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Student perceptions of a mathematics major for prospective elementary teachers with an inquiry-based philosophy

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Abstract: In this paper we present data from one-on-one interviews conducted with students who have taken intermediate and advanced inquiry-based mathematics courses in a program that prepares future preK-8 teachers. Many of these students entered college with a fear of math but then gained confidence from a required introductory math course and chose to pursue a major in Mathematics for Teaching. These interviews help explain their choice to pursue this major and their general experiences in IBL math classes. These insights were used to improve our program and may also help other mathematics instructors and programs that hope to implement an inquiry-based approach in their departments.

1. INTRODUCTION

1.1 Preface

Wheelock College was founded as “Miss Wheelock’s Kindergarten Training Institute” in 1888 and became a four-year degree granting college in 1941. Wheelock’s history of using active, engaged pedagogies goes back to its founding, but mathematics has mostly been seen as marginal to the college’s mission. The college has no history of teaching a traditional mathematics curriculum, so the department has had great freedom to experiment with active pedagogies.

The two authors each left traditional graduate math programs with a strong interest in undergraduate teaching, particularly in helping students see that there is more to math than what most have seen in school and that they could succeed in math, no matter their previous experience. The second author came to Wheelock College in 1994, as the first mathematician on the full time faculty; she was hired to help develop a Math and Science major for prospective elementary teachers after the state of Massachusetts started requiring prospective elementary teachers to have an Arts and Sciences Major. She was drawn by opportunities to have the

freedom to experiment pedagogically, to teach future teachers and help develop a program for them, and to collaborate with and learn from the many education faculty members who were passionately involved in changing math and science education. At the time, very few mathematicians were involved in educating pre-service Pre-K to 8 teachers, and she spent more time helping develop and run the program than writing about it.

A generation later, when the first author, a statistician, became interested in mathematics education, he found a robust community doing research on undergraduate education and on inquiry-based pedagogy. He came to Wheelock in 2011, as the third full time math faculty member and was drawn to the college for many of the same reasons the second author was. Before long, he saw many opportunities to study a program that had grown organically from the needs of our students, but that had the potential to add new insight to important issues in the research literature.

This paper represents the first collaboration of the authors on a formal study of Wheelock College's unusual inquiry based mathematics major for prospective K-8 teachers. Our personal goals in conducting this study were to improve our program, to share interesting and/or useful results with others, and to lay the groundwork for future studies and future collaborations.

1.2 Introduction to the Study

Inquiry-based learning has seen an increased presence in the undergraduate mathematics classroom in recent years, as inquiry materials to support classes such as Linear Algebra [21], Abstract Algebra [8, 11] and Differential Equations [16] have been developed. However, despite empirical evidence indicating that undergraduate students who take inquiry-based mathematics courses have increases in learning gains [9, 13, 14, 17, 19], increases in mathematics teaching self-confidence [6] and evidence of increases in science literacy self-confidence [2], instructors

are still apprehensive about implementing inquiry-based strategies for reasons including lack of coverage, [22] trouble with assessment [7, 12], lack of materials [7], and lack of training [4]; Many of these obstacles to implementing inquiry teaching are being addressed. Materials have been and are under development [8, 11, 16, 21], and coverage issues have been theorized to not exist as deeper conceptual knowledge may lead to stronger thinkers who can compensate for a lack of coverage [9,22]. However, it is still not common for courses for math majors to be taught from an inquiry perspective, and students who do experience this type of class typically experience them as anomalies in a curriculum where most pedagogy is directive.

Of the various inquiry-based pedagogies in the literature, the pedagogy at Wheelock College is best described as “inquiry-oriented,” according to Rasmussen and Kwon’s description, where the two primary components are students inquiring into mathematics and instructors inquiring into students’ thinking [17]. As part of this pedagogy, students routinely ask mathematical questions and seek their own answers to them. For many of our students, this pedagogy is completely new to them when they take our introductory courses. Eight of the nine math majors we interviewed had not intended to be math majors when they started college but then discovered they enjoyed the introductory courses enough to declare a math major. Eight of these nine students had intended to pursue an education major when they started college and were pursuing this degree during their introductory math course.

Wheelock College has both an unusual demographic and an unusually strong commitment to inquiry-based pedagogy within the mathematics major, and these qualities have enabled us to be somewhat of a laboratory for designing a non-traditional math major that attracts students who otherwise would not typically elect a math major. From the mathematics major’s first inception in the early 1990s, Wheelock’s math major has been founded on the department’s longstanding commitment to active, hands-on pedagogies emphasizing problem solving, reasoning, collaboration, communication, and conceptual understanding.

