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The persistence of air temperature, a statistical study

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THE PERSISTENCE OF AIR TEMPERATURE, A STATISTICAL STUDY
by

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THE PERSISTENCE OF AIR TEMPERATURE

1. INTRODUCTION

The present thesis discusses the outlines of a mathematical model for meteorological temperature which includes elements of persistence. It presents and discusses the results of exploring some statistically determined characteristics of a proposed measure of persistence, namely, non-circular coefficients of lagged correlation, and concludes with suggestions for further work toward deriving and applying such a model.

Persistence of air temperature at a point may be defined as any direct numerical measure of the tendency of temperature at any level to remain near that level over a definite period of time. The reference level itself may be a location (number) on the Fahrenheit temperature scale, or a derived location such as a simple or complex assumed temperature normal at the point.

Climatic analysts can readily obtain from the United States Weather Bureau the means and extremes of air temperature at standard sites in the United States.* A more restricted temperature variability measure is obtained from averages of the daily maximum and of the daily minimum. Their difference is the mean daily range. Hourly temperature readings are published. Departures from normal (i.e., long term) values are also listed, and days with extreme temperatures above or below arbitrary levels are tallied.

*For example, see References 16 and 17

Finally, the tendency of temperature to persist at levels which require heating for human comfort is measured indirectly by published degree days. As is explained on page 4 of Reference 19, one heating degree day (HDD) is tallied for each Fahrenheit degree by which a daily mean temperature falls below 65°F. In other words,

$$HDD = \begin{cases} \theta_{base} - T_{mean}, & \theta_{base} > T_{mean} \\ 0, & \text{otherwise,} \end{cases}$$

where T_{mean} is the mean temperature for each successive day in turn throughout the heating year, and

$$\theta_{base} = 65^{\circ}F.$$

In United States Weather Bureau practice,

$$T_{mean} = \frac{\text{daily maximum} + \text{daily minimum}}{2}.$$

The daily values of HDD are accumulated through the heating season, and departures from normal also are published.

Except for special studies, the temperature statistics published as discussed above are essentially all that are readily available for United States stations. It is possible to obtain special tabulations and calculations of climatic temperature data from the National Weather Records Center at Asheville, North Carolina, on funded request.[3] However, no standardized computation and publication of direct calculations of temperature persistence for a wide selection of points, either from this or any other source, are known to the writer.

2. BACKGROUND

Other indirect measures of persistence are known to exist. One such measure is the seasonal accumulation of growing degree days. The number of growing degree days (GDD) is defined[4] as

$$GDD = \begin{cases} T_{mean} - \theta_{base}, & \theta_{base} < T_{mean} \\ 0, \text{ otherwise,} \end{cases}$$

where now T_{mean} is the mean temperature for each day throughout the growing season, and

$$\theta_{base} = \begin{cases} 40^{\circ}F. \text{ for peas} \\ 50^{\circ}F. \text{ for corn.} \end{cases}$$

An accumulation of degree days during a growing season has been compared with readings of an instrument called the tenderometer to establish satisfactory ranges of GDD for marketable table peas.[4] An application is to guide and advise farmers concerning the growth of peas to be expected in untried localities.

Nuttonson (see, for example, pages 60-61 and 208-211 of Reference 10) has developed sophisticated applications of the degree-day measure based on 40°F to the growth of rye. However, 40°, 50°, and 65° are not the only temperature bases that might be deemed critical for such applications. For example, Nuttonson (see pages 11ff of Reference 10) quotes Russian findings that

> "For the transition, into the reproductive stage, winter rye requires a longer exposure to low temperatures (between 34° and 39°F for 20 to 55 days) than spring rye, which requires exposure to 41° to 51°F for 8 to 10 days."

Nuttonson also considers that $\sum$ GDD can be approximated satisfactorily from monthly rather than daily temperature means, with correspondingly less calculation.

Cooling degree days (CDD) are defined[19] as

$$CDD = \begin{cases} T_{mean} - \theta_{base}, & \theta_{base} < T_{mean} \\ 0, \text{ otherwise,} & \end{cases}$$

where θ_{base} is 80°F.

It appears that degree days above a variety of levels, degree days within particular levels and degree days below various specified levels all are of interest. Daily means and monthly means of temperature, among other variates, are suggested for the calculations. Rather than such heterogeneous treatment of essentially similar applications, it seems in order to consider the possibility of applying a single measure of temperature persistence to solve the variety of problems related to the duration of temperature in specified ranges.

Another reason for interest in a measure of persistence different from cumulative degree days is the aforementioned indirectness with which degree days express the tendency of temperature to remain above (or below, or within) a specified level. Consider a crop requiring consecutive hours above θ_0 degrees during a certain phenological stage. At one site, the temperature might remain within a reasonable zone above θ_0 degrees for much of the day and drop mildly below θ_0 for much of the night, so that the favorable plant environment suggested by the GDD measure is, in fact, realized. At another, much more variable site the temperature might rise severely above θ_0 during the day to an unfavorably hot environment, and drop correspondingly low at night to a level of cold that could inhibit or damage the crop. Even so, the numerical GDD measure could be the same at both sites.

3. SOME ELEMENTS OF A GENERAL TEMPERATURE MODEL

Since persistence may be thought of as the tendency of temperature at a point and time to be closely associated with temperature at an earlier time, there naturally comes to mind some form of lag correlation coefficient, or possibly Markov transition-probability matrix, as a measure of the tendency of temperature to persist in ranges of interest. The continuous nature of the temperature variate, and the extreme variety of ranges of interest, pose a difficulty in determining suitable discrete ranges which could be used as mutually exclusive Markov states. Furthermore, intuitive reflections on the duality of variance and covariance suggest first interest in correlation measures. Thus, the outline of a general temperature model, based on variance and covariance both through space and through time, stands on the horizon as a goal toward which fruitful research might be directed.

Figure 1 presents a conceptual a conceptual breakdown of components of temperature variability at a point. It is not the only breakdown which could be devised, but it is considered to be a serviceable framework for a point of departure in seeking to construct a matching breakdown of components of temperature autocovariance such that useful relationships can be constructed among variance elements and autocovariance elements, both through time and through space.

Isolated bits of calculation and research have been published on temperature variance (e.g., Reference 1 and page 38 of Reference 19) and on temperature autocovariance (e.g., References 8 and 14). Reference 1 maps a component of temperature variability, the year to year standard deviation of the monthly mean. Reference 14 charts a related set of components of temperature autocovariation through time at a point, namely, the day to day lag correlation with lags of 1 (1) 30 days.

Reference 14 also presents some interrelationships among temperature variance and autocovariance, from which the following are adapted:

$$\text{a.} \quad \sigma_d^2 = (d)\,\sigma_y^2 - (d-1)\,\bar{C}_d\,, \qquad (1)$$

where σ_d^2 is the variance of the daily temperature average during a month over the years, σ_y^2 is variance of the monthly temperature average over the years, $\bar{C}_d$ is the mean lag covariance of one daily temperature average with each other daily temperature average during the month, and d is the number of days in the month. As an example, in practice, if the month is June, then

$d = 30$

and June daily and monthly averages for a sufficient number of years are used to estimate σ_d^2, σ_y^2 and $\bar{C}_d$. Obviously, some adjustment in d is necessary for February, such as ignoring February 29 on leap years. At this stage of the investigation, the present writer can only suggest that a "sufficient number of years" for daily data is

$$n \geq 20 \text{ years},$$

in the absence of thorough knowledge of the numerical behavior of the autocovariances and of the precision to be expected from typical interpolation from climatic maps, and with due regard for the ultimate limitation in precision imposed by Weather Bureau temperature readings to whole Fahrenheit degrees. Available estimates of σ_y are believed frequently to be precise to within one Fahrenheit degree.[1]

$$\text{b.} \quad \sigma^2_{max} = 4\sigma_y^2 - 2\,C_{max-min} - \sigma^2_{min}, \qquad (2)$$

where σ^2_{max} is the variance of the daily maximum temperature, σ^2_{min} that of the daily minimum, and $C_{max-min}$ is the covariance of the daily maximum and minimum temperatures.

Relationships (1) and (2) are postulated to hold only for normally distributed variates, without delineation of their robustness, or relative validity, if the temperature variate departs in various ways from normality. The relationships hold in the absence of year to year correlation of the monthly variate provided no year to year trend exists. Concerning these assumptions, Reference 1 has this to say on page 2:

> "The first assumption...that monthly temperatures follow the normal distribution...is supported directly by U.S. Weather Bureau contract studies of temperature normals...and checked for about 50 stations by Bumer(3)...
>
> "The second assumption is that monthly temperatures in successive years are independent in the probability sense. Bumer found no evidence to the contrary.

> "The third assumption...is that no long term temperature trend affects the applicability of past records."

This is not a conclusive basis for admitting the normality assumption and related assumptions. Roschke[11] found evidence of a small but definite third moment in actual temperature distributions. One also could argue intuitively for a dislocation or skewness of the temperature frequency curve near water areas. A drop in the temperature of turbulent air to below the freezing point of water would be retarded noticeably while surfaces of the freezing water impart latent heat of fusion to the adjacent atmosphere. What role evaporation may play in causing further skewness (or possibly partially balancing out the freezing dislocation) in the temperature frequency curve is not known.

4. MODEL FOR ANALYSIS

Despite realization that troublesome complications caused by small but consistent departures of hourly temperature from normality may arise, a model is chosen for analysis which would be much facilitated if temperatures were to follow closely the normal probability distribution law. It is felt that practical adjustments for skewness can probably be developed if the first steps are fruitful. After all, it is a rare child who starts by running without having first learned to crawl.

In view of the goal of matching the variability structure in Figure 1 with a related autocovariance structure, component by component, the model chosen for analysis of autocovariance is

$$T = \mu_T + \tau_a + \tau_d + e_T ,$$

Where T is an instantaneous temperature reading, μ_T is the overall temperature mean at that locality; τ_a is the annual temperature trend (i.e., the appropriate point on a cyclic curve of the difference between long term daily temperature means and μ_T); τ_d is the diurnal, or average daily, temperature trend; and e_T is the residual, or net departure of T from the sum of the three norms.

Substituting

$$\tau_a' = \mu_T + \tau_a$$

we have

$$T = \tau_a' + \tau_d + e_T .$$

The model is sketched in Figure 2.

5. TOPICS TO BE EXPLORED

Pertinent characteristics of temperature persistence are to be evaluated by constructing coefficients of lag correlation, r, which are functions of the e_T, and performing statistical tests of hypothesis about the r.

In meteorology, it is expected that temperature does, indeed, tend to persist, or remain nearly the same with respect to its norms, for short periods of time. It is expected that temperature will tend to persist less strongly over longer periods as synoptic changes (i.e., short period weather changes) set in. If a measure, r, of persistence is used that varies between -1 and +1, the situation expected by the writer is that sketched in A of Figure 3.

The sketch, A, disregards the very real possibility that typical alternation of weather situations and sequences, occurring naturally a few days apart, would result in "negative persistence" over such intervals (B of Figure 3). The writer knows of no evidence that alternation of weather patterns is that regular and consistent. Thom[14] found no such evidence in analyzing unadjusted temperature readings over 24-, 48-, 72-, 96-hour and longer lags. Consideration of negative persistence would greatly complicate follow-on studies based on the type of general temperature model which is discussed above. In any event, effects of negative persistence on hourly lags is not expected to be significant.

The ideal measure of persistence might prove to be the limiting value to which r, the numerical persistence, converges as the length of the period over which r is measured tends to zero (assuming such a limiting value exists and is not unity). No such calculation will be attempted here. Not only is the standard interval for repetition of temperature measurements a minimum of one hour at first order United States weather stations (and historically longer at many other weather stations), but also the United States Weather Bureau practice is to read temperatures only to whole Fahrenheit degrees.

Any notion that finer readings are realistic will soon be dispelled by experience in rejoining separated thermometer columns at a climatological headquarters of the Bureau, or calibrating first-order remote reading temperature bridges. Vagaries of small air pockets also affect instrumental responses at the instant of reading, and no benefit is seen in complicating wide area studies by such microclimatic considerations at this stage of preliminary exploration of the general model discussed above.

The resulting precision, at best

$$T = \text{"true" instantaneous temperature} \pm 0.5\ F.^{o},$$

is considered too wide for any attempt to determine a limiting value as r is taken over shorter and shorter periods that tend to zero. The practical limitation of one-hour persistence is accepted for this exploratory inquiry, and this is the only lag to be analyzed.

A measure of persistence, r, is desired which may be used fruitfully as a supplementary independent variable in the regression of phenological measures on GDD. Since r must be computed from some particular series of readings, freedom of secondary correlation of any computed r with r computed from another series nearby in time will avoid complications in statistical inference. For example, a situation such as is depicted in A or B of Figure 4 could severely limit the mathematical tractability of r. An r whose behavior is random with respect to time (C in Figure 4) would seem more promising.

The tendency of a particular formulation of r to behave like A or B in Figure 4 is to be investigated here. Either situation could be expected to result in reducing the number of runs of successive values of r, computed from readings evenly spaced in time, above or below a reference value of r. The objective of this analysis is to isolate a formulation of r which does not show this second-order persistence.

The assumption that the population from which r is sampled has Pearsonian

$$H_o: \quad \rho = 0$$

also is to be tested statistically from an adequate series of actual temperature data.

6. TEST STATISTICS

A possible statistic for this time series analysis is the straightforward coefficient of correlation, r_N, between N separate pairs of 2N values

$$X = \hat{e}_T$$

derived from individual successive readings of hourly temperature, namely,

$$r_N = C_{2N} / \sqrt{V_{2N-1} \quad V_{2N}}$$

$$= \frac{X_1X_2 + X_3X_4 + \ldots + X_{2N-1}X_{2N} - \sum_{i=1}^{N} X_{2i-1} \sum_{i=1}^{N} X_{2i} / N}{\sqrt{\left[\sum_{i=1}^{N} X_{2i-1}^2 - \left(\sum_{i=1}^{N} X_{2i-1}\right)^2 / N\right]\left[\sum_{i=1}^{N} X_{2i}^2 - \left(\sum_{i=1}^{N} X_{2i}\right)^2 / N\right]}},$$

where N is the number of sample pairs; C and V are the sample covariance and (subset) variances, respectively; and the X's are considered to be normally distributed with the same variance about the same mean (independently distributed also if there were no persistence). If

$$N \geqslant 500$$

(see page 336 of Reference 7*), the null assumption could be made that

$$r_N = NID\ (0,\ 1/N).$$

By means of this statistic, the existence of persistence could be tested.

