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Form as meter: metric forms through Fourier space

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FORM AS METER:

METRIC FORMS THROUGH FOURIER SPACE

by

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challenge the epistemological foundation of my methods (I refer to your Foucault-phase). The one thing that remains the same is the type of pizza you eat during these conversations.

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For all your support and for the honor of calling you friends, family, colleagues and mentors, thank you for everything.

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ABSTRACT

The Discrete Fourier Transform, which was initially mentioned in the music theory domain by David Lewin, is an analytical tool developed by Ian Quinn, and later expanded by theorists such as Jason Yust, William Sethares, and Andrew Milne. Though it was originally designed for pitch-class spaces, Emmanuel Amiot has explored the DFT's implementation into the rhythmic domain, and has recently used it to unravel mathematical problems in music. An explanation of the DFT model will be made available here to a reader requiring only fundamental arithmetic. Throughout this thesis, I intend to explore the DFT in the music of various composers to demonstrate applicability, and will argue for a metric conception of form.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 4: REBECCA CLARKE: "SHY ONE"

1. INTRODUCTION

The title *Form as Meter* is a reference to Christopher Hasty's seminal work *Meter as Rhythm*, in which he, by rejecting traditional conceptions of meter, avers that meter is an ongoing process. The process, as a result, projects complex patterns of periodic groupings that configure *form* from rhythm. In other words, meter is rhythmic in that it too is a perceptually creative and ongoing process.

Several significant music theorists have directly addressed, or alluded to, meter's relationship with form. Among them, was David Lewin, who, in his succinct article "On Harmony and Meter in Brahms Op.76, No.8," proposed a connection between the tonal and metric functions within the piece, after which a subsequent line of theorists extended his concepts. Lewin's work inspired metric theories such as Richard Cohn's in "Complex Hemiolas, Ski-Graphs, and Metric Spaces;" Scott Murphy's in "Metric Cubes in Some Music of Brahms;" and Samuel Ng's in "The Hemiolic Cycle and Metric Dissonance in the First Movement of Brahms's *Cello Sonata* in F major, Op.99." *Form as Meter* provides a renewed perspective on meter by addressing epistemological disputes in the rhythmic domain by demonstrating a few of the analytical utilities of the Discrete Fourier Transform (DFT) in the rhythmic domain. This paper also shows how the DFT can simultaneously incorporate and complement metric-formal theories from Lewin, Cohn, Murphy and Ng.

The DFT, which was initially mentioned in the music theory domain by David Lewin, is an analytical tool developed by Ian Quinn, and later expanded by theorists such as Jason Yust, William Sethares, and Andrew Milne. Though it was originally designed for pitch-class spaces, Emmanuel Amiot has explored the DFT's implementation into the rhythmic domain, and has recently used it to unravel mathematical problems in music (such

as tiling). An explanation of the DFT model will be made available here to a reader requiring

only fundamental arithmetic—I offer "propaedeutic orientation tools in order to make the

reader *understand why* certain conceptual mechanisms or definitions are built."[1] Throughout

this thesis, I intend to explore the DFT in the music of various composers to demonstrate

applicability, and will argue for a metric conception of form.

1.1 THE DISCRETE FOURIER TRANSFORM

The Fourier Transform (FT) is a method of decomposing a waveform into its

constituent sinusoidal parts—it analyzes the frequency domain and extracts sinusoidal

parameters. A sinusoid (or sine wave) is a smooth oscillating curve, as shown in Figure 1-1.

Fundamentally, the FT's task is to break down and retrieve information from the signal, and

ours, as interpreters, is to relate these individual elements to the whole.[2] The FT can be

thought of in terms of a painter's palette: given a mix of colors blended together, the FT has

the ability to disassemble the mixture into its primary colors and examine the influence of

those primary colors to the whole. Essentially, the tints and shades that amalgamate into a

mixture each contribute to the overall color, and by inspecting the respective contribution,

you can learn how they work in reference to the mixture. (The FT also has the ability to

reassemble the original mixture, given a collection of color.)[3] This paper focuses on the

Discrete Fourier Transform (DFT), as opposed to the *Continuous Fourier Transform* (CFT). The

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[1] Guerino Mazzola Stefan Göller, Stefan Müller, and Carlos Agon, *The Topos of Music: Geometric Logic of Concepts, Theory, and Performance* (Basel: Birkhäuser Verlag, 2002), 3.

[2] For the exact retrieval process, see the script in appendix 1.

[3] One feature of the FT that will not be examined extensively in this paper is the ability to reconstruct a whole from a set of given parts. The FT is known as an "invertible, information preserving transformation;" It is an *automorphism*.

CFT is a continuous transformation of a continuous signal, while the DFT calculates a

continuous transformation of a discrete signal—a list of discrete numbers.[4]
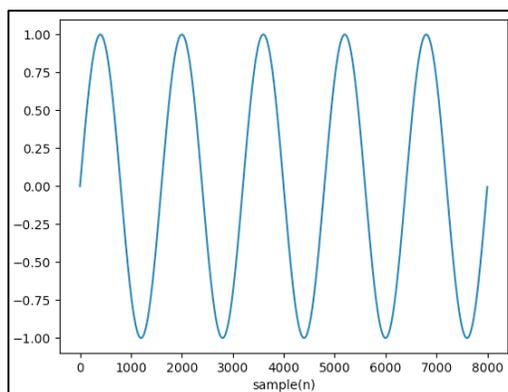
Figure 1-1. Sine Wave

It was David Lewin, in the closing remarks of his 1959 article "Re: Intervallic

Relations between Two Collections of Notes," who first proposed that the DFT be used in

music theory.[5] After Lewin touched on it again in 2001, Ian Quinn further developed the

DFT's application in music theory in his dissertation, which he summarizes and expands on

in two articles published in *Perspectives of New Music*.[6] Since then, the DFT has been applied

and written about in various contexts, analytical and theoretical alike, including, but not

limited to, creating harmonic geometries and other spaces (Amiot 2013; Yust, 2015); relating

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[4] Though I do not go into particulars here, the DFT can be seen as a specialization of the FT; the DFT deals in sums and is meant to approximate the complex integrals of the FT. Essentially, the DFT is taking discrete "samples" of time. In my full thesis, a more elaborate explanation of *why* I use the DFT is included. The actual calculation for the DFT is the following equation (continued on the following page…):

$$X[n] = \sum_{K=0}^{N-1} x[k]e^{-\frac{j2\pi nk}{N}} \quad n = 0, 1, 2, ..., N-1$$

$$= \langle x[k], e^{\frac{j2\pi nk}{N}} \rangle \text{ (Though the Python code implemented here actually uses a Fast Fourier Transform—}$$
its algorithmic equivalent).

[5] David Lewin, "Re: Intervallic Relations between Two Collections of Notes," *Journal of Music Theory* 3, no. 2 (1959): 298-301.

[6] Ian Quinn, "A Unified Theory of Chord Quality in Equal Temperaments" (PhD Dissertation, University of Rochester 2004); Ian Quinn, "General Equal-Tempered Harmony: Introduction and Part I," *Perspectives of New Music* 44, no. 2 (2006): 114-58; Ian Quinn, "General Equal-Tempered Harmony: Parts 2 and 3," *Perspectives of New Music* 45, no. 1 (2007): 4-63.

the Fourier Transform to a voice-leading approach (Tymoczko, 2008); and

defining/constructing balanced collections (Milne, et al., 2015).

Quinn (2006, 2007) confronts disagreements on theories of chord quality by using

the DFT. As Quinn defines them, there are two camps, *warp* and *weft*, which evaluate chord

quality by different criteria. *Warp* prioritizes "distance and taxonomy," while *weft* focuses on

"interval content, subset structure, and transformational symmetries."[7] Quinn uses the DFT

as a method to interweave both branches, providing a unification of the interval-content-

and distance-oriented modes of thought.[8]

1.2 DFT PROCESS

The DFT in music theory is, in simple terms, a method to parse a collection into

parts, returning information in its *Fourier components*. The DFT is a multi-faceted device that

has functioned as a method of evaluating musical quality, devising musical

spaces/topologies, measuring similarity between pitch-class sets, determining perfectly

balanced sets, and more. The transformation decomposes the interval content of a given

collection, returning a series of Fourier components—comprised of *magnitudes* and *phases*.

The following section elucidates the DFT's terminology and methodology.

The DFT, like any function, needs an input to generate an output. The input is a

vector and, in return, the DFT outputs the *magnitudes* and *phases* of different Fourier

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[7] Quinn, 2007: 120-121.

[8] The details of Quinn's operationalization will not, for brevity's sake, be explored in detail here—see Quinn's two articles in *Perspective of New Music* for a detailed account. The articles are very accessible; the reader can easily grasp the concepts without fully understanding the mathematical premises that fundamentally underlie the theory.

components f_n (all to be explained below). In the following demonstration, we examine the "tresillo-timeline." *Timeline* refers to a rhythmic pattern—also referred to as a guide pattern. The tresillo timeline, seen in Figure 1-2, is an even distribution of 3 notes within an 8-unit span.[9] The *unit* refers to the smallest divisional factor (1), and *span* is the length of the vector (8 in this case).[10] So, translated to music, the tresillo's unit-pulse equals an eighth note, and its span-pulse equals a whole note. Generally, the length of the span—or vector length—for the DFT corresponds to the modular space *n* of the pitch-class/metric cycle $\mathbb{Z}_n$. $\mathbb{Z}_n$ is determined by the cyclic space; for example, the vector length of a triad within the diatonic scale would be $\mathbb{Z}_{12}$ because there are (traditionally) 12 pitch-classes within the octave. Pertaining to rhythm, the vector length adapts to different possible subdivisions of a fixed metrical unit Thus the metric cycle of a $\frac{4}{4}$ meter divided into eighth notes would be $\mathbb{Z}_8$; that of $\frac{4}{4}$ divided into sixteenth-notes would be $\mathbb{Z}_{16}$; and that of $\frac{3}{4}$ divided into 32nd notes would be $\mathbb{Z}_{24}$ etc.[11] The coding retrieval program I have designed subdivides a given excerpt into the largest basic units such that each onset can be accounted for (note the 1s and 0s above figure 1-2). In the tresillo pattern, eighth notes subdivide the measure in order to specify each onset—therefore, the code detects that the eighth note is the unit-pulse:

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[9] The *tresillo* is known by many names and has been examined through various analytical lenses by music theorists and ethnomusicologists alike. Some of this music-theoretic work includes Euclidean algorithms (Godfried Toussaint, 2013), "platonic rhythms" as a result of 3-generation (Richard Cohn), non-isochronous meters, and maximal evenness (London, 2004).

[10] Richard Cohn, "Complex Hemiolas, Ski-Hill Graphs and Metric Spaces," *Music Analysis* 20, no. 3 (2001): 295-326.

[11] In these examples, the meter restricts the range of activity (a window), and the relationships of different subdivisions (made of the smallest unit) comprise the components.

| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

Figure 1-2. [10010010] Timeline

Because the tresillo conveys 4/4 meter, an 8-dimensional vector (or, interchangeably, an 8-dimensional array) can accommodate all of the onsets: [0, 0, 0, 0, 0, 0, 0, 0]. Each onset in the collection—positions [0, 3, 6] in relation to the range [0; 7]—occurs once, so each note (shown as '1') is added to the array in its respective position, resulting in the vector [1, 0, 0, 1, 0, 0, 1, 0]. Because we are dealing with an 8-dimensional array, the full DFT returns 8 *Fourier components* f_n: $f_0, f_1, f_2, \ldots f_7$. Each Fourier component f_n has a *magnitude* $|f_n|$, and a *phase* φ_n. Given our input (the tresillo, [1, 0, 0, 1, 0, 0, 1, 0]), the DFT returns the following output values for the magnitude and phase (Fourier component n = (magnitude$_n$, phase$_n$)) ≡ (f_n = ($|f_n|$, φ_n)):

$f_0 \approx (3.0, 0.0)$	$f_1 \approx (0.414, 0.785)$	$f_2 \approx (1.0, 1.571)$	$f_3 \approx (2.414, -0.785)$	$f_4 \approx (1.0, 0.0)$
	$f_5 \approx (2.414, 0.785)$	$f_6 \approx (1.0, -1.571)$	$f_7 \approx (0.414, -0.785)$	

Table 1-1. Tresillo Timeline: *Fourier Components, Magnitude and Phase*

The magnitude (x value) of f_0 returns the cardinality of the set—the tresillo timeline returns 3, a cinquillo timeline returns 5, etc.—so it provides no other information besides the number of onsets.[12] Also note that f_{8-n}, located in the table across from its mod$_8$ complement, indicated by the black arrows, has the same magnitude (x) and the opposite phase (y) as the

---

[12] A cinquillo timeline (also known as a bell pattern) is a traditional Cuban timeline, with the following timeline:
[1, 0, 1, 1, 0, 1, 1, 0].)

component it pairs with (with the exception of f_4, which is its own complement). Therefore, given an 8-dimensional array for input, the non-trivial information is in f_{1-4}. The number of trivial components (here being 4) changes based on the number of elements in the array; in a situation using a 12-dimensional vector, the non-trivial information would be f_{1-6}.[13] The following bar graphs—or as Amiot calls them, *Fourier profiles*—visually represent the respective magnitudes and phases of the 8 different Fourier components: gray represents f_0, blue represents f_{1-4}, and red represents the trivial f_{5-7}.[14]

	Magnitude	Phase
0	3	0
1	0.414213562	0.785398163
2	1	1.570796327
3	2.414213562	-0.785398163
4	1	0
5	2.414213562	0.785398163
6	1	-1.570796327
7	0.414213562	-0.785398163
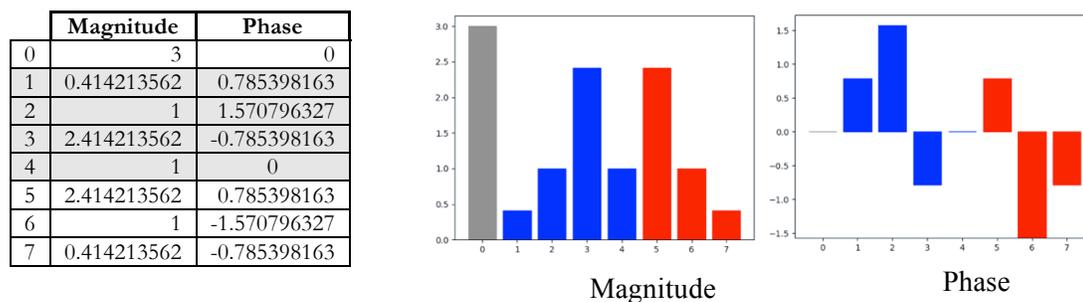
Magnitude   Phase

Figure 1-3. Tresillo Timeline: Magnitudes and Phase

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[13] This is called the *Nyquist critical frequency*; for any sampling interval Δ, the special frequency $\varpi_n = \frac{n}{N\Delta}$.

[14] Emmanuel Amiot, *Music Through Fourier Space: Discrete Fourier Transform in Music Theory* (Cham: Springer International Publishing, 2016), 12.

1.3 DIFFERENT 3-NOTE RHYTHMS IN AN 8-UNIT SPAN

To summarize, the DFT unpacks periodic elements from the given input—in our case, it exhaustively measures and expresses periodic elements within a given rhythm. Given a rhythm (in the form of onsets), the DFT returns Fourier components (f_n) comprised of magnitudes ($|f_n|$)and phases (φ_n). The following section demonstrates the benefits of viewing rhythms through a periodic perspective. As Sethares says, "Rhythm is one of the most basic ways that we understand and interact with time," [15] and, in a methodological sense, viewing it as periodicities triangulates rhythms temporally within those periodicities: in other words, does the rhythm "fit" into a specific period or not—if not, why? Are there regularities within the rhythm? Thinking about rhythm in terms of periodicity provides, as I will attempt to demonstrate, a renewed lens into metric structure, both local and global.

If we take the eighth note as the unit-pulse in a 4/4 measure, consider the intuitive difference between the rhythms shown in Figure 1-4 and Figure 1-5: [10101000] and [01010100]. Both timelines span 8-units and consist of 3 onsets, each separated by 2 units (otherwise known as quarter-note durations). The difference between the rhythms is their placement within the 8-unit span, so—to put it in set-theoretic terms—while both onsets have a (024) prime form (in a modular 8 space), they are separated by T_1 in the measure (see Figure 1-4 and Figure 1-5) so that they share no onset positions. Because these rhythms are transpositionally identical, the magnitudes $|f_n|$ for both of these timelines are identical as well—seen in the left images of both Figure 1-6 and Figure 1-7.

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[15] William Arthur Sethares, *Rhythm and transforms* (Springer-Verlag London, 2014), 1.

Figure 1-4. [10101000] Timeline
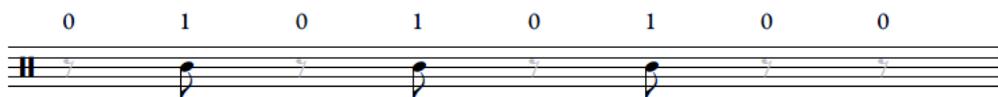
Figure 1-5. [01010100] Timeline

[10101000]		
Component	Magnitude	Phase
0	3	0
1	1	-1.5707
2	1	0
3	1	1.5707
4	3	0
5	1	-1.5707
6	1	0
7	1	1.5707
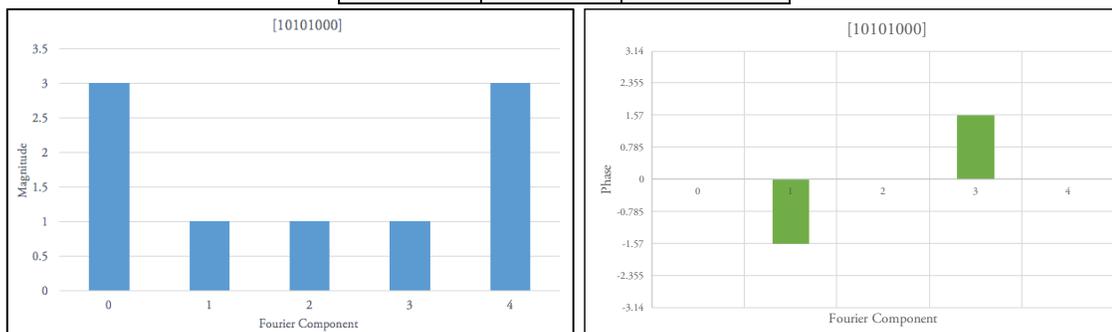
Figure 1-6. [10101000] Timeline: Magnitudes and Phases

[01010100]		
Component	Magnitude	Phase
0	3	0
1	1	-2.356
2	1	-1.5707
3	1	-0.7853
4	3	3.14
5	1	0.7853
6	1	1.5707
7	1	2.356
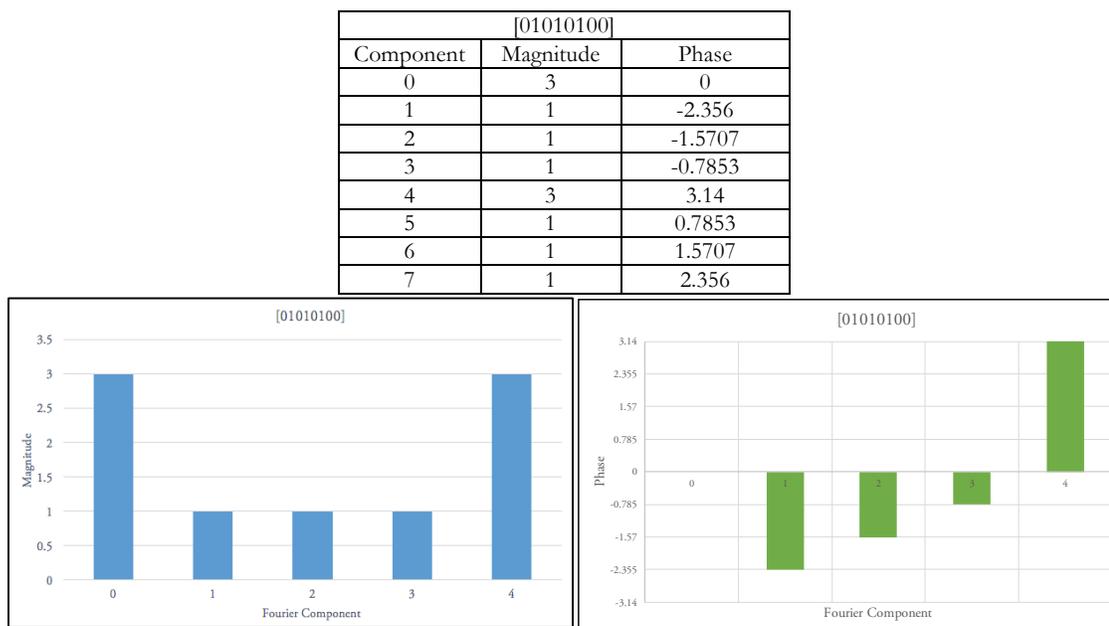
Figure 1-7. [01010100] Timeline: Magnitudes and Phases

In both rhythms f_0 is 3, meaning that the cardinality of the set is 3 and, by association, that the highest possible value for any given component in the span will also be 3. If a magnitude value is equivalent to the cardinality, I will refer to it as *perfectly maximal*. From that, it follows that both of these rhythms have, given a cardinality of 3, a perfectly-maximal magnitude value for f_4. This means that for each period, each onset lands in the same position within f_4's periodic phase. To figure out which subdivisional pulse the specific component (f_n) corresponds to—otherwise known as the period—one divides the span by the respective component. The period of f_n equals the span (s) divided by the component (n):

$$p = \frac{s}{n}$$

The span equals 8, and n equals 4, so $2 = \frac{8}{4}$; the period is 2 units and is, therefore, notationally equivalent to a quarter note (two times the eighth-note pulse unit). In this case, the subdivision is a quarter note—splitting the measure into four—meaning that each onset

is positioned in the same quarter-note projection (see Figure 1-8).
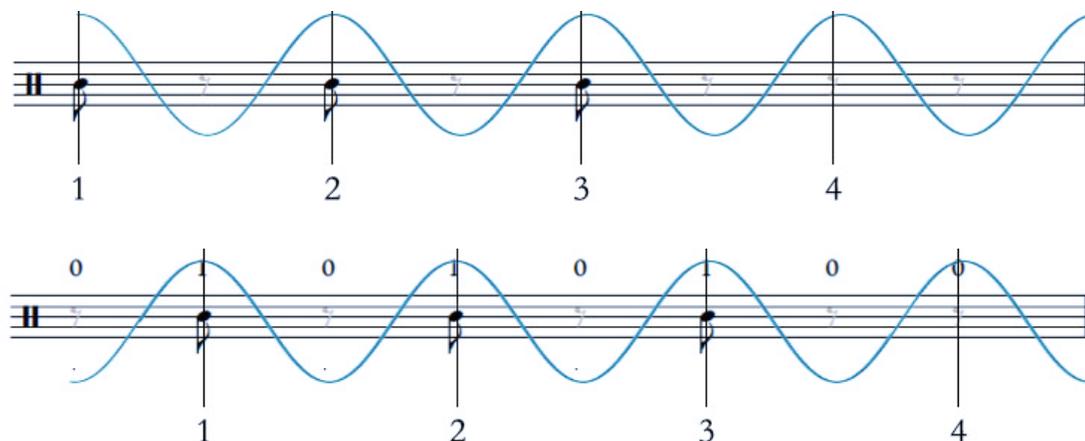
Figure 1-8. [10101000] and [01010100]: f_4

Though their magnitudes are identical, the two timelines are positioned differently

within the 8-note span. *Phase*, a component's positioning in the span or array, shows how

these positions differ. The phase value will always be a position that maximizes the

components magnitude. Figure 1-9 represents each phase as a unit circle. This representation

is possible because phases are cyclic. (The circumference of the unit circle is, by definition,

2π.) The spaces in Figure 1-9 are known as *phase spaces*. Phase spaces (Ph_n) show the phase of

a given component; Ph_n is simply the phase of f_n in the form of a circle, ignoring the

magnitude.[16] To produce all phase spaces, one multiplies the integers in Ph_1 by *n*. For

example, to derive Ph_2, multiply each integer in Ph_1 by 2 in mod_8: $(1*2)mod_8$, $(2*2)mod_8$, …

$(7*2)mod_8$. This process divides the space into *n* parts—integers may, and often will,

superimpose on the same modular position (Figure 1-9—these circles read

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[16] Developed in Yust (2015); each phase space corresponds to one of Quinn's *Fourier Balances*—see Quinn 2007, 2008.

counterclockwise).[17] The value of φ_n refers to the rhythm's orientation/position in the span.

