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The reorganization of secondary school mathemati

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THE REORGANIZATION OF SECONDARY SCHOOL MATHEMATICS.

SUBMITTED BY

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OUTLINE

The Reorganization of Secondary School Mathematics

A Need for Reorganization

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- II Criticisms on Curriculum Now in Use.
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  - (b) Reasoning deductive.
  - (c) Artificial.
  - (d) Traditional, not practical.
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  - (g) Subject difficult.

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- III To develop fundamental concepts.
- IV To enable child to carry on work of future occupation.
- V To serve as basis for future preparation.
- VI To find new and better ways of doing work.
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- VIII To enable us to determine whether child will profit by future study.
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THE REORGANIZATION OF SECONDARY SCHOOL MATHEMATICS.

Efficient teachers of mathematics admit that this subject is not taught in an ideal manner. No subject in the entire curriculum is so organized or so taught as mathematics in our public schools today. The mathematics of the school has not been sufficiently related to the mathematics of the life outside of school. The roots of mathematics in the schools have not been sufficiently "embedded in the soil of reality."

But the rapid increase in the number of Junior High Schools is one of the most significant facts in the recent educational progress. The organization and development of the Junior High School "seems sufficiently in accord with experience and with common sense to give some promise of permanence and hence to justify serious consideration."

In the report of the "Committee of Ten on Secondary School Studies," a recommendation was found concerning the articulation of elementary and seconday school education. Several subjects now reserved for high school - such as algebra, natural science, and foreign language - should be begun earlier than now and therefore within the schools classified as elementary: or as an alternative, the secondary school period should be made to begin two years earlier than at present, leaving six years instead of eight for the elementary school period. Under the present organization, elementary methods and subjects are kept in use too long.

To give a very clear idea about the Junior High School, we are going to list the purposes, under no special subject but as a complete unit.

I To enrich the curriculum of pupils from 12 to 16.

II To provide a more gradual transition than at present from elementary school to high school.

III To provide material and methods that will encourage initiative on the part of the pupils.

IV To provide a gradual transition to the departmental teaching, characteristic of the secondary school.

V To promote by subject rather than by grade.

VI To provide systematic educational guidance for each pupil.

VII To seek to retain pupils longer in school.

VIII To provide vocational or trade training for pupils who will assuredly leave school at the end of the compulsory education period.

IX To make possible the accelerated promotion of the pupils of greater ability.

X To organize the material and methods of the junior high school with reference to the capacities and needs of the adolescent boy and girl.

The changes in material and methods of instruction of the presen four year high school, contemplated in the above listed purposes, present great difficulties when it is sought to put them into practice. There are striking inadequacies in our traditional course of study, both in what we teach and how we teach it.

I Mathematics, as taught, is too mechanical and formal.

II Reasoning is deductive while practical living demands inductive.

III It is artificial.

IV The time at which it is studied, the time devoted to it, the manner in which it is taught, the amount to be covered are determined by tradition and not in general, by any consideration of the needs of the child.

V The subject itself is difficult.

VI The average child is not mathematically inclined.

VII More than half the conventional first year course will never be used by a vast majority of our pupils.

VIII The claims which we make that we are training pupils "to think intelligently" will be difficult, it not impossible, to prove. The very content and organization of the course tends to inhibit this. Most of it provides little or no opportunity for training in problem solving. Our courses of study emphasize habit, formation and rote memory, and these courses are almost exactly determined by the textbook.

X Lastly, the standardized tests given to more than 100 high schools show clearly that neither in securing formal skill nor in developing powers of analytical thinking is our instruction satisfactory.

After reading that list of bitter and bold criticisms (bold, in that it revolutionizes the whole idea of high school mathematics) we would seem to ask, "Well! how would you teach mathematics?" The answer is according to the purposes outlined below which have the Junior high aims clearly in mind, and also which have the life of the child nearer the heart.

I We should give each child such a knowledge of the subject as will enable him to understand the exactness and the force with which mathematics works and the part it plays in solving the problems of nature.

II We should endeavor to give to each child such a knowledge of mathematics as will enable him to understand the exactness and force of the subject which makes it possible for a man to turn the elements of nature to his own use.

III We should, in so far as is consistent with the more distinct aims of mathematical education develop such fundamental concepts as will enable the child to express his thoughts more clearly and to understand the written and spoken language more readily.

