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ABSTRACT

This thesis consists of three essays in microeconomic theory. The first two are about the formation of organizations, and the third is about individual or organizational decision making in ambiguous settings.

In the first essay I explore the implications of costs associated with binding agreements on equilibrium agreement structures. Establishing binding agreements is often costly in real world economies. These contracting costs are usually regarded as harmful by economists as the costs decrease the gains from cooperation. They affect which agreements form by changing the incentives of agents, potentially prevent the establishment of efficient contracts. Using an alternating offers bargaining model of coalition formation I show that the presence of transaction costs can lead to an efficient outcome in situations where inefficiency arises in equilibrium without these costs. These

results provide new insights for policies targeting transaction costs.

There are many situations in Economics and Political Science that involve limited possibilities for firms or parties to organize themselves into groups, mostly due to regulatory restrictions. In addition, in these settings the surplus of a given group often depends on the organizational structures formed outside of the group. The second essay introduces a coalition formation model that is able to analyze markets with both restricted cooperation and externalities across coalitions. This concept allows a more realistic modeling, opening the possibility to use this framework to analyze the welfare effects of mergers.

In the third essay I propose a new model of decision making under uncertainty with multiple priors that is, unlike the well-known model of Gilboa and Schmeidler (1989), able to express attitude towards ambiguity. In addition, the decision does not necessarily depend on the two extreme (worst case and best case) priors as in the model of Ghirardato et al. (2001). I use choice correspondences by lexicographic semiorders that are generalizations of the choice functions defined in Manzini and Mariotti (2012). I also provide a method constructing lexicographic semiorders for choosing from ambiguous acts.

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Chapter 1

Coalition Formation with Costly Binding Agreements

1.1 Introduction

Binding agreements are widespread in everyday life. Economic activities requiring the collaboration of multiple people are often regulated by legally enforceable contracts between the participants, such as employment contracts or agreements between firms specifying a transaction. The purpose of these agreements is to ensure that the collaborating parties act in a way that is collectively beneficial for them as a group, preventing situations where agents seek to maximize their own benefits, disregarding the interests of others. There are numerous situations where these contracts are not available for free: for instance, the contracting parties have to hire and pay a lawyer to ensure that the correct legally enforceable contract is written.

Various fields of Economics have different approaches regarding binding agreements and the costs associated with them. Economic theory in general considers the presence of contracting costs to be harmful for the overall surplus of agents. Non-cooperative game theory and its applications are usually based on the assumption that parties cannot even make binding agreements. In cooperative game theory and in the theory of coalition formation, while the fundamental assumption is that binding agreements are feasible, typically the costs of establishing them are not modeled explicitly. In this paper I build a model where binding agreements are feasible and costly, and I show that in a wide range of situations the presence of agreement costs *increases* the total surplus compared to the outcome that would arise in the absence of these costs.

Costs associated with binding agreements have a significant impact on the formation of agreements as agents' incentives change, resulting in potentially different agreement structures in equilibrium compared to a costless environment. This paper investigates how the costs of establishing binding agreements influence the negotiation about entering into contracts and the efficiency of the resulting outcomes. In such a setting a natural hypothesis is that contracting costs lead to efficiency problems. Although the efficient contracts would be written in an environment where agreements are free, as these costs reduce the gains from cooperation, agents fail to reach the efficient outcome in a costly environment. While this hypothesis is correct in some settings, surprisingly the opposite phenomenon is also possible: *costly binding agree-*

ments may help reaching the efficient outcome when efficiency is not reached in the absence of these costs. These interesting cases are the focus of this paper.

Basic economic intuition suggests that the presence of costs related to establishing or enforcing binding agreements has a negative impact on the economy, as these costs decrease the gains from the economic activity specified by the agreement. Since the costs of establishing binding agreements are not directly related to production or any kind of economic activity, these costs are essentially *transaction costs*. The "Coase Theorem" (originating from Coase (1960)), one of the best known ideas in Economics, states that in the absence of transaction costs agents always reach an efficient outcome - an outcome that maximizes the total surplus across all agents - via negotiation. According to this argument, transaction costs serve as an obstruction to negotiation, and if they are sufficiently high, parties may fail to reach the surplus-maximizing outcome through bargaining. There is a large literature analyzing the effect of transaction costs on two-player Coasean bargaining and the consensus is that transaction costs reduce efficiency (see Anderlini and Felli (2001, 2006), Bolton and Faure-Grimaud (2010) and Lee and Sabourian (2007) among others).

When the logic of the Coase Theorem is applied to the formation of agreements among agents, it is expected that players reach the surplus-maximizing outcome when they negotiate without transaction costs. In addition, sufficiently high transaction costs impede negotiating and does not allow parties to reach efficiency. While this Coasean logic is accurate in some settings, as suggested above, the opposite phenomenon can also happen.

In situations with more than two agents, even in the absence of transaction costs, it is possible that agents fail to establish the contracts leading to the highest overall surplus. This phenomenon is already known in the coalition formation literature, see for example Ray and Vohra (1997, 2001), Diamantoudi and Xue (2007) or Hyndman and Ray (2007). In a Coasean world, the only effect transaction costs can have is to obstruct negotiating partners from reaching the efficient outcome that would arise in a frictionless setting. I show that this is not always the case: in some situations, the presence of transaction costs leads to the efficient outcome which would never be reached in a setting free of transaction costs.

The existence of surplus-increasing transaction costs has important policy implications. Despite the common belief among economists, in some situations the welfare-improving action regarding transaction costs is to keep them high. In this paper I show that under some conditions, an environment with lower transaction costs is not necessarily desirable, as it can lead to social welfare loss when the formation of small groups is a potential issue. Therefore, when policy-makers decide about policies targeted at the reduction of transaction costs, a more careful approach is necessary and industry structures should be taken into account.

The intuition and the mechanism through which the presence of costs associated with establishing agreements restores efficiency is different based on the source of the inefficiency arising without transaction costs. I present two conceptually different types of situations where coalition formation leads to an inefficient outcome via costless bargaining, and I show how the presence of transaction costs helps restoring efficiency.

The first source of inefficiency I consider is when agents establish agreements to maximize their joint payoff disregarding the payoffs of others outside of their agreement. Even if the efficient outcome - where the combined surplus of all agents is maximal - is a single contract among all agents, it is possible that the surplus per capita is higher for a specific contract within a smaller group. In this situation agents have an incentive to form that smaller group and ensure themselves higher payoffs than they could expect in the efficient outcome. In the presence of transaction costs this incentive is weaker as the transaction cost "taxes" the gains from excluding others from the agreement, therefore it can help reaching the efficient outcome. It is important to note that the efficient outcome is also subject to the same transaction cost. However, since the total surplus is higher in the efficient outcome, the same cost results in a lower relative loss.

The second possible source of inefficiency occurs in settings with externalities among contracting groups. In these situations the well-being of an agent does not only depend on the contract she establishes, but also on what agreements others, who are not part of the agent's group, form. A notable example is free-riding in public good provision, as analyzed in Ray and Vohra (2001). Inefficiency due to free-riding arises because, even if it is known that some players will be free-riders and do not contribute to public good provision, the rest of the players are still better off if they make a binding agreement specifying a high level of contribution in order to maximize their own payoff. Due to the non-excludable consumption of public goods, the contributing players increase the free-riders' payoff even more than their own as a side effect. Free-riders, in some sense, are "forcing" other players to form these binding agreements on high contribution by declaring that they will not contribute. Introducing transaction costs makes the formation of agreements harder for the contributing players, therefore free-riders can no longer expect others to contribute. For this reason, when deciding about whether to free-ride or join a contributing group, the potential free-riders rather choose to contribute. That is, the presence of transaction costs can prevent free-riding despite that free-riders themselves are not subject to these costs.

This paper is organized as follows. Section 1.2 introduces some important concepts and provides a review of the related literature. Section 3.2 defines the coalition formation game I use in my analysis. Then I turn to the two different situations described above where transaction costs can restore efficiency. First, in Section 1.4 I analyze games without externalities among coalitions; then in Section 1.5 I study situations with externalities. In Section 1.6 I discuss the influence of some important assumptions on my results. Section 1.7 concludes the paper.

1.2 Background

In this section I start by introducing some important concepts related to binding agreements among groups and I provide different possible interpretations for the general transaction cost term used throughout the paper. Then I summarize the related literature.

1.2.1 Coalition formation

This paper studies how transaction costs affect the formation of agreements among agents. Agents establish these agreements to formalize the conditions of an economic activity that is beneficial to every participant. A natural framework to analyze such situations is cooperative game theory.

Cooperative game theory focuses on how the members of coalitions divide the coalitional value - the surplus of the coalition - taking the coalition structure as given. In coalition formation models agents negotiate with each other about entering into binding agreements, therefore the resulting coalition structure is an equilibrium of an explicitly or implicitly modeled bargaining game. Coalitions are groups of players that maximize the joint surplus of the entire group. A binding agreement among the members of a coalition ensures that players will indeed act in a way that increases the joint surplus, and not seek to maximize their own benefits instead.

Traditionally, coalition formation models do not explicitly separate the costs of establishing or enforcing coalitional agreements from the value derived from them. However, in economic or political applications of coalition theory it is very important to decompose the value achieved via cooperation from the costs of establishing or maintaining the coalition. The reason why an explicit modeling of these costs is important is that there are situations where the costs associated with agreements change while the economic or political activity specified by the agreements are unaffected. Moreover, governments or other institutions are often capable of influencing transaction costs via regulation.

To decide whether a policy that modifies transaction costs associated with establishing agreements is desirable or not, we need to understand how people adjust their decisions about entering into contracts in the new environment. This paper provides a method to predict the consequences of these policies.

1.2.2 Costly binding agreements

Costs associated with binding agreements have several potential interpretations. First, they can be interpreted as *contracting costs* which are simply the monetary costs of writing the contract. Another possible interpretation is that the costs of binding agreements are *enforcement costs* related to enforcing the actions specified by the agreement, such as the distribution of surplus. These costs can be also interpreted as fees of a supervisor actively monitoring that the contracting parties keep their end of the bargain. Alternatively, these costs can arise due to difficulty of coordination among agents. Depending on the analyzed situation, the search for potential coalition partners can also be a source of agreement costs.

Although the possible interpretations are numerous, the costs associated with binding agreements have two defining properties in my model. First, these costs arise only if there is cooperation among at least two agents. An agent who acts on its own without cooperating with anyone else - in the terminology of cooperative game theory, a player in a singleton coalition - is never affected by these costs. The second property is that these costs are not directly related to the economic activity specified by the agreement. For example, consider a contract between an upstream and downstream firm specifying the delivery of goods of a given quantity and quality. The fee of the lawyer writing the contract does qualify as an agreement cost, while the price of fuel, wage of the truck driver and any costs directly related to the delivery are not agreement costs. In summary, costs associated with binding agreements reduce the gains from potential agreements without directly affecting the economic activity specified by these agreements.

1.2.3 Related literature

I use a cooperative game theory framework to build a model that shows the potential efficiency benefits of transaction costs in the formation of agreement structures among groups. The theory of cooperative games originates from von Neumann and Morgenstern (1944). Games in partition function form, are a specific class of cooperative games first introduced by Thrall and Lucas (1963). The model introduced in the next chapter is based on this class of games.

There is a growing literature on coalition formation, see Ray and Vohra (2015) for a survey. The papers most closely related to my model are Bloch (1996) and Chatterjee et al (1993) as they also use an extensive form bargaining game to determine the outcome coalition structures. Similarly to this paper, Ray and Vohra (1997), Hyndman and Ray (2007) and Diamantoudi and Xue (2007) also emphasize the efficiency dimension of coalition formation outcomes and find that the coalition formation negotiation process does not always lead to an efficient outcome in the absence of transaction costs.

Several other papers use a coalition formation framework to analyze public good provision games, most notably Ray and Vohra (2001), Furusawa and Konishi (2011). Dixit and Olson (2000) and Ellingsen and Paltseva (2016) use a somewhat different negotiation framework but has a similar inefficiency result as Ray and Vohra (2001) and this paper.

The effect of transaction costs on Coasean negotiation is extensively discussed in the literature. Anderlini and Felli (2001, 2006), Bolton and Faure-Grimaud (2010) and Lee and Sabourian (2007) assume different types of transaction costs affecting negotiation and find that the presence of transaction costs generally causes efficiency problems. White and Williams (2009), Mackenzie and Ohndorf (2013) and Robson and Skarpedas (2008) shows that costly enforcement of property rights leads to potential inefficiency.

There are multiple core ideas in the Organization Economics literature that are closely related to this paper. Legros and Newman (1996, 2013) are based on the idea that due to some non-contractible production decisions, firms that are willing to cooperate with each other have to hire a professional manager to ensure the efficiency of the cooperation. This argument uses the same logic as the starting point of this paper: binding agreements are not available by default, there is a cost associated with them, possibly due to a third party not participating in the economic activity for which the coalition is formed. Legros et al (2018) uses the organizational framework described above to provide a model of endogeneous market structure.

1.3 Coalition Formation with Costly Binding Agreements

In this section I introduce the model I use to analyze which binding agreements arise in an environment with transaction costs. I use a sequential coalition formation game similar to Chatterjee et al (1993) and Bloch (1996). I define the formal model I use in my analysis, then I discuss the main assumptions imposed.

The presence of externalities between coalitions is an important feature of my model. Games of partition function form are cooperative games where a value of a coalition depends on how the rest of the players are partitioned into coalitions. For example in the case of three players, the value of a singleton coalition can be different when the other two players are in a two player coalition or are in separate singleton coalitions. Therefore, in order to capture externalities between players in my model, the values of coalitions are given by a partition function. Throughout the paper I assume that all players are symmetric in a sense that all coalitions of the same size have the same value, that is, the payoffs only depend on the size of the coalition but not on the identity of its members.

Definition 1. Let be $N = \{1, .., n\}$ the set of players with $n > 2$. The cooperative game (V, N) of partition function form is a function V defined on pairs of $S \subseteq N$ and $\pi \in \Pi(N)$ where $\Pi(N)$ denotes the set of possible partitions of N. The value of each coalition S, if the current partition is π, is given by $V(S, \pi) \in \mathbb{R}$, $S \in \pi$.

V is symmetric if for all π ,π' and $S \in \pi$, $S' \in \pi'$ we have $V(S, \pi) = V(S', \pi')$ as long as $|S| = |S'|$ and $|\pi| = |\pi'|$ where $|\pi| = (|S_1|, ..., |S_k|) = (s_1, ..., s_k)$ with numbers arranged into a descending order.

The collection $(s_1, ..., s_k)$ is referred as *numerical coalition structure* (Ray and Vohra, 1999).

The assumption that $n > 2$ is maintained for all games analyzed in this paper as in the framework I use two player coalition formation problems are trivial.

In this paper I focus on a specific class of partition function games where the total surplus is the highest when the grand coalition forms. This property is called *cohesiveness*. That is, since I focus on cohesive games, the efficient outcome - where the total surplus of all players is maximal - is always the grand coalition.

An important special case of the cooperative game defined above is the *characteristic function* game where there are no externalities between coalitions. That is, the value of a coalition depends only on the members - or in the symmetric case only on the size of the coalition - and does not depend on the coalition structure formed by the rest of the players. In this case the value function reduces to $v : 2^N \to \mathbb{R}$ and cohesiveness is equivalent to superadditivity (that is, for all disjoint $S, T \subset N$ we have $v(S \cup T) \geq v(S) + v(T)$). Section 1.4 will focus on this special case.

The outcome coalition structure π, that assigns values to coalitions using the function V defined above is determined by a noncooperative alternating offer bargaining game in the spirit of Rubinstein (1982), which is a common framework in the coalition formation literature using the non-cooperative approach, see Ray and Vohra (2015).

The process of the game is the following. At the beginning of the game no players are assigned to any coalitions. At the start of the game, a *protocol* randomly selects a player to be the first proposer. The role of the protocol is the same as the role of Nature in games with incomplete information. The selected player i makes an offer to a set of players not yet assigned to any coalition to form coalition S. All players in S (not including player i) answer this offer in a randomly determined order by accepting or rejecting it. If everyone accepts the offer, coalition S forms and the players in S leave the game. Then the game continues with $N \setminus S$ as the set of players, and a new proposer is picked randomly.

If there is a player in S that rejects the offer, then S does not form and all players return to the pool of players without coalitions. The game then continues with the protocol randomly selecting a new proposer from this pool. The game ends when every player is assigned to coalitions. The difference between a player in a singleton coalition and a player not yet assigned into coalitions is important.

Similarly to Bloch (1996), I assume that there is no discounting between the rounds of bargaining, and if the bargaining game continues infinitely players receive a payoff of zero. The intuition is the following: I model economic situations where binding agreements are necessary to engage in a long-term economic activity creating the coalitional value. Once binding agreements are formed, the activity continues indefinitely, making the time spent on bargaining negligible as long as the bargaining process ends in finite rounds.

Note that coalitional agreements are assumed to be irreversible in a sense that once a player is assigned to a coalition, she can no longer receive another offer to be a part of a different coalition instead. This irreversibility assumption is crucial for the results presented in Section 1.4 and Section 1.5. I discuss the implications of relaxing this assumption in Section 1.6.

Another important assumption is that the coalitional surplus is divided equally. This also excludes the possibility of side payments within the coalition. Depending on the modeled situation at hand this assumption might be quite strong, however to keep the analysis as focused on the role of agreement costs as possible, I will disregard the possibility of side payments. If the surplus is divided equally, then the offers made by players during the bargaining game simply contain the proposed coalition, there is no need to specify a distribution of coalitional surplus in the offer.

The formal definition of the coalition formation game is given below:

Definition 2. The coalition formation game $(V, N, N^*, \pi_{-N^*}, \Sigma, \rho)$ consists of the following:

- $N = \{1, ..., n\}$ is the set of players, $n > 2$
- N^* is the set of players not yet assigned to a coalition, $N^* = N$ at the beginning of the game
- π_{-N^*} is a partition of $N \setminus N^*$
- V is a symmetric partition function
- $\sigma_P \in \Sigma_P : (N^*, \pi_{-N^*}) \to \Pi(N^*)$ is a strategy of the proposing player
- $\sigma_R \in \Sigma_R : (\Pi(N^*), \pi_{-N^*}) \to \{\text{Accept, Reject}\}$ is a strategy of a responding player
- ρ is a protocol selecting a random player in N^* to be the proposer if currently there is no proposing player

When the protocol selects a player to be the proposer, the player chooses a subset S of N^* including the player herself according to her strategy σ_P. If there are other players in this selected subset, they have to choose whether to Accept or Reject the offer to form coalition S. If all players choose Accept, S is formed and $N^* \setminus S$ becomes the new N^*.

If $N^* = \emptyset$, all players receive payoffs according the following rule: for all players $i \in S \in \pi$, $u_i(S, \pi) = \frac{V(S,\pi)}{|S|}$.

The outcome of a game is a coalition structure π that is a partition of N. Throughout the paper I will focus on outcomes rather than equilibrium strategies as I am interested in what coalition structures form. Note that the strategies of players are *stationary* as they do not depend on histories, only on payoff-relevant information such as the coalitions already formed, the set of players that are still in the game and the current proposal.

Due to the externalities captured by the partition function, when players decide whether to form a specific coalition S they have to consider how the remaining players are going to organize themselves into coalitions. This is modeled by having σ_P and σ_R dependent on both π_{-N^*} and N^*, the coalitional structure formed by players that are already in coalitions and the set of players yet to form into coalitions, respectively.