However, it is in the implicit comparison of pairs of X's that the above statistic utilizes the information about persistence that is present in the sample data. It takes 2N X's to provide N pairs. Potentially, there are 2N - 1 pairs of successive values among 2N X's, and in routine computer processing, at least, the bulk of the work is in faithfully obtaining the X's, processing them for entry and entering them in the computer.

For this and other reasons, a better statistic for the present kind of analysis would be a lagged correlation coefficient. Anderson[2] ascribes the circular definition of such a coefficient to Hotelling, namely,

$$R_{LN} = C_{LN} / V_N$$

$$= \frac{X_1 X_{L+1} + X_2 X_{L+2} + \cdots + X_N X_L - \left(\sum_{i=1}^{N} X_i\right)^2 / N}{\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2 / N},$$

* However, Mood (page 314 of Reference 9) suggests that $N \geq 25$ is sufficient for valid use of Fisher's z, and Snedecor (see page 150 of Reference 12) apparently places no lower limit on N for valid application of Fisher's z.

where L is the lag, here, in hours; C and V are the lag covariance and variance, respectively; and the X's are considered to be normally distributed about the same mean. The central large-sample distribution for ${}_{L}R_{N}$ is found (page 6 of Reference 2) to be

$${}_{L}R_{N} = \text{NID}\left[-1/(N-1),\ (N-2)/(N-1)^2\right]$$

if H_0 is true. For data series with

$$N = 5\ (1)\ 15\ (5)\ 75,$$

a table of critical values for L = 1 and

$$\gamma = 1 - \alpha = \begin{cases} .95 \\ .99 \end{cases},$$

where γ is the level of confidence and α is the risk of a wrong decision in testing null hypotheses, is provided on page 8 of Reference 2.

To provide a simplification in calculations, the noncircular lag correlation coefficient is used for the present exploratory study, namely,

$${}_{L}r_{N} = {}_{L}C_{N+L} / \sqrt{V_N V_{N+L}}$$

$$= \frac{X_1X_{L+1} + X_2X_{L+2} + \cdots + X_NX_{L+N} - \sum_{i=1}^{N} X_i \sum_{i=L+1}^{L+N} X_i / N}{\sqrt{\left[\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2/N\right]\left[\sum_{i=L+1}^{L+N} X_i^2 - \left(\sum_{i=L+1}^{L+N} X_i\right)^2/N\right]}}, \quad (4)$$

where ${}_{L}C_{N+L}$ is the autocovariance. Anderson's distribution for ${}_{L}R_{N}$ is believed to hold more closely for ${}_{L}r_{N}$ than any other distribution which the writer has found in available literature.

However, $E({}_L r_N)$ will be taken as zero instead of $-1/(N-1)$ in the interest of applying a sufficiently severe test criterion.

For the reasons discussed in Section 5, L is taken as one hour. In the interest of sufficient degrees of freedom to make the test statistic also tractable for the kind of large sample analysis which is desirable* in climatological work, without at the same time overlapping into a different season, a six week period during the flat winter portion of the annual trend is selected, namely, January 1 (to avoid overlapping separate years for practical reasons) through February 11. This provides 42 days, or 1008 hours, of sequential temperature values.

The statistic by means of which the existence of persistence is to be tested becomes

$$ {}_1r_{1008} = \frac{\sum_{i=1}^{1008} x_i x_{i+1} - \sum_{i=1}^{1008} x_i \sum_{i=2}^{1009} x_i \Big/ 1008}{\sqrt{\left[\sum_{i=1}^{1008} x_i^2 - \left(\sum_{i=1}^{1008} x_i\right)^2 \Big/ 1008\right]\left[\sum_{i=2}^{1009} x_i^2 - \left(\sum_{i=2}^{1009} x_i\right)^2 \Big/ 1008\right]}} . \quad (4a) $$

* In meteorological work, fields of climatic expectancy often are approximated by plotting site values on maps and drawing isopleths as a tool to interpolate or even extrapolate to other localities of concern. Interest in the usual statistical confidence limits tends to be replaced by interest in a tolerable physical error in locating the isopleths. This suggests only an underlying need that the point values against which isopleths are drawn be definitely more precise, such as by an order of magnitude, than the physical considerations guiding the isopleth analysis, so that the (physical) precision of the isopleth field can control the error inherent in such a chart. One reason for interest in correlation of the temperature variate through space, mentioned above, is to buttress by statistical controls the physical understanding, or lack of understanding, which governs precision of most isopleths. For isopleth charts in which time is a coordinate, lag correlations through time can play a like role.

For testing possible second-order persistence, subsets are selected as follows

$${}_1r_{24} \qquad (4b)$$

$${}_1r_{72} \qquad (4c)$$

The reason for interest in 4c is that the writer has the opinion from applied work with a fair amount of temperature data that temperature spells of two to four days' length tend to occur in winter in middle latitudes. As a guide to further research on a variance-autocovariance model of temperature, it is desirable to isolate a relatively homogeneous unit of statistical aggregation.

7. TESTS

In order to verify with high confidence that the strongly suspected persistence exists, the following one-tailed test is made on the statistic (4a).

$$\begin{aligned} H_o&: \; {}_1\rho_{1008} = 0 \\ H_1&: \; {}_1\rho_{1008} > 0 \end{aligned} \qquad (5)$$

Since

$$N = 1008 > 75$$

(see page 6 of Reference 2), the assumption is that ${}_1r_N$ is distributed under random sampling as

$${}_1r_N = \text{NID} \left[0, (N-2)/(N-1)^2 \right]$$

when persistence is absent. H_o of (5) is rejected, verifying H_1 at significance level α, provided

$${}_1r_N \geq 0 + {}^{+}t(\alpha)\sqrt{N-2}/(N-1) = {}_1r_N^{*}.$$

Since the establishment of real physical persistence is basic to further exploration of a variance-autocovariance model, the confidence level,

$$\gamma = 1 - \alpha \ ,$$

is set very high at

$$\gamma = .999.$$

Accordingly,

$$\alpha = 1 - \gamma = .001.$$

From page 473 of Reference 9,

$${}^{+}t(.001) = 3.090.$$

The test criterion becomes

$${}_1r_{1008}^{*} = 3.090 \ \sqrt{1006}/1007 = .0973. \qquad (6)$$

In attempting to detect any second-order persistence which may exist, such that ${}_1r_{24}$ for a particular 24-hour period is correlated with ${}_1r_{24}$ for the preceding 24-hour period, a test using runs above and below the median is made on a sequence of the statistics (4b). The run test developed on pages 177-182 of Reference 6 is applied, supplemented by the more nearly exact significance values tabulated in Reference 13. This nonparametric test is selected since the ${}_1r_N$ are not expected to be distributed normally, as will appear in the sequel (see Figure 6).

Because interest here is in detecting any important second-order persistence if reasonably possible, the significance level is set much higher than for test (6), at

$$\alpha = .2.$$

The hypothesis of randomness is rejected if

$$p\ (u \leq u_o) \leq .2$$

for

$$n_a = n_b = 21, \qquad (7)$$

where n_a is the number of values of $_1r_{24}$ above and n_b below the median, and u_o is the number of runs counted. The achieved significance level will be determined from those in Reference 13 after u_o has been found.

In attempting to determine whether $_1r_{72}$ may provide a more homogeneous unit for further research on a variance-autocovariance model of temperature, a sequence of statistics (4c) leads similarly to rejection of the hypothesis of randomness if

$$p(u \leq u_o) \leq .2$$

for

$$n_a = n_b = 7. \qquad (8)$$

7. DATA SOURCES

Hourly temperature data for Los Angeles Weather Bureau Office from 00 hours, January 1, 1961 through 00 hours, February 12, 1961 were taken from References 17 and 18. See Table 1.

Daily temperature normals were taken from Reference 15. See Table 2.

8. DATA EVALUATION AND PROCESSING

It is, of course, not possible to state that all published Weather Bureau data are free of reading, transcription, and keypunching errors. However, the routines used to avoid such errors are very good, in the present writer's judgment, and the published data are of high quality. In any event, economics will dictate that these be the data for most climatological analyses.

The normals are simply daily averages taken over the period 1921 through 1950, during which period the official thermometer location was moved four times[16]. However, it is the definite orally stated opinion of S. Miller, Meteorologist at the Los Angeles Weather Bureau Office, that these station breaks do not affect the normals by more than 0.2 F° at most. Since the precision of the readings is at best ±0.5 F°, this is not deemed critical.

The steps used in processing the data are those shown in Tables 3 through 5.

All data processing was accomplished by desk calculator. This is an extensive processing based on over 1000 data, and it is recognized that the risk of error is sizeable, while the tolerance for error is small. Techniques used to control the errors include:

a. The body of Table 4 was filled out and partially checked by computing the first, or 1 January, entry on a Marchant calculator, differencing mentally for successive entries until the annual trend value changed on 7 January, and recomputing the 6 January value on the calculator as a check. A like procedure was followed from 7 January through 26 January and from 27 January through 11 February. Other values, such as those where the sign of the residual changed, also were checked by recomputing in individual cases of possible doubt. Among other things, a running mental check was maintained on the reasonable vertical continuity of the residuals as they were determined, and suspicious values recomputed. Several errors were detected by these means and corrected.

b. While collecting sums of squares on a Marchant calculator, the residual was first entered on the keyboard, then deliberately "forgotten" and copied from the keyboard dial into the multiplier column of keys in an effort to make the sum of residuals accurately reflect the numbers entered in the keyboard as a check on the sum of squares. The sums of residuals were then computed independently on a ten-key adder and compared with those copied from the calculator dials.

c. While collecting sums of lagged products, a reverse procedure was used. The lagged residual was entered into the multiplier, then transferred by memory into the keyboard and then checked back against the tabulated value before forming the next cross product. Also, one erroneous residual was detected and corrected by attention to reasonableness during the calculation of lagged products.

d. Only 24 squares or products were accumulated at a time, it being felt that to include more would invite error with high probability. The 42 partial sums were then combined on a ten-key tape adding machine.

e. ΣX, $\Sigma|X|$ and "Σ" X_{i+1} were reconciled in that order to detect errors (7 were found), and the cross-products checked for the possibility of erroneously exceeding the corresponding sums of squares.

f. The 42 partial ΣX, $\Sigma X_i X_{i+1}$, "Σ" X_{i+1}, ΣX^2 and $\Sigma|X|$

were summed in turn on a ten-key tape adder and the tapes checked against the tabulated values.

g. Table 5 was calculated on a Friden calculator with square root feature. Each keyboard entry for subtraction, multiplication, square root and division was mentally read back from the keyboard against the tabulated value before depressing the appropriate operation key. Multiplier entries and dividends were read back against tabulated values after reaching the lower and upper dials, respectively. After tabulating each result, a few seconds were allowed to lapse, after which the new tabulated value was read back against the appropriate calculator dials before clearing the machine in the next operation.

h. All tests and transcriptions of data were checked at least once, and all tabulations scanned for figures that might be out of line.

9. ERRORS

Regarding the seven particular errers in sums of products, mentioned in paragraph 8e above, six were ascribed to inversions by casting out nines from the amount of the net correction. The other error in sums of products was ascribed to entry of an erroneous digit into the calculator.

Assuming all such digit-entry errors were detected,

$$p\ (\text{error}) = 1/2017 \doteq 5 \times 10^{-4},$$

since 1008 individual cross products and 1009 squares were calculated. The same probability is assumed for another such error, which may or may not compensate for the first error. By cursory inspection of

Table 4, the number of digits in the residuals is found to average at least 3, yielding probability

$$p\ (\text{compensation}) \leq 10^{-3}$$

that the second erroneous digit would compensate if made in calculating the same subsidiary sum of 24 products. This could occur only in one of the other 23 products accumulated in the subsidiary sum. There are 42 such sums of cross products and an additional 42 sums of squares.

Therefore, the probability may be estimated to be

$$p\ (\text{undetected compensation}) \doteq 5^2 10^{-(4+4+3)} 23(84) + \text{higher-order terms}$$

$$\doteq 5 \times 10^{-7}$$

that two digit-entry errors, affecting one ${}_1r_{24}$, one ${}_1r_{72}$ and one ${}_1r_{1008}$, would occur which would permit the reconciliation of $\sum X$, $\sum |X|$ and "$\sum$" X_{i+1} to be satisfied without correction of the errors. This probability is low with respect to the significance levels chosen for the present statistical tests and is entirely negligible with respect to the assumptions and approximate distributions used.

The probability of undetected error increases as longer data series increase the factor, 84. Therefore, it could well rise above the probability of data-entry errors into digital computers, especially with verification of the data-entry punched cards. The probability of accidentally compensating an error of inversion must be too small to be worth estimating, and no attempt will be made to evaluate the risk of undetected errors like the unreasonable residual. However, other errors may well have crept in to which no thought was given, and it is generally recognized that overly long human calculations often foster excessive errors.

These observations tend to suggest the practical desirability of digital computers in order to control errors in large scale application of such a temperature model. Considerations of economy and time schedules reinforce this opinion.

11. RESULTS

The result of test (5) as quantified in (6) is (see Table 5) that

$$ {}_1r_{1008} = .965 >> .0973 = {}_1r^*_{1008}. $$

Consequently, H_0 is rejected.

