected adjustment costs given by:

$$r(l_{t-1}, z_t; w) = [1 - X(l_t, z_t; w)] \int g(n_{t+1}, l_t) dF(z_{t+1}, z_t) + X(l_t, z_t; w) g(0, l_t), \tag{1.21}$$

where $n_{t+1} = L(l_t, z_{t+1}; w)$. Integration yields aggregate adjustment costs given by $R(\mu, M; w)$. I assume proceeds from the regulation are rebated uniformly to all households as a lump-sum payment to households by the government. In fact, severance payments make up for the largest part of firing costs in Chile, and are paid entirely to the workers when they are fired. Aggregate adjustment costs do not appear in the resource constraint as they appear in both sides of the equation.

The demand curve in Section 1.3 is replaced by the resource constraint:

$$C = Y - Mc_e - F \tag{1.22}$$

where output is given by:

$$Y = \int_{z^*} [f(L(l,z;p), S(n,z;p), z) - c_f] d\mu(z,l) + M \int_{z^*} f(L(0,z;p), S(n,z;p), z) d\nu(z), \tag{1.23}$$

and the fees paid by the firms for the subcontracted workers are given by:

$$F = fw \left[\int_{z^*} S(n,z;w) d\mu(z,l) + M \int_{z^*} S(n,z;w) d\nu(z) \right]. \tag{1.24}$$

Finally, the clearing condition for the labor market is given by:

$$N^s(\mu, M; w) = \int_{z^*} [L(l,z;w) + S(n,z;w)] d\mu(z,l) + M \int_{z^*} [L(0,z;w) + S(n,z;w)] d\nu(z) \tag{1.25}$$

A ***stationary industry equilibrium*** with positive entry and exit is a set of value functions and decision rules, a wage w^*, a stationary distribution of firms μ^*, and a mass of entrants M^* such that:

1. Given prices, the value functions of the firms and the policy functions are consistent with firms optimization.

2. There is an invariant distribution over firms: $\mu^* = T(\mu^*, M^*; w^*)$.

3. The resource constraint (equation 1.22) and the labor market clearing conditions

(equation 1.25) are satisfied.

4. The free entry (equation 1.10) is satisfied.

1.6.2 Results

In this section I present the results for the policy analysis. Few things to consider before presenting the results: first, I only compare steady-state values, and do not discuss the transitional dynamics. Second, I need to parametrize labor supply preferences: I set the elasticity of labor $\phi = 0.84$ (see Medina and Soto (2007) for estimations for Chile), and $B = 11.62$ so that total employment is 0.25.

1.6.2.1 Aggregate outcomes, prices and labor market

Table 1.7 reports the steady-effects of reducing firing costs in the four estimated models. The column label Full model/Slow tenure shows the effect of reducing firing costs in the benchmark model. Output goes up 3.54 percent when firing costs are eliminated, both due to an increase in productivity (+1.02 percent) coming from the better allocation of resources, and in total employment (+2.49 percent). The increase in permanent workers is even larger, as all the jobs previously assigned to subcontracted workers are reallocated to workers inside the firm. In the absence of firing costs, the wage of permanent workers goes up 5.75 percent as the distortions coming from firing costs disappear.

One of the main findings of the paper comes from the comparison of the effect of reducing firing costs between my benchmark model and the model without subcontracting/quick tenure. The column labeled No subcontracting/quick tenure in Table 1.7 presents the effect of reducing firing costs in a model that is equivalent to the framework in Hopenhayn and Rogerson (1993). In this case, eliminating the regulation has also a positive impact on output, labor productivity and employment, though the effect is substantially larger in comparison to the effect of the same reform applied to my benchmark economy. In particular, the effect is larger on labor productivity, as subcontracting workers firms circumvent the regulation, and improve the allocation of labor in heavily regulated environments. The allocation of resources, therefore, in an economy where firms cannot subcontract is more inefficient, and the benefits of removing the regulation are larger. Even when firing costs (as a percentage of the wage bill of permanent workers) are substantially larger in the benchmark economy (i.e. 6.1 versus 3.4 percent) the firms in this economy use resources more efficiently, and better allocate labor due to the presence of subcontracted workers.

For completeness, the table also presents the results of removing the regulation in the benchmark economy, but when permanent workers get tenure quickly, and in the model without subcontracting when permanent workers slowly get tenure. The results are still consistent with the fact that firms manage risk better in the presence of subcontracted workers, as they buffer the regular workforce from economic fluctuations to avoid workers' firing costs by subcontracting workers.

Table 1.7: Steady-State Effects of Eliminating Firing Costs

	Full model		No subcontracting	
	Quick tenure	Slow tenure	Quick tenure	Slow tenure
Output	2.87	3.54	4.20	2.24
Consumption	2.81	3.59	2.90	2.28
Average labor productivity	0.88	1.02	2.49	0.32
Total employment	1.97	2.49	1.67	1.92
Permanent	3.09	3.73	1.67	1.92
Wage permanent workers	4.51	5.75	4.34	3.92
Layoff costs/wage bill (before)	0.036	0.061	0.034	0.040
Subcontracting costs/wage bill (before)	0.087	0.092	-	-

Notes: The table reports the steady-state percentage change if the firing costs are eliminated starting from each of the different estimated models.

Table 1.8 shows the results from the comparison between the benchmark economy (with quick and slow tenure), and the new stationary equilibria associated with eliminating subcontracted workers.[33] The results show that output, employment and productivity go down when subcontracted workers are prohibited, as this change eliminates a margin that firms exploit to adjust to productivity shocks; firms fire subcontracted workers as a response to a negative shocks without paying firing costs. Instead, in the model without subcontracted workers, firms are forced to smooth their employment level over time to reduce firing costs. In the benchmark model, the lower output comes more from a decrease in the number of workers than from a decrease in average labor productivity. Instead, in the model with quick tenure the effects comes from a slow down in the reallocation of workers, and a decrease in productivity, and not so much from a decrease in employment. Firms in the economy with slow tenure use resources more efficiently, and already allocate employment better (i.e. subcontracted costs/wage bill are 0.092% in the economy with slow tenure versus 0.087% in the quick tenure economy). These results come against the common view that subcontracted jobs are of lower quality, and that they decrease productivity. We see that the winners from this policy are permanent workers which increase in their hirings.

Row 5 reports the change in the wage of permanent workers in both models. When subcontracted workers are eliminated, there is a decrease in the wage of permanent workers coming from the increase in the number of permanent workers which lowers average labor productivity. As productivity decreases a lot more in the model with quick tenure, the effect on wages is also substantially larger. This lower wage compensates firms for the higher average adjustment cost of labor (i.e layoff costs/wage bill increase 1.74 and 0.67 percent).

[33]In the model, to eliminate subcontracted workers I assume the fee charged by the subcontract firm becomes sufficiently high.

Table 1.8: Steady-State Effects of Eliminating Subcontracted Workers

	Quick tenure	Slow tenure
Output	-0.146	-0.082
Average labor productivity	-0.095	-0.020
Mass of firms	-0.620	-0.227
Layoff costs/wage bill	1.738	0.674
Total employment	-0.051	-0.062
permanent	1.073	1.150
Wage permanent workers	-0.058	-0.018
Layoff costs/wage bill		
before	0.036	0.061
after	0.037	0.062
Subcontracting costs/wage bill	0.087	0.092

Notes: The table reports the steady-state percentage change if subcontracted work was eliminated from both of models or, equivalently, if the wage premium on subcontracted workers was prohibitively high.

1.7 Conclusion

In this paper, I analyze the effect of firing costs on aggregate outcomes when firms can circumvent the regulation subcontracting as a substitute for hiring full-time workers. In countries with strict job security regulations firms use flexible staffing arrangements to buffer the regular workforce from economic fluctuations and avoid workers' firing costs. I set up an industry equilibrium model in the tradition of Hopenhayn and Rogerson (1993) with heterogeneous firms and endogenous entry and exit, where firms can hire two types of workers: permanent workers that entail random firing costs, and subcontractors that are totally flexible, but carry a wage premium above the compensation permanent workers demand.

The results for the model estimations show that to match plant-level employment dynamics in the manufacturing sector in Chile subcontracted workers are needed. Put differently, a model that ignores this adjustment margin yields firing costs that are too low and very much at odds with empirical data. In the model with subcontracted workers firing costs are equivalent to seven months' wages, and permanent workers get tenure after approximately 4 years in the job. Firms, in this framework, are willing to pay a rather large wage premium on subcontracted workers to be able to substitute for hiring workers (10 percent). Instead, in the model without subcontracted workers, firing costs are equivalent to only one month's wage.

These findings are consistent with the results from the policy experiments which show that allowing firms to subcontract workers increases output, employment and productivity. Subcontracted workers allow firms to respond more aggressively to productivity shocks, which enhances the allocation of labor across firms and hence total factor productivity (TFP). Further, when firms can subcontract, the negative effects of firing costs in aggregate outcomes are less than previously estimated in the literature.

A.1 Solution algorithm

A.1.1 Partial equilibrium model

In this section, I present the solution algorithm for the partial-equilibrium model. Basically, the algorithm consists of two steps: 1) find the unique price p^* that is consistent with the free entry condition; 2) second, find the fixed point of .

Step 1 Iterate over p_i until the entry condition is satisfied at p^*:

(a) For each p_i, compute $V_i(l, z; p_i)$ and $V_i(0, z; p_i)$

(b) Let $EC(p_i) \equiv \int V(0, z; p) d\nu(z)/p_i - c_e$. If $EC(p_i) > 0$, then set $p_{i+1} < p_i$, otherwise set $p_{i+1} > p_i$.

Step 2 Iterate over (μ_i, M_i) until $Q^d = Q^s$ at (μ^*, M^*):

(a) Letting $M_0 = 1$, solve for the stationary distribution μ_0^{ss} using the law of motion for the distribution of firms (equation 1.12)

(b) Let $EQ(\mu_i, M_i) \equiv Q^d - Q^s(\mu_i^{ss}(M_i), M_i; p^*)$. If $EQ(\mu_i, M_i) > 0$, then set $M_{i+1} > M_i$, otherwise set $M_{i+1} < M_i$. When $EQ(\mu_{i+1}, M_{i+1}) \approx 0$ then $(\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)$

A.1.2 General equilibrium model

To solve the general equilibrium model as explained in Section 1.6, the algorithm starts with Step 1 as before, but solving for the wage of permanent workers w_i instead of p_i. Then, I continue on to Step 2a:

Step 2a Iterate over (μ_i, M_i) until the resource constraint $C = Y - Mc_e - F$ and the labor market clearing condition $L^d = N^s$ are satisfied at (μ^*, M^*):

(a) Letting $M_0 = 1$, solve for the stationary distribution μ_0^{ss} using the law of motion for the distribution of firms (equation 1.12)

(b) Let $LMC(\mu_i, M_i) \equiv L^d(\mu_i^{ss}(M_i), M_i; w^*) - N^s[w^*, \Pi(\mu_i^{ss}(M_i), M_i; w^*)]$. If $LMC(\mu_i, M_i) > 0$, then set $M_{i+1} < M_i$, otherwise set $M_{i+1} > M_i$. When $LMC(\mu_{i+1}, M_{i+1}) \approx 0$ then $(\mu_{i+1}, M_{i+1}) = (\mu^*, M^*)$

Chapter 2

Business cycle dynamics and asset prices with capital adjustment costs

2.1 Introduction

Standard business cycle models (RBC) are successful in accounting for the main statistical features of macroeconomic aggregate quantities.[1] However, their success is relatively modest in terms of their ability to match asset-pricing facts. In an attempt to unite business cycle dynamics and asset returns, Tallarini (2000) adapts Epstein-Zin (EZ) preferences to a standard RBC model to disentangle individual attitudes towards risk and towards intertemporal substitution. By increasing risk aversion, without changing the intertemporal elasticity of substitution (IES), the model's asset pricing predictions improve significantly without affecting quantity dynamics. When risk aversion increases, agents can only increase precautionary savings to lessen the effects of technology shocks, as substituting across states of nature is not possible. Instead, when the IES changes, agents choose different consumption paths affecting quantity dynamics.

Even when this framework is regarded as promising to line up quantity dynamics and asset prices in production economies, Tallarini's model lacks relevant features. First, the model is incapable of generating any significant equity premium as the risk in standard RBC models comes entirely from technology shocks. Without adjustment costs, capital can be transferred instantaneously to and from consumption, making the relative price of capital always equal to one. Capital adjustment costs, instead, make it costly for agents

[1] Standard RBC models refer to frameworks with agents maximizing expected discounted utility under complete markets and no frictions in capital accumulation. The main statistical features are that investment is almost three times more volatile than output, while consumption is less volatile than output, and hours worked show almost the same volatility. In general, macroeconomic variables tend to be strongly procyclical, and they show substantial persistence.

to smooth fluctuations in consumption through the production sector, inducing them to take more consumption risk and increasing the equity premium. Second, the higher risk in an economy with capital adjustment costs could potentially affect investment and consumption decisions, and business cycle dynamics breaking down Tallarini's "separation theorem". Finally, Tallarini (2000) only considers the unity IES case ignoring the model's ability to improve aggregate quantity predictions through a better calibration of this parameter.

To address these issues, this paper analyzes the effects of convex capital adjustment costs on business cycle and asset pricing when consumers have recursive preferences. In a standard one-sector stochastic growth model I unite the success of Epstein and Zin (1989) non-expected utility preferences with a fleshed-out production technology with convex capital adjustment costs in the style of Jermann (1998). The proposed model accounts for the main statistical features of macroeconomic aggregate quantities. At the same time, adjustment costs increase the equity risk premium, with the mean stock return and its standard deviation in the order of magnitude that are closer to the data. Further, for a plausible calibration the model produces a stable and rather low risk-free rate.

In this paper I also produce a comprehensive study of the asset pricing and business cycle implications of a one-sector stochastic growth model with capital adjustment costs and Epstein-Zin preferences. For this purpose, I first investigate the impact on quantity dynamics of changes in agents' attitude towards risk for different calibrations of the IES and the capital adjustment costs parameter. The results show that regardless of the calibration for the IES and the level of adjustment costs on capital, the behavior of aggregate quantities does not depend on attitudes towards risk. The endogenous amount risk coming from productivity shocks in this model economy with capital adjustment costs is not enough to generate a different response from consumption, investment or hours worked when risk aversion is increased. Even when capital adjustment costs increase the amount of risk in the model, this is still not enough to generate a significant difference in the second moment of the quantity variables.

Second, I examine the model's asset-pricing predictions under different calibrations of the risk aversion, the IES and the level of capital adjustment costs. This exercise is interesting because Epstein-Zin preferences disentangle the IES from risk aversion, which implies agents have a preference for the timing of resolution of uncertainty: expected returns do not only depend on the covariance of return with consumption growth (as in the power utility case), but also on the covariance with future consumption growth.[2] Further, capital adjustment costs introduce a wedge between the market value of installed capital and production cost today, eventually inducing agents to take more consumption risk. Hence, asset prices are volatile not only because productivity is random but also because capital prices change when investment changes.

In the main result of the model, I show both features contribute to broaden the asset-pricing facts the model can match. In contrast to the results in Tallarini (2000), when risk aversion increases, both excess return volatility and the equity premium increase in the model with capital adjustment costs, coming closer to match the data. Further, as precautionary savings increase mean capital stock, the risk-free rate decreases also contributing to the results. The comparative statistic exercise shows that to generate a reasonable volatility for the equity return when shocks to technology are permanent, the model requires a rather high calibration for the IES. This also helps to match the mean risk-free rate and its volatility to the data. A lower IES makes agents highly averse to substitute consumption over time, increasing the volatility of the marginal rate of substitution, and generating an excessive risk-free rate variation.[3]

[2]These two sources of volatility have become known as short- and long-run risk factors, see Bansal and Yaron (2004) for further details.

[3]Kaltenbrunner and Lochstoer (2010) and Croce (2014) show that the nature of the exogenous technology shock (i.e. transitory or permanent) and individual's preferences for the timing of resolution of uncertainty plays a crucial role in terms of the contribution of the long-run risk to the price of risk. In the case of a permanent productivity shock, the endogenous correlation between shocks to realized and

Finally, I investigate whether Tallarini's "separation theorem" breaks down when risk is time-varying. To address this issue while preserving the simplicity of the model, I introduce an alternative specification for the conditional volatility of productivity growth rates following the literature on time-varying volatility in finance. In this specification of the model, I assume the standard deviation of productivity follows an AR(1) process. I find that when risk is time-varying Tallarini's "separation theorem" breaks down, and macroeconomic quantities are affected by attitudes towards risk. Since asset prices also respond to the increased risk, there is a correlation between quantity dynamics and risk premia.

Related Literature This paper is at the intersection of two of the broad strands of literature that try to reconcile business cycle and asset pricing facts in a general equilibrium framework. First, it is related to the collection of papers that resort to different specifications of preferences to account for asset market implications.[4] Jermann (1998) and Boldrin et al. (2001) introduce habit formation into agents' preferences in a standard RBC model. Habit-formation preferences raise the variation of marginal rates of substitution across states, and hence generate the equity premium. However, they also increase the marginal rate of substitution over time inducing an excessive risk-free rate variation. Epstein-Zin preferences address this issue as they disentangle individual attitudes towards risk and towards intertemporal substitution; Tallarini (2000) shows that by increasing risk aversion (without changing the IES), RBC model's asset pricing predictions

expected consumption growth is positive. If individuals have preferences for early resolution of uncertainty (i.e. dislike shocks to realized and expected consumption growth) long-run risk carries a positive price of risk. Instead, with a mean-reverting shock expected consumption growth is negatively correlated with realized consumption growth, acting as a hedge and decreasing the overall price of risk. They show that both shocks can be calibrated to match the high price of risk, and the low risk-free rate with a low coefficient of risk aversion.

