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ESSAYS ON INSTITUTIONAL INVESTORS, CENTRAL BANKS AND ASSET PRICING

by

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Essentially, all models are wrong, but some are useful.
George E. P. Box

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ABSTRACT

The objective of this dissertation is to investigate the impact of important market participants such as Mutual Funds, Hedge Funds and the Federal Reserve Bank on the equilibrium equity premium, risk free rate and asset volatility and to analyze the effect of these institutions on risk shifting, portfolio allocation and financial stability. Specific features of institutional investors and central banks as well as their role in financial markets are reviewed and analyzed in Chapter 1.

In Chapter 2, it is shown that the competitive pressure to beat a benchmark may induce institutional trading behavior that exposes retail investors to tail risk. In our model, institutional investors are different from a retail investor because they derive higher utility when they outperform the benchmark. This forces institutions to take on leverage to over-invest in the benchmark. Institutions execute fire sales when the benchmark asset experiences negative shocks. This behavior increases market volatility, raising the tail risk exposure of the retail investor. Nevertheless, ex-post, tail risk is only short lived, all investors survive in the long run under standard conditions, and the most patient investor dominates in the sense that she has the highest consumption wealth ratio. Ex-ante, however, benchmarking is welfare reducing for the retail investor, and beneficial only to the impatient institutional investor.

Chapter 3 presents an analysis on how monetary authorities seeking to stabilize inflation, output and smooth interest rates distort the term structure of interest rates and prices of risk relative to an economy where central authorities adjust the money supply without taking into consideration the slope of the yield curve. Closed-form expressions for all equilibrium quantities are presented and the impact of quantitative easing on prices, risk premium and volatility of financial markets instruments, such as stocks and bonds, are evaluated. The changes in macroeconomic variables such as consumption, money demand and investment policies are also investigated. Under the adopted parametrization, quantitative easing is welfare improving. In addition, quantitative easing increases nominal bond and equity volatility, while reducing both real and nominal bond yields for all maturities.

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List of Abbreviations

BOJ	....	Bank of Japan
CARA	....	Constant Absolute Risk Aversion
CRRA	....	Constant Relative Risk Aversion
ECB	....	European Central Bank
ETF	....	Exchange-Traded Fund
FED	....	Federal Reserve Bank
FSOC	....	Financial Stability Oversight Council
HARA	....	Hyperbolic Absolute Risk Aversion
HJB	....	Hamilton-Jacobi-Bellman
ICI	....	Investment Company Institute
IMF	....	International Monetary Fund
OFR	....	Office of Financial Research
PBC	....	People's Bank of China
PDF	....	Probability Distribution Function
QE	....	Quantitative Easing
SDE	....	Stochastic Differential Equation

Chapter 1

Introduction

In September of 2008, global financial markets experienced the worst disruption since the Great Depression of 1929. The collapse of the fourth largest investment bank in U.S. spread panic across the financial system, leading to subsequent failures of commercial and investment banks, insurance companies and non-financial corporations. In an effort to understand the main factors and actors responsible for the crisis, many regulators and academics started to investigate the role played by financial intermediaries. Their main concern was that the sheer size of these companies and their trading pattern could potentially overexpose their companies to systemic risk and trigger fire sales events, constituting a threat to the stability of financial markets. Nearly a decade after the crisis, some of these investment companies have an even bigger balance sheet and concerns about systemic risk prevail.

According to the 2015 report[1] released by the Investment Companies Institute (ICI), the U.S.-registered investment companies managed U$ 18.2 trillion dollars at year-end of 2014. Only U.S. mutual fund and exchange-traded fund (ETF) account for U$ 17.8 trillion of these assets. In fact, since 1997, the assets under management of these firms grew 287%. The substantial growth of these financial institutions in the past quarter of century is often justified by the accumulation of wealth of the Baby Boom Generation, the aging of the U.S. population and the gradual change of employer-based retirement systems.

[1] https://www.ici.org/pdf/2015_factbook.pdf.

The colossal size of these companies is better put into perspective when we analyze the share of the equity and debt market owned by them. According to the same report, by the end of 2014, investment companies hold roughly 30% of the outstanding shares of U.S. equity market and 46% of the outstanding commercial papers. In addition, other characteristics present in the industry, such as benchmarking incentives and misalignment of interest between investors and managers, could potentially induce trading behavior that is different from what is predicted by mean-variance theory. Consequently, these firms' trades could have a significant impact on prices and volatilities of financial securities.

To investigate the effects induced by investment companies on financial markets and their consequences for financial stability, I present in Chapter 2 an asset-pricing model that allows to capture how the industry's incentives lead to holdings that deviate from mean-variance portfolios and study the effects on volatility. The chapter is divided in five main parts. Section 2.1 has an introduction on the topic of institutional investors as well as a literature review. Section 2.2 has the description of the model. We indicate the available assets in the economy and describe agents' preference. Specifically, the section contains a discussion on how we capture the benchmarking incentives of investment companies. In Section 2.3, we describe the concept of an exchange economy equilibrium in the presence of institutional investors. We characterize the effect of benchmarking incentives on assets, portfolio plans and consumption policies. The fourth part, Section 2.4, studies the systemic implications of benchmarking incentives. Using the general equilibrium expressions derived in previous sections, we measure the impact of benchmarking on investors' tail risk. We rely on portfolio returns' value-at-risk of financial institutions to access their exposure to disaster events. In addition, we show what are the survival implications for survival of agents that have benchmarking incentives. We also investigate which market participants are better off in the presence of investment companies that have incentives to benchmark. Section 2.5 concludes.

Another key agent that played a crucial role during the crisis' unwind was the Federal Reserve Bank of United States (FED). In the aftermath of the financial crisis of 2007-2009, the FED relied on unconventional and untested measures, such as acquisition of mortgages securities, commercial papers and direct lending to non-banks institutions, to restore financial stability and to prevent the collapsed of the entire financial system. In essence, all these measures intend to *ease* loan conditions for non-financial firms by supplying banks with a *quantity* of new money equal to the value of the purchased assets. These measures are commonly referred to as *Quantitative Easing* (QE).

The consecutive rounds of quantitative easing in U.S. extended the FED's balance sheet from less than $1 trillion dollars in 2007 to more than $4 trillion dollars in 2015. Other central banks around the world, such as the European Central Bank (ECB), the People's Bank of China (PBC) and the Bank of Japan (BOJ), have also adopted similar policies in order to restore liquidity in financial markets. However, several questions about the effectiveness of QE remain unsettled. First, it is still unclear wether or not this massive flood of new cash has become another source of uncertainty as central banks start selling these assets and reducing the money supply, increasing market volatility. Second, it is not clear wether or not the large and cheap money supply has stimulated reckless behavior of markets' participants, contributing to riskier exposure of financial institutions. Third, despite the relative success in battling short-term deflation, central banks still struggle to boost markets' confidence, making firms reluctant to invest and hire. Consequently, the persistent unemployment and sluggish economic recovery present a challenge for the central bank's aim of keeping low levels of medium and long-term inflation. Fourth, the massive inflow of money into emerging markets could potentially be a source of destabilization of these financial markets, adding more global uncertainty and potentially feeding this risk back to developed economies.

To address some of these questions, Chapter 3 presents continuous-time production based monetary economy that investigate how central banks distort risk by seeking to smooth financial variables through monetary policy. The chapter is divided in four parts. Section 3.1 has an introduction on the topic monetary asset pricing as well as a review literature. Section 3.2 has the description of the model. The role of the central bank is outlined in this section. It also contains the complete characterization of the equilibrium with closed-form expressions for the macro-financial variables. The third part, Section 3.3, contains the numerical analysis of the model. It discusses how quantitative easing impacts volatility, equity and term premium. A welfare analysis shows that central banks' interventions that smooth yield curve is welfare improving. Last, the analysis of dynamics of output and price level quantify how these macroeconomic variables are impacted by monetary shocks. Section 3.4 concludes.

Chapter 2

The Systemic Effects of Benchmarking

2.1 Introduction

The asset management industry has grown exponentially over the past decades. Figure 3·1 displays the time series of assets under management by hedge funds worldwide, which have grown from $100 billion in 1997 to roughly $2.5 trillion in 2014. Other types of funds, such as mutual funds and index funds, have experienced similar growth. Because of their sheer size, the trading behavior of funds can impact prices and risks in financial markets in non-standard ways. This has sparked a discussion among regulators and the media about the potential risks that asset management may pose to financial stability. In this paper, we provide some first theoretical answers to questions about potential threats that the asset management industry may pose to the stability of financial markets.

One key feature of the asset management industry is that a manager is compensated for the performance of the managed portfolio in excess of a predetermined benchmark.[1] This so called "benchmarking" incentivizes managers to invest in ways that deviate from the standard theory. For example, Basak and Pavlova (2013), Brennan (1993), Buffa et al. (2015), Cuoco and Kaniel (2011), Roll (1992), and others show

[1]Ma et al. (2015) document that about three-quarters of all mutual fund managers in the U.S. receive performance-linked compensation. Indeed, when an individual investor delegates the management of her portfolio to a manager that can exert effort to become informed about the distribution of returns, then the optimal contract in this principal-agent setting is one that compensates the manager for her performance in excess of a benchmark; see, e.g., van Binsbergen et al. (2008), Cvitanić et al. (2014), Cvitanić et al. (2006), Li and Tiwari (2009), and Stoughton (1993).

that benchmarking incites managers to overinvest in stocks that are highly correlated with their benchmarks. Given this specific feature of the asset management industry, the Financial Stability Oversight Council (FSOC) asks in a Notice released in December 2014 to which extent benchmarking can "create incentives to alter portfolio allocation in ways that [...] do not take into account risks to the investment vehicle or the broader financial markets?"[2]

We answer this question by analyzing a pure-exchange economy consisting of a retail and two institutional investors who can invest in a benchmark and a non-benchmark stock, and who can borrow and lend from each other. Stocks are in positive net supply, and have price processes that may jump. Time runs continuously from 0 to infinity, and investors consume at all periods of time. The retail investor derives standard log-utility from consumption, and can thus be interpreted as an individual mean-variance investor. The institutional investors, on the other hand, derive higher log-utility from consumption in states of the world in which the benchmark stock outperforms relative to the non-benchmark stock. These preferences incentivize our institutional investors to gain higher exposures to the benchmark stock than the retail investor, consistent with the behavior of managers whose performances are evaluated relative to a benchmark. Consequently, our institutional investors can be interpreted as asset managers who derive log-utility from the compensation they obtain for managing portfolios.

[Figure 1 about here.]

Investors in our model are heterogenous because they differ from each other through their benchmarking and their time preferences. Despite the complexity of our model, we can solve for all general equilibrium quantities in semi-closed form

[2] Notice Seeking Comment on Asset Management Products and Activities, Docket No. FSOC-2014-0001, pp. 11-12.

using the Fourier inverse methodology of Martin (2013). This allows us to run comparative statistics to analyze the systemic effects of benchmarking by measuring the tail risk exposures of our retail and institutional investors in economies with different benchmarking incentives. It also allows us to carry out a survival and a welfare analysis.

Our main finding can be summarized as follows: Stronger benchmarking incentives lead to higher (lower) tail risk exposure of the retail investor and the aggregate market when the benchmark stock underperforms (outperforms). More precisely, we find that stronger benchmarking incentives incite institutional investors to take on leverage to overexpose themselves to the benchmark stock. This results in consumption and portfolio plans of institutional investors that are highly sensitive to the relative performance of the benchmark in states of the world in which the benchmark underperforms. Institutional investors react strongly to news in such states of the world. They trade large amounts of the stocks, resulting in large market volatility. If very bad news about the benchmark arrive in the form of a jump, then institutional investors initiate fire sales. That is, they sell large amounts of the benchmark stock at a discounted price, and buy large amounts of the non-benchmark stock at a premium price. This constitutes a flight-to-quality phenomenon. Even though this behavior reduces the tail risk exposure of the institutional investors, higher volatility and fire sales increase the tail risk exposure of the retail investor and the aggregate market in bad states of the benchmark. In contrast, institutional investors carry out buy-and-hold strategies in states of the world in which the benchmark outperforms. This behavior reduces market volatility and therefore also the tail risk exposure of the retail investor and the aggregate market in states of the world in which the benchmark outperforms. However, because institutional investors barely react to news in good states of the benchmark, they end up exposed to higher tail risk. Overall, with large benchmarking incentives, the retail investor and the aggregate market are exposed to high tail risk in states of the world in which the institutional investor is exposed to low tail risk, and viceversa. Benchmarking therefore introduces a channel through which the trading behavior of institutional investors can impact the tail risk exposure of the retail investor and the market.

We extend our analysis by considering the costs and benefits of benchmarking ex post and ex ante. We find that benchmarking does not affect the long term performance of our investors. All investors survive and become infinitely rich in the long run under the mild condition that at least one stock has positive expected dividend rate. We also find that the most patient investor dominates in the long run, regardless of the benchmarking incentives of our institutional investors. As a result, the exposure to tail risk borne by the retail investor is only short lived, and does not affect her ex post performance in the long run. Ex ante, however, benchmarking is disadvantageous to the retail investor, and beneficial only to the impatient institutional investor. We establish this fact by measuring the equivalent variation of consumption for economies with and without benchmarking incentives. This analysis reveals that the retail investor always needs to consume less in a world without benchmarking incentives to achieve the same utility as in a world with benchmarking incentives. The opposite result only holds for the impatient institutional investor. Consequently, benchmarking is welfare reducing for the retail investor and potentially also for the patient institutional investor ex ante, even though it does not affect the long term performance of our investors ex post.

The results of our theoretical study have important implications for the regulation of the asset management industry. Our findings indicate that benchmarking can create incentives for asset managers to alter portfolios in ways that do not fully take into account their effects on the tail risk exposure of individual investors and the aggregate market. Our results suggest that stronger benchmarking incentives generally make individual investors and potentially also patient fund managers worse off ex ante due to an increased tail risk exposure when compared to a world without benchmarking incentives. Still, tail risk does not materialize in the long run. These results indicate that it is imperative for regulators to formulate precise objectives for a potential regulation of the asset management industry. If the regulator is concerned about investor failure, then our results suggest that there may not be any scope for regulation. On the other hand, if the regulator is concerned about the tail risk exposure of retail investors, then regulating the compensation packages offered to fund managers may be one viable option. However, this option comes at the cost of making fund managers worse off ex ante.

Our results also indicate that the regulation of the asset management industry needs to be designed differently than the regulation of banks. We find that in an economy with benchmarking, the retail investor and the aggregate market are only exposed to low tail risk in states of the world in which institutional investors are exposed to high tail risk. Consequently, standard regulatory tools for banks that target their tail risk exposure, such as value-at-risk measurements and stress testing, may not be able to identify scenarios in which retail investors and the aggregate market are at risk of tail events. Because the higher tail risk exposure of the retail investor and the aggregate market in our model is induced by frequent and large trades by institutional investors, one potential tool for controlling for this effect may be transaction taxes. However, transaction taxes may also affect the retail investor in negative ways. Our results highlight that there is a need for further research that carefully analyzes the costs and the benefits of a potential regulation of the asset management industry.

This chapter is organized as follows. The remainder of this Section discusses the related literature. Section 2.2 introduces our model and discusses its main features. We solve for the general equilibrium of our model in Section 2.3. Section 2.4 contains our main results on the tail risk, survival, and welfare effects of benchmarking. Section 2.5 concludes. Appendix A.1 contains the proofs of our results, and one adapting the Fourier inverse methodology of Martin (2013) for our setting.

2.1.1 Related literature

The model we formulate in this paper is closely related to the reduced-form model of Basak and Pavlova (2013). Unlike Basak and Pavlova (2013), our institutional investors consume at all times, and their marginal utility of consumption is increasing in the dividend ratio attributed to the benchmark rather than in the dividend level. Our institutional investors are heterogenous, allowing us to also analyze the impact of time discount parameters on prices and risks in our market. We also allow for jumps in the dividend and stock price processes. By incorporating jumps, we can evaluate the impact of unanticipated negative news on portfolio allocations, stock prices, volatilities, and tail risk. This is a key ingredient of our analysis of the systemic effects of benchmarking. Despite these differences, asset prices in our model behave comparably to asset prices in Basak and Pavlova (2013), and we can capture similar features of managerial compensation.

Our results contribute to several strands of literature. First, we contribute to the literature on institutional investors and their impact on asset prices.[3] Consistent with the existing literature, our institutional investors face incentives that incite them to tilt their portfolios towards their benchmarks; see Brennan (1993), Gómez and Zapatero (2003), Kapur and Timmermann (2005), and Roll (1992), among others. This leads to an asset class effect that raises the price of the benchmark stock relative to a similar non-benchmark stock (Cuoco and Kaniel (2011), Basak and Pavlova

[3]Stracca (2006) provides an extensive survey of this literature.

(2013), and Hodor (2014)). It also induces an increase in the volatility of our stocks (Basak and Pavlova (2013) and Buffa et al. (2015)). However, we differ from and extend this literature in several ways. We do not consider the principal-agent problem underlying the decision of an individual investor to delegate the management of her wealth to a portfolio manager. We also neither derive nor model the optimal contract between a principal (investor) and an agent (manager) in this setting. Instead, we follow Basak and Pavlova (2013) and take a reduced-form approach to modeling the incentives faced by institutional investors. This choice introduces sufficient tractability to be able to solve the general equilibrium of our model is semi-closed form and to carry out important comparative statics. In contrast to the existing literature, we also allow for dividend jumps in our model. We find that even when just one asset has an underlying dividend process that jumps, jump risk may spread to stocks whose dividend processes do not jump. This is due to the fact that all investors update their portfolio holdings when a jump occurs, changing the demand for all stocks and affecting all stock prices. We also find that jumps in asset prices become less severe as institutional investors have stronger benchmarking incentives. This is primarily driven by the fact that institutional investors hold on to their stocks when benchmarking incentives are high.

Second, our results contribute to the literature on the costs and benefits of benchmarking. Admati and Pfleiderer (1997) analyze the use benchmarks in compensation packages for managers, and find that benchmarking may result in suboptimal risk sharing and portfolio choices. However, van Binsbergen et al. (2008) argue that these negative effects may be offset by the benefits of benchmarking in aligning diversification and investment horizon incentives. Das and Sundaram (2002) compare two types of performance-based compensation structures for managers, linear fulcrum fees and option-like incentive fees, and find that investor welfare tends to be higher under option-like fees. Carpenter (2000) and Cuoco and Kaniel (2011) find that option-like fees can push the manager to reduce the managed portfolio's volatility if the manager has HARA utility from terminal wealth. In contrast to this literature, we do not explicitly model the type of performance-based compensation that a manager derives from managing a portfolio. Still, our reduced-form analysis indicates that benchmarking may introduce systemic effects that the literature was unaware of. We show that when managers face strong benchmarking incentives, their trading behavior may increase the tail risk exposure of retail investors, which reduces their welfare.

Third, we contribute to the literature on heterogenous agents with additively separable utility functions. Gollier and Zeckhauser (2005) analyze models with heterogenous time preferences, and show that the shares of aggregate consumption attributed to each agent vary dynamically over time. Consistent with Gollier and Zeckhauser (2005), the consumption shares of our retail and institutional investors also change as time evolves. However, in our model this is not only driven by heterogenous time preferences but also by the fact that the consumption shares of our institutional investors depend on the relative performance of the benchmark, which fluctuates stochastically. Tran and Zeckhauser (2014) consider continuous-time, finite-horizon economies driven by Brownian motions in which agents have heterogeneous risk and time preferences, as well as heterogenous beliefs. These authors show that more risk tolerant investors take on more volatile consumption plans.[4] Compared to Tran and Zeckhauser (2014), we allow for jumps in our dividend processes but not for heterogenous beliefs. In our model, the risk tolerance of institutional investors is time-dynamic because it depends on the performance of the benchmark. This yields another channel driving the volatility of consumption plans of institutional investors. Yan (2008) analyzes infinite-horizon Brownian motion economies in which agents have heterogenous time preferences and beliefs, and shows that the investor with the lowest survival index

[4]Risk tolerance is understood as the marginal propensity to consume out of aggregate wealth.

dominates in the long run.[5] We extend the findings of Yan (2008) by showing that the investor with the lowest survival index also dominates, in the sense of consumption wealth ratio, in the long run in homogenous belief economies in which dividend processes may jump, and in which institutional investors face benchmarking incentives. Cvitanić et al. (2012) solve for the general equilibrium of an infinite-horizon Brownian motion economy in which agents have CRRA utility functions and heterogenous risk and time preferences, as well as heterogeneous beliefs. These authors find that the agents' optimal portfolios exhibit substantial heterogeneity in equilibrium. Extending these results, we find that benchmarking may undo some of the portfolio heterogeneity across heterogenous institutional investors. This is due to the fact that strong benchmarking incentives force institutional investors to strongly tilt their portfolios towards the benchmark, irrespective of their other preferences.

Finally, our results also contribute to the literature on financial stability. Most of the current debate has focused on the influence of banking on financial stability; see, e.g., the speech by former Federal Reserve Bank Chairman Ben Bernanke at the 2012 Federal Reserve Bank of Atlanta Financial Markets Conference.[6] There are a few papers that like us focus on the influence of asset management on financial stability. Bank for International Settlements (2003) discusses how benchmarking may affect financial markets, and concludes that benchmarking can negatively impact market efficiency and volatility at most over the short term. We carry out a formal general equilibrium analysis of the effects of benchmarking on financial markets, and find that benchmarking can induce a short-term rise in tail risk exposure, but it cannot affect the long-term performance of financial investors. Garbaravicius and Dierick (2005)

[5]The survival index of an investor is the sum of her time discount parameter, her optimism bias, and the risk-adjusted expected dividend growth rate. Our investors do not have any optimism bias because they are perfectly rational.

[6]A transcript of this speech is available at `http://www.federalreserve.gov/newsevents/speech/bernanke20120409a.pdf`.

analyze the Lipper TASS hedge fund data base, and identify three channels through which hedge funds may pose threats to financial stability: *(i)* through hedge fund failures, *(ii)* through banks exposures to hedge funds, and *(iii)* through the impact of hedge funds' trading behavior on financial markets. Our theoretical analysis shows that benchmarking may enable the latter channel. Garbaravicius and Dierick (2005) also document empirically that hedge funds tend to take on leverage, consistent with the behavior of our institutional investors. Daníelsson et al. (2005) survey the theoretical and empirical literature available at the time, and argue that hedge funds can have systemic effects on financial markets primarily because of the market impact of large hedge fund failures. Extending these results, we show that the trading behavior of hedge fund managers may also have systemic implications because it can raise the tail risk exposure of retail investors.

Daníelsson and Shin (2003) coined the term "endogenous risk" as additional financial risk that arises from the interactions and trading behavior of agents in a financial system. Our results show that benchmarking by institutional investors may give rise to endogenous risk. This occurs because benchmarking forces institutional investors to react strongly to news about their benchmarks in states of the world in which the benchmark underperforms. Our endogenous risk channel is similar to the one posited by Daníelsson and Zigrand (2008) and Daníelsson et al. (2010). In these papers, endogenous risk arises because financial institutions face value-at-risk contraints, reducing their risk appetite whenever the value-at-risk constraint becomes binding. Similarly as in Daníelsson et al. (2013), endogenous risk in our model manifests itself systemically in the form of increased tail risk exposures.

2.2 Model

We analyze a pure-exchange economy populated by heterogenous agents with CRRA utilities. There are three investors of two different types: a *retail* investor and two *institutional* investors. Our economy is comprised of two risky assets and one safe asset. Time is measured continuously and runs from zero to infinity. All dividends and prices are modeled by stochastic processes that are measurable with respect to a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and adapted to the complete information filtration $(\mathcal{F}_t)_{t\geq 0}$ that represents the flow of information over time.

2.2.1 Assets

There are two risky assets with prices $S_1 = (S_{1,t})_{t\geq 0}$ and $S_2 = (S_{2,t})_{t\geq 0}$. These assets are in net positive supply. We normalize the supply of each asset to one without loss of generality. The risky assets produce streams of dividends that satisfy the following stochastic differential equations:

$$\frac{\mathrm{d}D_{1,t}}{D_{1,t-}} = \mu_1 \mathrm{d}t + \sigma_1 \mathrm{d}Z_t + J_1 \mathrm{d}N_t$$
$$\frac{\mathrm{d}D_{2,t}}{D_{2,t-}} = \mu_2 \mathrm{d}t + \sigma_2 dZ_t + J_2 \mathrm{d}N_t$$

Here, $Z = (Z_t)_{t\geq 0}$ is a standard Brownian motion and $N = (N_t)_{t\geq 0}$ is a Poisson process with constant arrival rate $\lambda > 0$ that is independent of Z. For simplicity and to ensure market completeness, we restrict to the case of two independent sources of risk. However, generalizations to additional sources of risk are possible. The parameters governing the evolution of dividends are assumed to be positive scalars with the exception of J_1 and J_2, which are constants strictly larger than -1.

There is also a safe asset with price $B = (B_t)_{t\geq 0}$. This asset has net zero supply. The safe asset allows investor to borrow and lend in the money markets. Borrowing and lending occurs at the interest rate r_t, which is stochastic and satisfies

$$\mathrm{d}B_t = r_t B_t \mathrm{d}t.$$

2.2.2 Investors

There are three investors in our market: a *retail* investor and two *institutional* investors. Let the superscript "R" denote all variables corresponding to the retail investor, and the superscripts "A" and "B" denote all variables corresponding to the institutional investors A and B, respectively.

Agents have non-negative initial endowments W_0^R, W_0^A, and W_0^B that correspond to fractions of the total wealth of the economy at the time 0:

$$W_0 = W_0^R + W_0^A + W_0^B.$$

Institutional investor $j \in \{A, B\}$ owns a fraction α^j of the total assets in positive supply of the economy, while the retail investor R owns the remainder $\alpha^R = 1 - \alpha^A - \alpha^B$ of total wealth. At time 0, agents maximize their expected lifetime discounted utility stream subject to their budget constraints, and commit to fixed consumption and portfolio plans. We refer toAppendix A.1 for details.

We list some notation. Let W_t^j denote the wealth of investor $j \in \{R, A, B\}$ at time $t > 0$. Further, let c_t^j denote the amount of wealth consumed by investor $j \in \{R, A, B\}$ at time $t > 0$. Finally, let $\pi_t^j = (\pi_{1,t}^j, \pi_{2,t}^j, \pi_{l,t}^j)$ denote the portfolio of investor $j \in \{R, A, B\}$ at time $t > 0$. This portfolio consists of a fraction $\pi_{1,t}^j$ of wealth invested in asset 1, a fraction $\pi_{2,t}^j$ of wealth invested in asset 2, and a fraction $\pi_{l,t}^j$ of wealth offered as lending on the money markets. We assume that any wealth not consumed is invested in either the stock or the money markets. As a result, we have $\pi_{1,t}^j + \pi_{2,t}^j + \pi_{l,t}^j = 1$.

We assume that agents have log preferences with respect to consumption as this yields a simple framework in which our results can be easily illustrated. However, generalizations to other CRRA formulations are possible. Further, we illustrate our results in a setting with only one retail and two institutional investors for simplicity. Generalization to settings with more retail and institutional investors are also possible.

Retail investor

Investor R derives log-utility from intermediate consumption. She chooses a consumption plan $(c_t^R)_{t\geq 0}$ and a portfolio plan $(\pi_t^R)_{t\geq 0}$ that maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-\rho_R t}\log c_t^R dt\right] \tag{2.1}$$

subject to the budget constraint that the present value of her consumption plan does not exceed her initial wealth; see Appendix A.1 for details. The retail investor discounts time exponentially with rate $\rho_R > 0$.

Institutional investors

It is well-known that the performance of an institutional investor is measured in relation to a benchmark. As a result, institutional investors have incentives to post high returns in scenarios in which the underlying benchmark is posting high returns. In order to capture this unique feature of an institutional investor's incentives, we assume that investors A and B benchmark against stock 2, and that their marginal utilities of consumption are increasing in the performance of stock 2.

More concretely, we assume that investor $j \in \{A, B\}$ chooses a consumption plan $(c_t^j)_{t\geq 0}$ and a portfolio plan $(\pi_t^j)_{t\geq 0}$ as to maximize

$$\mathbb{E}\left[\int_0^\infty e^{-\rho_j t}(1 + I_j s_t)\log c_t^j \mathrm{d}t\right] \tag{2.2}$$

subject to the budget constraint that the present value of the consumption plan does not exceed the corresponding initial wealth. Here, $I_j \geq 0$ is a benchmark importance parameter, and

$$s_t = \frac{D_{2,t}}{D_{1,t} + D_{2,t}}$$

is the ratio of dividends attributed to asset 2. Our institutional investors discount time exponentially with rates $\rho_A > 0$ and $\rho_B > 0$, respectively.

2.2.3 Discussion

Our model of institutional investors can be viewed as a reduced-form model of benchmarking portfolio managers. As we show in Section 2.3, large values of I_j incite institutional investor j to strongly tilt her portfolio towards the benchmark stock. This behavior is consistent with the behavior of portfolio managers who derive compensation from the excess performance of the managed portfolio above a benchmark; see Basak and Pavlova (2013), Brennan (1993), Buffa et al. (2015), Cuoco and Kaniel (2011), Gómez and Zapatero (2003), and Roll (1992), among others. Consequently, we can interpret our institutional investors as portfolio managers. We also show in Section 2.3 that the institutional investors' consumption plans in equilibrium reflect several key features of compensation for managers whose performances are evaluated relative to a benchmark. Therefore, consumption of our institutional investors can be interpreted as managerial compensation, and I_A and I_B as measures of the benchmarking incentives derived from managers' compensation packages.

Our model is closely related to the model of Basak and Pavlova (2013), who consider a pure-exchange economy with one retail and one institutional investor. Stock prices in Basak and Pavlova (2013) do not jump. The agents in the model of Basak and Pavlova (2013) derive utility from terminal wealth at the finite maturity $T < \infty$; i.e., there is no intermediate consumption. As a result, the agents in Basak and Pavlova (2013) have stronger incentives to consume as time-to-maturity decreases. In contrast, we are interested only in changes in the portfolio-consumption policies of different agents generated by their particular attitudes towards risk. By choosing an infinite time horizon we eliminate incentives to consume stronger as time-to-maturity decreases. In addition, by allowing for intermediate consumption we can study how benchmarking affects borrowing and lending in the money markets. By introducing jumps we can analyze how large shocks in one asset spread to other assets, and how agents hedge against jump risk in a dynamic setting with benchmarking. Finally, we allow for two heterogeneous institutional investors. As a result, we can study the differences and similarities between the portfolios of our institutional investors, and their impact prices and risks in financial markets. The answers to these questions have important implications for policy making. Despite the differences between our model and the one of Basak and Pavlova (2013), we can capture similar features of managerial compensation. To be precise, our model of institutional investors captures properties (i) and (iii) of Proposition D1 of Appendix D of Basak and Pavlova (2013), as does the model of Basak and Pavlova (2013).

