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On the analysis of multivariate time series with specializations on multi-output regression models and two-sample testing

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ON THE ANALYSIS OF MULTIVARIATE TIME SERIES WITH SPECIALIZATIONS ON MULTI-OUTPUT REGRESSION MODELS AND TWO-SAMPLE TESTING

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ABSTRACT

The dissertation aims to develop new statistical inference protocols and their associated asymptotic theory for multivariate time series. Firstly, we address the problem of variable labeling and estimation for multi-output time-varying coefficient models. In these models, the regression coefficient can change over time as a nonparametric function, capturing the time-varying nature of the data. As a result, variables can be classified as time-varying, time-constant, or irrelevant within a nested structure. A variable is considered time-constant or irrelevant if its coefficient function is consistently a constant or zero, respectively, for all outputs. We propose a stratified penalization method that achieves accurate tricategory labeling in a single step. We establish the theoretical properties of this method, including its estimation and labeling consistency. Monte Carlo simulations demonstrate that our proposed method improves the accuracy of labeling and estimation compared to existing methods.

Next, we proceed to investigate two-sample inference for time series, specifically considering cases where the compared time series may have staggered observation

periods and exhibit joint dependence. We introduce a warped self-normalized subsampling test for strongly mixing data, showing that this approach can be utilized not only for comparing means between two samples but also for comparing other quantities such as variances or quantiles in a unified manner. The associated asymptotic theory is established. We conduct numerical experiments, including Monte Carlo simulations and a real data analysis, to further illustrate the effectiveness of our proposed method.

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List of Abbreviations

i.i.d.	.....	Independent and Identically Distributed
LCR	.....	Labeling Consistency Ratio
MSE	.....	Mean Squared Error
NI	.....	Northern Ireland
NwEW	.....	Northwest England and Wales
SOI	.....	Southern Oscillation Index
SPLL	.....	Stratified Penalized Local Linear
UK	.....	United Kingdom
WSNS	.....	Warped Self-Normalized Subsampling

Chapter 1

Introduction

Many important statistical results are initially developed for independent and identically distributed (i.i.d.) observations. However, in practice, data of interest is often neither independent nor identically distributed. For instance, the price of a stock today generally depends on its price yesterday, violating the independent assumption. Additionally, the volatility of a stock may change over time, violating the identically distributed assumption. Much research has focused on relaxing the strong i.i.d. assumption, and time series analysis is one area of study in this regard. However, generalizing results from the i.i.d. assumption to time series data can be a challenging problem. In this chapter, we provide two examples to illustrate this issue, which will also motivate the problems we solve in this dissertation. In Section 1.1, we will consider generalizing the classical linear regression model to regression on time series. In Section 1.2, we will consider generalizing the statistical inference on the population mean under the i.i.d. setting to the time series setting. Section 1.3 outlines the dissertation.

1.1 Linear Regression Model of Time Series

Linear regression models are widely recognized as basic yet powerful statistical tools for examining the relationship between a response variable and a set of explanatory variables. For the classical linear regression model, the relationship can be represented

as:

$$y_i = \boldsymbol{x}_i^\top \boldsymbol{\beta} + e_i, \quad i = 1, \ldots, n, \tag{1.1}$$

where the symbol $^\top$ denotes the transpose operator, $(y_i, \boldsymbol{x}_i), i = 1, \ldots, n$, are i.i.d. samples from the population, $\boldsymbol{\beta}$ is the regression coefficient vector, and (e_i) are random errors. To perform statistical analysis of the linear model, such as t-tests for the significance of each coefficient and F-tests for the overall significance of the model, we assume that there is no multicollinearity and the errors e_i are independent of $\boldsymbol{x}_i$, i.i.d. with a normal distribution with zero mean and constant variance. It seems straightforward to generalize the classical linear regression models (1.1) by performing regression on two time series, which leads to the following static time series linear regression model, see, for example, Part 2 of Wooldridge (2015):

$$y_t = \boldsymbol{x}_t^\top \boldsymbol{\beta} + e_t, \quad t = 1, \ldots, T, \tag{1.2}$$

where t denotes the time period, $(\boldsymbol{x}_t)$ and (y_t) are two time series dated contemporaneously, $\boldsymbol{\beta}$ is the regression coefficient vector, and (e_t) are random errors. However, since both $(\boldsymbol{x}_t)$ and (y_t) are time series where the past values may have effect on the current value, the i.i.d. assumption on e_i in the classical models (1.1) no longer holds in (1.2). In Chapter 11 of Wooldridge (2015), the assumptions of models (1.1) are generalized to the assumptions of the static linear regression model (1.2) for stationary time series as follows:

- Weak dependence: $\{(y_t, \boldsymbol{x}_t) : t = 1, \ldots, T\}$ is weakly dependent.
- Zero conditional mean: $\mathrm{E}(e_t \mid \boldsymbol{x}_t) = 0, \quad t = 1, \ldots, T,$
- Homoskedasticity: $\mathrm{var}(e_t \mid \boldsymbol{x}_t) = \nu^2, \quad t = 1, \ldots, T,$
- No Serial Correlation: $\mathrm{cov}(e_t, e_s \mid \boldsymbol{x}_s, \boldsymbol{x}_t) = 0$ for all $t \neq s$.

If all four aforementioned assumptions are met, and there is no multicollinearity, the classical linear model's results remain valid. In other words, the least square estimator of (1.2) is asymptotically normally distributed. This allows for the use of common statistical tests like the t-test and F-test mentioned earlier. However, these assumptions often fail in time series analysis due to its inherent characteristics. For example, in economic data, it is always the case that current events are related to past events, causing serial correlation. To address this issue, more effort should be made, such as modeling the error term e_t as an autoregressive (AR) model. In addition, in many real-world cases, homoscedasticity is often not met, like in stock price returns. For such situations, heteroscedasticity models are necessary, which can handle the nonconstant variance. Various approaches have been studied in the litereature, including ARCH and GARCH models. In summary, the generalization from classical linear regression to time series is not straightforward and comes with several limitations.

Another limitation of model (1.2) is that the regression coefficient $\boldsymbol{\beta}$ remains constant. However, this assumption may prove to be limiting when applying linear regression models to time series data. Empirical examples have demonstrated that the regression coefficient does not necessarily remain as a constant and may vary over time. For example, Fan and Zhang (1999) conducted a study on the relationship between the number of daily hospital admissions and the levels of multiple pollutants in Hong Kong, revealing a time-varying relationship. Additionally, Gao and Hawthorne (2006) performed a regression analysis of the global temperature series on the Southern Oscillation Index (SOI) and proposed that at least the intercept term should be considered as time-varying in a nonparametric manner due to a lack of knowledge about the change. Besides, Zhang and Wu (2015) investigated the modeling of U.S. treasury yields and provided statistical evidence for a time-varying linear

drift in the yield curve rates with a six-month maturity. Such applications motivated the following time-varying coefficient model:

$$y_{i,n} = \boldsymbol{x}_{i,n}^{\top}\boldsymbol{\beta}(t_{i,n}) + e_{i,n}, \quad i = 1, \ldots, n, \tag{1.3}$$

where $\boldsymbol{\beta} : [0,1] \to \mathbb{R}^p$ is the coefficient function, $t_{i,n} = i/n$ represents the time, and $(e_{i,n})$ is a sequence of random noise. In model (1.3), the regression coefficient is no longer assumed to be a constant but is modeled as a nonparametric function of time to capture the time-varying feature. As a result, variables can be classified as time-varying (the coefficient function is a non-parametric function), time-constant (the coefficient function is a constant), or irrelevant (the coefficient function is zero) within a nested structure.

With the explanatory variables, a large amount of data to be analyzed may not always have just one response variable. For example, in the field of biology, one might examine the trend of life expectancy by gender; in finance, one might be interested in how the prices of two different stock markets change over time. Hence, we may generalize (1.3) to the following multi-output time-varying coefficient model:

$$\boldsymbol{y}_{i,n} = B(t_{i,n})^{\top}\boldsymbol{x}_{i,n} + \boldsymbol{e}_{i,n}, \quad i = 1, \ldots, n. \tag{1.4}$$

Here, $\boldsymbol{y}_{i,n} \in \mathbb{R}^d$ represents the multi-output response vector, $\boldsymbol{x}_{i,n} \in \mathbb{R}^p$ is the set of explanatory variables, $B : [0,1] \to \mathbb{R}^{p\times d}$ is the coefficient function matrix, where the kth row is the coefficient function vector for the kth variable, and $(\boldsymbol{e}_{i,n})$ is a sequence of random vectors. The model (1.4) includes the model (1.3) as a special case by setting $d = 1$. In model (1.4), a variable is considered time-constant or irrelevant if its coefficient function is consistently a constant or zero, respectively, for all outputs. In Chapter 2, we consider the tricategory labeling problem of partitioning variables into time-varying, time-constant, and irrelevant categories for multi-output

time-varying coefficient model (1.4), and propose a stratified penalization method to achieve this goal in a single step.

1.2 Inference on the Population Mean of Time Series

We start with the classical one-sample inference. Assuming we have a random sample of size n_z, $Z_1, \ldots, Z_{n_z}$, from a population Z with an unknown mean $\mu_z = \mathrm{E}(Z)$ and variance $\sigma_z^2 = \mathrm{var}(Z)$, the sample mean $\bar{Z}_{n_z} = n_z^{-1} \sum_{i=1}^{n_z} Z_i$ is a natural estimator for the population mean μ_z. If the population Z is not normally distributed, to perform statistical inference on the population mean, such as hypothesis testing or constructing confidence intervals, we rely on the following central limit theorem:

$$n_z^{1/2}(\bar{Z}_{n_z} - \mu_z) \to_d N(0, \sigma_z^2) \tag{1.5}$$

where $\to_d$ denotes convergence in distribution. Once we have (1.5), we can conduct statistical inference on μ_z. For example, let $q_{z,1-\alpha/2}$ denote the $100(1-\alpha/2)\%$ percentile of standard normal distribution, an asymptotic $100(1-\alpha)\%$ two-sided confidence interval of μ_z is given by

$$(\bar{Z}_{n_z} - n_z^{-1/2}\sigma_z q_{z,1-\alpha/2}, \bar{Z}_{n_z} + n_z^{-1/2}\sigma_z q_{z,1-\alpha/2}). \tag{1.6}$$

Here, the unknown parameter σ_z is always estimated by the sample standard derivation $\hat{\sigma}_z = \{(n_z-1)^{-1} \sum_{i=1}^{n_z} (Z_i - \bar{Z}_{n_z})^2\}^{1/2}$, which is a consistent estimator of σ_z.

Nevertheless, applying the above results to time series data can be challenging, even when the time series process is stationary. In particular, instead of sampling $Z_1, \ldots, Z_{n_z}$ from population Z where $Z_1, \ldots, Z_{n_z}$ are i.i.d., we obtain a sample $Z_1, \ldots, Z_{n_z}$ from a stationary process $(Z_i)_{i \in \mathbb{Z}}$ with mean $\mu_z = \mathrm{E}(Z_0)$. The mean can still be estimated by $\bar{Z}_{n_z} = n_z^{-1} \sum_{i=1}^{n_z} Z_i$. We need to develop an asymptotic theory similar to (1.5) for $\bar{Z}_{n_z}$ in order to conduct statistical inference in μ_z. Let

$g_z = \sum_{k\in\mathbb{Z}} \mathrm{cov}(Z_0, Z_k)$ represent the long-run variance, which is defined as the sum of autocovariances of all orders. Under certain short-range dependence conditions, such as the strong mixing condition of Rosenblatt (1956), or the m-dependent condition of Hoeffding and Robbins (1948), a central limit theorem similar to (1.5) can be derived as

$$n_z^{1/2}(\bar{Z}_{n_z} - \mu_z) \to_d N(0, g_z) \tag{1.7}$$

However, even if we can similarly obtain the confidence interval from (1.7), estimating the nuisance parameter g_z poses a nontrivial problem in terms of achieving consistency and may involve choosing additional tuning parameters.

More issues arise when we generalize from one-sample time series inference to a two-sample problem. Let's suppose we have samples $X_1, \ldots, X_{n_x}$ and $Y_1, \ldots, Y_{n_y}$ from two stationary processes $(X_i)_{i\in\mathbb{Z}}$ and $(Y_j)_{j\in\mathbb{Z}}$ with means $\mu_x = \mathrm{E}(X_0)$ and $\mu_y = \mathrm{E}(Y_0)$, respectively. If we assume that X_i and Y_j are independent and both stationary m-dependent sequences, and under certain standard conditions (see, for example, Brockwell and Davis 1991), we can derive that $\bar{X}_{n_x} - \bar{Y}_{n_y}$ is asymptotically normal with a mean of $\mu_x - \mu_y$ and a variance of $n_x^{-1}g_x + n_y^{-1}g_y$, where g_x and g_y are the long-run variances of $\{X_i\}_{i\in\mathbb{Z}}$ and $\{Y_j\}_{j\in\mathbb{Z}}$, respectively. We can use this result to conduct two-sample statistical inference on $\mu_x - \mu_y$, after obtaining consistent estimations of the unknown parameters g_x and g_y that appear in the asymptotic expression, which is a nontrivial problem. It is worth noting that the above results are based on assumptions that may be questionable in practice. Firstly, these results only apply when comparing independent sequences, but it would be more appropriate to account for the dependence between two time series. For example, when comparing the returns of two stocks, we may need to consider the interdependence between the two series, even if they belong to different markets, given the interconnected nature of financial markets. Likewise, when analyzing the effects of COVID-19 across two

countries, assuming independence may not hold true due to potential correlations in the positive case numbers of both countries, as argued in Sulyok et al. (2021). Therefore, an approach that accounts for the dependence between the two series may be preferred when conducting two-sample inferences in time series. In Chapter 3, we delve into two-sample inference for time series, examining scenarios where the compared time series display joint dependence. It is worth noting that we permit the compared time series to have different lengths with nonidentical start and end observation time, resulting in staggered observation periods.

1.3 The Outline of the Dissertation

The dissertation aims to develop novel statistical inference protocols and their associated asymptotic theory for multivariate time series. Specifically, we focus on a tricategorical labeling problem for multi-output time-varying coefficient models and a self-normalized subsampling procedure for inference of staggered time series that may exhibit joint dependence. The rest of this dissertation is organized as follows.

In Chapter 2, we firstly present a concise overview of the time-varying coefficient model (1.3) and the related tricategorical labeling task, while reviewing the methodology that inspired our proposed method. Subsequently, in Section 2.3, we introduce our method within the multi-output framework, focusing on the multi-output time-varying coefficient models with a single explanatory variable. This scenario is a special case of (1.4) when $p = 1$. We propose a stratified penalization method that can achieve accurate tricategorical labeling in a single step. Theoretical properties of this method, including its estimation and labeling consistency, are established. Then, we in Section 2.4 generalize the proposed method to the general multi-output time-varying coefficient models (1.4), where the associated theoretical properties are also provided. Monte Carlo simulation and a real-data analysis are conducted to examine

its finite-sample performance. Technical proofs are deferred to the end of the chapter.

In Chapter 3, we introduce a warped self-normalized subsampling test for the two-sample inference problem on time series with staggered observations and joint dependence. We demonstrate that our proposed test is not only applicable to comparing means between two samples but can also be used to compare other quantities such as variances or quantiles in a unified manner. The associated theoretical asymptotic justification of the proposed warped self-normalized subsampling method are provided. In addition, we conduct numerical experiments, including a Monte Carlo simulation and a real data analysis, to further illustrate the effectiveness of the proposed method. Technical proofs are presented in Section 3.7. Additional simulation results are deferred to the end of this chapter.

Chapter 2

Variable Labeling of Multi-Output Time-Varying Coefficient Models

In Chapter 2, we consider the tricategory labeling problem of partitioning variables into time-varying, time-constant, and irrelevant categories for multi-output time-varying coefficient model. This chapter is based on Zhang et al. (2023).

2.1 Introduction

The time-varying coefficient model (1.3) has been widely studied in the literature, with references including Robinson (1989), Hoover et al. (1998), Orbe et al. (2005), Cai (2007), Zhou and Wu (2010), Zhang and Wu (2012), Chen and He (2018), Truquet (2018), Li et al. (2020), among many others. It covers a wide range of time series models. For instance, when setting $x_{i,n} = 1$ (i.e., only an intercept term in the time-varying coefficient model contaminated by random errors), the model reduces to the mean nonstationary model $y_{i,n} = \beta(t_{i,n}) + e_{i,n}$. This model is useful for exploring the mean nonstationary process and has been widely used in nonparametric trend estimation and testing problems, as seen in Johnstone and Silverman (1997), Wu and Zhao (2007), Zhang and Wu (2011), and Zhang (2016). Additionally, if we use lagged values as potential explanatory variables (i.e., set $\boldsymbol{x}_{i,n} = (y_{i-1,n}, \ldots, y_{i-p,n})^\top$), the time-varying coefficient model becomes the influential time-varying autoregressive model:

$$y_{i,n} = \beta_1(t_{i,n})y_{i-1,n} + \ldots \beta_p(t_{i,n})y_{i-p,n} + e_{i,n}, \quad i = 1, \ldots, n, \tag{2.1}$$

This model has been studied by Rao (1970), Dahlhaus et al. (1999), Moulines et al. (2005), and Van Bellegem and Dahlhaus (2006), among many others. It is important to note that the model (2.1) is nonstationary due to the time-varying autoregressive coefficients. The methodology we proposed in this chapter allows for a general class of nonstationary processes that includes the model (2.1) as a special case.

For time-varying coefficient models (1.3) with nonstationary time series observations, Zhou and Wu (2010) constructed simultaneous confidence bands for the coefficient functions, and Zhang and Wu (2012) considered an integrated squared test that can be more suitable for detecting smooth and dense changes. In addition to estimating the coefficient functions and testing the associated hypotheses, an important problem is the labeling or partitioning of variables into time-varying, time-constant, and irrelevant categories. This tricategory labeling task for the time-varying coefficient model (1.3) has been extensively studied using various approaches. One commonly used method involves a two-step approach, where bicategory labeling is performed separately at each step. For instance, Li and Liang (2008) assumed prior knowledge on the partition between time-varying and time-constant components and applied a penalized quasi-likelihood method to label the time-constant variables in the parametric part, and a separate generalized likelihood ratio test to label the time-varying variables in the nonparametric part. Zhang and Wu (2012) first utilized an information criterion to label zero and nonzero variables, and then further applied an integrated squared nonparametric test among the labeled nonzero variables to label time-constant variables. In a different study, Zhang (2015) used a penalized local linear method to initially label irrelevant variables, then applied an information criterion on the remaining variables to further label the time-varying ones.

Motivated by the discussion in Section 2.2, we propose a penalized local linear method that can simultaneously achieve successful tricategory labeling and semipara-

metric estimation in a single step. It is worth noting that we consider the multi-output time-varying coefficient model (1.4) that includes the univariate output model (1.3) as a special case by setting $d = 1$. Here, $(\boldsymbol{e}_{i,n})$ is a sequence of random vectors that form a triangular array of multivariate nonstationary processes and it can depend on $\boldsymbol{x}_{i,n}$ to accommodate heteroscedastic errors. Compared with the single-response setting (1.3), variable selection in the multi-output setting typically requires additional effort; see, for example, Turlach et al. (2005), Rothman et al. (2010), Chen and Huang (2012), and Lee and Liu (2012), among others. The aforementioned studies consider variable selection for multi-output regression models in the traditional setting when the regression coefficient is assumed to be a constant. Here, we consider the time-varying setting (1.4), in which the regression coefficient can change over time as a nonparametric function. In this case, one is interested in labeling not only relevant and irrelevant variables, but also time-varying and time-constant variables. Here, a variable is said to be time constant or irrelevant if its coefficient function is uniformly a constant or zero, respectively, for all outputs. This cannot be achieved by performing variable selection separately on each of the response variables.

2.2 Motivation

In this section, we present the motivation of our proposed method using the univariate output time-varying coefficient model (1.3). Intuitively, the coefficient function in model (1.3) can be estimated using the local constant estimator, which at each time point $t \in [0, 1]$ can be obtained by

$$\breve{\boldsymbol{\beta}}(t) = \underset{\boldsymbol{\eta}(t)\in\mathbb{R}^p}{\operatorname{argmin}} \sum_{i=1}^{n} \{y_{i,n} - \boldsymbol{x}_{i,n}^{\top}\boldsymbol{\eta}(t)\}^2 K\{(t_{i,n} - t)/b_n\}, \tag{2.2}$$

where $K(\cdot)$ is a kernel function and b_n is the bandwidth. For the kth component of $\boldsymbol{\eta}(\cdot)$, let $|\eta_k|_{[0,1]} = \{\int_0^1 |\eta_k(t)|^2 dt\}^{1/2}$ denote its norm. Wang and Xia (2009) proposed

the penalized local constant estimator that minimizes

$$\int_0^1 \sum_{i=1}^{n} \{y_{i,n} - \boldsymbol{x}_{i,n}^\top \boldsymbol{\eta}(t)\}^2 K\{(t_{i,n} - t)/b_n\} dt + \sum_{k=1}^{p} f_{\lambda_k}(|\eta_k|_{[0,1]}), \tag{2.3}$$

where $f_{\lambda_k}(\cdot)$ is the penalty function for the kth variable, with the tuning parameter λ_k that controls the degree of penalization. The penalized estimator in (2.3) can achieve parameter estimation and variable selection simultaneously where the associated coefficient function of irrelevant variables will be penalized to zero. However, for the time-varying coefficient model, it is often the case that one is interested in distinguishing not only relevant and irrelevant variables, but also time-varying and time-constant components; see, for example, Cai et al. (2000), Fan and Zhang (2000), Li and Liang (2008), Zhang and Wu (2012), and Zhang (2015), among others. This makes it a tricategory labeling problem, in which each variable is labeled as time-varying, time-constant, or irrelevant if the associated coefficient function is a time-varying nonparametric function, a uniform constant, or zero, respectively.

