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SCHOOL OF EDUCATION

Thesis

A COMPARATIVE STUDY OF MATHEMATICAL UNDERSTANDINGS POSSESSED BY TEACHERS IN-SERVICE

Submitted by

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In Partial Fulfillment of Requirements for the Degree of

Master of Education

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TABLE OF CONTENTS

LIST OF TABLES

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CHAPTER I

THE PROBLEM

Purpose

It was the purpose of this study to:

1. Determine the mathematical understandings possessed by a group of teachers in-service.
2. Compare the levels of mathematical understandings possessed by the different members of the group with the various grade levels of teaching experience within the group.
3. Compare the levels of mathematical understandings of the members of the group with their length of service.
4. Compare the levels of mathematical understandings of those members who have taken courses in the teaching of arithmetic with those who have taken no courses in this area.
5. Make a comparison between the levels of mathematical understandings of those members with Bachelor's degrees and those with Master's degrees.

Source and Justification of Problem

As emphasis on the teaching of arithmetic in a meaningful way continues to increase, attention is also being focused on those whose task it is to carry out such a program. A majority of the prospective teachers and teachers in-service

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are products of the "rote-learning school." Therefore some place in their training must come an evaluation of their understandings of arithmetic and a program of reteaching where it is needed.

This can be approached more easily at the pre-service level where the standards can be high and requirements must be met. But there are still many beyond the reaches of pre-service training who will be influencing the minds of children for many years to come. Some of these teachers realize their own shortcomings in the field of arithmetic and seek help through workshops and in-service courses. But there are still many who apparently avoid arithmetic courses because their own experiences in this field have been unsuccessful.

It may well be that some will never need a very extensive background in arithmetic because of the grade levels they are teaching. These teachers will not be hindering their own pupils then to any great extent.

On the other side may be those who have sufficient understanding and are qualified to teach the arithmetic of the grade levels to which they are assigned.

Although teachers in-service may be products of the "mechanical school", they might have a natural insight into the meanings and understandings of the arithmetic as it is now being taught. These same teachers may have had an opportunity to broaden and clarify their thinking through teaching experience.

This study, therefore, attempts to find an answer to some of these questions as a means of strengthening instruction and learning in the arithmetic curriculum.

Scope and Limitations

The present investigation involved 58 teachers in-service in the elementary grades, kindergarten through grade six. This group consisted of teachers, who participated on a voluntary basis, from seven schools in the same system.

The basic data for the study were obtained through the use of "A Test of Basic Mathematical Understandings" constructed and validated by Dr. Vincent J. Glennon[1] of the School of Education at Syracuse University, and used with his permission. This test was administered to the teachers in their schools by the writer. The validated instrument, described more fully in later chapters and included in the Appendix, tested the teachers' understandings in relation to the nature of the number system and to the rationale of computation involving the four fundamental processes (addition, subtraction, multiplication, and division) with whole numbers, common fractions, and decimal fractions.

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1. Vincent J. Glennon, A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels, Unpublished Doctor's Dissertation, Harvard University, Graduate School of Education, Cambridge, 1948.

For administrative reasons, a working-time limit of 63 minutes was imposed on the examinees. To insure that all teachers had opportunity to work on each of the five sections, the total working time was divided proportionately among them (Sections I-III, V limited to 12 minutes, and Section IV to 15 minutes.)

A review of the related literature and research follows in Chapter II. The collected data are presented in the third chapter. An analysis of the data is also included in this chapter. The fourth and final chapter presents a summary of the study, its findings, and conclusions.

CHAPTER II

REVIEW OF RELATED LITERATURE AND RESEARCH

Along with the all important problem of improving instruction and achievement in arithmetic in the elementary grades is the consideration of the qualifications of those teachers who will have the responsibility for such a program. In recent years much has been written, in professional books and children's text books relating to arithmetic instruction, about making arithmetic mathematically meaningful for the child. Professional yearbooks, monographs, and periodicals have treated various aspects of this problem. Prospective teachers may benefit from the increased attention which is being given to the mathematical background, training and preparation on the pre-service level. But still another area of concern is the background of mathematical understandings possessed by teachers in-service.

Studies have been conducted on both the pre-service and in-service levels. A pioneer study by Glennon[1] pointed to the seriousness of the problem. Similar investigations substantiate the findings and conclusions of the pioneer study. This study will be discussed in detail later in the chapter.

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1. Vincent J. Glennon, A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels, Unpublished Doctor's Dissertation, Harvard University, Graduate School of Education, Cambridge, 1948.

As Weaver[2] states: "Cumulative evidence adds to the apparent seriousness of the situation and increases the concern we must have regarding it."

Related literature and research on the pre-service level have been extensively reviewed in two recent studies, Cristiani[3] and Moran[4], concerned chiefly with this aspect of the problem. Therefore the writer will consider only the literature and research concerning teachers in-service. There is definitely a lack of arithmetic understandings among teachers in-service. Robinson[5] observed that elementary school teachers have at best only a mechanical knowledge of arithmetic. There may be legitimate reasons for this condition,

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2. J. Fred Weaver, "A Crucial Problem in the Preparation of Elementary-School Teachers," The Elementary School Journal (February, 1956), 56:255.

3. Vincent A. Cristiani, Nicholas J. Giacobbe, and Joseph G. Thibeautl, A Study of the Mathematical Understandings Possessed by Prospective Elementary School Teachers, Master's Thesis, Boston University, School of Education, Boston, 1954.

4. Marcella Moran, Growth in Arithmetic Understandings Developed During the Course "Methods in Teaching Arithmetic," Master's Thesis, Boston University, School of Education, Boston, 1955.

5. E.A. Robinson, The Professional Education of Elementary Teachers in the Field of Arithmetic. Contributions to Education, No. 672, N.Y.: Bureau of Publications, Teachers College, Columbia University, 1936. p. 193.

but they must be rectified. As Weaver[6] points out in his summary of the studies made by Grossnickle, Layton, and Rhoads, the requirements in the subject matter of mathematics for those preparing to teach in the elementary schools are extremely low. Somewhere in the curriculum of arithmetic instruction is an important link which must be strengthened before results can be seen.

When Dutton[7] investigated the attitudes of prospective teachers toward arithmetic, he found the reasons for both favorable and unfavorable feelings. Favorable responses included the following: enjoyment because of proficiency, good teachers who explained it and made it meaningful, appreciation of arithmetic as a vital subject, a challenging experience, and its numerous practical applications. In contrast to these responses were the following: lack of understanding, teaching disassociated from life, pages of word problems, boring drill, poor teaching, lack of interest and poor motivation, and the fear of making mistakes. Dutton[8] concludes that the perpetuation of dissatisfaction and uneasiness toward arithmetic and the importance of breaking this "vicious circle" of unfavorable attitudes must be considered.

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6. J. Fred Weaver, "Teacher Education in Arithmetic," Review of Educational Research (October, 1951), 21:317-318.

7. W.H. Dutton, "Attitudes of Prospective Teachers Toward Arithmetic," Elementary School Journal (October, 1951), 52:87.

8. Ibid., p. 89.

This establishes a definite and serious need for the increased preparation of teachers of arithmetic. In fact Grossnickle[9] states "A teacher should not be certified to teach arithmetic in the elementary school who has not had at least one good course in the teaching of the subject." It is extremely important to have a good background in content mathematics as well. Newsom[10] cites the fact that all too frequently teachers in the elementary grades are hardly a jump ahead of their alert students, and many teachers have confided in him that they lack confidence before their classes in approaching various arithmetical concepts.