IV We should as nearly as possible, give each child, such a knowledge of mathematics as will enable him to carry on the work of his future occupation as it is now conducted.

V We should give to each child such a knowledge of mathematics as will serve as a basis for future preparation if progress in his work should demand it.

VI We should, as nearly as possible, give to each child such a knowledge of mathematics as will enable him to find new and better ways of doing his work.

VII We should give him such a knowledge of the subject as will enable him to recognize a mathematical situation when it arises, and if he cannot solve it, he can take it to an expert.

VIII We should require of each pupil sufficient mathematics to determine whether he will profit by further study of the subject and to select those who will probably be leaders in mathematical thought.

IX To those who show marked ability, we should give enough mathematics to keep the door of specialization in mathematics and in fields of work dependent on mathematics, open.

Here we have the aims of the mathematics of the secondary schools laid before us, but in order to be more concrete, we will give a very brief outline of the mathematical course of a standard high school.

The traditional course is a succession of partial courses or branches taken through the high school period; that is, we have an alternation of topics in rather large units of years and half years - one year of algebra, one year of geometry, and one half or one year of advanced algebra with the more advanced students following up geometry and solid geometry. Here there is a question of a unifying or coördinating principle - What is it in the traditional course?

The principle of geometry seems to be logical sequence, for the most part and a fair degree of completeness as the final goal; while the principle of unification in algebra seems to have bee the historical order of the development of the operations of arithmetic with generalized numbers.

Many who have not studied the subject from every angle and have not probed to its very depth will ask, "What is to be gained in changing from a time honored course?" Several advantages seem to be gained by such a procedure:

I It is possible so to arrange a course in a sequence that the most useful (Mathematically and practically) can be presented in cumulative order - while acknowledged difficult parts can be postponed to a later stage, when they will be more easily approached.

II By bringing algebraic and geometric methods into close relation from the start, a longer period of training in correlating these methods is gained.

III For students who will not go far in mathematics either on account of taste or ability, a combined course seems to offer more that is useful (or Usable) and within grasp of weaker students (mathematically) than is possible under the old plan.

IV Those who may desire to take more advanced courses will approach them with better perspective of their content and purpose. As a consequence, they will make more rapid progress in the more specialized branches, such as formal algebra and demonstrative geometry.

V Saving of time has been offered as a distinct advantage. Teachers who have tried it claim that both plane and solid geometry can be satisfactorily completed in a single year following a good course in combined mathematics.

It might be well to outline an example of procedure for definiteness. For example, one book starts out with the simple equation as the instrument in solving concrete problems within the understanding of the pupils. Incidently, but with definite purpose, through the first chapter, some of the terms and fundamental processes relating to equations are used and explained; such as:

1. Equal numbers may be added to both sides of an equation, or

2. If both numbers of an equation be multiplied by the same or equal numbers, the results will be equal.

In the next chapter, some definite fundamental geometric facts and principles are introduced. Then the equation is applied to a new set of problems involving the geometric knowledge just gained.

In the following chapters, some further algebraic and geometric principles are developed and applied to problems of a more advanced type. In general, the year's work is developed systematically and with as much logical soundness as pupils are capable of appreciating. There is thus accumulated a stock of intuitional and experimental geometry correlated with a working knowledge of the fundamentals of algebra in use of simple forms. Such a foundation will, with little doubt, form a good basis for a study of a course in formal algebra or geometry.

The five distinct advantages as described above have distinct factors involved in gaining satisfaction in the change from the old standards. First, we consider the students; their needs, interests and capacities. Subject matter must function throughout the process of learning and the present mathematics does not seem to function even in the hands of skilled teachers. Not only that but we must consider the mental characteristics of adolescent students. Adolescence is not a period of formal drill, the old time method of mathematics; an adolescent studen can grasp mathematical concepts but the concepts must be brought in to the range of his experience.

The second factor involved is one including social needs which the high school must meet. The high school is being rapidly reorganized to adapt itself to changing social needs, and in so doing is becoming an institution for all the people. It must discover broad zones for special talents of individuals and the curricula should be differentiated to parallel broad zones of adult activity.