The result of the test of randomness as quantified according to (7) is (see Table 6) that the median value of ${}_1r_{24}$ is such that

$$ .843 < \text{median } ({}_1r_{24}) < .863 $$

and that

$$ u_o = 18. $$

In order to evaluate this result, significance values tabulated in Reference 13 are extrapolated graphically in Figure 7, indicating that

$$ p\ (u \leq 18) \doteq .14 < .2 $$

for

$$ n_a = n_b = 21. $$

Consequently, the hypothesis that successive ${}_1r_{24}$ are random is rejected.

The result of the test of randomness as quantified according to (8) is (see Table 6) that the median of $_1r_{72}$ is such that

$$.898 < \text{median } ({}_1r_{72}) < .901$$

and that

$$u_o = 9.$$

It is found[18] that

$$p\ (u \leq 9) = .79 >> .2$$

for

$$n_a = n_b = 7.$$

Consequently, the hypothesis of randomness is not rejected.

12. DISCUSSION OF RESULTS

The high significance of $_1r_{1008}$ is no surprise. Thom[14] found persistence into the fourth day which he considered significant, whereas the statistics (4) consider persistence only for one hour. It should be mentioned that Thom used raw temperatures and not residuals from annual and diurnal trend. Persistence has not been verified heretofore in the residuals to the writer's knowledge.

However, the statistic appears to have a defect. Table 7 compares each $_1r_{72}$ with the average of the three relevant $_1r_{24}$, and also with the maximum of the three relevant $_1r_{24}$. In all cases except two, $_1r_{72}$ is closer to the maximum than to the average, and in seven instances $_1r_{72}$ exceeds the maximum of its constituent $_1r_{24}$. In all fourteen cases, $_1r_{72}$ exceeds the average of its constituent $_1r_{24}$. It is suggested that this results from geometric averaging in the denominator and implicit arithmetic averaging in the numerator of the correlation coefficient. The value of the statistic ${}_L r_N$ calculated from readings equally spaced in time tends to rise as N increases. The same effect is present in ${}_1 r_{1008}$ as compared with its constituent ${}_1 r_{72}$ and ${}_1 r_{24}$.

This effect leads the writer to wonder whether the large N reduces the efficacy of the test made by means of ${}_1 r_{1008}$. The table on page 8 of Reference 2 provides the exact significance values, ${}_1 R_N^*$, for

$$\alpha = 1 - \gamma = .01$$

for ${}_1 R_{20}$ and ${}_1 R_{25}$ as follows.

$${}_1 R_{20}^* = .432$$

$${}_1 R_{25}^* = .398.$$

Interpolating linearly,

$${}_1 R_{24}^* = .405.$$

Making the modification proposed in Section 6,

$${}_1 r_{24}^* = {}_1 R_{24}^* + 1/(N-1) = .405 + .043 = .448.$$

All of the calculated values for ${}_1 r_{24}$ listed in Table 5 except one are significant by this test. The exception is .439 for 9 February. Apparently, there may be individual days when weather variability suffices to cancel out the net persistence of air temperature as measured by ${}_1 r_{24}$, at least in winter, although .439 is significant at the .05 level by this test.

For

$${}_1r_N = {}_1r_{72},$$

N is sufficiently close to Anderson's recommended 75 that his large-sample approximation will be used here, with the modification proposed in Section 6. We have

$${}_1r^*_{72} = 3.090\ \sqrt{70}/71 = .364.$$

The smallest value of ${}_1r_{72}$, namely, .794 for 4-6 February, is well above ${}_1r^*_{72}$.

The above tests suggest strongly that positive persistence from hour to hour over 72-hour periods is a real and universal property of temperature residuals from diurnal and annual trend. It would be desirable to test this assumption for a wide selection of sites and seasons. If the suggested persistence is verified and if lagged correlations are, indeed, found to be a manageable and useful measure of persistence, then maps should be developed showing isopleths of constant persistence, where the measure of persistence is some function of ${}_1r_N$ and N is in the neighborhood of 72 hours.

A recommendation cannot be made at this time as to the exact function to be used. One factor will be the determination of that function which best serves as a supplementary independent variable in regressions of phenological measure on GDD. Another will be the statistical tractability of the measure used. In developing the measure of persistence, some central value of ${}_1r_{72}$ might be obtained such as a geometric mean or a median of individual $_1r_{72}$ over an entire season for as many as 6 or 8 years.

Perhaps, despite the evident tendency of $_1r_N$ to rise with length of the data base, $_1r_{1008}$ may prove as useful as shorter-period measures in the suggested regression calculation. Perhaps the square root of the autocovariance will prove useful and will match the standard deviation of temperature in deriving a pair of supplementary measures for phenological studies and other applications.

In developing isopleth maps for selected "seasons", confidence limits on the persistence measure that is adopted should be derived as a guide to how closely the isopleths need to follow site values, especially where the data sites are sparsely located. As to the "season", the use of monthly values may induce minor difficulties since

$$28 \leq d \leq 31,$$

where d is the number of days in a month. However, practically all climatological analyses are based either on months or on trimonthly seasons. While the winter trend is small*, the longer and nonlinear spring and autumn trends suggest a possible difficulty in using $_Lr_N$ based on entire seasons. An example of the anticipated difficulty is that $_Lr_N$ might prove less valuable in producing regressions of crop growth on a series of independent variables including $_Lr_N$ if the base for $_Lr_N$ is taken too long, because the

* So small, in fact, that it probably could have been ignored in the present calculations with negligible effect.

persistence characteristic itself may have a directed trend during times of progressive weather advancement such as spring and autumn. Perhaps "seasons" of two months would be a practical compromise. At least, there is no obvious trend in either ${}_1r_{24}$ or ${}_1r_{72}$ over the six weeks calculated here (see Figure 5), albeit most of the values are far from that of ${}_1r_{1008}$.

Whereas the test (7) indicated that second-order persistence probably exists in the ${}_1r_{24}$, the test (8) showed no indication of second-order persistence in the ${}_1r_{72}$ as measured in terms of too few runs above and below the median. In fact,

$$p\ (u \leq u_o) > .5.$$

If too many runs are found consistently in further studies along this line, a meteorological reason for this should be sought. However, this single weak result would not appear to warrant the further inquiry.

The present analysis suggests that ${}_1r_{72}$ is a better candidate than ${}_1r_{24}$ for the basic temperature autocovariance statistic in an attempt to construct a variance-autocovariance model of air temperature, as well as for studies of regression of phenological measures on temperature expectancy, including measures of temperature location, variability and persistence.

On the whole, the results encourage the view that a matrix of mean, variance and autocovariance of air temperature at a point (in many ways like the variance-covariance matrix of regression methods), can be developed and its linkages, or interrelationships, can be found. Further work for other sites and periods is desirable to test the present results against independent data and to improve or extend them.

13. CONCLUSIONS

Hour to hour persistence of temperature has definitely been shown to exist over 72-hour periods at Los Angeles Weather Bureau Office site during the first six weeks of 1961.

Correlation coefficients of hourly Weather Bureau temperature readings, adjusted for annual and diurnal trend, with readings for the next succeeding hours may be useful measures of temperature persistence if the number of readings used for calculating the coefficients is in the neighborhood of 72 successive readings.

Numbers of successive readings in the neighborhood of 24 probably admit second-order persistence into further analysis.

From these preliminary results, further work toward construction of a variance-autocovariance model of air temperature at a point appears feasible.

Use of digital computers is recommended for follow-on studies, to control errors as well as to reduce costs and the time required.

14. SUMMARY

Lagged correlations of hourly residuals of air temperature at a single point were calculated from data for a single six-week period near the flat portion of the annual temperature curve. A sub-period was determined for which successive coefficients appear to be randomly distributed through time about their median value. Convincing evidence of positive lag correlation over a sub-period of approximately three days indicated that appropriate auto-covariance statistics can be used to measure persistence of temperature residuals through time in developing a more complete mathematical model of climatological temperature.