> "Moreover," he continues, "it must be emphasized that an incorrect presentation by the elementary school teacher of concepts in arithmetic may handicap a student for the rest of his life; many secondary school and college teachers have been forced to labor at great lengths to rectify arithmetical understandings that students have been taught on more elementary levels."[11]

Therefore "If you wish to qualify for such a position (one involving the teaching of arithmetic in an elementary school),

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9. Foster E. Grossnickle, "The Training of Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education, 1951, p. 229.

10. C.V. Newsom, "Mathematical Background Needed by Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education, 1941, p. 233.

11. Ibid.

the main requirement would be that you understand arithmetic. You cannot teach what you do now know."[12] As Weaver[13] states, in describing the need for a broader understanding of mathematical concepts than that which will be taught,

> "It is an impossibility for teachers to emphasize and direct attention to the development of meanings which they themselves do not understand, or of which they are not cognizant. Furthermore, no teacher can expect her instruction to be most meaningful to pupils until her own breadth and depth of meaning transcends that which she expects to develop in her pupils."

He suggests the following solution to a three-fold or three-sided problem so far as present and future teachers of arithmetic are concerned:

> "(1) They must recognize the necessity of meaningful instruction as a prerequisite to functional competence.
>
> (2) They must have an understanding of meanings to be developed, both from the level of experience and maturity of the pupil being taught and from that of the teacher herself.
>
> (3) They must be conscious of the psychological and methodological aspects of a meaningful instructional program."[14]

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12. J. Fred Weaver, "A Crucial Problem in the Preparation of Elementary-School Teachers," The Elementary School Journal (February, 1956) 56:255.

13. J. Fred Weaver, "A Crucial Aspect of Meaningful Arithmetic Instruction," Mathematics Teacher (March, 1950), 43:112.

14. Ibid.

As far as the in-service development of teachers is concerned, teachers' primary interests are practical. They are concerned with improving their teaching so children will get a better understanding of arithmetic. Wilburn and Wingo[15] emphasize that in nothing which is taught in the elementary school is it more important that teachers have an adequate understanding of the content itself than in arithmetic. Another report[16] further substantiates the belief that teachers must understand the content of arithmetic before they can teach it effectively. Schaff[17] also supports the argument that there is a definite need for improvement among the teachers by stating that for one reason or another, an inexcusably large number of experienced elementary school teachers simply do not know as much arithmetic as they should in order to teach it effectively.

To overcome this deficiency Grossnickle[18] feels that all teachers, irrespective of grade levels at which they teach,

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15. D. Banks Wilburn, and Max G. Wingo, "In-Service Development of Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education, 1941, p. 253.

16. National Council of Teachers of Mathematics, "Final Report of the Commission on Postwar Plans," Mathematics Teacher (November, 1947), 40:324.

17. William L. Schaaf, "Arithmetic for Arithmetic Teachers," School Science and Mathematics (October, 1953), 53:537.

18. Grossnickle, op. cit., p. 213.

should have basic courses in mathematics which will give them the opportunity to acquire backgrounds, appreciations, and understandings. The question of methods of teaching arithmetic is an important one in both pre-service and in-service training, but the understanding of the subject itself is of at least equal importance. Teachers will not see the importance of changing their methods of teaching unless they have sufficient understanding of the number system to enable them to see the deficiencies in their methods.[19] Of course, as Wilburn and Wingo[20] point out "..... the status of the professional education of the teachers will be a significant factor in determining the organization of the in-service program for improving instruction."

There is an increased awareness of the need for teaching and evaluating for growth in these understandings. Glennon is among those who feel needed redirection in the program of in-service development of teachers of arithmetic. He suggests that curriculum revision of the professional courses must be concerned with emphasizing the subject matter as well as with the principles of teaching the subject-matter.[21]

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19. Wilburn, *op. cit.*, p. 253.

20. *Ibid.*, p. 261.

21. Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," *The Mathematics Teacher* (December, 1949), 42:389.

Because no appropriate instrument existed, Glennon constructed and validated an eighty-item test of basic understandings relating to the arithmetic content of the first six grades of the elementary school.[22] By the normative method of investigation he was able to obtain an index of prevailing conditions within seven educational levels, ranging from children in the elementary schools to teachers in-service. His test embraced the following important points: all direct computation was minimized or eliminated, objectivity gained through multiple choice items was obtained from the administration of a completion test, one hundred thirty-six items were reduced to eighty, and test covered five essential areas.[23] These areas included: I. The decimal system of notation (15 items), II. Basic understandings of integers and processes (15 items), III. Basic understandings of fractions and processes (20 items), and V. Basic understandings of the rationale of computation (15 items).

It was a study, in part, of the degree to which teachers in-service have acquired the basic mathematical understandings and meanings. There have been few precise studies in this

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22. Vincent J. Glennon, A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels, Unpublished Doctor's Dissertation, Harvard University, Graduate School of Education, Cambridge, 1948.

23. Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," The Mathematics Teacher (December, 1949), 42:391.

area of teacher education in the last thirty years.

Glennon used one hundred sixty teachers from Kindergarten through grade eight on the in-service level of the study. The length of service for these teachers ranged from one year to thirty-four years. In an analysis of the data he found there were only three items that were answered correctly by all (100%) of the teachers in-service. Another important finding revealed that the teacher in-service understands about 55% (slightly more than half) of the understandings that are basic to the computational processes commonly taught in grades one through six.[24]

An hypothesis related to the group under discussion states there is no significant difference in achievement of basic mathematical understandings between a teacher in-service who has done graduate work in The Psychology and Teaching of Arithmetic and one who has not done graduate work in The Psychology and Teaching of Arithmetic. The two groups represented the same statistical population in terms of the number of courses in mathematics taken previously in college and high school. The hypothesis was supported and within the limitations of the study, the data indicated that graduate work in The Psychology and Teaching of Arithmetic

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24. Ibid., p. 393.

did not contribute to growth in basic mathematical understandings.[25]

Glennon also investigated the question of what degree of relationship existed, among teachers in-service in the elementary grades, between the numbers of years of experience the teacher had had in teaching arithmetic and her achievement of the basic mathematical understandings. For the sample tested, there was almost no relationship between the number of years a person had taught arithmetic and her achievement of basic mathematical understandings. From the findings we may conclude that the experience of teaching arithmetic is no guarantee that the teacher will grow in her understandings of the subject.[26]

Glennon's findings are substantiated by the findings of a study conducted by Orleans.[27] Lack of understanding of arithmetic evidenced by the teachers tested is more a function of the way arithmetic has been learned than of the inherent difficulty of the subject itself.[28] Orleans makes it clear that arithmetic is nothing more than a series of short cuts

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25. Ibid.

26. Ibid., p. 395.

27. Jacob S. Orleans, The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic, Office of Research and Evaluation, Division of Teacher Education, Publication No. 12, New York: College of the City of New York, 1952. p. 59.

28. Ibid., p. 37.

which have been developed by mathematicians over the span of centuries to facilitate computation. As a result, the teaching of arithmetic in our schools today consists of introducing children to short cuts which by-pass basic concepts and processes.[29] One of the major hypotheses of Orleans' study[30] was that perhaps a major factor in the failure of children to get closer to an understanding of arithmetic, than is possible through their present exposure to short cuts, is that their teachers are ignorant of the processes and concepts represented by the short cuts. He adds: "It would seem reasonable to assume that if the teachers do not themselves understand underlying processes and concepts they can not get pupils to learn with understanding." Two important inferences of this study are:

> "(1) People in general, teachers and educated laymen, have difficulty in verbalizing their explanations of arithmetic processes, concepts, and relationships.
>
> (2) There are apparently few processes, concepts, or relationships in arithmetic which are understood by a large per cent of teachers."[31]

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29. Ibid., p. 1.