There are three key benefits of our model specification. First, our model can be seen as a simple but powerful generalization of a Markowitz portfolio selection model with retail and institutional investors that allows us to answer important questions about the systemic effects of benchmarking. Second, our model captures several features of portfolio managers' incentives that have been established by the extant literature. Third, we can solve for the general equilibrium in our model in semi-closed form up to numerical integration (see Section 2.3). This allows us to perform important comparative statics. Still, a more realistic objective function for our institutional investors would be

$$\mathbb{E}\left[\int_0^\infty e^{-\rho_j t}\left(1 + I_j \frac{S_{2,t}}{S_{1,t} + S_{2,t}}\right) \log c_t^j \mathrm{d}t\right]. \tag{2.3}$$

Because the prices $S_{1,t}$ and $S_{2,t}$ are determined endogenously in equilibrium, the above formulation of the institutional investors' incentives is intractable. Given the Markovian structure of our model, we show in Section 2.3 that there is a one-to-one mapping between prices and dividends of the form $S_{n,t} = g_n(D_{1,t}, D_{2,t})$ for $n \in \{1, 2\}$. As a result, our formulation of the institutional investors' objective functions may be seen as a first-order approximation of (2.3).

2.3 Equilibrium

An equilibrium at time 0 in our model consists of:

- Consumption plans $(c_t^R)_{t\geq 0}$, $(c_t^A)_{t\geq 0}$, and $(c_t^B)_{t\geq 0}$ and portfolio plans $(\pi_t^R)_{t\geq 0}$, $(\pi_t^A)_{t\geq 0}$, $(\pi_t^B)_{t\geq 0}$ for the retail investor and each institutional investor that maximize (2.1) and (2.2) for $j \in \{A, B\}$ subject to each investor's budget constraints, and

- Prices $(S_{1,t})_{t\geq 0}$ and $(S_{2,t})_{t\geq 0}$, as well as an interest rate process $(r_t)_{t\geq 0}$ such that markets are cleared:

$$\begin{aligned}
0 &= \pi_{l,t}^R W_t^R + \pi_{l,t}^A W_t^A + \pi_{l,t}^B W_t^B, \\
S_{1,t} &= \pi_{1,t}^R W_t^R + \pi_{1,t}^A W_t^A + \pi_{1,t}^B W_t^B, \\
S_{2,t} &= \pi_{2,t}^R W_t^R + \pi_{2,t}^A W_t^A + \pi_{2,t}^B W_t^B.
\end{aligned}$$

An important property of our equilibrium is that we can solve for all relevant quantities in semi-closed form up to certain integrals which need to be computed numerically. As a result, our model formulation ensures tractability and allows us to precisely pin down the different mechanisms driving prices and risks in our market. This property also allows us to carry out comparative statics relative to the different model parameters. We refer to Appendix A.1 for a precise characterization of the equilibrium.

Proofs of the results are given in Appendix A.1.1.

Before proceeding, we introduce some notation that will be used throughout this section. The stock prices $(S_{1,t})_{t\geq 0}$ and $(S_{2,t})_{t\geq 0}$ are determined endogenously in equilibrium. Let $\sigma_{n,t}$ denote the instantaneous diffusive volatility of stock $n \in \{1,2\}$ at time $t > 0$ when no jump occurs. That is, if no jump occurs at time $t > 0$, then the conditional time-t variance of the n-th stock return over the small time period $\Delta \approx 0$ is

$$\mathrm{Var}_t\left(\log \frac{S_{n,t+\Delta}}{S_{n,t}}\right) \approx \sigma_{n,t}^2 \Delta.$$

In addition, let $J_{n,t}$ denote the jump size of stock $n \in \{1,2\}$ if a jump occurs at time $t > 0$; that is,

$$\frac{S_{n,t} - S_{n,t-\Delta}}{S_{n,t-\Delta}} \approx J_{n,t-\Delta}$$

for $\Delta \approx 0$ if a jump occurs at time t. Define the exposure matrix Σ_t that is composed of the stocks' exposure to each source of risk as

$$\Sigma_t = \begin{bmatrix} \sigma_{1,t} & \sigma_{2,t} \\ J_{1,t} & J_{2,t} \end{bmatrix}.$$

The exposure matrix Σ_t plays a key role in the consumption-portfolio plans in equilibrium. Finally, define the following moments of the benchmark dividend ratio for $j \in \{R, A, B\}$ and $k \in \{1,2,3\}$:

$$M_{k,t}^j = \mathbb{E}_t\left[\int_t^\infty e^{-\rho_j(v-t)} s_v^k \mathrm{d}v\right],$$

$$\Delta M_{k,t}^j = \mathbb{E}_t\left[\int_t^\infty e^{-\rho_j(v-t)} \left(s_v - s_{v-}\right)^k |_{\text{Jump at time } v}\ \mathrm{d}v\right].$$

$M_{k,t}^j$ measures the long-term conditional k-th moment of s_t given all information at time t when discounted with the discount rate of investor j, while $\Delta M_{k,t}^j$ gives the long-term conditional k-th moment of the jump magnitude of s_t when a jump occurs given all information at time t after discounting with the discount rate of investor j. We derive closed-form expressions for these moments in Appendix A.1 using the Fourier inverse methodology of Martin (2013). These expressions can be easily computed via numerical integration.

We fix the model parameters as in Table 3.2 unless specified otherwise. Our parameter choice is inspired by the parameter estimates of Backus et al. (2011) for U.S. equities derived from a Merton model of stock returns. Unlike Backus et al. (2011), though, we assume that jumps are less frequent but more severe in order to illustrate the impact of severe negative shocks on asset prices, risks, and portfolio allocations. We choose our model parameters as to match the unconditional means and variances of the growth rates of dividends 1 and 2, while allowing dividend 2 but not dividend 1 to jump. In other words, we choose

$$J_1 = 0, \quad \mu_1 - \frac{\sigma_1^2}{2} = \mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2, \quad \text{and} \quad \sigma_1^2 = \sigma_2^2 + \lambda J_2^2.$$

By allowing jumps in $D_{2,t}$ but not in $D_{1,t}$ we can study how negative benchmark shocks impact non-benchmark assets as well as portfolio allocations. By equalizing the means and variances of the dividend growth rates we make a mean-variance optimizing investor indifferent between holding stock 1 and stock 2. Therefore, unequal demands for benchmark and non-benchmark assets under our parametrization can be entirely attributed to the benchmarking incentives of institutional investors and their impact on asset prices and risks. Our parameter choice allows us to focus exclusively on the impact of benchmarking on asset prices, risks, and portfolio allocations without having to worry about the influence of idiosyncratic risks. Still, we have experimented with other parameter choices and find that the effects of benchmarking are significantly amplified under a more realistic parametrization of our model.

[Table 1 about here.]

Given the structure of financial markets with two risky assets and two independent sources of uncertainty, we can show that our model has a unique state price density to value assets in equilibrium, and our market is complete. Let $\xi = (\xi_t)_{t\geq 0}$ denote the unique state price density process. Define $(\theta_t)_{t\geq 0}$ as the process of market prices of diffusion risk, and $(\psi_t)_{t\geq 0}$ as the process of market prices of jump risk. These processes represent the compensation that agents request for bearing volatility and jump risk, respectively. The market prices of risks are also determined endogenously in equilibrium.

2.3.1 State price density

We begin by characterizing the state price density in our market.

Proposition 2.3.1. *Define*

$$\phi_j = \left(\frac{1}{\rho_j} + I_j M_{1,0}^j\right)^{-1} > 0.$$

The unique state price density of the economy is

$$\xi_t = \frac{Q_t/Q_0}{D_t/D_0}, \tag{2.4}$$

where

$$\begin{aligned} D_t &= D_{1,t} + D_{2,t}, \\ Q_t &= \alpha^R \rho_R e^{-\rho_R t} + \alpha^A \phi_A e^{-\rho_A t}(1 + I_A s_t) + \alpha^B \phi_B e^{-\rho_B t}(1 + I_B s_t). \end{aligned}$$

The state price density has several interesting features. If there are no benchmarking incentives ($I_A = I_B = 0$) and all agents have the same time discount parameter, we obtain the standard state price density that is inversely proportional to aggregate dividends. When institutional investors have positive benchmarking incentives ($I_A > 0$ or $I_B > 0$), the pricing kernel is sensitive to the performance of the benchmark. In such cases, the benchmark influences the pricing kernel in two ways. The first influence comes through Q_t in the numerator of (2.4), which encompasses the pricing contribution of each agent in the market and is an increasing function of benchmark dividend ratio s_t. This channel captures the index effect of Basak and Pavlova (2013). In states of the world in which the benchmark is outperforming, institutional investors are exposed to the risk of underperforming relative to the benchmark. These states of the world carry high marginal utility for institutional investors, and this is reflected in the pricing kernel. This effect is illustrated in Figure 3·2(a), which shows that the pricing kernel is an increasing function of s_t.

[Figure 2 about here.]

A second channel through which benchmarking affects the pricing kernel is through aggregate dividends in the denominator of (2.4). This channel reflects the fact that states of the world in which the benchmark is outperforming *relative* to alternative investment opportunities are more risky for institutional investors. In such states it is hard for institutional investors to match the performance of the benchmark. To understand this effect, consider the opposite scenario: If the benchmark level is high whenever the alternative stock level is also high, then it is easy for institutional investors to post high returns when the benchmark is posting high returns. Thus, states of the world in which both the benchmark and the alternative investment opportunity are booming are low risk states for our institutional investors. Such states carry low marginal utility for institutional investors, and this is also reflected in the pricing kernel. This is a *relative index effect* through which benchmarking affects prices, and it complements the absolute index effect of Basak and Pavlova (2013). Figure 3·2(b) illustrates the relative index effect. It shows that the pricing kernel decreases if the benchmark dividend level increases and the non-benchmark dividend level increases by the same amount.

2.3.2 Prices of Risk

The state price density of Proposition 2.3.1 yields the prices of risk as well as the interest rate in the market.

Proposition 2.3.2. *The drift, volatility, and jump size functions of D_t and Q_t are:*

$$\begin{aligned}
\mu_{d,t} &= (1-s_t)\mu_1 + s_t\mu_2, \quad \sigma_{d,t} = (1-s_t)\sigma_1 + s_t\sigma_2, \quad J_{d,t} = (1-s_t)J_1 + s_t J_2, \\
\mu_{q,t} &= -\rho_R \alpha^R e^{-\rho_R t} - \alpha^A \phi_A \rho_A e^{-\rho_A t}(1 + I_A s_t) - \alpha^B \phi_B \rho_B e^{-\rho_B t}(1 + I_B s_t) \\
&\quad + (I_A \alpha^A \phi_A e^{-\rho_A t} + I_B \alpha^B \phi_B e^{-\rho_B t}) s_t (1-s_t) \big[\mu_2 - \mu_1 \\
&\quad + (\sigma_1 - \sigma_2)(\sigma_1(1-s_t) + \sigma_2 s_t)\big], \\
\sigma_{q,t} &= \frac{I_A \alpha^A \phi_A e^{-\rho_A t} + I_B \alpha^B \phi_B e^{-\rho_B t}}{Q_t}(\sigma_2 - \sigma_1) s_t (1-s_t), \\
J_{q,t} &= \frac{I_A \alpha^A \phi_A e^{-\rho_A t} + I_B \alpha^B \phi_B e^{-\rho_B t}}{Q_t} \frac{s_t(1-s_t)(J_2 - J_1)}{1 + (1-s_t)J_1 + s_t J_2}.
\end{aligned}$$

The market prices of volatility and jump risk are given by:

$$\theta_t = \sigma_{d,t} - \sigma_{q,t}, \tag{2.5}$$

$$\psi_t = \frac{1 + J_{q,t}}{1 + J_{d,t}}. \tag{2.6}$$

The interest rate is

$$r_t = \mu_{d,t} - \mu_{q,t} - \sigma_{d,t}^2 + \sigma_{d,t}\sigma_{q,t} + \lambda(1 - \psi_t). \tag{2.7}$$

Our prices of risk are not constant despite the fact that our dividend processes have constant coefficients. There are two effects driving the stochasticity of market prices of risk. First, the dividend ratio s_t is a state variable as in Cochrane et al. (2008) and Martin (2013). Thus, fluctuations of s_t are mapped onto fluctuations of risk prices. When the dividend ratio changes, agents change their portfolio holdings, and this is reflected in fluctuating market prices of risks. Second, fluctuations of Q_t are also mapped onto fluctuations of market prices of risk when institutional investors have benchmarking incentives ($I_A > 0$ or $I_B > 0$). In such scenarios, changes in the performance of the benchmark lead to changes in the institutional investors' portfolios, yielding stronger fluctuations in market prices.

[Figure 3 about here.]

Figure 3·3 illustrates the behavior of market prices of volatility and jump risk as the benchmark dividend ratio and the benchmark importance parameter vary. From the expressions in Proposition 2.3.1 we see that the market price of volatility risk is increasing in the benchmark importance parameters I_A and I_B if $\sigma_2 < \sigma_1$ (as in our parametrization in Table 3.2), and decreasing in I_A and I_B otherwise. Similarly, the market price of jump risk is decreasing in I_A and I_B if $J_2 < J_1$ (as in our parametrization in Table 3.2), and increasing otherwise. The stronger the benchmarking incentives are, the higher demand the institutional investors will have for the benchmark asset, as we show below. Therefore, institutional investors request less compensation to assume the risks associated with the benchmark asset. Consequently, compensation for volatility risk decreases if the benchmark dividend has higher volatility than the non-benchmark dividend, while compensation for jump risk decreases if the benchmark dividend has more severe jumps than the non-benchmark dividend.

Similar effects also lead to a time-varying interest rate in our model. Figure 3·4 plots the interest rate r_t as a function of the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. When the benchmark dividend ratio is low, the interest rate decreases as institutional investors' incentives to benchmark increase. On the other hand, the interest rate increases as institutional investors' incentives to benchmark increase when the benchmark dividend ratio is high.[7] As we will demonstrate below, this result is due to the institutional investors' benchmark hedging needs. Institutional investors reduce their leverage when the benchmark underperforms, resulting in a lower interest rate. This phenomenon is exacerbated when

[7] We find similar interest rate dynamics for alternative parametrizations of our model.

benchmarking incentives are large.

[Figure 4 about here.]

2.3.3 Asset prices

Next, we pin down the stock prices $S_{1,t}$ and $S_{2,t}$, and their volatilities and jump sizes.

Proposition 2.3.3. *The price of the benchmark stock is*

$$S_{2,t} = \frac{D_t}{Q_t}\left(\alpha^R \rho_R e^{-\rho_R t} M^R_{1,t} + \alpha^A \phi_A e^{-\rho_A t}(M^A_{1,t} + I_A M^A_{2,t}) + \alpha^B \phi_B e^{-\rho_B t}(M^B_{1,t} + I_B M^B_{2,t})\right),$$

and the price of the non-benchmark stock is

$$S_{1,t} = \frac{D_t}{Q_t}\left(\alpha^R e^{-\rho_R t} + \alpha^A \phi_A e^{-\rho_A t}(\rho_A^{-1} + I_A M^A_{1,t}) + \alpha^B \phi_B e^{-\rho_B t}(\rho_B^{-1} + I_B M^B_{1,t})\right) - S_{2,t}.$$

The stock volatility functions are

$$\begin{aligned}
\sigma_{2,t} =& \theta_t + \frac{D_t(\sigma_2 - \sigma_1)}{Q_t S_{2,t}}\Big[\alpha^R \rho_R e^{-\rho_R t}(M^R_{1,t} - M^R_{2,t}) + \alpha^A \phi_A e^{-\rho_A t}(M^A_{1,t} - M^A_{2,t}) \\
&+ \alpha^B \phi_B e^{-\rho_B t}(M^B_{1,t} - M^B_{2,t}) + 2 I_A \alpha^A \phi_A e^{-\rho_A t}(M^A_{2,t} - M^A_{3,t}) \\
&+ 2 I_B \alpha^B \phi_B e^{-\rho_B t}(M^B_{2,t} - M^B_{3,t})\Big], \\
\sigma_{1,t} =& \theta_t\left(1 + \frac{S_{2,t}}{S_{1,t}}\right) + \frac{D_t(\sigma_2 - \sigma_1)}{Q_t S_{1,t}}\Big[I_A \alpha^A \phi_A e^{-\rho_A t}(M^A_{1,t} - M^A_{2,t}) \\
&+ I_B \alpha^B \phi_B e^{-\rho_B t}(M^{2B}_{1,t} - M^{2B}_{2,t})\Big] - \sigma_{2,t}\frac{S_{2,t}}{S_{1,t}}.
\end{aligned}$$

The jump sizes of the stocks are

$$\begin{aligned}
J_{2,t} = \psi_t^{-1} - 1 + \frac{D_t}{\psi_t Q_t S_{2,t}}\Big(&\alpha^R \rho_R e^{-\rho_R t}\Delta M^R_{1,t} + \alpha^A \phi_A e^{-\rho_A t}\Delta M^A_{1,t} + \alpha^B \phi_B e^{-\rho_B t}\Delta M^B_{1,t} \\
&+ I_A \alpha^A \phi_A e^{-\rho_A t}\Delta M^A_{2,t} + I_B \alpha^B \phi_B e^{-\rho_B t}\Delta M^B_{2,t}\Big), \\
J_{1,t} = \psi_t^{-1}\Big(1 + \frac{S_{2,t}}{S_{1,t}} &+ \frac{1}{S_{1,t}}\frac{D_t}{Q_t}(I_A \alpha^A \phi_A e^{-\rho_A t}\Delta M^A_{1,t} + I_B \alpha^B \phi_B e^{-\rho_B t}\Delta M^B_{1,t})\Big) \\
&- 1 - \frac{S_{2,t}}{S_{1,t}}(1 + J_{2t}).
\end{aligned}$$

Figure 3·5 displays the stock prices as functions of the benchmark dividend ratio and the benchmark importance parameters. Naturally, the benchmark stock is more expensive than the non-benchmark stock whenever the benchmark is outperforming; i.e., whenever s_t is large. On the other hand, the benchmark stock is cheaper than the non-benchmark stock when s_t is low. We also see that the price $S_{1,t}$ of the non-benchmark stock is highly sensitive to the benchmark importance parameters I_A and I_B, while the price $S_{2,t}$ of the benchmark stock is not very sensitive to the benchmark importance parameters. These price effects are driven by the institutional investors' needs to hedge against underperforming relative to the benchmark as we will demonstrate in the next sections.

[Figure 5 about here.]

Figure 3·6 displays stock volatilities and jumps as functions of the benchmark dividend ratio and the benchmark importance parameters. The volatilities of both stocks are increasing functions of the benchmark importance parameter. Jumps sizes are also increasing functions of the benchmark importance parameters so that jumps become less severe as institutional investors have stronger incentives to benchmark. In addition, we see that stock volatilities are highest when the benchmark dividend ratio is low to intermediate, while jumps are most severe when the benchmark dividend ratio is extremely high or extremely low. Again, these effects are driven by the institutional investors' needs to hedge against underperformance relative to the benchmark, as we will show below. Figure 3·6 also illustrates the feedback effect from benchmark shocks to the non-benchmark stock: even though the non-benchmark dividend process does not jump, the stock price associated with the non-benchmark dividend stream does jump. This is due to the fact that after a jump occurs, all investors update their portfolios and alter the demand for all stocks, which affects all stock prices.

[Figure 6 about here.]

2.3.4 Optimal portfolio plans

We are now in a position to describe the optimal portfolio plans of institutional and retail investors. An important assumption is that the exposure matrix Σ_t is of full rank. This assumption is closely related to market completeness as indicated by Bardhan and Chao (1996) and Hugonnier et al. (2012). Sufficient conditions ensuring this assumption are ad-hoc and technical. However, our simulations show that this assumption is satisfied is most scenarios.

Proposition 2.3.4. *Suppose that Σ_t has full rank for all $t \geq 0$. For the retail investor, the optimal portfolio plan is given by*

$$\begin{pmatrix} \pi_{1,t}^R \\ \pi_{2,t}^R \end{pmatrix} = \Sigma_t^{-1} \begin{pmatrix} \theta_t \\ \psi_t^{-1} - 1 \end{pmatrix}.$$

For institutional investor $j \in \{A, B\}$, the optimal portfolio plan is given by

$$\begin{pmatrix} \pi_{1,t}^j \\ \pi_{2,t}^j \end{pmatrix} = \underbrace{\Sigma_t^{-1} \begin{pmatrix} \theta_t \\ \psi_t^{-1} - 1 \end{pmatrix}}_{\text{mean-variance}} + \underbrace{\Sigma_t^{-1} \begin{pmatrix} (\sigma_2 - \sigma_1)(M_{1,t}^j - M_{2,t}^j) \\ \Delta M_{1,t}^j \psi_t^{-1} \end{pmatrix} \frac{I_j}{\rho_j^{-1} + I_j M_{1,t}^j}}_{\text{benchmark hedge}}.$$

Finally, we have

$$\pi_{l,t}^j = 1 - \pi_{1,t}^j - \pi_{2,t}^j,$$

for $j \in \{R, A, B\}$.

At any point of time, the portfolio held by the retail investor coincides with the standard mean-variance portfolio given the log-utility formulation. The portfolio of an institutional investors at time t is decomposed into two components: a component that accounts for the standard mean-variance portfolio (the first summand) and arises due to the institutional investors' needs to hedge against volatility and jump risk, and a component that hedges against fluctuations of the benchmark dividend ratio s_t (the second summand). Our results indicate that institutional investors have hedging motives beyond the mean-variance motives of Duffie and Richardson (1991).

The additional hedging affects the demand for all assets. When an institutional investor has small incentives to benchmark ($I_j \approx 0$), her optimal portfolio is close to the optimal portfolio of the retail investor. Large incentives to benchmark shift the institutional investors' portfolios away from the retail investor's portfolio, and closer towards the benchmark stock. Figure 3·7 illustrates the portfolio plans of our retail and institutional investors, and showcases that institutional investors tilt their portfolios towards the benchmark stock as their incentives to benchmark grow large. It is noticeable that institutional investors take on leverage to invest in the benchmark stock as the benchmarking incentives grow large.

[Figure 7 about here.]

How do the portfolios of our institutional investors compare to each other? Our institutional investors are heterogeneous and differ from each other through their benchmark importance parameters and their time preferences. Thus, different institutional investors hold different portfolios. Note that, as $I_j \to \infty$, the portfolio of institutional investor j converges to

$$\begin{pmatrix} \pi^j_{1,t} \\ \pi^j_{2,t} \end{pmatrix} = \Sigma_t^{-1} \begin{pmatrix} \theta_t \\ \psi_t^{-1} - 1 \end{pmatrix} + \Sigma_t^{-1} \begin{pmatrix} (\sigma_2 - \sigma_1)(1 - M^j_{2,t}/M^j_{1,t}) \\ \psi_t^{-1} \Delta M^j_{1,t}/M^j_{1,t} \end{pmatrix}$$

given that $I_j/(\rho_A^{-1} + I_j M^j_{1,t}) \to 1/M^j_{1,t}$. The formulation of the moments of s_t in Appendix A.1 implies that $M^j_{2,t}/M^j_{1,t} \approx s_t$ and $\Delta M^j_{1,t}/M^j_{1,t} \approx \Delta s_t$ conditional on a jump at time t in the realistic setting $\rho_j \approx 0$. Therefore, Proposition 2.3.4 tells us that the differences between the portfolios of our institutional investors are only very small whenever the benchmark importance parameters are large.

It is well understood that benchmarking incentives lead to deviations from the standard mean-variance portfolio allocation (see Basak et al. (2007), Brennan (1993), Gómez and Zapatero (2003), Jorion (2003), and Roll (1992), among many others).

Our findings on the institutional investors' portfolios complement the existing literature by allowing asset prices to jump, by allowing agents to consume at intermediate times, and by allowing for heterogenous institutional investors. Basak and Pavlova (2013) find that institutional investors that benchmark against the absolute level of a benchmark and consume only at terminal time have additional demand for stocks that are highly correlated with the benchmark. Similarly, our institutional investors have additional demand for the benchmark stock. Nevertheless, the additional demand for the benchmark stock depends on the risk preferences of each institutional investor, as well as on the risk profiles of each stock. Consistent with risk aversion, Figure 3·8 shows that the institutional investors' demand for the benchmark stock decreases and their demand for the non-benchmark stock increases as benchmark dividend jumps become more severe. Figure 3·8 also indicates that, against the standard mean-variance intuition, increases in the volatility of a dividend stream raise the institutional investors' demand for the corresponding risky security. Both of these effects are driven by the need of institutional investors to hedge against fluctuations of the benchmark.

[Figure 8 about here.]

2.3.5 Consumption plans

The following proposition characterizes the optimal consumption policies of our agents.

Proposition 2.3.5. *The optimal consumption plan for institutional investor* $j \in \{A, B\}$ *is*

$$c_t^j = \rho_j \frac{1 + I_j s_t}{1 + I_j \rho_j M_{1,t}^j} W_t^j = \frac{\alpha^j W_0 \phi_j e^{-\rho_j t}(1 + I_j s_t)}{\xi_t}.$$

For the retail investor, the optimal consumption plan is

$$c_t^R = \rho_R W_t^R = \frac{\alpha^R W_0 \rho_R e^{-\rho_R t}}{\xi_t}.$$

The consumptions plans of our investors are increasing functions of wealth. As Figure 3·9(a) indicates, consumption is also nonlinearly increasing in the benchmark dividend ratio s_t, and linearly increasing in the benchmark dividend level $D_{2,t}$ when s_t is kept fixed. Consistent with our log-utility formulation, the retail investor consumes a constant fraction of her wealth at any point of time. Institutional investors, on the other hand, consume a time-varying fraction of their wealths. The wealth-consumption ratios of institutional investors are nonlinear functions of the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. Figure 3·9(b) displays the wealth-consumption ratios of institutional investors as functions of s_t, I_A, and I_B. We see that the wealth-consumption ratio of an institutional investor is increasing in her benchmark importance parameter. An institutional investor consumes the largest fraction of her wealth when the benchmark slightly overperforms and her benchmark importance parameter is large.

[Figure 9 about here.]

Consumption by our institutional investors can be interpreted as compensation for benchmarking portfolio managers. The literature has extensively studied optimal contracts for portfolio managers in principal-agent settings in which an investor (principal) delegates the management of her portfolio to a manager (agent) who must exert effort to become informed about the distributions of returns (see Stracca (2006) for a literature survey). There is consensus that optimal contracts specify compensation packages that are nonlinear and increasing in the performance of the managed portfolio over and above of a predetermined benchmark (see, e.g., Li and Tiwari (2009), and Stoughton (1993)). The compensation plans of our institutional investors reflect this feature. In Figure 3·10 we plot the consumption of institutional investor A in excess of what is consumed by the retail investor. In order to make the consumption of both investors comparable, we temporarily neglect institutional investor B and set $\alpha_B = 0$,

$\alpha_A = \alpha_R = 0.5$, and $\rho_A = \rho_R = 0.02$ for this figure. We see that the institutional investor consumes more than the retail investor if $I_A > 0$. In particular, the institutional excess consumption is increasing in the benchmark importance parameter. We also see that the institutional excess consumption is nonlinearly increasing in the benchmark dividend ratio s_t, and linearly increasing in the benchmark dividend level $D_{2,t}$ when s_t is kept fixed. We know from Figure 3·7 that institutional investors allocate large fractions of their wealths on the benchmark when the benchmark importance parameters are large. In such cases, the performance of an institutional investor is highly linked to the performance of the benchmark stock. As a result, institutional investors perform well when the benchmark performs well, and in such scenarios they also consume more than a comparable retail investor. We conclude that institutional investors in our model enjoy a consumption bonus when their portfolios perform well, which is consistent with the optimal contacts for benchmarking portfolio managers established by the literature.

[Figure 10 about here.]

2.4 Systemic Effects

Based on the general equilibrium of Section 2.3, we now study the systemic implications of benchmarking incentives. We proceed as follows. First, we measure the impact of benchmarking on tail risk for investors in our market. To this extent we measure *value-at-risk* of the portfolio returns of each institutional and retail investor, as well as of the aggregate market. Value-at-risk is the percentage loss we can forecast for a future period of time with a 1% probability. It gives a measure of the risk of large systemic losses faced by our investors and the aggregate market. In a second step we analyze what trading patterns of institutional investors expose retail investors to large systemic losses. In particular, we study how benchmarking incentives may lead to large trading volumes and fire sales by institutional investors, resulting in large volatilities. We then study how benchmarking affects the long-term survival of our investors. In a final step we measure the impact of benchmarking on welfare.

2.4.1 Value-at-risk

We compute values-at-risk for our institutional investors, our retail investor, and the aggregate market via exact Monte Carlo simulation. Figure 3·11 plots the 1-year values-at-risk against the benchmark importance parameter I_j and the dividend ratio s_0. We see that values-at-risk across the board are highest in periods of extremely positive or extremely negative benchmark performance; i.e., in periods in which the dividend ratio s_0 is very large or very small. However, we see that a shifting of tail risk occurs as the benchmark dividend ratio falls from 1 to 0. Institutional investors are most exposed to value-at-risk when s_0 is large and the benchmark overperforms. This is primarily due to the fact that institutional investors invest large fractions of their wealths in the benchmark stock whenever s_0 is large. The benchmark stock has large negative jumps when s_0 is large (Figure 3·6). Figure 3·7 suggests that the institutional investors' portfolio weights for stocks are least sensitive to fluctuations in the dividend ratio s_0 when the benchmark importance parameters I_A and I_B are large and s_0 is large. As a result, institutional investors carry out buy-and-hold strategies when s_0, I_A, and I_B are large, with large exposures to tail risk arising from the large negative jumps of the benchmark.

[Figure 11 about here.]

In contrast, the retail investor is most exposed to value-at-risk when s_0 is small and the benchmark importance parameters are large. We see that value-at-risk for the retail investor strongly increases when the benchmark importance parameters I_A and I_B grow large in scenarios in which the dividend ratio s_0 is low. The retail investor reduces her investment in the safe asset when s_0 falls beyond a certain threshold (Figure 3·7). Consequently, the retail investor is more exposed to the stock market in scenarios in which s_0 is low. When s_0 is low and I_A and I_B are large, the volatilities of both stocks are also large and their jumps are severe; see Figure 3·6. As a result, the retail investor is more exposed to tail risk whenever s_0 is small.

Aggregate market capitalization is equal to the sum of the wealths of all investors because of market clearing. Because of the strong investment of institutional investors in the benchmark when s_0 is high, value-at-risk for the aggregate market is highest when the benchmark dividend ratio is high. Still, we see that aggregate market value-at-risk is increasing in the benchmark importance parameters I_A and I_B whenever s_0 is low. The large tail risk assumed by the retail investor when the benchmark dividend ratio is low dominates the tail risk exposures of the institutional investors. Consequently, the aggregate market is more exposed to tail risk when benchmark incentives grow stronger in scenarios in which the benchmark underperforms.