A natural approach to achieve this goal, proposed by Gao (2019) and Chan et al. (2021), is to consider a penalized local linear method. The local linear method proposed by (Fan and Gijbels, 1996) estimates not only the coefficient function, but also its derivative:

$$\{\tilde{\boldsymbol{\beta}}(t), \tilde{\boldsymbol{\beta}}'(t)\} = \underset{\boldsymbol{\eta}(t), \boldsymbol{\eta}'(t) \in \mathbb{R}^p}{\operatorname{argmin}} \sum_{i=1}^{n} \{y_{i,n} - \boldsymbol{x}_{i,n}^\top \boldsymbol{\eta}(t) - \boldsymbol{x}_{i,n}^\top \boldsymbol{\eta}'(t)(t_{i,n} - t)\}^2 K\{(t_{i,n} - t)/b_n\}.$$

Then, Gao (2019) and Chan et al. (2021) proposed the following penalized version that minimizes

$$\Omega_n(\{\boldsymbol{\eta}(t), \boldsymbol{\eta}'(t)\}_{t \in [0,1]}) + \sum_{k=1}^{p} f_{\lambda_k}(|\eta_k|_{[0,1]}) + \sum_{k=1}^{p} f_{\tau_k}(|\eta_k'|_{[0,1]}), \tag{2.4}$$

where

$$\Omega_n(\{\boldsymbol{\eta}(t), \boldsymbol{\eta}'(t)\}_{t\in[0,1]}) = \int_0^1 \sum_{i=1}^{n} \{y_{i,n} - \boldsymbol{x}_{i,n}^\top \boldsymbol{\eta}(t) - \boldsymbol{x}_{i,n}^\top \boldsymbol{\eta}'(t)(t_{i,n} - t)\}^2 K\{(t_{i,n} - t)/b_n\} dt.$$

The first penalty f_{λ_k} penalizes the coefficient estimator and shrinks it to zero for irrelevant variables, whereas the second penalty g_{τ_k} penalizes the derivative estimator and shrinks it to zero for time-constant variables. The penalized local linear estimator in (2.4) is intuitively straightforward in the sense that it exploits the derivative estimator from the local linear method and takes advantage of the mathematical connection between a function being constant and its derivative being zero. However, Zhang et al. (2023) argue that directly imposing a penalty on the derivative may not work as intended. Intuitively, it is because under the local linear estimation framework, the $\boldsymbol{x}_{i,n}(t_{i,n} - t)$ and $\boldsymbol{x}_{i,n}$ are treated as two separated explanatory variables, then the coefficient of $\boldsymbol{x}_{i,n}(t_{i,n} - t)$ (i.e. the derivative term) being zero does not necessarily guarantee the coefficient of $\boldsymbol{x}_{i,n}$ will be a constant for different time points. The Theorem 1 in Zhang et al. (2023) shows that imposing a direct penalization on the derivative may cause ambiguity at least when labeling time-varying and time-constant variables. Our goal is to propose a stratified fix that is able to automatically produce nonparametric coefficient estimators for time-varying variables, constant estimators for time-constant variables, and zero estimators for irrelevant variables, thus achieving the goals of tricategory labeling and semiparametric estimation at the same time.

2.3 Stratified Penalization: Single Explanatory Variable Case

In this section, we focus on the multi-output time-varying coefficient model with single explanatory variable, which is a special case of model (1.4) by setting $p = 1$. We state the proposed method in Section 2.3.1. In Section 2.3.2, we establish the

associated theoretical properties, including its estimation and labeling consistency for a general class of nonstationary processes. Section 2.7 provides technical proofs of the theorem in Section 2.3.2. The methodology and theoretical results are generalized to adapt the general multi-output time-varying coefficient model (1.4) in Section 2.4.

2.3.1 Methodology

Consider

$$\boldsymbol{y}_{i,n} = \boldsymbol{\beta}(t_{i,n})x_{i,n} + \boldsymbol{e}_{i,n}, \quad i = 1, \ldots, n, \tag{2.5}$$

where $\boldsymbol{y}_{i,n} \in \mathbb{R}^d$ is the multi-output response vector, $x_{i,n} \in \mathbb{R}$ is the explanatory variable, $\boldsymbol{\beta} : [0,1] \to \mathbb{R}^d$ is the coefficient function, and $(\boldsymbol{e}_{i,n})$ is a sequence of random vectors. Model (2.5) covers the aforementioned mean nonstationary model in Section 2.1. For each $j \in \{1, \ldots, d\}$, we define $a_{0,j}(t) = \beta_j(t)$, $a_{1,j}(t) = b_n\beta_j'(t)$ and denote $\boldsymbol{a}_{0,\cdot}(t) = \boldsymbol{\beta}(t)$, $\boldsymbol{a}_{1,\cdot}(t) = b_n\boldsymbol{\beta}'(t)$ and $\boldsymbol{a}_{\cdot,j}(t) = \{\beta_j(t), b_n\beta_j'(t)\}^\top$. Therefore, $A(t) = \{a_{l,j}(t)\}_{l,j}$ denotes a matrix whose the first row is $\boldsymbol{\beta}^\top(t)$ and second row is $b_n\boldsymbol{\beta}'^\top(t)$. Denote the norm of $A(t)$ as $|A|_{[0,1]} = \{\sum_{l,j}\int_0^1 a_{l,j}(t)^2dt\}^{1/2}$. We can write the multi-output kernel criterion function as

$$\Upsilon_n(\{A(t)\}_{t\in[0,1]}) = \int_0^1 \sum_{i=1}^n \left|\boldsymbol{y}_{i,n} - \boldsymbol{a}_{0,\cdot}(t)x_{i,n} - \boldsymbol{a}_{1,\cdot}(t)x_{i,n}\left(\frac{t_{i,n}-t}{b_n}\right)\right|^2 K\left(\frac{t_{i,n}-t}{b_n}\right)dt.$$

To construct appropriate penalty structures that can achieve simultaneous tricategory labeling and semiparametric estimation, instead of imposing penalties directly on the coefficient part and the derivative part, as in (2.4), we propose decomposing the norm of A according to the different strata implied by the nested tricategory labeling structure. In particular, the irrelevant label stratum has a projection of zero, the time-constant label stratum has a projection of $\bar{\boldsymbol{a}} = \int_0^1 \boldsymbol{a}_{0,\cdot}(t)dt$; and the time-varying label stratum has a projection of $\boldsymbol{a}_{0,\cdot}(t) - \bar{\boldsymbol{a}}$ together with $\boldsymbol{a}_{1,\cdot}(t)$. This motivates us to consider the stratified penalized local linear (SPLL) estimator $\hat{A}(t) = \{\hat{A}_{l,j}(t)\}_{l,j}$,

$t \in [0,1]$, that minimizes

$$\Upsilon_n(\{A(t)\}_{t\in[0,1]}) + f_{\lambda_{0,n}}(|\bar{\boldsymbol{a}}|) + g_{\tau_{0,n}}\left(\left[\int_0^1 \{|\boldsymbol{a}_{0,\cdot}(t) - \bar{\boldsymbol{a}}|^2 + |\boldsymbol{a}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right),$$

where $f_{\lambda_{0,n}}(\cdot)$ and $g_{\tau_{0,n}}(\cdot)$ are penalty functions with nonnegative tuning parameters $\lambda_{0,n}$ and $\tau_{0,n}$ respectively. The two terms $|\bar{\boldsymbol{a}}|$ and $\int_0^1 |\boldsymbol{a}_{0,\cdot}(t)-\bar{\boldsymbol{a}}|^2dt$ in the penalty have their own and separate goals. In particular, the term $\int_0^1 |\boldsymbol{a}_{0,\cdot}(t)-\bar{\boldsymbol{a}}|^2dt$ is mainly used to merge with the derivative $\int_0^1 |\boldsymbol{a}_{1,\cdot}(t)|^2dt$ as a group structure. This ensures that, for a time-constant variable, both will be penalized to zero, making the coefficient function $\boldsymbol{a}_{0,\cdot}(t) \equiv \bar{\boldsymbol{a}}$ be a constant and its derivative $\boldsymbol{a}_{1,\cdot}(t) \equiv 0$, thus solving the issue of the direct derivative penalization approach. The other additional term $|\bar{\boldsymbol{a}}|$ ensures that once the regression coefficient function is shrunken to its constant projection $\bar{\boldsymbol{a}}$, it can further penalize the coefficient to zero to correctly label irrelevant variables. We in the Section 2.3.2 establish the theoretical properties of the proposed method for a general class of nonstationary processes.

2.3.2 Theoretical Results

In (2.5), we allow nonstationary time series observations, which have been studied extensively in the literature; see, for example, Priestley (1965), Dahlhaus (1997), Cheng and Tong (1998), Mallat et al. (1998), Giurcanu and Spokoiny (2004), Ombao et al. (2005), Zhou and Wu (2010), and Zhang (2013), and the references therein. Let $(\boldsymbol{\epsilon}_i)$ be a sequence of i.i.d. innovations and denote its shift process by $\boldsymbol{\mathcal{F}}_i = (\ldots, \boldsymbol{\epsilon}_{i-1}, \boldsymbol{\epsilon}_i)$, we shall here follow Zhang (2015) and assume that

$$\max_{1\le i\le n} \|x_{i,n} - G(t_{i,n}, \boldsymbol{\mathcal{F}}_i)\| = O(n^{-1}), \quad \max_{1\le i\le n} \|\boldsymbol{e}_{i,n} - \boldsymbol{H}(t_{i,n}, \boldsymbol{\mathcal{F}}_i)\| = O(n^{-1}), \quad (2.6)$$

for some measurable functions G and $\boldsymbol{H}$ such that $(x_{i,n})$ and $(\boldsymbol{e}_{i,n})$ are proper sequences of random vectors with $E(\boldsymbol{e}_{i,n} \mid x_{i,n}) = \mathbf{0}$. Let $\boldsymbol{\epsilon}_0^\star$ be identically distributed as $\boldsymbol{\epsilon}_0$ but independent of the sequence $(\boldsymbol{\epsilon}_i)$, we can define the coupled shift process $\boldsymbol{\mathcal{F}}_i^\star = (\ldots, \boldsymbol{\epsilon}_{-1}, \boldsymbol{\epsilon}_0^\star, \boldsymbol{\epsilon}_1, \ldots, \boldsymbol{\epsilon}_i)$, then for any collection of processes $\{\boldsymbol{J}(t; \boldsymbol{\mathcal{F}}_i)\}_{i\in\mathbb{Z}}$ on $t \in [0,1]$, the functional dependence measure of Wu (2005) can be written as

$$\delta_{i,q}(\boldsymbol{J}) = \sup_{t\in[0,1]} \|\boldsymbol{J}(t; \boldsymbol{\mathcal{F}}_i) - \boldsymbol{J}(t; \boldsymbol{\mathcal{F}}_i^\star)\|_q, \quad \Delta_{0,q}(\boldsymbol{J}) = \sum_{i=0}^{\infty} \delta_{i,q}(\boldsymbol{J}),$$

where $\delta_{i,q}(\boldsymbol{J})$ measures the dependence of $\boldsymbol{J}(t; \boldsymbol{\mathcal{F}}_i)$ on the innovation $\boldsymbol{\epsilon}_0$, and its cumulative effect is measured by $\Delta_{0,q}(\boldsymbol{J})$. Let $\boldsymbol{L} = G \times \boldsymbol{H}^\top$, we assume the following conditions.

(A1) The coefficient function $\boldsymbol{\beta} \in \mathcal{C}^3[0,1]$, the class of three times continuously differentiable functions on $[0,1]$;

(A2) The underlying process satisfies $\Delta_{0,4}(G) + \Delta_{0,2}(\boldsymbol{L}) < \infty$;

(A3) There exists a constant $0 < C < \infty$ such that

$$\|G(t_1, \boldsymbol{\mathcal{F}}_i) - G(t_2, \boldsymbol{\mathcal{F}}_i)\| + \|\boldsymbol{L}(t_1, \boldsymbol{\mathcal{F}}_i) - \boldsymbol{L}(t_2, \boldsymbol{\mathcal{F}}_i)\| \leq C|t_1 - t_2|$$

holds uniformly for all $t_1, t_2 \in [0,1]$;

(A4) $\mathrm{E}\{G(t, \boldsymbol{\mathcal{F}}_i)^2\}$ is bounded away from zero on $[0,1]$.

Condition (A1) is a smoothness condition on the regression coefficient function, which is a common assumption for nonparametric kernel estimation; see, for example, Fan and Gijbels (1996). Condition (A2) is a short-range dependence condition quantified by the functional dependence measure of Wu (2005); see also Zhou and Wu (2010) and Zhang and Wu (2012). Condition (A3) is a stochastic Lipschitz continuity condition,

under which the underlying process can be locally approximated by a stationary process within a small window; see, for example, the discussion in Zhang and Wu (2011). Condition (A4) is a regularity condition that prevents singularity when performing local kernel estimation. Throughout this dissertation, we assume that the kernel function $K \in \mathcal{K}$, the collection of symmetric functions in $\mathcal{C}^1[-1,1]$, with $\int_{-1}^{1} K(v)dv = 1$. Examples include the Epanechnikov kernel $K(v) = 0.75\max(1-v^2, 0)$, the Bartlett kernel $K(v) = \max(1-|v|, 0)$, and the rectangle kernel $K(v) = 0.5\mathrm{I}(|v| \leq 1)$, with $\mathrm{I}(\cdot)$ being the indicator function, among many others. We also assume the following conditions on the penalty functions.

(P1) $f_\lambda(0) = 0$ and $g_\tau(0) = 0$.

(P2) $\lambda^{-1}\sup_{x\in\mathbb{R}}|f'_\lambda(x)| < \infty$ and $\tau^{-1}\sup_{x\in\mathbb{R}}|g'_\tau(x)| < \infty$.

(P3) $\lambda^{-1}\liminf_{x\to 0+}|f'_\lambda(x)| > 0$ and $\tau^{-1}\liminf_{x\to 0+}|g'_\tau(x)| > 0$.

Conditions (P1)–(P3) are natural requirements for good penalty functions (Fan and Li, 2001), and are satisfied by many popular choices, such as the LASSO penalty of Tibshirani (1996), the hard thresholding penalty of Antoniadis (1997), and the SCAD penalty of Fan and Li (2001). Let $\mathcal{D}_v$, $\mathcal{D}_c$, and $\mathcal{D}_0$ denote subsets of time-varying, time-constant, and irrelevant variables, respectively. Theorems 1 and 2 provide the estimation consistency and labeling consistency for the proposed SPLL estimator.

Theorem 1. *Assume (A1)–(A4), (P1), (P2),* $b_n \to 0$ *and* $nb_n \to \infty$. *If* $\{(nb_n)^{-1/2} + b_n^2\}(\lambda_{0,n} + \tau_{0,n}) = O(1)$, *then the norm*

$$|\hat{A} - A|_{[0,1]} = O_p\{(nb_n)^{-1/2} + b_n^2\}.$$

Theorem 2. *Assume (A1)–(A4), (P1)–(P3),* $b_n \to 0$ *and* $nb_n^2 \to \infty$. *If* $\{(nb_n)^{-1/2} + b_n^2\}(\lambda_{0,n} + \tau_{0,n}) = O(1)$, $\lambda_{0,n}/\{(nb_n)^{1/2} + nb_n^3\} \to \infty$ *and* $\tau_{0,n}/\{(nb_n)^{1/2} + nb_n^3\} \to \infty$.

Then if $\boldsymbol{\beta} \in D_0 \cup D_c$*, we have*

$$\text{pr}\left\{\sup_{t\in[0,1]}\left|\hat{\boldsymbol{a}}_{0,\cdot}(t)-\int_0^1\hat{\boldsymbol{a}}_{0,\cdot}(s)ds\right|=0 \text{ and } \sup_{t\in[0,1]}|\hat{\boldsymbol{a}}_{1,\cdot}(t)|=0\right\}\to 1,$$

and further if $\boldsymbol{\beta} \in D_0$*, we have*

$$\text{pr}\left\{\sup_{t\in[0,1]}|\hat{\boldsymbol{a}}_{0,\cdot}(t)|=0 \text{ and } \sup_{t\in[0,1]}|\hat{\boldsymbol{a}}_{1,\cdot}(t)|=0\right\}\to 1.$$

By Theorem 2, for time-constant or irrelevant variables, the SPLL estimator proposed in Section 2.3.1 automatically produces a constant coefficient function with a zero derivative estimator. In addition, for irrelevant variables nested within $\mathcal{D}_0 \cup \mathcal{D}_c$, the coefficient function is regularized to zero uniformly over time. Therefore, it achieves the simultaneous tricategory variable labeling and semiparametric estimation in a single step.

2.4 Stratified Penalization: The General Case

In Section 2.3, we explore the specific scenario of (1.4) where there is only one explanatory variable by setting $p = 1$. We establish the SPLL method and its associated theoretical properties. However, our interest always lies in the regression problem involving more than one explanatory variables. This section demonstrates the applicability of our method to the broader multi-output time-varying coefficient model (1.4) for any fixed $p \geq 1$. In Section 2.4.1, we will extend the SPLL method introduced in Section 2.3.1. Recall that $\mathcal{D}_v$, $\mathcal{D}_c$, and $\mathcal{D}_0$ denote subsets of time-varying, time-constant, and irrelevant variables, respectively. Here, rather than labeling the single explanatory variable in (2.5) into $\mathcal{D}_v$, $\mathcal{D}_c$, or $\mathcal{D}_0$, we address the general (1.4) and partition the set of explanatory variables into the above three subsets, that in a nested structure. A variable is considered time-constant or irrelevant if its coefficient

function is consistently a constant or zero, respectively, for all outputs. The theoretical properties are established in Section 2.4.2, with the proof deferred to Section 2.7.

2.4.1 Methodology

Recall that the multi-output time-varying coefficient model (1.4) is given by

$$\boldsymbol{y}_{i,n} = B(t_{i,n})^\top \boldsymbol{x}_{i,n} + \boldsymbol{e}_{i,n}, \quad i = 1, \ldots, n.$$

In comparison to model (2.5) that only permits single explanatory variables $x_{i,n}$, we have $\boldsymbol{x}_{i,n} \in \mathbb{R}^p$ representing the set of explanatory variables in (1.4) for any fixed $p \geq 1$. Moreover, $B : [0,1] \to \mathbb{R}^{p \times d}$ denotes the coefficient function matrix, where each row k corresponds to the coefficient function vector for the kth variable. Let $\Theta(t) = \{\theta_{l,k,j}(t)\}_{l,k,j}$ be a three-way tensor function. Here, we define $\Theta_{0,\cdot,\cdot}(t) = \{\theta_{0,k,j}(t)\}_{k,j} = B(t)$ and $\Theta_{1,\cdot,\cdot}(t) = \{\theta_{1,k,j}(t)\}_{k,j} = b_n B'(t)$. The norm of $\Theta(t)$ is denoted as $|\Theta|_{[0,1]}$ and is defined as $\{\sum_{l,k,j} \int_0^1 \theta_{l,k,j}(t)^2 dt\}^{1/2}$. Based on this, we can express the multi-output kernel criterion function as follows:

$$\begin{aligned} \Upsilon_n(\{\Theta(t)\}_{t\in[0,1]}) \;=\; & \int_0^1 \sum_{i=1}^n \left| \boldsymbol{y}_{i,n} - \Theta_{0,\cdot,\cdot}(t)^\top \boldsymbol{x}_{i,n} - \Theta_{1,\cdot,\cdot}(t)^\top \boldsymbol{x}_{i,n} \left(\frac{t_{i,n} - t}{b_n} \right) \right|^2 \\ & \times K\left(\frac{t_{i,n} - t}{b_n} \right) dt. \end{aligned}$$

Following the same strategy in Section 2.3.1, we propose decomposing the norm of Θ according to the different strata implied by the nested tricategory labeling structure. The irrelevant label stratum has a projection of zero, the time-constant label stratum has a projection of $\bar{\boldsymbol{\theta}}_{k,\cdot} = \int_0^1 \boldsymbol{\theta}_{0,k,\cdot}(t) dt$, where $\boldsymbol{\theta}_{0,k,\cdot}(t) = \{\theta_{0,k,1}(t), \ldots, \theta_{0,k,d}(t)\}^\top$, and the time-varying label stratum has a projection of $\boldsymbol{\theta}_{0,k,\cdot}(t) - \bar{\boldsymbol{\theta}}_{k,\cdot}$, together with $\boldsymbol{\theta}_{1,k,\cdot}(t) = \{\theta_{1,k,1}(t), \ldots, \theta_{1,k,d}(t)\}^\top$. The SPLL estimator $\hat{\Theta}(t) = \{\hat{\theta}_{l,k,j}(t)\}_{l,k,j}$, for

$t \in [0,1]$, minimizes

$$\Upsilon_n(\{\Theta(t)\}_{t\in[0,1]}) + \sum_{k=1}^{p} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}|) + \sum_{k=1}^{p} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}_{0,k,\cdot}(t) - \bar{\boldsymbol{\theta}}_{k,\cdot}|^2 + |\boldsymbol{\theta}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right),$$

where $f_{\lambda_{k,n}}(\cdot)$ and $g_{\tau_{k,n}}(\cdot)$ are penalty functions with nonnegative tuning parameters $\lambda_{k,n}$ and $\tau_{k,n}$, respectively. We establish its theoretical properties, including the estimation and labeling consistency, in Section 2.4.2 for a general class of nonstationary processes, and our results are directly applicable to vector time-varying autoregressive models. An iterative algorithm that can be used to compute the SPLL estimator was described in Zhang et al. (2023).