[4] In standard RBC models, with time and state separable preferences, the coefficient of relative risk aversion is equal to the reciprocal of the IES. Increasing risk aversion to increase the curvature of the utility function does not increase the volatility of the stochastic discount factor, but has the perverse effect of making consumption even smoother: a higher risk aversion decreases the IES, producing individuals highly averse to substitute consumption over time. They prefer to change investment instead of making any adjustment in consumption.

improve significantly without affecting quantity dynamics.[5]

The second strand of this literature incorporates capital adjustment costs to improve asset market predictions. In production economies, agents can easily adjust investment, hours worked or any other margin to obtain a smooth consumption paths when a shock hits the economy.[6] Capital adjustment costs make it costly for capital to adjust rapidly, introducing a wedge between the market value of installed capital and production cost today, eventually inducing agents to take more consumption risk. For instance, in the case of a concave cost of adjustment function, a higher investment rate implies a higher price of capital and so the wedge increases when investment increases. In a such an economy, asset prices are volatile not only because productivity is random but also because capital prices change when investment changes (Jermann, 1998; Boldrin et al., 2001; Kuehn, 2010).

The studies closest to mine are Kaltenbrunner and Lochstoer (2010), Campanale et al. (2010) and Croce (2014); the three studies investigate asset pricing implications of a one-sector stochastic growth model with recursive preferences and convex capital adjustment costs. We differ in that Kaltenbrunner and Lochstoer (2010) assume households experience no disutility of labor, and hours worked in their framework is always fixed at the maximum possible; not allowing hours worked to vary over the cycle increases the volatility of consumption growth contributing to improve the asset pricing predictions of the model. Campanale et al. (2010), on the other hand, works with broader set of preferences belonging to Chew-Dekel class which nests the particular case of Epstein-Zin preferences. Finally, Croce (2014) introduces a long-run component in the productivity growth process, and shows that it contributes to improve both business cycle statistics and asset pricing predictions.

[5]In a similar framework, Kaltenbrunner and Lochstoer (2010) show that a model with permanent technology shocks and a high IES can match the high price of risk and the low volatility of the risk-free rate with a low coefficient of relative risk aversion.

[6]For a discussion on the role of labor/leisure as an adjustment margin see, for instance, Lettau and Uhlig (2000), Uhlig (2007), and Jaccard (2010).

The remainder of the paper is organized as follows: Section 2.2 presents the basic model, and outlines the calibration. Section 2.3 presents the results for the comparative statistic exercises changing risk aversion, IES and capital adjustment costs, and discusses the quantitative implications of these findings. Section 2.4 concludes. The Appendix provides details on the computational methods used, and additional derivations.

2.2 Model

In this section I present the model which considers a frictionless standard RBC model—one-sector stochastic growth model. I start by motivating the elements in the model. First, agents have non-expected utility preferences over consumption and leisure in the class of recursive preferences considered by Epstein and Zin (1989). I consider a very general specification for preferences that nests the specific case of unity IES since it allows the performance of interesting comparative analysis parameterizing the model for different values of the IES parameter and the risk aversion. Asset pricing implications of endogenous long-run risk significantly depend on agents' preferences for the timing of the resolution of uncertainty–whether the coefficient of risk aversion is higher, smaller or equal to the inverse of the IES parameter.[7] Hence, the importance of considering this different calibrations for the IES.

Second, I assume costs of adjustment to capital in the form of Jermann (1998). Adjustment costs make it costly for capital to adjust rapidly, introducing a wedge between the market value of installed capital and production cost today. Given that the cost of adjustment function is concave, a higher investment rate implies a higher price of capital and so the wedge increases when investment increases. In this model, therefore, asset prices are volatile not only because productivity is random but also because capital prices change when investment changes.

[7] See, for instance, Kaltenbrunner and Lochstoer (2010)

Finally, in my paper I only consider permanent technology shocks. Several studies point to the relevance of considering the interactions between different technology processes and long-run risk. Hence, I leave for future work the study of transitory productivity shocks in a model like this.

2.2.1 Basic setup

The economy is populated by a large number of identical and infinitely lived households, who maximize recursive utility over consumption and hours worked. These households trade in the stock and bond market, and perceive dividends for holding the firm's stocks. There is also a continuum of identical price-taking firms that own the capital, produce output, invest and pay wages to the households. They finance the purchase of capital selling claims to their cash flows to households. Labor is supplied elastically by households.

2.2.1.1 Representative household

The representative household maximizes recursive utility over consumption C_t and hours worked N_t following Epstein and Zin (1989) such that:

$$V_t = \left((1-\beta) u(C_t, N_t)^{1-\gamma} + \beta E_t (V_{t+1}^{1-\theta})^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}} \tag{2.1}$$

where the per period utility index is $u(C, N) = C^{\nu}(1-N)^{1-\nu}$.

Given that u is homogeneous of degree one, $\gamma \geq 0$ is the inverse of the IES ψ over the consumption-leisure bundle, and $\theta \geq 0$ controls risk aversion towards static gambles over the bundle. The term $E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}$ is called a "risk-adjustment" or the "certain equivalent" of future utility. The discount factor is β and the time-period is assumed to be one quarter.[8]

This specification for preferences allows for a separation between attitudes towards risk, which govern portfolio choice, and attitudes towards intertemporal substitution which lead to smooth consumption. This framework produces agents with preferences for the timing of the resolution of uncertainty: they prefer an early resolution of uncertainty if $\theta > \frac{1}{\psi}$, a late resolution of uncertainty if $\theta < \frac{1}{\psi}$, and they are indifferent if one is the reciprocal of the other. In the latter case, when $\theta = \frac{1}{\psi}$, recursive preferences collapse to the standard time-separable expected discounted utility case.

The household budget constraint is

$$W_{t+1} = (W_t + w_t N_t - C_t) R^W_{t+1}, \tag{2.2}$$

where W_t denotes wealth and $R^W_{t+1} = s_t R^e_{t+1} + (1 - s_t) R^f_{t+1}$ the return on the wealth portfolio. Households invest a proportion s_t of their wealth in a risky claim on the firm's dividend stream that returns R^e_{t+1}, and the rest $(1 - s_t)$ in a risk-free bond with return R^f_{t+1}.

From the household optimality condition the equity return satisfies the standard asset pricing formula

$$E_t[M_{t,t+1} R^e_{t+1}] = 1, \tag{2.3}$$

where $R^e_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$. Similarly, the return on the risk-free rate can also be expressed in terms of the stochastic discount factor as

$$R^f_{t+1} = \frac{1}{E_t[M_{t,t+1}]}. \tag{2.4}$$

[8]It can be shown that when IES=1 preferences simplify to: $V_t = u(C_t, N_t)^{1-\beta} E_t(V_{t+1}^{1-\theta})^{\frac{\beta}{1-\theta}}$ as specify in Tallarini (2000).

In this economy the stochastic discount factor (SDF) $M_{t,t+1}$ is given by[9]

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{v(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t} \right)^{(1-v)(1-\gamma)} \left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta} \tag{2.5}$$

When $\gamma \neq \theta$ an extra term $\left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{\gamma-\theta}$ appears in the expression for the old power formula for the SDF: expected returns not only depend on the covariance of returns with consumption growth as in the standard consumption-based model, but also on the covariance of returns with the utility index. Since the utility index is function of the distribution of *future* consumption, news about agents' future perspectives matter as well as *current* consumption conditions.

2.2.1.2 Representative firm

The representative firm produces a single good using a standard constant returns to scale production technology

$$Y_t = K_t^{\alpha}(Z_t N_t)^{1-\alpha}, \tag{2.6}$$

where Y_t is aggregate output, K_t is the fixed stock of capital carried into date t, and Z_t is an exogenous, labor-enhancing technology level.

The firm accumulates capital subject to adjustment costs according to

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t, \tag{2.7}$$

where $I_t \geq 0$ is aggregate investment and takes one period to be reflected in capital, δ is the rate of depreciation of capital and $\phi(\cdot)$ is a weakly concave function that describes the form of convex capital adjustment costs in the model. Following Jermann (1998), this

[9] See Appendix A.1 for a short derivation.

function is defined as

$$\phi\left(\frac{I_t}{K_t}\right) = a_1 + \frac{a_2}{(1-\eta)}\left(\frac{I_t}{K_t}\right)^{(1-\eta)}, \tag{2.8}$$

where η is the inverse of the elasticity of the investment rate to Tobin's q. Parameters a_1 and a_2 are normalized to be $a_1 = \frac{\eta}{1-\eta}(1-\delta-e^{\mu})$ and $a_2 = (e^{\mu}-1+\delta)^{\eta}$ so that capital adjustment costs are zero in the steady-state.

The technology process is defined as

$$Z_t = e^{\mu t + \tilde{z}_t} \tag{2.9}$$

$$\tilde{z}_t = \rho \tilde{z}_{t-1} + \sigma \varepsilon_t, \tag{2.10}$$

where ε_t are standard normally distributed i.i.d random shocks, and ρ determines the persistence of the shock process.

Each period the firm optimally chooses N_t and I_t to maximize the present discounted value of all current and future expected cash flows:

$$E_t \sum_{s=0}^{\infty} M_{t,t+s}(F(K_{t+s}, Z_{t+s}N_{t+s}) - w_{t+s}N_{t+s} - I_{t+s}) \tag{2.11}$$

where $M_{t,t+s}$ is the stochastic discount factor of the households, the owners of the firm in this economy.

The firm's equilibrium conditions for investment can be characterized by

$$E_t\left[M_{t,t+1}R^I_{t+1}\right] = 1, \tag{2.12}$$

where the investment return R^I_{t+1} is defined as

$$R_{i,t+1} \equiv \phi'(I_t/K_t)\left\{\alpha\left(\frac{Z_{t+1}}{K_{t+1}}\right)^{1-\alpha} + \frac{1-\delta+\phi(I_{t+1}/K_{t+1})}{\phi'(I_{t+1}/K_{t+1})} - \frac{I_{t+1}}{K_{t+1}}\right\}. \tag{2.13}$$

Given that the production technology and the cost of adjustment function are linearly homogeneous the return to capital is equivalent to the market return to an unlevered firm claim (dividends). Capital and labor are paid their marginal product such that wages and dividends are defined by $w_t = (1-\alpha)\frac{Y_t}{N_t}$ and $D_t = \alpha Y_t - I_t$, respectively.

Finally, the representative agent is endowed with one unit of time which can be used for leisure or labor such that

$$L_t + N_t \leq 1, \tag{2.14}$$

and the resource constraint is given by

$$C_t + I_t \leq Y_t. \tag{2.15}$$

2.2.2 Stationary recursive representation

In this section I specify the recursive representation of the model used for the numerical solution. The household's problem is indexed by two state variables K_t and Z_t, two independent control variables C_t and N_t, and one (normal) shock ε_t.

Given the first welfare theorem holds the problem of this economy can be formulated in terms of the social planner's problem:

$$V(K,Z) = \max_{C,N,I} \left\{ \left[(1-\beta)(C^{v}(1-N)^{1-v})^{1-\gamma} + \beta \left(E_{\varepsilon'} V(K',Z')^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right]^{\frac{1}{1-\gamma}} \right\}, \tag{2.16}$$

subject to

$$\begin{aligned} C + I &= K^{\alpha}(ZN)^{1-\alpha}, \\ K' &= (1-\delta)K + \phi\left(\frac{I_t}{K_t}\right)K, \\ Z' &= Ze^{\mu+\sigma\varepsilon'}. \end{aligned}$$

To have a stationary economy the problem is normalized by Z_t. Given the homotheticity of the utility function, the value function is homogeneous of degree v in K and Z. Hence, the value function is redefine as $V(K,Z) = Z^v g(k)^{\frac{1}{1-\gamma}}$ where g satisfies the following Bellman equation and $k = \frac{K}{Z}$, $c = \frac{C}{Z}$ and $i = \frac{I}{Z}$:[10]

$$g(k) = \max_{c,i,N} \left\{ (1-\beta)(c^v(1-N)^{1-v})^{1-\gamma} + \beta e^{\mu v(1-\gamma)} \left(E_{\varepsilon'} \left[e^{\sigma\varepsilon' v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right] \right)^{\frac{1-\gamma}{1-\theta}} \right\} \tag{2.17}$$

subject to

$$\begin{aligned} c + i &= k^\alpha N^{1-\alpha}, \\ k' &= \frac{(1-\delta)k + \phi(\frac{i}{k})k}{e^{\mu+\sigma\varepsilon'}}. \end{aligned}$$

The model has a steady state solution in the transformed variables. I worked directly on the appropriately set of normalized equations and then "renormalized" once solved the model. See the Appendix for a detailed explanation on the computational method.

2.2.3 Calibration

The set of parameters necessary to compute the model is:

$$\Theta = \{\beta, \alpha, \delta, \rho, \mu, \sigma_\varepsilon, \nu, \theta, \gamma, \eta\} \tag{2.18}$$

where β is the discount rate, α is capital share in the production function, δ is the depreciation rate of capital, μ, and σ_ε are the parameters that define the productivity process, ν is the exponent on consumption, θ is risk aversion, γ is the inverse of IES, and η defines the curvature of the capital adjustment cost function.

For the calibration of the model I use standard values in the macroeconomic literature. Since the interest of this paper lies in the effects on asset returns and business cycle

[10] Note that the problem needs to be transformed into a min if $\gamma > 1$.

variables of changing the risk aversion, the IES and the curvature of the adjustment cost function I present results for the model under different calibrations of these parameters. It is worth noting that the calibrated capital adjustment costs used in this paper are within the range of such costs reported in the literature.[11] Table 2.1 summarizes the parameters' values.

Table 2.1: Parameter Assignments

Parameter	Description	Value
β	Discount rate	0.9926
α	Capital share	0.34
δ	Depreciation rate	0.021
μ	Technology drift	0.004
ν	Exponent on consumption	0.25
λ	Leverage	2
σ_ε	Std. dev. productivity shock	0.0115
θ	Risk aversion	$(1, 10, 25, 100)$
$1/\gamma$	IES	$(0.1, 1, 1.5)$
η	Adjustment costs	$(0, 0.2, 0.6)$

2.3 Main results

In this section I present the main results of the paper. First, I discuss the implications of increasing risk aversion on the business cycle properties of the model, and analyze the implications for business cycle dynamics of changes in the IES and the capital adjustment costs. Second, I present the asset pricing implications of introducing costs of adjustment to the standard real business cycle model, and perform several comparative exercises changing the IES and the η as well. Finally, I consider the effect in macroeconomic dynamics of increasing risk aversion when the model incorporates time-varying volatility for TFP shocks.

[11] See [to be completed]

2.3.1 Business cycle statistics

Table 2.2 reports the steady-state mean values of capital, output, consumption, investment and hours worked for different values of the relative risk aversion coefficient and the capital adjustment costs parameter η when the IES equals $1.5, 1$ and 0.2. In the case of IES=1 (Panel B), an increase in the coefficient of relative risk aversion from 1 (expected utility case) to 100 increases the mean value of all macro variables between 1 percent (hours worked) to 8 percent (capital), approximately. The effect of increased risk aversion in mean quantities is smaller for larger capital adjustment costs.

An increase in risk aversion increases precautionary savings as standard production technologies do not allow agents to hedge themselves against technology shocks. The only action they can take to mitigate the effects of these shocks is to increase their savings to augment wealth next period. The increase in precautionary savings is smaller when there are adjustments costs in the economy because adjustment costs induce firms to invest less aggressively in response to technology shocks reducing the need for higher precautionary savings. The same conclusions hold when the elasticity of substitution is greater or smaller than unity (see Panel A and C in Table 2.2).

Along the same lines, the effects of increased risk aversion on aggregate quantities' volatility and cross-correlations is almost negligible independent on the calibration for IES and the capital adjustment costs in the model. Tables 2.3, 2.4 and 2.5 report the standard deviation, and cross-correlations of consumption, investment and hours worked with output for increased risk aversion for IES= $[0.2, 1, 1.5]$ and $\eta = [0, 0.2, 0.6]$. The endogenous amount of risk coming from productivity shocks in this economy is not enough to generate a different response from consumption, investment or hours worked when risk aversion is increased. Even when adjustment costs increase the risk, this is still not enough to generate a significant difference in the second moment of the quantity variables. Intro-

Table 2.2: Steady-State Mean Values Changing $1/\gamma$, θ and η

Panel A: $1/\gamma = 1.5$

	$\eta=0$					$\eta=0.2$					$\eta=0.6$				
θ	k	y	c	i	N	k	y	c	i	N	k	y	c	i	N
1	8.17	0.77	0.57	0.20	0.23	8.17	0.77	0.57	0.20	0.23	8.11	0.77	0.57	0.20	0.23
100	8.79	0.80	0.58	0.22	0.23	8.55	0.79	0.58	0.21	0.23	8.22	0.78	0.57	0.21	0.23

Panel B: $1/\gamma = 1$

	$\eta=0$					$\eta=0.2$					$\eta=0.6$				
θ	k	y	c	i	N	k	y	c	i	N	k	y	c	i	N
1	8.01	0.77	0.57	0.20	0.23	8.01	0.77	0.57	0.20	0.23	7.95	0.77	0.57	0.20	0.23
100	8.62	0.79	0.58	0.22	0.23	8.42	0.78	0.57	0.21	0.23	8.10	0.77	0.57	0.20	0.23

Panel C: $1/\gamma = 0.2$

	$\eta=0$					$\eta=0.2$					$\eta=0.6$				
θ	k	y	c	i	N	k	y	c	i	N	k	y	c	i	N
1	6.50	0.70	0.54	0.16	0.22	6.50	0.70	0.54	0.16	0.22	6.36	0.70	0.53	0.16	0.22
100	7.15	0.73	0.55	0.18	0.23	7.04	0.73	0.55	0.18	0.23	6.77	0.71	0.54	0.17	0.23

Notes: the table shows the steady-state mean values for different values of the adjustment cost parameter η, and IES $1/\gamma$ for high and low risk aversion. The rest of the parameters are given in Table 2.1.

ducing adjustment costs to the model does not change the main conclusions of Tallarini's "separation" between quantity dynamics and risk aversion.[12]

Even when the increased risk induced by capital adjustment costs does not affect the separation between quantities and asset prices in the RBC model, it is interesting to investigate the effects on the second moments of aggregate variables. With frictions on capital adjustments, the cost of adjusting investment increases and, hence, the adjustment when a shock hits the economy falls on consumption. This generates an increase in the volatility of consumption along with a decrease in the volatility of investment (compare columns (2) & (3) across Tables 2.3, 2.4 and 2.5).