2.4.2 Volatility and fire sales

Figure 3·7 indicates that the sensitivities of the institutional investors' portfolio weights for stocks are highest whenever the benchmark importance parameters I_A and I_B are large and the dividend ratio s_0 is small. As a result, small fluctuations of the benchmark dividend ratio can result in large changes in the portfolios of institutional investors. Institutional investors sell (buy) large amounts of the benchmark and buy (sell) large amounts of the non-benchmark asset as a result of a small drop (rise) in the dividend ratio when s_0 is low. This behavior generates large trading volumes, which induces large volatilities for both stocks and severe jumps for the benchmark stock in scenarios in which s_0 is small and I_A and I_B are large (Figure 3·6). As a result, the portfolio volatilities of institutional investors decrease and the portfolio volatility of the retail investor increases with increasing benchmarking importance parameters when the benchmark dividend ratio is small; see Figure 3·12. This naturally increases the retail investor's exposure to tail risk.

[Figure 12 about here.]

Large portfolio sensitivities open up the possibility of fire sales by institutional investors after a jump occurs. Investors experience a cashflow shock when the benchmark dividend process jumps, prompting them to adjust their portfolio holdings. The degree to which investors' portfolios are adjusted critically depends on the benchmark importance parameters and the dividend ratio. Consider first a market in which there are no institutional investors; i.e., $I_A = I_B = 0$. Because the benchmark dividend falls drastically when a jump occurs, the cashflows of the benchmark stock become less profitable than the cashflows of the nonbenchmark stock. This results in a substitution effect in which investors reduce their exposure to the benchmark stock and increase their exposure to the nonbenchmark stock. The outlook of smaller cashflows also induces an income effect. Because investors expect less cashflows from the benchmark stock, they become less risk tolerant and reduce their exposure to both risky stocks. Figure 3·13 indicates that the substitution effect dominates when there are no institutional investors and when the dividend ratio is low. All investors reduce their exposure to the benchmark.

[Figure 13 about here.]

Figure 3·13 indicates that the income effect for institutional investors gets exacerbated when benchmarking incentives are large and the dividend ratio is low. When the benchmark importance parameters I_A and I_B are large, institutional investors are under strong pressure to beat the benchmark. After a jump, the outlook of lower cashflows incites institutional investors to strongly adjust their portfolios. They will further reduce their exposure to the benchmark asset, and increase their exposure to the non-benchmark asset. They also slightly increase their exposure to the safe asset; i.e., institutional investors reduce their leverage after a jump. These are *flight-to-quality* phenomena.

Because of the large portfolio sensitivities, institutional investors sell off large fractions of the benchmark stock and buy large fractions of the non-benchmark stock after a jump when their benchmarking incentives are large and the dividend ratio is low. They can only sell the benchmark stock at a discount and buy the non-benchmark stock at a premium after a jump; see Figure 3·6. This phenomenon constitutes *fire sales* in our model. Retail investors perceive the fire sales as a good opportunity to acquire the benchmark stock and sell the non-benchmark stock. The substitution effect for the retail investor is weak when benchmarking incentives are large. The income effect incites the retail to buy the benchmark stock. These effects combined reduce the degree to which the retail investor cuts down her exposure to the benchmark stock after a jump when benchmarking incentives are large. Consequently, the retail investor is more exposed to the benchmark stock after a jump when the dividend ratio is low and benchmarking incentives are large. This results in a higher exposure to tail risk.

2.4.3 Survival

How does the exposure to tail risk impact the long term survival of our investors? We can answer this question by looking at the long-term consumption plans and shares.[8] We can rewrite the consumption c_t^j of institutional investor $j \in \{A, B\}$ as

$$(1 + I_j s_t)\frac{Q_0 \alpha^j W_0 \phi_j}{\bar{c}(t)} \cdot$$
$$\cdot \left((1 - s_0) e^{\left(\mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 - \rho_j + \bar{\rho}\right) t + \sigma_1 W_t + J_1 \bar{N}_t} + s_0 e^{\left(\mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 - \rho_j + \bar{\rho}\right) t + \sigma_2 W_t + J_2 \bar{N}_t} \right)$$

[8]Given our log-utility formulation, consumption at any point of time is a fraction of total wealth.

where $\bar{I} = \max\{I_A, I_B\}$, $\bar{\rho} = \max\{\rho_R, \rho_A, \rho_B\}$, $\bar{N}_t = N_t - \lambda t$ is the compensated Poisson jump martingale, and

$$\bar{c}(t) = \alpha^R \rho_R e^{(\bar{\rho}-\rho_R)t} + \alpha^A \phi_A (1 + I_A s_t) e^{(\bar{\rho}-\rho_A)t} + \alpha^B \phi_B (1 + I_B s_t) e^{(\bar{\rho}-\rho_B)t}.$$

By construction, we have $0 < \lim_{t\to\infty} \bar{c}(t) < \infty$ and $0 < s_t < 1$ almost surely for all $t > 0$. As a result, the asymptotic behavior of c_t^j as $t \to \infty$ is driven only by the asymptotic behavior of discounted stock prices. From the expression above it becomes obvious that $c_t^j \to \infty$ as $t \to \infty$ almost surely if

$$\bar{\rho} + \mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 > \rho_j \quad \text{or} \quad \bar{\rho} + \mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 > \rho_j. \tag{2.8}$$

In this case we also have $W_t^j \to \infty$ as $t \to \infty$ almost surely and institutional investor j becomes infinitely rich in the long run through her investments. On the other hand, we have $c_t^j \to 0$ as $t \to \infty$ almost surely if

$$\bar{\rho} + \mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 < \rho_j \quad \text{and} \quad \bar{\rho} + \mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 < \rho_j. \tag{2.9}$$

In this case, institutional investor j becomes extinct in the long run. Given that $\bar{\rho} \geq \rho_j$, an inspection of (2.9) reveals that extinction can only if both dividend process have negative expected growth rate.

Similarly, we can rewrite the consumption c_t^R of the retail investor as

$$\frac{Q_0 \alpha^R W_0 \rho_R}{\bar{c}(t)} \Big[(1 - s_0) e^{\left(\mu_1 - \frac{\sigma_1^2}{2} + \lambda J_1 - \rho_R + \bar{\rho}\right)t + \sigma_1 W_t + J_1 \bar{N}_t} + s_0 e^{\left(\mu_2 - \frac{\sigma_2^2}{2} + \lambda J_2 - \rho_R + \bar{\rho}\right)t + \sigma_2 W_t + J_2 \bar{N}_t} \Big].$$

We see that the retail investor becomes infinitely rich in the long run under condition (2.8), and she goes extinct almost surely if condition (2.9) holds. As for institutional investors, the retail investor can only go extinct in the long run if both dividend processes have negative expected growth rate.

The analysis of consumption plans indicates that, in the long run, either all investors survive and become infinitely rich, or all investors fail. By studying consumption shares we can determine which investor dominates in the long run. The consumption share of institutional investor A is

$$\frac{c_t^A}{c_t^R + c_t^A + c_t^B} = \frac{1}{1 + \frac{\alpha^R \rho_R}{\alpha^A \phi_A (1+I_A s_t)} e^{(\rho_A - \rho_R)t} + \frac{\alpha^B \phi_B (1+I_B s_t)}{\alpha^A \phi_A (1+I_A s_t)} e^{(\rho_A - \rho_B)t}}.$$

An analogous representation can be derived from the consumption share of institutional investor B. The consumption share of the retail investor is

$$\frac{c_t^R}{c_t^R + c_t^A + c_t^B} = \frac{1}{1 + \frac{\alpha^A \phi_A (1+I_A s_t)}{\alpha^R \rho_R} e^{(\rho_R - \rho_A)t} + \frac{\alpha^B \phi_B (1+I_B s_t)}{\alpha^R \rho_R} e^{(\rho_R - \rho_B)t}}.$$

Given that $1 \leq 1 + I_j s_t \leq 1 + \bar{I} < \infty$ for $j \in \{A, B\}$, consumption shares are only driven by the relation between the time discount coefficients of our investors. The investor that dominates in terms of relative consumption is the one with the smallest time discount coeffient; i.e., the investor with $\rho_j = \min\{\rho_R, \rho_A, \rho_B\}$. All other investors will have consumption ratios that converge to zero. The most patient investor will be the richest in the long run.

The conditions for survival we derive do not depend on the benchmarking importance parameters I_A and I_B. As a result, survival occurs independently of the benchmarking behavior of our institutional investors. In the same vein, the most patient investor dominates in the long run independently of the behavior of institutional investors. We conclude that albeit benchmarking exposes the retail investor to tail risk in the short term, tail risk does not materialize in the long run. The long term performance of our investors is unaffected by the benchmarking incentives of our institutional investors. These results extend the findings of Yan (2008), who shows that in a general class of infinite-horizon models driven by Brownian motions, the investor with the lowest survival index dominates in the long run.[9] We show that this result also holds when dividends are allowed to jump, and when institutional investors have benchmarking incentives.

Condition (2.8) is satisfied in our numerical case study parametrized in Table 3.2. All investors – institutional and retail – become infinitely rich in the long run in our example. However, the retail investor has the smallest time discount coefficient. Therefore, the retail investor will be the richest investor in the long run. Although the retail investor is exposed to large tail risk when the benchmark stock underperforms due to the trading behavior of institutional investors, these tail risks are only short-lived. Our institutional investors take on large bets when the benchmark underperforms (fire sales as in Figure 3·13), and they consume large fractions of their wealths (Figure 3·9). This behavior reduces the wealth of institutional investors relative to the retail investor.

2.4.4 Welfare

Although benchmarking does not affect the long term performance of our investors, ex ante it may be disadvantageous because of the higher exposure to tail risk. Next, we analyze the welfare implications of benchmarking.

For fixed benchmark importance paramters I_A and I_B, the preferences formulated in (2.1) and (2.2) are locally nonsatiated, the utility possibility set is convex, and the equilibrium described in Section 2.3 is Pareto efficient. However, the utility possibility set may no longer be convex as I_A and I_B change. As a result, we cannot analyze the impact of benchmarking on welfare in our model by measuring changes in social welfare as responses to changes in I_A and I_B. In order to circumvent this issue, we adopt the notion of equivalent variation and evaluate the impact of benchmarking on

[9] The ranking of survival indices in our model is equal to the ranking of time discount parameters of our investors given that our investors are perfectly rational and do not have optimism biases.

welfare by computing how much additional consumption our investors would have to consume at any given point of time in a world in which there are no benchmarking incentives ($I_A = I_B = 0$) in order to achieve the same level of utility as in a world with positive benchmarking incentives ($I_A > 0$ or $I_B > 0$). Because for $I_A = I_B = 0$ all investor have log preferences, positive equivalent variations of consumption for investor j imply welfare gains for investor j, while negative equivalent variations imply welfare losses for investor j.

Figure 3·14 displays the equivalent variations of consumption for our different investors at inception. The equivalent variation of consumption of the retail investor is negative for every value of the benchmark dividend ratio s_0. The same result hold for all other times $t > 0$. As a result, the retail investor is worse off in a world in which institutional investors have benchmarking incentives. The analysis is subtle for our institutional investors. We see that institutional investor A with the highest discount rate has positive equivalent variation of consumption in every state of the world. This implies that the most impatient institutional investor is better off in a world with benchmarking incentives. However, institutional investor B with a low discount rate has negative equivalent variation in states of the world in which the benchmark underperforms. Unlike for the retail investor, for whom benchmarking is always welfare reducing, benchmarking may be welfare reducing or increasing for institutional investors depending on how patient they are.

We obtain similar results for alternative choices of discount parameters. The retail investor is always worse off when institutional investors have benchmarking incentives.

[Figure 14 about here.]

2.5 Conclusion

We show that benchmarking may induce trading behavior by institutional investors which exposes a retail investor and the aggregate market to tail risk. To show this we solve in semi-closed form the general equilibrium of a pure-exchange economy with one retail investor and two institutional investors who can invest in a benchmark stock, a non-benchmark stock, and a safe asset. Institutional investors have marginal utility of consumption that increases in the relative performance of their benchmark. This incites them to tilt their portfolios towards the benchmark stock. It also increases their portfolio sensitivities with respect to the relative performance of the benchmark. As a result, institutional investors trade large volumes of the stocks in states of the world in which the benchmark underperforms, raising market volatility. This naturally increases the tail risk exposure of the retail investor and the aggregate market.

Inspite of the higher tail risk exposure, we find that benchmarking does not affect the long term performance of our investors. In the long run, all investors survive if at least one stock has positive expected dividend rate, and the most patient investor dominates in terms of relative wealth. Still, benchmark is welfare reducing for the retail investor ex ante, and is only welfare increasing for the impatient institutional investor.

Our results have important implications for the regulation of the asset management industry. In December 2014, the FSOC released a notice asking if competitive pressures faced by portfolio managers may incentivize them to trade in ways that do not internalize risks to the broad financial markets.[10] We find that the answer to this question is yes – benchmarking may indeed incentivize portfolio managers to invest in ways that induce additional risks in financial markets. We find that retail investors may be worse off in economies with benchmarking than in economies without bench-

[10]Notice Seeking Comment on Asset Management Products and Activities, Docket No. FSOC-2014-0001, pp. 11-12.

marking due to a higher tail risk exposure, even though their long term performance is unaffected by the trading behavior of institutional investors. These results indicate that it is imperative for regulators to formulate precise objectives for a potential regulation of the asset management industry. Do regulators wish to control for the tail risk exposure of the retail investor or the aggregate market? Do regulators with to prevent investor failures? These questions need to be answered first before any effective regulatory policy can be formulated.

In any case, our results indicate that the regulation of the asset management industry will require different regulatory tools than the regulation of banks. We find that the retail investor and the aggregate market are exposed to high tail risk whenever benchmarking institutional investors are exposed to low tail risk, and vice versa. As a result, standard regulatory tools that measure tail risk exposures of financial institutions, such as value-at-risk and stress tests, may be unable to detect situations in which retail investors and the aggregate market are at risk of tail events.

This paper is the first to analyze the systemic implications of benchmarking from a normative point of view by considering its general equilibrium effects. Still, our model is of reduced form. We do not address the principal-agent problem underlying the decision of individual investors to allocate their wealth among portfolio managers. We also do not address the optimal contracting problem between individual investors and portfolio managers that gives rise to benchmarking. We leave it to future research to analyze the systemic effects of benchmarking in a principal-agent setting in which these problems can be addressed. Furthermore, we do not analyze potential regulatory tools for controlling for the systemic effects of benchmarking. Our results indicate that more research is needed to be able to carefully evaluate the effectiveness of a potential regulation of benchmarking.

Chapter 3

Central Bank and Asset Pricing

3.1 Introduction

The recent turmoil in global financial markets triggered by the fear that the Chinese economy is slowing down faster than what analysts predicted brought public attention to central banks' reaction to volatile financial markets. In an attempt to contain the financial meltdown of 32% from June 12, 2015 to July 7, 2105, the Chinese central bank, the People's Bank of China, made a 25 basis point interest rates cut, bringing rates to their record low on June 27, 2015. In addition, the central bank issued a loan of $42 billion dollars to 21 brokerage firms, so they could purchase stocks directly. Despite the intense effort to ease financial markets, it is still not well understood how these types of government interventions could potentially generate more risk and contribute further to market destabilization with consequences to the real economy. In order to understand the effects of such interventions to macroeconomic variables, this chapter investigates the following question: how does monetary authorities seeking to stabilize inflation, output and smooth the slope of the yield curve distort the term structure of interest rates and prices of risk relative to an economy where central authorities adjust the money supply without taking into consideration the slope of the yield curve?

The idea that monetary policy could target financial indicators goes back to McCallum (1994). The author shows that a monetary policy aiming to smooth interest rates and the yield curve could explain the rejection of the expectation hypothesis[1]. Despite the important insights provided by McCallum (1994) on rationalizing Fama and Bliss (1987) empirical puzzle, the exogenous specification of the risk premium adopted by the author prevents one from understanding the feedback effects caused by the central bank targeting the slope of the yield curve on prices of risk and interest rate. In his concluding remarks, McCallum (1994) highlights the limitation of a monetary policy seeking solely to smooth short-rate and the slope of the yield curve, i.e., level and slope but not curvature (volatility):

> *"(It) represents a simplification relative to the actual behavior of the FED, which almost certainly responds to recent inflation and output or employment movements as well as the spread. So, if one were to attempt to econometrically estimate actual reaction functions, then measures of inflation and output gaps would need to be included. But in that case values of these variables would need to be explained endogenously, so the system of equations in the model would have to be expanded. (...) In short, this type of study would require specification and estimation of a complete dynamic macroeconometric model."*.

In order to circumvent the limitations highlighted in McCallum (1994), this chapter develops a macro-finance model where monetary authorities target the slope of the yield curve, inflation and output. Contrary to New Keynesian models that assume an exogenous process for inflation or the short term rate, all three quantities involved in the central bank's decision to supply money to the economy are *endogenous* variables linked to the production side of the economy. Thus, the model provides a clear

[1]The simple version of the Expectation Hypothesis relates the forward rate with the expected future short-rate and a risk premium. When confronted with data, the regression coefficient on the term premium is significant smaller than predicted by the model.

exposition on the mechanism through which monetary stabilization and capital gain taxes change the conditional variance of the pricing kernel.

Other papers also attempt to address McCallum (1994)'s challenge by endogenizing the bond risk premium. Gallmeyer et al. (2005) resort to a specific dynamics of the state variables and no-arbitrage condition on bonds to endogenize the risk premium. As the authors observe, despite the fact that the exogenous specification of the risk premium serves as a good descriptive tool and provides the desirable properties a model should have in order to match the data, it does not explain how factors driving the risk premium could also be related to interest rates movements. In order to match empirical regularities, Gallmeyer et al. (2005) test two different model specifications: one with a stochastic volatility factor and another with a stochastic price of risk. These specifications are supported by the findings of Dai and Singleton (2003), who argue in favor of models with state-dependent prices of risk instead of state variables with stochastic volatility by showing that the former is equally tractable and tend to deliver better empirical results. Gallmeyer et al. (2005) also provide a theoretical set of restrictions on asset prices and macroeconomic variables such that the Taylor's rule is equivalent to the McCallum's rule.

Following Gallmeyer et al. (2005) and Buraschi and Jiltsov (2005), the model introduced in Section 3.2 generates a one factor term structure model where the state variable with stochastic volatility is a persistent component embedded in the money supply. Besides the clear economic interpretation of this state variable, the model's prices of risk also differs from Gallmeyer et al. (2005) in the sense that the *endogenous* price of risk here is state-dependent. Therefore, some empirical regularities of the term structure highlighted in Atkeson and Kehoe (2009), such as (1) the association between movements in the yield spread - the difference between long and short-rate - and movements in risk and (2) the fact that part of the short-rate movements are associated to changes in the conditional variance of the log marginal utility of consumption, are also present in the model.

Another important term structure regularity emphasized by Atkeson and Kehoe (2009) is the fact that, over the business cycle, the Federal Reserve Bank responds to shocks in real risk. The authors show that factors identified by a principal component analysis of the yield curve are associated with movements in the conditional mean and variance of the pricing kernel. In particular, the model presented in this chapter shows that the factor driving the yield curve is the time varying persistent component of the money supply. As a result, the conditional moments of the pricing kernel move according to changes in this factor. Atkeson and Kehoe (2009) highlight that most of the existing literature traditionally assumes a constant conditional variance of the pricing kernel which culminates in empirical inconsistencies that prevents one from understanding the linkages between monetary policy, macroeconomic variables and financial markets. Recently, some studies in the macro-finance literature support the idea that the volatility of the pricing kernel is not constant. In particular, Bansal et al. (2014) show that volatility risks carries a sizable positive risk premium, suggesting that volatility risk is an important factor to understand the mechanics of asset prices and macroeconomics dynamics. Boguth and Kuehn (2013) also find empirically that the time varying conditional volatility of consumption is a priced risk factor.

The tractability of factor models for interest rates and their empirical fit are studied in Rudebusch and Wu (2008). The authors develop a macroeconomic model to study the impact of a monetary policy targeting inflation and output gap on the term premium and term structure dynamics. Similar to the papers aforementioned, their analysis relies on an exogenous specification of the prices of risk and the short-rate, which are driven by unobservable factors linked to the slope and level of the term structure. They conclude that the latent factors have important macroeconomics and monetary policy underpinnings. Similar to their analysis of output and price level dynamics, the impulse response functions derived in this chapter help in the understanding of how inflation and the nominal term structure shift as monetary and productivity shocks realize. In addition to the limitations imposed by the assumption of an exogenous specification of prices of risk, short-rate and output, Rudebusch and Wu (2008) emphasize[2] that their interpretation of the level factor as expected inflation is conflicting with their assumption on the inflation dynamics. In the model introduced in this chapter, the inconsistency is not present since both quantities are endogenous variables emerging from equilibrium. Importantly, interest rates, yield curve slope, prices of risk, inflation and output are linked together under a theoretical description of preferences and production technology, resulting in an asset pricing kernel which is consistent with the dynamics of macro-financial variables.

This work is closely related to Buraschi and Jiltsov (2005). In their paper, the authors develop a real business cycle model with taxes and endogenous monetary policy to analyze nominal and real risk premium of the yield curve. They are able to deliver endogenous state-dependent prices of risk which results in the rejection of the expectation hypothesis. Nevertheless, the authors do not take into account central banks' objective of smoothing interest rates, which prevent them to address the concerns raised by McCallum (1994) and Rudebusch and Wu (2008).

This chapter is organized as follows. Section 3.2 describes the model, characterizes the equilibrium and the term structure of interest rate. Section 3.3 provides a numerical analysis of the model. Section 3.3.1 analyzes the impact of quantitative easing on the welfare of the representative agent. Section 3.3.2 analyzes the dynamics of price level and output by investigating the impulse response function of these quantities to monetary shocks. Section 3.4 concludes.

[2]See Rudebusch and Wu (2008), footnote 9.

3.2 Model

Consider a continuous time economy on $[0, \infty)$ where the uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with augmented filtration $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ generated by two independent Brownian motions: $Z^k = (Z_t^k)_{t \geq 0}, Z^m = (Z_t^m)_{t \geq 0}$.

All the stochastic processes are assumed to be progressively measurable with respect to $\mathbb{F}$ and all the equalities of random variables presented are considered to hold $\mathbb{P}$-a.s.

3.2.1 Households

There is a continuum of infinitely lived households that maximize the expected lifetime utility J_t by choosing the amount of consumption c_t and real cash balances m_t. Each household maximizes the following expected utility function:

$$J_t = \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} (\alpha \log c_s + (1-\alpha) \log m_s) ds \right]. \tag{3.1}$$

In this formulation, β is the subjective rate of time preference and α is the expenditure share on consumption. Throughout the text, it is assumed that $0 < \beta$ and $0 < \alpha < 1$.

The presence of money in the utility function is usually justified by the argument that cash reduces the associated costs of obtaining a higher net consumption good. These expenditures are commonly identified as transaction or liquidity costs. Examples of monetary asset pricing models that rely on this formulation are Sidrauski (1967), Danthine and Donaldson (1986), Stulz (1986), Bakshi and Chen (1996), Buraschi and Jiltsov (2005) and references therein. An alternative class of general equilibrium asset pricing models that justify the presence of money is the one that encompass a cash-in-advance type of constraint. This approach, adopted by Clower (1967), Day (1984), Feenstra (1985), Lee (1989), Bakshi and Chen (1997) and others, assumes that certain types of goods require cash in order to be consumed. However, the presence of cash-in-advance constraints limits the rate at which cash goods can be purchased. Lucas and Stokey (1987) and Balduzzi (2007) combine features of both cash-in-advance and money in the utility function approaches by allowing the representative agent to have some flexibility in allocating consumption between cash goods and goods that can be acquire directly in exchange for securities, the so called credit goods.

Balduzzi (2007) analyzes an endowment economy where the representative agent derives utility on cash goods, $c_1(t)$, and credit goods, $c_2(t)$, and maximizes the following expected utility function:

$$J_t = \mathbb{E}_t \left[\int_t^\infty e^{-\beta(s-t)} (\alpha \log c_2(s) + (1-\alpha) \log c_1(s)) ds \right], \tag{3.2}$$

where the cash good $c_1(s)$ is subject to the cash-in-advance constraint[3]:

$$m(t) \geq c_1(t). \tag{3.3}$$

Similar to Bakshi and Chen (1997), the author focuses on the analysis of the equilibrium where the constraint is always binding, i.e.,

$$c_1(t) = m(t). \tag{3.4}$$

Sidrauski (1967) also relies on (3.4) to justify the presence of money in the utility function. In addition, Bakshi and Chen (1997) points out that the empirical findings of Hodrick et al. (1991) support the idea that the constraint on cash goods almost always bind.

[3]Formally, the constraint takes the form $m(t) \geq \int_t^{t+\varepsilon} c_1(s)ds \approx \varepsilon c_1(t)$. Normalizing ε to one, we obtain the cash-in-advance constraint in (3.3).

3.2.2 Firms and Technology

In this economy, capital K_t is the only factor of production and the final good Y_t is generated according to the following AK technology:

$$Y_t = AK_t, \tag{3.5}$$

where A is a constant production technology factor.

The aggregate physical capital stock has its dynamics represented by:

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t^k - \tau K_t \frac{dp_t}{p_t}. \tag{3.6}$$

The first term on the right hand side of (3.6) illustrates that whenever the firm's investment I_t surpass depreciated capital δK_t, there is a positive contribution to the expected capital accumulation rate. The second term shows that the accumulation of capital is risky and it is exposed the Brownian shock Z^k. The exposure of capital to this shock is exogenously set at the scalar σ, which is assumed to be positive. While negative shocks have the natural interpretation of capital destruction in equation (3.6), positive innovations can be interpreted as gains generated by the employment of new capital relative to the old capital in place. The third component in (3.6) affecting capital accumulation can be interpreted as a constant capital gain tax τ imposed by fiscal authorities on capital gains. Thus, positive innovations on the variation of the price level p_t result in more tax collection and consequently less capital available. The price level and its dynamics are an *endogenous* outcome of the equilibrium.

Other studies like Rebelo and Xie (1999), Buraschi and Jiltsov (2005), Wälde (2011) and Posch (2011) also rely on a risky capital accumulation process. Rebelo and Xie (1999) and Wälde (2011) highlight that despite the fact that equation (3.6) does not represent the usual locally deterministic capital evolution commonly adopted in neoclassical growth models, both capital evolution formulation become indistinguishable once the market clearing condition and resources constraints are imposed[4].

3.2.3 Monetary Authorities

Monetary authorities control the money supply, M_t^s, and conduct the monetary policy targeting the following variables: (1) the short-rate to the long-term yield level $\bar{r}$, (2) inflation to a level $\bar{\pi}$, and (3) capital accumulation to a level $\bar{k}$. Contrary to $\bar{k}$ and $\bar{\pi}$ that are taken exogenously, the long-term yield level $\bar{r}$ is an outcome from the equilibrium. In addition to the targeting objectives, the money supply is also affected by an exogenous persistent component g_t that evolves according to the Feller diffusion:

$$dg_t = \kappa_g(\bar{g} - g_t)dt + \sigma_g\sqrt{g_t}dZ_t^m, \tag{3.7}$$

where $\kappa_g, \bar{g}$ and σ_g are positive constants.

Under these assumptions, the endogenous money supply can be summarized by the following stochastic differential equation:

$$\frac{dM_t^s}{M_t^s} = q_1\left(dr_t - \bar{r}dt\right) + q_2\left(\frac{dp_t}{p_t} - \bar{\pi}dt\right) + q_3\left(\frac{dK_t}{K_t} - \bar{k}dt\right) + dg_t \tag{3.8}$$

where q_1, q_2 and q_3 are parameters that correspond to the authorities' weights on deviations from the target. If $q_1 = q_2 = q_3 = 0$, the monetary policy is set exogenously. The monetary policy described in (3.8) embodies the characteristics of the widely adopted Taylor's rule and it also contains a term that reflects the willingness of the central bank to flat the term structure of interest rate.

Taylor (1993) proposes a monetary policy that consist in adjusting the short-rate in response to variations to inflation and output from the central bank pre-established targets. According to Orphanides (2003a), the Taylor's rule is relatively successful in explaining the Federal Reserve intervention during Paul Volcker and

[4]See Eaton (1981).

Alan Greenspan's tenure as chairman of the Federal Reserve. Nevertheless, McCallum (1993) argues that information on inflation and output gap are usually not available to policy makers at the moment of the short-rate setting. A similar argument is used by Orphanides (2003b) to show that, when accounting for informational limitations, the adoption of the Taylor's rule by policymakers could have led to a worse economic performance during the Great Inflation period in the seventies. Orphanides (2003a) and Meyer (2002) also highlight that the classic Taylor's rule does not encompass forecast and expectations about economic activity, even thought both indicators are crucial information for central bankers to shape monetary policy.

To overcome the difficulties of low frequency data, McCallum (1994) proposes a monetary rule in which central authorities adjust the short-rate to smooth interest rates across time and the yield curve. This specification is interesting from the standpoint of implementation since high-quality financial market data is readily available and it is usually reliable. McCallum (1994) framework is also able to rationalize the empirical puzzle of Fama and Bliss (1987), also known as the failure of the expectation hypothesis, by specifying an exogenous persistent process for risk premium. Along with the yield curve smoothing, the risk premium persistence is able to explain the empirical findings of Fama and Bliss (1987). Despite the fact that McCallum's rule serves as a useful statistical description of reality, the factors driving risk premium and short-rate lack a clear economic interpretation.

Equation (3.8) is able to capture the essence of both rules described above. By setting $q_1 = 0$, monetary authorities respond to deviation of inflation and output dynamics from their targets, which is how central banks behave according to Taylor (1993). On the other hand, by setting $q_2 = q_3 = 0$, money is supplied whenever the short-rate deviates from the endogenous long-term rate that the central bank targets. The idea of yield curve smoothing relates with the policy advocated by McCallum

(1994). Other financial indicators, such as bond and equity volatility, can be incorporated to (3.8) as a proxy of financial stability at the cost of simple equilibrium expressions.

3.2.4 Equilibrium

This section presents the definition of an equilibrium in this continuous-time production based monetary economy. The characterization of the equilibrium relies on the construction of a representative agent[5].

Definition 3.2.1. *The representative agent equilibrium is defined as a set of prices (interest rate, prices of risk and price level) given by the functions $r_t, \theta_t^k, \theta_t^m, p_t$, respectively, a value function $J(K_t, g_t)$ and a set of decision rules on consumption, money demand and investment, represented by the functions $\{c_t, m_t, I_t\}$, such that:*

(i) The representative agent maximize expected utility described in (3.1) *over consumption and money demand subject to the resource constraints* (3.6) *and* (3.5).

(ii) Money supply equals money demand: $M_t^s = m_t p_t$.

(iii) Resource constraint: $Y_t = c_t + m_t + I_t$.

The next proposition presents the equilibrium state price density.