2.4.2 Theoretical Results

Similar to (2.6) in Section 2.3.2, to adjust the general case (1.4), we follow Zhang (2015) and assume that

$$\max_{1\le i\le n} \|\boldsymbol{x}_{i,n} - \boldsymbol{G}(t_{i,n}, \boldsymbol{\mathcal{F}}_i)\| = O(n^{-1}), \quad \max_{1\le i\le n} \|\boldsymbol{e}_{i,n} - \boldsymbol{H}(t_{i,n}, \boldsymbol{\mathcal{F}}_i)\| = O(n^{-1}), \quad (2.7)$$

for some measurable functions $\boldsymbol{G}$ and $\boldsymbol{H}$, such that $(\boldsymbol{x}_{i,n})$ and $(\boldsymbol{e}_{i,n})$ are proper sequences of random vectors with $E(\boldsymbol{e}_{i,n} \mid \boldsymbol{x}_{i,n}) = \boldsymbol{0}$. Assumption (2.7) will reduce to (2.6) if $p = 1$. Compared with the exact representation in Wu (2005), the approximate framework in (2.7) allows the popular time-varying autoregressive model and covers a wide range of linear and nonlinear processes; see also the discussion in Zhang and Wu (2012). Let $\boldsymbol{L} = \boldsymbol{G} \times \boldsymbol{H}^\top$. Corresponding to conditions (A1)–(A4), we assume the following conditions.

(B1) The coefficient matrix function $B \in \mathcal{C}^3[0,1]$, the class of three times continuously differentiable matrix-valued functions on $[0,1]$.

(B2) The underlying process satisfies $\Delta_{0,4}(\boldsymbol{G}) + \Delta_{0,2}(\boldsymbol{L}) < \infty$.

(B3) There exists a constant $0 < C < \infty$ such that

$$\|\boldsymbol{G}(t_1, \boldsymbol{\mathcal{F}}_i) - \boldsymbol{G}(t_2, \boldsymbol{\mathcal{F}}_i)\| + \|\boldsymbol{L}(t_1, \boldsymbol{\mathcal{F}}_i) - \boldsymbol{L}(t_2, \boldsymbol{\mathcal{F}}_i)\| \leq C|t_1 - t_2|$$

holds uniformly for all $t_1, t_2 \in [0, 1]$.

(B4) The smallest eigenvalue of $E\{\boldsymbol{G}(t, \boldsymbol{\mathcal{F}}_i)\boldsymbol{G}(t, \boldsymbol{\mathcal{F}}_i)^\top\}$ is bounded away from zero on $[0, 1]$.

Recall that $\mathcal{D}_v$, $\mathcal{D}_c$, and $\mathcal{D}_0$ denote subsets of time-varying, time-constant, and irrelevant variables, respectively, and we further write $\mathcal{D}_v = \mathcal{D}_{v0} \cup \mathcal{D}_{v1}$, where $\mathcal{D}_{v0}$ is the set of time-varying variables with $|\bar{\boldsymbol{\theta}}_{k,\cdot}| = 0$ and $\mathcal{D}_{v1} = \mathcal{D}_v \setminus \mathcal{D}_{v0}$. Theorems 3 and 4 provide the estimation consistency and labeling consistency for the proposed SPLL estimator. The proof will be presented in Section 2.7.

Theorem 3. *Assume (B1)–(B4), (P1), (P2),* $b_n \to 0$, *and* $nb_n \to \infty$. *If* $\{(nb_n)^{-1/2} + b_n^2\}(\max_{k \in \mathcal{D}_c \cup \mathcal{D}_{v1}} \lambda_{k,n} + \max_{k \in \mathcal{D}_v} \tau_{k,n}) = O(1)$, *then the norm*

$$|\hat{\Theta} - \Theta|_{[0,1]} = O_p\{(nb_n)^{-1/2} + b_n^2\}.$$

Theorem 4. *Assume (B1)–(B4), (P1)–(P3),* $b_n \to 0$, *and* $nb_n^2 \to \infty$. *If* $\{(nb_n)^{-1/2} + b_n^2\}(\max_{k \in \mathcal{D}_c \cup \mathcal{D}_{v1}} \lambda_{k,n} + \max_{k \in \mathcal{D}_v} \tau_{k,n}) = O(1)$, $\min_{k \in \mathcal{D}_0} \lambda_{k,n} / \{(nb_n)^{1/2} + nb_n^3\} \to \infty$, *and* $\min_{k \in \mathcal{D}_v^c} \tau_{k,n} / \{(nb_n)^{1/2} + nb_n^3\} \to \infty$, *then*

$$\text{pr}\left\{ \max_{k \in \mathcal{D}_0 \cup \mathcal{D}_c} \sup_{t \in [0,1]} \left| \hat{\boldsymbol{\theta}}_{0,k,\cdot}(t) - \int_0^1 \hat{\boldsymbol{\theta}}_{0,k,\cdot}(s) ds \right| = 0 \text{ and } \max_{k \in \mathcal{D}_0 \cup \mathcal{D}_c} \sup_{t \in [0,1]} |\hat{\boldsymbol{\theta}}_{1,k,\cdot}(t)| = 0 \right\} \to 1,$$

and

$$\text{pr}\left\{ \max_{k \in \mathcal{D}_0} \sup_{t \in [0,1]} |\hat{\boldsymbol{\theta}}_{0,k,\cdot}(t)| = 0 \text{ and } \max_{k \in \mathcal{D}_0} \sup_{t \in [0,1]} |\hat{\boldsymbol{\theta}}_{1,k,\cdot}(t)| = 0 \right\} \to 1.$$

Similar to the discussion in Section 2.3.2, Theorem 4 provides theoretical justification that the proposed SPLL method achieves the simultaneous tricategory variable

labeling and semiparametric estimation of multi-output coefficient model (1.4), without having to decompose the problem into two bicategory labeling subproblems, as in Li and Liang (2008) and Zhang and Wu (2012). Note that in Li and Liang (2008), prior knowledge is assumed on the partition between the time-varying and time-constant variables, and therefore their labeling problem is supervised. This study concerns the unsupervised setting, where we do not assume we have this prior knowledge. In the following section, we describe the implementation of the proposed SPLL method, and examine its finite-sample performance using a Monte Carlo simulation study and a real-data analysis.

2.5 Implementation

In this section, we present the implementation of the proposed method in Section 2.4, which includes the single explanatory variable case in Section 2.3 as a special case. We propose a data-driven tuning parameter selection method, conduct a Monte Carlo simulation study and present a real-data analysis.

2.5.1 Tuning Parameter Selection

Implementing the proposed SPLL method requires two sets of tuning parameters. The first ($\tau_{k,n}$) controls the degree of regularization from the time-varying stratum to the time-constant stratum, and the second ($\lambda_{k,n}$) controls the degree of regularization from the time-constant stratum to the irrelevant label stratum. Here, we adopt the idea of the adaptive LASSO (Zou, 2006), and set

$$\begin{aligned}
\lambda_{k,n} &= \lambda_n \cdot \left| \int_0^1 \tilde{\boldsymbol{\theta}}_{0,k,\cdot}(t)dt \right|^{-1}, \\
\tau_{k,n} &= \tau_n \cdot \left[\int_0^1 \left\{ \left| \tilde{\boldsymbol{\theta}}_{0,k,\cdot}(t) - \int_0^1 \tilde{\boldsymbol{\theta}}_{0,k,\cdot}(s)ds \right|^2 + |\tilde{\boldsymbol{\theta}}_{1,k,\cdot}(t)|^2 \right\} dt \right]^{-1/2},
\end{aligned} \tag{2.8}$$

for some tuning parameters λ_n and τ_n that do not depend on k, where $\tilde{\Theta}(t) = \{\tilde{\theta}_{l,k,j}(t)\}_{l,k,j}$, for $t \in [0,1]$, can be taken as the unpenalized local linear estimator. Note that the norm $[\int_0^1 \{|\tilde{\boldsymbol{\theta}}_{0,k,\cdot}(t) - \int_0^1 \tilde{\boldsymbol{\theta}}_{0,k,\cdot}(s)ds|^2 + |\tilde{\boldsymbol{\theta}}_{1,k,\cdot}(t)|^2\}dt]^{1/2}$ is relatively small for time-constant variables and relatively large for time-varying variables. Thus, the choice in (2.8) can lead to adaptive tuning for different variables. When the bandwidth $b_n = cn^{-1/5}$, for some $0 < c < \infty$, has the asymptotic mean squared error optimal rate, following a similar discussion to that in Zhang (2015), one can simply use the asymptotic choice $\lambda_n = n^{1/5}$ and $\tau_n = n^{1/5}$. For any set $\mathcal{D}$, we use $|\mathcal{D}|$ to denote its cardinality. To provide a finite-sample data-driven choice of the pair (λ_n, τ_n), we consider a natural extension of the information criterion used in Zhang (2015) to the current setting, and minimize

$$\mathrm{EIC}(\lambda,\tau) = \log\{\Upsilon_n(\{\hat{\Theta}(t;\lambda,\tau)\}_{t\in[0,1]})\} + \frac{\log n}{nb_n}\cdot|\hat{\mathcal{D}}_0^c(\lambda,\tau)| + \frac{\log n}{nb_n}\cdot|\hat{\mathcal{D}}_v(\lambda,\tau)|,$$

which can also be viewed as a semiparametric extension of the traditional BIC. The simulation study in Section 2.5.2 shows that this data-driven tuning selector performs reasonably well for the current tricategory variable labeling and semiparametric estimation.

2.5.2 Simulation Results

We conduct Monte Carlo simulations to examine the finite-sample performance of the proposed SPLL method. For this, let $\boldsymbol{\epsilon}_k = (\epsilon_{k,1}, \ldots, \epsilon_{k,p-1})^\top \in \mathbb{R}^{p-1}$, for $k \in \mathbb{Z}$, be independent innovation vectors with independent Rademacher components, and let $P_j(t)$ be the jth order Legendre polynomial. Let $\boldsymbol{M}^\diamond = (0.2^{|i-j|})_{1\le i,j\le p-1}$ and $\boldsymbol{P}(t) \in \mathbb{R}^{(p-1)\times(p-1)}$ be a diagonal matrix with the jth diagonal element $P_j(2t-1)/4$. Then, the vector $\boldsymbol{\xi}_k = \boldsymbol{M}^\diamond\boldsymbol{\epsilon}_k$ has dependent components, and we form $\boldsymbol{x}_{i,n} = \sum_{j=0}^{\infty}\boldsymbol{P}(i/n)^j\boldsymbol{\xi}_{i-j}$, which is a nonstationary process, owing to the coefficients being

time varying. Let $\varepsilon_{k,l}$, for $k \in \mathbb{Z}$, $l \in \{1, \ldots, d\}$, be an array of independent standard normal random variables that is also independent of the process $(\boldsymbol{\epsilon}_k)$. We then form the nonstationary nonlinear process $\boldsymbol{\zeta}_{i,n} = (\zeta_{i,1,n}, \ldots, \zeta_{i,d,n})^\top$, with $\zeta_{i,l,n} = \varepsilon_{i,l} + 2(i/n - 0.5)^2\{|\varepsilon_{i-1,l}| - (2/\pi)^{1/2}\} + \sum_{j=1}^{\infty} j^{-2}\varepsilon_{i-j,l}$. We consider the following multi-output time-varying coefficient model with heteroscedastic errors:

$$\boldsymbol{y}_{i,n} = \boldsymbol{\beta}_0(t_{i,n}) + \sum_{k=1}^{p-1} \boldsymbol{\beta}_k(t_{i,n}) x_{i,k,n} + 0.5\sigma(x_{i,2,n}^2 + x_{i,3,n}^2)^{1/2}\boldsymbol{\zeta}_{i,n}, \quad i = 1, \ldots, n.$$

Let $n = 500$. We consider the following configurations on the variable labeling, where we use *a*-*b*-*c* to indicate a configuration with a time-varying variables, b nonzero time-constant variables, and c zero variables.

(2-2-16) Two time-varying variables: $\boldsymbol{\beta}_0(t) = \{3(2t-1)^2, 2(2t-1)^3\}^\top$ and $\boldsymbol{\beta}_2(t) = \{2\sin(2\pi t - 1), 2\cos(2\pi t + 1)\}^\top$; two time-constant variables: $\boldsymbol{\beta}_1(t) = (-1, \pi/3)^\top$ and $\boldsymbol{\beta}_3(t) = (1.5, -2^{1/2})^\top$; and 16 zero variables $\boldsymbol{\beta}_4(t) = \cdots = \boldsymbol{\beta}_{19}(t) = (0,0)^\top$.

(5-5-10) Five time-varying variables: $\boldsymbol{\beta}_0(t) = \{3(2t-1)^2, 2(2t-1)^3\}^\top$, $\boldsymbol{\beta}_2(t) = \{2(2t-1), 2\cos(2\pi t - 1)\}^\top$, $\boldsymbol{\beta}_4(t) = \{2\cos(2\pi t), 2(2t-1)^2\}^\top$, $\boldsymbol{\beta}_6(t) = \{\cos(\pi t) + 1, 2\sin\{\exp(\pi t - 2)\}\}^\top$, and $\boldsymbol{\beta}_8(t) = [\exp(-2t+1), 2\{\sin(-2\pi t + 1)\}^3]^\top$; five time-constant variables: $\boldsymbol{\beta}_1(t) = (2, -1.5)^\top$, $\boldsymbol{\beta}_3(t) = (1.5, -\pi/3)^\top$, $\boldsymbol{\beta}_5(t) = (\pi/2, \pi^{1/2})^\top$, $\boldsymbol{\beta}_7(t) = (-3^{1/2}, 2^{1/2})^\top$, and $\boldsymbol{\beta}_9(t) = (-e^{1/2}, 3/\pi)^\top$; and 10 zero variables $\boldsymbol{\beta}_{10}(t) = \cdots = \boldsymbol{\beta}_{19}(t) = (0,0)^\top$.

(2-8-10) Two time-varying variables: $\boldsymbol{\beta}_0(t) = \{3(2t-1)^2, 2(2t-1)^3\}^\top$ and $\boldsymbol{\beta}_2(t) = \{2\sin(2\pi t), 2\cos(2\pi t)\}^\top$; eight time-constant variables: $\boldsymbol{\beta}_1(t) = (1, 5^{1/2}/2)^\top$, $\boldsymbol{\beta}_3(t) = (\pi/2, -1.3)^\top$, $\boldsymbol{\beta}_4(t) = (e^{1/2}, -1.5)^\top$, $\boldsymbol{\beta}_5(t) = (1.5, 4/\pi)^\top$, $\boldsymbol{\beta}_6(t) = (1.2, 3^{1/2})^\top$, $\boldsymbol{\beta}_7(t) = (0.8, 7^{1/3})^\top$, $\boldsymbol{\beta}_8(t) = (-2^{1/2}, 1)^\top$, and $\boldsymbol{\beta}_9(t) = (-5/\pi, \pi/3)^\top$; and 10 zero variables $\boldsymbol{\beta}_{10}(t) = \cdots = \boldsymbol{\beta}_{19}(t) = (0,0)^\top$.

The three configurations above represent cases with a small number of nonzero variables, a balanced number of time-varying and time-constant variables, and an unbalanced number of time-varying and time-constant variables, respectively. For each configuration, we consider two noise levels $\sigma \in \{1, 2\}$, and apply the proposed SPLL method for the semiparametric variable labeling and estimation. We also make a comparison with the two-step procedure of Zhang (2015), denoted by Zhang15, and the direct derivative penalization method of Gao (2019), denoted by Gao19. Throughout our numerical experiments, we use the LASSO penalty function. Note that a case is considered to be under-labeled if at least one component of one variable is mislabeled from time varying to time constant, from time constant to zero, or from time varying to zero. On the other hand, a case is considered to be over-labeled if there is no under-labeling, and at least one variable is mislabeled from zero to time constant, from time constant to time varying, or from zero to time varying. We also report the labeling consistency ratio (LCR), defined as the proportion of correctly labeled variables, along with the mean squared error (MSE) of the associated semiparametric estimates. Overall, we can observe our proposed SPLL method performs reasonably well, because for most of the cases considered, it produces the highest proportion of correctly labeled models, lowest MSE and highest LCR, especially for the challenging cases when $\sigma = 2$. Its performance is also reasonably robust to different choices of the bandwidth. More detailed simulation analysis can be found in Zhang et al. (2023).

2.5.3 Data Analysis

In this section, we apply the results to study the influence of El Niño-Southern Oscillation, characterized by the Southern Oscillation Index (SOI), on temperature anomalies, which is an important problem in climate science, and has been studied by Privalsky and Jensen (1995), Zheng and Basher (1999), Gao and Hawthorne (2006), McLean (2014), and Zhang (2015), among others. In this data analysis, we focus

Table 2.1: Simulation results for $\sigma = 1$, based on 100 realizations for each configuration.

Model	b_n	Method	Under-label	Correct	Over-label	MSE	LCR
2-2-16	0.1	SPLL	0.00	1.00	0.00	0.0093	1.0000
		Zhang15	0.04	0.93	0.03	0.0327	0.9920
		Gao19	0.00	0.72	0.28	0.0138	0.9835
	0.2	SPLL	0.00	1.00	0.00	0.0110	1.0000
		Zhang15	0.05	0.92	0.03	0.0311	0.9920
		Gao19	0.00	0.29	0.71	0.0106	0.9295
	0.3	SPLL	0.00	1.00	0.00	0.0232	1.0000
		Zhang15	0.06	0.91	0.03	0.0386	0.9920
		Gao19	0.00	0.15	0.85	0.0213	0.8820
5-5-10	0.1	SPLL	0.00	1.00	0.00	0.0197	1.0000
		Zhang15	0.04	0.94	0.02	0.0970	0.9805
		Gao19	0.93	0.03	0.04	0.1734	0.9315
	0.2	SPLL	0.00	1.00	0.00	0.0219	1.0000
		Zhang15	0.04	0.93	0.03	0.0989	0.9820
		Gao19	0.00	0.31	0.69	0.0229	0.9290
	0.3	SPLL	0.00	1.00	0.00	0.0376	1.0000
		Zhang15	0.04	0.87	0.09	0.1117	0.9800
		Gao19	0.00	0.17	0.83	0.0384	0.8870
2-8-10	0.1	SPLL	0.00	0.99	0.01	0.0110	0.9995
		Zhang15	0.04	0.94	0.02	0.0857	0.9825
		Gao19	0.00	0.80	0.20	0.0159	0.9880
	0.2	SPLL	0.00	1.00	0.00	0.0127	1.0000
		Zhang15	0.05	0.92	0.03	0.0841	0.9815
		Gao19	0.00	0.47	0.53	0.0127	0.9520
	0.3	SPLL	0.00	1.00	0.00	0.0244	1.0000
		Zhang15	0.05	0.91	0.04	0.0919	0.9810
		Gao19	0.00	0.22	0.78	0.0235	0.9145

Table 2.2: Simulation results for $\sigma = 2$, based on 100 realizations for each configuration.