30. Ibid., p. 4.

31. Jacob S. Orleans, and Edwin Wandt, "The Understandings of Arithmetic Possessed by Teachers," The Elementary School Journal (May, 1953), 53:507.

The professional research and literature in this Chapter has indicated the great concern over the mathematical backgrounds possessed by teachers in-service. From all indications it appears that it is rather weak. There is, therefore, an urgent need for improved preparation on the in-service level as well as on the pre-service level. The present study attempts to investigate further conditions which have not been elaborated on in previous research.

CHAPTER III

COLLECTION, PRESENTATION, AND ANALYSIS OF DATA

In the winter of 1956 the Glennon "Test of Basic Mathematical Understandings" was administered to 58 teachers in their respective schools.

The validity and reliability of the instrument had been established for this purpose in connection with a previous research study.[1] However, two of the items in the original test were modified slightly by Prof. J. Fred Weaver with the permission of the author.

The test, as administered, consisted of 80 items which were grouped into five sections as follows:

Section I - The Decimal System of Notation

Section II- Basic Understandings of Integers and Processes

Section III-Basic Understandings of Fractions and Processes

Section IV- Basic Understandings of Decimals and Processes

Section V - Basic Understandings of the Rationale of Computation

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1. Vincent J. Glennon, A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels, Unpublished Doctor's Dissertation, Harvard University, Graduate School of Education, Cambridge, 1948.

For administrative purposes it was necessary to impose a time limit of 63 minutes working time for the test. This was divided proportionately among the five sections. The time limit was adequate to answer all sections. The subjects were asked not to identify themselves by writing their names on the test papers. It was felt that having the questions answered anonymously would make the subjects feel more at ease in taking the test. It was also felt that some of the teachers would co-operate more willingly if they knew their papers could not be identified. That was particularly true of teachers in a school who were asked by their principal to co-operate in the study.

To facilitate checking of the scoring which was done, the following were tabulated on each paper for each section and for the test as a whole: items right, items wrong, items omitted. Each teacher's raw score, for each section and for the test as a whole, was expressed as _items right_.

The data sheet, which was on the reverse side of the answer sheet, included the following information: years of experience, levels of experience, type of college training, degrees held, and courses taken in arithmetic methods. Test, answer and data sheet are included in the Appendix.

The basic data from the administration of Glennon's _Test of Basic Mathematical Understandings_ are presented below (Tables I-III) in the form of comparative frequency distributions of the raw scores (number of items answered correctly) on each section of the test and on the test as a whole, and a comparison chart for each section of the test and for the test as a whole. The chart has been prepared to show the mean and standard deviation, and the mean expressed as a percent of the maximum possible score.

TABLE I

Distribution of Scores on _A Test of Basic Mathematical Understandings_: Sections I-V

	SECTIONS				
Items Correct	I	II	III	IV	V
19				3	
18				2	
17				5	
16				1	
15	5	4		1	
14	13	10		2	2
13	9	13	7	3	5
12	11	11	7	3	7
11	9	9	3	1	6
10	2	5	5	4	4
9	3	3	6	5	11
8	2		8	8	7
7	1		5	10	5
6	1	1	7	2	4
5	1	1	6	1	3
4	1	1	2	2	1
3			1		
2			1	4	2
1				1	
0					1
N =	58	58	58	58	58
M =	11.90	11.93	8.55	9.93	8.97
σ =	2.46	2.26	3.46	4.81	3.03

The mean and standard deviation for each section of the test may be noted in Table I. In Section I (The Decimal System of Notation), which consisted of 15 items, the mean score was 11.90 and the standard deviation was 2.46. Section II (Basic Understandings of Integers and Processes) with 15 items had a mean score of 11.93 and a standard deviation of 2.26. The third section of 15 items (Basic Understandings of Fractions and Processes) showed a mean score of 8.55 and a standard deviation of 3.46.

It may be observed in Section IV (Basic Understandings of Decimals and Processes) consisting of 20 items that the mean score was 9.93 and the standard deviation 4.81. In the fifth and final section of 15 items which tested Basic Understandings of the Rationale of Computation, the mean score was 8.97 and the standard deviation 3.03.

TABLE II

Distribution of Scores on *A Test of Basic Mathematical Understandings*: Whole Test

Items Correct	f
72-74	4
69-71	2
66-68	3
63-65	3
60-62	4
54-56	6
51-53	9
48-50	8
45-47	6
42-44	4
39-41	1
36-38	2
33-35	2
27-29	1
18-20	1
15-17	2

$N = 58$

$M = 51.28$

$\sigma = 13.00$

It is of interest to note that on the test as a whole (80 items) the mean score was 51.28 and the standard deviation was 13.00.

In Table III which follows is a summary of the data given in Tables I and II. The mean scores, standard deviations, and the means expressed as a percent of the total number of items is presented for each section of the test and for the test as a whole.

TABLE III

Comparison of Means and Standard Deviations on each Test Section and on the Test as a Whole

Section of Test	Number of Items	Mean Score	Mean Expressed as % of the Total Items	S.D.
Section I (The Decimal System of Notation)	15	11.90	79%	2.46
Section II (Basic Understandings of Integers and Processes)	15	11.93	80%	2.26
Section III (Basic Understandings of Fractions and Processes)	15	8.55	57%	3.46
Section IV (Basic Understanding of Decimals and Processes)	20	9.93	50%	4.81
Section V (Basic Understandings of the Rationale of Computation)	15	8.97	60%	3.03
Entire Test	80	51.28	64%	13.00

On Sections I and II the group's level of understanding was higher than on Sections III, IV, and V. For the entire test, the group as a whole responded correctly to an average of only 64% of all the items in the test. The highest mean score (11.93) was made on Section II (Basic Understandings of Integers and Processes) and represents 80% of the possible maximum score of 15. The lowest mean score (9.93) was made on Section IV (Basic Understandings of Decimals and Processes), approximately 50% of a possible score of 20.

In Glennon's investigation, he found that the group of teachers-in-service made an average raw score of 43.81. This represents 54.77% of the total number of items (80) on the test. It must be remembered that the scores in Glennon's study were corrected for chance and those of this study were not. Therefore, the difference between his findings and those of the present study is not as great as would appear to be the case.

The data in the present study were analyzed further in relation to the following factors: instructional levels of experience, length of service, courses taken in arithmetic methods, and degrees held.

A comparison of the means and standard deviations based on the total possible score in relation to three levels of experience is presented in Table IV below.

TABLE IV

Comparison of Performance on Glennon *Test of Basic Mathematical Understandings* for Teachers-in-Service at Three Instructional Levels

Instructional Level	n	Mean Rights	SD
Kdg. - Grade 2	18	42.50	13.53
Grades 3 and 4	17	50.00	11.37
Grades 5 and 6	23	59.09	8.07
XXX	58	XXX	XXX

It is interesting to note that the mean score increases with the instructional level of experience. The mean score (42.50) at the Kdg-Grade 2 level represents 53% of the total possible score. The mean score (50.00) at the Grades 3 and 4 level is 63% of the possible maximum score; and at the Grades 5 and 6 level, the mean score (59.09) is 74% of the total. The mean score of the groups combined was 51.28.

Null Hypothesis to be tested: There is no significant difference in the level of mathematical understanding (as measured by the Glennon test) among teachers at three instructional levels: Kindergarten through Grade 2, Grades 3 and 4, and Grades 5 and 6.

TABLE V

Variance Table for Data of Performance at Three Instructional Levels

Source of Variance	SS	df	Variance Estimates	F
Between groups	2,817.26	2	1408.63	11.06
Within groups	7,004.33	55	127.35	
Total	9,821.59	57		

For df = 2 and 55, *F* must reach a value of 5.01 to be statistically significant at the 1%-level. Since this point is exceeded by the computed value of F, the null hypothesis may be rejected with a high degree of confidence.