Proposition 3.2.2. *The state price density satisfies the following stochastic differential equation:*

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \theta_t^k dZ_t^k - \theta_t^m dZ_t^m; \;\; \xi_0 = 1,$$

where the short-rate, r_t, and prices of risk, θ_t^k and θ_t^m, are given by

$$r_t = r_0 + r_1 g_t, \qquad \theta_t^k = \sigma - \tau\sigma_k, \qquad \theta_t^m = -(\tau\sigma_m + \mu_1\sigma_g)\sqrt{g_t},$$

[5]Buraschi and Jiltsov (2005) argued that despite the fact that externalities like taxes are generally responsible for the failure of the Second Welfare Theorem, this is not the case when taxes are linear in output and production technology has constant return to scale. See Stokey and Lucas (1989), p.547, example 18.2.

with

$$r_0 = A - \delta - \tau\Pi_0 - (\sigma - \tau\sigma_k)^2, \qquad \sigma_k = \frac{q_3 - 1}{1 - q_2 + \tau(q_3 - 1)}\sigma,$$

$$\sigma_m = \frac{q_1 r_1 + 1}{1 - q_2 + \tau(q_3 - 1)}\sigma_g, \qquad \omega_n = (\tau - 1)\frac{\sigma_m}{\sigma_g},$$

$$\mu_1 = \frac{-2\Pi_1\tau - \sigma_m^2\tau^2}{2\beta(\beta + \kappa_g)}, \qquad \Pi_1 = \frac{\tau\sigma_m^2 - (q_1 r_1 + 1)\kappa_g}{1 - q_2 + \tau(q_3 - 1)},$$

$$a_1 = (d_1 + c_1)\omega_n - 1, \qquad c_1 = -\frac{\kappa_g + \sqrt{\kappa_g^2 + 2\sigma_g^2\rho_1^n}}{2\rho_1^n},$$

$$b_1 = \frac{-d_1(\kappa_g + 2\rho_1^n c_1) + a_1(\sigma_g^2 - \kappa_g)}{a_1 c_1 - d_1}, \quad \rho_1^n = (1 - \tau)\Pi_1 - (1 + \tau^2)\frac{\theta_m^2}{2} + (1 - \tau)\frac{\sigma_m}{\sigma_g}\kappa_g,$$

$$d_1 = (1 - c_1)\omega_n \frac{\sigma_g^2\omega_n - \kappa_g + \sqrt{(\sigma_g^2\omega_n - \kappa_g)^2 - \sigma_g^2(\sigma_g^2\omega_n^2 - 2\omega_n\kappa_g - 2\rho_1^n)}}{\sigma_g^2\omega_n^2 - 2\omega_n\kappa_g - 2\rho_1^n},$$

$$\begin{aligned}
r_1 = &\frac{(q_2 + \tau(1 - q_3) - 1)^3}{2q_1^2\sigma_g^2\tau^2(2 - q_2 + \tau(q_3 - 1))}\left(q_1\tau\frac{2\sigma_g^2\tau(q_2 - 1 + \tau(1 - q_3))}{(q_2 - 1 + \tau(1 - q_3))^3}\right.\\
&+ q_1\tau\frac{(q_2 - 1 + \tau(1 - q_3))^2 - 2\sigma_g^2\tau}{(q_2 - 1 + \tau(1 - q_3))^3} + 1\\
&- \left(\frac{\tau^2\left((q_2 - 1)(q_1 - 3q_3 + 3)(q_1 - q_3 + 1) + 4q_1(q_2 - 2)\sigma_g^2\right)}{(q_2 - 1 + \tau(1 - q_3))^3}\right.\\
&+ \frac{(q_2 - 1)^2\tau(2q_1 - 3q_3 + 3)}{(q_2 - 1 + \tau(1 - q_3))^3}\\
&\left.\left.- \frac{(q_3 - 1)\tau^3\left((q_1 - q_3 + 1)^2 + 4q_1\sigma_g^2\right) - (q_2 - 1)^3}{(q_2 - 1 + \tau(1 - q_3))^3}\right)^{1/2}\right),
\end{aligned}$$

$$\Pi_0 = \frac{(q_1 r_1 + 1)\kappa_g\bar{g} + (q_3 - 1)(A - \delta - \beta) - (q_1\bar{r} + q_2\bar{\pi} + q_3\bar{k}) - \sigma_k(\sigma - \tau\sigma_k)}{1 - q_2 + \tau(q_3 - 1)}$$

$$\begin{aligned}
\bar{r} = &\frac{1 - q_2 + \tau(q_3 - 1)}{1 - q_2 + q_1 + \tau(q_3 - q_1 - 1)}\left(A - \delta + (\tau - 1)\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g} + (\sigma - \tau\sigma_k)^2\right.\\
&\left.+\sigma_k^2 + \sigma_k(\sigma - \tau\sigma_k) - \kappa_g\bar{g}\frac{a_1}{d_1}\right)\\
&+(1 - \tau)\frac{(q_1 r_1 + 1)\kappa_g\bar{g} + (q_3 - 1)(A - \delta - \beta) - (q_2\bar{\pi} + q_3\bar{k}) - \sigma_k(\sigma - \tau\sigma_k)}{1 - q_2 + q_1 + \tau(q_3 - q_1 - 1)},
\end{aligned}$$

Proof. See Appendix A.2 □

The expressions for prices of risk in Proposition 3.2.2 show that the price of risk associated with shocks to capital, Z^k, is constant and independent of the targeting weight q_1, also referred here as the quantitative easing parameter. In addition, if the inequality

$$\frac{1-q_2}{1-q_2+\tau(q_3-1)} > 0$$

is satisfied, then the price of risk θ_t^k is always positive. However, the price of risk associated with monetary shocks, Z^m, is time varying and depends on the state variable g_t. Moreover, its non-linear dependence on q_1 does not allow one to unequivocally pin down its sign. Here, the *endogenous* time-varying price of risk generates a time-varying conditional variance of the pricing kernel, which, as discussed earlier, is essential to explain the business cycle component of the term structure, as pointed by Atkeson and Kehoe (2009). From the expression for the pricing kernel stated in Proposition 3.2.2, it is clear that monetary authorities' weights impact prices of risk, with consequences to real risk.

Gallmeyer et al. (2005) also notice the importance of a time-varying price of risk in explaining the term structure movements, but they rely on an exogenous specification of the price of risk. Contrary to their framework, the model presented in Section 3.2 has short-rate (r_t), price level (p_t) and prices of risk (θ_t^k, θ_t^m) as *endogenous* quantities, which allow one to further investigate the co-movements between prices of risk and expected inflation (π_t).

Another important observation is that the short-rate r_t is an affine function of the state variable g_t. Nevertheless, due to the highly non-linear coefficient r_1 as a function of the quantitative easing parameter q_1, one cannot determine if monetary shocks would increase or decrease the short-rate.

Next, optimal policies, inflation, the endogenous evolution of capital and price level are characterized. Proposition 3.2.3 presents the results.

Proposition 3.2.3. *The value function of the representative agent is*

$$J(K_t, g_t) = \frac{1}{\beta}\log(\beta K_t) + \mu_1 g_t + \mu_0, \tag{3.9}$$

where

$$\begin{aligned} \mu_1 &= \frac{-2\Pi_1\tau - \sigma_m^2\tau^2}{2\beta(\beta+\kappa_g)}, \\ \mu_0 &= \frac{(1-\alpha)\log(1-\alpha) + \alpha\log\alpha + A - \beta - \delta + \beta\bar{g}\kappa_g\mu_1 - \Pi_0\tau - \frac{(\sigma-\tau\sigma_k)^2}{2}}{\beta^2}. \end{aligned} \tag{3.10}$$

The optimal policies for consumption, money demand and investment are, respectively,

$$c_t = \alpha\beta K_t, \quad m_t = (1-\alpha)\beta K_t, \quad I_t = (A-\beta)K_t.$$

The equilibrium price level can be written as

$$\frac{dp_t}{p_t} = \pi_t dt + \sigma_k dZ_t^k + \sigma_m\sqrt{g_t}dZ_t^m,$$

where expected inflation π_t is a linear function of the state variable g_t, expressed as

$$\pi_t = \Pi_0 + \Pi_1 g_t.$$

The equilibrium capital accumulation satisfies the following stochastic differential equation:

$$\frac{dK_t}{K_t} = (A - \beta - \delta - \tau\Pi_0 - \tau\Pi_1 g_t)dt + \sigma_k(\sigma - \tau\sigma_k)dZ_t^k - \tau\sigma_m\sqrt{g_t}dZ_t^m.$$

Proof. See Appendix A.2 □

Note that all optimal policies - consumption, money demand and investment - are linear in capital due to the structure of the utility function. Similar to the short-rate, expected inflation, π_t, is also an affine function of the state variable g_t and the effect of quantitative easing cannot be unequivocally determined since the coefficient Π_1 is a highly non-linear function of q_1.

The next proposition shows the expression for bond prices and yields.

Proposition 3.2.4. *The nominal and real bond price are, respectively,*

$$\begin{aligned} B^n(t,T) =& e^{-\rho_0^n\cdot(T-t)+\eta_0^n(T-t)+(\eta_1^n(T-t)-\omega_n)g_t}, \\ B^r(t,T) =& e^{-\rho_0^r\cdot(T-t)+\eta_0^r(T-t)+(\eta_1^r(T-t)-\omega_r)g_t}, \end{aligned} \tag{3.11}$$

where

$$\begin{aligned}
\eta_1^n(T-t) =& \frac{1+a_1e^{b_1(T-t)}}{c_1+d_1e^{b_1(T-t)}}, \qquad \eta_1^r(T-t) = \frac{1+a_1^re^{b_1^r(T-t)}}{c_1^r+d_1^re^{b_1^r(T-t)}}, \\
\eta_0^r(T-t) =& \kappa_g\bar{g}\frac{a_1^rc_1^r-d_1^r}{b_1^rc_1^rd_1^r}\log\left(\frac{c_1^r+d_1^re^{b_1^r(T-t)}}{c_1^r+d_1^r}\right)+\frac{\kappa_g\bar{g}}{c_1^r}(T-t), \\
\rho_0^n =& A-\delta+(1-\tau)\Pi_0+(\tau-1)\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g}+(\sigma-\tau\sigma_k)^2+\sigma_k^2+\sigma_k(\sigma-\tau\sigma_k), \\
\eta_0^n(T-t) =& \kappa_g\bar{g}\frac{a_1c_1-d_1}{b_1c_1d_1}\log\left(\frac{c_1+d_1e^{b_1(T-t)}}{c_1+d_1}\right)+\frac{\kappa_g\bar{g}}{c_1}(T-t), \\
d_1^r =& (1-c_1^r)\omega_r\frac{\sigma_g^2\omega_r-\kappa_g+\sqrt{(\sigma_g^2\omega_r-\kappa_g)^2-\sigma_g^2(\sigma_g^2(\omega_r)^2-2\omega_r\kappa_g-2\rho_1^r)}}{\sigma_g^2(\omega_r)^2-2\omega_r\kappa_g-2\rho_1^r}, \\
\rho_0^r =& A-\delta-\tau\Pi_0-\tau\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g}-(\sigma-\tau\sigma_k)^2, \qquad \omega_n = (\tau-1)\frac{\sigma_m}{\sigma_g}, \\
\rho_1^n =& (1-\tau)\Pi_1-(1+\tau^2)\frac{\theta_m^2}{2}+(1-\tau)\frac{\sigma_m}{\sigma_g}\kappa_g, \quad \omega_r = \tau\frac{\sigma_m}{\sigma_g}, \\
\rho_1^r =& -\left(\tau\frac{\sigma_m}{\sigma_g}\kappa_g+\tau\Pi_1+\frac{(\tau\sigma_m)^2}{2}\right), \qquad c_1^r = -\frac{\kappa_g+\sqrt{\kappa_g^2+2\sigma_g^2\rho_1^r}}{2\rho_1^r}, \\
a_1^r =& (d_1^r+c_1^r)\omega_r-1, \qquad b_1^r = \frac{-d_1^r(\kappa_g+2\rho_1^rc_1^r)+a_1^r(\sigma_g^2-\kappa_g)}{a_1^rc_1^r-d_1^r}.
\end{aligned}$$

The nominal and real yield rate are, respectively,

$$\begin{aligned}
R^n(t,T) =& -\frac{1}{T-t}\log B^n(t,T) = \rho_0^n-\frac{\eta_0^n(T-t)}{T-t}+\frac{\omega_n-\eta_1^n(T-t)}{T-t}g_t, \\
R^r(t,T) =& -\frac{1}{T-t}\log B^r(t,T) = \rho_0^r-\frac{\eta_0^r(T-t)}{T-t}+\frac{\omega_r-\eta_1^r(T-t)}{T-t}g_t.
\end{aligned}$$

Proof. See Appendix A.2.1. □

Contrary to the findings of Bakshi and Chen (1996), the money supply impacts directly not only the short-rate but also the nominal and real term structure. The main reason is that the capital evolution is affected explicitly by the tax on nominal value of capital, which makes price variability impact real output. This mechanism is absent in Bakshi and Chen (1996). Consequently, the real pricing kernel is impacted by the capital fluctuation generated by changes in price level, which end up impacting the short-rate and the term structure of nominal and real bonds.

The expressions for the nominal and real yield, indicated in Proposition 3.2.4, show that both term structures belong to the class of the one factor model. Given that the short-rate is also an affine function of the state variable g_t, it follows that both term premium are an affine function of g_t. Consequently, all yields are perfectly correlated across maturities. This result represents a shortcoming inherent to all single factor models as pointed out by Cox et al. (1985).

To conclude the equilibrium characterization, expressions for nominal and real claims over consumption are presented in Proposition 3.2.5.

Proposition 3.2.5. *The price of a contingent claim over consumption are, in nominal and real terms, respectively,*

$$\begin{aligned} S_t^n =& \alpha\beta K_t \int_t^{\infty} B^n(t,s)ds, \\ S_t^r =& \alpha K_t. \end{aligned} \tag{3.12}$$

The dynamics of (3.12)*, the nominal and real risk premium are shown in Appendix A.2.2.*

Proof. See Appendix A.2.2. □

3.3 Numerical Analysis

In order to investigate the behavior of the equilibrium quantities derived in Section 3.2.4, this section presents a comparative statics analysis of the model's outcomes relative to the quantitative easing parameter q_1. Table 3.2 summarizes the parametrization adopted and it follows the plausible values used by Buraschi and Jiltsov (2005) and Matthys (2014).

[Table 2 about here.]

The top panels in Figure 3·15 shows the effect of flattening the term structure by adjusting the weight q_1. As monetary authorities increase the targeting intensity, the short-rate moves towards the long-term yield, shortening the gap between both rates. This results indicates that an increase in money supply due to smoothing the yield curve could increase the short-term rate while dumping the long-term yield. Note that depending on how intense the targeting weight q_1 is, it could potentially invert the slope of the yield curve.

The effects of quantitative easing on expected inflation is illustrated in the bottom left panel of Figure 3·15. As monetary authorities increase the weight q_1 to flatten the term structure, the short-rate exposure r_1 to monetary shocks increases. The negative relationship between the short-rate exposure r_1 and expected inflation exposure Π_1 to monetary shocks[6], results in a lower expected inflation.

The bottom right panel of Figure 3·15 depicts the effects of quantitative easing on the price of risk associated with the monetary shocks. By increasing the weight q_1, the price level volatility increases and, as a result, the price of risk associated with the monetary shocks also increases. In fact, the expression for σ_m shown in Proposition 3.2.2 reveals that the price of risk θ_m is increasing in q_1 when the short-rate exposure to g_t, r_1, responds positively to an increase in q_1.

[Figure 15 about here.]

The weight q_1 impacts the nominal and real term structure in different ways. As it is illustrated in Panel (a) of Figure 3·16, the nominal yield curve decreases as q_1

[6] See the expression for Π_1 in Proposition 3.2.2

increases. As discussed previously, quantitative easing drives expected inflation down. Therefore, nominal bonds yields also decrease with the increase of q_1. In addition, the term structure is decreasing in the bond maturity. The reasoning behind this result is the following: nominal long term bonds are exposed to expected inflation for a longer period and, consequently, will collect the benefits of it for a longer period of time. Thus, short term bonds should offer a higher rate to become attractive to risk averse agents.

The effects for real bond yields are the opposite to the ones described for nominal bond yields. First, Panel (b) of Figure 3·16 shows that the model generates an upward real yield curve, which is consistent with the empirical findings of Alvarez and Jermann (2005). Contrary to the case of nominal bonds, in which both the pricing kernel and price level are affected by expected inflation, here only the pricing kernel is affected by expected inflation through changes in the capital accumulation growth rate, resulting in higher real rates.

According to Piazzesi and Schneider (2007) and Beeler and Campbell (2009), long-run risk models have problems generating the upward sloping yield curves. As explained in Albuquerque et al. (2012), the downward sloping curve is generated by risk averse agents trying to protect against bad states, when consumption is drastically low. Given that bonds' payoff are certain in these states, agents are willing to pay high prices to obtain such securities. If agents want to be insured for longer periods of time, the price of the securities should be higher, which implies in lower returns. Nevertheless, in the presented model, the longer the maturity, the longer the exposure of capital accumulation to inflation risk. For this reason, risk averse agents require a compensation for this exposure, which results into higher real premium.

[Figure 16 about here.]

Figure 3·17 displays the term premium for bonds maturing in 1, 5 and 10 years as a function of the quantitative easing parameter q_1. As it is shown, both premiums are negative, indicating that the claims serve as a hedge to inflation risk. The equilibrium capital evolution stated in Proposition 3.2.3 shows that monetary shocks decrease the current capital level but it increases expected capital growth if the exposure to the persistent component g_t is negative, i.e., $\Pi_1 < 0$. Thus, a positive monetary shock would drive expected future capital growth upward, leading the short-rate down and bond prices up. Consequently, bonds hedge inflation risk and have a negative real term premium.

[Figure 17 about here.]

Figure 3·18 illustrates the behavior of nominal and real bond variance as a function of quantitative easing for the bonds maturing in 1, 5 and 10 years. As it is shown, the nominal bond variance is an increasing function of the maturity and the quantitative easing parameter q_1. These results come from the fact that a higher weight q_1 increases the volatility of both the real pricing kernel and price level. Thus, while real bond variance decreases with q_1, the effect on the price level volatility dominates the reduction of the real pricing kernel volatility, resulting in a more volatile nominal pricing kernel.

[Figure 18 about here.]

A similar analysis can be conducted to investigate the nominal and real equity premium. The expressions derived in Proposition 3.2.5 show that real stock is a fraction of the capital good while the nominal stock is linear in capital but it also depends on the expressions for nominal bonds. Thus, the same reasoning can be applied to explain the same pattern of results displayed in Figure 3·19. Similarly to the analysis of the nominal bond yields, the more volatile nominal pricing kernel makes the nominal premium more sensible to quantitative easing variations.

[Figure 19 about here.]

Similar to the findings of Stulz (1986), the model is able to generate a positive relation between expected nominal returns on consumption claims and expected inflation, while a mild negative relation between expected inflation and expected real rate of return.

3.3.1 Welfare Analysis

This section presents the welfare effects of quantitative easing. In order to assess the impact of quantitative easing on the welfare of the representative agent, a equivalent variation measure is adopted.

Consider the definition for welfare cost/gain of Dibooglu and Kenc (2009):

Definition 3.3.1. *The welfare cost/gain is defined as the percentage of capital the representative agent is ready to give up in period zero to be as well off in a world with quantitative easing intensity q_1, as she is in a world with no quantitative easing, i.e., $q_1 = 0$.*

The next proposition present the welfare cost/gain of quantitative easing in closed form.

Proposition 3.3.2. *Let γ be the welfare cost/gain of quantitative easing as in Definition 3.3.1. Then,*

$$
\begin{aligned}
\gamma(q_1) = & 1 - \exp\Bigg\{ - \frac{1}{\beta^2}\Bigg(\tau(\Pi_0(q_1) - \Pi_0(0)) + \tau \frac{\sigma_m(q_1) - \sigma_m(0)}{\sigma_g}\kappa_g \bar{g} \\
& + \left(\tau \frac{\sigma_m(q_1) - \sigma_m(0)}{\sigma_g}\kappa_g + \tau(\Pi_1(q_1) - \Pi_1(0)) + \tau^2 \frac{\sigma_m^2(q_1) - \sigma_m^2(0)}{2} \right) \bar{g} \Bigg) \\
& - \left(\tau(\Pi_1(q_1) - \Pi_1(q_0)) + \tau^2 \frac{\sigma_m^2(q_1) - \sigma_m^2(q_0)}{2} \right) \frac{g_0 - \bar{g}}{\beta(\beta + \kappa_g)} \Bigg\}.
\end{aligned}
\tag{3.13}
$$

Figure 3·20 illustrates the welfare cost/gain of quantitative easing following the parametrization presented in Table 3.2. As it is shown, an increase in money supply generated by quantitative easing results in a welfare gain for the representative agent. The deflationary effects induced by quantitative easing lead to higher capital accumulation and, consequently, to a higher consumption ratio.

[Figure 20 about here.]

3.3.2 Analysis of Dynamics

In order to understand how shocks to monetary policy and capital accumulation impact equilibrium output Y_t and the price level p_t, an analysis of the impulse response function these quantities is provided. The analysis is similar to the one presented by Rudebusch and Wu (2008). The next proposition characterizes these elasticities.

Proposition 3.3.3. *The normalized impulse response functions for output and price level are, respectively,*

$$\varepsilon_{t,T}^{Y,k} = \sigma, \qquad \varepsilon_{t,T}^{p,k} = \sigma_k,$$

$$\begin{aligned}\varepsilon_{t,T}^{Y,m} =& \frac{\tau\sigma_m\sigma_g g_t}{\mathbb{E}_t\left[K_T\right]}\mathbb{E}_t\left[K_T\left(\int_t^T\left(\tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2}\right)e^{-\int_t^v\left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}}\right)du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}}dZ_u^m} dv\right.\right.\\ &\left.\left. + \tau\sigma_m\int_t^T \frac{1}{2\sqrt{g_v}}e^{-\int_t^v\left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}}\right)du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}}dZ_u^m} dZ_v^m\right)\right],\end{aligned}$$

$$\begin{aligned}\varepsilon_{t,T}^{p,m} =& \frac{\sigma_m\sigma_g g_t}{\mathbb{E}_t\left[p_T\right]}\mathbb{E}_t\left[p_T\left(\int_t^T\left(\Pi_1 - \frac{\sigma_m^2}{2}\right)e^{-\int_t^v\left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}}\right)du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}}dZ_u^m} dv\right.\right.\\ &\left.\left. + \sigma_m\int_t^T \frac{1}{2\sqrt{g_v}}e^{-\int_t^v\left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}}\right)du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}}dZ_u^m} dZ_v^m\right)\right].\end{aligned}$$

Proof. See Appendix A.2.4. □

The top two panels in Figure 3·21 illustrate the behavior of the elasticities for $q_1 = \frac{1}{2}$ with their respective 95% confidence bounds. The middle panels illustrate the elasticities for $q_1 = 1$. The bottom panels display the elasticities for $q_1 = 1$, in dashed line, and for $q_1 = \frac{1}{2}$, in solid line.

As it can be observed, the impact of monetary shocks is amplified when quantitative easing is higher. In addition, a monetary shock has a positive permanent effect on output while the effect on price level is negative and transient.

[Figure 21 about here.]

3.4 Conclusion

A macro-finance model where monetary authorities adjust the money supply targeting not only output and inflation gap but also the slope of the yield curve is presented in this work. Under a continuous-time production based monetary economy, the persistent component present in the money supply becomes the factor driving the term structure movements. Moreover, the *endogenous* characterization of expected inflation, short-rate and prices of risk in closed-form allow us to study the feedback effects of quantitative easing on macroeconomic and financial variables.

Under the parametrization adopted, quantitative easing tends to flatten the term structure, to generate deflation and to increase the time-varying prices of risk. In addition, the model generates a downward sloping nominal term structure and an upward sloping real term structure. The numerical exercise that analyzes the impact of quantitative easing on the agent's welfare, for $q_1 \in [0, 5]$, indicates that quantitative easing could be potentially welfare improving.

Figures

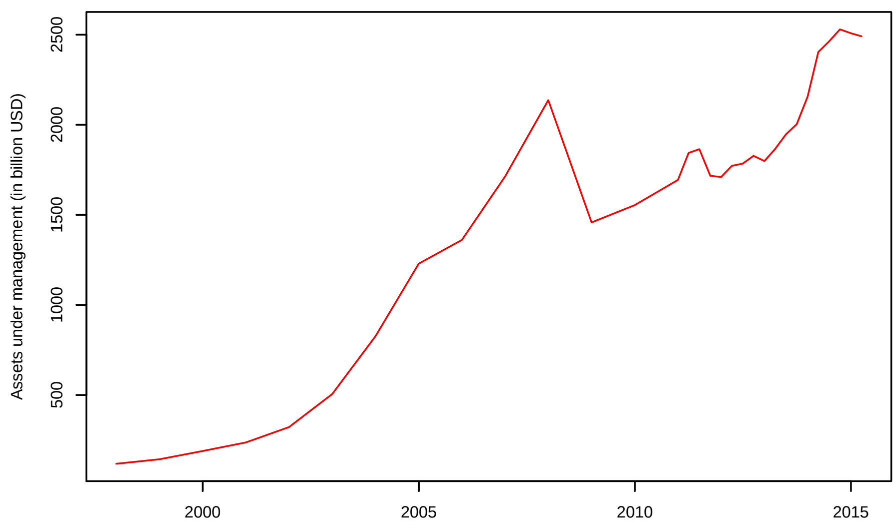
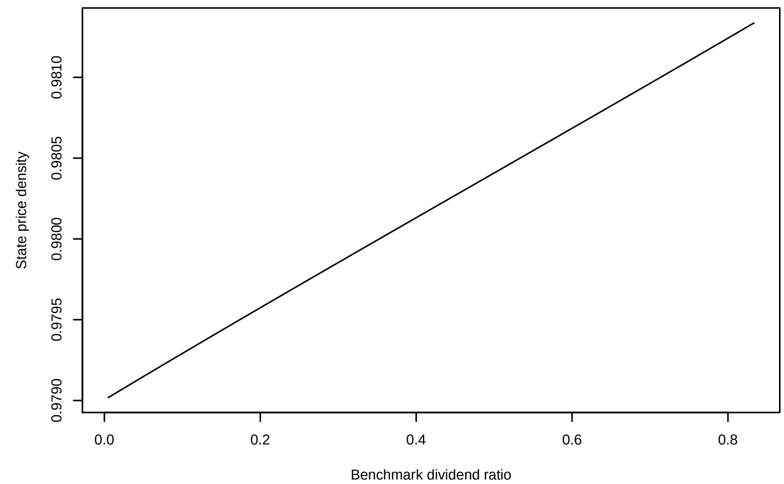
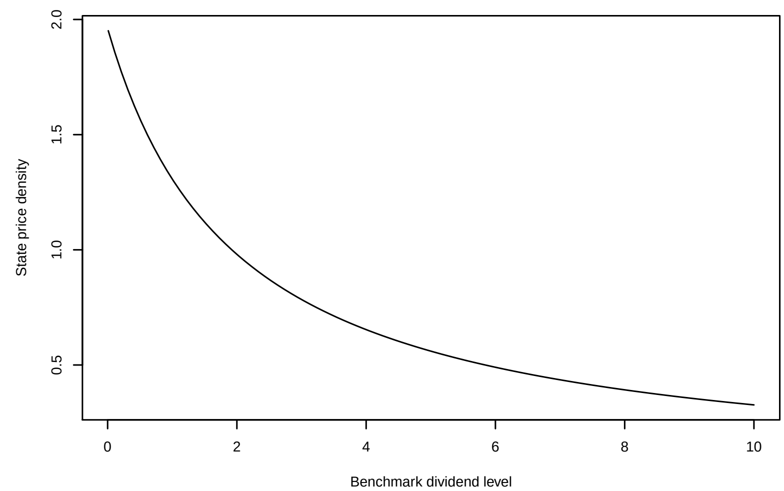


Figure 3·1: *Assests under management in the global hedge fund industry.* Source: Barclay Hedge.

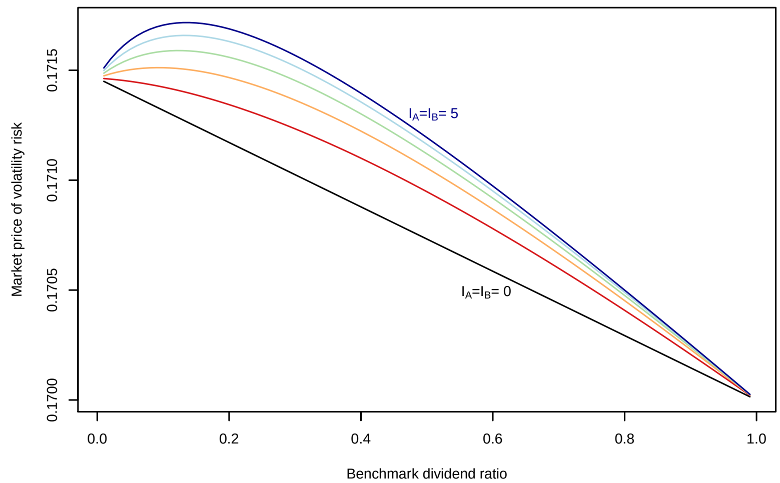


(a) State price density ξ_t versus benchmark dividend ratio s_t at $t = 1$ for $I_A = I_B = 2$.

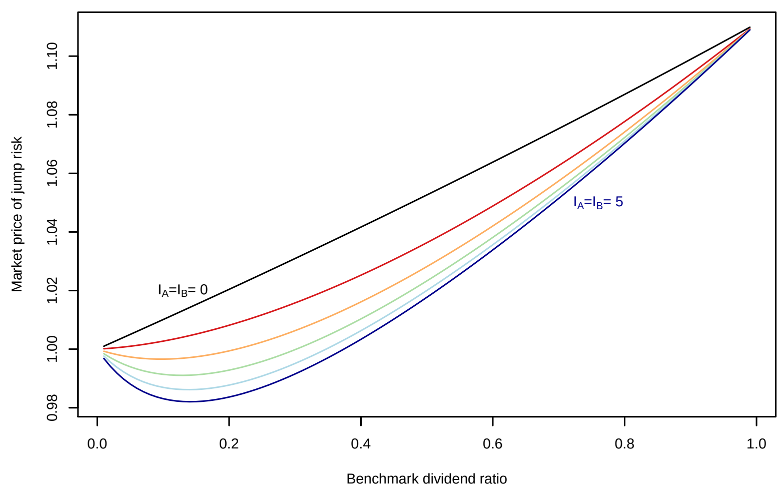


(b) State price density ξ_t versus benchmark dividend level $D_{2,t}$ at $t = 1$. Here, we take $I_A = I_B = 2$ and $D_{1,t} = D_{2,t}$ so that $s_t = 0.5$ is kept fixed.

Figure 3·2: *State price density.* These figures plot the state price density ξ_t against the benchmark dividend ratio s_t and the benchmark dividend level $D_{2,t}$.



(a) Market price of volatility risk θ_t at time $t = 0$.