Model	b_n	Method	Under-label	Correct	Over-label	MSE	LCR
2-2-16	0.1	SPLL	0.00	0.99	0.01	0.0335	0.9995
		Zhang15	0.09	0.66	0.25	0.0765	0.9730
		Gao19	0.89	0.06	0.05	0.1931	0.9355
	0.2	SPLL	0.00	1.00	0.00	0.0283	1.0000
		Zhang15	0.10	0.81	0.09	0.0652	0.9810
		Gao19	0.00	0.26	0.74	0.0366	0.9085
	0.3	SPLL	0.00	1.00	0.00	0.0375	1.0000
		Zhang15	0.10	0.75	0.15	0.0702	0.9785
		Gao19	0.00	0.09	0.91	0.0414	0.8425
5-5-10	0.1	SPLL	0.04	0.94	0.02	0.0795	0.9970
		Zhang15	0.46	0.44	0.10	0.2510	0.9375
		Gao19	1.00	0.00	0.00	0.4982	0.7855
	0.2	SPLL	0.00	0.96	0.04	0.0610	0.9980
		Zhang15	0.12	0.79	0.09	0.1926	0.9635
		Gao19	0.61	0.14	0.25	0.1504	0.9385
	0.3	SPLL	0.00	0.98	0.02	0.0714	0.9990
		Zhang15	0.12	0.74	0.14	0.1857	0.9650
		Gao19	0.02	0.14	0.84	0.0848	0.8795
2-8-10	0.1	SPLL	0.00	0.98	0.02	0.0403	0.9990
		Zhang15	0.10	0.71	0.19	0.1682	0.9595
		Gao19	0.76	0.11	0.13	0.2111	0.9420
	0.2	SPLL	0.00	1.00	0.00	0.0352	1.0000
		Zhang15	0.12	0.81	0.07	0.1570	0.9650
		Gao19	0.00	0.41	0.59	0.0450	0.9405
	0.3	SPLL	0.00	1.00	0.00	0.0443	1.0000
		Zhang15	0.13	0.79	0.08	0.1593	0.9665
		Gao19	0.00	0.18	0.82	0.0501	0.9020

on determining what lags of the SOI should be used, and whether they should be treated as time-varying explanatory variables. For this, we consider the multi-output time-varying coefficient model

$$\boldsymbol{y}_i = \boldsymbol{\beta}_0(t_{i,n}) + \sum_{j=1}^{25} \boldsymbol{\beta}_j(t_{i,n}) x_{i,j-13} + \boldsymbol{e}_i, \quad i = 1, \ldots, n, \tag{2.9}$$

where $\boldsymbol{y}_i = (y_{i,1}, y_{i,2})^\top$ are temperature anomalies from the northern and southern hemispheres, and x_k is the k-month-ahead SOI for $k \in \{-12, \ldots, 12\}$. For comparison with existing results, we use monthly data from 01/1936 to 12/2019, which can be downloaded from the Climatic Research Unit website [1]. In this case, the sample size $n = 984$, and time series plots are provided in Figure 2·1. We then apply the SPLL method proposed in Section 2.4.1 for the semiparametric variable labeling and estimation of (2.9), where the tuning parameters are selected by using the extended information criterion described in Section 2.5.1. For the bandwidth, we use a two-step selection procedure. We first use the asymptotic bandwidth $b_n^\circ = n^{-1/5}$ to obtain an initial labeling. Then, we apply the dependence-adjusted generalized cross-validation described in Section 4.3 of Zhang and Wu (2012) to the selected nonzero variables to obtain a data-driven bandwidth; see also Section 4.1 of Zhang (2015). Our analysis results show that, among the 25 lags considered, $x_{i,-2}$ and $x_{i,0}$ are labeled as time-constant variables, and all other lags are labeled as zero variables. In addition, the intercept is labeled as a time-varying variable, suggesting the following semiparametric multi-output model:

$$\boldsymbol{y}_i = \boldsymbol{\beta}_0(t_{i,n}) + \boldsymbol{\beta}_{11} x_{i,-2} + \boldsymbol{\beta}_{13} x_{i,0} + \boldsymbol{e}_i, \quad i = 1, \ldots, n,$$

where the estimated time-varying coefficients $\hat{\boldsymbol{\beta}}_0(\cdot) = \{\hat{\beta}_{0,1}(\cdot), \hat{\beta}_{0,2}(\cdot)\}^\top$ are plotted in Figure 2·2 for the northern and southern hemispheres, and the estimated time-

[1] https://crudata.uea.ac.uk/cru/data/temperature/

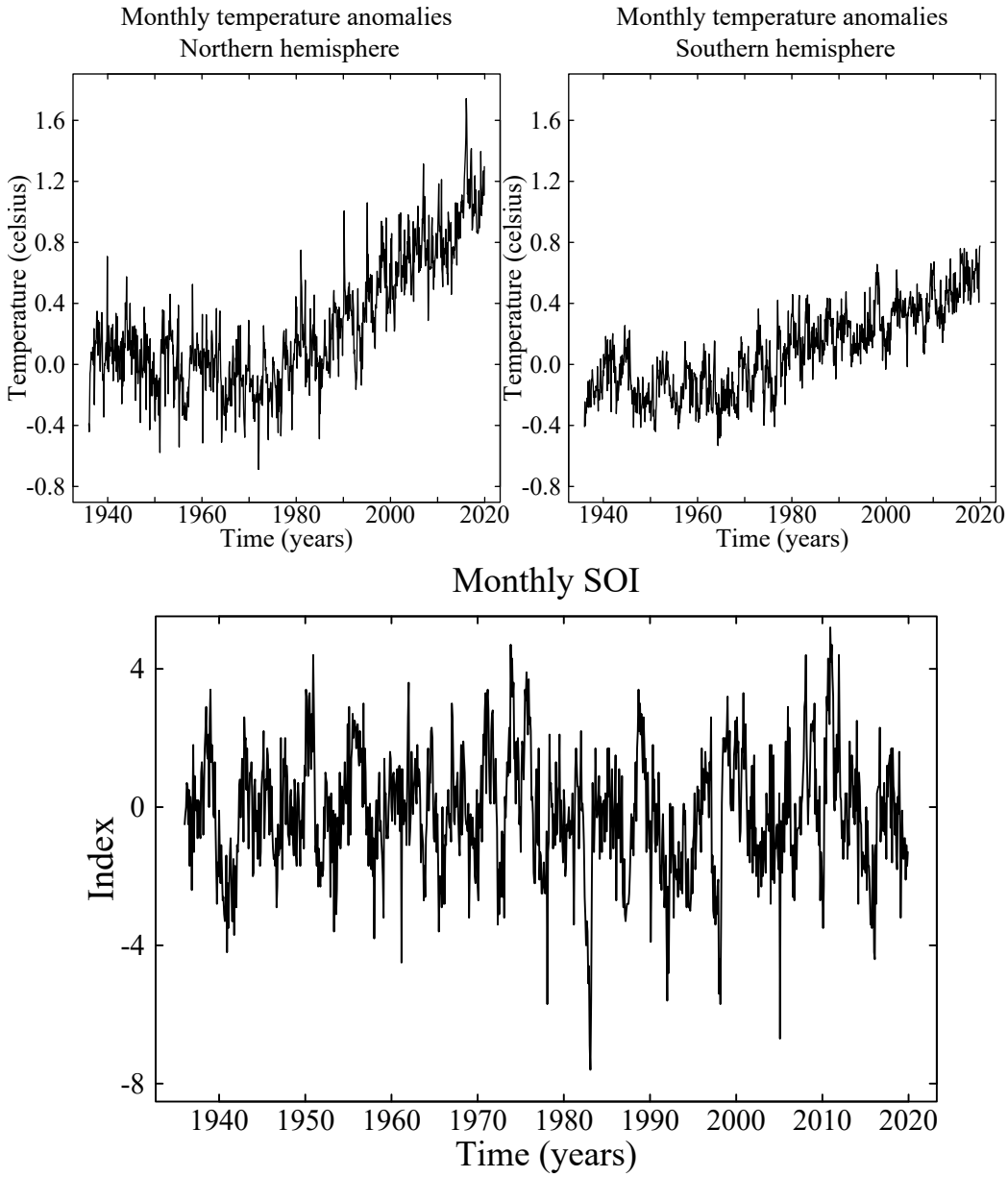

Figure 2·1: Time series plots for monthly temperature anomalies from the northern hemisphere (top left), monthly temperature anomalies from the southern hemisphere (top right), and the monthly SOI (bottom) during the period 01/1936–12/2019.

constant coefficients are $\hat{\boldsymbol{\beta}}_{11} = (-0.011, -0.015)^\top$ and $\hat{\boldsymbol{\beta}}_{13} = (-0.015, -0.014)^\top$.

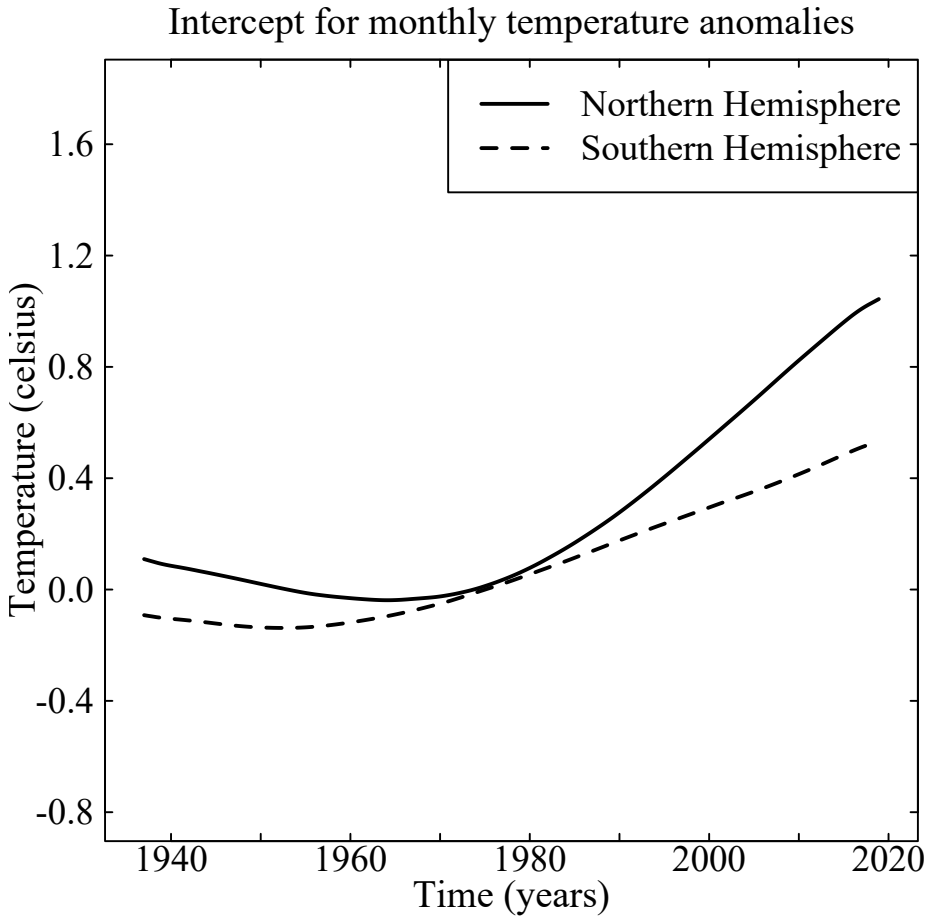

Figure 2·2: Estimated time-varying coefficients $\hat{\boldsymbol{\beta}}_0(\cdot) = \{\hat{\beta}_{0,1}(\cdot), \hat{\beta}_{0,2}(\cdot)\}^\top$ for the northern (solid) and southern (dashed) hemispheres.

Note that the method of Zhang (2015) leads to the multi-output model

$$\boldsymbol{y}_i = \boldsymbol{\beta}_0(t_{i,n}) + \boldsymbol{\beta}_{10} x_{i,-3} + \boldsymbol{\beta}_{11} x_{i,-2} + \boldsymbol{\beta}_{12} x_{i,-1} + \boldsymbol{\beta}_{13} x_{i,0} + \boldsymbol{e}_i, \quad i = 1, \ldots, n.$$

Compared with the model selected by the proposed SPLL method, it shares the same insight that the effect of the SOI on temperature anomalies can be viewed as time-constant, whereas the intercept should be treated as time varying. This provides a data-driven approach to verify the semiparametric assumption commonly

used in the climate science literature; see, for example, McLean (2014), who poses the possibility of a time-varying relationship, but does not explore statistical tools other than the simple linear regression to investigate further. On the other hand, the proposed SPLL method selects only $x_{i,-2}$ and $x_{i,0}$ as important lags, which differs from the aforementioned model selected by the method of Zhang (2015). The simulation results reported in Table 2.2 show that the method of Zhang (2015) can have a higher probability of producing over-labeled models than the proposed SPLL method does, especially when there are a lot of zero coefficients, as in the 2-2-16 configuration. Therefore, we believe the model produced by the SPLL method is more reasonable. The climate science literature tends to find a lag between the SOI and temperature anomalies, mainly by relying on the correlation of annual averages calculated starting from different months. In contrast, the current analysis allows for the possibility that temperature anomalies may be associated with multiple lags of the SOI.

2.6 Conclusion

As discussed in Section 1.1, the extension of classical linear regression models to time series poses challenges. A common extension is the time-varying coefficient model (1.3). This study focuses on variable labeling and estimation of multi-output time-varying coefficient models (1.4) which encompasses the (1.3) as a special case. In Section 2.3, we introduce a stratified fix to the method proposed by Gao (2019) and Chan et al. (2021). By incorporating information from the coefficient estimator and combining it with the derivative into the same stratum, we achieve successful labeling and estimation at the same time. Theoretical justification is provided for the application of our proposed SPLL method to (2.5) containing single explanatory variables. Section 2.4 extends the SPLL method to general multi-output time-varying coefficient models (1.4) with possible multiple explanatory variables. Numerical experi-

ments demonstrate that our method enhances the labeling and estimation accuracy compared to existing approaches.

We anticipate that the proposed SPLL method can be extended and beneficial in other scenarios involving multi-category labeling with a nested structure. For instance, one might consider a scenario where the time-constant label is substituted with a more general parametric label. In such cases, a version of the proposed stratified penalization method is anticipated to be useful. Furthermore, the consideration of multiple parametric labels in a nested structure, like polynomials with nested orders, could be explored. Nevertheless, the detailed formulation of these problems and the development of stratified penalization variations as solutions are outside the scope of this dissertation and are left for future exploration.

2.7 Technical Proofs

Here, we introduce the following notations for the proof in Section 2.7. For any $m, n \in \mathbb{R}$, we define $m \wedge n = \min(m, n)$ and $m \vee n = \max(m, n)$. For any matrix $\boldsymbol{M} = (m_{i,j})_{i,j}$, we use $\boldsymbol{m}_{\cdot,j}$ to denote its jth column and $\boldsymbol{m}_{i,\cdot}$ to denote the transpose of its ith row, and we write $\bar{\rho}(\boldsymbol{M}) = \sup\{|\boldsymbol{M}\boldsymbol{u}| : |\boldsymbol{u}| = 1\}$ and $\tilde{\rho}(\boldsymbol{M}) = \inf\{|\boldsymbol{M}\boldsymbol{u}| : |\boldsymbol{u}| = 1\}$. We use the matrix norm $|\boldsymbol{M}| = (\sum_{i,j} m_{i,j}^2)^{1/2}$, and for any matrix-valued function $\boldsymbol{f} : [0,1] \to \mathbb{R}^{s_1 \times s_2}$, we write $|\boldsymbol{f}|_{[0,1]} = \{\int_0^1 |\boldsymbol{f}(t)|^2 dt\}^{1/2}$. For the multi-output response vector $\boldsymbol{y}_{i,n} \in \mathbb{R}^d$, we write $\boldsymbol{y}_{i,n} = (y_{i,1,n}, \ldots, y_{i,d,n})^\top$, and similarly $\boldsymbol{e}_{i,n} = (e_{i,1,n}, \ldots, e_{i,d,n})^\top$. Let C be a constant whose values may vary from place to place. For Theorem 1 and 2, let $\tilde{x}_{i,n} = G(t_{i,n}; \boldsymbol{\mathcal{F}}_i)$, $M(G,t) = E\{G(t, \boldsymbol{\mathcal{F}}_i)^2\}$, and we denote the true value by $\{A^{\text{true}}(t)\}_{t\in[0,1]}$. For Theorem 3 and 4, let $\tilde{\boldsymbol{x}}_{i,n} = \boldsymbol{G}(t_{i,n}; \boldsymbol{\mathcal{F}}_i)$, $\boldsymbol{M}(\boldsymbol{G},t) = E\{\boldsymbol{G}(t, \boldsymbol{\mathcal{F}}_i)\boldsymbol{G}(t, \boldsymbol{\mathcal{F}}_i)^\top\}$ and we denote the true value by $\{\Theta^{\text{true}}(t)\}_{t\in[0,1]}$.

2.7.1 Proof of Theorem 1

Proof. Throughout the proof of this theorem, let

$$\begin{aligned}\Psi_n(\{A(t)\}_{t\in[0,1]}) &= \Upsilon_n(\{A(t)\}_{t\in[0,1]}) + f_{\lambda_{0,n}}(|\bar{\boldsymbol{a}}|) \\ &\quad + g_{\tau_{0,n}}\left(\left[\int_0^1 \{|\boldsymbol{a}_{0,\cdot}(t) - \bar{\boldsymbol{a}}|^2 + |\boldsymbol{a}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right), \quad (2.10)\end{aligned}$$

$\phi_n = (nb_n)^{-1/2} + b_n^2$ and $V(t) = \{v_{\cdot,\cdot}(t)\}_{l,j}$ be a matrix function. Then, by the proof in Theorem 1 of Fan and Li (2001), it suffices to show that, for any $\epsilon > 0$, there exists a large constant $Q^\star$ such that

$$\text{pr}\left\{\inf_{|V|_{[0,1]}=Q^\star} \Psi_n(\{A^{\text{true}}(t) + \phi_n V(t)\}_{t\in[0,1]}) > \Psi_n(\{A^{\text{true}}(t)\}_{t\in[0,1]})\right\} \geq 1-\epsilon. \quad (2.11)$$

By (2.10), we can write

$$\begin{aligned}&\Psi_n(\{A^{\text{true}}(t) + \phi_n V(t)\}_{t\in[0,1]}) - \Psi_n(\{A^{\text{true}}(t)\}_{t\in[0,1]}) \\ =\ & I_n(\{V(t)\}_{t\in[0,1]}) + II_n(\{V(t)\}_{t\in[0,1]}) + III_n(\{V(t)\}_{t\in[0,1]}),\end{aligned}$$

where

$$\begin{aligned}I_n(\{V(t)\}_{t\in[0,1]}) &= \Upsilon_n(\{A^{\text{true}}(t) + \phi_n V(t)\}_{t\in[0,1]}) - \Upsilon_n(\{A^{\text{true}}(t)\}_{t\in[0,1]}), \\ II_n(\{V(t)\}_{t\in[0,1]}) &= f_{\lambda_{k,n}}(|\bar{\boldsymbol{a}}^{\text{true}} + \phi_n\bar{\boldsymbol{v}}|) - f_{\lambda_{k,n}}(|\bar{\boldsymbol{a}}^{\text{true}}|), \\ III_n(\{V(t)\}_{t\in[0,1]}) &= g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{a}_{0,\cdot}^{\text{true}}(t) + \phi_n\boldsymbol{v}_{0,\cdot}(t) - \bar{\boldsymbol{a}}^{\text{true}} - \phi_n\bar{\boldsymbol{v}}|^2 \right.\right. \\ &\qquad \left.\left. + |\boldsymbol{a}_{1,\cdot}^{\text{true}}(t) + \phi_n\boldsymbol{v}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right) \\ &\quad - g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{a}_{0,\cdot}^{\text{true}}(t) - \bar{\boldsymbol{a}}^{\text{true}}|^2 + |\boldsymbol{a}_{1,\cdot}^{\text{true}}(t)|^2\}dt\right]^{1/2}\right).\end{aligned}$$

Here, we provide bounds for $I_n(\{V(t)\}_{t\in[0,1]})$, $II_n(\{V(t)\}_{t\in[0,1]})$ and $III_n(\{V(t)\}_{t\in[0,1]})$.

We first deal with the term $I_n(\{V(t)\}_{t\in[0,1]})$. For this, by elementary calculation,

we have

$$
\begin{aligned}
& I_n(\{V(t)\}_{t\in[0,1]}) \\
= & \int_0^1 \sum_{i=1}^n \Big| \boldsymbol{y}_{i,n} - \{\boldsymbol{a}_{0,\cdot}^{\text{true}}(t) + \phi_n \boldsymbol{v}_{0,\cdot}(t)\} x_{i,n} \\
& \qquad -\{\boldsymbol{a}_{1,\cdot}^{\text{true}}(t) + \phi_n \boldsymbol{v}_{1,\cdot}(t)\} x_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right)\Big|^2 K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
& - \int_0^1 \sum_{i=1}^n \left| \boldsymbol{y}_{i,n} - \boldsymbol{a}_{0,\cdot}^{\text{true}}(t) x_{i,n} - \boldsymbol{a}_{1,\cdot}^{\text{true}}(t) x_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right)\right|^2 K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
= & \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \big[y_{i,j,n} - \{a_{0,j}^{\text{true}}(t) + \phi_n v_{0,j}(t)\} x_{i,n} \\
& \qquad -\{a_{1,j}^{\text{true}}(t) + \phi_n v_{1,j}(t)\} x_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right)\Big]^2 K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
& - \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \left\{ y_{i,j,n} - a_{0,j}^{\text{true}}(t) x_{i,n} - a_{1,j}^{\text{true}}(t) x_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right)\right\}^2 K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
= & \phi_n^2 \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \left[x_{i,n} \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n}-t}{b_n}\right)\right\}\right]^2 K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
& - 2\phi_n \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \left[x_{i,n} a_{0,j}^{\text{true}}(t_{i,n}) + e_{i,j,n} - x_{i,n} \left\{ a_{0,j}^{\text{true}}(t) + a_{1,j}^{\text{true}}(t) \left(\frac{t_{i,n}-t}{b_n}\right)\right\}\right] \\
& \qquad \times \left[x_{i,n} \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n}-t}{b_n}\right)\right\}\right] K\left(\frac{t_{i,n}-t}{b_n}\right) dt,
\end{aligned}
$$

Let $\kappa_s = \int_{-1}^1 u^s K(u) du$. By Lemma 6 of Zhou and Wu (2010), for $s \in \{0,1,2\}$, we have

$$
\left\| \frac{1}{nb_n} \sum_{i=1}^n \tilde{x}_{i,n}^2 \left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right) - \kappa_s M(G,t) \right\| \le C\{(nb_n)^{-1/2} + b_n\}.
$$

In addition, by (2.6), we have

$$
\sup_{t\in[0,1]} \left| \sum_{i=1}^n (x_{i,n}^2 - \tilde{x}_{i,n}^2) K\left(\frac{t_{i,n}-t}{b_n}\right)\right| = O_p(1), \tag{2.12}
$$

and

$$\sup_{t\in[0,1]}\left|\sum_{i=1}^{n}(x_{i,n}\boldsymbol{e}_{i,n}-\tilde{x}_{i,n}\tilde{\boldsymbol{e}}_{i,n})K\left(\frac{t_{i,n}-t}{b_n}\right)\right|=O_p(1). \tag{2.13}$$

Note that $\kappa_0=1$, $\kappa_1=0$, we can obtain

$$\begin{aligned}
&\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{n}v_{0,j}(t)^2x_{i,n}^2K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
=\;&\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)^2\left\{\sum_{i=1}^{n}(x_{i,n}^2-\tilde{x}_{i,n}^2)K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt\\
&+nb_n\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)^2\left\{\sum_{i=1}^{n}\frac{1}{nb_n}\tilde{x}_{i,n}^2K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt\\
=\;&nb_n\sum_{j=1}^{d}\int_0^1\kappa_0M(G,t)v_{0,j}(t)^2dt\\
&+nb_n\left\{\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)^2dt\right\}O_p\{(nb_n)^{-1}+(nb_n)^{-1/2}+b_n\}\\
=\;&nb_n\sum_{j=1}^{d}\int_0^1 M(G,t)v_{0,j}(t)^2dt+nb_n|\boldsymbol{v}_{0,\cdot}|_{[0,1]}^2O_p\{(nb_n)^{-1/2}+b_n\}.
\end{aligned}$$