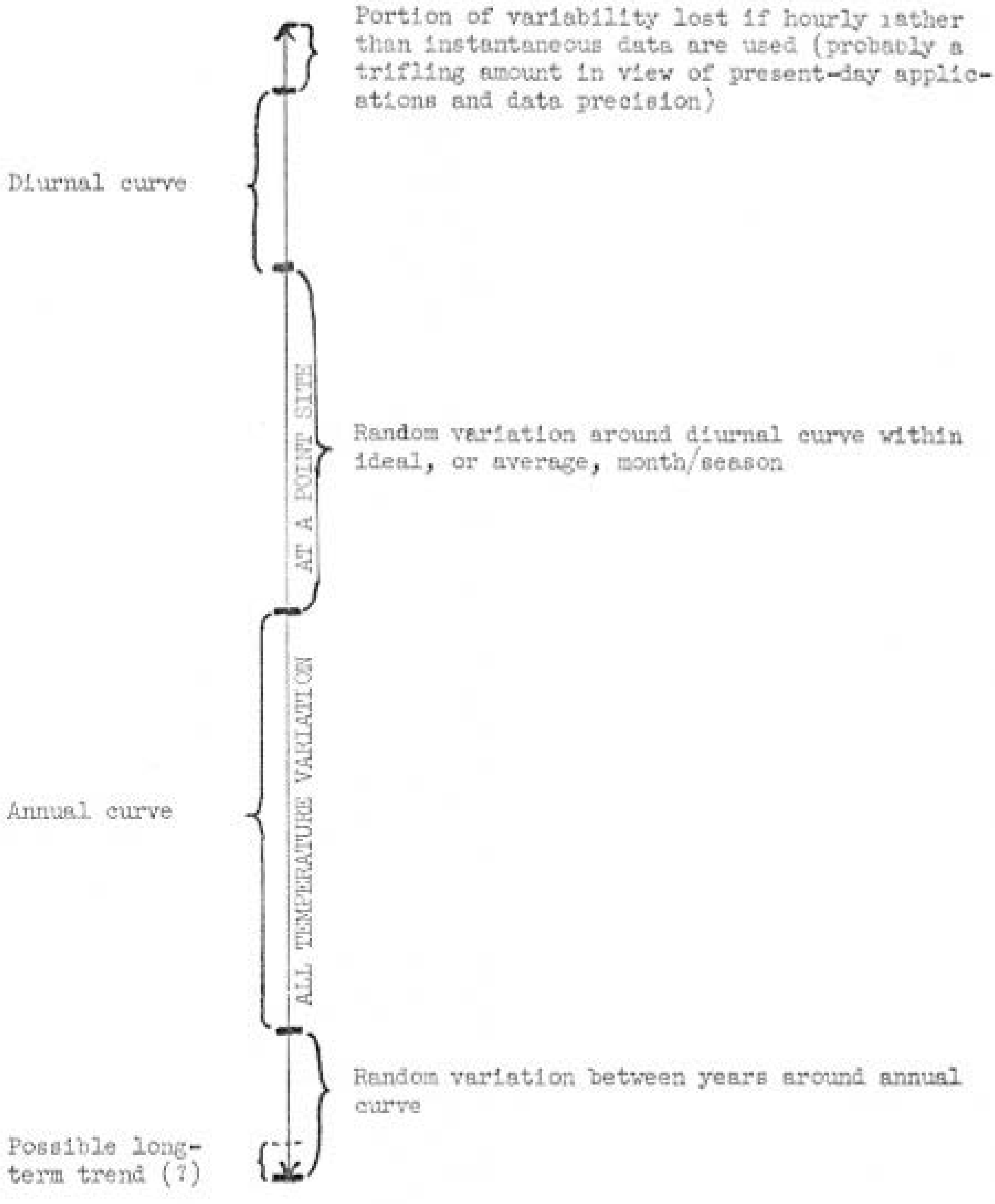


Figure 1 - Schematic of Temperature Variation Model

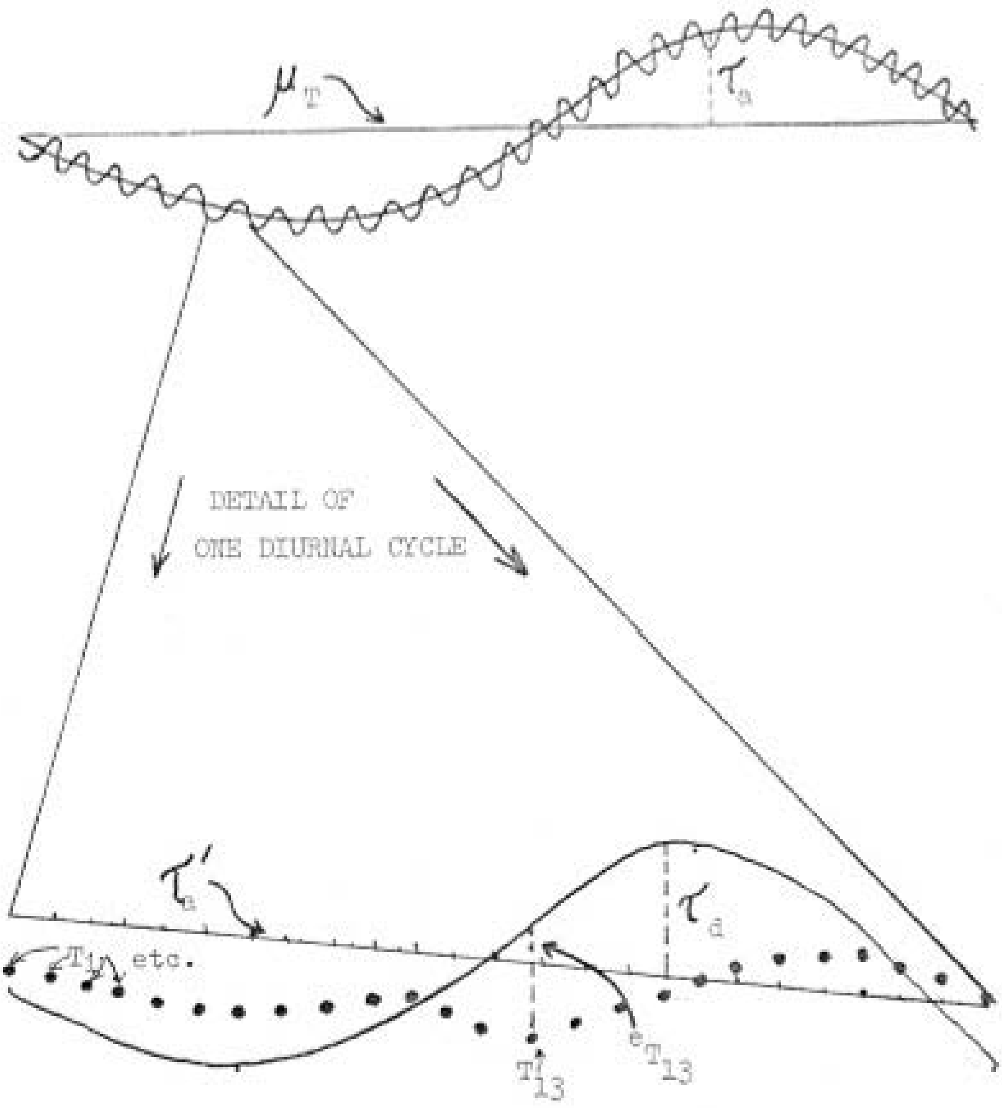


Figure 2 - Schematic of Model for Analysis

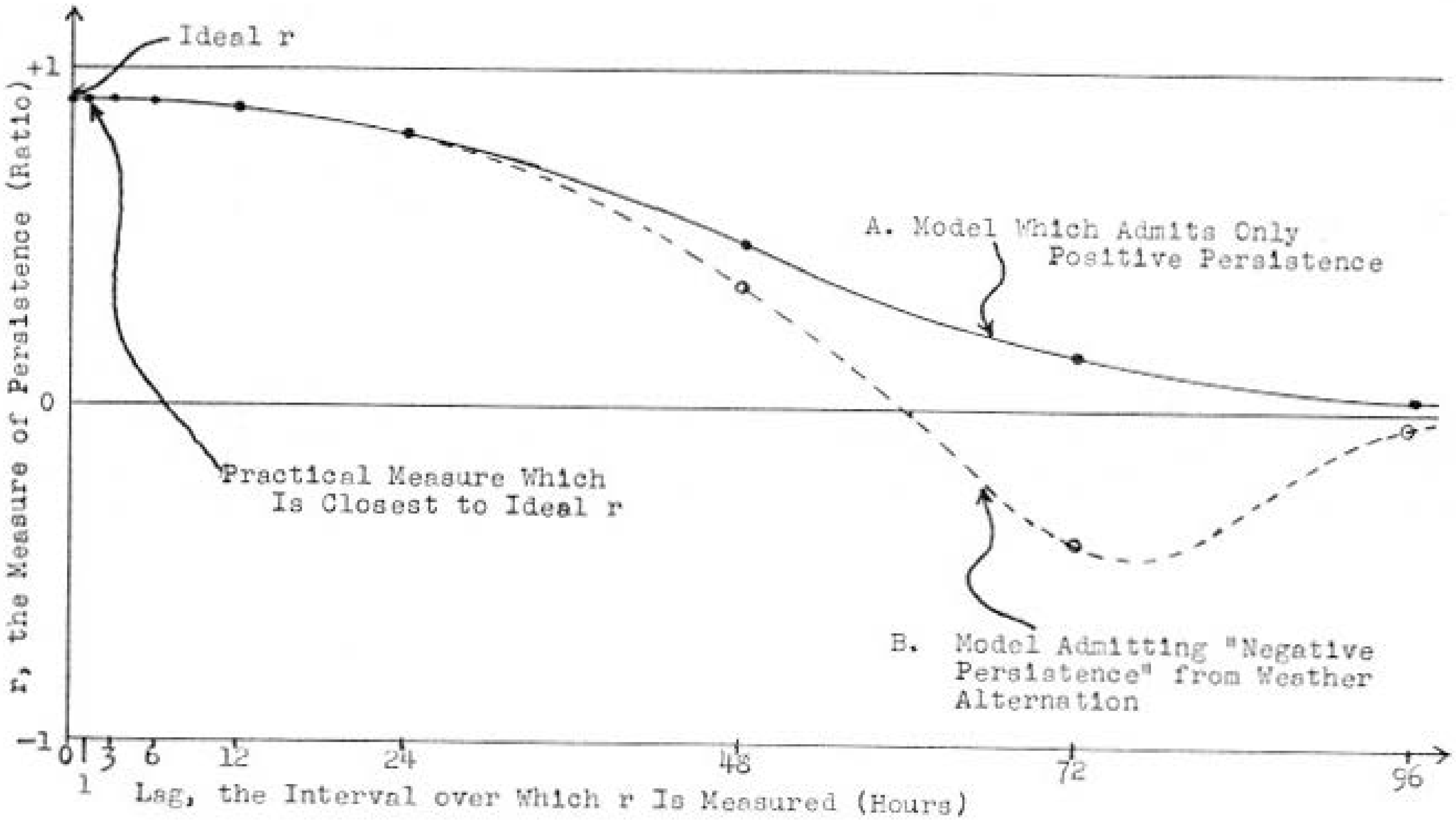
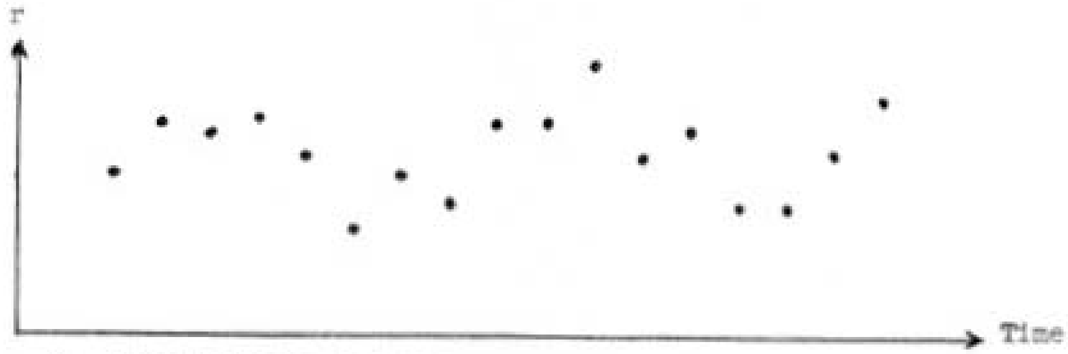
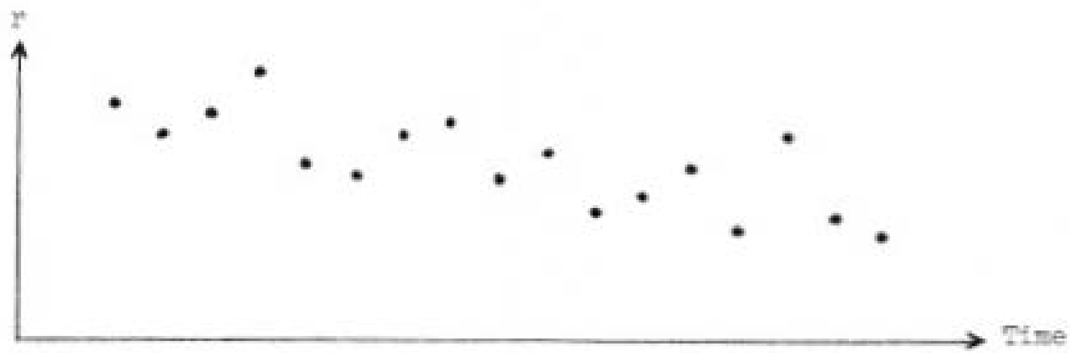


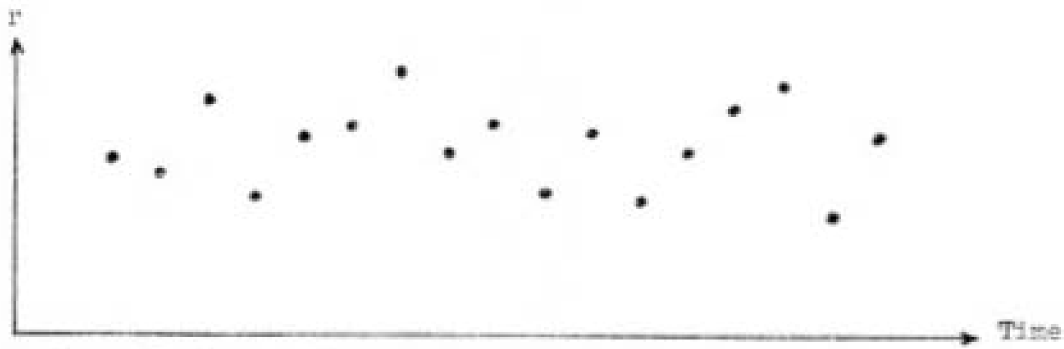
Figure 3 — Idealization of Persistence as a Function of Lag