Similarly to Bloch (1996), Ray and Vohra (1997) and Kóczy (2007), I assume that the players can make a rational prediction about the other players' actions, therefore the equilibrium concept is (stationary) subgame perfect equilibrium in the sequential game defined above. In the rest of the paper I will use the notation $\sigma(V, N)$ to denote the sequential coalition formation game where the payoffs are given by the cooperative game (V, N).

This paper modifies the framework defined above by introducing *transaction costs* to the model. These costs are assigned to coalitions and they simply decrease the value of the given coalition.

It is possible that larger coalitions are subject to higher transaction costs, therefore transaction costs are non-decreasing in the size of the coalition. Outside of this monotonicity, no further structure is assumed about the costs in this paper. Similarly to the value of the coalition, the transaction cost depends only on the size of the coalition and it is independent of which players are in that given coalition. Singleton coalitions are not subject to transaction costs since they do not need a binding agreement ensuring that they maximize the coalition's surplus instead of their own personal profit as the coalitional surplus coincides with the individual benefit.

A coalition formation game with costly binding agreements adds one more element to the game defined in Definition 2: a vector $\tau = \{t_1, t_2, ..., t_n\}$ with $t_i \leq t_j$ for all $i \leq j$. The i-th element of the vector represents the transaction cost that has to be paid by any coalition with i players in it. The transaction cost for singleton coalitions, t_1 is always equal to zero. Due to transaction costs, the payoff of player i in coalition S, when partition π is formed, changes to the following:

$$u_i(S, \pi) = \frac{V(S, \pi) - t_{|S|}}{|S|}.$$

The assumption that transaction costs are non-decreasing in coalition size implies that $t_i \leq t_j$ for all $i < j$. I will use the notation (V_t, N) and $\sigma(V_t, N)$ to refer to games (V, N) and $\sigma(V, N)$ augmented with the vector of transaction costs t.

In the next two sections I show how introducing transaction costs changes the equilibrium outcome of coalition formation games and how it can help restore efficiency.

1.4 Games without externalities

There are many real-world situations where there are gains from cooperating with others. The problems I analyze in this section have the feature that cooperation is beneficial for all participating parties and the efficient outcome is the one where all players choose to cooperate, that is, the grand coalition of all players forms. Furthermore, the activity of a given coalition does not affect agents outside of that group. Examples of this type of games are situations where the joint value originates from technological synergies - such as economies of scale - or from the provision of excludable (for example, local) public goods.

This section focuses on situations where while the most efficient outcome is the grand coalition, it is not possible to divide the surplus of the grand coalition in a way that each possible combination of players gets at least as high payoff as they could ensure for themselves in a smaller coalition. These games represent the first possible reason why the formation of coalitions can lead to an inefficient outcome: a subset of players refuses to participate in the efficient grand coalition if they can achieve a higher payoff in a smaller coalition. However, this deviation reduces the total surplus of all players.

First I look at the equilibrium outcome of this type of games in the absence of transaction costs and show that for a class of games, bargaining without transaction costs leads to an inefficient equilibrium. Then I point out how introducing transaction costs can restore efficiency while increasing the total surplus of the players.

1.4.1 No transaction costs

As a higher degree of cooperation is beneficial for each player, it seems plausible that without transaction costs, players reach the most efficient outcome. This is certainly true when there are only two players because there is only one possible contract between agents. However, games with three or more players open the possibility to multiple potential contracts among players. In these situations the efficient outcome will be reached in equilibrium only if the marginal returns to cooperation are either constant or increasing as further players join the coalition. Instead, if there are decreasing marginal gains from cooperation, even in the absence of transaction costs, it is no longer guaranteed to reach the efficient outcome through a coalition formation game described in the previous section. This paper shows that in that case it is possible that the presence of transaction costs helps reaching the efficient outcome.

To demonstrate a situation where agents fail to reach the efficient outcome in the absence of transaction costs consider the following scenario. There are three manufacturers at the same location, operating in the same industry. The manufacturers can produce separately, but they can also choose to horizontally integrate with one or two other manufacturers. Integration is beneficial due to economies of scale: the total surplus of an industry structure consisting of two integrated firms and a single firm is higher than the combined surplus of three single firms; and the surplus produced by the three-firm integration is higher than the combined surplus of the two-firm integration and one single firm industry structure. However, the gains from integration are higher when moving from producing alone to operating as a two-manufacturer integration than the gains from moving to the full integration from the two-firm integration. Note that in this situation gains from integration, as there are no externalities among firms, are purely technological, there are no market power effects. This game can be captured by the following numerical example.

Example 1.

Consider a game with three players. The singleton coalition has a surplus of 20, the two-player coalition has a surplus of 70 and the grand coalition has a surplus of 102. The efficient outcome is the grand coalition since its total surplus, 102, is higher than 90 or 60, the total surplus when the numerical coalition structure is (2,1) and the combined surplus when all players are in singleton coalitions, respectively. However, the equilibrium of the bargaining game described in Section 3.2 leads to an outcome with a two-firm integration and a single firm because the payoff players can expect from the grand coalition is 34, while in a two-player coalition they can get a payoff of 35. Therefore, when the first player makes her proposal, she offers the possibility of a two player coalition to one of the players with an equal split of the surplus, and the proposed player accepts it.

In this example the efficient outcome is not reached in equilibrium because the two players in the small coalition maximize their own benefits instead of the joint surplus, and they are better off when deviating from the efficient outcome.

Farrel and Scotchmer (1988) studies three-player games similar to the example above and proposes a solution to these kind of problems by promoting one of the players to a "ringleader" who has some power to capture a part of the surplus, without sharing it with the other players. According to their result, if the ringleader has enough power, the efficient outcome forms. Note that the distribution of the payoffs will be asymmetric as the ringleader takes a high portion of the total surplus. In this paper I propose a different solution to this problem that preserves the symmetry of players.

1.4.2 Introducing transaction costs

Now I introduce a transaction cost to Example 1. Running the horizontal integration of multiple manufacturing firms requires a professional manager who charges a fee for her services. Assume that this fee is equal to 9. Introducing this transaction cost changes the surplus available for players in the two and three firms coalitions to 61 and 93 respectively. Now the equal division of the surplus in the grand coalition gives 31 to each player, while the two player coalition gives only 30.5. Therefore, there is no incentive any more to form a two firm coalition as it is no longer possible to get higher payoff than in the efficient outcome, hence the resulting equilibrium outcome is the efficient grand coalition.

It is important to point out that the transaction cost restores efficiency despite that both the grand coalition and the frictionless equilibrium structure (2,1) are subject to the cost. As the same cost has a relatively higher effect on the players' payoff in the frictionless outcome compared to the efficient outcome, players' incentives change and it becomes desirable to form the grand coalition. As a result, the two-player coalition is no longer advantageous for the player making the first proposal.

In addition, the total payoff of all players is higher in the presence of transaction costs even if we account for the cost itself. This property implies that the expected payoff of a player is higher in that game as well, therefore ex ante every player is strictly better off when there are transaction costs. That is, if players can choose which game they want to play before the order of proposers is drawn, they all prefer the game where cooperation is costly compared to the one when forming coalitions is free. The interpretation of this result in the manufacturing industry example is the following: before the game starts and players can choose managers that operate the integrated firm and work for free or managers who work for a strictly positive wage, *they prefer to pay the manager instead of getting her services for free.*

Note that this result is *not* achieved in a setting where transaction cost eliminates the inefficient equilibrium outcome by discriminatively targeting it and making it more costly. Instead, the efficient outcome is subject to the same transaction cost. Moreover, even in cases where the transaction cost is slightly higher for the grand coalition (up to 12 compared to the 9 associated with the two player coalition), the same result still holds with the efficient outcome being the unique equilibrium and the presence of transaction costs is preferable by the players ex ante. Intuitively, the inefficient outcome is no longer an equilibrium because the total surplus is lower in that case, therefore the same transaction cost feels more costly from the point of view of a given player.

In summary, when cooperation is not associated with additional costs, the efficient outcome is not reached since the deviating two players can be better off than they would be in the efficient outcome at the expense of the third player. However, if transaction costs are introduced to the model and cooperation is costly enough, the advantage of forming the two-player coalition disappears. Contrary to the traditional perception, instead of hindering the economy from reaching the efficient state, transaction costs are pushing the economy towards efficiency.

1.4.3 Surplus improving transaction costs for games without externalities

Now I formalize a general result regarding the situations described above. First I state the conditions when the absence of transaction costs leads to an inefficient equilibrium of the coalition formation game. In the case of symmetric superadditive characteristic function games these conditions are equivalent to the emptiness of the core. Then I characterize the cases when there exists a vector of *surplus-increasing transaction costs* that ensures the formation of the grand coalition in equilibrium, while still low enough to make the sum of all payoffs higher than in the frictionless game. The formal definition of surplus-increasing transaction costs are the following:

Definition 3. Let (V, N) be a cohesive game where N is not an outcome of a stationary SPE in $\sigma(V, N)$. Then, t is a vector of surplus-increasing transaction costs if there is a stationary SPE in $\sigma(V_t, N)$ with N as an outcome and for all π^* arising as an outcome of $\sigma(V, N)$, we have

$$V(N) - t_n \geq \sum_{S \in \pi^*} V(S).$$

Note that the transaction costs defined in Definition 3 do not include all possible transaction cost vector t that increase the total surplus of players. Surplus-increasing transaction costs are defined as transaction costs that both restore the efficient outcome N and increase the total surplus of the players. To find out what games have potential surplus-increasing transaction costs, the first step is to identify the set of games that do not reach the efficient outcome in an equilibrium without transaction costs.

Definition 3 has an important implication: if the efficient outcome is reached in the absence of transaction costs, then the Coasean argument is valid and transaction costs indeed hurt the economy. The potential surplus-improving effect of transaction costs is originating from the fact that the efficient outcome is not always reached in an environment free of these costs.

In Example 1 the efficient grand coalition does not form because it is impossible to divide the value 102 in a way that any two players get at least 70 combined. Using the terminology of cooperative game theory, this feature means that the game has an empty core. Below I provide a formal definition of the core of a characteristic function game.

Definition 4. Let (v, N) be a characteristic function game. The core of the game is the set $C(v, N)$ of vectors $x \in \mathbb{R}^n$ such that $\sum_{i \in N} x_i = v(N)$ and for all $S \subseteq N$,

$$\sum_{i \in S} x_i \geq v(S).$$

That is, the core is the set of possible distributions of the value of the grand coalition that guarantees every subcoalition to have at least as high payoff as they could earn if they formed the given subcoalition instead. If the core is nonempty - it is possible to divide the grand coalition's worth in a desirable way - then the grand coalition is expected to be stable. A natural question to ask is whether the grand coalition arises as the equilibrium of the sequential bargaining game when the core of the game is nonempty. Chatterjee et al (1993) shows that the statement is not true if the players are not symmetric. Here I show that the statement is true in the case of symmetric games.

Proposition 1. *Let (v, N) be a symmetric characteristic function form game and $\sigma(v, N)$ is a coalition formation game with value function v and player set N where the core of (v, N) is nonempty. Then there is a stationary SPE of $\sigma(v, N)$ that gives N as outcome.*

Proof. Since the core of (v, N) is nonempty, there is no coalition S such that

$$\frac{v(S)}{s} > \frac{v(N)}{n}. \tag{1.1}$$

Condition (1.1) means that there is no coalition S that is able to ensure higher average payoff to its members than the grand coalition. Given that, when player i proposes to form N, for all other players $j \neq i$ it is an equilibrium strategy to accept it. By (1.1) it is clear that if any player declines the formation of N, she cannot expect higher payoff than $\frac{v(N)}{n}$, therefore there is no profitable deviation from accepting the offer to form N.

In addition, for any proposing player, when the set of remaining players is N, it is an equilibrium strategy to propose N if the responders accept it. If the proposer proposes N and the proposal gets accepted, the proposer receives a payoff of $\frac{v(N)}{n}$. Due to condition (1.1), $\frac{v(N)}{n}$ is the highest possible payoff a player can receive in the game, so no profitable deviation form proposing the grand coalition. $\square$

The converse of Proposition 1 is also true: for all symmetric game (v, N) such that there is a stationary SPE in $\sigma(V, N)$ such that the grand coalition is formed, then the core of the game must be nonempty (this implies that equal split of $v(N)$ is in the core).

Proposition 2. *Consider a symmetric superadditive characteristic function game (v, N) where there is a stationary SPE in $\sigma(v, N)$ with N as equilibrium outcome. Then, the core of (v, N) is nonempty.*

I prove this proposition in the Appendix A.1.1.

Below I formulate that for every symmetric superadditive characteristic function game (v, N) there exists a vector t of transaction costs such that the core of (v_t, N) is nonempty.

Lemma 3. *For every symmetric superadditive characteristic function game with empty core there is a cost t associated with each non-singleton coalition such that the game (v_t, N) has a nonempty core.*

The proof can be found in Appendix A.2.

Combining the results of Propositions 1, 2 and Lemma 3 leads to the following result.

Corollary 4. *Let (v, N) b a symmetric superadditive game with characteristic functions where N is not an outcome in any stationary SPE of $\sigma(v, N)$. Then, there exist a vector of transaction costs t such that N is the outcome of a stationary SPE of $\sigma(v_t, N)$.*

Corollary 4 ensures that the vector of transaction costs that restore N as the outcome of (v_t, N). However, it does not imply anything about the total surplus of players. The next result characterizes the class of games for which *surplus-increasing transaction costs* exist.

Proposition 5. *Let (v, N) be a symmetric superadditive game with an empty core and let π^* be the SPE of $\sigma(v, N)$ and S^* is the coalition with highest average value in π^* with $|S^*| = s$. If*

$$v(N) - \sum_{S \in \pi^*} v(S) \geq \frac{n \cdot v(S^*) - s \cdot v(N)}{n - s},$$

then there is a surplus-increasing t.

Proposition 5 is a direct consequence of Corollary 4 and Definition 3.

1.5 Games with externalities

This section analyzes situations with externalities among coalitions. There are numerous examples of these situations. Cartels and non-excludable public good provision exhibit *positive externalities*. Cartels are able to raise market prices in order to increase their revenues by colluding. However, firms outside of the cartel also benefit from the high market price. In public good provision settings, as the consumption is non-excludable, every individual benefits from the public good even if they do not participate in its production. Externalities are positive in these settings because the larger the cartel is, or the larger the group providing the public good is, the higher is the surplus of individuals *outside* of these groups.

In case of *negative externalities* this mechanism works backwards: the larger a given coalition is, the lower is the surplus of agents outside of the coalition. Political competition is a good example of negative externalities among groups.

In the remainder of this section first I apply the coalition formation framework defined in Section 3.2 to a problem of public good provision introduced by Ray and Vohra (2001). I use public good provision games to demonstrate how the formation of coalitions can lead to inefficient outcomes in the absence of transaction costs when there are externalities among players. I start by summarizing the main findings of

Ray and Vohra (2001), then I show how the introduction of costly binding agreements affects the outcome predicted by the model and restores efficiency.

Following the public good provision application, I introduce some general results characterizing the existence of surplus-improving transaction costs in settings with positive or negative externalities.

1.5.1 Public good provision

Traditionally public goods are viewed as goods that cannot be efficiently provided by competitive markets due to the problem of free-riding. The reasoning is the following: in markets involving public goods no one can be excluded from consuming them regardless whether the consumers paid for them or not, which leads to free-riding problems. While Lindahl (1919) and Samuelson (1954) characterized the prices based on individual valuations that lead to efficient public good provision, in practice there are several problems that makes the implementation of Lindahl-Samuelson prices difficult.

One of these problems is that the agents' true valuation for the public good is private information, and agents are not willing to disclose it if they expect to be charged based on them. The economic literature usually focuses on mechanisms that are able to provide public goods efficiently, usually by proposing solutions for extracting the private information about the true valuation of the public good (see Clarke (1971) and Groves (1973) among others).

However, even if the valuations are common knowledge and the correct Lindahl-

Samuelson prices can be determined, implementing them is a completely different problem. Agents still have the incentive not to contribute and free-ride. The actual payment of the Lindahl-Samuelson prices has to be forced by a government or a binding agreement among agents that specifies the contribution levels (potentially based on the Lindahl-Samuelson prices). In the example analyzed in this section agents are able to implement efficient levels of public good provision if they form coalitions where the members enter into an agreement that specifies contribution levels maximizing the joint surplus of the coalition. Ray and Vohra (2001) shows that a coalition formation game similar to Bloch (1996) can lead to inefficient provision of public goods.

Below I summarize the model of Ray and Vohra (2001) to analyze public good provision games with no transaction costs and to illustrate the inefficiency problem[1] in this setting. Then I introduce transaction costs to this model to show how the equilibrium outcome and its properties change.

No transaction costs

Let $N = \{1, .., n\}$ be the set of players. Each player i has access to a technology to produce z_i amount of public good at $c(z_i)$ cost, which is assumed to be convex in z_i. Every unit of the public good contributes to the payoff of all players, regardless of who produced the public good. The payoff of player i is given by

$$u_i = Z - c(z_i),$$

[1]Note that Ray and Vohra (2001) define two versions of this public good provision game. Here I refer to the version they label as "restricted game".

where $Z = \sum_{i \in N} z_i$. Players can form coalitions among each other, and within coalitions they can make binding agreements such that the members of the coalition maximize the payoff of the entire coalition, not just the payoff of the given player. While the members of a coalition cooperate with each other, the cross-coalition interaction is noncooperative: players ignore the payoffs of any other player outside of their coalition. That is, a coalition S of s players solves the following maximization problem:

$$\max \sum_{i \in S} z_i - c(z_i).$$

Since $c(\cdot)$ is convex, the coalition will choose a production plan where each member produces the same quantity z_S. Therefore the maximization problem is essentially simplifies to

$$\max s \cdot z_S - c(z_S).$$

After each coalition S made its respective production decision, the next step is to calculate the payoff of each player. The payoff of player i in coalition S is given by

$$u_i = s \cdot z_S - c(z_S) + \sum_{S' \neq S} s' \cdot z_{S'},$$

where s' is the number of players in coalition S'.

The coalitions are formed as a subgame-perfect equilibrium of a bargaining game similar to the coalition formation game described in Section 3.2. There is a random order in which players not yet assigned to coalitions make offers to other players to form a coalition. The proposed players respond to the offer in a random order. If everyone accepts the offer, the coalition forms and the players leave the game. If a player refuses the offer, she becomes the next proposer. The game continues until each player is assigned to a coalition. The resulting coalitions decide about the amount of public good to be produced, and these production decisions determine the payoffs. Players are assumed to have a payoff of zero if the bargaining never ends.

When players decide about what kind of offer to propose or whether to accept or reject a particular offer, they make a rational prediction about which coalition structure the remaining players will form in later stages of the game.

To illustrate how this coalition formation game leads to an inefficient outcome, consider a case with $n = 4$ and $c(z) = \frac{z^2}{2}$. Table 1 table below lists the possible numerical coalition structures and the payoffs associated with them.