(b) Market price of jump risk ψ_t at time $t = 0$.

Figure 3·3: *Market prices of risk.* These figures give comparative statics of the market prices of volatility and jump risk, θ_t and ψ_t relative to changes in the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$.

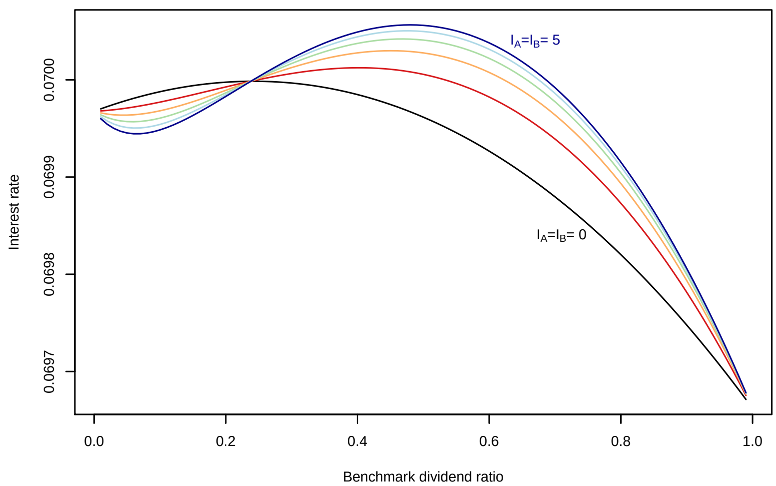
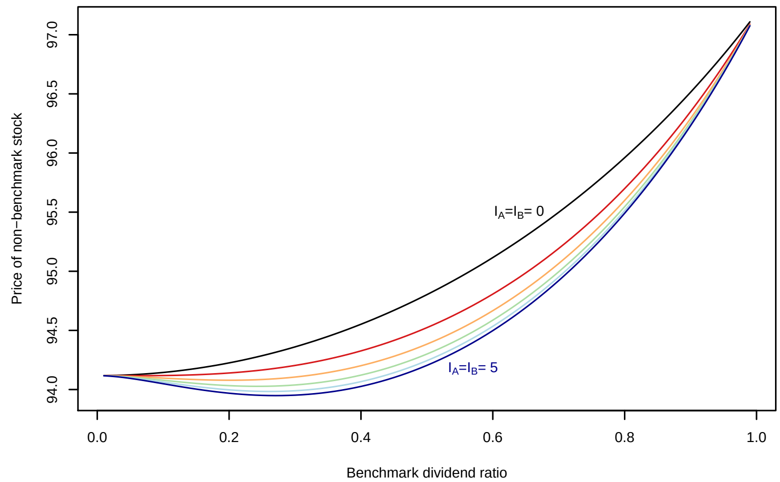
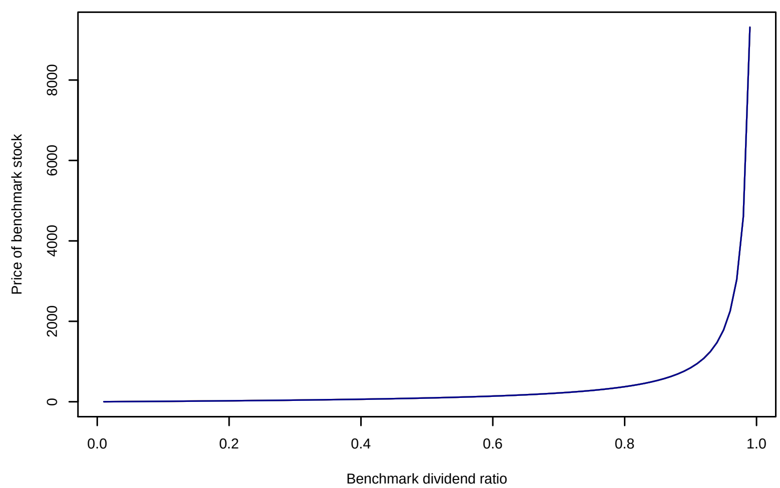


Figure 3·4: *Interest rate.* This figure gives comparative statics of the interest rate r_t relative to changes in the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$.

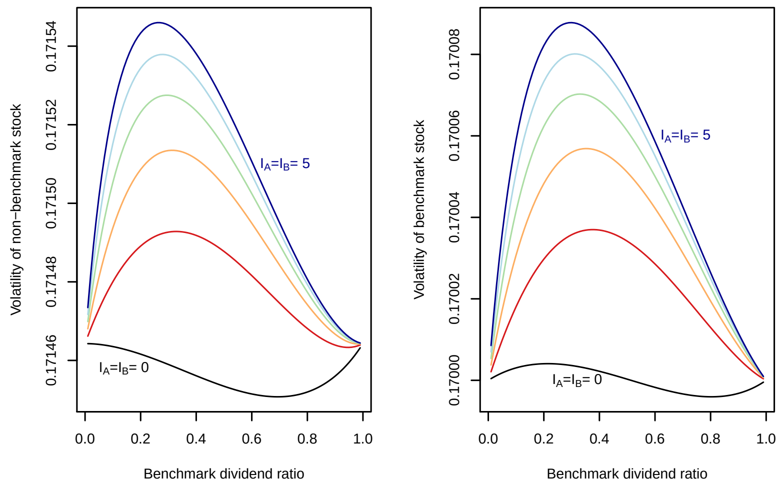


(a) Price of the non-benchmark stock, $S_{1,t}$, for different values of I_A and I_B.

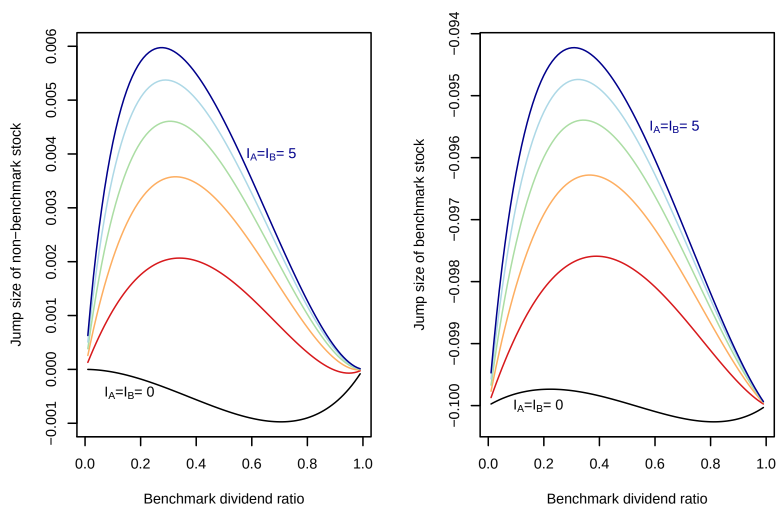


(b) Price of the benchmark stock, $S_{2,t}$, for different values of I_A and I_B. The price is close to insensitive to the benchmark importance parameters so that the lines for different values of I_A and I_B lie on top of each other.

Figure 3·5: *Stock prices.* These figures give comparative statics of the prices of the benchmark and the non-benchmark stocks, $S_{1,t}$ and $S_{2,t}$, relative to changes in the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$.

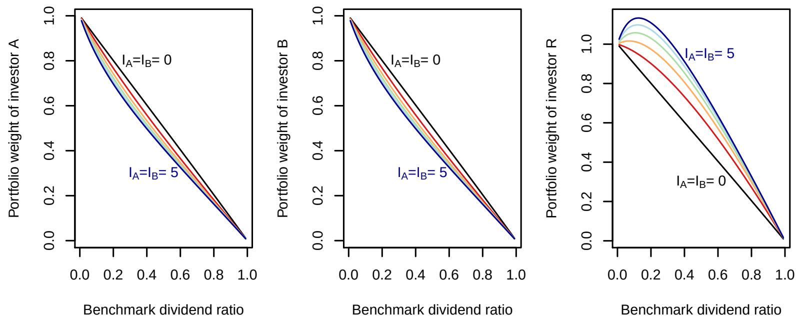


(a) Stock volatilities, $\sigma_{1,t}$ and $\sigma_{2,t}$, for different values of I_A and I_B.

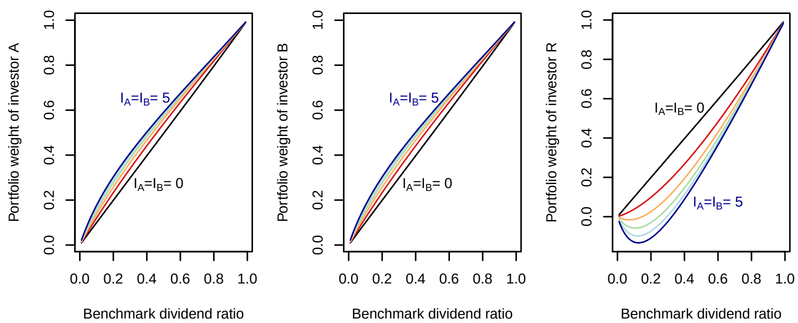


(b) Stock jump sizes, $J_{1,t}$ and $J_{2,t}$, for different values of I_A and I_B.

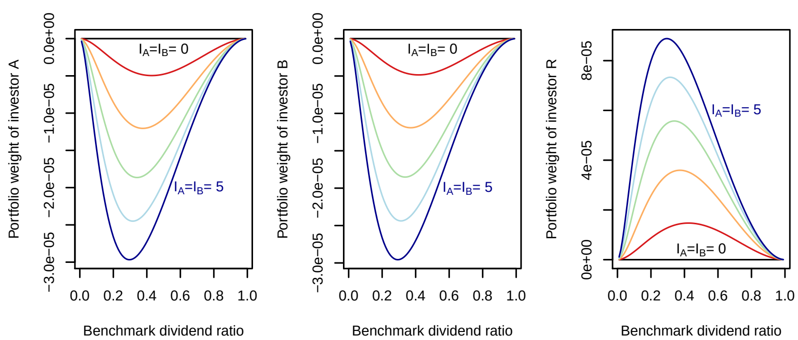
Figure 3·6: *Stock volatilities and jump sizes.* These figures give comparative statics of the volatilities ($\sigma_{1,t}$ and $\sigma_{2,t}$) and jump sizes ($J_{1,t}$ and $J_{2,t}$) relative to changes in the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$.



(a) Portfolio weights for the non-benchmark stock for different values of I_A and I_B.

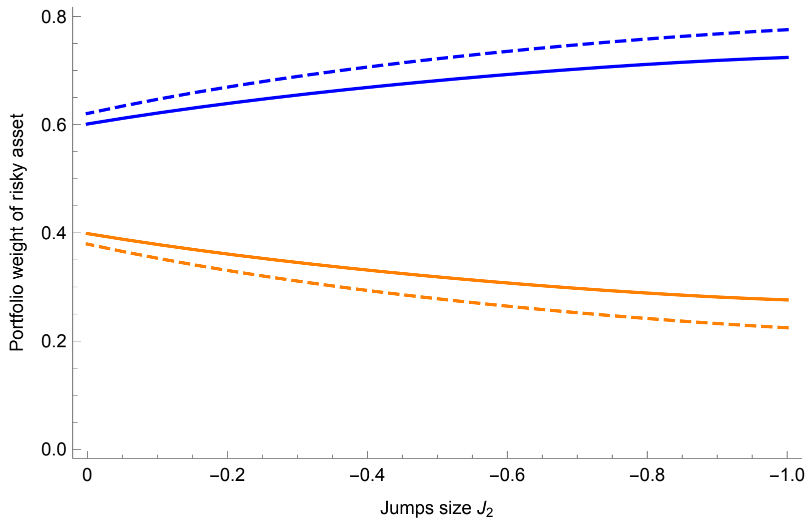


(b) Portfolio weights for the benchmark stock for different values of I_A and I_B.

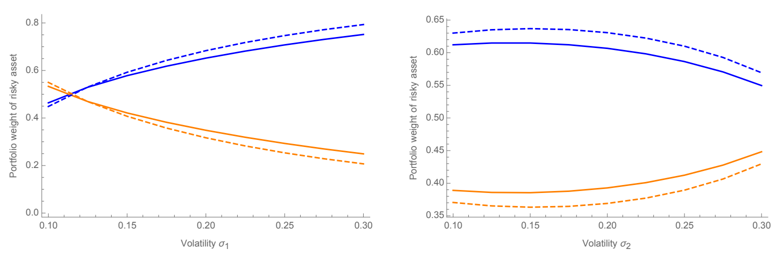


(c) Portfolio weights for the safe asset for different values of I_A and I_B.

Figure 3·7: *Portfolios.* These figures show the portfolio plans of different investors for different values of the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is done at time $t = 0$ under the assumption that $I_A = I_B$.

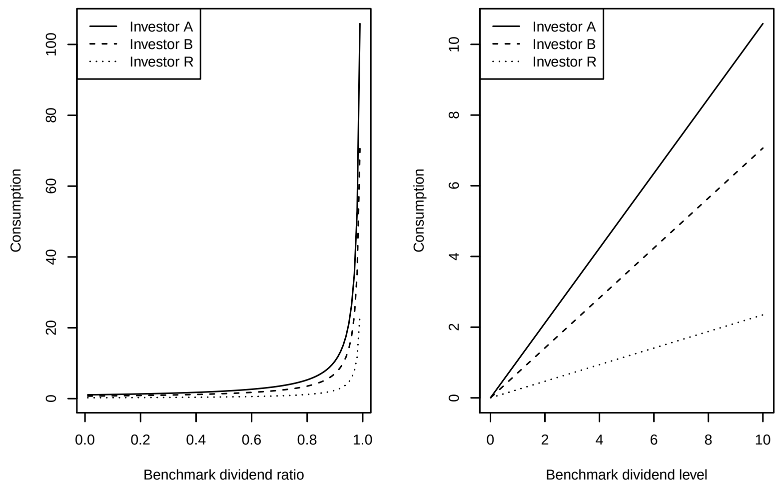


(a) Institutional investors' portfolio weights for the non-benchmark stock, $\pi_{1,t}^A$ (solid blue) and $\pi_{1,t}^B$ (dashed blue), and the benchmark stock, $\pi_{2,t}^A$ (solid red) and $\pi_{2,t}^B$ (dashed red), as functions of the benchmark dividend jump size J_2.

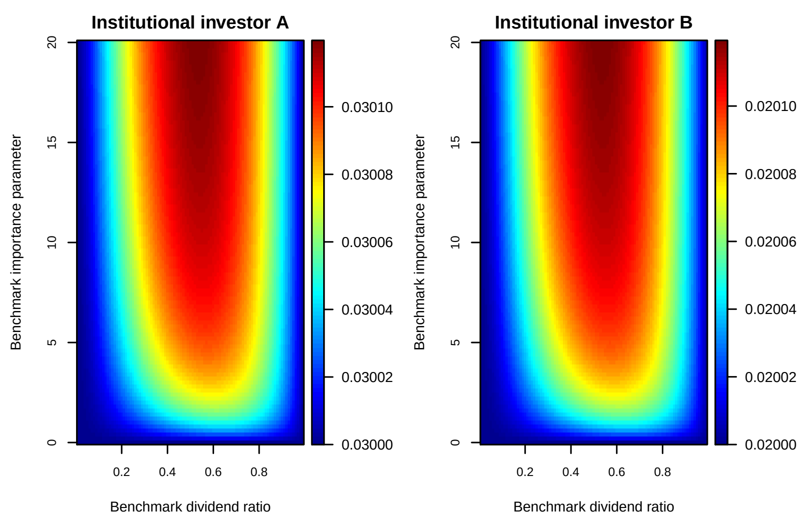


(b) Institutional investors' portfolio weights for the non-benchmark stock, $\pi_{1,t}^A$ (solid blue) and $\pi_{1,t}^B$ (dashed blue), and the benchmark stock, $\pi_{2,t}^A$ (solid red) and $\pi_{2,t}^B$ (dashed red), as functions of the non-benchmark and benchmark dividend volatilities, σ_1 and σ_2.

Figure 3·8: *Comparative statics of porfolios of institutional investors.* These figures are generated under the assumption that $t = 0$, $I_A = I_B = 2$, and $s_0 = 0.5$. They reflect ceteris paribus changes relative to the parametrization of Table 3.2.



(a) Consumption c_t^j for $j \in \{R, A, B\}$ as a function of the benchmark dividend ratio s_t and level $D_{2,t}$. We take $t = 0$ and $I_A = I_B = 2$. For the right plot, we assume $D_{1,t} = D_{2,t}$.



(b) Wealth to consumption ratio c_t^j/W_t^j for $j \in \{R, A, B\}$ as a function of the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. We take $t = 0$ and $I_A = I_B$.

Figure 3·9: *Consumption plans.* These charts plot consumption as functions of the dividend ratio s_t, the dividend level $D_{2,t}$, and the benchmark importance parameters I_A and I_B.

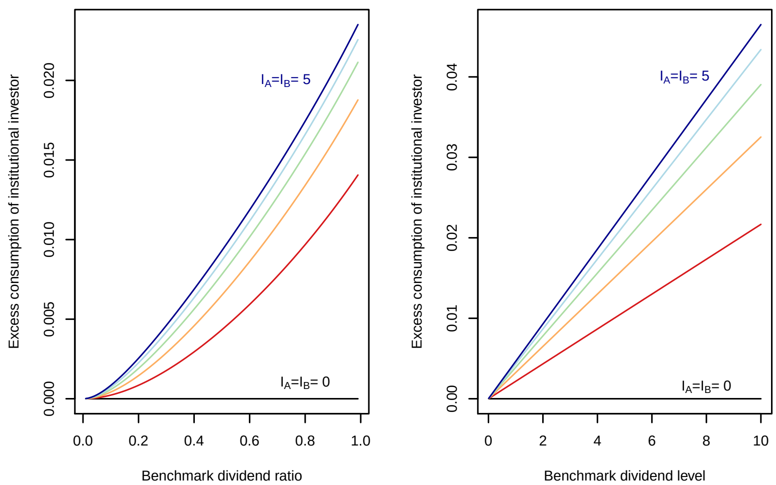


Figure 3·10: *Institutional excess consumption.* These figures plot the excess consumption of institutional investor, $c_t^A - c_t^R$, as functions of the benchmark dividend ratio s_t, the dividend level $D_{2,t}$, and benchmark importance parameter I_A. We take $t = 0$. To make the retail and institutional investors comparable, we assume here that $\alpha_A = \alpha_R = 0.5$, $\alpha_B = 0$, and $\rho_A = \rho_R = 0.02$. For the right plot, we also assume $D_{1,t} = D_{2,t}$.

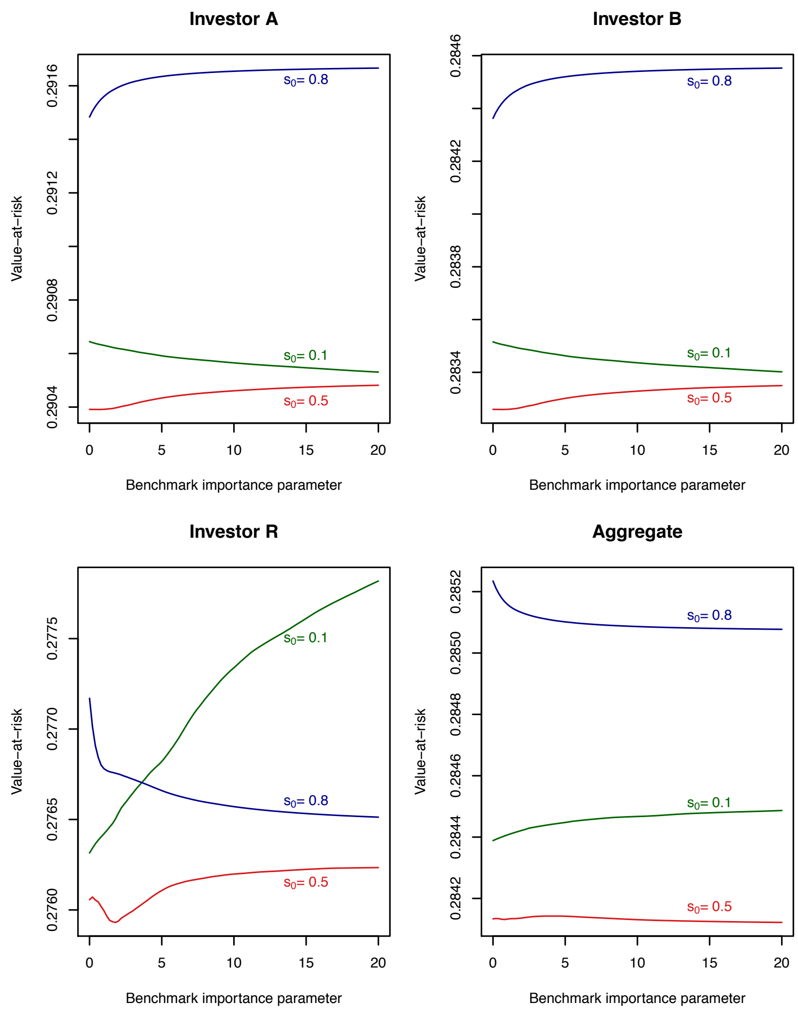


Figure 3·11: *Value-at-Risk.* These figures plot the 1-year value-at-risk of the institutional and retail investors, and of the aggregate market against the benchmark dividend ratio s_t and the benchmark importance parameters I_A and I_B. The analysis is carried out at time $t = 0$ under the assumption that $I_A = I_B$. We simulate 10^5 exact samples of $D_{1,t}$ and $D_{2,t}$ for $t = 1$, and use these to construct samples of the benchmark dividend ratio s_t, as well as wealths W_t^j and realized portfolio returns $(W_t^j - W_0^j)/W_0^j$ for $j \in \{R, A, B\}$ at $t = 1$. Value-at-risk is the 1%-quantile of the simulated realized return distribution times -1.

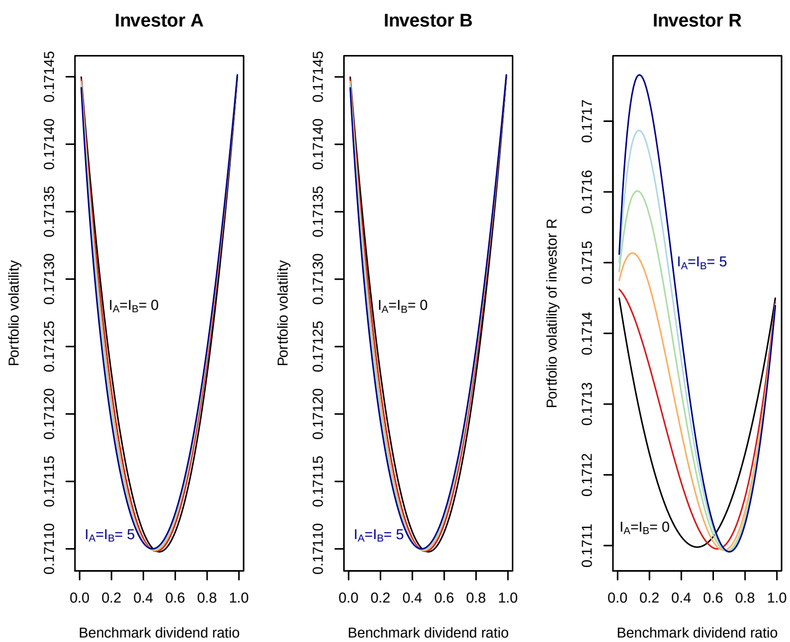
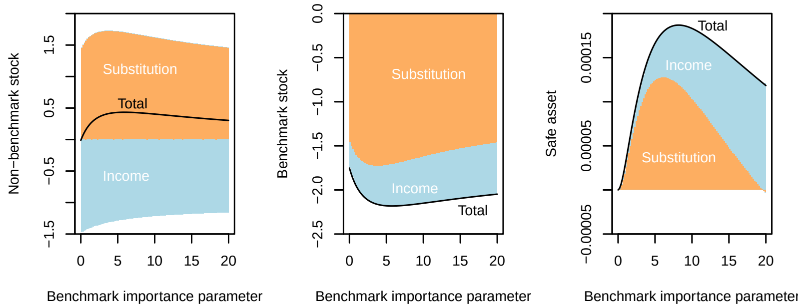
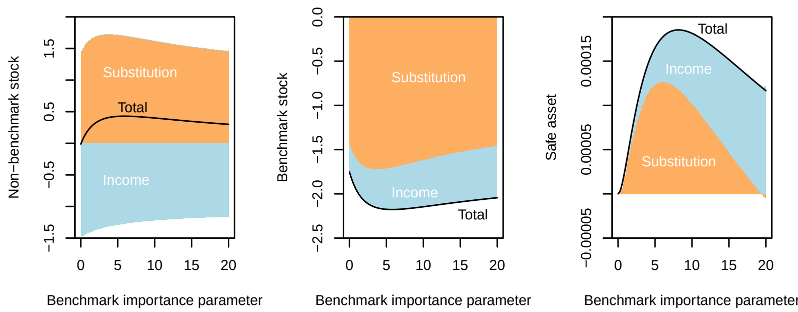


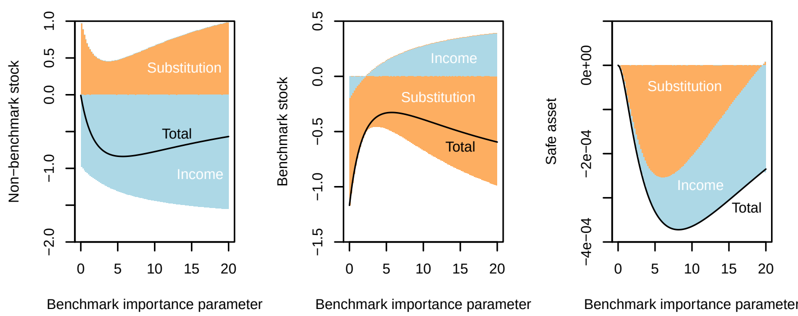
Figure 3·12: *Portfolio volatilities.* This figure plots portfolio volatilities for investors A, B, and R at time $t = 0$ as a function of the benchmark dividend ratio s_0 and the benchmark importance parameters I_A and I_B. The portfolio volatility of investor $j \in \{R, A, B\}$ can be computed as the square root of $(\sigma_{1,t}\pi^j_{1,t} + \sigma_{2,t}\pi^j_{2,t})^2 + \lambda(J_{1,t}\pi^j_{1,t} + J_{2,t}\pi^j_{2,t})^2$.



(a) Portfolio changes of institutional investor A.

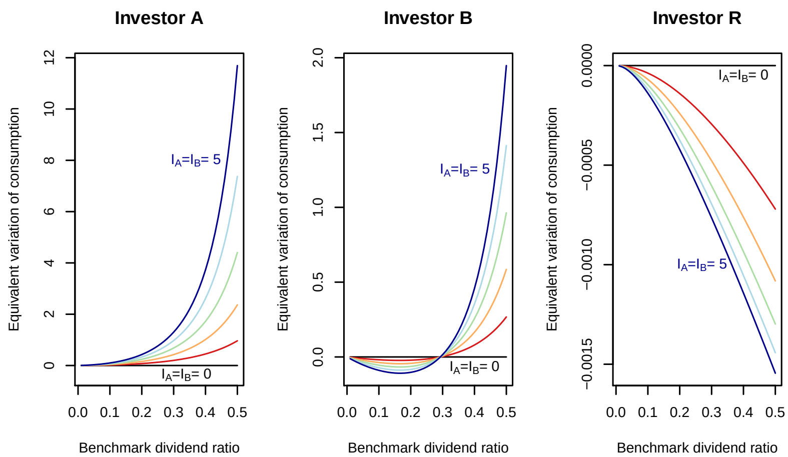


(b) Portfolio changes of institutional investor B.

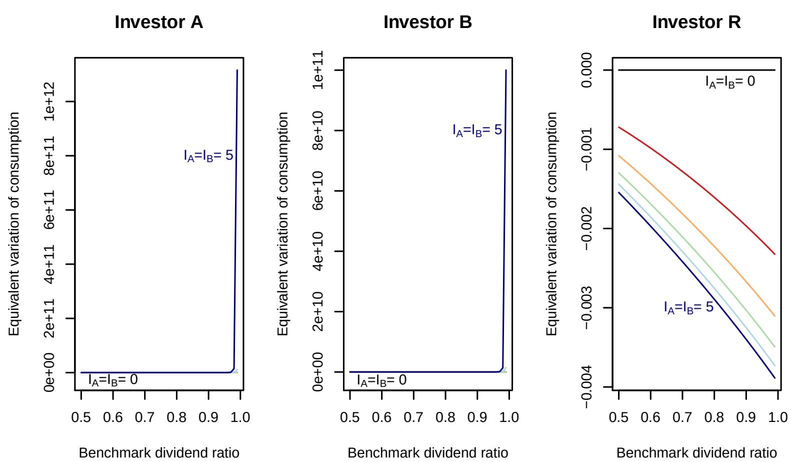


(c) Portfolio changes of retail investor R.

Figure 3·13: *Portfolios changes after a jump when the dividend ratio is low.* These figures plot portfolio changes as functions of the benchmark importance parameter when a jump occurs. We assume $s_0 = 0.2$ and $I_A = I_B$. If a jump occurs at time t, then the portfolio change of investor j is measured as $\pi_t^j W_t^j - \pi_{t-}^j W_{t-}^j$, where π_{t-}^j and W_{t-}^j are the portfolio weights and wealths of investor j right before the jump occurs. Portfolio changes are decomposed into substitution effects $((\pi_t^j - \pi_{t-}^j)W_{t-}^j)$ and income effects $(\pi_t^j(W_t^j - W_{t-}^j))$.



(a) Equivalent variation of consumption for $s_0 \leq 0.5$.



(b) Equivalent variation of consumption for $s_0 \geq 0.5$.

Figure 3·14: *Equivalent variation of consumption.* These figures show the equivalent variation of consumption; i.e., the amount of additional consumption that investor $j \in \{R, A, B\}$ has to consume at any point of time in a world in which $I_A = I_B = 0$ in order to achieve the same level of utility as in a world with $I_A > 0$ or $I_B > 0$. Formally, letting $c^j_{0,t}$ denote the consumption at time t of investor j when $I_A = I_B = 0$, the equivalent variation of consumption (EV^R_t, EV^A_t, EV^B_t) satisfy $\mathbb{E}[\int_0^\infty e^{-\rho_R t} \log c^R_t \mathrm{d}t] = \mathbb{E}[\int_0^\infty e^{-\rho_R t} \log(c^R_{0,t} + EV^R_t)\mathrm{d}t]$, $\mathbb{E}[\int_0^\infty e^{-\rho_A t}(1 + I_A s_t) \log c^A_t \mathrm{d}t] = \mathbb{E}[\int_0^\infty e^{-\rho_A t} \log(c^A_{0,t} + EV^A_t)\mathrm{d}t]$, and $\mathbb{E}[\int_0^\infty e^{-\rho_B t}(1 + I_B s_t) \log c^B_t \mathrm{d}t] = \mathbb{E}[\int_0^\infty e^{-\rho_B t} \log(c^B_{0,t} + EV^B_t)\mathrm{d}t]$. We only plot EV^j_t for $t = 0$; analogous plots hold for all $t > 0$.

