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ESSAYS ON BARRIERS TO ENTREPRENEURSHIP AND INFORMALITY

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ABSTRACT

In Chapter 1, we develop a dynamic heterogeneous agents model that endogenously links racial wage gaps to racial asymmetries in entrepreneurship. In the model, white and nonwhite workers with different education levels choose between employment and entering entrepreneurship, given their accumulated assets and race-specific wages. Calibrating the model to the Brazilian economy, we show that in general equilibrium reducing the racial wage gap translates into aggregate economic gains as it reduces the misallocation of talent in the economy.

Chapter 2 studies the implications of tax evasion in the context of a size-based tax system in Brazil that alleviates the tax burden for small firms. Because eligibility is based on a size cutoff, the tax system may provide incentives for firm owners to keep their businesses small. I document firm splitting patterns that are suggestive of strategic tax evasion by firm owners to gain access to the tax system. States charging higher taxes for companies that are not in the system feature a larger measure of firm splits. I develop a model to rationalize the choice of firm splitting, calibrated to match empirical facts describing the Brazilian retail trade sector. The model predicts that tax evasion via firm splitting have negligible effects on TFP at the cost of large tax revenue losses.

In Chapter 3, we study the role of the informal sector as an employment and consumption buffer at times of economic distress. We document its pro-cyclical nature: the share of informal workers increases when GDP per capita falls and unemployment rates soar. We also estimate the earnings premium from formalization and how it varies with workers' educational level. Then, we develop a heterogeneous agents model with endogenous job search that incorporates the features of the Brazilian formal and informal labor markets. We show that informality may increase average marginal propensities to consume, amplifying aggregate consumption responses to economic shocks, in spite of reducing equilibrium unemployment rates.

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CHAPTER 1

Racial Wage Gaps and Barriers to Black Entrepreneurship

1.1 INTRODUCTION

Racial inequalities are predominant features in developed and developing countries. There has been a lot of research focusing on the determinants of economic disparities by race (Lang & Lehmann, 2012; Lang & Kahn-Lang Spitzer, 2020) and its consequences in many areas of society such as education, crime, and health. However, there is significantly less understanding of the macroeconomic consequences of racial inequality. Hsieh et al. (2019) makes the argument that labor market barriers defined by race and gender generate a misallocation of talent in the economy, leaving talented workers out of more productive occupations. It is natural to think that labor market barriers to specific groups can also have consequences on the decision to open businesses. Understanding the decisions of workers in entrepreneurship opens up different dimensions through which racial disparities can affect the economy.

In this paper, we investigate the interplay between racial disparities in the labor market and entrepreneurship and their macroeconomic outcomes. We develop a dynamic heterogeneous agents model in which white and nonwhite individuals choose between employment and entrepreneurship and have the same distribution of productivity conditional on education levels, but are remunerated differently in the labor market. We calibrate the model to match moments from the Brazilian economy, drawn from unique owner-employer-employee administrative data that allows us to track the race of entrepreneurs and workers. The model endogenously links racial wage gaps to racial asymmetries in entrepreneurship and

allows us to create counterfactuals through which we investigate the role of racial disparities in macroeconomic outcomes. In our main findings, we show that eliminating racial wage gaps, thus reducing the misallocation of talent in the economy, increases output in 1.26%.

We begin by highlighting stylized facts from the Brazilian economy. Despite corresponding to 45% of the population in the formal labor force, nonwhite individuals only correspond to 30% of entrepreneurs. The nonwhite population is also much less likely to be college educated and have wages on average 30% lower than white workers. When adjusting for observable characteristics such as age, gender, and geographic location, the racial wage gap falls to 16.2% for college-educated workers and 6.64% for non-college-educated workers. Following Hsu Rocha & Dias (2021), we also show differences in the racial wage gap in firms with white and nonwhite entrepreneurs. On average, the racial wage gap in firms with nonwhite entrepreneurs is 25% smaller than in firms with only white entrepreneurs. This is true for both college-educated and non-college-educated workers.

We then outlay our model, designed to replicate the stylized facts, and understand the interplay between racial wage gap and racial disparities in entrepreneurship. In our model, agents can be white or nonwhite, and have a given education level. Within each education level, white and nonwhite workers have the same innate distribution of productivity. At every period, white and nonwhite individuals can choose whether to start their own businesses or to supply labor to other entrepreneurs. In the labor market, black workers face a wage penalty relative to white workers that we set exogenously to reflect wage gaps estimated using the Brazilian context. To open a business, workers must pay a fixed entry cost, which generates a barrier that workers who desire to venture into entrepreneurship need

to overcome by saving on a risk-free asset. Therefore, wage gaps can generate entrepreneurial gaps as black workers are not able to accumulate assets as intensely as white workers do.

The dynamic nature of our model incorporates new channels through which labor market disparities reflect in entrepreneurship differences. Static models of entrepreneurial choice that feature wage gaps in the labor market would generate entrepreneurial gaps that favor the groups of workers being discriminated against. Because black workers face a wage penalty relative to white workers, they would have a smaller opportunity cost of starting a business. In a dynamic model with asset accumulation, we still have the opportunity cost channel, but we also incorporate the savings channels, through which lower labor market earnings decrease entry into entrepreneurship.

We then calibrate the model in order to match empirical moments describing the Brazilian labor market and entrepreneurial environment. Through counterfactual exercises in which we reduce the labor market penalties by race, we quantify the role of racial wage gaps in explaining racial differences in business ownership. We also explore the role of differences in college education observed in the black and white populations and how that accounts for racial differences in entrepreneurship. Finally, we measure the potential macroeconomic gains in terms of GDP and TFP in counterfactual scenarios in which: wage gaps are removed; the entry cost of opening a business is reduced for black workers; and a larger share of black workers is college educated.

Removing wage gaps have heterogeneous effects on black business ownership rates depending on both the education level of the worker and the race of the employer. Removing wage gaps for black workers with at most a High School de-

gree have a bigger impact on black entrepreneurship than removing wage gaps for college-educated black workers. Moreover, closing the wage gaps for black workers working for white employers also have stronger effects on black entrepreneurship when compared to the group of black workers working for black employers. Lifting the wage penalty for black workers with a High School degree or less translate into GDP and TFP gains of 1.04% and 1.24%. Removing wage gaps for all black workers, regardless of their education status or employer's race, increases the GDP by 1.26% and the TFP by 1.27%, while the welfare gains in terms of consumption for black workers can be as high as 3.53%.

Increasing the share of college educated black workers is not as effective at boosting black entrepreneurship as reducing the entry fixed cost for black agents. However, similar increases in black business ownership induced by college education have stronger effects on the economy's GDP than the ones caused by alleviating the entry cost for potential black entrepreneurs. The stronger effects on aggregate output from closing the college gap relative to a reduction in entry costs arise from the fact that entry into black entrepreneurship exerts weaker externalities on white entrepreneurship. Because college educated workers have higher labor market earnings on average, increasing the share of college educated black workers increases entrepreneurship rates for that group only. Reducing fixed costs of entry for all black workers boosts entrepreneurship for college and non-college educated workers alike. The response in equilibrium wages is stronger in the second counterfactual scenario, leading to higher exit rates from entrepreneurship within the white population that are induced by smaller profits combined with a higher opportunity cost of opening a business.

We relate to a large literature studying the interplay between liquidity con-

straints and entrepreneurial choice. In Evans & Jovanovic (1989), one of the earliest attempts to estimate the degree of liquidity constraints in a static model of entrepreneurial choice, authors conclude that financial frictions matter not only for business ownership, but also generate significant sub-optimal allocations of capital conditional on entry into entrepreneurship. More recently, Cagetti & De Nardi (2006), Buera (2009) and Moll (2014) investigate the role of financial frictions for business ownership and optimal capital allocation in a dynamic setting, where wealth accumulation is endogenously determined and self-financing exerts a role at mitigating liquidity constraints. We contribute to this literature by considering racial disparities in the ability to accumulate resources and evaluating how wage gaps deter business ownership for individuals discriminated in the labor market. We also disentangle the role of capital constraints in the extensive and intensive margin of entrepreneurial activity, separating how entry costs matter for business ownership from the presence of credit constraints in the allocation of capital once a business is formed.

This paper also connects to a nascent literature in macroeconomics studying the aggregate implications and efficiency losses derived from markets that feature discrimination more generally. For instance, Chiplunkar & Goldberg (2021) develop a static multi-sector model to quantify barriers to female entrepreneurship in India and what are the potential output and welfare gains from removing the gender-based barriers altogether. In another effort to measure the extent to which gender disparities affects growth, Cavalcanti et al. (2022) develops a two period model of occupational choice in which fertility decisions disproportionately affect women's capacity to invest in human capital and enter the labor force. They conclude that about 30% of the US growth in the past decades can be accounted for by

the loosening of barriers to female labor force participation.

Lastly, this paper contributes to an active literature on racial disparities in labor markets and entrepreneurship. (see Lang & Lehmann (2012); Lang & Kahn-Lang Spitzer (2020) for reviews of the literature on racial labor market disparities). Many papers have focused on identifying discrimination (Bertrand & Mullainathan, 2004; Glover et al., 2017; Kline & Walters, 2021; Kline et al., 2021), and recent work has moved towards understanding more aggregate patterns of racial disparities in labor market outcomes Derenoncourt (2019); Derenoncourt & Montialoux (2021). On the entrepreneurship side, the literature's main focus has been on explaining the reasons why racial minorities are underrepresented in entrepreneurship (Fairlie, 1999; Fairlie & Robb, 2008; Chatterji & Seamans, 2012) and how businesses owned by URM perform relative to the average business owned by white individuals (Fairlie & Robb, 2008; Fairlie et al., 2020). Racial disparities have been carefully described in Latin America and Brazil as well. The seminal work in Oliveira et al. (1985) uses household survey data from 1976 to thoroughly describe the differences in labor market outcomes between white and nonwhite workers in Brazil. There is growing literature using the RAIS matched employer-employee to study racial differences in the Brazilian labor market (Hirata & Soares, 2020; Derenoncourt et al., 2021b,a).

This paper is structured as follows. In Section 1.2 we discuss the racial differences in the Brazilian labor market in terms of both wage gaps and entrepreneurial rates. We estimate wage gaps for non-white workers and show how they vary with respect to their education level and the race of their employer. In Section 1.3, we lay down a dynamic model of self-financing and entrepreneurial choice that accounts for the estimated patterns in the Brazilian labor market. Then, we describe

the model's parametrization and discuss our calibration strategy in Section 1.4. Finally, in Section 1.5, we provide the counterfactual exercises in which we quantify the aggregate implications of removing wage gaps, alleviating fixed entry costs for non-white workers and increasing the share of college education for the non-white population.

1.2 CONTEXT

Brazil is the largest country in Latin America, with 220 million inhabitants. Since colonial times, its history has been defined by racial inequalities as the country that received the most people through slavery institutions from sub-Saharan Africa and the last country in the Americas to abolish slavery. Brazil's official classification of race encompasses 5 categories: *pretos*, *pardos*, *indigenas*, *brancos*, and *amarelos*. For all purposes, in this paper, we group the first three categories as *nonwhite*, whereas the two latter are grouped as white[1].

Racial inequalities are observed in both employment and entrepreneurship outcomes. We show descriptive statistics of the Brazilian formal Labor Market between 2011 and 2015 in Table 1.1. The statistics were created using PNAD (*Pesquisa Nacional por Amostra Domicilar*), which is a household survey conducted by the Brazilian Institute of Geography and Statistics (*IBGE*). The average white worker has 1.5 more years of schooling than the average nonwhite worker. There are also large differences in the share of white and nonwhite workers that are highly educated. 32 % of white workers have some college education, while the share is only 15% for nonwhite workers.

[1]Despite large sociological and Political debates about the differential racial treatment between *Pretos* and *Pardos*, the two groups have similar economic outcomes. Hsu Rocha & Dias (2021) have an extensive discussion of why the 5 categories are grouped in these 2.

Table 1.1: Summary Statistics of Brazilian Economy

	(1) All	(2) White	(3) Nonwhite
Characteristics of all Individuals in the Labor Force			
Sh. with some College	0.247	0.326	0.157
Avg. Schooling Years	9.855	10.57	9.038
Sh. Female	0.378	0.413	0.339
Average Age	38.38	38.71	37.99
Sh. in North or Northeast Regions	0.216	0.119	0.328
Avg. Mo Earnings (2019 RS)	2695.2	3272.7	2033.6
1 if employer; 0 otherwise	0.0671	0.0923	0.0382
Characteristics of Full time Workers in the Formal Sector			
Sh. with some College	0.236	0.313	0.152
Avg. Schooling Years	9.754	10.46	8.988
Sh. Female	0.383	0.422	0.341
Average Age	38.04	38.25	37.81
Avg. Mo Wage (2019 RS)	2320.9	2762.6	1843.5
Characteristics of Formal Entrepreneurs			
Sh. with some College	0.412	0.460	0.278
Avg. Schooling Years	11.26	11.62	10.28
Sh. Female	0.313	0.326	0.278
Average Age	43.05	43.24	42.52
Avg. Mo Earnings (2019 RS)	7194.8	7664.3	5895.8
Observations	265101	131557	133544

Notes: This Table provides summary statistics of Brazilian Labor Markets constructed using PNAD data from 2011 to 2015. The sample is restricted to individuals between 25 and 60 years old working as employees or entrepreneurs in the private formal sector. Employees are defined as formal if they have a signed working permit, and entrepreneurs are defined as those who employ at least one worker and contribute to social security.

These differences are also observed in wages and in the probability of being an entrepreneur. Through Mincerian regressions, we can characterize these wage gaps controlling for observable characteristics such as education, age, gender and geographic location[2] We use data from 2011 to 2015 to estimate versions of the following regression:

[2]As shown in Table 1.1, nonwhite workers are more concentrated in the north and northeast regions, which are less economically developed than the southern regions of the country.

$$Y_i = \alpha + \beta \cdot Nonwhite_i + \Gamma' X_i + \epsilon_i \tag{1.1}$$

where $Nonwhite_i$ is a dummy that takes value one if the individual is nonwhite and X_i is a vector of controls that include state fixed effects, age, gender and education dummies.

We observe large differences in earnings and the likelihood of being an entrepreneur between white and nonwhite individuals. We show our estimates of coefficient β from equation 1.1 in Table 1.2. Nonwhite workers in the private formal sector on average earn 28% than white ones. A large share of the difference can be explained by observable characteristics, namely education. Nevertheless, an earnings gap of 10.5% and 15% for workers and entrepreneurs remain even after controlling for observables.

The same patterns are observed in the last two columns where β gives us the difference in the likelihood of becoming entrepreneurs. Column (5) indicates that 9.2% of white workers in the private-formal sector are entrepreneurs, where for nonwhite workers this share is 5.4 percentage points smaller, thus indicating that only 3.8% of nonwhite workers in the private formal sector are entrepreneurs. Controlling for observable characteristics nonwhite individuals are 4p.p. less likely to be entrepreneurs than white individuals.

Table 1.2: Mincerian Regression Coefficients

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable	Log (Wages)				I[Entrepreneur]	
Nonwhite Individual	-0.279	-0.105	-0.232	-0.148	-0.0541	-0.0403
	(0.00265)	(0.00224)	(0.0149)	(0.0152)	(0.00105)	(0.00108)
Workers' Observables	No	Yes	No	Yes	No	Yes
State FE	No	Yes	No	Yes	No	Yes
Dep. Var. Mean	7.61	7.61	8.51	8.51	.092	.092
Observations	247879	247879	17220	17220	265101	265101

Notes: This Table shows the coefficient β from equation 1.1. The sample is restricted to individuals between 25 and 60 years old working as employees or entrepreneurs in the private formal sector in PNAD data from 2011 to 2015. Employees are defined as formal if they have a signed working permit, and entrepreneurs are defined as those who employ at least one worker and contribute to social security.

1.2.1 Racial Disparities and the Race of Entrepreneurs

In this subsection we show the relation between racial gaps in labor market outcomes and racial composition of entreprenuers. We use identified data on firm owners and link it to matched employer-employee data on the universe of formal labor markets. The data procedures are all detailed in Appendix 3.2. With this combination, we are able to identify the race of entrepreneurs and of workers.

Sorting of Nonwhite Workers to Firms with Nonwhite owners: We begin by showing that firms with nonwhite entrepreneurs have a significantly higher share of nonwhite employees. This is not explained by other observable characteristics of either firms or workers. To describe and access the magnitude of this pattern, we estimate equations 1.2 and 1.3:

$$\text{Share Worker}_j^{NW} = \beta_0 + \beta_1 \cdot Entreprenuer_j^{NW} + \mathbf{X}_j^T \Gamma + \epsilon_j \tag{1.2}$$

Equation 1.2 is at the firm level. $\text{Share Worker}_j^{NW}$ is the share of nonwhite workers among employees in firm j. $Entreprenuer_j^{NW}$ indicates if at least one of

the entrepeneurs who own firm j is nonwhite. $\mathbf{X}_j$ is a set of control variables that include sector, municipality, average age, gender and education of workers and entrepreneurs, number of partners, firm entry year, and 10 firm size categories. β_1 gives a straightforward magnitude of the difference in the racial composition of firms with white and nonwhite entrepreneurs

We also provide evidence at the worker level. For this, we estimate the following equation.

$$Entrepreneur_{j(i)}^{NW} = \beta_0 + \beta_1 \cdot Worker_i^{NW} + \mathbf{X}_i^T \beta_3 + \epsilon_i \tag{1.3}$$

in which $Entreprenuer_{j(i)}^{NW}$ is an indicator that the owner of firm $j(i)$ is nonwhite, $Worker_i^{NW}$ is an indicator that worker i is nonwhite, and $\mathbf{X}_i$ is a vector of control variables. Similar to the firm-level analysis, we can add variables to the control vector to observe how or sorting estimates change with each set of controls.

In equation 1.3, coefficient β_1 is interpreted as the difference in the probability of being employed by a nonwhite entrepreneur if the worker is nonwhite relative to if she was white. We show our estimates for firm level regressions in columns (1)-(4) and for worker level regressions in columns(5)-(7) of Table 1.3.

We observe that firms with nonwhite entrepreneurs have a higher share of nonwhite employees. Comparing our estimates in columns (1) and (3), we find that sorting patterns are not explained by sector or other characteristics of workers or entrepreneurs. However, a large part of the difference is explained by geographic variation, as we observe in column (3) that the coefficient decreases to 6.3 p.p.. In column (4) we weight our estimates by the number of employees in each firm, and we observe no significant changes in the coefficient[3]. Overall, controlling for ge-

[3]Miller & Schmutte (2021) argues that larger firms hire more nonwhite workers, thus there could

ographic, firm and worker characteristics, our estimates suggest that firms with nonwhite owners have a 24% higher share of nonwhite employees than the average white owned firm.

Similar to firm level results, in columns (5)-(7) we observe that geographic variation explains a large share of the sorting of nonwhite workers and nonwhite entrepreneurs. Our estimates in column (7) suggests that nonwhite workers are 27% more likely to be employed by nonwhite entrepreneurs than white workers.

Table 1.3: Sorting of Nonwhite Workers and Nonwhite Entrepreneurs

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Firm Level				Worker Level		
Dependent Variable	Share of Nonwhite Employees				I[Nonwhite Entrepreneur]		
Nonwhite Entrepreneur	0.329	0.319	0.0631	0.0624			
	(0.000811)	(0.000815)	(0.000816)	(0.00302)			
Nonwhite Worker					0.207	0.193	0.0473
					(0.000221)	(0.000218)	(0.000232)
Dep. Var. Mean	.25	.25	.25	.25	.17	.17	.17
Worker and Firm Characteristics	No	Yes	Yes	Yes	No	Yes	Yes
Observations	No	No	Yes	Yes	No	No	Yes
N	1253066	1252998	1252854	1252854	17249458	17249450	17249422

Notes: This Table shows the coefficients β_1 from equation 1.2 and 1.3. In columns (1) - (4) the dependent variable mean is the average share of nonwhite workers in firms with white entrepreneurs. In columns (5)-(7) the dependent variable mean is the unconditional probability that a white worker is employed by a nonwhite entrepreneur. Sample includes all for-profit firms for whom we can identify the race of the owner. In column 4, observations are weighted by the number of employees in each firm. Robust standard errors in parenthesis.

Racial Wage Gap and Race of Entrepreneurs: Next, we re-examine the racial wage gap using our combination of administrative records. The advantage is that in this exercise we can look at how the racial wage gap varies according to the race of entrepreneurs.

Our main specification follows the following equation:

$$ln(wage)_i = \psi_{j(i)} + \beta_1 \cdot Worker_i^{NW} + \beta_2 \cdot Worker_i^{NW} \cdot Entrepreneur_{j(i)}^{NW} + \mathbf{X}_i^T \Lambda + \epsilon_i \quad (1.4)$$

be differences driven by weighting firms with different sizes.

in which $Entrepreneur_{j(i)}^{NW}$ is an indicator that the firm j where individual i is employed has at least one nonwhite entrepreneur, $Worker_i^{NW}$ is an indicator that worker i is nonwhite, $\psi_{j(i)}$ are Fixed Effects for the firm j where worker i is employed, and $\mathbf{X}_i$ is a vector of control variables at the worker level which include age, gender and education.

In this specification, β_1 captures racial wage gap adjusted for observable characteristics in firms without nonwhite entrepreneurs. The sum of β_1, β_2 is the racial wage gap in firms with nonwhite entrepreneurs.