Similarly we have

$$\begin{aligned}
&\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{n}v_{1,j}(t)^2x_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)^2K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
=\;&\sum_{j=1}^{d}\int_0^1 v_{1,j}(t)^2\left\{\sum_{i=1}^{n}(x_{i,n}^2-\tilde{x}_{i,n}^2)\left(\frac{t_{i,n}-t}{b_n}\right)^2K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt\\
&+nb_n\sum_{j=1}^{d}\int_0^1 v_{1,j}(t)^2\sum_{i=1}^{n}\left\{\frac{1}{nb_n}\tilde{x}_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)^2K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt\\
=\;&nb_n\sum_{j=1}^{d}\int_0^1\kappa_2M(G,t)v_{1,j}(t)^2dt+nb_n|\boldsymbol{v}_{1,\cdot}|_{[0,1]}^2O_p\{(nb_n)^{-1/2}+b_n\}
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{n} v_{0,j}(t)v_{1,j}(t)x_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)K\left(\frac{t_{i,n}-t}{b_n}\right)dt \\
= &\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)v_{1,j}(t)\left\{\sum_{i=1}^{n}(x_{i,n}^2-\tilde{x}_{i,n}^2)\left(\frac{t_{i,n}-t}{b_n}\right)K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt \\
&+nb_n\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)v_{1,j}(t)\sum_{i=1}^{n}\left\{\frac{1}{nb_n}\tilde{x}_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)K\left(\frac{t_{i,n}-t}{b_n}\right)\right\}dt \\
= &\ nb_n\left\{\sum_{j=1}^{d}\int_0^1 v_{0,j}(t)v_{1,j}(t)dt\right\}O_p\{(nb_n)^{-1/2}+b_n\}
\end{aligned}$$

Let $\bar{\rho}_M=\sup_{t\in[0,1]}M(G,t)$, $\tilde{\rho}_M=\inf_{t\in[0,1]}M(G,t)$. Combining the results above, we have

$$\begin{aligned}
&\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{n}\left[x_{i,n}\left\{v_{0,j}(t)+v_{1,j}(t)\left(\frac{t_{i,n}-t}{b_n}\right)\right\}\right]^2 K\left(\frac{t_{i,n}-t}{b_n}\right)dt \\
= &\sum_{j=1}^{d}\int_0^1\sum_{i=1}^{n}\left\{x_{i,n}^2v_{0,j}(t)^2+2x_{i,n}^2v_{0,j}(t)v_{1,j}(t)\left(\frac{t_{i,n}-t}{b_n}\right)\right. \\
&\left.+x_{i,n}^2v_{1,j}(t)^2\left(\frac{t_{i,n}-t}{b_n}\right)^2\right\}K\left(\frac{t_{i,n}-t}{b_n}\right)dt \\
= &\ nb_n\sum_{j=1}^{d}\int_0^1 M(G,t)v_{0,j}(t)^2+\kappa_2M(G,t)v_{1,j}(t)^2dt+nb_n|V|_{[0,1]}^2O_p\{(nb_n)^{-1/2}+b_n\} \\
\geq &\ nb_n|V|_{[0,1]}^2[(\kappa_2\wedge 1)\tilde{\rho}_M+O_p\{(nb_n)^{-1/2}+b_n\}].
\end{aligned}$$

On the other hand, by the Cauchy-Schwarz inequality and Lemma A.1 of Zhang and

Wu (2012),

$$
\begin{aligned}
& \left| \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} x_{i,n} \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} e_{i,j,n} K\left(\frac{t_{i,n}-t}{b_n} \right) dt \right| \\
\leq & \sum_{j=1}^{d} \left| \int_0^1 v_{0,j}(t) \left\{ \sum_{i=1}^{n} \left(x_{i,n} e_{i,j,n} - \tilde{x}_{i,n} \tilde{e}_{i,j,n} \right) K\left(\frac{t_{i,n}-t}{b_n} \right) \right\} dt \right| \\
& + \sum_{j=1}^{d} \left| \int_0^1 v_{0,j}(t) \left\{ \sum_{i=1}^{n} \tilde{x}_{i,n} \tilde{e}_{i,j,n} K\left(\frac{t_{i,n}-t}{b_n} \right) \right\} dt \right| \\
& + \sum_{j=1}^{d} \left| \int_0^1 v_{1,j}(t) \left\{ \sum_{i=1}^{n} \left(x_{i,n} e_{i,j,n} - \tilde{x}_{i,n} \tilde{e}_{i,j,n} \right) \left(\frac{t_{i,n}-t}{b_n} \right) K\left(\frac{t_{i,n}-t}{b_n} \right) \right\} dt \right| \\
& + \sum_{j=1}^{d} \left| \int_0^1 v_{1,j}(t) \left\{ \sum_{i=1}^{n} \tilde{x}_{i,n} \tilde{e}_{i,j,n} \left(\frac{t_{i,n}-t}{b_n} \right) K\left(\frac{t_{i,n}-t}{b_n} \right) \right\} dt \right| \\
\leq & \left(\sum_{j=1}^{d} \int_0^1 |v_{0,j}(t)| dt \right) O_p\{1 + (nb_n)^{1/2}\} + \left(\sum_{j=1}^{d} \int_0^1 |v_{1,j}(t)| dt \right) O_p\{1 + (nb_n)^{1/2}\} \\
\leq & 2d|V|_{[0,1]} O_p\{(nb_n)^{1/2}\}.
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
& \left| \int_0^1 \sum_{i=1}^{n} \tilde{x}_{i,n}^2 \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} K\left(\frac{t_{i,n}-t}{b_n} \right) dt \right| \\
\leq & \left(\int_0^1 |v_{0,j}(t)|^2 dt \right)^{1/2} \left(\int_0^1 \left| \sum_{i=1}^{n} \tilde{x}_{i,n}^2 K\left(\frac{t_{i,n}-t}{b_n} \right) \right|^2 dt \right)^{1/2} \\
& + \left(\int_0^1 |v_{1,j}(t)|^2 dt \right)^{1/2} \left(\int_0^1 \left| \sum_{i=1}^{n} \tilde{x}_{i,n}^2 \left(\frac{t_{i,n}-t}{b_n} \right) K\left(\frac{t_{i,n}-t}{b_n} \right) \right|^2 dt \right)^{1/2} \\
\leq & 2nb_n |V|_{0,1} [\bar{\rho}_M + O_p\{(nb_n)^{-1/2} + b_n\}]
\end{aligned}
$$

and

$$\begin{aligned} & \left| \int_0^1 \sum_{i=1}^n (x_{i,n}^2 - \tilde{x}_{i,n}^2) \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \\ \leq \; & 2nb_n |V|_{[0,1]} O_p\{(nb_n)^{-1}\} \end{aligned}$$

Since $-1 \leq (t_{i,n} - t)/b_n \leq 1$, we have

$$\begin{aligned} \left| a_{0,j}^{\text{true}}(t_{i,n}) - a_{0,j}^{\text{true}}(t) - a_{1,j}^{\text{true}}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right| &= \left| \frac{b_n^2}{2} \left(\frac{t_{i,n} - t}{b_n} \right)^2 \beta_j''(\xi) \right| \\ &\leq \frac{b_n^2}{2} \sup_{t \in [0,1]} |\beta_j''(t)| \end{aligned} \tag{2.14}$$

holds for some $\xi \in [0,1]$. Then, we can obtain that

$$\begin{aligned} & \left| \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \left\{ a_{0,j}^{\text{true}}(t_{i,n}) - a_{0,j}^{\text{true}}(t) - a_{1,j}^{\text{true}}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} \right. \\ & \qquad \left. \times x_{i,n}^2 \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \\ \leq \; & \sum_{j=1}^d \frac{b_n^2}{2} \sup_{t \in [0,1]} |\beta_j''(t)| \left[\left| \int_0^1 \sum_{i=1}^n \tilde{x}_{i,n}^2 \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \right. \\ & \qquad \left. + \left| \int_0^1 \sum_{i=1}^n (x_{i,n}^2 - \tilde{x}_{i,n}^2) \left\{ v_{0,j}(t) + v_{1,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \right] \\ \leq \; & \sum_{j=1}^d \frac{b_n^2}{2} \sup_{t \in [0,1]} |\beta_j''(t)| \left(2nb_n |V|_{[0,1]} [\bar{\rho}_M + O_p\{(nb_n)^{-1} + (nb_n)^{-1/2} + b_n\}] \right) \\ = \; & nb_n |V|_{[0,1]} \left[b_n^2 \left\{ \sum_{j=1}^d \sup_{t \in [0,1]} |\beta_j''(t)| \right\} \bar{\rho}_M + O_p\{b_n^2 (nb_n)^{-1/2} + b_n^3\} \right]. \end{aligned}$$

Combining the results above, we have

$$
\begin{aligned}
& I_n(\{V(t)\}_{t\in[0,1]}) \\
\geq\ & \phi_n^2 nb_n |V|^2_{[0,1]}[(\kappa_2 \wedge 1)\tilde{\rho}_M + O_p\{(nb_n)^{-1/2} + b_n\}] \\
& -2\phi_n nb_n |V|_{[0,1]} \left[2dO_p\{(nb_n)^{-1/2}\} + b_n^2 \left\{ \sum_{j=1}^{d} \sup_{t\in[0,1]} \left|\beta_j''(t)\right| \right\} \bar{\rho}_M \right. \\
& \qquad \left. + O_p\{b_n^2 (nb_n)^{-1/2} + b_n^3\} \right] \\
\geq\ & \phi_n^2 nb_n |V|^2_{[0,1]}[(\kappa_2 \wedge 1)\tilde{\rho}_M + O_p\{(nb_n)^{-1/2} + b_n\}] \\
& -2\phi_n nb_n |V|_{[0,1]} \left[b_n^2 \left\{ \sum_{j=1}^{d} \sup_{t\in[0,1]} |\beta_j''(t)| \right\} \bar{\rho}_M + O_p\{(nb_n)^{-1/2} + b_n^3\} \right].
\end{aligned}
$$

Now, we deal with the term $II_n(\{V(t)\}_{t\in[0,1]})$ and $III_n(\{V(t)\}_{t\in[0,1]})$. By the triangle inequality and Cauchy-Schwarz inequality, we have

$$
\begin{aligned}
|f_{\lambda_{0,n}}(|\bar{\boldsymbol{a}}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}|) - f_{\lambda_{0,n}}(|\bar{\boldsymbol{a}}^{\text{true}}|)| &= |f'_{\lambda_{0,n}}(\xi)(|\bar{\boldsymbol{a}}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}| - |\bar{\boldsymbol{a}}^{\text{true}}|)| \\
&\leq \left| f'_{\lambda_{0,n}}(\xi) |\phi_n \bar{\boldsymbol{v}}| \right| \\
&\leq \phi_n |V|_{[0,1]} \sup_{x\in\mathbb{R}} |f'_{\lambda_{0,n}}(x)|,
\end{aligned}
$$

where ξ is a real number between $|\bar{\boldsymbol{a}}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}|$ and $|\bar{\boldsymbol{a}}^{\text{true}}|$. Using a similar argument,

$$
\begin{aligned}
& \left| g_{\tau_{0,n}} \left(\left[\int_0^1 \{ |\boldsymbol{a}_{0,\cdot}^{\text{true}}(t) + \phi_n \boldsymbol{v}_{0,\cdot}(t) - \bar{\boldsymbol{a}}^{\text{true}} - \phi_n \bar{\boldsymbol{v}}|^2 \right. \right. \right. \\
& \qquad \left. \left. \left. + |\boldsymbol{a}_{1,\cdot}^{\text{true}}(t) + \phi_n \boldsymbol{v}_{1,\cdot}(t)|^2 \} dt \right]^{1/2} \right) \\
& \quad - g_{\tau_{0,n}} \left(\left[\int_0^1 \{ |\boldsymbol{a}_{0,\cdot}^{\text{true}}(t) - \bar{\boldsymbol{a}}^{\text{true}}|^2 + |\boldsymbol{a}_{1,\cdot}^{\text{true}}(t)|^2 \} dt \right]^{1/2} \right) \Bigg| \\
\leq\ & \phi_n \left[\int_0^1 \{ |\boldsymbol{v}_{0,\cdot}(t) - \bar{\boldsymbol{v}}|^2 + |\boldsymbol{v}_{1,\cdot}(t)|^2 \} dt \right]^{1/2} \sup_{x\in\mathbb{R}} |g'_{\tau_{0,n}}(x)| \\
\leq\ & \phi_n |V|_{[0,1]} \sup_{x\in\mathbb{R}} |g'_{\tau_{0,n}}(x)|.
\end{aligned}
$$

Note that $\phi_n^2 nb_n = 1 + nb_n^5$, $\phi_n nb_n^3 = (nb_n^5)^{1/2} + nb_n^5 = O(1 + nb_n^5)$, and $\{(nb_n)^{-1/2} + b_n^2\}(\lambda_{0,n} + \tau_{0,n}) = O(1)$. Based on the above observations, we can conclude that for

sufficiently large $Q^\star$, the dominate term of

$$\Psi_n(\{A^{\text{true}}(t)+\phi_n V(t)\}_{t\in[0,1]})-\Psi_n(\{A^{\text{true}}(t)\}_{t\in[0,1]})$$

is $\phi_n^2 n b_n |V|^2_{[0,1]}(\kappa_2\wedge 1)\tilde{\rho}_M>0$, and the result follows. □

2.7.2 Proof of Theorem 2

Proof. Throughout this proof, we rewrite $A_{0,\cdot}(t)$ as a sum of its center and an additional time-varying component. To be more specific, let $\mathcal{C}\{A(t)\}:=\bar{\boldsymbol{a}}=(\gamma_j)_j$ denote the center, then $\mathcal{V}_0\{A(t)\}:=\boldsymbol{a}_{0,\cdot}(t)-\bar{\boldsymbol{a}}=(\iota_{0,j})_{0,j}$ and $\mathcal{V}_1\{A(t)\}:=\boldsymbol{a}_{1,\cdot}(t)=(\iota_{1,j})_{1,j}$ represent the time-varying component with respect to the center. We can now rewrite (2.10) as

$$\begin{aligned}
&\Lambda_n(\{\mathcal{C}\{A(t)\}\}_{t\in[0,1]},\{\mathcal{V}_0\{A(t)\}\}_{t\in[0,1]},\{\mathcal{V}_1\{A(t)\}\}_{t\in[0,1]})\\
=\;&\int_0^1\sum_{i=1}^n\left|\boldsymbol{y}_{i,n}-\mathcal{C}\{A(t)\}x_{i,n}-\mathcal{V}_0\{A(t)\}x_{i,n}-\mathcal{V}_1\{A(t)\}x_{i,n}\left(\frac{t_{i,n}-t}{b_n}\right)\right|^2\\
&\qquad\times K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
&+f_{\lambda_{0,n}}(|\boldsymbol{\gamma}|)+g_{\tau_{0,n}}\left(\left[\int_0^1\{|\boldsymbol{\iota}_{0,\cdot}(t)|^2+|\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right)+\boldsymbol{\zeta}^\top\int_0^1\mathcal{V}_0\{A(t)\}dt\\
=\;&\sum_{j=1}^d\int_0^1\sum_{i=1}^n\left\{y_{i,j,n}-\gamma_j x_{i,n}-\iota_{0,j}(t)x_{i,n}-\iota_{1,j}(t)x_{i,n}\left(\frac{t_{i,n}-t}{b_n}\right)\right\}^2K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
&+f_{\lambda_{0,n}}(|\boldsymbol{\gamma}|)+g_{\tau_{0,n}}\left(\left[\int_0^1\{|\boldsymbol{\iota}_{0,\cdot}(t)|^2+|\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right)+\sum_{j=1}^d\zeta_j\int_0^1\iota_{0,j}(t)dt \qquad (2.15)
\end{aligned}$$

where $\boldsymbol{\zeta}=(\zeta_j)_j\in\mathbb{R}^d$ represents the Lagrange multiplier that concerns the constraint $\int_0^1\mathcal{V}_0\{A(t)\}dt=0$. The minimization problem (2.15) relates to the normal equations:

$$\begin{cases}
\partial\Lambda_n([\mathcal{C}\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_0\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_1\{A(t)\}]_{t\in[0,1]})/\partial\gamma_j=0;\\
\partial\Lambda_n([\mathcal{C}\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_0\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_1\{A(t)\}]_{t\in[0,1]})/\partial\iota_{0,j}(t)=0;\\
\partial\Lambda_n([\mathcal{C}\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_0\{A(t)\}]_{t\in[0,1]},[\mathcal{V}_1\{A(t)\}]_{t\in[0,1]})/\partial\iota_{1,j}(t)=0.
\end{cases} \qquad (2.16)$$

Throughout this proof, for $s \in \{0, 1\}$, let

$$U_{j,s,n}(t) = \sum_{i=1}^{n} \left\{ y_{i,j,n} - \gamma_j x_{i,n} - \iota_{0,j}(t) x_{i,n} - \iota_{1,j}(t) x_{i,n} \left(\frac{t_{i,n} - t}{b_n} \right) \right\} \times \left(\frac{t_{i,n} - t}{b_n} \right)^s x_{i,n} K \left(\frac{t_{i,n} - t}{b_n} \right).$$

Then, for any $j = 1, \ldots, d$, (2.16) can be represented as

$$\begin{cases} \int_0^1 2U_{j,0,n}(t)dt = f'_{\lambda_{0,n}}(|\boldsymbol{\gamma}|)\frac{\gamma_j}{|\boldsymbol{\gamma}|}; \\ 2U_{j,0,n}(t) = g'_{\tau_{0,n}}([\int_0^1 \{|\boldsymbol{\iota}_{0,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt]^{1/2}) \frac{\iota_{0,j}(t)}{[\int_0^1 \{|\boldsymbol{\iota}_{0,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt]^{1/2}} + \zeta_j; \\ 2U_{j,1,n}(t) = g'_{\tau_{0,n}}([\int_0^1 \{|\boldsymbol{\iota}_{0,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt]^{1/2}) \frac{\iota_{1,j}(t)}{[\int_0^1 \{|\boldsymbol{\iota}_{0,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt]^{1/2}}. \end{cases}$$

Therefore, by observing the constraint that $\int_0^1 \boldsymbol{\iota}_{0,\cdot}(t)dt = 0$, we can obtain that

$$4 \sum_{j=1}^{d} \left\{ \int_0^1 U_{j,0,n}(t)dt \right\}^2 = \{f'_{\lambda_{0,n}}(|\boldsymbol{\gamma}|)\}^2, \tag{2.17}$$

and

$$\begin{aligned} & 4 \sum_{j=1}^{d} \left\{ \int_0^1 U_{j,0,n}(t)^2 + U_{j,1,n}(t)^2 dt \right\} \\ = & \left\{ g'_{\tau_{0,n}} \left(\left[\int_0^1 \{|\boldsymbol{\iota}_{0,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt \right]^{1/2} \right) \right\}^2 + \sum_{j=1}^{d} \zeta_j^2. \end{aligned} \tag{2.18}$$

Note that, by the proof of Lemma 6 in Zhou and Wu (2010),

$$\sup_{t \in [0,1]} \left| \sum_{i=1}^{n} \tilde{x}_{i,n}^2 \left(\frac{t_{i,n} - t}{b_n} \right)^s K \left(\frac{t_{i,n} - t}{b_n} \right) \right| = O_p(n^{1/2} + nb_n),$$

and by (2.12) and (2.13),

$$\sup_{t \in [0,1]} \left| \sum_{i=1}^{n} (x_{i,n}^2 - \tilde{x}_{i,n}^2) \left(\frac{t_{i,n} - t}{b_n} \right)^s K \left(\frac{t_{i,n} - t}{b_n} \right) \right| = O_p(1),$$

we have

$$\sup_{t\in[0,1]}\left|\sum_{i=1}^{n} x_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right| = O_p(n^{1/2}+nb_n).$$

Then, by Theorem 1, we can obtain that

$$\begin{aligned}
&\int_0^1\left[\sum_{i=1}^{n} x_{i,n}^2\{\hat{a}_{0,j}(t)-a_{0,j}^{\text{true}}(t)\}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right]^2 dt\\
=&\int_0^1\left|\sum_{i=1}^{n} x_{i,n}^2\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right|^2 |\hat{a}_{0,j}(t)-a_{0,j}^{\text{true}}(t)|^2 dt\\
=&\ O_p\{(n^{1/2}+nb_n)^2\}O_p[\{(nb_n)^{-1/2}+b_n^2\}^2] = O_p(nb_n+n^2b_n^6),
\end{aligned}$$

and

$$\begin{aligned}
&\int_0^1\left[\sum_{i=1}^{n} x_{i,n}^2\left\{\hat{a}_{1,j}(t)-a_{1,j}^{\text{true}}(t)\right\}\left(\frac{t_{i,n}-t}{b_n}\right)^{s+1} K\left(\frac{t_{i,n}-t}{b_n}\right)\right]^2 dt\\
=&\ O_p(nb_n+n^2b_n^6).
\end{aligned}$$

In addition, by (2.14) and Lemma A.1 of Zhang and Wu (2012), we have

$$\left\|\sum_{i=1}^{n}\tilde{x}_{i,n}\tilde{e}_{i,j,n}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right\| \le C(nb_n)^{1/2},$$

and

$$\begin{aligned}
&\left\|\sum_{i=1}^{n}\tilde{x}_{i,n}^2\left\{a_{0,j}^{\text{true}}(t_{i,n})-a_{0,j}^{\text{true}}(t)-\left(\frac{t_{i,n}-t}{b_n}\right)a_{1,j}^{\text{true}}(t)\right\}\right.\\
&\qquad\left.\times\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right\| \le Cb_n^2\{(nb_n)^{1/2}+nb_n\}.
\end{aligned}$$

