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Unit organization of the topic Exponents and radicals in intermediate algebra

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Service Paper

UNIT ORGANIZATION OF THE TOPIC EXPONENTS AND RADICALS IN INTERMEDIATE ALGEBRA

Submitted by

William Robertson McGhee

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First Reader: Dr. Roy O. Billett

Second Reader: Dr. J. Wendell Yeo

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INTRODUCTION

The Problem

Purpose of this paper.-- Senior-high-school mathematics courses have long been taught by the traditional method of daily assignments. Most educators are in agreement that there is room for a great deal of improvement at this level. The writer, in attempting to improve the quality of his own teaching, has tried to apply some of the basic principles and procedures set forth by Professor Roy O. Billett of Boston University in the course, "The Unit Method of Teaching" and in his book, Fundamentals of Secondary School Teaching.1/ This paper is a record of the organization of the topic "Exponents and Radicals" and its presentation to one of the writer's classes in intermediate algebra.

The pupils.-- There were twenty-nine pupils in the eleventh-grade class taught by this method, twenty-eight of whom ranged in age from 15 years 3 months to 18 years 1 month. The remaining member was 25 years old and a veteran. They all came from homes of moderate circumstances. Mathematics marks were available for all but two from first-year algebra. Three received a rank of A; fifteen, a rank of B;

1/ Roy O. Billett, Fundamentals of Secondary School Teaching. Houghton Mifflin Company, Boston, 1940.

six, a rank of C; and three, a passing mark of D. Of the two remaining, the veteran had had algebra before, but no record was available; the other was a boy who had come to this country the previous year from Greece, but had not had algebra before. He required a great deal of individual help due to his difficulty in reading English. However, he was an eager lad and once he learned a process, he achieved top grades in the class. Intelligence quotients 1/ for all but six were available and ranged from 92 to 135. This information is presented in detail in Table 1 which follows:

Table 1. Description of the Class Members.

Pupil	Age a/	Intelligence Quotient	Mark in Algebra I	Reading Ability		Outside Interest
				Score	Norm	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	16-1	135	89	129	84	Music
2	15-4	129	84	112	82	Sports
3	16-5	129	70	86	84	Sports
4	15-3	126	85	100	84	Sports
5	15-7	121	90	116	84	Sports
6	15-6	120	89	103	84	Music
7	15-6	119	84	73	84	Sports
8	15-4	117	90	103	82	Dramatics
9	15-4	116	73	96	84	Sports
10	15-11	115	82	103	84	Chemistry
11	15-11	114	84	90	84	Sports
12	15-11	114	77	85	84	Sports
13	15-4	113	88	103	84	Sports
14	16-0	111	85	102	84	Sports
15	15-3	108	85	89	82	Music

1/ Derived from Otis, Quick Scoring Test of Mental Ability, Beta Form, given January 1946 to those in the ninth grade.

a/ Ages are given in years and months as of September 1.

Table 1. (concluded)

Pupil	Age	Intelligence Quotient	Mark in Algebra I	Reading Ability Score	Reading Ability Norm	Outside Interest
(1)	(2)	(3)	(4)	(5)	(6)	(7)
16	16-5	107	85	11 a/	9.9	Sports
17	15-11	107	80	8.8 a/	9.9	Music
18	16-3	107	75	70	84	Sports
19	15-3	106	92	87	82	Sports
20	16-4	105	87	100	84	Photography
21	18-1	104	65	83	81	Sports
22	16-11	95	80	73	84	Sports
23	15-11	92	78	86	84	Coins
24	16-0	--	89	115	81	Reading
25	16-4	--	84	110	81	Photography
26	17-1	--	97	118	81	Sports
27	16-11	--	93	89	81	Reading
28	25-0	--	--	--	--	Sports
29	17-9	--	--	58	81	Reading

<u>Testing</u>.-- A teacher-built test on the unit was given as a pre-test. Although some of the items tested retention of work done on exponents and radicals in first-year algebra, the results of the test were rather discouraging. Three pupils got none of the examples correct, and the highest scores were 20 points out of a possible 108. The same test was given as a final test to determine the gain for each pupil on the unit. The results on each test and the gains made are shown in the table on the following page.

a/ Scores and norms for these two were available in the form of years.

Table 2. Pre-test and Final Test Results.

Pupil (1)	Pre-test (2)	Final Test (3)	Gain (4)
1	10	82	72
2	4	72	68
3	2	88	86
4	8	68	60
5	8	74	66
6	14	72	58
7	20	88	68
8	20	100	80
9	12	82	70
10	6	100	94
11	12	84	72
12	0	52	52
13	14	78	64
14	2	108	106
15	0	60	60
16	4	100	96
17	0	80	80
18	6	74	68
19	16	108	92
20	10	52	42
21	6	64	58
22	14	48	34
23	8	46	38
24	2	96	94
25	2	94	92
26	6	96	90
27	6	80	74
28	10	92	82
29	6	94	88
Mean	7.9	80.8	72.3
Standard Deviation	5.6	17.5	17.8

Frequency distributions and histograms of the above data

will be found in Chapter III.

Seven self-administering progress tests and a second form of each were built, testing progressive phases of the unit. However, it was felt later by the writer that fewer of these would have been better; possibly only three or four. So much time was consumed, getting them corrected and passed back as quickly as possible so the pupils could be told to proceed with the next practice material or not, that not enough time was available to give all the individual help that might have been given. Very good results were achieved on all seven of the progress tests, with the exception of test 5, which tested simplification of radicals. This test contained so many difficult items that much additional individual and group work was necessary. In teaching the unit again, the writer would call the class together for a blackboard presentation before having them do practice work on this phase. Scores for each pupil on each progress test will be found in the table on the following page, while copies of each progress test will be found in the appendix.

Table 3. Progress Test Results.

Pupil	Test 1	1A a/	2	2A	3	4 b/	5	5A	6	6A	7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	7	10	13		13	14	10		12		14
2	7	11	14		14	19	10		11		13
3	6	11	15		14	14	10		10		13
4	14		14		14	14	7	9	12		13
5	14		15		14	17	12		13		15
6	13		15		14	15	10		13		12
7	12		15		13	16	12		14		12
8	13		15		13	17	9	13	14		11
9	10		15		14	12	9	11	11		10
10	11		15		12	18	10		10		15
11	10		12		13	16	11		12		12
12	10		15		13	18	9	11	12		14
13	15		15		14	17	11		15		14
14	12		15		11	18	13		13		14
15	6	13	14		12	15	9	11	6	9	14
16	11		15		15	17	11		13		14
17	11		13		12	19	11		11		12
18	13		15		14	17	12		12		14
19	13		14		13	18	10		10		15
20	11		15		12	13	13		8	11	10
21	12		14		12	16	11		11		13
22	13		14		13	14	9	12	12		12
23	10		9	15	12	17	10		12		12
24	13		15		11	19	9	11	10		14
25	13		14		13	16	10		9	12	14
26	14		15		12	19	10		14		14
27	15		13		12	17	9	12	15		14
28	15		15		15	18	10		12		14
29	14		15		15	20	15		12		15

Textbooks.-- A classroom library was set up, consisting

a/ Those getting less than 10 items correct were retested on an equivalent form. Form A is omitted in this tabulation when no one took that form.

b/ Tests 4 and 4A consisted of 20 items; all the others, 15.

of 30 copies of Engelhardt and Haertter, Second Course in Algebra and one copy of each of the other books as listed on page 22. It was felt that Wells, Hart Progressive Second Algebra, of which each pupil had a copy, best presented this particular topic and accordingly, the practice work was assigned in it.[1] Pupils having difficulty with any one process were urged to consult the other texts and do additional examples.

The classroom.-- The classroom was one of the traditional type, as may be seen in the drawing of the floor plan on page 8. The room contained forty hinged-top-desk and chair combinations which were portable, with the teacher's desk at the front left corner of the room. The front wall was composed of a door, a bulletin board, and the rest, blackboard. A blackboard extended along the entire left-hand wall, with the back wall of the room made up mostly of windows, equipped with single opaque shades. The fourth side of the room contained a book closet and recessed coat closets.