In φ_4, the phase returns either 0 (upward) or π (downward), which means an onset will be

on-the-beat oriented or syncopated; the north position 0, comprised of the onsets [0246],

corresponds to metric downbeats, while the south position π, comprised of the onsets

[1357], corresponds to upbeats. Turning back to Figure 1-4's timeline, since φ_4 of the

[10101000] timeline equals zero, the positioning of the onsets is downbeat-oriented. By

associating magnitude with phase, we can posit that every onset corresponds to a position

"on the beat:"

$|f_4|$/<u>Magnitude</u>: We know that every onset lies at the same point in the period because $|f_4|$
is perfectly maximal (equivalent to the cardinality).
φ_4/<u>Phase</u>: The phase value equals 0, meaning it is downbeat-oriented.
<u>Magnitude with Phase</u>: If $|f_4|$ is perfectly-maximal—meaning every onset corresponds to a
position that divides the span into 4 parts—and φ_4 equals 0 (downbeat-oriented), then every
onset must be on a downbeat.

This logic also applies to the [01010100] timeline; because $|f_4|$ is a perfectly-maximal value,

everything lies in the same position of the period, and because φ_4 equals 3.1415 (π), the

whole timeline is located "off the beat;" the rhythmic pattern is syncopated. While these

rhythmic examples are strictly pedantic, this example demonstrates how the DFT

component may qualify syncopated-ness/downbeat-ness.

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[17] If *n* is coprime with the cycle, then the complete phase spaces in modular space will not have
superimposed numbers.
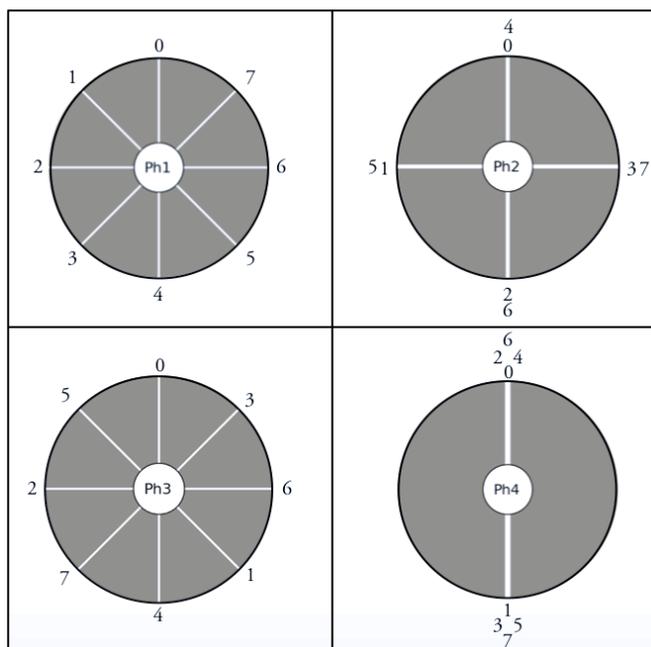
Figure 1-9. Phase Spaces for Z_8

Take the timelines [11000001] and [10010010], shown in Figures 1-10 and 1-11. Like timelines 1 and 2 (in Figures 1-4 and 1-5), their span and onset cardinalities are equivalent; however, the first timeline contains closely packed onsets (012), while the second distributes the onsets over the span (036)—i.e., the first rhythm has onsets separated by 1 unit and the second has onsets separated by 3. We can also view these rhythms in terms of their distribution across the span. If we were trying to generate a maximally-clustered timeline of 3-units, each with a unique location, within an 8-note span, the resulting timeline would include the (012) set; if we were trying to generate a maximally-even timeline of 3-units under the same conditions, the resulting timeline would include the (036) set.[18]

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[18] John Clough and Jack Douthett, "Maximally Even Sets," *Journal of Music Theory* 35, no. 1/2, (1991): 93-173.
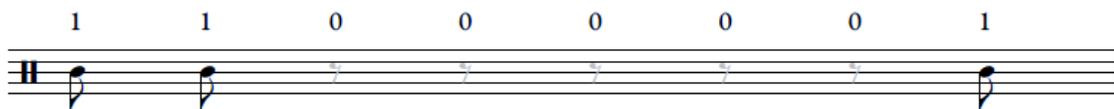
Figure 1-10. [11000001] Timeline
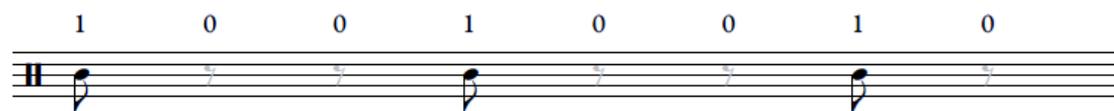
Figure 1-11. [10010010] Timeline

The magnitudes of the DFT components parallel our intuitive description of the timelines; Figures 1-12 and 1-14 show both the magnitudes and phases of Figures 1-10 and 1-11 respectively. In Figure 1-12, f_1 contains the highest magnitude: $|f_1| \approx 2.414$. There is a high value for f_1 because it approximates the division of the span (8) into n (1) parts, represented $8 = \frac{8}{1}$, which means that the high magnitude value for f_1 represents an onset cluster around a single point in the span (Figure 1-13). The tresillo rhythm ([10010010]) in Figure 1-11 is distributed maximally-evenly in an 8-note span—this is reflected by the high magnitude of f_3 in Figure 1-14. The high magnitude of f_3 means that this rhythm approximately divides the span into 3 parts. When we calculate $p = \frac{s}{n}$ for the tresillo timeline ($p = \frac{8}{3}$), the period value approximately equals 2.667—not a discrete value in an 8-span. Because the division (3) can not split the span (8) evenly, the highest possible magnitude for f_3 in Z_8 would have to be an approximated division of the space (and not equivalent to the cardinality)—hence the terminology *maximally* even. Figure 1-15 shows how this periodicity parses the space, and, given 3 onsets, how the tresillo timeline maximally divides this space into 3 parts. Both of these timelines are examples of a *magnitude maximization* of a specific component—this is made clearer when both graphs are compared next to each other (seen in Figure 1-16).

Component	Magnitude	Phase
0	3	0
1	2.414	0
2	1	0
3	0.414	3.14
4	1	3.14
5	0.414	3.14
6	1	0
7	2.414	0
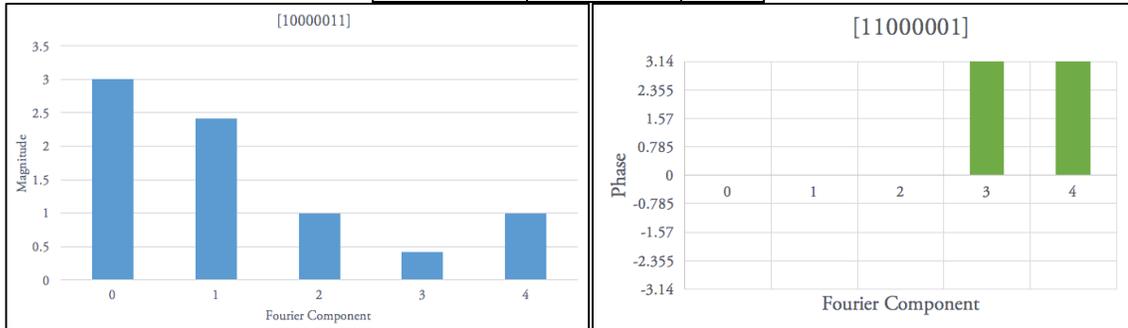
Figure 1-12. [11000001] Timeline: Magnitudes and Phases
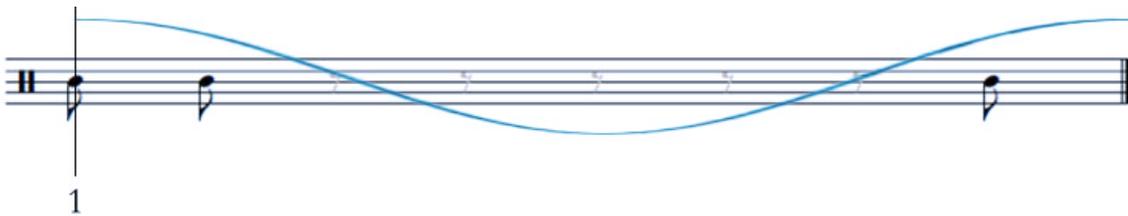
Figure 1-13. [11000001] Timeline: f_1

Component	Magnitude	Phase
0	3	0
1	0.414213562	0.785398163
2	1	1.570796327
3	2.414213562	-0.785398163
4	1	0
5	2.414213562	0.785398163
6	1	-1.570796327
7	0.414213562	-0.785398163
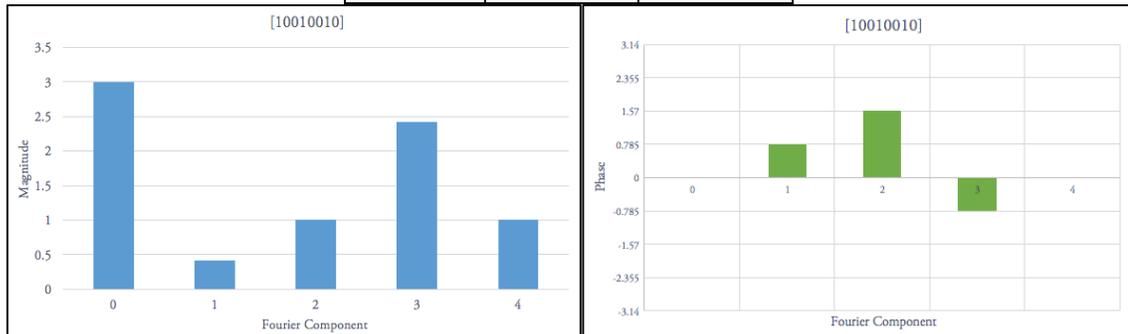
Figure 1-14. [10010010] Timeline: Magnitudes and Phases
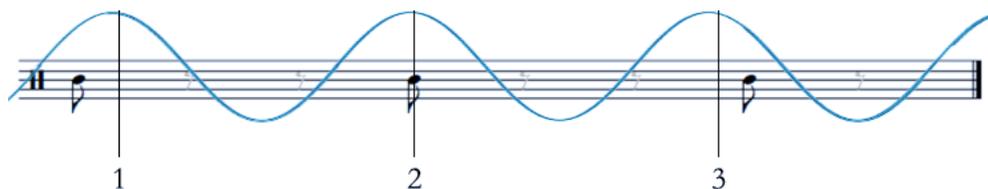
Figure 1-15. [10010010] Timeline: f_3
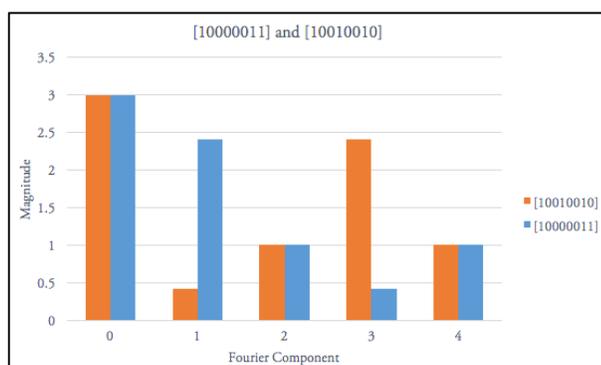
Figure 1-16. [10000011] and [10010010] Timelines: Magnitudes

If we embed the topological unit-circle of the phases within a Cartesian plane, we can represent phases *and* the magnitudes of a given profile. Each onset is therefore represented in complex space as a vector, which inherently involves magnitude and direction. The magnitude of a component is the distance from the center, and the phase is the angle. Figure 1-17 demonstrates the process of plotting the tresillo into Ph_1. The onsets [0, 3, 6] are charted as vectors in the left diagram, which are combined together in the right diagram, resulting in a visual representation which displays both magnitude and phase.[19] Figure 1-18 is a diagram of all components in complex space for the tresillo rhythm [10010010]. In the component-3 space, due to the onset positions [0, 3, 6], all vectors extend in a similar direction. In fact, this is the farthest distance that any three unique onsets can extend in one direction in Ph_3. Ph_1 for the timeline in Figure 1-10 would also extend the

---

[19] "Combination" here means that an order of operations does not apply—any order has the same result, in contradistinction to "permutation."

farthest distance that any three unique onsets can extend in one direction. In this way, the

DFT is able to quantify evenness and clustered-ness for a given collection.
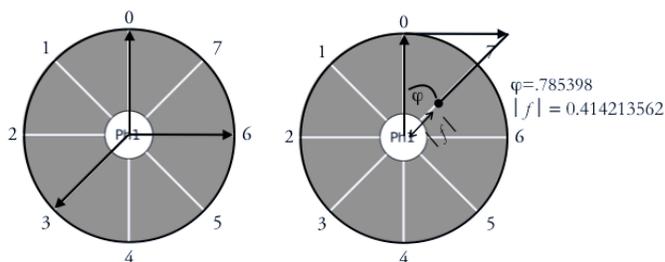
Figure 1-17. Tresillo Timeline: *Ph₁ Tutorial*
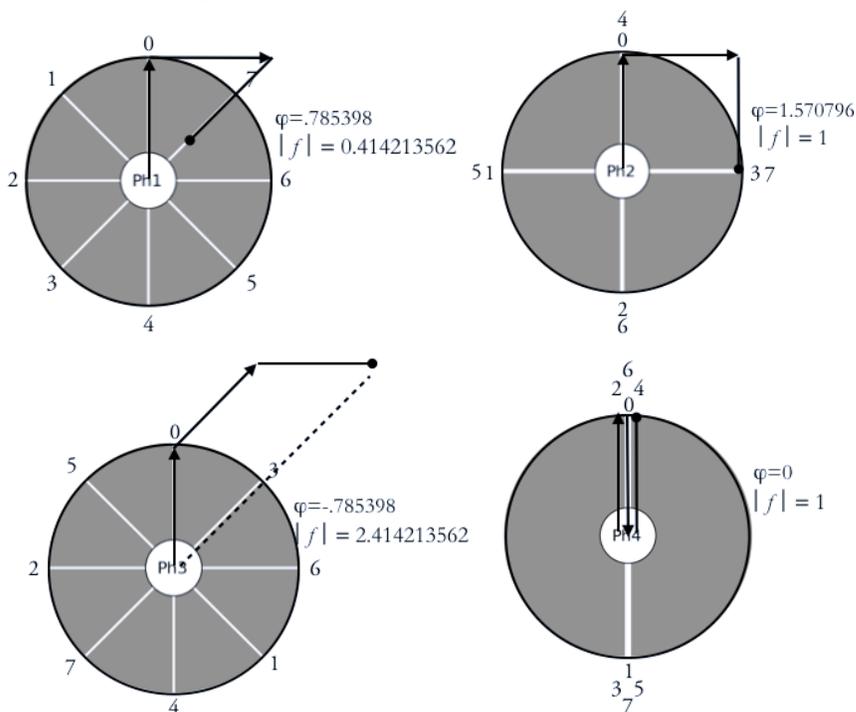
Figure 1-18. Tresillo Timeline: *Complex Spaces*

The timelines in Figure 1-19 and Figure 1-20 appear to be similar. One apparent

difference between them is that an eighth note is displaced; the voice leading between the

two timelines—[10001001] and [10001010]—is $[0,4,7] \xrightarrow[0,0,1]{} [0,4,6]$.[20] We can intuitively

---

[20] Dmitri Tymoczko, *Geometry of Music Harmony and Counterpoint in the Extended Common Practice* (Cary: Oxford University Press, USA), 2010.

describe Figure 1-19 as a rhythm that evokes half-note impulses, (dividing the space into two parts with one onset at the end [7],) and Figure 1-20 as a timeline that conveys quarter-note impulses. The magnitude profile for Figure 1-19 correlates with our depiction (shown in Figure 1-21). The component with the highest magnitude is f_2, which means that the pattern closely divides the span into two $p = \frac{8}{2} = 4$, the half-note value.[21]
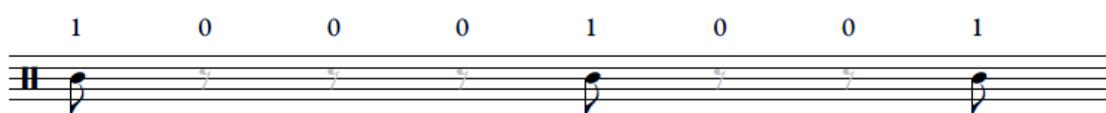
Figure 1-19. [10001001] Timeline

Figure 1-20. [10001010] Timeline

Component	Magnitude	Phase
0	3	0
1	1	0.785
2	2.236067977	0.463
3	1	2.356
4	1	0
5	1	-2.356
6	2.236067977	-0.463
7	1	-0.785
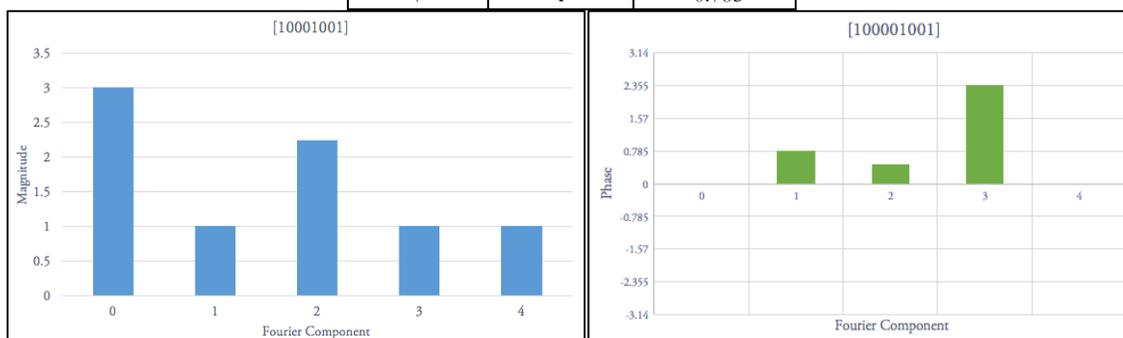
Figure 1-21. [10001001] Timeline: Magnitude and Phase

[21] The reason that $|f_2|$ is not equal to the cardinality $|f_0|$ is because of the final onset. The onset at position [7] is not reflected in the same half-note periodicity which contains [0, 4] (Figure 1-22). However, though the magnitude is not equal to the cardinality, for three unique onsets, this rhythm maximizes f_2 for any three unique onsets in an 8-note span.
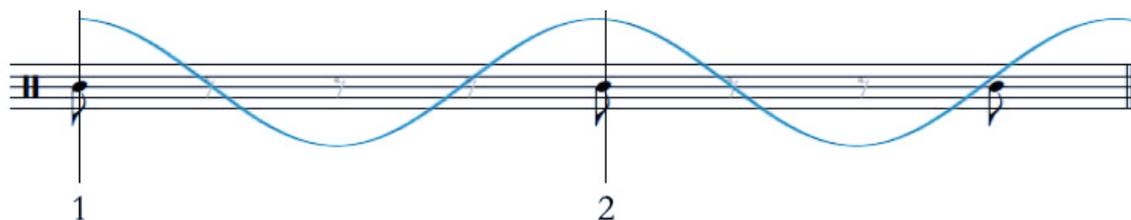
Figure 1-22. [10001001] Timeline: f_2

Despite their proximity in voice leading space, the [10001010] timeline has a

perfectly-maximal magnitude at f_4 (Figure 1-23), instead of an approximated maximization of

f_2—such as for [10001001]. The displaced onset changes the rhythm so that f_4 is perfectly-

maximized. In other words, with the displaced note's new position, every onset is accurately

positioned in terms of the quarter-note periodicity (Figure 1-24) (as opposed to the half

note). The [10001010] timeline shares the same magnitude profile as the timelines in Figure

1-4 and Figure 1-5. The difference, again, between this rhythm and the other two is its

position in the measure. The phase of f_4 is 0, which means the rhythm is downbeat-oriented.

Using our previous proof, we know that, by combining phase and the perfectly maximal

magnitude, everything lies on a downbeat. Both Figure 1-4 and Figure 1-24 have all onsets

on the downbeats—hence $\varphi_4 = 0$—however, their downbeat position in the measure is

different. The phases spaces of f_1 shows the phase of each pattern to be different (Figure 1-

25). The [10101000] timeline has the phase of the three onsets at position [2], hence why in

Figure 1-25, φ_1 locates the onset-collection at beat-class [2]; the [10001010] timeline has its

phase at position [6], hence why φ_1 locates the onset collection at beat-class [6].

Component	Magnitude	Phase
0	3	0
1	1	1.5707
2	1	0
3	1	-1.5707
4	3	0
5	1	1.5707
6	1	0
7	1	-1.5707
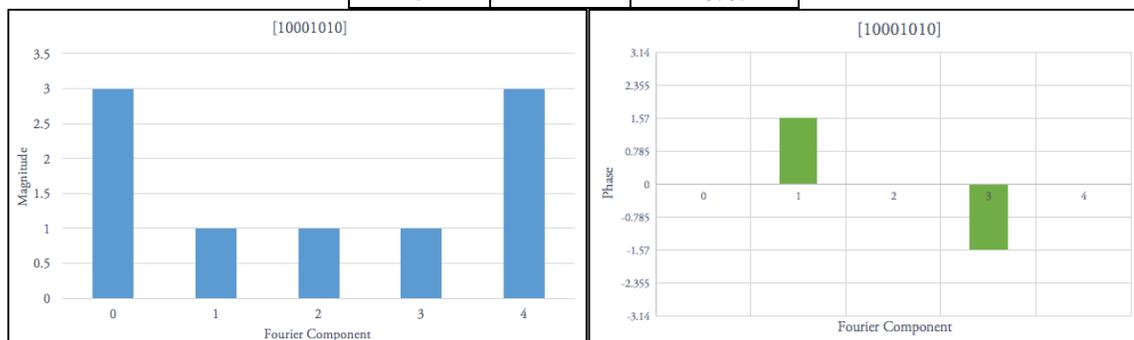
Figure 1-23. [10001010] Timeline: Magnitudes and Phases
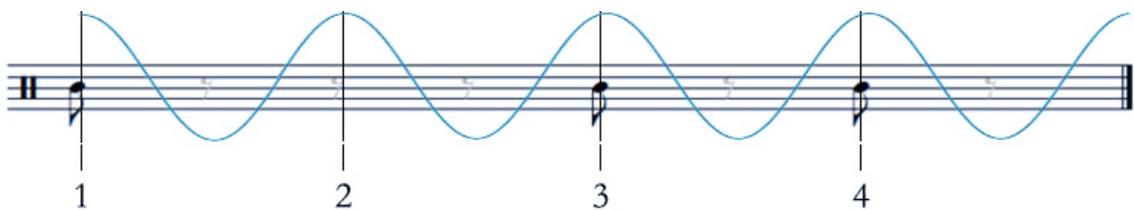
Figure 1-24. [10001010] Timeline: f_4
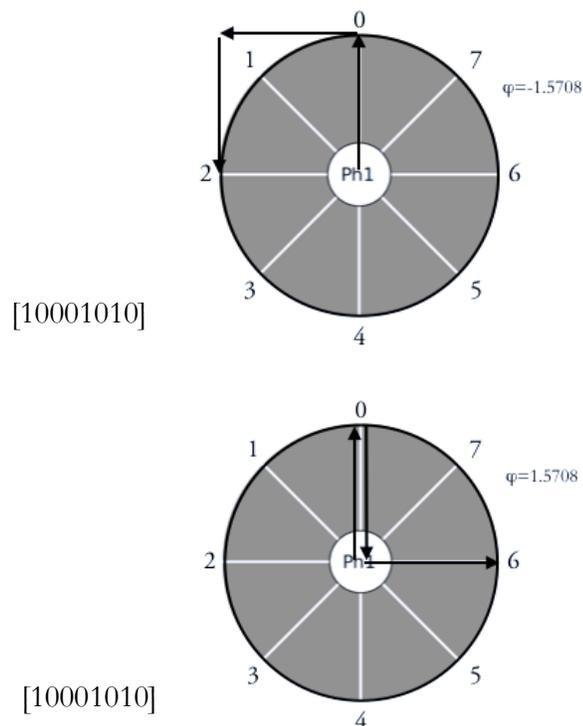
[10001010]

[10001010]

Figure 1-25. [10101000] and [10001010] Timelines: φ_1

By using the DFT, we view rhythms in relation to periodic structures. I have demonstrated how the Fourier components reflect our intuitive ideas about certain timelines while simultaneously qualifying rhythms in terms of certain concepts—such as syncopated-ness, evenness, and clustered-ness. As I argue here, by examining rhythms through Fourier space, we can make analytical claims about rhythm at a local level, and, as I extend the approach through the rest of this thesis, to broad metrical structures throughout a piece.

CHAPTER 2. ONSET RETRIEVAL WINDOWED DFT SCRIPT

The following chapter describes a coding procedure I implement in order to use the DFT analytically; thus far, we have only examined vectors with 8-unit spans, but in order to analyze entire pieces, consisting of thousands of onsets, a computer will expedite the process. Rather than count all note-onsets, and calculate the DFT for each piece of music, I chose to make a Python script to return the results. Python is a high-leveled program language that stresses readability.[22] Like any coding language, Python it is a powerful resource because it is efficient when dealing with quantities that would be otherwise cumbersome to do by hand. Coding languages are double-edged swords in that they perform the exact task that was coded; in the retrieval process, there is no leeway—they attempt to perform the script indiscriminately, returning either the desired result or an error.