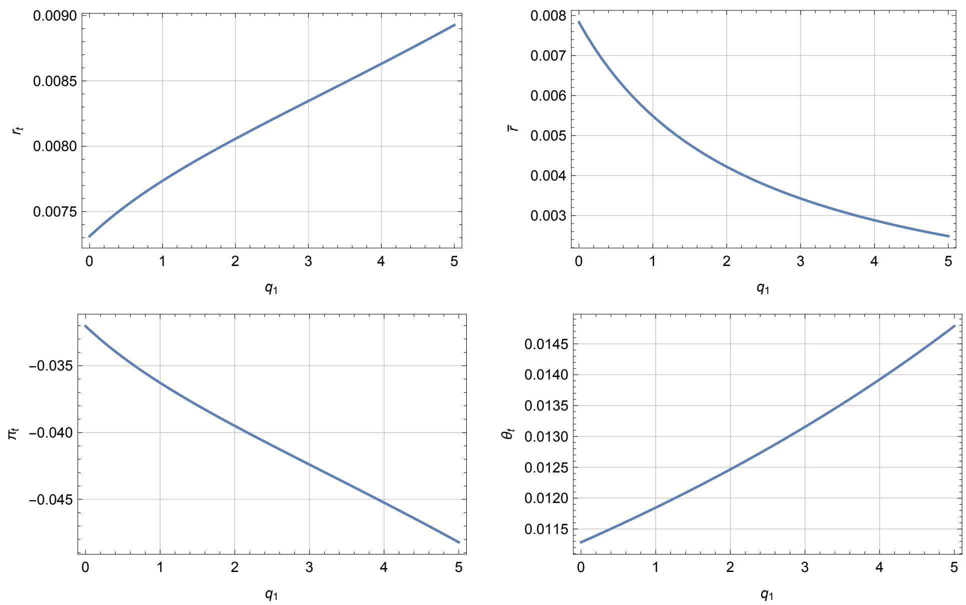
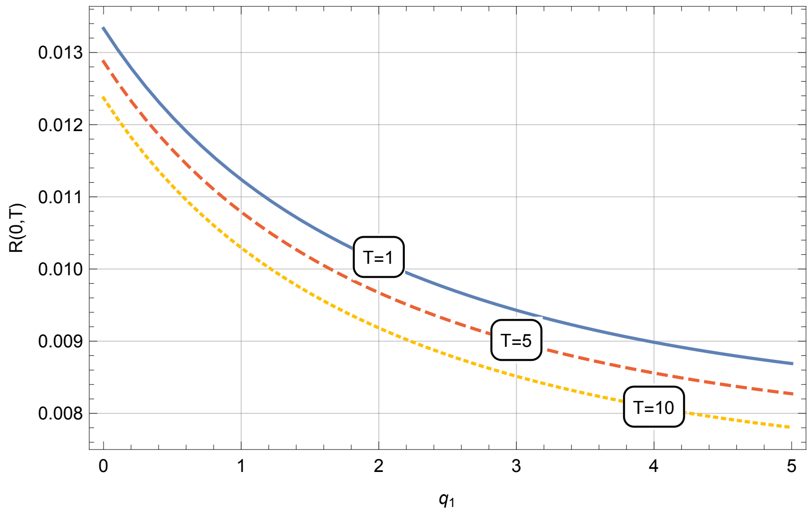
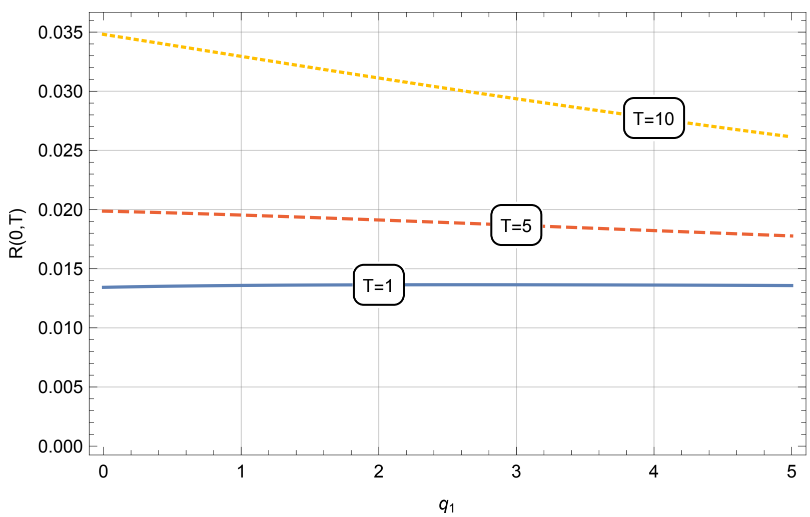


Figure 3·15: *Short-rate, long-rate, expected inflation and risk price.* These figures plot, respectively, the short-rate, long-rate, price of risk and expected inflation against the quantitative easing parameter q_1. The analysis is done at time $t = 0$.

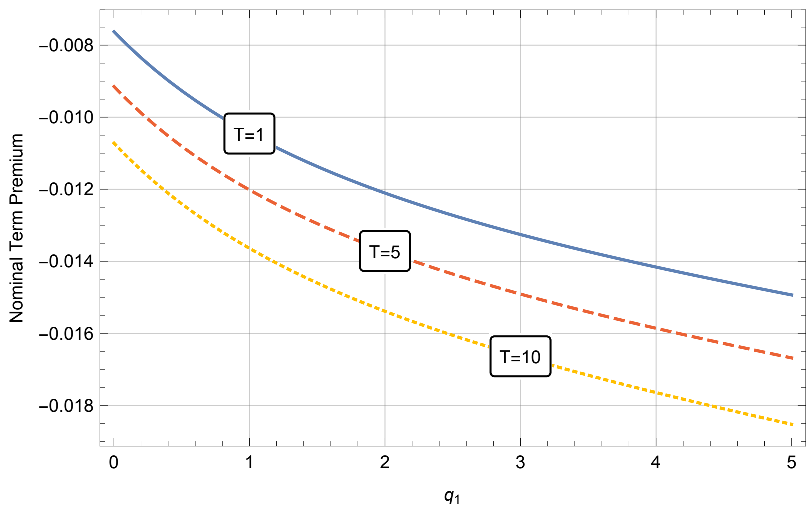


(a) Nominal Yields.

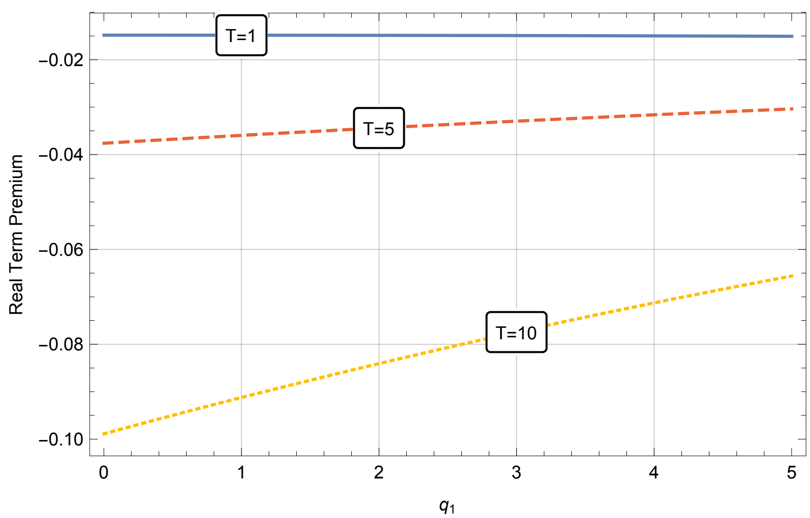


(b) Real Yields.

Figure 3·16: *Nominal and real yields.* Panel (a) plots the nominal yields against the quantitative easing parameter q_1 for nominal bonds maturing in 1, 5 and 10 years. Panel (b) plots the real yields against the quantitative easing parameter q_1 for real bonds maturing in 1, 5 and 10 years. The analysis is done at time $t = 0$.

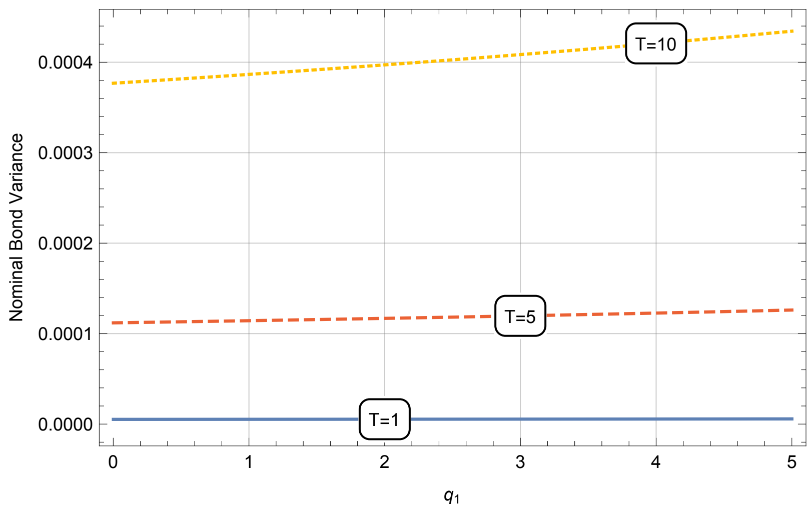


(a) Nominal Term Premium.

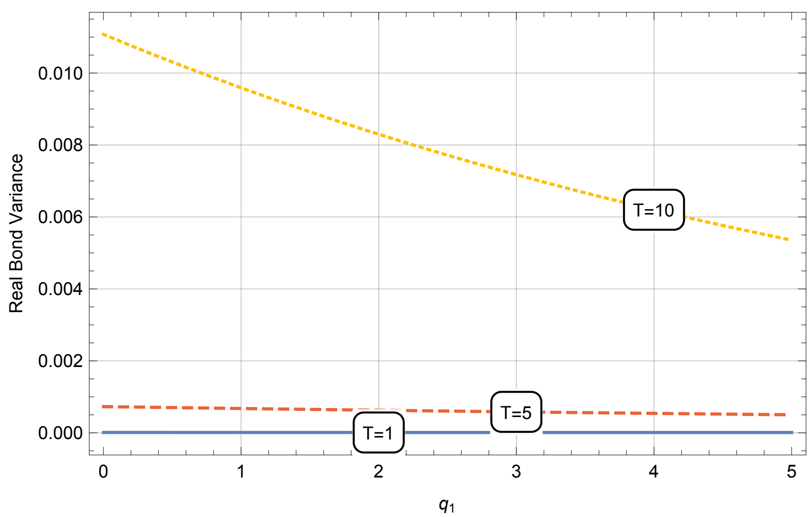


(b) Real Term Premium.

Figure 3·17: *Nominal and real term premium.* Panel (a) plots the nominal term premium against the quantitative easing parameter q_1 for nominal bonds maturing in 1, 5 and 10 years. Panel (b) plots the real term premium against the quantitative easing parameter q_1 for real bonds maturing in 1, 5 and 10 years. The analysis is done at time $t = 0$.

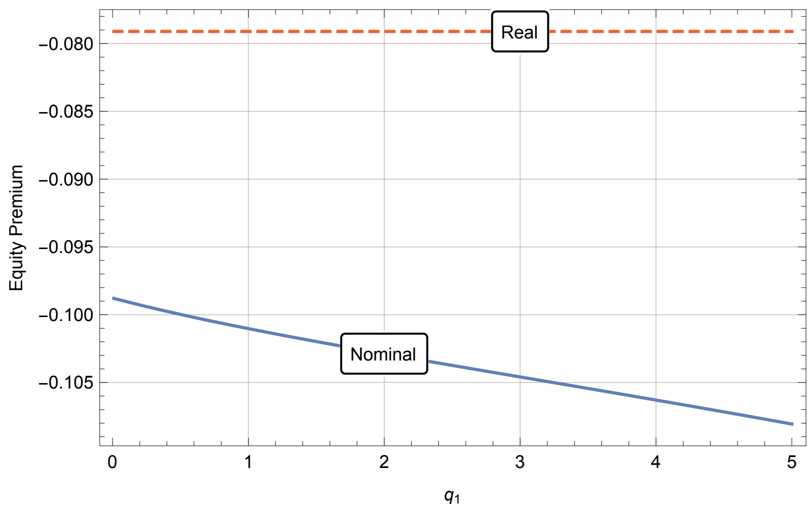


(a) Nominal Bond Variance.

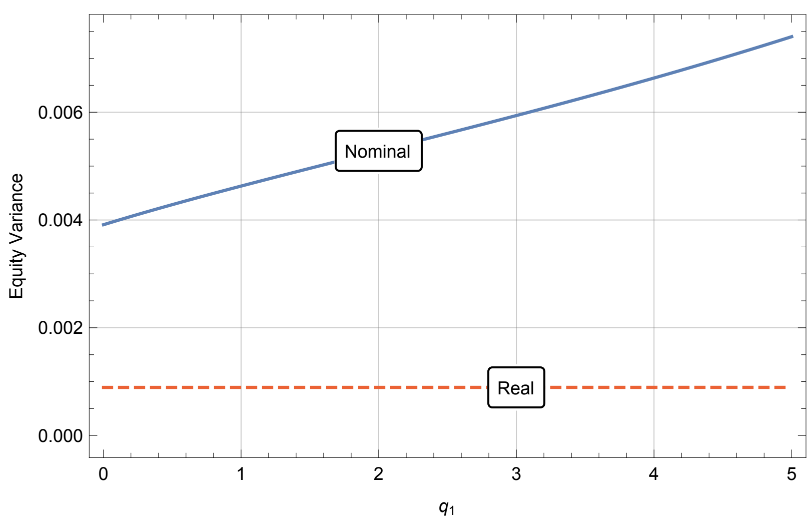


(b) Real Bond Variance.

Figure 3·18: *Nominal and real bond variance.* Panel (a) plots the nominal bond variance against the quantitative easing parameter q_1 for nominal bonds maturing in 1, 5 and 10 years. Panel (b) plots the real bond variance against the quantitative easing parameter q_1 for real bonds maturing in 1, 5 and 10 years. The analysis is done at time $t = 0$.



(a) Equity Premium.



(b) Equity Variance.

Figure 3·19: *Equity premium and variance.* Panel (a) plots the nominal and real equity premium against the quantitative easing parameter q_1. Panel (b) plots the nominal and real equity variance against the quantitative easing parameter q_1. The analysis is done at time $t = 0$.

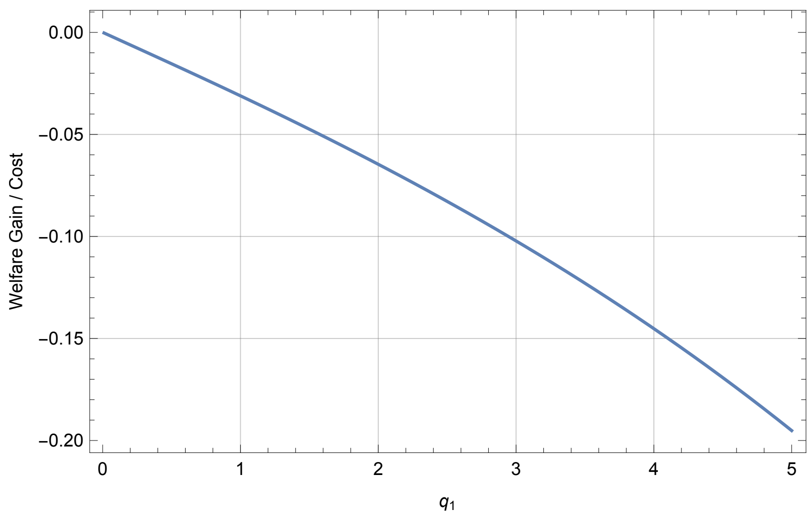


Figure 3·20: *Welfare Cost/Gain.* The figure plots the welfare cost/gain against the quantitative easing parameter q_1. While a positive variation indicates a loss of welfare generated by quantitative easing, a negative variation indicates a welfare gain relative to an economy without quantitative easing, i.e., with $q_1 = 0$. The analysis is done at time $t = 0$.

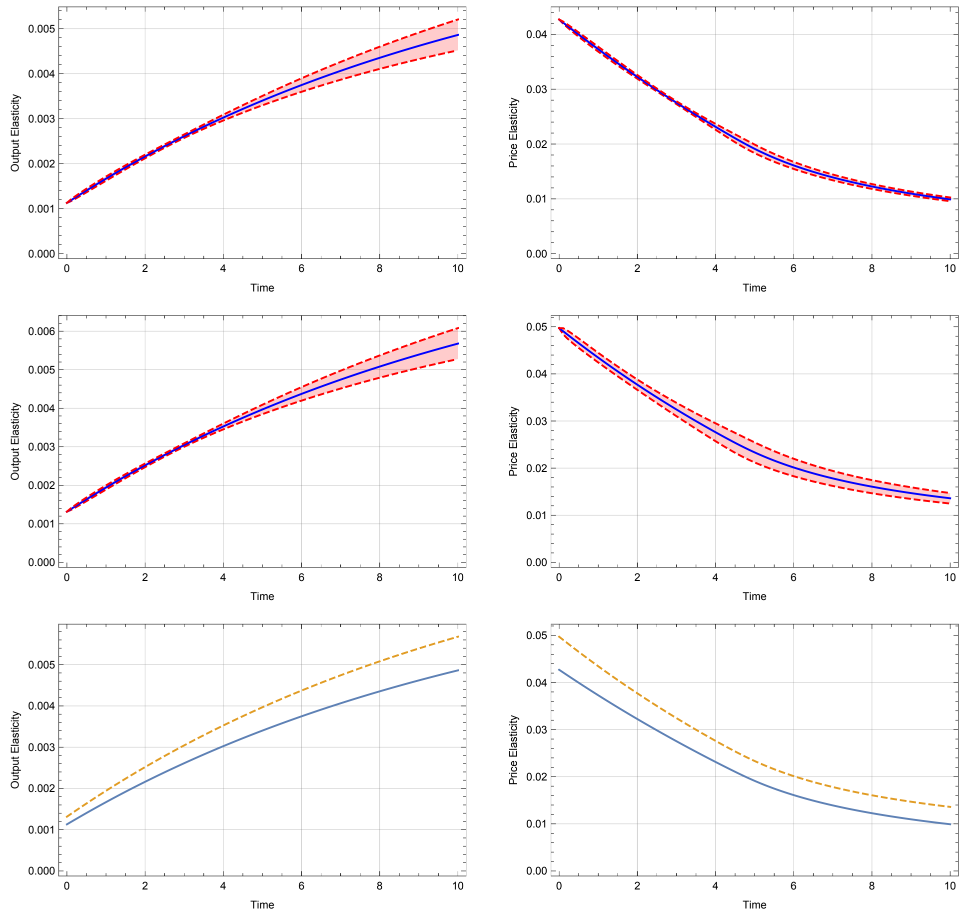


Figure 3·21: *Elasticities.* The figure illustrates the output elasticity against time, in the left column, and the price elasticity against time, in the right column. The elasticities in the top panels are done holding $q_1 = \frac{1}{2}$ and the 95% confidence bounds are also presented. The two panels in the middle of the figure show the elasticities when $q_1 = 1$. The panels in the bottom overlay the elasticities for the cases $q_1 = \frac{1}{2}$ and $q_1 = 1$ for a better comparison. The confidence bounds are suppressed for a better visualization. The time discretization adopted is $\Delta t = 0.1$ and the number of simulations is equal to 3000.

Tables

Preferences	$\rho_A = 0.03$	$\rho_B = 0.02$	$\rho_R = 0.01$
		$I_A = I_B \in [0, 100]$	
Dividends	$\mu_1 = 0.0753$	$\mu_2 = 0.08$	$\lambda = 0.05$
	$\sigma_1 = 0.1715$	$\sigma_2 = 0.17$	$J_1 = 0$
	$J_2 = -0.1$	$D_{1,0} = 2$	$D_{2,0} \in [0.01, 10]$
Initial Wealth	$\alpha^A = 0.375$	$\alpha^B = 0.375$	$\alpha^R = 0.25$

Table 3.1: *Model parameters.* These parameter are chosen to roughly match the parameters estimated by Backus et al. (2011) for U.S. equities and options. We assume that jumps are larger but less frequent than Backus et al. (2011). Similar parameters for jump sizes and jump frequencies of U.S. equities are estimated by Barro (2006) and Wachter (2013).

Preference and Technology	$\beta = 0.08$	$\alpha = 0.57$	$A = 0.08$	$\sigma = 0.03$
	$\tau = 0.1$	$\delta = 0.075$	$K_0 = 1$	
State Variable	$\kappa_g = 0.1$	$\sigma_g = 0.09$	$\bar{g} = 0.0004$	$g_0 = 0.07$
Monetary Policy	$q_1 \in [0, 5]$	$q_2 = -1.1$	$q_3 = 1.1$	$k = 0.034$
	$\bar{\pi} = 0.043$			

Table 3.2: *Parameters.* These parameters are chosen to roughly match the parameters estimated by Buraschi and Jiltsov (2005) and Matthys (2014).

Appendix A

Proofs

A.1 Chapter 2: Equilibrium formulation and solution

An equilibrium in our model consists of consumption plans $(c_t^R)_{t\geq 0}$, $(c_t^A)_{t\geq 0}$, and $(c_t^B)_{t\geq 0}$, portfolio plans $(\pi_t^R)_{t\geq 0}$, $(\pi_t^A)_{t\geq 0}$, $(\pi_t^B)_{t\geq 0}$, benchmark importance parameters I_A and I_B, stock prices $(S_{1,t})_{t\geq 0}$ and $(S_{2,t})_{t\geq 0}$, prices of diffusion $(\theta_t)_{t\geq 0}$ and jump $(\psi_t)_{t\geq 0}$ risk, and an interest rate process $(r_t)_{t\geq 0}$ such that the following conditions are satisfied:

- $(c_t^R)_{t\geq 0}$ and $(\pi_t^R)_{t\geq 0}$ solve

$$\sup\ \mathbb{E}\left[\int_0^\infty e^{-\rho_R t}\log c_t^R \mathrm{d}t\right],\ \text{subject to}\ \mathbb{E}\left[\int_0^\infty \xi_t c_t^R \mathrm{d}t\right] \leq \alpha^R W_0.$$

- For $j \in \{A, B\}$, $(c_t^j)_{t\geq 0}$ and $(\pi_t^j)_{t\geq 0}$ solve

$$\sup\ \mathbb{E}\left[\int_0^\infty e^{-\rho_j t}(1 + I_j s_t)\log c_t^j \mathrm{d}t\right], \text{subject to}\ \mathbb{E}\left[\int_0^\infty \xi_t c_t^j \mathrm{d}t\right] \leq \alpha^B W_0$$

- Asset markets are cleared:

$$W_t^R \pi_{l,t}^R + W_t^A \pi_{l,t}^A + W_t^B \pi_{l,t}^B = 0,$$
$$W_t^R \pi_{1,t}^R + W_t^A \pi_{1,t}^A + W_t^B \pi_{1,t}^B = S_{1,t},$$
$$W_t^R \pi_{2,t}^R + W_t^A \pi_{2,t}^A + W_t^B \pi_{2,t}^B = S_{2,t}.$$

Here, $\xi = (\xi_t)_{t\geq 0}$ is the state price density in the market so that the process

$$t \mapsto \xi_t W_t^j + \int_0^t c_s^j \xi_s \mathrm{d}s$$

is a martingale relative to the complete information filtration $(\mathcal{F}_t)_{t\geq 0}$ for each $j \in \{R, A, B\}$.

As showed by Martin (2013), all equilibrium quantities can be characterized based on a set of moments of s_t. We follow the approach of Martin (2013), and derive semi-analytical expressions for the dividend ratio moments of Section (2.3). These expressions can be computed exactly except for some integrals which need to be computed numerically.

Write $\tilde{y}_{2,v,t} = y_{2,v} - y_{2,t}$ for $y_{2,t} = \log D_{2,t}$. We can express the dividend share as

$$s_v = \frac{e^{\tilde{y}_{2,v,t}+y_{2,t}}}{e^{\tilde{y}_{2,v,t}+y_{2,t}} + e^{\tilde{y}_{1,v,t}+y_{1,t}}}$$

for $v \geq t$. Using this transformation, it follows that

$$\mathbb{E}_t\left[\int_t^\infty e^{-\rho_A(v-t)} s_v dv\right] = \mathbb{E}_t\left[\int_t^\infty e^{-\rho_A(v-t)} \frac{e^{\tilde{y}_{2,v,t}+y_{2,t}}}{e^{\tilde{y}_{2,v,t}+y_{2,t}} + e^{\tilde{y}_{1,v,t}+y_{1,t}}} dv\right]. \tag{A.1}$$

By multiplying and dividing by $e^{-(\tilde{y}_{2,v,t}+y_{2,t}+\tilde{y}_{1,v,t}+y_{1,t})/2}$ and denoting $u_{v,t} = \tilde{y}_{2,v,t} + y_{2,t} - \tilde{y}_{1,v,t} - y_{1,t}$, we can rewrite the integrand as

$$\frac{e^{u_{v,t}/2}}{2\cosh(u_{v,t}/2)}. \tag{A.2}$$

Fourier inversion implies that

$$\frac{1}{2\cosh(u_{v,t}/2)} = \int \mathcal{G}_1(z) e^{izu_{v,t}} \mathrm{d}z,$$

where $\mathcal{G}_1(z) = \frac{1}{2}\mathrm{sech}(\pi z)$. Because all integrands are bounded, the first moment can be expressed as

$$M^j_{1,t} = \mathbb{E}_t\left[\int_t^\infty e^{-\rho_j(v-t)}\int \mathcal{G}_1(z)e^{(iz+1/2)u_{v,t}}dzdv\right] = \int \frac{\mathcal{G}_1(z)e^{(iz+1/2)u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-1/2, iz+1/2)}dz$$

with the function $\boldsymbol{c}(\cdot,\cdot)$ defined as

$$\boldsymbol{c}(\theta_1,\theta_2) = \left(\mu_1 - \frac{1}{2}\sigma_1^2\right)\theta_1 + \left(\mu_2 - \frac{1}{2}\sigma_2^2\right)\theta_2 + \frac{1}{2}\sigma_1^2\theta_1^2 + \frac{1}{2}\sigma_2^2\theta_2^2 + \sigma_1\sigma_2\theta_1\theta_2 + \lambda\left((1+J_1)^{\theta_1} + (1+J_2)^{\theta_2} - 2\right),$$

and $j \in \{A, B, R\}$.

The second and third moments can be calculated in an analogous same way, leading to the expressions

$$M^j_{2,t} = \int \frac{\mathcal{G}_2(z)e^{(iz+1)u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-1, iz+1)}dz, \quad M^j_{3,t} = \int \frac{\mathcal{G}_3(z)e^{(iz+3/2)u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-3/2, iz+3/2)}dz,$$

where

$$\mathcal{G}_\gamma(z) = \frac{1}{2\pi}\frac{\Gamma(\frac{\gamma}{2}+iz)\Gamma(\frac{\gamma}{2}-iz)}{\Gamma(\gamma)}, \quad \gamma \in \{2,3\},$$

and $\Gamma(\cdot)$ represents the standard Euler gamma function. For the jump magnitude moments we can use a similar approach and obtain

$$\Delta M^j_{\gamma,t} = \int \frac{\mathcal{G}_\gamma(z)e^{(iz+\frac{\gamma}{2})u_t}}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})}\left(\left(\frac{1+J_2}{1+J_1}\right)^{iz+\frac{\gamma}{2}} - 1\right)dz, \ \ \gamma \in \{1,2\}.$$

A.1.1 Proofs

Proof of Proposition 2.3.1 and 2.3.2 . Since the state price density depends on aggregate dividends and the dividend share s_t, we derive its dynamics in terms of these state variables. An application of Ito's Lemma gives

$$
\begin{aligned}
dD_t =& D_t \underbrace{((1-s_t)\mu_1 + s_t\mu_2)}_{:=\mu_{d,t}} dt + D_t \underbrace{((1-s_t)\sigma_1 + s_t\sigma_2)}_{:=\sigma_{d,t}} dZ_t \\
&+ D_{t-} \underbrace{((1-s_{t-})J_1 + s_{t-}J_2)}_{:=J_{d,t-}} dN_t.
\end{aligned}
$$

Market clearing implies that, at any time t, aggregate consumption equal to aggregate dividend:

$$D_t = c_t^A + c_t^B + c_t^R.$$

As we show in Proposition 2.3.5,

$$c_t^j = \frac{e^{-\rho_A t}(1 + I_A s_t)\alpha^A W_0 \phi_A}{\xi_t}.$$

for $j \in \{A, B\}$ and

$$c_t^R = \rho_R W_t^R = \frac{\alpha^R W_0 \rho_R e^{-\rho_R t}}{\xi_t}.$$

By plugging these in, we have

$$D_t = W_0 \xi_t^{-1} \left(\gamma_{1t} + \gamma_{2t} + \gamma_{3t} s_t\right),$$

where

$$
\begin{aligned}
\gamma_{1t} &= \alpha^R \rho_R e^{-\rho_R t}, \\
\gamma_{2t} &= \alpha^A \phi_A e^{-\rho_A t} + \alpha^B \phi_B e^{-\rho_B t}, \\
\gamma_{3t} &= \alpha^A I_A \phi_A e^{-\rho_A t} + \alpha^B I_B \phi_B e^{-\rho_B t}.
\end{aligned}
$$

This gives the characterization of the state price density as

$$\xi_t = W_0 \frac{Q_t}{D_t}, \tag{A.3}$$

where we defined $Q_t = \gamma_{1t} + \gamma_{2t} + \gamma_{3t} s_t$. At $t = 0$ we have $\xi_0 = 1$, which implies that initial wealth can be expressed as $W_0 = \frac{D_0}{Q_0}$.