In this paper we examine how elementary education math majors experience inquiry in

mathematics courses designed for them, in the context of Wheelock's program. Besides helping the department improve our program, we believe that examining how these students experience these mathematics courses can provide valuable insight to those looking to implement an inquiry model in their class and to departments looking to significantly increase the number of courses taught under this model. Teaching is a two-way street, and as instructors we can prepare well, plan a thoughtful lesson, and develop engaging activities designed to develop mathematical skills and curiosities; but if students perceive these plans negatively, we will not accomplish what we set out to do. Therefore, this paper we investigate the following questions:

1. How does the student experience in inquiry-based mathematics and the teaching methods used in these courses compare to the student experience in other mathematics courses (either at other colleges or in high school)?
2. What do students in inquiry mathematics courses believe is different about these courses?
3. How do students' perceptions of themselves as mathematics students change after taking inquiry-based mathematics courses?

2. BACKGROUND

2.1 Wheelock College Context

In 1988 the Commonwealth of Massachusetts began requiring candidates for teacher licensure to have a 36-credit (9 course) undergraduate Arts and Sciences major to complement their education major, which led to the introduction of a major in Mathematics and Science in the 1990-1992 Wheelock College catalog. Presently, students can choose from 36-credit majors in Mathematics for Teaching, Science for Teaching, or Math/Science for Teaching; the department

also offers 16 credit minors in mathematics and a 48-credit Math for Teaching major (primarily taken by students who opt to finish their education coursework in graduate school). All of these majors and minors draw from the classes discussed below.

Wheelock College currently has approximately 1000 undergraduates, 38 of whom are majoring in math, science, or math/science for teaching, and 254 of whom are majoring in early childhood, elementary, or special education. All education majors are required to take an introductory *Concepts and Processes* sequence, modeled on the 2001 CBMS MET report [23]; the standard sequence consists of three courses. The introductory sequences are designed to teach mathematics content while modeling pedagogy. Since the default for many teachers is to teach as they were taught, one goal of the introductory sequence is have students experience inquiry-based pedagogy as part of their own mathematics learning, not just as something they are told in a methods class. These courses also include a required weekly study group; for all but the last course in a given sequence, study groups are led by an undergraduate “Math Leader,” a student who has taken and performed well in the course and who attends a biweekly supervision seminar. The math study groups and Math Leader program date back to 1998. We developed them to allow our students time to work on group activities outside of class, to help some of our majors start thinking about math pedagogy before they went into a practicum, and to take advantage of the strengths of our population, where students typically have strong interpersonal skills and enjoy helping others.

The data presented in this paper comes from one-on-one interviews with students about math major courses, and in many of the interviews, participants offered information about the introductory courses as well. We present a quote from the interviews here to illustrate the type of data this paper will use:

Blair: “The (introductory) classes are really interesting because most of them have study groups so you meet three days in the classes, and then you usually have a study group

where there's sometimes study group leaders, and you work on like some activity with a group of people, so you get to see what they're thinking, what you're thinking, and it's like a way to like cooperate with other people. So it's really interesting. I think it's really helpful because like I think a certain way, and when I see some of my friends, or other people, I'm like 'oh, now I understand the problem better.' ... We understand where (things) come from, and we really like derive where -- how the thinking of the process happens. ... Here I feel like they really show you how it's done rather than "I recognize the formula; I know what it is," but to actually understand like where it comes from, why we use it, and how many other ways we can solve it. I really think that's interesting"

2.2 Mathematics major Courses, Background:

While some students come to the college intending to major in Mathematics for Teaching, most prospective education majors come to the college far more committed to majoring in education than in a particular Arts and Sciences major. As will be illustrated below, the primary reason students give for choosing a Mathematics for Teaching major is a positive experience in the introductory math for teachers sequence; for many, the pedagogy in these classes is integral to increasing both their interest in and confidence in their ability to do math.

The 36-credit Mathematics for Teaching major begins with a three-semester math for elementary teachers sequence (or a two-semester honors version; only two classes count toward the major) modeled on the 2001 CBMS MET report [23]. Additional requirements include three intermediate courses: Algebra and Number Theory, Geometry, and Probability and Statistics, a capstone seminar, and three electives (at least one advanced). Intermediate electives include Calculus I, Regression, Math Through Crochet, Quilts, and Temari, and Math from Many Cultures. Advanced electives include Discrete Mathematics and the History of

Mathematics.

The main pedagogical influence on the design of the major were the five process standards included in the NCTM's *Principles and Standards for School Mathematics*: problem solving, reasoning and proof, communication, connections, and representations [25]. The major was designed to teach college-level courses with content most relevant to pre-K to 8 teachers, using pedagogy that modeled the NCTM process standards. When the major was designed, the term *constructivist* was more in vogue to describe its pedagogy; without significant changes in approach, the term *inquiry-based* now applies, and the major now models the mathematical practices in the Common Core curriculum [26], albeit less explicitly.