ORWDFT: Figure 2-1 shows a flow chart modeling how my code retrieves all the onsets, returns a series of overlapping windows throughout a piece, and then calculates the DFT on said windows (which will be defined below). Given the task—onset retrieval, windowing, and DFT calculation—the code will henceforth be referred to as the initialism ORWDFT.

---

[22] A high-level language (HLL) is easier to read and write than lower-leveled machine languages; HLLs mimic human language more closely.
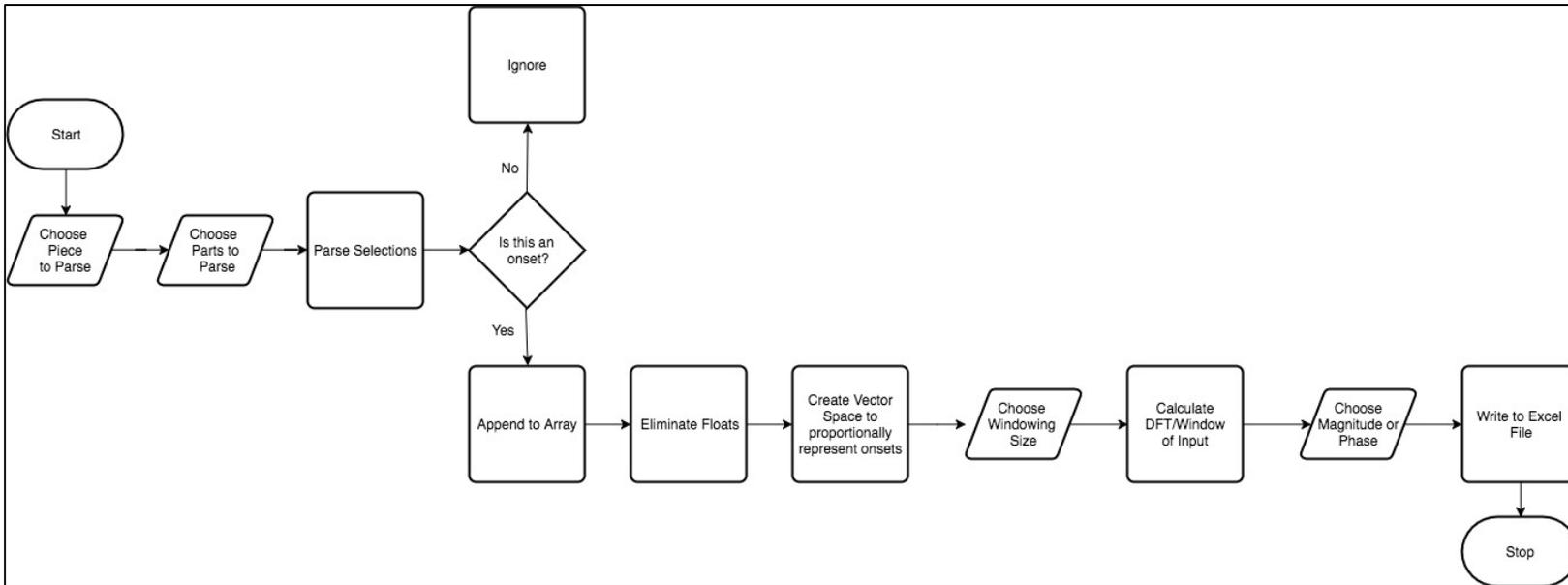
Figure 2-1. Onset-Retrieval/Hanning-Window/DFT Flowchart

Retrieval: The code uses the music21 package to unpack a score's note information, checking for *".Chord"* or *".Note"* types and monitoring the *"offset"* position of these occurrences.[23] In other words, it scans for situations where a note sounds and appends the "offset" position of where that note occurs in relation to the beginning. It then appends the *"multisetCardinality"*—the amount of notes sounding simultaneously—of each instance to a corresponding offset position in a list. The retrieval portion of the code returns an isomorphic vector that retains relative positioning of onsets within an XML file. Each onset is represented by 1, so for a multiset the onset density located at that position matches the cardinality—for a C major triad [C, E, G] the cardinality is 3; for a C major triad [C, E, E, G] the cardinality is 4.

Subdivisional Accommodation: The code creates *windows* as a vector space filled with zeros that occupy subdivisional positions corresponding to musical events. Therefore, a tresillo rhythm, corresponding to the input [10010010], can be padded with twice as many zeroes to expand the array to [100000010000001000].[24] This *zero padding* process allows for smaller subdivisions, and retains the input's proportions. The end result is a list comprised of proportionally-related onset values referred to as a *Discrete Time Series*.

Definition 1. *Discrete Time Series*: A time series $T = t_1,\dots t_l$ is an ordered set of l discrete-values variables.

Windowing: To simulate the progression of metric states over the course of a piece, and to track the changes of these states, the code proceeds to extract overlapping *Windows*

---

[23] Music21 is a Python package which has been supported by the School of Humanities, Arts, and Social Sciences at M.I.T. Michael Scott Cuthbert is the principal investigator. It, in simple terms, is a toolkit to aid in computational music studies. To read more about *".Chord,"* *".Note,"* and *"offset"* see the music21 documentation.

[24] Note that in Python, arrays are technically represented as "lists" of integers.

from the overall onset list (the *Discrete Time Series*). By overlapping the windows, we can see a

gradual progression from one metric state to the next.

Definition 2. *Windowing*: A subsequence S_y of T is of length $w < l$ of adjacent
positions from T. $S = t_y,...t_{y+w-1}$ for $1 <= y <= l-w+1$

Snapshots returned from the ORWDFT generate a series of windows with initial onsets

separated by the notated quarter note. The specified window size could potentially privilege

certain periodicities, so to prevent this bias I implement a *windowing function* to weight the

array towards the middle – a bell curve weighting called the *Hanning window*. Where N is the

length of the array, for *n* in N:

$$w(n) = .5 - .5 \cos\left(\frac{2\pi n}{N}\right), \quad 0 \leq n \leq N - 1$$

Hann window
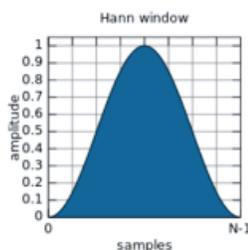
Phase Shifter: While the magnitude is unaffected by the ordering of the elements

within the list, phase is affected. Because phase involves positioning of the component in the

span, when discussing phase values in music, it is important to distinguish between *notated*

measures, and the *window*. As the *scanning window* shifts over through the onset list, the

resulting phase values will consistently change with each new array because the calculation

on its own does not account for the scanning movement—this requires a new calculation for

each window to see where the phase value is positioned in the span. Take Figure 2-2, which

clearly conveys $\frac{4}{4}$. The vector form of a four-measure group would assign 2's to all positions

that have a multiset (two notes sounding simultaneously) and 1's to positions having just

quarter notes, like this: [2111211121112111]. When the window moves over one position—a

quarter note—and continues to scan starting from the second beat of the notated measure,

the following vector would appear as: [1112111211121112]. The magnitudes of both of these

vectors would still be identical, but the phases would vary because of their different

positioning in the vector. As the window moves through the music, the phase value will be

aligned indiscriminately to the new window. This is not inherently negative; in fact, we can

still calculate the same value of phase from this newly situated position, but we would have

to calculate it for every new shift. Because it is easier to have a consistent reference point,

rather than recalculating where the phase is in every window, the code preemptively

implements a *phase shifter* to compensate for the *sliding* element of the *sliding window*.

Figure 2-2. *Phase Shifter* Example

To accommodate for the quarter note shifting, if, for each shift, we reposition each

element in the array "back" one quarter note, then the phase values will remain anchored to

the original notated barline: [1112111211121112] → [2111211121112111].[25] This way the

phase does not need to be recalculated for every shift the window makes, and we can instead

reference its position to the notated measure (based on the first scan); if the piece starts on a

notated downbeat, the phase values will always be calculated in reference to the notated

downbeat. It is easier to normalize the phase so that the notated downbeat always aligns with

---

[25] Where $\mathbb{Z}_n$ is a cyclic space of the vector input V, and T is a linear transformation mapping $V \to W$, the phase shifter operation is $V \xrightarrow[n+1(mod\,n)]{} W$. Therefore after the phase shifter, the second notated beat is normalized [1112111211121112] $\xrightarrow[16+1(mod\,16)]{}$ [2111211121112111].

one position, and in this way the phase remains anchored to the notated bar line.[26] Note that

the code does not recognize notated changes in meter and does not reset to new positions

for the barlines—it behaves as the music was notated in a consistent meter throughout. The

organization of the notated meter vis-a-vis the sliding window position will be relevant later

in this paper when examining a piece that has frequent shifts in meter, and resultant shifts in

phase-values.

2.1 Phase Tracking

As an example of how phase tracking works, take the exposition from Mozart's

Symphony No.41, K. 551, "Jupiter," seen in Figure 2-3. The meter is an unmistakable *pure*

duple, integrating periodicities at the quarter-note, half-note, and whole-note levels.[27] After

scanning with the ORWDFT, the magnitudes for these components are comparatively high,

so by isolating one of these components and following the phase, we will be able to track

how the music "moves" within the span. In other words, does the meter become displaced?

Are there onset groupings that challenge the previously established phase? By following the

phase of multiple musical lines, we can discuss theoretical ideas like *metric displacement* and

syncopation.[28]

---

[26] As a reminder, the *phase shifter* this does not affect the magnitude because only the placement in the list has changed.

[27] Richard L Cohn, "The Dramatization of Hypermetric Conflicts in the Scherzo of Beethoven's Ninth Symphony" (*19th-Century Music*) 15, no. 3 (1992): 188-206).

A *Pure* meter, by Cohn's definition, refers to when a meter consists of subdivisions that are a power of some prime integer. In the case where the *complex*—analogous to span—is 16, it is pure because it is only divisible by 2^4 (as opposed to a mix of two primes).

[28]Note that the phase shifter makes it easier to see all of these features in terms of an absolute reference point. Without the phase shifter, it would still be possible to compare DFT results and find these theoretical ideas, but each calculation would be isolated; the phase shifter effectively gives a reference point for phase values.

Figure 2-3. Mozart Symphony No. 41, K. 551 "Jupiter" Exposition, mm. 1-25

Figures 2-4 and 2-5, respectively, show the phase and magnitude of f_8—the component that corresponds to the half-note pulse. Rather than showing a metric profile of a single metric state, which visualizes every component at an immediate instance from one calculation, I chose to isolate f_8 and graph both the phase and magnitude *over time*. The *x*-axis represents the starting beat of the window, and each point on the phase line-graph corresponds to a point on the magnitude graph. To determine what measure the starting position of the window corresponds to, divide by 4 and add 1 (because in this example we have measures of 4 beats). The window position corresponds to that measure—if there is a floating integer remaining, it refers to the exact position within that measure. For the phase graph, "0" on the *y*-axis corresponds to the notated downbeats of the score. A positive value is "behind" the beat, and a negative value is "ahead" of the beat. The phase generally stays within the same range for the whole exposition. This can be interpreted as the half-note value being consistent in its placement in the phase, and, referring to the score in Figure 2-3, we realize these results make sense for the notated music; the notated music has strongly weighted onset groupings on beats 1 and 3. The phase in Figure 2-4 lies just above 0, which means that the phase is positioned just behind the beat. Given that, the plateau of phase values just above zero is pushed positive due to the grouping of sixteenth-note triplets that propel the music to the downbeats. Therefore, the phase graph displays how the music is in phase with the given downbeats, but also depicts how there is a solid group of onsets just before.
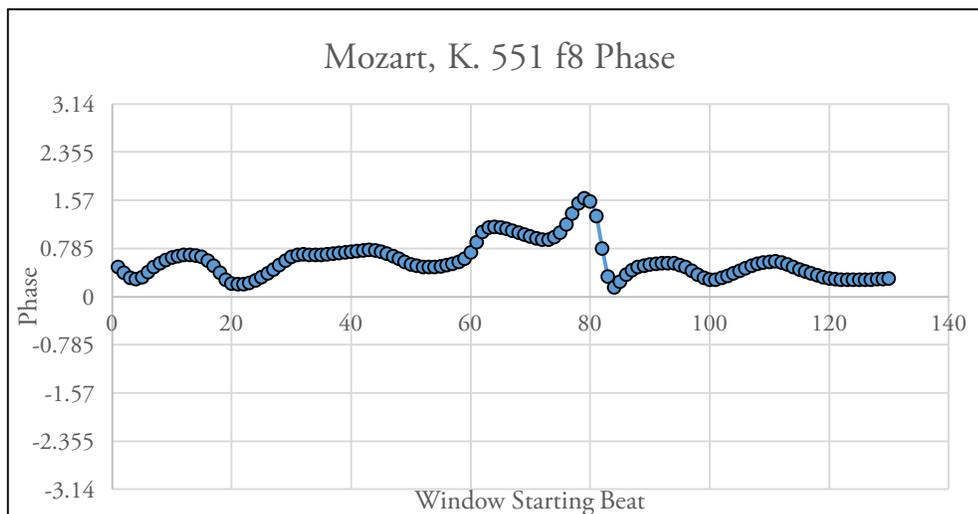
Figure 2-4. K. 551 "Jupiter" Exposition: f_8 Phase
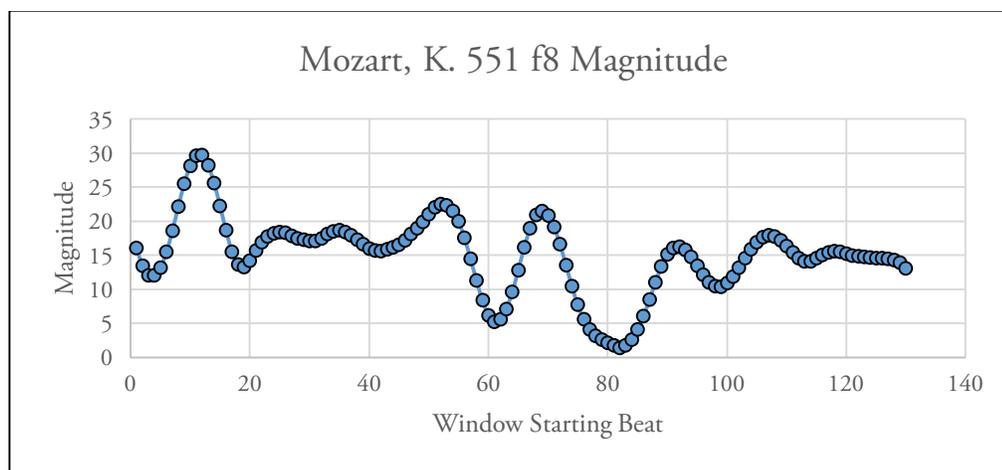
Figure 2-5. 551 "Jupiter" Exposition: f_8 Magnitude

In Figure 2-4, there is a change in phase around measures 60-85, approximately corresponding to mm. 15-–20. After a brief examination of the score, it is obvious that these alterations are caused by the syncopations in the flute, violin I, oboe and horn. Up until measure 15, the half-note projections have primarily been grouped with the notated downbeats. It is these syncopations placed on beats 2 and 4 that challenge the previously established projection stream—the projection on the off-beats has an opposing phase. By positioning themselves in a way that directly opposes the half-note, the magnitude value

drops significantly (seen in Figure 2-5), and as a result, destabilizes the phase, making it much more malleable.[29] What this means musically is that stimuli opposing the established magnitude—here the syncopation—makes the phase easier to manipulate. This captures what Krebs would call "preparation," or the more gradual manner of moving from consonance to dissonance; drops in magnitude and phase destabilization captures the process of introducing dissonance.

The influence of magnitude on phase can be portrayed in an analogy. Imagine a trip through the city: both Achilles (A) and the Tortoise (T) are trying to attend a musical performance at Symphony Hall (SH), located on Huntington Avenue (Figure 2-6a). They are both on Massachusetts Avenue, which is a street perpendicular to Huntington Avenue. Despite Achilles and the Tortoise being on the same perpendicular street, the Tortoise is closer to the hall itself. In this analogy, phase represents the streets/angle from the intersection, and magnitude is the distance from the intersection. The Tortoise is able to change his respective angle in relation to the intersection with relatively little movement. Figure 2-6b shows a series of angles from the intersection (which represents phase). The distance that each character needs to move to reach a point that shares the same angle from the intersection is not equivalent in movement through their pathways—Achilles must move significantly more than the Tortoise. A lower magnitude value means that the phase will be more susceptible to change.

---

[29] Because phase is easily influenced without a high magnitude, it is important to use both in tandem—a phase that shifts willy-nilly will have an insignificant magnitude.
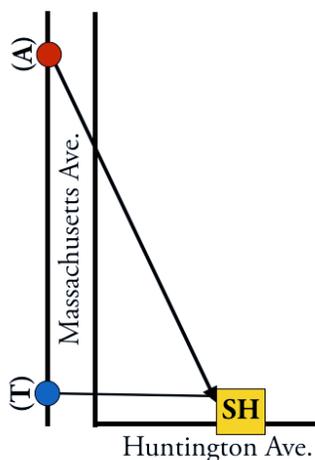
Figure 2-6a A Trip to Symphony
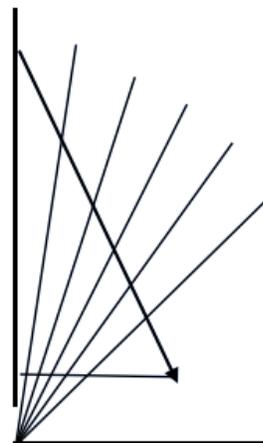
Figure 2-6b. Angles from the

Intersection

As discussed in the DFT walkthrough from the first chapter, we can map the phase and magnitude onto a unit circle by translating into its polar form on the complex plane.[30] Figure 2-7a shows the magnitude and phase of f_8 of Mozart's K. 551, visualizing both magnitude and phase in a complex plane. The cluster of points near "15" on the x-axis represents the average positioning of the phase for the exposition. To navigate the tangled paths, Figure 2-7b and Figure 2-7c divide the phase tracking into two visualizations—Figures 2-4 and 2-5 should be used in conjunction to understand how magnitude and phase interact. In Figure 7b, the red triangle corresponds to the first window. As expected, the phase there is located in the center of the point-cluster, and of our average phase value overall (as shown previously by the plateau in Figure 2-4). The red triangle that marks the ending of the scan (in Figure 2-7c) is also positioned in the same area, signifying that both the phase and magnitude are similar. What this means musically is that, after the end of the exposition the

---

[30] Given magnitude and phase, to return polar coordinates:
$(x = |f_n| \cdot \cos(\varphi_n), y = |f_n| \cdot \sin(\varphi_n))$

phase reverts back to what it was at the beginning—the music returns to a stable $\frac{4}{4}$.

Yellow squares are farther from the center-point of the circle, and refer to moments of high magnitude, while green squares are closer to the center-point and are areas of low magnitude. Each point refers to a quarter note in the score.[31] The two windows corresponding with beats 78–79 (bridging Figures 2-7b and 2-7c) are halfway through measure 19. The low magnitude at that moment is due to the reiterated eighth-note subdivisions. The even subdivisions flatten the magnitude, and create little preference for other components. As a result, these moments visually correspond to positions close to the circle in the phase space for f_8. The phase remains almost entirely in quadrant I—this is representative of a piece that remains consistent in its meter and placement of said meter. The sliding window and general changes will inevitably move the phase value around— hence the movement within quadrant I—but what is mainly relevant is if it clusters around the same area in that quadrant. The one point at which the graph dips into quadrant II can be explained by our discussion of Achilles and the Tortoise; with the already low magnitude, movement in phase is more dramatic—this makes it harder to locate a position of a metrical layer. While the phase in the Jupiter is straightforward, I will discuss more complex phase interpretations in later analyses.

---

[31] If a piece consistently maintains one notated meter, to find the musical beat corresponding to a given point, divide the point-number by the measure. The remainder is the beat in that position. In the Jupiter Symphony, we have 4 beats per measure. Say we are trying to find which musical position corresponds to the 70th window: 70 /4 = 17 with a remainder of 2. The position of the 70th window is measure 18 on the second beat.
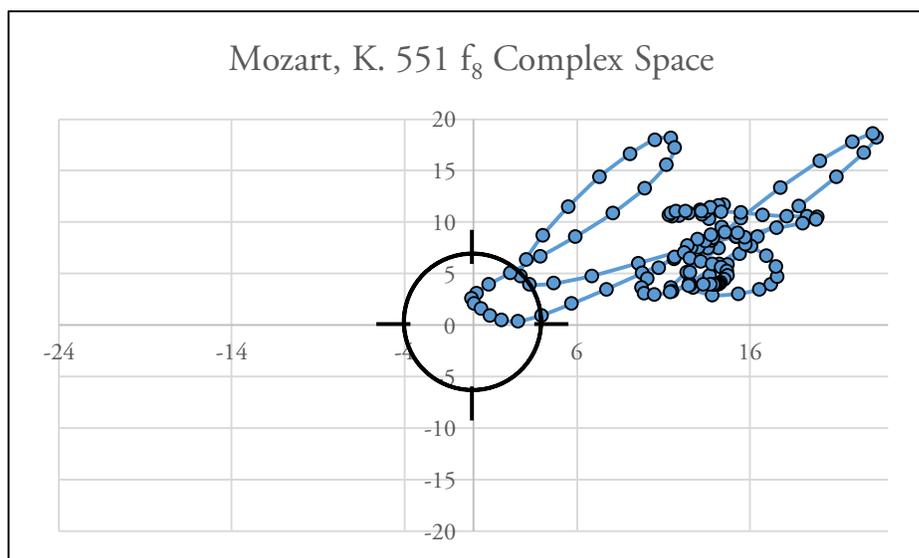
Figure 2-7a. Mozart K. 551, f_8 Complex Space
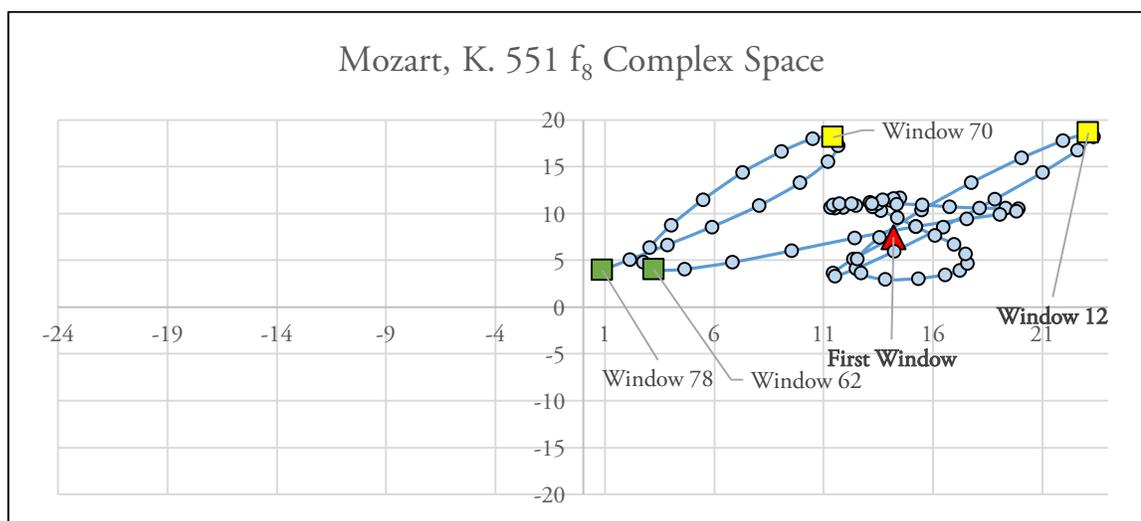
Figure 2-7b. Mozart K. 551, Complex Space for f_8, part 1
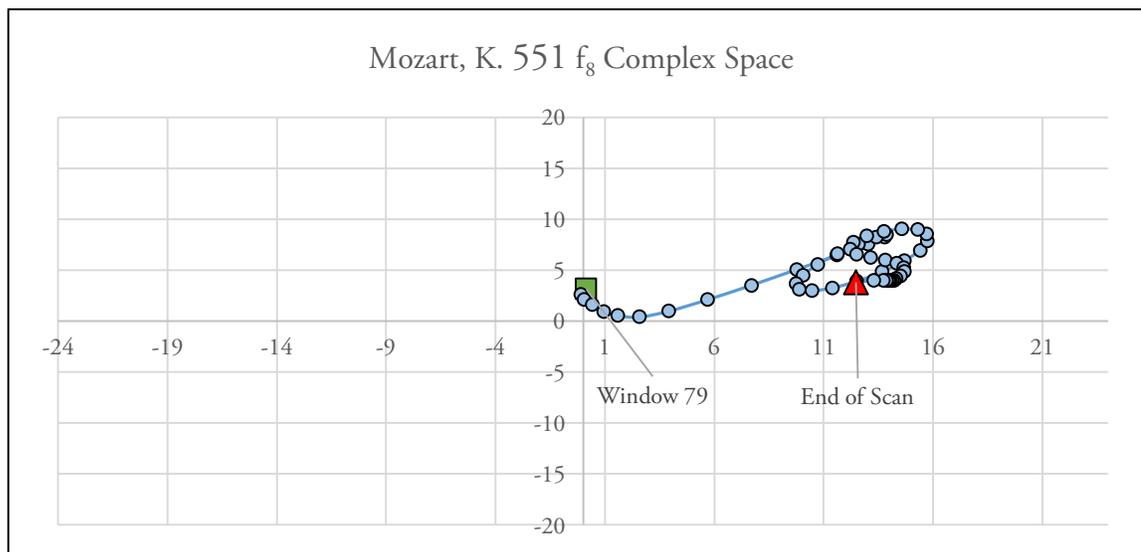
Figure 2-7c. Mozart K. 551, Complex Space for f_8, part 2

2.2 GENERAL REMARKS ON METHODOLOGY

Some aspects of the coding method are unfavorable; in many ways, the computer

ignores many salient features of the music such as harmony, melody, timbre, dynamics, or

anything that is not a note onset. The code isolates note-onsets, separating them from other

musical parameters. This method is idealist in many aspects; 1) it assumes that any musical

parameter can be isolated from a multitude of others; 2) it takes the notational system of

note-proportions to be accurate, where, by way of expressive timing/tempo changes, it often

is not;[32] and 3) it may inaccurately model the complex process of music perception involved

in rhythm because it isolates onsets as its only input. Alternatively, because the code returns

one parameter of the music, by examining rhythm and the relationship between local and

global timelines, we can assess its interaction with other parameters. The windowing aspect

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[32] Andreas C. Lehmann, "Expressive variants in the opening of Robert Schumann's Arlequin (from *Carnaval*, op. 9): 54 pianists' interpretations of a metrical ambiguity" in *Music and the Mind*, ed. by Iréne Deliège and Jane Davidson (Clarendon Press, 2011), 311-–324.

puts the music in terms of a gradual progression—onsets projected through time—which means that we have the ability to compare formal aspects of the piece to the Fourier components in order to examine how the components relate to form and how the components relate to each other.

