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# A psychophysical study of visual texture

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BOSTON UNIVERSITY  
GRADUATE SCHOOL

Dissertation

A PSYCHOPHYSICAL STUDY OF VISUAL TEXTURE

by

Herbert Kaizer

(A.B., Boston University, 1951; A.M., Boston University, 1955)

Submitted in partial fulfilment of the  
requirements for the degree of  
Doctor of Philosophy  
1956

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1956  
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## TABLE OF CONTENTS

	Page
I. INTRODUCTION.....	1
A. Background.....	1
B. Definitions and Statement of Problem.....	3
II. THE DETERMINATION OF A PHYSICAL MEASURE OF VISUAL TEXTURE.....	6
A. The Statistical Properties of Surfaces.....	7
B. A Proposed Index of Visual Texture.....	9
C. Relation of Autocorrelation Measure to Observer Discriminations.....	10
III. EXPERIMENTAL METHODS AND DESIGN.....	22
A. Stimulus Materials.....	22
B. Experimental Designs.....	25
1. Experiment I.....	25
2. Experiment II.....	28
IV. RESULTS.....	32
A. The Physical Measure.....	32
B. Experiment I.....	34
C. Experiment II.....	38
V. DISCUSSION OF RESULTS.....	42
A. The Physical Measure.....	42
B. Experiment I.....	44

C. Experiment II . . . . .	46
VI. SUMMARY AND CONCLUSION . . . . .	49
BIBLIOGRAPHY . . . . .	52
APPENDIXES	
A. Least Squares Computation for Photo 13-58 . . . . .	54
B. Proof of the Equivalence of $(1/e)$ th Values of Series B and C . . . . .	60
C. Total Response Matrices for Each Subject on Experiment I . . . . .	62
D. Tables of Comparative Judgment Rankings for Experiment II . . . . .	68
ABSTRACT . . . . .	70

## LIST OF TABLES

	Page
I. EXPERIMENTAL DATA FROM SPATIAL CORRELATOR .	16
II. $(1/e)$ th VALUE OF THE AUTOCORRELATION FUNCTION FOR EACH TARGET IN PRELIMINARY EXPERIMENT...	19
III. POOLED DATA ON RANKING EXPERIMENT.....	21
IV. DIAMETER OF IMAGE CIRCLE ON STIMULI USED IN EXPERIMENT I.....	33
V. SCALE VALUES FOR SUBJECTS.....	37
VI. RESULTS FOR PRE-TEST TO EXPERIMENT II.....	39
VII. RESULTS FOR EXPERIMENT II.....	41

## LIST OF FIGURES

	Page
1. Photograph and Schematic of Spatial Correlator.....	12
2. Stimuli Used for the Ranking Experiment.....	13
3. Spatial Correlator Trace of Photograph 13-58 .....	14
4. Experimental Curve Derived from Spatial Correlator for Photograph 13-58.....	17
5. Autocorrelation Function for Photograph 13-58 .....	18
6. Stimuli Used for Experiment I.....	24
7. Subject Seated at Experiment Apparatus.....	27
8. Stimuli Used in Experiment II.....	30
9. E.D-Scale Values as a Function of Autocorrelation Measure.....	35
10. Average of E.D-Scale Values for all Subjects as a Function of Autocorrelation Measure.....	36



# CHAPTER I

## INTRODUCTION

### A. Background

One of the major historical traditions in psychological research has been the investigation of how an observer discriminates various sensory events in relation to some objective dimension of the event. In a sense, this tradition can be interpreted as the search for relationships between response dimensions and some objectively defined dimensions. The tradition stems from Weber and Fechner, whose law or function represents a first attempt to generalize from experimentation a systematic and quantitative description of the relationship between some specified stimulus set and some ordered response set.

By and large the features of sensory events investigated in both the older and contemporary literature have been those properties for which physical measure correlates existed. Thus, in the area of vision, the major properties for which relationships have been experimentally specified have to do with brightness, hue, saturation, distance, and form. In these instances already existing physical measure dimensions offered easy and readily available correlates to which ordered response sets could be related. For instance, in brightness studies the units of measurement such as lamberts provided the basis for variation of the stimulus conditions in the psychophysical experiment.

While there are many examples in the literature which illustrate psychophysical research on these properties, there is nothing in the nature of sensory events which logically restricts one to the investigations of these features.

In a sense, the availability of particular physical measures resulted in the emphasis on such properties as brightness. It is possible, on the other hand, to formulate a view of the visual process which considers such properties as brightness irrelevant or at most secondary features of sensory events. This new point of view considers the immediate givens of visual experience to be surfaces. Accordingly the program of research calls for determining the psychophysical relationships for the properties of surfaces. This point of view has been formulated by Gibson in his recent book on perception of the visual world (11). For Gibson, the traditional analysis of visual phenomena did not lead to an increase of our knowledge of perception because when an unnecessary distinction was made between sensation and perception certain artificial properties were abstracted. We do not see or "sense" empty space, according to Gibson, but rather we see a series of continuous surfaces in some relation to one another where the basic relation is that of ground to horizon to sky. For Gibson, then, the basic problem of visual perception is: how does the observer discriminate between surfaces and relations among surfaces?

Gibson tentatively lists eight properties of surface which he feels need psychophysical investigation (12). These are: texture, color and

illumination, slant, distance, contour, shape-at-a-slant, and size-at-a-distance. In his total presentation he placed special emphasis on texture since he believes that gradients of texture on a surface are necessary stimulus correlates for many of the other listed properties.

This theoretical position is a compromise between Gestalt and empiricist approaches to the problems of visual phenomena and therefore emphasizes both phenomenological and psychophysical methods of investigation. While Gibson has utilized phenomenological methods of investigation, he nevertheless recognizes that the lack of objective measures of texture prevents psychophysical research, and he therefore stresses the need for the mathematical analysis of the texture dimension.

The present study is a quantitative and psychophysical investigation of a property of visual surfaces denoted as texture. It should be noted that this study does not constitute a test of Gibson's viewpoint. Such tests can only be meaningful after quantitative determinations have been made for the various properties of visual surfaces.

#### B. Definitions and Statement of Problem

Before specifying the details of the problem of this study it is necessary to provide some definition of the class of stimuli under investigation. Gibson defines the retinal correlate of texture as "adjacent cycles of intensity in the image." He further expands on this by saying that this correlate "corresponds to an image composed of speckled instead of homogeneous light, i. e., spots of alternating light and dark which are reportable

as 'grain' or microstructure." It is necessary for present purposes to modify this definition which is a product of a phenomenological analysis. Rather than giving a definition of texture in terms of events at the retina as Gibson does, it is necessary in psychophysical experimentation to define texture in terms of the stimulus object. This definition, however, will be analogous to the one quoted from Gibson above. Texture is defined in this study in terms of stimulus object as the distribution of alternations in light intensity over an object surface in which areas of equal intensity occupy a small portion of the surface relative to its total extent. It should be noted that the above is a purely nominal definition, and the "real" or operational definition of texture is given by some physical measure of the distribution of energies.

The problem of this investigation, then, in its general form, is to precisely specify the relationship between a set of physical measure operations and a set of observer responses on a particular class of stimuli. Specifically the major objective of this investigation is the determination of a psychophysical scale of visual texture. To accomplish this the following preliminary determinations must be made:

1. To specify one set of physical measure operations out of infinitely many possible sets, this set being relevant to observer discriminations.
2. To select stimuli representative of the class of objects under investigation.

3. To determine the psychophysical model (design) to be used in specifying the relationship.

The following chapter will be concerned primarily with solutions for the first of these preliminary objectives.

## CHAPTER II

### THE DETERMINATION OF A PHYSICAL MEASURE OF VISUAL TEXTURE

In the preceding chapter the stimulus definition of texture was given as a distribution of alternations in light intensities over an object surface where areas of equal intensity occupy a small portion of the surface relative to its total extent. The surfaces considered in this investigation will be photographic transparencies, and therefore the distribution will be a distribution of transmissions rather than reflection. This, aside from problems of instrumentation, results in no loss of generality.

There are infinitely many ways in which the distribution of intensities on a surface may be characterized. For instance, attention can be focussed on the type of geometric form made by the small areas of equal intensity or on the frequency of various absolute levels of intensity. However, these aspects of the distribution are rather limited for general descriptive purposes since they are too sensitive to change for different conditions (e.g., illumination) on the same surface. It appears that since Gibson's notion of texture suggests the possibility of plotting cyclic changes in intensity along two axes that a more general characterization of surfaces is possible from a consideration of alternations in intensity over distance along the surface. A visual examination of many surfaces, however, does not seem to support the suggestion that texture can be

characterized by any relatively simple graphic device. A more powerful quantitative tool must be found, and for this we turn to the consideration of the statistical properties of the distribution of intensity changes over distance.

#### A. The Statistical Properties of Surfaces

In the preceding chapter it was noted that no physical measure of texture was available in the psychological literature. There does exist, however, a body of research in physics and optics on problems relevant to the present one. This research concerns attempts to objectively characterize grain on photographic film. The various measuring techniques for grain have been reviewed by R. Clark Jones (15), and by a communication theory analysis he has shown these techniques to be special cases of the more general informational analysis of photographic grain. The techniques reviewed by Jones were based on scanning the film through the aperture of various types of photometric instruments. In his review he has demonstrated that these techniques are too dependent on the size and shape of the scanning aperture while the more general information measure has fewer restrictions in terms of scanning aperture. Fellget (7) has also proposed a communication theory analysis of photographic grain using a somewhat different mathematical treatment. Jones' approach leads to a concern with the power spectrum of the photographic film whereas Fellget deals with the Fourier transform of the spectrum, that is, the autocorrelation function. The definitions

of grain proposed in the course of this research in physics and optics seem almost identical to the definition of texture used here. Since the only difference in definitions is the size of the elements composing the distribution of light intensities, it appears that the autocorrelation function can be meaningfully employed for quantifying the texture of visual surfaces.

The important property of the distribution of alternations in light intensity on a surface is the fact that successive variations in intensity are not statistically independent. In fact, the distribution of intensity changes can be characterized as a Markov process.\* One technique useful in describing a Markov process is the autocorrelation function. While previously applied to time series, recent investigations by Kretzmer (16) and others at Bell Telephone Laboratories have shown this function to be equally applicable to the two-dimensional optical situation. The autocorrelation function in this case is defined as:

where  $f(x, y)$  represents the transmission at  $(x, y)$

$$(1) \phi(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A f(x, y) f(x+x_0, y+y_0) dy dx$$

$f(x+x_0, y+y_0)$  represents the transmission at  $(x+x_0, y+y_0)$  when the surfaces are shifted by the amount  $(x_0, y_0)$

---

\*A term of mathematical statistics denoting a process which produces a series of terms in which there is a statistical dependence of succeeding terms on preceding terms.



This expression is analogous to the more familiar covariance term of statistics. Essentially it is a measure of the degree of dependence of the succeeding terms in a series on the preceding ones.