As shown in Table 1.2, differences in education are a key factor in explaining the racial wage gap. Furthermore, the racial wage gap is bigger for employees with some college. We also investigate if there are differences in how firms with and without nonwhite entrepreneurs pay workers with or without college attainment. To do so, we also estimate the following equation:

$$\begin{aligned} ln(wage)_i \quad &= \psi_{j(i)} + \beta_1 \cdot Worker_i^{NW} + \beta_2 \cdot HS_i + \beta_3 \cdot HS_i \cdot Worker_{j(i)}^{NW} \\ &+ \beta_4 \cdot Coll_i \cdot Worker_{j(i)}^{NW} \cdot Entre_{j(i)}^{NW} \\ &+ \beta_5 \cdot HS_i \cdot Worker_{j(i)}^{NW} \cdot Entre_{j(i)}^{NW} + \mathbf{X}_i^T \Lambda + \epsilon_i \end{aligned} \tag{1.5}$$

where $Coll_i$ is an indicator if the worker has some college education, and HS_i is an indicator if the worker has at most a high school degree.

In this specification, the wages of white workers are the baseline value. β_1 gives us the racial wage gap for nonwhite workers with some college in firms with only white entrepreneurs. β_2 is the average difference in earnings between workers

with and without college. The sum $\beta_1 + \beta_3$ is the racial wage gap for workers without college in firms with white entrepreneurs. Coefficients β_4 and β_5 gives the difference in the racial gap between firms with white and nonwhite entrepreneurs for workers with High School and College respectively.

Results: In column (1) of Table 1.4, we show that the raw wage gap in our sample is of approximately 29%. In column (2), we add firm FE, and age and gender as controls. We observe that the racial wage gap decreases to 8.6% with the addition of these variables. Overall, we find that in our sample of administrative data, observable characteristics explain around two thirds of the total racial wage differentials. The results are similar to what we document with household survey data.

Next, in columns (3) we show how the racial wage gap differs according to the educational attainment of workers. The baseline value shows the racial wage gap for workers with college, whereas the sum of rows 1 and 3 give the racial wage gap for workers without college. We observe that the gap is significantly higher for college-educated workers.

In columns (4) and (5) we can look at the difference in the racial wage gap between firms with white and nonwhite entrepreneurs. When we pool workers with all education levels, we observe that the racial wage gap is on average 3.3 p.p. smaller in firms with nonwhite entrepreneurs. This represents a difference of 25%.

Lastly, in column (5), we can observe the same difference by education level. For workers with some college, the difference in the racial wage gap in firms with white and nonwhite owners is 5 p.p. whereas for workers with high school education or less is 2.8 p.p..

Table 1.4: Racial Wage Gap in Firms with White and Nonwhite Entrepreneurs

	(1)	(2)	(3)	(4)	(5)
			Log(Wages)		
Nonwhite Worker	-0.291	-0.0857	-0.162	-0.139	-0.184
	(0.000314)	(0.00408)	(0.0173)	(0.00730)	(0.0185)
Nonwhite Worker x Nonwhite Entr.				0.0336	
				(0.00423)	
Nonwhite worker x HS or less Education			0.0956		0.110
			(0.0182)		(0.0193)
NW worker x HS or less Education x NW Entr.					0.0280
					(0.00185)
NW worker x Some College x NW Entr.					0.0497
					(0.00884)
HS or less Educatoin			-0.548		-0.554
			(0.0179)		(0.0203)
Dep. Var. Mean	7.81	7.82	7.82	7.82	7.82
Firm F.E.	No	Yes	Yes	Yes	Yes
Worker and Firm Characteristics	No	Yes	Yes	Yes	Yes
Observations	17249458	16857233	16857233	16857234	16857234

Notes: This Table shows the coefficients from equation 1.4 and 1.5. Sample includes all for-profit firms for whom we can identify the race of the owner. In column 4, observations are weighted by the number of employees in each firm. Robust standard errors in parenthesis.

1.3 MODEL

1.3.1 Environment

Time is discrete with an infinite horizon. The economy is populated by workers (employees) and entrepreneurs (employers). Workers are heterogeneous with respect to their assets a, their labor productivity z, their race $g \in \{\text{white}, \text{black}\}$, the race of their employer $\tilde{g} \in \{\text{white}, \text{black}\}$ and their education status $e \in \{\text{HS,C}\}$.[4] Entrepreneurs also differ based on their assets, education status and race. We set the mass of white agents to be equal to one. Agents cannot change their education

[4]For exposition purposes, we will refer to non-white agents as black workers and black entrepreneurs. In addition, we refer to agents with some college as being college educated (C) and agents with at most a High School degree as agents with a High School degree (HS).

status; instead, for a given race g, there is a mass of agents with education status e given by $M(e, g)$, which is disciplined by the Brazilian micro-data .

Workers do not face the risk of unemployment. However, at every period, they can decide to run their own business and become entrepreneurs themselves. On the other hand, firms ran by entrepreneurs can exogenously close, changing their owners' employment status from employers to employees. We describe below the utility maximization problem of workers (W) and entrepreneurs (E).

Workers

Black workers are subject to a wage gap relative to white workers. It is not the purpose of this paper to provide an explanation for the sources of racial wage gaps. Instead, we take the wage gap as a primitive of the economy, allowing for it to capture features of the Brazilian labor market. For instance, we let the racial wage gap vary with respect to the education status of workers. Moreover, black workers of a given education status face different wage gaps depending on the race of their employer. As shown in Section **??**, black employees employed by black business owners face a smaller wage gap than black workers employed by white entrepreneurs. Therefore, racial imbalances in the composition of employers account for part of the average racial wage gap in the economy and thus form a link connecting entrepreneurial gaps to wage gaps.

Workers value current and future consumption and decide how much to save in the risk-free asset a at every period to maximize the present discounted value of lifetime utility. The return on the risk-free asset is given by the interest rate r, which is common for all agents. Workers choose whether to open a business and become employers or to continue supplying labor to other entrepreneurs. In order

to open a business, workers must pay a fixed entry cost κ_g that might be different for black and white workers.

Alternatively, workers can decide to stay in the labor market, but they would not know the race of their future employer.[5] The probability that a worker is employed by a black employer is a strictly increasing function $g(\cdot)$ of the share of black entrepreneurs in the economy p_b, satisfying $g(0) = 0$ and $g(1) = 1$. The share of black entrepreneurs in the economy is endogenously determined by workers' optimal occupational choices.

The recursive representation of workers' utility maximization problem is given below,

$$V^{\text{W}}(a, z, e, g, \tilde{g}) = \max_{a'} u(c) + \beta V^{\text{cont}}(a', z, e, g, \tilde{g}')$$

$$\text{subject to} \quad \log\ z' = \rho \log\ z + \varepsilon, \quad \varepsilon \sim N(\mu_e, \sigma^2),$$

$$c + a' = w(e, g, \tilde{g})z + (1 + r)a \quad \text{and} \quad a' \geq 0,$$

where the labor productivity z follows a log-normal distribution with a mean that depends only on the education status of the worker, and $w(e, g, \tilde{g})$ is the state-dependent wage, to be explicitly characterized later.

The continuation value of workers embeds the possibility of paying the fixed cost to start a business. Let $o \in \{0, 1\}$ denote the occupational choice of a worker, where $o = 1$ denotes a worker that decides to become an employer. If workers decide not to open a business ($o = 0$), they might be employed by a black en-

[5]For simplicity, we assume that a job spell lasts for a single period, so that there is a constant churn of workers across employers of different race. Alternatively, one could include an exogenous job separation rate so that only a fraction of employees change jobs. Our model choice is not only more parsimonious but also more conservative. Because there is a substantially larger share of white employers in the Brazilian economy, a richer model in which a fraction of employees do not have a chance to change jobs would lead to more black workers stuck at worse employment outcomes in terms of wages.

trepreneur with probability $g(p_b)$. Thus, the continuation value of a worker is given by

$$V^{\text{cont}}(a', z, e, g, \tilde{g}') = \max_{o \in \{0,1\}} oV^{\text{E}}(a' - \kappa_g) +$$

$$(1-o)\mathbb{E}\{g(p_b)V^{W}(a', z', e, g, \text{black}) + [1 - g(p_b)]V^{\text{W}}(a', z', e, g, \text{white})|z\},$$

where the expectation operator $\mathbb{E}[\cdot|z]$ is taken over the probability distribution of future labor productivity outcomes, conditional on the current productivity draw z.

Entrepreneurs

Entrepreneurs inherit the characteristics they possessed when they were workers. However, after incurring the racial-based fixed cost of opening a business, the entrepreneurs' demographic characteristics do not affect their utility maximization problem; the only variable that affects entrepreneurs' optimization problem is their assets. Entrepreneurs also value current and future consumption streams and their utility maximization problem has the recursive representation below,

$$V^{\text{E}}(a) = \max_{a'} u(c) + \beta(1-\eta)V^{\text{E}}(a')$$

$$\text{subject to} \quad c + a' = \pi(a, \theta) + (1+r)a \quad \text{and} \quad a' \geq 0,$$

where η is an exogenous exit probability from entrepreneurship[6], and $\pi(a, \theta)$ is the profits collected by an entrepreneur that runs a business with wealth level a and a

[6]By assumption, upon exit, entrepreneurs go back to the labor force with the same asset level, but they do not internalize this possibility in their utility maximization problem, attributing a zero value to this outcome. This allows for solving the entrepreneurial utility maximization problem independently from solving the Bellman Equation for workers. We also abstract from endogenous exit, where entrepreneurs would optimally choose whether to keep their companies or to go back to working for someone else.

constant and common productivity θ.

Entrepreneurs produce output by combining labor (n) and capital (k) according to a decreasing returns to scale production function $f(n, k; \theta)$. As in Moll (2014), entrepreneurs face a collateral constraint when choosing the optimal amount of capital to hire. The collateral constraint is based on the amount of assets owned by the entrepreneurs, so that wealthier entrepreneurs face looser restrictions and are therefore more likely to operate their business at its optimal size.

The profit maximization problem for entrepreneurs is given below

$$\pi(a, \theta) = \max_{n,k} f(n, k; \theta) - wn - (r + \delta)k$$

$$\text{subject to} \quad k \leq \lambda a,$$

where r is the risk-free interest rate, δ is the depreciation rate of capital, and λ characterizes the extent of the financial friction: the higher its value, the capital constraint is less binding . The presence of a binding financial friction induces an extra incentive to save for both employers and workers who are considering to become entrepreneurs.

Employers pay wage w per unit of labor, regardless of the race and education status of the employee. However, the employees take home only a fraction $\tau(g, e, \tilde{g}) \in (0, 1]$ of the wage paid by entrepreneurs. This fraction depends on the worker's race g, education status e and the race of the employer $\tilde{g}$. As mentioned before, we do not microfound $\tau(\cdot)$, but rather estimate it empirically using Brazilian labor market data. We set $\tau(\text{white}, \text{college}, \cdot) = 1$ so that college educated white workers receive the base wage of the economy. All other demographic groups face a wage gap relative to college-educated white workers. Therefore, the

per-productivity labor market earnings for a worker of race g, education status e and employed by an entrepreneur with race $\tilde{g}$ is given by $w(g, e, \tilde{g}) = \tau(g, e, \tilde{g})w$.[7]

In the model economy, entrepreneurs are not the only agents responsible for producing the final output. As in Cagetti & De Nardi (2006), there is a corporate sector that acts as a representative firm. It also combines capital and labor using the same production function $f(n, k; \theta_c)$, but it is subject to its own Hicks-neutral productivity parameter θ_c. The corporate sector takes as given the same input prices paid by the entrepreneurs.

1.3.2 Steady-State Equilibrium

In steady state, an equilibrium consists of a wage w and an interest rate r such that: (i) workers and entrepreneurs maximize their utility; (ii) the labor and capital demanded by the entrepreneurial and corporate sector match respectively the supply of labor and the assets owned by both workers and entrepreneurs; (iii) the distributions of agents over the state space, $\mu^i(\cdot),\ i \in \{\mathrm{W}, \mathrm{E}\}$, is time invariant and satisfy the following system of equations, where $\Omega_e(\cdot)$ is the ergodic distribution of workers' labor productivity with education e:

1. *Mass of workers with state space* (a, z, e, g) *employed by white entrepreneurs:*

$$\mu^{\mathrm{W}}(a, z, e, g, \text{white}) = [1 - g(p_b)]$$
$$\cdot \left\{ \sum_{\tilde{g}} \left[\int_A \int_Z \mathbf{1}\{a' = a \cap o = 0\} \text{Prob}[z' = z|Z] \mu^{\mathrm{W}}(A, Z, e, g, \tilde{g}) dZ dA \right] + \eta \Omega_e(z) \mu^{\mathrm{E}}(a, g, e) \right\};$$

2. *Mass of workers with state space* (a, z, e, g) *employed by black entrepreneurs:*

[7] This setting implies that there is a loss in economic resources as the entrepreneurs' expenditure on labor is not fully absorbed by the workers in the economy. We provide a lump-sum transfer of the unabsorbed share of wages to all agents, workers and entrepreneurs, so that in the aggregate the losses are undone.

$$\mu^{\mathrm{W}}(a,z,e,g,\text{black}) = g(p_b)\Big\{\sum_{\tilde{g}}\Big[\int_A\int_Z \mathbf{1}\{a' = a \cap o = 0\}$$

$$\cdot\,\text{Prob}[z' = z|Z]\mu^{\mathrm{W}}(A,Z,e,g,\tilde{g})dZdA\Big] + \eta\Omega_e(z)\mu^{\mathrm{E}}(a,g,e)\Big\};$$

3. *Mass of entrepreneurs with state space* (a,e,g)*:*

$$\mu^{\mathrm{E}}(a,g,e) = (1-\eta)\int_A \mathbf{1}\{a' = a\}\mu^{\mathrm{E}}(A,g,e)dA$$

$$+\sum_{\tilde{g}}\int_A\int_Z \mathbf{1}\{a' = a \cap o = 1\}\mu^{\mathrm{W}}(A,Z,e,g,\tilde{g})dZdA;$$

4. *Share of black entrepreneurs:*

$$p_b = \frac{\sum_e \int_A \mu^{\mathrm{E}}(A,e,\text{black})dA}{\sum_g\sum_e\int_A \mu^{\mathrm{E}}(A,e,g)dA};$$

5. *Mass of agents with education* e *and race* g*:*

$$\sum_{\tilde{g}}\Big[\int_A\int_Z \mu^{\mathrm{W}}(A,Z,e,g,\tilde{g})dZdA\Big] + \int_A \mu^{\mathrm{E}}(A,e,g)dA = M(e,g).$$

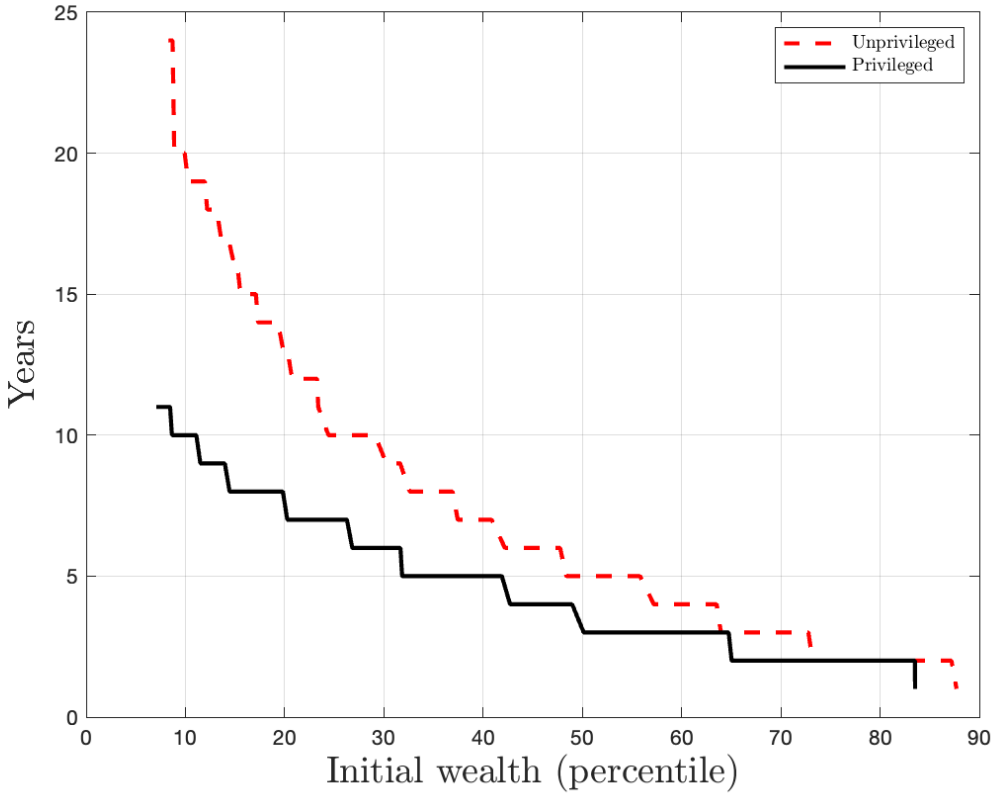
1.3.3 The link between wage gaps and the entrepreneurial gap

As usual in the heterogeneous agents literature, workers are subject to idiosyncratic risks in their labor market earnings due to the stochastic nature of labor productivity. Because of the incompleteness of the asset market, this risk is uninsurable. Savings are an increasing function of agents' own income. Workers with the same asset level and labor productivity, but with distinct wages, would carry different levels of assets to the next period. Therefore, the presence of a racial wage gap would imply that a white worker would acquire more assets than an otherwise identical black worker. In such a scenario, on average, white workers

are more likely to reach asset levels that would enable them to afford the entry cost to become an entrepreneur, endogenously generating a racial entrepreneurial gap.

To illustrate this point, consider a simpler version of the model where all workers have the same education status and the race of their employers does not matter for determining wage gaps. The only state variables that distinguish employees are their assets, labor productivity and race. Figure (1.1) plots how many years would it take for white (privileged) and black (unprivileged) workers to open their business, given an initial asset level. Workers in the lowest percentiles of the wealth distribution take longer to become entrepreneurs. Moreover, because the accumulation of assets is faster the higher the agents' income, racial wage gaps generate time gaps, i.e. black workers with the same asset level owned by a white worker would take more years to start a business.

Figure 1.1: Racial Time Gap to Open a Business

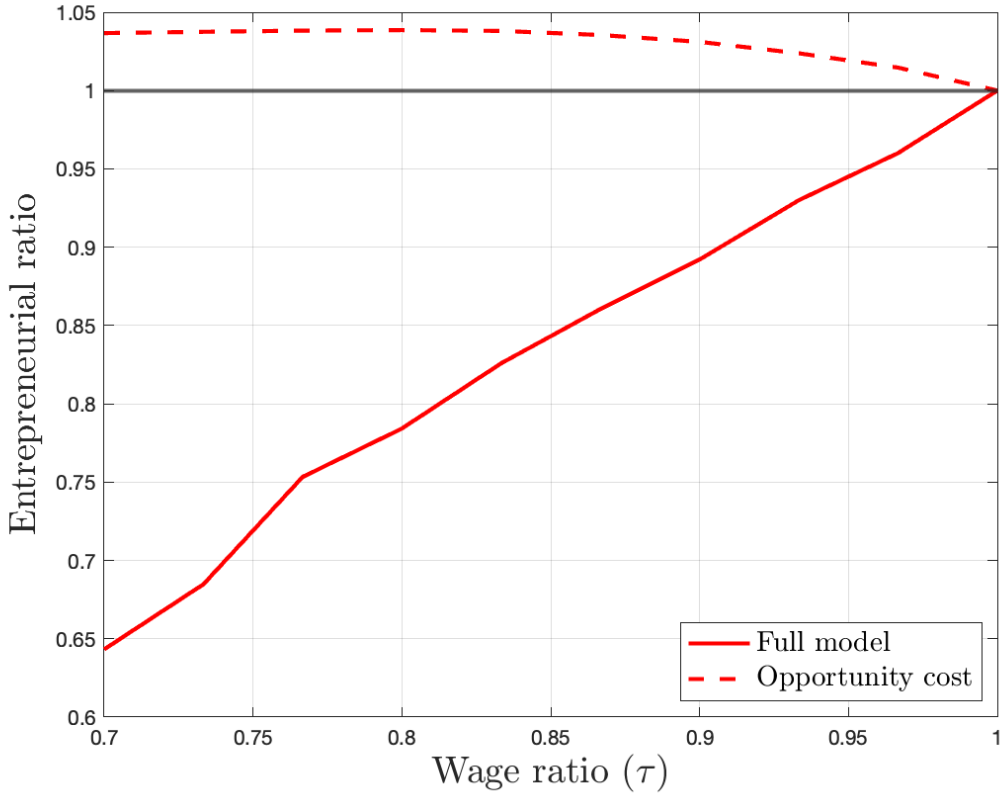


Although racial wage gaps create barriers to black entrepreneurship through the savings channel, they also lower black workers' opportunity cost to open a business. If a black worker expects to receive a relatively low wage in the labor market, she would be better off by starting a business an become an employer instead of being an employee with a smaller payoff than a white worker. This channel alone would predict that in labor markets with a higher racial wage gap, black workers would be willing to open a business more so than white workers, generating an entrepreneurial gap in favor of black business owners. Therefore, the presence of racial wage gaps has two opposing forces operating on the en-

trepreneurial gap.