In general, the efficiency of Python makes this project possible in the first place; if not for the computational aspect of the project, the windowed-onset-retrieval process, and DFT calculation would take months. This project is essentially exploratory—a way of testing if the DFT is a valid methodology for rhythmic evaluation. Whether the Fourier components (with regard to rhythm) parallel our intuitive thoughts about musical rhythm in larger formal contexts, or not, I explore the approach in hopes of expanding analytical tools for rhythmic/metric discourse.

CHAPTER III: BRAHMS, STRING SEXTET NO. 2 – PRESTO GIOCOSO

This paper implements the analytical instruments established earlier in the previous chapters. The analysis examines the relationship of rhythm and meter to the form of the Presto Giocoso Trio from the second movement of Brahms's Second String Sextet, comparing different metric models to the DFT's results. I will show, throughout this chapter, how the Discrete Fourier Transform (DFT) represents rhythmic characteristics in the music, and how these results are compatible with different metric reifications.

Figure 3-1 shows the anacrusis of measure 121 to measure 135 of the Presto Giocoso—the start of the 16-bar A section. The lowest subdivisional level in this excerpt is a quarter note, and everything lands on a quarter-note periodicity. This means that whichever Fourier component divides the span into quarter notes will have a perfectly maximum magnitude; Figure 3-2 shows the first four-measure span arranged as a DFT *metric profile*. As mentioned in Chapter 2, which discusses the ORWDFT process, the retrieval code will return a windowed series of profiles, with many having trivial components just by accounting for excessive subdivisions. In Figure 3-2, I exclude all but $f_{0\text{-}48}$, showing some of the trivial values (just to visualize how the full profile would repeat). Because the retrieval code accounts for lower subdivisions than what occurs in the music, the DFT will return a total of 288 components, but after accounting for the smallest subdivisional unit notated, the other components will be trivial. If the music contained smaller note values, fewer components would be trivial; the only non-trivial components are $f_{0\text{-}6}$, shown in Figure 3-2b. For the Brahms piece, the metric profile reads in a four-measure span comprised of 288 values.

The quarter-note periodicity corresponds to f_{12} because the window is four measures long and contains 12 quarters. f_{12} is perfectly maximal—equivalent to f_0—which, as stated in chapter 2, means that every onset aligns with the quarter-note subdivision.[33] The *x*-axis of Figure 3-2a and 3-2b show the components and which notated subdivision they correspond to.

Figure 3-1. Second String Sextet, II, Presto Giocoso: mm. 121-135
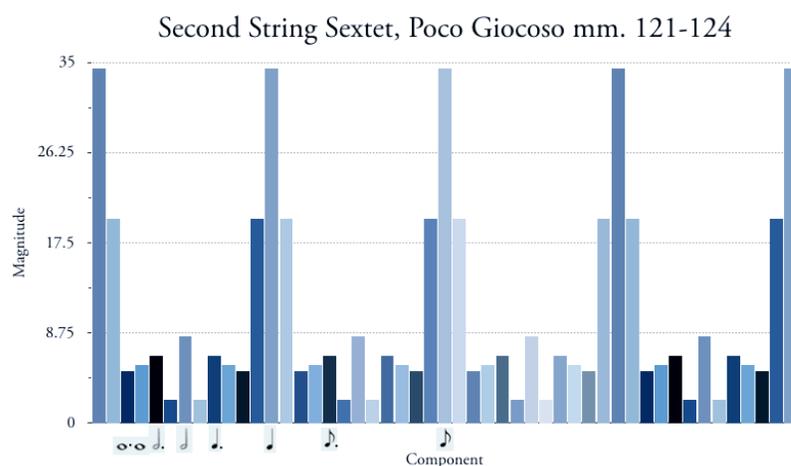
Figure 3-2a. Presto Giocoso: Magnitude for mm. 121–124

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[33] Note that, due to the weighting of measures, f_0 is different than the cardinality of onsets in the four-measure span. The mathematics behind the DFT/FFT treats the time-domain snapshot as periodic. This often causes poor representations at the ends of the snapshot. In order to accommodate for this discontinuity, "windowing" functions alter the edges of the snapshot to approach 0—eliminating the issue. See chapter 2 for windowing information.
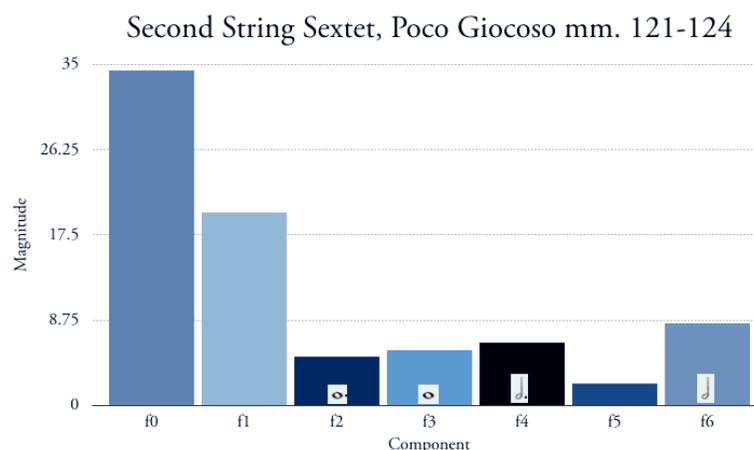
Figure 3-2b. Presto Giocoso: Magnitude for mm. 121–124, Non-Trivial

The Presto Giocoso introduces a metric conflict at the incipit which becomes enlarged and manipulated through the development of the piece. The conflict emerges between the upper three and lower three string parts; the upper strings iterate half-note subdivisions, while the lower strings iterate dotted half-note subdivisions—what Danuta Mirka would call a *split dissonance*—metrically dissonant because it occurs between the two lines.[34] This direct, *grouping* dissonance—metric dissonance in which the grouping of units is different—is prominently articulated by the opposition of subdivisions at the same metric level, between the half note and dotted-half note.[35] If the musical lines were separated, the upper strings would hypothetically convey a $\frac{3}{2}$ meter, while the lower strings would convey $\frac{6}{4}$.[36] We will examine the piece in DFT space below, but for the moment, let us examine the

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[34] Danuta Mirka, *Haydn and Mozart: Chamber Music for Strings, 1787-1791* (Oxford University Press, 2009).

[35] Harald Krebs, *Fantasy Pieces: Metrical Dissonance in the Music of Robert Schumann* (New York: Oxford University Press, 2003).

Peter Kaminsky, "Aspects of Harmony, Rhythm and Form in Schumann's *Papillons, Carnaval* and *Davidbündlertänze*" (Ph.D. Dissertation, University of Rochester, 1989).

[36] I use "hypothetically" because metric entrainment is far more complex than a series of onsets corresponding—this is idealistic. Justin London explores metric entrainment in depth in: Justin London, *Hearing in Time: Psychological Aspects of Musical Meter* (New York: Oxford University Press, 2012).

passage through the lens of Richard Cohn's metric ski-graphs. Subsequently, I will show how

the DFT accommodates an interpretation compatible with the ski-graphs.[37]

In his 1992 article, Cohn describes a *metric* interpretation as a series of pulses in terms

of integers that numerically represent their relative duration.[38] For example, a meter with a

quarter note, a half note, and dotted-whole note subdivision is <1, 2, 6>, conventionally

known as $\frac{3}{2}$ —and which Scott Murphy calls this *pulse representation*.[39] These integers can be

represented proportionally from largest pulse to smallest; the proportion of a dotted-whole

note to half note subdivision is $\frac{6}{2} = 3$ and the proportion half note to quarter note

subdivision is $\frac{2}{1} = 2$. This is represented as "[32]"—Murphy calls this *factor representation*. The

metric conflict in Brahms's *Presto Giocoso* is shown in both pulse and factor representation

below:

Upper 3 Strings: <1 2 6> or [32]
Lower 3 Strings: <1 3 6> or [23]

These relationships are visually depicted by a metric ski-graph in Figure 3-3. In a ski-hill

graph, each point is a specific, subdivisional pulse. Starting at the top of the graph, any

pathway down the ski-hill results in a meter, and any pathway down the mountain is viable.

A different pathway (or a different color) represents a different meter.

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[37] Richard Cohn, "Complex Hemiolas, Ski-Hill Graphs and Metric Spaces," *Music Analysis* 20, no. 3 (2001): 295–326.

[38] Richard Cohn, "Metric and Hypermetric Dissonance in the Menuetto of Mozart's Symphony in G Minor, K. 550," *Intégral* 6 (1992): 1-33.

[39] Scott Murphy, "Metric Cubes in Some Music of Brahms," *Journal of Music Theory* 53, no. 1 (2009): 1-56.
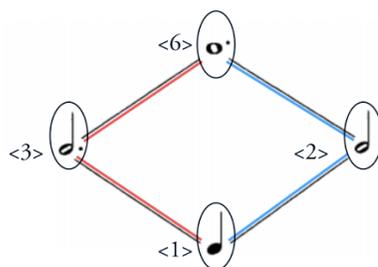
Figure 3-3. Presto Giocoso: Direct Metric Dissonance

Trained musicians often intuitively assign metric interpretations to excerpts based on periodic patterns, harmony, phrase, etc. However, a machine without a previously written schema or probabilistic model lacks the referential element that we, through statistical learning, naturally bring to the analytical process. So, if a computational or mathematical method reaffirms our interpretations, it, in a way, validates the method in part. Our interpretations are indeed reaffirmed by the DFT. Figure 3-2b shows both a prominent half-note subdivision (f_6) and a dotted-half-note subdivision (f_4). The metric profile also shows a prominent dotted-whole note (f_2), and a whole-note subdivision (f_3). When the code reads in the entire score, the dotted-half note pulse will inherently distribute the energy of the half-note pulse, so the rhythmic conflict is more apparent when the instrumental lines are run separately; the magnitudes for the above components are more pronounced when isolating the instrumental lines, as shown in Figure 3-4a and 3-4b. My original interpretation of the passage omitted a whole-note division because it is incompatible with the harmonic-melodic material—particularly the lower string pattern, which articulates the $\frac{3}{4}$ "oom-pah-pah" figure—and bar-line division; however, the ORWDFT code, which only takes onsets, ignores the harmonic-motivic implications and reports that the passage has an approximated whole-note presence. A metric-ski graph signifying the DFT's reported information looks

like Figure 3-5.[40]
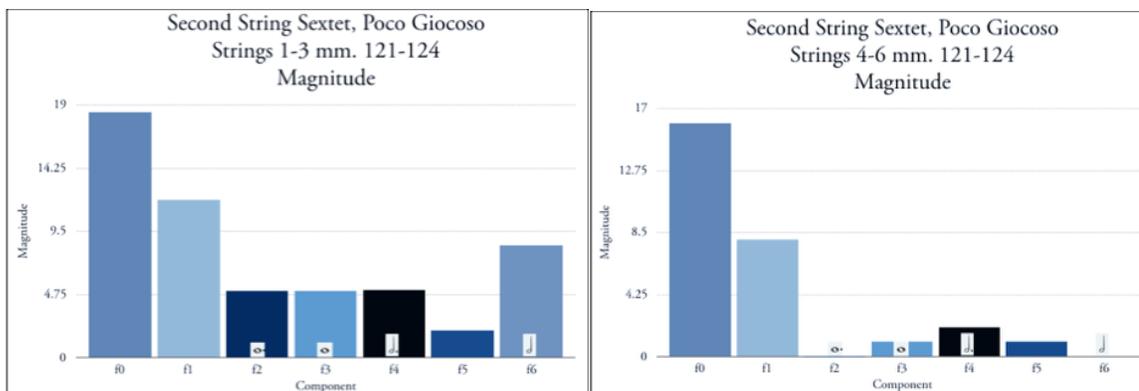
Figure 3-4a: Metric Profile for Upper Strings, mm. 121–124; Figure 3-4b: Metric Profile for Lower Strings, mm. 121–124
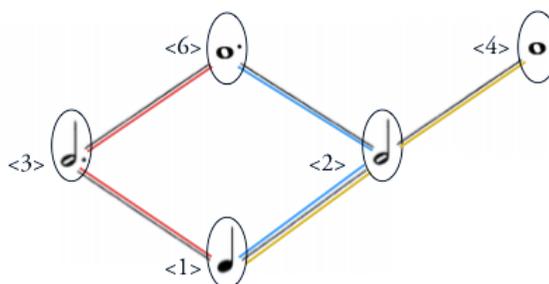
Figure 3-5. Presto Giocoso: Direct Metric Dissonance – DFT

The metric ski-graph does not include information about how those subdivisions are positioned in the span. But phase, as shown in the introduction and tutorial (in Chapter 1), does supply another element to interpret. Figure 3-6 shows the phase for measures 121–124—trivial components omitted. The half-note subdivision (f_6) divides the span into 6 periods, and since we know from $|f_{12}|$ that everything occurs on a quarter-note subdivision, every onset will be positioned in either one of two positions within φ_6— either the north or

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[40] One must be careful, however, not to assign components to subdivisions. The FT can distinguish higher subdivisional elements if they are differentiated in some manner, but otherwise it does not recognize nested structure. For example, a stream of straight quarter notes would return a 0 value for the components corresponding to any other subdivision. This will become important later.

south hemisphere in phase space. Musically, this means that if $\varphi_6 = 0$, then a majority of onsets are occurring an even number of beats from the initial anacrusis; if $\varphi_6 = \pi$, then the majority occur an odd number of beats. Because the periodicity cuts against the notated meter, no phase value consistently corresponds to the downbeats.

The value of φ_6 is 3.14 (or π), meaning that the majority of half notes occur an odd number of beats from the notated onset; that is, the majority of onsets fitting f_6 occur on the first notated downbeat. Referring to the score, we confirm that more half notes are accounted for in a pulse-stream starting on the notated downbeat.
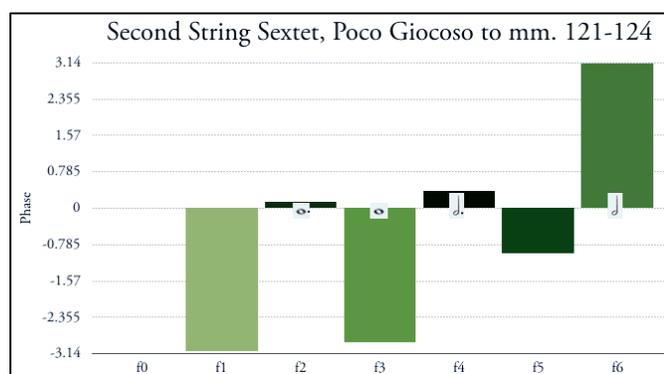
Figure 3-6. Presto Giocoso: Phase, mm. 121–124

f_4 corresponds to a division into 4 parts, so in a span of 4 measures, each period is a measure. I have represented Ph_4 in Figure 3-7.[41] The superimposed numbers are all in the same position of their respective period. The results actually oppose our intuition; the phase for f_4, is close to the top quadrant, which says that the starting onset (the anacrusis to measure 121) is a good fit for the division into 4 parts. Without considering harmony, register, parallelism, or any other parameter, the DFT has reported that, due solely to onset

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[41] Rather than represent the phase-space offsets with values of 24 (which is equivalent to the quarter note because of zero-padding), I have reduced each beat by a factor of 24. This means that 0 is the starting onset, 1 is a beat after, 2 is the third beat, etc.

information, the approximate start to the projected measure should be the anacrusis. If, say, an analyst was presented with the score and asked to label onset values and periodicities with those onsets, they would also get the same results. Even though the information seems in part counter-intuitive, it shows that the upper strings project a half-note division that goes over the barline, often omitting the notated downbeat. Instead, there are more onset values towards the pick-up to those bars, thus falling under the jurisdiction of the f_4 periodicity starting on the anacrusis to measure 121.
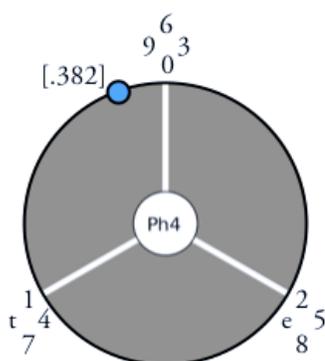
Figure 3-7. Presto Giocoso: Ph$_4$

A major benefit of the ORWDFT code is that it takes snapshots of the onsets and calculates a progressive metric profile as the piece proceeds.[42] This way, we can trace developments in the metric profile or isolate specific moments in the music.[43] Figure 3-8 is a sequential chart that shows how the metric profile changes (over time) in the passage. The following analysis extrapolates from Figure 3-8 by interpreting how these metric profiles relate to form, and their connection to other metric theories (such as Cohn's).

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[42] The ORWDFT code calculates overlapping windows in a piece of music—the specific overlap can be changed manually in the code. In this paper, I implement a window value of 24 (the quarter note), meaning that the window shifts over by a quarter note for each profile returned.

[43] The overlapping windowing technique is able to show metric progression; however, this technique inaccurately portrays a sectional shift when music drastically changes subdivisions. Either way, the results need interpretation.

Second String Sextet, Poco Giocoso mm. 121-124

Second String Sextet, Poco Giocoso mm. 132-135

Second String Sextet, Poco Giocoso mm. 153-156

Second String Sextet, Poco Giocoso mm. 165-168

Second String Sextet, Poco Giocoso mm. 169-172

Second String Sextet, Poco Giocoso mm. 181-184

Second String Sextet, Poco Giocoso mm. 193-196

Second String Sextet, Poco Giocoso mm. 204-207
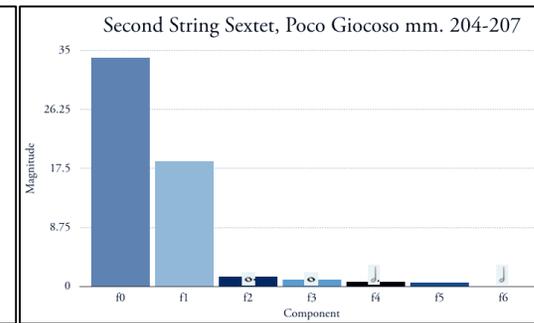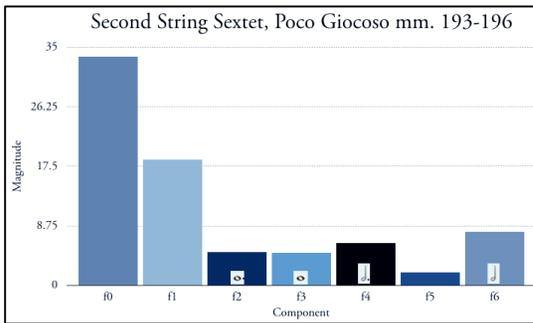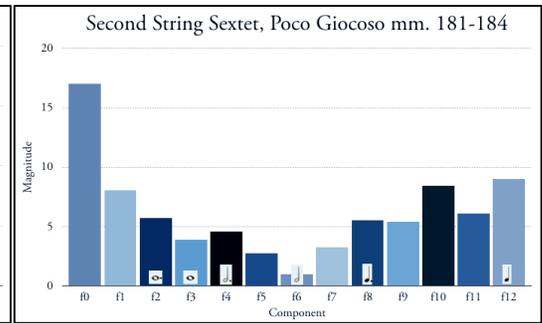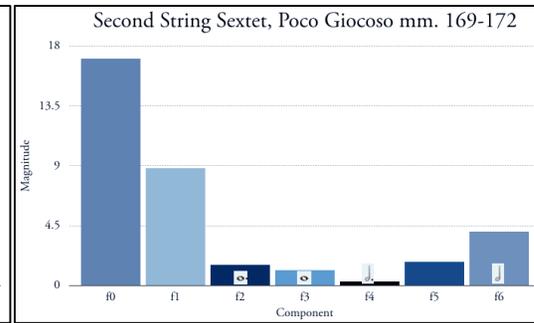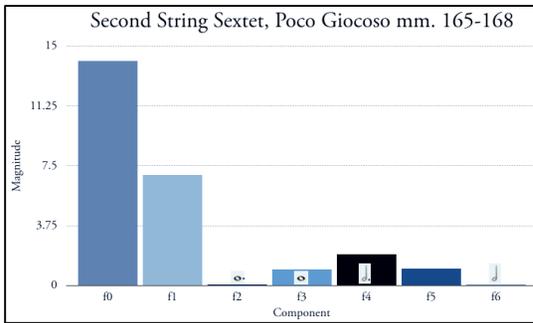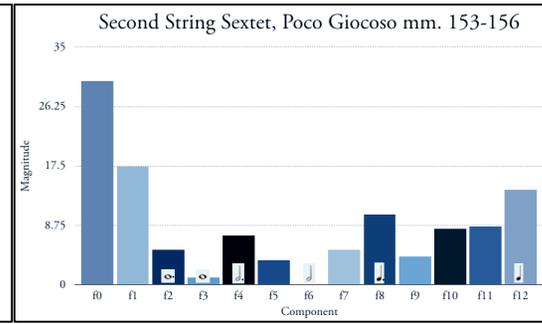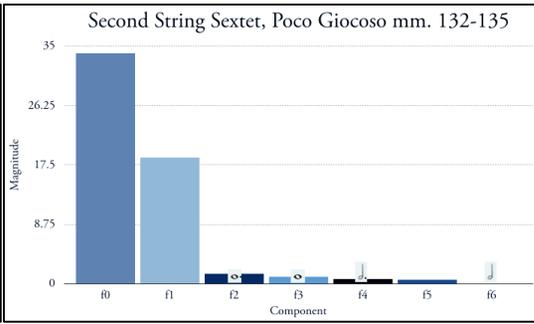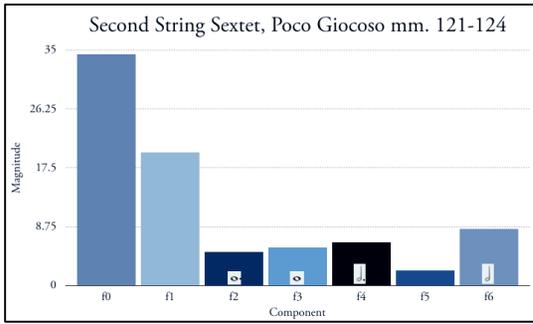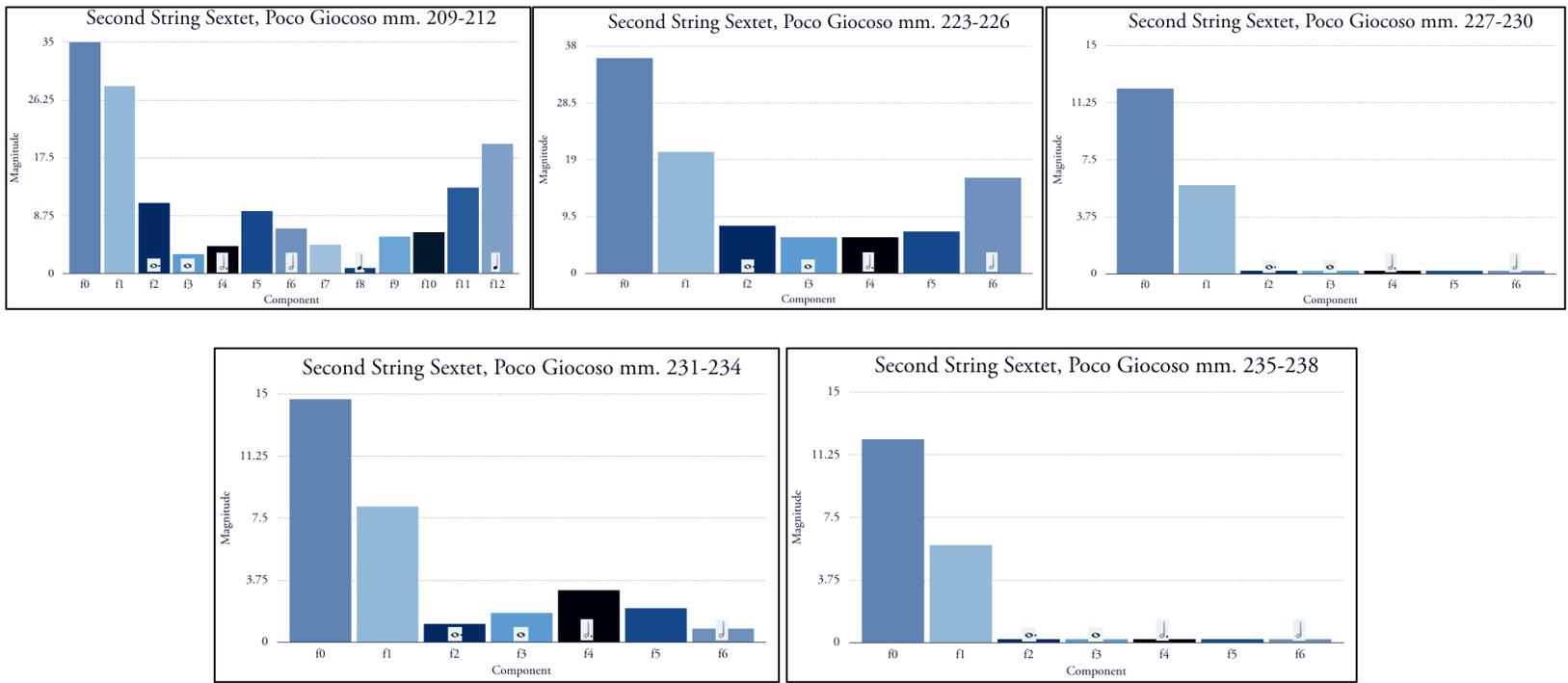
Figure 3-8. Presto Giocoso: Metric Form Progression, Magnitudes