[12] I also consider other alternative specifications for the cost of adjustment function such $\phi\left(\frac{I_t}{K_t}\right) = a_1 + \frac{a_2}{(1-\eta)}\left(\frac{I_t}{K_t} - \frac{\delta}{2}\right)^{(1-\eta)}$ and $\phi\left(\frac{I_t}{K_t}\right) = \frac{I_t}{K_t} + \frac{\eta}{2}\left(\frac{I_t}{K_t} - \delta - e^{\mu} + 1\right)^2$. In none of the cases the results changed.

Table 2.3: Business Cycle Statistics Changing $1/\gamma$ and θ for $\eta = 0$

	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
Data	0.99	0.54	2.75	0.93	0.48	0.61	0.74
	(0.06)	(0.03)	(0.17)	(0.06)	(0.06)	(0.05)	(0.04)

Panel A: $1/\gamma = 1.5$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	1.03	0.51	2.53	0.41	0.98	0.99	0.99
	(0.05)	(0.03)	(0.14)	(0.02)	(0.01)	(0.00)	(0.01)
100	1.03	0.50	2.45	0.41	0.98	0.99	0.99
	(0.05)	(0.03)	(0.14)	(0.02)	(0.01)	(0.00)	(0.01)

Panel B: $1/\gamma = 1$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.99	0.54	2.29	0.35	0.99	1.00	0.99
	(0.05)	(0.03)	(0.12)	(0.02)	(0.00)	(0.00)	(0.01)
100	0.99	0.54	2.21	0.35	0.99	1.00	0.99
	(0.05)	(0.03)	(0.12)	(0.02)	(0.00)	(0.00)	(0.01)

Panel C: $1/\gamma = 0.2$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.83	0.69	1.33	0.12	1.00	1.00	0.99
	(0.04)	(0.03)	(0.07)	(0.01)	(0.00)	(0.00)	(0.00)
100	0.83	0.69	1.28	0.11	1.00	1.00	0.99
	(0.04)	(0.03)	(0.07)	(0.01)	(0.00)	(0.00)	(0.00)

Notes: the table shows second moments implied by the model for different values of IES, $1/\gamma$, and risk aversion, θ, for the model without capital adjustment costs, $\eta = 0$. The rest of the parameters are given in Table 2.1. Series in growth rates, and standard errors in parenthesis.

Hours worked also decrease their volatility when there are capital adjustment costs; for instance, when a positive shock hits the economy the adjustment is smaller and so the required increase in hours worked (compare column (4) across Tables 2.3, 2.4 and 2.5). Finally, it is interesting to note that the cross-correlation between output and consumption, investment and hours worked, increase as the costs of adjustment in this economy increase (compare columns (5), (6) & (7) across Tables 2.3, 2.4 and 2.5).

Table 2.4: Business Cycle Statistics Changing $1/\gamma$ and θ for $\eta = 0.2$

Panel A: $1/\gamma = 1.5$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.88	0.64	1.53	0.18	1.00	1.00	0.99
	(0.04)	(0.03)	(0.08)	(0.01)	(0.00)	(0.00)	(0.01)
100	0.88	0.64	1.52	0.18	1.00	1.00	0.99
	(0.04)	(0.03)	(0.07)	(0.01)	(0.00)	(0.00)	(0.01)

Panel B: $1/\gamma = 1$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.86	0.66	1.46	0.16	1.00	1.00	0.99
	(0.04)	(0.03)	(0.07)	(0.01)	(0.00)	(0.00)	(0.01)
100	0.86	0.65	1.44	0.16	1.00	1.00	0.99
	(0.04)	(0.03)	(0.07)	(0.01)	(0.00)	(0.00)	(0.01)

Panel C: $1/\gamma = 0.2$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.80	0.72	1.05	0.06	1.00	1.00	0.99
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)
100	0.80	0.72	1.04	0.06	1.00	1.00	0.99
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: the table shows second moments implied by the model for different values of IES, $1/\gamma$, and risk aversion, θ, for the model with low capital adjustment costs, $\eta = 0.2$. The rest of the parameters are given in Table 2.1. Series in growth rates, and standard errors in parenthesis.

2.3.1.1 Quantity dynamics

Qualitatively the business cycle dynamics of the model are the ones of the standard stochastic growth model with permanent productivity shocks studied profusely in the literature. Figures 2.1, 2.2, and 2.3 show the impulse response for selected macroeconomic variables under the benchmark parameterization for three different values of the IES, and the adjustment cost parameter.[13]

In response to a positive permanent productivity shock, consumption and investment

[13] Given that ϕ is chosen so that costs of adjustment are zero in steady state, the average adjustment costs for $\eta = 0.2$ are around 0.01% of output and for $\eta = 0.6$ around 0.02% of output. In both cases the numbers are very small.

Table 2.5: Business Cycle Statistics Changing $1/\gamma$ and θ for $\eta = 0.6$

Panel A: $1/\gamma = 1.5$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.79	0.72	0.98	0.05	1.00	1.00	0.98
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)
100	0.79	0.72	0.97	0.05	1.00	1.00	0.98
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)

Panel B: $1/\gamma = 1$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.79	0.73	0.96	0.05	1.00	1.00	0.98
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)
100	0.79	0.73	0.95	0.04	1.00	1.00	0.99
	(0.04)	(0.04)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)

Panel C: $1/\gamma = 0.2$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$
1	0.76	0.75	0.78	0.01	1.00	1.00	0.98
	(0.04)	(0.04)	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)
100	0.76	0.75	0.78	0.01	1.00	1.00	0.98
	(0.04)	(0.04)	(0.04)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: the table shows second moments implied by the model for different values of IES, $1/\gamma$, and risk aversion, θ, for the model with high capital adjustment costs, $\eta = 0.6$. The rest of the parameters are given in Table 2.1. Series in growth rates, and standard errors in parenthesis.

jump as technology instantaneously adjusts to the new steady state. Investment and hours worked immediately increase to the expectation of a permanently higher productivity in the future. Investment increases at the expense of current consumption. This increase in capital stock allows agents to gradually increase (smoothly) consumption over time towards the new steady state. Neither consumption, nor capital or output adjust instantaneously, and the length of the adjustment process depends on preference parameters (i.e. the IES and the discount factor), the capital adjustment costs, and the persistence of the productivity shock.

The initial response of consumption is larger the lower the IES and the larger the adjustment costs in the economy consistent with the notion that the volatility of consumption growth is decreasing in the IES, and increasing in the adjustment cost parameter η.[14] Intuitively, agents with low IES prefer smooth consumption streams—they are strongly averse to intertemporal substitution— and they adjust consumption on impact. Instead, the initial response of investment is smaller for lower values of the IES. When there are adjustment costs, the initial response of investment is smaller and for consumption larger. Interestingly, in the case where adjustment costs are "too high" and the IES is "too low" there is a countercyclical behavior of employment. As expected, the length of the adjustment process is longer in the economy where agents have a lower IES and adjustment costs are higher.

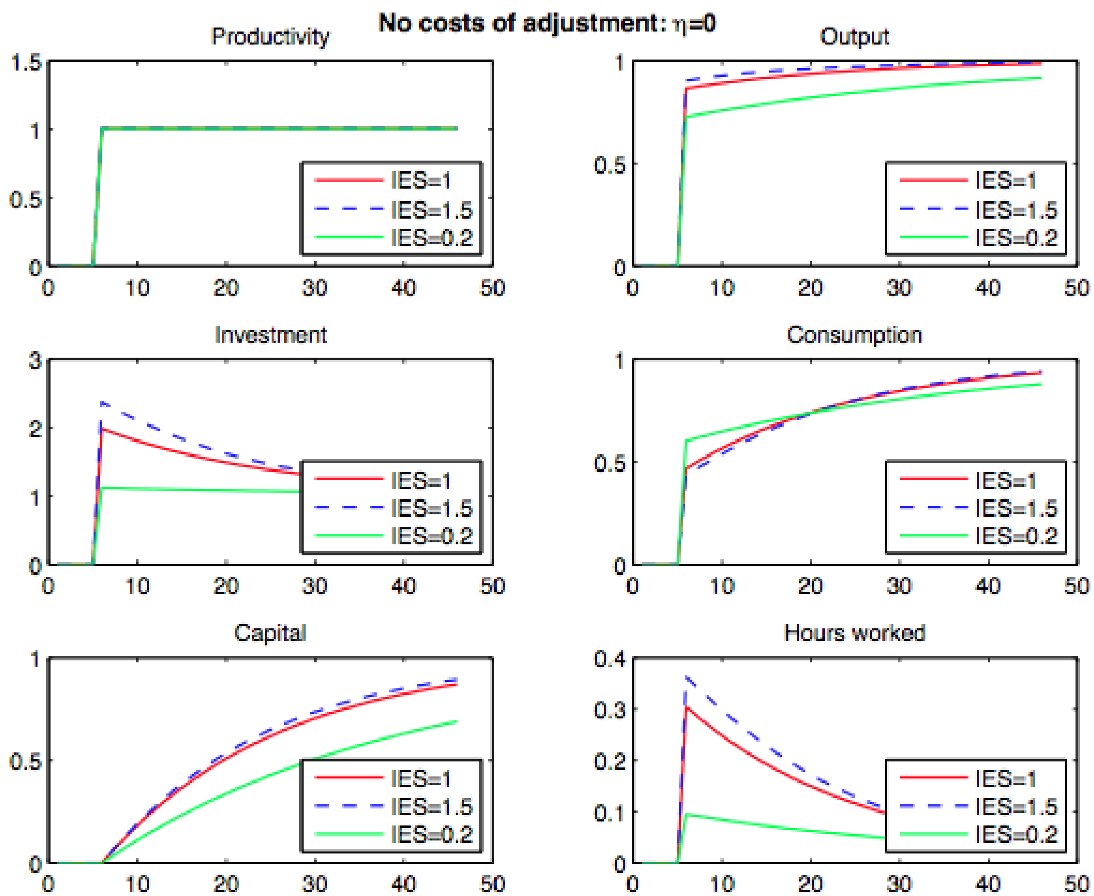
2.3.2 Asset returns

In this section I present results for the asset pricing implications of changing capital adjustment costs. In a similar framework, Tallarini (2000) matches the Sharpe ratio increasing the parameter of risk aversion. However, since his model lacks of capital adjustment costs, he does so for a low equity return and standard deviation. Adding capital adjustment costs tightens the comparison between the model prediction and the observed data along these lines.

Table 2.6 reports population moments for the risk-free rate, the equity return, the equity premium, and the Sharpe ratio for the excess return for different values of the IES and the adjustment costs parameter. First, note that increasing risk aversion increases the equity premium, decreasing both the equity return and (proportionally less) the risk-free rate. Precautionary savings increase the mean capital stock, reducing the return on equity and on the pure risk-free asset. The model with no adjustment costs and unity IES as in Tallarini (2000), still delivers a very low equity premium even when the risk aversion rises

[14] See, for instance, (Jermann, 1998; Kaltenbrunner and Lochstoer, 2010; Campanale et al., 2010).

Figure 2.1: Impulse Response Functions for Macroeconomic Quantities: No Capital Adjustment Costs

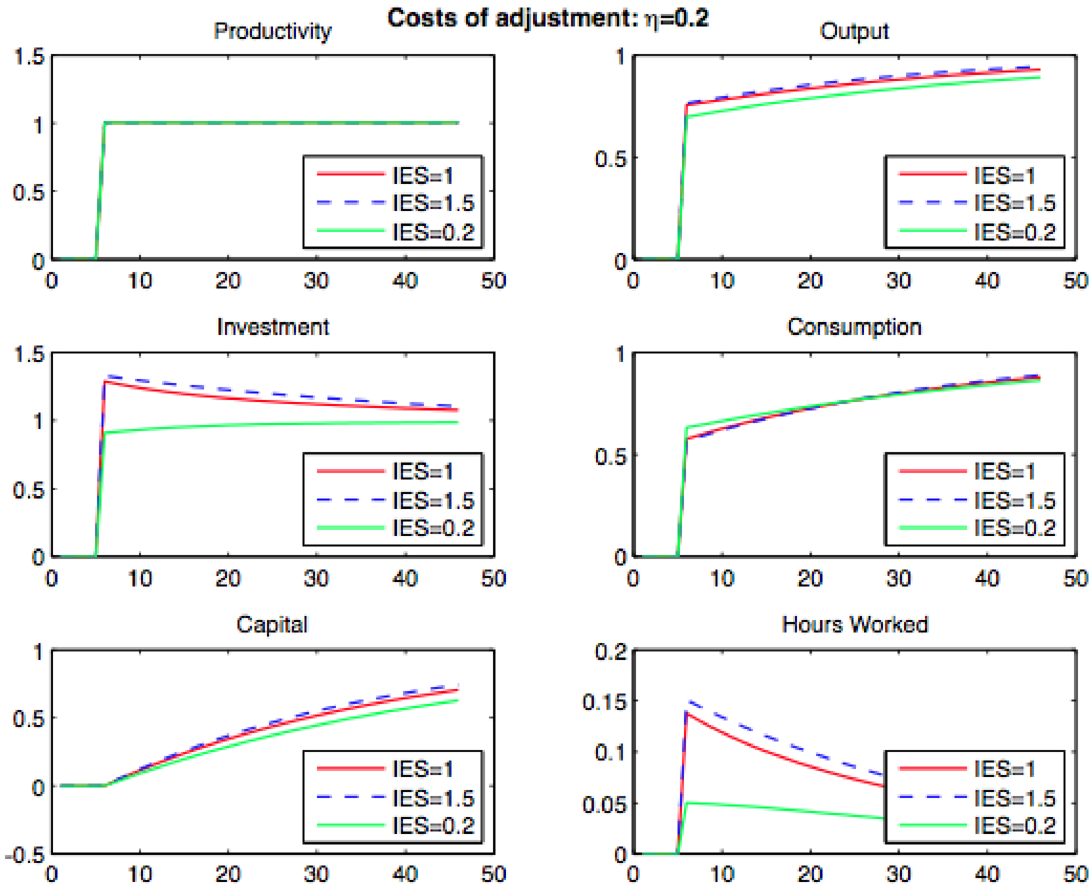


Notes: the figure shows the impulse response functions for selected macroeconomic quantities in the model without capital adjustment costs. The rest of the parameters are given in Table 2.1.

to 100 (0.01% per quarter) (see rows (1) & (2) of Panel B in Table 2.6). The reason behind the low equity premium is the low volatility in excess return: the only source of variability in the model without adjustment costs is variation in the marginal product of capital. With no frictions in the capital accumulation process the price of a unit of capital is always constant at 1. Increasing risk aversion the only asset pricing fact Tallarini can match is the Sharpe ratio.

Everything else equal, capital adjustment costs contribute to increase the mean excess

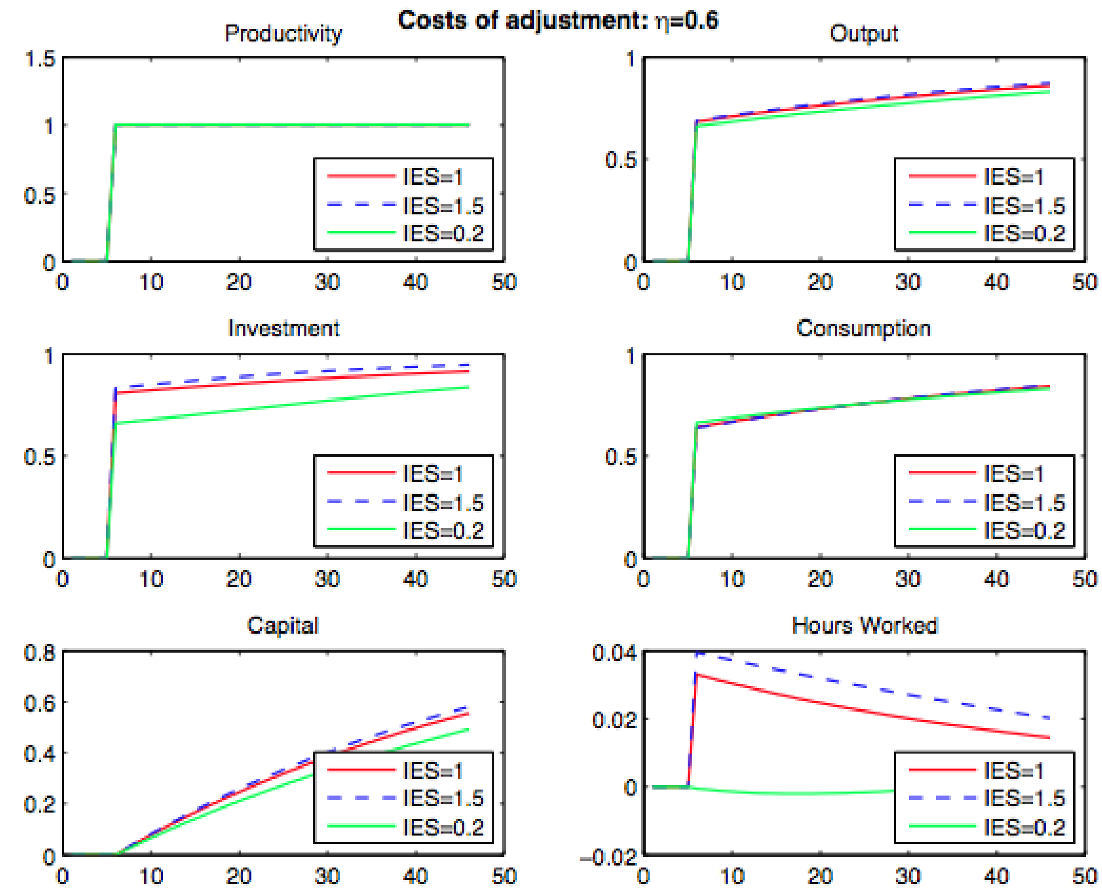
Figure 2.2: Impulse Response Functions for Macroeconomic Quantities: $\eta = 0.2$



Notes: the figure shows the impulse response functions for selected macroeconomic quantities in the model with low capital adjustment costs, $\eta = 0.2$. The rest of the parameters are given in Table 2.1.

return (see column (5) from panels A, B and C from Table 2.6). For a given change in the adjustment cost parameter, the resulting change in the risk-free rate as risk aversion increases does not depend of capital adjustment costs. The return on equity, however, exhibits a smaller decrease as risk aversion increases when the adjustment costs in the economy are higher, causing an increase in the mean equity premium. In this specification, where the price of capital is not constant, asset prices also exhibit an important amount of variation.