In Figure (1.2), we plot the equilibrium entrepreneurial ratio, measured as the mass of black entrepreneurs divided by the mass of white entrepreneurs, as a function of the ratio of black and white workers' wages. We decompose this relationship for a simple version of the model with a standard calibration in order to isolate the opportunity cost channel from a version of the model where both savings and opportunity cost are present. To get the opportunity cost channel, we turned off the importance of assets in opening up a business: we set the fixed cost of opening a firm κ_g to zero and the financial friction parameter λ to infinity. The opportunity cost channel predicts that the smaller the relative wage of black workers, in equilibrium, the majority of entrepreneurs would be black. When we let the fixed cost of opening a business to be high enough and having binding financial frictions, there is positive relationship connecting the wage ratio to the entrepreneurial ratio. The magnitude of the savings channel would be the distance between the dashed and solid lines.

Figure 1.2: Opportunity Cost versus the Savings Channel



1.4 CALIBRATION

Before proceeding to the quantitative results, we calibrate a subset of the model parameters to match empirical moments describing the Brazilian economy. In this section we make explicit our preferred functional forms and discuss how we identify the associated parameters.

In the data, we let a firm belong to the entrepreneurial sector if it is privately held, has at least one formally registered employee (other than the owner) and operates in the private sector. Within this group of firms, we exclude observations

above the 99th percentile of the firm size distribution, measured with respect to the number of employees. We gather information on the firm's owners to establish their race. If a firm is owned by multiple partners, we say its owner is black if at least one of the partners is black. We let a firm be part of the corporate sector if it operates in the private sector and is publicly listed in the stock market or if it is above the 99th percentile of the firm size distribution. Since we can only observe employer and employee matches in the formal sector, we also confine the workers in the data to be formally employed. We also exclude workers employed by firms in the public sector. Then we gather their demographic information, such as race, education status and the race of their employer.

We set the mass of the white population to be 1, so that the mass of black agents is given in relative terms. Within each race g, the share of college educated agents $M_{g,\text{coll.}}$ is also disciplined by the data. We set the mean of log labor productivity innovations for college-educated workers μ_{college} to be zero. Then, we identify the mean of log innovations for workers with high-school degree by matching the average skill premium of workers in terms of labor market earnings. To identify the autocorrelation of the productivity processes and the variance of the innovation term, we use panel data at the employee level on earnings to estimate an AR(1) process, controlling for workers' characteristics.

We let workers and entrepreneurs have identical CRRA preferences with parameter γ. The probability of a worker to be employed by a black entrepreneur is linear on the share of black employers in the economy, i.e., $g(p_f) = p_f$. The production function of sector $i \in \{entrepreneurial, corporate\}$ is given by a decreasing returns to scale Cobb-Douglas, with output elasticities for labor and capital given by α_n and α_k, satisfying $\alpha_n + \alpha_k < 1$, i.e., $f(n, k; \theta_i) = \theta_i n^{\alpha_n} k^{\alpha_k}$.

The sectorial Hicks-neutral productivities θ_i cannot be identified separately. We set the corporate sector productivity to unity and let the entrepreneurial sector productivity be such that the model matches the share of the labor force that is employed by firms we identify as belonging to the entrepreneurial sector in the data. We let the racial based fixed cost of opening a firm κ_g to be such that we match the share of business owners within the white and black populations.

Table 3.2 exhibits all the model parameters, broken down into two sets. The first set consists of the parameters we externally calibrate by either choosing standard values from the literature or pinning down directly from the data. The second set consists of four parameters we calibrate internally with the purpose of matching model generated moments to their empirical counterparts. The share of entrepreneurs within the white and black populations would discipline the fixed cost of entry parameter κ_g: the higher the parameter values, the harder it is the asset requirement for workers to start a business. The average earnings ratio between high school and college workers pins down the log mean productivity of a high school degree. Finally, the share of the labor force employed by the model entrepreneurs vis-a-vis the corporate sector disciplines the entrepreneurial sector productivity, since it increases each entrepreneur's labor demand.

Table 1.5: Calibrated Parameters

Externally Calibrated Parameters

Parameter	Description	Value	Source
β	Discount Factor	0.95	Standard
γ	CRRA Parameter	2	Standard
δ	Depreciation Rate of Capital	7%	Standard
α_n	Output Elasticity of Labor	0.4	Standard
α_k	Output Elasticity of Capital	0.2	Standard
$\theta_{\text{corporate}}$	Hicks-Neutral Productivity (Corporate Sector)	1	Numeraire
η	Exit Rate	4.63%	Annual Exit Rate from Entrepreneurship (IBGE,2015)
ρ_{college}	Autocorrelation for College Productivity	0.84	Panel Data on Annual Earnings for College
σ_{college}	Std Dev. for Productivity Innovations (College)	0.41	Panel Data on Earnings for College
μ_{college}	Mean Log Productivity for College	0	Numeraire
ρ_{HS}	Autocorrelation for High School Productivity	0.73	Panel Data on Earnings for High School
σ_{HS}	Std Dev. for Productivity Innovations (HS)	0.52	Panel Data on Earnings for High School
M_{white}	Mass of white agents	1	Set to 1 (to be the numeraire)
$M_{\text{white,college}}$	Mass of college (white)	33%	Share of college educated population (white)
M_{black}	Mass of black agents	0.8511	Measure of black agents
$M_{\text{black,college}}$	Mass of college (black)	16%	Share of college educated population (black)
$\tau_{\text{white,college}}$	Fraction of wages absorbed by college educated white workers	1	Panel Data on Earnings (Numeraire)
$\tau_{\text{white,HS}}$	Fraction of wages absorbed by high school educated white workers	1	Panel Data on Earnings (Numeraire)
$\tau_{\text{black,black,college}}$	Fraction of wages absorbed by blacks working for blacks (college)	0.86	Panel Data on Earnings
$\tau_{\text{black,white,college}}$	Fraction of wages absorbed by blacks working for whites (college)	0.82	Panel Data on Earnings
$\tau_{\text{black,black,HS}}$	Fraction of wages absorbed by blacks working for blacks (HS)	0.96	Panel Data on Earnings
$\tau_{\text{black,white,HS}}$	Fraction of wages absorbed by blacks working for whites (HS)	0.93	Panel Data on Earnings

Internally Calibrated Parameters

Parameter	Description	Value	Moment
κ_{white}	Fixed Cost of Entrepreneurship (white)	12.93	Share of Entrepreneurs (within the White Population)
κ_{black}	Fixed Cost of Entrepreneurship (black)	14.10	Share of Entrepreneurs (within the Black Population)
μ_{HS}	Mean Log Productivity for High School	-0.22	Average Earnings of High Schoolers (Relative to College Workers)
$\theta_{\text{entrepreneur}}$	Hicks-Neutral Productivity (Entrepreneurs)	3.36	Share of Labor Force Employed by Entrepreneurs

The calibrated value of the fixed cost parameter κ for white entrepreneurs is approximately 12.93 model units, which is close to 2 times larger than the average asset level of the economy. The same parameter for the black entrepreneurs is approximately 14.10 model units, which translates to an entry cost that is 9.04% higher for black agents. Once workers become entrepreneurs, they endow a constant productivity $\theta_{\text{entrepreneur}}$, inferred to be 3.36 times larger than the corporate sector's Hicks-Neutral productivity. Its value needs to be large in order for the model to simultaneously conciliate with two data facts: (i) the share of entrepreneurs in the economy is small (only 9.23% and 3.82% of the white and black population are respectively employers); and (ii) even though the number of firm owners vis-a-vis the number of workers is small, they employ a large share of the labor force (close to 56%) in the formal private sector. Finally, college educated workers are on average 22% more productive than workers with a high school degree.

To assess the model's performance, we compare in Table 1.6 model generated

moments to the data. The model is able to closely match the share of white and black entrepreneurs in the economy, the share of the labor force employed by the entrepreneurial sector and the average earnings of workers with a high school degree. The model gets relatively close to matching some untargeted moments such as the share of white entrepreneurs without a college degree and the economy's aggregate capital to output ratios. However it understates the fraction of black entrepreneurs with at most a high school degree and the average firm size in terms of number of employees.

Table 1.6: Model Performance

Targeted Moments		
	Model	**Data**
Share of Entrepreneurs (White)	9.49%	9.23%
Share of Entrepreneurs (Black)	3.81%	3.82%
Avg. High School Earnings to College Ratio	51.91%	52.89%
Share of Labor Force Employed by Entrepreneurs	55.35%	56.1%
Untargeted Moments		
Share of Entrepreneurs with High School (White)	50.43 %	53.97%
Share of Entrepreneurs with High School (Black)	59.53%	72.15%
Avg. Firm Size (number of employees per firm)	4.90	7.50
Capital to Output Ratio	2.70	2.5

1.5 QUANTITATIVE RESULTS

In this section, we quantify the impact of closing the wage gap on the share of black entrepreneurs in the economy and on other macroeconomic outcomes, such as the GDP and TFP. We also evaluate how closing the wage gap for different groups of black employees may have heterogeneous effects on the same outcomes. For example, we close the wage gap for college educated black workers employed by white entrepreneurs, while keeping unaltered the wage gaps faced by other groups

of black workers. Finally we quantify the aggregate role of racial differences in college education and the fixed costs of entry.

1.5.1 Closing the Wage Gaps

As discussed in Section 1.3, closing the wage gap exerts two competing forces on black entrepreneurship. First, there is the savings channel, through which closing the wage gap for black workers allows them to save more and reach asset levels high enough for them to pay the fixed cost of opening a business. The second force is the opportunity cost channel, in which closing the wage gap for black workers increases their value of working vis-a-vis- that of running a business, causing a decrease in black entrepreneurship. A general equilibrium model allows us to quantify which of the two forces dominate.

In the baseline economy, we set the model parameters as displayed in Table 3.2. Then we close the wage gap for each group of black workers: for each education status e (high school and college) and employer's race $\tilde{g}$ (black and white), we set $\tau(\text{black}, \tilde{g}, e) = 1$. Finally we close the wage gap for all black workers at once.

In Table 1.7, we present the results of the proposed exercises. In column (1) we show model statistics describing the entrepreneurial activity for the baseline economy. Then, in columns (2) to (5), we close the wage gap for each group of black workers. Finally, in column (6), we close the wage gap for all black workers simultaneously.

Table 1.7: Effects of Closing Wage Gaps on Entrepreneurship

		High School		College		
	(1)	(2)	(3)	(4)	(5)	(6)
Outcome	**Baseline**	**Black Employer**	**White Employer**	**Black Employer**	**White Employer**	**All**
Share of Entrep. (within Black)	3.81%	3.70%	4.16%	3.83%	3.77%	4.10%
Share of Entrep. (within White)	9.49%	9.49%	9.56%	9.49%	9.70%	9.63%
Average Wage Gap	7.74%	6.77%	3.75%	7.15%	5.83%	0%

Closing the wage gap for distinct groups of black workers have heterogeneous aggregate impacts on the entrepreneurial activity. Because black workers employed by white entrepreneurs face a larger wage penalty when compared to their peers that work for black employers, closing their wage gap has a larger effect overall on black entrepreneurship rates and also on the average wage gap in the economy. Moreover, closing the wage gap for black workers with a high school degree and employed by white entrepreneurs boosts black entrepreneurship from a baseline level of 3.81% to 4.16%. Closing the wage gap for other groups of black workers, such as those employed by black entrepreneurs, can have a negative, albeit small, impact on the share of entrepreneurs within the black population, highlighting the opportunity cost channel associated with exogenous increases in the labor market earnings. Overall, removing the wage gap for all black workers increases the share of entrepreneurs among black agents from 3.81% to 4.10%, with positive externalities on the share of entrepreneurs within the white population (it grows from 9.49% to 9.63%).

The increase in black entrepreneurship induced by the elimination of racial wage gaps may have large macroeconomic effects as shown in Table 1.8. Closing the wage gap for black workers with a high school degree working for white employers (column 2) increases the GDP of the entrepreneurial sector by nearly 2.61% (or 260.49 bps), which represents an increase in the economy's GDP of approximately 1.05%. Closing the wage gap for all black workers improves the economy's GDP by 1.27% (and the entrepreneurial sector's output by 2.07%). [8] This also translates into gains in TFP. Removing wage gaps for all black workers im-

[8]The effect of closing the wage gap on the economy's GDP is always smaller in magnitude to the isolated effect on the entrepreneurial sector's output. The reason is that the two sectors compete for the same inputs, so that any expansion in the entrepreneurial activity happens at the expense of the corporate sector.

proves the aggregate productivity by nearly 1.27%. In terms of welfare, we also measure the effects of lifting the wage penalty for black workers on black workers' aggregate consumption. Depending on the education status of the worker and the race of their employers, closing the racial wage gap can increase consumption for black workers by at least 0.73% (73.36 bps) and at most 3.53%, when it is closed for all black agents simultaneously.

Table 1.8: Macroeconomic Effects of Closing Wage Gaps

	High School		College		
	(1)	(2)	(3)	(4)	(5)
Outcome	**Black Employer**	**White Employer**	**Black Employer**	**White Employer**	**All**
Output (Entrepreneurs)	-80.78 bps	260.49 bps	8.97 bps	104.41 bps	206.53 bps
Output (All)	-25.41 bps	104.64 bps	6.59 bps	53.19 bps	126.32 bps
Consumption (Black Workers)	73.36 bps	126.88 bps	99.44 bps	140.82 bps	353.24 bps
TFP (Entrepreneurs)	-32.63 bps	124.37 bps	5.41 bps	49.09 bps	126.63 bps

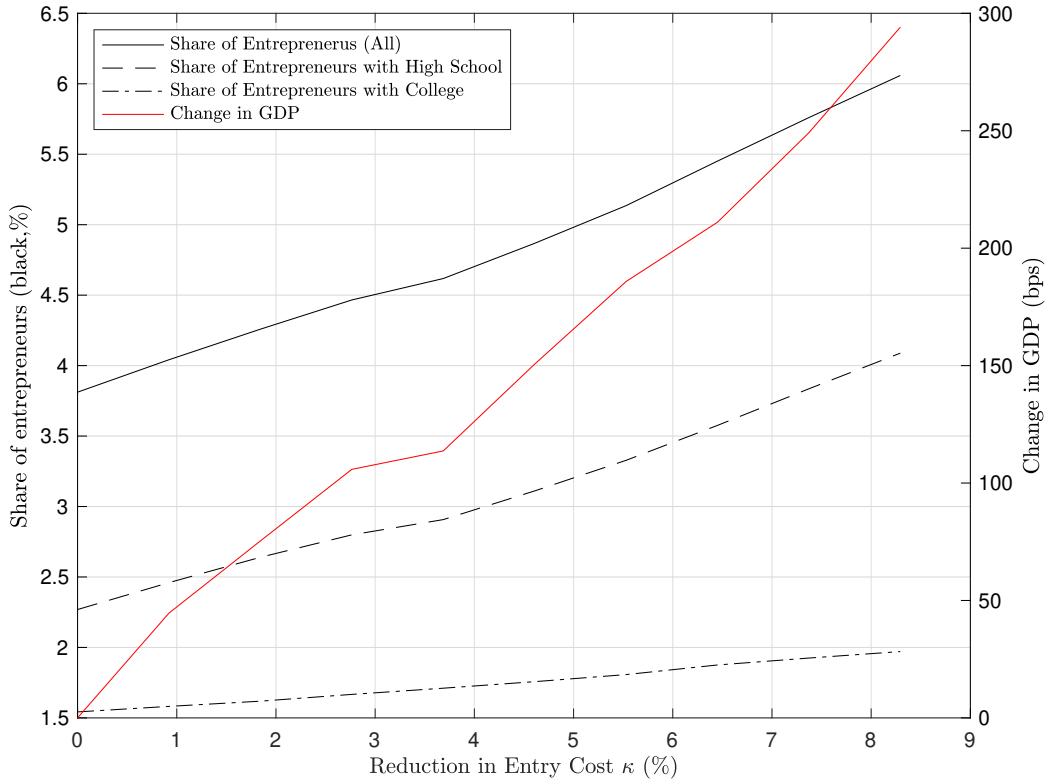
1.5.2 Reducing Entry Barriers for Black Entrepreneurs

In this exercise, we investigate how the fixed cost κ of opening a business affects the entrepreneurial activity. Specifically, we reduce the fixed cost κ for potential black entrepreneurs while keeping unchanged its value for the white population. Then we quantify its implications for the overall entrepreneurial activity and macroeconomic outcomes. The usefulness of this exercise is to measure the possible benefits and externalities associated with public policies designed to foster entrepreneurship targeted at unprivileged groups via start-up capital relief.

In Figure 1.3, we plot both the response of entrepreneurship as a share of the black population and the associated changes in the economy's GDP as result of percent reductions of the fixed cost κ, from its baseline value up until it reaches the same value as the cost faced by white entrepreneurs. Such a policy is highly effective at fostering the ownership of businesses by the black population. A reduction of 8% in the fixed cost of opening a business for black entrepreneurs (which is

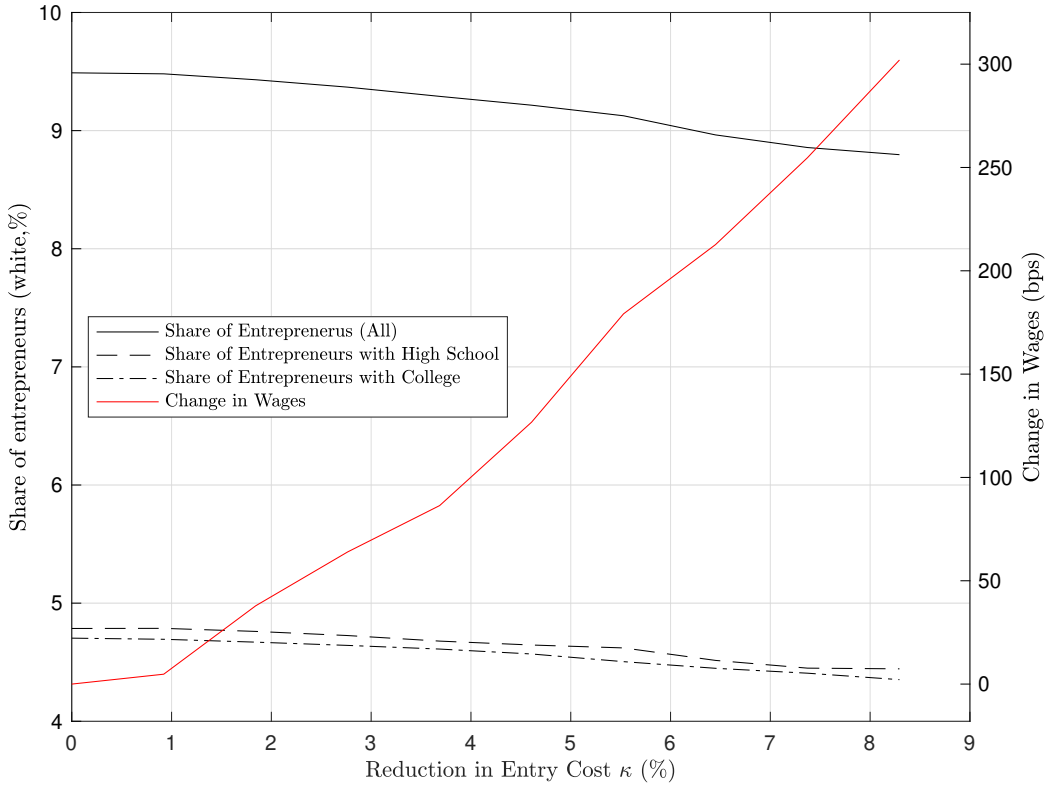
equivalent to equating κ_g for all races) increases the share of entrepreneurs within the black population to 6%. This effect is stronger for entrepreneurs with a High School education. Moreover, reducing the entry cost for black owned business is associated with an increase in GDP of nearly 3% (or 300 bps).

Figure 1.3: Reducing Entry Fixed Cost for Black Population



However, there are externalities generated by the entry of black workers into employer status exerted on entrepreneurship rates within the white population. As more black workers choose to open a business, labor demand increases at the same time as labor supply falls, promoting an increase in the equilibrium wages. Such a change in wages increases the value of working, diminishing the incentive for white workers to become business owners themselves. The opportunity cost channel is stronger for college educated employers, decreasing their participation

Figure 1.4: White Entrepreneurship and the Opportunity Cost Channel



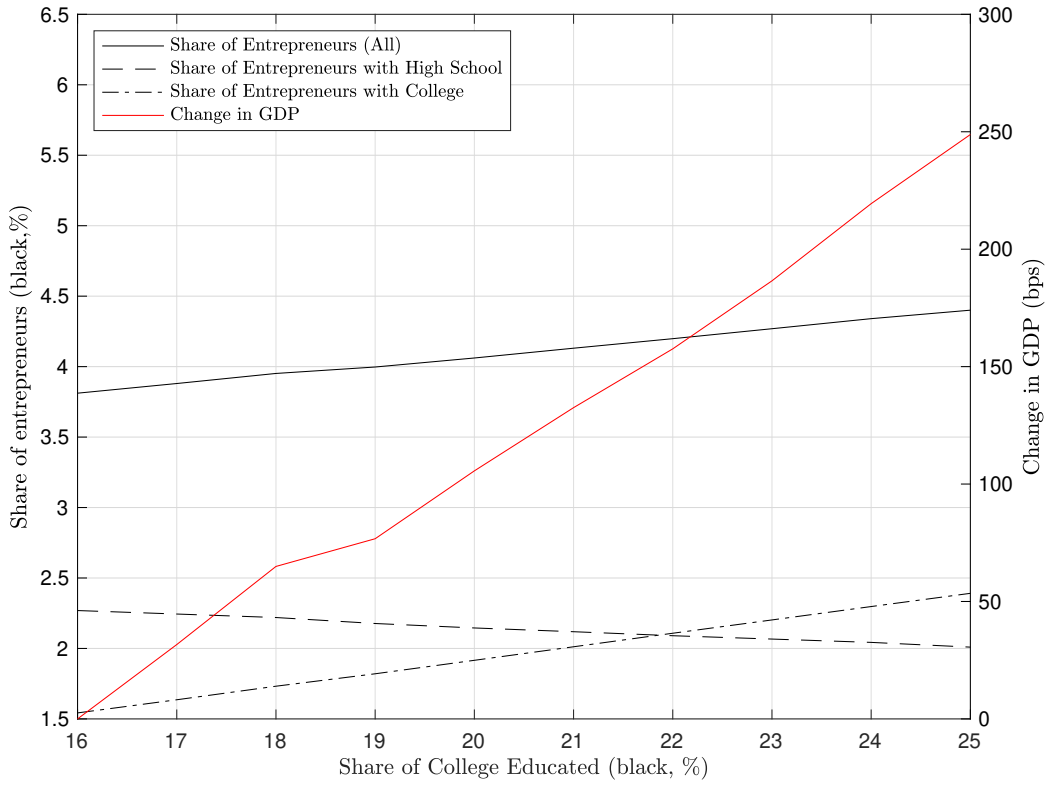
in the entrepreneurial sector as shown in Figure 1.4.