Table 1.1: Public good provision with no transaction costs

Numerical coalition structure	Payoffs of players			
(4)	8	8	8	8
(3,1)	5.5	5.5	5.5	9.5
(2,2)	6	6	6	6
(2,1,1)	4	4	5.5	5.5
(1,1,1,1)	3.5	3.5	3.5	3.5

As the Table 1 shows, the total payoff is the highest in the case when the grand coalition of all the four players forms, therefore that is the efficient outcome. However, the equilibrium of the game is the coalition structure (3,1). If the player making the first offer chooses to form a singleton coalition, the remaining players cannot achieve higher payoff than 5.5 in any possible numerical coalition structure, hence these players have no incentive to deviate from forming the three-player coalition after the first player left the game. Since this outcome gives the highest possible payoff for the first proposer, she has no incentive to deviate from this strategy either. Therefore (3,1) is an equilibrium coalition structure but it is not efficient.

Note that the player in the singleton coalition is free-riding: she produces the least amount of the public good of all players and has the highest payoff due to enjoying the benefits of the high level of provision by others.

The intuition behind this outcome is the following. The player who has the opportunity to make the first offer realizes that even if she free-rides, the remaining players cannot do better than cooperating with each other and producing a large amount of public good. By declaring that she contributes only the minimum amount, the first player "forces" the remaining players to a situation when the best they can do is to produce the highest possible amount of public good to maximize their own payoffs. However, due to the presence of externalities, this high level of public good provision benefits the free-riding player as well. Since the player in the singleton coalition bears lower cost than players in the three person coalition, the free-rider has a higher payoff than the other players.

Introducing transaction costs

This section shows how the presence of transaction costs change the outcome described above. In the previous example it was possible to enter into binding agreements within coalitions without any additional costs. Now consider a case when establishing binding agreements costs 0.3 for any non-singleton coalitions. Singleton coalitions are not subject to this transaction cost as there is no need of binding agreements in this case. Introducing this transaction cost modifies the payoffs as presented in Table 2:

Table 1.2: Public good provision with transaction costs

Numerical coalition structure	Payoffs of players			
(4)	7.925	7.925	7.925	7.925
(3,1)	5.4	5.4	5.4	9.5
(2,2)	5.85	5.85	5.85	5.85
(2,1,1)	3.85	3.85	5.5	5.5
(1,1,1,1)	3.5	3.5	3.5	3.5

Notice that in this game the unique equilibrium outcome is the grand coalition. Why is this outcome different from the frictionless game? Now if the player making the first offer decides to form a singleton coalition, the remaining players no longer have any incentive to form the three player coalition, as they had in the game without transaction costs. When the second player makes her offer, she will realize that if she also decides to form a singleton coalition she will be better off than if she is in the three person coalition provided that the last two players are going to form the two person coalition. Since the last two players are indeed better off forming the two player coalition, the player making the second offer anticipates this move and declares to form its own singleton coalition if the first player chose to do so. That is, if the first proposer forms a singleton coalition, the resulting numerical coalition structure is going to be (2,1,1) compared to the (3,1) outcome without transaction costs.

Now the first player foresees this scenario and realizes that she ensures the maximal payoff to herself when she offers everyone to form the grand coalition. Since all players are able to make the same prediction and conclude that this outcome maximizes their payoff, they accept the offer.

In the game without transaction costs the efficient outcome (the grand coalition) failed to form in equilibrium, however in the game with transaction costs the equilibrium outcome is the efficient one. Similarly to the partnership games in Section 1.4, efficiency is attained not despite but because of the presence of transaction costs.

Moreover, the combined payoff of all players is higher in the game with transaction costs. In the first game the combined payoff is 26, in the second game it is 31.7. That is, if players have the opportunity to choose between the two regimes ex ante (before the identity of the first proposer is revealed), they strictly prefer the situation when making binding agreements is costly to the one when they are completely free.

It is also important to note that a special feature of efficiency restoring-mechanism in the previous example is that transaction costs help eliminate free-riding not because the free-rider is punished by the transaction cost. The free-rider is completely unaffected by the transaction cost because the cost is only paid by the players that are not free-riding.

The intuition behind this result is that in the case without transaction costs, the free-rider is able to guarantee himself a higher payoff than she would get in the efficient outcome. The reason is that even when she declares herself to be a free-rider, the remaining players still maximize their payoff when they form the three player coalition. However, if forming the three-player coalition is less lucrative due to the transaction costs, the remaining three players will opt out of forming it, making the free-rider worse off than she would be in the grand coalition.

The existence of the transaction costs decreases the power of the potential free-rider to force the remaining players into a position where their best available option is to produce the most possible amount of public good, which also helps the free-rider. Hence the presence of transaction costs eliminates free-riding, despite that only the contributing players are directly affected by the cost. The *indirect* effect of the costs restores efficiency and improves the surplus of all agents.

1.5.2 Surplus improving transaction costs for games with externalities

Below I characterize the circumstances where there are surplus-improving transaction costs for coalition formation problems with externalities, then I discuss important differences between direct and indirect surplus-improving transaction costs and implications to situations with positive and negative externalities.

To identify the circumstances when there are surplus-increasing transaction costs for games with externalities I introduce the notion of responsible coalitions.

Definition 5. Let (V, N) be a cohesive game where N is not formed in any of the stationary SPE of $\sigma(V, N)$. Then, coalition $S \subset N$ is *responsible for not forming* N if S is formed after the first proposal in $\sigma(V, N)$.

It is easy to see that for any S responsible for not forming N

$$\frac{V(S, \pi)}{s} > \frac{V(N)}{n}$$

where π is the stationary SPE of $\sigma(V, N)$, that is, the average payoff of the responsible coalition has to be higher than the average payoff of the grand coalition. This is the only reason why the members of S are not willing to join the grand coalition which maximizes the total surplus of all players.

In order to be surplus-increasing, a vector of transaction costs t has to directly or indirectly decrease the value of any possible responsible coalition S such that in the resulting game $\sigma(V_t, N)$ with the new equilibrium outcome π_t

$$\frac{V(S, \pi_t)}{|S|} \leq \frac{V_t(N, N)}{n}.$$

Proposition 6 characterizes necessary and sufficient conditions for existence of surplus-improving transaction costs.

Proposition 6. *Let (V, N) be a cohesive partition function game where N is not the outcome in any stationary SPE of $\sigma(V, N)$. Then, there exists a surplus-improving transaction cost vector t if and only if for all potential responsible coalition S and for all π_S that is an outcome of a stationary SPE in the game $\sigma(V, N)$ conditional on S is formed after the first offer, we have*

$$\frac{V(S, \pi_t) - t_{|S|}}{|S|} \leq \frac{V(N, N) - t_n}{n} \tag{1.2}$$

for all π_t outcome of $\sigma(V_t, N)$ conditional on S is formed after the first offer and

$$V(N, N) - t_n \geq \sum_{T \in \pi_S} V(T, \pi_S). \tag{1.3}$$

Proposition 6 is proven in Appendix A.3.

The nature of surplus-increasing transaction costs can be quite different in games with externalities compared to the case with no external effects. Proposition 5 characterizes the cases when it is possible to achieve the condition above in the case of games without externalities. In those situations the transaction cost t is able to restore N by imposing a high enough cost on the responsible S such that in the game with transaction costs the average payoff in S is no longer above the average payoff of N. This is a *direct* method of restoring N as an outcome using transaction costs.

This method of restoring efficiency does not always work in games with externalities. In Section 1.5.1 the coalition responsible for not forming the efficient outcome was a singleton. Since singleton coalitions are never subject to transaction costs, it is impossible to restore the grand coalition directly using transaction costs.

However, free-riding was only profitable if the numerical coalition structure $\pi = (3,1)$ is formed in equilibrium. If the outcome is $\pi_t = (2,1,1)$ for some transaction cost vector t, then the free-rider no longer has incentive to stay out of the grand coalition. The transaction cost vector $t = (0, 0.3, 0.3, 0.3)$ is able to restore N as the outcome of the coalition formation game *indirectly*, without imposing any costs on the responsible free-rider by "breaking up" the three-player coalition and hence decreasing the payoff of the free-rider.

In summary, in games with externalities transaction costs can restore efficiency directly by imposing costs on coalitions that are not willing to join the grand coalition or indirectly through external effects by changing the reaction of players outside the deviating coalition.

There are two important differences between direct and indirect surplus-improving transaction costs. First, the existence of direct surplus-improving transaction costs requires that every potential responsible coalition has at least two members. Indirect costs do not impose this restriction. The second difference is that direct costs do not rely on changing the coalition structure outside of the responsible coalition S, while indirect transaction costs do. Changing the coalition structure outside of S is not always possible: for example if $|N \setminus S| = 1$, the coalition structure outside of S is impossible to change.

In games without externalities it is impossible to have indirect surplus-increasing transaction costs due to the lack of external effects among coalitions. In the general case it is uncertain what are the conditions that determine whether direct or indirect transaction costs will be able to restore efficiency in a given game. However, for games with positive or negative externalities there are some rules that help determine which type of transaction costs we have to look for. In the rest of this section I analyze games with specific types of externalities.

Definition 6 of games with positive externalities captures the idea that for every coalition S, if players outside of S organize themselves into bigger coalitions, the value of S increases. The public good example presented above has this property.

Definition 6. A game (V, N) is a game with positive externalities if for all $\pi, \pi' \in \Pi(N)$ and all $S \in \pi$,

$$V(S, \pi) \leq V(S, \pi')$$

when $\pi' = (\pi \setminus (S_i, S_j), S_i \cup S_j)$, $S_i, S_j \neq S$.

As we have seen in Section 1.5.1, if there are positive externalities, then it is possible to have a singleton coalition to be responsible for not forming the efficient grand coalition. As singleton coalitions are never subject to transaction costs, there are no surplus-improving direct transaction costs.

On the other hand, it is generally simple to have indirect surplus-improving transaction costs: if the coalition structure has smaller coalitions, then the values of coalitions (including the one responsible for not forming the efficient outcome) are lower.

That is, the indirect transaction cost has to break up coalitions outside of the deviating one to suppress the incentives to deviate. Breaking up coalitions by adding transaction costs to them is simple. Hence in situations with positive externalities indirect transaction costs are more likely to succeed in restoring efficiency and increasing the surplus of all players.

Contrary to the case with positive externalities, the presence of negative externalities implies that the value of S will be lower if players outside of S are in bigger coalitions.

Definition 7. A game (V, N) is a game with negative externalities if for all $\pi, \pi' \in \Pi(N)$ and all $S \in \pi$,

$$V(S, \pi) \geq V(S, \pi')$$

when $\pi' = (\pi \setminus (S_i, S_j), S_i \cup S_j)$, $S_i, S_j \neq S$.

As discussed above, with positive externalities it is not always possible to have direct suprlus-improving transaction costs as there are situations where singleton coalitions are deviating from the grand coalition. This is not the case with negative externalities. In Proposition 7 I show that in a game with negative externalities no singleton coalition can be responsible for not forming N.

Proposition 7. *Let (V, N) be a cohesive symmetric game with negative externalities where N is never the outcome in a stationary SPE of $\sigma(V, N)$. Then, no coalition S with $|S| = 1$ can be responsible.*

Proof. Assume that in a stationary SPE of $\sigma(V, N)$ with outcome π, the coalition responsible for not forming N is S with $|S| = 1$. Then,

$$V(S, \pi) > \frac{V(N, N)}{n}.$$

Now consider the partition π' of n singleton coalitions. Due to negative externalities,

$$V(S, \pi) \leq V(S, \pi').$$

This implies

$$n \cdot V(S, \pi') > V(N, N),$$

which contradicts the cohesiveness of V. □

Proposition 7 shows that games with negative externalities share the feature that no singleton coalition can achieve higher payoff than its payoff in the grand coalition with games without externalities (in that case this feature is a simple consequence of superadditivity). For this reason, it is always possible to have transaction costs that directly restore the formation of the grand coalition and the condition stated in Proposition 5 for characteristic function games is sufficient for the existence of surplus-improving transaction costs in games with negative externalities.

However, in the case of negative externalities the condition in Proposition 5 is no longer a necessary condition for the existence of surplus-improving transaction costs. Due to externalities, there is another way of restoring the efficient outcome besides imposing a transaction cost on the "deviating" coalition such that the average payoff will be lower than in the grand coalition. The presence of negative externalities implies that a given coalition's payoff decreases when larger coalitions form outside of that coalition. As shown in Example 1, the existence of transaction costs can result in formation of larger coalitions. That is, if there is a coalition S responsible for not forming the grand coalition, and the set $N \setminus S$ of the remaining players is not organized into a single coalition, then it is possible to have indirect surplus-improving transaction costs. However if $N \setminus S$ forms a single coalition, then unlike in the case with positive externalities, indirect transaction costs cannot work.

Generally, contrary to the case with positive externalities, in games with negative externalities it is more likely to have a direct surplus increasing transaction costs than an indirect one.

1.6 Robustness of results

In the previous sections I analyzed coalition formation games using an alternating offers bargaining framework defined in Section 3.2. One of the important assumptions of this model is that binding agreements are irreversible, and once a set of players forms a coalition they leave the game and other players cannot propose them to form another coalition. In this section I discuss how the choice of the coalition formation model and the irreversibility assumption influences the results presented in my paper.

1.6.1 Alternative models of coalition formation

The characterization of situations with surplus-improving transaction costs in Section 1.4 and 1.5 does not depend on the sequential structure of the bargaining game described in Definition 2.

It is possible to replace the coalition formation model with one of the "blocking" approach. This direction of the coalition formation research does not use dynamic bargaining models to predict what coalitions arise in equilibrium. Instead, the blocking approach focuses on stable coalition structures where there are no profitable deviations by any group of players. Due to externalities, when considering a deviation it is crucial what reaction they expect from the rest of the players. There are several different models in the literature that use the blocking approach, differing in their assumptions about the reactions to deviations.

In the recursive core of Kóczy (2007), the reactions to deviations are "rational" in a sense that when players decide about what coalitions to form in response to a deviation, they choose outcomes where their payoffs are maximized. If the sequential bargaining model replaced with the recursive core, all of the previous results would still hold.[2] Overall, the sequential structure of the model is not important, what is crucial is the rationality - best response property - of the reactions.

[2] This is not surprising, see Kóczy (2009) for relations between the recursive core and Bloch (1996).

1.6.2 Reversible agreements

Agreements in this paper are assumed to be irreversible, that is, once a set players agrees to be in a coalition S, they can no longer receive offers to form another coalition and cannot be selected as proposers.

The model described in Section 3.2 is not suitable to deal with situations with reversible agreements. In this case the feature that no payoff is received by any player before the negotiation process ends can easily lead to agents bargaining forever in equilibrium and never reaching an agreement.

However, my model is designed to analyze the formation of long-term, thus irreversible agreements. Reversible, temporary agreements need a different theoretical framework. It is an interesting question whether the phenomenon of surplus-increasing transaction costs is unique to long-term agreements.

One direction of the coalition formation research analyzes temporary agreements, see for example Ray and Vohra (1999), Diamantoudi and Xue (2007) or Hyndman and Ray (2007). While the complete characterization of situations with surplus-increasing transaction costs in the case of temporary agreements is outside of the scope of this paper, here I present an example of a coalition formation game with temporary agreements by Ray and Vohra (2015). This game yields inefficiency in equilibrium and I show how the presence of transaction cost helps restore efficiency while all players are better off ex ante.

Example 2.

Consider the following version of the public good game presented in Section 1.5.1.

Table 1.3: Reversible agreements with no transaction costs

Numerical coalition structure	Payoffs of players		
(3)	12	12	12
(2,1)	7	7	19
(1,1,1)	6	6	6

Ray and Vohra (2015) shows that in the equilibrium outcome of this game players will cycle between the numerical coalition structures (3), (2,1) and (1,1,1) and the average payoff of the players will be 9.67. Now consider a transaction cost of 2.4 applied to any non-singleton coalition. The payoffs in the game with transaction costs are given by Table 4.

Table 1.4: Reversible agreements with transaction costs

Numerical coalition structure	Payoffs of players		
(3)	11.2	11.2	11.2
(2,1)	5.8	5.8	19
(1,1,1)	6	6	6

Now similarly to the example in Section 1.5.1, the unique equilibrium outcome of the coalition formation game is (3). The coalition structure (2,1) never forms, not even temporarily. The reason is that if one player announces that she forms a singleton coalition, the remaining players also chose to form singleton coalitions to maximize their own payoffs. Therefore no player wants to form any other coalition outside of the grand coalition and this outcome stays stable in every period. The average payoff for each player is 11.2, which is higher than the 9.67 from the game without transaction costs.

Based on the analysis of this section it can be concluded that the existence of a surplus-increasing transaction cost is quite a robust phenomenon and is not simply the product of the specific modeling choices made in this paper.

1.7 Conclusion

In this paper I analyzed coalition formation games with the presumption that binding agreements are costly. I used a simple model of coalition formation to show that despite the common intuition, the costly nature of binding agreement can be beneficial. I demonstrated this argument in two different settings.

First I considered simple superadditive games without externalities among coalitions. In these games there are always gains from cooperation so the efficient outcome is the grand coalition, however, for a given range of payoffs all equilibria of the coalition formation game is different from the efficient one. I showed that in some cases there is a range of strictly positive transaction costs that helps players reach the efficient outcome and the combined payoff of players is higher than in a game without transaction costs. Proposition 5 characterizes the conditions for situations where the efficient outcome is not reached without transaction costs due to a subset of players seek to maximize their own payoff at the expense of the players outside of their subset, however adding a strictly positive transaction cost is able to suppress the incentives leading to the inefficient outcome and restores efficiency.

Next I analyzed situations with externalities among coalitions. Similarly to the case without externalities, bargaining without transaction cost can lead to inefficient equilibria, while adding transaction cost can help restoring efficiency. In this setting, the intuition behind this result is different than it was for games with no externalities. Inefficiency in public good provision arises when potential free-riders can abuse the situation that even after they declared that they free-ride, other players are forced to contribute the highest amount to public good production in order to maximize their own payoff, substantially increasing the free-riders' payoff as a side effect. The presence of transaction costs makes contribution harder for non-free-riders, therefore takes away some power from the free riders to "force" the rest of the players into a situation that benefits free-riders the most. Surprisingly, positive transaction costs can reduce the potential gains from free-riding despite that free-riders themselves are not subject to transaction costs.

Finally I investigated the consequences of using alternative coalition formation models or dropping the assumption of irreversible agreements. I concluded that the phenomenon of surplus-increasing transaction costs is robust to these modeling choices.

The most important implication of the results concerns the approach towards transaction costs in general. The belief of harmful nature of transaction costs is widespread in Economics, therefore most economists encourage policies reducing transaction costs. However, the results of this paper suggest that an environment with lower transaction costs is not necessarily desirable, as it can lead to social welfare loss when the formation of small groups is a potential issue. Therefore, when policy-makers decide about policies targeted at the reduction of transaction costs, a more careful approach is necessary and the industry structures should be taken into account.

The indirect surplus-improving effect of transaction costs has another interesting policy implication. The fact that the increased transaction cost affecting the *contributors* are able to stop *free-riders* and restore the efficient outcome suggests that it is possible to regulate markets *indirectly* if there are external effects among firms, organizations or other groups of agents, . Consider an undesirable situation on a given market - such as extreme market power or a plant causing heavy pollution - but it is prohibitively costly to mitigate it via direct regulation due to technical or legal constraints. If the policy-maker succesfully identifies that this situation is the result of the interaction of agents in a given market and manages to change the environment in a way that under the new circumstances a more desirable outcome emerges as a result of interaction between agents, it is possible to regulate the situation while avoiding the prohibitive costs of direct intervention.