After measures 121–124, the next significant difference in profile occurs from measures 132–135, when all strings iterate a quarter-note periodicity (Figure 3-9). The profile is flat, aside from f_{0-1}, because it lacks periodic information. The metric ambiguity of the rhythm within this window leaves the state of metric conflict open; for a few measures, the meter is actively suspended, replaced by a blank canvas onto which we, as listeners, can impose our preferred meter.
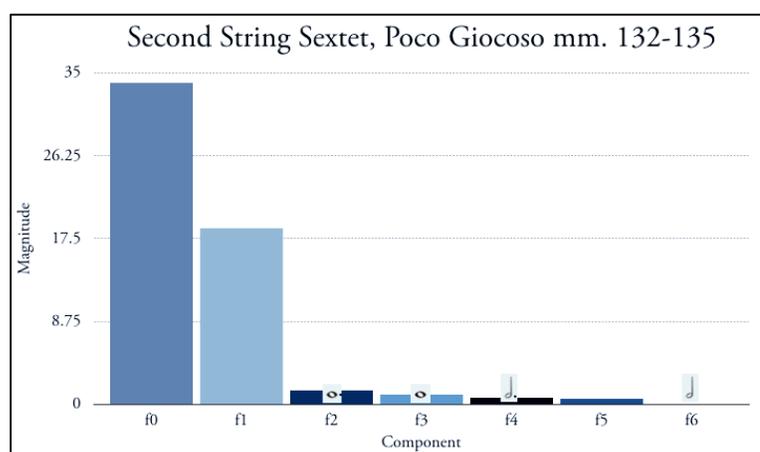
Figure 3-9. Presto Giocoso: Magnitude for mm. 132–135

Before measure 153, the metric state toggles between the profiles in Figure 3-2 and Figure 3-9. The recurrence of profiles has formal implications. The opening phrase from measures 121–136 is a 16-bar nested-sentence: 4 + 4 + 8 (2 + 2 + 4), followed by a parallel phrase. The nested continuations of the phrases (133–136 or 149–152) are the shifts in metric profile to from Figure 3-2 to Figure 3-9. In context, the metric profile in Figure 3-9 is a case of *metric liquidation*. Liquidation—first mentioned in Schoenberg's *Fundamentals of Musical Composition*—occurs in the continuation portion of the presentation-continuation model of phrases.[44] William Caplin defines *liquidation* as "the systematic elimination of

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[44] Arnold Schoenberg, *Fundamentals of Musical Composition* (Faber, 1970).

characteristic motives."[45] Liquidation is normally a melodically-oriented element, but here the metric conflict also dissolves, due to the ambiguity at the end of the phrase.

The profile from mm. 153–156—the transition—shows various changes from the previous profiles. Most visually apparent is the expansion of non-trivial components from f_{0-6} to f_{0-12}. There is a significant reduction in $|f_{12}|$, which, since the start of the piece, has been equivalent to $|f_0|$; because of the eighth notes in measure 153, not every note lands on a quarter-note periodicity. Figure 3-10 has a high magnitude for the quarter-note, dotted-half-note, and dotted-whole-note periodicities, while the magnitude of the half-note periodicity (f_6) is now practically 0. The dotted-quarter-note periodicity (f_8) is also significantly larger—now, non-trivial. The newly surfaced eighth notes add a smaller subdivisional unit, and, consequently, occupy another level in the metric ski-hill graph, seen in Figure 3-11.
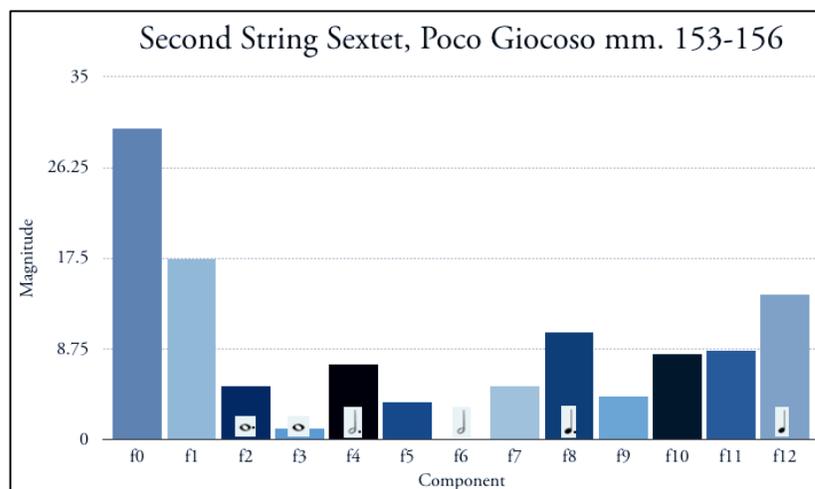
Figure 3-10. Presto Giocoso: Magnitude for mm. 153–156

[45] William E. Caplin, *Classical Form: A Theory of Formal Functions for the Instrumental Music of Haydn, Mozart, and Beethoven* (Oxford: Oxford University Press, 1998), 11.
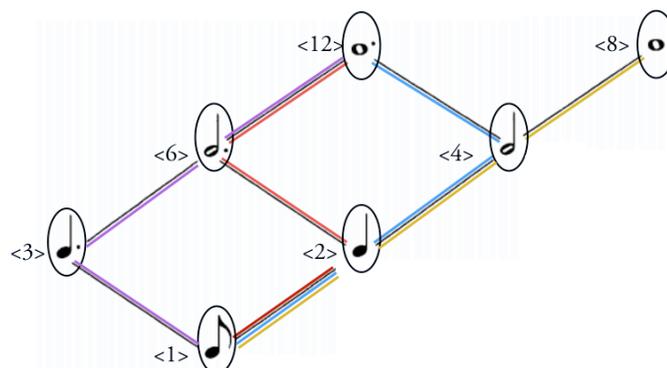
Figure 3-11. Presto Giocoso: Ski-Hill Graph for mm. 153–156

The profile for measures 153–156 (Figure 3-10) indicates a metric preference towards $\frac{6}{4}$ [232] (the red slope in Figure 3-11) in the metric ski-graph, and also presents $\frac{12}{8}$ [223] (the purple slope) as an option. A dotted-quarter note followed by three consecutive eighths may, and often does, convey either $\frac{6}{8}$ or $\frac{12}{8}$; however, when harmony is factored in, the prevailing meter is clearly either $\frac{3}{4}$ or 6/4 (see Figure 3-12). In measure 153, the Bs in the second violin and first cello are upper neighbors to the fifth scale degree in the V^7 chord (D^7). $\frac{3}{4}$ (or $\frac{6}{4}$) is further enforced by the quarter note at the end of each 2-measure grouping. The phrase structure for the transition is a 12-bar phrase, comprised of two 6-bar subphrases—contrasting the normative 4-bar phrases that have been implemented thus far.

Figure 3-12. Presto Giocoso, mm. 151–162

Each pathway down the ski-hill in Figure 3-11 results in a different meter

(represented in factor notation here): [222], [322], [232], and [223]. The factor notations

represent the meters $\frac{4}{4}$, $\frac{3}{2}$, $\frac{6}{4}$, and $\frac{12}{8}$ respectively. Scott Murphy's metric cubes represent the

proportional relationships so that each point on a cube represents a hypothetical meter.[46]

Each point on the metric cube shown in Figure 3-13 corresponds to a path down the metric

ski-graph in Figure 3-11. Though they can both represent meter, proximity in these spaces

varies: the ski-hill diagram represents meter through a pulse-representation model,

connecting literal meters that differ by one pulse level; the metric-cube diagram represents

meter through factor representation, connecting meters whose *factors* differ by 1. Because the

factor representation examines the underlying proportional relationships of subdivisions, it

may show structural connections in a piece differently. For example, in the ski-graph, the

initial conflict (blue to red) is one pulse-movement away, but in the metric cube they are

shared by the same cube-face, yet are still two point-adjacencies away. A metric cube

includes more possibilities because it does not lock top or bottom subdivisional values as the

ski hills do—direct movement in the cube will always change the top or bottom

subdivisional values.

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[46] Scott Murphy, "Metric Cubes in Some Music of Brahms," *Journal of Music Theory* 53, no. 1 (2009): 1-56.
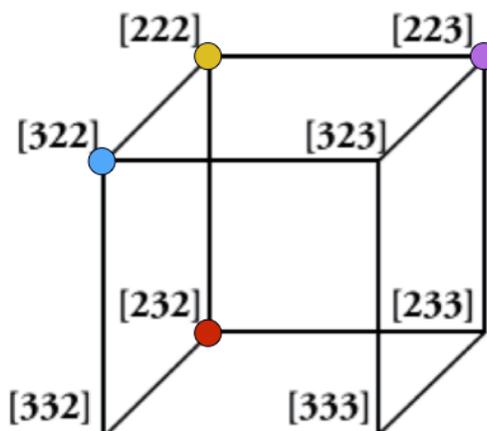
Figure 3-13. Presto Giocoso: Metric Cube for mm. 153–156

The music thins in the B section (measure 165), both texturally and rhythmically

(Figure 3-14). The profiles for mm. 165–168 (Figure 3-15) and mm. 169–172 (Figure 3-16)

both have significantly lower f_0 due to the lower frequency of onsets in this passage.[47] While

the initial conflict in mm. 121-124 directly juxtaposed the dotted-half-note subdivision (f_3)

with the half-note subdivision (f_6) in different *auditory streams*, mm. 165-172 presents these

durational projections consecutively in the same stream. Because they are in the same

stream, Mirka considers this a *merged* dissonance.[48] The product is an alternation between two

4-bar hypermeasures of the two different periodicities f_4 and f_6. This expresses the hemiolic

character of the initial conflict in a new way: horizontally (indirectly dissonant)—as opposed

to vertically (directly dissonant). Even the slur markings in the other lines group 6 quarter

notes together, a span divisible by both 3 or 2, underscoring the grouping dissonance.

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[47] The y-axis is scaled down. In cases where the magnitudes are relatively low, it may be beneficial to normalize the values: $\frac{f_n}{f_0}$.

[48] Danuta Mirka, *Haydn and Mozart: Chamber Music for Strings, 1787-1791* (Oxford University Press, 2009).

Figure 3-14. Presto Giocoso, mm. 163–176
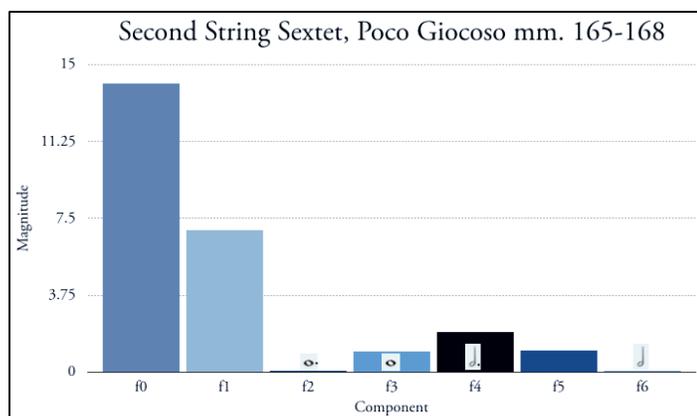
Figure 3-15. Presto Giocoso, mm. 165–168
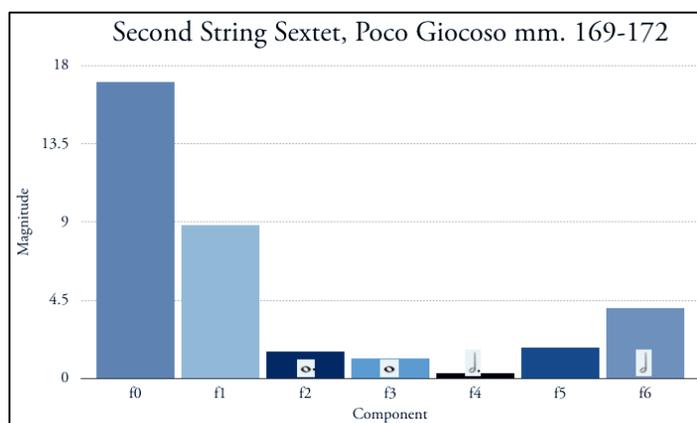
Figure 3-16. Presto Giocoso, mm. 169–172

The half-note pulse (f_6) that dominates the second 4-bar gesture begins misaligned with the upbeat to measure 169, causing one of Krebs's metrical dissonance types: *displacement dissonance*. Displacement dissonance occurs when the "position of the first unit is shifted either forward or back in time."[49] In this case, the displacement dissonance is *indirect*; the dissonance occurs within a single line and, thus inherently, the dissonance exists between two consecutive, differing meters. Because displacement has to do with metric positioning, phase values will provide us with information. However, with only quarter notes, φ_6 will only have values at π or 0—not very informative for phase tracking. Additionally, because the magnitude of f_6 is close to 0 for mm. 165–168, the phase value there is essentially meaningless. To concentrate on the displacement, I focus strictly on the upper three string lines in Figure 3-17 from mm. 165–179. The magnitude/phase values are shown over time in Figures 3-18a and 3-18b below; every beat in Figure 3-17 corresponds to a "point" in Figure 3-18's graphs. The plateaued phase is locked on 0—corresponding with the upbeat of the whole piece. Here, 0 is also the upbeat to measure 165.[50] φ_6 provided us with a steady phase throughout the passage because it has two options, but it tells us nothing about the displacement dissonance that starts in the upbeat to measure 169.

[49]Nicole Biamonte, "Fomal Functions of Metric Dissonance in Rock Music," *Music Theory Online* 20.2.

[50] The spike in measure 24 is related to its negligible magnitude, along with an onset grouping on pi (on beat 24).

Figure 3-17. Presto Giocoso, measures 165–179
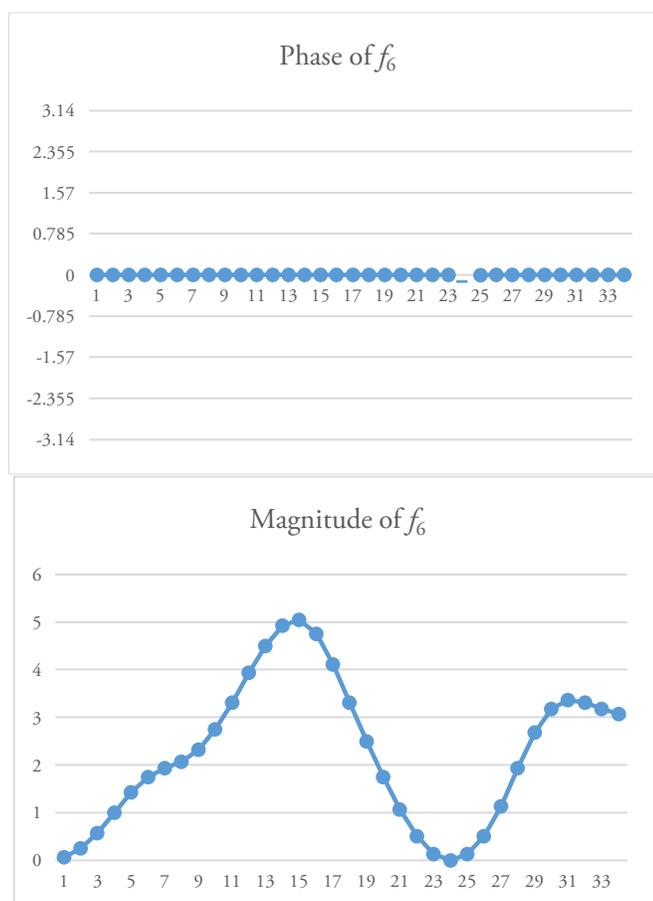
Figure 3-18a. and 3-18b. Phase and Magnitude for f_6: windows from mm. 165–179

By tracking φ_3, we can build on the information gained from f_6. This, at first, seems counterintuitive when thinking in terms of the DFT; the DFT on a consistent half-note stream would have nothing for the whole-note component, because it would be accounted for by the half note's component.[51] I argue that looking at φ_3 here, in addition to f_6, has analytical benefits. f_6 gives us relatively little information on positioning because it will either return 0 or π, but we can locate the position of streams more accurately with the help of f_3. Though the magnitude for f_3 should be relatively low throughout, the phase value, while unstable, should still be able to provide us with worthwhile positioning information. The following three visuals are used in conjunction to show the phase tracking in this passage:

Figure 3-17 - Shows a score of the passage.

Figure 3-19 - Phase tracking results for φ_3. Every beat in Figure 3-17 corresponds to a point in Figure 3-19's graph; the brief plateaus and windows discussed are marked with color.

Figure 3-20 - The phase pathway of Figure 3-18 on a circle. The discussed moments are marked in color, coordinating with Figure 3-18 as well.

The phase starts at –1.57 (or –π/2), marked by the green circle in Figure 3-19. Remember that, because the entire piece starts on an upbeat, the phase value is relative to the first window—the anacrusis to the first beat. Therefore, in Figure 3-19b, the value 0 is the anacrusis, and –1.57 corresponds with *notated* downbeat. As the window continues to scan, and displacement is introduced, the phase moves clockwise (or upward in Figure 3-19a). The windows starting on the sixth beat plateau pick up the half-note pulses in the center of the

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[51] More aspects of how this the DFT epistemologically relates to meter will be taken up later in the conclusion.

window, returning a value around 1.57.[52] The positive value 1.57 is metrically equivalent to

beats {3, 11, 7} in the 4-bar window. In this way, φ_3 is showing the displacement dissonance

starting in the upbeat to measure 165. Starting in the thirteenth window, the phase plateaus

again at π, this time for a longer string of windows. This point corresponds to measure 169,

where the whole-note periodicity is maintained for the next (notated) 4 bars. The

approximate phase value for these windows (π) positions the onsets—which divide the 4-bar

span into 3 parts—at {2, 6, 10}. A look to the score confirms this as the continuation of

half-note impulses. The phase returns to its original position as the parallel phrase begins

again. While φ_6 told us that the phase is steady throughout this passage, at a larger metric

level, φ_3 tells us about the different phase positioning of the "$\frac{4}{4}$" meter groups.
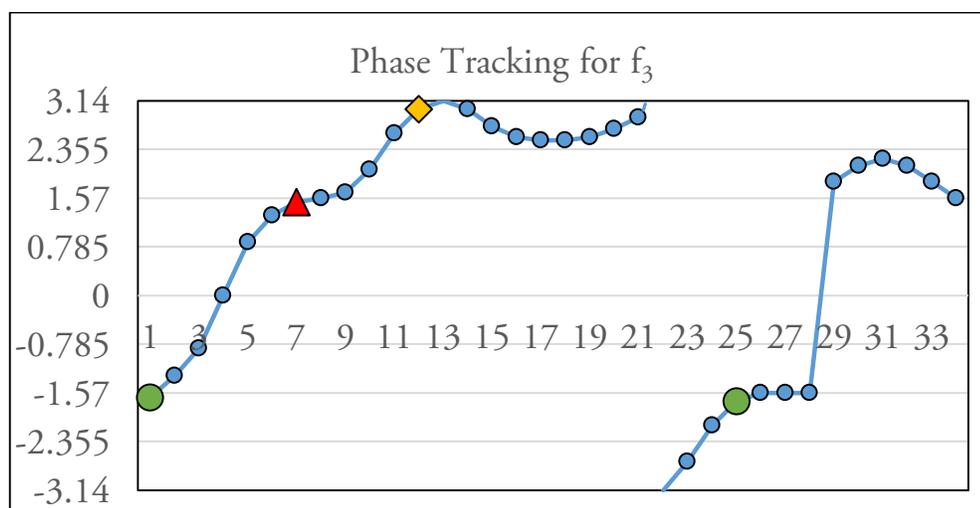
Figure 3-19. Phase Tracking for f_3: Upper String Lines, mm. 165–179

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[52] The center of the window is weighted the highest.
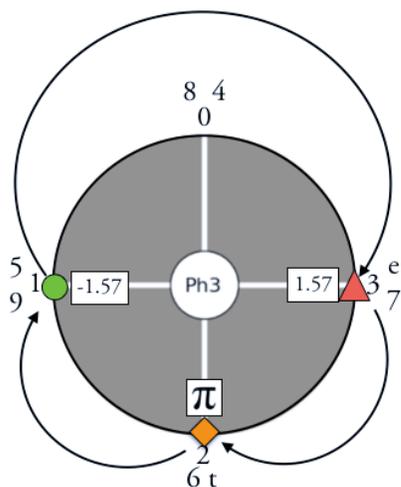
Figure 3-19b. Phase Tracking (Circle) for f_3: Upper String Lines, mm. 165–179

In mm. 181–184, the retransition starts like the previous transition section by incorporating eighth notes. In fact, it mirrors the profile of the previous transition section; mm. 181–184 and mm. 153–156 have similar metric profiles; both convey $\frac{6}{4}$ despite the a dotted-quarter-note periodicity; and both use inverted forms of the same motivic material (running eighth notes leading to a quarter at the end of a two-bar gesture), applying the same 6-bar phrase rhythm. The A section returns, altered, in measure 193 after the retransition. The reprise of the A section initiates the expected 8-bar phrase, but quickly deviates from our anticipation of a subsequent phrase (mm. 209–212); instead, it uses textural material from the transition sections. Despite the clear reminiscence of the transition material, the section is manipulated in a way that changes the metric profile (Figure 3-20); f_8, which has a high magnitude for the previous transition sections, is practically non-existent, and the opposite applies for f_6. Instead of the usual 6-bar hypermetric phrases accompanying this material, the music remains in 4-bar hypermeasures that govern primary thematic sections. In other words, the established paradigmatic transitions that have consisted of returning

metric states and subdivisions are altered. The profile correlates with the "blue" meter in ski-

hill/metric cube representation $\frac{6}{4}$.[53]
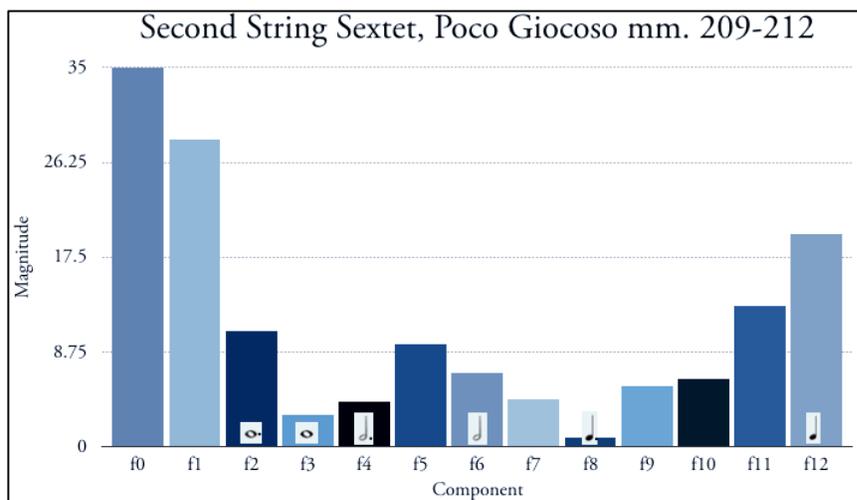
Figure 3-20. Presto Giocoso: Magnitudes for mm. 209–212

The return of the B section (measure 227) is also altered, imbued with the metric

conflict that has pervaded the movement (see Figure 3-21). This final section rotates its

material between a grouping of descending quarter notes—drawn from the metric

liquidation material in the A section—paired in twos that convey $\frac{3}{2}$, and the original B motive

(from measure 165) conveying $\frac{6}{4}$. The proximity of these conflicting meters concurrently

utilizing different thematic material highlights the conflict area and heralds the end of the

formal unit. The formal section concludes after this tension. Disappointingly for this

analysis, the profile does not exactly reflect the metric conflict, because, for the initial 4-bar

subphrases (such as mm. 227–231), the metric conflict stems from the melodic parallelism

and is not reflected in the plain onset information. Thus, even though these metric profiles

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[53] The excerpt begins with a dotted half note, but in mm. 210–212 there are more projections of the half-note pulse.

show promise for depicting meter and metric phenomena such as metric dissonance, there is still a fundamental issue surrounding the model: meter is more than just a series of idealized onset values. This point will be addressed later.