Figure 2.3: Impulse Response Functions for Macroeconomic Quantities: $\eta = 0.6$



Notes: the figure shows the impulse response functions for selected macroeconomic quantities in the model with high capital adjustment costs, $\eta = 0.6$. The rest of the parameters are given in Table 2.1.

Further, a lower IES not only increases the mean of the risk-free rate and its volatility, but also the mean equity return for any parameterization of the adjustment cost function reducing the equity premium. In contrast to previous studies with habit preferences (Jermann, 1998; Boldrin et al., 2001) or with Epstein-Zin preferences as (Campanale et al., 2010), my results suggest that a rather high IES is needed to generate a reasonable volatility of equity return, and a larger equity premium when there are adjustment costs in the economy. Interestingly, a rather high IES is also needed in this type of models to lower the volatility of the risk-free rate in order to match the data. A low IES implies the agent is highly averse to substituting consumption over time, increasing the volatility of the marginal rate of substitution, producing an excessive risk-free rate variation.

Table 2.6: Financial Statistics Changing $1/\gamma$, θ and η

θ	$E(R_f)$	$\sigma(R_f)$	$E(R_e)$	$\sigma(R_e)$	$E(R_e - R_f)$	$\sigma(R_e - R_f)$	S.R.
Data	0.23	0.80	2.20	7.92	1.97	7.82	0.25

Panel A: $1/\gamma = 1.5$

θ	$E(R_f)$	$\sigma(R_f)$	$E(R_e)$	$\sigma(R_e)$	$E(R_e - R_f)$	$\sigma(R_e - R_f)$	S.R.
				$\eta = 0$			
1	1.11	0.09	1.11	0.09	0.00	0.04	0.01
100	0.98	0.08	0.99	0.09	0.01	0.03	0.26
				$\eta = 0.2$			
1	1.11	0.05	1.12	0.33	0.00	0.33	0.01
100	0.95	0.05	1.04	0.33	0.09	0.33	0.26
				$\eta = 0.6$			
1	1.12	0.03	1.12	0.61	0.01	0.61	0.01
100	0.93	0.03	1.10	0.61	0.16	0.61	0.27

Panel B: $1/\gamma = 1$

θ	$E(R_f)$	$\sigma(R_f)$	$E(R_e)$	$\sigma(R_e)$	$E(R_e - R_f)$	$\sigma(R_e - R_f)$	S.R.
				$\eta = 0$			
1	1.15	0.09	1.15	0.09	0.00	0.03	0.01
100	1.01	0.09	1.02	0.09	0.01	0.03	0.26
				$\eta = 0.2$			
1	1.15	0.05	1.15	0.3	0.00	0.32	0.01
100	0.98	0.05	1.06	0.32	0.08	0.32	0.26
				$\eta = 0.6$			
1	1.15	0.03	1.16	0.60	0.01	0.60	0.01
100	0.97	0.03	1.13	0.60	0.16	0.60	0.26

Panel C: $1/\gamma = 0.2$

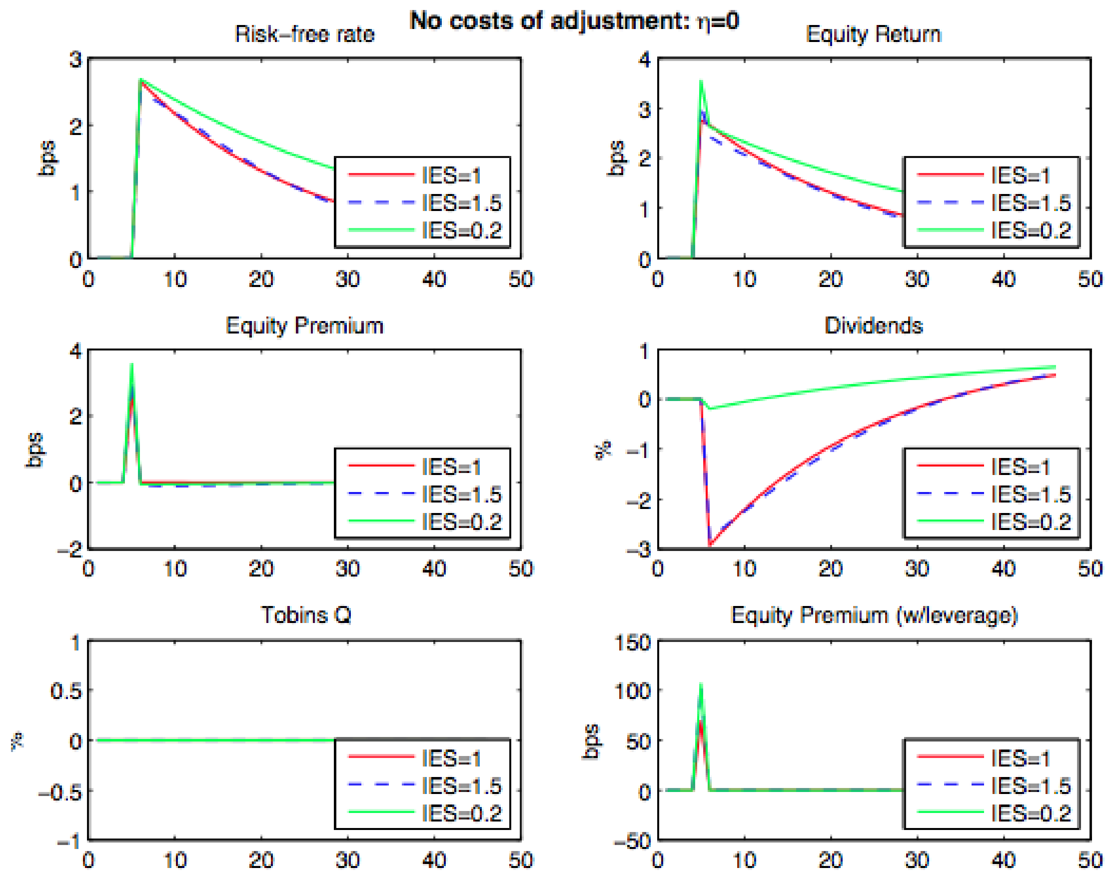
θ	$E(R_f)$	$\sigma(R_f)$	$E(R_e)$	$\sigma(R_e)$	$E(R_e - R_f)$	$\sigma(R_e - R_f)$	S.R.
				$\eta = 0$			
1	1.56	0.10	1.56	0.11	0.00	0.03	0.01
100	1.36	0.10	1.37	0.10	0.01	0.02	0.23
				$\eta = 0.2$			
1	1.56	0.08	1.56	0.26	0.00	0.25	0.01
100	1.34	0.08	1.40	0.24	0.06	0.23	0.25
				$\eta = 0.6$			
1	1.58	0.06	1.59	0.51	0.00	0.51	0.01
100	1.33	0.06	1.46	0.50	0.13	0.50	0.26

Notes: the table shows mean and standard deviation of returns implied by the model for a claim to dividends and pure risk-free asset for different values of IES, $1/\gamma$, risk aversion, θ, and capital adjustment costs, η. The rest of the parameters are given in Table 2.1.

Figures 2.4, 2.5 and 2.6 illustrate the impulse responses of asset returns for different values of the IES and the costs of adjustment parameter. As expected, the dynamic of the risk-free rate is dictated by the pattern of consumption growth showed in figures 2.1, 2.2, and 2.3. On impact the response of the risk-free rate is positive, but then it falls back again as consumption transits towards the new steady state. In addition, the response is higher for lower values of the IES independent whether there are costs of adjustment to capital in the economy. As discussed before, a higher IES and larger adjustment costs significantly increase the volatility of equity return and excess return. In contrast, when there are no frictions in capital accumulation a rather low IES is required to generate a higher volatility of equity return and of the risk premia.

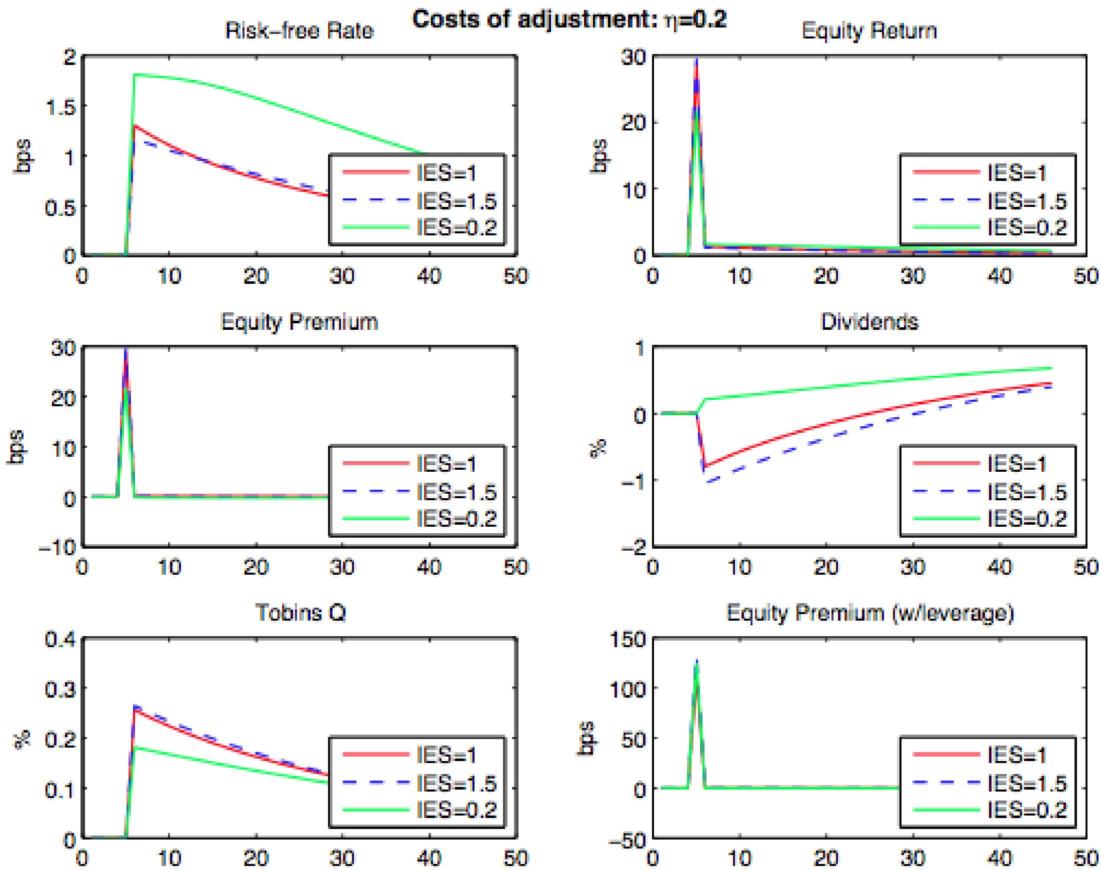
The bottom-left panel of figures 2.4, 2.5 and 2.6 reproduce the response of the price of capital, i.e. the cost of redirecting the marginal unit from current consumption to capital accumulation. In the model without capital adjustment costs, Tobin's Q is always exactly equal to 1 and the only source of variability in asset prices comes from the marginal product of capital (see Figure 2.4). In contrast, with convex capital adjustment costs Tobin's Q depends on $\phi'(\cdot)$ and hence on investment dynamics. Given the costs of adjustment function is concave, a higher investment rate implies a higher price of capital, and an increase in the cost of transforming consumption into capital, increasing the volatility of asset prices.

Figure 2.4: Impulse Response Functions for Financial Variables: No Capital Adjustment Costs



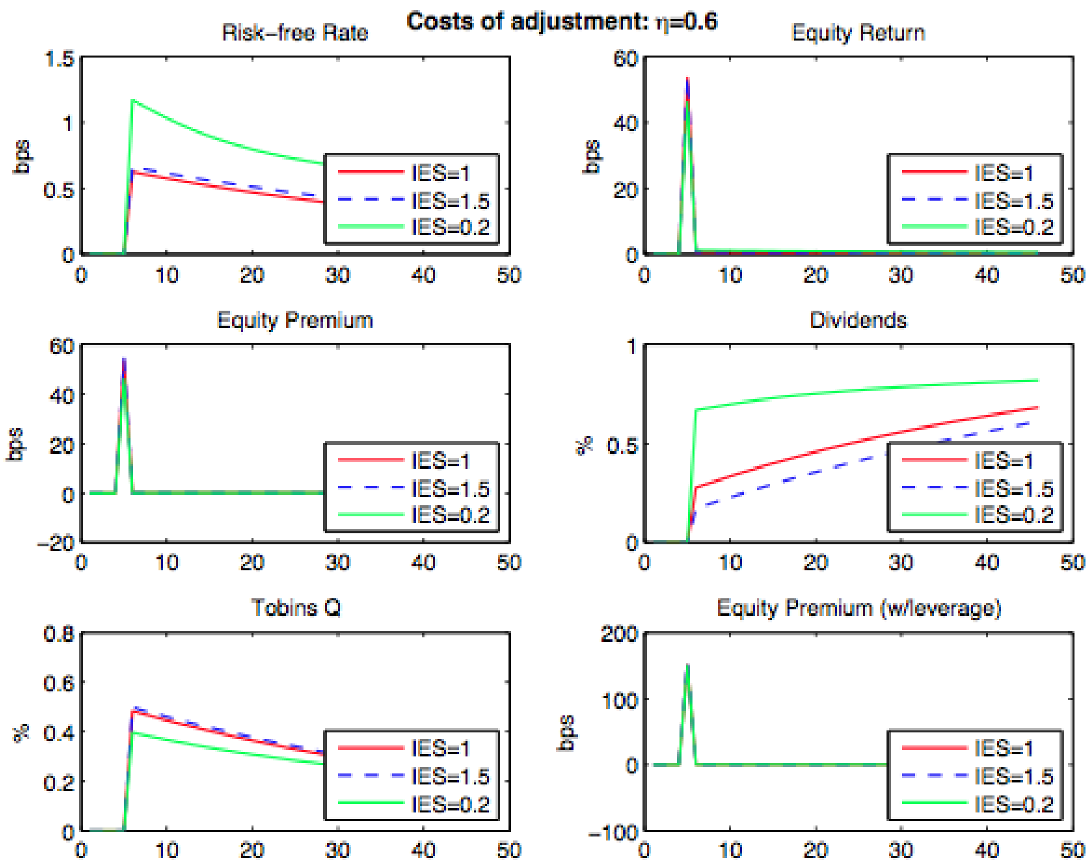
Notes: the figure shows the impulse response functions for selected financial variables in the model without capital adjustment costs. The rest of the parameters are given in Table 2.1.

Figure 2.5: Impulse Response Functions for Financial Variables: $\eta = 0.2$



Notes: the figure shows the impulse response functions for selected financial variables in the model with low capital adjustment costs, $\eta = 0.2$. The rest of the parameters are given in Table 2.1.

Figure 2.6: Impulse Response Functions for Financial Variables: $\eta = 0.6$



Notes: the figure shows the impulse response functions for selected financial variables in the model with high capital adjustment costs, $\eta = 0.6$. The rest of the parameters are given in Table 2.1.

Table 2.7 presents the volatility of dividend growth and its correlation with output growth. Figures 2.4, 2.5 and 2.6 display dividends impulse response for the three calibrations of the IES and the adjustment costs. Consistent with the results in Jermann (1998), for low IES (equivalent to having habit formation) dividends are procyclical with capital adjustment costs. Without capital adjustment costs they even turn to be countercyclical on impact. In the case when the IES is greater or equal to unity, higher costs of adjustment (i.e. $\eta = 0.6$ in this case) are required for dividends to display a cyclical behavior on impact. Intuitively, with adjustment costs the investment/capital ratio deviates less from its steady-state value leading to more procyclical dividends and a larger premia for payout uncertainty.

Table 2.7: Dividends Changing $1/\gamma$, θ and η

θ	σ(Div)	$\rho D, Y$	σ(Div)	$\rho D, Y$	σ(Div)	$\rho D, Y$
	$1/\gamma = 0.2$		$1/\gamma = 1$		$1/\gamma = 1.5$	
			$\eta = 0$			
1	0.28	0.22	3.47	0.13	4.39	0.10
	(0.03)	(0.07)	(0.30)	(0.08)	(0.39)	(0.08)
100	0.36	0.24	4.17	0.13	5.31	0.10
	(0.02)	(0.07)	(0.40)	(0.08)	(0.55)	(0.08)
			$\eta = 0.2$			
1	0.26	-0.25	1.17	0.22	1.48	0.21
	(0.02)	(0.07)	(0.11)	(0.07)	(0.13)	(0.07)
100	0.21	-0.23	1.37	0.22	1.72	0.21
	(0.03)	(0.07)	(0.13)	(0.07)	(0.18)	(0.07)
			$\eta = 0.6$			
1	0.72	-0.30	0.24	-0.23	0.18	-0.19
	(0.04)	(0.07)	(0.04)	(0.07)	(0.02)	(0.07)
100	0.71	-0.30	0.24	-0.25	0.18	-0.17
	(0.04)	(0.07)	(0.02)	(0.07)	(0.02)	(0.08)

Notes: the table shows standard deviation of dividends and the correlation with output growth implied by the model for different values of IES, $1/\gamma$, risk aversion, θ, and capital adjustment costs, η. The rest of the parameters are given in Table 2.1. Series in growth rates, and standard errors in parenthesis.