The chief division of which the high school has many members have educational foundations in (1) Homemaking and housekeeping, (2) Agriculture (3) Mechanical and engineering pursuits (4) Commerce (5) Professions thru a college preparatory curriculum. Each one of these special divisions must have the appropriate mathematics.

Another important question under the social needs which a high school must meet is the moral purpose. Mathematics, as such, does, at present contribute little to moral training, contrary to what some enthusiastic mathematicians claim. Essential morality is a question of relations of individuals in society, and that all we mean by moral education is the adjustment of the pupils to the standards of life in a society sanctioned by the highest social ideals of his time. It is not a matter of book learning but rather arises from an interaction of various personalities composing the school; - but in the very nature of the subject matter, mathematics is as devoid of personality as possible. The only traing we can claim for mathematics is exact thinking.

The third and last advantage in the change from the old standards is the critical evaluation of material available in mathematics. Given a body of pupils in the adolescent period, whose development proceeds according to natural law, the secondary school has its function first to direct their several individual native abilities into the proper broad zones of adult activity and then to equip them with an organized system of ideas which will enable them to interpret the new and strange situations in which they will be placed. The question there arises, what has mathematics to offer which is essential and valuable in this process?

The most important use of mathematics is as a tool in scientific thinking and to provide opportunities to use the tool of mathematics, differentiated courses are needed;

The first, which really requires the minimum of mathematics, is the Domestic Arts course where the student needs only arithmetic and mechanical drawing. The second, the Agricultural course needs three phases of mathematics, Geometry, Trigonometry, and Algebra; but the geometry is to be taught constructively, the trigonometry is to include no theory but only a practical use of logarithms; and the algebra is to teach a mastery of the equation and subordinate processes.

The third curriculum, the commercial, presents special problems and opportunities in securing educative content. Here we become involved in rational interpretation of statistics, in records of complicated transactions, an understanding of backing, currency and kindred questions. So the problem in this light is to give the pupil as far as possible, a working knowledge of the science of accounting, the principles of statistics, and of the higher arithmetic and algebra of business processes, which, of course, take in insurance, returns or investments, annuities and bonds.

The fourth, the mechanic arts, offers the broadest scope for methematical teaching. The curriculum ordinarily embraces woodworking of a somewhat advanced type; forgin, pattern-making; molding and casting; general machine shop practice with engine lathe, drill press, bed planer and milling machines. To this must be added, mechical drawing of a more advanced type than that found elsewhere. The underlying mathematics which will interpret the subject matter of this curriculum and which also answers for college work is (1) Geometry, plane, solid, and descriptive. (2) Elements of plane trigonometry and (3) Elements of analytic geometry and calculus.

So much for the gains and the factors involved in gains in a change from the old standards. As has been pointed out, we want to teach the child what he himself will need in his work in life. So it seems to be a practical and altogether desirable thing to outline a course of mathematics for the three years at least in which mathematics is required.

We would begin in the first half of the seventh grade with a study of arithmetic, reviewing briefly all the essential topics of previous grades for accuracy and against future needs. This subject relates to the immediate mathematical interests of the pupils; it connects directly with the mathematics that has preceded and it will enable the pupil to maintain or increase his efficiency in the computation which he has acqmired. The work in arithmetic should be organized about certain large topics of practical value which challenge the interests and needs of the pupils. Such topics as percentage, and its various operations, profit and loss, commission, discount, and taxes open a wide field for problems that link up with life.

The second half of the seventh grade should be devoted to constructive and intuitional geometry. The subject is more concrete than algebra and it admits of more simple illustration; it relates to the arithmetic immediately preceding; it challenges the interest; of the pupil and it can be made very practical. Problems correlating with this work everyday activities such as plastering, carpeting, paingting, paving as well as with the work of the department of arts and the shop are constantly presented for solution. The mere formal solution is not accepted in the performance of any of the above problems unless accompanied by some expression of the mode of procedure applicable to each case.

Many who have not gone very deeply into this subject would say, "Teach geometry to seventh grade pupils! How absurd! " But not if one goes into the teaching of it as a practical tool. Take it first as "Geometry of Form: What shape is it?" Here we find angles, triangles, quadrilaterals and common polygons. With common drawing instruments, such as compasses, ruler protractor and right triangle, we can construck various kinds of triangles, and discover the angle sum of any triangle; we can construck perpendiculars and bisectors; we can construct an angle equal to a given angle and a triangel with various parts given; we can construct parallels and develop principles relating to the angles formed by transcutting two parellels. We can draw to scale and apply the problems of the builder, the farmer, the engineer, the designer, and the geographer.