There is plenty of room for future research on the topic of this paper. Besides investigating the effects of transaction costs under different coalition formation models, another interesting extension of the model is a version when the transaction costs are endogeneous. For example, there is a set of players - lawyers, professional managers or supervisors - that provide binding agreements and the price is determined by the supply and demand of these services. Studying coalition formation with endogeneous agreement costs would lead to a better understanding of situations where government interventions targeted at the reduction of transaction costs are justified.

Chapter 2

Games in Partition Function Form with Restricted Cooperation

2.1 Introduction

In Industrial Organization it is well known that the profit of a given firm depends on the structure of the industry in which the firm operates. Large firms have market power and are able to influence the price. Increases in the market power of a single firm affect the entire industry, not just that given firm. In addition, due to restrictions placed by competition authorities, not every market structure is feasible, the firms in a given industry cannot merge arbitrarily with each other.

Similarly, in political competition the success of a given coalition depends not only on how large and powerful that coalition, but also the size and power of the rivaling coalitions and political parties cannot choose their coalition partners freely due to ideological differences.

In both of these situations externalities across groups are crucial: firms' profits depend on the entire market structure constituted by other firms and the success of a given coalition in political competition depends on its rivals' power. Furthermore, in both market and political competition it is not possible for players to form groups without restrictions.

In the cooperative game theory literature there are models that are able to deal with externalities across coalitions, and models that focus on restricted cooperation. However, up to my knowledge there is no cooperative game model that is able to express both of these features. In this paper I define a class of cooperative games with both externalities across coalitions and restricted cooperation and generalize the recursive core of Kóczy (2007) to this class of games.

The reason why I choose the recursive core from the several solution concepts for partition function games is because that solution concept is the most fitting to the applications I have in mind for my model. Solution concepts of PFF games are looking for coalition structures that are stable against deviations. Due to the externalities across coalitions, reactions to deviations are crucial in determining what coalition structures are stable. Different solution concepts in the literature make different assumptions about these reactions.

The recursive core assumes that when reacting to deviations players will choose their actions to maximize their own payoff given the situation. In other solution concepts, for example the α-core, the players are not concerned with their own payoffs when choosing their deviations, they react in a way that hurts the deviators the most. In the case of mergers and political competition it is natural to assume that players are primarily concerned with their own surplus when choosing their deviations, not with the surplus of the deviators. Therefore I choose the recursive core as the solution concept for the class of cooperative games I introduce in this paper. As the construction of the recursive core in Kóczy (2007) relies heavily on the feature that every coalition is allowed to form, I generalize the concept to situations with restricted cooperation.

After defining the restricted recursive core I present a possible empirical application of our model using the merger simulation method of Nevo (2000a) to determine the stable industry structures for and predict expected mergers on markets.

The paper is organized as follows. In Section 2.3 I overview the baseline cooperative game model and games in partition function form (PFF), which are able to express the externalities across coalitions. Then I introduce restricted cooperation in Section 2.4 where I define the $\Lambda(N)$-restricted recursive core and discuss some of its properties. Section 2.5 demonstrates how to apply the concept of restricted recursive core to predict stable industry structures after mergers.

2.2 Related literature

The model in this paper is built to deal with situations whith both externalities and restricted cooperation. The standard model of cooperative game theory, the game with transferable utility (TU game) is not able to express any externalities between players. However, games in partition function form (PFF games) first introduced by Thrall and Lucas (1963) are able to do that. There are various solution concepts in the literature for PFF games, see for example Aumann and Peleg (1960); Myerson (1977b); Shenoy (1979); Huang and Sjöström (2003) and Kóczy (2007) among others. In contrast to the TU games, regarding the PFF games one of the important questions is what partition will be formed by the players. In other words, the PFF games often focus on the problem of *coalition formation*.

There is also an important non-cooperative approach in the study of the formation of coalitions in the presence of externalities, important contributions include Bloch (1996); Ray and Vohra (1999) and Yi (1997). Another part of the literature studies the relationship between the cooperative and non-cooperative solution concepts, for example Kóczy (2009) and Huang and Sjöström (2006).

Focusing on the cooperative approach I highlight the recursive core of Kóczy (2007) among the several solution concepts. It has a "central position" in the literature being a refinement between a somewhat weak and an extremely strong stability concepts, the α-core (Aumann and Peleg, 1960) and the ω-core (Shenoy, 1979). The stability concept of α-core assumes that residual players hurt deviators as much as they can, while the concept ω-core assumes that residuals will help deviators as much as possible. Since these concepts assume that residuals make efforts to hurting (or helping) the deviators even if they hurt themselves as well. This assumption is counterintuitive, and the concept of recursive core assumes that residuals make only "credible" reactions to every possible deviation. Another attractive purpose of the recursive core, as Kóczy (2009) shows, is that its pessimistic (or large, according to the term we use) version of the recursive core coincides with the set order-independent sequential equilibrium coalition structures of Bloch (1996). In this paper I generalize the recursive core of Kóczy (2007) to the newly introduced PFF games with restricted cooperation.

In practical situations it is often the case that some coalitions cannot form due to e.g. communication problems, hierarchical relations among the players, or institutional restrictions (for example by competition authorities regarding mergers). For TU games, restricted cooperation is extensively discussed by the literature. Aumann and Dreze (1974) study the implications of restricted cooperation on various, widely used solution concepts of TU games. Other authors (Myerson, 1977a; Owen, 1986; Derks and Peters, 1993) propose different frameworks for restricted cooperation.

Owen (1986) and Myerson (1977a) use graph structures to define the set of permitted coalitions. They consider a graph where the nodes are the players, and coalition S is permitted if all players in S are connected in the graph. Derks and Peters (1993) use a different approach. They introduce a restriction function, a monotonic projection of 2^N to 2^N which assigns a subcoalition to each coalition. There is another part of the literature which studies hierarchical permission structures where some agents are not free to cooperate without the presence of other agents; see Gilles, Owen, and Brink (1992); Brink (1997) among others.

Another direction of research regarding restricted cooperation assumes a certain structure of the feasible coalitions. This structure can be convex geometries (see e.g. Bilbao and Losada (1998)), antimatroids (see e.g. Algaba, Bilbao, Brink, and Losada (2004)), union stable systems, also called weakly union-closed systems (see e.g. Algaba, Bilbao, Borm, and López (2000); Faigle, Grabisch, and Heyne (2010); Faigle and Grabisch (2011)), matroids (see e.g. Bilbao, Driessen, Losada, and Lebrón (2001)), regular set systems (see e.g. Honda and Grabisch (2008) and Lange and Grabisch (2009)), distributive lattices (see e.g. Grabisch and Xie (2011); Grabisch (2011)), various ordered structures (see e.g. Grabisch (2009), and symmetric ordered structures (see e.g. Grabisch (2004)) among others.

2.3 TU games and PFF games

In this section I overview the model of TU games, which is the baseline model of cooperative games. After that I consider the games in partition function form.

2.3.1 TU games

First of all I show the simplest, and in previous works the most frequently used cooperative game theory model, the coalitional TU game (shortly TU game, see for instance

Peleg and Sudhölter (2003)).

Definition 8. Let N be a non-empty finite set of the players, the cooperative transferable utility game (henceforth TU game) is the function $v : 2^N \to \mathbb{R}$ such that $v(\emptyset) = 0$. Moreover, let $\mathcal{G}^N$ be the class of TU games with players set N.

In cooperative games players form *coalitions*: that is, coalitions are subsets of N, therefore 2^N denotes the class of the possible coalitions. A TU game is given by the coalitional function (also called characteristic function) v, where v assigns a real number – the value of the coalition – to each coalition. The fact that the formation of any coalition is allowed must be emphasized (in other words, any elements of 2^N can arise as a coalition).

The most frequently used solution concept of TU games is the *core* (Gillies, 1959). The *core* is a set of *imputations*, which are payoff vectors, such that all players get at least as they could achieve by themselves alone; and the sum of all player's payoff is equal to the value of the grand coalition. Formally:

Definition 9. Let $v \in \mathcal{G}^N$, then the set of imputations of v is defined as

$$I(v) = \{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N) \text{ and } x_i \geq v(\{i\}),\ i \in N\}$$

.

The core is the set of imputations, where for all coalitions the members of the given coalition get at least as much as the value of the coalition.

Definition 10. Let $v \in \mathcal{G}^N$, then the core of v is defined as

$$C(v) = \{x \in I(v) : \sum_{i \in S} x_i \geq v(S), \text{ for all } S \subseteq N\}$$

.

2.3.2 PFF games and the recursive core

Games in partition function form differ from TU games in that they can model externalities between players. So in the case of PFF games it is possible, that a certain coalition can have different depending on the realized partition (in effect, depending on the coalitional structure formed by other players).

We adopt the definition of Myerson (1977b) which uses the set of embedded coalitions (ECL) to define the PFF game.

Definition 11. Let ECL_N be defined as follows:

$$ECL_N = \{(S, \pi) : S \in \pi \in \Pi(N)\}.$$

Then, PFF game V with players set N is a point in $\mathbb{R}^{ECL_N}$, that is, V is a function from ECL_N to $\mathbb{R}$.

Let Γ^N denote the class of partition function games with player set N.

Note, that this definition differs from the definition in Kóczy (2009). The discrete partition function in Kóczy (2009) exactly determines the payoff of each player in a given partition, while the definition above only determines the payoff of each coalition in a given partition. Example 3 demonstrates how the partition function is able to express externalities between coalitions.

Example 3.

Let $N = \{1, 2, 3\}$ be the set of players, and for simplicity consider only the partitions $\pi_1 = (\{1\}, \{2\}, \{3\})$ and $\pi_2 = (\{1\}, \{2, 3\})$.

Look first at TU game v, $v(\{1\}) = 2$, $v(\{2\}) = 3$, $v(\{3\}) = 4$, $v(\{2, 3\}) = 8$. Since v is defined on the coalitions only, $v(\{1\}) = 2$ regardless of whether the other two players are in one coalition or not. Therefore TU games cannot express external effects.

Consider now the PFF game V. Now the realization of π_1 yields $V(\{1\}, \pi_1) = 2$, $V(\{2\}, \pi_1) = 3$, $V(\{3\}, \pi_1) = 4$, and the realization of π_2 yields $V(\{1\}, \pi_2) = 1$, $V(\{2, 3\},\ \pi_2) = 8$. So in the case of PFF game the payoff of players who create a one-player coalition depend on whether player 2 and player 3 coalesce or not; so the PFF game can model externalities.

Notice, that since in TU games only the payoffs are relevant, in PFF games besides the payoffs the realized partitions are also important. Therefore in the definition below both dimensions are expressed.

Definition 12. Let $V \in \Gamma^N$ be PFF game, then the set of outcomes of PFF game V is as follows:

$$\Omega(V) = \left\{ (\pi, x) \in (\Pi(N), \mathbb{R}^N) : \forall S \in \pi : \sum_{i \in S} x_i = V(S, \pi) \right\}.$$

The definition of residual game helps to understand the recursive core. Here we define the situation of players who stay in the game.

Definition 13. Let $V \in \Gamma^N$ be a PFF game and consider a set of players $R \subset N$. Assume that $N \setminus R$ have committed to form partition $\pi_{N\setminus R} \in \Pi(N \setminus R)$. Then the residual game $V_{\pi_{N\setminus R}}$ is the PFF game over player set R, with partition function $V_{\pi_{N\setminus R}}$ such that for all $\pi_R \in \Pi(R)$, $S \in \pi_R$:

$$V_{\pi_{N\setminus R}}(S, \pi_R) = V(S, \pi_R \cup \pi_{N\setminus R}) \ .$$

In the following part I present the small (large) recursive core. This definition differs from the one in Kóczy (2009) in several points. First, there are no fixed payoffs for the players in each given partition, since the payoffs are assigned to the coalitions. Second, in this model I use the same approach as in Definition 10, instead of defining the core by undominated outcomes. Finally I use the terms *small* and *large* instead of *optimistic* and *pessimistic* recursive core, because I think that these expressions are more fitting (the optimistic recursive core is smaller than the pessimistic one).

Definition 14. Small (large) recursive core (RC).

1. Trivial game: The recursive core of $V \in \Gamma^{\{i\}}$ is the only outcome with the trivial partition:

$$RC(V) = (\{i\}, V(\{i\}, \{\{i\}\}))\ .$$

2. Induction assumption: Assume that RC has been defined for all games with at most k players. Then for each PFF game $V \in \Gamma^R$, $|R| \leq k$, let

$$A(V) = \begin{cases} RC(V) & \text{if } RC(V) \neq \emptyset, \\ \Omega(V) & \text{otherwise,} \end{cases}$$

   where $\Omega(V)$ is the set of all possible outcomes of V.

3. The small (large) recursive core: The small (large) core of PFF game $V \in \Gamma^N$, $|N| = k + 1$ is as follows:

$$RC_S(V) = \Big\{(\pi, x) \in \Omega(V) : \forall\ R \subseteq N,\ \forall \pi'_{N\setminus R} \in \Pi(N \setminus R),$$
$$(\pi'_{N\setminus R}, x'_{N\setminus R}) \in A(V_{\pi'_R}), \sum_{i\in R} x_i \geq \sum_{S\in\pi'_R} V(S, \pi'_R \cup \pi'_{N\setminus R})\Big\},$$

$$\Big(RC_L(V) = \Big\{(\pi, x) \in \Omega(V) : \forall R \subseteq N,\ \exists \pi'_{N\setminus R} \in \Pi(N \setminus R),$$
$$(\pi'_{N\setminus R}, x'_{N\setminus R}) \in A(V_{\pi'_R}), \forall \pi'_R \in \Pi(R),$$
$$\sum_{i\in R} x_i \geq \sum_{S\in\pi'_R} V(S, \pi'_R \cup \pi'_{N\setminus R})\Big\}\Big).$$

The idea behind the definition is that the recursive core consists of outcomes that are stable in some sense. An outcome is stable if there is no subset of players which can get higher payoffs by deviating from the outcome. (Note that in my model the payoffs are assigned to coalitions not to players, however if a coalition gets higher payoff, then its members can divide the surplus in a way that each of them gets more than before.) Since there are externalities between coalitions, when some players want to deviate from a given partition, they have to take into account the possible reaction by others.

When reacting to the action of the deviating players the rest of the players face a residual game according to Definition 13. This residual game is also a PFF game with fewer players. These residual players will form a stable partition in their own residual game while the partition of the players outside of the residual game is fixed. This stable outcome will be the recursive core of this residual game (or any outcome, if the residual core is empty).To check whether an outcome is in the recursive core or not, we have to consider the possible deviations. For a given deviation we have to consider the reaction of the residual players which also will be an element of the recursive core of the residual game. To check whether an outcome of the smaller residual game is stable, we have to check the possible deviations, and so on. Therefore the definition above starts with defining the recursive core of the *trivial game* (one player game), and builds recursively from that outcome.

Now I discuss the difference between the small and large recursive core. The small recursive core consists the outcomes that are extremely stable in a sense that for any deviations, *whatever credible outcome of the residual game* realizes (credible outcomes are elements of $A(V)$), the deviators cannot be better off. Contrarily, the large recursive core is less restrictive. An outcome is an element of the large recursive core, if for any deviations there *exists a credible outcome of the residual game* for which the deviators cannot be better off.

2.4 Restricted cooperation

In this section I introduce the notion of restricted cooperation: I relax the assumption that every subset of 2^N can form as a coalition. First we define our concept of PFF games with restricted cooperation then generalize the recursive core to this class of games and I discuss the properties of the newly defined $\Lambda(N)$-restricted recursive core.

2.4.1 PFF games with restricted cooperation

As I mentioned in the introduction, the concept of restricted cooperation in TU games has a developed literature. However, I have to use a new concept of restricted cooperation to be consistent with applications. For example, consider the situation when there are three players, and we allow all coalitions to form except the grand coalition. Myerson (1977a)'s graph structures cannot describe such a situation: each player should be connected by the graph since we allow all possible two-member coalitions, but if all players are connected by the graph then the grand coalition also must be allowed. In the case described above I cannot use the restriction function of Derks and

Peters (1993) either, since we cannot assign a definite subcoalition to the infeasible grand coalition.

In addition, the set of feasible coalitions must depend on the given partition. Consider the following Industrial Organization example. Suppose there are four firms, $N = \{A, B, C, D\}$. The competition authority approves the merger of firms A and B if firms C and D do not merge, and also permits the merger of firms C and D if firms A and B do not merge. The two mergers jointly are prohibited. In this case the coalition $S = \{A, B\}$ is feasible if C and D are not in one coalition and is not permitted otherwise. It is clear that the definitions of restricted cooperation in TU games listed above are not able to model these kind of feasible structures as in TU games when we look at a given coalition, it does not matter wat coalitions the other players form.

Therefore, in Definition 15 of restricted cooperation in PFF games the restriction is on the set of *partitions*, not the set of coalitions. It is a more general method to impose restrictions on the coalitional structure (for example, listing allowed partitions with coalitions that can be described with a graph structure discussed above, we exactly get the concept of Myerson (1977a)).

Definition 15. Let $\Lambda(N) \subseteq \Pi(N)$ denote the set of all feasible partitions, where $\Lambda(N) \neq \emptyset$. We call $\Lambda(N)$ as a *restriction* of $\Pi(N)$.

Given this concept of restricted cooperation, I generalize the Definition 11 and define the $\Lambda(N)$-restricted PFF games.

Definition 16. Let $ECL_{\Lambda(N)}$ be defined as follows:

$$ECL_{\Lambda(N)} = \{(S, \lambda) : S \in \lambda \in \Lambda(N)\}.$$

Then, PFF game with restricted cooperation $V^{\Lambda}(N)$ with player set N and set of feasible partitions $\Lambda(N)$ is a point in $\mathbb{R}^{ECL_{\Lambda(N)}}$, that is, $V^{\Lambda}(N)$ is a function from $ECL_{\Lambda(N)}$ to $\mathbb{R}$.

Let $\Gamma^{\Lambda(N)}$ denote the class of partition function games with player set N and set of feasible partitions $\Lambda(N)$ (sometimes I also refer these games as $\Lambda(N)$-restricted PFF games hereafter).

It is easy to see that if $\Lambda(N) = \Pi(N)$, then we get exactly our definition of PFF games, so Definition 16 is indeed a generalization. In the rest of the paper if it does not make confusion I simply refer to $V^{\Lambda}(N)$ as V^{Λ} without mentioning the player set.

Similarly to the PFF games, the outcomes of the $\Lambda(N)$-restricted PFF games are also pairs consisting of a partition and a payoff vector with feasible payoffs.

Definition 17. Let $V^{\Lambda(N)} \in \Gamma^{\Lambda(N)}$ be PPF game with restricted cooperation, then the set of outcomes of PFF game with restricted cooperation $V^{\Lambda}(N)$ is as follows:

$$\Omega(V^{\Lambda}(N)) = \left\{(\lambda, x) \in (\Lambda(N), \mathbb{R}^N) : \forall S \in \lambda : \sum_{i \in S} x_i = V(S, \lambda)\right\}.$$

2.4.2 The $\Lambda(N)$-restricted recursive core

In the previous section I introduced the $\Lambda(N)$-restricted PFF games, and in this section I define the solution concept $\Lambda(N)$-restricted recursive core for this class of games. As in the previous chapter, first I define the residual game which is crucial in the definition of the recursive core.