To make the room more suitable for presentation of the unit, the following things were done:

(1) Four of the forty desk-and-chair combinations were moved out of the room to provide more space for group work

1/ For writer's later feeling, see comment in Chapter IV, page 37.

at a table in the corner between the windows and coat closets.

(2) One of the unused blackboards was used as a bulletin board by attaching papers for display with scotch tape.

(3) The portable desks were pushed together at times to allow other pupils to work together on various activities.

(4) A 4-drawer filing cabinet was borrowed to provide storage space for the tests, optional materials, and the supplementary books. A cardboard box, placed on top, was used to hold the 5" by 8" supplementary cards for the optional related activities.

<u>Duplication of materials</u>.-- A duplicating machine was available to the mathematics and science departments, and by typewriting the stencils at home it was an easy matter to run off the materials needed for distribution to the class.

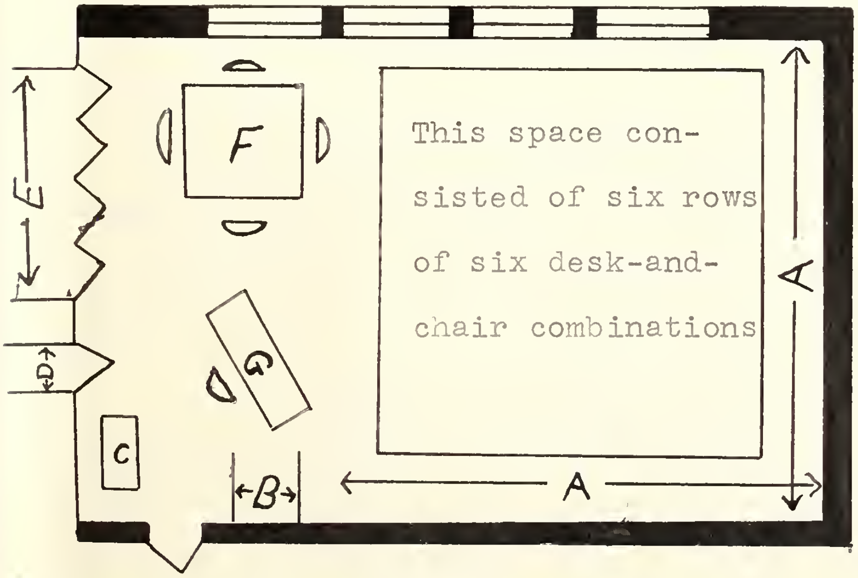

Figure 1. Floor Plan of Classroom.

<u>KEY</u>

A - Blackboard

B - Bulletin Board

C - Filing Cabinet

D - Book Closet

E - Coat Closets

F - Table

G - Teacher's Desk

CHAPTER II

UNIT ORGANIZATION OF THE TOPIC

EXPONENTS AND RADICALS IN INTERMEDIATE ALGEBRA

The Unit 1/

A proper understanding of exponents and radicals is necessary for the continued study of algebra and is particularly important in the study of logarithms.

Delimitation 2/ of the Unit

1. An exponent is a small number written above and to the right of a number and indicates how many times the number is used as a factor. For example: $5^3 = 5 \cdot 5 \cdot 5 = 125$

2. The law of multiplication of exponents directs us to add exponents when we multiply a power of a number by another power of the same number. For example: $4^3 \cdot 4^2 = 4^5$

3. The law of division of exponents directs us to subtract exponents when we divide a power of a number by another power of the same number. For example: $3^6 \div 3^2 = 3^{6-2} = 3^4$

4. In finding the power of a power of a number, we

1/ This term and others to follow have special meanings in connection with the unit method of teaching. Roy O. Billett, Fundamentals of Secondary School Teaching. Houghton Mifflin Company, Boston, 1940, p. 505.

2/ Roy O. Billett, op. cit., p. 505.

multiply the two exponents together. For example: $(4^2)^3 = 4^{2\cdot 3} = 4^6$

5. In finding the power of a product of two or more numbers, we raise each factor to that power and multiply them together, e.g. $(ab)^2 = a^2b^2$

6. In finding the power of a quotient of two numbers, we raise both the numberator and denominator to that power. e.g. $(8/3)^2 = 8^2/3^2 = 64/9$

7. The principal root of a positive number is positive. For example: the principal square root of 4 is 2.

8. The principal odd root of a negative number is negative. For example: the principal cube root of -8 is -2.

9. Roots of numbers may be indicated also by fractional exponents. For example: $16^{1/4}$ stands for $\sqrt[4]{16}$.

10. The numerator of a fractional exponent indicates the power of the number; and the denominator, the root to be extracted. For example: $25^{3/2} = (\sqrt{25})^3 = 5^3 = 125$.

11. Any quantity (except zero) raised to the zero power has a value of 1. For example: $8^0 = 1$.

12. Any quantity raised to a negative power is equal to 1 divided by the same quantity raised to the same positive power. As an example, $5^{-3} = 1/5^3 = 1/125$.

13. Any factor of one term of a fraction may be transferred to the other term provided the sign of its exponent is changed. For example: $x^{-2}/y^{-3} = y^3/x^2$.

14. When an indicated root can be obtained, the result is a rational number; $\sqrt{25} = 5$. When the root cannot be obtained, the result is an irrational number; $\sqrt{5}$ is irrational.

15. The denominator of the fractional exponent, or the small number written in the angle of the radical sign, is the index. The "4" of $\sqrt[4]{16}$ is the index.

16. Radicals can be indicated by fractional exponents, which obey all the laws of positive integral exponents.

17. In simplifying a radical:

a. A radical is usually written with as low an index as possible.

b. Factors are removed from the radicand when possible.

c. The radicand is written without a denominator.

18. In order to add or subtract radicals, they must be similar radicals; which means they must have the same index and the same radicand.

19. In multiplication of radicals, multiply the numbers outside the radical sign and multiply the radicands, if they have the same index: $2\sqrt{3} \cdot 3\sqrt{5} = 6\sqrt{15}$; if not, write each down successively: $3\sqrt{3} \cdot 4\sqrt[3]{5} = 12\sqrt{3}\ \sqrt[3]{5}$.

20. Radicals of the same kind can be divided in two ways:

a. When the radicand of the dividend contains the

radicand of the divisor, place under one radical sign and divide: $\sqrt{8} / \sqrt{2} = \sqrt{8/2} = \sqrt{4} = 2$

b. Rationalize the denominator: $\sqrt{8} / \sqrt{5} = 2\sqrt{10} / 5$

Probable Indirect and Incidental Learning Products 1/

1. An appreciation of mathematics as a tool.

2. An appreciation of how negative and fractional exponents, as well as positive exponents, obey prescribed laws just as do the heavenly bodies in the universe.

3. An attitude of cooperation through group work.

4. An attitude of self-reliance through working at one's own speed and solving problems as encountered.

References for Teacher's Use

1. Billett, Roy O., Fundamentals of Secondary School Teaching. Houghton Mifflin Company, Boston, 1940.

2. Breslich, Ernest R., Problems in Teaching Secondary-School Mathematics. The University of Chicago Press, Chicago, 1931.

3. Butler, Charles H., and F. Lynwood Wren, The Teaching of Secondary Mathematics. McGraw-Hill, New York, 1941.

4. Hassler, Jasper O., and Rolland R. Smith, The Teaching of Secondary-School Mathematics. Macmillan, Boston, 1930.

5. National Council of Teachers of Mathematics, Yearbooks. Bureau of Publications, Teachers College, Columbia University, New York.

6. Sanford, Vera, A Short History of Mathematics. Houghton Mifflin Company, Boston, 1930.

1/ Roy O. Billett, op. cit., p. 506.