Next we take the subject as the "Geometry of Size; How long is it."

Here we introduce numerous practical problems involving the estimates and measurements of heights, distances, and areas. Make the pupils estimate the dimensions of the school room, the school ground and then check the estimate by measurement. By the use of congruent triangles, we measure such heights as trees, buildings, flagpoles and widths of rivers. The pupil is delighted to find that he has sufficient mathematical knowledge to determine inaccessible heights and distances. He has the knowledge and should feel the power.

Here we may introduce the study of area by the use of squared paper and develop formulae for areas of common figures which apply to numerous practical situations such as those which most commonly confront the farmer, housewife, engineer, business man, and artisan.

The third topic, "Geometry of Position, Where is it?" may be very nicely introduced by problems such as locating the the proper position for second base on a basball diamond after the other bases have been located. This especially will be facinating because there are more arguments on this point in a "scrub" game than any other in any organized play. You can also challenge the interest of the pupil by introducing problems dealing with attempts to locate buried treasure. Thus you will develop four important principles; (1) a practical method for determining points equidistant from two given points. (2) distance of a point from a line. (3) points which are equidistant from two lines and (4) the points at a given distance from a given point.

Experience seems to indicate that the arithmetic and constructive and intuitional geometry of the seventh grade should be continued on through the eighth year. We should develop the geometric sense by introducing mensuration of figures, including surface and volume. We should derive formulas for and apply to problems arising from the (a) Trapezoid (b) Circle (c) Equilateral triangle (d) cylinder (e) prism

(f) cone (g) pyramid (h) sphere.

This, of course, should not be burdened with technical phraseology but should be utilitarian in the largest sense. In deriving the formulas for the above, we are teaching the algebra of formulas and equations, the knowledge of which is needed in reading many books and articles; the graph is used in many lines of business, the equation is helpful in manipulating formulae and the knowledge of negative numbers is necessary to the equipment of every reader of current literature as well as of scientific books.

But, during this time we must continue the study of arithmetic. Fundamental operations with integers, common fractions and decimals should be reviewed for speed and accuracy. Then we take up for detailed study those practical topics of arithmetic for which the pupil's maturity has prepared him; among these topics should be a complete study of percentage and interest, taking up successive discount, the arithmetic of trade, transportation, industry, building, banking, home life, farming, problems including investments, loans, mortgages, stocks and bonds, insurance, taxes, internal revenue, and tariff. As one will notice this year is a continuation, after a review, of the seventh grade and merely takes the intuitional and practical side of the three subjects in a unified manner.

It is not improbable that the ninth school year will be the last year of <u>required</u> mathematics. It is very desirable, therefore, that the pupil have some knowledge of formal mathematics. The time, then, in the ninth grade is devoted to a study of demonstrative algebra and geometry. By omitting the non-essentials, it has been proved that pupils can complete algebra through quadratics during the first half of the year. This means the pupils are introduced to factoring, fractions, fractional equations, graphs, simultaneous equations, square roots, and quadratics. In some of the schools where the plan has been tried, the trigonometry of similar triangles was introduced.

There should be, however, a gradual introduction to demonstrative geometry. Independent deductions should precede formal proofs and a large number of practical exercises should follow each proposition studied. Extreme formality of treatment should be avoided and originality, clearness, and conciseness should be emphasized. The way to do this, in my opinion, is to follow the method used in the Girl's High School, Boston, for first year demonstrative geometry-use no textbook . The teacher dictates the theorem or perhaps, only the hypothesis, and the class draws the figure and puts down in their notebooks the hypothesis and what is to be proved. The proff is left to them. This method leaves the pupil to her own ingenuity and originality of proof and any proof is acceptable, providing it is right, although sometimes the proofs are long and complicated. It seems to me, however, an excellent way to avoid rote learning and gives opportunity for fine work.

With the work thus laid down by the seventh, eighth, and ninth grades, which are commonly called the junior high school grades, the question arises as to how we are to articulate the work of mathematics of the junior and senior high schools. We may get continuity in the work of the secondary schools in three way:

I By the continuity of the plan; which is secured by having one man, or group of me in conference, prescribe the work in any given locality for both the junior and senior high schools.