### B. A Proposed Index of Visual Texture

The selection of the autocorrelation function as a characterization of texture still necessitates the determination of some single parameter of this function to be used as a basis for ordering a series of surfaces to be used in a psychophysical experiment. The definition of the autocorrelation function provides no ready parameter for this purpose. It is therefore necessary to restrict the surfaces considered in this investigation to those where some analytic expression can be specified for the autocorrelation function which will then yield some parameter for the purpose of ordering surfaces. This is possible by restricting the surfaces considered to those in which the distribution of intensities is random. The following equation represents a least squares estimate of the autocorrelation function in this case.

$$(2) \quad y = a + b e^{-\frac{a}{b}x}$$

where  $y$  = transmission

$a$  = square of the mean

$b$  = mean square minus square of the mean at zero delay

$x = (x_0, y_0)$ , i. e., distance of shift

Since it is desirable to have an expression which is independent of the square of the mean the equation can be normalized to produce the following function:

$$(3) \quad \phi'(x_0, y_0) = \frac{y-a}{b} = e^{-\left(\frac{x}{c}\right)^2}$$

The parameter to be used in this investigation is the  $(1/e)$ th value of the autocorrelation function. The  $(1/e)$ th value is defined as that value of  $x$  where  $\phi'(x_0, y_0) = 1/e$ . For equation (3) the  $(1/e)$ th value is simply equal to the parameter  $c$ .

The  $(1/e)$ th value is a measure of how fast the autocorrelation function drops off to its asymptote. It is therefore inversely related to the breadth of the power spectrum. In turn the breadth of the power spectrum is related to the size of the detail on a surface. Thus for large  $(1/e)$ th values there is a correspondingly narrow spectrum and therefore a lack of fine detail. On the other hand, for small  $(1/e)$ th values there is a correspondingly broad spectrum and therefore fine detail. Accordingly this led to the suggestion that the observer reports be directed toward discrimination along the continuum of "fine to coarse" detail.

### C. Relation of Autocorrelation Measure to Observer Discriminations

A preliminary experiment was carried out to determine whether there is a uniform relationship between the  $(1/e)$ th value of the autocorrelation function and observer discrimination of texture on the dimension from fine to coarse. An analog computer, designed by the Boston

University Physical Research Laboratories, was used for the purpose of determining the autocorrelation function of photographic transparencies. This instrument consists primarily of two tubes, a lens system of two collimating lenses, a light source, a photomultiplier, and a continuous recording ammeter. If two identical photographic transparencies are placed in the two coordinate system and are perfectly matched, a reading may be taken of the transmission of light through these surfaces onto the photomultiplier and recorded by the micro-ammeter. A photograph and schematic diagram of the instrument are shown in Figure 1.

The resultant reading is analogous to the autocorrelation function at zero delay (i. e. , the transmission of each point multiplied by itself and integrated over the entire surface). If one of the photographs is now shifted by a given amount the resultant reading will be the analog of the autocorrelation function with delay equal to the amount of shift. This procedure can be continued until the values recorded on the ammeter reach an asymptote. This asymptote is the analog of the square of the mean of the distribution and should vary according to the average transmission of the surface.

This procedure was used to determine the autocorrelation functions of the series of aerial photographs of terrain shown in Figure 2. Figure 3 shows a typical product of the spatial correlator (Boston University analog computer). These are the data for photo 13-58 as shown in Figure 2. Since it is not possible to calibrate the spatial correlator so that the

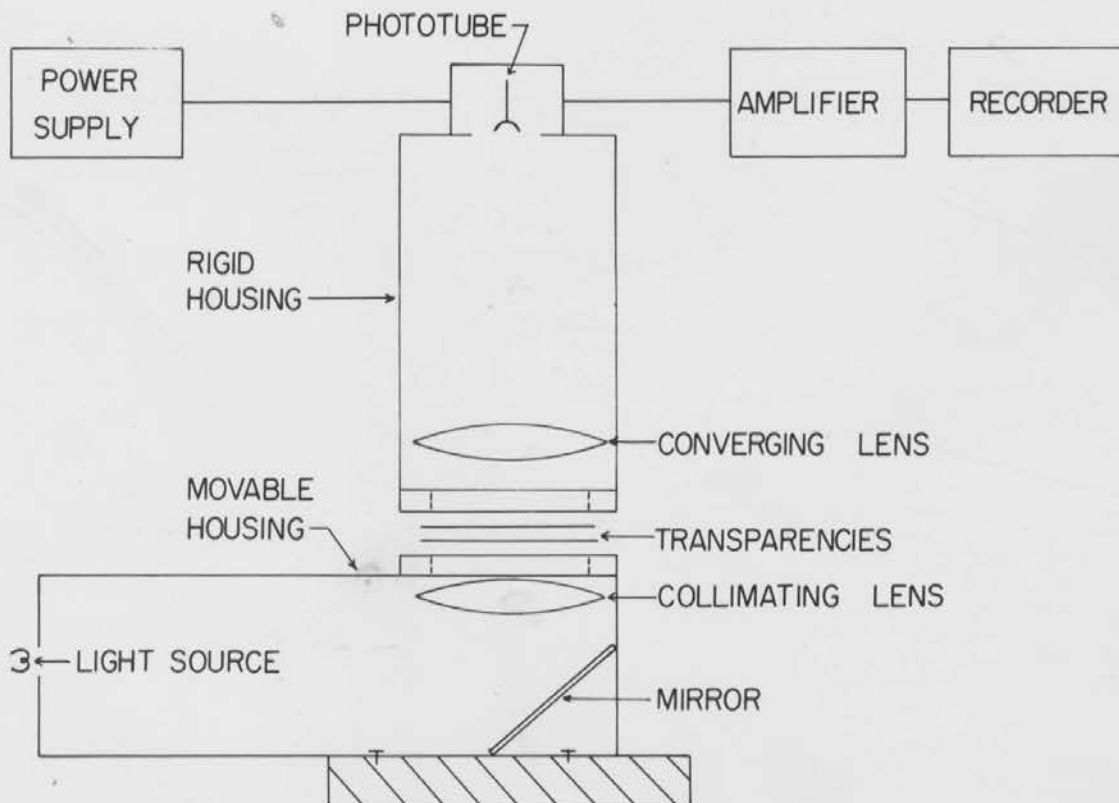


Figure 1. Photograph and Schematic of Spatial Correlator



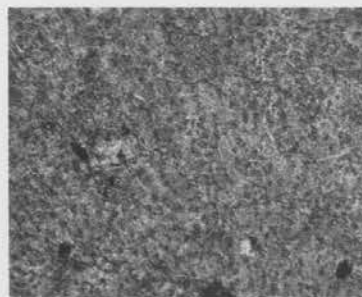
13-62



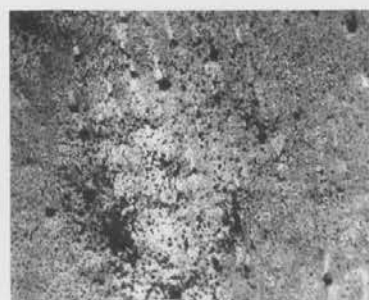
14-49



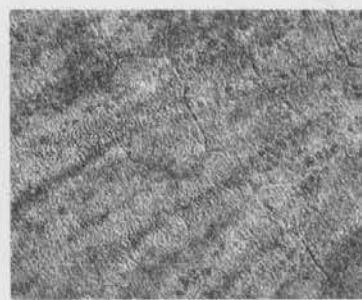
13-55



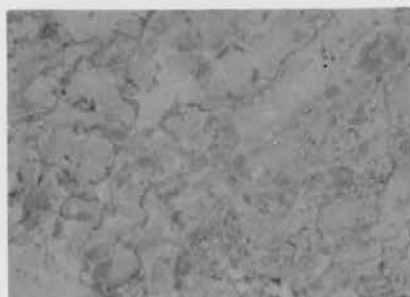
13-58



23-43

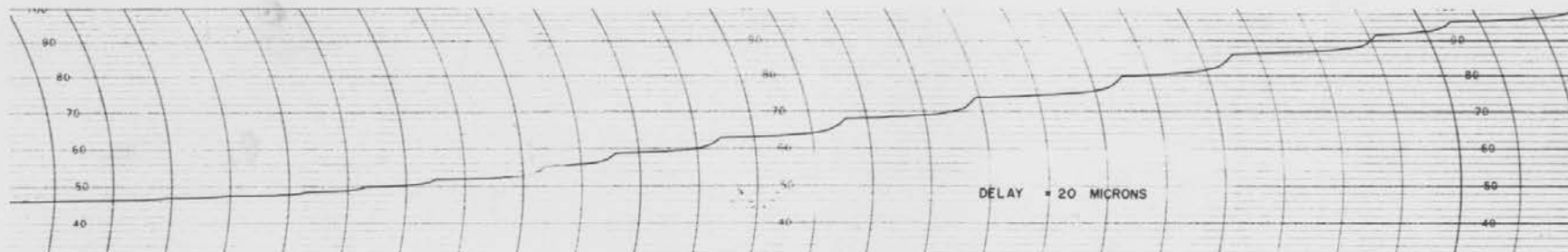


14-31



15-71

Figure 2. Stimuli Used for the Ranking Experiment



Each dip of curve represents a shift of  $20 \mu$

Figure 3. Spatial Correlator Trace of Photograph 13-58

reading at zero delay is exactly 1.00, these data must be adjusted using the normalization:

$$(4) \quad X_{(x_o, y_o)} = \frac{O_{(x_o, y_o)}}{O_{(o)}}$$

where  $O_{(o)}$  = reading  
at zero delay

$O_{(x_o, y_o)}$  = reading at  
 $(x_o, y_o)$  delay

$X_{(x_o, y_o)}$  = normalized  
value at  $(x_o, y_o)$  delay

Table I shows the normalizations for photo 13-58, and Figure 4 is the graph of the experimental curve as derived from the spatial correlator.

The analytical form of the autocorrelation function was stated in equation (2). Since only the mean square minus the square of the mean portion of the function is of interest, the data were normalized by use of equation (3). An illustration of the computation schedule is shown in Appendix A giving the computations for photo 13-58 utilizing the least square method of curve fitting. For illustration, Figure 5 shows the theoretical curve and the experimental points for photo 13-58. For all cases these curves are reasonable fits, well within the limits of instrument error. Table II gives the least square estimates of the  $(1/e)$ th value for all targets.