1.5.3 Increasing the Share of College Educated Black Agents

In Brazil, there is a gap in college education between the black and the white population. While 33% of the latter group possess a college degree, only 16% of non-white Brazilians finish their higher education. In the model, as expressed in Table 1.6, there is an over-representation of college educated agents in the entrepreneurial sector for both races. The reason is that college educated workers have a higher labor productivity on average and thus face higher labor market earnings, save more and are more likely to reach an asset level that enables them to open a business.

Thus, part of the entrepreneurial gap in the rate of entrepreneurship between black and white agents can be attributed to the aforementioned differences in college education. In order to understand the role of college education on entrepreneurial participation, we gradually increase the share of college educated black agents and quantify its effects on black entrepreneurship and the economy's GDP. The results are presented in Figure 1.5 below.

Figure 1.5: Overall Effects of Increasing College Education for Black Agents

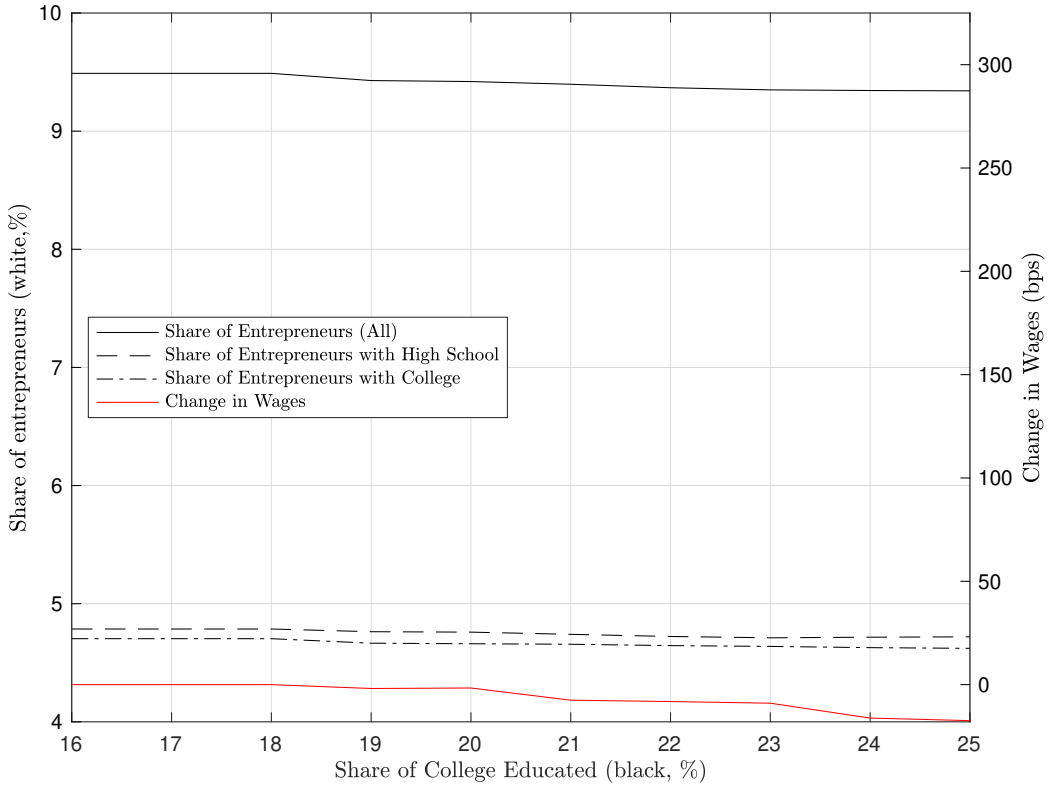


Two notable distinctions stand-out from this exercise when compared to reducing the fixed costs of opening a business for the black population. First, increasing the share of college education from 16% to 25% has a positive effect on the share of entrepreneurs in the black population (from 3.81% to nearly 4.5%). This increase in entrepreneurial activity is modest compared to the effects of reducing the fixed cost of opening a business (see Figure 1.3).

However, the gains in terms of GDP associated with the entry of black workers into entrepreneurship induced by increasing their college education is significantly higher than the gains in GDP induced by reductions in the entry fixed cost. For example, to reach a 2.5% gain in GDP through increasing the college education, black entrepreneurship needs to increase from 3.81% to a share just below 4.5%. To get the same gains in GDP through reductions in the entry cost for black entrepreneurs, the share of black employers need to grow to 5.5%.

The reason for the different response in GDP between both exercises can also be attributed to the externalities caused upon white entrepreneurs in the economy. A relatively small entry of black workers into entrepreneurship do not exert as much upward pressure on equilibrium wages. Therefore the opportunity cost channel that would inhibit white entrepreneurship is nearly silent which would help preserving higher production in the entrepreneurial sector. The effects of increasing the share of college educated black agents on white entrepreneurship and equilibrium wages are shown in Figure 1.6.

Figure 1.6: White Entrepreneurship and the Opportunity Cost Channel



1.6 CONCLUSION

In this paper we study the aggregate consequences of racial discrepancies in labor market earnings and business ownership in the context of large developing economy, Brazil. We show that non-white workers face wage gaps that vary not only with their education status, but also with the race of their employers. Non-white workers with at most a High School degree earn between 92% and 96% of what similar white workers do, depending on the race of their employer. Those with some college education earn from 82% to 86% of college educated white workers' wages in the formal sector. Moreover, we showed that only 3.8% of the non-white population owns a business in the private formal sector, as opposed to an entrepreneurial rate of 9.2% within the white labor force.

In order to quantitatively assess the importance of racial wage gaps for business formation, we develop a heterogeneous agents model of occupational choice calibrated to match moments from the Brazilian labor market and business environment. We show that removing wage gaps increases the rate of business ownership with the non-white population, leading to a GDP and TFP growth of 1.26% and 1.28%, respectively. Moreover, we quantified the positive association between entrepreneurial rates in the non-white population and the economy's GDP caused by (i) reducing entry costs for non-white agents and (ii) increasing the share of college education within the non-white population. We conclude that exercise (ii) leads to a steeper curve connecting entrepreneurial rates and GDP, due to smaller negative externalities exerted on business ownership within the white population.

This paper provides a tractable framework for analysing business formation in the context of unequal labor markets, endogenizing the link between wage gaps and business ownership. Its dynamic nature allows for studying the short and

long run implications of policies destined to foster entrepreneurial participation for minority groups and discriminated populations. Future research could help us better understand how labor productivity correlates with entrepreneurial talent. A testable implication of our model is that more productive workers are more likely to open a business. Further, if their productivity as workers is predictive of their productivity as entrepreneurs once they open a firm, the removal of labor market barriers can have positive and substantial effects for GDP, TFP and welfare.

CHAPTER 2

The Aggregate Implications of Firm Splits

2.1 INTRODUCTION

Size dependent tax regimes are widely spread across the world. In developing countries they usually provide tax incentives for smaller firms with the expectation that it would boost firm growth, promote job creation and increase formalization of businesses and employees. At the same time, empirical evidence in developed countries suggests that size dependent tax regimes can create misallocation that negatively impacts an economy's GDP.[1]

In this paper I study the implications for aggregate TFP and output of a tax system that alleviates the fiscal burden and red tape for small companies, based on a size threshold. I ask the following question: can tax evasion mitigate misallocation due to distortions induced by such size-dependent tax regimes? This size-dependent taxation might provide incentives for managers to keep their firms smaller than they would be in its absence, negatively affecting aggregate output. On the other hand, some managers might make their companies look artificially small, by splitting them into several independent units, all of them able to enjoy eligibility status in the tax system.

I built a model with heterogeneous firms whose owners choose their optimal labor allocation in order to maximize profits net of revenue taxes and fixed costs of tax compliance. A decreasing returns to scale technology implies an optimal firm size, which enables small enough firms to participate into a less stringent tax system. For firms too big to benefit from this taxation regime, their manager can

[1] See Bachas et al. (2018) and Garicano et al. (2016).

incur in additional costs to split up the company by founding several smaller firms eligible to participate in the tax system.

In Brazil, firms with a revenue of at most R$ 3.6 million within a calendar year are qualified[2] to opt in the Simples tax system[3], under which they face smaller revenue tax rates in conjunction to an unification of federal, state and municipal taxes, alleviating the tax burden, compliance costs and bureaucracy in paying taxes.[4] I explore a data set made publicly available by the Brazilian IRS. This data contains information on every partner in every formally registered firm in the country. By matching a firm to its owner, this data allows the identification of individuals who owns multiple firms and could be potentially gaming the tax system by splitting a larger enterprise into multiple smaller firms, with different tax identification numbers, and thus avoiding to cross the taxation threshold for large firms.

This paper is closely related to Garicano et al. (2016), in which the authors quantify the inefficiencies of regulatory burdens that bind for French firms with more than 50 employees. The main source of misallocation in their model is that by imposing costly labor regulation constraints in firms with more than 50 employees, the French legislators might unintentionally make firm owners hesitant to cross the legislation threshold, avoid hiring of more employees and keep their businesses smaller than they would be in the absence of such costs. They estimate welfare

[2]Technically, the federal government establishes that a firm with a revenue below R$4,800,000 is eligible to opt in the program according to the 2016's *Lei Complementar nº 155*. However, Brazilian states kept the revenue threshold at R$3,600,000, so in practice that is the revenue threshold that matters.

[3]I refer to Monteiro & Assunção (2012) for a detailed explanation of how the Simples system works.

[4]To get an idea of how cumbersome it is for Brazilian firms to comply with tax schedules, the World Bank's 2020 Doing Business Report estimated that firm managers in Brazil spend 1501 hours per year to prepare, file and pay several different taxes, including corporate, labor, sales and value added taxes. On average, managers in Latin America and Caribbean take 317.1 hours.

losses equivalent to 3.4% of GDP due to the size-contingent regulations.

Monteiro & Assunção (2012) investigated potential benefits stemming from the Simples system in Brazil. The authors argued that the program alleviated formalization costs for small entrepreneurs, causing a decrease in informality and an increase of firm entry due to the benefits offered to small firms. More recently, Rocha et al. (2018) show that the Simples program increased firm entry in Brazil exclusively due to formalization of informal firms, with no significant effect on the transition from employees or unemployment to entrepreneurship. However, the increased number of formally registered firms do not compensate the tax benefits granted under the program, so that the Simples system faces net tax revenue losses.

The main contribution of my paper is to quantify a potential source of tax evasion, namely the splitting-up of businesses that slices the firm annual revenues so that fiscal authorities will observe annual revenues that are smaller as a consequence of the firm division. I then link this Public Finance contribution to the Macroeconomics of Misallocation literature. The main idea is as follows. On the one hand, a size-dependent legislation that benefits smaller firms in detriment of larger ones might inhibit firm growth, reducing the average firm size in the economy and thus hurting aggregate TFPs and output. On the other hand, in an environment where firm owners split their companies, the actual average firm size in the economy is larger than what it is observed by fiscal authorities. This might undo the misallocation effects of such size-dependent regimes, boosting aggregate TFPs and output. In fact, my model predicts that output and TFP are both next to 0.04% higher when allowing managers to divide their businesses, compared to a model lacking this form of tax evasion.

This paper is structured as follows. Section 2 presents the empirical analysis, where I detail my strategy for flagging firms I believe are spin-offs of larger companies. I also show that the Brazilian states charging higher taxes for companies outside the Simples system also are places that host a larger share of split-up firms, suggesting that managers might be splitting their business with the purpose of opting in the program to avoid the tax burden outside of it. In Section 3 I discuss the theory that allows me to rationalize the choice of firm splitting and understand its implications for the aggregate economy. In section 4 I provide a quantitative exercise and counter-factual analysis, in which I quantify that tax evasion via firm splitting can slightly improve the economy's TFP and output, albeit at a relatively large blow to tax revenues.

2.2 EMPIRICAL ANALYSIS

To examine the phenomena of firm splitting in Brazil, I am using data made publicly available by the *Receita Federal do Brasil*[5] (RFB), which contains information on every formally registered firm as well as each firm's owner information. Among others, for each firm I observe variables such as the firm's registry number (*Cadastro Nacional de Pessoa Jurídica*, or CNPJ), its zip code, its 7-digit sector following the country's National Classification of Economic Activities (CNAE) and, more importantly, whether the firm is a participant in the Simples System. On the owners' side, I observe pieces of information such as their names, their Cadastro de Pessoa Física[6] (CPF) and their equity share on each firm they own.

With the purpose of identifying whether a firm owner decided to split her business into multiple companies, I narrow the data into the universe of firms that are registered in the Simples System and avoid ownership status describing firms owned by other businesses, such as holding companies and large conglomerates in general. I also exclude companies that have a reported equity value of zero.

I flag firms that I believe are a byproduct of a divided business based on four distinct concepts, with different degrees of conservatism. In the first demarcation I consider firms that are owned by the same person and located in the same city as being part of a larger business. In the second one, I flag companies that are owned by the same person and located in the same zip code. In the third one I consider a firm as being part of a larger one if there is at least another one owned by the same person, located in the same city and belonging to the same 7-digit sector. The fourth and final one involves tagging firms owned by the same individual, located

[5]The Brazilian Internal Revenue Service.
[6]The Brazilian equivalent of a Social Security Number.

in the same zip code and classified in the same 7-digit sector.

Of all the firms registered in the Simples system, 20.08% fall into the first and looser category; 9.27% belong to the second category; 6.02% belong to the third; and, finally, 2.58% are in the most restrictive definition of a split-up. Note that regardless of the definition of a divided firm, in all of them I only consider firms that are owned by the same person as being part of a split-up. I ignore the instances in which the firm owner registers a fragment of the firm under the name of another person (e.g. a family member, a friend, etc.). Therefore, most likely, I am underestimating the measure of divided firms within the Simples system.[7] From now on, whenever I mention a split-up firm, I will consider the third category, i.e. a firm whose fragments are owned by the same person, located in the same city and in the same 7-digit sector.

The main point of this paper is that firm owners split their businesses, and thus their annual revenues, in order to qualify for the Simples system. The ideal data set would be one that provides the econometrician with firm-level data on annual revenues. With this information and the splitting definition I discussed before, one could add up each individual firm's annual revenue and check whether the aggregation yields a revenue that is larger than the program's threshold. Unfortunately, I do not observe annual revenues at the firm level.

With the data I have, I explore the state-level variation of the ICMS tax to check whether places charging a higher tax rate for firms outside the Simples system also exhibit a higher measure of firms being split-up and registered in the program. The

[7]It might be the case that managers are splitting their businesses with purposes other than to gain eligibility status in the Simples regime. For example, firm owners might divide their businesses' activities to insulate their assets in case of bankruptcy. Throughout the empirical section I provide some validation exercises that bring my definition of split-ups closer to the first motive, i.e. splitting for tax evasion purposes rather than due to limited liability concerns.

ICMS tax is a value added tax on any transaction of goods and services, and each Brazilian State has the discretionary power to decide how much to charge its tax payers. Under the Simples system, the effective tax rate destined to the payment of the ICMS tax is not only decided at the federal level, but also substantially smaller than the tax rate charged by each state to firms outside the Simples system. The tax aliquots for the former range from 1.25 to 3.95%, depending on the annual revenue of the firm, while those for the latter vary from 17 to 20% depending on the state (see table 7 in the Appendix).

States charging a relatively large ICMS tax rate for firms that are not participating in the Simples system make opting in the program somewhat more attractive. If that's the case, in such states one might see more managers having their businesses split to gain eligibility status and enjoy lower ICMS tax rates under the program. Thus, I regress the dummy variable indicating whether a firm is a byproduct of a split-up decision (as previously defined) on the ICMS tax rate charged by the state in which the firm is located, as captured by the linear probability model below,

$$\text{Split-up Dummy}_i = \beta_0 + \beta_1 \text{ICMS}_{s(i)} + \varepsilon_i,$$

where the null hypothesis is $H_0 : \beta_1 > 0$.

As reported by Table (2.1), there is suggestive evidence of more abundant business splitting activities in states charging relatively large tax rates for firms outside the Simples program. When considering the universe of formally registered firms, a Δ increase in the ICMS tax rate translates into the same Δ increase in the probability of a firm split-up, as the coefficients reported in columns (1) and (2) are close to 1. When I restrict the sample to firms in the retail trade sector (column 3), the covariance between state taxes and the splitting dummy is twice as large, so that a

Δ change in the ICMS tax rate predicts 2Δ change in the probability of a firm split.

Table 2.1: ICMS tax and the Probability of a Firm Split-Up

Sector	All	All	Retail Trade
	(1)	(2)	(3)
ICMS Tax	1.039***	0.979***	2.182***
	(0.0139)	(0.0139)	(0.0289)
Observations	5,463,558	5,463,558	1,512,360
Industry Fixed Effect	No	Yes	-

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

To provide a validation exercise for my definition of a split-up firm, I test whether managers engaging in splitting their companies are *savvier* than its peers. If a manager is splitting his firm with the main purpose of fragmenting gross revenues and thus become eligible to participate in the Simples program, one could expect that such a manager would take less time to register in the tax system. Such managers are more likely to hire accountants and lawyers to help with the legal and tax implications of owning multiple companies and thus they are better informed about the Brazilian tax environment. I constructed a variable that counts how many days since its founding date it takes for a firm to be registered in the Simples program.[8] Then I performed a linear regression of this variable on the dummy flagging whether the enterprise is part of a split-up firm, as detailed before.

The estimation was conducted including all firms in the data and also restricting only to firms in the retail trade sector, with and without controlling for state fixed effects. Table (2.2) displays the results. Columns (3) and (4) refer to the retail

[8]For example, if a firm is founded on 01/01/2020 and is officially registered in the Simples system on 01/02/2020, it took 1 day for this firm to be registered in the Simples system. The date differences is inclusive of weekends.

trade subgroup. In both specifications, retailers that are flagged as being part of a split-up company are registered in the program almost 1 year sooner (285 days to be exact) when compared to other Simples participants. For the overall Brazilian economy, managers who split their businesses do also register in the program relatively fast, as depicted in columns (1) and (2), although the date difference is not as large as in the retail trade sector (it lies in between 55 and 62 days).

Table 2.2: Days Until Simples Registration

	(1)	(2)	(3)	(4)
	All Sectors		Retail Trade	
Split-Up Firm	-61.48***	-55.06***	-285.1***	-258.5***
	(8.279)	(8.236)	(15.26)	(15.19)
Observations	2,663,372	2,663,372	733,871	733,871
State Fixed Effects	No	Yes	No	Yes

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In the next section, I will describe an off-the-shelf model that can rationalize the choice of splitting a company in order to take advantages of a tax system that benefits small enough firms. I will show that this model captures the facts described in this section. The first feature of the model contemplates the suggestive evidence that the harsher the tax rate outside the size-dependent legislation, the higher the measure of firm splitting. The second feature captures the idea that splitting is costly, rationalizing the fact that some managers do not engage in splitting up their companies and thus end up running larger businesses outside the Simples program. Finally, I will discuss the implications of this form of tax evasion in alleviating the misallocation potentially introduced by size-dependent rules and show that there is a trade-off between foregone tax revenues and higher output and TFP

levels.

2.3 THEORY

2.3.1 The Environment

Agents are heterogeneous and indexed by their managerial talent z. They can either open a firm and manage it or work for one, supplying inelastically one unit of labor. Should they run a business, their revenue[9] is determined according to

$$f(n; z)$$

where n is the amount of labor employed and z the agent's productivity at managing her workforce.

Assumption 1. *$f(n; z)$ satisfies the following properties:*

a. f is continuously differentiable with respect to n and z;

b. The first and second partial derivatives with respect to labor obey $f_n > 0$ and $f_{nn} < 0$;

c. The first and second partial derivative with respect to talent z is such that $f_z > 0$ and $f_{zz} \geq 0$.

d. The cross partial derivative satisfy $f_{nz} > 0$

Assumption (1-a) is standard and guarantees a nice characterization of optimal labor inputs through First Order Conditions. Assumption (1-b) imposes a decreasing returns to scale (DRS) technology as in Lucas (1978), so that managers with

[9] I are abstracting from output prices, so that revenue and production are equivalent in this context.

productivity z has an optimal firm size target. Assumption (1-c) ensures that, *ceteris paribus*, more productive managers have higher output. Finally, Assumption (1-d) introduces a complementarity between the manager's talent and the labor allocation.