Usually in the literature equity market dividends are defined as a levered claim to output such that $D_t^{LEV} = Y_t^{\lambda}$, where $\lambda > 1$ is the leverage parameter. This, with the aim of improving the fit of dividend growth volatility, the high equity return volatility and the equity risk premium. For instance, with a leverage factor of 2 on the output claim, the resulting equity return in the model with IES=1, no adjustment costs and a risk aversion of 10 would be around 2.3% quarterly (instead of 1.1%), with a return volatility of 0.8% (instead of 0.01%). In the same way, the response of the equity return, the equity premium and dividends to a positive technology shock gets amplified substantially. There is an important trade-off between leverage and dividend growth volatility that should be taken into account when calibrating this type of models (see Table 2.8).

Table 2.8: Financial Statistics Changing θ and η for $1/\gamma = 1$

θ	$E(R_{e,LEV})$	$\sigma(R_{e,LEV})$	$E(R_{e,LEV} - R_f)$	$\sigma(R_{e,LEV} - R_f)$	$\sigma(\text{Div}_{LEV})$	$\rho_{\text{Div}_{LEV},Y}$
			$\eta = 0$			
1	1.16	1.45	0.01	1.45	2.37	0.12
10	2.33	0.79	1.20	0.79	2.45	0.12
25	3.16	0.43	2.04	0.43	2.57	0.12
			$\eta = 0.2$			
1	1.16	1.65	0.01	1.65	0.32	0.00
10	2.36	1.25	1.23	1.25	0.32	0.01
25	3.24	1.06	2.13	1.06	0.32	0.05
			$\eta = 0.6$			
1	1.17	1.82	0.02	1.82	1.35	-0.28
10	2.39	1.70	1.26	1.71	1.39	-0.28
25	3.32	1.64	2.21	1.65	1.35	-0.28

Notes: the table shows mean and standard deviation of returns implied by the model for a claim on levered output for different values of risk aversion, θ, capital adjustment costs, η, and $1/\gamma = 1$. The rest of the parameters are given in Table 2.1.

2.3.3 Predictability of asset returns

The empirical finance literature has widely documented that a series of variables are capable of forecasting asset returns. If this is the case, then is must be true that expected returns are not constant but vary over time.[15] Moreover, these variables typically show a strong correlation with the business cycle, i.e. expected returns tend to be higher in recession and higher in economic booms. Therefore, if stock prices change because expected returns change and not because expected future dividend growth changes, then market movements must be related to macroeconomics through time-varying risk premia rather than cash flows.

Table 2.9 reports the volatility of the time-varying equity premium for different values of the IES, risk aversion and the elasticity of the cost of adjustment function. The results show the model is incapable of generating economically significant time-variation in the equity premium. In fact, even when we observe an increase in the volatility as risk aversion

[15] For a review of the literature see Cochrane (2007).

increases and also as adjustment costs increase, in every case they are extremely low. In this type of models, variability in expected returns is driven by the risk-free rate variation, which is substantially lower than what is observed in the data.

Table 2.9: Expected Equity Premium Changing $1/\gamma$, θ and η

	$\sigma(E_t[R_e - R_f])$					
	No levarage			With leverage		
	$1/\gamma = 0.2$	$1/\gamma = 1$	$1/\gamma = 1.5$	$1/\gamma = 0.2$	$1/\gamma = 1$	$1/\gamma = 1.5$
			$\eta = 0$			
1	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.001	0.000	0.000
25	0.001	0.000	0.000	0.001	0.004	0.001
100	0.002	0.001	0.001	0.002	0.001	0.001
			$\eta = 0.2$			
1	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000
25	0.001	0.000	0.000	0.001	0.003	0.000
100	0.003	0.001	0.001	0.003	0.001	0.001
			$\eta = 0.6$			
1	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.001	0.001	0.000
25	0.001	0.000	0.000	0.001	0.001	0.000
100	0.002	0.001	0.001	0.003	0.001	0.001

Notes: the table shows the standard deviation of the expected equity premium implied by the model for different values of IES, $1/\gamma$, risk aversion, θ, and capital adjustment costs, η. The rest of the parameters are given in Table 2.1.

2.3.4 Time-Varying Risk Premia

In this section I investigate whether introducing some form of time-varying risk premia to the standard real business cycle model presented in the previous section affects consumption, investment, output and employment dynamics. The aim is to investigate whether Tallarini's "separation theorem" still hold in this setting.

To allow for a time-varying risk premia I introduce changes in the conditional volatility of productivity growth rates following the literature on long-run consumption risk (Bansal and Yaron, 2004). In particular, to account for economic uncertainty I model volatility of TFP shocks as being time-varying. The dynamics for the TFP process now are represented by

$$\begin{aligned} \log z_{t+1} &= \mu + \log z_t + \sigma_{t+1}\varepsilon_{t+1} - \frac{\sigma_{t+1}^2}{2}, \\ \sigma_{t+1} &= \rho\sigma_t + \sigma_w w_{t+1}, \end{aligned} \tag{2.19}$$

where w_{t+1} is standard normally distributed i.i.d random shocks and σ_{t+1} represents the time-varying economic uncertainty incorporated in the TFP growth.

Table 2.10 presents the standard deviations, cross-correlations and relative volatilities of consumption, investment and hours worked with output for the modified model with IES=1, θ=$\{1,10,100\}$ and different values for the adjustment cost parameter. Clearly, in this case the effect of increased risk aversion on consumption, investment, hours worked and output is significant, particularly for the case without adjustment costs in the economy. On the one hand, costs of adjustment contribute to increase the amount of risk in the economy determining a different response for increased levels of risk aversion, but on the other hand they restrict the degree of adjustment in the economy when a shock hits. Hence, matching asset-pricing moments is not anymore just a matter of increasing the degree of risk aversion, while maintaining the IES unchanged as in Tallarini (2000). Quantity dynamics are affected by the degree of risk aversion in the economy.

Not surprisingly, incorporating some form of time-varying risk premia also contributes to improve the asset-pricing facts of the model. In the same way as before, increasing risk aversion increases the equity premium, decreasing the equity return and the risk-free rate. Even when for both calibrations of the model, no adjustment costs and $\eta = 0.2$, equity premium is still low, it is higher than in the benchmark model. Variation in expected returns is not only driven by variation in the risk-free rate but also by the stochastic volatility. The results in Table 2.11 shows that the modified model is capable of generating economically significant time-variation in the equity premium.

Table 2.10: Business Cycle Statistics Changing θ and η for $1/\gamma = 1$

θ	$\sigma(Y)$	$\sigma(C)$	$\sigma(I)$	$\sigma(N)$	$\rho_{C,Y}$	$\rho_{I,Y}$	$\rho_{N,Y}$	$\frac{\sigma(C)}{\sigma(Y)}$	$\frac{\sigma(I)}{\sigma(Y)}$	$\frac{\sigma(N)}{\sigma(Y)}$
					$\eta = 0$					
1	1.08	0.59	2.52	0.39	0.99	0.99	0.99	0.55	2.32	0.36
	(0.07)	(0.04)	(0.17)	(0.02)	(0.00)	(0.00)	(0.01)			
10	0.97	0.53	2.24	0.35	0.99	0.99	0.98	0.55	2.31	0.36
	(0.06)	(0.03)	(0.15)	(0.02)	(0.00)	(0.00)	(0.01)			
100	1.13	0.64	2.69	0.47	0.90	0.96	0.90	0.56	2.38	0.42
	(0.07)	(0.04)	(0.16)	(0.03)	(0.01)	(0.00)	(0.01)			
					$\eta = 0.2$					
1	0.91	0.69	1.54	0.17	1.00	1.00	0.99	0.76	1.69	0.19
	(0.06)	(0.04)	(0.10)	(0.01)	(0.00)	(0.00)	(0.01)			
10	0.93	0.70	1.57	0.18	1.00	1.00	0.99	0.76	1.69	0.19
	(0.06)	(0.04)	(0.10)	(0.01)	(0.00)	(0.00)	(0.01)			
100	0.94	0.71	1.60	0.20	0.99	0.99	0.93	0.75	1.71	0.21
	(0.06)	(0.04)	(0.10)	(0.01)	(0.00)	(0.00)	(0.01)			

Notes: the table shows second moments implied by the model with time-varying risk premia for different values of risk aversion, θ and capital adjustment costs, η for $1/\gamma = 1$. The rest of the parameters are given in Table 2.1. Series in growth rates, and standard errors in parenthesis.

Table 2.11: Financial Statistics Changing θ and η for $1/\gamma = 1$

Panel A: $1/\gamma = 1.5$

θ	$E(R_f)$	$\sigma(R_f)$	$E(R_e)$	$\sigma(R_e)$	$E(R_e - R_f)$	$\sigma(R_e - R_f)$	S.R.	$\sigma(E_t[R_e - R_f])$
					$\eta = 0$			
1	1.14	0.09	1.14	0.10	0.00	0.03	0.01	0.000
10	1.13	0.09	1.13	0.09	0.00	0.03	0.02	0.000
100	0.97	0.10	0.98	0.10	0.01	0.04	0.27	0.004
					$\eta = 0.2$			
1	1.14	0.06	1.14	0.34	0.00	0.34	0.01	0.001
10	1.12	0.06	1.13	0.35	0.01	0.34	0.03	0.005
100	0.94	0.10	1.03	0.35	0.10	0.35	0.28	0.047

Notes: the table shows mean and standard deviation of returns implied by the model for a claim to dividends and pure risk-free asset for different values of risk aversion, θ, and capital adjustment costs, η for $1/\gamma = 1$. The rest of the parameters are given in Table 2.1.

2.4 Conclusion

In this paper I analyze the equilibrium effects of convex capital adjustment costs on quantity dynamics and asset prices when individuals have Epstein-Zin preferences. The introduction of both features in an otherwise standard RBC is successful: the model can account for the main statistical features of macroeconomic aggregate quantities, while at the same generate a sizable risk premium, with the mean stock return and its standard deviation in orders of magnitude closer to the data. Further, it is possible to obtain a stable and rather low risk-free rate.

I also show that regardless of the calibration for the IES and the adjustment costs elasticity, the behavior of aggregate quantities does not depend on attitudes towards risk. Moreover, several calibrations for the costs of adjustment elasticity are analyzed and alternative specifications for the cost of adjustment function are used, but in none of the cases risk aversion has an effect in quantity dynamics.

Despite the improvements of the model in terms of matching quantity dynamics and asset returns, the model is incapable of generating economically significant time-variation in risk premia and, consequently, replicate the predictability of excess returns as observed in the data. I introduce changes in the conditional volatility of productivity growth rates in the spirit of the literature of long-run consumption risk to allow for a time-varying risk premia. In this case, the model's asset-pricing predictions improve and when risk is time-varying macroeconomic quantities are affected by attitude towards risk.

A.1 Derivation stochastic discount factor

Appendix A.1 contains a short derivation of the stochastic discount factor (SDF) under Epstein and Zin (1989) preferences.

To compute the stochastic discount factor in this economy:

$$M_{t,t+1} = \frac{\partial V_t/\partial c_{t+1}}{\partial V_t/\partial c_t}, \tag{1}$$

note that the derivative of the value function today with respect to the consumption today:

$$\frac{\partial V_t}{\partial c_t} = (1-\beta)V_t^{\gamma} v \frac{(C_t^v(1-N_t)^{1-v})^{1-\gamma}}{C_t}, \tag{2}$$

and the derivative of the value function with respect to consumption tomorrow:

$$\frac{\partial V_t}{\partial c_{t+1}} = \beta V_t^{\gamma} E[V_{t+1}^{1-\theta}]^{\frac{\theta-\gamma}{1-\theta}} V_{t+1}^{-\theta} \left((1-\beta)V_{t+1}^{\gamma} v \frac{(C_{t+1}^v(1-N_{t+1})^{1-v})^{1-\gamma}}{C_{t+1}} \right). \tag{3}$$

Note that in the last step I used the result regarding $\partial V_t/\partial c_t$ forwarded one period. Then, canceling extra terms I get:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{v(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-v)(1-\gamma)} \left(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta}. \tag{4}$$

This equation shows how the pricing kernel is affected by recursive preferences–as long as the inverse of the IES coefficient γ is different from the risk aversion coefficient θ the SDF will be affected by this extra term.

In the limiting case when risk aversion $\theta = 1$ the stochastic discount factor for the economy can be computed following the same strategy. In this case, preferences for the representative agent will be given by

$$V_t = \Big((1-\beta)u(C_t, N_t)^{1-\gamma} + \beta \exp((1-\gamma)E_t(\log V_{t+1}))\Big)^{\frac{1}{1-\gamma}}, \tag{5}$$

with the per period utility function defined in the same way as before. $\gamma \geq 0$ is still defined as the inverse of the IES ψ over the consumption-leisure bundle, but now since $\theta = 1$ the certainty equivalent of tomorrow's utility term, $E_t(V_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}$, collapses to $\exp((1-\gamma)E_t(\log V_{t+1}))$. The same way as before, the pricing kernel of the economy can be computed using the results for the derivative of the value function today with respect to the consumption today:

$$\frac{\partial V_t}{\partial c_t} = (1-\beta)V_t^{\gamma} v \frac{(C_t^v(1-N_t)^{1-v})^{1-\gamma}}{C_t}, \tag{6}$$

and for the derivative of the value function with respect to consumption tomorrow:

$$\frac{\partial V_t}{\partial c_{t+1}} = \beta V_t^{\gamma} \exp\left((1-\gamma)E_t[\log V_{t+1}]\right)(1-\beta)V_{t+1}^{\gamma-1} v \frac{(C_{t+1}^v(1-N_{t+1})^{1-v})^{1-\gamma}}{C_{t+1}}. \tag{7}$$

Then, canceling extra terms we have

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{v(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-v)(1-\gamma)} \left(\frac{V_{t+1}}{\exp(E_t[\log V_{t+1}])}\right)^{\gamma-1}. \tag{8}$$

A.1.1 Stochastic discount factor unity IES case

By definition when the IES ψ over consumption-leisure bundle is restricted to equal one as in Tallarini (2000), the parameter γ will also equal unity in the general specification for preferences presented. In this case, preferences will be given by

$$V_t = u(C_t, N_t)^{1-\beta} E_t(V_{t+1}^{1-\theta})^{\frac{\beta}{1-\theta}}, \tag{9}$$

with the per period utility function given by $u(C, N) = C^{\nu}(1-N)^{1-\nu}$ as before. The risk aversion towards static gambles over the bundle is still determined by $\theta \geq 0$. To compute the SDF in this economy we follow the same approach–compute the derivative of the value function today with respect to the consumption today as

$$\frac{\partial V_t}{\partial c_t} = (1-\beta)\frac{V_t}{C_t}v, \tag{10}$$

and the derivative of the value function with respect to consumption tomorrow using the result in $\partial V_t / \partial c_t$ forwarded one period

$$\frac{\partial V_t}{\partial c_{t+1}} = (1-\beta)\frac{V_t}{C_{t+1}}v\beta E[V_{t+1}^{1-\theta}]^{-1}V_{t+1}^{1-\theta}. \tag{11}$$

The SDF will be given by the formula

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} \frac{V_{t+1}^{1-\theta}}{E_t(V_{t+1}^{1-\theta})}. \tag{12}$$

In the particular case when $\theta = 1$ preferences collapse to the familiar expected utility case. The utility function for the representative agent will be given by

$$V_t = u(C_t, N_t)^{1-\beta} \exp\left(\beta E_t[\log V_{t+1}]\right), \tag{13}$$

with the per period utility function defined as before. Following the same approach described before we end up with the old power formula from the standard stochastic neoclassical growth model. Note that the derivative of the value function today with respect to the consumption today is given by

$$\frac{\partial V_t}{\partial c_t} = (1-\beta)v\frac{V_t}{C_t}, \tag{14}$$

and the derivative of the value function with respect to consumption tomorrow by

$$\frac{\partial V_t}{\partial c_{t+1}} = (1-\beta)\beta \frac{V_t}{C_{t+1}} v. \tag{15}$$

Thus, using these results the SDF will be given by the more familiar formula

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1}. \tag{16}$$

In this case, expected returns will be proportional to the covariance between returns and consumption growth, with the risk aversion coefficient $\theta = 1$ as the constant of proportionality.

B.2 Computational method

Appendix B.2 contains a brief explanation of the computational method utilized to solve the presented model. The computational method used to solve the model is exactly as in Gourio (2012). In this section I will explain the numerical method in extent for both the cases of permanent and transitory shocks, only for the more general case of preferences and the case when the IES ψ equals unity. The specific derivations required to solve the model for the limiting case when risk aversion $\theta = 1$, will be just stated in footnotes.

B.2.1 Permanent shocks

The modified Bellman equation for the stationary problem is:

$$g(k) = \max_{c,i,N} \left\{ (1-\beta)(c^v(1-N)^{1-v})^{1-\gamma} + \beta e^{\mu v(1-\gamma)} \left(E_{\varepsilon'} \left[e^{\sigma \varepsilon' v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right] \right)^{\frac{1-\gamma}{1-\theta}} \right\}, \tag{17}$$

subject to

$$\begin{aligned} c + i &= k^{\alpha} N^{1-\alpha} \\ k' &= \tfrac{(1-\delta)k + \phi(\frac{i}{k})k}{e^{\mu+\sigma\varepsilon'}} \end{aligned}$$

Note that when the calibration indicates $\gamma > 1$ the max needs to be changed to a min because we take a power $1/1-\gamma$. The numerical approximation to the solution of the model is obtained as follows:

1. **Discretize the space for the state and control variables**: pick a grid for k, i and the normal shock ε. Compute $\pi(\varepsilon)$ associated to each possible realization of the shock in the grid. I used 120 points for the grid for k, 1,200 points for the grid for i, and 10 points for the grid for ε.