Because there is a significant difference in the level of understanding among teachers at three instructional levels, a further study of the data (Table VI) was necessary to determine more specifically where the significant differences were located.

TABLE VI

T-tests for Pairs of Mean Differences Based on Instructional Levels

Group Differences	$M_1 - M_2$	SE_{diff}	t
$\overline{3-4}$ - $\overline{K-2}$	7.50	3.82	1.96
$\overline{5-6}$ - $\overline{3-4}$	9.09	3.61	2.52 (*)
$\overline{5-6}$ - $\overline{K-2}$	16.59	3.55	4.67 (**)

When df = 55, t must equal 2.01 or more to be significant at the 5%-level (*) and must equal or exceed 2.67 to be significant at the 1%-level (**). These data show that there is no significant difference between the mean scores made by the Grades 3 and 4 and the Kindergarten through Grade 2 levels. There is significant difference at the 5%-level between the mean scores of the Grades 5 and 6 group and the Grades 3 and 4 group. There is also significant difference at the 1%-level between the mean scores of the Grades 5 and 6 group and the Kindergarten through Grade 2 Group.

A comparison of the means and standard deviations based on the total possible score in relation to length of service based on three levels: 1-2 years, 3-4 years, and 5 or more years of teaching experience is presented in Table VII below.

TABLE VII

Comparison of Performance on
Glennon *Test of Basic Mathematical Understandings*
for Teachers-in-Service at Three Levels of Teaching
Experience

Length of Service	n	Mean Rights	SD
1-2 years	20	43.85	11.19
3-4 years	11	54.27	12.16
5-more years	27	55.56	12.15
xxx	58	xxx	xxx

The observation may be made that the mean score increases with the length of service. Those with 1-2 years of service attained a mean score which represents approximately 55% of the maximum possible score; those with 3-4 years of service, 68%; and those with 5 or more years of service, 69%.

<u>Null Hypothesis to be tested</u>: There is no significant difference in the level of mathematical understanding (as measured by the Glennon test) among teachers who differ in length of service, classified by three levels: 1 or 2 years, 3 or 4 years, 5 or more years of teaching experience.

TABLE VIII

Variance Table for Data of Performance According to Length of Service

Source of Variance	SS	df	Variance Estimates	F
Between groups	1,696.19	2	848.10	5.74
Within groups	8,125.40	55	147.73	
Total	9,821.59	57		

For df = 2 and 55, *F* must reach a value of 5.01 to be statistically significant at the 1%-level. Since this point is exceeded by the computed value of F, the null hypothesis may be rejected.

Because there is a significant difference in the level of understanding among teachers according to the length of service, a further study of the data (Table IX) was necessary to determine more specifically where the significant differences were located.

TABLE IX

t-tests for Pairs of Mean Differences
Based on Length of Service

Group Differences	$M_1 - M_2$	SE_{diff}	t
$\overline{3-4} - \overline{1-2}$	10.42	4.56	2.29 (*)
$\overline{5-5^+} - \overline{3-4}$	1.29	4.35	.30
$\overline{5-5^+} - \overline{1-2}$	11.71	3.59	3.26 (**)

When df = 55, t must equal 2.01 or more to be significant at the 5%-level (*) and must equal or exceed 2.67 to be significant at the 1%-level (**).

These data show that there is no significant difference between the mean scores attained by the group which has 5 or more years of service and the group which has 3-4 years of service. It may be observed that there is significant difference at the 5%-level between the mean scores of the 3-4 year group and the 1-2 year group. At the 1%-level there is significant difference between the mean scores of the 5 or more years group and the 1-2 year group.

The most inexperienced group, those with 1-2 years of service, was responsible for the fact that there was significant difference in the level of mathematical understandings among teachers who differ in length of service.

Glennon found no significant difference in his study of the relationship between the number of years of experience teachers-in-service have had in teaching arithmetic and their levels of basic mathematical understandings.

A further comparison was made between the scores of those teachers who have had courses in arithmetic methods and the scores of those who have not had courses in arithmetic methods. Table X includes data which determine whether there is significant difference between the scores of the two groups.

TABLE X

t Ratio of Significance between the Scores of Teachers-in-Service Who Have Had Courses in Arithmetic Methods and Teachers Who Have Not Had Courses

	N	M	SD	M_1-M_2	SE_{diff}	t
Have Had Courses	46	53.63	11.10	11.38	4.05	2.81(*)
Have Not Had Courses	12	42.25	15.62			

It is important to note that the mean score (53.63) made by the group who had taken arithmetic methods courses was higher than the mean score (42.25) attained by the group who had not taken arithmetic courses.

Null Hypothesis to be tested: There is no significant difference in the level of mathematical understanding (as measured by the Glennon test) between teachers who have had courses in arithmetic methods and teachers who have not had courses in arithmetic methods.

When df = 56, t must equal 2.01 or more to be significant at the 5%-level and must equal or exceed 2.67 to be significant at the 1%-level (*). It may be observed that there is significant difference at the 1%-level between the mean scores of those who have had arithmetic courses and those who have not had arithmetic courses. No distinction was made between courses on the Bachelor's degree level and courses on the Master's degree level.

These findings are contrary to the findings made by Glennon in comparing the mathematical understandings of two groups, one of which had done graduate work in the *Psychology and Teaching of Arithmetic* and one which had *not* done such graduate work. His data indicated, for the group tested, that graduate work in the *Psychology and Teaching of Arithmetic* did *not* contribute to growth in basic mathematical understandings.

Comparison of performance on the Glennon Test of Basic Mathematical Understandings was made on the basis of those who held a Bachelor's degree and those who held a Master's degree. Table XI includes data which determines if there is significant difference between the scores made by the two groups.

TABLE XI

t Ratio of Significance between the Scores of Teachers-in-Service Who Have Bachelor's Degrees and Teachers Who Have Master's Degrees

	N	M	SD	M_1-M_2	Se_{diff}	t
Bachelor's	38	49.03	12.44	5.91	3.95	1.50
Master's	16	54.94	13.98			

In this phase of the study, there were only two who held no degrees and two who had a Master's degree plus thirty credits. Because of the small number, these four were not included in this part of the study. The mean score (49.03) of the group holding Bachelor's degrees represents 61% of the total possible score, while the mean score of the group with Master's degrees is 69%.

Null Hypothesis to be tested: There is no significant difference in the level of mathematical understanding (as measured by the Glennon test) between teachers who have Bachelor's degrees and teachers who have Master's degrees.

When df = 52, t must equal 2.01 or more to be significant at the 5%-level. Since the t ratio 1.50 is less than this amount, the difference in scores made by the Bachelor's degree group and the Master's degree group cannot be considered a significant one.

CHAPTER IV

FINDINGS, CONCLUSIONS, AND SUGGESTIONS FOR FURTHER STUDY

The basic data for this study were obtained through the use of Dr. Vincent J. Glennon's "Test of Basic Mathematical Understandings." The test was administered to 58 teachers-in-service in the elementary grades, Kindergarten through Grade 6, in the same school system. The present study was concerned with the basic mathematical understandings possessed by teachers-in-service in relation to instructional levels of teaching experience, length of service, courses taken in arithmetic methods, and degrees held. The major findings and conclusions are summarized below.

Purpose I

This study was undertaken to determine the level of basic mathematical understanding (as measured by the Glennon test) possessed by teachers-in-service.

1. The group as a whole had a relatively low level of understanding of concepts basic to the computational processes commonly taught in grades one through six. The mean number of items correct on the entire test was 64% of the total number of items.
2. The performance of the group as a whole was especially poor in basic understandings of decimals and processes,

fractions and processes, and the rationale of computation.