Next, an application of Ito's Lemma on s_t leads to the following stochastic differential equation,

$$ds_t = \underbrace{s_t(1-s_t)\left[\mu_2 - \mu_1 + (\sigma_1 - \sigma_2)\left(\sigma_1(1-s_t) + \sigma_2 s_t\right)\right]}_{=\mu_{s,t}} dt + \underbrace{s_t(1-s_t)(\sigma_2 - \sigma_1)}_{=\sigma_{s,t}} dZ_t$$
$$+ \underbrace{\frac{s_{t-}(1-s_{t-})(J_2 - J_1)}{1 + (1-s_{t-})J_1 + s_{t-}J_2}}_{=J_{s,t-}} dN_t.$$

In order to use the method of undetermined coefficients to pin down market prices of risk and interest rate, we also need the dynamics of D_t^{-1}. Ito's Lemma yields:

$$dD_t^{-1} = (\sigma_{d,t}^2 - \mu_{d,t})D_t^{-1}dt - \sigma_{d,t}D_t^{-1}dZ_t - \frac{J_{d,t-}}{1 + J_{d,t-}} D_{t-}^{-1}dN_t.$$

Using the dynamics derived for s_t, we can express the dynamics of Q_t as

$$dQ_t = Q_t\mu_{q,t}dt + \underbrace{\gamma_{3t}\sigma_{s,t}}_{:=Q_t\sigma_{q,t}} dZ_t + \underbrace{\gamma_{3t}J_{s,t-}}_{:=Q_{t-}J_{q,t-}} dN_t,$$

where

$$\mu_{q,t} = -\rho_R\gamma_{1t} - \alpha^A\phi_A\rho_A e^{-\rho_A t}(1 + I_A s_t) - \alpha^B\phi_B\rho_B e^{-\rho_B t}(1 + I_B s_t) + \gamma_{3t}\mu_{s,t}.$$

An additional application of Ito's Lemma on the right hand side of (A.3) gives

$$d\xi_t = -\xi_t\left(\mu_{d,t} - \mu_{q,t} - \sigma_{d,t}^2 + \sigma_{d,t}\sigma_{q,t} + \left[1 - \frac{1 + J_{q,t}}{1 + J_{d,t}}\right]\lambda\right)dt$$
$$- \xi_t(\sigma_{d,t} - \sigma_{q,t})dZ_t + \xi_{t-}\left[\frac{1 + J_{q,t-}}{1 + J_{d,t-}} - 1\right](dN_t - \lambda dt).$$

By matching the coefficients we obtain the following expressions for risk prices:

$$\theta_t = \sigma_{d,t} - \sigma_{q,t}\ ;\quad \psi_t = \frac{1 + J_{q,t}}{1 + J_{d,t}}\ ;$$
$$r_t = \mu_{d,t} - \mu_{q,t} - \sigma_{d,t}^2 + \sigma_{d,t}\sigma_{q,t} + (1 - \psi_t)\lambda.$$

□

Proof of Proposition 2.3.3. Market completeness implies that $\xi_t S_{2,t} = \mathbb{E}_t\left[\int_t^\infty \xi_v D_{2v} dv\right]$.

Therefore,

$$\frac{Q_t}{D_t}S_{2,t} = \mathbb{E}_t\Big[\int_t^{\infty}\Big(\alpha^R\rho_R e^{-\rho_R v}s_v + (\alpha^A\phi_A e^{-\rho_A v} + \alpha^B\phi_B e^{-\rho_B v})s_v$$
$$+ (I_A\alpha^A\phi_A e^{-\rho_A v} + I_B\alpha^B\phi_B e^{-\rho_B v})s_v^2\Big)\mathrm{d}v\Big]$$
$$= \alpha^R\rho_R e^{-\rho_R t}M^R_{1,t} + \alpha^A\phi_A e^{-\rho_A t}M^A_{1,t} + \alpha^B\phi_B e^{-\rho_B v}M^B_{1,t}$$
$$+ I_A\alpha^A\phi_A e^{-\rho_A t}M^A_{2,t} + I_B\alpha^B\phi_B e^{-\rho_B t}M^B_{2,t}. \quad \text{(A.4)}$$

We reformulate and obtain:

$$S_{2,t} = \frac{D_t}{Q_t}\left(\alpha^R\rho_R e^{-\rho_R t}M^R_{1,t} + \alpha^A\phi_A e^{-\rho_A t}(M^A_{1,t} + I_A M^A_{2,t}) + \alpha^B\phi_B e^{-\rho_B t}(M^B_{1,t} + I_B M^B_{2,t})\right).$$

We apply the same approach for asset $S_{1,t}$ and obtain:

$$S_{1,t} = \frac{D_t}{Q_t}\left(\alpha^R e^{-\rho_R t} + \alpha^A\phi_A e^{-\rho_A t}(\rho_A^{-1} + I_A M^A_{1,t}) + \alpha^B\phi_B e^{-\rho_B t}(\rho_B^{-1} + I_B M^B_{1,t})\right) - S_{2,t}.$$

Next, we compute the volatility and jump size functions for both asset prices by the method of coefficient matching. Define the function f as

$$f(s_t; z, \gamma) \equiv e^{(iz+\frac{\gamma}{2})\log\left(\frac{s_t}{1-s_t}\right)} = \left(\frac{s_t}{1-s_t}\right)^{iz+\frac{\gamma}{2}},$$

such that

$$M^j_{\gamma,t} = \int \frac{\mathcal{G}_\gamma(z)e^{(iz+\frac{\gamma}{2})u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})}dz = \int \frac{\mathcal{G}_\gamma(z)f(s_t; z, \gamma)}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})}dz.$$

Ito's formula implies that

$$\begin{aligned}
\mathrm{d}M^j_{\gamma,t} = & (\cdots)\,\mathrm{d}t + \int \frac{\mathcal{G}_\gamma(z) f'(s_t;z,\gamma)\sigma_{s,t}}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} dz \mathrm{d}Z_t \\
& + \int \frac{\mathcal{G}_\gamma(z)\left(f(s_t;z,\gamma) - f(s_{t-};z,\gamma)\right)}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} dz \mathrm{d}N_t \\
= & (\cdots)\,\mathrm{d}t + \int \frac{\mathcal{G}_\gamma(z)(\frac{\gamma}{2}+iz)e^{(iz+\frac{\gamma}{2})u_{t,t}}(\sigma_2-\sigma_1)}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} dz \mathrm{d}Z_t \\
& + \int \frac{\mathcal{G}_\gamma(z)e^{(iz+\frac{\gamma}{2})u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} \left[\left(\frac{1+J_2}{1+J_1}\right)^{iz+\frac{\gamma}{2}} - 1\right] dz \mathrm{d}N_t.
\end{aligned}$$

Because we are only matching the uncertainty coefficients, the drift is irrelevant to our calculations. We have for $\gamma \in \{1,2\}$:

$$\int \left(iz+\frac{\gamma}{2}\right) \frac{\mathcal{G}_\gamma(z)e^{(iz+\frac{\gamma}{2})u_{t,t}}}{\rho_A - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} dz = \gamma\left(M^j_{\gamma,t} - M^j_{\gamma+1,t}\right),$$

$$\int \frac{\mathcal{G}_\gamma(z)e^{(iz+\frac{\gamma}{2})u_{t,t}}}{\rho_j - \boldsymbol{c}(-iz-\frac{\gamma}{2}, iz+\frac{\gamma}{2})} \left[\left(\frac{1+J_2}{1+J_1}\right)^{iz+\frac{\gamma}{2}} - 1\right] \mathrm{d}z = \Delta M^j_{\gamma,t}.$$

Therefore,

$$\mathrm{d}M^j_{\gamma,t} = (\cdots)\,\mathrm{d}t + \gamma(\sigma_2-\sigma_1)\left(M^j_{\gamma,t} - M^j_{\gamma+1,t}\right)\mathrm{d}Z_t + \Delta M^j_{\gamma,t}\mathrm{d}N_t.$$

On the other hand, the product rule implies that

$$\mathrm{d}\xi_t S_{2,t} = (\cdots)\,\mathrm{d}t + \xi_t S_{2,t}\left(\sigma_{2,t} - \theta_t\right)\mathrm{d}Z_t + \xi_{t-}S_{2,t-}((1+J_{2,t-})\psi_{t-} - 1)\mathrm{d}N_t.$$

A match of the coefficients on both sides of (A.4) and using the relations above results in:

$$\sigma_{2,t} =\theta_t + \frac{D_t(\sigma_2-\sigma_1)}{Q_tS_{2,t}}\Big[\alpha^R\rho_R e^{-\rho_R t}(M^R_{1,t}-M^R_{2,t}) + \alpha^A\phi_A e^{-\rho_A t}(M^A_{1,t}-M^A_{2,t})$$
$$+ \alpha^B\phi_B e^{-\rho_B t}(M^B_{1,t}-M^B_{2,t}) + 2I_A\alpha^A\phi_A e^{-\rho_A t}(M^A_{2,t}-M^A_{3,t})$$
$$+ 2I_B\alpha^B\phi_B e^{-\rho_B t}(M^B_{2,t}-M^B_{3,t})\Big],$$
$$J_{2,t} =\psi_t^{-1} - 1 + \frac{D_t}{\psi_t Q_t S_{2,t}}\Big(\alpha^R\rho_R e^{-\rho_R t}\Delta M^R_{1,t} + \alpha^A\phi_A e^{-\rho_A t}\Delta M^A_{1,t}$$
$$+ \alpha^B\phi_B e^{-\rho_B t}\Delta M^B_{1,t} + I_A\alpha^A\phi_A e^{-\rho_A t}\Delta M^A_{2,t} + I_B\alpha^B\phi_B e^{-\rho_B t}\Delta M^B_{2,t}\Big).$$

An analogous approach for the price of asset 1 yields:

$$\sigma_{1,t} =\theta_t\left(1+\frac{S_{2,t}}{S_{1,t}}\right) + \frac{D_t(\sigma_2-\sigma_1)}{Q_tS_{1,t}}\Big[I_A\alpha^A\phi_A e^{-\rho_A t}(M^A_{1,t}-M^A_{2,t})$$
$$+ I_B\alpha^B\phi_B e^{-\rho_B t}(M^{2B}_{1,t}-M^{2B}_{2,t})\Big] - \sigma_{2,t}\frac{S_{2,t}}{S_{1,t}}$$
$$J_{1,t} =\psi_t^{-1}\Big(1+\frac{S_{2,t}}{S_{1,t}} + \frac{1}{S_{1,t}}\frac{D_t}{Q_t}(I_A\alpha^A\phi_A e^{-\rho_A t}\Delta M^A_{1,t}$$
$$+ I_B\alpha^B\phi_B e^{-\rho_B t}\Delta M^B_{1,t})\Big) - 1 - \frac{S_{2,t}}{S_{1,t}}(1+J_{2t})$$

□

Proof of Proposition 2.3.4. With the optimal consumption policies and prices of risk determined, we turn to the characterization of the optimal allocation rules. To simplify the exposition of the result, let us introduce the matrix notation:

$$\pi^A_t = [\pi^A_{1,t}\ \pi^A_{2,t}]',\ \pi^B_t = [\pi^B_{1,t}\ \pi^B_{2,t}]',\ \pi^R_t = [\pi^R_{1,t}\ \pi^R_{2,t}]',$$
$$\boldsymbol{\sigma}_t = [\sigma_{1,t}\ \sigma_{2,t}]',\ \boldsymbol{J}_t = [J_{1,t}\ J_{2,t}]',$$
$$\Sigma_t = [\boldsymbol{\sigma}_t\ \boldsymbol{J}_t]',\ \boldsymbol{\mu}_t = [\mu_{1,t}\ \mu_{2,t}\ ]',\boldsymbol{r}_t = [r_t\ r_t]'.$$

Then, the wealth process for agent j, with $j \in \{R, A, B\}$, satisfies

$$dW^j_t = (W^j_t(r_t + [\boldsymbol{\mu}_t - \boldsymbol{r}_t]'\pi^j_t) - c^j_t)dt + W^j_t[dZ_t\ dN_t]\cdot(\Sigma_t\pi^j_t).$$

Given that the process $\xi_t W_t^j + \int_0^t c_s^j \xi_s ds$ is a martingale with dynamics

$$d(\xi_t W_t^j) + c_t^j \xi_t dt = \left(\boldsymbol{\sigma}'_t \pi_t^j - \theta_t\right) dZ_t + \left((1 + \boldsymbol{J}'_t \pi_t^j)\psi_t - 1\right) (dN_t - \lambda dt), \tag{A.5}$$

we use the equality $\xi_t W_t^j + \int_0^t \xi_s c_s^j ds = \mathbb{E}_t \left[\int_0^\infty \xi_s c_s^j ds\right]$ and the optimal policies c_t^j characterized above, to match uncertainty loadings and solve for the optimal allocation. Starting with insitutional investor A, we have the following equality

$$\begin{aligned} \xi_t W_t^A =& \mathbb{E}_t \left[\int_t^\infty \xi_s c_s^A ds\right] = \mathbb{E}_t \left[\int_t^\infty e^{-\rho_A(v-t)}(1 + I_A s_v) dv\right] \alpha^A W_0 \phi_A e^{-\rho_A t} \\ \xi_t W_t^A =& \alpha^A W_0 \phi_A e^{-\rho_A t} \left(\rho_A^{-1} + I_A M_{1,t}^A\right), \end{aligned} \tag{A.6}$$

where the last equality follows from $M_{1,t}^A = \mathbb{E}_t \left[\int_t^\infty e^{-\rho_A(v-t)} s_v dv\right]$.

Since we are only interested in the loadings on the uncertainty sources, we focus on the last term on the right hand side in the expression above and match the coefficients with the one in (A.5). Proceeding as we did in the previous section, we apply Ito's Lemma and match the uncertainty loadings. It follows that

$$\begin{aligned} \xi_t W_t^A (\boldsymbol{\sigma}'_t \pi_t^A - \theta_t) =& I_A \phi_A \alpha^A W_0 e^{-\rho_A t} \mathbb{E}_t \left[\int_t^\infty e^{-\rho_A(v-t)} (\sigma_2 - \sigma_1) s_v (1 - s_v) dv\right] \\ =& I_A \phi_A \alpha^A W_0 e^{-\rho_A t} (\sigma_2 - \sigma_1) \left(M_{1,t}^A - M_{2,t}^A\right), \end{aligned}$$

We can further simplify by substituting (A.6) in the expression above to obtain

$$\boldsymbol{\sigma}'_t \pi_t^A = \theta_t + \frac{I_A(\sigma_2 - \sigma_1)\left(M_{1,t}^A - M_{2,t}^A\right)}{\rho_A^{-1} + I_A M_{1,t}^A}.$$

To conclude the characterization, we need to calculate the hedging against the Poisson shock. Proceeding as before, we have

$$\xi_t W_t^A ((1 + \boldsymbol{J}'_t \pi_t^A)\psi_t - 1) = I_A \phi_A \alpha^A W_0 e^{-\rho_A t} \Delta M_{1,t}^A,$$

which can be simplified to

$$\boldsymbol{J}'_t \pi_t^A = \psi_t^{-1} - 1 + \frac{I_A \Delta M_{1t}^A}{\psi_t(\rho_A^{-1} + I_A M_{1,t}^A)}.$$

The optimal portfolio is characterized by

$$\pi_t^A = \Sigma_t^{-1} \begin{bmatrix} \theta_t \\ \psi_t^{-1} - 1 \end{bmatrix} + \Sigma_t^{-1} \begin{bmatrix} (\sigma_2 - \sigma_1)(M_{1,t}^A - M_{2,t}^A) \\ \Delta M_{1,t}^A \psi_t^{-1} \end{bmatrix} \frac{I_A}{\rho_A^{-1} + I_A M_{1,t}^A}.$$

To conclude the proof, we solve the investor's problem. From the martingale relation, we have

$$\begin{aligned} \xi_t W_t^R = \mathbb{E}_t \left[\int_t^\infty \xi_s c_s^R dv \right] =& \mathbb{E}_t \left[\int_t^\infty \alpha^R W_0 \rho_R e^{-\rho_R v} dv \right] \\ \xi_t W_t^R =& \alpha^R W_0 e^{-\rho_R t}. \end{aligned} \tag{A.7}$$

An application of Ito's Lemma on the last term on (A.7) and matching the uncertainties coefficients as done before, leads to the following characterization of the optimal weights:

$$\pi_t^R = \Sigma_t^{-1} \begin{bmatrix} \theta_t \\ \psi_t^{-1} - 1 \end{bmatrix}.$$

□

Proof of Proposition 2.3.5. We start solving the institutional investors' problems. We focus on investor A; the derivation for investor B is analogous. From the first order condition we have

$$e^{-\rho_A t}(1 + I_A s_t)(c_t^A)^{-1} = y\xi_t \Rightarrow c_t^A = e^{-\rho_A t}(1 + I_A s_t)\xi_t^{-1} y^{-1}, \tag{A.8}$$

where y is the Lagrange multiplier obtained by substituting (A.8) in the budget constraint:

$$\begin{aligned} \mathbb{E} \left[\int_0^\infty \xi_t c_t^A dt \right] =& \mathbb{E} \left[\int_0^\infty e^{-\rho_A t}(1 + I_A s_t) y^{-1} dt \right] = \alpha^A W_0 \\ y^{-1} =& \frac{\alpha^A W_0}{\mathbb{E} \left[\int_0^\infty e^{-\rho_A t}(1 + I_A s_t) dt \right]}. \end{aligned}$$

By denoting

$$\phi_A = \left(\frac{1}{\rho_A} + I_A \mathbb{E}\left[\int_0^\infty e^{-\rho_A t} s_t dt\right]\right)^{-1},$$

and substituting y^{-1} in (A.8), we have the following expression for the optimal consumption level for institutional investors:

$$c_t^A = \frac{e^{-\rho_A t}(1 + I_A s_t)\alpha^A W_0 \phi_A}{\xi_t}.$$

Now, we analyze the optimization problem for the retail investor. This agent's first order condition is

$$e^{-\rho_R t}(c_t^R)^{-1} = y_R \xi_t \Rightarrow c_t^R = e^{-\rho_R t} \xi_t^{-1} y_R^{-1}. \tag{A.9}$$

Plugging (A.9) in the investor's budget constraint gives that

$$\mathbb{E}\left[\int_0^\infty \xi_t c_t^R dt\right] = \mathbb{E}\left[\int_0^\infty e^{-\rho_R t} y_R^{-1} dt\right] = \alpha^R W_0 \Rightarrow y_R^{-1} = \frac{\alpha^R W_0}{\mathbb{E}\left[\int_0^\infty e^{-\rho_R t} dt\right]} = \alpha^R W_0 \rho_R.$$

Substituting y_R^{-1} back into (A.9) yields:

$$c_t^R = \frac{e^{-\rho_R t} \alpha^R W_0 \rho_R}{\xi_t}.$$

□

A.2 Chapter 3: Equilibrium formulation and solution

The methodology I used can be summarized in the following steps:

(i) Given the affine structure of the problem, conjecture an affine structure for the dynamics of the price level.

(ii) Using the evolution of the price level, obtain the dynamics for capital that only depends on the capital itself and the state variable g_t.

(iii) Set the Hamilton-Jacobi-Bellman equation for the representative agent using the capital accumulation equation (3.5) and the resources constraint present in Definition 3.2.1, respectively.

(iv) Using the first order conditions, obtain the optimal policies for consumption and money demand.

(v) Plugging the controls back into the Hamilton-Jacobi-Bellman equation and using the traditional guess for the value function, $J(K_t, g_t) = \frac{1}{\beta}\log(\beta K_t) + \mu_1 g_t + \mu_0$, obtain a system that it is linear on the state variable g_t.

(vi) Using the method of undetermined coefficients, solve for μ_0 and μ_1.

(vii) Substitute the conjectured solution back into the Hamilton-Jacobi-Bellman equation and verified that it is in fact the true solution.

(viii) Obtain the optimal policies for consumption, money and investment as a linear function on capital.

(ix) Conjecture an affine structure in the state variables for the short rate and use it with the market clearing for money market to obtain a new equation representing the dynamics of the price level p_t.

(x) Use the method of undetermined coefficients to pin down the price level exposures to each shock.

(xi) Using household preferences and the capital accumulation equation, derive the pricing kernel expression for the economy.

(xii) Apply Ito's lemma to recover the short rate and solve the fixed point problem for the exposures of r_t to state variables.

(xiii) Derive the expression for the nominal bond using derived pricing kernel.

(xiv) Derive the explicit solution for the nominal yield and take the limit on the maturity to obtain the long-term rate and conclude the characterization of the equilibrium.

There are two state variables in the problem: capital K_t and the money growth persistence component g_t. Capital accumulates according to the equation:

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t^k - \tau K_t \frac{dp_t}{p_t}. \tag{A.10}$$

Using firm's output equation

$$Y_t = AK_t,$$

the resources constraint

$$Y_t = I_t + c_t + m_t,$$

I rewrite (A.10) as

$$dK_t = ((A - \delta - \tau\pi_t)K_t - c_t - m_t)dt + \sigma K_t dZ_t^k - \tau K_t \frac{dp_t}{p_t}. \tag{A.11}$$

Following Buraschi and Jiltsov (2005), I assume the following affine structure for the evolution of price level p_t, expected inflation π_t and the short term rate r_t:

$$\frac{dp_t}{p_t} = \pi_t dt + \sigma_k dZ_t^k + \sigma_m \sqrt{g_t} dZ_t^m, \tag{A.12}$$

$$\pi_t = \Pi_0 + \Pi_1 g_t, \tag{A.13}$$

$$r_t = r_0 + r_1 g_t, \tag{A.14}$$

where $\sigma_k, \sigma_m, \Pi_0, \Pi_1, r_0$ and r_1 are determined in equilibrium.

By substituting the evolution of p_t and π_t in (A.11), we have

$$dK_t = ((A - \delta - \tau(\Pi_0 + \Pi_1 g_t))K_t - c_t - m_t)dt + (\sigma - \tau\sigma_k)K_t dZ_t^k - \tau\sigma_m\sqrt{g_t}K_t dZ_t^m. \tag{A.15}$$

We solve the investor's utility maximization problem subject to (A.15) and the evolution of the state variables g_t, which satisfy the following stochastic differential equation:

$$dg_t = \kappa_g(\bar{g} - g_t)dt + \sigma_g\sqrt{g_t}dZ_t^m.$$

The Hamilton-Jacobi-Bellman (HJB) equation for the social planner is

$$\begin{aligned} 0 = \sup_{c_t, m_t} \Bigg\{ & -\beta J + \alpha \log c_t + (1-\alpha)\log m_t \\ & + J_K((A - \delta - \tau(\Pi_0 + \Pi_1 g_t))K_t - c_t - m_t) + K_t^2 J_{KK}\left(\frac{(\sigma - \tau\sigma_t^k)^2}{2} + \frac{(\tau\sigma_t^m)^2}{2}g_t\right) \\ & + J_g\kappa_g(\bar{g} - g_t) + \frac{\sigma_g^2}{2}g_t J_{gg} - \tau\sigma_m\sigma_g g_t K_t J_{Kg}\Bigg\}. \end{aligned} \tag{A.16}$$

The first order condition with respect to c_t and m_t gives

$$\begin{aligned} c_t =& \frac{\alpha}{J_K}, \\ m_t =& \frac{1-\alpha}{J_K}. \end{aligned}$$

Plugging the optimal controls back and the conjecture

$$J(K_t, g_t) = \frac{1}{\beta}\log(\beta K_t) + \mu_1 g_t + \mu_0, \tag{A.17}$$

where μ_0 and μ_1 are constants to be determined, in (A.16), we obtain

$$\begin{aligned} 0 =& -\frac{(1-\alpha)\log(1-\alpha) + \alpha\log\alpha + 2A - 2\beta(\beta\mu_0 - \theta_g\kappa_g\mu_1 + 1)}{\beta} \\ & - \frac{2\delta - 2\Pi_0\tau - (\sigma - \sigma_k\tau)^2}{\beta} + g_t\frac{(2\beta^2\mu_1 + 2\beta\kappa_g\mu_1 + 2\Pi_1\tau + \sigma_m^2\tau^2)}{\beta} \end{aligned}$$

For this equation to be satisfied at all time, we use the method of undetermined coefficients, set all coefficients to zero and solve for μ_0 and μ_1. It follows that

$$\begin{aligned} 0 =& 2\beta^2\mu_1 + 2\beta\kappa_g\mu_1 + 2\Pi_1\tau + \sigma_m^2\tau^2, \\ \mu_1 =& \frac{-2\Pi_1\tau - \sigma_m^2\tau^2}{2\beta(\beta + \kappa_g)}. \end{aligned}$$

The expression for μ_0 reduces to

$$\mu_0 = \frac{(1-\alpha)\log(1-\alpha) + \alpha\log\alpha + A - \beta - \delta + \beta\bar{g}\kappa_g\mu_1 - \Pi_0\tau - \frac{(\sigma - \tau\sigma_k)^2}{2}}{\beta^2}.$$

With the conjectured solution completely characterized, I plug (A.17) into (A.16) and verify that it is indeed the true solution. At this stage, I proved that, given constants $\sigma_k, \sigma_m, \Pi_0, \Pi_1, r_0, r_1, y_0$ and y_1, (A.17) is the true solution associated with the Hamilton-Jacobi-Bellman equation in (A.16). The following steps demonstrate how to pin down these constants.

First, I use the expression derived for (A.17) to obtain the following expressions for the optimal policies on consumption, money demand and investment, respectively,

$$c_t = \alpha\beta K_t, \tag{A.18}$$

$$m_t = (1-\alpha)\beta K_t, \tag{A.19}$$

$$I_t = (A - \beta)K_t. \tag{A.20}$$

By replacing the optimal policies into (A.15), we have the following *endogenous* capital evolution in equilibrium

$$\frac{dK_t}{K_t} = (A - \delta - \beta - \tau(\Pi_0 + \Pi_1 g_t))dt + (\sigma - \tau\sigma_k)dZ_t^k - \tau\sigma_m\sqrt{g_t}dZ_t^m, \tag{A.21}$$

with solution given by

$$\begin{aligned} K_T =& K_t \exp\left\{ \int_t^T \left(A - \delta - \beta - \tau\Pi_0 - \frac{(\sigma - \tau\sigma_k)^2}{2} - \left(\tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2} \right) g_v \right) dv \right. \\ & \left. + (\sigma - \tau\sigma_k)(Z_T^k - Z_t^k) - \tau\sigma_m \int_t^T \sqrt{g_v} dZ_v^m \right\}. \end{aligned} \tag{A.22}$$

Next, I turn to the money market clearing. By imposing money demand equals to money supply, it follows that:

$$M_t^s = m_t p_t = p_t(1 - \alpha)\beta K_t,$$

where the last equality follows from replacing m_t by the optimal cash demand expression derived in (A.19). Applying Ito's lemma and rearranging terms, we can express the price level evolution as

$$\frac{dp_t}{p_t} = \frac{dM_t^s}{M_t^s} - \frac{dK_t}{K_t} - \text{cov}_t\left(\frac{dp_t}{p_t}, \frac{dK_t}{K_t}\right) dt. \tag{A.23}$$

By using the expression in (A.12), I rewrite the covariance term above as

$$\sigma_t^{mk} \equiv \text{cov}_t\left(\frac{dp_t}{p_t}, \frac{dK_t}{K_t}\right) = \sigma_k(\sigma - \tau\sigma_k) - \tau\sigma_m^2 g_t. \tag{A.24}$$

In addition, we used the money demand equation:

$$\frac{dM_t^s}{M_t^s} = q_1\left(dr_t - \bar{r}dt\right) + q_2\left(\frac{dp_t}{p_t} - \bar{\pi}dt\right) + q_3\left(\frac{dK_t}{K_t} - \bar{k}dt\right) + dg_t, \tag{A.25}$$

with the affine conjecture that the equilibrium short interest rate is given by:

$$r_t = r_0 + r_1 g_t.$$

Substituting (A.21), (A.25) and (A.24) in (A.23), I have the following dynamics for the price level p_t:

$$\begin{aligned}\frac{dp_t}{p_t} = & \left(\frac{(q_1 r_1 + 1)\kappa_g \bar{g} + (q_3 - 1)(A - \delta - \beta - \tau \Pi_0) - (q_1 \bar{r} + q_2 \bar{\pi} + q_3 \bar{k}) - \sigma_k(\sigma - \tau\sigma_k)}{1 - q_2} \right. \\ & \left. + \frac{\tau\sigma_m^2 - (q_1 r_1 + 1)\kappa_g - \tau(q_3 - 1)\Pi_1}{1 - q_2} g_t \right) dt + \frac{q_3 - 1}{1 - q_2}(\sigma - \tau\sigma_k) dZ_t^k \\ & + \frac{(q_1 r_1 + 1)\sigma_g - \tau(q_3 - 1)\sigma_m}{1 - q_2} \sqrt{g_t} dZ_t^m.\end{aligned}$$

Matching the coefficients above with the ones in (A.12), it follows that

$$\begin{aligned}\sigma_k &= \frac{q_3 - 1}{1 - q_2 + \tau(q_3 - 1)}\sigma, \qquad \sigma_m = \frac{q_1 r_1 + 1}{1 - q_2 + \tau(q_3 - 1)}\sigma_g, \\ \Pi_1 &= \frac{\tau\sigma_m^2 - (q_1 r_1 + 1)\kappa_g}{1 - q_2 + \tau(q_3 - 1)}, \\ \Pi_0 &= \frac{(q_1 r_1 + 1)\kappa_g \bar{g} + (q_3 - 1)(A - \delta - \beta) - (q_1 \bar{r} + q_2 \bar{\pi} + q_3 \bar{k}) - \sigma_k(\sigma - \tau\sigma_k)}{1 - q_2 + \tau(q_3 - 1)}.\end{aligned} \tag{A.26}$$

which gives the results presented in Proposition 3.2.3.

In order to complete the characterization of the equilibrium, we need to determine the coefficients r_0, r_1 and $\bar{r}$. For this reason, I turn to the characterization of the state price density in this economy, which is given by

$$\xi_t = e^{-\beta t}\frac{K_0}{K_t}. \tag{A.27}$$

An application of Ito's formula leads to

$$\begin{aligned}\frac{d\xi_t}{\xi_t} =& - \left(A - \delta - \tau\Pi_0 - (\sigma - \tau\sigma_k)^2 - \tau(\Pi_1 + \tau\sigma_m^2)g_t\right) dt \\ & - (\sigma - \tau\sigma_k)dZ_t^k + \tau\sigma_m\sqrt{g_t}dZ_t^m.\end{aligned}$$

The structure above implies that the prices of risk are given by

$$\begin{aligned}\theta_k =& \sigma - \tau\sigma_k, \\ \theta_t^m \equiv& \theta_m\sqrt{g_t} = -\tau\sigma_m\sqrt{g_t}.\end{aligned}$$

The expression for the short interest rate reduces to

$$r_t = A - \delta - \tau\Pi_0 - (\sigma - \tau\sigma_k)^2 - \tau(\Pi_1 + \tau\sigma_m^2)g_t.$$

Observing that the short rate is an affine function of g_t, we can rewrite it as $r_t = r_0 + r_1 g_t$, where the coefficients r_0 and r_1 solve the equations:

$$\begin{aligned}r_1 =& -\tau(\Pi_1 + \tau\sigma_m^2), \\ r_0 =& A - \delta - \tau\Pi_0 - (\sigma - \tau\sigma_k)^2.\end{aligned}$$

Substituting the expressions for Π_1 and σ_m derived in (A.26), we obtain the following quadratic equation for r_1:

$$\begin{aligned}0 =& r_1^2 \frac{q_1^2\tau^2}{(q_2 - q_3\tau + \tau - 1)^3} \\ & + r_1 \left(\frac{q_1\tau}{1 - q_2 - \tau(q_3 - 1)} \left(\kappa_g - \tau \left(1 + \frac{2}{(1 - q_2 - \tau(q_3 - 1))^2} \right) \right) + 1 \right) \\ & + \frac{\tau}{1 - q_2 - \tau(q_3 - 1)} \left(\kappa_g + \tau \left(1 - \frac{1}{(1 - q_2 - \tau(q_3 - 1))^2} \right) \right),\end{aligned}$$

with solution given by

$$
\begin{aligned}
r_1 = & \frac{(q_2+\tau(1-q_3)-1)^3}{2q_1^2\sigma_g^2\tau^2(2-q_2+\tau(q_3-1))} \cdot \\
& \cdot \left(\frac{q_1\tau\left(2\sigma_g^2\tau(q_2-1+\tau(1-q_3))+(q_2-1+\tau(1-q_3))^2-2\sigma_g^2\tau\right)}{(q_2-1+\tau(1-q_3))^3} + 1 \right. \\
& - \left(\frac{\tau^2\left((q_2-1)(q_1-3q_3+3)(q_1-q_3+1)+4q_1(q_2-2)\sigma_g^2\right)}{(q_2-1+\tau(1-q_3))^3} \right. \\
& + \frac{(q_2-1)^2\tau(2q_1-3q_3+3)-(q_3-1)\tau^3\left((q_1-q_3+1)^2+4q_1\sigma_g^2\right)}{(q_2-1+\tau(1-q_3))^3} \\
& \left. \left. + \frac{(q_2-1)^3}{(q_2-1+\tau(1-q_3))^3} \right)^{1/2} \right).
\end{aligned}
$$

With the characterization of r_1, I turn to the characterization of the nominal bond price and its long term yield $\bar{r}$. Once this quantity is determined, all other equilibrium quantities are completely determined.