7. Schorling, Raleigh, The Teaching of Mathematics. The Ann Arbor Press, Ann Arbor, Michigan, 1936.

The Unit Assignment 1/

A tentative time allotment of three weeks, 15 periods, was planned for the unit on exponents and radicals. The following pre-test was administered to the class to determine what the pupils retained about the topic from their study of first-year algebra. The same test was given again as a final achievement test to calculate gains made by each pupil. One period at the beginning and one at the end of the unit was allowed for the test.

Pre-test and Final Test on Exponents and Radicals 2/

General Directions.-- Perform the work on the back of the sheet and place the answer on the indicated line in the vertical column to the right.

1. Give the numerical value of: 1.

   a. $25^{1/2}$ a. ....

   b. $(-64)^{1/3}$ b. ....

   c. $(1/8)^0$ c. ....

   d. 3^{-2} d. ....

2. Give the simplest form of: 2.

   a. $\sqrt{27x^3}$ a. ....

1/ Roy O. Billett, op. cit., p. 173-180; p. 281; p. 464; p. 506.

2/ Titled only "Test on Exponents and Radicals" when administered to the pupils.

b. $\sqrt[3]{8xy^3}$ b. ....

c. $\sqrt[4]{2x^8y}$ c. ....

d. $\sqrt[3]{-1/27\ x^6y}$ d. ....

3. Give the simplest form of:

a. $\sqrt{\frac{5}{3a}}$ a. ....

b. $\sqrt{\frac{4}{5b^2}}$ b. ....

c. $\sqrt[3]{\frac{1}{9x^6}}$ c. ....

d. $\sqrt{\frac{-27}{x^4y^3}}$ d. ....

4. Simplify: 4.

a. $\sqrt{50} - \sqrt{18}$ a. ....

b. $\sqrt[3]{32x} - \sqrt[3]{108x}$ b. ....

c. $\sqrt{5/a} + \sqrt{7\ 1/5\ a}$ c. ....

d. $\sqrt[3]{9/4x} - \sqrt[3]{2/3x}$ d. ....

5. Find the following products: 5.

a. $\sqrt{5} \cdot \sqrt{3}$ a. ....

b. $2\sqrt{6x} \cdot 3\sqrt{3x}$ b. ....

c. $(\sqrt{3} + x)(\sqrt{3} - x)$ c. ....

d. $(\sqrt{5} + 3b)(\sqrt{5} - 2b)$ d. ....

6. $1 + \sqrt{2}$ satisfies one of the following equations. Place the correct letter in the blank at the right. 6. ....

a. $x^2 - 2x + 1 = 0$; b. $x^2 - x - 2 = 0$;

c. $x^2 - 2x - 1 = 0$; d. $x^2 - x + 2 = 0$

7. Find the following quotients: 7.

a. $\dfrac{8 - \sqrt{40}}{4 - \sqrt{5}}$ a. ....

b. $\dfrac{12}{\sqrt{6}}$ b. ....

c. $\dfrac{\sqrt[3]{18}}{\sqrt[3]{6}}$ c. ....

d. $\dfrac{\sqrt[4]{64}}{\sqrt[4]{4}}$ d. ....

8. Find the value of $\dfrac{3x - 4}{x - 2}$ when

$x = \sqrt{5} - 2$ 8. ....

9. Perform the indicated operations: 9.

a. $(a^{-4})\ (a^{3})$ a. ....

b. $(b^{1/4})\ (b^{1/3})$ b. ....

c. $(x^{3/4})\ (x^{-1/4})$ c. ....

d. $(a^{-1/2})\ (a^{0})$ d. ....

e. $10^{1.5} \cdot 1000$ e. ....

f. $(a^{-3} + y^{-3})\ (a^{-3} + y^{-3})$ f. ....

10. Perform the indicated operations: 10.

a. $(a^{-3}) \div (a^{-4})$ a. ....

b. $(b^{3}) \div (b^{-4})$ b. ....

c. $(a^{2/3}) \div (a^{1/3})$ c. ....

d. $(x^{-3/4}) \div (x^{1/4})$ d. ....

e. $5^{2x} \div 5^{x}$ e. ....

f. $10^{1.38} \div 10^{.87}$ f. ....

11. Give the simplest form for: 11.

a. $(10^{-1.2})^2$ a. ....

b. $(a^5)^{-3}$ b. ....

c. $(a^{3/4})^4$ c. ....

d. $(x^5)^{2/5}$ d. ....

e. $(a^4b^6)^{-1/2}$ e. ....

f. $(a^{-3}b^{-3})^{1/6}$ f. ....

12. Give the simplest form for: 12.

a. $81^{3/4}$ a. ....

b. $36^{3/2}$ b. ....

c. $(-27)^{5/3}$ c. ....

d. $(-8)^{4/3}$ d. ....

e. $(1/8\ a^6)^{2/3}$ e. ....

f. $(-1/27\ a^3)^{4/3}$ f. ....

13. Rewrite with only positive ex- 13.

ponents:

a. $\frac{4a^{-3}b^2}{5c^2d^{-3}}$ a. ....

b. $\frac{x^2}{(2y)^{-2}}$ b. ....

14. Write without any denominator: 14.

a. $\frac{x^3y^{-2}}{z^4}$ a. ....

b. $\frac{5a^3b^{-2}}{2^{-1}c^{-2}}$ b. ....

Introduction of the Unit.-- An introductory talk and

discussion, seeking to tie up what was learned in first-year algebra with what was to come, was based on the following points:

(1) Positive and negative exponents are used in scientific notation to facilitate writing large numbers: 8.5×10^6 for 8,500,000 and 3.1×10^{-6} for .0000031.

(2) The whole foundation of logarithms rests on exponents. Logarithms are in reality exponents and follow the laws of exponents.

(3) Illustrate the five laws of exponents. (See delimitation: items 2-6)

(4) What do we mean by roots of numbers?

(5) What are the meanings of zero and negative exponents?

(6) What do fractional exponents indicate?

(7) How may radicals be added, subtracted, multiplied, or divided?

The study guides were given out. The pupils read them through and any questions they had were answered at the end of their reading. The uses of the practice material and the progress tests and retests were explained.

The optional related activities were explained, a list of which was posted on the bulletin board and described more fully on the supplementary cards kept in the 5" x 8" file.

The pupils were told that two periods would be set aside at the end of the unit for the sharing of class experiences, for discussion of the optional activities performed by the pupils, and an opportunity provided to clear up any questions still remaining.

The final day was devoted to an achievement test on the unit.

Core Activities.[1/]- The Core activities were presented to the pupils in the form of the following Study-and-Activity Guide:

Study-and-Activity Guide

General Directions.-- In performing the work on the unit, you will have an opportunity to work at your own speed. However, do not take this to mean that you can waste time. Do the assigned examples in the text for each new understanding and process with exponents. If necessary, do more than those assigned if you feel you need the practice. When you feel sure of the process, hand in your practice work so that the teacher can be sure you are ready to take the progress test on it. Do not start a new process until you have taken the test and show satisfactory progress. If, after taking a progress test, the teacher tells you the test results are unsatisfactory, do

1/ Roy O. Billett, op. cit., p. 507.

additional practice work either in your own text or in those additional texts which are available. Then take a second progress test on the same process. A final achievement test will be given at the end of the unit. In addition to the assigned examples, most of you will wish to work on one or more of the optional activities listed. You do not have to do the problems and practice work in the order listed, but secure the teacher's permission if you desire to make a change.

Problems and Practice Work   Text: Wells, Hart Progressive Second Algebra

1. a. What is an exponent and what does it indicate? What are the five laws of exponents? Be familiar with their proofs for positive integers.

   b. Practice work

      P. 186: any 4 of 1-12; any 4 of 13-24; any 4 of 25-36

      P. 187: any 8 of 37-60; any 2 of 61-64; any 4 of 67-78

   c. Progress test 1 1/

2. a. How is the principal nth root of a number indicated? What are the names of the symbols used? Roots may also be indicated by fractional exponents. What does the numerator and denominator each indicate?

   b. Practice work

      P. 189: Ex. 110: any 10 of 1-24; any 6 of 25-36

1/ Copies of the progress tests as administered will be found in the appendix.