3. Glennon found the mean score for a comparable group of teachers to be approximately 55% of the maximum score. Because the scores on Glennon's investigation were corrected for chance and those in the present study were not, the findings are in closer agreement than they appear to be.

4. The conclusion may be drawn from the above findings that there is a definite need for improvement in the level of basic mathematical understanding possessed by teachers-in-service, particularly in the areas of decimals, fractions, and rationale of computation.

Purpose II

The present study investigated the level of basic mathematical understanding in relation to instructional levels of teaching experience.

1. There is significant difference in the level of mathematical understanding among teachers at three instructional levels: Kindergarten through Grade 2, Grades 3 and 4, and Grades 5 and 6.

2. The most significant difference was between those who had experience at the Grades 5 and 6 level and those at the Kdg. - Grade 2 level.

3. Next in order of significance was the difference between those who had experience in Grades 5-6 and those teachers with experience at the Grades 3 and 4 level.
4. However, there was no significant difference in the mean scores made at the Kdg. - 2 level and the Grades 3 - 4 level.
5. It may be concluded from the above statements that among the factors influencing a higher level of understanding is that of having experience in the grades (5-6) where higher content material is taught.

Purpose III

The present study investigated the level of mathematical understanding in relation to length of service.

1. The data indicated that there was a difference in the level of mathematical understanding among teachers who differ in length of service, classified by three levels: 1 or 2 years, 3 or 4 years, 5 or more years of teaching experience.
2. There was a significant difference between those in the 1-2 year group and those in the 3-4 year group, but no significant difference between the 3-4 year group and those who had taught 5 or more years.
3. There was a highly significant difference in level o f

mathematical understanding between the least experienced, 1-2 years, and the teachers who had taught 5 or more years.

4. In conclusion, it is evident that the relatively low level of mathematical understanding for the inexperienced (1-2 year) group was responsible for the finding that there was a relationship between the number of years a person had taught arithmetic and the level of understanding.
5. Glennon found in his study that there was no relationship between length of service and achievement in basic mathematical understanding.

Purpose IV

In this study a comparison was made between the level of mathematical understanding possessed by teachers-in-service and courses taken in arithmetic methods.

1. The mean score made by the group who had courses in arithmetic methods was significantly higher than the score made by the group who had not taken courses in arithmetic methods.
2. Glennon, however, found that graduate work in the <u>Psychology and Teaching of Arithmetic</u> did <u>not</u> contribute to growth in basic mathematical understandings.

3. In the present study the difference may be attributed to the fact that many different courses were included, some students had taken several courses in arithmetic, and previous mathematical training in high school was not investigated. In Glennon's study the two groups represented the same statistical population in terms of number of courses in mathematics taken previously in college and high school and were compared on the basis of one course, the Psychology and Teaching of Arithmetic.
4. An important conclusion to be drawn is that for the group of teachers in this study, courses in arithmetic methods definitely help to increase the level of mathematical understanding.

Purpose V

It was the purpose of this study to compare the level of mathematical understanding possessed by teachers-in-service with the degrees held.

1. The data indicated that there is no difference in the level of basic mathematical understanding between those who have Bachelor's degrees and those who have Master's degrees.
2. The conclusion may be drawn that, for the sample tested, the degree held had no bearing on the level of achievement on the test.

It should be kept in mind that although the reported findings are deemed valid for the group tested, they would not necessarily hold true for other groups who might differ from that of the present study.

Suggestions for further study

1. A similar study could be made on a larger population of teachers-in-service and the findings compared with those of the present study, which was limited to 58.
2. It would be advantageous to test a group on a compulsory basis. The present study consisted primarily of volunteers, who may have had reasonable confidence in their level of basic mathematical understanding.
3. An analysis could be made of the scores on each section of the test in relation to length of service, instructional level, and courses in arithmetic methods. This would tend to point out more closely where further emphasis should be placed in the in-service training of teachers.
4. A study could be made of the level of mathematical understanding possessed by a teacher in relation to the achievement in arithmetic made by the class with which she worked during the school year.

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APPENDIX

A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS

Directions

This is a test to see how well you understand arithmetic. You do not have to do any written work to find the answers. In fact, you will not be permitted to work out any written computations whatsoever.

The test is divided into five parts:

I. The Decimal System of Notation.
II. Basic Understandings of Integers and Processes.
III. Basic Understandings of Fractions and Processes.
IV. Basic Understandings of Decimals and Processes.
V. Basic Understandings of the Rationale of Computation.

Read each statement or question carefully and decide which of the suggested answers is the correct one. Then write the letter for this answer on the proper line on the answer sheet. All answers are to be recorded in this way on the separate answer sheet. MAKE NO WRITTEN MARKS WHATSOEVER ON ANY OF THE TEST SHEETS.

Sample Item

Which of the following numbers has the largest value?

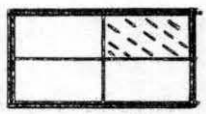
A. 23   B. 9   C. 35   D. 45   E. 11

Since 45 is the correct answer, you would write the letter D on the proper line on the answer sheet.

Try each example but do not stay too long on any one statement or question. If you cannot find the answer you may go on to the next example and come back to the one which you omitted if time permits.

You may go all the way through the test without stopping. When you finish the examples in one section, go right on to the next section.

In Section III you will find shaded diagrams similar to the one at the right of this page. This diagram should be read as 3/4 (i.e., three-fourths). Read all diagrams in this way. Remember: The value of the fraction is indicated by the white or unshaded part of the diagram.



When you are told to do so, begin at the top of the next page and proceed thru the test in the manner which has been indicated.

Remember:- DO NO WRITTEN WORK TO FIND THE ANSWERS. Make no written marks on any of the test sheets. Record the letter of your choice for each correct answer on the proper line on the answer sheet.

Note:- This test has been copyrighted (1947) by Dr. Vincent J. Glennon, School of Education, Syracuse University. The test has been reproduced, and is being used, by permission of the author.

Section I. The decimal system of notation.

1. If you rearranged the figures in the number 43,125 which of the following arrangements would give the smallest number?
A. 54,321 B. 21,345 C. 12,345
D. 14,532 E. 13,245

2. If you rearranged the figures in the number 53,429 which of the following arrangements would give the largest number?
A. 95,324 B. 95,432 C. 59,432
D. 95,234 E. 95,243

3. Which of the following has a 3 in the hundreds' place?
A. 23,069 B. 86,231 C. 49,563
D. 39,043 E. 42,304

4. In the number 2,222 the 2 on the left represents a value how many times as large as the 2 on the right?
A. 1 (same value) B. 10 C. 100
D. 200 E. 1,000

5. About how many tens are there in 6542?
A. 6.5 B. $65\frac{1}{2}$ C. 654
D. 6,540 E. 65,000

6. If the figures in 23,469 were rearranged, which of the following would place the smallest figure in the tens' place?
A. 46,932 B. 96,432 C. 69,234
D. 34,629 E. 92,346

7. In the number 7,255 the 5 on the left represents a value how many times as large as the 5 on the right?
A. 1 (same value) B. 2 C. 5
D. 10 E. 100

8. Which of the following statements <u>best</u> tells why we write a zero in the number 4,039 when we want it to say "four thousand thirty-nine"?
A. Because the number would say 'four hundred thirty-nine' if we wrote a zero in some other place.
B. Writing a zero helps us to read the number.
C. Writing a zero tells us to read the hundreds' figure carefully.
D. Because the number would be wrong if we left out a zero some place.
E. Because we use zero as a place-holder to show that there is no amount to record in that place.