2.3.2 The Simples Tax Regime and Firm Splitting

The Simples Tax Regime is a size-dependent regulatory environment that benefits firms below a revenue threshold $\bar{Y}$. Managers that select into the Simples program face a tax regime summarized by the pair (τ_S, T_S). If a firm is not registered in the Simples program, it is subject to a taxation environment described by the pair (τ_L, T_L). The first element of the pairs, τ_i, $i \in \{S, L\}$ represents the marginal tax rate on revenues, while the second entry, T_i, captures a fixed-cost of compliance. The marginal tax rates under the Simples regime and outside it satisfy $\tau_S < \tau_L$, so that firms under the Simples framework pay smaller revenue tax rates.

Since the benefits from the Simples tax system kick in only for firms with revenue below $\bar{Y}$, it might provide managers with incentives to cap revenue at the legislation threshold. Managers can also decide to divide their firms into smaller subsidiaries, registering them as independent entities. The latter option, however, comes at a cost. Let $g(f(n; z))$ denote the losses incurred in dodging taxes.

Assumption 2. *$g(\cdot)$ is continuously differentiable, satisfying $g'(\cdot) > 0$ and $g''(\cdot) \geq 0$.*

Assumption (2) introduces a size-dependent cost of splitting a company. The larger the firm, in term of its revenue streams, the heavier the burden to split the business and evade taxes. One may view it as a higher probability of being caught

by fiscal authorities.[10] Furthermore, when splitting the company, hiring a large number of employees might prove to be cumbersome, since the manager need to decide how to register each new hire within the firm's smaller pieces. The function $g(\cdot)$ intend to only capture the existence of such costs. I do not intend to explicitly model the intensive margin of firm splitting, that is, conditional on splitting, how many pieces the firm is fragmented into. I assume that the number of firms the owner register in the program is given by the ceiling function

$$\text{ceiling}\left(\frac{f(n,z)}{\bar{Y}}\right)$$

so that it is guaranteed that each individual firm originated from the split-up has a revenue that is smaller than the Simples threshold. For example, if the target revenue of the company is 5 model units and the revenue threshold is 2 model units, the manager will have to split his business in 3 smaller enterprises.

2.3.3 The Profit Maximization Problem

A manager indexed by productivity z and subject to a tax system $i \in \{S, L\}$ decides how many employees n to hire at wage level w in order to maximize the firm's profits net of taxes and compliance costs. Let $n^i(z)$ denote the optimal factor allocation decision for such a manager. Then

$$n^i(z) \in \arg\max_n (1-\tau_i) f(n,z) - wn - T_i. \tag{2.1}$$

Proposition 1. *The labor demand schedule $n^i(z)$ is strictly increasing in z.*

Proof. Fix a pair of managers indexed by productivity z_1 and z_2 such that $z_1 < z_2$.

[10] Bachas et al. (2018) provide evidence in favor of higher rates of tax compliance and tax enforcement for larger firms.

Let n_1 and n_2 be the associated optimal employment decision for each manager. Assumptions (1-a) and (1-b) guarantees that the First Order Condition associated to the optimization problem in (2.1) is not only necessary but also sufficient to pin down the optimal labor allocation. Then I must have

$$f_n(n_1, z_1) = \frac{w}{1-\tau_i} = f_n(n_2, z_2)$$

$$> f_n(n_2, z_1),$$

where the inequality is granted by Assumption (1-d).

Finally, Assumption (1-c) requires that $n_1 < n_2$. □

Proposition (1) indicates that more talented managers, subject to a given tax regime i, will hire more workers than less productive managers. Moreover, coupled with Assumptions (1-b) and (1-c), Proposition (1) also ensures that the target revenue is also strictly increasing with respect to productivity z. Let $y^i(z) = f(n^i(z), z)$ denote the target revenue for a manager with talent z facing a tax system $i \in \{S, L\}$.

Corollary 1. *The target revenue $y^i(z)$ is strictly increasing with respect to z.*

Corollary (1) provides us with a productivity cutoff below which the target revenue is smaller than the maximum revenue $\bar{Y}$ allowed for a manager to be able to opt in the Simples system. Let z_S denote such a cutoff. Then

$$y^S(z_S) = \bar{Y}. \tag{2.2}$$

Let $\pi^i(z) = (1-\tau_i)y^i(z) - wn^i(z) - T_i$ be the profits collected by manager with productivity z and under the tax regime $i \in \{S, L\}$. Then, whenever $z \leq z^S$, a manager can choose to be part of the Simples system and will do so whenever $\pi^S(z) \geq \pi^L(z)$. Note that since in such a case the manager's productivity is small enough, she does not need to misreport revenues or split its company in order to benefit from the Simples regime.

Now, consider a manager with talent $z > z^S$. By Corollary (1), the firm's target revenue will be higher than the Simples threshold $\bar{Y}$. It might be optimal for such a manager to split the firm into smaller subsidiaries with the purpose of evading taxes. Let $n^D(z)$ be the optimal labor allocation of a manager that considers dividing the company in order to opt in the Simples system. Then

$$n^D(z) \in \arg\max_n (1-\tau_S)f(n,z) - wn - T_S - g(f(n;z)). \tag{2.3}$$

Proposition 2. *If $g''(\cdot) = 0$, the labor demand schedule of a splitting firm, $n^D(z)$, is strictly increasing with the managers productivity.*

Proof. Similar to the proof of Proposition (1). Note that a manager with talent z now will choose its optimal labor demand according to the following First Order Condition

$$f_n(n,z) = \frac{w + g'(f(n,z))f_n(n,z)}{1-\tau_S}.$$

$$\Leftrightarrow f_n(n,z) = \frac{w}{1-\tau_S - g'(\cdot)}$$

The Left Hand Side (LHS) is the same as before, decreasing in n and increasing

in z. The Right Hand Side (RHS) is constant in both n and z. A higher z shifts the LHS upwards, requiring a larger n to yield an equality with the RHS. □

In a similar spirit as in Garicano et al. (2016), it might be optimal for a productive enough manager to choose an employment decision so that her firm produces a revenue exactly equal to the Simples threshold $\bar{Y}$ and be subject to the program's taxation[11]. Let $\bar{n}(z)$ be such that

$$f(\bar{n}(z), z) = \bar{Y},$$

for some $z > z^S$. Then, it will be optimal for a manager to cap its revenue at the Simples threshold whenever $\bar{\pi}(z) \geq \pi^D(z)$, where the latter objects may be defined as

$$\begin{aligned} \bar{\pi}(z) &= (1 - \tau_S)\bar{Y} - w\bar{n}(z) - T_S \\ \pi^D(z) &= (1 - \tau_S)f(n^D(z), z) - wn^D(z) - T_S - g(n^D(z)). \end{aligned} \tag{2.4}$$

This can potentially generate an additional productivity cutoff $z_D > z_S$ below which a firm caps its revenue at $\bar{Y}$, and above which it is optimal for a manager to split the firm. Either way, the firm will have a target revenue small enough so that it will enable it to opt in the Simples system.

Proposition 3. *The optimal profits $\pi^i(z)$, $\forall i \in \{S, L\}$ and $\pi^D(z)$ are an increasing and convex function of the manager's productivity z. The profit stemming from a revenue-capped firm, $\bar{\pi}(z)$, is increasing in the manager's talent.*

Proof. To show that profits are increasing, fix $i \in \{L, S\}$. By definition

[11] This is the main source of misallocation induced by a size-dependent legislation that benefits smaller entities, since in the absence of such a regulatory environment, firms would not have the incentive to stay small.

$$\pi^i(z) = (1-\tau_i)f(n^i(z), z) - wn^i(z) - T_i.$$

Taking a derivative with respect to z yields

$$\partial_z \pi^i(z) = (1-\tau_i)f_z(n^i(z), z) + \partial_z n^i(z)((1-\tau_i)f_n(n^i(z), z) - w).$$

The second term on the RHS is zero, since it is the FOC of the profit maximization problem. Thus

$$\partial_z \pi^i(z) = (1-\tau_i)f_z(n^i(z), z) > 0$$

where the inequality is a gift from Assumption (1-c).

To show convexity, consider

$$\partial_{zz} \pi^i(z) = (1-\tau_i)[\partial_z n^i(z) f_{nz}(n^i(z), z) + f_{zz}(n^i(z), z)].$$

By Assumptions (1-c) and (1-d), coupled with Proposition (1) we have

$$\partial_{zz} \pi^i(z) > 0$$

.

The proof for $\pi^D(z)$ is analogous, provided that $g''(\cdot) = 0$ and $1 - \tau_S - g'(\cdot) > 0$.

To show that $\bar{\pi}(z)$ is increasing in productivity, one may note that taking Assumptions (1-b) and (1-c) accompanied by the definition of $\bar{n}(z)$, one must get that the latter is decreasing in z (i.e. $\bar{n}'(z) < 0$). Then

$$\bar{\pi}'(z) = -w\bar{n}'(z) > 0.$$

□

Proposition (3) is important, because an increasing and convex profit function for firms outside the Simples program, i.e. $\pi^L(z)$, allows for the existence of a further productivity cutoff z_L, above which it is optimal to be large and incur in higher taxation costs instead of splitting the company and evade taxes.

The proposed structure for profit maximization can be summarized in a less convoluted way. It implies the existence of several cutoffs in the managerial talent, which define intervals in the productivity space where managers are classified as follows:

- If $z \leq z_W$, the manager won't run a business and instead will be a worker;
- If $z_W < z \leq z_S$, the manager will run a firm small enough to freely opt in the Simples system, without having to split the firm;
- If $z_S < z \leq z_D$, the firm will be in the Simples and the manager will cap its revenue at the program revenue's threshold $\bar{Y}$;
- If $z_D < z \leq z_L$, the manager will split the firm and opt in the Simples system;
- Finally, if $z_L < z$, the firm will be large and outside the Simples program.

2.3.4 The Competitive Equilibrium

Let $\Phi(z)$ denote the cumulative distribution function of productivity z. Then, given $\Phi(z)$, a revenue function $f(n, z)$, a tax regime given by (τ_i, T_i), $\forall i \in \{S, L\}$, a revenue threshold $\bar{Y}$ and a splitting cost function $g(f(n, z))$, a competitive equilibrium is defined by (i) a wage level w; productivity cutoffs (z_W, z_S, z_D, z_L); (iii) labor demand functions $n^i(z)$, $\forall i \in \{S, L\}$, $\bar{n}(z)$, $n^D(z)$ such that: (C1) agents do not wish to

change their occupation; (C2) labor demands are profit maximizers; and (C3) labor demanded by active firms equal to the mass of agents that choose to be a worker,

$$\int_{z>z_W} n^*(z)d\Phi(z) = \Phi(z_W),$$

where

$$n^*(z) = \begin{cases} n^S(z), & \text{if } z_W < z \leq z_S \\ \bar{n}(z), & \text{if } z_S < z \leq z_D \\ n^D(z), & \text{if } z_D < z \leq z_L \\ n^L(z), & \text{if } z > z_L. \end{cases}$$

2.4 QUANTITATIVE EXERCISE

In this section I present the functional forms describing the economy and then discuss the calibration strategy. Then I provide the counter-factual analysis that works as follows. I will have two model departures from a benchmark economy in which the Simples system is ruled out. In both model departures the Simples system will be introduced in the environment. In the first model departure, tax evasion via firm splitting is shut down. In the second one, firm splitting is turned on. Then I compare aggregate output, implied TFP and tax revenues in both model departures to the same variables in the benchmark economy.

2.4.1 Parametrization

Consider a revenue (production) function given by

$$f(n, z) = zn^{\alpha},$$

where $\alpha < 1$ is the span of control parameter guaranteeing decreasing returns to scale. It satisfies Assumption (1).

Let the costs of dividing a company be given by

$$g((f(n, z))) = bf(n, z),$$

where $b > 0$. Note that such a function trivially satisfies Assumption (2). It implies that a dividing manager will have to pay an informal marginal tax on revenues given by b in case she decides to split her business[12]. Finally let the cross-sectional

[12]To see this, think of the objective function of a manager considering a firm split-up

$$(1 - \tau_S)f(n, z) - wn - bf(n, z)$$

distribution of the productivity z be given by a Truncated Pareto, with a probability distribution function given by $\phi(z) = \frac{\beta z_{\min}^{\beta}}{z^{\beta+1}}$, where $\beta, z_{\min} > 0$.

Such model specification yields the following labor demand schedules

$$\begin{aligned} n^i(z) &= \left[\frac{(1-\tau_i)\alpha z}{w}\right]^{\frac{1}{1-\alpha}}, \ \forall i \in \{S, L\}, \\ \bar{n}(z) &= \left[\frac{\bar{Y}}{z}\right]^{\frac{1}{\alpha}}, \\ n^D(z) &= \left[\frac{(1-\tau_S-b)\alpha z}{w}\right]^{\frac{1}{1-\alpha}} \end{aligned} \tag{2.5}$$

and profit functions

$$\begin{aligned} \pi^i(z) &= (1-\alpha)\left[\frac{\alpha}{w}\right]^{\frac{\alpha}{1-\alpha}}[(1-\tau_i)z]^{\frac{1}{1-\alpha}} - T_i, \ \forall i \in \{S, L\}, \\ \bar{\pi}(z) &= (1-\tau_S)\bar{Y} - w\left[\frac{\bar{Y}}{z}\right]^{\frac{1}{\alpha}} - T_S, \\ \pi^D(z) &= (1-\alpha)\left[\frac{\alpha}{w}\right]^{\frac{\alpha}{1-\alpha}}[(1-\tau_S-b)z]^{\frac{1}{1-\alpha}} - T_S. \end{aligned} \tag{2.6}$$

Equations (2.5) and (2.6) imply the following productivity cutoffs[13]

$$\begin{aligned} z_W &= \frac{1}{1-\tau_S}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{w+T_S}{1-\alpha}\right)^{1-\alpha} \\ z_S &= \left[\frac{w}{\alpha(1-\tau_S)}\right]^{\alpha}\bar{Y}^{1-\alpha} \\ z_L &= [(1-\alpha)^{\alpha-1}(T_L - T_S)^{1-\alpha}\left\{\left[\frac{\alpha}{w}\right]^{\frac{\alpha}{1-\alpha}}(1-\tau_L)^{\frac{1}{1-\alpha}} - \left[\frac{\alpha}{w+b}\right]^{\frac{\alpha}{1-\alpha}}(1-\tau_S-b)^{\frac{1}{1-\alpha}}\right\}^{\alpha-1} \end{aligned} \tag{2.7}$$

$$\Leftrightarrow (1-\tau_S-b)f(n,z) - wn.$$

Hence, her effective marginal tax rate is given by $\tau_s + b$.

[13] It is not possible to analytically derive the cutoff z_D, above which it is worth for managers to split a firm.

The analytical characterization of the first cutoff z_W sheds some light on the effect of the Simples program on firm entry. This cutoff determines the point above which a manager is talented enough to run a small business instead of working for one. The smaller the pair (τ_S, T_S) under the Simples regime, the less talented a manager needs to be to run a profitable enterprise, **ceteris paribus**.[14]

It is possible to see how the splitting decision cost parameter b affects the cutoff z_L above which managers decide not to split their businesses and opt out the Simples regime. The higher the splitting costs, the smaller z_L, so that more managers would choose to run a large enterprise outside the tax alleviating program. On top of that, the larger the fixed costs of tax compliance outside the Simples system, that is, the larger T_L, the higher it is the cutoff z_L, so that more managers would consider the splitting decision a more appealing alternative. Again, all the aforementioned considerations are true in a partial equilibrium comparative statics.

To illustrate the profit maximization structure, Figure (2.1) displays the payoffs for each productivity level, namely the profits from: (i) running a small business under the Simples regime ($\pi^S(z)$); (ii) a business with revenue capped at the program's threshold $\bar{Y}$ ($\bar{\pi}(z)$); (iii) splitting a firm and entering the Simples system ($\pi^D(z)$); and, (iv), running a large business without being part of the Simples regime ($\pi^L(z)$). It also delineates the productivity intervals in which each of the previous types of business are the most profitable. The dashed-line stresses the most profitable type of firm for each productivity level.

[14]Of course, in a general equilibrium setting, one needs to take into account the effect on wages of a change in the tax system under the Simples regime. For example, lower taxes and compliance costs under the Simples system might increase wages in equilibrium, which would then make the transition from employee to entrepreneurship less appealing. This is consistent with the findings in Rocha et al. (2018).

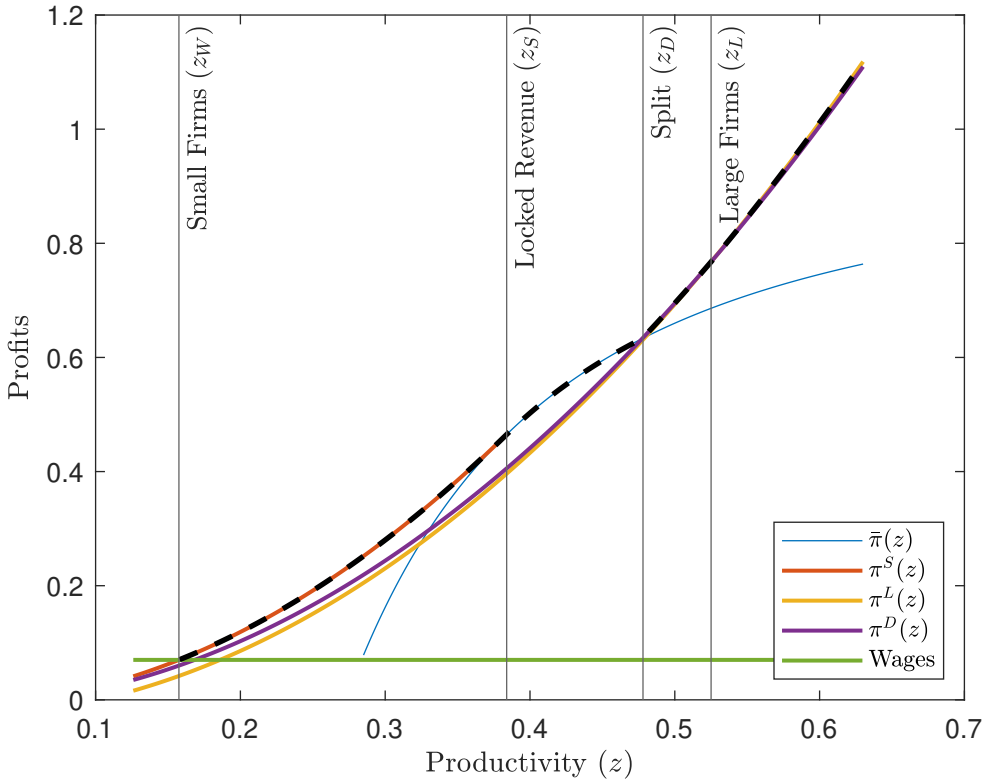


Figure 2.1: Profits and Productivity Cutoffs

2.4.2 Calibration

Before running the counter-factual exercises, it is important to carefully pin down the parameters values shaping this experimental economy. I divided the parameter vector $\theta = [\alpha, z_{\text{min}}, \tau_s, \tau_L, \bar{Y}, b, T_L, \beta]'$ into two components[15]. The first component, $\theta_1 = [\alpha, z_{\text{min}}, \tau_s, \tau_L, \bar{Y}]'$ is the one I am externally calibrating based on previous work and tax statutory values. The second component, $\theta_2 = [b, T_L, \beta]'$, is calibrated with the purpose of matching model generated moments to its empirical counterparts.

There are five moments I am targeting in the calibration: (i) the average state

[15] I am setting the fixed cost of tax compliance under the Simples program, T_S, to zero. Thus, one might reinterpret the fixed cost of tax compliance outside the program, T_L, as being a cost relative to the Simples compliance costs.

level measure of split-up firms in the retail trade sector; (ii) the average state-level Simples participation in the retail sector; (iii) the average firm gross revenue in the retail sector relative to the Simples program revenue threshold[16]; (iv) the workers to firms ratio in the retail trade sector; and (v) the slope coefficient from regressing the measure of firm splitting on state-level ICMS taxes in the retail trade sector[17]. Table (2.3) provides the calibrated values and the model performance relative to the data.

Note that the splitting decision cost b will mainly affect the first moment, while the fixed cost of tax compliance outside the Simples, T_L, disciplines Simples participation. The scale parameter of the truncated Pareto, β, disciplines moments (iii) and (iv), since it determines the relative weight of productivity realizations in the right tail of the distribution. In other words, the higher its value, a smaller probability is assigned to high levels of productivity, decreasing average firm revenues and labor allocation.

Table 2.3: Parameter Values and Model Performance

Parameter	Decription	Value	Source
Externally Calibrated Parameters			
α	Cobb-Douglas Parameter	0.6050	Ulyssea (2018)
$z_{\min}$	Pareto's Location Parameter	7.70	Ulyssea (2018)
τ_S	Simples' Marginal Tax on Revenues	3.95%	Statutory Value
τ_L	Marginal Tax on Revenues Outside Simples	17.76%	Average State ICMS tax
$\bar{Y}$	Simples Revenue Threshold	36	2018's Threshold (in R$100,000)
Internally Calibrated Parameters			
b	Cost of Splitting	19.87%	
T_L	Fixed Cost of Tax Compliance Outside Simples	6.0548	
β	Pareto's Scale Parameter	2.6098	

Model Performance			
Moment	Model	Data	Data Source
Split-up Firms Measure	7.95%	7.56%	2018 RFB's Data (Retail Trade Only)
Simples Participation	84.03%	82.10%	2018 RFB's Data (Retail Trade Only)
Average Firm Revenue Relative to Simples Threshold	0.3852	0.3658	PAC 2018
Workers to Firms Ratio	8.19	6.4309	PAC 2018
Partial Effect of Taxes on Split-up Measure	2.119	2.182	OLS Slope Estimate

In the model, moment (v) is implemented as follows. For a given vector of pa-

[16] In 2018, the gross annual threshold was R$ 3,600,000.
[17] As documented in column (3) of table (2.1).

rameter values, including the average state-level taxes outside the Simples System (τ_L), I solve for the wage level that clears the labor market. Given the equilibrium wages, I calculate the measure of firm splits for each Brazilian state by solving the model again, changing only the marginal tax rate outside the Simples system from its average rate to each particular state tax statutory value. Then I regress the state-level split-up measure implied by the model on the state level ICMS taxes, normalizing each model observation with the number of firms contained in each state.[18]

[18]The normalization is necessary, because the slope coefficient in the data was obtained from an OLS regression at the firm-level. In the model I am using simulated data aggregated at the state-level. One can show that the estimates of firm-level data are equivalent to using the aggregated data at the state-level after the proposed normalization. This greatly simplifies the number of times I have to solve the model for calibration purposes.