2. **Value function iteration**:

   (a) Compute for each k and i in the grid the value for $N(k,i)$ that solves:

   $$R(i,k) = \max_{N}(k^{\alpha}N^{1-\alpha} - i)^{\nu(1-\gamma)}(1-N)^{(1-\nu)(1-\gamma)} \tag{18}$$

   (b) Now that the state and action space are discrete this problem can be solved as a standard discrete dynamic programming problem. The Bellman equation can be written as

   $$g(k) = \max_{i}\left\{ R(i,k) + \beta e^{\mu v(1-\gamma)} \left(\sum_{\varepsilon'} \pi(\varepsilon') e^{\sigma\varepsilon' v(1-\theta)} g(k'(i,k,\varepsilon')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}, \tag{19}$$

   subject to

   $$k' = \frac{((1-\delta)k + \phi(\frac{i}{k})k)}{e^{\mu+\sigma\varepsilon'}}$$

   When computing the expectation term the value function will lie outside the grid thus the *spline* interpolation method is used to approximate it.[16]

3. **Computing the value and policy functions**: computed g, the value function for the original non-stationary specification can be recovered $V(K,Z) = Z^v g(k)$, and the policy functions for $C = c(k)Z$, $I = i(k)Z$, $N = N(k)$ and $Y = k^\alpha N(k)^{1-\alpha} Z$ can be computed.

4. **Computing asset prices**: for this purpose we first need to compute the SDF for this economy. We know the SDF is given by equation (2) and using homogeneity, the SDF between two states of the world $s = (k)$ and $s = (k')$ can be expressed as

$$\begin{aligned} M(s,s',\varepsilon') &= \beta \left(\frac{Z'c(k')}{Zc(k)}\right)^{v(1-\gamma)-1} \left(\frac{1-N(k')}{1-N(k)}\right)^{(1-v)(1-\gamma)} \left(\frac{Z'^v g(k')^{\frac{1}{1-\gamma}}}{E_{z'}\left(Z'^{v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta} \\ &= \beta \left(\frac{Z'}{Z}\right)^{(\gamma-\theta)v+v(1-\gamma)-1} \left(\frac{c(k')}{c(k)}\right)^{v(1-\gamma)-1} \left(\frac{1-N(k')}{1-N(k)}\right)^{(1-v)(1-\gamma)} \times \ldots \\ &\ldots \left(\frac{g(k')^{\frac{1}{1-\gamma}}}{E_{z'}\left((\frac{Z'}{Z})^{v(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta} \end{aligned} \tag{20}$$

From here it is easy to obtain the price (return) of any asset with payoff $d(k',\varepsilon')$ using the well-know asset pricing formula $P(k) = E_{\varepsilon'} M(s,s',\varepsilon') d(k',\varepsilon')$[17].

[16] In the case when $\theta = 1$ the stationary recursive formulation for this economy will be given by:

$$g(k) = \max_i \left\{(1-\beta)(c^v(1-N)^{1-v})^{1-\gamma} + \beta \exp\left(E_{\varepsilon'}\left[\log\left(\exp((\mu+\sigma\varepsilon')(1-\gamma)v) g(k')\right)\right]\right)\right\},$$

subject to

$$\begin{aligned} c + i &= k^\alpha N^{1-\alpha} \\ k' &= \frac{(1-\delta)k + \phi(\frac{i}{k})k}{e^{\mu+\sigma\varepsilon'}} \end{aligned} .$$

Note that for the transformation we need to redefine $V(K,z) = Z^\nu g(k)^{\frac{1}{1-\gamma}}$

[17] In the limiting case when $\theta = 1$, the SDF will be given by the derived equation in the previous section of this appendix

$$\begin{aligned} M(s,s',\varepsilon') &= \beta \left(\frac{Z'c(k')}{Zc(k)}\right)^{v(1-\gamma)-1} \left(\frac{1-N(k')}{1-N(k)}\right)^{(1-v)(1-\gamma)} \left(\frac{Z'^v g(k')^{\frac{1}{1-\gamma}}}{\exp\left(E_{z'}\left[\log\left(Z'^v g(k')^{\frac{1}{1-\gamma}}\right)\right]\right)}\right)^{\gamma-1} \\ &= \beta \left(\frac{Z'}{Z}\right)^{-1} \left(\frac{c(k')}{c(k)}\right)^{v(1-\gamma)-1} \left(\frac{1-N(k')}{1-N(k)}\right)^{(1-v)(1-\gamma)} \left(\frac{g(k')^{\frac{1}{1-\gamma}}}{\exp\left(E_{z'}\left[\log\left((\frac{Z'}{Z})^v g(k')^{\frac{1}{1-\gamma}}\right)\right]\right)}\right)^{\gamma-1} \end{aligned}$$

(a) Using the formula the price for a one-period pure risk-free asset with $d = 1$ is given by

$$P(k) = E_{\varepsilon'} M(s, s', \varepsilon')$$
$$= \frac{\left(\begin{array}{l} \beta \sum_{\varepsilon'} \pi(\varepsilon') e^{((\gamma-\theta)v + v(1-\gamma)-1)(\mu+\sigma\varepsilon')} c(k')^{v(1-\gamma)-1} \times ... \\ (1-N(k'))^{(1-v)(1-\gamma)} g(k')^{\frac{\gamma-\theta}{1-\gamma}} \end{array} \right)}{c(k)^{v(1-\gamma)-1}(1-N(k))^{(1-v)(1-\gamma)} \left(\sum_{\varepsilon'} \pi(\varepsilon') e^{v(1-\theta)(\mu+\sigma\varepsilon')} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{\gamma-\theta}{1-\theta}}} \tag{21}$$

Then, the risk-free asset return is simply $R^f(k) = \frac{1}{P(k)}$.

(b) The price of a claim to the representative firm's stream of dividends $\{D_t\}$ can be computed similarly using the standard formula. Define $D_t = Y_t - w_t N_t - I_t$ and let P_t denote the price of this equity which satisfies the standard recursion

$$P_t = E_t(M_{t,t+1}(P_{t+1} + D_{t+1})) \tag{22}$$

Given that dividends can be expressed as $D = Zd(k)$, where $d(k) = \alpha k^{\alpha} N^{1-\alpha} - i$, the firm value recursion can be written as $P = Zf(k)$ with

$$f(k) = E_{s'|s} \left(M(s', s) \times (\frac{Z'}{Z})(d(k') + f(k')) \right) \tag{23}$$

Using the expression for the SDF,

$$
\begin{aligned}
f(k) &= \beta E_{\varepsilon'}\left[\begin{array}{c} (\frac{Z'}{Z})^{(\gamma-\theta)v+v(1-\gamma)}(\frac{c(k')}{c(k)})^{v(1-\gamma)-1}(\frac{1-N(k')}{1-N(k)})^{(1-v)(1-\gamma)} \times \ldots \\ \left(\frac{g(k')^{\frac{1}{1-\gamma}}}{E_{\varepsilon'}\left((\frac{Z'}{Z})^{v(1-\theta)}g(k')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta} (d(k')+f(k')) \end{array}\right] \\
&= \frac{\left(\begin{array}{l} \beta\sum_{\varepsilon'}\pi(\varepsilon')e^{((\gamma-\theta)v+v(1-\gamma))(\mu+\sigma\varepsilon')}c(k')^{v(1-\gamma)-1}\times \\ (1-N(k'))^{(1-v)(1-\gamma)}g(k')^{\frac{\gamma-\theta}{1-\gamma}}(d(k')+f(k')) \end{array}\right)}{c(k)^{v(1-\gamma)-1}(1-N(k))^{(1-v)(1-\gamma)}(\sum_{\varepsilon'}\pi(\varepsilon')e^{v(1-\theta)(\mu+\sigma\varepsilon')}g(k')^{\frac{1-\theta}{1-\gamma}})^{\frac{\gamma-\theta}{1-\theta}}}
\end{aligned}
$$

Then, the equity return can be calculated as

$$
\begin{aligned}
R^e_{t,t+1} &= \frac{P_{t+1}+D_{t+1}}{P_t} \\
&= \frac{Z_{t+1}}{Z_t}\frac{f(k_{t+1})+d(k_{t+1})}{f(k_t)}
\end{aligned}
$$

To solve the recursion we have to iterate until convergence starting from an initial guess for instance $f(k)=0$.[18] To compute the price of the leveraged claim (i.e. claim to the levered output), define $D_t^{LEV}=Y_t^{\lambda}$ and let P_t denote the price of this levered equity satisfying the standard price recursion. Note that dividends can now be written as $D=Z^{\lambda}d(k)$, and so the firm value recursion as $P=Z^{\lambda}f(k)$ with

$$
f(k)=E_{s'|s}\left(M(s',s)\times(\frac{Z'}{Z})^{\lambda}(d(k')+f(k'))\right). \tag{24}
$$

5. **Simulation**. The model statistics are obtained by simulating 1,000 samples of length 400, starting from an arbitrary guess for capital, and cutting the first 200 periods in each sample.

[18] This same formulations can be used to compute asset prices in the limiting case when $\theta=1$ just by using the appropriate expression for the SDF as derived in the first section of this appendix and the corresponding transformation for the original value function $V(K,Z)$.

B.2.1.1 Computational method in the unity IES case

In the case when IES equals unity the computational method is exactly the same explained previously in this appendix. For this reason I just present briefly the key formulations required to solve the model. The stationary recursive formulation for this economy is given by

$$\begin{aligned} \log g(k) &= \max_i\{(1-\beta)(v\log c+(1-v)\log(1-N))+ \\ &\quad \tfrac{\beta}{1-\theta}\log\left(E_{\varepsilon'}e^{v(1-\theta)(\mu+\sigma\varepsilon')}g(k')^{1-\theta}\right)\}, \end{aligned} \tag{25}$$

subject to

$$\begin{aligned} c+i &= k^\alpha N^{1-\alpha}, \\ k' &= \tfrac{(1-\delta)k+\phi(\frac{i}{k})k}{e^{\mu+\sigma\varepsilon'}}. \end{aligned}$$

where g satisfies $V(K,Z)=Z^v g(k)$ and all the non-stationary variables have been detrended by the stochastic technology level Z.[19]

In the same way as before, to compute asset prices in this economy we first need to calculate the SDF. Using the derived expression and homogeneity, the pricing kernel between two states of the world $s=(k)$ and $s=(k')$ can be expressed as

$$\begin{aligned} M(s,s',\varepsilon') &= \beta\left(\frac{Z'c(k')}{Zc(k)}\right)^{-1}\frac{Z'^{v(1-\theta)}g(k')^{1-\theta}}{E_{\varepsilon'}\left(Z'^{v(1-\theta)}g(k')^{1-\theta}\right)} \\ &= \beta\left(\frac{Z'}{Z}\right)^{(1-\theta)v-1}\left(\frac{c(k')}{c(k)}\right)^{-1}\left(\frac{g(k')^{1-\theta}}{E_{\varepsilon'}\left((\frac{Z'}{Z})^{v(1-\theta)}g(k')^{1-\theta}\right)}\right). \end{aligned}$$

Next, using the standard asset pricing formula $P_t = E_t(M_{t,t+1}(P_{t+1}+D_{t+1}))$ we can compute the price/return on the risk-free asset and on the equity claim.[20]

[19] In the case when $\theta=1$ the stationary recursive formulation for this economy will be given by

$$\log g(k) = \max_i\left\{(1-\beta)\left(v\log c+(1-v)\log(1-N)\right)+\beta E_{\varepsilon'}\log\left(g(k')e^{v(\mu+\sigma\varepsilon')}\right)\right\}$$

subject to

$$\begin{aligned} c+i &= k^\alpha N^{1-\alpha} \\ k' &= \tfrac{(1-\delta)k+\phi(\frac{i}{k})k}{e^{\mu+\sigma\varepsilon'}} \end{aligned}.$$

Note that for the transformation we need to redefine $V(K,Z)=Z^\nu g(k)$.

[20] In the limiting case when $\theta=1$, the SDF will be given by the expression derived in the previous

The price on the risk-free asset with payout $d = 1$ will be given by

$$\begin{aligned} P(k) &= E_{\varepsilon'} M(s, s', \varepsilon') d(k', Z', \varepsilon') \\ &= \frac{\left\{\beta \sum_{\varepsilon'} \pi(\varepsilon') e^{((1-\theta)v-1)(\mu+\sigma\varepsilon')} c(k')^{-1} g(k')^{1-\theta}\right\}}{c(k)^{-1} \sum_{\varepsilon'} \pi(\varepsilon') e^{v(1-\theta)(\mu+\sigma\varepsilon')} g(k')^{1-\theta}}. \end{aligned}$$

Similarly, the equity is simply a claim to the representative firm's stream of dividends $\{D_t\}$. Let $D_t = Y_t - w_t N_t - I_t$ and P_t denote the price of this equity satisfying the standard asset pricing formula. Given that dividends can be expressed as $D = Zd(k)$, where $d(k) = \alpha k^{\alpha} N^{1-\alpha} - i$, the firm value recursion can be written as $P = Zf(k)$ with

$$\begin{aligned} f(k) &= E_{s'|s} \left(M(s', s) \times (\tfrac{Z'}{Z})(d(k') + f(k')) \right) \\ &= \beta E_{\varepsilon'} \left[(\tfrac{Z'}{Z})^{(1-\theta)v} (\tfrac{c(k')}{c(k)})^{-1} \left(\frac{g(k')^{1-\theta}}{E_{\varepsilon'}\left((\frac{Z'}{Z})^{v(1-\theta)} g(k')^{1-\theta} \right)} \right) (d(k') + f(k')) \right]. \end{aligned}$$

or equivalently,

$$f(k) = \frac{\left\{\beta \sum_{\varepsilon'} \pi(\varepsilon') e^{(1-\theta)v(\mu+\sigma\varepsilon')} c(k')^{-1} g(k')^{1-\theta} (d(k') + f(k'))\right\}}{c(k)^{-1} \sum_{\varepsilon'} \pi(\varepsilon') e^{v(1-\theta)(\mu+\sigma\varepsilon')} g(k')^{1-\theta}}.$$

Again, this same formulations can be used to compute asset prices (including the levered claim to output) in the limiting case when $\theta = 1$ using the appropriate expression for the SDF as derived in the first section of this appendix and the corresponding transformation for the original value function $V(K, Z)$.

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section of this appendix

$$\begin{aligned} M(s, s', \varepsilon') &= \beta \left(\frac{Z' c(k')}{Z c(k)} \right)^{-1} \\ &= \beta \left(\frac{Z'}{Z} \right)^{-1} \left(\frac{c(k')}{c(k)} \right)^{-1} \end{aligned}$$

Chapter 3

Do Firm-Level Shocks Generate Aggregate Fluctuations?: A Cross-Country Analysis

3.1 Introduction

Business cycle fluctuations are often thought to have caused by aggregate shocks, since uncorrelated sector- or firm-level shocks average out in the aggregate due to the law of large numbers. However, a number of studies in recent years show the diversification of idiosyncratic shocks breaks down when sectoral linkages or firm size distributions are highly skewed. These studies provide the insight that in an economy where few sectors serve as major input suppliers or few firms account for a disproportionate share of production, shocks to these sectors or firms can propagate to generate aggregate fluctuations.

While the sector-based story has a long theoretical tradition accompanied by empirical evidence,[1] the relevance of firm-level shocks to aggregate fluctuations remains to be acknowledged and quantified. Gabaix (2011) was the first study to formally show that in an economy with fat-tailed size distribution of firms, idiosyncratic shocks to firms diversify at a milder rate that leads to nontrivial effects on aggregate fluctuations. Further, output volatility originated from micro-shocks is an increasing function of the Herfindahl

[1] Sector-based stories date back to the seminal work of Long and Plosser (1983) where they show input-output linkages can propagate sector-level shocks within a six-sector dynamic stochastic general equilibrium model. Later on, Dupor (1999) discredits this view arguing that the results rely on the number of sectors being too small. Around the same time Horvath (1998, 2000) argues the cancellation of sector-specific shocks is affected by the existence of factor demand linkages; whether shocks diversify does not depend on the number of sectors, but whether some of them are important inputs in the production processes of many others. Recently, Acemoglu et al. (2012) generalize Horvath's view and show that if there are only few input supplier sectors then the convergence of aggregate volatility to zero slows down considerably. A productivity shock in one major sector has a "cascading" effects in the economy, affecting this way not only the direct downstream sectors but also the series of interconnected sectors.

index of firms' sales shares, or in other words, of how concentrated the economy is.[2] The theoretical framework is then accompanied by an empirical analysis where the author shows shocks to largest firms in the US statistically explains GDP growth over the period 1952-2008.

In light of Gabaix's theory, countries that are more concentrated than the US should observe stronger effects of firm-level shocks on aggregate fluctuations. However, since the theory is relatively new and comprehensive panel data are not readily available for many countries, studies investigating the effects of idiosyncratic shocks outside the US are scarce.

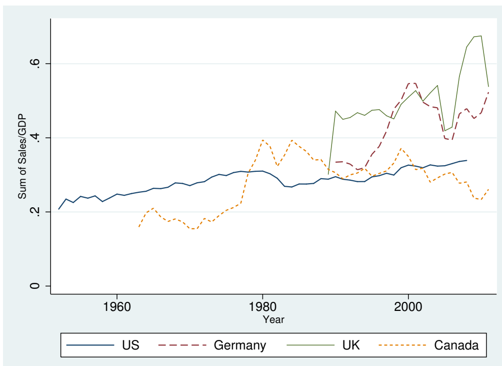
This paper contributes to the existing literature by examining the relevance of idiosyncratic shocks to output fluctuations for three OECD countries (Germany, Canada and the UK) in addition to the US. Figure 3.1a presents the relative size of the top 100 firms in terms of their sales to GDP ratio in these four economies, and Figure 3.1b the Herfindahl index of firms' sales share among the largest 100 firms. Comparing with the US, the top 100 firms in other countries not only account for a larger share of their corresponding GDP, they are also much more concentrated. These descriptive statistics suggest large firms in these countries could be an important source of variability, and the effects of idiosyncratic shocks on the aggregate should be stronger as a result.