These photographs were then ranked by twenty observers, all employees of Boston University Physical Research Laboratories. They were presented the transparencies on a light table with all seven photos in view at the same time. Each observer was asked to order the photos with

TABLE I  
EXPERIMENTAL DATA FROM SPATIAL CORRELATOR

Delay in microns	Observed	Normalized
0	0.985	1.000
20	0.960	0.975
40	0.920	0.934
60	0.860	0.873
80	0.800	0.812
100	0.740	0.751
120	0.680	0.690
140	0.630	0.640
160	0.590	0.599
180	0.555	0.563
200	0.520	0.528
220	0.500	0.508
240	0.485	0.492
260	0.475	0.482
280	0.470	0.477
300	0.465	0.472



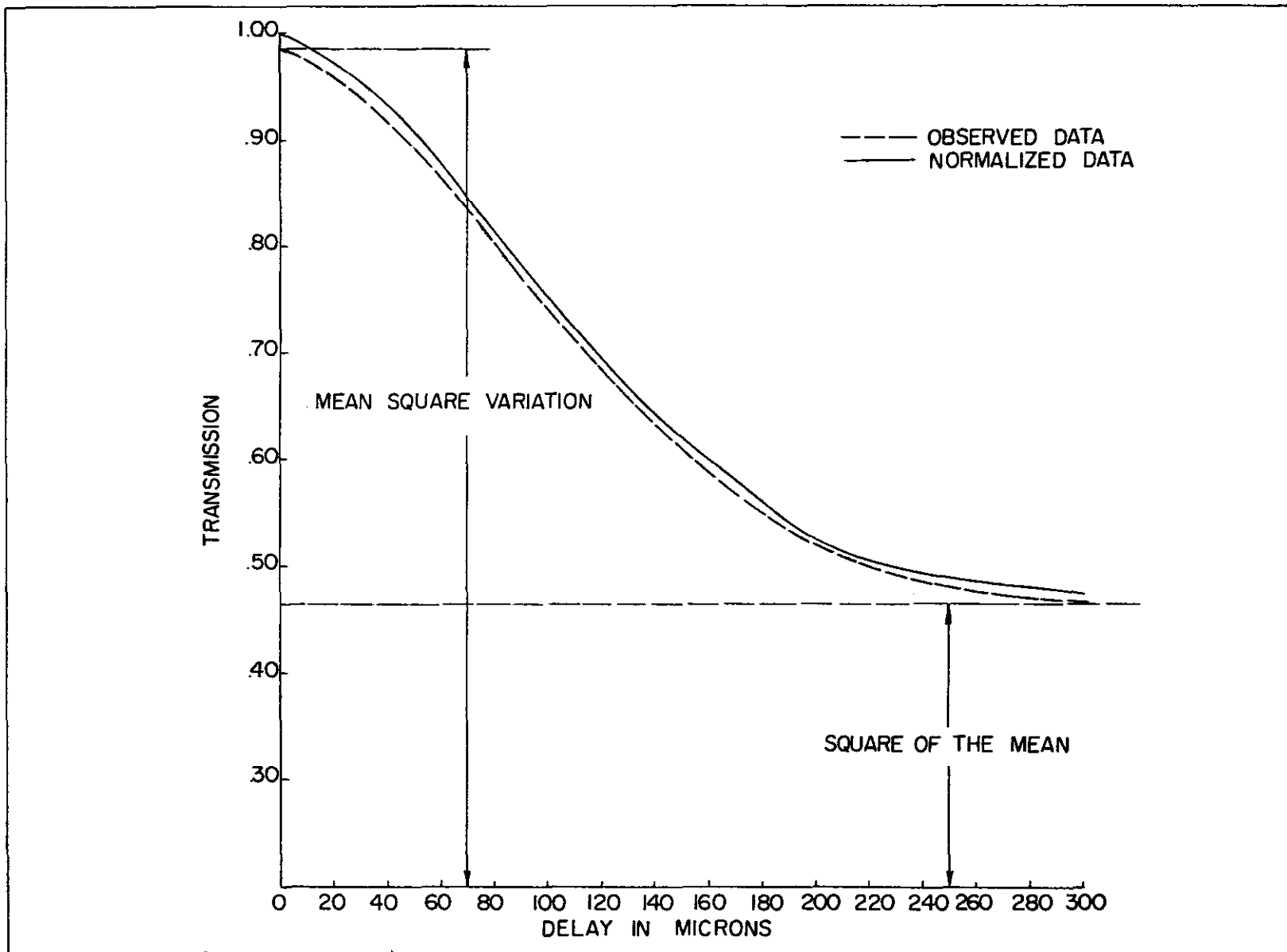


Figure 4. Experimental Curve Derived from Spatial Correlator for Photograph 13-58

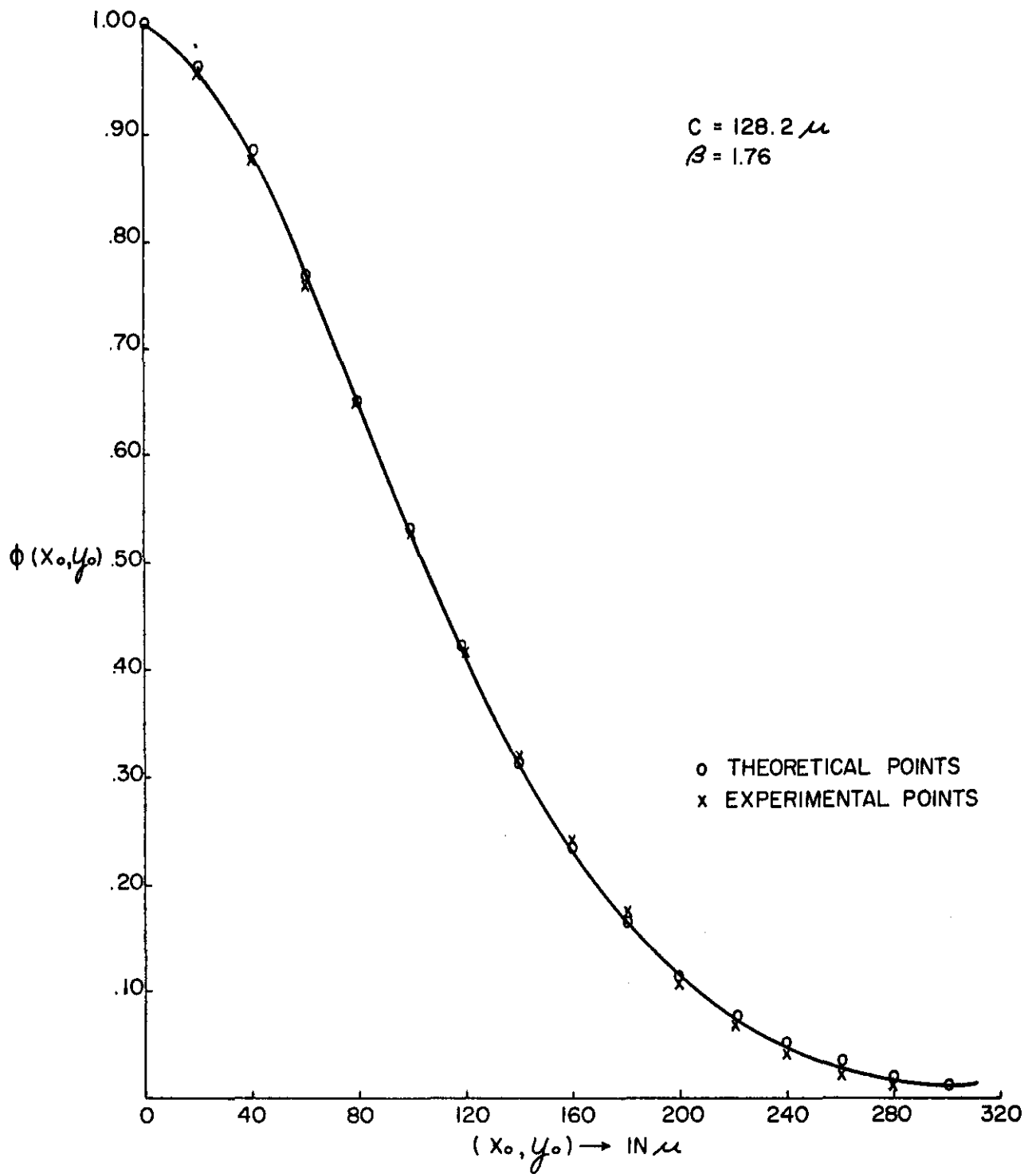


Figure 5. Autocorrelation Function for Photograph 13-58

TABLE II  
(1/e)th VALUE OF THE AUTOCORRELATION FUNCTION  
FOR EACH TARGET IN PRELIMINARY EXPERIMENT

Photograph	(1/e)th Value (in $\mu$ )
13-62	72.4
13-55	77.3
14-49	78.6
13-58	128.2
23-43	185.0
14-31	241.7
15-71	895.8

respect to the fineness and coarseness of their texture. References to fabrics and photographic grain were used to suggest the fine-to-coarse continuum. The ranks assigned by these subjects were averaged and tabulated as shown in Table III. The rank correlation coefficient between the order assigned on the basis of the  $(1/e)$ th value and the average ranks was computed. The rank correlation coefficient was found to be .96, which is sufficient demonstration of the relevance of the autocorrelation analysis of texture to observer discriminations.

Certain sources of variability were found in the assignment of ranks by observers. This variability in ranking was the result of two uncontrolled features in the preliminary experiment. These are (1) the nonuniformity of the distribution on some of the surfaces used in the preliminary experiment and (2) the ambiguity of the instructions given to the observers because of the lack of visual examples of the continuum the observer was asked to discriminate along. Methods for controlling these sources of variability will be discussed in the next chapter.

TABLE III  
 POOLED DATA ON RANKING EXPERIMENT

Photograph	(1/e)th Value	Rank by (1/e)th Value	Pooled Rank by Subjects
13-62	72.4	1	1
13-55	77.3	2	3
14-49	78.6	3	2
13-58	128.2	4	4
23-43	185.0	5	5.5
14-31	241.7	6	5.5
15-71	895.8	7	7

(rank correlation = 0.96)

## CHAPTER III

### EXPERIMENTAL METHODS AND DESIGN

This chapter describes the design of two major experiments. The first of these is designed to determine the form of the relationship between observer discriminations of surfaces and the physical parameter selected. This is a psychophysical experiment using the method of single stimuli. The second experiment is a test of the proposition that observer discrimination can be expressed as a single valued function of the physical parameter when the average transmission of the surfaces is varied. The experimental design for this is the method of comparison by pairs.

#### A. Stimulus Materials

In the preliminary study the need for more precisely controlled stimulus materials was made apparent since an appreciable amount of variability was found to result from the non-uniformity of the surfaces studied. It was therefore decided to construct artificial stimulus surfaces whose properties could be precisely controlled. The technique used was developed by K. Aschenbrenner (1) and yields a series of surfaces with random but uniform distributions of light and dark elements. The production of these surfaces can be controlled to give a series over which the  $(1/e)$ th value varies in a regular linear fashion.

Briefly, the technique consists of dropping a large number of light and dark paper elements of a given shape onto a background until the background is completely covered. If we then photograph this target from varying distances the resultant photographic surfaces will vary in  $(1/e)$ th value directly as a function of the minimization scale of the photograph, and thus a linear minimization scale in the photography will produce a linear scale of  $(1/e)$ th values.

It was decided to restrict the surfaces investigated to the simplest mathematical case, and therefore only black and white elements were used to generate the surfaces studied here. Furthermore, the shapes of these elements were restricted to circles and squares of equal areas since in this way surfaces with a wide variety of edge contours could be obtained. Three different series of surfaces were generated. In all cases the proportion of circle elements to square elements was equal. The three series differing in respect to the proportion of white to black elements were: 25-75, 50-50, and 75-25. Thus these three series differed in the average transmission of light through the surface. In the first case the average transmission is 75 per cent, in the second 50 per cent, and in the third 25 per cent. The reason for the use of 25 per cent and 75 per cent transmission series will be discussed in the second experiment.