3. a. In dealing with fractional exponents, one may either extract the root first, or raise the number to the indicated power. In doing the practice work, use whichever method seems easier.

   b. Practice work

      P. 189: Ex. 111: any 4 of 1-10; any 4 of 11-20; any 4 of 21-30

   c. Progress test 2

4. a. What meaning has the exponent zero? What meaning has a negative exponent?

   b. Practice work

      P. 190: the odd examples of 1-44

5. a. What is the rule for transferring a factor of a term of a fraction to the other term?

   b. Practice work

      P. 191: any 5 of 1-12; any 6 of 13-28

   c. Progress test 3

6. a. The five laws of exponents for positive integral exponents are equally valid for fractional, zero, and negative exponents. Perform the indicated operations with the following exercises.

   b. Practice work

      P. 192: the odd examples of 1-60

7. a. Secure additional practice on the work covered so far by doing the miscellaneous examples involving exponents which follow:

b. Practice work

P.193: any 10 of 1-32; any 6 of 33-50; any 12 of 51-87

c. Progress test 4

8. a. What are the meanings of rational and irrational numbers? What is an index of a radical? How else may radicals be indicated? What are the three steps in simplifying radicals?

b. Practice work

P. 195: any 6 of 1-16; any 3 of 17-24; any 3 of 25-32; any 3 of 33-40

c. Progress test 5

9. a. When and how may we add or subtract radicals?

b. Practice work

P. 196: any 8 of 1-24; any 8 of 25-44

c. Progress test 6

10. a. How can radicals be multiplied?

b. Practice work

P. 197: any 5 of 1-15; any 4 of 16-27; any 5 of 28-42; any 5 of 43-53; any 3 of 54-57

11. a. In what two ways can radicals be divided?

b. Practice work

P. 198: any 6 of 1-15; any 5 of 16-27; any 6 of 28-43

c. Progress test 7

12. General review of the unit - P. 199

13. Achievement test on the entire unit

List of References for the Pupils 1/

Engelhardt, F., and L. D. Haertter, Second Course in Algebra. John C. Winston Company, Philadelphia, 1940.

Hawkes, H. E., W. A. Luby, and F. C. Touton, New Second Course in Algebra. Ginn and Company, Boston, 1926.

Leventhal, M. J., C. Salkind, and F. E. Seymour, Adventures in Algebra, Second Course. Globe Book Company, New York, 1941.

Schorling, R., J. R. Clark, and R. R. Smith, Second Year Algebra. World Book Company, New York, 1942.

Schorling, R., J. R. Clark, and S. A. Lindell, Modern Algebra, Second Course. World Book Company, New York, 1929.

Stone, J. C., and V. S. Mallory, A Second Course in Algebra. Benjamin H. Sanborn Company, Chicago, 1937.

VanLeuven, E. P., General Trade Mathematics. McGraw-Hill Book Company, New York, 1942.

Wells, W., and W. W. Hart, Progressive Second Algebra. D. C. Heath and Company, Boston, 1943.

Optional Related Activities 2/

1. On page 283 of Schorling, Clark and Lindell's Modern Algebra you will find a problem entitled, "The Power of Compound Interest". Does this problem have anything to do with exponents? Can you find the appropriate formula and work the problem out? It asks the question, "How much of New York City do you think you could buy today with this sum?" See if you can find out the present valuation of New York City and use the informa-

1/ Roy O. Billett, op. cit., p. 509

2/ Roy O. Billett, op. cit., p. 507

tion to answer the question. Prepare a neat paper for display on the bulletin board working out this activity.

2. Does the quadratic formula make use of anything learned in this course? What specific things? See if you can read ahead enough to solve a quadratic equation unsolvable except by the formula. Prepare a paper to be displayed on the bulletin board illustrating this; or, you may choose to present it at the blackboard during our pooling-of-experiences phase.
3. Four or five of you may wish to prepare an achievement test on the unit, which the class can use as a basis for review before taking the final test. Use any of the available texts for help.
4. Prepare a graph, using one set of axes, on which you can draw the lines representing the equations: $y=x$, $y = x^2$, $y = x^3$, and $y = x^4$. Prepare a brief statement of your conclusions in regard to this.
5. Examine the portion of your text not yet studied and prepare a list of topics which make use of exponents or radicals. Select one of these and explain, either for an oral or written report, what use might be made of it.
6. Four or five of you may elect to work as a group to prepare a list of difficulties which you or other pupils have met in working with exponents and radicals. Write down examples and show their solutions.
7. Prepare a progress test on one or more of the main

processes for this unit. Prepare a corresponding set of answers to accompany it. Consult 3 or more of the algebra texts available.

8. Make a collection of illustrations, which involve exponents and radicals, from science or trade books. If possible, show the meanings or simplifications of these illustrations. You may use science books which you or your classmates have. Consult any in our room library, the school library, or the city library.
9. Make a collection of five formulas involving exponents and radicals, such as those for the area of a circle or the volume of a sphere. Write down a problem for each and show its solution by means of substituting in the formula.
10. Read ahead on the topic logarithms and prepare an oral report explaining how work with logarithms is related to exponents.
11. Five of you may work together, each preparing a demonstration at the blackboard, showing the proofs of the five laws of exponents for positive integral exponents.
12. Using Sanford's A Short History of Mathematics and Schorling, Clark, and Smith's Second Year Algebra, p. 363, prepare a written report on the background of exponents. How did they come into being? At what time in civilization? Could we have gotten very far without the system of exponents? What about logarithms?

13. Prepare a talk or write a paper on any worthwhile activity connected with exponents and radicals suggested by a pupil. Secure the teacher's approval before proceeding too far.

CHAPTER III

TEACHING THE UNIT

Report based on log.-- On the first day the class was told that their work for the next three weeks was in the nature of an experiment to find a better way of teaching. It was explained that they would now be given a pre-test and a similar test at the conclusion of the work to determine how much learning had taken place. Then they were given the rest of the period for the test. Since they were unable to answer many of the items, there was some mumbling and smiling at each other, but it was reiterated that the marks on this pre-test would not affect their final marks and their attitudes toward the test from then on were much improved.

On the second day, an introduction was given to start them thinking and to relate the work on exponents and radicals learned in first-year algebra with the present course. They were given a summary of the procedures to be followed. Study guides were passed out and they were given an opportunity to read them through and after that to ask questions on points not quite clear to them. The optional activities were mentioned and, since some seemed self-conscious at the idea of oral reports, it was

suggested that some pupils might do written reports for display on the bulletin board. The remainder of the period was devoted to work on the first process.

On the following day, the class seemed eager, and practically all said they were ready to take progress test one, so they passed in their practice work and while they took the test help was given to the others. By the end of the period, all had taken the test and it was decided that those getting less than 10 of the 15 items correct could take another test on the process before proceeding with the next process.

By the end of the first week, only two had not taken test 3, while twelve had successfully passed test 4.

From time to time small groups, and occasionally the whole group, were called together for a discussion or presentation of items found to be giving trouble.

By the end of the second week, thirteen had completed progress test 6 and only four had not successfully passed test 5. These four, who were having so much trouble on this process, took seats at a corner of the room while the writer worked with them as a group and re-presented the material at the blackboard. When there seemed to be no further questions, they took test 5A.

By the twelfth day, eight had successfully completed progress test 7 and were either working on a review of the unit or the optional activities.

A group of four worked together on one of the activities at the table. Throughout the teaching of the unit, many worked together in twos or threes at their seats on points which were puzzling. This was not discouraged, but it meant keener supervision was necessary on the writer's part to see that they were really discussing work and not just chatting.

Quite a number consulted the other texts seeking additional examples of the type missed, or a better explanation of some process. One boy in particular, working on an optional activity relating exponents to logarithms, did considerable extra reading and work in its execution.