Definition 18. Let $V^{\Lambda} \in \Gamma^{\Lambda(N)}$ be a PFF game with $\Lambda(N)$-restricted cooperation and consider coalition $R \subset N$ such that there exists $\lambda \in \Lambda(N)$, $\lambda = \pi_R \cup \pi_{N\backslash R}$, where $\pi_R \in \Pi(R)$ and $\pi_{N\backslash R} \in \Pi(N\backslash R)$. Assume that $N\backslash R$ have committed to form partition $\pi_{N\backslash R} \in \Pi(N \setminus R)$ such that there exists $\lambda \in \Lambda(N)$, $\pi_{N\backslash R} \subseteq \lambda$. Then the residual game $V^{\Lambda}_{\pi_{N\backslash R}}(R)$ is the PFF game with $\Lambda(R)$-restricted cooperation over player set R such that for all $\pi_R \in \Lambda(R)$, $S \in \pi_R$:

$$V^{\Lambda}_{\pi_{N\backslash R}}(R)(S, \pi_R) = V^{\Lambda}(N)(S, \pi_R \cup \pi_{N\backslash R}) \ ,$$

where $\Lambda(R) = \{\pi \in \Pi(R) : \exists\lambda \in \Lambda(N),\ \pi_R \cup \pi_{N\backslash R} = \lambda\}$.

This definition is a modified version of Definition 13. It is important to note that the set of the "leaving" players $(N \setminus R)$ is not an arbitrary set of players: there exists a permitted partition consisting of partitions of R and $N \setminus R$. In addition, the partition formed by the leaving players must be a subset of a partition of $\Lambda(N)$. Furthermore, the partition of the residual players (R) must be such a partition that is together with the partition of the leaving players constitute a feasible partition. These restrictions ensure that all of the resulting coalition structures are in the set of feasible coalitions.

Definition 19 below defines the small and the large $\Lambda(N)$-restricted recursive cores. The setup of the following definition is similar to Definition 14, but in the case of restricted cooperation we cannot start from the one player games as trivial games, since it is possible that not all single-player coalitions are permitted. The definition of the residual core (Definition 14) is based on the recursion on the number of the players, but in the restricted cooperation case we cannot do so. The key of the definition below is that instead of the number of players, the recursion is on the number of feasible coalitions. Now even a game with three players can be a trivial game if there is only one feasible partition of the three players. It is important to emphasize that as in Definition 18, all partitions in the definition below are feasile partitions.

Definition 19. The small (large) $\Lambda(N)$-restricted recursive core (RC^{Λ}).

1. Trivial game: The $\Lambda(N)$-restricted recursive core of $V^{\Lambda}_{\pi_{N\setminus R}}(R)$ such that $|\Lambda(R)| = 1$ is the only outcome with the trivial partition and with any x satisfying

$$\sum_{i \in S} x_i = V(S, \lambda), \qquad S \in \lambda \in \Lambda(R) \ .$$

2. Induction assumption: Assume that RC^{Λ} has been defined for all games with number of allowed partitions at most k . Then for each PFF game $V^{\Lambda}_{\pi_{N\setminus R}}(R)$, $|\Lambda(R)| \leq k$, let

$$A^{\Lambda}(V^{\Lambda}_{\pi_{N\setminus R}}(R)) = \begin{cases} RC^{\Lambda}(V^{\Lambda}_{\pi_{N\setminus R}}(R)), & \text{if } RC^{\Lambda}(V^{\Lambda}_{\pi_{N\setminus R}}(R)) \neq \emptyset \\ \Omega(V^{\Lambda}_{\pi_{N\setminus R}}(R)) & \text{otherwise} \end{cases},$$

3. The small (large) recursive core: The small (large) $\Lambda(N)$-restricted recursive core of game $V^{\Lambda(N)} \in \Gamma^{\Lambda(N)}$, $|\Lambda(N)| = k+1$ is as follows:

$$RC^{\Lambda}_{S}(V^{\Lambda}) = \Big\{(\lambda, x) \in \Omega(V^{\Lambda}) : \forall R \subseteq N,\ \forall \pi'_{N\setminus R} \in \Lambda(N \setminus R),$$
$$(\pi'_{N\setminus R}, x'_{N\setminus R}) \in A(V^{\Lambda}_{\pi'_R}(N \setminus R)),\ \sum_{i\in R} x_i \geq \sum_{S\in\pi'_R} V^{\Lambda}(S, \pi'_R \cup \pi'_{N\setminus R})\Big\}, \tag{2.1}$$

$$\Big(RC^{\Lambda}_{L}(V^{\Lambda}) = \Big\{(\lambda, x) \in \Omega(V^{\Lambda}) : \forall R \subseteq N,\ \exists \pi'_{N\setminus R} \in \Lambda(N \setminus R),$$
$$(\pi'_{N\setminus R}, x'_{N\setminus R}) \in A(V^{\Lambda}_{\pi'_R}(N \setminus R)),\ \forall \pi'_R \in \Pi(R), \tag{2.2}$$
$$\sum_{i\in R} x_i \geq \sum_{S\in\pi'_R} V^{\Lambda}(S, \pi'_R \cup \pi'_{N\setminus R})\Big\}\Big).$$

Again, if all partitions are feasible – $\Pi(N) = \Lambda(N)$ – we get Definition 14. Moreover, if all partitions are feasible, then the games with $|\Lambda(R)| = 1$ are exactly the games with $|R| = 1$. The recursive structure and the concepts of small and large recursive core are the same as in Definition 14.

It is important to note that it is possible that the restricted recursive core is empty for some games and set of feasible partitions. Proposition 10 provides an example of an empty recursive core.

2.4.3 Properties of the $\Lambda(N)$-restricted recursive core

In this subsection I discuss some of the properties of the $\Lambda(N)$-restricted recursive core. First I show that the pair of the small and the large $\Lambda(N)$-restricted recursive core is between the $\Lambda(N)$-restricted α- and ω-cores. Then introducing the notion of the restriction of a PFF game, I relate the $\Lambda(N)$-restricted recursive cores to the recursive cores.

One of the results of Kóczy (2007) is that the pair of the recursive cores is a refinement of the α- and the ω-core pair, i. e. $C_\omega(V) \subseteq RC_S(V) \subseteq RC_L(V) \subseteq C_\alpha(V)$. Here we argue that it is also true for the $\Lambda(N)$-restricted recursive core. To show this, first I define the $\Lambda(N)$-restricted equivalents of the α- and ω-cores. From now fix the set of feasible coalitions $\Lambda(N)$.

The α-core (Aumann and Peleg, 1960) is stable against the deviations where the residuals hurt the deviating players as much as they can. In other words, an outcome is an element of the α-core if for any set of deviating players, for any deviations, there exists a reaction of the residual players when the deviators are worse off. Observe that the existence of only one outcome like that is enough since the residuals will form the partition which hurts the deviators the most. In the $\Lambda(N)$-restricted case it means the following:

Definition 20. Let $V^\Lambda \in \Gamma^{\Lambda(N)}$ be a PFF game with restricted cooperation. The

$\varLambda(N)$-restricted α-core is as follows:

$$C_{\alpha}^{\varLambda} = \Bigg\{ (\lambda, x) \in \Omega(V^{\varLambda}) : \forall R \subseteq N \ \exists \pi'_{N \setminus R} \in \varLambda(N \setminus R),$$

$$(\pi'_{N \setminus R}, x'_{N \setminus R}) \in \Omega(V^{\varLambda}_{\pi'_R}(N \setminus R)), \ \forall \pi'_R \in \Pi(R), \ \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^{\varLambda}(S, \pi'_R \cup \pi'_{N \setminus R}) \Bigg\} \tag{2.3}$$

One can see that the difference between $C_{\alpha}^{\varLambda}$ and $RC_L^{\varLambda}(V^{\varLambda})$ is that in the case of $C_{\alpha}^{\varLambda}$ the outcome realized by the residual players is an element of $\Omega(V^{\varLambda}_{\pi'_R}(N \setminus R)$, instead of $A(V^{\varLambda}_{\pi'_R}(N \setminus R)$ as it was in the case of the $RC_L^{\varLambda}(V^{\varLambda})$.

In contrast to the α-core, the ω-core (Shenoy, 1979) contains outcomes that are stable against all possible deviations given that the residuals will help the deviating players as much as possible (that is, the ω-core contains extremely stable outcomes). The definition of $\varLambda(N)$-restricted ω-core is the following:

Definition 21. Let $V^{\varLambda} \in \Gamma^{\varLambda(N)}$ be a PFF game with restricted cooperation. The $\varLambda(N)$-restricted ω-core is as follows:

$$C_{\omega}^{\varLambda} = \Bigg\{ (\lambda, x) \in \Omega(V^{\varLambda}) : \forall R \subseteq N, \ \forall \pi'_{N \setminus R} \in \varLambda(N \setminus R),$$

$$(\pi'_{N \setminus R}, x'_{N \setminus R}) \in \Omega(V^{\varLambda}_{\pi'_R}(N \setminus R)), \ \sum_{i \in R} x_i \geq \sum_{S \in \pi'_R} V^{\varLambda}(S, \pi'_R \cup \pi'_{N \setminus R}) \Bigg\} \tag{2.4}$$

The difference between $C_{\omega}^{\varLambda}$ and $RC_S^{\varLambda}(V^{\varLambda})$ is the same as it was between $C_{\alpha}^{\varLambda}$ and $RC_L^{\varLambda}(V^{\varLambda})$ above. From Definitions 20 and 21 can be seen that $C_{\omega}^{\varLambda} \subseteq C_{\alpha}^{\varLambda}$.

When I relate the $\varLambda(N)$-restricted recursive cores to the $\varLambda(N)$-restricted α- and

ω-cores I also use the following corollary that immediately follows from Definition 19.

Corollary 8. $RC_S^\Lambda(V^\Lambda) \subseteq RC_L^\Lambda(V^\Lambda)$.

Now we can relate these solution concepts to each other. The result generalizes Corollary 7 of Kóczy (2007) to the class of PFF games with restricted cooperation.

Proposition 9. *Let $V^\Lambda \in \Gamma^{\Lambda(N)}$ be a PFF game with restricted cooperation. Then we have*

$$C_\omega^\Lambda(V^\Lambda) \subseteq RC_S^\Lambda(V^\Lambda) \subseteq RC_L^\Lambda(V^\Lambda) \subseteq C_\alpha^\Lambda(V^\Lambda).$$

Proof. The proof has two parts, first I prove that $C_\omega^\Lambda(V^\Lambda) \subseteq RC_S^\Lambda(V^\Lambda)$. Assume that outcome $(\lambda, x) \in \Omega(V^\Lambda)$ satisfies (2.4), hence $(\lambda, x) \in C_\omega^\Lambda(V^\Lambda)$. Since $A(V^\Lambda) \subseteq \Omega(V^\Lambda)$, then (λ, x) also satisfies (2.1), which means that $(\lambda, x) \in RC_S^\Lambda(V^\Lambda)$.

Now I show that $RC_L^\Lambda(V^\Lambda) \subseteq C_\alpha^\Lambda(V^\Lambda)$. Assume that outcome $(\lambda, x) \in \Omega(V^\Lambda)$ satisfies (2.2), hence $(\lambda, x) \in RC_L^\Lambda(V^\Lambda)$. Similarly as above, since $A(V^\Lambda) \subseteq \Omega(V^\Lambda)$, and we even had a $(\pi'_{N\setminus R}, x'_{N\setminus R}) \in A(V^\Lambda_{\pi'_R}(N \setminus R))$, then we also have such an outcome in $\Omega(V^\Lambda_{\pi'_R}(N \setminus R))$. This condition implies that (λ, x) also satisfies (2.3), therefore $(\lambda, x) \in C_\alpha^\Lambda(V^\Lambda)$. $\square$

Now I introduce the notion of the restriction of a PFF game in order to make implications about a recursive core and the $\Lambda(N)$-restricted recursive core of two somewhat related games. Consider a player set N. In a PFF game with the player set N the function V is defined on each embedded coalition and assigns a value each of them. In the $\Lambda(N)$-restricted PFF game with player set N we have a coarser set of embedded coalitions on which the function V^{Λ} is defined. However, V is also defined on each embedded coalition on which V^{Λ} is defined. Taking this fact into account, I define the *restriction* of a PFF game.

Definition 22. Let be N the player set, $\Lambda(N)$ is the set of feasible partitions, V is a PFF game and V^{Λ} is a $\Lambda(N)$-restricted PFF game with players set N. Let also be $S \in \lambda\Lambda(N)$. We say that V^{Λ} is a restriction of V if

$$V^{\Lambda}(S, \lambda) = V(S, \lambda).$$

for all $\lambda \in \Lambda$.

So if V^{Λ} is a restriction of V, then every outcome of V^{Λ} is also an outcome of V. Given this, it does make sense to compare the recursive core of V with the $\Lambda(N)$-restricted recursive core of V^{Λ}. We formulate and prove the proposition below for the small recursive core (and small $\Lambda(N)$-restricted recursive core) only, an analogous formulation and proof arises for the large objects.

Proposition 10. *Let be N the player set, $\Lambda(N)$ is the set of permitted partitions, V is a PFF game and V^{Λ} is a restriction of V. Then, neither $RC_S(V) \subseteq RC_S^{\Lambda}(V^{\Lambda})$ or $RC_S^{\Lambda}(V^{\Lambda}) \subseteq RC_S(V)$ is true.*

Proof. We prove the proposition by counterexamples. Let the player set be $N = \{1, 2, 3\}$. Denote the elements of $\Pi(N)$ as follows:

$\pi_1 = \Big\{\{1\}, \{2\}, \{3\}\Big\}$,

$$\pi_2 = \Big\{ \{1,2\}, \{3\} \Big\},$$
$$\pi_3 = \Big\{ \{1,3\}, \{2\} \Big\},$$
$$\pi_4 = \Big\{ \{2,3\}, \{1\} \Big\},$$
$$\pi_5 = \Big\{ \{1,2,3\} \Big\}.$$

1.

Consider the following PFF game V_1:

- $V_1(S, \pi_1) = 2$ for all $S \in \pi_1$,
- $V_1(S, \pi_2) = V_1(S, \pi_3) = V_1(S, \pi_4) = \begin{cases} 1 & \text{if } |S| = 1, \\ 6 & \text{otherwise.} \end{cases},$
- $V_1(S, \pi_5) = 18$ for all $S \in \pi_5$.

Consider the outcome $(\pi_5, (6,6,6))$. No one-player coalition can improve its payoff by deviating, since deviation yields a payoff of 1 instead of the original 6 (the payoff 1 is because the remaining two players will form a coalition in their residual game ensuring themselves a total payoff of 6 and they be better off with the 2–2 payoff in π_1). Neither any two-player coalition can improve its payoff by deviating, since deviation yields a payoff of 6 to the given coalition instead of the initial 12. So the small recursive core of V_1 is nonempty.

Take V_1^{Λ}, the restriction of V_1 with $\Lambda(N) = \{\pi_1, \pi_2, \pi_3, \pi_4\}$. The $\Lambda(N)$-restricted small recursive core of V_1^{Λ} is empty. The only possible outcome with partition π_1 cannot be in $RC_S^{\Lambda}(V^{\Lambda})$, since any two player coalition can improve its total payoff by forming a two-player coalition. Consider now the partitions π_2, π_3 and π_4. In any outcome with one of these partitions (let us say π_1), Player 3 has a payoff of 1. The other two players have 6 payoff, so at least one of them (let us say, Player 2) is getting $x \leq 3$. Then, this outcome is not in $RC_S^\Lambda(V_1^\Lambda)$, since Player 3 can form a coalition with Player 2 and change the partition of players to π_4. Player 1 can offer a payoff of $3+\varepsilon$ to Player 2, while she have $3-\varepsilon$ which is still greater than her original 1 ($0 < \varepsilon < 1$). But given this outcome, now Player 1 can make the same offer to Player 3 and force her to deviate and so on. So the $\Lambda(N)$-restricted small recursive core of V_1^Λ is indeed empty. We proved that $RC_S(V) \subseteq RC_S^\Lambda(V^\Lambda)$ is not true.

2.

Consider the following PFF game V_2:

- $V_2(S, \pi_1) = 2$ for all $S \in \pi_1$,
- $V_2(S, \pi_2) = V_2(S, \pi_3) = V_2(S, \pi_4) = \begin{cases} 1 & \text{if } |S| = 1, \\ 6 & \text{otherwise.} \end{cases}$,
- $V_2(S, \pi_5) = 6.5$ for all $S \in \pi_5$.

Now $RC_S(V_2)$ is empty. Any outcome with partitions π_1, π_2, π_3, π_4 cannot be in $RC_S(V)$ because of the arguments stated above. Any outcome with partition π_5 also cannot be stable since either a single player deviates if her payoff is below 1, or a two-player coalition deviates if their payoff is below 6, and since $6 + 1 < 6.5$ at least one of these two cases will always happen, $RC_S(V_2)$ is empty.

Take now V_2^Λ, the restriction of V_2 with $\Lambda(N) = \{\pi_1\}$. Now the game is a trivial one, and its $\Lambda(N)$-recursive core contains the outcome $(\pi_1, (2, 2, 2))$. In this case we see that $RC_S^{\Lambda}(V^{\Lambda}) \subseteq RC_S(V)$ is not true.

Putting together the two statements above we proved the proposition. $\square$

2.5 Guidelines for an empirical application

In this section I show a possible empirical application of the model introduced above. The area of application is Industrial Organizations, particularly the prediction of possible mergers. The method is presented here relies on the merger simulation method of Nevo (2000a). The consideration of other methods of merger simulation is out of this paper's focus, refer to Budzinski and Ruhmer (2009) for a survey of the alternative models.

Using this method presented below, with the proper data, one can calculate the profits of the firms before and after any mergers. These profits are the payoffs of the coalitions in question. Using the results of the estimated econometric model and the observations about the past merger control decisions in the relevant economy, one can obtain the set of feasible partitions. The feasible partitions are firm ownership structures that can be achieved by approved mergers. Given the set of feasible partitions and the payoff for each embedded coalitions, one can give the $\Lambda(N)$-restricted recursive core of the resulting PFF game with restricted cooperation.

In this particular case the partitions realized in the outcomes of the recursive core are interpreted as stable industry structures. Knowing what structures are stable, if we observe that the actual industry structure is unstable we can predict the future mergers in the given industry.

In Section 2.5.1 I provide a method to determine the payoffs for the embedded coalitions. Section 2.5.2 is about how to construct the set of feasible partitions. In Section 2.5.3 we provide an example of the search for the $\Lambda(N)$-restricted recursive core in a given game.

2.5.1 Determining the payoffs of coalitions

Now I introduce the merger simulation method described in Nevo (2000a). We give here an overview of the paper, and do not go much into the details, the interested reader should consult with Nevo (2000a). The framework assumes a realistic market structure with relatively high concentration and differentiated products.

The first task during a merger simulation is to estimate the demands produced by the competing firms. Nevo (2000a) uses a random coefficient logit specification for the demand estimations. This framework is very flexible since it allows to assume a heterogeneity in consumer preferences for the different product characteristics and also heterogeneous sensitivity to price. The firms compete according to the Bertand-Nash model. For further details about the model he uses, one can also see Berry et al (1995) and Nevo (2000b).