From the 50 per cent transmission series (series A) eighteen surfaces were selected. These are shown in Figure 6. The thirteen surfaces

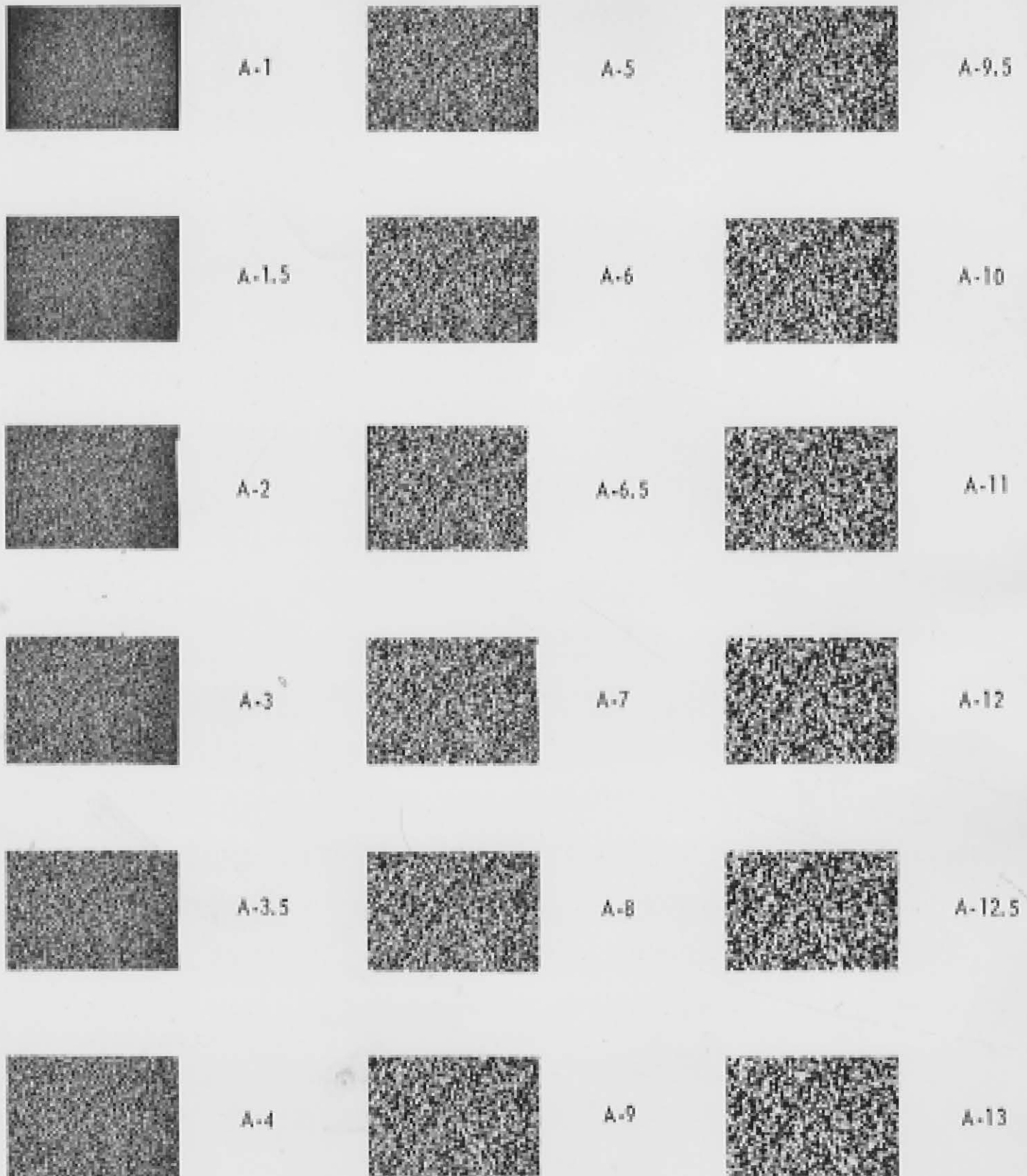


Figure 6. Stimuli Used for Experiment I



labeled by integers are equally spaced with respect to the  $(1/e)$ th value, The five surfaces labeled A1.5, A3.5, A6.5, A9.5, and A12.5 have  $(1/e)$ th values intermediate to the two adjacent integer numbered surfaces. That is, the  $(1/e)$ th value of A1.5 is half-way between A1 and A2 and so on. These five surfaces served as external controls on the stability of the derived scale values.

## B. Experimental Designs

### 1. Experiment I

Since we are not primarily interested here in a precise threshold determination but rather in some statement about the precise form of the relationship between the two sources of measurement (physical and observer), the psychophysical design chosen was dictated by the requirements of economy and a rational method for determining scale values and variability. For these requirements it was decided that Garner's model for absolute judgments (10) would be suitable. Essentially the model determines a scale in which the scale units are a measure of the confusability or discriminability of the points on a stimulus continuum. In addition the method yields a measure of information transmission which is in a sense a measure of variability or error in judgment. The information measure is an index of the number of stimulus objects in the continuum which can be discriminated without error. Unfortunately the Garner model contains no internal check on the stability of the derived scale values and thus necessitates some external check. The check method used in this

study was the predictability of scale values for stimuli intermediate in physical measure to the experimental stimuli.

In the major experiments all stimuli were viewed on a light table under constant and even illumination. Figure 7 shows a picture of a subject seated at the experimental apparatus. In all experiments a frame was placed on the light table such that only that area of the surface on which the judgment was to be based was exposed. A total of six subjects was used in experiment I, all male between ages of 21-28. Each S was first presented with the extremes and mid-points of the stimulus continuum to be judged. This is represented by photo's A-1, A-13, and A-7 on Figure 6. The entire range of stimuli was then presented to the S in a random order. The extremes and mid-point were then shown again. This initial familiarization or anchoring procedure was repeated four times. Each S was then asked to assign a whole number from one to thirteen representing the relative fineness or coarseness of a surface with one representing the finest and thirteen the coarsest. Each experimental and control surface was presented to the subject 100 times for judgment making a total of 1800 responses spread over four two-hour experimental sessions. Each S was allowed a short break at the end of the half hour and a ten minute rest period at the end of each hour. At each break or rest period the subject was shown the anchoring points again and at the beginning of each experimental session and the end of each long rest period S repeated the familiarization procedure.



Figure 7. Subject Seated at Experimental Apparatus

## 2. Experiment II

This experiment was designed to test the proposition that texture discrimination can be expressed as a single valued function of the auto-correlation function when the average transmission of the surfaces is varied. For this purpose two additional series of stimuli were produced, one having an average transmission of 75 per cent, called series B, and another with an average transmission of 25 per cent, called series C. Thus, for the proposition to be verified, surfaces photographed at exactly the same distance (i. e., with the same minimization scale) should be discriminated only on the basis of their  $(1/e)$ th values and independent of changes in average transmission.

This experiment, was designed as a "comparison by pairs" or, in statistical language, as an incomplete block design. In this design the three series A, B, and C represent different treatments, the preferences for which are the major experimental concern. The statistical model adopted for this experiment is one developed by Bradley and Terry (5) and is an expansion of the binomial probability model. Essentially this model states that it is possible to associate to each treatment a number  $\pi_i$ , such that  $\pi_i \geq 0$ , and  $\sum_i \pi_i = 1$ , where  $\frac{\pi_i}{\pi_i + \pi_j}$  represents the probability that treatment i will be preferred when compared to treatment j. In this experiment, preference represents the judgment that a surface from series A, for instance, is finer than a surface from series B and so on. The null hypothesis is that all  $\pi_i$ 's are equal and thus the probability that

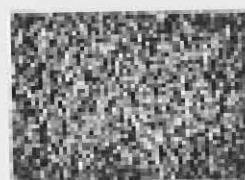
one treatment is preferred to another is one half. The alternative hypothesis states that the  $\pi_i$ 's are different and provides a maximum likelihood estimate of the  $\pi_i$ 's and a statistic for determining the significance of differences from a relatively small sample.

Using this model necessitated a pre-test to determine whether any significant difference in preference values exists between treatments or photographs of the three series taken at different minimization scales. This pre-test, a test of the significance of difference between levels, is provided by the model. The pre-test was performed using one of the six subjects who had participated in the previous experiment. The subject was presented with each possible treatment comparison within each level, and treatment comparisons were made on six different levels. The replications were performed for each block in a random fashion. Levels were also randomized within the design. Since no significant difference was found between  $\pi_i$  values for each level, the major part of the experiment was performed with the following procedure.

Two levels were selected representing either half of the continuum of each series. All stimuli used are shown in Figure 8. All possible treatment comparisons were made by each subject within each of the levels, and a change of level was simply considered another replication of a block. Ten replications were made for each block:



A-4



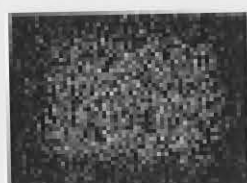
A-9



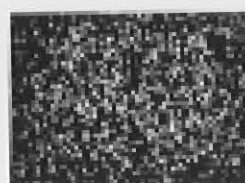
B-4



B-9



C-4



C-9

Figure 8. Stimuli Used for Experiment II

five on each level. Four of the remaining subjects who had participated in Experiment I were used. The results of this experiment are recorded in the following chapter.

## CHAPTER IV

## RESULTS

A. The physical measure

The method of construction for the stimuli for experiments I and II eliminated the necessity for the tedious measurement procedure used in the preliminary experiment. For experiment I relative distance between the  $(1/e)$ th values for the stimulus points on the continuum can be obtained analytically. As stated in the preceding chapter the  $(1/e)$ th value of any stimulus in the series is a function of its reduction or magnification in photographic scale from some given stimulus point. Specifically the  $(1/e)$ th value for any given surface is given as a function of the value for some reference stimulus by the following expression.

$$(4) \quad c_j = \frac{d_j}{d_i} c_i \quad \text{where}$$

$$c_j = (1/e)\text{th value of } j^{\text{th}} \text{ stimulus on continuum}$$

$$c_i = (1/e)\text{th value of } i^{\text{th}} \text{ stimulus on continuum}$$

$$d_j = \text{diameter of a image circle on stimulus } j$$

$$d_i = \text{diameter of a image circle on stimulus } i$$

The diameter of image circles for all stimuli is recorded on Table IV. An examination of this table indicates that the distance between adjacent experimental stimuli is equal to 10% of the  $(1/e)$ th value of the mid-point stimulus A-7. Control stimuli then differ from the preceding experimental stimuli by one-half this value. For all practical purposes



TABLE IV  
DIAMETER OF IMAGE CIRCLE ON STIMULI  
USED IN EXPERIMENT I

Stimulus	Diameter (in mm)
A-1	.16
A-1.5	.18
A-2	.20
A-3	.24
A-3.5	.26
A-4	.28
A-5	.32
A-6	.36
A-6.5	.38
A-7	.40
A-8	.44
A-9	.48
A-9.5	.50
A-10	.52
A-11	.56
A-12	.60
A-12.5	.62
A-13	.64

then, the numerical index of each stimulus can be considered an index of both its order in the series and its distance from the other stimuli in the series with respect to the physical measure.

For experiment II the relevant information for the three transmission series is the comparative value of the physical measure for stimuli with given image circle diameters. Appendix B contains an analytic proof that the variational portion of the correlation function for series B and C is identical for photographs of the same size image circle. While no neat analytic scheme is known at present for demonstrating the relationship of series A to B and C, an examination of the data of experiment II, which will be given below, will show that the equivalence of series B and C is sufficient for rejecting the proposition tested.

#### B. Experiment I

Figure 9 presents the graph of each subjects' equal discriminability (E.D) scale values plotted against the autocorrelation indices for the given surfaces. Figure 10 is the average of scale values plotted against the  $(1/e)$ th indices. Table V shows the E.D scale values for each subject on each stimulus and the average of these scale values. Table V also gives the average amount of information transmitted for each subject and the mean information transmission over the six subjects. In Appendix C the total response matrix of each subject for all stimuli has been included.