Figure 3-21. Presto Giocoso, mm. 215–244

We can view the form of Brahms's trio through a phrase-rhythm and metric-formal lens. Reflecting on the excerpts here, the music parses into clear A and B sections with transition sections between them:

A - TR - B - TR - A' - tr - B'

Each primary action zone—A and B—incorporates the same fundamental rhythmic characteristics: 1) the metric conflict foreshadowed at the incipit; and 2) a consistent 4-bar

hypermeter (see Table 3-1 for a phrase-rhythm chart). Although, per section and as discussed, the conflict is elaborated differently, there is a constant conflict concerning the half-note pulse and dotted-half-note pulse (otherwise known as the "blue"/"red" regions in the ski-graph/metric cube) in both the A and the B sections. The A sections present the conflict as a split between auditory streams (occurring in more than one part) and as a direct opposition, contrasting the subdivisions simultaneously, while the B section decreases onset frequency and presents the conflict as an indirect dissonance occurring in a single part. The phrase rhythm in the primary thematic sections of the piece consists of 4-bar hypermeasures, and phrases are comprised of 2–4-bar subphrases. The primary transition sections counter both of these fundamental rhythmic characteristics: the metric profiles preference $\frac{6}{4}$ (the red-coded slopes/points) and $\frac{12}{8}$ (the blue-coded slopes/points) while, hypermetrically, the music for these sections hast 6-bar groupings.

Table 3-1. Brahms _Second String Sextet, II - Trio Presto Giocoso_, Op. 36 Phrase Rhythm Form Chart[54]

Formal Unit	Measures (length)	Phrase / Grouping	Hypermeasures
A Section:	**mm. 121 (120 anacrusis)-152 (32)**		
Antecedent	mm. 121–136 (16):	4+4+8 (2+2+4) nested sentence	
	mm. 121-128 (8)	8 (4 [2 + 2] + 4 [2 + 2])	**1 2 3 4 1 2 3 4**
	mm. 129-136 (8)	8 (4 [2 + 2] + 4)	**1 2 3 4 1 2 3 4**
Consequent	mm. 137–152 (16)	4+4+8 (2+2+4) nested sentence	
	mm. 137-144 (8)	8 (4 [2 + 2] + 4 [2 + 2])	**1 2 3 4 1 2 3 4**
	mm. 145-153^ (9)	9^ (4 [2 + 2] +2 + 3^)	**1 2 3 4 1 2 3 4 (1)**
Transition:	mm. 153–164 (12)	6 (2 + 2 + 2) + 6 (1 + 1 + 2 + 2)	**1 2 3 4 5 6 1 2 3 4 5 6**
B Section:	**mm. 165–180 (16)**		
Phrase 1	mm. 165–172 (8)	8 (4 **[**2+2] + 4)	**1 2 3 4 1 2 3 4**
Phrase 2	mm. 173–180 (8)	8 (4 + 4)	**1 2 3 4 1 2 3 4**
Transition:	mm. 181–192 (12)	6 (2 + 2 +2) + 6 (3 + 3)	**1 2 3 4 5 6 1 2 3 4 5 6**
A Section (Reprise):	**mm. 193–209^(17^)**		
Phrase 1	mm. 193–200 (8)	8 (4 [2 + 2] + 4 [2 + 2])	**1 2 3 4 1 2 3 4**
Phrase 2	mm. 201–209^ (9^)	9^(4 [2 + 2] + 2 +3^)	**1 2 3 4 1 2 3 4**
Transition:	mm. 209–227^	(19^) 4 (2 + 2) + 11^(4 [2 + 2] + 7^) + 5^	**1 2 3 4 1 2 3 4 5 6 1 2 3 4**
B Section (Return): mm. 227–250 (24)			
Phrase 1	mm. 227–234 (8)	8 (4 [2 + 2] + 4 [2 + 2])	**1 2 3 4 1 2 3 4**
	mm. 235–243^ (9^)	9^ (4 [2 + 2] + 5^ [2 + 2^])	**1 2 3 4 1 2 3 4**
	mm. 243–250 (8)	8 (4 [2 + 2]+ 4)	**1 2 3 4 1 2 3**

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[54] '^' represents a phrase overlap.

Thus far, I have represented magnitudes in *metric profiles* that display a single window. While this format visually conveys all magnitudes in a straightforward manner, a depiction in which symbolic *time* is represented would better represent how these components tie metric profiles to the overarching form. By isolating and tracking the two components that have maintained the highest magnitudes—f_4 and f_6—a visual representation will capture the metric conflict discussed. In Figure 3-22, the purple line is f_6 and the yellow is f_4, while the *x*-axis represents time—as in each individual window moving beat by beat through the piece—and the *y*-axis represents magnitude. The sections running across the graph align the starting position of the window with the beginning of the section. It is apparent where the A sections and TR sections reappear because they exhibit a similar metric conflict with similar textural density. One developmental change in the transition sections is that, from the first TR to the Re-TR, and then finally to the third TR, the magnitude for f_6 grows until overtaking the strength of f_4 in the final TR. This can be developmentally described as a metric progress of *pulse saliency*—which pulse is most strongly conveyed through onsets. The dotted-half-note pulse initially has a high magnitude for the TR sections, but in the final TR section the half-note pulse competes for *saliency*. Turning to the score, we see that in the final TR (measure 209 in Figure 3-23), the original conflict is inserted into the TR section. The conflict of pulse saliency has made its way from the A theme into the TR sections, which, since the final TR, have articulated a dotted-half-note pulse; like an infection, the pulse disagreement has pervaded the sanctum of the section which once held the highest magnitude for f_4 and that clearly conveyed triple meter.
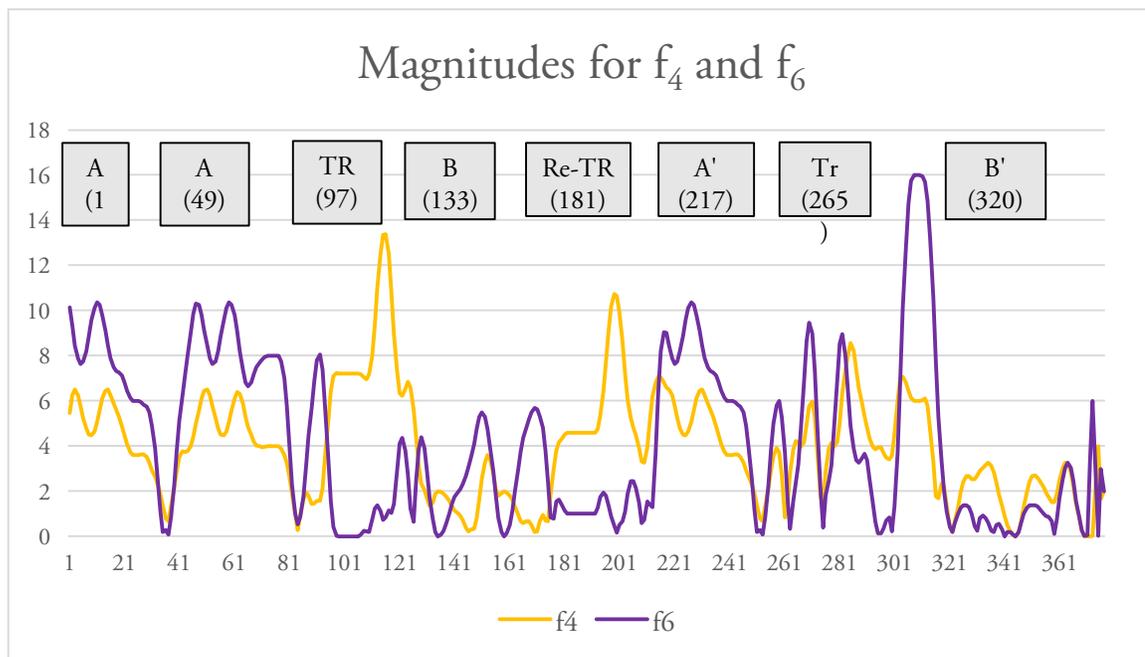
Figure 3-22. Magnitudes for f_4 and f_6 over time

Figure 3-23. Presto Giocoso, mm. 202–214, Transition Section (m. 209)

Because the values of magnitudes are based on cardinality, the graph above is highly dependent on textural density. Having a texture-based graph is a double-edged sword: it may capture how textural structures relate to form, but for periods of music that have low onset density—as in section B—it may, on a glance, disguise which pulse is salient during these

sections. *Normalizing* the magnitudes to $\frac{f_n}{f_0}$ scales everything to the common cardinality of 1,

and thus accommodates for the cardinality/textural-density issue. In the normalized graph in

Figure 3-24, whichever magnitude takes precedence in those sections will be pronounced

regardless of cardinality. From this, the phrasing of the B sections becomes apparent; the tail

end of each B section emphasizes the half-note pulse. If we refer back to Figures 3-14–3-16,

we confirm that this is where the pulses are presented horizontally in succession, as opposed

to how they were originally introduced as directly juxtaposed. Another way to produce

relevant magnitudes would be to graph the share of a *power spectrum* showing components

and their respective share of the overall "power" (Figure 3-25).[55] The Perceval-Plancherel

identity theorem says that, under the FT, the total power is preserve; because power is the

sum of squared weights, the procedure is similar to normalization by cardinality. The first

graph omits other magnitudes, so one large difference between the charts is that the power

spectrum represents the other components. According to the graph, f_2 has a large magnitude

at the retransition section. This is because all parts but Violin 1 play held durations and have

very little rhythmic activity. The parts that hold the durational values enter again with the

upbeat and downbeat two measures later. Being that our window is 4 measures long, this

activity would boost the magnitude for f_2.

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[55] Emmanuel Amiot, *Music Through Fourier Space* (Switzerland: Springer International Pu, 2016), 7.
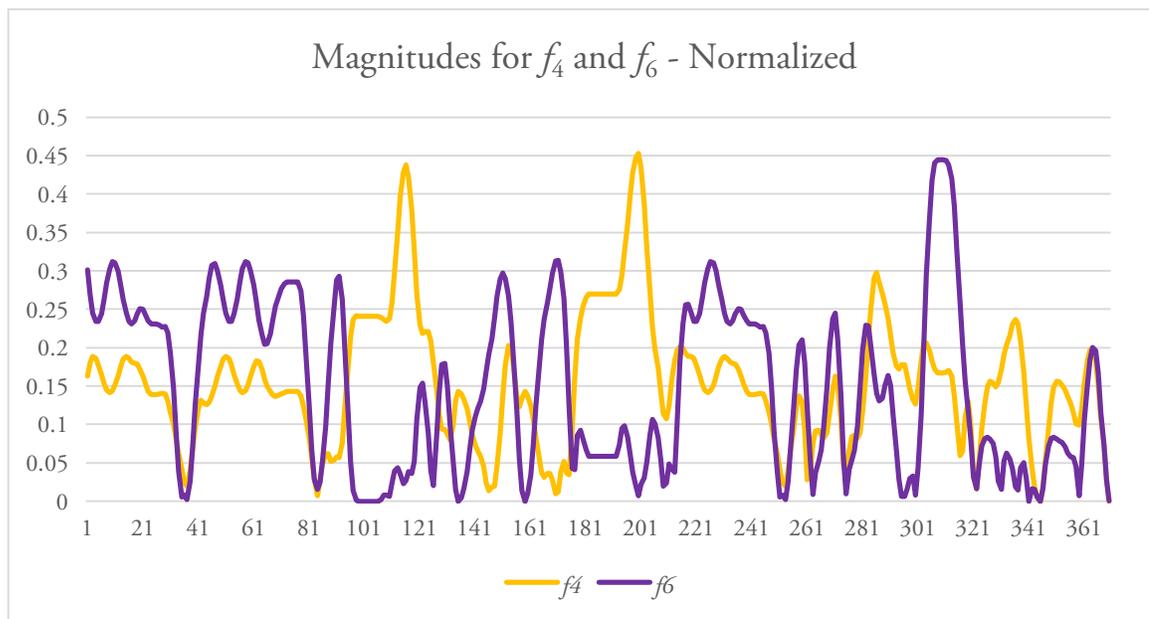
Figure 3-24. Normalized Magnitudes for f_4 and f_6 over time
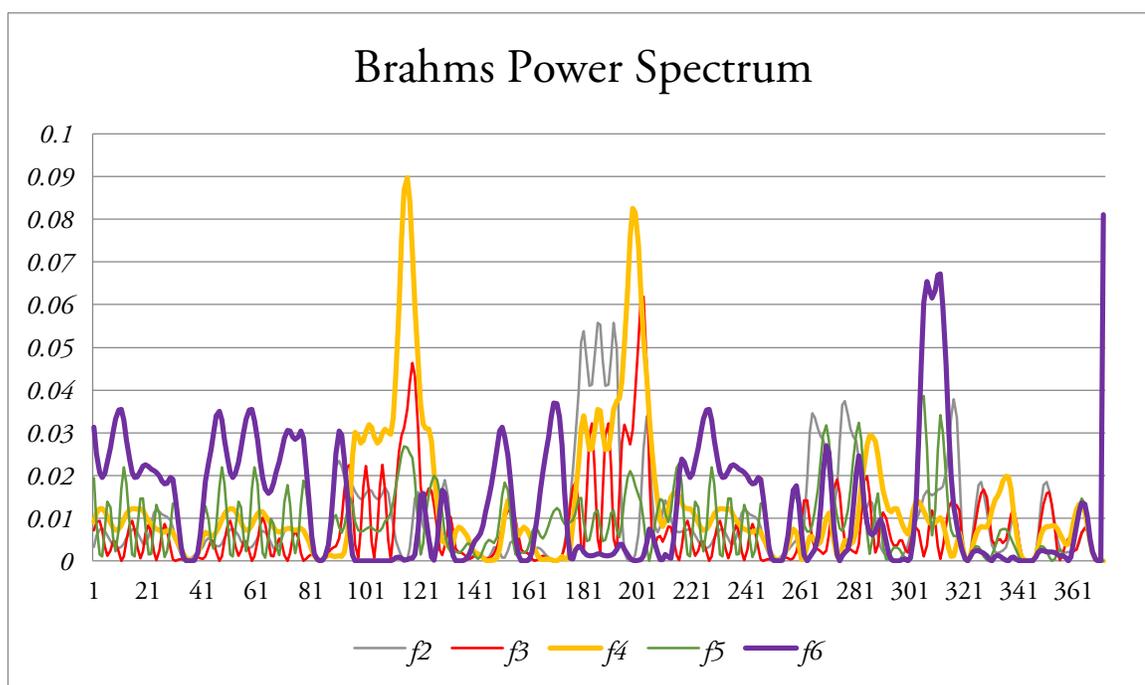
Figure 3-25. Presto Giocoso: Power Spectrum

Throughout this thesis, I have explored the DFT and how it can be implemented in metric analysis. Through my current research, and during the continuation of this project, I hope to continue metric-analytical discourse—particularly with regard to formal structures. In an attempt to bring form-defining aspects of rhythm to the foreground, I have intentionally forgone harmonic analysis. Leaving the other musical parameters in subsidiary positions, though unrealistic in terms of perception, isolates a single variable to determine, as a result, how it interacts with our intuitive musical analysis. The ORWDFT code I have implemented in this paper isolates onsets as the only input, which is idealistic and neglects to incorporate other factors that amount to meter as a phenomenon: culture, melody, harmony, texture, and innumerable parameters contribute to the concept and effect of meter. In applying the DFT to metrical analysis, my goal is, like Gottfried Toussaint's, exploratory: "Exploring the extent to which the comparative analysis … provides insight that can be transferred from one modality to the other is a fruitful endeavor… and if some concepts do not transfer successfully, these provide us with insight about their differences."[56] Though this project is only in its first stages, I have examined features of, and endeavored to articulate a renewed perspective on, meter and rhythm, while simultaneously incorporating the ideas of previous metric theorists. Metric form is relatively unexplored—only studied by a handful of modern music theorists. I can foresee potential for the ORWDFT code to incorporate phrase rhythm if different musical parameters were accounted for, or, after sufficiently developing the metric profiles, including a probabilistic model of metric states. This research, in my opinion, contains significant potential for advancing discourse on

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[56] Godfried T. Toussaint, *Geometry of Musical Rhythm: What Makes a "Good" Rhythm Good?* (S.1: CRC Press, 2017), 54.

meter, and in developing it further, I hope to bring more attention to the exploration of the

DFT, rhythm, and metric form

CHAPTER IV. ANALYSIS OF "SHY ONE" BY REBECCA CLARKE

In the following chapter I analyze "Shy One," a song by Rebecca Clarke. Thus far, our examples have only consisted of music with notated meters that are both regular and unchanging. The frequent changes in meter between $\frac{5}{4}$ and $\frac{6}{4}$ in "Shy One" present new epistemological and theoretical challenges to the concept of meter. In the last century the definition of meter—and, even more polemically, hypermeter—has been disputed in academic literature. Scholars organize into various camps; some distinguish *grouping* from *meter*, while others consider the two concepts under one umbrella. The operationalization that Lerdahl and Jackendoff lay out in *A Generative Theory of Tonal Music*'s metric well-formedness rule 4 states that "each metrical level of music consist of equally spaced beats."[57] Therefore, such a definition implies that a meter that has an *asymmetrical division*, such as $\frac{5}{4}$, is demoted to *non-metrical*. Later in the book, the authors claim that "the resulting metrical structure follows the irregularities of local detail."[58] In GTTM, the strict, periodic regularities which construct meter are therefore related to the deeper levels of metric organization. Initially agreeing with the distinction between meter and grouping, David Huron says:

> "We can see that it is not simply the strict hierarchical metrical frameworks that influence a listener's temporal expectations. In addition to these *metric* expectations, listeners also form distinctly *rhythmic* expectations, which need not employ strictly periodic pulse patterns."[59]

He argues further that grouping or rhythmic patterns form expectation—for example, the rate at which a bouncing ball hits the ground accelerates as it loses kinetic energy, but

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[57] Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge, MA: MIT Press, 1983), 69.

[58] Ibid., 297.

[59] David Huron, *Sweet Anticipation: Music and the Psychology of Expectation* (Cambridge, MA: MIT Press, 2008), 187.

humans are still able to predict the rate of change. This example is one of many that Huron discusses to demonstrate that predictability may play a vital role in meter. Huron says, "Periodicity is not necessary for the formation of such expectations… It is important only that the listener be experienced with the temporal structure and that some element of the temporal pattern be predictable."[60] Expectation is built through statistical learning and a subject's exposure to a stimulus. As Huron defines it, the periodic case of meter is therefore what one could call a *predictable half-truth*: periods are inherently regular, and therefore conform to a predictable norm. Rhythms, however, can still conform to a prediction model whether they are periodic or not. Expectation influences meter and rhythms alike, but meter will inherently be predictable, while rhythms must conform to a *template* of sorts—an established rhythmic schema.

When presented with asymmetrical meters, internal groupings can no longer be evenly distributed and are therefore obviously not periodic. Is viewing something that is partially non-regular in terms of periodicity—as the DFT does—counterintuitive, or does it provide a reliable grounding from which to interpret the rhythms? I analyze the piece "Shy One" below with the DFT as an analytical tool, demonstrating how the DFT, an equation that deconstructs an input into its purely periodic, sinusoidal components, works equally well with music that implements changing, non-regular meters. I later reflect on how a predictive definition of rhythmic templates not only reintegrates previous definitions of meter, but also integrates "irregular" meters such as 5/4 and 7/8.

In Yeats's "To an Isle in the Water" (seen in Figure 4-1), the poetry depicts a woman

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[60] David Huron, *Sweet Anticipation: Music and the Psychology of Expectation* (MIT Press, 2008), 188.

moving around a room in preparation for some sort of ritual. The shy woman sets dishes out and lights candles around a "curtained room." There is no explicit reference to what the woman could be doing; the pithy poem is ambiguous in its lack of description, leaving room for the reader to interpret the ceremony. Perhaps she is preparing for a pagan ritual, or maybe she is simply setting the dinner table. The "Isle" in the poem may refer to Innisfree—the subject of Yeats's famous "Lake Isle of Innisfree" (written after "To an Isle in the Water")—or, knowing Yeats's interest in historic legends, Avalon, an island where King Arthur's wounds were treated after battle. Regardless of the specific reference, the narrator longs to go with the woman, seen in the parallel lines: "With her would I go," and "With her would I fly." The auxiliary function of "would" in both contexts reinforces the narrator's unrequited desire; "would" plays between its two meanings, signifying both "want" and conditional. Yeats conjures contrasting imagery involving fire and water—a classic antithesis—to capture the underlying passion that the narrator feels on observing the woman. Water and fire represent the dichotomy between the characters, the water symbolic of the reserved woman and the fire representing the narrator's passion. The following analysis examines the harmonic and rhythmic parameters that Rebecca Clarke manipulated when setting the poem in her 1920 musical rendition of Yeats' poem ("Shy One" score attached).

> Shy one, shy one,
> Shy one of my heart,
> She moves in the firelight
> Pensively apart
>
> She carries in the dishes,
> And lays them in a row.
> To an isle in the water
> With her would I go.

She carries in the candles,
And lights the curtained room,
Shy in the doorway
And shy in the gloom;

And shy as a rabbit,
Helpful and shy.
To an isle in the water,
With her would I fly.

Figure 4-1. Yeats, "To an Isle in the Water"

The piece groups the stanzas into higher divisions, splitting the poem into two larger sections each comprised of two stanzas—a strophic $(AB)(A'B)$. The modified strophic form thereby adheres to the poetic scheme, that recalls "with her would I…". Subsections A and A' do not have any apparent repetitions of text, so the variation also embodies that formal layout. The shift in notated meters—fluctuating between $\frac{5}{4}$ and $\frac{6}{4}$—also bolsters my argument in favor of the piece's formal organization into two large A sections: x represents one measure of $\frac{5}{4}$, and y represents one measure of $\frac{6}{4}$. The notated-meter scheme is shown below:

Stanza I [mm. 1–4]: $(x\,x\,x\,y)$ // Stanza II [mm. 5–8] $(x\,y\,x\,y)$
Stanza III [mm. 9–12]: $(x\,x\,x\,y)$ // Stanza IV [mm. 13–17] $(x\,y\,x\,y)$
Figure 4-2. "Shy One": Formal Organization of Meters

Each 4-line stanza of the poem is set to a corresponding 4-measure phrase comprised of $\frac{5}{4}$ and $\frac{6}{4}$-bar units. Harmonically, the music in stanza II (mm. 5–8) is restated again in the accompanying music of stanza IV (mm. 13–17), reflecting the poetic structure which rearticulates the departure idea: "To an isle in the water."