To assess the empirical relevance of idiosyncratic shocks for output fluctuations in these countries, we regress the growth of GDP per capita on the "granular residual" and its different lags. The granular residual (GR), proposed by Gabaix (2011), is a transparent statistic that summarizes the importance of firm idiosyncratic shocks using the weighted sum of firm-level shocks, where the weights are calculated as the firms' sales to GDP ratio to reflect their relative importance. This empirical strategy allows us to work with

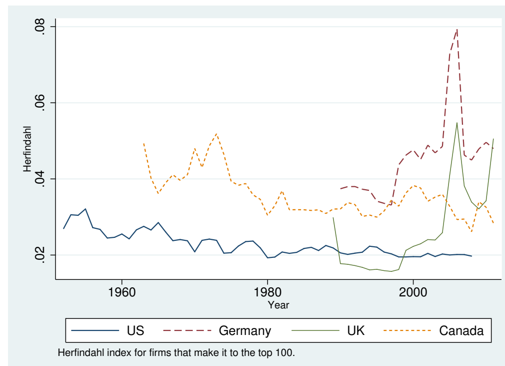
[2]The results in Gabaix (2011) only require economies with sufficiently large herfindahls. The central case of firms' size being Zipf-distributed is justified because of its tractability and clean exposition. Also, this distribution generates very high herfindahls.

a broader sample of countries as we do not require input-output matrices to measure the importance of firm-level productivity shocks for the economy, only annual firms' sales and employment, and countries' GDP per capita are necessary.[3]

Figure 3.1: Relative Size and Degree of Concentration of Top 100 Firms by Country



(a) Top 100 firms' sales to GDP ratio



(b) Herfindahl index of top 100 firms

Notes: the figure presents the relative size of the top 100 firms in terms of their sales to GDP ratio in the US, Germany, the UK and Canada (on the left), and the Herfindahl index of firms' sales share among the largest 100 firms in these same countries (on the right).

Our results show that idiosyncratic shocks to large firms are of little relevance in the UK or Canada but explain roughly 1/3 of the output fluctuations in the US and Germany. The top ranking firms are indeed the most important contributors to granular effects, but because diversification is at work still in the UK and Canada these firms are not able to explain GDP growth. Initial investigation seems to confirm the conjecture that a strong transportation sector drives the granular effects in the US and Germany. Our results suggests that while firm size distribution is found to be highly skewed in many economies[4], the ability of the largest firms to transmit shocks may not be universal and thus not to be taken for granted.

[3]Despite using an empirical strategy that is lenient on data requirements, we still face some data limitations along the time-series dimension. In the end, we are able to construct the GR using the top 100 firms for at least 20 years for all of our countries.

[4]Di Giovanni and Levchenko (2012) find in a set of 43 countries, firm size distributions are highly skewed and on average close to Zipf.

The studies closest to ours are Di Giovanni et al. (2014) and Foerster et al. (2011); both studies investigate the importance of firm level shocks, but they focus on one country each and differ in methodology. While the first finds support for the hypothesis that microeconomic shocks drive aggregate fluctuations, like ours the second paper casts doubts on the validity of this hypothesis. Using detailed French firm level data, Di Giovanni et al. (2014) find firm-specific shocks to be almost twice as important as the combined effect of sectoral and macroeconomics shocks in driving aggregate sales growth. Further, they find that most of the effect is not coming directly from shocks to individual firms, but input-output linkages. Foerster et al. (2011) uses factor methods to decompose US industrial production and explore the plausibility of the granular- and network-origins hypothesis of aggregate fluctuations. They find no support for the granular hypothesis, and aggregate volatility is better explained by co-variability among sectors. Furthermore, co-variability is explained by common factors, very little of which is the result of the transmission via input-output linkages.

The rest of the paper is organized as follows. Section 3.2 describes the empirical specification used to estimate the importance of firm-level idiosyncratic shocks. In Section 3.3 we present the regression results for the countries in our sample, and in Section 3.4 we deconstruct the differences in the findings for the US and Germany versus those for the UK and Canada. Finally our conclusions are summarized in Section 3.5.

3.2 Empirical Strategy

In this section, we first summarize the theoretical framework from which the GR is derived and how to construct it. We then specify the empirical strategy and describe the data.

3.2.1 Granular residual

In an economy with input-output linkages and endogenous input response, aggregate productivity growth is a weighted sum of firms' Hicks-neutral productivity shocks, ε_i:

$$g_{TFP} = \sum_i \frac{\text{sales of firm } i}{GDP} \varepsilon_i, \tag{3.1}$$

where the weights $\sum_i \frac{\text{sales of firm } i}{GDP}$ capture the propagation effect from firm-level shocks to the rest of the economy. Domar (1961) and Hulten (1978) show that these are the appropriate weights to measure the total effect of firm-level productivity changes on aggregate productivity. The intuition is that an increase in productivity in one firm increases output in all those other firms that use this final good as an intermediate input, which in turn increases output again, and so on.

Without disturbances, growth in GDP is proportional to the growth in TFP:

$$g_{GDP} = \mu \; g_{TFP}, \tag{3.2}$$

where $\mu \geq 1$ is the factor usage intensity that is a combination of the elasticity of substitution of labor and output elasticities with respect to production inputs.[5]

Combining (3.1) and (3.2), the impact of idiosyncratic shocks on aggregate output is captured by the following relationship:

$$g_{GDP} = \mu \sum_i \frac{\text{sales of firm } i}{GDP} \varepsilon_i, \tag{3.3}$$

[5] Consider an economy with the production technology $Y = A_t L_t^{\alpha}$ and consumer preference $U_t(C_t, L_t) = \log(C_t - \frac{L_t^{1+1/\phi}}{1+1/\phi})$. With competitive markets for output and labor, the equilibrium is characterized by $N_t = (\alpha A_t)^{\frac{1}{1+1/\phi-\alpha}}$ and $Y_t = \alpha A_t^{\frac{1+1/\phi}{1+1/\phi-\alpha}}$. In this economy, output growth is proportional to TFP growth $\hat{Y} = \mu \hat{A}$, where $\mu = \frac{1+1/\phi}{1+1/\phi-\alpha}$ is the factor usage.

where $\Gamma^* = \sum_i \frac{\text{sales of firm } i}{GDP}\varepsilon_i$ is Gabaix's "granular residual". Equation (3.3) provides the basis for the regression framework to test the granular hypothesis: granular effects are said to be present when the weighted sum of idiosyncratic shocks to large firms statistically explains GDP fluctuations as measured by growth in GDP per capita.

Intuitively, in an economy where shocks are uncorrelated across firms and all of them have the same variance, from equation (3.3) we see that output growth volatility from micro-shocks reduces to a function of the firms' sales herfindahl:

$$\sigma_{GDP} = \mu\sigma h; \quad h = \left(\sum_{i=1}^{N}\left(\frac{S_{it}}{Y_t}\right)^2\right)^{1/2}, \tag{3.4}$$

where h is the modified Herfindahl (Herfindahl$^{1/2}$) of the economy. Hence, the more fat-tailed is the firm-size distribution or, the higher the firms' sales herfindahl, larger is the effect on aggregate volatility of firm-level shocks. For instance, if firm size is Zipf-distributed, output growth volatility is proportional to $\sigma/\ln N$, and not only it does not converge to zero as N goes to infinity, but the rate of decay is much lower. Instead, if all the firms are of equal size, output growth volatility in a country with N firms is proportional to $\sigma/\sqrt{N}$, and so as N goes to infinity output volatility converges to zero.[6]

The only detailed calculation of this empirical strategy is the construction of firm level shocks. To avoid data availability issues, we also estimate firm-level productivity using labor productivity of firm i:[7]

$$z_i = \ln \frac{\text{sales of firm } i \text{ in year } t}{\#\text{ of workers in firm } i \text{ in year } t}. \tag{3.5}$$

To compute the shocks to firms' productivity growth, we model firms' labor productivity

[6] See Gabaix (2011) for a detailed proof.

[7] We implement the Olley and Pakes (1996) method to estimate the firm-level Solow residual and contrast it with our measure of labor productivity. We find that the two methods yield similar estimates for the productivity growth in the US.

growth, g, as depending on a set of firm's characteristics, X_{ijt}, and an idiosyncratic shock, ε_{ijt}:

$$g_{ijt} = \beta' X_{ijt} + \varepsilon_{ijt}. \tag{3.6}$$

For simplicity, we use year and industry dummies to proxy for firm's characteristics:

$$g_{it} = c + d_t + \varepsilon_{it} \tag{3.7}$$

$$g_{ijt} = c + d_t + \text{IND}_{jt} + \varepsilon_{ijt}, \tag{3.8}$$

and calculate firm level shocks as the demeaned labor productivity growth rates, where the mean is computed over all firms of the year (the case when we use only year dummies), or of the year and industry (when both year and industry dummies are used in the regressions).[8]

Using both estimations for the idiosyncratic shocks, we construct two versions of the "granular residual":

$$\Gamma_{t,v1} = \sum_{i=1}^{K=100} \frac{S_{i,t-1}}{Y_{t-1}} \hat{\varepsilon}_{it}; \quad \Gamma_{t,v2} = \sum_{i=1}^{K=100} \frac{S_{i,t-1}}{Y_{t-1}} \hat{\varepsilon}_{ijt}. \tag{3.9}$$

Since we are interested in the effect of the largest firm we only work with the largest top $K = 100$ firms ranked by their sales to output ratio in the previous period.[9]

[8] The challenge with correctly identifying ε_{it} remains apparent as in the data it is hard to identify ε_{it} because aggregate shocks could cause firm i's volatility or reflect it. We do not directly address the reflection problem but we perform robustness checks to also control for the common factors so to prove the explanatory power of the GR is not coming from aggregate shocks (e.g. oil, monetary, fiscal policy shocks and etc).

[9] To verify the empirical methodology we simulated a simple economy, with exogenous production and without linkages. With the true idiosyncratic shocks, we construct the GR and regress output on this measure. The empirical strategy tends to bias the GR downwards in its magnitude and volatility, decreasing its explanatory power. Further, it seems to capture well the idiosyncratic components of a firm's productivity growth in a granular world without inducing any spurious bias. Interestingly, the results also show that the explanatory power of the model decreases when the idiosyncratic shocks to large firms are less volatile.

3.2.2 Econometric specification

To test for the effect of firm-level idiosyncratic shocks on aggregate fluctuations we regress our measure of idiosyncratic shocks, Γ, on growth of GDP per capita using the following specification:

$$g_{Y_t} = \alpha + \sum_{i=0}^{2} \mu_i \Gamma_{t-i} + u_t. \tag{3.10}$$

The R-squared from regressing GDP growth g_{Y_t} on Γ_t and its different lags allows one to assess the extent to which idiosyncratic shocks explain the variability of GDP growth. If we recall equation (3.3), the coefficient on the granular residual provides an estimation of the factor usage in these different countries.[10]

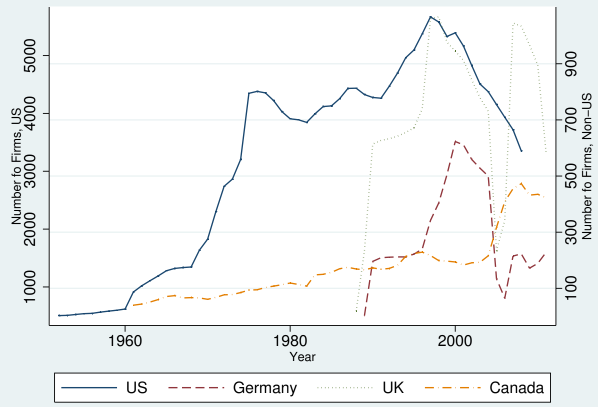
3.2.3 Data

Firm-level data for the US and Canada, UK and Germany are from Compustat North America, Computstat Global. The length of coverage varies and it spans from 1950 to present and from 1987 to present for NA and international companies. We keep only firms incorporated *and* headquartered in their home country so to exclude foreign firms to the best of our ability. The oil industry is excluded due to difficulty in teasing out real firm-level shocks from the aggregate commodity price shocks.[11] Figure 3.2 summarizes the number of firms with valid data required by our regression specification by country and year.

[10] Gabaix (2011) takes $\mu = 2.6$ as the benchmark to compare the regression coefficient with for the US.

[11] We also exclude financial firms as in Gabaix (2011), "because the nature of their sales are not in line with the meaning of 'gross output' in the paper".

Figure 3.2: Number of Firms With Valid Sales and Employee Data



Number of firms with valid sales and employee data, excluding oil and financial industries.

The advantage of using Compustat is the comparability of information reported across countries, but the coverage on the number of firms is limited in Germany and Canada for some of the years. Further, since the analysis is done over the top 100 firms in the economy we lose additional observations because for some years there are less than 100 firms. However, since sales of the top 50 and 100 firms as a percentage of GDP track each other closely for Germany and Canada, we restrict our samples to years with at least 50 rather than 100 firms. This means for years less than 100 (but more than 50) firms we use all firms in our empirical exercises.

Macroeconomic data (GDP, GDP per capita and GDP deflators) are taken from the World Bank's Development Indicators database. GDP deflators are used to convert sales into real terms.[12] Some of the sales figures in Compustat are denominated in non-local currencies, and we look to the respective country's central bank website to obtain the exchange rates.[13]

[12]Ideally one would like to use industry production indexes but they are not readily available so we use GDP deflators instead.

[13]This could introduce potential measurement errors since the exchange rates were matched to the fiscal years, rather than the period covered by a firm's financial statements.

3.3 Estimation Results

In this section, we begin by presenting some basic statistics of the variables used in the regressions. Then, we present the estimation results for the US and Canada, UK and Germany as specified in regression (3.10) for different lags for the GR.

Table 3.1 provides a summary of country characteristics in terms of the variables used in the regression model. The average and the volatility of per capita GDP growth across the countries are very similar (columns 1 and 2), but firm-level productivity growth is much more volatile in non-US countries (column 4). Column 5 shows the correlation between growth rates across firms is small for all of the countries, suggesting the measure we use for firm level shock does capture idiosyncratic variation to the firms. Finally, the average and standard deviation of the GR are smaller for version 2 than for 1 (columns 6 and 7 for version 1 and 8 and 9 for version 2). This shows when we demean by year-industry, we further get rid of industry-specific shocks that could confound our results.

Table 3.1: Country Characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	μ_{g_Y}	σ_{g_Y}	μ_{ε}	σ_{ε}	$\rho_{\varepsilon_i,\varepsilon_j}$	$\mu_{\Gamma_{v1}}$	$\sigma_{\Gamma_{v1}}$	$\mu_{\Gamma_{v2}}$	$\sigma_{\Gamma_{v2}}$
US	0.020	0.019	0.017	0.110	0.023	0.0003	0.0037	0.0003	0.0032
Canada	0.017	0.021	0.017	0.208	0.016	0.0006	0.0055	-0.0001	0.0044
Germany	0.019	0.019	0.015	0.179	0.020	0.0034	0.0092	-0.0001	0.0072
UK	0.019	0.022	0.011	0.258	0.010	0.0031	0.0144	-0.0016	0.0081

Notes: μ_{g_Y} is the average annual per capita GDP growth; σ_{g_Y} is the standard deviation of annual per capita GDP growth; μ_{ε} is the average annual firm-level productivity shocks; σ_{ε} is the standard deviation of annual firm-level productivity shocks; ρ_{g_i,g_j} is the average annual sample correlation of firm-level productivity shocks; $\mu_{\Gamma_{v1}}$ is the average annual granular residual with year-demeaning; $\sigma_{\Gamma_{v1}}$ is the standard deviation of the annual granular residual; $\mu_{\Gamma_{v2}}$ is the standard deviation of the annual granular residual with industry demeaning; $\sigma_{\Gamma_{v2}}$ is the standard deviation of annual granular residual with industry demeaning.

3.3.1 Impact of idiosyncratic firm-level shocks

Table 3.2 presents the regression results for the US, Germany, the UK and Canada. Taking the US as the benchmark, we see in columns (1) & (2) of Panel A that the granular residual explains 24% (35%) of the fluctuations in GDP growth using 1 (2) lag(s) when demeaning by year. When we control for the contemporaneous granular residual and its two lags, their coefficients are significant and hovering around 2.6—the theoretical value of μ Gabaix (2011) uses for comparison. The presence of granular effects is confirmed in these results.

When we control for industry-year specific shocks by demeaning at the 2-digit industry level, we observe an increase in the granular effects for the US in Panel B columns (1) & (2) of the same table. The resulting firm idiosyncratic shocks are closer to the true ε_{it} when industry specific shocks are controlled for, hence if the granular hypothesis holds we should expect to explain more of GDP fluctuations.[14]

The results for the other countries are mixed. For Germany, we find the GR to explain GDP fluctuations even better than for the US (35% (36%) when demeaning by year, and 24% (34%) when demeaning by year-industry). However, regardless of demeaning the GR by year or industry, we do not find granular effects in Canada or the UK. The adjusted R^2 is essentially zero and the coefficients are insignificant and often negative. Even though all countries meet the sufficient conditions, we find idiosyncratic shocks translate to aggregate fluctuations only in the US and Germany.

The natural progression at this point is to implement factor methods to examine whether any residual common shocks give rise to our findings, and to use principal component

[14] We also experimented with demeaning at the 3-digit and 4-digit level but the granular effects are weakened as a result. Increasing the level of disaggregation should improve the explanatory power further theoretically but at the same time inducing attenuation bias empirically because the mean would then be estimated with fewer firms. These two forces work against each other and complicate the interpretations of the regression results, hence we focus on year and year-industry at the 2-digit level demeaning in this paper as mentioned in the Introduction.