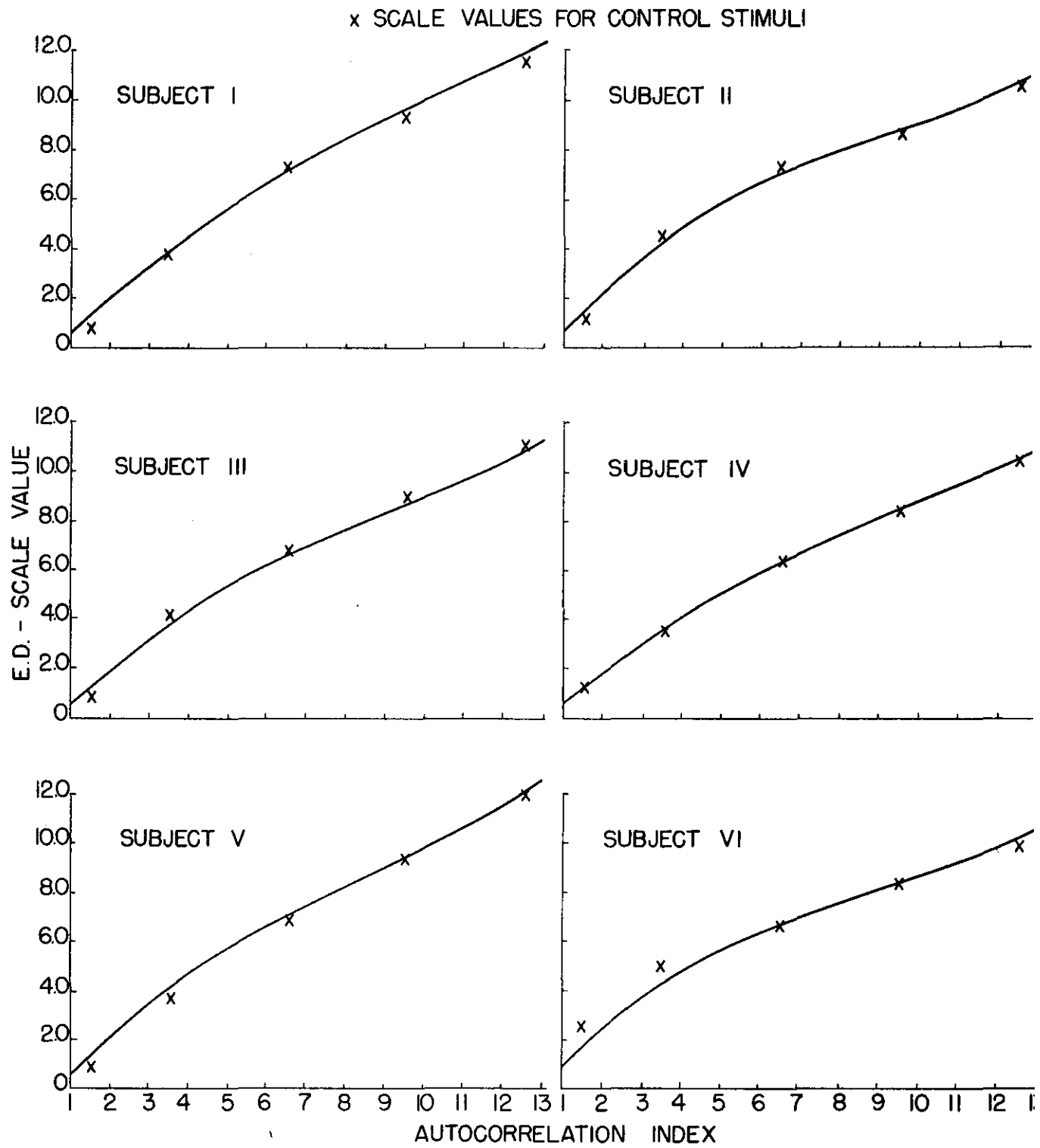


Figure 9. E. D. - Scale Values as a Function of Autocorrelation Measure for Each Subject

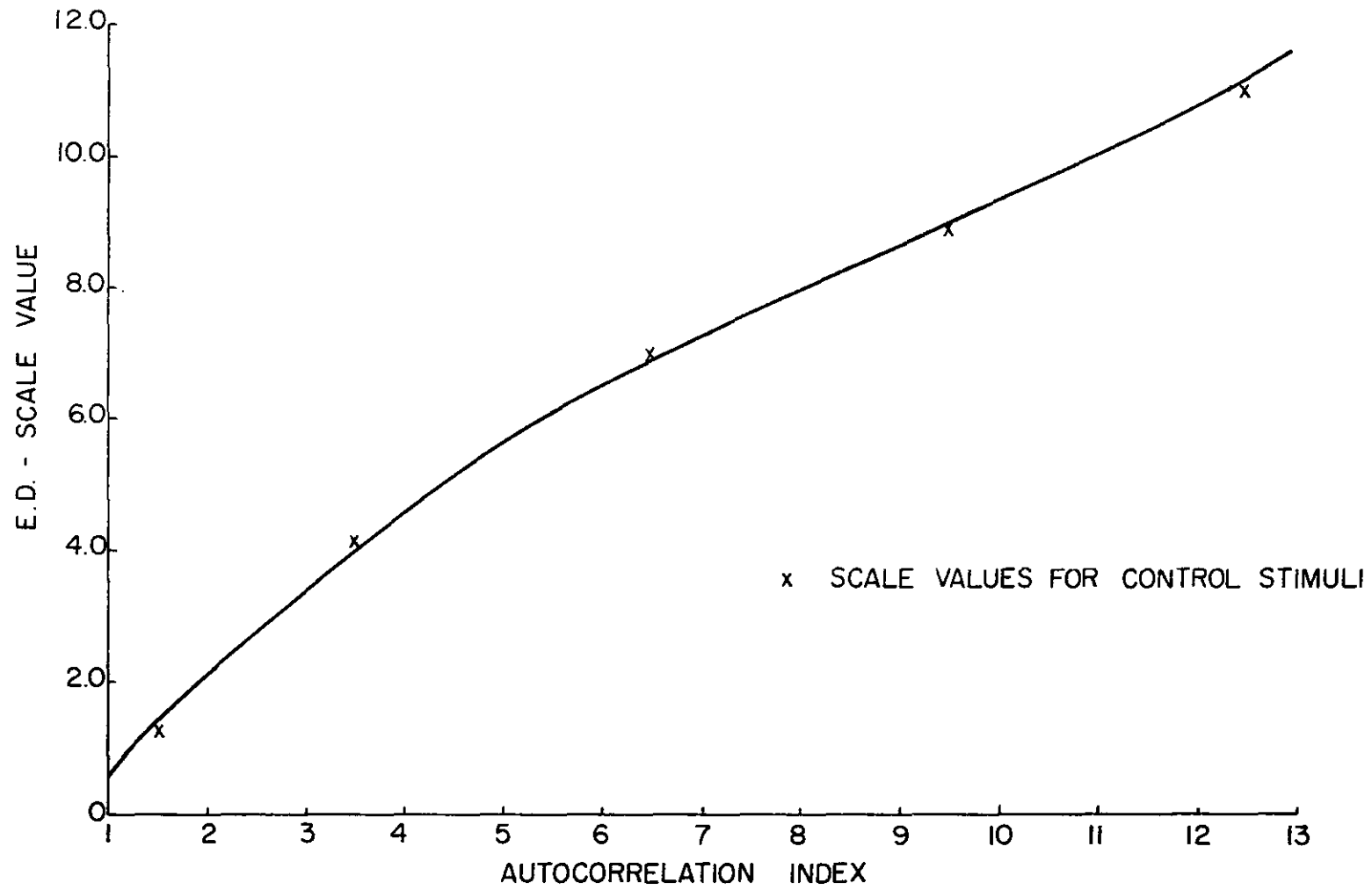


Figure 10. Average of E. D. - Scale Values for all Subjects as a Function of Autocorrelation Measure

TABLE V  
SCALE VALUES FOR SUBJECTS

Target	Subjects						Average
	I	II	III	IV	V	VI	
1	.56	.65	.58	.68	.57	.79	.64
1.5	.88	1.24	.76	1.33	.83	2.57	1.27
2	1.90	2.19	1.90	1.90	2.02	2.35	2.04
3	3.28	3.03	2.99	2.87	3.55	3.78	3.25
3.5	3.83	4.48	4.15	3.47	3.71	5.07	4.12
4	4.75	4.97	4.34	4.11	4.51	4.74	4.57
5	5.89	5.89	5.40	5.03	5.86	5.76	5.64
6	6.76	6.65	6.24	5.85	6.70	6.44	6.44
6.5	7.37	7.20	6.80	6.38	7.00	6.74	6.92
7	7.75	7.12	6.99	6.57	7.53	7.04	7.17
8	8.53	7.96	7.66	7.58	8.26	7.62	7.94
9	9.18	8.50	8.40	8.20	9.02	8.16	8.58
9.5	9.35	8.54	9.07	8.35	9.43	8.39	8.86
10	9.85	8.98	9.05	8.81	10.23	8.54	9.24
11	10.73	9.75	9.75	9.29	10.83	9.20	9.93
12	11.54	10.55	10.66	10.22	11.75	10.08	10.80
12.5	11.54	10.56	11.20	10.57	12.03	9.81	10.95
13	12.37	11.04	11.38	10.91	12.61	10.55	11.48
Information transmitted (in bits)	1.90	1.73	1.84	1.79	1.84	1.54	1.77

### C. Experiment II

Table VI summarizes the results of the pre-test to experiment II. Of special interest, in the pre-test results, is the statistic  $B_1 - B_1^c$ . This has a  $\chi^2$  of 17.06 which for 10 degrees of freedom falls below the .05 significance level. This result provides a rationale for pooling the results across levels in the main part of this experiment. These results are summarized in Table VII. It should be noted on this table that the  $B_1 - B_1^c$  statistic does indicate significant differences between subjects. The general trend, however, is still apparent from the pooled  $p_i$  values indicating that a significant preference difference exists between treatments with A being preferred as the finest texture, B the next and C the coarsest texture. The complete response tables for both the pre-test and the main experiment are shown in Appendix D.

TABLE VI  
RESULTS FOR PRE-TEST TO EXPERIMENT II

Levels		Treatments		
		A	B	C
2	$\Sigma r^*$	29	21	40
	$p_i^*$	.10	.90	0
	$B_1^2$	1.412		
4	$\Sigma r$	28	22	40
	$p_i$	.20	.80	0
	$B_1^4$	2.173		
6	$\Sigma r$	25	25	40
	$p_i$	.50	.50	0
	$B_1^6$	3.010		
8	$\Sigma r$	24	26	40
	$p_i$	.60	.40	0
	$B_1^8$	2.923		
10	$\Sigma r$	22	28	40
	$p_i$	.80	.20	0
	$B_1^{10}$	2.173		
12	$\Sigma r$	23	27	40
	$p_i$	.70	.30	0
	$B_1^{12}$	2.653		

TABLE VI, continued

		Treatments		
		A	B	C
	$\sum r$	151	149	240
Pooled	$p_i$	.50	.50	0
	$B_1$	18.048		

$$B_1^c = 14.344$$

$$\chi_{B_1^c}^2 = 183.5 \quad p < .00001$$

$$\chi_{(B_1 - B_1^c)}^2 = 17.06 \quad p > .05$$

$$\chi_{B_1^c}^2 = \text{statistic for difference between treatments}$$

$$\chi_{(B_1 - B_1^c)}^2 = \text{statistic for difference between levels}$$

\* $\sum r$  = Sum of ranks where rank of 1 assigned to member of comparison pair judged finer

\* $p_i$  = maximum likelihood estimate of  $\pi_i$



TABLE VII  
RESULTS FOR EXPERIMENT II

Subjects		Treatments		
		A	B	C
I	$\sum r$	22	29	39
	$p_i$	.81	.18	.02
	$B_1^I$	3.684		
II	$\sum r$	25	28	37
	$p_i$	.57	.35	.08
	$B_1^{II}$	6.525		
III	$\sum r$	26	24	40
	$p_i$	.40	.60	0
	$B_1^{III}$	2.923		
IV	$\sum r$	22	28	40
	$p_i$	.80	.20	0
	$B_1^{IV}$	2.173		
Pooled	$\sum r$	95	109	156
	$p_i$	.64	.33	.02
	$B_1$	19.67		

$$B_1^c = 15.31$$

$$\chi^2_{B_1^c} = 95.83 \quad p < .0001$$

$$\chi^2_{(B_1 - B_1^c)} = 20.08 \quad p < .25$$

## CHAPTER V

### DISCUSSION OF RESULTS

The discussion of results of this investigation is divided into three subsections, each of which represents a major portion of the problem area. The three sections concern the physical measure of surfaces selected in this study, experiment I, and experiment II. Since this study must be classed for the most part as an exploratory investigation of the problem area, the major portion of the discussion concerns the implications for future research which derive from the results. It seems premature at this stage of research to attempt theoretical integration of the results.