We have $t = 1, ..., T$ markets, $i = 1, ..., I$ consumers and $j = 1, ..., J$ products. The indirect utility of consumer i from product j at market t is given by the following equation:

$$u_{ijt} = x_{jt}\beta_i^* + \alpha_i^* p_{jt} + \xi_{jt} + \varepsilon_{ijt} \doteq V_{ijt} + \varepsilon_{ijt}, \tag{2.5}$$

where x_{jt} is a vector of observed product characteristics, p_{jt} is the price of good j at market t. The coefficients β_i^* and α_i^* are the consumers' taste parameters for the product characteristics and the price. These coefficients are conditional of the observed demographic characteristics of the consumers. ξ_{jt} is a market-specific product characteristic which is observed by the consumers but not observed by the econometrician, and ε_{ijt} is the error term. The demands for a given product are calculated using a discrete-choice model based on the utility function given by (2.5), and using the demands we can calculate the market share (s_j) for each good.

On the demand side there are F firms producing the set of products $\mathcal{F}_f$. The firms' profit maximization problem is given by

$$\Pi_f = \sum_{j \in \mathcal{F}_f} (p_j - mc_j) I s_j(p) - C_f, \tag{2.6}$$

where mc_j is the constant marginal cost of product j and C_f is firm f's fixed cost of production. Now assuming Bertrand-Nash equlibrium and strictly positive prices, the price-cost markups can be calculated as follows:

$$p - mc = \Psi^{pre}(p)^{-1} s(p). \tag{2.7}$$

The estimated marginal costs can be recovered using (2.7):

$$mc = p - \Psi^{pre}(p)^{-1} s(p). \tag{2.8}$$

$\Psi^{pre}(p)$ is the pre-merger ownership matrix and is given by

$$\Psi^{pre}_{j,r}(p) = \begin{cases} -\partial s_j(p)/\partial p_r, & \text{if } \exists f: \{r,j\} \subset \mathcal{F}_f, \\ 0 & \text{otherwise.} \end{cases} \tag{2.9}$$

Now if the matrix Ψ^{post} is defined in the same way as (2.9) but given by the post-merger ownership matrix, we can obtain the post-merger price p^* which satisfies

$$p^* = mc + \Psi^{post}(p^*)^{-1} s(p*). \tag{2.10}$$

There is three implicit assumptions in (2.10): the Bertrand-Nash model of firm conduct; the same pre- and post-merger marginal costs and that Ψ^{pre} and Ψ^{post} use the same demand estimates with the different ownership structures. Nevo (2000a) discusses the validity of these assumptions.

With the proper Ψ^{post} we can simulate the post-merger prices for every possible ownership structure, and if we have the prices from (2.10), then using the estimated marginal costs from (2.8), we can substitute these values into (2.6) and can calculate the profits for the firms for all ownership structure. In the terms of PFF games, the (possibly merged) firms are the coalitions and the ownership structures are the partitions, and the profits are the values of the coalitions. There are obviously externalities between the players (one firm's payoff depend on whether the other firms are merged or not), so the PFF games are coherent models of this situations.

2.5.2 Determining the set of feasible partitions

I demonstrated above how to calculate the profits for the firms for all possible ownership structure, however obviously it is not the case that all possible ownership structure can realize due to the rulings of the competition authorities. In game theoretic terms, the players cannot form all possible partitions, so we have a game (in addition, a PFF game as we seen above) with restricted cooperation.

Competition authorities do not allow mergers when it is possible that the post-merger market will be significantly worse for the consumers than the pre-merger market was. The structural merger simulation framework of Nevo (2000a) described above (using the estimated demand functions) is also able to analyze the change in the consumer welfare in each simulated post-merger equilibrium. The compensating variation[1] for consumer i is given by

$$CV_i = \frac{\ln\left[\sum_{j=0}^{J} V_{ij}^{post}\right] - \ln\left[\sum_{j=0}^{J} V_{ij}^{pre}\right]}{\alpha_i^*}, \tag{2.11}$$

where V_{ij}^{pre} and V_{ij}^{post} are the pre- and post-merger utilities for consumer i given by

[1]The compensating variation is a welfare evaluation tool for economic changes, see for example Mas-Colell et al (1995). It is the amount that must be paid to the consumer after the price change to granting her original utility level.

(2.5). Summing up the compensating variations we can obtain the total consumer welfare change.

The tolerated level of the negative welfare change can vary across countries, see for example Bergman et al (2011) for a comparison of merger control decisions in the European Union and the United States. Determining the cut-off value for the approved mergers is out of the focus of this paper, but in an empirical application one can figure out what mergers would be approved by the given competition authority and what mergers would not. The ownership structures reached via approved mergers constitute the set of permitted partitions in the analyzed game.

2.5.3 Solution of the game

Now I illustrate how to find the $\Lambda(N)$-recursive core of a PFF game with restricted cooperation using an example. Assume that we have a market with 4 major firms (A, B, C, D) with the method described above, and we found out that the permitted ownership structures are the following:

$$\lambda_1 = \Big\{\{A\},\{B\},\{C\},\{D\}\Big\},$$
$$\lambda_2 = \Big\{\{A,D\},\{B\},\{C\}\Big\},$$
$$\lambda_3 = \Big\{\{B,D\},\{A\},\{C\}\Big\},$$
$$\lambda_4 = \Big\{\{C,D\},\{A\},\{B\}\Big\},$$
$$\lambda_5 = \Big\{\{B,C,D\},\{A\}\Big\}.$$

The values of the different coalitions conditional to the realized partitions are given by the table below.

	λ_1	λ_2	λ_3	λ_4	λ_5
{A}	8	–	7	7	5
{B}	4	3	–	3	–
{C}	3	2	2	–	–
{D}	1	–	–	–	–
{A,D}	–	10	–	–	–
{B,D}	–	–	6	–	–
{C,D}	–	–	–	5	–
{B,C,D}	–	–	–	–	9.5

To find the recursive core, we have to check each feasible partition whether it satisfies the conditions of Definition 19 or not. In this example there will be no difference between the small and large recursive cores since $A^A(V^A_{\pi_{N\setminus R}}(R))$ will always have only one element.

Consider λ_1 first. In this case A and D forming $\{A, D\}$ can achieve a total payoff of 10 instead of their original 1+8=9, (B and C face a trivial residual game as the reaction to this deviation), so λ_1 is not in the recursive core.

For λ_3, players B, C and D forming $\{B, C, D\}$ can earn a total payoff of 9.5 instead of their original 6+2=8 (A face a trivial game as reaction to this deviation), so λ_3 is not in the recursive core.

For λ_4, players B, C and D forming $\{B, C, D\}$ can earn a total payoff of 9.5 instead of their original 5+3=8 (A face a trivial game as reaction to this deviation), so λ_3 is not in the recursive core.

Consider now λ_5. B must have at least a payoff of 3 and C must get at least 2 from the total 9.5 of the coalition of B, C, and D, because if they don't, then they will deviate forming one-player coalitions. In this case they can ensure themselves the payoffs 3 and 2 respectively, whatever is the reaction of the other players. So the maximum payoff that D can get from the total 9.5 of the coalition of B, C, and D is 4.5. Given this, players A and D forming $\{A, D\}$ can achieve a total payoff of 10 instead of their original 5+4.5=9.5, (B and C face a trivial residual game as the reaction), so λ_5 is not in the recursive core.

Finally consider λ_2. Players B and C cannot deviate from this partition (neither jointly or one-by-one). The only possible deviation of A and D is to form one-player coalitions. If D form an one-player coalition, then she will get a payoff of 1 and player A will get a payoff of 8 (the residuals face a trivial game). So it is not worth to D forming the one-player coalition since she and A can get more If player A forms a one-player coalition, then the residual players form $\{B, C, D\}$ (because they are better of than they can be with any other partition, see the reasons above), and A will get a payoff of 5 and D will get at most a payoff of 4.5. So first, forming one-player coalitions are not worth for any players of the coalition $\{A, D\}$. Second, as we seen above, if player D decides that she deviates together with B and C, then A will get only a payoff of 5. Knowing this, A is willing to offer any y to D from the total 10 payoff of their coalition as long as $y < 5$. All remaining possible deviations involve D (A do not want to deviate in itself, B and C cannot deviate), but no possible deviation can give her more payoff than 4.5. If she commits to form coalitions $\{B, D\}$, $\{C, D\}$ or $\{B, C, D\}$ given that B and C will always demand at least 3 and 2 respectively, the maximum payoff D can achieve is 3 (for $\{B, D\}$ and $\{C, D\}$) or 4.5 (for $\{B, C, D\}$). So if the payoff she receives from the total 10 of $\{A, D\}$ is greater than 4.5, D will never deviate neither alone, neither with other players. We can conclude that the partition λ_2 is stable with the proper payoff vectors.

The $\Lambda(N)$ recursive core of the game described above is (x, λ_2), where $x = (5.5 - \varepsilon, 3, 2, 4.5 + \varepsilon)$ such that $0 < \varepsilon < 0.5$.

2.6 Conclusion

In this paper I introduced the concept of games in partition function form with restricted cooperation. After defining the recursive core using Myerson (1977b)'s definition of PFF games, I developed a framework to deal with both externalities and restricted cooperation. Since in the PFF environment the well-being of one group of players may depend on what actions the other group of the players take, instead of restricting the set of feasible coalitions (as it is usual in the literature), the restriction is placed on the set of coalitions.

I generalized the concept of recursive core to this new class of games, and after defining the $\Lambda(N)$-restricted versions of α- and ω-core, I proved (for fixed restriction) the same relationship between these solution concepts (i. e. α-, ω-, and recursive core) as Kóczy (2007) did. We also showed that if we consider a restriction of a PFF game there is no containing relation in neither directions between the recursive cores of the original and the restricted PFF game.

For further research, the phenomenon of restricted cooperation to *non-cooperative* coalition formation games is still not analyzed yet, together with the links between the cooperative and noncooperative approach in this environment. Another interesting open question is that what are the conditions for the nonemptiness of the $\Lambda(N)$-restricted recursive core.

Chapter 3

Lexicographic Semiorders and Ambiguity

3.1 Introduction

Since the seminal paper of Ellsberg (1961), choice problems with ambiguity (that is, choice problems with uncertainty when the probabilities of the possible outcomes are not known) received large attention in decision theory. Two of the most important models of decision with ambiguity are the Multiple Priors or maxmin model by Gilboa and Schmeidler (1989) and the α-maxmin model by Ghirardato et al (2004). Gilboa and Schmeidler (1989) provide a model of a strict ambiguity averse decision maker, while Ghirardato et al (2004) incorporate attitude towards ambiguity and allow for less ambiguity averse (or more ambiguity loving) choices.

Both of the above models use the extreme (the worst or the best possible) priors to evaluate a given act, however, sometimes it is counter-intuitive to make decisions based on these extremes.

To see why decision making based on extremities might be problematic, consider a thought experiment. Imagine a driver driving on a mountain while she is in a hurry. She can drive fast or slow and she does not know whether the road on the mountain is dangerous or it is not, she has different priors over the safety of the roads and does not know which prior is the true (i. e. the driver faces an ambiguous decision problem). Driving fast on a dangerous road might result in very bad outcomes, while driving slow on safe roads is a waste of time. An ambiguity averse driver prefers driving slow since she picks her decision according to the worst possible prior where the probability of the roads being dangerous is the highest. However, if that mountain is an erupting volcano, then according to the worst possible prior the driver will die now matter how she drives (if she drives slowly the lava catches her, if she drives fast she dies in an accident due to the extremely dangerous roads). In this case no matter how ambiguity averse the decision maker is, it is not reasonable to make decisions according to the worst possible prior. It is more reasonable to ignore the possibility of extremely dangerous roads (when choices do not really matter) and just drive fast.

In the light of this intuition, I propose a model of decision making under ambiguity which uses a different notion of attitude towards ambiguity than Ghirardato et al (2004). In my argument, the parameter of ambiguity aversion (or ambiguity love) is a threshold value which determines whether the agents decide according to a given prior or just ignore that prior since the difference between payoffs is too small to care about ambiguity. The level of ambiguity aversion can be interpreted as a measure of payoff differences originating from ambiguity the decision maker is willing to tolerate. A more ambiguity averse agent tolerates smaller differences than a more ambiguity loving agent. Formally, the choice from a set of acts is determined lexicographically using not just the worst case and best case utilities of acts, but a finite sequence of i-th worst case utilities and the ambiguity aversion parameter ε which is equal to zero in the case of an extreme ambiguity averse agent and tends to infinity in the case of extreme ambiguity love.

In this paper I show how to construct the finite sequence of i-th worst case utilities for ambiguous acts using the multiple priors of the agents and how to use them together with the ambiguity aversion parameter as a lexicographic semiorder (see Manzini and Mariotti (2012)) to induce choices which can be different than the ones predicted by the models in Gilboa and Schmeidler (1989) and Ghirardato et al (2004).

The remainder of the paper is organized as follows: Section 3.2 introduces choice correspondences by lexicographic semiorders. Section 3.3 shows how to construct lexicographic semiorders for Anscombe-Aumann acts, Section 3.4 illustrates the differences in implied behavior by my model and by the models of Gilboa and Schmeidler (1989) and Ghirardato et al (2004) with two simple examples, Section 3.5 discusses the restrictions of the i-th worst case utilities. Section 3.6 concludes the paper.

3.2 Choice correspondences by lexicographic semiorders

In this section I present the tools that are necessary to introduce Manzini and Mariotti (2012)'s notion of choice functions by lexicographic semiorders. Then I extend this notion and define choice correspondences by lexicographic semiorders. After that I provide a characterization for the choice rule.

3.2.1 The Model

First I define semiorders introduced by Luce (1956). A semiorder P on set X is a binary relation which is irreflexive and satisfies the following two properties:

1. $(x, y), (w, z) \in P$ implies $(x, z) \in P$ or $(w, y) \in P$,
2. $(x, y) \in P$ and $(y, z) \in P$ imply $(x, w) \in P$ or $(w, z) \in P$.

Irreflexivity together with properties 1 and 2 above imply that a semiorder is transitive. The Scott-Suppes Theorem (Scott and Suppes, 1958) shows that semiorders represent a simple threshold model, i. e. an object is preferred to another object if the utility from the first object exceeds the utility of the other object by a fixed threshold: P is a semiorder on a finite set X if and only if there is a utility function f and a positive number α such that aPb if and only if $f(a) > f(b) + \alpha$.

Now I present the choice function by lexicographic semiorders defined in Manzini and Mariotti (2012). The primitives are a set X of alternatives, a domain of choice problems Σ whose elements S are nonempty subsets of X. A choice set is trivial if

$\#S = 1$, a collection $\mathcal{C}$ is trivial if S is trivial for all $S \in \mathcal{C}$. A choice function defined on Σ is a function $c : \Sigma \to X$ such that $c(S)$ is an element of S for every $S \in \Sigma$.

Now consider an ordered sequence $f = (f_i)_{i \in I}$, where I can be a finite sequence of numbers $\{1, ..., n\}$ or the entire set $\mathbb{N}$ of natural numbers. The elements f_i of f are numerical representations of semiorders. The sequence f together with a positive $\sigma \in \mathbb{R}$ is called a *lexicographic semiorder*[1] on X, denoted $(f_i, \sigma)_{i \in I}$.

These semiorders represent the characteristics used to evaluate the elements of the choice sets. There is an order of importance of the characteristics, hence the ordered sequence of semiorders f_i. The decision maker starts with considering the first characteristic and evaluates the alternatives in choice set S based on that characteristic. The numerical representation f_1 assigns a real number to each alternatives $s \in S$. The decision maker does not care about small differences, an alternative s is superior to an alternative s' only if $f(s)$ exceeds $f(s')$ by a fixed threshold. This fixed threshold is equal to σ. That is, if there is an alternative s' in the choice set S for which we have $f_1(s') + \sigma < f_1(s)$, where $s \in S$ is another feasible alternative, then s' will not be chosen since it is clearly inferior to another alternative according to the first (most important) characteristic. Thus, the decision maker will eliminate s' from the choice set when she starts to evaluate the remaining alternatives according to the second characteristic, and this process is repeated until only one alternative remains or the decision maker is indifferent between all of the remaining alternatives. In this case we

[1]The sequence is called 'lexicographic semiorder' despite that it is not a sequence of semiorders but numerical representations of semiorders.

have set-valued choice.

To capture this process of elimination, Manzini and Mariotti (2012) define the so-called "survivor sets" $M_i(S)$ recursively for all $i > 0$:

$$M_0(S) = S$$

$$M_i(S) = \{s \in M_{i-1}(S) | \text{for all } s' \in M_{i-1}(S) : f_i(s) + \sigma \geq f_i(s')\}.$$

That is, if there is an $s \in M_{i-1}(S)$ such that there exists $s' \in M_{i-1}(S)$ with $f_i(s) + \sigma < f_i(s')$, then s is inferior to s' according to the lexicographic semiorder $(f_i, \sigma)_{i \in I}$, therefore s gets eliminated and does not advance into $M_i(S)$.

Now we have all the tools to present the definition of the choice function by lexicographic semiorder.

Definition 23 (Manzini and Mariotti, 2012)**.** A choice function c is a choice by lexicographic semiorder if there exists a lexicographic semiorder $(f_i, \sigma_i)_{i \in I}$ such that for all $S \in \Sigma$ there is a $j \in I$ for which $\{c(S)\} = M_j(S) = M_k(S)$ for all $k \geq j$. In this case, we say that $(f_i, \sigma)_{i \in I}$ induces c.

Notice that since I is allowed to be $\mathbb{N}$ (that is, an infinite set), Manzini and Mariotti (2012) need to assume that the elimination process always stops after finite number of steps (this finite number does not have to be fixed across all choice problems S).

However, this choice function is not able to deal with ambiguous choices without modifications. The main problem with the function as defined above that if we have a choice set S and a lexicographic semiorder $(f_i, \sigma)_{i \in I}$ we might have that $c(S)$ does not exist. For example, consider $S = \{a, b\}$ with $|f_i(a) - f_i(b)| < \sigma$ for all $i \in I$. Then $M_j(S)$ will be $\{a, b\}$ for all $j \in I$, that is, we can never have $c(S) = M_j(S)$ as an element of S.

Since we need a decision rule which always provides a choice from a set of ambiguous acts, I modify the notion of choice by lexicographic semiorder to obtain this attractive feature and introduce the choice correspondences by lexicographic semiorder.

The primitives of the new choice rule are the same as before with two exceptions. First, I consider choice correspondences $c^* : \Sigma \rightsquigarrow \mathcal{P}(\mathcal{X})$ where $\mathcal{P}$ is the power set of X and the index set I of any lexicographic semiorder has to be a finite sequence $\{1, ..., n\}$.[2]

Definition 24. A choice correspondence c^* is a choice by lexicographic semiorder if there exist a lexicographic semiorder $(f_i, \sigma)_{i \in I}$ such that for all $S \in \Sigma$ one of the following conditions hold:

1. There is $j \in I$ such that $\# M_j(s) = 1$ and $c^*(S) = M_j(s)$,
2. There is $s \in M_n(S)$ such that $c^*(S) = s$ and $f_n(s) > f_n(s')$ for all $s \neq s' \in M_n(S)$,
3. There are $s_1, s_2, ..., s_k \in M_n(S)$ with $k \geq n$ such that $c^*(S) = \{s_1, ..., s_n\}$, $f_n(s_1) = f_n(s_2) = ... = f_n(s_k)$, and for all $s' \in M_n(S)$, $s' \notin C(S)$ we have $f_n(s') < f_n(s)$ for all $s \in c^*(S)$.

[2] In Section 3.3 I will show that one can construct a finite sequence of semiorders for the analyzed ambiguous choice problems.

In this case, we say that $(f_i, \sigma)_{i \in I}$ induces c^*.