On the other hand, by a similar argument as in the proof of Theorem 1, we have

$$\left\|\sum_{i=1}^{n} x_{i,n}e_{i,j,n}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right\| \le C(nb_n)^{1/2},$$

and

$$\left\| \sum_{i=1}^{n} x_{i,n}^2 \left\{ a_{0,j}^{\text{true}}(t_{i,n}) - a_{0,j}^{\text{true}}(t) - \left(\frac{t_{i,n}-t}{b_n} \right) a_{1,j}^{\text{true}}(t) \right\} \right.$$
$$\left. \times \left(\frac{i/n-t}{b_n} \right)^s K\left(\frac{i/n-t}{b_n} \right) \right\| = C(nb_n^3).$$

Combining the above results, we can obtain that

$$\begin{aligned}
\int_0^1 \hat{U}_{j,s,n}(t)^2 dt &= \int_0^1 \sum_{i=1}^{n} \left[\left\{ y_{i,j,n} - \hat{a}_{0,j}(t) x_{i,n} - \hat{a}_{1,j}(t) x_{i,n} \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right. \\
&\qquad \left. \times \left(\frac{t_{i,n}-t}{b_n} \right)^s x_{i,n} K\left(\frac{t_{i,n}-t}{b_n} \right) \right]^2 dt \\
&= \int_0^1 \sum_{i=1}^{n} \left(\left[e_{i,j,n} - x_{i,n}\{\hat{a}_{0,j}(t) - a_{0,j}^{\text{true}}(t)\} \right. \right. \\
&\qquad - x_{i,n}\{\hat{a}_{1,j}(t) - a_{1,j}^{\text{true}}(t)\} \left(\frac{t_{i,n}-t}{b_n} \right) \\
&\qquad \left. + x_{i,n} \left\{ a_{0,j}^{\text{true}}(t_{i,n}) - a_{0,j}^{\text{true}}(t) - \left(\frac{t_{i,n}-t}{b_n} \right) a_{1,j}^{\text{true}}(t) \right\} \right] \\
&\qquad \left. \times \left(\frac{t_{i,n}-t}{b_n} \right)^s x_{i,n} K\left(\frac{t_{i,n}-t}{b_n} \right) \right)^2 dt \\
&= O_p(nb_n + n^2 b_n^6),
\end{aligned}$$

and, as a result, the left-hand-side of (2.18) becomes

$$4 \sum_{j=1}^{d} \left\{ \int_0^1 U_{j,0,n}(t)^2 + U_{j,1,n}(t)^2 dt \right\} = O_p(nb_n + n^2 b_n^6).$$

If $\boldsymbol{\beta} \in \mathcal{D}_0 \cup \mathcal{D}_c$, then, by Theorem 1,

$$\left[\int_0^1 \{ |\hat{\boldsymbol{\iota}}_{0,\cdot}(t)|^2 + |\hat{\boldsymbol{\iota}}_{1,\cdot}(t)|^2 \} dt \right]^{1/2} = O_p\{(nb_n)^{-1/2} + b_n^2\}.$$

Under (P3) and the assumed conditions, we have

$$\frac{\left\{g'_{\tau_{0,n}}\left(\left[\int_0^1\{|\boldsymbol{\iota}_{0,\cdot}(t)|^2+|\boldsymbol{\iota}_{1,\cdot}(t)|^2\}dt\right]^{1/2}\right)\right\}^2}{nb_n+n^2b_n^6}\to\infty.$$

Therefore, the solution will be at the point where the differentiability does not hold, which leads to

$$\mathrm{pr}\left\{\sup_{t\in[0,1]}|\hat{\boldsymbol{\iota}}_{0,\cdot}(t)|=0 \text{ and } \sup_{t\in[0,1]}|\hat{\boldsymbol{\iota}}_{1,\cdot}(t)|=0\right\}\to 1.$$

For the second claim, note that

$$\begin{aligned}
\int_0^1\hat{U}_{j,0,n}(t)dt &= \int_0^1\sum_{i=1}^n\left\{y_{i,j,n}-\hat{a}_{0,j}(t)x_{i,n}-\hat{a}_{1,j}(t)x_{i,n}\left(\frac{t_{i,n}-t}{b_n}\right)\right\}\\
&\qquad\times x_{i,n}K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
&= \int_0^1\sum_{i=1}^n\Big[e_{i,j,n}-x_{i,n}\{\hat{a}_{0,j}(t)-a_{0,j}^{\mathrm{true}}(t)\}\\
&\qquad -x_{i,n}\{\hat{a}_{1,j}(t)-a_{1,j}^{\mathrm{true}}(t)\}\left(\frac{t_{i,n}-t}{b_n}\right)\\
&\qquad +x_{i,n}\left\{a_{0,j}^{\mathrm{true}}(t_{i,n})-a_{0,j}^{\mathrm{true}}(t)-\left(\frac{t_{i,n}-t}{b_n}\right)a_{1,j}^{\mathrm{true}}(t)\right\}\Big]\\
&\qquad\times x_{i,n}K\left(\frac{t_{i,n}-t}{b_n}\right)dt\\
&= O_p\{nb_n^3+(nb_n)^{1/2}\}.
\end{aligned}$$

Thus, by Theorem 1, $|\hat{\boldsymbol{\gamma}}|=O_p\{(nb_n)^{-1/2}+b_n^2\}$ holds for $\boldsymbol{\beta}\in\mathcal{D}_0$. On the other hand, note that

$$\frac{f'_{\lambda_{0,n}}(|\boldsymbol{\gamma}|)}{nb_n^3+(nb_n)^{1/2}}\to\infty,$$

the second claim follows by (2.17) and a similar argument. □

2.7.3 Proof of Theorem 3

Proof. Let

$$\begin{aligned}\Psi_n(\{\Theta(t)\}_{t\in[0,1]}) &= \sum_{k=1}^{p} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}_{k,\cdot}|^2+|\boldsymbol{\theta}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right)\\ &\quad+\Upsilon_n(\{\Theta(t)\}_{t\in[0,1]})+\sum_{k=1}^{p} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}|), \qquad (2.19)\end{aligned}$$

$\phi_n=(nb_n)^{-1/2}+b_n^2$, and $V(t)=\{v_{\cdot,\cdot,\cdot}(t)\}_{l,j,k}$ be a 3-way tensor function. Similar to the proof of Theorem 2, it suffices to show that, for any $\epsilon>0$, there exists a large constant $Q^\star$ such that

$$\text{pr}\left\{\inf_{|V|_{[0,1]}=Q^\star}\Psi_n(\{\Theta^{\text{true}}(t)+\phi_nV(t)\}_{t\in[0,1]})>\Psi_n(\{\Theta^{\text{true}}(t)\}_{t\in[0,1]})\right\}\geq 1-\epsilon.$$

and we write

$$\begin{aligned}&\Psi_n(\{\Theta^{\text{true}}(t)+\phi_nV(t)\}_{t\in[0,1]})-\Psi_n(\{\Theta^{\text{true}}(t)\}_{t\in[0,1]})\\ =\;& I_n(\{V(t)\}_{t\in[0,1]})+II_n(\{V(t)\}_{t\in[0,1]})+III_n(\{V(t)\}_{t\in[0,1]}),\end{aligned}$$

where

$$\begin{aligned}I_n(\{V(t)\}_{t\in[0,1]}) &= \Upsilon_n(\{\Theta^{\text{true}}(t)+\phi_nV(t)\}_{t\in[0,1]})-\Upsilon_n(\{\Theta^{\text{true}}(t)\}_{t\in[0,1]}),\\ II_n(\{V(t)\}_{t\in[0,1]}) &= \sum_{k=1}^{p} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}+\phi_n\bar{\boldsymbol{v}}_{k,\cdot}|)-\sum_{k=1}^{p} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}|),\end{aligned}$$

and

$$\begin{aligned}III_n(\{V(t)\}_{t\in[0,1]}) &= \sum_{k=1}^{p} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)+\phi_n\boldsymbol{v}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}-\phi_n\bar{\boldsymbol{v}}_{k,\cdot}|^2\right.\right.\\ &\qquad\left.\left.+|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)+\phi_n\boldsymbol{v}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right)\\ &\quad-\sum_{k=1}^{p} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}|^2+|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right).\end{aligned}$$

We first deal with the term $I_n(\{V(t)\}_{t\in[0,1]})$. By Lemma 6 of Zhou and Wu (2010), for $s \in \{0,1,2\}$, we have

$$\left\| \frac{1}{nb_n} \sum_{i=1}^{n} \tilde{\boldsymbol{x}}_{i,n} \tilde{\boldsymbol{x}}_{i,n}^\top \left(\frac{t_{i,n}-t}{b_n} \right)^s K\left(\frac{t_{i,n}-t}{b_n} \right) - \kappa_s \boldsymbol{M}(\boldsymbol{G},t) \right\| \leq C\{(nb_n)^{-1/2} + b_n\}.$$

In addition, by (2.7), we have

$$\sup_{t\in[0,1]} \left| \sum_{i=1}^{n} (\boldsymbol{x}_{i,n}\boldsymbol{x}_{i,n}^\top - \tilde{\boldsymbol{x}}_{i,n}\tilde{\boldsymbol{x}}_{i,n}^\top) K\left(\frac{t_{i,n}-t}{b_n} \right) \right| = O_p(1), \tag{2.20}$$

and

$$\sup_{t\in[0,1]} \left| \sum_{i=1}^{n} (\boldsymbol{x}_{i,n}\boldsymbol{e}_{i,n}^\top - \tilde{\boldsymbol{x}}_{i,n}\tilde{\boldsymbol{e}}_{i,n}^\top) K\left(\frac{t_{i,n}-t}{b_n} \right) \right| = O_p(1). \tag{2.21}$$

Let $\bar{\rho}_{\boldsymbol{M}} = \sup_{t\in[0,1]} \bar{\rho}\{\boldsymbol{M}(\boldsymbol{G},t)\}$, $\tilde{\rho}_{\boldsymbol{M}} = \inf_{t\in[0,1]} \tilde{\rho}\{\boldsymbol{M}(\boldsymbol{G},t)\}$. Then, following a similar argument as in the proof of Theorem 1, we have

$$\begin{aligned}
& \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} \left[\boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right]^2 K\left(\frac{t_{i,n}-t}{b_n} \right) dt \\
= \;& nb_n \sum_{j=1}^{d} \int_0^1 \{\boldsymbol{v}_{0,\cdot,j}(t)^\top \boldsymbol{M}(\boldsymbol{G},t)\boldsymbol{v}_{0,\cdot,j}(t) + \kappa_2 \boldsymbol{v}_{1,\cdot,j}(t)^\top \boldsymbol{M}(\boldsymbol{G},t)\boldsymbol{v}_{1,\cdot,j}(t)\} dt \\
& + nb_n |V|_{[0,1]}^2 O_p\{(nb_n)^{-1/2} + b_n\} \\
\geq \;& nb_n |V|_{[0,1]}^2 [(\kappa_2 \wedge 1)\tilde{\rho}_{\boldsymbol{M}} + O_p\{(nb_n)^{-1/2} + b_n\}].
\end{aligned}$$

Similarly, by the Cauchy-Schwarz inequality and Lemma A.1 of Zhang and Wu (2012), we have

$$\begin{aligned}
& \left| \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} \boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} e_{i,j,n} K\left(\frac{t_{i,n}-t}{b_n} \right) dt \right| \\
\leq \;& \left(\sum_{j=1}^{d} \int_0^1 |\boldsymbol{v}_{0,\cdot,j}(t) + \sum_{j=1}^{d} \int_0^1 |\boldsymbol{v}_{1,\cdot,j}(t)| dt \right) O_p\{1 + (nb_n)^{1/2}\} \\
\leq \;& 2d|V|_{[0,1]} O_p\{(nb_n)^{1/2}\}.
\end{aligned}$$

Since $-1 \leq (t_{i,n} - t)/b_n \leq 1$, we have

$$\left| \boldsymbol{\theta}^{\text{true}}_{0,\cdot,j}(t_{i,n}) - \boldsymbol{\theta}^{\text{true}}_{0,\cdot,j}(t) - \boldsymbol{\theta}^{\text{true}}_{1,\cdot,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right| \leq \frac{b_n^2}{2} \sup_{t \in [0,1]} |\boldsymbol{b}''_{\cdot,j}(t)| \tag{2.22}$$

holds for some $\xi \in [0,1]$. Then, we can obtain that

$$\begin{aligned}
&\left| \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} \left\{ \boldsymbol{\theta}^{\text{true}}_{0,\cdot,j}(t_{i,n}) - \boldsymbol{\theta}^{\text{true}}_{0,\cdot,j}(t) - \boldsymbol{\theta}^{\text{true}}_{1,\cdot,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\}^{\top} \boldsymbol{x}_{i,n} \right. \\
&\qquad \left. \times \boldsymbol{x}_{i,n}^{\top} \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \\
\leq \; & \sum_{j=1}^{d} \frac{b_n^2}{2} \sup_{t \in [0,1]} |\boldsymbol{b}''_{\cdot,j}(t)| \\
&\quad \times \left[\left| \int_0^1 \sum_{i=1}^{n} \tilde{\boldsymbol{x}}_{i,n} \tilde{\boldsymbol{x}}_{i,n}^{\top} \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \right. \\
&\qquad + \left| \int_0^1 \sum_{i=1}^{n} (\boldsymbol{x}_{i,n} \boldsymbol{x}_{i,n}^{\top} - \tilde{\boldsymbol{x}}_{i,n} \tilde{\boldsymbol{x}}_{i,n}^{\top}) \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n} - t}{b_n} \right) \right\} \right. \\
&\qquad\qquad \left. \left. \times K \left(\frac{t_{i,n} - t}{b_n} \right) dt \right| \right] \\
\leq \; & \sum_{j=1}^{d} \frac{b_n^2}{2} \sup_{t \in [0,1]} |\boldsymbol{b}''_{\cdot,j}(t)| \left(2nb_n |V|_{[0,1]} [\bar{\rho}_{\boldsymbol{M}} + O_p\{(nb_n)^{-1} + (nb_n)^{-1/2} + b_n\}] \right) \\
= \; & nb_n |V|_{[0,1]} \left[b_n^2 \left\{ \sum_{j=1}^{d} \sup_{t \in [0,1]} |\boldsymbol{b}''_{\cdot,j}(t)| \right\} \bar{\rho}_{\boldsymbol{M}} + O_p\{b_n^2 (nb_n)^{-1/2} + b_n^3\} \right].
\end{aligned}$$

Combining the results above, we have

$$
\begin{aligned}
& I_n(\{V(t)\}_{t\in[0,1]}) \\
= \;& \phi_n^2 \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} \left[\boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right]^2 K\left(\frac{t_{i,n}-t}{b_n} \right) dt \\
& -2\phi_n \sum_{j=1}^{d} \int_0^1 \sum_{i=1}^{n} \left[\boldsymbol{x}_{i,n}^\top \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t_{i,n}) + e_{i,j,n} - \boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t) + \boldsymbol{\theta}_{1,\cdot,j}^{\text{true}}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right] \\
& \qquad \times \left[\boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{v}_{0,\cdot,j}(t) + \boldsymbol{v}_{1,\cdot,j}(t) \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right] K\left(\frac{t_{i,n}-t}{b_n} \right) dt \\
\geq \;& \phi_n^2 n b_n |V|_{[0,1]}^2 [(\kappa_2 \wedge 1)\tilde{\rho}_{\boldsymbol{M}} + O_p\{(nb_n)^{-1/2} + b_n\}] \\
& -2\phi_n n b_n |V|_{[0,1]} \left[2dO_p\{(nb_n)^{-1/2}\} + b_n^2 \left\{ \sum_{j=1}^{d} \sup_{t\in[0,1]} \left| \boldsymbol{b}_{\cdot,j}''(t) \right| \right\} \bar{\rho}_{\boldsymbol{M}} \right. \\
& \qquad \left. + O_p\{b_n^2 (nb_n)^{-1/2} + b_n^3\} \right] \\
\geq \;& \phi_n^2 n b_n |V|_{[0,1]}^2 [(\kappa_2 \wedge 1)\tilde{\rho}_{\boldsymbol{M}} + O_p\{(nb_n)^{-1/2} + b_n\}] \\
& -2\phi_n n b_n |V|_{[0,1]} \left[b_n^2 \left\{ \sum_{j=1}^{d} \sup_{t\in[0,1]} |\boldsymbol{b}_{\cdot,j}''(t)| \right\} \bar{\rho}_{\boldsymbol{M}} + O_p\{(nb_n)^{-1/2} + b_n^3\} \right].
\end{aligned}
$$

Now, we deal with the term $II_n(\{V(t)\}_{t\in[0,1]})$. For this, note that for $k \in \mathcal{D}_0 \cup \mathcal{D}_{v0}$, we have $|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}}| = 0$, and thus

$$
II_n(\{V(t)\}_{t\in[0,1]}) \geq \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}_{k,\cdot}|) - \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}}|).
$$

Then, by the triangle inequality and Cauchy-Schwarz inequality, we have

$$
\begin{aligned}
& \left| \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}_{k,\cdot}|) - \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f_{\lambda_{k,n}}(|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}}|) \right| \\
= \;& \left| \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f'_{\lambda_{k,n}}(\xi_k) \left(|\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}} + \phi_n \bar{\boldsymbol{v}}_{k,\cdot}| - |\bar{\boldsymbol{\theta}}_{k,\cdot}^{\text{true}}| \right) \right| \\
\leq \;& \left| \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} f'_{\lambda_{k,n}}(\xi_k) |\phi_n \bar{\boldsymbol{v}}_{k,\cdot}| \right| \leq \phi_n |V|_{[0,1]} \sum_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}} \sup_{x\in\mathbb{R}} |f'_{\lambda_{k,n}}(x)|,
\end{aligned}
$$

where ξ_k is a real number between $|\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}+\phi_n\bar{\boldsymbol{v}}_{k,\cdot}|$ and $|\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}|$. Using a similar argument, for $k \in \mathcal{D}_0 \cup D_c$, we have $\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t) = \bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}$ and $|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)| = 0$, and thus

$$
\begin{aligned}
III_n(\{V(t)\}_{t\in[0,1]}) \;\geq\; & \sum_{k\in\mathcal{D}_v} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)+\phi_n\boldsymbol{v}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}-\phi_n\bar{\boldsymbol{v}}_{k,\cdot}|^2\right.\right. \\
& \left.\left. +|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)+\phi_n\boldsymbol{v}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right) \\
& -\sum_{k\in\mathcal{D}_v} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}|^2+|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
& \left|\sum_{k\in\mathcal{D}_v} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)+\phi_n\boldsymbol{v}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}-\phi_n\bar{\boldsymbol{v}}_{k,\cdot}|^2\right.\right.\right. \\
& \qquad \left.\left.\left. +|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)+\phi_n\boldsymbol{v}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right) \\
& \quad -\sum_{k\in\mathcal{D}_v} g_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\theta}^{\text{true}}_{0,k,\cdot}(t)-\bar{\boldsymbol{\theta}}^{\text{true}}_{k,\cdot}|^2+|\boldsymbol{\theta}^{\text{true}}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right)\Bigg| \\
\leq\; & \phi_n|V|_{[0,1]}\sum_{k\in\mathcal{D}_v}\sup_{x\in\mathbb{R}}|g'_{\tau_{k,n}}(x)|.
\end{aligned}
$$

Note that $\phi_n^2 nb_n = 1+nb_n^5$, $\phi_n nb_n^3 = (nb_n^5)^{1/2}+nb_n^5 = O(1+nb_n^5)$, and $\{(nb_n)^{-1/2}+b_n^2\}(\max_{k\in\mathcal{D}_c\cup\mathcal{D}_{v1}}\lambda_{k,n}+\max_{k\in\mathcal{D}_v}\tau_{k,n}) = O(1)$. The result follows by a similar argument as the proof in Theorem 1. □

2.7.4 Proof of Theorem 4

Proof. Similar to the proof of Theorem 2, we rewrite $\Theta_{0,\cdot,\cdot}(t)$ as a sum of its center and an additional time-varying component. Let $\mathcal{C}\{\Theta(t)\} := (\bar{\boldsymbol{\theta}}_{1,\cdot},\ldots,\bar{\boldsymbol{\theta}}_{p,\cdot})^\top = (\gamma_{k,j})_{k,j}$ denote the center, then $\mathcal{V}_0\{\Theta(t)\} := \{\boldsymbol{\theta}_{0,1,\cdot}(t)-\bar{\boldsymbol{\theta}}_{1,\cdot},\ldots,\boldsymbol{\theta}_{0,p,\cdot}(t)-\bar{\boldsymbol{\theta}}_{p,\cdot}\}^\top = (\iota_{0,k,j})_{0,k,j}$ and $\mathcal{V}_1\{\Theta(t)\} := \{\boldsymbol{\theta}_{1,1,\cdot}(t),\ldots,\boldsymbol{\theta}_{1,p,\cdot}(t)\}^\top = (\iota_{1,k,j})_{1,k,j}$ represent the time-varying compo-

nent with respect to the center. We can now rewrite (2.19) as

$$
\begin{aligned}
& \Lambda_n(\{\mathcal{C}\{\Theta(t)\}\}_{t\in[0,1]}, \{\mathcal{V}_0\{\Theta(t)\}\}_{t\in[0,1]}, \{\mathcal{V}_1\{\Theta(t)\}\}_{t\in[0,1]}) \\
= & \int_0^1 \sum_{i=1}^n \left| \boldsymbol{y}_{i,n} - \mathcal{C}\{\Theta(t)\}^\top \boldsymbol{x}_{i,n} - \mathcal{V}_0\{\Theta(t)\}^\top \boldsymbol{x}_{i,n} - \mathcal{V}_1\{\Theta(t)\}^\top \boldsymbol{x}_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right) \right|^2 \\
& \qquad \times K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
& + \sum_{k=1}^p f_{\lambda_{k,n}}(|\boldsymbol{\gamma}_{k,\cdot}|) + \sum_{k=1}^p g_{\tau_{k,n}} \left(\left[\int_0^1 \{|\boldsymbol{\iota}_{0,k,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,k,\cdot}(t)|^2\} dt \right]^{1/2} \right) \\
& + \mathrm{tr}\left(\boldsymbol{\zeta}^\top \int_0^1 \mathcal{V}_0\{\Theta(t)\} dt \right) \\
= & \sum_{j=1}^d \int_0^1 \sum_{i=1}^n \left\{ y_{i,j,n} - \boldsymbol{\gamma}_{\cdot,j}^\top \boldsymbol{x}_{i,n} - \boldsymbol{\iota}_{0,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} - \boldsymbol{\iota}_{1,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right) \right\}^2 \\
& \qquad \times K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
& + \sum_{k=1}^p f_{\lambda_{k,n}}(|\boldsymbol{\gamma}_{k,\cdot}|) + \sum_{k=1}^p g_{\tau_{k,n}} \left(\left[\int_0^1 \{|\boldsymbol{\iota}_{0,k,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,k,\cdot}(t)|^2\} dt \right]^{1/2} \right) \\
& + \sum_{k=1}^p \sum_{j=1}^d \zeta_{k,j} \int_0^1 \iota_{0,k,j}(t) dt, \qquad (2.23)
\end{aligned}
$$

where $\mathrm{tr}(\cdot)$ denotes the trace operator, and $\boldsymbol{\zeta} = (\zeta_{k,j})_{k,j} \in \mathbb{R}^{p\times d}$ represents the Lagrange multiplier that concerns the constraint $\int_0^1 \mathcal{V}_0\{\Theta(t)\} dt = 0$. For $s \in \{0,1\}$, let

$$
\begin{aligned}
U_{k,j,s,n}(t) = & \sum_{i=1}^n \left\{ y_{i,j,n} - \boldsymbol{\gamma}_{\cdot,j}^\top \boldsymbol{x}_{i,n} - \boldsymbol{\iota}_{0,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} - \boldsymbol{\iota}_{1,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} \left(\frac{t_{i,n}-t}{b_n}\right) \right\} \\
& \times \left(\frac{t_{i,n}-t}{b_n}\right)^s x_{i,k,n} K\left(\frac{t_{i,n}-t}{b_n}\right).
\end{aligned}
$$