#### A. The Physical Measure

The selection of the autocorrelation function, as has been pointed out previously, is an arbitrary choice. The rationale for its selection is that it represents a kind of average (which is measurable) of all the statistical information of surfaces. The significant property of the autocorrelation function for this study is its relationship to the average size of the elements of a surface or more precisely the spatial frequency represented by the transform of the autocorrelation function. One possible index of this property is the parameter we have specified as the  $(1/e)$ th value of the function. The selection of this parameter is based on the ease with which it may be

determined once the function is known and the fact that it is traditionally used as an index of the speed at which the correlation drops off. It is important to note, however, that the analytic expression which was used in this study and to which the empirical data from the spatial correlator was fitted, contains two parameters. The rationale for this analytic expression lies in the fact that the surfaces considered here were random with respect to the distribution of energies. The precise physical significance of the exponent  $\beta$  in this expression is not at present known. Indeed this gap in our knowledge represents one important area for investigation. In order to apply the methodology developed in this study with no loss in generality it must be possible to analytically determine analogous expressions for other varieties of surfaces or in general provide some model whereby the autocorrelation function can be analytically determined from information about the properties of the distribution of energies on the surface. This procedure would then enable one to specify the physical referent of any parameter occurring in the resultant expression. Research along this direction is at present being carried out in Boston University Physical Research Laboratories and will represent an important contribution to the general problem area of image evaluation.

## B. Experiment I

This experiment was designed to determine the precise form of the relationship between the autocorrelation measure and observer judgments of texture. In general the results of this experiment follow quite closely the kinds of relationships Garner has found in his investigation of the loudness continuum (8). That is the relationship between the derived E. D. scale and the physical measure seems to be almost linear in the middle of the range whereas at the extremes of the continuum there seems to be a slightly higher discriminability. Some unpublished research by Hake and Erikson (14) suggest, however, that this may be, to some extent, an artifact of the method rather than a property of the discrimination abilities of the observer. Whichever inference one wishes to make in this respect has little bearing on the main consideration of this experiment which is that surfaces specified by the autocorrelation function seem to be ordered with respect to subject discrimination in much the same way as any other 'sensory continuum.' Indeed it can be seen from the individual or average functions that the general Weber-Fechner function is approximated. That is the higher discriminability phenomena is less pronounced at the high extreme of the continuum where the  $\frac{\Delta I}{I}$  fraction is small with respect to the low end of the continuum.

The Garner model unfortunately does not contain any internal check on the reliability of the derived scale values. In order to have

some indication of scale stability five control stimuli were introduced in the experiment with physical values intermediate to the initial 13 stimulus items. If the scale is relatively stable these stimuli should have predictable scale values. The points denoted on the psychophysical functions are the scale values of these stimuli. As can be seen these values for the most part fall close to their predicted positions indicating a relatively stable scale. However, there is no statistical method readily available to check the significance of those deviations that do occur for the control points. To this end it would seem worthwhile in future research to provide a precise index of the reliability or stability of the derived scale.

Table V, containing the amount of information transmitted for each subject, is of some interest. The relatively small variance of these values from subject to subject supports a contention long made in the literature that the information handling capacities of the human organism under given conditions is relatively stable. A rather interesting area of research is that of what changes in conditions can give rise to larger information transmission possibilities. The interpretation of the information transmission measure given by Garner is that it is the log to the base 2 of the number of alternative categories of events that may be discriminated without error. This would mean that under our experimental conditions only a little less than four

stimulus categories can be discriminated without error. Thus the conditions under which information transmission can be increased would be of great interest to any investigators who wished to incorporate these findings into a texture scale for the purpose of describing any visual surfaces.

One further problem area is suggested by the results of this experiment: an investigation of the extremes of the stimulus continuum studied here. What is meant by extremes in this case is represented by stimuli which are not clearly distinguishable from a flat gray surface and stimuli where the number and shape of elements become readily apparent. This would enable us to set the precise limits on where the notion of texture and the fine to coarse continuum cease to be meaningful to an observer.

### C. Experiment II

This experiment was designed to test the proposition that texture discrimination could be expressed as a single valued function of the autocorrelation function independent of variations in average transmission. In the previous chapter it was noted that the autocorrelation functions of series B and C were identical. To accept the proposition, it must at least be shown that series B and C are equally preferred by observers for the comparative judgment of 'finer than.' The results of this experiment, however, do not support

this notion. Indeed an examination of Table VII will show that on the average the probability of an observer calling B finer than C is about .94 while the probability of C being judged finer than B is only about .06. It therefore becomes apparent that texture discrimination must be considered at least a bivariate function, dependent on the slope of the autocorrelation function (the  $(1/e)$ th value) and the average transmission.

It should be noted that the term autocorrelation function used above actually refers only to the variational portion of the function (the normalized function). The non-normalized function does include a representation of the average transmission (the constant "a" in equation (2) is actually equal to  $\bar{t}^2$ ). Indeed this line of thought represents the rationale for examining average transmission as a possible relevant variable. Since the autocorrelation function as defined in equation (1) is a description of all the statistical information on the surface, then the only relevant information not included in the normalized function of equation (3) is the average transmission. This does not mean, however, that it is not possible for non-statistical features to be relevant for texture discrimination. With the design used in this experiment it should be possible to quickly test the relevance of many of the obvious variables such as color and the shape of generating elements.

In addition to these variables there remains for investigation the important statistical variable produced by the introduction of partial ordering on the surface. Although this may prove to be suitably described by the autocorrelation function it will probably be necessary to derive some more complex index of the function than the  $(1/e)$ th value. Some such parameter might be the area included under the function from zero delay to some arbitrary value.

The most immediate implication of this experiment would lead to research on the specification of the bivariate function for discrimination of texture. For this purpose it would be necessary to include more points on the transmission continuum and utilize the more powerful factorial design for experimentation.

Before closing this discussion, it should further be noted that the results obtained in this experiment are not unexpected from a phenomenological point of view. However, this or any other single explanatory principle appears to be insufficient to subsume the results of all the comparisons. For instance, an obvious inference from these results might be that the subject is responding to the stimuli as if they were black detail on a white background. While this inference, phenomenologically speaking, would enable us to predict the results of comparison A, C and B, C, it would predict results contrary to those obtained in comparison A, B. Thus, once more, before the results of this section can be completely understood at least some of the research suggested above must be carried out.



## CHAPTER VI

### SUMMARY AND CONCLUSIONS

The problem of this dissertation has been the investigation of the relationship between individuals' texture judgments and certain objective or physical dimensions of surfaces. The study was undertaken to provide some quantitative framework for the evaluation of Gibson's notion of visual texture.

One of the major tasks involved in this investigation was the determination of some physical measure which could be shown to be related to a set of judgments about texture. The autocorrelation function, in particular its  $(1/e)$ th value, an index of the speed at which this function drops off, was found to have a high correlation to judgments along the continuum of fine to coarse texture. Accordingly a psychophysical experiment was carried out using Garner's model for absolute judgments in order to determine the precise nature of this relationship. In addition another experiment was carried out using a comparative judgment design in order to test whether this relation was independent of variation in average transmission.

The major findings of these experiments were: (1) The psychophysical function for texture discrimination is essentially of the same form as the functions determined for other properties of sensory events using the same psychophysical model. Specifically the

relationship between judgments of fine to coarse texture and the physical measure is linear in the mid-range and slightly steeper at the extremes of the range of stimuli used in the experiment.

(2) Texture discriminations cannot be expressed as a single valued function of the autocorrelation measure but are dependent on at least one other variable, that of average transmission. Specifically it was found that, when two stimuli are equal in autocorrelation measure but differ in average transmission (75% vs. 25%), the 75% stimuli were judged finer with a probability of .94.

The results of these investigations have suggested a number of questions for future investigation. These are:

- (1) How can the autocorrelation function be analytically determined from the properties of the distribution of energies on a surface?
- (2) What precise measure of texture scale reliability can be devised?
- (3) What methods of scale construction can increase reliability for the purpose of applying this scale as a descriptive device? This question would involve the proper selection of stimuli and the determination of the conditions under which the greatest amount of information can be transmitted.
- (4) What is the precise nature of the psychophysical function at the extremes of the continuum?

(5) What is the relevance of other non-statistical variables such as color to texture discrimination?

(6) What is the precise nature of the bivariate relationship found between texture judgments on the one hand and the autocorrelation measure and the average transmission on the other hand.

(7) What is the applicability of the autocorrelation analysis when extended to the less restricted population of surfaces in which some partial order is allowed in the distribution.

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**APPENDIX A**

**Least Squares Computation for Photo 13-58**

## STEPS IN COMPUTATION SCHEDULE FOR PHOTO 13-58

Constants:

$$\bar{\beta} = 1.63 \qquad \bar{a}/\bar{c} = 1.273 \times 10^{-2}$$

$$\bar{c} = 128\mu$$

Steps:

1. Delay =  $x(\text{in}\mu)$
2.  $x/\bar{c} = (1)/\bar{c}$
3.  $\log x/\bar{c} = \log (2)$
4.  $\bar{\beta} \times \log x/\bar{c} = \bar{\beta} \times (3)$
5. - antilog (4)
6. antilog (5) = 1st estimate of  $y = \bar{y}$
7. - antilog (4)  $\times \bar{y} = (5) \times (6)$
8. - (7)  $\times \bar{a}/\bar{c} = a$
9. (3)  $\times$  (7) = b
10. Observed pts. = y obs.
11.  $\bar{y} - y \text{ obs} = (6) - (10) = 1$
12. (8)<sup>2</sup> = aa
13. (8)  $\times$  (9) = ab
14. (8)  $\times$  (11) = a1
15. (9)<sup>2</sup> = bb
16. (9)  $\times$  (11) = b1
17. (11)<sup>2</sup> = 11
18. [aa] =  $\sum (12)$

## STEPS IN COMPUTATION SCHEDULE FOR PHOTO 13-58 Continued

19.  $[ab] = \Sigma(13)$

20.  $[a1] = \Sigma(14)$

21.  $[bb] = \Sigma(15)$

22.  $[b1] = \Sigma(16)$

23.  $[11] = \Sigma(17)$

24.  $[vv] = [aa] (d\bar{c})^2 + 2 [ab] d\bar{\beta} d\bar{c} + [bb] d\bar{\beta}^2 + 2 [a1] d\bar{c} + 2 [b1] d\bar{\beta} + [11]$