With the DFT thus far we have only examined examples that use a single notated meter. "Shy One" shifts between $\frac{5}{4}$ and $\frac{6}{4}$, posing a problem for the previously selected

window size. In the Brahms piece, the ORWDFT code used a window of 288 units, in

which the quarter note is equivalent to 24 units (f_{12}), the half note is 48 units (f_6), the whole

note is 96 units (f_3), etc… This window size privileges subdivisions that are in the duple and

triple path in the ski-hill graph. It may not be so apparent on first glance, but because the

span is 12 quarter notes long, there is no Fourier component associated *directly* with the $\frac{5}{4}$-

measure length—the component which divides the span into $\frac{5}{4}$ lies in between the dotted-

whole note pulse and the whole note pulse (seen in Figure 4-3).[61] To accommodate a 5-

quarter-note periodicity (qua $\frac{5}{4}$), we simply change the window to a size which is divisible by

5. Figure 4-4 shows the opening profile from the ORWDFT with a window expanded to 15

beats—the disadvantage now lies with the other higher-level divisions (the whole note and

dotted-whole note), which are now reduced to the subsidiary position between two Fourier

components (held previously by $\frac{5}{4}$). One explanation is that the "energy" of a particular

frequency is no longer carried by a single component, but distributed between adjacent

components. As we switch from a 12-beat window to a 15-beat window, the "energy" from

the half-note subdivision f_6 (in the 12-beat window) is distributed between f_7 and f_8 (in the

15-beat window). Regardless of what periodicities we choose to represent between the two

windows, the nature of the problem is the same. Note that it is possible to accommodate

both higher-level divisions, but to do so would mean expanding a window to 30 beats. By

expanding the window, however, the location of the corresponding component becomes less

certain; there is a tradeoff between window length and temporal resolution. The tradeoff is

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[61] The fact that it lies between the dotted-whole note and whole-note pulse is important and will become relevant later.

based on the Heisenberg uncertainty principle—inherent in the properties of all wave-like systems. Though this may prove to be fruitful exploration of the music, for now I have chosen to restrain the window to a smaller span to pinpoint components at local levels. When referring to metric profiles, I will use an additional subscript to signify which graph I refer to: $f_{n(s)}$, where s is the number of quarter notes in the window. For example, the third component of a window covering 15 beats is: $f_{3(15)}$.
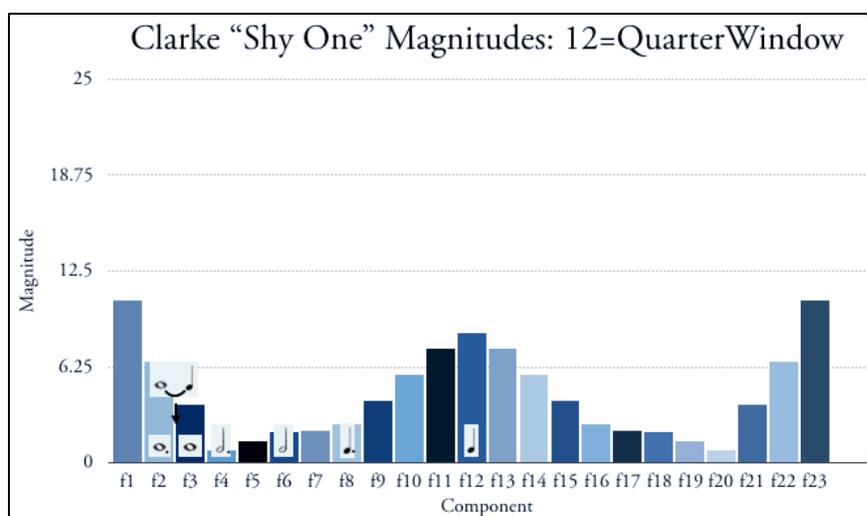
Figure 4-3. "Shy One" Magnitudes, beats 1–6: 288 Units
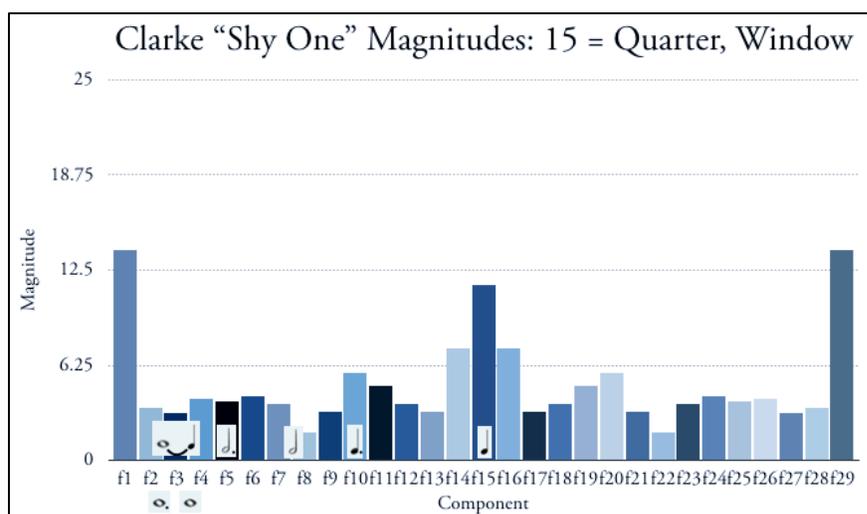
Figure 4-4. "Shy One" Magnitudes, beats 1–8: 360 Units

In the first $\frac{5}{4}$ measure, the opening minor third on the words "Shy One" is repeated immediately after, with the head of the second gesture extended to create a 2 + 3 *tactus* grouping. Figure 4-5 shows the score for the opening 3 measures/15 beats. Despite the clear parallelism—aurally cued by the starting arpeggiation figures in the low register, or melodic contour—the components associated with the length of the full $\frac{5}{4}$ measure ($f_{2(12)}$ and $f_{3(12)}$ in Figure 4-3, and $f_{3(15)}$ in Figure 4-4) start relatively low. The profile starts generally balanced, but by the time the window span gets to beats 1–12 (or 1–15), the onsets convey a strong dotted-half note periodicity—shown by $\left|f_{4(12)}\right|$ in Figure 4-6. There is also a strong $\left|f_{5(12)}\right|$, which would correspond to a *roughly even* division of the $\frac{5}{4}$ measure—this will be addressed later. The organization of the onsets weights the dotted-half note strongly for a lower-leveled component because of the internal construction of the measures: the $\frac{5}{4}$ in the opening measures group as (2 + 3) (3 + 2) (2 + 3).

Figure 4-5. "Shy One:" mm. 1–3

Figure 4-6. "Shy One:" Window from beats 1–12

Turning back to $|f_{5(12)}|$: despite not having a subdivisional unit, a component which corresponds to an approximately even division of the $\frac{5}{4}$ measure has a high magnitude. One potential explanation is that this component is an indication that subdivisions of 3 and 2 are present, and the presence of both would boost a roughly even division of $\frac{5}{4}$. Though an in-depth study would be required to determine anything conclusive, these results show promise that the DFT can fuzzify even divisions of meters that are inherently asymmetric.[62] (This division corresponds approximately to $|f_{6(15)}|$.)

For the following analysis, the shifting meters means that there are conflicts at multiple subdivisional levels. *Relativistic* metrical theories discuss metric levels as arbitrary relations, in some cases relating pulse levels *ad infinitum*, but the level at which these metric changes occur is very relevant to the metrical organization of the piece. Mirka, on the other hand, takes a historical perspective, drawing on previous theorists such as Koch, Riepel and

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[62]A cursory study of asymmetrical meters with DFT shows high magnitude levels for divisions into roughly even divisions. For more on this discussion, see the conclusion.

Türk to define different metrical levels. Mirka discusses meter in terms of *absolute levels*, as

opposed to more relativistic metrical theories like Cohn's, Krebs's, Murphy's and Yeston's.

She classifies three different absolute levels with the labels *takt*, *takteile*, and *taktglieder*—these

levels from "the metrical hierarchy form the pulse levels and interpretive level of the primary

sense of these words."[63] These subdivisions are equivalent to three recursively related values:

the meter or *takte*, followed by the next two subdivisional constituents *takteile* and *taktglieder*

respectively. These levels can be reassigned during the course of a piece. For example, a

piece which originally appointed the whole note to the *takte* can, at some point, reassign the

takte value to the half-note pulse depending on subdivisions present. The underlying notion

of separating primary interpretive levels implies that metric conflicts or disagreements at

different levels may also carry different interpretable effect. With recent developments in

cognitive science, one theory states that humans *attend* to a periodic pulse based on sonic

events and, in processing these events, we not only learn when those events occur but also

predict future onset occurrences. The manner in which we synchronize with a periodic

stimulus involves "phase-locking"—a physiological concept of neural oscillation matching

with the stimulus. [64] This is called *entrainment*.[65] Our behavioral entrainment has a bandwidth

of preferred tempi, meaning that entraining to a pulse is highly dependent on the temporal

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[63] Danuta Mirka, *Metric Manipulations in Haydn and Mozart: Chamber Music for Strings, 1787–1791* (New York: Oxford University Press, 2009), 134.

[64] Justin London, *Hearing in Time: Psychological Aspects of Musical Meter* (New York: Oxford University Press), 2012.

[65] Other models discuss dynamic rhythmic mechanisms as a trichotomy: (1) "temporally selective anticipation in rhythmic streams, mediated by oscillatory entrainment;" (2) "anticipation in nonrhythmic streams, which requires sustained vigilance;" and (3) memory-based prediction.

Assaf Breska, and Leon Y. Deouell, "Neural Mechanisms of Rhythm-based Temporal Prediction: Delta Phase-locking Reflects Temporal Predictability but Not Rhythmic Entrainment" (*PLOS Biology* 15, no. 2), 2017.

length of *timespans* (the durations between rhythmic onsets).[66] This means that a listener

might not perceive fractionally equivalent pulses as the same; an eighth-note pulse imbedded

in a quarter-note pulse is different than a half-note pulse imbedded in a whole-note pulse

even if they are both 1:2 related. In the case of "Shy One," acknowledging different levels of

metrical interpretation provides a profitable and noteworthy approach to metric analysis. In

"Shy One," the *taktteile*, or quarter-note pulse, is consistent throughout. It is the upper

metrical-level—the *takte*—that is in constant flux, constantly switching subdivisional values.

The notated meter $\frac{5}{4}$ is a compound meter comprised of two different subdivisions

exchanging the *takt*. Though the DFT says nothing explicit about Mirka's theories, by

applying the DFT, the results give us information about *pulse saliency*—essentially, the degree

a specific component characterizes a rhythm. In turn, we interpret these results in terms of

Mirka's absolute distinctions; the DFT will provide us with a profile describing pulse

saliency, and from that we identify which absolute level corresponds with which pulse.

Because the metric organization of the first two measures splits the 5/4 groupings so

that pairs of "3s" and "2s" are adjacent—alternating 3+2 with 2+3 groupings—we would

anticipate the resulting $f_{4(12)}$ and $f_{6(12)}$ (in Figure 4-3) to have relatively low magnitudes; a

stream of "2s" would reduce the magnitude for $f_{4(12)}$, and a stream of "3s" would reduce the

magnitude for $f_{6(12)}$.[67] As a result of the inherent asymmetry in dividing $\frac{5}{4}$, a preference for a

consistent periodicity is not possible at the *takte* level - it will shift between $f_{4(12)}$ and $f_{6(12)}$. In

Figure 4-7, I marked any onset grouping with 5 or more notes coinciding, marked in

between the staves. With the exception of measure 3's downbeat, every other strongly

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[66] Jason Yust, *Organized Time*, (New York: Oxford University Press), forthcoming 2018.
[67] The 3+2 rhythm should correspond most closely with $f_{5(12)}$, as explained.

weighted onset-grouping falls into an ongoing dotted-half note projection (illustrated below the music). Complementing our previous assertion, which made use of Figure 4-6, there appears to be a stronger dotted-half note projection in the $\frac{5}{4}$ setting. In measure 3, the dotted-half note projection continues past the downbeat, and reaches beat 2, which also contains a high concentration of onsets. To summarize the subdivisional conflict, there is a disagreement at the *takte* level between the half note and the dotted-half note, with a preference for the dotted-half note ($f_{4(12)}$).

Figure 4-7. "Shy One": mm. 1–3, dotted-half note stream

The following phrase, starting in the upbeat to measure 3, emphasizes a projection of the half note (Figure 4-8.). The emphasized half-note subdivision initiates its stream on the second beat which is, interestingly enough, when the dotted half note stops. In other words,

there is a metrical adjustment in the *takte*, substituting the half note for the dotted-half note.

Figure 4-9 shows a metric profile from beats 9–20, confirming the relevance of $f_{6(12)}$ (the half-note pulse).[68] (The profiles from now on isolate subdivisional values of interest—the *takte* and compound *takte*.) Because window positioning is extremely important with regard to windowing, "bb" will refer to the beat positions. The convergence and transferal of subdivisional pulses embodies a movement forward, a subtly conveyed notated accelerando via *takte*-shift. This junction represents the text "she moves in the firelight," which occurs concurrently; a pulse change subtly increases temporal entrainment, physically capturing an increase in movement. Or, perhaps the increase in frequency of the *takte* symbolically shows the psychological state of the narrator—illustrating the narrator's excitement while observing the "shy one." An increase in the *takte* pulse might be likened to a quickening of the heart rate. Looking back, the initial gesture and its echo (which repeated the descending third) can also be interpreted as the narrator's excitement. Based on Huron's theories of expectation, we would expect a parallel gesture which used the same text to have similar characteristics; however, by drawing out the C a beat longer, our initial expectation is denied. Instead, a *tension* response builds in anticipating how long this new event lasts before returning to the pitch A.[69] This is an irregular rhythmic ebb of expectations built up from aperiodic pulses. The consistently irregular pulse is analogous to palpitations; the aperiodicities (quite literally) take the narrator's breath away.

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[68] Remember, stated in chapter 2, that because of the windowing procedure, the middle of the window is weighted heavier than the farther removed portions—to avoid wraparound aliasing. This means that in order to examine measure 3 more accurately, the optimal window begins before the start of the piece.

[69] David Huron, *Sweet Anticipation: Music and the Psychology of Expectation* (MIT Press, 2008), 9.

Figure 4-8. "Shy One": mm. 2–5, half note stream

Figure 4-9. "Shy One": Metric Profile for bb. 9–20

Similar to the *takte* level, the alteration between $\frac{5}{4}$ and $\frac{6}{4}$ adds an additional metrical

conflict—the pulse which is designated the compound *takt* fluctuates back and forth. Longer

timespans are fundamentally more volatile due to the human entrainment capacity; on

average, the range of entrainment is between 100 ms and 5–6 seconds.[70] Both recordings, by

Patricia Wright (piano), with Kathron Sturrock (voice), and Philipp Vogler (piano) with

Hélène Lindqvist (voice), take a tempo of ♩ ≈ 60 bpm—both generously expressive for the

marked *moderato grazioso*. Both pulse streams, comprised of 5 quarter notes or 6 quarter notes,

fall within the band of possible entrainment behaviors.[71] Therefore, these compound *takte*

occupy positions as structural measures, and/or as potential entrainment behaviors.

The metrical layer for the dotted-whole note pulse is shown in Figure 4-10, and the

metric profile in Figure 4-11a. (Figure 4-11b shows the profile for $f_{(15)}$).[72] Figure 4-11a

corresponds to a 12-unit window spanning beats 4 to 15 in the score. The magnitude for $f_{2(12)}$

is large, corresponding to onsets grouping the span into 2—that is, the dotted-whole note.

So while our original profile (in Figure 4-9) weighed the dotted-whole-note and 5-quarter-

note streams evenly, this window favors $f_{2(12)}$. The two 5-onset clusters grouped next to each

other at the start of the third measure reinforce the division of the span into two.[73] There is a

reversal in compound *takt* preference in the window covering beats 12–26. Figure 4-12b

shows that the 15-beat window contains a high magnitude for $f_{3(15)}$, meaning that the

compound *takt* switches from the dotted-whole-note to the 5-quarter-note pulse. By beat 4,

the dotted-whole-note pulse—$f_{2(12)}$—is more prominent. The 5-quarter-note onset groupings

start on beat 14 (Figure 4-13).

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[70] Justin London, *Hearing in Time: Psychological Aspects of Musical Meter* (New York: Oxford University Press), 2012.

[71] Ibid., 46.

Though Justin London reports that metric entrainment can occur within a range from "about 100 ms to about 5 or 6 seconds," we do have a preference for periodicities around 600 ms (or .6 seconds).

[72] I have only visualized components of immediate importance to make things more clear.

[73] Clustering on a point will increase the magnitude of that division—review chapter 1 on clustering/maximally even sets.

Figure 4-10. "Shy One": mm. 1–3, 6-quarter-note stream
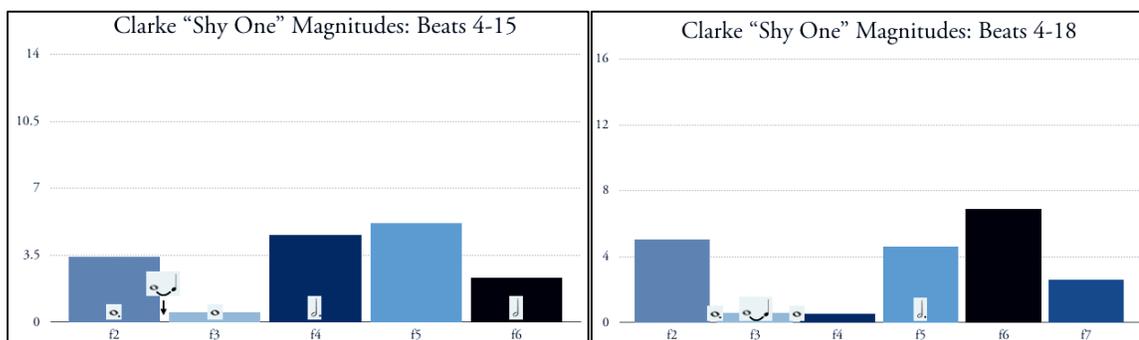
Figure 4-11a. "Shy One": Metric Profile, Window Size 12 quarter notes for bb. 4–15;
and 4-11b. Window Size 15 quarter notes for bb. 4–18
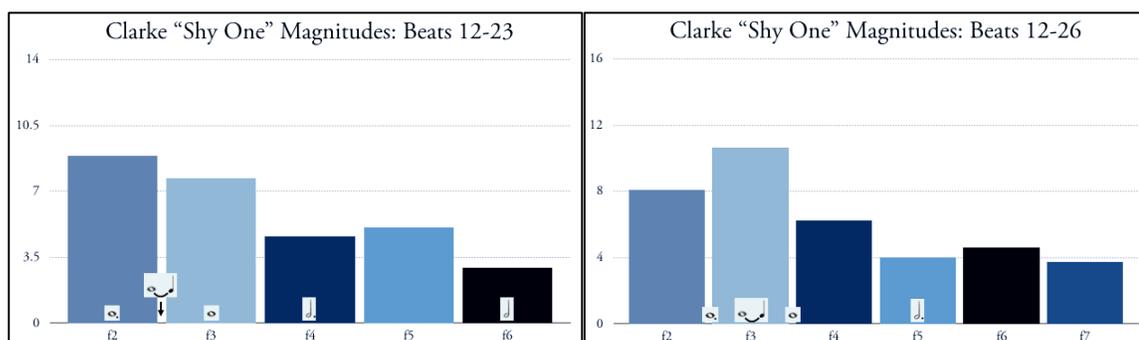
Figure 4-12a. Shy One": Metric Profile, Window Size 12 for bb. 12–23;
and 4-12b. Window Size 15 for bb. 12–26

Figure 4-13. "Shy One": mm. 2-5, 5-quarter-note stream

The following section, which switches between $\frac{5}{4}$ and $\frac{6}{4}$ (*xyxy* in measures 5-8), is primarily governed by the dotted-half-note pulse. Because $\frac{6}{4}$ groupings pair into <3+3>, and the $\frac{5}{4}$ are <2+3> (or <3+2>), music that groups onsets at meter-defining beats would emphasize the dotted-half-note pulse. This is not to say that the half-note subdivision is absent—it intrudes in $\frac{5}{4}$ measures, constantly denying the dotted-half-note projection, to destabilize the meter briefly—it is just less frequent in the 'b' sections (of the A(ab)A'(a'b) form). The magnitudes of $f_{4(12)}$ and $f_{6(12)}$ can be normalized and plotted across the whole piece, as shown in Figure 4-14, to demonstrate the constantly changing state of the *takte*. Figure 4-14 are *normalized* (f_n/f_0) so as to discount the effect of overall number of onsets on the magnitude values.[74] Each number on the *x*-axis corresponds to the center of the window, so

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[74] The first and last six windows are cut from the profiles because, as the sliding window moves into the beginning of the piece, the onset number will be significantly less, meaning the results are insignificant.

"point-1" is centered in the window, and the left tail takes no input because it is before the piece starts. According to our tracked magnitudes, the piece initial conveys a strong dotted-half-note periodicity (in blue). At points 13–16, the magnitude for the half-note periodicity (f_6) is significantly larger than f_4—this reconfirms our previous discussion (see Figure 4-8). f_6 resurfaces again around point 35, where both magnitudes are significant. f_6 is larger at this moment because the end of measure 7 has a chord two units away from the downbeat of the next measure (8), which implements another large chord—large as in more onset-packed. Not only that, but the upbeat to measure 8 is a chord, further reinforcing that f_6 periodicity. At point 44, the A' section re-presents the initial inflections of f_4 and f_6. The blue path takes dominance once more, until 12–13 points later. By tracking the magnitudes over time, it shows *pulse saliency*, and by association, captures certain phrase patterns.
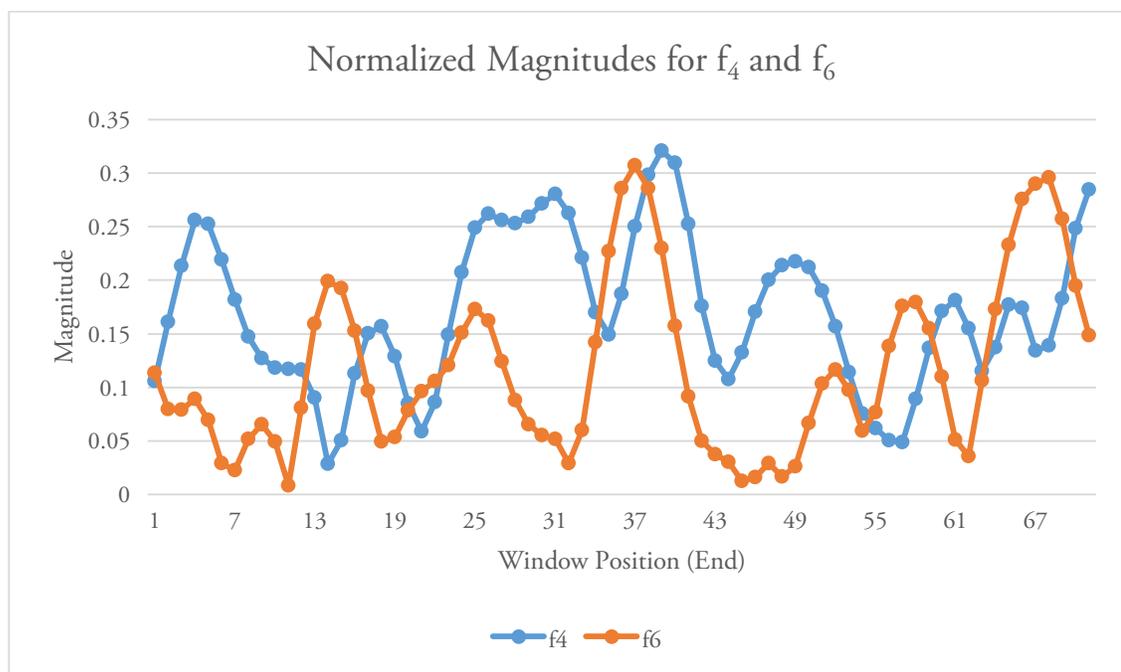
Figure 4-14. "Shy One": Normalized Magnitudes (Adjusted)

Phase Tracking

Due to the phase shifting procedure (explained in chapter 2, which describes how

the ORWDFT operates), the resulting phase is relative to the starting measure. Because

phase relates to positioning, I refer to the meter in the score as the *notated meter* and otherwise

as the *phase position*.

Given a consistent, projected meter from the score that evenly divides the window

size, with the phase shifting procedure, phase is *anchored* to the notated barline. The phase

position would hypothetically remain level for a piece, with few metric changes throughout.

Changes in phase may indicate displacements from the previously established and sounding

meter. In a strophic piece with parallel structure—such as "Shy One"—we would expect the

DFT's components to mirror one another. It intuitively makes sense that a reprise of

material would represent a reprise of DFT profiles. As discussed earlier, the notated meter in

"Shy One" changes between $\frac{5}{4}$ and $\frac{6}{4}$—see Figures 4-2 and 4-5. Because the large A section

repeats, we would expect the phase positioning of the resurfaced A section to equal its

counterpart, but because of how the notated meters are organized (Figure 4-2), the phase

value, which represented the initial barline grouping, is changed. The A section lasts 43

beats, followed immediately by A' on beat 44. 43 is a prime number and thus not divisible by

the components we have been examining for "Shy One"—$f_{\underline{3}(15)}$, $f_{\underline{5}(15)}/f_{\underline{4}(12)}$, and $f_{\underline{6}(12)}$. This

means that the phase value which corresponds to the initial gesture is not equal to the value

of the restated material. in measure 44 (notated).