2.4.3 Counter-Factual Analysis

With the calibration strategy discussed, I now proceed to quantifying the implications of firm splitting on aggregate outcomes. For this exercise, I consider the benchmark economy as the one in which the Simples system does not exist, so that, regardless of firm revenues, every manager is subject to the same marginal tax rate τ_L. Using this economy as the main reference, I consider two departures from the benchmark economy. In both departures the Simples system is introduced, so that there is a revenue threshold $\bar{Y}$ below which managers pay a marginal tax rate given by τ_S and above which they pay τ_L.

In the first model departure, which I will refer as Model 1, the splitting decision is turned off. In the second departure from the benchmark, managers are able to split their businesses as described in the theory section of this paper. I will call it Model 2. In Model 1, I set the splitting cost parameter to $b = 1 - \tau_S$, so that the objective function of a manager considering splitting his business will always have negative profits, so that in equilibrium the measure of splitting firms is zero. In Model 2, the parameter values are the ones described in Table (2.3).

The first aggregate outcome I will consider when comparing the three versions of the model is aggregate output,

$$Y = \int_{z_w}^{\infty} y^*(z)\phi(z)dz,$$

where $y^*(z)$ is the optimal revenue of a firm managed by an agent with productivity z. The second aggregate outcome I will pay attention to is the model's implied aggregate TFP defined as

$$Z = \frac{Y}{N^{\alpha}}$$

where N is the measure of workers in the economy. Remember that an agent will choose to be an employee if his managerial talent z is below z_w, the cutoff above which it is optimal to run a business. Thus N is an endogenous object, just as aggregate output Y, and can be obtained according to $N = \Phi(z_w)$. Finally, I calculate the aggregate tax revenues in all models. In the benchmark economy, Model 1 and Model 2, the aggregate tax revenues might be respectively calculated as

$$T_{\text{benchmark}} = \tau_L Y,$$

$$T_1 = \tau_S(Y_S + Y_C) + \tau_L Y_L$$

and

$$T_2 = \tau_S(Y_S + Y_C + Y_D) + \tau_L Y_L$$

where Y_S, Y_C, Y_D and Y_L are the aggregate revenues of small firms (S), firms that cap their revenues (C), firms that divide their business (D) and large firms (L).

I document the aggregate results in Table (2.4) below. Aggregate outcomes for Models 1 and 2 were calculated relative to their benchmark counterparts. Allowing managers to split their businesses slightly mitigates the misallocation induced by the introduction of the Simples system, both in terms of aggregate output, as well as in terms of implied TFPs. In the version with firm splitting (Model 2), aggregate output is only 1.12% smaller than the benchmark economy's output, whereas in

the model where the splitting decision is turned off, aggregate output is 1.16% smaller.

In both model departures the introduction of the Simples system increases the implicit aggregate TFP as previously defined. The reason for that is that the aggregate labor demand in an economy without the Simples system (where all firms pay the high marginal tax rate τ_L) is higher than the aggregate labor demand in both model departures, in which the Simples system is introduced. The reason for that is that when all firms pay a high marginal tax rate, it is less appealing to be an employer (business owner) instead of being an employee. Thus the cutoff z_w in the absence of the Simples system is higher, implying a larger aggregate labor allocation and a smaller TFP. In the model version where the splitting decision is turned on, the aggregate TFP is 0.91 % higher than the benchmark's TFP, compared to an increase of 0.87% in the model where splitting a business is not allowed. Therefore, through the lens of the model presented in this paper, tax evasion mitigate output losses introduced by the Simples system and improve even further the economy's TFP.

Table 2.4: Aggregate Outcomes Relative to Benchmark Model

	Agg. Output	Agg. TFP	Tax Revenue Losses
Departure 1 (w/o Split-Ups)	-1.16%	0.87%	-14.31%
Departure 2 (with Split-Ups)	-1.12%	0.91%	-15.18%

The clear drawback of an economy where agents are able to cheat the fiscal authorities by splitting their businesses is in tax revenue losses. Relative to the benchmark economy, where every active firm pays a high marginal tax rate τ_L, when I allow managers to split their companies and join the Simples program and pay the smaller marginal tax rate τ_S, the tax revenue collected by fiscal authorities

is 15.18% smaller than the tax collected without the Simples system. In the model specification where I rule out firm splitting, the percentage decrease in aggregate tax revenue is 14.31%.

2.5 CONCLUSION

In this paper I study the implications for aggregate TFP and output of the Brazilian Simples tax system. The Simples program allows for firms that have small enough annual gross revenues to pay relatively small marginal tax rates while also reducing the red tape involved in preparing, filing and paying taxes. Such size-contingent tax scheme might inhibit firms to grow beyond the legislation threshold, potentially hurting the aggregate GDP of the economy. At the same time, it might provide incentives for managers to game the system by splitting up the firm into smaller units, registering them as independent tax payers and collect the benefits of being artificially small.

I then asked the following question: can tax evasion mitigate misallocation due to distortions induced by such size-dependent tax regimes? To answer this question, I described a model where heterogeneous firms can register in a tax program if their target revenue is smaller than the program threshold; if its managers cap their revenue at the Simples threshold; or if the manager split the business in small enough units. The model predicts that allowing managers to split their firms can increase GDP and TFP by 0.04% [19], when compared to a scenario in which the splitting decision is turned off. However, this form of tax evasion reduces aggregate tax revenues by 0.86% compared again to the economy without the splitting decision.

[19] Subtract row 1 from row 2 numbers in Table (2.4).

CHAPTER 3

Informality as a Buffer for Heterogeneous Workers?

3.1 INTRODUCTION

Informality is a common feature of most developing countries.[1] It is of frequent interest to policymakers as informal firms are typically less productive and are a source of tax evasion. At the same time, informality is seen as a buffer for workers in periods of crisis.[2] The informal sector would thus act as an automatic stabilizer: in times of recessions, some employees would transition from formal to informal jobs, increasing the relative importance of the informal sector at the expense of government tax revenues.

In this paper, we revisit the idea that informality can act as an employment buffer for displaced workers, who, in the absence of an informal sector, would find themselves unemployed or even out of the labor force. By decreasing unemployment rates, informality would also serve as an income buffer and could potentially mitigate reductions in workers' consumption during bad states of the world. We then ask the following question: does informality attenuate the response of aggregate consumption following negative economic shocks?

There are two mechanisms through which informality would asymmetrically affect the sensitivity of aggregate consumption in the business cycle. Informality could attenuate consumption responses by providing an alternative to unemployment during bad states of the world. A worker employed in the informal sector would be able to mitigate earnings losses induced by job separations during an economic downturn, compared to workers that become unemployed.

[1] See La Porta & Shleifer (2014).
[2] See Dix-Carneiro & Kovak (2019) and Ponczek & Ulyssea (2022).

On the other hand, informality can also amplify aggregate consumption responses in the business cycle. If the presence of an informal sector reduces average wages, it would lead to an economy that features higher levels of workers' average marginal propensity to consume.[3] Thus, negative economic shocks would cause larger aggregate consumption responses, relative to a counterfactual scenario where informality is fully repressed.

Our main contribution is to develop a heterogeneous agents model with endogenous job search and that realistically depicts recently documented heterogeneity in marginal propensities to consume[4], providing a better suited framework to analyse the behavior of aggregate consumption along the business cycle. In our model, workers are distinct from each other along four dimensions: their wealth, labor productivity, education and employment status. At every period, unemployed and informally employed workers decide how much effort to put into finding a job in the formal and informal sectors. The higher their effort, the higher the probability of finding a job, at the cost of disutility from searching. The education level of the worker determines the efficiency in finding a job in both formal and informal sectors. Because poorer workers have high marginal propensity to consume, they also have higher utility gains from finding a better paying job.

In the production side of the economy, firms are heterogeneous with respect to their productivity and endogenously choose whether to operate in the formal of informal sector. Formal firms pay taxes on their wage bill and fixed costs of production associated to compliance to regulatory obligations. Informal firms do not pay taxes and do not have a fixed cost of operation. However, they face a size

[3] In a high income economy, savings' rates are larger and the economy would feature a larger number of workers with active borrowing constraints.

[4] See Johnson et al. (2006), Jappelli & Pistaferri (2014), Parker (2017) and Fagereng et al. (2021).

penalty as they become more visible to government authorities the larger they get.

We calibrate the model to conciliate with documented facts about the Brazilian formal and informal labor market: (i) informality is pro-cyclical; (ii) formal jobs pay more than informal jobs. Then we study the aggregate response of some macroeconomic outcomes following a negative productivity shock in a baseline scenario and in a counterfactual economy where informality is absent. Our preliminary results are the following. In the baseline economy, average marginal propensities to consume are larger than in an economy without informal jobs. Therefore, similar negative productivity shocks cause larger drops in consumption in a world where informality is present. On the other hand, even though unemployment increases in both scenarios, it does more so in a world without informality.

There has been an extensive effort to identify the response of informality to aggregate shocks using micro-data variation. More specifically, the literature has been particularly interested in establishing the causal link connecting trade liberalization episodes[5] in developing countries to local labor market outcomes, among which the size of informal employment, labor force participation and unemployment rates. For instance, Dix-Carneiro & Kovak (2019) finds that regions in Brazil more affected by its trade liberalization policy had higher share of informally employed workers even two decades after the shock, while unemployment rates soared in the short-run only, suggesting that displaced workers find refuge in the informal sector.[6]

Using the same natural experiment in Brazil, Ponczek & Ulyssea (2022) highlights the enforcement of labor market regulations and its role in shaping the re-

[5]Perhaps for they offer a more plausible exogeneity than other aggregate shocks.

[6]See Goldberg & Pavcnik (2003) for an early attempt to answer the same question for Brazil and Colombia.

sponse of informal employment and formal firms' exit following trade shocks. In particular, the authors explore regional variation in regulation enforcement and show that locations with less strict enforcement witnessed a larger growth of the informal sector, following trade shocks. We contribute to this literature by linking the micro evidence to macroeconomic outcomes and worker's welfare, providing a framework under which it is possible to analyse how informality affects the sensitivity of worker's labor market earnings to aggregate shocks (informality as an employment buffer) and the sensitivity of their consumption with respect to exogenous variations in income (i.e. the marginal propensity to consume).

The rest of this paper is structured as follows. In Section 3.2 we present stylized facts about informality in Brazil. In Section 3.3, we describe our modelling strategy taking into account the empirical evidence. In Section 3.4 we calibrate the model and perform quantitative exercises. Finally, in Section 3.5, we conclude.

3.2 STYLIZED FACTS

In this section we document two key facts about informality in Brazil that will help us discipline our modeling choice. The first fact refers to the cyclical behavior of the share of informal workers. The second fact is the suggestive evidence that there is large and significant earnings gaps between formal and informal workers. Moreover we document how this earnings gap varies with respect to the workers' education level.

The data source is PNAD (National Household Survey), an annual survey conducted by the Brazilian statistical agency (IBGE). It provides detailed demographic and labor market information at the household level. Up until 2016, it worked as a repeated cross-section, after which it was discontinued. From 2012 onward, IBGE

also carried out a version of the survey (*PNAD Continua*) that featured a rotating panel. It replaced the original PNAD after 2016. On average, every survey-year contains 300,000 observations. We include every survey released since 1995, a year after which Brazil implemented the successful inflationary stabilization plan (namely, *Plano Real*).

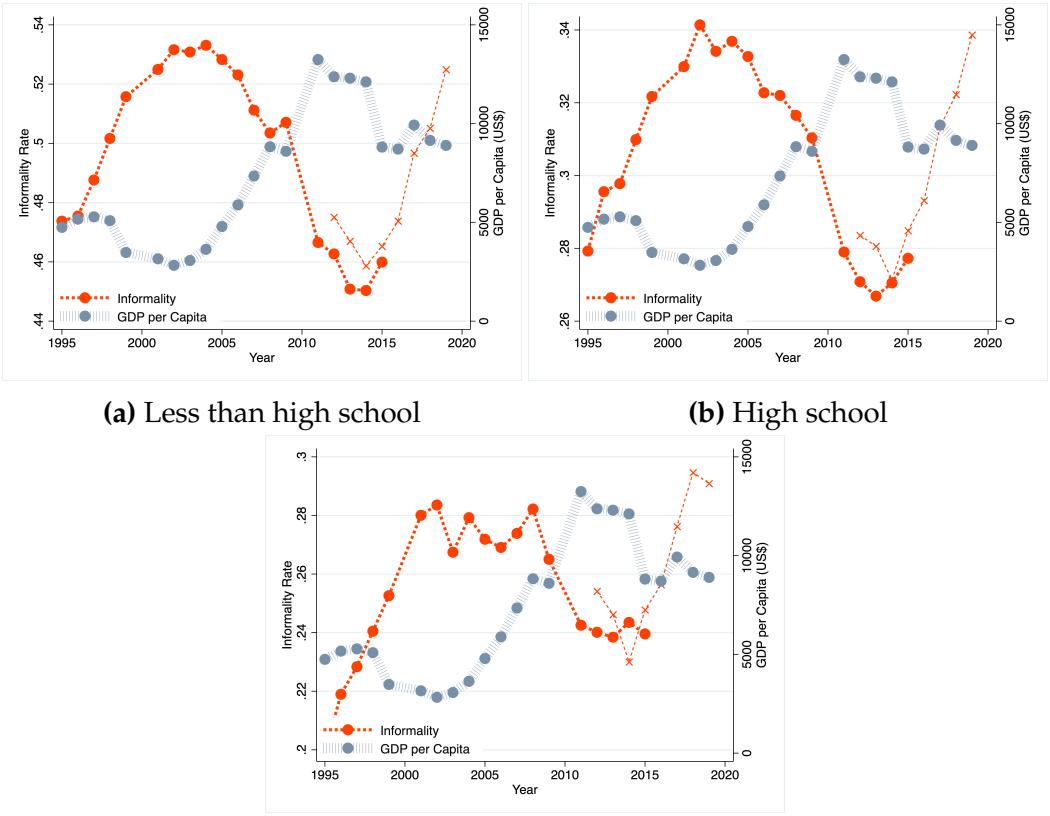
In the current version of this paper, we define an informal worker as a household member who claims to be a salaried employee that does not have a work permit or if the member is an unregistered self-employed individual. The group of formal workers include household members that are salaried and possess a valid work permit. We keep workers whose age is above 25 and below 50 years. We also divide workers into three education groups: workers with less than 7 years of education are in the group *Less than High School*; workers with exactly seven years of education belong to the group *High School Degree*; finally workers with *Some College Education* are those with more than 7 years of education. Our informality measure thus is defined per education group and consists of the share of salaried employees that have an informal job.

To document the aggregate behavior of informality in Brazil, we use both versions of the survey. To estimate the earnings gap between informal and formal workers, we restrict our analysis to *PNAD Continua*.

3.2.1 The Cyclicality of Informality

The first stylized fact refers to the time series behavior of informality and how it compares to other measures of economic activity such as the economy's GDP per capita and the unemployment rate. In Figure 3.1, we plot the evolution of informality since 1995 in conjunction with the time series for the economy's GDP per

capita measured in US$. Panels (3.1a), (3.1b) and (3.1c) breaks down informality shares by education level.



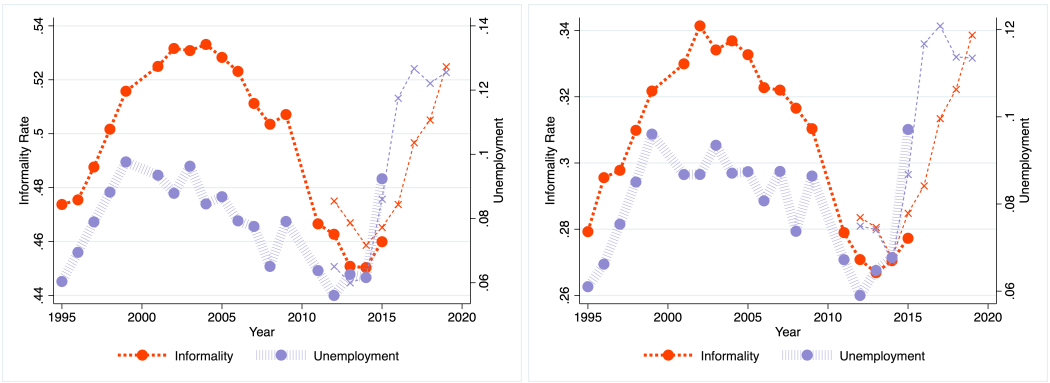
(a) Less than high school **(b)** High school

(c) Some College Education

Figure 3.1: Informality and GDP per Capita by Education Group

By inspection, it is possible to conclude that informality displays a counter-cyclical behavior, as it tends to decline when the economy is booming and vice-versa. Its correlation with the GDP per capita series is relatively high for workers with less education. The informality share within workers with less than a high school degree and workers with a high school degree have correlations of -0.74 and -0.77 respectively with the GDP per capita time series. The share of informal workers with some college education are less sensitive to the aggregate state of the economy, displaying a correlation of -0.18.

The same conclusion holds if we use the unemployment rate of the economy to measure business cycle fluctuations. Informality seems to increase whenever unemployment levels are higher. The informality shares have positive correlations with unemployment of 0.49, 0.55 and 0.52 for workers with less than high school education, with high school degree and some college education, respectively.



(a) Less than high school

(b) High school

(c) Some College Education

Figure 3.2: Informality and Unemployment by Education Group

3.2.2 Earnings Gap Between Formal and Informal Workers

This is not the first paper to document premiums in formal job's earnings relative to informal jobs.[7] Our goals is to estimate the size of the formal earnings gap

[7]See El Badaoui et al. (2010) and Bargain & Kwenda (2011).

for each group of workers based on their education level in order to explicitly discipline our calibration strategy. Pinning down the formal wage premium is thus important for our counterfactual analysis, where we repress the informal sector from the economy.

To estimate the earnings gap between formally and informally employed workers, we restrict our sample to years where we have a longitudinal data set (i.e. from 2012 to 2019). We specify the following linear model,

$$\log \text{earnings}_{i,t} = \sum_{j,s} \beta_{j,s} \mathbf{1}\{\text{job status} = j\}\mathbf{1}\{\text{educ. type} = s\} + X_{i,t} + \varepsilon_{i,t}, \quad (3.1)$$

where $j \in \{\text{formal, informal}\}$ and $s \in \{\text{less than HS, HS degree, some college}\}$, and $X_{i,t}$ is a set of controls such as race, gender and state fixed effects. We use workers with less than a high school degree and formally employed as the comparison group, so that $\beta_{e,s}$ would capture the earnings premium with respect to that group of workers.

In Table 3.1 we display the regression results for three different specifications regarding what we include in the control variable. In column (1), we just obtain mean differences in earnings relative to the comparison group. In column (2), we control for state fixed effects. Finally, in column (3), our preferred specification, we additionally explore racial, age and gender variation in the data. Standard errors are clustered at the state level.

Table 3.1: Formal to Informal Earnings Gap

	Log Earnings		
	(1)	(2)	(3)
<High School x Formal	-	-	-
<High School x Informal	-0.608	-0.507	-0.512
	(0.0609)	(0.0521)	(0.0544)
High School x Formal	0.148	0.145	0.218
	(0.00798)	(0.00499)	(0.00446)
High School x Informal	-0.242	-0.194	-0.107
	(0.0490)	(0.0329)	(0.0309)
Some College x Formal	0.769	0.739	0.828
	(0.0496)	(0.0413)	(0.0383)
Some College x Informal	0.462	0.467	0.566
	(0.0859)	(0.0723)	(0.0638)
State F.E.	No	Yes	Yes
Race, Age, Gender	No	No	Yes
Observations	569,618	569,618	569,618
Adj. R-squared	0.300	0.348	0.411

From this exercise, we conclude that formal earnings gap are large and varies with respect to the education level of workers. Workers that did not complete high school earn 51.2% more if they are formally registered. Workers with a high school degree have a formal earnings premium of 32.5% ($0.218 + 0.107$). Finally workers with some college education receive a 26.2% ($0.828 - 0.566$).

3.3 MODEL

In this section, we present a model where heterogeneous workers endogenously search for jobs in the formal and informal sector. The economy also features heterogeneous firms that self-select into the formal or informal sector based on their productivity. Below, we describe the utility maximization problem of workers and the profit maximization decision of firms. Finally, we define the steady state equi-

librium of the economy.

3.3.1 Workers

Time is discrete and with an infinite horizon. Workers value current and future consumption, receiving a per-period CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. They discount the future at rate β and transfer resources over time by saving in a risk free asset a that pays the interest rate r. Workers are heterogeneous in four dimensions: asset holdings, a persistent and exogenous labor productivity shock z, a permanent skill type $s \in \{L, M, H\}$[8] and an employment status $j \in \{$formal (f), informal (i), unemployed (u)$\}$. At every period, workers decide how much effort e to exert to find a job in the formal and the informal sector, which might affect their future employment status. Workers employed by the formal and informal sector face an exit probability rate $\eta_{j,s}$ that depends on their skill type s and the sector they are currently employed j. Upon exit, workers become unemployed.