The fifteenth day was given over to the pooling of experiences. To get ready for this, a committee of three met with the teacher to plan the program. All the pupils of the class had an opportunity to make suggestions to this committee. Almost half the class did optional related activities. Throughout the pooling-of-experiences phase, the teacher made pertinent remarks on points either missed or not stressed by those participating. The program, as finally planned, followed this agenda:

(1) Introduction by teacher.

(2) Have each of the four doing activity 5 demonstrate one formula and its solution at the blackboard.

(3) Oral report relating logarithms to exponents.

(4) Oral report on the background of exponents.

(5) Have the problem on N. Y. C. put on blackboard and explained.

(6) Have the student who plotted the four graphs quickly sketch their curves at the blackboard.

(7) Give those who prepared test questions a chance to present them to the class.

(8) The teacher will answer any questions on exponents and radicals and summarize the work of the unit.

The final achievement test was given on the next day. Forty-five minutes was allowed for the test and about ten minutes for recording their reactions on the questionnaires.

Class reactions from the questionnaires.-- Only five of the class stated that they did not prefer the unit method to the traditional. Some of their reasons were:

(1) "More can be learned by the more constant instruction of the teacher."

(2) "There was too much cheating and waste of time."

(3) "The material is more easily understood when the teacher explains it at the board, instead of having us get it from the book."

Those who expressed preference for the unit method gave as some of their reasons:

(1) "A person can learn more by himself."

(2) "I could work at my own speed."

(3) "It gives time for other activities."

(4) "I understood the problems better than I did before."

(5) "I can do what I feel like doing in one day."

(6) "I got extra practice by doing examples in other books."

In answer to the second question, "What did you like most about this unit?", some of them repeated their reasons for preferring the unit method. Others liked working by themselves at their own speed, while two or three liked having little or no home work.

To question three, about half answered there was nothing about the unit they disliked. One said, "I disliked everything about it", while two or three said they disliked progress test 5. Another thought there weren't thorough enough explanations. One didn't like having so many examples to do, while another disliked the optional activities.

All who did optional activities agreed that they were helpful in letting them see the importance of the topic and its relation to others.

All but two answered that they were able to keep better track of their progress, while all but two others liked the freedom of doing the work at their own speed.

In answer to item seven, on suggestions for improving this type of assignment, the following were given:

(1) "Have more class instruction."

(2) "There were too many progress tests; 4 or 5 better."

(3) "Please cross out the optional activities."

(4) "The final test should be taken when the pupil is ready and not all together on the same day."

(5) "Let's have unit assignments on other topics, too."

(6) "I suggest we go back to the old method."

(7) The rest had no suggestions for improvement.

Results of testing.-- While only four had to retake progress test one, due to getting less than 10 items correct, the marks on the whole were lower than for the rest of the tests. It was noticed particularly, as time went on, that they were doing a more thorough job on the practice material before attempting to take a test.

However, progress test five caused the most trouble and it was necessary for eight to take the test over. In addition, the whole class did so poorly on it that the writer re-presented the material to the entire class. After that, they went on to tests 6 and 7 and did good work on them.

It was mentioned earlier, and can bear restatement, that fewer progress tests than seven would work out better. The unit could be split up into three or four parts and a test given on each. This would give the pupils a chance to assimilate more material before being tested, neces-

sitating less class time to testing and the accompanying correction, and enabling the teacher to give more time to supervision. On the other hand, as it turned out, the pupils had a chance to really dig out the information themselves, while those who needed help, received it.

The test used for the pre-test was given as a final achievement test. The range in the scores on the pre-test was 0-20. The highest possible score was 108. The range on the same test, when given as a final achievement test, was 46-108. All pupils showed improvement. The gains ranged from 34 points to 106 points as shown in the table on page 4.

As can be noted from the frequency distributions and histograms which follow, the three sets of data show fairly close adherence to the normal curve of distribution. If anything, the items could have been made a little more difficult, and increasing the number of items would tend to give a better spread of scores.

Table 4. Frequency Distributions on Three Sets of Data.

Pre-test		Final Test		Gains	
Interval	Frequency	Interval	Frequency	Interval	Frequency
19-20	2	105-109	2	101-106	1
17-18	0	100-104	3	95-100	1
15-16	1	95-99	2	89-94	5
13-14	3	90-94	3	83-88	2
11-12	2	85-89	2	77-82	3
9-10	3	80-84	5	71-76	3
7-8	3	75-79	1	65-70	5
5-6	6	70-74	4	59-64	3
3-4	2	65-69	1	53-58	2
1-2	4	60-64	2	47-52	1
0	3	55-59	0	41-46	1
		50-54	2	35-40	1
		45-49	2	29-34	1
Mean	7.9		80.8		72.3
S. D.	5.6		17.5		17.8

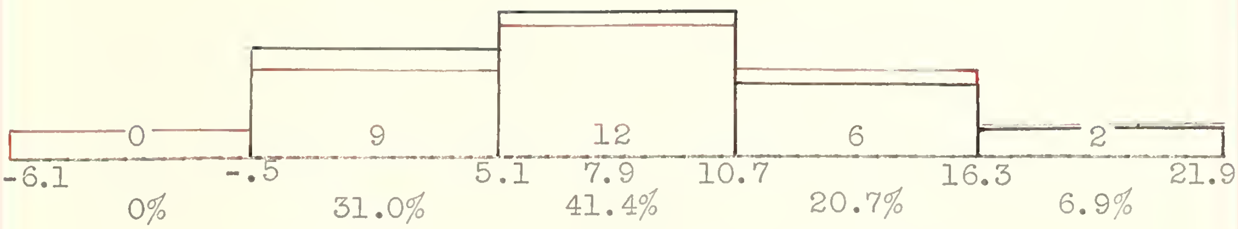

Figure 2. Histogram of Pre-test Scores

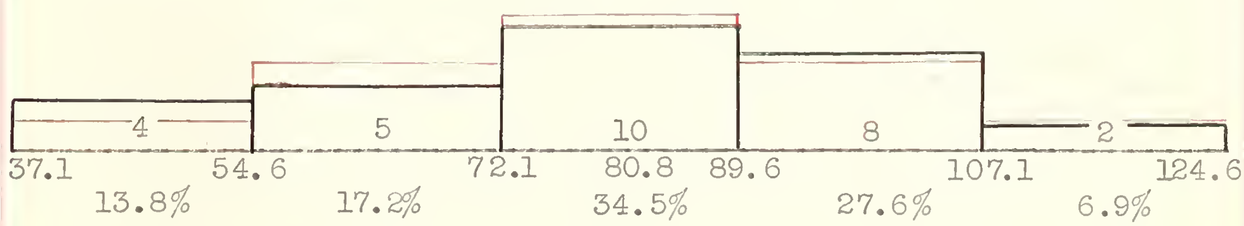

Figure 3. Histogram of Final Test Scores

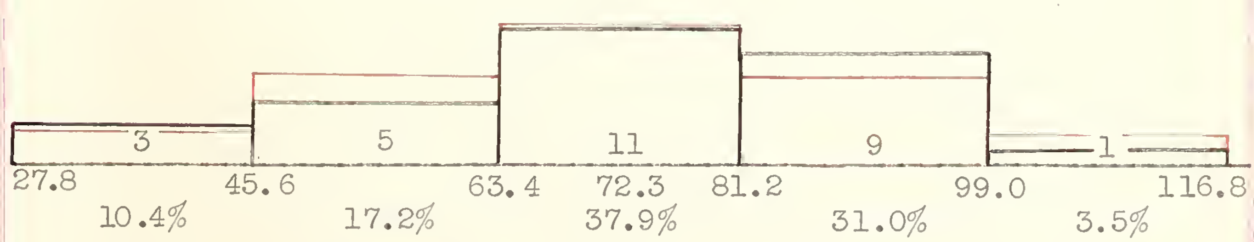

Figure 4. Histogram of Gains

(Note: Black lines indicate actual scores - red lines, normal distribution.)