A. Cyclical Change in r



B. Time Trend in r



C. r Random in Time

Figure 4 - Some Possible Characteristic Behaviors of a Persistence Measure, r

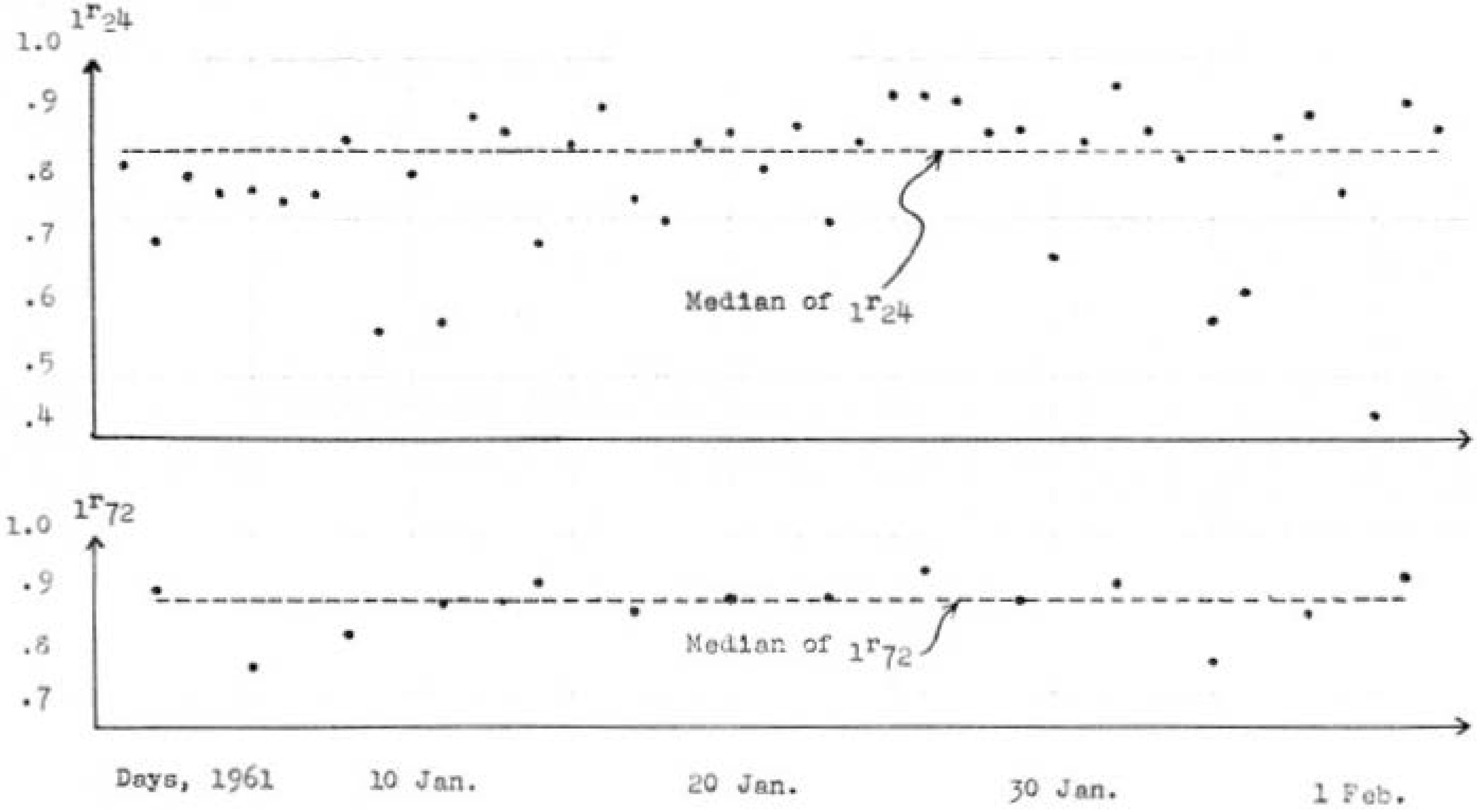


Figure 5 – Successive Lag Correlations $_1r_{24}$ and $_1r_{72}$

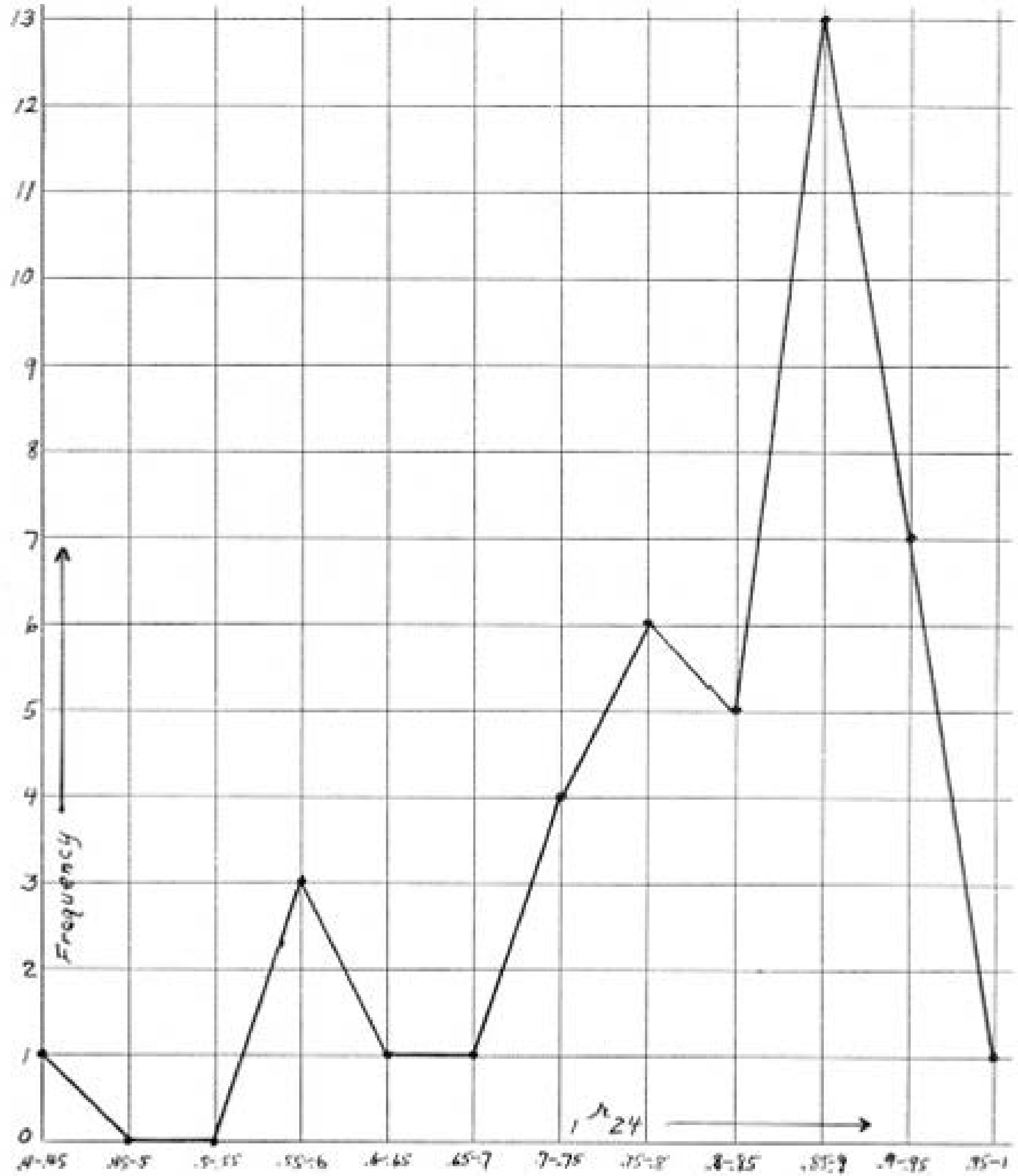
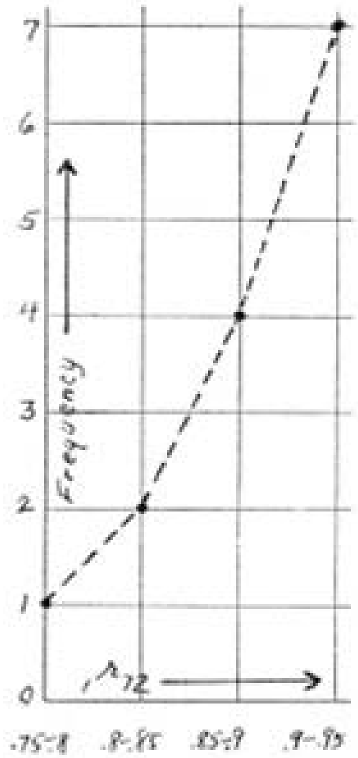


Figure 6 — Frequency Polygons for Lag Correlations $_1r_{24}$ and $_1r_{72}$



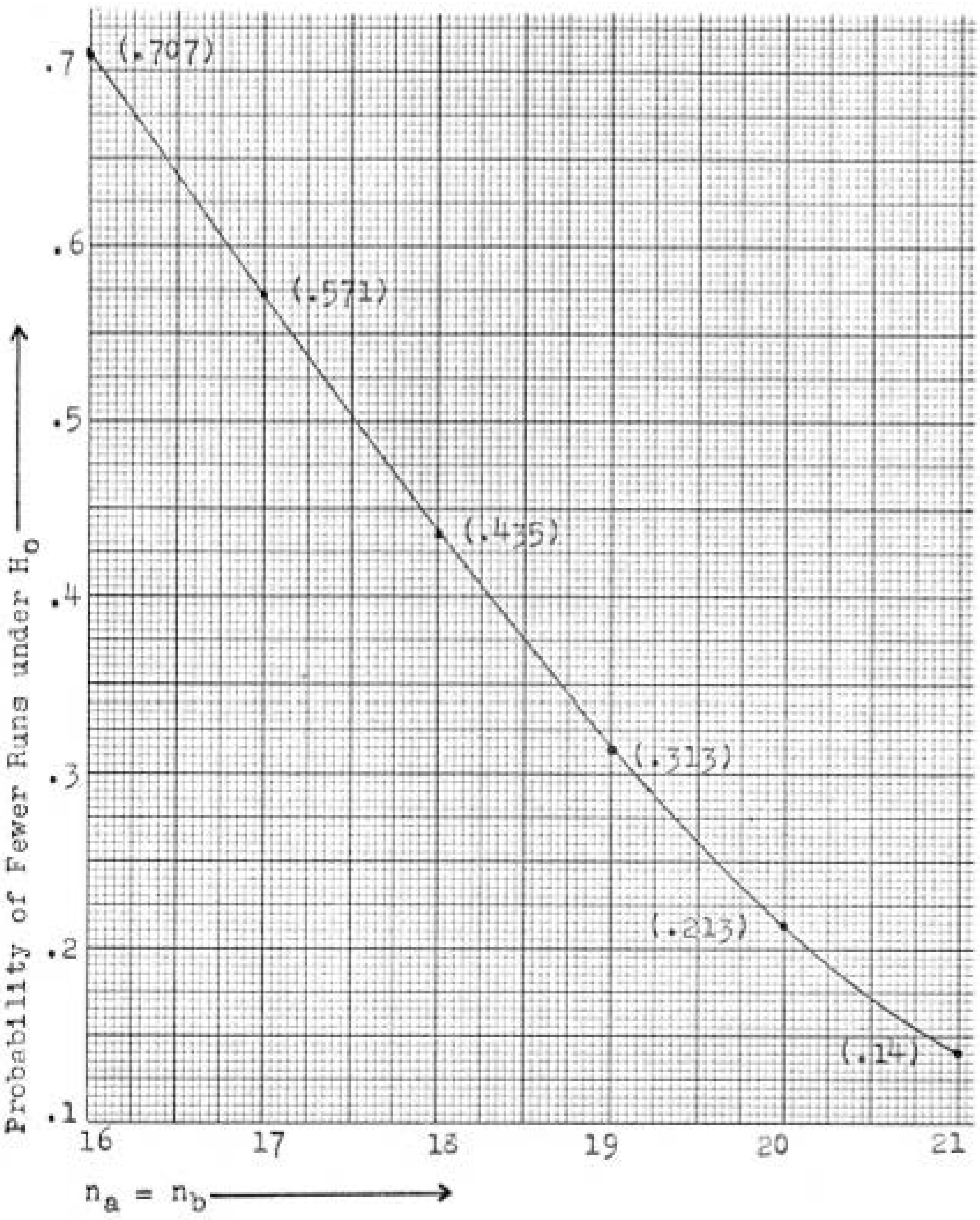


Figure 7 - Extrapolation of Swed-Eisenhart Criterion for 18 Runs

Table 1 - Los Angeles Hourly Temperatures,
January to 12 February 1961

DATE, 1961	1 Jan	2 Jan	3 Jan	4 Jan	5 Jan	6 Jan	7 Jan
HOUR							
0000	48	48	51	53	54	56	54
0100	48	47	52	54	51	53	52
0200	49	45	53	52	50	50	50
0300	50	45	50	52	53	52	52
0400	50	44	48	51	49	49	52
0500	49	43	48	50	48	52	51
0600	49	43	46	51	48	53	50
0700	48	43	48	50	46	54	50
0800	47	41	51	50	46	54	50
0900	48	45	57	55	58	60	57
1000	51	51	61	59	62	66	63
1100	55	58	64	63	66	70	68
1200	57	59	66	67	68	72	70
1300	60	63	69	69	71	73	73
1400	62	65	70	69	72	70	73
1500	62	63	69	69	71	69	69
1600	59	60	69	68	70	67	67
1700	56	58	65	66	67	64	65
1800	54	55	62	63	65	62	61
1900	53	55	60	61	62	60	58
2000	52	53	58	60	58	58	57
2100	52	52	56	58	55	55	57
2200	50	52	55	57	54	53	57
2300	49	49	54	56	57	55	57

Table 1, Continued

DATE, 1961	8 Jan	9 Jan	10 Jan	11 Jan	12 Jan	13 Jan	14 Jan
HOUR							
0000	59	57	60	64	60	53	61
0100	55	57	56	62	59	52	59
0200	54	56	59	61	58	53	57
0300	55	54	58	61	58	52	56
0400	54	55	53	61	59	50	56
0500	53	57	51	59	57	50	60
0600	51	54	52	59	56	49	57
0700	52	53	50	58	57	49	56
0800	55	58	55	63	58	54	60
0900	59	64	62	66	61	59	65
1000	66	65	67	70	66	66	70
1100	71	69	72	73	71	70	74
1200	73	71	75	76	73	73	77
1300	76	74	77	78	74	75	80
1400	77	75	77	77	72	73	80
1500	75	73	77	77	70	70	79
1600	72	70	75	75	69	70	79
1700	69	67	70	73	66	68	74
1800	67	66	68	70	61	66	72
1900	65	63	68	67	60	63	72
2000	62	59	67	65	58	61	69
2100	60	58	66	64	57	59	66
2200	59	57	64	63	57	58	65
2300	59	59	63	63	56	61	63

Table 1, Continued

DATE, 1961	15 Jan	16 Jan	17 Jan	18 Jan	19 Jan	20 Jan	21 Jan
HOUR							
0000	62	61	64	66	61	61	63
0100	61	59	63	62	60	64	63
0200	59	57	64	63	59	64	61
0300	56	55	64	60	59	62	60
0400	56	55	62	58	56	62	60
0500	55	54	62	60	55	64	59
0600	55	55	60	58	56	63	57
0700	54	55	59	60	55	64	57
0800	58	59	62	62	58	64	57
0900	63	66	68	66	65	65	59
1000	71	73	72	71	72	68	64
1100	75	78	76	76	74	71	70
1200	78	79	80	79	77	73	71
1300	80	82	83	82	78	72	73
1400	81	83	82	82	79	72	73
1500	79	81	80	81	79	68	72
1600	77	81	78	76	79	67	69
1700	77	78	76	75	77	63	65
1800	76	74	74	72	75	63	64
1900	72	70	70	69	72	62	64
2000	68	68	67	67	71	62	62
2100	67	68	67	66	68	58	61
2200	66	66	66	65	66	59	59
2300	62	68	66	63	67	61	59

Table 1, Continued

DATE, 1961	22 Jan	23 Jan	24 Jan	25 Jan	26 Jan	27 Jan	28 Jan
HOUR							
0000	59	54	54	58	57	55	55
0100	59	54	54	58	57	54	54
0200	59	54	53	57	58	53	53
0300	58	53	52	57	57	53	53
0400	58	52	51	56	56	52	51
0500	57	52	50	56	56	53	51
0600	57	53	50	56	55	52	51
0700	57	51	50	56	56	51	50
0800	58	55	52	56	57	53	52
0900	59	59	54	57	57	56	54
1000	62	63	57	58	57	59	57
1100	64	66	60	58	57	61	59
1200	66	68	61	60	57	62	62
1300	68	70	64	60	59	63	63
1400	69	69	65	60	59	62	65
1500	64	66	66	60	59	61	62
1600	65	63	63	61	59	60	62
1700	64	61	59	61	58	59	60
1800	62	59	59	60	57	57	58
1900	60	59	58	59	57	57	58
2000	60	59	59	58	57	56	57
2100	58	57	59	58	55	56	56
2200	56	56	59	58	54	55	55
2300	55	55	59	57	55	55	54

Table 1, Continued

DATE, 1961	29 Jan	30 Jan	31 Jan	1 Feb	2 Feb	3 Feb	4 Feb
HOUR							
0000	53	55	54	59	54	54	59
0100	54	54	53	59	54	54	58
0200	53	53	54	58	54	53	57
0300	52	52	52	58	53	53	57
0400	51	52	51	57	53	52	55
0500	51	50	51	56	52	52	53
0600	51	50	51	56	53	52	53
0700	51	51	51	56	54	52	56
0800	52	54	53	58	55	52	55
0900	54	58	54	59	55	55	62
1000	57	62	57	61	56	57	66
1100	60	64	60	63	58	61	69
1200	62	67	63	65	60	70	72
1300	63	69	65	67	62	71	74
1400	63	70	67	66	63	72	76
1500	62	70	63	63	61	71	73
1600	62	68	61	63	62	70	71
1700	61	64	60	62	61	68	70
1800	59	62	59	61	60	66	68
1900	60	60	59	60	59	67	66
2000	58	58	59	59	58	64	63
2100	57	57	59	58	57	63	61
2200	57	56	59	56	55	62	59
2300	56	55	59	55	54	60	60