Many students who choose the Mathematics for Teaching major have gaps in their pre-college preparation that would make it hard for them to complete a more traditional mathematics major within four years, although they are often creative problem solvers and problem posers, who are eager to learn more math. We cannot and do not seek to cover all of the topics in a traditional major, as some courses are less relevant for pre-K to 8 teaching. Most notably, calculus is an elective, which many students do not take. With only six or seven math courses in the program beyond the introductory sequence, it never made sense to us to use three or four of them for calculus (and possibly pre-calculus), when one of our goals is to show students how much more there is to math than what is in the traditional school curriculum.

A few examples from a recent section of the Algebra and Number Theory course illustrate some of the pedagogy and content choices involved in Wheelock's courses for math majors, as well as some of the ways that Wheelock's courses differ from many standard math major courses. On the first day of class, the students played a clapping game to introduce modular arithmetic and to review least common multiples and greatest common factors: they clapped their names (sometimes two names simultaneously) with claps for consonants and knee slaps for vowels [18]. After practicing modular arithmetic in various contexts, students made multiplication tables in different mods. As they were gaining fluency in these new number

systems, they also engaged their curiosity by looking for patterns and thinking about why the patterns they saw occurred and why they might continue. A discussion about patterns led to insights about units, zero divisors, and related concepts. Later in the class, students explored patterns in repeating decimals, which led to a discussion Fermat's and Euler's theorems. All these activities had the potential to deepen students' understanding of the elementary curriculum, while reinforcing the connection between curiosity, exploration, and mathematics understanding. Many activities can also be adapted to use with children, which further engages students pursuing an education major.

The capstone seminar includes significant problem posing, as well as individual and group work. Instructors generally begin the course with a set theme but no specific course schedule, which leaves room for students to engage their curiosity and follow their interests. As the capstone is taken late in the program, we presume that most of the students in the study had not yet taken it.

3. METHODS

Data for this study came from 11 participants drawn from a population of approximately 45 students. The student population included any current student who had completed a course designated by the researchers as a "math major" course during the previous three academic years, including Mathematics for Teaching majors/minors, Mathematics/Science majors, Science majors, and a small number of students who took a "math major" class for either general education credit or transfer credit for another school in the college's consortium. While the purpose of this study was to investigate the experiences and perceptions of those using the classes for their major or minor, we did not want exclude anyone who has participated in these classes as students taking the courses for other purposes may have different and valuable insights. Participants were only informed that they were being interviewed about their

experiences in their mathematics courses and were not told that the researchers were collecting data related to inquiry-based instruction.

Data was collected through semi-structured interviews conducted by a neutral interviewer hired from a local university. The entire population was invited to participate, and those who chose to participate were informed that every effort would be taken to keep their identity anonymous. The participants were made aware that Wheelock College would be named in the study and that with the small number of students in the population, anonymity could not be guaranteed. In addition, if the participant volunteered a response that named another member of the population, that name was changed. Finally, participants given the option of allowing the researchers to know if they had participated in the study two years after their graduation date, so that the researchers might follow up in a longitudinal study.

The semi-structured interview design allowed the interviewer to ask consistent, integral questions while also allowing for follow up questions to clarify and build a rapport. The semi-structured design has advantages when data collection is conducted during only one interview, without the possibility of a follow-up interview [1]. To prepare for this study, the interviewer was given a sample interview conducted by one of the researchers with a graduate of the program.

Interviews were transcribed and forwarded to the researchers; the audio recordings were then deleted once the researcher indicated there were no questions about the transcription.

After all interviews had been conducted, the interviewer contacted the entire population and requested they complete a questionnaire that asked them to estimate the division between lecture and non-lecture activities in each math-major course they had taken. The researchers did not know whether a participant answering the questionnaire did or did not participate in the interviews. The participants were able to answer a division rounded to the nearest 5%, and all possible divisions rounded to the nearest 5% were offered as possible responses. The questionnaire was deliberately sent to the population after the interviews so that the participants were not biased in how they responded to interview questions.

The interview revolved around 13 base questions, and this paper will focus on the following six questions:

- Interview Question 1 (IQ1) - If you were going to describe the math major classes to a person who does not know about Wheelock math major classes, how would you describe them?
- IQ2 - When did you decide to become a math major? (or minor, or science major/minor; question molded to participant)
- IQ6 - How have your intermediate/advanced mathematics courses been different from your introductory courses?
- IQ10: How, if at all, has your opinion of yourself as a mathematics student changed since you have taken mathematics courses at Wheelock?
- IQ11: How, if at all, do you think mathematics classes at Wheelock are different than mathematics classes at other colleges? What makes them better? What makes them worse?
- IQ13: If I told you the math department has a department wide teaching philosophy, what do you think that philosophy would be?