9. Which of the following has a 4 in the ten thousands' place?
A. 423,102 B. 643,142 C. 438,116
D. 374,942 E. 763,420

10. If the figures in 86,473 were arranged differently, which of the following would place the largest figure in the thousands' place?
A. 73,648 B. 38,467 C. 76,483
D. 87,643 E. 86,734

11. In the number 3,944 the 4 on the right represents a value how many times as large as the 4 on the left?
A. 1/10 B. 1/2 C. 5
D. 1 (same value) E. 10

12. In the number 5,492 the 4 represents a value how many times as large as the 2?
A. 2 B. 10 C. 20
D. 100 E. 200

13. About how many hundreds are there in 34,820?
A. $3\frac{1}{2}$ B. 35 C. 350
D. 3,500 E. 35,000

14. Which of the following methods is the best for determining the value of a figure in a number? for example, the value of the 7 in 3748.
A. Its position in the number.
B. Its value when compared with other figures in the number.
C. Its value in the order from 1 to 9.
D. Its value when compared with the whole of the number.
E. Its position in the number and its value.

15. In the number 7,843 the 4 represents a value how many times as large as the 8?
A. 1/10 B. 1/20 C. 1/2
D. 2 E. 20

(Go right on to Section II)

Section II Basic understandings of integers and processes.

1. If you had a bag of 365 marbles to be shared equally by 5 boys, which would be the quickest way to determine each boy's share?
   A. counting B. adding C. subtracting
   D. multiplying E. dividing

2. When a whole number is multiplied by a whole number other than 1, how does the answer compare with the whole number multiplied?
   A. larger B. smaller C. same
   D. 10 times as large E. can't tell

3. When a whole number is divided by a whole number other than 1, how does the answer compare with the whole number divided?
   A. larger B. smaller C. same
   D. one-half as large E. can't tell

4. Which of the following is the quickest way to find the sum of several numbers of the same size?
   A. by counting B. by adding C. by subtracting
   D. by multiplying E. by dividing

5. If the zeros in the two numbers in this example were left off, how would the answer be changed?
   A. The answer would be ten times as large.
   B. The answer would be one hundred times as large.
   C. The answer would be one-tenth as large
   D. The answer would be one-hundredth as large.
   E. The answer would not change.

   60/ 3720

6. Here is an example in subtraction in which letters have been used instead of figures. Which statement is true:
   A. AFGB and CXU added together equal TWMY.
   B. CXU and TWMY added together equal AFGB.
   C. AFGB and TWMY added together equal CXU.
   D. TWMY subtracted from CXU equals AFGB.
   E. CXU subtracted from TWMY equals AFGB.

   AFGB
   \- CXU
   TWMY

7. How would the answer to this example be changed, if a zero were added (annexed) to the right of each number?
   A. The answer would be ten times as large.
   B. The answer would be one hundred times as large.
   C. The answer would not change.
   D. Cannot tell until you add both ways.
   E. The answer would be one thousand times as large.

   364
   2936
   14
   438

8. Adding (annexing) two zeros to the right of a whole number has the same effect as:
   A. Adding ten to the number.
   B. Adding one hundred to the number.
   C. Multiplying the number by ten.
   D. Multiplying the number by one hundred.
   E. Dividing the number by one hundred.

9. What would be the effect on the answer if you added (annexed) two zeros to 439 and took away the zero from 450?

   439
   x 450

   A. The answer would be ten times as large.
   B. The answer would be one hundred times as large.
   C. The answer would remain the same.
   D. The answer would be one-tenth as large.
   E. The answer would be one-hundredth as large.

10. Crossing off a zero from the right side of a number has the same effect as:
   A. Subtracting ten
   B. Subtracting one hundred
   C. Multiplying by ten
   D. Multiplying by one
   E. Dividing by ten

11. What would be the effect on the answer if you added (annexed) two zeros to 92 and changed 4500 to 450?

   92/ 4500

   A. The answer would be ten times as large.
   B. The answer would be one-tenth as large.
   C. The answer would be one hundred times as large.
   D. The answer would be one-hundredth as large.
   E. The answer would be one-thousandth as large.

12. Which one of the following methods could be used to find the answer to this example?

   17/ 612

   A. Multiply 17 by the quotient.
   B. Add 17 six hundred twelve times. Answer would be the sum.
   C. Subtract 17 from 612 as many times as possible. Answer would be number of times you were able to subtract.
   D. Add 612 seventeen times. Answer would be the sum.
   E. Multiply 17 by 612. Answer would be the product.

13. If the numbers in a large addition example were changed so that the top number was placed at the bottom and the bottom number was placed at the top, how would the answer be affected?
   A. Answer would be larger.
   B. Answer would be smaller.
   C. Answer would not change.
   D. Could not do the example.
   E. Cannot tell until you add both ways and compare.

14. How would the example be affected if you put the 29 above 4306?

   4306
   x 29

   A. The answer would be larger.
   B. The answer would be smaller.
   C. The answer would be the same.
   D. Cannot tell until you multiply both ways.
   E. You cannot do the example when the large number is on the bottom and the small number on top.

15. What would be the effect on the answer if you added (annexed) two zeros to 39?

   39/ 859

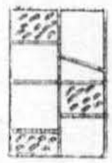
   A. The answer would be one hundred times as large.
   B. The answer would be one-hundredth as large.
   C. The answer would be one-thousandth as large.
   D. The answer would not change.
   E. You could not do the example.

(Go right on to Section III)

Section III. Basic understandings of fractions and processes.

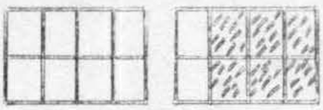
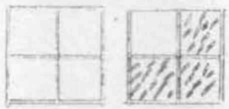
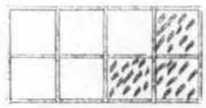
1. Which of the following fractions is the largest?
A. 1/7 B. 5/7 C. 3/7 D. 11/7 E. 6/7

2. Which of these statements best tells why we cannot say that the unshaded parts of this picture represent 5 "eighths"?
A. Because more than 5/8 of it is unshaded.
B. Because the unshaded parts are not together.
C. Because all the unshaded parts are not the same size.
D. Because less than 5/8 of it is unshaded.
E. Because the parts are not the same shape.



3. Which of the following fractions is the smallest?
A. 1/9 B. 1/5 C. 1/2 D. 1/7 E. 1/3

4. Which picture shows how the result would look if you divided the numerator and denominator of 10/8 by 2?

A. 
B. 
C. 
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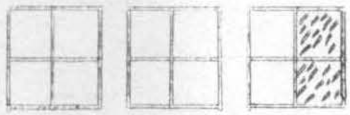
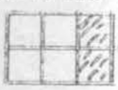
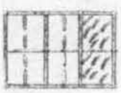
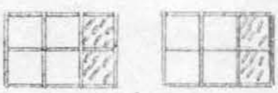
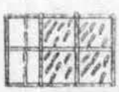
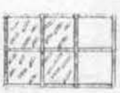
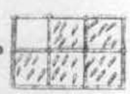
E. 

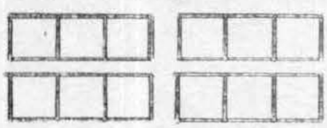
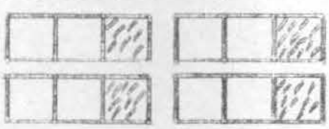
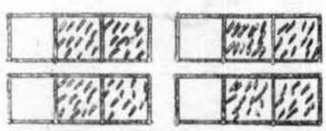
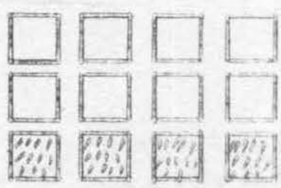
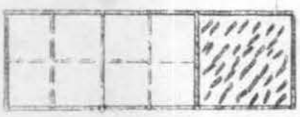
5. When a whole number is multiplied by a common (proper) fraction, how does the answer compare with the whole number?
A. larger B. smaller C. same D. cannot tell E. half as large

6. Which picture shows how the result would look if you divided the numerator of this fraction by 2? →



A. 
B. 
C. 
D. 
E. 

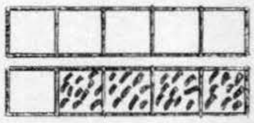
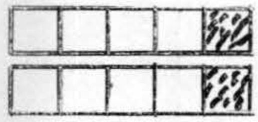
7. Which picture best shows the example, 4 x 2/3 ?