Table 1, Continued

DATE, 1961	5 Feb	6 Feb	7 Feb	8 Feb	9 Feb	10 Feb	11 Feb	12 Feb
HOUR								
0000	57	58	55	54	55	58	58	58
0100	56	58	55	54	55	58	57	
0200	54	55	54	52	54	58	54	
0300	53	55	55	52	54	58	54	
0400	53	53	55	51	54	57	54	
0500	54	54	54	51	53	57	54	
0600	52	53	53	51	53	57	56	
0700	56	54	52	51	53	58	56	
0800	56	56	54	53	55	60	56	
0900	62	58	57	56	58	65	57	
1000	66	60	60	58	62	70	59	
1100	69	62	62	63	65	74	61	
1200	71	62	63	64	68	75	64	
1300	73	63	63	65	71	75	64	
1400	72	65	65	65	73	75	65	
1500	71	64	64	64	68	72	64	
1600	71	64	62	61	67	69	71	
1700	68	63	60	60	64	66	59	
1800	65	61	59	59	62	63	58	
1900	64	58	59	58	61	62	58	
2000	63	58	58	58	61	60	58	
2100	62	57	57	57	59	59	58	
2200	60	56	56	57	60	59	58	
2300	58	56	55	57	59	58	58	

Table 2-Los Angeles Daily Normal Temperature, 1921-1950

Period	Mean = τ_a'
Dec. 26-Jan. 6	56
Jan. 7-Jan. 26	55
Jan. 27-Feb. 12	56
Feb. 13-Feb. 25	57
Feb. 26-Mar. 5	58
Mar. 6-Mar. 19	59
Mar. 20-Mar. 30	60
Mar. 31-Apr. 11	61
Apr. 12-Apr. 22	62
Apr. 23-May 1	63
May 2-May 10	64
May 11-May 19	65
May 20-May 29	66
May 30-Jun. 13	67
Jun. 14-Jun. 20	68
Jun. 21-Jun. 25	69
Jun. 26-Jun. 29	70
Jun. 30-Jul. 4	71
Jul. 5-Jul. 11	72
Jul. 12-Jul. 25	73
Jul. 26-Aug. 6	74
Aug. 7-Aug. 28	73
Aug. 29-Sept. 14	72
Sept. 15-Sept. 24	71
Sept. 25-Sept. 29	70
Sept. 30-Oct. 4	69
Oc. 5-Oct. 10	68
Oct. 11-Oct. 17	67
Oct. 18-Oct. 25	66
Oct. 26-Nov. 1	65
Nov. 2-Nov. 6	64
Nov. 7-Nov. 14	63
Nov. 15-Nov. 19	62
Nov. 20-Nov. 26	61
Nov. 27-Dec. 2	60
Dec. 3-Dec.9	59
Dec. 10-Dec. 15	58
Dec. 16-Dec. 25	57

Table 3 -- Calculation of Los Angeles Diurnal Temperature Trend for the Period 1 January through 11 February 1961

Hour (LST)	$\sum_{d=1}^{42} X_d(h)$	$\bar{X}_h$	$\Delta\bar{X}_h$
0	2391	56.93	-3.65
1	2358	56.14	-4.44
2	2324	55.33	-5.25
3	2305	54.88	-5.70
4	2264	53.91	-6.67
5	2254	53.67	-6.91
6	2237	53.26	-7.32
7	2240	53.33	-7.25
8	2322	55.29	-5.29
9	2474	58.91	-1.67
10	2639	62.83	2.25
11	2780	66.19	5.61
12	2876	68.48	7.90
13	2954	70.33	9.75
14	2970	70.72	10.14
15	2901	69.07	8.49
16	2842	67.67	7.09
17	2747	65.41	4.83
18	2664	63.43	2.85
19	2605	62.03	1.45
20	2543	60.55	-0.03
21	2490	59.29	-1.29
22	2451	58.36	-2.22
23	2437	58.02	-2.56
Σ	61,068	1454.03	
$\bar{X}_d$	60.58	60.58	
N^{-1}	.02381		
24N	1008		

Table 4 - Residuals from Annual and Diurnal Trend

DATE, 1961	1 Jan.	2 Jan.	3 Jan.	4 Jan.	5 Jan.	6 Jan.	7 Jan.
HOUR, LST.							
0000	-4.35	-4.35	-1.35	0.65	1.65	3.65	2.65
0100	-3.56	-4.56	0.44	2.44	-0.56	1.44	2.44
0200	-1.75	-5.75	2.25	1.25	-0.75	-0.75	0.25
0300	-0.30	-5.30	-0.30	1.70	2.70	1.70	2.70
0400	0.67	-5.33	-1.33	1.67	-0.33	-0.33	3.67
0500	-0.09	-6.09	-1.09	0.91	-1.09	2.91	2.91
0600	-0.32	-5.68	-2.68	2.32	-0.68	4.32	2.32
0700	-0.75	-5.75	-0.75	1.25	-2.75	5.25	2.25
0800	-3.71	-9.71	0.29	0.29	0.29	3.29	2.29
0900	-6.33	-9.33	2.67	0.67	3.67	5.67	3.67
1000	-7.25	-7.25	2.75	0.25	3.75	7.75	5.75
1100	-6.61	-3.61	2.39	1.39	4.39	8.39	7.39
1200	-6.90	-4.90	2.10	3.10	4.10	8.10	7.10
1300	-5.75	-2.75	3.25	3.25	5.25	7.25	8.25
1400	-4.14	-1.14	3.86	2.86	5.86	3.86	7.86
1500	-2.49	-1.49	4.51	4.51	6.51	4.51	5.51
1600	-4.09	-3.09	5.91	4.91	6.91	3.91	4.91
1700	-4.83	-2.83	4.17	5.17	6.17	3.17	5.17
1800	-4.85	-3.85	3.15	4.15	6.15	3.15	3.15
1900	-4.45	-2.45	2.55	3.55	4.55	2.55	1.55
2000	-3.97	-2.97	2.03	4.03	2.03	2.03	2.03
2100	-2.71	-2.71	1.29	3.29	0.29	0.29	3.29
2200	-3.78	-1.78	1.22	3.22	0.22	-0.78	4.22
2300	-4.44	-4.44	0.56	2.56	3.56	1.56	4.56
$\sum_{i=1}^{24} x_i$	-86.11	-107.11	37.89	59.39	61.89	82.89	95.89
$\sum_{i=1}^{24} x_i x_{i+1}$	410.90	548.45	145.65	188.12	315.69	410.10	488.10
$"\sum_{i=1}^{24}"x_{i+1}$	84.43	104.11	50.13	60.39	68.45	69.55	100.89
$\sum_{i=1}^{24} x_i^2$	430.36	591.14	164.90	197.26	349.46	449.50	486.42
$\sum_{i=1}^{24} \lvert x_i \rvert$	88.09	107.11	52.89	59.39	74.21	86.61	95.89

Table 4, Continued

DATE, 1961	8 Jan	9 Jan	10 Jan	11 Jan	12 Jan	13 Jan	14 Jan
HOUR, LST.							
0000	7.65	5.65	8.65	12.65	8.65	1.65	9.65
0100	4.44	6.44	5.44	11.44	8.44	1.44	8.44
0200	4.25	6.25	9.25	11.25	8.25	3.25	7.25
0300	5.70	4.70	8.70	11.70	8.70	2.70	6.70
0400	5.67	6.67	4.67	12.67	9.67	1.67	7.67
0500	4.91	8.91	2.91	10.91	8.91	1.91	11.91
0600	3.32	6.32	4.32	11.32	8.32	1.32	9.32
0700	4.25	5.25	2.25	10.25	9.25	1.25	8.25
0800	5.29	8.29	5.29	13.29	8.29	4.29	10.29
0900	5.67	10.67	8.67	12.67	7.67	5.67	11.67
1000	8.75	7.75	9.75	12.75	8.75	8.75	12.75
1100	10.39	8.39	11.39	12.39	10.39	9.39	13.39
1200	10.10	8.10	12.10	13.10	10.10	10.10	14.10
1300	11.25	9.25	12.25	13.25	9.25	10.25	15.25
1400	11.86	9.86	11.86	11.86	6.86	7.86	14.86
1500	11.51	9.51	13.51	13.51	6.51	6.51	15.51
1600	9.91	9.91	12.91	12.91	6.91	7.91	16.91
1700	9.17	7.17	10.17	13.17	6.17	8.17	14.17
1800	9.15	8.15	10.15	12.15	3.15	8.15	14.15
1900	8.55	6.55	11.55	10.55	3.55	6.55	15.55
2000	7.03	4.03	12.03	10.03	3.03	6.03	14.03
2100	6.29	4.29	12.29	10.29	3.29	5.29	12.29
2200	6.22	4.22	11.22	10.22	4.22	5.22	12.22
2300	6.56	6.56	10.56	10.56	3.56	8.56	10.56
$\sum_{i=1}^{24} x_i$	177.89	170.89	221.89	284.89	171.89	133.89	287.89
$\sum_{i=1}^{24} x_i x_{i+1}$	1437.52	1283.80	2302.19	3354.51	1314.30	981.64	3641.14
"$\sum_{i=1}^{24}$" x_{i+1}	175.89	173.89	225.89	280.89	164.89	141.89	290.89
$\sum_{i=1}^{24} x_i^2$	1470.32	1296.58	2306.90	3412.18	1365.40	962.42	3639.70
$\sum_{i=1}^{24} \lvert x_i \rvert$	177.89	170.89	221.89	284.89	171.89	133.89	287.89

Table 4, Continued

DATE, 1961	15 Jan	16 Jan	17 Jan	18 Jan	19 Jan	20 Jan	21 Jan
HOUR, LST.							
0000	12.65	9.65	12.65	14.65	9.65	9.65	11.65
0100	10.44	8.44	12.44	11.44	9.44	13.44	12.44
0200	9.25	7.25	14.75	13.25	9.25	14.25	11.25
0300	6.70	5.70	14.70	10.70	9.70	12.70	10.70
0400	7.67	6.67	13.67	9.67	7.67	13.67	11.57
0500	6.91	5.91	13.91	11.91	6.91	15.91	10.91
0600	7.32	7.32	12.32	10.32	8.32	15.32	9.32
0700	6.25	7.25	11.25	12.25	7.25	16.25	9.25
0800	8.29	9.29	12.29	12.29	8.29	14.29	7.29
0900	9.67	12.67	14.67	12.67	11.67	11.67	5.67
1000	13.75	15.75	14.75	13.75	14.75	10.75	6.75
1100	14.39	17.39	15.39	15.39	13.39	10.39	9.39
1200	15.10	16.10	17.10	16.10	14.10	10.10	8.10
1300	15.25	17.25	18.25	17.25	13.25	7.25	8.25
1400	15.86	17.86	16.86	16.86	13.86	6.86	7.86
1500	15.51	17.51	16.51	17.51	15.51	4.51	8.51
1600	14.91	18.91	15.91	13.91	16.91	4.91	6.91
1700	17.17	18.17	16.17	15.17	17.17	3.17	5.17
1800	18.15	16.15	16.15	14.15	17.15	5.15	6.15
1900	15.55	13.55	13.55	12.55	15.55	5.55	7.55
2000	13.03	13.03	12.03	12.03	16.03	7.03	7.03
2100	13.29	14.29	13.29	12.29	14.29	4.29	7.29
2200	13.22	13.22	13.22	12.22	13.22	6.22	6.22
2300	9.56	15.56	13.56	10.56	14.56	8.56	6.56
$\sum_{i=1}^{24} X_i$	289.89	304.89	344.89	318.89	297.89	231.89	201.89
$\sum_{i=1}^{24} X_i X_{i+1}$	3734.42	4335.94	5044.83	4259.34	3928.07	2612.38	1743.78
$\sum_{i=1}^{24} X_{i+1}$	286.89	307.89	346.89	313.89	297.89	233.89	197.89
$\sum_{i=1}^{24} X_i^2$	3810.64	4337.66	5034.62	4350.54	3963.66	2634.02	1799.66
$\sum_{i=1}^{24} \lvert X \rvert$	289.89	304.89	344.89	318.89	297.89	231.89	201.89

Table 4, Continued

DATE, 1961	22 Jan	23 Jan	24 Jan	25 Jan	26 Jan	27 Jan	28 Jan
HOUR (LST)							
0000	7.65	2.65	2.65	6.65	5.65	2.65	2.65
0100	8.44	3.44	3.44	7.44	6.44	2.44	2.44
0200	9.25	4.25	3.25	7.25	8.25	2.25	2.25
0300	8.70	3.70	2.70	7.70	7.70	2.70	2.70
0400	9.67	3.67	2.67	7.67	7.67	2.67	1.67
0500	8.91	3.91	1.91	7.91	7.91	3.91	1.91
0600	9.32	5.32	2.32	8.32	7.32	3.32	2.32
0700	9.25	3.25	2.25	8.25	8.25	2.25	1.25
0800	8.29	5.29	2.29	6.29	7.29	2.29	1.29
0900	5.67	5.67	0.67	3.67	3.67	1.67	-0.33
1000	4.75	5.75	-0.25	0.75	-0.25	0.75	-1.25
1100	3.39	5.39	-0.61	-2.61	-3.61	-0.61	-2.61
1200	3.10	5.10	-1.90	-2.90	-5.90	-1.90	-1.90
1300	3.25	5.25	-0.75	-4.75	-5.75	-2.75	-2.75
1400	3.86	3.86	-0.14	-3.14	-6.14	-4.14	-1.14
1500	0.51	2.51	2.51	-3.49	-4.49	-3.49	-2.49
1600	2.91	0.91	0.91	-1.09	-3.09	-3.09	-1.09
1700	4.17	1.17	-0.83	1.17	-1.83	-1.83	-0.83
1800	4.15	1.15	1.15	2.15	-0.85	-1.85	-0.85
1900	3.55	2.55	1.55	2.55	0.55	-0.45	0.55
2000	5.03	4.03	4.03	3.03	2.03	0.03	1.03
2100	4.29	3.29	5.29	4.29	1.29	1.29	1.29
2200	3.22	3.22	6.22	5.22	1.22	1.22	1.22
2300	2.56	2.56	6.56	4.56	2.56	1.56	0.56
$\sum_{i=1}^{24} X_i$	133.89	87.89	47.89	74.89	45.89	10.89	7.89
$\sum_{i=1}^{24} X_i X_{i+1}$	878.40	357.26	207.29	658.28	631.87	126.41	62.45
$\sum_{i=1}^{24} X_{i+1}$	128.89	87.89	51.37	106.29	105.11	49.83	34.61
$\sum_{i=1}^{24} X_i^2$	922.90	369.26	205.74	689.40	676.98	135.40	74.96
$\sum_{i=1}^{24} \lvert X_i \rvert$	133.89	87.89	56.85	114.85	109.71	51.11	38.37