Table 3.2: Explanatory Power of the Granular Residual

Panel A: GR with Year Demeaning

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	USA		Germany		UK		Canada	
Γ_t	1.83**	2.51***	1.19**	0.93**	0.08	-0.00	-0.22	-0.37
	(0.69)	(0.69)	(0.42)	(0.44)	(0.32)	(0.35)	(0.58)	(0.60)
Γ_{t-1}	2.58***	2.88***	1.00**	1.15**	0.31	0.35	0.38	0.55
	(0.71)	(0.67)	(0.43)	(0.43)	(0.32)	(0.34)	(0.58)	(0.59)
Γ_{t-2}		2.13***		-0.23		-0.14		0.02
		(0.71)		(0.44)		(0.34)		(0.59)
Intercept	0.02***	0.02***	0.01	0.01	0.01***	0.02***	0.02***	0.02***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
N	56	55	21	20	22	21	48	47
R^2	0.27	0.38	0.42	0.46	0.05	0.07	0.01	0.03
adj. R^2	0.24	0.35	0.35	0.36	-0.05	-0.09	-0.03	-0.04

Panel B: GR with Year-Industry at the 2-digit Level Demeaning

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	USA		Germany		UK		Canada	
Γ_t	2.79***	3.70***	-0.04	0.00	0.21	0.00	-0.29	-0.57
	(0.76)	(0.78)	(0.61)	(0.55)	(0.61)	(0.68)	(0.70)	(0.72)
Γ_{t-1}	3.34***	3.92***	1.60**	1.95***	0.71	0.71	0.73	0.78
	(0.74)	(0.71)	(0.60)	(0.58)	(0.59)	(0.64)	(0.70)	(0.70)
Γ_{t-2}		2.07***		0.48		-0.24		0.32
		(0.75)		(0.55)		(0.61)		(0.70)
Intercept	0.02***	0.02***	0.01***	0.01***	0.02***	0.02***	0.02***	0.02***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)
N	56	55	21	20	22	21	48	47
R^2	0.37	0.48	0.31	0.44	0.07	0.10	0.03	0.05
adj. R^2	0.35	0.45	0.24	0.34	-0.03	-0.06	-0.02	-0.02

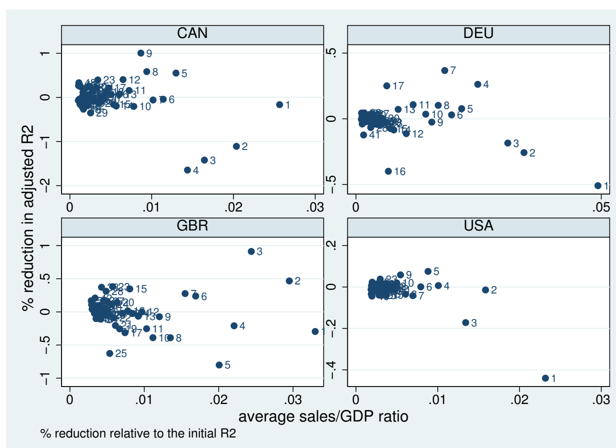
Notes: * for p<.1, ** for p<.05, and *** for p<.01. Standard errors in parenthesis. Per capita GDP growth is regressed on the granular residual with year demeaning (Panel A), and 2-digit industry demeaning (Panel B) calculated over the top 100 firms.

analysis to investigate which firms explain the aggregate fluctuations the most. However, since the identities of the largest firms change from year to year, we cannot use conventional methods to examine whether our findings are driven by common factors or specific firms. Thus in the next section we deconstruct our findings utilizing still the GR measure demeaned at the industry level, since this level of demeaning yields the most robust proxy for idiosyncratic shock as shown in Table 3.1.

3.4 Deconstructing the empirical results

We first examine the relative importance of top ranking firms amongst themselves in explaining GDP fluctuations. Since the ranking of these top firms changes every year, we drop observations that have the same ranking to construct the new GR. The results are presented in Figure 3.3 where we drop a top ranked "composite firm" one at a time and plot the percentage reduction in adjusted-R^2's against its average weights over the sample period. The figure shows lower ranks bunched together around zero in all four countries, suggesting their relative importance is small. However, the impact of the largest firms is prominent in all countries with the starkest contrast in the case of the US.

Figure 3.3: Reduction in Adjusted-R^2 After Dropping Top Ranked Firms



Notes: the figure plots the percentage reduction in adjusted-R^2's from dropping one at a time a top ranked "composite firm" against its average weights over the sample period. The identity of the "composite firm" changes every year as the ranking of the top firms also does.

In light of this finding, we construct the granular residual with just the top 10 ranks to find that the positive findings in the US and Germany can be explained just as well (see Panel A in Table 3.3). Panel B in the same table additionally shows dropping top 10 ranks reduces the explanatory power significantly and the coefficients are no long significant. In the UK and Canada, however, firms of the same ranks did not play a significant role and dropping them leaves the R^2 as low as before and the coefficients remain insignificant.[15] Since summing up the GRs for the top ranks dilute their explanatory power in the UK and Canada, we suspect the diversification mechanism may be at work still in these two countries.

Table 3.3: Explanatory Power of the Modified Granular Residual

Panel A: GR With Only the Top 10 Ranks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	USA		Germany		UK		Canada	
Γ_t	2.71***	3.30***	-0.01	0.01	0.24	0.17	-0.46	-0.65
	(0.74)	(0.74)	(0.67)	(0.62)	(0.90)	(0.91)	(0.69)	(0.71)
Γ_{t-1}	3.03***	3.34***	1.77**	2.03***	0.99	1.14	0.40	0.39
	(0.72)	(0.70)	(0.66)	(0.64)	(0.88)	(0.92)	(0.69)	(0.70)
Γ_{t-2}		1.77**		0.30		-0.92		0.44
		(0.73)		(0.62)		(0.88)		(0.70)
Intercept	0.02***	0.02***	0.01***	0.01**	0.02***	0.02**	0.02***	0.02***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)
N	56	55	21	20	22	21	48	47
R^2	0.36	0.44	0.30	0.40	0.06	0.14	0.02	0.04
adj. R^2	0.34	0.41	0.22	0.29	-0.03	-0.01	-0.03	-0.03

Panel B: GR without the top 10 ranks

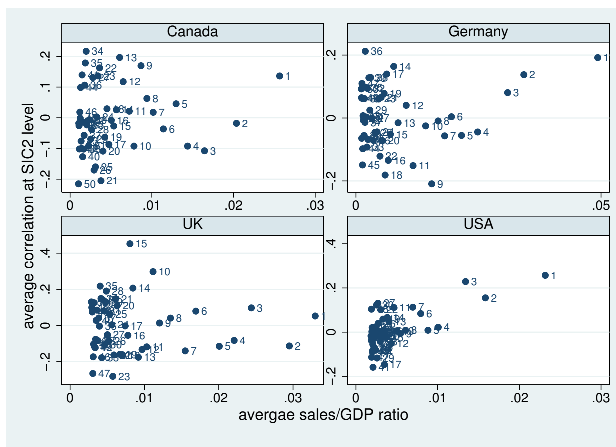
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	USA		Germany		UK		Canada	
Γ_t	-1.92	-1.90	-4.34	-3.76	0.23	-0.11	0.50	0.25
	(2.50)	(2.53)	(3.10)	(3.09)	(1.13)	(1.58)	(1.33)	(1.34)
Γ_{t-1}	-0.12	-0.33	2.39	3.58	0.81	0.94	0.97	1.37
	(2.49)	(2.56)	(3.12)	(3.54)	(1.13)	(1.52)	(1.32)	(1.34)
Γ_{t-2}		-2.07		1.03		0.43		-0.85
		(2.54)		(3.09)		(1.26)		(1.33)
Intercept	0.02***	0.02***	0.01	0.01	0.01***	0.02**	0.02***	0.02***
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)
N	56	55	21	20	22	21	48	47
R^2	0.01	0.02	0.21	0.25	0.03	0.04	0.02	0.03
adj. R^2	-0.03	-0.03	0.13	0.11	-0.07	-0.13	-0.03	-0.04

Notes: * p<.1, ** p<.05, and *** p<.01. Standard errors in parenthesis. In Panel A, per capita GDP growth is regressed on the granular residuals calculated over the top 10 ranks. In Panel B, per capita GDP growth is regressed on the granular residuals calculated over the top 91-100 firms.

[15] This conclusion is robust to dropping the top 15 ranks.

To further examine where the differences in the contribution of top firms with comparable importance could be coming from, we construct the rank specific weighted idiosyncratic shock and plot its correlation with GDP growth in Figure 3.4.[16] The right column of the figure shows unambiguously that the top ranks in Germany and the US are also positively correlated with GDP growth, but the picture is much more nuanced for Canada and the UK. We conclude that the higher tendency of high ranking firms moving in opposite directions in the UK and Canada neutralizes the effect of idiosyncratic shocks, thus it is not surprising the final measure of granular residual–the sum–do not explain aggregate fluctuations for these two countries.

Figure 3.4: Correlation Between Firm Specific Granular Residual and GDP Growth



Notes: the figure plots the average of the correlation between per capita GDP growth and the contemporaneous rank-specific GR, with its one lag and two lags, respectively. The rank-specific GR corresponds to rank specific weighted firm idiosyncratic shock.

The last piece of the puzzle remains as to why shocks diversify away in the case of the UK and Canada but not in the US or Germany. We conjecture that granular effects in the US and Germany are driven by the fact that several top ranked firms in the US and Germany belong to the Transportation Equipment sector (SIC37), which in both

[16] We compute the correlation between GDP growth and the contemporaneous rank-specific GR, with its one lag and then two lags. In the end we plot the average of the three correlations against the rank's average sales to GDP ratio.

countries represents an important component of output. With our data and methodology we are only able to shed some light to this hypothesis. To do so, we first show a snapshot of the top 10 firms and sectors as ranked by sales to GDP in year 2002 in Table 3.4 and 3.5, respectively. In both countries, 3 out of 10 firms belong to the SIC37, and this is the top-ranked sector in both countries as well.[17] Second, we construct two modified versions of the GR (with year and year-industry demeaning) dropping the firms that belong to each 2-digit industry one at the time. Figure 3.5 plots the volatility and correlation between this modified GR and GDP growth for the US, Germany, the UK and Canada. The horizontal line indicates the volatility ratio between the economy-wide GR and per capita GDP growth, and the vertical line indicates the correlation between the two.

Table 3.4: Top 10 Firms, 2002

	Canada		Germany		UK		US	
	SIC2	S/Y	SIC2	S/Y	SIC2	S/Y	SIC2	S/Y
1	36	2.45	37	7.28	20	3.09	53	2.16
2	36	2.45	37	4.22	10	2.54	37	1.73
3	54	2.23	99	4.14	28	2.35	37	1.60
4	48	1.96	99	3.33	53	2.32	99	1.24
5	37	1.95	53	2.36	48	2.24	73	0.85
6	54	1.94	48	2.30	48	2.02	21	0.72
7	33	1.76	37	1.83	28	2.01	48	0.66
8	37	1.54	33	1.81	54	1.68	42	0.65
9	36	1.40	42	1.59	20	1.26	37	0.57
10	48	1.29	28	1.55	21	1.12	48	0.52

Notes: the table reports the top 10 firms as ranked by sales in 2001, and the 2-digit sector they belong to.

When demeaning by year (top panel of Figure 3.5), we see in the US that when firms belonging to SIC37 are dropped the volatility and correlation of the GR in relation to GDP growth becomes zero and negative, while dropping other sectors made relatively little changes. We arrive at the same conclusion upon examining the results from demeaning by year-industry for the US in the bottom panel of Figure 3.5. In the case of Germany,

[17] We redo this same exercise for the rest of the years and the same conclusions hold.

Table 3.5: Top 10 Sectors in 2002

	Canada			Germany		
Rank	SIC2	# Top Firms	S/Y	SIC2	# Top Firms	S/Y
1	Elec.	5	6.47	Trans.Eq.	8	16.12
2	Food Strs.	5	5.34	Conglmrts.	3	7.52
3	Comm.	8	4.19	Chem.	14	6.82
4	Trans.Eq.	4	3.64	Comm.	5	3.59
5	Metal Ind.	7	2.57	G.Mechdis.Strs	2	2.59
6	G.Mechdis.Strs.	2	1.28	Metal Ind.	5	2.23
7	Chem.	5	1.16	Trans.Svcs	2	1.83
8	Paper and Allied	5	0.98	Machne.Eq.	14	1.60
9	Lumber	7	0.89	Motr.Warehsing.	1	1.59
10	Railroad Trans.	2	0.84	Whsl Tr.	4	1.30
	UK			**US**		
Rank	SIC2	# Top Firms	S/Y	SIC2	# Top Firms	S/Y
1	Food Prd.	8	6.41	Trans.Eq.	8	5.09
2	Chem.	7	6.33	G.Mechdis.Strs	8	4.27
3	Comm.	7	5.88	Comm.	11	3.48
4	Food Strs.	5	3.76	Chem.	10	2.64
5	G.Mechdis.Strs	4	3.53	Food Strs.	6	1.71
6	MetalMining	2	3.10	Conglmrts.	2	1.63
7	Trans.Eq.	4	2.18	Food Prd.	7	1.60
8	Biz.Svcs.	6	1.74	Machne.Eq.	6	1.52
9	Whsl Tr.	3	1.51	Biz.Svcs.	4	1.52
10	Prntn.Pub.	4	1.44	Whsl Tr.	4	1.34

Notes: the table reports the top 10 sectors as ranked by sector sales of top 100 firms in 2001.

SIC37 also stands out along with SIC99 (nonclassified establishments, including industrial conglomerates) among others when demeaning by year. However, dropping each of the two sectors has the opposite effect on the correlation between the GR and GDP growth. When we demean by year-industry and so controlling for industry-wide shocks, SIC37 clearly stands out to be the one that dominates the negative correlation between the GR and GDP since dropping it turns the correlation between the GR and GDP growth positive (see bottom panel of Figure 3.5).[18]. Overall, idiosyncratic shocks at the firm level

[18]From the results in Section 3.3 we know the coefficient on the contemporaneous GR demeaning by

in Germany do explain aggregate fluctuations with the effects coming also mostly from SIC37.

The overall message for Canada and the UK is mixed. When we demean by year or by year-industry, we do see much more dispersion which may explain why the economy-wide GR does not explain the year-to-year GDP fluctuations. Though there were sectors that stand out in these countries, they are scattered around the near zero correlation between the economy-wide GR and GDP growth counteracting each other's contribution.

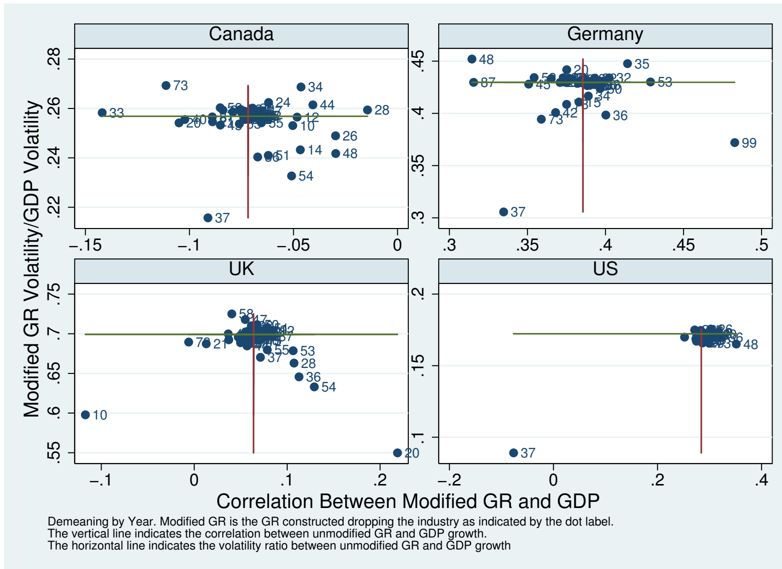
3.5 Conclusion

This paper quantifies the importance of firm-level idiosyncratic shocks in explaining aggregate fluctuations. It is motivated by the theoretical framework (Gabaix, 2011) that shows in an economy with fat-tailed size distribution of firms, law of large numbers breaks down and idiosyncratic shocks to firms diversify at a much milder rate and lead to non-trivial effects on aggregate fluctuations. In the data, firm level shocks are estimated as the demeaned productivity growth rates, where the mean is calculated over top firms of the year or of the year and industry. These shocks are then weighted by firms' sales to GDP ratio, and the empirical strategy tests whether the sum of these weighted shocks statistically explains GDP fluctuations.

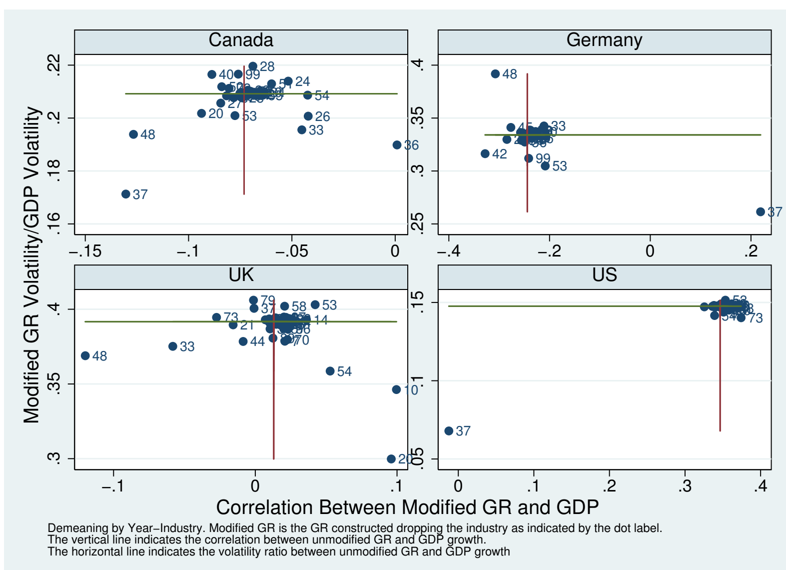
We find shocks to large firms are of little relevance in the UK or Canada but explain roughly 1/3 of the output fluctuations in the US and Germany. While top ranking firms contribute the most to granular effects, they do not always sum up to play a significant role in every country. We conclude that the reason they did not explain GDP growth in the UK and Canada is because diversification is at work still in these two countries. In the US and Germany, preliminary evidence suggest that the granular residual are driven

year-industry is negative when regressing this variable and its lags on per capita GDP growth in Germany.

Figure 3.5: Volatility and Correlation Between Modified Granular Residual and Per Capita GDP Growth



(a) Granular Residual With Year Demeaning



(b) Granular Residual With Year-Industry at the 2-Digit Level Demeaning by firms that belong to the transportation industry. Our results suggest that while firm size distribution is found to be highly skewed in most economies, the ability of the largest firms to transmit shocks is not universal. Other transmission mechanisms are at work, and the importance of them differs on a country-by-country basis.

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