The participants all have gender-neutral aliases assigned by the interviewer, and for consistency, will all be assigned feminine pronouns in the discussion. In addition, all students who are majoring in Elementary Education, Early Childhood Education or Special Education will be referred to as Education majors. The participants include nine students who would be considered members of the mathematics program (as either Mathematics for Teaching majors, or Mathematics and Science for Teaching majors) and two students pursuing different majors: a student from a different school majoring in Economics and a non-education Science major who was required to take one math class. Based on the information collected during the interview process, the researchers constructed the following backgrounds for the interview participants:

- Alex - Education and Mathematics for Teaching major who decided to major in math after taking the introductory elementary math content courses. She is also a Math Leader, so presumably a stronger student.

- Aubrey - Education and Mathematics and Science interdisciplinary major who decided to major in something with a science background because she thinks it will help her if she decides to pursue speech pathology.
- Blair - A Mathematics for Teaching major who is not majoring in education. As a non-education major she takes twelve additional mathematics credits along with some more traditional mathematics curriculum courses at some of the consortium colleges.
- Cameron - Education and Mathematics for Teaching major who came to college planning on majoring in mathematics and has aspirations to go into educational administration. She is also a math leader, so presumably a stronger student.
- Logan - Education and Mathematics for Teaching major who decided to major in math to make her a more marketable person in education.
- Parker - A Science major who appears to be concentrating in Life Science. She is not an education major.
- Peyton - Education and Mathematics and Science major. Took an advanced math track in high school that included AP Calculus.
- Quinn - Education and Mathematics for Teaching major who decided to major in math to make her a more marketable person in education. She is a former Math Leader, so presumably a stronger student.
- Robin - Education and Mathematics for Teaching major who decided to major in math after taking the introductory elementary math content courses.
- Skyler - A non-Wheelock student from a different college in the consortium taking a course at Wheelock for her major at her college. She is confident in her mathematical abilities.
- Tanner - Education and Mathematics for Teaching major who came to college hating math. Did not complete all of the interview questions.

3.1 Data Analysis

Transcripts of interviews were coded independently by each researcher using a method most similar to grounded theory [20]. Interview questions were designed around the research questions; however, no pre-conceived codes were used: each researcher read through the transcribed answer to each interview question from all participants and then created codes based on the repeated themes he or she identified. In addition, another member of the mathematics faculty participated in coding to aid the investigators and provide an additional perspective. After researchers independently coded the responses to each question, codes were compared for reliability. If themes were present across interview questions, the researcher tried to use a consistent code; examples of themed codes that were present across questions were: problem solving, confidence, explanations, and group work.

4. DATA & RESULTS

4.1 Program and Class Perceptions

In IQ1, participants were asked to describe mathematics classes to a hypothetical person who did not know anything about the classes; ten of the participants offered a response that revolved around four themes: “group oriented,” “not typical,” “problem solving,” and “reasoning and asking why.” Students consistently observed these classes as something different than they previously knew or expected when taking a math class. When telling a hypothetical person about the classes, “not typical” was the most prevalent response, with six out of ten respondents. One of these responses came from Tanner:

“I would describe them as very different from, especially from, your stereotypical math class especially high school, and from other universities. Just because they’re really focused on understanding the material versus just kind of like memorizing formulas so

there's a lot of um, at first like frustration when trying to explain like why things work, like things that you've known your entire life. ... They push you to understand and like explain everything that you know, so it's really challenging but I feel like, um, I'm incredibly more confident in like what I'm saying and I could explain it to anybody if anybody asked me. So it's kind of like a different, like skill set that they target."

In this response, Tanner provides insight into a few of themes we observed. The first observation is that students actively view the program as different and unique. The second is that students are able to observe instructors focusing on understanding and reasoning. That is, the classes are experienced as places where students can take ownership of mathematical concepts and their learning. The third is a theme observed in other parts of the interview, which will be discussed in the next section: increased confidence.

The next most common theme was "problem solving," with five responses. Alex specifically mentioned the words "problem solving" when she said:

"We work together, problem solving, figuring things out from point A to point B to point Z, everything you could possibly know about the topic." Robin also mentioned problem solving when she said: "When people think of math classes, they think of a teacher up at the board giving equations, and then you solving them and learning processes. It's not like that, it is like you do your own discoveries, you know, they might give you a topic and you work on that topic yourself and kind of go through it yourself. Nothing is ever just told to you."

Both Tanner and Alex's responses were representative of others who emphasized that they perceive students in the classes as being responsible for solving problems themselves using their own approaches.

A third major theme in the student data was the importance of "reasoning and asking why." Participants describe the importance of being able to explain answers and indicated that stating a correct answer is not sufficient. For example, Cameron said:

“Courses are more like going further in depth into things ... I feel like one question they always asked was ‘Why?’ Like, if there is a certain formula (that) works a certain way, or a certain pattern appears. It is not enough to just notice that it happens and to acknowledge that, you need to sort of prove that it will continue to happen and explain why that happens. And, that is not really something that I’ve encountered in high school and throughout middle school or anything.”