25.  $d\bar{\beta} = \frac{[ab][a1] - [aa][b1]}{[aa][bb] - [ab]^2}$

26.  $d\bar{c} = \frac{[bb][a1] - [aa][b1]}{[ab]^2 - [aa][bb]}$

27.  $\beta = \bar{\beta} + d\bar{\beta} = \bar{\beta} + (24)$

28.  $c = \bar{c} + d\bar{c} = \bar{c} + (25)$

29.  $x/c = (1)/(28)$

30.  $\log x/c = \log(29)$

31.  $\beta \times \log x/c = (27) \times (30)$

32.  $- \text{antilog}(\beta \times \log x/c) = - \text{antilog}(31)$

33.  $\text{antilog}(32) = \text{theoretical point} = y_{\text{theor}}$



## COMPUTATION SCHEDULE FOR PHOTO 13-58

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	- ∞	- ∞	0	1.000	0	$0 \cdot 10^{-3}$	$0 \cdot 10^{-1}$
20	.156	-1.859	-3.024	.049	.952	- .047	.60	.87
40	.312	-1.165	-1.899	.150	.861	- .129	1.64	1.50
60	.469	- .757	-1.201	.301	.740	- .223	2.84	1.69
80	.625	- .470	- .766	.415	.628	- .292	3.72	1.37
100	.781	- .247	- .403	.668	.513	- .342	4.35	.84
120	.937	- .065	- .106	.899	.407	- .366	4.66	.24
140	1.093	.089	.145	1.156	.315	- .366	4.66	- .33
160	1.250	.223	.363	1.438	.237	- .341	4.34	- .76
180	1.407	.341	.555	1.742	.175	- .305	3.88	-1.04
200	1.562	.439	.715	2.045	.129	- .264	3.36	-1.16
220	1.720	.542	.884	2.420	.089	- .215	2.74	-1.17
240	1.875	.628	1.023	2.784	.062	- .173	2.20	-1.09
260	2.030	.708	1.152	3.168	.042	- .133	1.70	- .94
280	2.186	.781	1.273	3.575	.028	- .100	1.27	- .78
300	2.342	.850	1.385	4.000	.018	- .072	.92	- .61

## COMPUTATION SCHEDULE FOR PHOTO 13-58 Continued

(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
1.000	0	$0.10^{-6}$	$0.10^{-4}$	$0.10^{-5}$	$0.10^{-2}$	$0.10^{-3}$	$0.10^{-4}$
.953	- .000	.36	.52	- .06	.76	- .09	.01
.875	- .014	2.69	2.46	-2.30	2.25	-2.10	1.96
.759	- .019	8.07	4.80	-5.40	2.86	-3.21	3.61
.644	- .016	13.84	5.10	-5.96	1.88	-2.19	2.56
.528	- .015	18.92	3.65	-6.53	.71	-1.26	2.25
.413	- .006	21.72	1.12	-2.80	.06	- .44	.36
.318	- .003	21.72	-1.54	-1.40	.11	.10	.09
.241	- .004	18.84	-3.30	-1.74	.58	.30	.16
.172	.003	15.05	-4.04	1.16	1.08	- .31	.09
.106	.023	11.29	-3.90	7.73	1.35	-2.67	5.29
.068	.021	7.51	-3.21	5.75	1.37	-2.46	4.41
.038	.024	4.84	-2.40	5.28	1.19	-2.62	5.76
.019	.023	2.89	-1.60	3.74	.88	-2.16	5.29
.009	.019	1.61	- .99	2.41	.61	-1.48	3.61
.000	.018	.85	- .56	1.65	.37	-1.10	3.24

## COMPUTATION SCHEDULE FOR PHOTO 13-58 Continued

(18)  $1.502 \times 10^{-4}$

(19)  $-3.89 \times 10^{-4}$

(20)  $.154 \times 10^{-4}$

(21)  $16.06 \times 10^{-4}$

(22)  $-213.9 \times 10^{-4}$

(23)  $38.69 \times 10^{-4}$

(24)  $10.11 \times 10^{-4}$

(25) .13

(26) .2

(27) 1.76

(28) 128.2

## COMPUTATION SCHEDULE FOR PHOTO 13-58 Continued

(29)	(30)	(31)	(32)	(33)
0	- ∞	- ∞	0	1.000
.156	-1.858	-3.270	- .038	.963
.312	-1.165	-2.050	- .129	.879
.468	- .759	-1.336	- .263	.769
.624	- .472	- .831	- .436	.646
.780	- .248	- .436	- .647	.524
.936	- .066	- .116	- .890	.411
1.092	.088	.155	-1.168	.311
1.248	.221	.389	-1.476	.229
1.404	.339	.597	-1.817	.162
1.560	.445	.783	-2.188	.112
1.716	.540	.950	-2.586	.075
1.872	.627	1.104	-3.017	.049
2.028	.707	1.244	-3.470	.031
2.184	.781	1.375	-3.955	.019
2.340	.850	1.496	-4.465	.011

## APPENDIX B

Proof of the Equivalence of  $(1/e)$ th Values of Series B and C

It is important to note for purposes of this proof the manner in which series B and C were constructed. Briefly series C is the contrast reversal or negative of series B.

A surface from series B can be denoted by the function  $f_B(x, y)$  where:

$$(1) \quad f_B(x, y) = \begin{cases} 0 & \text{when } (x, y) \text{ is opaque} \\ 1 & \text{when } (x, y) \text{ is transparent} \end{cases}$$

It is obvious then that a surface from series C which is simply the negative of the above can be denoted as:

$$(2) \quad f_C(x, y) = 1 - f_B(x, y)$$

The autocorrelation function of a surface from series B is defined as:

$$(3) \quad \phi_B(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A f_B(x, y) f_B(x+x_0, y+y_0) dy dx$$

Similarly the autocorrelation function of the negative of the above surface is:

$$(4) \quad \phi_C(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A f_C(x, y) f_C(x+x_0, y+y_0) dy dx$$

but by eq. (2)

$$(5) \quad \phi_C(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A [1 - f_B(x, y)][1 - f_B(x+x_0, y+y_0)] dy dx$$

Multiplying under the integral we get:

$$(6) \quad \phi_C(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A [1 - f_B(x, y) - f_B(x+x_0, y+y_0) + f_B(x, y) f_B(x+x_0, y+y_0)] dy dx$$

$$(7) \quad \text{then } \phi_C(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \left[ \iint_A dy dx - \iint_A f_B(x, y) dy dx - \iint_A f_B(x+x_0, y+y_0) dy dx + \iint_A f_B(x, y) f_B(x+x_0, y+y_0) dy dx \right]$$

The first integral in these brackets obviously contributes at most a constant. Since it makes no difference where the integration is started the next two integrals both represent the mean transmission of the surface which is again a constant. The only non-constant term in the expression is the last. But by eq. (3) this is equal to  $\phi_B(x_0, y_0)$

$$(8) \therefore \phi_c(x_0, y_0) = \text{Const.} + \phi_B(x_0, y_0)$$

Since the  $(1/e)$ th value is measured only on the normalized function with all constants subtracted out, eq. (8) demonstrates that the  $(1/e)$ th values for photographs from series B and C on the same level are identical.

## APPENDIX C

Total Response Matrices for Each Subject on Experiment I



## TOTAL RESPONSE MATRIX - SUBJECT I

		Response												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Stimulus	1	91	9											
	1.5	57	41	2										
	2	21	77	2										
	3		52	43	5									
	3.5		34	48	13	5								
	4		7	43	36	13	1							
	5		1	12	28	34	23	2						
	6				10	32	40	16	1	1				
	6.5				3	14	38	44	1					
	7				4	6	30	57	2	1				
	8					3	6	58	27	4	1	1		
	9					2	2	33	40	10	7	4	2	
	9.5						1	20	51	19	6	3		
10						3	10	27	25	19	9	6	1	
11						1	5	7	15	17	24	26	5	
12							2	1	3	9	16	46	23	
12.5									4	12	20	34	30	
13								1	1	3	1	5	22	67

## TOTAL RESPONSE MATRIX - SUBJECT II

Stimulus	Response													
	1	2	3	4	5	6	7	8	9	10	11	12	13	
1	77	23												
1.5	45	55												
2	15	78	6	1										
3	1	64	29	4	1	1								
3.5		16	47	25	9	3								
4		7	36	36	17	4								
5			14	28	31	23	4							
6			2	18	22	37	18	1	1	1				
6.5				3	18	36	42	1						
7				4	16	41	38	1						
8					3	20	56	17	2	1	1			
9					3	7	43	27	16	2	2			
9.5							52	27	15	6				
10					2	3	18	40	20	8	6	3		
11						2	6	16	25	22	17	8	4	
12						1	2	4	8	19	29	28	9	
12.5							2	1	5	24	32	31	5	
13								1	1	5	11	24	38	20

## TOTAL RESPONSE MATRIX - SUBJECT III

		Response												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Stimulus	1	86	14											
	1.5	66	33	1										
	2	19	69	11	1									
	3		50	37	10	3								
	3.5		17	30	37	13	3							
	4		9	29	40	17	4	0	0	0	1			
	5			5	19	35	33	5	0	1	2			
	6			1	5	9	45	31	4	3	2			
	6.5			1	3	4	25	39	20	5	3			
	7				2	3	15	45	16	9	8	2		
	8				1	1	2	24	36	16	13	6	1	
	9						2	5	16	31	28	12	5	1
	9.5						1	1	6	19	27	28	18	
10								5	15	37	28	15		
11						1	0	0	3	19	37	32	8	
12									3	2	11	57	27	
12.5									1	2	8	36	53	
13											7	30	63	

## TOTAL RESPONSE MATRIX - SUBJECT IV

Stimulus	Response												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	74	21	5										
1.5	38	54	6	2									
2	15	57	20	6	1	1							
3	2	28	45	19	4	1	1						
3.5		14	32	31	21	2							
4		4	14	38	34	6	2	2					
5			3	16	36	32	8	3	2				
6				1	19	45	20	8	5	2			
6.5					7	32	45	16					
7					1	26	48	19	3	1	1	1	
8						5	16	41	26	7	4	0	1
9							4	31	32	23	8	1	1
9.5								16	51	24	8	1	
10								14	28	27	17	7	7
11								2	17	31	28	14	8
12								3	1	11	22	46	17
12.5									1	6	19	34	40
13									1	3	10	29	57

## TOTAL RESPONSE MATRIX - SUBJECT V

Stimulus	Response												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	87	13											
1.5	60	40											
2	15	66	15	4									
3		34	43	18	4	1							
3.5		17	49	23	10	1							
4		5	29	32	23	9	0	0	2				
5			5	13	36	36	7	3					
6				1	16	58	18	6	1				
6.5				1	15	43	40	1					
7					4	37	41	7	7	2	2		
8						11	49	25	9	6			
9						3	31	34	13	15	4		
9.5							1	54	44	1			
10							7	23	26	22	21	1	
11							3	10	26	15	29	17	
12								1	13	17	23	37	9
12.5									6	12	28	39	15
13									3	3	19	41	34

## TOTAL RESPONSE MATRIX - SUBJECT VI

		Response												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Stimulus	1	63	37											
	1.5	7	55	37	1									
	2	5	67	23	5									
	3		17	59	15	7	1	1						
	3.5			24	30	40	6							
	4		3	29	39	22	6	1						
	5		1	4	19	45	23	3	3	1	1			
	6			3	11	27	31	15	5	6	2			
	6.5			1	3	22	39	27	7	1				
	7				3	11	38	26	15	7				
	8					3	26	37	16	11	1	5	1	
	9					1	6	36	35	10	7	5		
	9.5						4	32	26	22	12	4		
10						4	28	24	11	19	8	6		
11						1	9	14	28	19	19	10		
12								8	15	14	22	31	10	
12.5							1	8	16	22	24	19	10	
13							1	8	7	16	14	28	26	