In common-practice period classical music, changes in notated meter are not very

common, so the definition of phase being anchored to the barline holds. However, due to

the frequent meter changes in "Shy One", this is no longer a sufficient definition. A

reorientation of how we view phase is in order. With the phase shifting procedure, the phase value should be thought of in terms of its positioning to an *absolute temporal grid*. Phase values, then, correspond to an absolute positioning established at the initiation of the first onset. Therefore, phase values are abstracted away from the barlines and, instead, should be viewed in relation to each other and to the temporal grid. This redefinition integrates the original view of phase being anchored to the barline, except that this only applies when the barline is consistent in relation to the absolute grid.

Figures 4-15a and 4-15b demonstrate how meter changes may influence phase. Figure 4-15a repeats the same model six times: 4 measure groupings of $\frac{5}{4}$ followed by one bar of $\frac{6}{4}$. A repetitive $\frac{5}{4}$ pattern will elicit a high magnitude for $f_{3(15)}$ because it divides the span of 15 into 3 parts. Phase tracking is best enacted on components with generally higher magnitudes; otherwise, as shown in Chapter 2, the phase will shift more rapidly.[75] Phase tracking a component with low magnitude means that the element is less salient, and the information gained from phase tracking will probably be less relevant in the music. Stability in phase is represented as a *plateau* in the graphs; if the line relatively levels out, it means that the orientation of the implicit downbeat—with respect to an absolute temporal grid—is stable. As shown in the phase-tracking representation of 4-15a, 4-15b forms five plateaus and starts a sixth, with each plateau representing a steady pulse projection. The changes in y-axis value are due to the "added beat" in the $\frac{6}{4}$ measures. Each $\frac{6}{4}$ measure pushes the phase down, or counterclockwise on a phase clock (see Figure 4-16). To correct for the extra beat

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[75] This is not to say that a component that has a relatively high magnitude throughout should not be trusted because of a dip in energy—in fact, this should inform us more about what is going on in the music. A feature which was once prevalent is somehow less present.

in the $\frac{6}{4}$ measures, if we were to think of phase shifting in terms of our old definition, the phase values are pushed back and reassigned to new positions in the notated meter. Thinking about phase in terms of notated meter is potentially confusing here—the phase values do not readjust to align with the notated downbeats. Instead, viewing the music in terms of an absolute temporal grid allows the results to speak for themselves. Stability in the phase's *x*-axis represents a stable pulse, and, in this example, the changing phase means the music was pushed "back a beat."

Figure 4-15a. $\frac{5}{4}$ interrupted regularly

$$\varphi_3(15)$$
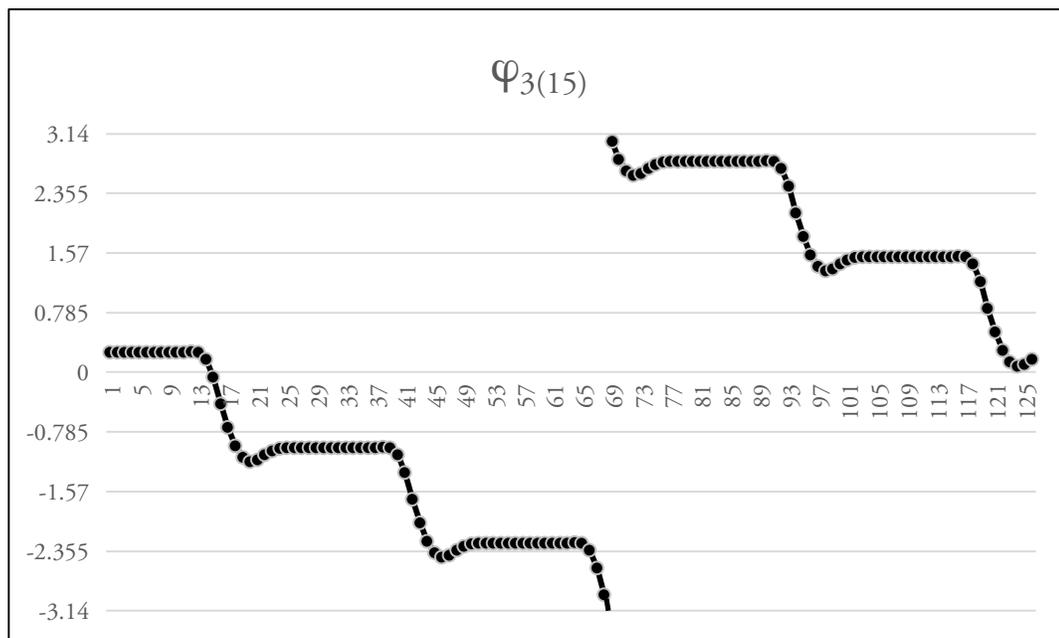
Figure 4-15b. $\frac{5}{4}$ Phase Tracking
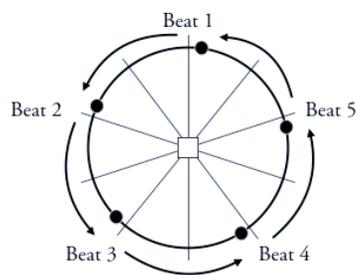
Figure 4-16. $\frac{5}{4}$ phase space rotation

For each $\frac{6}{4}$ in "Shy One," the notated meter's downbeats are offset from the absolute

grid of $\frac{5}{4}$ below it. This means that the $\frac{5}{4}$ grid used to gauge will be off for every measure of $\frac{6}{4}$,

like so:

Notated Music Beats	**1**	2	3	4	5	**1**	2	3	4	5	6	**1**	2	3	4	5	6	**1**
$\frac{5}{4}$ Absolute Grid	**1**	2	3	4	5	**1**	2	3	4	5	**1**	2	3	4	5	**1**	2	3	

The notated "position 1" in the representation above, would then be positioned where beat 3 is in the grid. Figure 4-17 shows this in standard musical notation. The top musical line shows an example in which the notated meter changes from $\frac{5}{4}$ to $\frac{6}{4}$ and then back. Against the absolute grid (located below it), the downbeats of the changing meter become misaligned as the $\frac{6}{4}$ continues. The phase value would begin to plateau when the meter returns to $\frac{5}{4}$, but by then the phase value at which this stability occurs is different than at the start. Just like our hypothetical table above, the notated meter now starts on the third "beat position" of the absolute temporal grid.
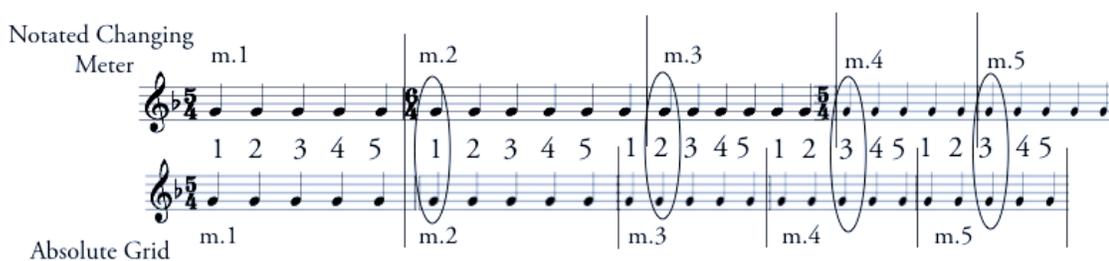
Figure 4-17. Example of Notated Meter Changes to an Absolute Temporal Grid

By tracking the phase over the course of "Shy One," we can show where these phase changes correspond to in the music. Figure 4-18a shows $\varphi_{3(15)}$, which corresponds to measures of $\frac{5}{4}$. When tracking the placement of $\frac{5}{4}$ measures in relation to the grid, imposing a measure of $\frac{6}{4}$ *should*, as just discussed, push the $\frac{5}{4}$ back a beat. Overall, there is downward motion in the tracked phase profile, or, equivalently, counterclockwise motion on a phase-space clock face (Figure 4-18b). In relation to the music, this rotational motion corresponds to what we expected; the phase is pushed back to another position in the temporal grid. This naturally makes sense based on what we know from Figures 4-15–4-16.
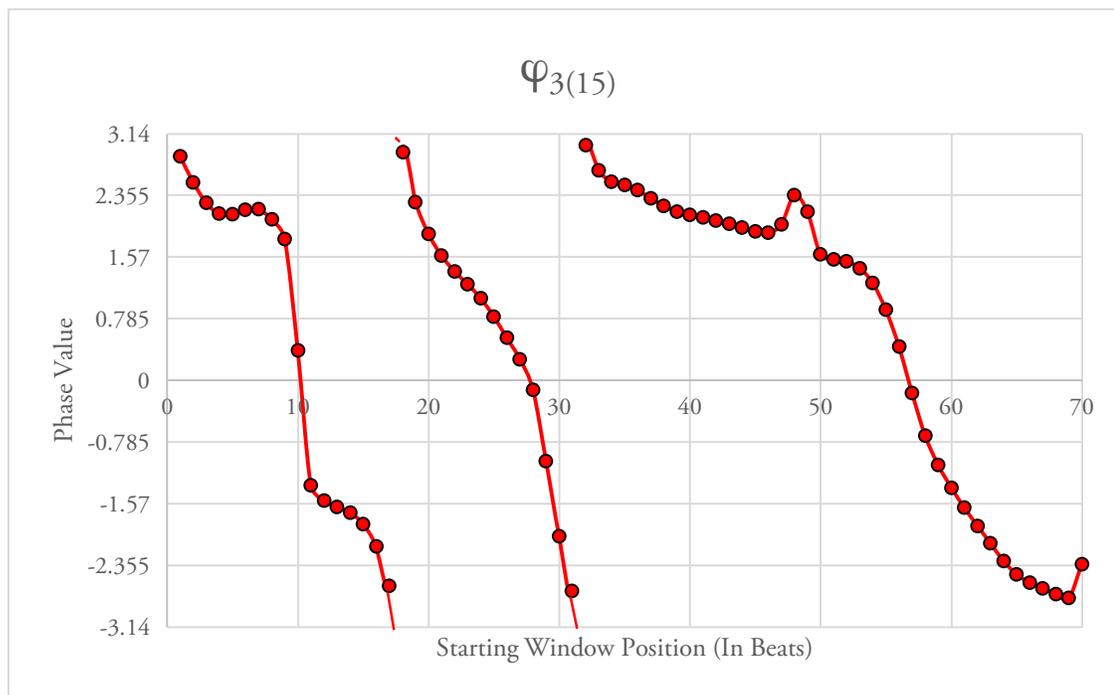
Figure 4-18a. "Shy One": $f_{3(15)}$ Phase Tracking
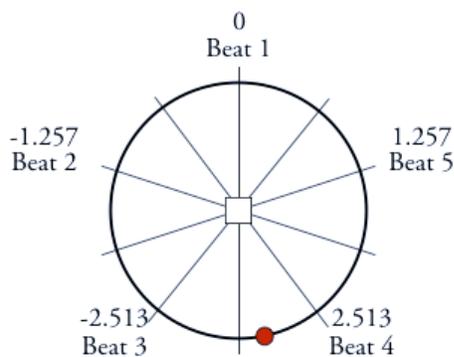
Figure 4-18b. $\frac{5}{4}$ Phase Space

There are very few plateaus in Figure 4-18a, meaning there are few consistent streams of $\frac{5}{4}$. The phase value of the shifting notated meter should hypothetically manifest plateaus when the meter rests long enough to establish some salient pulse; against the absolute temporal grid of unwavering $\frac{5}{4}$, changes in meter alter the relative phase value. The longest $\frac{5}{4}$ stream begins with the window corresponding to point 33 in Figure 4-18a. This

plateau corresponds to "*yxy*" in the first strophe A–B: A[*xxx**y***]B[***xy****xy*].[76] Figure 4-19a shows where this stream occurs (in mm. 4–7), and Figure 4-19b shows how the absolute grid pairs with the notated score. The phase-value range in the plateau starting around window 33 is approximately 1.89–2.6, which, according to the absolute grid, corresponds to around beat 4 in the $\frac{5}{4}$. A single $\frac{6}{4}$ measure necessitates a compensation for the beat and reorientation of the phase value back a notated beat, but at this moment in the music, there have not been enough $\frac{6}{4}$ measures to push the phase back to beat 4. The phase value at point 33—a window starting in beat 18 of the piece—positions $f_{3(15)}$ halfway through the measure, even though the first $\frac{6}{4}$ has yet to complete. The full measure of $\frac{6}{4}$ is not responsible for the stream displacement. The consistent stream for $f_{3(15)}$ starts halfway through measure 4 and is connected into the notated third beat of the subsequent $\frac{5}{4}$ measure (which would be the fourth beat in the absolute grid—Figure 4-19b). In other words, the majority of onsets in this section are not clustered around the metric downbeats. This is an example in which clustering reinforces a stream; the eighth notes at the end of the second projection group around the value even beat 3 of that measure reinforces the phase. As the phase tracker proceeds the value drifts towards beat 5, which makes sense with the imposition of $\frac{6}{4}$ in measure 6.

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[76] In order to capture initial events, for the coding procedure in "Shy One" each point represents the *end* of the window, not the beginning. Because the piece is so short, this procedure starts scanning gradually into the piece rather than starting its 15-span window on the starting onset. Therefore, the window which corresponds to point 33 actually starts on beat 18 of the piece.

Figure 4-19a. "Shy One": mm. 4-7, 5-quarter-note stream

Figure 4-19b. "Shy One:" mm. 4-7, Notated Measures vs. $\frac{5}{4}$ Absolute Grid

Both $\varphi_{5(15)}$ and $\varphi_{4(12)}$—corresponding to the dotted-half note of their respective windows—are also in a state of flux. The frequent phase change of $\varphi_{5(15)}$ and $\varphi_{4(12)}$ is due to the constant shifting of the *takte* pulse. The phase-tracking graphs representing the dotted-half note pulse are in Figure 4-20, directly juxtaposing $\varphi_{5(15)}$ and $\varphi_{4(12)}$ to show how they are

essentially equivalent. There are brief plateaus starting around points 9, 22, 29, 44, and 53. The window starts prior to when the piece begins, so that the highest-weighted portion in the Hanning window incorporates the start of the piece. Because of this step, one subtracts half of the associated window from the "point" value to locate the center of the window. Therefore, point 9 is equivalent to beat 3 in the first measure; this phase stream thus represents a stream identified in Figure 4-7. Points 22 and 29 capture a stair-stepping effect from shifting back and forth between $\frac{6}{4}$ and $\frac{5}{4}$—the **_yxy_** portion of the first b subsection. The ascent in phase space is the directly opposite phase-tracking response from our previous $\frac{5}{4}$ tracking; because we are tracking dotted-half note streams, a $\frac{5}{4}$ measure functions as *interrupting* a stream as opposed to extending it by a beat. As for the windows around point 44, a clear projection of dotted-half notes in the $\frac{6}{4}$ measure (m. 8) weighs strongly in favor of that phase. The plateau around beat 53 relates to a brief projection from measures 9–10 from the inner-metric organization of <23> → <32>, so that the dotted-half note is projected farther. With the frequent changing of meter, each one of stable plateaus for the *takte* is brief, never lasting more than a few beats.
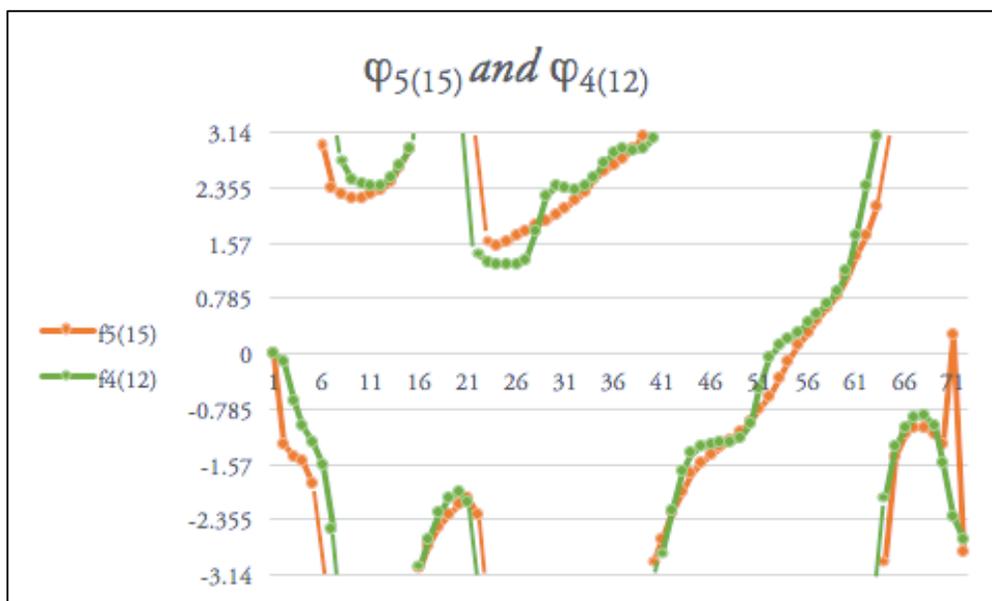
Figure 4-20. "Shy One": $f_{5(15)}$ and $f_{4(12)}$ Phase Tracking

In the case of "Shy One," the phase of the *takte* changes very frequently, while the compound *takt* level is more predictable. Even on large-scheme progressions, the phase of the compound *takt* is pushed back, extending a concept that Krebs would call *subliminal dissonance*—a phenomenon where the placement of a previously established meter clashes with an ongoing one.[77] The concept of subliminal dissonance is, quite literally, extended in that the absolute temporal grid tracks the continual accrual of subliminal dissonance in relation to the starting phase. The effect of the numerous metric changes creates a general state of opacity; with reliance on the compound *takt* being unreliable, and the *takte* even more so, there is no other definitive pulse to entrain to a higher level than the *taktteile* (the quarter note). The metric state is thus dialectic at two primary levels: the local level in which the asymmetrical meter $\frac{5}{4}$ inherently contrasts subdivisions of 2 and 3, and the compound

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[77] Harald Krebs, *Fantasy Pieces: Metrical Dissonance in the Music of Robert Schumann* (New York: Oxford Univ. Press), 2003.

takt in which $\frac{5}{4}$ alternates with $\frac{6}{4}$. Though my work here strictly concerns the DFT, in terms of interpretation these irregular metric changes may symbolize the narrator's emotional state. In this way, it is possible that the DFT can be utilized in formal-narrative theories.

CHAPTER V: CONCLUSION

This paper ends where it began: how can the DFT provide theoretical and analytical insights to the rhythmic domain? To conclude this thesis, I open a discussion on the latent potential of the DFT's future applications and what this means for the future of theorizing about rhythm and meter.

Regarding pitch, Quinn described the magnitudes of components in terms of *saliency*. Quinn defined saliency in terms of quality; if a set is closer to a maximally-even set, then it has those characteristics. In other words, the magnitude levels describe certain characteristics of the input. I extend that, in rhythm, magnitudes relate to rhythms in terms of *pulse saliency*. In this way, when there are multiple rhythmic lines juxtaposed simultaneously in the music, the DFT has the ability to examine each one of those separately and describe their character, and subsequently describe the character of the unified form. The resulting metric profile has the benefit of describing *how much* of a specific component is conveyed through the rhythmic surface; Cohn's ski-hill graphs can represent subdivisional presence, but the DFT can, in addition, show the degree of influence it has on the profile.

At the local level, phase can identify differences in rhythmic positioning. This is useful in that the magnitude of a profile will return identical results to another rhythm which is positioned differently—phase can distinguish the rhythms from one another. The scanning procedure and phase shifter component assist the interpretive process needed in order to track phase through a piece. By thinking about phase values in terms of an absolute temporal grid—as discussed earlier—metric displacements and syncopations in the music are placed in reference to one another at a global level. I foresee that phase, with additional weighting procedures, has the potential for locating downbeats of meters, and showing

form-defining rhythmic gestures; tracking phase throughout and locating cadential displacements such as cadential syncopations may show statistical trends in compositional styles or compositional periods.

TOWARDS A THEORY OF METER

Horlacher has discussed a problem of defining meter in terms of periodicities, saying that "metric irregularities may gain a certain privileged or normative status independent of a fixed point of reference in the background."[78] Horlacher's view encourages a flexible listening of rhythmic patterns that may not be included in the definition of *meter* for many theorists. The difficulty with labeling meter in a piece like "Shy One" becomes controversial: do we consider the non-periodic rhythmic irregularities as meter? The strophic form certainly allows us to expect the recurring $\frac{5}{4}$ contrasted with $\frac{6}{4}$ in the consequent section. Or, alternatively, do we accept that meters like $\frac{5}{4}$ are are not equally divisible, and may thus only be understood in relation to a regularity? The difference lies in the definition of meter. The term *meter* is primarily used to convey two different phenomena: 1) The cognitive concept linked to *entrainment*; and 2) nested, equally spaced divisions of a space. These two ideas are separated as such, with the formal term meter referencing the ecological adaptation to attend to a regular pulse and the latter as a structural organization into a recursive structure of equal-spaced events.

Rhythms can either support or oppose the meter; they can either reinforce the meter or, in some cases, articulate structures that contradict it. The differentiation between rhythm

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[78] Gretchen Horlacher, "Metric Irregularity in "Les Noces: The Problem of Periodicity" (*Journal of Music Theory* 39, no. 2, 1995: 285-309), 290.

and meter allows us to acknowledge both the presence of surface rhythms and a deeper periodic structure, that is, meter. In the words of Cohn, "Meter arrives unbidden to the projecting mind and entraining body, and takes up permanent residence by default."[79] In this way, Cohn is saying that entrainment is an unconscious reaction; meter as entrainment is formed through statistical learning, and expectation through that. Huron uses expectation to frame the equally spaced events that construct the common definition of meter: "Periodic events are predictable for the simple reason that they establish a regular time interval that acts as a predictive template."[80] Expectations of periods are intrinsic to the very makeup of the structure, so meter will, by association, inherently contain forms of expectation. I return to a previous quotation in which Huron goes on to say that "Although periodicity helps listeners to form temporal expectations, periodicity is not necessary for the formation of such expectations. It is important only that the listener be experienced with the temporal structure, and that some element of the temporal pattern be predictable."[81] He goes on to argue that meters are *predictive schema*—a model of some framework based on previous exposure and statistical learning. In this way, the definition of meter can be extended from a set of isochronous event onsets—a definition which invalidates asymmetrical meters like $\frac{5}{4}$ and $\frac{7}{8}$—to that of general schema. Few musicians would doubt that $\frac{5}{4}$ is not a meter, but according to a widely accepted definition, it would be relegated to a recurring rhythmic pattern. I expand upon Huron's predictive-based model by positing that meter is the way in which familiarity through exposure influences the natural systems of human expectation.

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[79] Richard Cohn, "A Platonic Model of Funky Rhythms," (*Music Theory Online*, 1 June 2016), 1.6 mtosmt.org/issues/mto.16.22.2/mto.16.22.2.cohn.html.

[80] David Huron, *Sweet Anticipation: Music and the Psychology of Expectation* (MIT Press, 2008), 175.

[81] Ibid., 187.

This definition integrates the periodic model in that periods are easy to form expectations about.

As I have shown here, the DFT captures metric profiles that exhibit characteristic features of a pattern in terms of saliency. In moving forward with my research, I anticipate that these templates will provide a method of qualifying specific degrees of metricality. In fact, these templates describe the pulse saliency and this can, in turn, be thought of in terms of expectancy. Amiot describes interval vectors in terms of the "probability of hearing a given interval in a given (pc-)set,"[82] and saliency can be thought of in similar terms; the likelihood of hearing a given pulse in a span is based on its magnitude.[83] Just as if we were to hear a sound world comprised of a diatonic scale, we would expect sounds to be from that sound world. Similarly, the DFT's metric profiles capture a state of probability. Based on Huron's model of expectation, the DFT's rhythmic templates share some similarities with conventional ideas about meter—I do not claim that the DFT profiles display meter, but merely that they exhibit rhythmic and metric qualities that enhance metric discourse. I hope that, through this thesis project, I have shown fruitful products of an alternative methodology and its contribution to theories of rhythm and meter. With more research, I intend to explore the use of the DFT and how it may be deployed towards a broader, yet more precise theory of meter.

Following Quinn's (2005) revival of Lewin's work (1959) concerning the DFT, its use has continued to clarify a range of analytical and theoretical questions posed by the

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[82] Mathematics and Computation 2017 Amiot 153

[83] In fact, DFT of the interval vector is the convolution of a set with its inverse. The squares of DFT of the original set is then the interval vector.

music theory community. In this thesis, I aim to add to the recent scholarship which has

extended the seemingly universal applications of DFT by applying it to rhythm and meter. I

hope to have established a method that has the capabilities of examining local and global

rhythmic structures alike. This thesis project prompts further questioning, and in doing so,

will hopefully provide some inspiration to theorists of all kinds to explore the immense

provinces made accessible through the DFT.

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