II By continuity of the supervision; which is secured by having as supervisors those who have had experience as classroom teachers in both elementary and high school.

III By continuity of the instruction, which is secured by compulsory frequent exchanges of visits, and so of ideas, by classroom teachers in the two schools.

The method which seems to be the best for securing continuity of instruction is by means of the function. A rather recent European book, Scheffer's "Lehrbuck der Mathematick" (Leipsic, 1911) on mathematics for non-technical students is divided into chapters with heading like this; I, Linear Function, II. Quadratic Functions, III. Algebraic Function, IV. Logarithmic Functions, V. Exponential Functions, VI. Trigonometric Functions. In each chapter, the author carries the reader from the very elements of the subject up through the graphical representations, the analytic geometry and calculus.

This idea, then, is the one which is sufficient in scope to unify a whole course. The concept of a variable and the dependence of this variable upon another is of great importance to everyone. Of course, it is true that the abstract form of these concepts can become significant to the pupil only after long experience and training in mathematics; but, we can present specific concrete examples and illustrations of dependence even in the early parts of the course. That is, we are not to teach even the element of the <u>theory</u> of functions and the work "function" itself should be used sparingly, if at all, and no formal definition of the word should be insisted upon.

"What is desired is the growth in the mind of the pupil of the idea of <u>relationship</u> between quantities, of the <u>dependence</u> of one quantity upon another, of the <u>correspondence</u> that exists between related quantities either of an arithmetical or geometric character. It is desired further that the pupil form the habit of seeking to understand such relations when they present themselves and of thinking clearly in terms of them. "Such is the requirement of the functional training made byt "The National Committee on Mathematical Requirements."

The first relationship which we meet are those which occur in algebra. The very first illustrations given are the uses of letters in the place of numbers, which offer valuable material for the discussion of quantitative relations. Thus, such relations as $A = \frac{1}{2}bh$ (area of a triangle) $I = rpt$ (simple interest) and $A = \pi r$, are statements of general relationships, the nature of which should be understood. These should be used to accustom the student not only to the use of letter in the place of numbers and to the solution of simple numerical problems, but, also, to the idea, for example that changes in "r" affect the value of A. "If "r" is doubled, what will happen to A? If "r" is doubled, what will happen to I?" Appreciation of the meaning of such relationships will tend to clarify the entire subject under consideration.

The second topic in the algebra field which relies on the function is the equation. Every simple problem leading to an equation in the first part of an algebra would be better understood for just such a discussion as that mentioned above.

The third topic which will relate more closely to the first is the function in formulas of pure science and of practical affairs. In such formulas as those for falling bodies, levers, etc., the manner in which changes in one quantity cause changes in another, are of prime importance and then discussion need cause no difficulty whatever. Take for a specific example, the formula $F = \frac{Wv^2}{32r}$. When such a formula is used, the teacher should get intelligent answers to any question of this sort: "If the weight is twice as heavy, what is the effect upon the force? If the speed is twice as great what is the effect upon the force?

Another type of relationship is found in the formulas of pure algebra. For instance, the square of the sum of two numbers, $(x + h)^2$ will be better understood and appreciated if accompanied by a discussion of the manner in which changes in "H" cause changes in the total result. This can be accomplished safely by attaching such a discussion to such concrete realties as the error made in computing the area of a square field or of a square room when an error has been made in measuring the side of the room.

In connection with the formula for a product, we may introduce the functional idea in tables. For example, a table of squares is constructed from a functional relation, $y = x^2$. The differences in such a table are the differences caused by changes in the value of the independent variable. An appreciation of the facts in such tables is desirable and all necessary for their proper use.

The sixth and last type of functional relation is that which exists in graphical representations. This is so obvious that no detailed explanation seems to be necessary. However, it is desirable to point out the functional relations may be studied directly by means of graphs without the intervention of any algebraic formule.

Thus far the instances taken up have been largely algebraic, though certain mensuration formulas of geometry have been mentioned. There are five distinct relationships in geometry which illustrate the functional concept.