In this statement, Cameron indicates that these courses are both not typical and offers that the reason they are not typical is the emphasis on understanding (i.e. the why).

Another participant in the study, Skyler, is a student from a different college who took one math major course at Wheelock. Her responses provide insight from an outsider who comes from a college that structures math courses in a traditional lecture-oriented manner. Her overall impressions of the class were mixed, leaning toward the negative:

“Of course, questions are encouraged; office hours are very encouraged, which of course, my professor (at Wheelock) did. And my college does that as well ... But, I would not philosophize about whether $2+2$ is actually 4, and how you got to that answer. Just take it, get it wrong, bad. Get it right, great. Move on.”

In this response Skyler equates discussing why an answer is correct with “philosophizing,” something she clearly views as negative. Throughout her interview, she indicated that she did not like having to reason about answers and emphasized her belief that you should learn how to do something and then do it.

In her answer to IQ1, Skyler says, “It’s very relaxed, very laid back. A good environment. Lots of opportunities for questions. ... It was more of a group kind of class than just an individual class.” Despite her negative view of the inquiry-based pedagogy, she does indicate that the class is a good environment. Skyler’s opinion of the class may provide some insight into difficulties that could arise when implementing an inquiry teaching strategy in a single course within an entire mathematics program that has a directive approach. Her opinion is likely

shared by some students in the first semester of the introductory math for teachers sequences at Wheelock. However, Wheelock students who maintained that view of IBL after such a strong exposure throughout the introductory sequence would be unlikely to take more mathematics courses at Wheelock, and hence would not be included in the study.

Study participants who are members of the mathematics program frequently mentioned the influence of their introductory mathematics courses in their responses to IQ2, which asks why they chose a math major. The required sequence for education majors is taught from an inquiry perspective and resembles elementary math content courses in many colleges and universities. These courses strongly emphasize collaborative group work, and concepts are almost always presented in an inquiry-based or a guided problem based structure. Students are then able to transition from these classes to math major classes that are also designed for future PreK-8 teachers.

Participants were able to provide direct feedback on these classes in IQ6, where the nine participants who took these introductory courses were asked how the “math major” courses differ from their introductory courses. Some participants answered in terms of content, indicating that the material is more advanced, something that would be true in any program; if the participant answered in terms of content, the interviewer followed up and redirected them to answer in terms of class structure. The prevalent theme of these answers was that there was less group work and more individual work in the math major courses, and six of nine participants were coded into this theme. Participants indicated that, while group activities and collaboration were still a part of the structure in these classes, they felt they more individually responsible for their own understanding.

Blair indicated in her response that this change was structural because the courses did not have the scheduled study group; when the out-of-class group aspect was lost, out-of-class learning was more likely to be individual. Quinn also pointed to changes in class structure, stating there was “less scaffolding” and “more figuring things out on your own,” but went on to

say “in all classes there is a lot of students teaching other students,” indicating that collaboration is an important part of all math classes even when that collaboration has not been specifically planned. A theme observed in several interviews was students thinking the math major classes were not as structured as the introductory classes. Further investigation is needed to determine why students believe that the class was not as structured despite the careful design of each class, but the researchers believe that the changes in the structural elements of the class, such as fewer scheduled meeting times and no explicitly scheduled study groups, obscured the consistent elements in the pedagogical approach for many students. If the researchers are correct, this demonstrated that structure and pedagogy cannot be cleanly separated in students’ perspectives on IBL classes.

We were also interested in student perceptions of the differences between their Wheelock math major classes and math classes at other schools. Of the ten participants who answered this question, only two have actually taken math classes at other schools: Skyler, who is from a different school, and Blair, who is a Wheelock Mathematics for Teaching without an education major taking twelve additional mathematics credits, including a class taken at a different college inside the consortium. We first highlight responses to IQ11 from these two participants, including this response from Blair:

“(The classes here are) better, I think you -- one, you can talk to the teacher whenever because the classes are so small. Um also, it’s really just understanding—you’re really diving into a topic, whereas in other courses at other colleges, you just learn the process and how to get it. You really understand the concept behind it here.”

In this response, Blair indicates that the class emphasizes conceptual understanding and notices that in her class at the other college emphasizes procedure and answers. While that is likely not the emphasis the instructor has planned, it is the perceived emphasis for the course. Blair also points to a structural element of the Wheelock course: the smaller class sizes. Blair clearly sees an inquiry structure as a positive and a directive structure as a negative; however,

Skyler has the opposite view:

Skyler: “Well, I’ve only taken one other math class in my respective college, and ... there were all like first-years in our class, and like no one paid attention. No one, you know, asked any questions. So I was always person that’s like raising my hand.”