<u>The teacher's reaction and conclusions</u>.-- Throughout the teaching of the unit, the writer gained the impression, through observation and indirect questioning, that a large share of the class really enjoyed this type of assignment. They seemed to delight in being free to start a new process as soon as their scores on a progress test indicated they were ready for it, without having to wait for the rest of the class. Also, there was a regular hum of activity as they worked independently or in groups on their immediate problems. There seems little doubt that this method of teaching enables the pupil to grow by intelligently directing his own thinking, and he is constantly aware at each step of the way of his progress toward his goals.

No real conclusions can be drawn in this paper for two reasons:

(1) A small group - 29 students - was taught by this method.

(2) The writer feels that more than is evidenced by this report can be accomplished by better use of the unit method. Some of the shortcomings which he feels should be corrected in subsequent teaching of this unit will be found in the following chapter.

However, the writer feels that, despite the great amount of planning that goes into the construction of a unit and its corresponding unit assignment, the pupils'

mastery of the core activities and the richness of experience they gain through the optional related activities make teaching by the unit method worthwhile.

CHAPTER IV

CRITICISMS AND SUGGESTIONS FOR IMPROVING THE UNIT

The writer feels, after this first attempt of his to build a unit and teach it, that he has fallen short of the mark in some instances.

In the study guide.-- The study guides, as presented to the pupils, did not give them specific enough help. Examples similar to those under each item of the delimitation could have been inserted to make the work clearer. It seems now that it was a mistake to assume that one textbook was adequate. A complete set of Engelhardt and Haertter's Second Course in Algebra was on hand and could have been charged out to each pupil at the beginning of the unit. Then assignments for the core activities could have been taken from each text, as well as from the extra copies of the other texts that were available. All this would have done much to enrich the core activities and make them easier to understand.

In the optional related activities.-- Also, not enough specific help was given with the optional related activities. The author realizes all too well the difficulties entailed in setting up optional activities for many topics of mathematics, including this one, but a great many

good optional activities should be listed so as to interest as many of the pupils as possible. To improve the list already given, the following two would be added:

(1) Back in the 16th century, a French lawyer by the name of Francois Vieta did much to standardize the notation of algebra. On page 363 of Schorling, Clark and Smith's Second Year Algebra you will find he used terms like "x quadratus" for x^2. Prepare a list of similar terms he used and their meanings, which you can find in Latin, French, or Greek dictionaries. Also you might like to consult Sanford's Short History of Mathematics and identify, after listing, comparable ways that have been used to represent the above term through the ages. Prepare a neat paper for display on the bulletin board.

(2) The following problem based on geometric progression may cause you some wonder! If a person were able to save one cent the first day, and every day thereafter to save double the amount saved the preceding day, how much money would he have saved at the end of thirty days? See the formula for this on page 247 of Wells and Hart's Progressive Second Algebra and prepare to demonstrate the finding of the answer at the blackboard during our pooling-of-experiences phase.

Practically all members of the class who did optional activities finished their core activities first. The author, in teaching the unit again, would encourage some of the pupils to begin work on the optional activities earlier, since, by doing so, it seems they would get more out of the unit.

From Table 1, pages 2-3, it can be seen that the main interests of the class members are music, sports, dramatics, photography, coin collecting, chemistry, and reading. It would be extremely desirable to get optional activities based on these. However, since it doesn't seem possible to achieve this goal on this particular unit, these interests will be kept in mind and worked in wherever possible on subsequent units.

In the classroom.-- Even though the classroom was one of the traditional type, it is felt that what changes were made didn't transform the situation enough to allow the unit method full play. The following changes would be attempted before presenting the unit again:

(1) For twenty-nine pupils a larger room is needed, so the writer would try to procure a larger room even if it were available only at the time a class were to be taught by this method.

(2) If possible, permission should be obtained to substitute one or two more tables for some of the single desks now in the room.

(3) Four sections of blackboard are used little, if at all. An attempt should be made to convert some of this space for bulletin board use.

The author has been unsuccessful in attempting to locate motion pictures, slides, or film strips dealing with exponents and radicals. However, some twenty wooden models of solids such as cones, pyramids, spheres, and cylinders are on hand in the classroom, and definite use could be made of them in pointing out the various formulas for their volumes and areas which involve exponents.

In the progress tests.-- According to reports, some cheating went on during the taking of the progress tests. This is rather hard to control, since the teacher is busy helping those not taking the tests. Each pupil was handed a progress test to self-administer whenever he handed in his completed practice work on a phase of the unit. Let me point out here that the answers were not necessarily obtained when the corrected papers were returned to the students. Rather, it seems that it might have been an easy matter for the first ones taking each test to jot down the examples and/or the answers and pass them on to others in the class. However, scores of the progress tests were not used to determine pupils' grades, but only to show them that they were ready to proceed on another process, so actually the students themselveswere the ones who suffered. This fact could be stressed at numerous intervals during

the teaching of the unit. This situation may easily be avoided with respect to the pre-test and final test by the construction of equivalent forms.

As stated before, the writer would reduce the seven progress tests to four. The material covered by progress tests 1, 2, and 3 would be tested by a single test composed of 30 items. Test 4 would be left as is. Tests 5 and 6 would becombined into one test of 25 items and test 7 left intact. A blackboard presentation would be given to the entire class before they began work on the material covered by tests 5 and 6.

In the final test.- In rebuilding the final test, the author would add two more items similar to number 6. He would increase section 4 from 4 items to 10 items, in order to thoroughly test the processes of simplifying and combining radicals. Three items similar to number 8 would be added and two each would be added to sections 13 and 14 with the hope that a better spread of scores might result.

In conclusion, it seems evident that some topics like graphs and systems of first degree equations are easier than others to organize by the unit method, but since it is hoped eventually to teach all the topics of the course by this method, the author is not a little glad to have started with this particular topic.

APPENDIX

A. Sample Questionnaire Filled Out by Each Pupil

1. Do you prefer this type of assignment to the old? Why?

2. What did you like most about this unit?

3. What did you dislike about it?

4. Did you do any optional activities and were they helpful in letting you see the importance of this topic and how it is related to others?

5. Were you able to keep track of your progress better with this assignment?

6. Did you like the freedom of doing the work at your own speed rather than by the cut-and-dried daily assignment?

7. Have you any suggestions for improving this type of assignment?

B. Progress Tests and Retests

Progress Test 1

In the following examples, find the result of the indicated operations.

1. a. $x^9 \cdot x^3$ — 1. a. ....
   b. $x^2 \cdot x^r$ — b. ....
   c. $x^{3n} \cdot x^{2n+1}$ — c. ....

2. a. $x^6 \div x^n$ — 2. a. ....
   b. $x^{4n} \div x^{2n}$ — b. ....
   c. $m^{r+1} \div m^2$ — c. ....

3. a. $(x^4)^2$ — 3. a. ....
   b. $(-a^3b^2)^3$ — b. ....
   c. $(4ab^2)^4$ — c. ....

4. a. $(-x^a)^4$ — 4. a. ....
   b. $\frac{(x^n)^3}{(y^a)}$ — b. ....
   c. $\left(-\frac{a^{n-1}}{b^{m-1}}\right)^2$ — c. ....

5. a. $(x - y)^5 \div (x - y)^2$ — 5. a. ....
   b. $3^3 \cdot 4^2$ — b. ....
   c. $\frac{a^{xy}}{b^{xy}}$ — c. ....

Progress Test 1A

In the following examples, find the result of the indicated operations.