A.2.1 Nominal and Real Bonds

The expression for the nominal bond $B^n(t,T)$ is represented by

$$
B^n(t,T) = \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{p_t}{p_T}\right] = \mathbb{E}_t\left[e^{-\beta(T-t)}K_{T,t}p_{T,t}\right], \tag{A.28}
$$

where I defined

$$
p_{T,t} \equiv \frac{p_t}{p_T}, \qquad K_{T,t} \equiv \frac{K_t}{K_T}.
$$

Using the evolution described in (A.12), the solution for the stochastic differential equation for p_t can be expressed as

$$
p_T = p_t \exp\left\{ \int_t^T \left(\pi_s - \frac{\sigma_k^2}{2} - \frac{\sigma_m^2}{2}g_s\right)ds + \sigma_k(Z_T^k - Z_t^k) + \sigma_m \int_t^T \sqrt{g_s}\,dZ_s^m \right\} \tag{A.29}
$$

Rewriting the solution for g_t in (3.7) as

$$\sigma_g \int_t^T \sqrt{g_s} dZ_s^m = g_T - g_t - \kappa_g \bar{g}(T-t) + \kappa_g \int_t^T g_s ds,$$

and substituting it on (A.29), along with $\pi_t = \Pi_0 + \Pi_1 g_t$, I write the ratio $p_{T,t} \equiv \frac{p_t}{p_T}$ as

$$\begin{aligned} p_{T,t} =& \exp\left\{ \int_t^T \left(\frac{\sigma_k^2}{2} - \Pi_0 + \frac{\sigma_m}{\sigma_g}\kappa_g \bar{g} + \left(\frac{\sigma_m^2}{2} - \Pi_1 - \frac{\sigma_m}{\sigma_g}\kappa_g \right) g_s \right) ds \right. \\ & \left. - \sigma_k (Z_T^k - Z_t^k) + \frac{\sigma_m}{\sigma_g} (g_T - g_t) \right\}. \end{aligned} \tag{A.30}$$

Similarly, I write $K_{T,t} \equiv \frac{K_t}{K_T}$ as

$$\begin{aligned} K_{T,t} =& \exp\left\{ \int_t^T \left(\delta + \beta - A + \tau \Pi_0 - \tau \frac{\sigma_m}{\sigma_g}\kappa_g \bar{g} + \frac{(\sigma - \tau\sigma_k)^2}{2} \right. \right. \\ & \left. \left. + \left(\frac{(\tau\sigma_m)^2}{2} + \tau\Pi_1 + \tau \frac{\sigma_m}{\sigma_g}\kappa_g \right) g_s \right) ds - (\sigma - \tau\sigma_k)(Z_T^k - Z_t^k) + \tau \frac{\sigma_m}{\sigma_g} (g_T - g_t) \right\}. \end{aligned} \tag{A.31}$$

Using that shocks are independent and substituting (A.30) and (A.31) in (A.28), it follows that

$$\begin{aligned} B^n(t,T) =& \mathbb{E}_t \left[\exp\left\{ \int_t^T \left(\delta - A + (\tau - 1)\Pi_0 + (1-\tau) \frac{\sigma_m}{\sigma_g}\kappa_g \bar{g} + (\sigma - \tau\sigma_k)^2 \right. \right. \right. \\ & \left. + \sigma_k^2 + \sigma_k(\sigma - \tau\sigma_k) + \left[\frac{\sigma_m^2}{2}(\tau^2 + 1) + (\tau - 1)\Pi_1 + (\tau - 1)\frac{\sigma_m}{\sigma_g}\kappa_g \right] g_s \right) ds \\ & \left. \left. + (\tau - 1)\frac{\sigma_m}{\sigma_g}(g_T - g_t) \right\} \right] \\ =& e^{-\omega_n g_t - \rho_0^n \cdot (T-t)} \mathbb{E}_t \left[e^{-\int_t^T \rho_1^n g_s ds + \omega_n g_T} \right], \end{aligned} \tag{A.32}$$

where

$$\rho_0^n = A - \delta + (1-\tau)\Pi_0 + (\tau - 1)\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g} + (\sigma - \tau\sigma_k)^2 + \sigma_k^2 + \sigma_k(\sigma - \tau\sigma_k), \quad \text{(A.33)}$$

$$\rho_1^n = (1-\tau)\Pi_1 - (1+\tau^2)\frac{\theta_m^2}{2} + (1-\tau)\frac{\sigma_m}{\sigma_g}\kappa_g, \qquad \omega_n = (\tau - 1)\frac{\sigma_m}{\sigma_g}. \quad \text{(A.34)}$$

Following Cox et al. (1985) and Duffie et al. (2000), the conditional expectation in (A.32) can be expressed as a function of g_t, taking the form

$$\mathbb{E}_t\left[e^{-\int_t^T \rho_1^n g_s ds + \omega_n g_T}\right] = e^{\eta_0^n(T-t) + \eta_1^n(T-t) g_t},$$

where $\eta_0^n(\cdot)$ and $\eta_1^n(\cdot)$ are functions of time to maturity and they are determined by solving the associated Riccati equation:

$$\eta_1^{n\prime}(t) = -\eta_1^n(t)\kappa_g + \frac{1}{2}(\eta_1^n(t))^2\sigma_g^2 - \rho_1^n,$$
$$\eta_0^{n\prime}(t) = \eta_1^n(t)\kappa_g\bar{g},$$

with boundary conditions given by

$$\eta_1^n(0) = \omega_n,$$
$$\eta_0^n(0) = 0.$$

I follow Duffie (2005) and express η_1^n and η_0^n as

$$\eta_1^n(T-t) = \frac{1 + a_1 e^{b_1(T-t)}}{c_1 + d_1 e^{b_1(T-t)}},$$
$$\eta_0^n(T-t) = \kappa_g\bar{g}\frac{a_1c_1 - d_1}{b_1c_1d_1}\log\left(\frac{c_1 + d_1 e^{b_1(T-t)}}{c_1 + d_1}\right) + \frac{\kappa_g\bar{g}}{c_1}(T-t), \quad \text{(A.35)}$$

where

$$
\begin{aligned}
c_1 =& -\frac{\kappa_g + \sqrt{\kappa_g^2 + 2\sigma_g^2\rho_1^n}}{2\rho_1^n}, \\
d_1 =& (1-c_1)\omega_n \frac{\sigma_g^2\omega_n - \kappa_g + \sqrt{(\sigma_g^2\omega_n - \kappa_g)^2 - \sigma_g^2(\sigma_g^2(\omega_n)^2 - 2\omega_n\kappa_g - 2\rho_1^n)}}{\sigma_g^2(\omega_n)^2 - 2\omega_n\kappa_g - 2\rho_1^n}, \\
a_1 =& (d_1 + c_1)\omega_n - 1, \\
b_1 =& \frac{-d_1(\kappa_g + 2\rho_1^n c_1) + a_1(\sigma_g^2 - \kappa_g)}{a_1 c_1 - d_1}.
\end{aligned}
\tag{A.36}
$$

Using the characterization above, the nominal bond can be written as

$$B^n(t,T) = e^{-\rho_0^n\cdot(T-t) + \eta_0^n(T-t) + (\eta_1^n(T-t) - \omega_n)g_t}, \tag{A.37}$$

and the nominal yield rate as

$$R^n(t,T) = -\frac{1}{T-t}\log B^n(t,T) = \rho_0^n - \frac{\eta_0^n(T-t)}{T-t} + \frac{\omega_n - \eta_1^n(T-t)}{T-t} g_t. \tag{A.38}$$

To conclude the equilibrium, we take the limit of (A.38) as T goes to infinity to obtain the long term rate:

$$\bar{r} = \lim_{T\to\infty} R^n(t,T).$$

Assuming that the limits exist and using L'Hôpital's rule, I obtain:

$$
\begin{aligned}
\lim_{T\to\infty} \frac{\eta_0^n(T-t)}{T-t} &= \kappa_g \bar{g}\frac{a_1}{d_1}, \\
\lim_{T\to\infty} \frac{\omega_n - \eta_1^n(T-t)}{T-t} &= 0.
\end{aligned}
$$

Thus, the long-term rate is given by

$$\bar{r} = \rho_0^n - \kappa_g \bar{g}\frac{a_1}{d_1}. \tag{A.39}$$

Substituting the expression for ρ_0^n derived in (A.33) in (A.39), we have

$$\bar{r} = A - \delta + (1-\tau)\Pi_0 + (\tau - 1)\frac{\sigma_m}{\sigma_g}\kappa_g \bar{g} + (\sigma - \tau\sigma_k)^2 + \sigma_k^2 + \sigma_k(\sigma - \tau\sigma_k) - \kappa_g \bar{g}\frac{a_1}{d_1}.$$

Using the expression for Π_0 derived in (A.26) and solving for $\bar{r}$, I obtain:

$$\begin{aligned}\bar{r} =& \frac{1 - q_2 + \tau(q_3 - 1)}{1 - q_2 + q_1 + \tau(q_3 - q_1 - 1)}\left(A - \delta + (\tau - 1)\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g} + (\sigma - \tau\sigma_k)^2\right. \\ &\left. + \sigma_k^2 + \sigma_k(\sigma - \tau\sigma_k) - \kappa_g\bar{g}\frac{a_1}{d_1}\right) + (1-\tau)\frac{(q_1 r_1 + 1)\kappa_g\bar{g}}{1 - q_2 + q_1 + \tau(q_3 - q_1 - 1)} \\ &+ (1-\tau)\frac{(q_3 - 1)(A - \delta - \beta) - (q_2\bar{\pi} + q_3\bar{k}) - \sigma_k(\sigma - \tau\sigma_k)}{1 - q_2 + q_1 + \tau(q_3 - q_1 - 1)}\end{aligned}$$

This complete the characterization of all equilibrium quantities in my model.

Next, I determine the dynamics of B_t^n, which can be recovered by applying Ito's lemma on (A.37). It follows that

$$\begin{aligned}\frac{dB_t^n}{B_t^n} =& (\rho_0^n - \eta_0^{n\prime} - \eta_1^{n\prime} g_t)dt + (\eta_1^n(T-t) - \omega_n)(\kappa_g(\bar{g} - g_t)dt \\ &+ \sigma_g\sqrt{g_t}dZ_t^m) + (\eta_1^n(T-t) - \omega_n)^2\frac{\sigma_g^2}{2}g_t dt \\ =& \Bigg(\rho_0^n - \eta_0^{n\prime} + (\eta_1^n(T-t) - \omega_n)\kappa_g\bar{g} \\ &+ \left((\eta_1^n(T-t) - \omega_n)\left((\eta_1^n(T-t) - \omega_n)\frac{\sigma_g^2}{2} - \kappa_g\right) - \eta_1^{n\prime}\right)g_t\Bigg)dt \\ &+ (\eta_1^n(T-t) - \omega_n)\sigma_g\sqrt{g_t}dZ_t^m.\end{aligned}$$

The bond risk premium, also know as the term premium, TP_t, can be expressed as:

$$\begin{aligned}TP_t =& \rho_0^n - \eta_0^{n\prime} + (\eta_1^n(T-t) - \omega_n)\kappa_g\bar{g} - r_0 \\ &+ \left((\eta_1^n(T-t) - \omega_n)\left((\eta_1^n(T-t) - \omega_n)\frac{\sigma_g^2}{2} - \kappa_g\right) - \eta_1^{n\prime} - r_1\right)g_t\end{aligned} \qquad \text{(A.40)}$$

The expressions for the real bonds is easier to obtain. Using the expression for the state price density in (A.27), we have

$$\begin{aligned} B^r(t,T) &= \mathbb{E}_t[\xi_{t,T}] = \mathbb{E}_t\left[e^{-\beta(T-t)}\frac{K_t}{K_T}\right] \\ &= \mathbb{E}_t\Bigg[\exp\Bigg\{\int_t^T\Bigg(\delta+\beta-A+\tau\Pi_0-\tau\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g}+\frac{(\sigma-\tau\sigma_k)^2}{2} \\ &+\left(\frac{(\tau\sigma_m)^2}{2}+\tau\Pi_1+\tau\frac{\sigma_m}{\sigma_g}\kappa_g\right)g_s\Bigg)ds-(\sigma-\tau\sigma_k)(Z_T^k-Z_t^k) \\ &+\tau\frac{\sigma_m}{\sigma_g}\left(g_T-g_t\right)\Bigg\}\Bigg] \\ &= e^{-\rho_0^r\cdot(T-t)+\eta_0^r(T-t)+(\eta_1^r(T-t)-\omega_r)g_t}. \end{aligned} \tag{A.41}$$

where

$$\begin{aligned} \rho_0^r =& A-\delta-\tau\Pi_0-\tau\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g}-(\sigma-\tau\sigma_k)^2 \\ \rho_1^r =& -\left(\tau\frac{\sigma_m}{\sigma_g}\kappa_g+\tau\Pi_1+\frac{(\tau\sigma_m)^2}{2}\right), \\ \omega_r =& \tau\frac{\sigma_m}{\sigma_g}. \end{aligned}$$

Invoking Cox et al. (1985) and Duffie et al. (2000) one more time and proceeding as in Appendix A.2.1, I rewrite the conditional expectations as a function of the state variable g_t:

$$\mathbb{E}_t\left[e^{-\int_t^T\rho_1^r g_s ds+\omega_r g_T}\right] = e^{\eta_0^r(T-t)+\eta_1^r(T-t)g_t},$$

where $\eta_0^r(\cdot)$ and $\eta_1^r(\cdot)$ are characterized by solving the associated Riccati equations in the exact same way as in Appendix A.2.1.

Using the characterization above, the real bond can be written as

$$B^r(t,T) = e^{-\rho_0^r\cdot(T-t)+\eta_0^r(T-t)+(\eta_1^r(T-t)-\omega_r)g_t}. \tag{A.42}$$

The real yield rate is

$$R_r(t,T) = -\frac{1}{T-t}\log B^r(t,T) = \rho_0^r - \frac{\eta_0^r(T-t)}{T-t} + \frac{\omega_r - \eta_1^n(T-t)}{T-t} g_t.$$

The dynamics of $B^r(t,T)$ can be recovered by applying Ito's lemma on (A.42). It follows that

$$\begin{aligned}\frac{dB_t^r}{B_t^r} =& \Bigg(\rho_0^r - \eta_0^{r\prime} + (\eta_1^r(T-t) - \omega_r)\kappa_g \bar{g} \\ &+ \left((\eta_1^r(T-t) - \omega_r)\left((\eta_1^r(T-t) - \omega_r)\frac{\sigma_g^2}{2} - \kappa_g\right) - \eta_1^{r\prime}\right) g_t\Bigg) dt \\ &+ (\eta_1^n(T-t) - \omega_r)\sigma_g \sqrt{g_t} dZ_t^m.\end{aligned}$$

The bond real risk premium is denoted by:

$$\begin{aligned}TP_t^r =& \rho_0^r - \eta_0^{r\prime} + (\eta_1^r(T-t) - \omega_r)\kappa_g \bar{g} - r_0 \\ &+ \left((\eta_1^r(T-t) - \omega_r)\left((\eta_1^r(T-t) - \omega_r)\frac{\sigma_g^2}{2} - \kappa_g\right) - \eta_1^{r\prime} - r_1\right) g_t.\end{aligned}$$

A.2.2 Nominal Asset Prices

The characterization of the state price density allows us to compute the price of a continent claim on consumption in nominal terms. The price can be expressed as

$$\begin{aligned}S_t =& \mathbb{E}_t\left[\int_t^\infty \xi_{t,s} p_{s,t} c_s ds\right] = \mathbb{E}_t\left[\int_t^\infty e^{-\beta(T-t)}\frac{K_t}{K_s}\frac{p_t}{p_s}\alpha\beta K_s ds\right] \\ =& \alpha\beta K_t \int_t^\infty \mathbb{E}_t\left[e^{-\beta(T-t)}\frac{p_t}{p_s}\right] ds = \alpha\beta K_t \int_t^\infty B^n(t,s) ds \\ =& \alpha\beta K_t \int_t^\infty e^{-\rho_0^n\cdot(T-t) + \eta_0^n(T-t) + (\eta_1^n(T-t) - \omega_n)g_t} ds = \alpha\beta K_t h(g_t),\end{aligned}$$

where I defined

$$h(g_t) = \int_t^\infty e^{-\rho_0^n \cdot (T-t) + \eta_0^n (T-t) + (\eta_1^n (T-t) - \omega_n) g_t} ds.$$

Note that the expression for real stock price can only be solve in closed form up to an integral.

To obtain the evolution of the real stock price, I apply Ito's lemma in (3.12) and obtain the following dynamics for S_t:

$$\begin{aligned}
\frac{dS_t}{S_t} =& \frac{dh(g_t)}{h(g_t)} + \frac{dK_t}{K_t} + \operatorname{cov}\left(\frac{dh(g_t)}{h(g_t)}, \frac{dK_t}{K_t}\right) \\
\frac{dS_t}{S_t} =& \left(\frac{\partial_g h}{h}\kappa_g(\bar{g} - g_t) + \frac{\sigma_g^2}{2}\frac{\partial_{gg} h}{h} g_t\right) dt + \frac{\partial_g h}{h}\sigma_g \sqrt{g_t} dZ_t^m + (A - \delta - \beta) dt \\
& + (\sigma - \tau\sigma_k) dZ_t^k - \tau\sigma_m \sqrt{g_t} dZ_t^m - \tau\sigma_m\sigma_g \frac{\partial_g h}{h} g_t dt \\
\frac{dS_t}{S_t} =& \left(A - \delta - \beta + \frac{\partial_g h}{h}\kappa_g \bar{g} + \left(\frac{\sigma_g^2}{2}\frac{\partial_{gg} h}{h} - (\tau\sigma_m\sigma_g + \kappa_g)\frac{\partial_g h}{h}\right) g_t\right) dt \\
& + (\sigma - \tau\sigma_k) dZ_t^k + \left(\frac{\partial_g h}{h}\sigma_g - \tau\sigma_m\right)\sqrt{g_t} dZ_t^m.
\end{aligned}$$

The nominal equity premium can be expressed by

$$RP_t = A - \delta - \beta + \frac{\partial_g h}{h}\kappa_g \bar{g} - r_0 + \left(\frac{\sigma_g^2}{2}\frac{\partial_{gg} h}{h} - (\tau\sigma_m\sigma_g + \kappa_g)\frac{\partial_g h}{h} - r_1\right) g_t.$$

The characterization of a claim on consumption in real terms follows directly from the relation:

$$S_t^r = \mathbb{E}_t\left[\int_t^\infty \xi_{t,s} c_s ds\right] = \mathbb{E}_t\left[\int_t^\infty e^{-\beta(s-t)} \frac{K_t}{K_s} \alpha\beta K_s ds\right] = \alpha K_t.$$

Thus, the evolution of the real stock price is equal the evolution of capital.

The real equity premium can be expressed by

$$RP_t^r = A - \delta - \beta - \tau\Pi_0 - r_0 - (\tau\Pi_1 + r_1) g_t.$$

A.2.3 Welfare Analysis

Let $J(q_1, K_0)$ be the expected utility of the representative investor in an economy where monetary authorities target the slope with intensity q_1. By denoting $1-\gamma$ the fraction of capital K_0 that the representative agent should receive or waive in order to obtain the same utility level from the economy with no short-rate targeting, i.e., $q_1 = 0$. Thus, we want to find γ such that the following identity holds:

$$J(q_1, K_0) = J(0, (1-\gamma)K_0). \tag{A.43}$$

Using the expressions for the optimal consumption and money demand derived in (A.18) and (A.19), respectively, we have

$$\begin{aligned} J(q_1, K_0) =& \mathbb{E}_t \left[\int_0^\infty e^{-\beta t} \left(\alpha \log c_t + (1-\alpha) \log m_t \right) dt \right] \\ =& \mathbb{E}_t \left[\int_0^\infty e^{-\beta t} \left(\alpha \log(\alpha \beta K_t) + (1-\alpha) \log(1-\alpha)\beta K_t \right) dt \right] \\ =& \frac{\alpha \log \alpha + (1-\alpha)\log(1-\alpha) + \log \beta}{\beta} + \mathbb{E}_t \left[\int_0^\infty e^{-\beta t} \log K_t dt \right] \end{aligned} \tag{A.44}$$

Using the expression for capital stock in (A.22), it follows that

$$\begin{aligned} \mathbb{E}[\log K_t] =& \log K_0 + \left(A - \delta - \beta - \tau \Pi_0 - \tau \frac{\sigma_m}{\sigma_g} \kappa_g \bar{g} - \frac{(\sigma - \tau \sigma_k)^2}{2} \right) t \\ & - \left(\tau \frac{\sigma_m}{\sigma_g} \kappa_g + \tau \Pi_1 + \frac{(\tau \sigma_m)^2}{2} \right) \int_0^t \mathbb{E}[g_v] dv - \tau \frac{\sigma_m}{\sigma_g} (\mathbb{E}[g_t] - g_0). \end{aligned} \tag{A.45}$$

Substituting the expression

$$\mathbb{E}[g_v] = g_0 e^{-\kappa_g v} + \bar{g}(1 - e^{-\kappa_g v}),$$

into (A.45), we obtain the following expression for $\mathbb{E}[\log K_t]$

$$\begin{aligned}\mathbb{E}[\log K_t] =& \log K_0 + \left(A - \delta - \beta - \tau\Pi_0 - \tau\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g} - \frac{(\sigma - \tau\sigma_k)^2}{2}\right. \\ & \left. - \left(\tau\frac{\sigma_m}{\sigma_g}\kappa_g + \tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2}\right)\bar{g}\right)t - \left(\tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2}\right)\cdot \\ & \cdot\left(\frac{1 - e^{-\kappa_g t}}{\kappa_g}\right)(g_0 - \bar{g})\end{aligned} \tag{A.46}$$

It follows that the final expression for the expected utility is

$$\begin{aligned}J(q_1, K_0) =& \frac{\alpha\log\alpha + (1-\alpha)\log(1-\alpha) + \log\beta}{\beta} + \frac{\log K_0}{\beta} \\ & + \frac{1}{\beta^2}\left(A - \delta - \beta - \tau\Pi_0 - \tau\frac{\sigma_m}{\sigma_g}\kappa_g\bar{g} - \frac{(\sigma - \tau\sigma_k)^2}{2}\right. \\ & \left. - \left(\tau\frac{\sigma_m}{\sigma_g}\kappa_g + \tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2}\right)\bar{g}\right) \\ & - \left(\tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2}\right)\frac{g_0 - \bar{g}}{\beta(\beta + \kappa_g)}\end{aligned} \tag{A.47}$$

Note that in the expression above, only Π_0, Π_1 and σ_m are functions of $q1$. I make this dependence on q_1 explicit in the welfare cost/gain expression.

Using the identity (A.43) and solving for γ, I obtain the following expression for the welfare cost/gain:

$$\begin{aligned}\gamma(q_1) =& 1 - \exp\left\{ -\frac{1}{\beta^2}\left(\tau(\Pi_0(q_1) - \Pi_0(0)) + \tau\frac{\sigma_m(q_1) - \sigma_m(0)}{\sigma_g}\kappa_g\bar{g}\right.\right. \\ & \left. + \left(\tau\frac{\sigma_m(q_1) - \sigma_m(0)}{\sigma_g}\kappa_g + \tau(\Pi_1(q_1) - \Pi_1(0)) + \tau^2\frac{\sigma_m^2(q_1) - \sigma_m^2(0)}{2}\right)\bar{g}\right) \\ & \left. - \left(\tau(\Pi_1(q_1) - \Pi_1(q_0)) + \tau^2\frac{\sigma_m^2(q_1) - \sigma_m^2(q_0)}{2}\right)\frac{g_0 - \bar{g}}{\beta(\beta + \kappa_g)}\right\}.\end{aligned} \tag{A.48}$$

A.2.4 Impulse Response

To calculate the output and price level elasticities, I follow Detemple et al. (2003). First, at time $t < T$, I calculate the Malliavin derivative of output Y_T in the direction of the persistent monetary shock Z^m. Given that output is an AK model with constant productivity factor A, we can focus on the Malliavin derivative of capital and I denote this quantity by $\mathcal{D}_t^m K_T$. By Malliaving differentiating the expression in (A.22), we have

$$\mathcal{D}_t^m K_T = - K_T \left(\int_t^T \left(\tau \Pi_1 + \frac{(\tau\sigma_m)^2}{2} \right) \mathcal{D}_t^m g_v dv + \tau\sigma_m \int_t^T \frac{\mathcal{D}_t^m g_v}{2\sqrt{g_v}} dZ_v^m + \tau\sigma_m \sqrt{g_t} \right). \tag{A.49}$$

The Malliavin derivative of the state variable g_t, $\mathcal{D}_t^m g_v$, can be solved in closed form. By Malliavin differentiating the dynamics of g_t in (3.7),

$$d\mathcal{D}_t^m g_u = - \kappa_g \mathcal{D}_t^m g_u du + \mathcal{D}_t^m g_u \frac{\sigma_g}{2\sqrt{g_u}} dZ_u^m, \qquad \mathcal{D}_t^m g_t = \sigma_g \sqrt{g_t}.$$

and integrating from t to T, we obtain:

$$\mathcal{D}_t^m g_T = \sigma_g \sqrt{g_t} \mathbb{E}_t \left[e^{-\int_t^T \left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_v}} \right) dv + \int_t^T \frac{\sigma_g}{2\sqrt{g_v}} dZ_v^m} \right]. \tag{A.50}$$

Using Clark-Hausmann-Ocone representation and the expression in (A.50) for $\mathcal{D}_t^m g_T$, we can write the monetary shock elasticity of output as:

$$\begin{aligned}
\varepsilon_{t,T}^{Y,m} = & \frac{\mathbb{E}_t\left[\mathcal{D}_t K_T\right]}{\mathbb{E}_t\left[K_T\right]} = -\frac{\sigma_g \sqrt{g_t}}{\mathbb{E}_t\left[K_T\right]} \cdot \\
& \cdot \mathbb{E}_t \left[K_T \left(\int_t^T \left(\tau\Pi_1 + \frac{(\tau\sigma_m)^2}{2} \right) e^{-\int_t^v \left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}} \right) du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}} dZ_u^m} dv \right.\right. \\
& \left.\left. + \tau\sigma_m \int_t^T \frac{1}{2\sqrt{g_v}} e^{-\int_t^v \left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}} \right) du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}} dZ_u^m} dZ_v^m \right) \right] - \tau\sigma_m\sigma_g g_t.
\end{aligned}$$

A similar calculation gives the expression for the monetary shock elasticity of price level. By Malliavin differentiating the solution of (A.12), I obtain:

$$\mathcal{D}_t^m p_T = p_T \left(\int_t^T \left(\Pi_1 - \frac{\sigma_m^2}{2} \right) \mathcal{D}_t^m g_v dv + \sigma_m \int_t^T \frac{\mathcal{D}_t^m g_v}{2\sqrt{g_v}} dZ_v^m + \sigma_m \sqrt{g_t} \right), \tag{A.51}$$

which results into the following monetary shock elasticity of price:

$$\begin{aligned}
\varepsilon_{t,T}^{p,m} =& \frac{\mathbb{E}_t \left[\mathcal{D}_t p_T \right]}{\mathbb{E}_t \left[p_T \right]} = \frac{\sigma_m \sigma_g g_t}{\mathbb{E}_t \left[p_T \right]} \cdot \\
& \cdot \mathbb{E}_t \left[p_T \left(\int_t^T \left(\Pi_1 - \frac{\sigma_m^2}{2} \right) e^{-\int_t^v \left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}} \right) du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}} dZ_u^m} dv \right.\right. \\
& \left.\left. + \sigma_m \int_t^T \frac{1}{2\sqrt{g_v}} e^{-\int_t^v \left(\kappa_g + \frac{\sigma_g^2}{8\sqrt{g_u}} \right) du + \int_t^v \frac{\sigma_g}{2\sqrt{g_u}} dZ_u^m} dZ_v^m \right) \right] + \sigma_g \sigma_m g_t.
\end{aligned}$$

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Degrees

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2008 B.S. in Applied Mathematics, UFRJ, Brazil

Research Fields

Asset Pricing, Macro-Finance

Professional Experience

2015 Boston University Questrom School of Business, Boston, MA
Lecturer
- Investment Analysis and Portfolio Management

2010 BTG Pactual, Rio de Janeiro, Brazil
Market Risk Analyst
- Implemented risk management models to track bank's exposure to risk factors
- Generated risk reports to the board of directors

2009 IMPA-Petrobras Research Group, Rio de Janeiro, Brazil
Research Assistant
- Established investment strategies for Petrobras based on real option analysis

Distinguished Academic Experience

2015: Participant, Financial Intermediation and Contracting
2015: Participant, Wolfram Science Summer School 2015
2014: Presenter, SIAM Conference on Financial Mathematics & Engineering
2013: Participant, MIT Capital Markets Research Workshop

Teaching Experience

2013-2015: Teaching Assistant for *Fixed Income*, taught by Prof. Prieto
2013-2014: Teaching Assistant for *Derivatives*, taught by Prof. Detemple
2012: Teaching Assistant for *Stochastic Calculus*, taught by Prof. Lyasoff
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Research Grants, Fellowships and Awards

BU Center for Finance, Law and Policy Research Grant, 2014
BU PhD Fellowship, 2010-2015
Outstanding Teaching Award - Best Math Finance Teaching Assistant, 2013 and 2014
IMPA-Petrobras Full Scholarship, 2009 and 2010
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