Table 4, Continued

DATE, 1961	29 Jan	30 Jan	31 Jan	1 Feb	2 Feb	3 Feb	4 Feb
HOUR (LST)							
0000	0.65	2.65	1.65	6.65	1.65	1.65	6.65
0100	2.44	2.44	1.44	7.44	2.44	2.44	6.44
0200	2.25	2.25	3.25	7.25	3.25	2.25	6.25
0300	1.70	1.70	1.70	7.70	2.70	2.70	6.70
0400	1.67	2.67	1.67	7.67	3.67	2.67	5.67
0500	1.91	0.91	1.91	6.91	2.91	2.91	3.91
0600	2.32	1.32	2.32	7.32	4.32	3.32	4.32
0700	2.25	2.25	2.25	7.25	5.25	3.25	7.25
0800	1.29	3.29	2.29	7.29	4.29	1.29	4.29
0900	-0.33	3.67	-0.33	4.67	0.67	0.67	7.67
1000	-1.25	3.75	-1.25	2.75	-2.25	-1.25	7.75
1100	-1.61	2.39	-1.61	1.39	-3.61	-0.61	7.39
1200	-1.90	3.10	-0.90	1.10	-3.90	6.10	8.10
1300	-2.75	3.25	-0.75	1.25	-3.75	5.25	8.25
1400	-3.14	3.86	0.86	-0.14	-3.14	5.86	9.86
1500	-2.49	5.51	-1.49	-1.49	-3.49	6.51	8.51
1600	-1.09	4.91	-2.09	-0.09	-1.09	6.91	7.91
1700	0.17	3.17	-0.83	1.17	0.17	7.17	9.17
1800	0.15	3.15	0.15	2.15	1.15	7.15	9.15
1900	2.55	2.55	1.55	2.55	1.55	9.55	8.55
2000	2.03	2.03	3.03	3.03	2.03	8.03	7.03
2100	2.29	2.29	4.29	3.29	2.29	8.29	6.29
2200	3.22	2.22	5.22	2.22	1.22	8.22	5.22
2300	2.56	1.56	5.56	1.56	0.56	6.56	6.56
$\sum_{i=1}^{24} X_i$	14.89	66.89	29.89	90.89	18.89	106.89	168.89
$\sum_{i=1}^{24} X_i X_{i+1}$	91.38	202.00	146.32	522.69	177.81	676.24	1209.87
$\sum_{i=1}^{24} X_{i+1}$	45.01	65.89	47.73	86.71	56.51	100.91	166.89
$\sum_{i=1}^{24} X_i^2$	98.40	212.50	142.20	553.48	199.06	689.04	1246.28
$\sum_{i=1}^{24} \lvert X_i \rvert$	44.01	66.89	48.39	94.33	61.35	110.61	168.89

Table 4, Continued

DATE, 1961	5 Feb	6 Feb	7 Feb	8 Feb	9 Feb	10 Feb	11 Feb	12 Feb
HOUR (LST)								
0000	4.65	5.65	2.65	1.65	2.65	5.65	5.65	5.65
0100	4.44	6.44	3.44	2.44	3.44	6.44	5.44	
0200	3.25	4.25	3.25	1.25	3.25	7.25	3.25	
0300	2.70	4.70	4.70	1.70	3.70	7.70	3.70	
0400	3.67	3.67	5.67	1.67	4.67	7.67	4.67	
0500	4.91	4.91	4.91	1.91	3.91	7.91	4.91	
0600	3.32	4.32	4.32	2.32	4.32	8.32	7.32	
0700	7.25	5.25	3.25	2.25	4.25	9.25	7.25	
0800	5.29	5.29	3.29	2.29	4.29	9.29	5.29	
0900	7.67	3.67	2.67	1.67	3.67	10.67	2.67	
1000	7.75	1.75	1.75	-.25	3.75	11.75	0.75	
1100	7.39	0.39	0.39	1.39	3.39	12.39	-0.61	
1200	7.10	-1.90	-0.90	0.10	4.10	11.10	0.10	
1300	7.25	-2.75	-2.75	-0.75	5.25	9.25	-1.75	
1400	5.86	-1.14	-1.14	1.14	6.86	8.86	-1.14	
1500	6.51	-0.49	-0.49	-0.49	3.51	7.51	-0.49	
1600	7.91	0.91	-1.09	-2.09	3.91	5.91	-2.09	
1700	7.17	2.17	-0.83	-0.83	3.17	5.17	-1.83	
1800	6.15	2.15	0.15	0.15	3.15	4.15	-0.85	
1900	6.55	0.55	1.55	0.55	3.55	4.55	0.55	
2000	7.03	2.03	2.03	2.03	5.03	4.03	2.03	
2100	7.29	2.29	2.29	2.29	4.29	4.29	3.29	
2200	6.22	2.22	2.22	3.22	6.22	5.22	4.22	
2300	4.56	2.56	1.56	3.56	5.56	4.56	4.56	
$\sum_{i=1}^{24} X_i$	141.89	58.89	42.89	26.89	99.89	178.89	56.89	
$\sum_{i=1}^{24} X_i X_{i+1}$	881.72	256.05	176.42	71.32	438.48	1468.97	318.62	
"$\sum_{i=1}^{24}$" X_{i+1}	142.89	62.83	54.19	33.91	102.89	178.89	63.39	
$\sum_{i=1}^{24} X_i^2$	898.28	286.04	189.10	79.94	439.28	1479.10	340.78	
$\sum_{i=1}^{24} \lvert X_i \rvert$	141.89	71.45	57.29	37.99	99.89	178.89	74.41	

Table 5 - Open-Ended Lag Correlations $_1r_{24}$, $_1r_{72}$ and $_1r_{1008}$.

DATE, 1961	1 Jan	2 Jan	3 Jan	4 Jan	5 Jan
$\sum_{i=1}^{24} x_i x_{i+1}$	410.90	548.45	145.65	188.12	315.69
$\sum_{i=1}^{24} x_i$	-86.11	-107.11	37.89	59.39	61.89
$\sum_{i=2}^{25} x_i$	-86.11	-104.11	39.89	60.39	63.89
$\sum_{i=1}^{24} x_i \sum_{i=2}^{25} x_i / 24$	308.96	464.63	62.98	149.44	164.76
Numerator	101.94	83.82	82.67	38.68	150.93
$\sum_{i=1}^{24} x_i^2$	430.46	591.14	164.90	197.26	349.46
$(\sum_{i=1}^{24} x_i)^2/24$	308.96	478.02	59.82	146.97	159.60
$\sum_{i=1}^{24} x_i^2 - (\sum_{i=1}^{24} x_i)^2/24$	121.50	113.12	105.08	50.29	189.86
$\sum_{i=2}^{25} x_i^2$	430.46	574.04	163.50	199.56	360.06
$(\sum_{i=2}^{25} x_i)^2/24$	308.96	451.62	66.30	151.96	170.08
$\sum_{i=2}^{25} x_i^2 - (\sum_{i=2}^{25} x_i)^2/24$	121.50	122.42	97.20	47.60	189.98
Denominator^2		13,848. 1504	10,213. 7760	2,393. 8040	36,069. 6028
Denominator	121.50	117.68	101.06	48.93	189.92
$_1r_{24}$	.839	.712	.818	.791	.795

Table 5, Continued

DATE, 1961	6 Jan	7 Jan	8 Jan	9 Jan	10 Jan
$\sum_{i=1}^{24} X_i X_{i+1}$	410.10	488.10	1,437.52	1,283.80	2,302.19
$\sum_{i=1}^{24} X_i$	82.89	95.89	177.89	170.89	221.89
$\sum_{i=2}^{25} X_i$	81.89	100.89	175.89	173.89	225.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i/24$	282.83	403.10	1,303.71	1,238.17	2,088.45
Numerator	127.27	85.00	133.81	45.63	213.74
$\sum_{i=1}^{24} X_i^2$	449.50	486.42	1,470.32	1,296.58	2,306.96
$(\sum_{i=1}^{24} X_i)^2/24$	286.28	383.12	1,318.54	1,216.81	2,051.47
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	163.22	103.30	151.78	79.77	255.49
$\sum_{i=2}^{25} X_i^2$	443.20	537.92	1,443.72	1,339.48	2,392.16
$(\sum_{i=2}^{25} X_i)^2/24$	279.42	424.12	1,289.05	1,259.91	2,126.10
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	163.78	113.80	154.67	79.51	266.06
Denominator^2	26,732.1716	11,755.5400	23,475.8126	6,347.2989	67,975.6694
Denominator	163.50	108.42	153.22	79.67	260.72
$_1r_{24}$	.778	.784	.873	.573	.820

Table 5, Continued

DATE, 1961	11 Jan	12 Jan	13 Jan	14 Jan	15 Jan
$\sum_{i=1}^{24} X_i X_{i+1}$	3,354.51	1,314.30	981.64	3,641.14	3,734.42
$\sum_{i=1}^{24} X_i$	284.89	171.89	133.89	287.19	289.89
$\sum_{i=2}^{25} X_i$	280.89	164.89	141.89	290.19	286.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	3,334.28	1,180.96	791.57	3,472.49	3,465.27
Numerator	20.23	133.34	190.07	141.65	269.15
$\sum_{i=1}^{24} X_i^2$	3,412.18	1,365.40	962.42	3,629.70	3,810.64
$(\sum_{i=1}^{24} X_i)^2 / 24$	3,381.76	1,231.09	746.94	3,436.57	3,501.51
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2 / 24$	30.42	134.31	215.48	203.13	309.13
$\sum_{i=2}^{25} X_i^2$	3,326.98	1,293.30	1,052.82	3,706.60	3,743.74
$(\sum_{i=2}^{25} X_i)^2 / 24$	3,287.47	1,132.86	838.87	3,508.76	3,429.41
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2 / 24$	39.51	160.44	213.95	197.84	314.33
Denominator^2	1,201.8942	21,548.6964	46,101.9460	40,187.2392	97,168.8329
Denominator	34.67	146.79	214.71	200.47	311.72
$_1r_{24}$	.584	.908	.885	.707	.863

Table 5, Continued

DATE, 1961	16 Jan	17 Jan	18 Jan	19 Jan	20 Jan
$\sum_{i=1}^{24} X_i X_{i+1}$	4,335.94	5,044.83	4,259.34	3,928.07	2,612.38
$\sum_{i=1}^{24} X_i$	304.89	344.89	318.89	297.89	231.89
$\sum_{i=2}^{25} X_i$	307.89	346.89	313.89	297.89	233.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	3,911.36	4,984.95	4,170.68	3,697.44	2,259.86
Numerator	424.58	59.88	88.66	230.63	352.52
$\sum_{i=1}^{24} X_i^2$	4,337.66	5,034.62	4,350.54	3,963.66	2,634.02
$(\sum_{i=1}^{24} X_i)^2/24$	3,873.25	4,956.21	4,237.12	3,697.44	2,240.54
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	464.41	78.41	113.42	266.22	393.48
$\sum_{i=2}^{25} X_i^2$	4,404.56	5,089.22	4,229.04	3,963.66	2,676.62
$(\sum_{i=2}^{25} X_i)^2/24$	3,949.84	5,013.86	4,105.29	3,697.44	2,279.36
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	454.72	75.36	123.75	266.22	397.26
Denominator^2	211,176.5152	5,908.9776	14,035.7250		156,313.8648
Denominator	459.54	76.87	118.47	266.22	395.37
$_1r_{24}$	.924	.779	.748	.866	.892

Table 5, Continued

DATE, 1961	21 Jan	22 Jan	23 Jan	24 Jan	25 Jan
$\sum_{i=1}^{24} X_i X_{i+1}$	1,743.78	878.40	357.26	207.29	658.28
$\sum_{i=1}^{24} X_i$	201.89	133.89	87.89	47.89	74.89
$\sum_{i=2}^{25} X_i$	197.89	128.89	87.89	51.89	73.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i/24$	1,664.67	719.05	321.86	103.54	230.57
Numerator	79.11	159.35	35.40	103.75	427.71
$\sum_{i=1}^{24} X_i^2$	1,799.66	922.90	369.26	205.74	689.40
$(\sum_{i=1}^{24} X_i)^2/24$	1,698.32	746.94	321.86	95.56	233.69
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	101.34	175.96	47.40	110.18	455.71
$\sum_{i=2}^{25} X_i^2$	1,722.46	871.40	369.26	242.94	677.10
$(\sum_{i=2}^{25} X_i)^2/24$	1,631.69	692.19	321.86	112.19	227.49
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	90.77	179.21	47.40	130.75	449.61
Denominator^2	9,198.6318	31,533.7916		14,406.0350	204,891.7731
Denominator	95.91	177.58	47.40	120.03	452.65
$_1r_{24}$	.825	.897	.747	.864	.945

Table 5, Continued

DATE, 1961	26 Jan	27 Jan	28 Jan	29 Jan	30 Jan
$\sum_{i=1}^{24} X_i X_{i+1}$	631.87	126.41	62.45	91.38	202.00
$\sum_{i=1}^{24} X_i$	45.89	10.89	7.89	14.89	66.89
$\sum_{i=2}^{25} X_i$	42.89	10.89	5.89	16.89	65.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	82.01	4.94	1.94	10.48	183.64
Numerator	549.36	121.47	61.51	80.90	18.36
$\sum_{i=1}^{24} X_i^2$	676.98	135.40	74.96	98.40	212.50
$(\sum_{i=1}^{24} X_i)^2/24$	87.75	4.94	2.59	9.24	186.43
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	589.23	130.46	72.37	89.16	26.07
$\sum_{i=2}^{25} X_i^2$	652.08	135.40	68.36	105.00	208.20
$(\sum_{i=2}^{25} X_i)^2/24$	76.65	4.94	1.45	11.89	180.90
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	575.43	130.46	66.91	93.11	27.30
Denominator^2	339,060.6189		4,842.2767	8,301.6876	711.7110
Denominator	582.29	130.46	69.59	91.11	26.68
$_1r_{24}$	.944	.931	.884	.888	.688