Interviewer: “What about how it was taught? Were there differences in the teaching style?”

Skyler: “Well, in my respective college, there was a lot—we went over a lot more things. So we had a schedule that we stuck to. We finished actually earlier than the syllabus. So I learned a lot more in the (course at my home institution).”

Interviewer: “There was no syllabus here?”

Skyler: “There was, but I never really referred to it. It was sort of like ‘What are we doing today?’ Oh we’re talking about this.”

In this exchange, Skyler indicates that the courses at Wheelock have less structure and the courses at her home institution students ask very few questions. Skyler also believes she learned more at her institution. In other interview responses, she indicates that, for her, learning more means learning more topics. She also indicates that she does not like exploring why a procedure is what it is; she just wants to learn how to execute the procedure. Of course, the math major courses at Wheelock do have a structure; Skyler indicates that the structure is not what she is accustomed to and, therefore, feels unstructured. The attitude that the unfamiliar is not legitimate is one that anyone making changes to an established program is likely to

encounter.

At Wheelock, we have been using active pedagogies in the math program since its inception, and over the years, these pedagogies have gone by different names. Thus, while faculty are likely to make explicit that we are trying to model effective pedagogy, we have been less explicit identifying that pedagogy as “inquiry-based” (a phrase without a widely accepted definition) or anything else. At the end of each interview in the study, participants were told that their math classes are designed under a common philosophy and were then asked what they thought that philosophy was (IQ13). This question may help identify how students in inquiry-based classrooms interpret an inquiry structure. Participants offered varying responses that included “exploring,” “asking why,” “communication,” and “not rushing.” However, none of these answers appeared in more than two responses. The common theme expressed by five of the ten participants who answered the question (all five were members of the mathematics program) was that the instructors design courses and structure class activities so that “everybody understands.” Alex specifically says the philosophy is “something about everybody understanding and something like very social,” and Blair states,

“I would say it has little to do with how many topics (students) cover, and what types of things you cover, whereas (students) understand the basic idea of the course, and if like they can relate it to everyday things. (The instructors) focus a lot on that. ... And I think that, they really take the time for every topic that you go over, that you really understand it, which I haven’t seen in other places—making sure everyone’s on the same page. I think that’s kind of their philosophy.”

Of course, instructors who do not use inquiry-based pedagogy also design their courses with widespread understanding as a goal. However, the finding that students in these inquiry-based classes believe they are designed with the intention that “everybody understands” is consistent with Fukawa-Connelly’s study that had students compare their experiences in inquiry-based courses to their experiences in courses that were taught directly. In Fukawa-Connelly’s

study, students thought their inquiry-based course instructors were more concerned about their understanding [3].

Finally, the researchers were interested in students' perceptions of how often instructors in their courses used directive teaching and how often they used non-traditional, active, or inquiry-based strategies. Seventeen members of the population answered a questionnaire asking them to estimate the percentage of class time that was lecture versus non-lecture as descriptors of directive and nondirective. Lecture was defined as times the instructor was addressing the entire class and explaining a topic, and non-lecture included time spent on group work, guided activities, student presentations, class discussions, and any other class activities. The members of the faculty who teach each class self-assessed how often they use directive strategies in each course. These percentages varied based on the class, but all percentages were under 50% with the majority of estimates in the 10%-20% range. For classes more conducive to hands-on activities, such as geometry, the percentage was low (about 10%) while courses such as probability and statistics were estimated to have more directive instruction (about 40%). In classes the instructors assessed as low directive instruction, participant responses were consistent, all under 30%; however, when instructors assessed classes as more directive, the student responses were inconsistent with ranges of responses for the same class, falling between 0% and 70%. In probability and statistics, the most directly taught course according to instructor assessment, student responses averaged about 60% but ranged uniformly from 20% to 100%. This latter result raises interesting questions for further investigation into how factors such as previous experience with directive teaching, performance in the course, personality, or learning disabilities affect students' perception of pedagogical strategies in the classroom. This result also suggests that instructor perception may not be the best measure of time allocation.

4.2 Self Perceptions

In the researcher's experience, many students come to classes with a view that mathematics is an artificial, complicated body of facts and algorithms they have to memorize and perform. These experiences are represented throughout much of the data reported in section 4.1, where students recall viewing math as a set of procedures designed to obtain a correct answer. In section 4.1, we found that students perceive their math classes at Wheelock as different from their earlier perceptions of a math class. They refer to their earlier perceptions as “typical” and their perceptions of Wheelock math classes as “not typical.” Among the nine participants in the mathematics program, one, Cameron, entered college planning on majoring in mathematics and another, Peyton, indicated she was on the advanced math track in high school. The other seven participants reported negative attitudes about math before entering college, ranging from the belief that they would not ever major in math to a hatred of mathematics. We investigated whether the participants’ perceptions of themselves as mathematics students had changed and why the participants decided to major in mathematics (or mathematics and science).