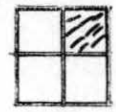
A. 
B. 
C. 
D. 
E. 

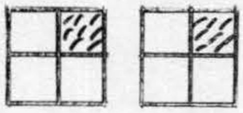
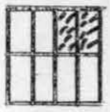
8. When a common (proper) fraction is divided by a common fraction, how does the answer compare with the fraction divided?
A. larger B. smaller C. same D. cannot tell E. twice as large

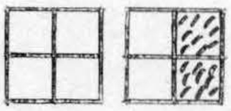
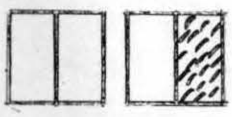
9. Which picture shows how the result would look if you multiplied the numerator and denominator of 3/5 by 2?

A.  B.  C. 

D.  E. 

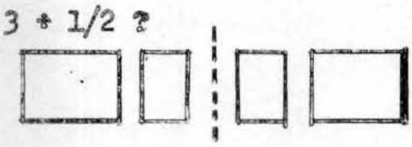
10. Which picture shows how the result would look if you multiplied the denominator of this fraction by 2? ——————→ 

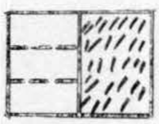
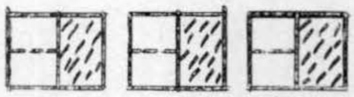
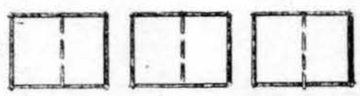
A.  B.  C. 

D.  E. 

11. When a whole number is divided by a common (proper) fraction, how does the answer compare with the whole number?
A. larger B. smaller C. same D. cannot tell E. varies

12. Which picture looks like this example: 3 ÷ 1/2 ?

A.  B. 

C.  D.  E. 

13. Which sentence best tells why the answer is larger than the 5?

$$5 \div \frac{3}{4} = 6\frac{2}{3}$$

A. Because inverting the divisor turned the 3/4 upside down.
B. Because multiplying always makes the answer larger.
C. Because the divisor 3/4 is less than 1.
D. Because dividing by proper and improper fractions makes the answer larger than the number divided.
E. Inverting a fraction puts the larger number on top.

14. Which sentence is shown by this picture? 

A. Fractions with common denominators may be added.
B. The value of a fraction is changed if a number is subtracted from the numerator and denominator.
C. Dividing the numerator and denominator of a fraction by the same number does not change the value of the fraction.
D. Fractions with the same denominators are equal.
E. Fractions with the same numerators are equal.

15. When a common (proper) fraction is multiplied by a common fraction, how does the answer compare with the fraction multiplied?
A. larger B. smaller C. same D. cannot tell E. varies

Section IV. Basic understandings of decimals and processes.

1. How should you write the decimal, "eighty and eight hundredths"?
(A) .8008 (B) 80.800 (C) 80.08 (D) 80.008 (E) 8008.08

2. How should you read this decimal: .0309 ?
A. Three and nine hundredths.
B. Three hundred nine thousandths.
C. Three hundred nine ten-thousandths.
D. Thirty-nine thousands.
E. Three hundred nine hundredths.

3. Which decimal tells how long line Y is when compared with line X?
line X  line Y 
(A) .5 (B) .625 (C) 1.25 (D) 75 (E) 33

4. About how many tenths are there in 1.25?
(A) .13 (B) 1.3 (C) 13 (D) 125 (E) 1250

5. About how many hundredths are there in .635?
(A) 1/2 (B) 6.35 (C) 63.5 (D) 635 (E) 6350

6. What would be the effect on the answer if you dropped the zero from 23.90?

```
  23.90
x 2.75
```

A. The answer would have the same value.
B. The answer would be one-tenth as large.
C. The answer would be ten times as large.
D. You would point off three places.
E. It would be the same as subtracting zero from the answer.

7. How would the answer be changed if you changed 6.5 to .65 and 84.5 to 845?

6.5/ 84.5

A. The answer would be the same.
B. The answer would be ten times as large.
C. The answer would be one hundred times as large.
D. The answer would be one-tenth as large.
E. The answer would be one-hundredth as large.

8. Which seems to be the correct answer to this example:
ten divided by five-tenths.
(A) 1/2 (B) 2 (C) 10 (D) 20 (E) 50

9. Which decimal tells how long line Y is when compared with line X?
line X  line Y 
(A) 1.25 (B) 1.50 (C) 2 (D) 2.40 (E) 2.50

10. Which of the following decimals has the largest value?
(A) 30.3 (B) 30.03 (C) 30.0333 (D) 30.303 (E) 30.003

11. What would be the effect on the answer if you changed 368 to 3680 and 24 to 2.4?

```
 368
x 24
```

A. The answer would be smaller.
B. It would not change the answer.
C. It would be the same as adding a zero to the answer.
D. The answer would be one-tenth as large.
E. Cannot tell until you do the example both ways.

12. Which decimal has the smallest value?
(A) .3 (B) .09 (C) .048 (D) .693 (E) .0901

13. How would the answer be affected if you moved the point one place to the left in both numbers?

$$\begin{array}{r} 43.5 \\ \times\ \underline{4.8} \end{array}$$

A. The answer would be one-tenth as large.
B. The answer would be one-hundredth as large.
C. The answer would be one hundred times as large.
D. It would be the same as subtracting 100 from the answer.
E. The answer would have the same value.

14. How would the answer be changed if you moved the point two places to the right in both numbers?

$$\begin{array}{r} 43.6 \\ \times\ \underline{2.45} \end{array}$$

A. The answer would have the same value.
B. The answer would be one thousand times as large.
C. You would point off differently.
D. You cannot move the point in the top number two places.
E. The answer would be 10,000 times as large.

15. How would the answer be affected if you moved the point in 485.3 one place to the right?

$62\overline{)485.3}$

A. The answer would be ten times as large.
B. The answer would be 10 larger.
C. The answer would be one-tenth as large.
D. The answer would have a zero at the right.
E. The value of the answer would be the same.

16. How would the answer be affected if you changed 7.3 to 73 and 1390 to 13.90?

$7.3\overline{)1390}$

A. The answer would be one hundred times as large.
B. The answer would be one-tenth as large.
C. The answer would be one thousand times as large.
D. The answer would be one-hundredth as large.
E. The answer would be one-thousandth as large.

17. About how many tenths are there in .055?
(A) 0 (B) 1/2 (C) 5 (D) 10 (E) 50

18. About how many thousandths are there in 16.5?
(A) 1.7 (B) 17 (C) 170 (D) 1,700 (E) 17,000

19. Why is the answer smaller than the top number?

$$\begin{array}{r} 8 \\ \times\ \underline{.5} \\ 4.0 \end{array}$$

A. Because 8 is more than .5
B. Because you are finding how many .5's in 8.
C. Because .5 is less than 8.
D. When you multiply by a decimal the answer is always smaller than the top number.
E. Because multiplying by .5 is the same as finding half of the number.