The timing of workers' job search effort and consumption plan decisions is the following. In the beginning of the period, unemployed workers choose how much effort to place at finding a job in the informal and formal sectors; workers in the informal sector decide their effort in finding a job in the formal sector.[9] Workers dislike exerting search effort, attributing to it a convex disutility $\xi(\cdot)$ that satisfies Inada conditions. In the middle of the period (the interim), workers observe whether they obtained a job, and if so, in which sector. After the employment

[8] Skill types in the model reflect the years of education of a worker in the data, where skill type L represents workers with less than 7 years of education, skill type M refers to workers with exactly 7 years of education (i.e. workers with a high school degree), and skill type H maps into workers with more than 7 years of education.

[9] Formal sector workers would also look for jobs in the informal sector, but in equilibrium their search effort is zero due to a formal sector wage premium that emerges endogenously in the model. To avoid unnecessary exposition, we abstract from search for this group of workers.

status is realized, workers decide how much to consume and save for the next period, when employed workers may exogenously be fired and another job search starts.[10] The recursive representation of unemployed and informal workers search optimization problem as well as workers' consumption decision at the interim are represented below:

Unemployed workers (u) search problem:

$$\begin{aligned} V^u(a,z,s) = \max_{e_i,e_f} & -\xi(e_i) - \xi(e_f) + p(e_i)V^i_{\text{int}}(a,z,s) + p(e_f)V^f_{\text{int}}(a,z,s) \\ & + [1 - p(e_i) - p(e_f)]V^u_{\text{int}}(a,z,s), \\ & \text{s.t. } p(e_i) + p(e_f) \leq 1 \end{aligned} \tag{3.2}$$

Informal workers (i) search problem:

$$\begin{aligned} V^i(a,z,s) = \max_{e_f} & -\xi(e_f) + p(e_f)V^f_{\text{int}}(a,z,s) + [1 - p(e_f)]V^i_{\text{int}}(a,z,s), \\ & \text{s.t. } p(e_f) \leq 1 \end{aligned} \tag{3.3}$$

Interim Consumption Decisions:

[10] Note that even if an informal worker's job is not destroyed, she will look for a formal job in the next period.

$$
\begin{aligned}
V^u_{\text{int}}(a,z,s) &= \max_c u(c) + \beta \mathbb{E}[V^u(a',z',s)|z] \\
&\text{s.t. } c + a' = e^z y_{u,s} + (1+r)a; \\
V^i_{\text{int}}(a,z,s) &= \max_c u(c) + \beta(1-\eta_{i,s})\mathbb{E}[V^i(a',z',s)|z] + \eta_{i,s}\mathbb{E}[V^u(a',z',s)|z] \\
&\text{s.t. } c + a' = e^z y_{i,s} + (1+r)a; \\
V^f_{\text{int}}(a,z,s) &= \max_c u(c) + \beta(1-\eta_{f,s})\mathbb{E}[V^f(a',z',s)|z] + \eta_{f,s}\mathbb{E}[V^u(a',z',s)|z] \\
&\text{s.t. } c + a' = e^z y_{f,s} + (1+r)a \\
&\text{productivity process: } z' = \rho z + \varepsilon,\ \varepsilon \sim \mathcal{N}(0,\sigma^2_\varepsilon).
\end{aligned}
\tag{3.4}
$$

Workers that enter unemployment receive a basic income $\bar{\omega}$ that is common to all.[11] Formal workers who become unemployed receive an unemployment insurance that is equal to a fraction b of their wages in the formal sector. Workers in the informal sector that are pushed to unemployment do not qualify for such unemployment benefits. However, informal workers do not pay income taxes, as opposed to those employed by the formal sector. Thus, earnings per employment and education status can be decomposed as follows:

$$
\begin{aligned}
y_{u,e} &= \bar{w} + b w_{f,e} \mathbf{1}\{\text{previously formal}\}, \\
y_{f,e} &= (1+\tau_w) w_{f,e}, \\
y_{i,e} &= w_{i,e}.
\end{aligned}
\tag{3.5}
$$

The unconstrained optimal search effort exerted by unemployed and informal workers depends on the difference between the interim value function obtained if they get the job they are looking for and the interim value function of their current

[11] This may capture home production, food stamps and other government transfers.

employment status. Since value functions are increasing in the workers' cash on hand, the wage premium at searched jobs versus their current wage dictates the level of search effort.

The search effort affects the probability of finding the searched job. We let this probability be a linear and increasing function of the level of search, with a slope that varies with the workers' skill s and the job their are looking for $j \in \{f, i\}$. Let $m_{s,j}$ denote the slope (or search efficiency) of the probability of finding job j for worker with skill s, such that $p_{s,j}(e) = m_{s,j}e$. Let the disutility of effort be $\xi(e) = \Lambda\frac{e^{1+\eta}}{1+\eta}$, where $\Lambda, \eta > 0$.

The first order condition of the search problem for unemployed and informal workers looking for jobs that yield employment status $j \in \{f, i\}$ yield the below solutions:

Unconstrained Unemployed Workers' Optimal Search Effort

$$\begin{aligned} e_f^*(a,z,s) &= \left\{ \frac{m_{s,f}[V_{\text{int}}^f(a,z,s) - V_{\text{int}}^u(a,z,s)]}{\Lambda} \right\}^{\frac{1}{\eta}} \\ e_i^*(a,z,s) &= \left\{ \frac{m_{s,i}[V_{\text{int}}^i(a,z,s) - V_{\text{int}}^u(a,z,s)]}{\Lambda} \right\}^{\frac{1}{\eta}} \end{aligned} \tag{3.6}$$

Unconstrained Informal Workers' Optimal Search Effort

$$e_f^*(a,z,s) = \left\{ \frac{m_{s,f}[V_{\text{int}}^f(a,z,s) - V_{\text{int}}^i(a,z,s)]}{\Lambda} \right\}^{\frac{1}{\eta}} \tag{3.7}$$

Three conclusions can be drawn from the optimal search levels. First, the higher the search efficiency $m_{s,j}$ the higher the level of effort exerted by workers with skill s to find a job at sector j. Therefore, the search efficiency parameter have a direct and indirect effect on the probability of finding a job. Second, as noted before, wage premiums between searched and current employment status indirectly

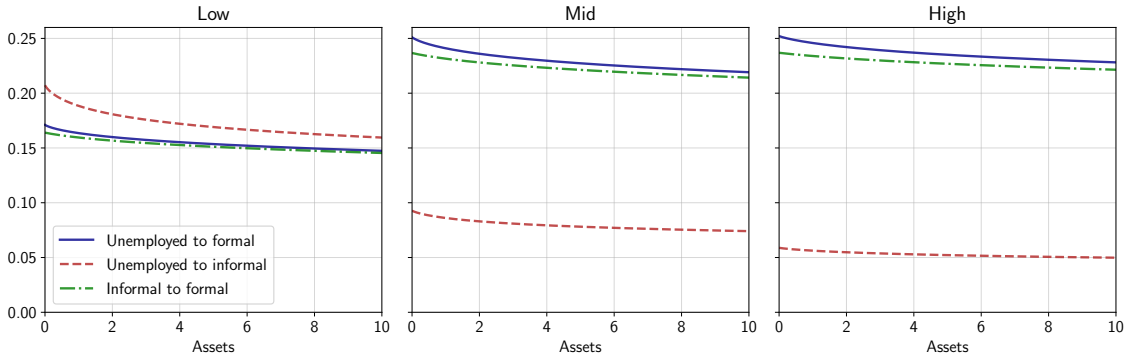


Figure 3.3: Heterogeneity in Job Finding Probabilities

increases the level of effort through value function differentials. Third, for a given wage premium, poorer agents exert higher levels of effort than richer ones, as they are more likely to be hand-to-mouth: thus, for poor agents, finding a better paying job substantially affects their level of consumption. This generates heterogeneity in search over the asset level (see Figure 3.3) and having a model that correctly captures the distribution of marginal propensities to consume are better suited to capture the benefits of having informal jobs as an alternative to unemployment.

3.3.2 Firms

Firms are heterogeneous with respect to their productivity θ, which is distributed according to a truncated Pareto distribution $G(\cdot)$ with a closed support $[\theta_{\min}, \theta_{\max}]$ and shape parameter δ. Each firm can choose to operate in the formal or informal sector. Firms in the formal sector pay taxes τ on their wage bill and incur on a fixed cost of production κ_f that captures regulation costs such as tax compliance and adequacy to government quality standards. Firms in the informal sector do not pay taxes nor face fixed costs of production. However, they might be audited by fiscal authorities and thus incur in an expansion cost, for the larger an informal

firm is, the higher is its visibility to the government.

Regardless of their formality status, firms combine low, medium and high skilled labor according to the labor aggregator $L = \left(\sum_s \Omega_s^{1/\sigma} n_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ where Ω_s disciplines skill s marginal productivity (and thus the skill premium) and σ captures the elasticity of substitution between skills. Their profit maximization is static and given below:

Formal Firms

$$\begin{aligned} \pi^f(\theta, A) = \max_{\{n_s\}_{s\in L,M,H}} & A\theta L^\alpha - (1+\tau)\sum_s w_s n_s - \kappa_f \\ \text{s.t.} L = & \left(\sum_s \Omega_s^{1/\sigma} n_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{3.8}$$

Informal Firms

$$\begin{aligned} \pi^i(\theta, A) = \max_{\{n_s\}_{s\in L,M,H}} & A\theta L^\alpha - \sum_s w_s n_s - \chi L \\ \text{s.t. } L = & \left(\sum_s \Omega_s^{1/\sigma} n_s^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \end{aligned} \tag{3.9}$$

The output elasticity of the labor aggregator α is smaller than unity, so that firms operate through a decreasing returns to scale technology and thus have an optimal size. The parameter A captures the aggregate state of the economy. We will arbitrarily change its value in the quantitative section of this paper to capture expansion and recession in the economy. The parameter χ captures the government enforcement capacity and it disciplines the wage premium of formalization. To see this, consider the wage ratio for skill s in the formal and informal sectors as implied by their first order conditions:

$$\frac{w_{s,f}}{w_{s,i}} = \frac{\alpha\theta L^{\alpha-1}}{(1+\tau)(\alpha\theta L^{\alpha-1} - \chi)}. \tag{3.10}$$

The higher is government enforcement, the costlier it is for informal firms to expand. Therefore they would only be willing to employ more workers of any given skill the smaller the wages they pay for that worker. Hence, the formality premium is increasing in χ. Moreover, the higher the tax rate faced by formal firms, the smaller the formality premium is.

Because formal firms face a fixed cost of operation and informal firms face a size penalty, there is selection into the formal sector: for a given tuple of wages in sector j, there is a productivity cutoff θ^* such that if $\theta > \theta^*$, the firm will be formal. The mass of informal firms in the economy is given by $G(\theta^*)$. Negative productivity shocks (an exogenous decline in A) affects the size of the informal sector through two channels: it can increase the formality cutoff such that the mass of informal firms increases. However, by increasing average productivity of formal firms, negative productivity shocks might shrink the intensive margin of informality, i.e. the relative labor demand in the informal sector. In partial equilibrium, the dominant force is determined by the kurtosis of $\frac{dG}{d\theta}(\cdot)$.

In general equilibrium, the effect of negative aggregate shocks on the size of the informal sector in terms of the share of informal workers is also determined by the relative search efficiencies for jobs in the formal and informal sector (i.e $m_{f,s}$ and $m_{i,s}$). In our quantitative exercises, we will measure how much the search efficiency in the formal sector for each skill need to decline to rationalize an increase in the informal sector during an economic downturn. In future versions of the paper, we plan to endogenize the search efficiency parameters, so that it directly depends on firms' actions. Models of vacancy posting and asymmetric labor market frictions in the formal and informal sectors are able to generate an increase in

the informality share following a negative economic shock.[12]

3.3.3 Government

The government issues a risk-free bond B, which is the asset that workers save on. It collects formal firms' and workers' pay-roll taxes in order to finance the payment of unemployment insurance to displaced formal workers, the unconditional unemployment payment $\bar{\omega}$ paid to all unemployed agents regardless of their previous formality status, and the interest on its debt. The government surplus T is then given by

$$\begin{aligned} T = B &+ \tau_w \sum_s \int_a \int_z e^z w_{s,f} \mu^f(a,z,s) dz da + \tau \sum_s w_s \int_{\theta^*}^{\theta_{\max}} n_s(\theta, A) dG(\theta) \\ &- b \sum_s \int_a \int_z e^z w_{s,f} \mu^f(a,z,s) dz da - \bar{\omega} \sum_j \sum_s \int_a \int_z \mu^j(a,z,s) dz da \\ &- (1+r)B, \end{aligned} \tag{3.11}$$

where $\mu^j(a,z,s)$ is the ergodic distribution of a worker with employment status j, asset level a, labor productivity z and skill s and $n_s(\theta, A)$ is the optimal labor demand of a firm with productivity θ at a state of the world with aggregate productivity A.

3.3.4 General Equilibrium

A general equilibrium consists of a tuple of wages $(w_L, w_M, w_H)^j$ for sector $j \in$ {formal, informal}, outstanding bond issuance B, savings and effort policy functions such that: (i) labor markets clear for all skills at the formal and informal

[12] See Dix-Carneiro et al. (2021).

sector; (ii) aggregate savings is equal to outstanding government debt; (iii) distributions of agents over the wealth, productivity, skill and employment status are time-invariant and its law of motion is determined by optimal search efforts and savings decisions.

3.4 CALIBRATION STRATEGY AND QUANTITATIVE EXERCISES

In this section we present the model calibration, which involves setting the values of a subset of the parameters in order to fit model generated moments to the stylized facts about informality in Brazil described in Section 3.2. With the calibrated model, we investigate differences in the long-run response of output, TFP, workers' welfare and the government budget following an exogenous decline in aggregate firm productivity in a world where informality exists and in another where it is ruled out. We also measure how much the parameters dictating search efficiency of jobs in the formal sector for all skills ($m_{s,f}$) need to adjust in order to account for the observed levels of informality in Brazil when they were at its peak.

3.4.1 Calibration

A model period represents a quarter. We externally calibrate a subset of the parameters, some of which we borrow from the related literature, some we take from statutory values, and others are chosen ad-hoc. The remaining parameters we internally calibrate in order to minimize the distance between model and data statistics. The internally calibrated parameters as well as the moments that identify them are listed below:

1. Discount factor (β): We let the discount factor β discipline the average marginal propensity to consume in the economy. Higher values of β increases the in-

centive to save, moving a larger mass of agents away from the borrowing constraint $a \geq 0$. Thus the mass of agents whose Euler Equations hold with equality increase, decreasing the marginal propensity to consume. We target a quarterly average marginal propensity to consume of 0.25, which is a little bit higher than the level estimated for the US economy.[13]

2. Fixed Cost for Formal Firms (κ_f): We let the share of informal firms in Brazil discipline the fixed cost of operating a formal business. The larger its value, the higher is the productivity cutoff above which firms formalize and, thus, the smaller the share of informal firms.

3. Size penalty for informal firms (χ): We let the average formal wage premium discipline the size penalty parameter for informal firms. The higher its value, the costlier it is for informal firms to expand, and thus they would be willing to pay smaller wages to their employees. Therefore, the formal wage premium increases with χ.

4. Search efficiency for jobs at the informal sector for every skill ($\{m_{i,s}\}_{s\in\{L,M,H\}}$): The share of low, medium and high skill workers employed by the informal sector disciplines the search efficiency of informality for each skill. The higher the search efficiency in the informal sector, the higher it is the probability of finding a job in that sector, which in turn dictates the share of informal workers in the labor force for every skill.

In Table 3.2, we display the chosen parameter values for the external calibration, but also the internally calibrated values followed by the moments used in

[13] See Kekre (2021)'s calibration strategy, where the target average marginal propensity to consume is 0.21 approximately.

Table 3.2: Calibrated Parameters

Externally Calibrated Parameters

Parameter	Description	Value
γ	CRRA Parameter	2
r	Interest Rate	1.5%
η	Search Curvature	0.9
Λ	Search Linear Cost	1
$\bar{\omega}$	Universal Unemployment Benefit	0.05
σ_ε^2	Variance of Labor Productivity Innovations	0.33
ρ	Auto-correlation of Log Labor Productivities	0.95
τ_w	Formal Workers' Income Tax	10%
b	Unemployment Insurance for Formal Workers	30%
$\{\eta_{s,i}\}_{s\in\{L,M,H\}}$	Separation Rate from Informality	(9%,9%,6%)
$\{\eta_{s,f}\}_{s\in\{L,M,H\}}$	Separation Rate from Formality	(3%,3%,2%)
$\{m_{s,f}\}_{s\in\{L,M,H\}}$	Search Efficiency for Formal Jobs	(0.02,0.03,0.032)
σ	Elasticity of Substitution of Skills	1.92
α	Output Elasticity of Labor	0.6
Ω_L	Low Skill Parameter on Labor Aggregator	0.07
Ω_M	Medium Skill Parameter on Labor Aggregator	0.10
Ω_H	High Skill Parameter on Labor Aggregator	0.22
τ	Formal Sector Payroll Tax	20%
δ	Shape Parameter of Pareto Distribution	2.5
θ_{min}	Location Parameter of Pareto Distribution	0.7
θ_{max}	Maximum Productivity (99th percentile)	4.4167

Internally Calibrated Parameters

Parameter	Description	Value	Moment
β	Discount Factor	0.934	Average MPCs
κ_f	Fixed Cost for Formal Firms	0.138	Share of Informal Firms
χ	Size Penalty for Informal Firms	0.343	Average Formal Wage Premium
$m_{I,L}$	Search Efficiency for Informal Jobs	0.132	Share of Informal Workers (Low Skilled)
$m_{i,M}$	Search Efficiency for Informal Jobs	0.052	Share of Informal Workers (Medium Skilled)
$m_{i,H}$	Search Efficiency for Informal Jobs	0.057	Share of Informal Workers (High Skilled)

identifying them. In Table 3.3 below, we display the targeted data versus the model generated moments in the calibration procedure. We chose informality data for the year 2013 as the baseline, since it was when the economy in Brazil was at its peak in terms of GDP per capita and informality was at its lowest levels for all workers' education groups.

Table 3.3: Targeted Moments and Model Performance

Moment	**Data**	**Model**	**Source**
Average MPC	0.25	0.249	Standard
Share of Informal Firms	69.8%	70.41%	Ulyssea (2018)
Average Formal Wage Premium	41.25%	42.08%	PNAD (All years)
Share of Informal Labor Force (Low Skill)	45%	44.74%	PNAD (2013)
Share of Informal Labor Force (Medium Skill)	29%	28.44%	PNAD (2013)
Share of Informal Labor Force (High Skill)	29%	28.49%	PNAD (2013)

Additionally, we provide in Table 3.4 the model generated transition matrices dictating, for each skill group, the probability of moving to different employment statuses conditional on their current employment status. It is also provided for comparison the same transition matrices we estimated in the data. In future versions of the paper, we will internally calibrate the full set of parameters dictating the search efficiency in order to target the data transition matrices for each skill group.

Table 3.4: Untargeted Moments

Data Transition Matrix

	Low Skill			Medium Skill			High Skill		
	F	I	U	F	I	U	F	I	U
Formal	0.93	0.04	0.03	0.95	0.03	0.03	0.96	0.03	0.02
Informal	0.17	0.74	0.09	0.28	0.64	0.09	0.31	0.64	0.06
Unemployed	0.15	0.20	0.65	0.20	0.11	0.70	0.18	0.08	0.74

Model Transition Matrix

	Low Skill			Medium Skill			High Skill		
	F	I	U	F	I	U	F	I	U
Formal	0.97	0.01	0.02	0.97	0	0.02	0.98	0	0.02
Informal	0.03	0.91	0.06	0.03	0.9	0.07	0.02	0.93	0.05
Unemployed	0.03	0.26	0.71	0.06	0.12	0.82	0.07	0.15	0.78

3.4.2 Quantitative Exercise

As discussed in the previous subsection, the model was calibrated so that the share of workers in the informal sector would match the one observed in 2013, when informality in Brazil was at its lowest level in our data set for all education groups, and when GDP per capita was at its peak (see Figure 3.1). We refer to this calibration as the baseline model.

Our quantitative exercises will be conducted as follows. First, we will decrease the aggregate productivity parameter A. At the same time, we recalibrate the search efficiency parameters associated to job search in the formal sector (i.e. $\{m_{f,s}\}_{s\in\{L,M,H\}}$) such that the share of informal workers in the model is identical to the share of informal workers in the data when informality in Brazil was at its peak and the GDP per capita at its lowest level (this corresponds to informality rates in 2004). Table 3.5 depicts the necessary changes in the search efficiency parameters for the model to match the levels of informality rates observed in 2004, following a negative productivity shock of 10%.

Table 3.5: Change in Search Efficiency After 10% Decrease in Productivity

Parameter	Percentage Change	Target Moment After Shock	Data	Model
$m_{L,f}$	-33%	Share of Informal Workers (L)	53%	52.34%
$m_{M,f}$	-32%	Share of Informal Workers (M)	37%	36.31%
$m_{H,f}$	-28%	Share of Informal Workers (H)	36%	35.36%

The current version of the model lacks endogenous responses in search efficiency in finding jobs in both the formal and informal sectors. Therefore, negative (positive) aggregate shocks in firms' productivity would not necessary lead to an increase (decrease) in the size of the informal sector, contradicting its pro-cyclical behavior. For the model to match the cyclicality of informality, it is necessary that the relative search efficiency in the formal sector declines following a negative shock, so that it becomes harder for workers to find a job in the formal sector during recessions.[14] In our calibration, a 10% decrease in aggregate productivity requires large reductions in the job finding efficiency in the formal sector of 33, 32 and 28% for low, medium and high skilled workers respectively.