Participants were directly asked in IQ10 how their opinion of themselves as a mathematics student had changed. Eight of the nine participants in the mathematics program eight were asked this question (Tanner had to end her interview early.), and six provided responses that talked about increased confidence, with four using the exact phrase “increased confidence” or “it has increased my confidence.” In addition, while Tanner was not asked IQ10, she did indicate in her response to IQ1 (quoted in section 4.1) that her confidence in math had increased. Similarly, Aubrey states, in her response:

“It boosted my confidence as even just a person taking math, regardless of my math major. It boosted my confidence like a ton in math. And it made me understand different math concepts in totally different ways than what had been presented to me before, and I think that’s one of the biggest takeaways from Wheelock math courses, is that there’s not one way to solve a math problem. It’s about the process that um you go through to

solving it. And there can be multiple different processes to that. So that was like a huge thing that boosted my confidence”

Aubrey emphasizes her increased confidence and attributes it to being a better problem solver. Problem solving is emphasized in inquiry-based learning, and this response may be evidence that better understanding problem solving can lead to increased confidence, as does Quinn’s response:

“I never thought of myself as being like great at math, or like really understanding math, and I think that since I’ve been here, since I took my beginning math classes, and started—and became a major, I think my self-concept as a math learner has increased. I learn math differently and more efficiently that I used to. ... I think that I definitely—you know, I never really thought of learning by doing before I came here. And now I really realize that, you know, that whole piece of doing explorations of things to derive formulas, and I’ve never thought of myself when I was in high school of, you know, deriving my own formula. I was like “Oh, just give me the piece of paper with the formulas on it,” you know. Um, so yeah, I think I’ve become more of a hands-on math learner.”

We coded this response as increased self-confidence and noted that Quinn also emphasizes exploration leading to mathematics learning. Increased mathematical self-confidence is consistent with findings in the self-confidence of children learning math and science in IBL settings [5, 10, 15].

Those who did not explicitly state that their confidence increased still mentioned experiencing a shift from thinking like a student to thinking like a teacher. Answering IQ10, Logan said:

“I think of math as more of a teacher and how you would explain something. If I don’t know something I try and think about how it might be explained,” and Peyton echoed her thoughts with the following: “In high school I was on the advanced math track and I was

only concerned with getting the answer, I just wanted the answer ... So now, I think of myself as a math teacher, someone who can explain things and have different methods.”

In addition, the participants were asked what helped them decide to major in mathematics (IQ2). Of the nine participants, one entered college planning on majoring in mathematics. Among the other eight, two themes were prevalent, six coded under “introductory courses” and four coded under “more marketable.” In addition, Cameron, who decided on a math major prior to enrolling, also cited marketability, bringing that total to five of nine responses. Among those who cited the introductory courses as a motivation for choosing a mathematics major, listing the environment of the class and an increased confidence in mathematics as motivations for choosing the math major emerged as common themes. Many participants believed that having a math or science degree to accompany their education degree made them more marketable; this perception, combined with increased confidence and the support present in the class structure led to a decision to try the math major. Tanner specifically states that she came to Wheelock not wanting to do math but changed her mind in the introductory courses:

“If you had asked me in high school if I was ever going to be a (math) major I would have said no because I used to hate math. But, my freshman year I took the intro class and I had an amazing teacher. Again, I was never confident in it, but I think that was like part of the problem. (The instructor) really like pushed me and was just like ‘you’re really smart, believe in yourself’ and like just like had a lot of confidence in it. It just shows like how important it is, especially for an elementary major to like be strong and confident in math and sciences just because they’re a really important subject. So, I think it was just a really good intro course experience.”

In this quote, Tanner gives much of the credit to her teacher and the intro course. We do note that these students do have fewer barriers to becoming mathematics majors than students in traditional programs do and have the support needed to major in the subject, despite their weaker high school backgrounds.

5. DISCUSSION

5.1 Findings

This qualitative study of the perceptions of students from Wheelock's mathematics program that uses an inquiry-based pedagogy in all of its classes indicates that inquiry-based classes may help boost student confidence. Participants in the study largely entered college with a negative view of mathematics and, through the program, gained confidence. They primarily attributed this increase to taking control of their own learning and becoming better problem solvers, both central pieces of an inquiry-based mathematical class structure. The researchers noted that for both of us, our confidence decreased as we progressed through our undergraduate mathematics programs. Perhaps we started out over-confident while our students started out under-confident, and the programs are too different to meaningfully compare, but it is possible that an inquiry-based structure would have boosted our confidence as undergraduates.