1. a. $x^2 \cdot x^5$ — 1. a. ....
   b. $x^{2n-1} \cdot x^{3n+4}$ — b. ....
   c. $x^5 \cdot x^m$ — c. ....
2. a. $x^9 \div x^3$ — 2. a. ....
   b. $x^{3n} \div x^n$ — b. ....
   c. $x^{5a+2} \div x^{3a-3}$ — c. ....
3. a. $(x^8)^2$ — 3. a. ....
   b. $(-5a^2b^5)^3$ — b. ....
   c. $(3a^2bc^3)^4$ — c. ....
4. a. $(-\frac{2a}{b})^2$ — 4. a. ....
   b. $(\frac{x^{n-1}}{y^{n+2}})^n$ — b. ....
   c. $(2a^x)^n$ — c. ....
5. a. $(c-d)^4 \div (c-d)$ — 5. a. ....
   b. $2^4 \cdot 5^3$ — b. ....
   c. $-\frac{x^{2a}}{y^{2a}}$ — c. ....

Progress Test 2

Give the principal root indicated:

1. a. $25^{1/2}$ — 1. a. ....

   b. $\sqrt[3]{-\frac{1}{27}}$ — b. ....

   c. $\sqrt[3]{1/8\ x^6}$ — c. ....

   d. $(.16)^{1/2}$ — d. ....

   e. $(-a^6b^9)^{1/3}$ — e. ....

2. a. $\sqrt[3]{\frac{x^9}{y^9}}$ — 2. a. ....

   b. $\sqrt{\frac{x^6}{y^6}}$ — b. ....

   c. $\sqrt{\frac{x^{4a}}{4}}$ — c. ....

   d. $\sqrt[4]{\frac{a^8}{b^4}}$ — d. ....

   e. $\sqrt[3]{\frac{-27}{a^3}}$ — e. ....

3. a. $(x^5)^{3/5}$ — 3. a. ....

   b. $64^{2/3}$ — b. ....

   c. $(-27)^{5/3}$ — c. ....

   d. $(\frac{-27}{8})^{5/3}$ — d. ....

   e. $(-.125)^{2/3}$ — e. ....

Progress Test 2A

1. a. $27^{1/3}$

   b. $(\frac{1}{27} a^6)^{1/3}$

   c. $\sqrt{.49}$

   d. $\sqrt[6]{x^{18}}$

   e. $(-16a^8)^{1/4}$

2. a. $\sqrt{\frac{x^{2a}}{16}}$

   b. $\sqrt[3]{\frac{x^{12}}{y^6}}$

   c. $(\frac{-64}{a^6b^3})^{1/3}$

   d. $(\frac{a^{4b}}{16})^{1/2}$

   e. $\sqrt[5]{\frac{x^{5a}}{y^{10}}}$

3. a. $(y^6)^{4/3}$

   b. $(-27)^{4/3}$

   c. $(9y^4)^{3/2}$

   d. $(.25)^{3/2}$

   e. $(-a^3)^{5/3}$

1. a. ....
   b. ....
   c. ....
   d. ....
   e. ....

2. a. ....
   b. ....
   c. ....
   d. ....
   e. ....

3. a. ....
   b. ....
   c. ....
   d. ....
   e. ....

Progress Test 3

Evaluate or simplify:

1.	$(-2)^{-4}$	1. ....
2.	16×10^{-3}	2. ....
3.	$8^{0} \cdot 8^{-1}$	3. ....
4.	8^{-3}	4. ....
5.	$(+2)^{-3}$	5. ....
6.	$100^{0} \cdot 3^{-4}$	6. ....
7.	$(-9)^{0}$	7. ....
8.	$(27)^{-1/3}$	8. ....
9.	$(25)^{-1/2}$	9. ....
10.	$5^{0} \cdot x^{2}$	10. ....

Rewrite with only positive exponents:

11.	$\dfrac{5a^{1/3}b^{-1/4}}{c^{-1/3}}$	11. ....
12.	$\dfrac{6a^{-3}b^{4}}{7c^{3}d^{-5}}$	12. ....

Write without any denominator:

13.	$\dfrac{5a^{6}b^{-3}}{2^{-2}c^{-3}}$	13. ....
14.	$\dfrac{x^{4}y^{-3}}{z^{5}}$	14. ....
15.	$\dfrac{5x^{2}y^{-3}z^{2}}{a^{-3}b^{4}}$	15. ....

Progress Test 3A

Evaluate or simplify:

1. $x^{1/2} \cdot x^0$ — 1. ....
2. $x^a \cdot x^{-1/2}$ — 2. ....
3. $a^2 \cdot a^{-4}$ — 3. ....
4. $(x^0yz^2)^{1/2}$ — 4. ....
5. $(x^{-2})^{1/2}$ — 5. ....
6. $-\sqrt{x^0} \cdot 3x^2$ — 6. ....
7. $(-\sqrt{5x})^0 \cdot 3(ab)^0$ — 7. ....
8. $x^{-3} \cdot x^4 \cdot x^0$ — 8. ....
9. $(-\sqrt{64})^{-2/3}$ — 9. ....
10. $(64)^{-1/3}$ — 10. ....

Rewrite with only positive exponents:

11. $\frac{6x^{-2/3}}{5y^{-4}}$ — 11. ....
12. $\frac{x^{-2}y^{-2/3}}{a^3b^{-3}}$ — 12. ....

Write without any denominator:

13. $\frac{a^3b^{-1/2}}{x^{-2}y}$ — 13. ....
14. $\frac{x^{-1}y}{z^{-2}}$ — 14. ....
15. $\frac{x^3y^{-2}}{z^4}$ — 15. ....

Progress Test 4

Perform the indicated operation:

1. $10^{2.5} \div 100$ — 1. ....
2. $x^{3/4} \div x^{1/4}$ — 2. ....
3. $(10^4)^{1/2}$ — 3. ....
4. $(10^{2.76})^{1/3}$ — 4. ....
5. $10^{.308} \cdot 10^{.192}$ — 5. ....
6. $x^{2/3} \div x^{1/3}$ — 6. ....
7. $x^{5/6} \div x^{2/3}$ — 7. ....
8. $x^{-4} \div x^{-2}$ — 8. ....
9. $x^{-2} \div x^{-1/2}$ — 9. ....
10. $(x^2y^3)^2$ — 10. ....
11. $(x^{2/3}y^{1/3})^2$ — 11. ....
12. $x^{-2/3} \cdot x^{-1/3}$ — 12. ....
13. $(-.027)^{1/3}$ — 13. ....
14. $8^0 \cdot 8^{-1/3}$ — 14. ....
15. $7^{-1} \div 7^2$ — 15. ....
16. If $x = 3$, what is the value of $x^{-2} - x^{-1} + 1$? — 16. ....
17. $(a^x \div b^y)^n$ — 17. ....
18. $5a^n \cdot 3a^3$ — 18. ....
19. $(-.2)^3 \cdot 10^4$ — 19. ....
20. $\{(-4x)^2\}^{1/2}$ — 20. ....

Progress Test 4A

Perform the indicated operation:

1. $10^{.568} \cdot 10^{.132}$	1. ....
2. $(10^{.836})^{1/2}$	2. ....
3. $(10^6)^{1/3}$	3. ....
4. $x^{9/4} \div x^{3/4}$	4. ....
5. $10^{3.5} \div 1000$	5. ....
6. $(x^3y^4)^3$	6. ....
7. $x^{-4} \div x^{-5/4}$	7. ....
8. $x^{-8} \div x^{-2}$	8. ....
9. $x^{7/8} \div x^{3/4}$	9. ....
10. $x^{5/3} \div x^{2/3}$	10. ....
11. $14^{-3} \div 14^2$	11. ....
12. $7^0 \cdot 7^{-1/4}$	12. ....
13. $(-.064)^{1/3}$	13. ....
14. $x^{-3/8} \cdot x^{-1/8}$	14. ....
15. $(x^{3/4}y^{1/8})^2$	15. ....
16. If x = 4, what is the value of $x^{1/2} - 4x^{-1/2} - 1$?	16. ....
17. $\{(-8x)^2\}^{1/2}$	17. ....
18. $(-3.5)^2 \cdot 10^2$	18. ....
19. $6a^n \cdot 3a^5$	19. ....
20. $(a^{2x} \div b^{3y})^n$	20. ....