20. How would the answer be changed if you changed 1.47 to 147?

$1.47\overline{)34.75}$

A. You would get the same answer.
B. The answer would be ten times as large.
C. The answer would be one hundred times as large.
D. The answer would be one-tenth as large.
E. The answer would be one-hundredth as large.

(Go right on to Section V)

Section V Basic understandings of the rationale of computation.

1. Why do we find a common denominator when adding fractions with unlike denominators?
   A. You cannot add together things that are different.
   B. It is easier to add fractions when they have a common denominator.
   C. The denominators have to be the same in order to add.
   D. We learned to add unlike fractions that way.
   E. So that all the fractions will have the same value.

2. When dividing by a decimal, why do we move the point to the right?
   A. Multiplying by a multiple of ten is a quick way of changing a decimal to a whole number.
   B. It places the decimal point in the quotient correctly.
   C. You can only divide by a whole number.
   D. To make the divisor equal to the dividend.
   E. It is easier to divide by a whole number than a decimal.

3. Which one of the following would give the correct answer to this example? 2.1 x 21
   A. The sum of 1 x 2.1 and 21 x 2.1
   B. The sum of 10 x 2.1 and 2 x 2.1
   C. The sum of 10 x 2.1 and 20 x 2.1
   D. The sum of 1 x 2.1 and 20 x 2.1
   E. The sum of 1 x 2.1 and 2 x 2.1

2.1
21

4. Which statement best tells why we "invert the divisor and multiply" when dividing a fraction by a fraction?
   A. It is an easy method of finding a common denominator and arranging the numerators in multiplication form.
   B. It is an easy method for dividing the denominators and multiplying the numerators of the 2 fractions.
   C. It is a quick, easy and accurate method of arranging two fractions in multiplication form.
   D. Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
   E. It is a quick method of finding the reciprocals of both fractions and reducing to lowest terms (cancelling).

5. Why do we move the second partial product one place to the left when we multiply by the 6?

   ```
     729
    x 68
   ```

   A. Because the answer has to be larger than 729.
   B. Because the six means six tens.
   C. Because 6 is the second figure in 68.
   D. Because we learned to multiply that way.
   E. Because the 6 represents a greater value than the 8 represents.

6. Which statement best tells why we arrange numbers in addition the way that we do?
   A. It is an easy way to keep the numbers in straight columns.
   B. It helps us to add correctly.
   C. It helps us add only those numbers in the same position.
   D. It helps us to carry correctly from one column to another.
   E. It would be harder to add if the numbers were mixed.

7. When you multiply by the 4 in 48 you will get a number that is how large compared with the final answer?

```
  485
x  48
```

A. One-twelfth as large.
B. One-tenth as large.
C. One-half as large.
D. Five-sixth as large.
E. Twice as large.

8. The answer to this example will be how large when compared with the 69?

```
  827
x  69
```

A. Twice as large.
B. Sixty-nine times as large.
C. One sixty-ninth as large.
D. Eight hundred twenty-seven times as large.
E. $\frac{1}{827}$ as large.

9. Which statement best tells why it is necessary to 'borrow' in this example?

```
  567
- 392
```

A. Because the top number is smaller than the bottom number.
B. You cannot subtract 92 from 67.
C. You cannot subtract 9 tens from 6 tens.
D. You cannot subtract 39 tens from 56 tens.
E. You cannot subtract 9 from 6.

10. Which statement best tells why we carry 2 from the second column?

```
251
161
252
271
```

A. The sum of the second column is 23 which has two figures in it. We have room for the 3 only, so we put the 2 in the next column.
B. The sum of the second column is more than 20, so we put the 2 in the next column.
C. Because we learned to add that way.
D. The value represented by the figures in the second column is more than 9 tens, so we put the hundreds in the next column.
E. If we do not carry the 2, the answer will be 20 less than the correct answer.

11. In this example you multiply by the 6, then by the 3. How do the two results (partial products) compare?

```
  749
x  36
```

A. The second represents a number one-half as large as the first.
B. The second represents a number twice as large as the first.
C. The second represents a number five times as large as the first.
D. The second represents a number ten times as large as the first.
E. The second represents a number twenty times as large as the first.

12. Which would give the correct answer to 439 x 563?
A. Multiply 439 x 3; 439 x 6; 439 x 5 - then add answers.
B. Multiply 439 x 3; 439 x 63; 439 x 563; then add answers.
C. Multiply 563 x 9; 563 x 3; 563 x 4 - then add answers.
D. Multiply 563 x 9; 563 x 39; 563 x 439 - then add answers.
E. Multiply 439 x 3; 439 x 60; 439 x 500 - then add answers.

13. Which statement best explains the 4 in the answer?

```
     48
26/ 1248
    104
     208
     208
```

A. The 4 means that there are forty-eight 26's in 1248.
B. The 4 in the answer means that there are four 26's in 1248.
C. The 4 means that 2 goes into 12 four times, and 5 would be too large.
D. The 4 means that there are at least forty 26's in 1248.
E. The 4 means that the answer will come out even.

14. Here is an example in subtraction of mixed numbers in which it is necessary to "borrow." Which statement best explains the borrowing.

$$\begin{array}{r} 5\frac{3}{8} \\ -\,2\frac{5}{8} \\ \hline \end{array}$$

A. You cannot subtract 5/8 from 3/8, so you take 1 from the 5 and put it in front of the 3 making 13.
B. You cannot subtract 5/8 from 3/8 so you add the 3 and the 8 making 11/8.
C. You cannot subtract 5/8 from 3/8, so you turn them around and subtract 3/8 from 5/8.
D. You cannot subtract 5/8 from 3/8, so you take 1 from the 5 and add it to 3/8 making it 4/8.
E. You cannot subtract 5/8 from 3/8, so you take 1 from the 5 and change it to 8/8; then add the 8/8 to 3/8 making 11/8.

15. Which statement best explains what happens when you reduce a fraction to lowest terms?
A. The size of the terms and the value of the fraction become smaller.
B. The value of the fraction does not change. The size of the part represented by the new denominator is smaller, and the number of parts represented by the new numerator is less.
C. The value of the fraction does not change. The terms are smaller, but they represent more parts of larger size.
D. The value of the fraction does not change, but the parts of the fraction represented by the new numbers become fewer in number and larger in size.
E. The value of the fraction changes because the new numbers are smaller.

End

ANSWER SHEET for A TEST OF BASIC MATHEMATICAL UNDERSTANDINGS

Name ______________________ Date ______________________

Section I	Section II	Section III	Section IV	Section V
1. ______	1. ______	1. ______	1. ______	1. ______
2. ______	2. ______	2. ______	2. ______	2. ______
3. ______	3. ______	3. ______	3. ______	3. ______
4. ______	4. ______	4. ______	4. ______	4. ______
5. ______	5. ______	5. ______	5. ______	5. ______
6. ______	6. ______	6. ______	6. ______	6. ______
7. ______	7. ______	7. ______	7. ______	7. ______
8. ______	8. ______	8. ______	8. ______	8. ______
9. ______	9. ______	9. ______	9. ______	9. ______
10. ______	10. ______	10. ______	10. ______	10. ______
11. ______	11. ______	11. ______	11. ______	11. ______
12. ______	12. ______	12. ______	12. ______	12. ______
13. ______	13. ______	13. ______	13. ______	13. ______
14. ______	14. ______	14. ______	14. ______	14. ______
15. ______	15. ______	15. ______	15. ______	15. ______
			16. ______	
			17. ______	
			18. ______	
			19. ______	
			20. ______	

Do Not Write Below This Line

O = ______	O = ______	O = ______	O = ______	O = ______
W = ______	W = ______	W = ______	W = ______	W = ______
R = ______	R = ______	R = ______	R = ______	R = ______

- -

TOTAL	O = ______	W = ______	R = ______	TOTAL