In the second exercise, we simulate an identical decrease in aggregate productivity, but in a world where informality is completely wiped out of the economy. In this counterfactual economy, we keep the same parameter values as those in the baseline economy, but also allow the search efficiency parameters of finding a job in the formal sector to change following the negative aggregate shock.[15] We then compare changes in macroeconomic outcomes induced by the negative aggregate shock in an economy where informality exists to the macroeconomic consequences in an economy where informality levels are lower. Results are displayed in Table

[14]For instance, Bosch et al. (2012) finds that the rise of informality in Brazil in the 1990s and early 2000s were caused by smaller job finding rates in the informal sector.

[15]We take the same recalibrated values for the search efficiency parameters obtained in the first quantitative exercise.

3.6 below.

Table 3.6: Informality and Negative Aggregate Shocks

	Baseline Informality			No Informality		
	(1)	(2)	(3)	(4)	(5)	(6)
Outcomes	**Pre-shock**	**After-shock**	**Change**	**Pre-shock**	**After-shock**	**Change**
Consumption (L)	1	0.86	-14%	1	0.89	-11%
Consumption (M)	1	0.85	-15%	1	0.88	-12%
Consumption (H)	1	0.83	-17%	1	0.84	-16%
Government Surplus (T)	1	0.66	-34%	1	0.7	-30%
Unemployment Rate	12.72%	19.79%	7.07 p.p.	28.95%	42.12%	14.17 p.p.
Average MPC	0.2491	0.2491	-	0.1878	0.2046	1.09%

The model predicts that in an economy where there is no informality, unemployment rates are larger and also are more sensitive to negative shocks.[16] However, when informality is present, negative aggregate shocks generate larger drops in aggregate consumption, regardless of the skill level of the worker.

The mechanisms through which in general equilibrium informality amplifies long-run consumption responses are the following. First, following a negative supply shock, labor demand for both informal and formal sectors fall. On itself, it would exert a downward pressure on wages for all skills and sectors. However, the search efficiency in finding jobs in the formal sector also falls, which decreases the labor supply of all skills in the formal sector only. Therefore, the fall in wages in the formal sector is not as large as the fall in wages in the informal sector. Because we recalibrate finding rates so that the share of informal workers increase for all skills (to mimic Figure 3.1's peak informality levels), the vast majority of workers end up employed at the sector that suffers the largest hit on wages.

In an economy where informality does not exist, a negative productivity shock also decreases labor demand and job finding rates in the formal sector. However,

[16]This is consistent with Dix-Carneiro et al. (2021) that showed that in regions where there is a better government enforcement (and thus informality size is relatively small), negative trade shocks induce larger unemployment rates.

firms that would be informal in spite of being profitable in the formal sector would now have to formalize. Upon formalization, labor demand for previously informal firms would be higher on average, since they do not face a size penalty. Hence, the equilibrium level of wages for all employed workers is larger when firms are required to formalize. Higher levels of income pushes more agents away from the borrowing constraint, diminishing the economy's average marginal propensity to consume from 0.2491 to 0.1878 in our calibration. Thus, the consumption response to negative income shocks for workers do not decrease consumption as much as it does in a world where informality exists, average wages are lower and the marginal propensity to consume is higher.

3.5 CONCLUSION

In this paper, we investigate whether informality can act as an employment and consumption buffer following recessionary economic shocks. We document its pro-cyclical nature: the share of informally employed workers increases at times of economic distress, as measured by declining GDP per capita and soaring unemployment rates. We also estimate the formality earnings premium for workers of distinct levels of education.

In light of these stylized facts, we developed a heterogeneous agents model with endogenous job search that allows for a realistic distribution of marginal propensities to consume and that captures features of the Brazilian labor market. A realistic description of the economy's distribution of MPCs matters for the response of aggregate consumption to negative shocks.

For the current model parametrization, we showed that in an economy calibrated to match the size of the Brazilian informal sector, average marginal propen-

sities to consume are higher than in a counterfactual scenario where informality is fully repressed. Even though the model predicts that informality absorbs displaced workers during times of distress, it amplifies the effect of negative shocks on consumption.

.1 APPENDIX TO CHAPTER 2

.1.1 Validation Exercises

The ICMS tax is particularly relevant within the Trade Sector, with potentially heterogeneous effects on the wholesale and retail trade sectors. In order to attract private investment and promote job creation, some states in Brazil offer tax benefits to some industries, including the wholesale trade sector. Those benefits usually include a discount in the ICMS marginal tax rate in transactions settled within the state's borders. Some states also prolong the deadline for the ICMS tax payment. Those benefits are not extended to the retail trade sector, so that retailers face the official ICMS marginal taxes. Table (7) breaks down each state's Simples system participation and also the percentage of the Simples participants that is a byproduct of a dividing firm, within the trade sector. It is also reported the ICMS tax rate charged by each state.

The fraction of the firms participating in the Simples system is larger in the retail trade sector (column 3 of table 7) when compared to the wholesale trade sector (column 2). This could be partially explained by the fact that the average firm size in the wholesale sector is larger than the average retailer[17], and thus fewer firms would be eligible to the Simples benefits. However, conditional on being a participant in the Simples system, the proportion of firms that are splitting-up is ubiquitously higher in the retail trade sector vis-à-vis the wholesale trade (columns 4 and 5).

[17] According to the 2018's Annual Trade Survey (PAC 2018), the average annual revenue of the wholesale trade sector was R$ 7,400,000, while the same statistic for the retail trade sector was R$1,317,000.

Table 7: State Level Simples Participation and Firm Split-up within the Trade Sector

	(1)	(2)	(3)	(4)	(5)
		Simples Participation		Measure of Firm Splitting	
State	ICMS Tax Rate	Wholesale	Retail	Wholesale	Retail
AC	17.00%	45.33%	71.30%	1.51%	5.52%
AL	18.00%	43.21%	78.02%	4.31%	9.13%
AM	18.00%	51.52%	77.14%	2.02%	6.25%
AP	18.00%	58.63%	80.97%	1.38%	7.00%
BA	18.00%	48.71%	83.83%	3.56%	8.04%
CE	18.00%	46.41%	77.22%	3.12%	7.57%
DF	18.00%	59.31%	92.17%	5.98%	15.96%
ES	17.00%	42.17%	88.99%	4.22%	8.30%
GO	17.00%	55.60%	86.21%	3.72%	7.15%
MA	18.00%	53.53%	82.50%	4.37%	7.24%
MG	18.00%	50.49%	89.91%	3.11%	8.12%
MS	17.00%	43.79%	78.79%	2.80%	5.82%
MT	17.00%	46.63%	81.94%	2.95%	5.73%
PA	17.00%	55.33%	82.44%	4.84%	6.46%
PB	18.00%	45.80%	78.05%	3.06%	7.49%
PE	18.00%	47.30%	78.18%	4.23%	7.65%
PI	18.00%	41.04%	76.04%	3.85%	7.61%
PR	18.00%	53.26%	87.71%	4.11%	6.79%
RJ	20.00%	61.76%	86.90%	4.32%	12.63%
RN	18.00%	46.97%	79.53%	3.91%	6.90%
RO	17.50%	55.59%	80.22%	3.76%	5.45%
RR	17.00%	51.96%	75.95%	4.06%	6.22%
RS	18.00%	51.56%	85.40%	2.38%	4.46%
SC	17.00%	55.66%	88.36%	3.24%	4.43%
SE	18.00%	52.14%	80.90%	3.70%	7.33%
SP	18.00%	50.63%	85.74%	3.89%	6.83%
TO	18.00%	46.87%	82.24%	2.85%	6.09%

The retail trade sector is not only more prone to firm fragmentation on the extensive margin (i.e. whether the firm owner split his business or not), but also on the intensive margin, (conditional on having fragmented a firm, into how many pieces were the firm split). Table (8) exhibits the distribution of the number of firms splits conducted by firm owners, within the trade sector. The retail trade sector has business owners who splits their business beyond five units, the maximum amount done by any manager in the wholesale sector. One can speculate the reasons why not only the number of retailers that split their business is higher than the number of wholesale traders that do the same thing, but also why a retailer slashes his companies into more pieces. It could be the case that larger firms are more visible to fiscal authorities, inhibiting owners of relative large business to split their companies and evade taxes by registering in the Simples system. Since the average revenue in the wholesale trade sector is larger[18], this prediction could be tested with an appropriate data set.

.1.2 Identification of Calibrated Parameters

To check how the target moments vary with respect to the calibrated parameters, Figure (4) portraits how the model moments change when each of the referred parameters move up and down by 1 p.p. relative to their calibrated values. For example, in Panel A one can see that the measure of split-up firms (*Split Sh.*) is the model moment that is most sensitive to changes in the cost of splitting parameter b. Holding all of the remaining parameters at their calibrated values from Table (2.3), when b is increased by 1pp, the measure of firms splits decrease by next to 20% (from 7.55% to 6.04%). Notice that the Simples Participation rate (*Simples Sh.*)

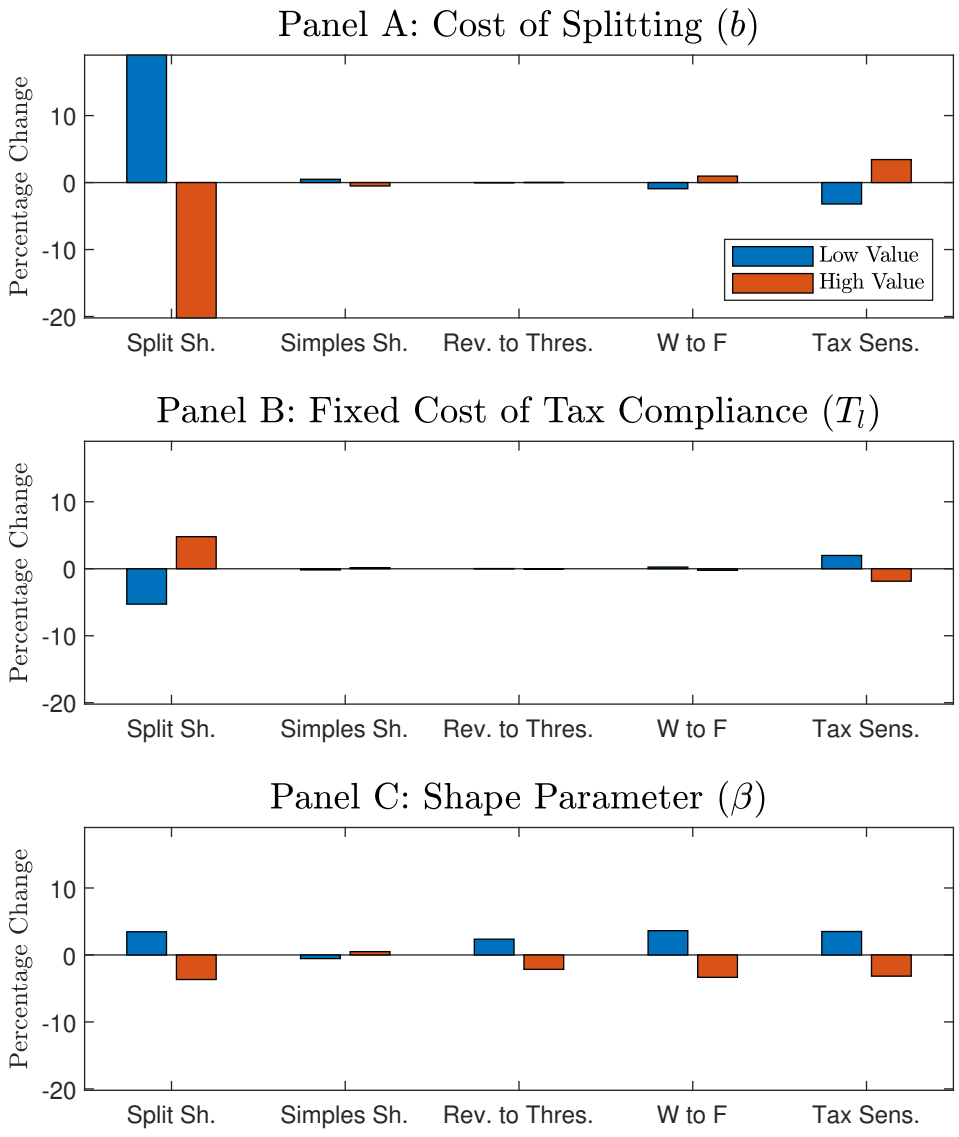
[18]Check footnote 17.

Table 8: Firm Splitting within the Trade Sector

Wholesale		**Retail**	
Number of Divisions	Percentage of Firms	Number of Divisions	Percentage of Firms
0	97.12	0	92.53
1	2.67	1	5.87
2	0.18	2	1.03
3	0.02	3	0.3
4	0.01	4	0.12
5	0	5	0.06
		6	0.03
		7	0.02
		8	0.01
		9	0.01
		10	0.01
		11	0
		12	0
		13	0
		15	0
		17	0
		18	0
		19	0

behaves as if it is invariant to all three internally calibrated parameters. This might explain why this is one of the moments that the model struggled the most to match.

Figure 4: Moment Sensitivity to Calibrated Parameters



.1.3 Context and Data

Brazil tax system divides firm into three major groups: individual Micro-entrepreneurs, Simples firms and regular firms. The micro-entrepreneur category is intended for

self-employed workers allowing them to collect government benefits such as emitting official receipts and retirement benefits. They are allowed to have up to one employee earning the minimum wage and must pay an yearly fixed tax. They can have up to 81,000 R$ of total revenue per year.

The Simples tax regime, the object of interest in this proposal, encompasses small firms with yearly revenue up to 3,600,000 R$. There is no sector or number of employees restriction to enter Simples. An individual can be the a major stakeholder (more than 10% of the firms equity) in as many Simples firms as she wants as long as the sum of their total revenue doesn't surpasses the 3,600,000 R$ threshold. Firms with larger revenue are included in the Regular Tax system.

Table 9: Distribution of Firms According to their Tax Regime

	Number of Firms	%
Micro-Entrepreneurship	7,964,258	43.07 %
Simples-firms	4,626,897	23.58%
Regular Tax system	7,028,977	33.35%
Total	19,620,132	100

BIBLIOGRAPHY

Bachas, P., Jaef, R. N. F., & Jensen, A. (2018). *Size-dependent tax enforcement and compliance: global evidence and aggregate implications*. The World Bank.

Bargain, O., & Kwenda, P. (2011). Earnings structures, informal employment, and self-employment: New evidence from brazil, mexico, and south africa. *Review of income and wealth*, *57*, S100–S122.

Bertrand, M., & Mullainathan, S. (2004). Are emily and greg more employable than lakisha and jamal? a field experiment on labor market discrimination. *American economic review*, *94*(4), 991–1013.

Bosch, M., Goñi-Pacchioni, E., & Maloney, W. (2012). Trade liberalization, labor reforms and formal–informal employment dynamics. *Labour Economics*, *19*(5), 653–667.

Buera, F. J. (2009). A dynamic model of entrepreneurship with borrowing constraints: theory and evidence. *Annals of finance*, *5*, 443–464.

Cagetti, M., & De Nardi, M. (2006). Entrepreneurship, frictions, and wealth. *Journal of political Economy*, *114*(5), 835–870.

Cavalcanti, T., Fernandes, L., Rachter, L., & Santos, C. (2022). Women and men at work: Fertility, occupational choice and development.

Chatterji, A. K., & Seamans, R. C. (2012). Entrepreneurial finance, credit cards, and race. *Journal of Financial Economics*, *106*(1), 182–195.

Chiplunkar, G., & Goldberg, P. K. (2021). Aggregate implications of barriers to female entrepreneurship. Tech. rep., National Bureau of Economic Research.

Derenoncourt, E. (2019). Can you move to opportunity? evidence from the great migration. *Unpublished working paper*.

Derenoncourt, E., Gérard, F., Lagos, L., & Montialoux, C. (2021a). Collective bargaining, wage floors, and the racial earnings gap. *Working Paper*.

Derenoncourt, E., Gérard, F., Lagos, L., & Montialoux, C. (2021b). Racial inequality, minimum wage spillovers, and the informal sector. *Working Paper*.

Derenoncourt, E., & Montialoux, C. (2021). Minimum wages and racial inequality. *The Quarterly Journal of Economics*, *136*(1), 169–228.

Dix-Carneiro, R., Goldberg, P. K., Meghir, C., & Ulyssea, G. (2021). Trade and informality in the presence of labor market frictions and regulations. Tech. rep., National Bureau of Economic Research.

Dix-Carneiro, R., & Kovak, B. K. (2019). Margins of labor market adjustment to trade. *Journal of International Economics*, *117*, 125–142.

El Badaoui, E., Strobl, E., & Walsh, F. (2010). The formal sector wage premium and firm size. *Journal of development economics*, *91*(1), 37–47.

Evans, D. S., & Jovanovic, B. (1989). An estimated model of entrepreneurial choice under liquidity constraints. *Journal of political economy*, *97*(4), 808–827.

Fagereng, A., Holm, M. B., & Natvik, G. J. (2021). Mpc heterogeneity and household balance sheets. *American Economic Journal: Macroeconomics*, *13*(4), 1–54.

Fairlie, R. W. (1999). The absence of the african-american owned business: An analysis of the dynamics of self-employment. *Journal of Labor Economics*, *17*(1), 80–108.

Fairlie, R. W., Robb, A., & Robinson, D. T. (2020). Black and white: Access to capital among minority-owned startups. Tech. rep., National Bureau of Economic Research.

Fairlie, R. W., & Robb, A. M. (2008). Race and entrepreneurial success. *Cambridge, MA: The MIT Press*.

Garicano, L., Lelarge, C., & Van Reenen, J. (2016). Firm size distortions and the productivity distribution: Evidence from france. *American Economic Review*, *106*(11), 3439–79.

Glover, D., Pallais, A., & Pariente, W. (2017). Discrimination as a self-fulfilling prophecy: Evidence from french grocery stores. *The Quarterly Journal of Economics*, *132*(3), 1219–1260.

Goldberg, P. K., & Pavcnik, N. (2003). The response of the informal sector to trade liberalization. *Journal of development Economics*, *72*(2), 463–496.

Hirata, G., & Soares, R. R. (2020). Competition and the racial wage gap: Evidence from brazil. *Journal of Development Economics*, *146*, 102519.

Hsieh, C.-T., Hurst, E., Jones, C. I., & Klenow, P. J. (2019). The allocation of talent and us economic growth. *Econometrica*, *87*(5), 1439–1474.

Hsu Rocha, R., & Dias, M. (2021). Racial gaps in the labor market: The role of nonwhite entrepreneurship. *Available at SSRN 3935982*.

Jappelli, T., & Pistaferri, L. (2014). Fiscal policy and mpc heterogeneity. *American Economic Journal: Macroeconomics*, *6*(4), 107–136.

Johnson, D. S., Parker, J. A., & Souleles, N. S. (2006). Household expenditure and the income tax rebates of 2001. *American Economic Review*, *96*(5), 1589–1610.

Kekre, R. (2021). Unemployment insurance in macroeconomic stabilization. Tech. rep., National Bureau of Economic Research.

Kline, P., & Walters, C. (2021). Reasonable doubt: Experimental detection of job-level employment discrimination. *Econometrica*, *89*(2), 765–792.

Kline, P. M., Rose, E. K., & Walters, C. R. (2021). Systemic discrimination among large us employers. Tech. rep., National Bureau of Economic Research.

La Porta, R., & Shleifer, A. (2014). Informality and development. *Journal of economic perspectives*, *28*(3), 109–126.

Lang, K., & Kahn-Lang Spitzer, A. (2020). Race discrimination: An economic perspective. *Journal of Economic Perspectives*, *34*(2), 68–89.

Lang, K., & Lehmann, J.-Y. K. (2012). Racial discrimination in the labor market: Theory and empirics. *Journal of Economic Literature*, *50*(4), 959–1006.

Lucas, R. E. (1978). On the size distribution of business firms. *The Bell Journal of Economics*, (pp. 508–523).

Miller, C., & Schmutte, I. (2021). The dynamics of referral hiring and racial inequality: Evidence from brazil. *Working Paper*.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, *104*(10), 3186–3221.

Monteiro, J. C., & Assunção, J. J. (2012). Coming out of the shadows? estimating the impact of bureaucracy simplification and tax cut on formality in brazilian microenterprises. *Journal of Development Economics*, *99*(1), 105–115.

Oliveira, L. d., Porcaro, R. M., & Araujo, T. C. N. (1985). O lugar do negro na força de trabalho. *Rio de Janeiro: IBGE*.

Parker, J. A. (2017). Why don't households smooth consumption? evidence from a $25 million experiment. *American Economic Journal: Macroeconomics*, *9*(4), 153–83.

Ponczek, V., & Ulyssea, G. (2022). Enforcement of labour regulation and the labour market effects of trade: Evidence from brazil. *The Economic Journal*, *132*(641), 361–390.

Rocha, R., Ulyssea, G., & Rachter, L. (2018). Do lower taxes reduce informality? evidence from brazil. *Journal of Development Economics*, *134*, 28–49.

Ulyssea, G. (2018). Firms, informality, and development: Theory and evidence from brazil. *American Economic Review*, *108*(8), 2015–47.

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