Progress Test 5

Simplify the following radicals:

1.	$\sqrt{72}$	1. ....
2.	$\sqrt[4]{243}$	2. ....
3.	$\sqrt[3]{256}$	3. ....
4.	$\sqrt[4]{49c^4}$	4. ....
5.	$\sqrt{56c^2d}$	5. ....
6.	$\sqrt{\frac{7x}{32y^4}}$	6. ....
7.	$\sqrt[3]{3/5}$	7. ....
8.	$\sqrt[3]{81x^6}$	8. ....
9.	$\sqrt[4]{3/4x^2}$	9. ....
10.	$\sqrt{45x^6}$	10. ....
11.	$\sqrt[3]{64x^4}$	11. ....
12.	$\sqrt[3]{1/8x^4}$	12. ....
13.	$\sqrt{5/32}$	13. ....
14.	$\sqrt[3]{\frac{8c^2}{9d}}$	14. ....
15.	$\sqrt[3]{\frac{ab}{54}}$	15. ....

Progress Test 5A

Simplify the following radicals:

1.	$-\sqrt{28c^4d^2}$	1. ....
2.	$-\sqrt[4]{36x^4}$	2. ....
3.	$-\sqrt[3]{81}$	3. ....
4.	$-\sqrt[4]{625x^5}$	4. ....
5.	$-\sqrt{98}$	5. ....
6.	$-\sqrt[3]{16x^3}$	6. ....
7.	$-\sqrt[3]{4/9}$	7. ....
8.	$-\sqrt{\frac{7x}{50y^8}}$	8. ....
9.	$-\sqrt[4]{7/8\ x^7}$	9. ....
10.	$-\sqrt{28x^4}$	10. ....
11.	$-\sqrt[3]{1/27a^5b^6}$	11. ....
12.	$-\sqrt[3]{125x^5}$	12. ....
13.	$-\sqrt{7/8}$	13. ....
14.	$-\sqrt[3]{\frac{7a^2}{12b^4}}$	14. ....
15.	$-\sqrt[3]{\frac{a^2b^3}{40}}$	15. ....

Progress Test 6

Simplify and combine similar radicals:

1. $\sqrt{72} - \sqrt{50}$ 1. ....
2. $2\sqrt{5} + \sqrt{45}$ 2. ....
3. $\sqrt{32} - \sqrt{18}$ 3. ....
4. $\sqrt[4]{243} - \sqrt[4]{3}$ 4. ....
5. $\sqrt{7/8} + \sqrt{8/7}$ 5. ....
6. $\sqrt[3]{3/2} + \sqrt[3]{3/16}$ 6. ....
7. $\sqrt[4]{4/9} + \sqrt[4]{9/4}$ 7. ....
8. $\sqrt{4/5} + \sqrt{45}$ 8. ....
9. $6\sqrt{1/8x^5} - \sqrt{9/8x^5}$ 9. ....
10. $\sqrt[3]{128x} - \sqrt[3]{54x}$ 10. ....
11. $\sqrt[3]{16} + \sqrt[3]{2} - \sqrt[3]{250}$ 11. ....
12. $\sqrt{2x^2} - 3/4x\sqrt{8} + \sqrt{18x^2}$ 12. ....
13. $\sqrt{8x^2} - 2x\sqrt{18} + 3\sqrt{50x^2}$ 13. ....
14. $4\sqrt[3]{2/9x} - 3\sqrt[3]{6x} + \sqrt[3]{3/4x}$ 14. ....
15. $4x\sqrt[3]{1/9x} + 8\sqrt[3]{3x^4}$ 15. ....

Progress Test 6A

Simplify and combine similar radicals:

1. $2\sqrt{3} + 2\sqrt{12}$ 1. ....
2. $\sqrt{48} - \sqrt{12}$ 2. ....
3. $\sqrt{18} + \sqrt{8}$ 3. ....
4. $2\sqrt[3]{48} - 3\sqrt[3]{6}$ 4. ....
5. $2\sqrt{8} - 5\sqrt{50}$ 5. ....
6. $\sqrt{3/8} + \sqrt{3/2}$ 6. ....
7. $3\sqrt{72} - 5\sqrt{50}$ 7. ....
8. $\sqrt{2/3} + \sqrt{54}$ 8. ....
9. $12\sqrt[3]{4} - 2\sqrt[3]{16}$ 9. ....
10. $\sqrt[3]{\frac{x^2}{9}} + \sqrt[3]{\frac{3x}{x^2}} - \sqrt[3]{3x^2}$ 10. ....
11. $\sqrt[3]{3/4x} - \sqrt[3]{2/9x}$ 11. ....
12. $\sqrt{48} - 5\sqrt{12} + 10\sqrt{1/3}$ 12. ....
13. $4\sqrt{9/2\ a} - 2\sqrt{8a} + 3\sqrt{18a}$ 13. ....
14. $2\sqrt[3]{1/25\ a} - \sqrt[3]{5/27\ a}$ 14. ....
15. $4\sqrt{9/2\ a} - 3\sqrt{8a} + 2\sqrt{18a}$ 15. ....

Progress Test 7

Find the following products:

1. $4\sqrt{5} \cdot \sqrt{15}$ 1. ....
2. $3\sqrt[3]{5} \cdot 2\sqrt[3]{50}$ 2. ....
3. $(4\sqrt[3]{x})^3$ 3. ....
4. $\sqrt[3]{4x^2} \cdot \sqrt[3]{2x^2}$ 4. ....
5. $\sqrt[4]{6a} \cdot \sqrt[4]{8a^3}$ 5. ....
6. $(5 - 3\sqrt{2})(5 + \sqrt{2})$ 6. ....
7. $(8x - \sqrt{5x})(3x + \sqrt{2x})$ 7. ....
8. Does $1 + 2\sqrt{2}$ satisfy the equation $x^2 - 2x + 3 = 0$? 8. ....

Find the following quotients:

9. $\sqrt{36} \div \sqrt{6}$ 9. ....
10. $\sqrt{16x^4} \div \sqrt{2x^3}$ 10. ....
11. $\sqrt[3]{48} \div \sqrt[3]{6}$ 11. ....
12. $\dfrac{\sqrt{21}}{\sqrt{3}}$ 12. ....
13. $\dfrac{\sqrt[5]{15x^7}}{\sqrt[5]{3x^2}}$ 13. ....
14. $\dfrac{16x^2y}{\sqrt{4xy}}$ 14. ....
15. $\dfrac{6m}{\sqrt[3]{9m^2}}$ 15. ....

Progress Test 7A

Find the following products:

1. $3\sqrt{7} \cdot 2\sqrt{21}$ — 1. ....
2. $10\sqrt[3]{15} \cdot 2\sqrt[3]{5}$ — 2. ....
3. $(2\sqrt[4]{x})^4$ — 3. ....
4. $2\sqrt[3]{9x^2} \cdot 3\sqrt[3]{3x}$ — 4. ....
5. $\sqrt[5]{8a^2} \cdot \sqrt[5]{8a^3}$ — 5. ....
6. $(3 - 2\sqrt{3})\ (3 + 2\sqrt{3})$ — 6. ....
7. $(4x - \sqrt{3x})\ (2x + 3\sqrt{2x})$ — 7. ....
8. Does $3 + \sqrt{2}$ satisfy the equation $x^2 - 6x + 7 = 0$? — 8. ....

Find the following quotients:

9. $\sqrt{56} \div \sqrt{7}$ — 9. ....
10. $\sqrt{28x^3} \div \sqrt{7x}$ — 10. ....
11. $\sqrt[4]{96} \div \sqrt[4]{6}$ — 11. ....
12. $\dfrac{\sqrt{27}}{\sqrt{3}}$ — 12. ....
13. $\dfrac{\sqrt[4]{35}}{\sqrt[4]{5}}$ — 13. ....
14. $\dfrac{24xy^2}{\sqrt{8xy}}$ — 14. ....
15. $\dfrac{3x}{\sqrt[4]{27x^2}}$ — 15. ....

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Mary C Foo

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