The first is the relationship in the congruence of triangles. In any such theorems, the parts necessary to establish congruence evidently determine completely the size of each other part. Thus, two sides and the included angle of a triangle evidently determine the length of the third side. If the student clearly grasps this fact, the meaning of this case of congruence will be more vivid to him and he will be able to use his understanding of the situation in practical cases.

In theorems regarding unequalities, the functional character is very pronounced. If two sides remain the same, as the angle between them grows, the side opposite it becomes larger. A full realization of the fact would envolve a real grasp of the functional connection between the angle and the side opposite it.

The third class of theorems in which the functional concept plays a large part is the circle. We imagine variations of the figure through all the intermediate stages from one case to another. The angle between two lines that cut a circle is measured by a proper combination of the two arcs cut out of the circle by the two lines. Such observations are essentially functional in character, for they consist in careful observations of the relationships between the angle to be measured and the arcs that measure it.

That discussion leads to the fourth class, actual motion. As figures move, either in whole or in part, the relationships between the quantities may change. To note these changes is to study the functional relationships between the parts of the figures. Without the functional idea, geometry would be wholly static.

The fifth and last class of theorems, perhaps is the most obvious one - the theorems of propertion. Even the simplest theorems on rectangles assert that the area of a rectangle is directly proportional to its height if the base is fixed.

The concept of the function in geometry leads very naturally to the function in trigonometry, which is nothing more nor less than the ratio of the sides of a right triangle. Thus the side of an angle is a definite ratio, whose value depends upon and is determined by the size of the angle to which it refers.

In reading over this rather long discussion on the importance of the function idea in the mathematics of the secondary schools, some might say that we have not followed our creed of practical knowledge and usefulness in future activities. But, the problem of insurance takes into consideration the relation of time, interest and chance; the great questions of tariff, rates of postage and express, freight rates, regulation of insurance rates; income taxes and many other public questions involve relationships between quantities that require habits of functional thinking for intelligent decisions. The training in such habits of thinking is therefore a vital element toward the creation of good citizenship.

So far, this paper has dealt only with the topics and courses that we should teach. There are many topics which have been recommended for omission in a course which would follow the ideal unified method. Many of these, of course, could be taken up later when the pupil has left the required mathematics class for the specialized or elective courses. Those to be omitted are:-

1. Parentheses more complicated than one within the other. (2.) Multiplication and division of polynomials having literal exponents or fractional coefficients. (3) Special rules for square of polynomials (x plus y plus z)^2. (4) Factoring leading to irrational or imaginary factors. (5) Transformation of proportions by alternation, etc. (6) In simultaneous equations, method of elimination by comparisons and the the solution of examples with more than three unknowns. (7) Radicals

of order higher than 3rd. Rationalization of binomial denominations. Radical equations. (8) Theory of exponents except sufficient work with monomials to make clear the meaning of fractional and negative exponents. (9) In quadratic equations, the "Hindu" method of completing the square. (10) Imaginary numbers except for a brief mention when encountered in the quadratic formula. (11) Equations in quadratic form. Simultaneous quadratics except when one equation is linear and the example can be easily solved by substitution. (12) Theory of quadratics

Now that we have designated what topics ought to be eleminated and having carefully and in detail covered the work which is to be taken or omitted in the first three years of the secondary schools, that which we will name as "required mathematics" the question now arises as to what the student who wishes to go on farther in that field should have open to him the last three years of high school.

First, having the elementary notions of demonstrative geometry, a continuation of this course should be presented. If the student has had a real, valuable course in intuitional geometry, this will simply be a review plus a great number of originals and exercises based directly on the theorems, as is done in Girl's High School.

An advanced course in algebra should also be open to them. "Advanced" in that it is a course in formal algebra, a continuation of the ninth year. Here we should include I, Illustrations and problems involving linear, quadratic and other simple functions of one variable; II. Various methods for solving a quadratic equation in one unknown. In this connection, we should include a very brief discussion of complex numbers, III. The algebraic solution of linear equation in two or three unknowns and the graphic solution of linear equations in two unknowns. IV. A study of exponents, radicals and logarithms, giving the definitions of negative, zero and fractional exponents and making clear that these definitions must be adopted if we wish such exponents to conform with the laws for positive integral exponents. Reduction of radical expressions to those involving fractional exponents should be given as well as the inverse transformation. The rules for performing the fundamental operations on expressions involving radicals. In close connection with the work on radicals and exponents there should be given as much of the theory of logarithms as is involved in their application to computation and sufficient practice in their use in computation to assure a fair degree of facility. V. Arithmetical and geometric progressions should be studied to the extent of finding the formula for the n^{th} term and the sum of n terms and the application to significant problems. VI. Binomial theorem with a proof for positive integral exponents; it may be stated that the formula applies to the case of negative and fractional exponents under suitable restrictions, and the problems may include the use of the formula in these cases as well as in the case of positive integral exponents.