Table 5, Continued

DATE, 1961	31 Jan	1 Feb	2 Feb	3 Feb	4 Feb
$\sum_{i=1}^{24} X_i X_{i+1}$	146.32	522.69	177.81	676.24	1,209.87
$\sum_{i=1}^{24} X_i$	29.89	90.89	18.89	106.89	168.89
$\sum_{i=2}^{25} X_i$	34.89	85.89	18.89	111.89	166.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	43.45	325.27	14.87	498.33	1,174.42
Numerator	102.87	197.42	162.94	177.91	35.45
$\sum_{i=1}^{24} X_i^2$	142.20	553.48	199.06	689.04	1,246.28
$(\sum_{i=1}^{24} X_i)^2/24$	37.23	344.21	14.87	476.06	1,188.49
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	104.97	209.27	184.19	212.98	57.79
$\sum_{i=2}^{25} X_i^2$	183.70	511.98	199.06	730.54	1,223.68
$(\sum_{i=2}^{25} X_i)^2/24$	50.72	307.38	14.87	521.64	1,160.51
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	132.98	204.60	184.19	208.90	63.17
Denominator^2	13,958.9106	42,816.6420	33,925.9561	44,491.5220	3,650.5943
Denominator	118.15	206.92	184.19	210.93	60.42
$_1r_{24}$	.871	.954	.885	.843	.587

Table 5, Continued

DATE, 1961	5 Feb	6 Feb	7 Feb	8 Feb	9 Feb
$\sum_{i=1}^{24} X_i X_{i+1}$	881.72	256.05	176.42	71.32	438.48
$\sum_{i=1}^{24} X_i$	141.89	58.89	42.89	26.89	99.89
$\sum_{i=2}^{25} X_i$	142.89	55.89	41.89	27.89	102.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	844.78	137.14	74.86	31.25	428.24
Numerator	36.94	118.91	101.56	40.07	10.24
$\sum_{i=1}^{24} X_i^2$	898.28	286.04	189.10	79.94	439.28
$(\sum_{i=1}^{24} X_i)^2 / 24$	838.87	144.50	76.65	30.13	415.75
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2 / 24$	59.41	141.54	112.45	49.81	23.58
$\sum_{i=2}^{25} X_i^2$	908.58	261.14	184.80	84.24	464.18
$(\sum_{i=2}^{25} X_i)^2 / 24$	850.73	130.15	73.12	32.41	441.10
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2 / 24$	57.85	130.99	111.68	51.83	23.08
Denominator^2	3,436.8685	18,540.3246	12,558.4160	2,581.6523	543.0724
Denominator	58.62	136.16	112.06	50.81	23.30
$_1r_{24}$	.630	.873	.906	.789	.439

Table 5, Continued

DATE, 1961	10 Feb	11 Feb
$\sum_{i=1}^{24} X_i X_{i+1}$	1,468.97	318.62
$\sum_{i=1}^{24} X_i$	178.89	56.89
$\sum_{i=2}^{25} X_i$	178.89	56.89
$\sum_{i=1}^{24} X_i \sum_{i=2}^{25} X_i / 24$	1,333.40	134.85
Numerator	135.57	183.77
$\sum_{i=1}^{24} X_i^2$	1,479.10	340.78
$(\sum_{i=1}^{24} X_i)^2/24$	1,333.40	134.85
$\sum_{i=1}^{24} X_i^2 - (\sum_{i=1}^{24} X_i)^2/24$	145.70	205.93
$\sum_{i=2}^{25} X_i^2$	1,479.10	340.78
$(\sum_{i=2}^{25} X_i)^2/24$	1,333.40	134.85
$\sum_{i=2}^{25} X_i^2 - (\sum_{i=2}^{25} X_i)^2/24$	145.70	205.93
Denominator	145.70	205.93
$_1r_{24}$	.930	.892

Table 5, Continued

DATE, 1961	1-3 Jan	4-6 Jan	7-9 Jan	10-12 Jan	13-15 Jan
$\sum_{i=1}^{72} X_i X_{i+1}$	1,105.00	913.91	3,209.42	6,971.00	8,357.20
$\sum_{i=1}^{72} X_i$	-155.33	204.17	444.67	678.67	710.97
$\sum_{i=2}^{73} X_i$	-150.33	206.17	450.67	671.67	718.97
$\sum_{i=1}^{72} X_i \sum_{i=2}^{73} X_i / 72$	324.32	584.64	2,783.33	6,331.14	7,099.53
Numerator	780.68	329.27	426.09	639.86	1,257.67
$\sum_{i=1}^{72} X_i^2$	1,186.50	996.22	3,253.32	7,084.54	8,412.76
$(\sum_{i=1}^{72} X_i)^2/72$	335.10	578.96	2,746.27	6,397.12	7,020.53
$\sum_{i=1}^{72} X_i^2 - (\sum_{i=1}^{72} X_i)^2/72$	851.40	417.26	507.05	687.42	1,392.23
$\sum_{i=2}^{73} X_i^2$	1,168.00	1,002.82	3,321.12	7,012.44	8,503.16
$(\sum_{i=2}^{73} X_i)^2/72$	313.88	590.36	2,820.88	6,265.84	7,179.41
$\sum_{i=2}^{73} X_i^2 - (\sum_{i=2}^{73} X_i)^2/72$	854.12	412.46	500.24	746.60	1,323.75
Denominator^2	727,197.7680	172,103.0596	253,646.6920	513,227.7720	1,842,964.4625
Denominator	852.76	414.85	503.63	716.40	1,357.56
$_1r_{72}$	.915	.794	.846	.893	.926

Table 5, Continued

DATE, 1961	16-18 Jan	19-21 Jan	22-24 Jan	25-27 Jan	28-30 Jan
$\sum_{i=1}^{72} X_i X_{i+1}$	13,640.11	8,284.23	1,442.95	1,416.56	355.83
$\sum_{i=1}^{72} X_i$	968.67	731.67	269.67	131.67	89.67
$\sum_{i=2}^{73} X_i$	968.67	729.67	268.67	127.67	88.67
$\sum_{i=1}^{72} X_i \sum_{i=2}^{73} X_i / 72$	13,032.24	7,414.97	1,006.28	233.48	110.43
Numerator	607.87	869.26	436.67	1,183.08	245.40
$\sum_{i=1}^{72} X_i^2$	13,722.82	8,397.34	1,497.90	1,501.78	385.86
$(\sum_{i=1}^{72} X_i)^2 / 72$	13,032.24	7,435.29	1,010.03	240.79	111.68
$\sum_{i=1}^{72} X_i^2 - (\sum_{i=1}^{72} X_i)^2 / 72$	690.58	962.05	487.87	1,260.99	274.18
$\sum_{i=2}^{73} X_i^2$	13,722.82	8,362.74	1,483.60	1,464.58	381.56
$(\sum_{i=2}^{73} X_i)^2 / 72$	13,032.24	7,394.70	1,002.55	226.38	109.20
$\sum_{i=2}^{73} X_i^2 - (\sum_{i=2}^{73} X_i)^2 / 72$	690.58	968.04	481.05	1,238.20	272.36
Denominator^2		931,302.8820	234,689.8635	1,561,357.8180	74,675.6648
Denominator	690.58	965.04	484.45	1,249.54	273.27
$_1r_{72}$	.880	.901	.901	.947	.898

Table 5, Continued

DATE, 1961	31 Jan - 2 Feb	3-5 Feb	6-8 Feb	9-11 Feb
$\sum_{i=1}^{72} X_i X_{i+1}$	846.82	2,767.83	503.79	2,226.07
$\sum_{i=1}^{72} X_i$	139.67	417.67	128.67	335.67
$\sum_{i=2}^{73} X_i$	139.67	421.67	125.67	338.67
$\sum_{i=1}^{72} X_i \sum_{i=2}^{73} X_i / 72$	270.94	2,446.10	224.58	1,578.91
Numerator	575.88	321.73	279.21	647.16
$\sum_{i=1}^{72} X_i^2$	894.74	2,833.60	555.08	2,259.16
$(\sum_{i=1}^{72} X_i)^2 / 72$	270.94	2,422.89	229.94	1,564.92
$\sum_{i=1}^{72} X_i^2 - (\sum_{i=1}^{72} X_i)^2 / 72$	623.80	410.71	325.14	694.24
$\sum_{i=2}^{73} X_i^2$	894.74	2,862.80	530.18	2,284.06
$(\sum_{i=2}^{73} X_i)^2 / 72$	270.94	2,469.52	219.35	1,593.02
$\sum_{i=2}^{73} X_i^2 - (\sum_{i=2}^{73} X_i)^2 / 72$	623.80	393.28	310.83	691.04
Denominator^2		161,524.0288	101,063.2662	479,747.6096
Denominator	623.80	401.90	317.90	692.64
${}_1r_{72}$	.923	.801	.878	.934

Table 5, Continued

$\sum_{i=1}^{1008} x_i x_{i+1}$	52,040.72
$\sum_{i=1}^{1008} x_i$	5,096.18
$\sum_{i=2}^{1009} x_i$	5,106.18
$\sum_{i=1}^{1008} x_i \sum_{i=2}^{1009} x_i / 1008$	25,815.49
Numerator	26,225.23
$\sum_{i=1}^{1008} x_i^2$	52,981.62
$(\sum_{i=1}^{1008} x_i)^2 / 1008$	25,764.93
$\sum_{i=1}^{1008} x_i^2 - (\sum_{i=1}^{1008} x_i)^2 / 1008$	27,216.69
$\sum_{i=2}^{1009} x_i^2$	52,994.62
$(\sum_{i=2}^{1009} x_i)^2 / 1008$	25,866.15
$\sum_{i=2}^{1009} x_i^2 - (\sum_{i=2}^{1009} x_i)^2 / 1008$	27,128.47
Denominator^2	738,347,158.2
Denominator	27,172.54
$_1r_{1008}$	.965

Table 6, Runs of N_a and N_b for $_1r_{24}$ and $_1r_{72}$ with Respect to Median

$_1r_{24}$	a=above	b=below	N_a	N_b	r_a	r_b	$U_o = \lambda_a + \lambda_b$
.839		b	21	21	9	9	18
.712		b					
.818		b					
.791		b					
.795		b					
.778		b					
.784		b					
.873	a						
.573		b					
.820		b					
.584		b					
.908	a						
.885	a						
.707		b					
.863	a						
.924	a						
.779		b					
.748		b					
.866	a						
.892	a						
.825		b					
.897	a	b					
.747							
.864	a						
.945	a						
.944	a						
.931	a						
.884	a						
.888	a						
.688		b					
.871	a						
.954	a						
.885	a						
.843		b					
.587		b					
.630		b					
.873	a						
.906	a						
.789		b					
.439		b					
.930	a						
.892	a						

Table 6, Continued

$_1r_{72}$	a=above	b=below	N_a	N_b	r_a	r_b	$U_o = r_a + r_b$
.915	a		7	7	5	4	9
.794		b					
.846		b					
.893		b					
.926	a						
.880		b					
.901	a						
.901	a						
.947	a						
.898		b					
.923	a						
.801		b					
.878		b					
.934	a						

Table 7 - Comparison of ${}_1r_{24}$, ${}_1r_{72}$ and ${}_1r_{1008}$

Date:	Average:	Maximum:		
1961	$\sum_{i=1}^{3} {}_1r_{24_i}/3$	$\underset{i=1}{\overset{3}{\mathrm{Max}}}\, {}_1r_{24_i}$	${}_1r_{72}$	${}_1r_{1008}$
1/1-3	.790	.839	.915	
4-6	.788	.795	.794	
7-9	.743	.873	.846	
10-12	.771	.908	.893	
13-15	.818	.885	.926	
16-18	.817	.924	.880	
19-21	.861	.892	.901	
22-24	.836	.897	.901	
25-27	.940	.945	.947	
28-30	.820	.888	.898	
31-2/2	.903	.954	.923	
2/3-5	.687	.843	.801	
6-8	.856	.906	.878	
9-11	.754	.930	.934	
Average	.813		.888	
				.965
Maximum		.954	.947	

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THE PERSISTENCE OF AIR TEMPERATURE, A STATISTICAL STUDY

by William B. Anderson

ABSTRACT

A conceptual model of meteorological temperature at a point is discussed in terms of components of variability, including hourly temperature trend, annual trend, and hourly, daily, monthly and yearly departures from trend. The notion is advanced that a matching conceptual model can be achieved in terms of components of persistence, or autocovariance of temperature. Some relationships between components of the two models are adapted from earlier work.

The elements comprising the generally published temperature statistics are summarized, and it is shown that no direct measures of persistence are included. A direct measure of persistence is then formulated as a possible candidate for use in constructing a more nearly complete mathematical model of temperature. Several possible measures are considered, and qualities appropriate to a suitable persistence statistic are discussed. The measure chosen is a non-circular coefficient of lag correlation of hourly temperature residuals from annual and diurnal trend. By means of this statistic, calculated from a six week period of record at Los Angeles, persistence is shown to exist at a high level of statistical significance.

The lag correlation based on 24 successive hourly temperature residuals at Los Angeles is, itself, found to be subject to

persistence from day to day. Lag coefficients taken over successive 72 hour periods appear to be free of this second order persistence. It also is found that lag correlations calculated over longer and longer periods tend to be higher and higher in numerical value. Therefore, the shortest-period coefficient which is free from second order persistence is recommended for applied studies that will prove its practical value and, if the value is so proved, also is recommended as a component of the complete variance-autocovarience model of temperature.

Considerations of error in the calculation of such lag correlation statistics are reviewed. Use of a digital computer is recommended for further studies of temperature persistence as measured by lag correlations.