Condition 1. coincides with the choice function definition. Condition 2 and 3 ensure that $c^*(S)$ is never empty for any $S \in \Sigma$, even if we have $|f_i(a) - f_i(b)| < \sigma$ for all $i \in I$ and all $a, b \in S$. Note that due to conditions 2 and 3 f_k is actually a representation of an order instead of a generic semiorder. I illustrate Definition 24 with an example.

Example 4.

Let $X = x_1, x_2, x_3, x_4$, $\Sigma = \{S_1, S_2, S_3\}$, $S_1 = \{x_1, x_2\}$, $S_2 = \{x_1, x_3\}$, $S_3 = \{x_1, x_3, x_4\}$, and c^* is such that $c^*(S_1) = \{x_1\}$, $c^*(S_2) = \{x_3\}$, $c^*(S_3) = \{x_3, x_4\}$. Let be $\sigma = 1$ and the numerical representations of semiorders f_1, f_2 are given by the following table:

	f_1	f_2
x_1	2	3
x_2	0	5
x_3	3	4
x_4	2	4

Then, c^* is a choice correspondence by lexicographic semiorder and $(f_i, \sigma)_{i=1,2}$ induces c^*. Consider $c^*(S_1)$: the decision maker has to choose between x_1 and x_2. First the decision maker looks at the first characteristic and compares the alternatives: $f_1(x_1) = 2$ and $f_1(x_2) = 0$. Since $\sigma = 1$, $f_1(x_1) > f_1(x_2)$, therefore condition 1 in Definition 24 holds and $c^*(S_1) = \{x_1\}$.

Now consider S_2. The decision maker is choosing from x_1 and x_3. Since $f_1(x_1) = 2$ and $f_1(x_3) = 3$, the decision maker cannot eliminate any alternatives using f_1, therefore she looks at f_2: $f_2(x_1) = 3 < 4 = f_2(x_3)$. That is, condition 2 holds and $c^*(S_2) = \{x_3\}$.

Finally look at $c^*(S_3)$. Since $f_1(x_3) = 3$ and $f_1(x_4) = 2$, there is no elimination and the decision maker looks at f_2. We have $f_2(x_3) = 4 = f_2(x_4)$ condition 3 holds and we have $c^*(S_3) = \{x_3, x_4\}$.

After defining my proposed choice rule I continue with the characterization of it.

3.2.2 Characterization

Manzini and Mariotti (2012) provide a characterization for choice functions by lexicographic semiorders. I extend their characterization to the correspondences introduced by Definition 24.

First I present the Reducibility axiom which is the single axiom that characterizes choice functions by lexicographic semiorders.

Definition 25 (Reducibility)**.** For every nonempty $\mathcal{C} \subset \Sigma$, there exists $S \in \mathcal{C}$ and $x, y \in S$ such that, for all $T \in \mathcal{C}$,

$$(T \setminus \{y\}) \in \mathcal{C}, x \in T \Rightarrow c(T) = c(T \setminus (T \setminus \{y\}_)).$$

In this case we say that x makes y irrelevant.

Reducibility is a weaker version of the Independence of Irrelevant Alternatives as it does not require independence from every irrelevant alternatives, instead, there should be at least a pair of alternatives x and y such that x makes y irrelevant: that is, the choice $c^*(T)$ from each set T which contains both x and y, the choice is the same as from the set $T \setminus \{y\}$, that is, if x is available, y is never chosen. I provide a simple example to illustrate reducibility.

Example 5.

Consider a choice problem $X = \{x_1, x_2, x_3\}$, $\Sigma = \{S_1 = \{x_1, x_2\}, S_2 = \{x_1, x_2, x_3\}, S_3 = \{x_2, x_3\}\}$, and let be $c^*(S_1) = \{x_2\}$, $c^*(S_2) = c^*(S_3) = x_3$. When x_1 and x_2 are available x_2 is chosen (that is, the choice from $\{x_1, x_2\}$ is the same as the choice from the trivial choice problem: $\{x_2\}$ is chosen from $\{x_2\}$). In addition, when $\{x_1, x_2, x_3\}$ are all available, the choice is $\{x_3\}$, the same as from $\{x_2, x_3\}$ Therefore this choice correspondence satisfies reducibility, x_2 makes x_1 irrelevant: every time when both x_1 and x_2 is available, the choice is the same as if x_1 is not available.

Now I first present Manzini and Mariotti (2012)'s characterization of their choice function by lexicographic semiorders, then I provide the characterization of c^* introduced in Definition 24.

Theorem 11 (Manzini and Mariotti, 2012)**.** *Let X be finite. Let c be a choice function defined on the domain Σ of all finite subsets of X. Then the function c is a choice by lexicographic semiorder if and only if c is reducible.*

For the proof, look at Manzini and Mariotti (2012). I do not include the proof here since the proof for characterization of c^* will be very similar. It is important to note that only the transitivity of semiorders is used in the proof. Therefore any transitive partial order can be used instead of semiorders. This is important for my model, since in Definition 24 at the last stage $i = k$ I used an order f_k instead of a semiorder. But since the order is also transitive, I can adapt the proof of Manzini and Mariotti (2012).

Now I reformulate the theorem for the choice correspondence c^* in Definition 24. The statement of the thorem is the same except for that there I consider choice correspondence instead of choice function. The proof follows Manzini and Mariotti (2012) with slight modifications.

Theorem 12. *Let X be finite. Let c^* be a choice correspondence described in Definition 24 defined on the domain Σ of all finite subsets of X. Then the correspondence c^* is a choice by lexicographic semiorder if and only if c is reducible.*

Proof. Necessity: Let c^* be induced by the lexicographic semiorder $(f_i, \sigma)_{i\in I}$. We will show that c^* is reducible. Let $\mathcal{C} \subseteq \Sigma$ be a collection of choice sets such that there is no $c^*(S) = S$ for all $S \in \mathcal{C}$. Let

$$j = \min\{i | M_i(S) \neq S \text{ for some } S \in C\}$$

(j is well defined since there is no $c^*(S) = S$ for all $S \in \mathcal{C}$). Let $T \in \{T \in C\}$ be such that $M_j(T) \neq T$. Let $x, y \in T$ such that $f_j(x) > f_j(y) + \sigma$. For every $S \in \mathcal{C}$, we have either $\{x, y\} \nsubseteq S$ (that is, reducibility holds vacuously) or we have $\{x, y\} \subseteq S$. If $\{x, y\} \subseteq S$, which is true at least for $S = T$, then for all $z \in S$, $f_j(y) > f_j(z)+\sigma$ implies

$f_j(x) > f_j(z) + \sigma$. Thus, $M_j(S) - M_j(S \setminus \{y\})$, which implies $c^*(S) = c^*(S \setminus \{y\})$.

Sufficiency: In this part of the proof we assume that c^* is reducible and we construct a lexicographic semiorder which induces c^*. The algorithm defines a sequence of collections $\{\mathcal{C}_i\}_{i \in I}$ and associates sequence of $m+1$-tuples $(x_{i1}, ..., x_{im}, y_i)_{i \in I}$ where $I = \{1, ..., n\}$. Let $\mathcal{C}_0 = \Sigma$ and let $x_{01}, ..., x_{0m}, y_0$ be any $m+1$ alternatives such that, for all $S \in \mathcal{C}_0$, $m = \#c^*(S)$, $x_{01}, ..., x_{0m}, y_0 \in S \Rightarrow c^*(S) = c^*(S \setminus \{y_0\})$. Alternatives like this exist by reducibility and by the fact that if $x \in c^*(S)$ makes $y \in S$ irrelevant, then any $x' \in c^*(S)$ makes y irrelevant. For $i > 0$, define recursively $x_{i1}, ..., x_{im}, y_i \in X$ as any $m+1$ alternatives such that $(x_{i1}, ..., x_{im}, y_i) \neq (x_{j1}, ..., x_{jm}, y_j)$ for any $j < i$ and

$$\text{for all } S \in \bigcap_{j<i} \mathcal{C}_j : \quad x_{i1}, ..., x_{im}, y_i \in S \Rightarrow c^*(S) = c^*(S \setminus \{y_i\})$$

and

$$\mathcal{C}_i = \bigcap_{j<i} \mathcal{C}_j \setminus \left\{ S \in \bigcap_{j<i} \mathcal{C}_j | \{x_{i1}, ..., x_{im}, y_i\} \subseteq S \right\}.$$

For every i, let $f_i(x_{i1}) = ... = f_i(x_{im}) = 1$, $f_i(y_i) = -1$ and $f_i(z) = 0$ for every other $z \in X \setminus \{x_{i1}, ..., x_{im}, y_i\}$. Since X is finite, for any i, unless $S \in \mathcal{C}_{i+1} \Rightarrow \#S = \#c^*(S)$, it is implied by reducibility that $\mathcal{C}_i \neq \mathcal{C}_{i+1}$. Therefore, $S \in \cap_{i \in I} \mathcal{C}_i \Rightarrow \#S = \#c^*(S)$.

This defines a lexicographic semiorder $f = (f_i)_{i \in I}$. I will show that f induces c^*. Fix $S \in \Sigma$. Suppose by induction that $c^*(S) \subseteq M_i(S)$, where $M_i(S)$ is a survivor set described previously. $M_i(S) \in \mathcal{C}_i$ has to hold, otherwise there was a $k \leq i$ such that $f_k(x_{1k}) = ... = f_k(x_{mk}) = 1, f_k(y_k) = -1$ and $\{x_{1k}, ..., x_{mk}, y_k\} \subseteq M_i(S) \in$

$\mathcal{C}_k$, contradicting the definition of survivor sets. If we also have $M_i(S) \in \mathcal{C}_{i+1}$, then $\{x_{1i+1}, ..., x_{mi+1}, y_{i+1}\} \nsubseteq M_i(S)$ and so we immediately have $c^*(S) \subseteq M_{i+1}(S)$. If $M_i(S) \notin \mathcal{C}_{i+1}$, then we must have $\{x_{1i+1}, ..., x_{mi+1}, y_{i+1}\} \subseteq S$. By the construction we cannot have $y_{i+1} \subseteq c^*(S)$ since $c^*(S) = c^*(S \setminus \{y_1\}) = ... = c^*(S \setminus \{y_1, ..., y_{i+1}\})$. Thus, $c^*(S) \subseteq M_{i+1}(S)$.

I now show that for all $s \in S \setminus c^*(S)$, there exists a k with $s \notin M_k(S)$. If this does not hold, let $\cap_{i \in I} M_i(S) = T$ and let $s \in T$. The definition of T implies that for all $i \in I$, $\{x_{i1}, ..., x_{im}, y_i\} \nsubseteq T$ since $f_i(x_{i1}) = ... = f_i(x_{im}) = 1$, $f_i(y_i) = -1$. Therefore $T \in \cap_{i \in I} \mathcal{C}_i$. Now this contradicts $s \notin c^*(S)$, $s \in T$ and $c^*(S) \subseteq T$, since $T \in \cap_{i \in I} \mathcal{C}_i$ implies $\#T = \#c^*(S)$. Now we constructed a lexicographic semiorder which induces c^*. □

After defining and characterizing the abstract decision rule, in the next section I focus on its application to choices between ambiguous acts.

3.3 Lexicographic semiorders for ambiguous choices

In this section I show how to construct lexicographic semiorders for choices from ambiguous acts. In this terminology the set of alternatives X will be the set of ambiguous acts, and we will construct the lexicographic semiorder using the multiple priors of the decision maker. First I briefly introduce the concept of ambiguity, then I look at a decision maker with finite priors, and finally I consider decision makers with infinite, closed and convex set of priors.

3.3.1 Ambiguity

Decisions under uncertainty are extensively studied in Economics. In the standard setting of choice under uncertainty, the decision maker has to choose from alternatives (so-called *lotteries*) which yield different outcomes with known probabilities. However, there are situations where the probabilities of possible outcomes of a lottery (or with another term, an *act*) are not known. In these situations we have *ambiguity* instead of the standard uncertainty. Since the seminal paper of Ellsberg (1961), ambiguity also received large attention from economists and decision theorists.

The most common method to model ambiguity is to assume that the decision maker has multiple priors about the probabilities of the possible outcomes (instead of a single probability distribution which is used in decisions under uncertainty). The primitives of the standard setting are the set of states of the world $\mathfrak{S}$, the set of possible outcomes Z, the set of Anscombe-Aumann acts $\mathcal{F}$ where an act $f : \mathfrak{S} \to \Delta(Z)$ is a mapping from $\mathfrak{S}$ to the set of probability measures on Z; and a bounded utility index $u : Z \to \mathbb{R}$ (see Anscombe and Aumann (1963)).

In details, we have a set $\mathfrak{S}$ of states of the world, these states will determine the outcomes of an ambiguous situation. The outcomes are given by the set Z. An Anscombe-Aumann act f specifies a lottery for each different state of the world: for $\mathfrak{s}_1, \mathfrak{s}_2 \in \mathfrak{S}$, $f(\mathfrak{s}_1)$ is a probability distribution on the set of outcomes, that is, a lottery with prizes from Z. Using the utility index u (which is defined on outcomes) one can construct a von Neumann-Morgenstern utility function for $f(\mathfrak{s})$, for simplicity I will denote this function by $u(\cdot)$.

That is, for a fixed state of the world $\mathfrak{s}$, the act f gives a simple lottery $f(\mathfrak{s})$. The ambiguity is coming from the fact that the decision makers do not have a single probability prior on the set of the state of the world, they have *multiple priors*, given by a set C. I use these multiple priors to construct lexicographic semiorders for choice problems with ambiguous acts. These semiorders are the i-th worst case utilities for a given act. The threshold parameter (which is denoted by ε) is the parameter of ambiguity aversion that shows how large utility differences the decision maker is willing to tolerate when evaluating an act considering the i-th worst case utility as a characteristic.

In the remainder of this section I show how to construct lexicographic semiorders for finite and infinite (closed and convex) set of priors.

3.3.2 Finite set of priors

Recall that the primitives of the ambiguity setting are the set of states of the world $\mathfrak{S}$, the set of possible outcomes Z, the set of Anscombe-Aumann acts $\mathcal{F}$ where an act $f : \mathfrak{S} \to \Delta(Z)$ is a mapping from $\mathfrak{S}$ to the set of probability measures on Z; and a bounded utility index $u : Z \to \mathbb{R}$ (see Anscombe and Aumann (1963)). Similarly to Gilboa and Schmeidler (1989), I assume that the decision maker has a subjective set of priors C. For now consider a finite C. We are looking for a decision rule which compares acts $f, g \in \mathcal{F}$.

First I define U_0, the worst case utility for each act, which will be the starting point for the decision maker in the evaluation of a given act. Let U_0 be given by

$$U_0(f) = \min_{p \in C} \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp, \tag{3.1}$$

that is, the utility value of lotteries $f(\mathfrak{s})$ according to the prior which yields the lowest possible utility. Note that equation (3.1) is exactly the maxmin utility function of Gilboa and Schmeidler (1989). Call U_0 as worst-case utility.

Now fix an act $f \in \mathcal{F}$ and consider the set

$$C_1^f = C \setminus \left\{ p : \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp = U_0(f) \right\}. \tag{3.2}$$

That is, C_1^f is the original set of priors C minus the priors that minimize $\int_{\mathfrak{S}} u(f(\mathfrak{s}))dp$. Now define

$$U_1(f) = \min_{p \in C_1^f} \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp. \tag{3.3}$$

Thus $U_1(f)$ is the maxmin utility of f with a set of priors where the original minimizers of $\int_{\mathfrak{S}} u(f(\mathfrak{s}))dp$ are excluded, that is, U_1 is the second worst utility. Note that since C is a finite set and u is bounded the minimum in equation (3.3) exists similarly as in equation (3.1).

Now assume that C_{i-1}^f (and therefore also $U_i(f)$) is already defined. Then let be

$$C_i^f = C_{i-1}^f \setminus \left\{ p : \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp = U_{i-1}(f) \right\} \tag{3.4}$$

and

$$U_i(f) = \min_{p \in C_i^f} \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp. \tag{3.5}$$

$U_i(f)$ is called as i-th worst case utility.[3]. If for some j we have that for all $p \in C_j^f$

$$\int_{\mathcal{S}} u(f(\int))dp = U_j(f), \tag{3.6}$$

then we denote C_j^f with C_*^f and $U_j(f)$ with $U_*(f)$. Notice that

$$U_*(f) = \max_{p \in C} \int_{\mathfrak{S}} u(f(s))dp.$$

$U_*(f)$ is the best case utility value of act f. So for a given act f we have a sequence $(U_0(f), U_1(f), ..., U_j(f) = U_*^f)$ of i-th worst case utilities. We want to use this sequence as a lexicographic semiorder to evaluate acts in a set $\mathcal{F}$. Notice that it is important to have sequences of the same length for each act. However, with the method I described above, it is possible that we have sequences of different lengths, that is, the value of j for which we have U_j as the best case utility might depend on f. To prevent this problem, consider a set of acts $\mathcal{F}$ and sequences $(U_0(f), U_1(f), ..., U_*^f)$ for each $f \in \mathcal{F}$ and let k be the length of the longest sequence. Then for each act, define a sequence

[3]Notice that the precise terminology would be $(i-1)$-th worst case utility, but for the sake of simplicity I call it i-th worst case utility

$(U_i(f))_{i \in I}$ where $I = \{1, ..., k\}$ and the elements of the sequences are given by

$$(U_i(f))_i = \begin{cases} U_i(f) & \text{if } i \leq j(f) \\ U_*(f) & \text{otherwise.} \end{cases} \tag{3.7}$$

That is, the elements of the sequences are the i-th worst case utilities as long as the index does not exceed $j(f)$ which is the rank of the best case utility, and repetitions of the best case utility if the index exceeds $j(f)$.

The construction of $(U_i(f))_{i \in I}$ guarantees that for all $f \in \mathcal{F}$ the number of elements of the sequences are the same. Thus for any $f \in \mathcal{F}$ we can define a sequence of semiorders $(U_i(f))_{i \in I}$, where $I = \{1, ..., k\}$. We will use this sequence as numerical representations of semiorders, and together with a real number $\varepsilon \geq 0$ we have a lexicographic semiorder $(U_i(f), \varepsilon)_{i \in I}$. My proposed decision rule is a choice correspondence by lexicographic semiorder as defined in 24 with $(U_i(f), \varepsilon)_{i \in I}$.

As an illustration, consider Example 4 formalized as a decision problem with ambiguous acts. Let $\mathfrak{S} = \{\mathfrak{s}_1, \mathfrak{s}_2\}$, the set of acts be $x = \{x_1, x_2, x_3, x_4\}$; the payoffs of the acts in the different states of the world are given by the following table:

	$\mathfrak{s}_1$	$\mathfrak{s}_2$
x_1	2	3
x_2	0	5
x_3	3	4
x_4	2	4

Let $\varepsilon = 1$ and the set of priors $c = \{\mu_1(\mathfrak{s}_1) = 1, \mu_2(\mathfrak{s}_1) = 0\}$. When the decision maker faces the choice set $S_1 = \{x_1, x_2\}$, first she looks at the worst case prior, μ_1. According to μ_1, the payoffs of x_1 and x_2 are 2 and 0 respectively. Since $2 > 0 + 1$, the decision maker chooses x_1. If we consider $S_3 = \{x_1, x_2, x_3\}$ then according to the worst case prior (which is μ_1 for each act), the payoffs are 0, 3 and 2 respectively. Since $0 + 1 < 2$, x_2 is eliminated, but neither of x_1 or x_3 since their worst case payoffs 2 and 3 are within ε. So the decision maker evaluates acts x_1 and x_3 according to the second worst measure, μ_2. The payoff is 4 for both acts, thus $c^*(S_3) = \{x_1, x_3\}$.