Another course which would rightly come into the higher mathematics of the high school is solid geometry. The aim of the work should be to exercise further the spatial imagination of the student and to give him both a knowledge of the fundamental spatial relationships and the power to work with them. The minimum of such a course should certainly include propositions dealing with perpendiculars to planes, dehedial angles and the simpler theorems on areas and volumes. It should be possible to complete such a minimum course in a third of a year. Some European schools have found it desirable to replace some of the work now usually given in solid geometry by certain important topics of descriptive geometry.

The work in elementary trigonometry begun in earlier years should be completed by including the logarithms solution of right and oblique triangles, radian measure, graphs of trigonometric functions, the derivation of the fundamental relations between the functions and their use in providing indentities and in solving easy trigonometric equations.

Beyond trigonometry in high school, there are very few who believe in teaching more but up to this time, mathematics has been required through the four years of high school training and now, we hope to have only those who are especially fond of mathematics or who really need it for use in higher education take the four year mathematic course. Hence, we have a relativly picked group. Why would it not be possible, then, to introduce the elements of college work?

For instance, why not include in a course the meaning and use of fundamental concepts of elementary statistics with simple frequency distributions with various graphic representations and measures of central tendency. In this group of electives we could also introduce a course in elementary calculus. The work should be largely graphic and closely related to that in physics: the necessary technique should be reduced to a minimum by basing it wholly or mainly on algebraic polynomials. The work should include (a) the general notion of a derivative . (b) Applications of derivatives to easy problems in rates and in maxima and minima. (c) Simple cases of inverse problems. (d) Approximate methods of summation leading up to integration as a powerful method of summation. (e) Applications to simple cases of motion, area, volume and pressure.

Throughout all these elective courses, historical and biographical material should be used to make the work more interesting and significant. Such additional electives as mathematics of investment, shop mathematics, surveying and navigation, descriptive or projective geometry will appropriately be offered by schools which have special needs or conditions. So, with these additions, it is possible to get in high school a knowledge of the elements of the subjects which are usually taught in college.

As a summary we wish to call to mind the fact that mathematics has really three aims: First, the practical aims which apply to the immediate or direct usefulness in life of a fact method or process in mathematics. The second, is the disciplinary aim, with proper restrictions-such training involves the development of certain more or less general characteristics and the formation of certain mental habits which, besides being directly applicable in the setting in which they are developed or formed, are expected to operate also in more or less closely related field. Third, is the cultural aim by which we mean the development or acquisition of the appreciation of beauty in the geometrical forms of nature, art and industry, in the ideals of perfection, as to logical structure, and the appreciation of the power of mathematics.

The course, which is still strong in traditional high schools is obviously inadequate for the needs and abilities of students. By a change from the old standard we gain many advantages which are not only helpful to those going on farther but also to those who are to leave school early or who enter business. The change to be made involves the question of differentiated courses so that the mathematics of a domestic arts course will be as adequate and useful to a pupil there as the mathematics given to students in an academic or college training course.

But, before we branch out to the differentiated courses, we have a three year course in unified mathematics which includes higher arithmetic, algebra, and contructive and intuitional geometry. From this last year of required mathematics, that is, the ninth year, we go on to the higher branches, bound throughout the whole course by the function concept which begins with first substitution of letters for numbers and continues on through calculus and statistics. So, we have the entire course developed, giving the pupil who does not wish to go on farther in mathematics, the elements of algebra and geometry and giving the pupil who wishes to go on, an opportunity to take the higher branches of mathematics.

In closing, we will quote a statement made by the National Committee on Mathematical Requirements, which seems the idea we should always have in mind in teaching mathematics: "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual." This is what a unified course in mathematics will aim to do.

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