Similar to the argument of obtain (2.17) and (2.18) in the proof of Theorem 2, by observing the constraint that $\int_0^1 \boldsymbol{\iota}_{0,k,\cdot}(t)dt = 0$, we have

$$4\sum_{j=1}^{d}\left\{\int_0^1 U_{k,j,0,n}(t)dt\right\}^2 = \{f'_{\lambda_{k,n}}(|\boldsymbol{\gamma}_{k,\cdot}|)\}^2, \tag{2.24}$$

and

$$\begin{aligned} & 4\sum_{j=1}^{d}\left\{\int_0^1 U_{k,j,0,n}(t)^2 + U_{k,j,1,n}(t)^2 dt\right\} \\ = & \left\{g'_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\iota}_{0,k,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,k,\cdot}(t)|^2\}dt\right]^{1/2}\right)\right\}^2 + \sum_{j=1}^{d}\zeta_{k,j}^2. \end{aligned} \tag{2.25}$$

Combining the results that

$$\sup_{t\in[0,1]}\left|\sum_{i=1}^{n}\tilde{x}_{i,k,n}\tilde{\boldsymbol{x}}_{i,n}^{\top}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right| = O_p(n^{1/2}+nb_n),$$

and

$$\sup_{t\in[0,1]}\left|\sum_{i=1}^{n}(x_{i,k,n}\boldsymbol{x}_{i,n}^{\top} - \tilde{x}_{i,k,n}\tilde{\boldsymbol{x}}_{i,n}^{\top})\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right| = O_p(1),$$

we have

$$\sup_{t\in[0,1]}\left|\sum_{i=1}^{n}x_{i,k,n}\boldsymbol{x}_{i,n}^{\top}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right| = O_p(1+n^{1/2}+nb_n) = O_p(n^{1/2}+nb_n).$$

Then, by Theorem 3 and the Cauchy-Schwarz inequality, we can obtain that

$$\begin{aligned} & \int_0^1\left[\sum_{i=1}^{n}x_{i,k,n}\boldsymbol{x}_{i,n}^{\top}\{\hat{\boldsymbol{\theta}}_{0,\cdot,j}(t) - \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t)\}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right]^2 dt \\ \leq & \int_0^1\left|\sum_{i=1}^{n}x_{i,k,n}\boldsymbol{x}_{i,n}^{\top}\left(\frac{t_{i,n}-t}{b_n}\right)^s K\left(\frac{t_{i,n}-t}{b_n}\right)\right|^2 |\hat{\boldsymbol{\theta}}_{0,\cdot,j}(t) - \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t)|^2 dt \\ = & O_p\{(n^{1/2}+nb_n)^2\}O_p[\{(nb_n)^{-1/2}+b_n^2\}^2] = O_p(nb_n + n^2b_n^6), \end{aligned}$$

and

$$\begin{aligned} & \int_0^1 \left[\sum_{i=1}^n x_{i,k,n} \boldsymbol{x}_{i,n}^\top \left\{ \hat{\boldsymbol{\theta}}_{1,\cdot,j}(t) - \boldsymbol{\theta}_{1,\cdot,j}^{\text{true}}(t) \right\} \left(\frac{t_{i,n}-t}{b_n} \right)^{s+1} K\left(\frac{t_{i,n}-t}{b_n} \right) \right]^2 dt \\ = \; & O_p(nb_n + n^2 b_n^6). \end{aligned}$$

Following a similar argument in the proof of Theorem 2, we can obtain that

$$\left\| \sum_{i=1}^n x_{i,k,n} e_{i,j,n} \left(\frac{t_{i,n}-t}{b_n} \right)^s K\left(\frac{t_{i,n}-t}{b_n} \right) \right\| \leq C(nb_n)^{1/2},$$

and

$$\begin{aligned} & \left\| \sum_{i=1}^n x_{i,k,n} \boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t_{i,n}) - \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t) - \left(\frac{t_{i,n}-t}{b_n} \right) \boldsymbol{\theta}_{1,\cdot,j}^{\text{true}}(t) \right\} \right. \\ & \qquad \left. \times \left(\frac{i/n-t}{b_n} \right)^s K\left(\frac{i/n-t}{b_n} \right) \right\| = C(nb_n^3). \end{aligned}$$

hold. Combining the above results, we have

$$\begin{aligned} \int_0^1 \hat{U}_{k,j,s,n}^2(t) dt &= \int_0^1 \sum_{i=1}^n \left[\left\{ y_{i,j,n} - \hat{\boldsymbol{\theta}}_{0,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} - \hat{\boldsymbol{\theta}}_{1,\cdot,j}(t)^\top \boldsymbol{x}_{i,n} \left(\frac{t_{i,n}-t}{b_n} \right) \right\} \right. \\ & \qquad \left. \times \left(\frac{t_{i,n}-t}{b_n} \right)^s x_{i,k,n} K\left(\frac{t_{i,n}-t}{b_n} \right) \right]^2 dt \\ &= \int_0^1 \sum_{i=1}^n \left(\left[e_{i,j,n} - \boldsymbol{x}_{i,n}^\top \{ \hat{\boldsymbol{\theta}}_{0,\cdot,j}(t) - \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t) \} \right. \right. \\ & \qquad - \boldsymbol{x}_{i,n}^\top \{ \hat{\boldsymbol{\theta}}_{1,\cdot,j}(t) - \boldsymbol{\theta}_{1,\cdot,j}^{\text{true}}(t) \} \left(\frac{t_{i,n}-t}{b_n} \right) \\ & \qquad \left. + \boldsymbol{x}_{i,n}^\top \left\{ \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t_{i,n}) - \boldsymbol{\theta}_{0,\cdot,j}^{\text{true}}(t) - \left(\frac{t_{i,n}-t}{b_n} \right) \boldsymbol{\theta}_{1,\cdot,j}^{\text{true}}(t) \right\} \right] \\ & \qquad \left. \times \left(\frac{t_{i,n}-t}{b_n} \right)^s x_{i,k,n} K\left(\frac{t_{i,n}-t}{b_n} \right) \right)^2 dt \\ &= O_p(nb_n + n^2 b_n^6), \end{aligned}$$

and, as a result, the left-hand-side of (2.25) becomes

$$4\sum_{j=1}^{d}\left\{\int_0^1 U_{k,j,0,n}(t)^2 + U_{k,j,1,n}(t)^2 dt\right\} = O_p(nb_n + n^2 b_n^6).$$

If $k \in \mathcal{D}_0 \cup \mathcal{D}_c$, Theorem 3 shows

$$\left[\int_0^1 \{|\hat{\boldsymbol{\iota}}_{0,k,\cdot}(t)|^2 + |\hat{\boldsymbol{\iota}}_{1,k,\cdot}(t)|^2\} dt\right]^{1/2} = O_p\{(nb_n)^{-1/2} + b_n^2\}.$$

Under (P3) and the assumed conditions, we have

$$\frac{\left\{g'_{\tau_{k,n}}\left(\left[\int_0^1 \{|\boldsymbol{\iota}_{0,k,\cdot}(t)|^2 + |\boldsymbol{\iota}_{1,k,\cdot}(t)|^2\} dt\right]^{1/2}\right)\right\}^2}{nb_n + n^2 b_n^6} \to \infty.$$

Therefore, the solution will be at the point where the differentiability does not hold, which leads to

$$\mathrm{pr}\left\{\max_{k\in\mathcal{D}_0\cup\mathcal{D}_c}\sup_{t\in[0,1]}|\hat{\boldsymbol{\iota}}_{0,k,\cdot}(t)| = 0 \text{ and } \max_{k\in\mathcal{D}_0\cup\mathcal{D}_c}\sup_{t\in[0,1]}|\hat{\boldsymbol{\iota}}_{1,k,\cdot}(t)| = 0\right\} \to 1.$$

Similarly,

$$\begin{aligned}
\int_0^1 \hat{U}_{k,j,0,n}(t)dt &= \int_0^1 \sum_{i=1}^{n}\Big[e_{i,j,n} - \boldsymbol{x}_{i,n}^\top\{\hat{\boldsymbol{\theta}}_{0,\cdot,j}(t) - \boldsymbol{\theta}_{0,\cdot,j}^{\mathrm{true}}(t)\} \\
&\qquad -\boldsymbol{x}_{i,n}^\top\{\hat{\boldsymbol{\theta}}_{1,\cdot,j}(t) - \boldsymbol{\theta}_{1,\cdot,j}^{\mathrm{true}}(t)\}\left(\frac{t_{i,n}-t}{b_n}\right) \\
&\qquad +\boldsymbol{x}_{i,n}^\top\left\{\boldsymbol{\theta}_{0,\cdot,j}^{\mathrm{true}}(t_{i,n}) - \boldsymbol{\theta}_{0,\cdot,j}^{\mathrm{true}}(t) - \left(\frac{t_{i,n}-t}{b_n}\right)\boldsymbol{\theta}_{1,\cdot,j}^{\mathrm{true}}(t)\right\}\Big] \\
&\qquad \times x_{i,k,n} K\left(\frac{t_{i,n}-t}{b_n}\right) dt \\
&= O_p\{nb_n^3 + (nb_n)^{1/2}\}.
\end{aligned}$$

Combining the fact $|\hat{\boldsymbol{\gamma}}_{k,\cdot}| = O_p\{(nb_n)^{-1/2} + b_n^2\}$ holds for $k \in \mathcal{D}_0$ and $\frac{f'_{\lambda_{k,n}}(|\boldsymbol{\gamma}_{k,\cdot}|)}{nb_n^3 + (nb_n)^{1/2}} \to \infty$, the second claim follows by (2.24) and a similar argument. □

Chapter 3

A Warped Self-Normalized Inference for Time Series with Staggered Observation Periods

Chapter 2 address the problem of variable labeling in multi-output time-varying coefficient models. In such models, the time series used for analysis should have the same lengths with the same start and end observation times. However, in practical scenarios, this contemporaneous assumption is frequently unfulfilled. For instance, in the CRUTEM data hosted by the Met Office Hadley Centre that contains hemispheric and global mean time series of land air temperature anomalies, the northern hemisphere has anomaly values started in 1850 while the southern hemisphere only has data started in 1857 mainly due to the poor land data coverage in the southern hemisphere before 1857. Also, for the United Kingdom (UK) precipitation data studied in Section 3.5.3, the monthly precipitation record started in January 1873 for the Northwest England and Wales (NwEW) region and January 1931 for the Northern Ireland (NI) region. More generally in meteorological science and climate science, it is often the case that one time series has a longer history of record than the other, and monitoring stations can be built or demolished at different times in different regions making their collected data on different time periods. In finance, portfolios can be created and abandoned at different times, leading to different recording periods. In economics, it is often the case that the economic variable of interest has published data available on different time horizons for different countries. In infectious dis-

eases research such as the most recent COVID-19, different countries often start and end the data collection at different times making their observation periods different. These applications have driven the development of methods that consider statistical inference for multiple time series where time series do not share the same observation period and may have different lengths. In Chapter 3, we explore the two-sample testing problem for time series with staggered observation periods. This chapter is based on Wang and Zhang (2024).

3.1 Introduction

Applications from various scientific problems often require the comparison of data from two populations, which can be, for example, clinical trial data from the treatment and control groups, temperature record data from different countries or regions, electricity usage data from different industries, among many others. This two-sample test problem in statistical analysis has been widely studied in the literature; see for example Hotelling (1931), Cressie and Whitford (1986), Hall and Martin (1988), Bai and Saranadasa (1996), Keselman et al. (2004), Chen and Qin (2010), Chen et al. (2013), Cai et al. (2014), Gregory et al. (2015), Xu et al. (2016), Städler and Mukherjee (2017), Chen et al. (2019), Zhang et al. (2020), and references therein. The aforementioned works mostly concerned the situation when the data can be viewed as independent samples from an underlying distribution. For time series data, Dette and Weißbach (2009) considered testing the difference between regression functions of two stationary conditional heteroskedastic autoregressive processes. Politis and Romano (2010) considered the use of subsampling in two-sample or multi-sample problems of time series data. Horváth et al. (2013) considered testing the equality of means in two functional time series by using a normal approximation for the functional sample mean and estimating the long-run covariance kernel. Dette et al. (2020)

considered the problem of testing relevant hypotheses in two functional time series. In the aforementioned works, however, it is often assumed that the two time series being compared are independent of each other. As discussed in Section 1.2, there is always a joint dependence exhibited between the two time series to be compared. In Chapter 3, we explore the problem of two-sample testing in statistical analysis under a time series setting, where the two time series being compared may have staggered observation periods and exhibit nonignorable joint dependence.

To address the issue of nuisance parameters mentioned in Section 1.2 when conducting two-sample inference, we propose utilizing the self-normalization framework introduced by Lobato (2001). The objective of Lobato (2001) was to test whether a dependent stochastic process is uncorrelated up to a fixed order. Previous methods for this test, prior to Lobato (2001), required the selection of user-chosen parameters, which could potentially affect the statistical inference. The test statistic proposed by Lobato (2001) based on self-normalization is asymptotically distribution-free under the null hypothesis that does not contain any unknown parameters. In order to illustrate the idea of self-normalization, Section 2.1 of Lobato (2001) considers a simple case, dealing with the equivalent problem of testing whether the mean of a process is zero. We shall here consider a similar problem as an example to illustrate the idea of self-normalization. Suppose we have a sample $Z_1, \ldots, Z_{n_z}$ from a stationary process $\{Z_i\}_{i\in\mathbb{Z}}$ with mean $\mu_z = \mathrm{E}(Z_0)$. Denote the long-run variance as g_z where $g_z = \sum_{k\in\mathbb{Z}} \mathrm{cov}(Z_0, Z_k)$. As mentioned in Section 1.2, the statistical inference for μ_z relies on (1.7), which requires estimating the nuisance parameter g_z. To avoid this issue, the self-normalization approach considers the following self-normalized statistic:

$$\frac{n_z(\bar{Z}_{n_z} - \mu_z)^2}{n_z^{-2}\sum_{k=1}^{n_z} k^2 \left(k^{-1}\sum_{i=1}^{k} Z_i - \bar{Z}_{n_z}\right)^2}.$$

Under appropriate weak dependence conditions, for example, Herrndorf (1984) and

Wu (2007), the following invariance principle holds:

$$n_z^{-1/2} \sum_{i=1}^{\lfloor n_z t \rfloor} (Z_i - \mu_z) \Rightarrow \sqrt{g_z} W(t), \quad t \in [0,1], \tag{3.1}$$

where $\Rightarrow$ denotes the weak convergence in the Skorokhod space and $W(t)$ is the standard Brownian motion. Then, by (3.1), under the null hypothesis that $\mu_z = 0$, one can demonstrate the weak convergence of:

$$n_z^{-2} \sum_{k=1}^{n_z} k^2 \left(k^{-1} \sum_{i=1}^{k} Z_i - \bar{Z}_{n_z} \right)^2 \to_d g_z \int_0^1 \{W(t) - tW(1)\}^2 dt.$$

Hence, by continuous mapping theorem, we can obtain the asymptotic distribution of the self-normalized statistic

$$\frac{n_z(\bar{Z}_{n_z} - \mu_z)^2}{n_z^{-2} \sum_{k=1}^{n_z} k^2 \left(k^{-1} \sum_{i=1}^{k} Z_i - \bar{Z}_{n_z} \right)^2} \to_d \frac{W(1)^2}{\int_0^1 (W(t) - tW(1))^2 dt}.$$

In this limiting distribution, the nuisance parameter g_z cancels out in both the numerator and denominator, rendering it independent of g_z.

The self-normalization framework is based on this idea, which was later expanded by Shao (2010) to incorporate time series inference. The method involves using an inconsistent normalizer to pivot the asymptotic distribution of the statistic being analyzed. This eliminates the need for estimating any nuisance parameter, resulting in a more efficient inference procedure. When joint dependence between the two time series to be compared, Zhang and Shao (2015) proposed the use of self-normalization to pivotalize the test statistic. Their method is based on the key assumption that the sample size of the two time series are the same, namely the balanced two-sample case. The extension to the unbalanced case can be highly nontrivial, as Shao (2015) demonstrated with counterexamples that self-normalization can work in two-sample problems only when the two time series are independent or when they are of the same

length. However, as we mentioned before, the two time series to be compared are not necessarily of the same length and can have different starting and ending observation times, which brings difficulty to accommodate their joint dependence. In this case, using independent random block subsamples from the two time series as in Politis and Romano (2010) may no longer preserve the same dependence structure as in the target global two-sample statistic. On the other hand, the limiting distribution of the self-normalized statistic as in Shao (2015) will be affected by the underlying joint dependence in this case and is no longer pivotalized unless the two time series are of the same length. To address this issue, we incorporated the block sampling method into the self-normalized framework.

Motivated by the bootstrap method proposed by Efron (1979) and Efron (1982), Carlstein (1986) first introduced a blocking scheme with non-overlapping blocks of the sequence to address the variance estimation problem, while Künsch (1989) and Liu and Singh (1992) later introduced the moving blocks bootstrap to time series, where the blocks are allowed to be overlapping. Various types of block sampling with its properties are then widely explored in the literature by, for example, Lahiri (1993), Politis and Romano (1994a), Politis and Romano (1994b), Li and Maddala (1996), Bühlmann (1997), Politis et al. (1997), Lahiri (1999), Politis et al. (1999), Paparoditis and Politis (2001), Politis et al. (2001), Bühlmann (2002), Lahiri (2003), Nordman and Lahiri (2012), Tewes et al. (2019), and references therein. Bai et al. (2016) proposed the self-normalized block sampling method to conduct statistical inference, such as hypothesis testing or the construction of confidence intervals, on the mean of a single time series. Although combining the strength of subsampling and self-normalization has been demonstrated to be useful in certain problems (Bai et al., 2016), its direct application still cannot handle the joint dependence under staggered observation periods in the two-sample test. To address this problem, we

proposed a warped self-normalized subsampling approach, which incorporates a time domain warping based on how the two time series are staggered to recover and match the original dependence structure.

In Sections 3.2 and 3.3, we revisit the subsampling method by Politis and Romano (2010) and the self-normalization approach by Shao (2015). We aim to understand why these methods were not suitable for comparing two time series with joint dependence and staggered observation periods. In Section 3.4, we introduce a warped self-normalized two-sample test. This test accommodates various staggering patterns. For instance, it can handle cases where the time series start simultaneously but end at different times, where one series is observed within a subset of the other's timeline, or where the observation intervals overlap partially or completely. A distinguishable feature of the proposed method is that it allows the length of nonoverlapping segments to increase with the sample size so that the data sample is not necessarily dominated by the overlapping segment. We demonstrate that this method is not only applicable to comparing means between two samples, but can also be used to compare other quantities such as variances or quantiles in a unified manner. In Section 3.5, we conduct numerical experiments, including a Monte Carlo simulation and a real data analysis, to further illustrate the effectiveness of our proposed method. Section 3.6 concludes this chapter. Technical proofs throughout this chapter are provided in Section 3.7.

3.2 Subsampling Approach: A Revisit

In this section, we revisit the subsampling approach of Politis and Romano (2010) to illustrate why such approach can become inapplicable in the current setting. For this, suppose we observe $X_k, \ldots, X_{k+N_x-1}$ and $Y_l, \ldots, Y_{l+N_y-1}$ from a stationary bivariate time series (X_i, Y_i), where the associated sample sizes are N_x and N_y respectively.