## APPENDIX D

Tables of Comparative Judgment Rankings for Experiment II

RESPONSE TABLE FOR COMPARATIVE JUDGMENTS ON PRE-TEST TO EXPERIMENT II

Replications	1	2	3	4	5	6	7	8	9	10
Treatments	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C
Level 2										
Pairs	A, B	2 1 -	2 1 -	2 1 -	1 2 -	2 1 -	2 1 -	2 1 -	2 1 -	2 1 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Level 4										
	A, B	2 1 -	2 1 -	2 1 -	2 1 -	2 1 -	2 1 -	1 2 -	2 1 -	1 2 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Level 6										
	A, B	2 1 -	2 1 -	2 1 -	2 1 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Level 8										
	A, B	1 2 -	2 1 -	1 2 -	2 1 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Level 10										
	A, B	1 2 -	2 1 -	1 2 -	1 2 -	1 2 -	2 1 -	1 2 -	1 2 -	1 2 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Level 12										
	A, B	2 1 -	2 1 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -	1 2 -	1 2 -
	A, C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B, C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2



RESPONSE TABLE FOR COMPARATIVE JUDGMENTS ON EXPERIMENT II

Replications	1	2	3	4	5	6	7	8	9	10
Treatments	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C	A B C
Subject I										
Pairs	A,B	1 2 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -	1 2 -	1 2 -	2 1 -
	A,C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B,C	- 1 2	- 2 1	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Subject II										
	A,B	2 1 -	2 1 -	1 2 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -	1 2 -
	A,C	2 - 1	1 - 2	1 - 2	1 - 2	1 - 2	2 - 1	1 - 2	1 - 2	1 - 2
	B,C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 2 1	- 1 2	- 1 2	- 1 2
Subject III										
	A,B	2 1 -	2 1 -	1 2 -	2 1 -	1 2 -	2 1 -	2 1 -	1 2 -	2 1 -
	A,C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B,C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2
Subject IV										
	A,B	1 2 -	1 2 -	2 1 -	1 2 -	1 2 -	1 2 -	1 2 -	2 1 -	1 2 -
	A,C	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2	1 - 2
	B,C	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2	- 1 2

A PSYCHOPHYSICAL STUDY OF VISUAL TEXTURE

Abstract of a Dissertation

Submitted in partial fulfilment of the requirements  
for the degree of Doctor of Philosophy

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## I. Introduction

One of the major historical traditions in psychological research has been the investigation of the ways in which an observer can differentiate or discriminate various sensory events in relation to some objective dimension of the event. Furthermore, the features of sensory events investigated have been abstract properties of the stimulus for which there were already existing physical-measure correlates such as brightness, hue, distance, etc. The nature of sensory events, however, does not necessarily restrict one to investigation of these features. Gibson has recently formulated a theoretical view of visual perception in which the status of these features is secondary. His position asserts that the immediate given of visual experience is surfaces and thus directs concern to the properties of surfaces as sensory events. Although the concept of texture of a surface is a central one in his scheme, there seems to be a complete lack of psychophysical investigation of this notion in the literature.

The above considerations led to the formulation of the present problem. Specifically, the relationship between a set of physical measure operations and a set of observer responses on a class of stimuli denoted as "textured surfaces" was investigated. Texture in this study is defined as the distribution of alternations in light intensity over a surface on which areas of equal intensity occupy small portions of the surface relative to its total extent.

## II. The Physical Measure

There are many ways in which the distribution of intensities denoted as texture in the above definition may be characterized. The most powerful general characterization arises from a consideration of the statistical properties of the distribution of changes in intensity over distance. On the basis of research on the measurement of photographic grain the important statistical property of the distribution on surfaces was found to be that successive variations in intensity are not statistically independent. These variations can be characterized as a Markov process. This property can be represented by the autocorrelation function in two dimensions defined as:

(1)

$$(1) \quad \phi(x_0, y_0) = \lim_{A \rightarrow \infty} \frac{1}{A} \iint_A f(x, y) f(x+x_0, y+y_0) dy dx$$

where  $f(x, y)$  represents the transmission at  $(x, y)$

$f(x+x_0, y+y_0)$  represents the transmission at  $(x+x_0, y+y_0)$  when the surfaces are shifted by the amount  $(x_0, y_0)$

In order to simplify the mathematical analysis, this investigation has only considered surfaces where the distributions of intensity with respect to distance are random. In this case we can predict the analytical form of the normalized autocorrelation function to be:

where  $y$  = transmission or intensity

$$(2) \quad \phi'(x_0, y_0) = \frac{y-a}{b} = e^{-\left(\frac{x}{c}\right)^2}$$

$a$  = square of the mean transmission

$b$  = mean square minus square of the mean at zero delay

$x = (x_0, y_0)$

The parameter of the autocorrelation function of principle interest in this study is the  $(1/e)$ th value or distance. This is defined as that value of  $x$  where  $\phi'(x, y_0) = e^{-1}$  and in the case where the function has the form of equation (2) this reduces to  $x=c$ . This parameter is a representation of the speed at which the autocorrelation function drops off to an asymptote. The use of the autocorrelation function and its  $(1/e)$ th value enables us to relate some physical measure to observer reports of textures varying from "fine" to "coarse."

A preliminary experiment was performed in order to check the usefulness of this method of analysis. A sample of surfaces was first obtained from aerial photographs of natural terrain from which seven were chosen. The set was then individually administered to twenty subjects who were asked to arrange the seven stimuli in a rank from fine to coarse texture. Following this the  $(1/e)$ th value was determined for each surface and the stimuli were ranked with respect to this measure. This was compared with the pooled ranks by the subjects and yielded a rank correlation coefficient of .96. This result favored the use of the  $(1/e)$ th parameter as the physical measure of surfaces.

### III. Experimental Methods and Design

Two main experiments were performed. The first was designed to determine the precise relationship between the  $(1/e)$ th value of the autocorrelation function and observers' discriminations of texture. The second experiment was a test of the proposition that observer discriminations can be expressed as a single valued function of the physical parameter independent of changes in average transmission.

In order to reduce subject variability it was necessary to construct a set of stimuli with a uniform and random distribution of white and black which would vary regularly with respect to the physical measure being used. Three series of surfaces were made differing with respect to the ratio of black to white elements. Series A contained a 50-50 mixture. Series B was 75% black, 25% white, and Series C was 25% white, 75% black.

In experiment I eighteen surfaces from Series A were selected, thirteen of these (the experimental stimuli) were equally spaced with respect to the  $(1/e)$ th value. The other five (the control stimuli) were intermediate to the experimental stimuli in  $(1/e)$ th value. All stimuli were randomly ordered. Six subjects were asked to judge each target 100 times and to assign a number from one to thirteen to represent the relative fineness or coarseness of the stimuli. A psychophysical function was computed from this data using the Garner model for absolute judgments. This model yields a scale of equal discriminability

units. In addition, the amount of information transmitted was computed for each subject as a measure of intra-subject variability. Since the Garner model offers no internal check on the reliability of the derived scale, five control stimuli were used to check the stability of the scale in terms of the predictability of intermediate scale values.

Experiment II was designed to test the proposition that texture discriminations can be expressed as a single valued function of the  $(1/e)$ th values independent of variations in average transmission. The experiment was set up as a comparison by pairs utilizing a model developed by Bradley and Terry which is a generalization of the binomial distribution model. In the design, Series A, B and C are considered as different treatments, and stimuli with the same size image circle represent a single level. All comparisons are made within a level. What is of interest here is the relative treatment preferences denoted as  $\pi_i$  : where the expression  $\frac{\pi_i}{\pi_i + \pi_j}$  represents the probability that treatment i will be judged finer than treatment j. A pre-test was performed which demonstrated no significant difference between levels with respect to treatment preference. It was therefore possible to design the main experiment considering changes in level as simply another replication.

Four of the six subjects used in experiment I participated in the main part of experiment II. A single block consisted of three comparisons, series A to B, A to C, and B to C. Each subject had ten replications of the block.

#### IV. Major Results

The psychophysical functions for experiment I show no difference from what would be expected from the literature for other properties of sensory events. In general the function tends to be linear in the mid-range and shows a somewhat higher discriminability at the extremes of the stimulus continuum. The scale values of the control stimuli show little variation from predicted positions and thus yield some positive evidence for the stability of the scale values.

The results of experiment II indicate in general that for observers series A is judged the finest texture, series B the next, and series C the coarsest. The differences between these treatments is statistically highly significant. However, it can be shown that the  $(1/e)$ th values for stimuli from series B and C on a given level are identical. It is apparent, therefore, that the proposition being tested must be rejected and texture discrimination expressed as at least a bivariate function of the autocorrelation measure and average transmission.



## V. Discussion

Three major areas of discussion are distinguishable following from the physical measurement procedure and from the results of experiment I and II. The implications of these results were examined primarily with respect to problems for further research.

The selection of the autocorrelation function as a basis for physical measurement has been in a sense an arbitrary choice. The rationale for its selection has been that it represents a kind of average of all the statistical information of the surface. Furthermore, the derivation of a single index for the function was made possible by restricting the surfaces considered to those which were random with respect to the distribution of energies. Before this method of analysis can be applied with no loss in generality, it must be possible to derive the analytic expression of the autocorrelation function for non-random surfaces. This would be equivalent to providing some explicit general model whereby the autocorrelation function can be analytically determined from the properties of the distribution of energies on a surface. Thus one could specify the physical referent of any parameter occurring in the analytic function. This procedure might be extremely useful in explicating other dimensions implicit in the common usage of term texture.

Experiment I was designed principally to determine the precise form of the relationship between the two sources of measurement. The results of this experiment have supported the meaningfulness of using the  $(1/e)$ th value as a physical measure of textured surfaces. For example, it was seen that, in at least its most general sense, the Weber-Fechner law holds since the higher discriminability expected at the extremes of the range of stimuli in this type of design is less pronounced at the coarse extreme where the  $\frac{\Delta I}{I}$  fraction is small with respect to the fine extreme. This suggests further research to determine how the Weber-Fechner law holds for the extremes of the stimulus continuum.

Experiment II was designed to test the proposition that texture discrimination can be expressed as a single valued function of the autocorrelation measure independent of variations in average transmission. The rejection of this proposition suggest that future investigations should, in part, be concerned with a systematic examination of this variable for its precise relevance to texture discrimination. Two additional areas of research are important if these results are to be usefully generalized as descriptive devices. These are first, the examination of other variables for their possible relevance to texture discrimination, and secondly, the extension of the autocorrelation analysis to the less restricted population of surfaces, that is those in which partial order occurs in the energy distribution.

## AUTOBIOGRAPHY

The author was born in Boston, Massachusetts on September 30, 1930, son of Celia and Harry Kaizer. He was graduated from Roxbury Memorial High School for Boys in June, 1947. From 1947-1949 he attended The College of the University of Chicago. He completed his undergraduate education at Boston University, College of Liberal Arts where he received his A. B. in August, 1951. In September, 1951 he entered the Graduate School of Boston University in the Department of Psychology and received his A. M. in psychology in August 1955. For the past three years the author has been engaged both part and full-time as a research assistant in Boston University Physical Research Laboratories.



THE AUTHOR