The parameter ε determines the decision maker's attitude towards ambiguity: ε represents how much ambiguity-related payoff difference she is willing to tolerate: the decision maker does not care about ε differences in the payoffs The larger $\frac{1}{\varepsilon}$ is, the more ambiguity averse the decision maker is. The extreme ambiguity averse case is when $\varepsilon = 0$. In this case the decision rule specified above coincides with the maxmin utility maximization of Gilboa and Schmeidler (1989) as long as $U_0(f) \neq U_0(g)$. The other extremity is when $\varepsilon \to \infty$, then the decision maker always decides according to $U_*(\cdot)$ as the α-maxmin decision maker in Ghirardato et al (2004) with $\alpha = 1$.

After describing how to construct lexicographic semiorders for finite sets of priors I show the construction method for infinite priors.

3.3.3 Closed and convex set of priors

In the case of finite priors above, I simply constructed the lexicographic semiorders by considering the priors (possibly) one-by one to obtain the i-th worst case utility values. It is not that simple if the set of priors is a closed and convex set with infinite elements, we cannot eliminate the acts using possibly every prior in set C, since that could easily lead to an infinite sequence of semiorders.

To avoid this problem I use the interpretation of ε: the decision maker does not care about ε differences in the payoffs of the acts. Since the utility function we used to evaluate the acts in each k-th worst case is continuous, it is possible to eliminate 'slices' from C instead of discrete elements as we did in the finite case. The details are provided below. The starting point U_0 is the same as it was in the finite case:

$$U_0(f) = \min_{p \in C} \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp. \tag{3.8}$$

Again, the equation (3.8) is exactly the maxmin utility function of Gilboa and Schmeidler. The minimum exists by Weierstrass Theorem.

Let $V(p, f)$ be the function

$$V(p, f) = \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp. \tag{3.9}$$

Since $\mathfrak{S}$ is finite, the integral in (3.9) reduces to a sum and then $V(\cdot)$ is continuous in p. That is, for a fixed act f, for every $\varepsilon > 0$ we have a $\delta > 0$ such that

$$d(p_1, p_2) < \delta \Rightarrow d(V(p_1, f), V(p_2, f)) < \varepsilon. \tag{3.10}$$

For $p \in \Delta(\mathfrak{S})$ define

$$B_\delta(p) = \{x \in \Delta : d(x,p) < \delta\}. \tag{3.11}$$

Now fix an act $f \in \mathcal{F}$ and consider the set

$$C_1^f = co\left(C \setminus \left(\bigcup_{p \in M_1^f} B_\delta(p)\right)\right), \tag{3.12}$$

where

$$M_1^f = \left\{p : \int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp = U_0(f)\right\}.$$

Now define

$$U_1(f) = \min_{p \in C_1^f} \int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp, \tag{3.13}$$

where δ is given by ε according to equation (3.10). Thus $U_1(f)$ is the maxmin utility of f with a set of priors where the original minimizers of $\int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp$ and all points which are within distance δ (that is, resulting in a utility within ε) are excluded.

Note that since C_1 is the convex hull of a closed set minus an open set, therefore it is closed. The minimum in equation (3.3) exists similarly as in equation (3.1).

Now assume that C_i^f (and therefore also $U_i(f)$ and M_i^f) is already defined. Then let be

$$C_{i+1}^f = co\left(C_i \setminus \left(\bigcup_{p \in M_i^f} B_\delta(p)\right)\right) \tag{3.14}$$

and

$$U_{i+1}(f) = \min_{p \in C^f_{i+1}} \int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp. \tag{3.15}$$

If for some j we have that for all $p \in C^f_j$

$$U_j(f) - \varepsilon \leq \int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp \leq U_j(f) + \varepsilon, \tag{3.16}$$

then instead of C^f_j we consider C^f_* with

$$U_*(f) = \max_{p \in C} \int\limits_{\mathfrak{S}} u(f(\mathfrak{s}))dp.$$

Proposition 13. *For all closed and convex set C, act f and $\varepsilon > 0$ we always arrive to C^f_* in finite steps using the elimination process described above.*

Proof. Since $u(\cdot)$ is bounded, then for all $f \in \mathcal{F}$ both $U_0(f)$ and $U_*(f)$ is finite. Furthermore,

$$k = \left[\frac{U_*(f) - U_0(f)}{\varepsilon}\right] + 1,$$

where $[x]$ is the integer part of the number x, is a finite natural number and it is easy to see that we need at most k steps to arrive to C^f_* from C using the elimination process described above. □

Now for all acts $f \in \mathcal{F}$ we have sequences $U(f) = (U_0(f), U_1(f), ..., U_*(f))$, but similarly to the finite priors case we might face the problem of sequences of different lengths for different acts. We can deal with this problem using exactly the same method as I used in the finite priors case. So even in the case of closed and convex priors we are able to construct a sequence of semiorders $(U_i(f))_{i \in I}$, where $I = \{1, ..., k\}$. We will use this sequence as numerical representations of semiorders, and together with a real number $\varepsilon \geq 0$ we have a lexicographic semiorder $(U_i(f), \varepsilon)_{i \in I}$. Similarly, my proposed decision rule is a choice correspondence by lexicographic semiorder as defined in 24 with $(U_i(f), \varepsilon)_{i \in I}$.

Now that I introduced my choice rule, in the next section I will provide simple examples to illustrate the behavior implied by my model.

3.4 Comparison with other models in the literature

In this section I provide two simple example to illustrate the differences in the behavior implied by my model, Gilboa and Schmeidler (1989)'s maxmin model and Ghirardato et al (2004)'s α-maxmin model. Regarding the smooth model by Klibanoff et al (2005) here, there is one major difference between the smooth model and my model: I do not assume that the decision maker can formulate second order probabilities about the different priors, while in Klibanoff et al (2005) these second order probabilities are vital.

Example 6.

Consider the following choice problem. There is an urn with one ball in it, the ball can be red, blue or green, however there is no information about the possible probability distribution colors (that is, the distribution of colors are ambiguous). The set of states is $\mathfrak{S} = \{R, B, G\}$ The decision maker is offered two acts, f and g, each act pays off an amount depending on the color of a ball drawn from the urn (the decision maker has to pick an act before the ball is drawn). These payoffs are presented in the table below.

	R	B	G
f	\$100	\$40	\$0
g	\$100	\$60	\$0

Assume that the set of the decision maker's priors is $C = \{\mu_1(G) = 1, \mu_2(B) = 1, \mu_3(R) = 1\}$. For each act, the worst possible case is the green ball, the best possible case is the red ball. Since the two acts yield exactly the same payoffs under these two priors, a decision maker using maxmin or α-maxmin utility function should be indifferent between acts f and g. However, there is a strong case to believe that agents might have a strict preference g over f. If the worst possible prior implies the same payoff for both acts, why anyone would decide about the act according to the worst prior? Why don't just ignore it and decide according to the second worst (where there is a difference)? A decision maker which follows the lexicographic semiorder choice rule will choose g over f for some range of ε, even with $\varepsilon = 0$.

Note that Example 6 has nothing to do with ambiguity aversion. No matter what the decision maker's attitude towards ambiguity is, it is simply unreasonable to be indifferent between f and g while in some states they give exactly the some payoffs and in other states g is strictly better. The next example addresses attitudes towards ambiguity.

Example 7.

Now consider the following modified choice problem. The setting is the same as in Example 6, but the payoffs are different.

	R	B	G
f	\$100	\$40	\$10
g	\$100	\$60	\$5

As in Example 6, the worst case is green ball, the best case is red ball. An extreme ambiguity averse decision maker would choose act f since it yields higher payoff according to the worst possible prior. An α-maxmin decision maker has the following utilities for each acts:

$$U(f) = \alpha \cdot u(\$100) + (1 - \alpha) \cdot u(\$10),$$

$$U(g) = \alpha \cdot u(\$100) + (1 - \alpha) \cdot u(\$5).$$

Now matter what α is (that is, no matter how ambiguity loving she is), an α-maxmin decision maker will always pick f since $U(f) > U(g)$. However, it might seem plausible that a moderately ambiguity loving decision maker prefers g to f. The intuition is the following: if we look at the worst possible prior (green ball), we are choosing between \$10 and \$5. Some ambiguity-loving decision maker might think that the difference between $u(\$10)$ and $u(\$5)$ is too small to be concerned about the consequences of ambiguity. Such decision maker might ignore the worst possible prior when making her decision and look at the second worst prior, the blue ball. Now she compares $u(\$40)$ with $u(\$60)$. Depending on u and the level of ambiguity love of the decision maker, she might think that the difference of these payoffs is large enough to care about the ambiguity. In this case our less ambiguity averse agent picks g over f. This choice can be accomodated with a choice rule by lexicographic semiorder given a ε such that $u(\$10) - u(\$5) < \varepsilon$ and $u(\$60) - u(\$40) > \varepsilon$. This example demonstrates clearly that the concept of attitude towards ambiguity is different in my model and in the α-maxmin model.

After illustrating the behavior implied by my model I discuss the restrictions on utility functions $U_i(f)$ that are necessary to impose if we want to ensure that ε is meaningful.

3.5 Discussion of utility measures

The sequence of i-th worst utilities and the parameter ε of ambiguity aversion are key concepts in the choice model I introduced above. Unfortunately, the model requires stronger restriction on $U_i(\cdot)$ than cardinality: we need that the differences between utility values (and therefore ε) be meaningful.

Proposition 14. *Let c^* be a choice correspondence defined on set of acts $\mathcal{F}$. Then, $(U_i(\cdot), \varepsilon)_{i=I}$ and $(V_i(\cdot), \varepsilon)_{i=I}$ are two different lexicographic semiorders which induce c for all choice set S if and only if for any $f, g \in \mathcal{F}$ and all $i \in I$, we have*

$$U_i(f) - U_i(g) = V_i(f) - V_i(g). \tag{3.17}$$

Proof. Necessity: If 3.17 holds, then the survival sets $M_i(S)$ described in Definition 24 are the same under both utility measures for all i and all S. Therefore the choice correspondences induced by the lexicographic semiorders constructed from these utility values have to be the same.

Sufficiency: I prove the contrapositive of the statement with a counterexample. Consider a simple decision problem with a choice set $S = \{a, b\}$ and lexicographic semiorder $(U(a) = 1, U(b) = 0, \varepsilon = 0.2)$. In this case the difference between the utilities of the two acts is equal to 1 and the choice function induced by this lexicographic semiorder is $c^*(S) = \{a\}$. However if we consider another lexicographic semiorder $(V(a) = 0, V(b) = 1, \varepsilon = 0.2)$ where the utility difference is equal to -1, we have that the choice from S is equal to $\{b\}$. That is, if the utility differences are not unchanged we might not get the same induced choice correspondence, which implies that if we two lexicographic semiorders imply the same choice correspondence for every S, then we must have that $U_i(f) - U_i(g) = V_i(f) - V_i(g)$ are the same for any $f, g \in \mathcal{F}$. $\square$

In some applications there is a natural measure for which the differences are meaningful. If we look at money lotteries, that is, if the elements of Z are amounts of moneys, then we can replace the utility values with certainty equivalents as a measure. These certainty equivalents are given by the expression

$$u(CE_i(f)) = \min_{p \in C_i^f} \int_{\mathfrak{S}} u(f(\mathfrak{s}))dp.$$

$CE_i(f)$ is a certain amount of money which gives the same utility as the ambiguous money lottery f. We call $CE_i(f)$ the i-th worst case certainty equivalent. Since these $CE_i(f)$ are amounts of money, the differences between them are meaningful, so a lexicographic semiorder with certainty equivalents can be applied in empirical studies.

3.6 Conclusion

This paper extends the notion of choice by lexicographic semiorders of Manzini and Mariotti (2012) to choice correspondences, and provides the characterization based on their work. In addition, it shows how to construct lexicographic semiorders for choice problems with ambiguous Anscombe-Aumann acts, and gives some illustrative examples with fixed set of priors to compare my model with two important models from the previous literature. From these examples it is clear that the model presented here implies different behavior that the standard models of Gilboa and Schmeidler (1989) and Ghirardato et al (2004).

There are still interesting opportunities for future work on this topic. The two most important addition to this paper are a second characterization and another example. The characterization should use axioms which are comparable with the axioms in Gilboa and Schmeidler (1989), Ghirardato et al (2004) and Klibanoff et al (2005). The second addition should be an intuitive example where the predictions of my model contradicts the predictions of Gilboa and Schmeidler (1989) and Ghirardato et al (2004) for every possible set of priors C.

Appendix A

Appendix

A.1 Balanced collections and the Bondareva-Shapley Theorem

To prove Proposition 2 and Lemma 3 I use the concept of balanced collections and apply the Bondareva-Shapley Theorem. For the Theorem, see Shapley (1967) and Bondareva (1963). In this Appendix I use the notation of Peleg and Sudhölter (2003) for balanced collections.

Definition 26. Let N be a set with $|N| = n$. Then for any $S \subseteq N$, the characteristic vector of S is $\chi_S \in \mathbb{R}^N$ such that

$$(\chi_S)_i = \begin{cases} 1, & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases}$$

Definition 27. A collection $\mathcal{B}$ of subsets of N is balanced if there exist positive balancing weights δ_S such that

$$\sum_{S\in\mathcal{B}} \delta_S \chi_S = x_N$$

Theorem (Bondareva-Shapley Theorem)**.** *Let (v, N) be a characteristic function game. Then the core of the game is nonempty if and only if there is no balanced collection $\mathcal{B}$ such that*

$$\sum_{S\in\mathcal{B}} \delta_S v(S) > v(N).$$

A.1.1 Proof of Proposition 2

Proof. If there is a stationary SPE in $\sigma(v, N)$ where the outcome is N, then there is no coalition $S \subset N$ such that (1.1) holds. If there is, then N cannot arise in a stationary SPE as the proposer has an incentive to propose S instead of N and in that subgame the responders' best reply is to accept it. Now I show that if there is no coalition S such that 1.1 holds, the core of the game must be nonempty.

The condition 1.1 implies that there is no coalition S with size $|S| = s$ such that for a balanced collection $\mathcal{B}$ with weights $(\delta_S)_{S\in\mathcal{B}}$ consisting only coalitions of size s, we have

$$v(N) < \sum_{S\in\mathcal{B}} \delta_S v(S). \tag{A.1}$$

The facts that the game is symmetric and A.1 holds for all coalition size s implies that there is no balanced collection $\mathcal{B}$ with weights δ_S such that A.1 holds. By the Bondareva-Shapley Theorem it means that the core of (v, N) is nonempty. $\square$

A.2 Proof of Lemma 3

Proof. By the Bondareva-Shapley Theorem, we know that for all games with an empty core there is a balanced collection $\mathcal{B}$ with a weight system $(\delta_S)_{S\in\mathcal{B}} \geq 0$ such that

$$v(N) < \sum_{S\in\mathcal{B}} \delta_S v(S). \tag{A.2}$$

First I show that there must be a balanced collection $\mathcal{B}$ with weight system $(\delta_S)_{S\in\mathcal{B}}$ satisfying A.2 such that $\delta_S = 0$ for all $|S| = 1$. Due to superadditivity, there is no $(\mathcal{B}, \delta_S)$ system satisfying A.2 where only singleton coalitions have positive weights. As a consequence, if there exists $(\mathcal{B}, \delta_S)$ satisfying A.2, it must have positive weight on a coalition S with $|S| = k > 1$. Now it is easy to see that for a balanced system $(\mathcal{B}', \delta'_S)$, where all $S \in \mathcal{B}'$ has $|S| = k$, we have

$$\sum_{S\in\mathcal{B}'} \delta'_S v(S) \geq \sum_{S\in\mathcal{B}} \delta_S v(S) > v(N).$$

That is, to prove the lemma it is enough to show that there is a cost t such that

$$v(N) - t \geq \sum_{S\in\mathcal{B}'} \delta_S (v(S) - t) \tag{A.3}$$

is true for all S with coalition size s. Given that $\mathcal{B}'$ only contains coalitions with a size of k, the balanced system $(\mathcal{B}', \delta'_S)$ consists of $\binom{n}{k}$ coalitions of size k and each player is exactly in $\binom{n-1}{k-1}$ different coalitions, therefore the weights are $\frac{1}{\binom{n-1}{k-1}}$. Condition A.3 takes the form

$$v(N) - t \geq \binom{n}{k} \cdot \frac{1}{\binom{n-1}{k-1}}(v(S) - t)$$

$$v(N) - t \geq \frac{n}{k}v(S) - \frac{n}{k}t$$

Even if $v(N) < \frac{n}{k}v(S)$ due to emptiness of the core, if

$$t \geq \frac{nv(S) - sv(N)}{n - s}, \tag{A.4}$$

then the core of the game (v_t, N) is nonempty. □

A.3 Proof of Proposition 6

Proof. Note that a vector of transaction costs t is surplus-improving if it restores the grand coalition as outcome and the total surplus in the outcome of game $\sigma(V_t, N)$ is higher than in the outcome of game $\sigma(V, N)$. It is easy to see that condition (1.3) is necessary and sufficient for the increase in total surplus. It is enough to show that condition (1.2) is necessary and sufficient for restoring N as the outcome of the coalition formation game $\sigma(V_t, N)$.

For necessity, assume that (1.2) does not hold, that is,

$$\frac{V(S,\pi_t) - t_{|S|}}{|S|} > \frac{V(N,N) - t_n}{n}$$

for some π_t outcome. This means that there is a SPE in $\sigma(V_t, N)$ where the outcome is π_t conditional on the formation of S after the first proposal such that member of S have higher payoff than they would get in N. Therefore the members of S still have an incentive to form S instead of the grand coalition and t does not restore N as the outcome of $\sigma(V_t, N)$. This proves necessity.

For sufficiency, I show that if (1.2) holds, then there is a t such that N is the outcome of $\sigma(V_t, N)$. Assume that (1.2) holds and consider the vector t of transaction costs such that

$$t_i = \begin{cases} 0 & \text{if } i = 1 \\ t_{|S|} & \text{if } 1 < i \leq |S| \\ t_n & \text{if } |S| < i \leq n \end{cases}$$

Now I show that there is no coalition T such that

$$\frac{V(T,\pi_t) - t_{|T|}}{|T|} > \frac{V(N,N) - t_n}{n}. \tag{A.5}$$

Assume there is a coalition T such that (1.2) holds. Due to symmetry and (1.2), $|T|$

cannot be equal to $|S|$. It is also impossible that $|T| > |S|$ since

$$\frac{V(T,\pi_t) - t_n}{|T|} > \frac{V(N,N) - t_n}{n}$$

implies

$$\frac{V(T,\pi_t)}{|T|} > \frac{V(N,N)}{n} + \left(\frac{t_n}{|T|} - \frac{t_n}{n}\right).$$

Since the term in brackets is positive, that implies that the average payoff in T is higher than in N, which means that T is a potential responsible coalitions. That contradicts the assumption that (1.2) holds for all potential responsible coalitions. $|T| < |S|$ is also impossible due to similar reasoning as above.

Therefore (1.2) is sufficient for restoring N as an outcome of $\sigma(V_t, N)$. □

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