The second finding is that students in Wheelock's inquiry-based courses recognize that the classes are atypical and view this difference as a positive; this finding is limited by interviewing mostly those who choose a Mathematics for Teaching major, as presumably students who did not enjoy their initial math experience did not select this major. Participants cited many characteristics as evidence of an atypical mathematical experience, including a strong student-teacher relationship, a collaborative classroom experience, group work in class, a lack of lectures, and an emphasis on conceptual understanding. Students understood that their experiences are not typical of what happens at other institutions. We acknowledge that these positive experiences are not exclusive to inquiry-based classes; however, we believe there is evidence to support that these students believe these positive attributes directly result

from inquiry-based classroom experiences.

The third finding is that when students take an inquiry class that involves material less conducive to hands-on activities, they do not perceive the teaching method of the class consistently. This finding is preliminary and based partially on self-assessment by instructors, and thus requires further investigation.

5.2 Takeaways

One of the many tenets of IBL is that participating in mathematics and becoming a producer of mathematics is more important than completing a list of course topics. As instructors we regularly “do math” and have an intrinsic understanding of the value and difficulties involved in that process. When we are presented with a new problem, we approach it with the goal of understanding it and learning from it rather than the goal of completion. When we use an IBL teaching approach, we want students to enjoy producing mathematics and reach a much deeper understanding of the topics we study together than they might in a class designed to “cover” many topics at a rapid pace. However, our study shows that while our students very often enjoy IBL, they also often believe that it is somehow inferior to classes their peers take at other schools, which are perceived to be harder and to cover more topics. We think there are several reasons why students may perceive their courses as easier. These courses cover fewer topics, and different topics, than classes at other institutions. In addition, many students believe that if they understand a topic, it must be less difficult than a topic they do not yet understand. They may also have outsized fears of commonly taught subjects that they have heard are hard

Disentangling these beliefs from the perceptions discussed in this paper would require another study, one that is important for us to do as a department. We don't typically see our students in the context of comparing their math major to those of friends at other schools, so the information about their negative perceptions of the major came as a surprise. This study

clearly indicates that students do perceive their Wheelock courses as inferior, and our department's needs to be proactive in trying to counter that perception, while we work to understand its roots. As Laursen has shown in her calculus study, instructors should not fear covering fewer topics [9]. However, if students perceive IBL courses as inferior, and coverage may be one of the reasons for that fear, then instructors need to address the issue of coverage directly with students. One resource for facilitating an early course conversation about IBL learning is Dana Ernst's "Setting the Stage" class activity described in the Mathematical Association of America's "Math Matters" blog [24].

Our second takeaway relates to students' perceptions of study groups in their introductory courses. We see the study groups as structural elements in the courses that support our IBL teaching philosophy but do not play a central role. However, each of the Math for Teaching majors indicated that they enjoyed the scheduled study groups and viewed this time with classmates as an important space for developing problem solving skills and making mistakes outside the instructor's presence. We strive to make our classrooms an environment where making mistakes is encouraged, but including a required study group led by a peer in the class structure may help students be more willing to risk mistakes and thus supports their curiosity and creativity in mathematics.

As a result of the study, we see the study groups as more central to teaching math majors than we had before, and we have implemented required study groups in other courses for math majors. Thus far, students have offered positive feedback, despite the extra time commitment. We encourage faculty in other programs to consider requiring study groups and giving group assignments for them that support the IBL pedagogy in class: the group time can be used for explorations to introduce a topic, which can be continued in class, as well as for homework assignments related to topics addressed in class. At Wheelock, having Math Leaders for the first two courses with study groups promotes a culture where students teach each other how to work together in a study group. Another priority for us is to further study our

Math Leader and study group programs to share more detailed information that can be useful to others in setting up similar programs.

Our final takeaway is that IBL approaches can increase student interest in mathematics and encourage students, including many students who would have been highly unlikely to choose a more traditional math major, to major in mathematics. At Wheelock, our major is designed for prospective elementary teachers, and the students come to college intending to major in education; at other schools, IBL-courses may provide a way to bring new students into mathematics-intensive majors or minors, perhaps related to other student interests. Also, students in our inquiry-based courses form a mathematics community that gets many excited about mathematics. An excited student base sells itself to prospective students and promotes mathematics to a population that might not have intended in pursuing mathematics beyond the required courses. We believe that these data show that introducing students to mathematics through inquiry can be an important step in making higher-level mathematics accessible to more students. Having a cohort of courses that utilize IBL may also help legitimize the practice in the eyes of the students.

5.3 Future Directions

It is our intention to follow up with all participants who agreed to allow the researchers know their identity and see how their perceptions of the program change after they graduated. The researchers also plan to investigate other programs that offer a mathematics degree for elementary educators and perhaps participate in a multi-site study. Some other areas that merit further investigation are the increase in student self-confidence, students' perceptions of IBL classes as less difficult, and the role of study groups in inquiry based learning. Finally, we would like to better quantify how often an inquiry instructor uses a directive approach and compare that data to reported student perceptions.

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