Here, the observation periods $[k, k+N_x-1]$ and $[l, l+N_y-1]$ for the two time series are not necessarily the same. As commented by Politis and Romano (2010), the literature on comparing time series of different lengths seems scarce, and Politis and Romano (2010) proposed to address the problem using subsampling. To illustrate, we consider the case of the mean, and we denote $\bar{X}_{k,k+N_x-1} = N_x^{-1}\sum_{i=k}^{k+N_x-1} X_i$ and $\bar{Y}_{l,l+N_y-1} = N_y^{-1}\sum_{j=l}^{l+N_y-1} Y_j$, then the method of Politis and Romano (2010) approximates the distribution of $\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1}$ by that of subsamples after a suitable scale adjustment. In particular, let $X_i, \ldots, X_{i+B_x-1}$ and $Y_j, \ldots, Y_{j+B_y-1}$ be subsamples of lengths B_x and B_y from the two time series, and we denote the underlying means and long-run variances by

$$\mu_x = \mathrm{E}(X_0), \quad \mu_y = \mathrm{E}(Y_0), \quad g_x = \sum_{k\in\mathbb{Z}} \mathrm{cov}(X_0, X_k), \quad g_y = \sum_{k\in\mathbb{Z}} \mathrm{cov}(Y_0, Y_k),$$

then by Politis and Romano (2010) the distribution of

$$Z_{\mathrm{PR10}} = (N_x^{-1}g_x + N_y^{-1}g_y)^{-1/2}(\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1})$$

under the null hypothesis of equal mean can be approximated by the empirical distribution $\hat{F}_{\mathrm{PR10}}(u)$, $u \in \mathbb{R}$, of the subsamples

$$H_{i,j} = (B_x^{-1}g_x + B_y^{-1}g_y)^{-1/2}(\bar{X}_{i,i+B_x-1} - \bar{Y}_{j,j+B_y-1}),$$

where $1 \le i \le q_x = N_x - B_x + 1$ and $1 \le j \le q_y = N_y - B_y + 1$. In particular, let $\mathrm{I}(\cdot)$ be the indicator function, then we can write

$$\hat{F}_{\mathrm{PR10}}(u) = q_x^{-1}q_y^{-1}\sum_{i=1}^{q_x}\sum_{j=1}^{q_y} \mathrm{I}(H_{i,j} \le u).$$

Assuming independence between (X_i) and (Y_j), Politis and Romano (2010) provided the theoretical guarantee of such a two-sample subsampling approach under strong

mixing. When the two time series are not independent of each other, however, the following theorem suggests that the subsampling approach of Politis and Romano (2010) may become inapplicable even for the simple case of m-dependent processes that share the same observation period.

Theorem 5. *Assume that (X_i, Y_i) is a m-dependent stationary process with finite second moment and we observe $X_k, \ldots, X_{k+N_x-1}$ and $Y_l, \ldots, Y_{l+N_y-1}$ with $k = l$ and $N_x = N_y$. If the long-run variances g_x and g_y are both bounded away from zero and the subsample lengths satisfy $1/B_x + B_x/N_x \to 0$ and $1/B_y + B_y/N_y \to 0$, then under the null hypothesis of $\mu_x = \mu_y$, as $N_x = N_y \to \infty$ we have*

$$\hat{F}_{\mathrm{PR10}}(u) \to_p \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy} u),$$

where $\to_p$ denotes the convergence in probability and

$$c_{xy} = \left(\frac{g_x + g_y - a_{xy}}{g_x + g_y} \right)^{1/2}, \quad a_{xy} = \sum_{k \in \mathbb{Z}} \{ \mathrm{cov}(X_0, Y_k) + \mathrm{cov}(Y_0, X_k) \}.$$

When the two time series (X_i) and (Y_j) are not independent as in Politis and Romano (2010), the cross term a_{xy} is generally nonzero which makes the constant $c_{xy} \neq 1$. As a result, the subsampling distribution $\hat{F}_{\mathrm{PR10}}$ in this case will become a distorted approximation to the distribution of $Z_{\mathrm{PR10}} = (N_x^{-1} g_x + N_y^{-1} g_y)^{-1/2} (\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1})$, which makes the method of Politis and Romano (2010) inapplicable to the current setting. Note that implementing the above discussed subsampling approach also requires estimating the long-run variances g_x and g_y, which can itself be a non-trivial problem and may involve the selection of additional tuning parameters. This motivates the use of self-normalization in two-sample problems as considered in Shao (2015) and Zhang and Shao (2015).

3.3 Self-Normalization Approach: A Revisit

The idea of self-normalization is to use a sequence of recursive estimators to pivotalize the asymptotic distribution of the two-sample statistic. Following Shao (2015), we let $N = N_x + N_y$ and consider the case when $k = l = 1$ so that the two time series (X_i) and (Y_j) share the same starting time. In this case, the recursive two-sample differences can be constructed as

$$D_{i,N} = \bar{X}_{1,\lfloor iN_x/N \rfloor} - \bar{Y}_{1,\lfloor iN_y/N \rfloor}, \quad i = 1, \ldots, N, \tag{3.2}$$

and the self-normalized two-sample statistic of Shao (2015) takes the form

$$Z_{\mathrm{S15}} = \frac{N D_{N,N}^2}{N^{-2} \sum_{i=1}^{N} i^2 (D_{i,N} - D_{N,N})^2}. \tag{3.3}$$

To derive the asymptotic distribution of Z_{S15}, we follow Shao (2015) and make the following assumption.

(IP) There exist $a, b \neq 0$ and c such that for any $M \to \infty$,

$$M^{-1/2} \sum_{i=1}^{\lfloor Mt \rfloor} \begin{pmatrix} X_i - \mu_x \\ Y_i - \mu_y \end{pmatrix} \Rightarrow \begin{pmatrix} a & 0 \\ -c & -b \end{pmatrix} \begin{Bmatrix} W_1(t) \\ W_2(t) \end{Bmatrix},$$

where $\Rightarrow$ denotes the weak convergence in the Skorokhod space and $W_1(t)$ and $W_2(t)$ are two independent standard Brownian motions.

Assumption (IP) is generally referred to as the invariance principle, which has been widely studied under various short-range dependence conditions; see for example Hannan (1979), Herrndorf (1984), Wu (2007), Berkes et al. (2014) and references therein. We also refer to Shao (2010), Zhang and Lavitas (2018) and Zhang et al. (2019) for additional references on the use of the invariance principle in self-normalization. By assumption (IP) and the continuous mapping argument as in Shao (2015) and Zhang and Lavitas (2018), one can show that as $N_x/N \to p_x$ and $N_y/N \to p_y$ for some

$p_x, p_y \in (0,1)$ the self-normalized two sample statistic

$$Z_{\mathrm{S15}} \to_d \frac{\{V_{p_x,p_y}(1;a,b,c)\}^2}{\int_0^1 \{V_{p_x,p_y}(t;a,b,c) - tV_{p_x,p_y}(1;a,b,c)\}^2 dt},$$

where

$$V_{p_x,p_y}(t;a,b,c) = \frac{a}{p_x}W_1(p_x t) + \frac{c}{p_y}W_1(p_y t) + \frac{b}{p_y}W_2(p_y t).$$

In the case when $p_x = p_y$ for which the two time series share the same length, we can write

$$V_{p_x,p_y}(t;a,b,c) = \frac{a+c}{p_x}W_1(p_x t) + \frac{b}{p_x}W_2(p_x t),$$

which then has the same distribution as the process $p_x^{-1/2}\{(a+c)^2 + b^2\}^{1/2}W_1(t)$ on $t \in [0,1]$. As a result, one can show that in this case,

$$Z_{\mathrm{S15}} \to_d \frac{\{W_1(1)\}^2}{\int_0^1 \{W_1(t) - tW(1)\}^2 dt},$$

where the asymptotic distribution is pivotal. When the two time series are not of the same length, however, the process $V_{p_x,p_y}(t;a,b,c)$, $t \in [0,1]$, is not necessarily a Brownian motion and the asymptotic distribution of Z_{S15} can then depend on the underlying unknown dependence structure; see also the discussion in Shao (2015). Note that even if the two time series are of the same length, when one is started recording earlier than the other, the aforementioned self-normalization of Shao (2015) can become inapplicable as well. To illustrate, we consider the case when $N_x = N_y$ but $k \neq l$, and we can generalize the recursive two-sample difference sequence in (3.2) to consider

$$D^{\star}_{i,N} = \bar{X}_{k,k+\lfloor iN_x/N \rfloor - 1} - \bar{Y}_{l,l+\lfloor iN_y/N \rfloor - 1}, \quad i = 1, \ldots, N. \tag{3.4}$$

Similar to (3.2), the quantity in (3.4) represents the recursive two-sample difference obtained by using the first i/N proportion of the data from both time series, where $D^{\star}_{N,N} = \bar{X}_{k,m} - \bar{Y}_{l,n}$ continues to represent the global two-sample difference. The

self-normalized two-sample statistic can then be constructed as

$$Z^{\star}_{\mathrm{S15}} = \frac{N(D^{\star}_{N,N})^2}{N^{-2}\sum_{i=1}^{N} i^2(D^{\star}_{i,N} - D^{\star}_{N,N})^2},$$

and its asymptotic distribution is given in Theorem 6.

Theorem 6. *Assume condition (IP) and $N_x = N_y$. If $k = 1$, $N \to \infty$, and $l/N \to \ell$ for some $\ell \in (0, 1/2)$, then under the null hypothesis of $\mu_x = \mu_y$ we have*

$$Z^{\star}_{\mathrm{S15}} \to_d \frac{\{V^{\star}_{\ell}(1; a, b, c)\}^2}{\int_0^1 \{V^{\star}_{\ell}(t; a, b, c) - tV^{\star}_{\ell}(1; a, b, c)\}^2 dt},$$

where

$$V^{\star}_{\ell}(t; a, b, c) = 2aW_1(t/2) + 2c\{W_1(\ell + t/2) - W_1(\ell)\} + 2b\{W_2(\ell + t/2) - W_2(\ell)\}.$$

By Theorem 6, the asymptotic distribution of the self-normalized statistic $Z^{\star}_{\mathrm{S15}}$ in this case can be affected by the recording lag in time represented by ℓ and is generally not pivotal unless $\ell = 0$. Therefore, a direct application of self-normalization as in Shao (2015) seems to be able to address the joint dependence only in the case when the two time series are observed during the same period, and such a method can become inapplicable for time series with staggered observation periods. We shall in the following propose a new method that can handle staggered observation periods in the presence of joint dependence.

3.4 Warped Self-Normalized Subsampling

3.4.1 The Mean Case: Illustration of the Idea

We first illustrate the idea by considering the mean case. Suppose we observe $X_k, \ldots, X_{k+N_x-1}$ and $Y_l, \ldots, Y_{l+N_y-1}$ from a stationary bivariate time series (X_i, Y_i), where the associated sample sizes are N_x and N_y respectively. The observation periods $[k, k + N_x - 1]$ and $[l, l + N_y - 1]$ for the two time series are not necessarily the

same. Write $N = N_x + N_y$, and without loss of generality, we assume that $1 = k \leq l$. Our idea is to use self-normalized statistics calculated from windows with appropriately warped times between the two time series to precisely capture the underlying dependence as in the global two-sample statistic. Let B_N be a sequence of nonnegative real numbers, we first construct a pair of index sets that represent the desired time warping as

$$\begin{aligned}\mathcal{I}_{x,i,B_N} = \{s \in \mathbb{Z} :\ & l + i - 1 - \lfloor B_N(l-1)/N \rfloor \leq s \\ & \leq l - \lfloor B_N(l-1)/N \rfloor + \lfloor B_N N_x/N \rfloor - 1 + i - 1\}\end{aligned}$$

and

$$\mathcal{I}_{y,i,B_N} = \{s \in \mathbb{Z} :\ l + i - 1 \leq s \leq l + \lfloor B_N N_y/N \rfloor - 1 + i - 1\}.$$

The time warping association between index sets of the two time series as proposed above is carefully constructed to preserve the dependence structure, and serves as a key component in self-normalized inference when the two time series have staggered observation periods. Intuitively, $\mathcal{I}_{x,i,j}$ and $\mathcal{I}_{y,i,j}$ are constructed to mimic the staggering pattern of the original data, so that the joint dependence between the original two time series with full length can be well approximated by that of the recursive pairs using the warped index sets when constructing the self-normalizer. To illustrate, we consider the simple example when $N_x = N_y = 100$ with $k = 1$ and $l = 51$. In this case, $N = N_x + N_y = 200$ and we observe $X_1, \ldots, X_{100}$ and $Y_{51}, \ldots, Y_{150}$, which have an overlap of length 50 equaling to half of their own length. If we apply the proposed time warping, then it can be seen that the recursive time-warped index pairs are $\mathcal{I}_{x,1,j} = \{50 - \lfloor j/4 \rfloor + 1, \ldots, 50 - \lfloor j/4 \rfloor + \lfloor j/2 \rfloor\}$ and $\mathcal{I}_{y,1,j} = \{50 + 1, \ldots, 50 + \lfloor j/2 \rfloor\}$, which continue to have an overlap equaling to approximately half of their own length. The idea is then to use $\mathcal{I}_{x,1,j}$ and $\mathcal{I}_{y,1,j}$ recursively over j to construct the time-warped self-normalizer, so that the associated recursive means will have asymptotically the

same joint dependence as the original $X_1, \ldots, X_{100}$ and $Y_{51}, \ldots, Y_{150}$. However, if we ignore the staggered observation periods and use $Z^{\star}_{\mathrm{S15}}$ in Section 3.3, then it amounts to the use of $X_1, \ldots, X_{\lfloor j/2 \rfloor}$ and $Y_{50+1}, \ldots, Y_{50+\lfloor j/2 \rfloor}$ when constructing the self-normalizer, which can have a different dependence structure when compared to the full data $X_1, \ldots, X_{100}$ and $Y_{51}, \ldots, Y_{150}$. Therefore, an appropriate time warping as carefully constructed in the current article plays an important role to preserve the dependence structure for valid self-normalized inference.

For any nonempty index set $\mathcal{I}$, we use $|\mathcal{I}|$ to denote its cardinality, and we define the averages $\bar{X}_{\mathcal{I}} = |\mathcal{I}|^{-1} \sum_{i \in \mathcal{I}} X_i$ and $\bar{Y}_{\mathcal{I}} = |\mathcal{I}|^{-1} \sum_{i \in \mathcal{I}} Y_i$. Then, based on the suitably constructed warped index sets $\mathcal{I}_{x,i,B_N}$ and $\mathcal{I}_{y,i,B_N}$ as proposed above, we can form the time-warped self-normalized statistic as

$$T_{i,B_N}(\Delta) = \frac{B_N(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}} - \Delta)^2}{B_N^{-2} \sum_{j=1}^{B_N} j^2 \{(\bar{X}_{\mathcal{I}_{x,i,j}} - \bar{Y}_{\mathcal{I}_{y,i,j}}) - (\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})\}^2}. \tag{3.5}$$

The presence of Δ allows the statistic to be used for assessing a more general null value of the difference $\mu_x - \mu_y$, and we write $T_{i,B_N} = T_{i,B_N}(0)$ for the null hypothesis of $\mu_x = \mu_y$. When $B_N = N$ and $i = 1$, we have $\mathcal{I}_{x,1,N} = \{j \in \mathbb{Z} : \ 1 \le j \le N_x\}$ and $\mathcal{I}_{y,1,N} = \{j \in \mathbb{Z} : \ l \le j \le l + N_y - 1\}$, in which case $T_{1,N}$ now represents the global time-warped self-normalized two-sample statistic.

We shall here provide a brief discussion on the connection and comparison with the conventional self-normalized statistic reviewed in Section 3.3. In particular, when the two time series share the same observation period with the same length for which $N_x = N_y$ and $k = l = 1$, then by Shao (2015) the self-normalized statistic $Z^{\star}_{\mathrm{S15}}$ in Section 3.3 will continue to work. In this case, there is no need to perform the additional time warping, and it can be shown that the proposed time-warped self-normalized statistic $T_{1,N}$ will automatically reduce to $Z^{\star}_{\mathrm{S15}}$, which can be an attractive feature. On the other hand, when the two time series to be compared do not share

the same observation period or when they are of different lengths, then the new self-normalized statistic $T_{1,N}$ can now be different from $Z^{\star}_{\mathrm{S15}}$ in Section 3.3 due to the new self-normalizer used in the construction of $T_{1,N}$ that incorporates an additional time warping between the two time series to preserve the underlying dependence structure. With the same time warping, the distribution of $T_{1,N}$ can then be well approximated by that of the time-warped self-normalized subsamples T_{i,B_N}.

We shall in the following present the detailed algorithm that describes and implements the proposed warped self-normalized two-sample test for time series with staggered observation periods.

(i) Given the observed time series $X_k, \ldots, X_{k+N_x-1}$ and $Y_l, \ldots, Y_{l+N_y-1}$ with $1 = k \leq l$, use (3.5) to compute the global warped self-normalized two-sample statistic $T_{1,N}$.

(ii) For a preselected B_N, use (3.5) to compute the warped self-normalized statistics for all subsamples of the designated size with Δ replaced by $\hat{\Delta} = \bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1}$ to neutralize the effect of the null for the subsamples, and denote them by $T_{i,B_N}(\hat{\Delta})$, $1 \leq i \leq M_N$, where

$$M_N = \min(k + N_x - l + 1 + \lfloor B_N(l-1)/N \rfloor - \lfloor B_N N_x/N \rfloor, N_y + 1 - \lfloor B_N N_y/N \rfloor).$$

(iii) Compute the $(1-\alpha)$-th quantile $\hat{q}_{T,1-\alpha}$ of $T_{i,B_N}(\hat{\Delta})$, $1 \leq i \leq M_N$, and reject the null hypothesis of $\mu_x = \mu_y$ at level α if $T_{1,N} > \hat{q}_{T,1-\alpha}$.

To provide theoretical justification of the proposed warped self-normalized method, we use the strong mixing framework of Rosenblatt (1956). In particular, let $\mathcal{F}_{a,b}$ be the σ-field generated by $(X_a, Y_a), \ldots, (X_b, Y_b)$ for $-\infty \leq a \leq b \leq \infty$, then the strong

mixing coefficient is defined as

$$\alpha(k) = \sup_{A \in \mathcal{F}_{-\infty,0},\ B \in \mathcal{F}_{k,\infty}} |\mathrm{pr}(A \cap B) - \mathrm{pr}(A)\mathrm{pr}(B)|.$$

Let Υ_k be the covariance matrix between the two vectors (X_i, Y_i) and (X_{i+k}, Y_{i+k}), then the long-run covariance matrix of the process (X_i, Y_i) is given by $\Upsilon = \sum_{k \in \mathbb{Z}} \Upsilon_k$. We make the following assumptions.

(S1) The process (X_i, Y_i) is strong mixing with

$$E(|X_i|^\iota) < \infty,\ E(|Y_i|^\iota) < \infty,\ \sum_{k=1}^{\infty} \{\alpha(k)\}^{1-2/\iota} < \infty$$

for some $\iota > 2$.

(S2) The long-run covariance matrix Υ is positive definite.

Assumptions (S1) and (S2) are standard primitive conditions for the invariance principle (IP) in Section 3.3; see for example Phillips and Durlauf (1986). The strong mixing condition has been widely used to study limit theorems of dependent processes, and we refer to the survey paper by Bradley (2005) and the book by Bradley (2007) for a detailed review and additional references. Theorem 7 provides the asymptotic justification of the proposed warped self-normalized subsampling method.

Theorem 7. *Assume (S1), (S2),* $N_x/N \to p_x \in (0,1)$, $N_y/N \to p_y \in (0,1)$ *and* $l/N \to \ell \in [0,1)$ *as* $N \to \infty$. *If* $B_N \to \infty$ *and* $B_N/N \to 0$, *then for any* $\alpha \in (0,1)$ *we have (i) under the null hypothesis of* $\mu_x = \mu_y$,

$$\mathrm{pr}(T_{1,N} > \hat{q}_{T,1-\alpha}) \to \alpha;$$

and (ii) under the alternative when $N^{1/2}|\mu_x - \mu_y| \to \infty$,

$$\mathrm{pr}(T_{1,N} > \hat{q}_{T,1-\alpha}) \to 1.$$

By Theorem 7, the asymptotic size of the proposed warped self-normalized two-

sample test is guaranteed to converge to its nominal level under the null, and the power of the proposed test will converge to one under the alternative when $N^{1/2}|\mu_x - \mu_y| \to \infty$. The proposed warped self-normalized two-sample test provides a unified solution to different staggering schemes of the two time series, for example when the two time series share the same starting time but have different ending times, when one time series is observed on a time subset of the other, or when the two time series are observed on time intervals that have overlapping and nonoverlapping segments. This makes it convenient to use in practice as one does not need to perform a case-by-case study and can simply apply the proposed method to obtain a meaningful p-value justified by a solid statistical theory. We shall in the following section consider extending the proposed test to quantities beyond the mean, such as the variance or quantiles.

3.4.2 The General Case: Quantities Beyond the Mean

Due to the incorporation of self-normalization, the proposed method can be easily extended to cases beyond the mean. For this, suppose one is interested in comparing the quantity $\theta_x = Q(F_{x,d})$ with $\theta_y = Q(F_{y,d})$, where Q is a functional that maps the d-dimensional marginal distributions $F_{x,d}$ and $F_{y,d}$ of the two time series to their respective quantities of interest. As illustrated in Shao (2010) and Zhang and Lavitas (2018), such a framework covers many commonly used quantities of interest as special cases. For example, if we set $d = 1$ and use $F_{x,1}$ to denote the marginal distribution, then taking $Q(F_{x,1}) = \int_{\mathbb{R}} uF_{x,1}(u)du$ leads us to the mean case considered in Section 3.4.1. As additional examples, we can set $Q(F_{x,1}) = \int_{\mathbb{R}} u^2F_{x,1}(u)du - \{\int_{\mathbb{R}} uF_{x,1}(u)du\}^2$ to focus on the variance case and $Q(F_{x,1}) = F_{x,1}^{-1}(q)$ for some $q \in (0,1)$ to focus on the quantile case; see also Shao (2010) and Zhang and Lavitas (2018) for further discussions. For index sets $\mathcal{I}_x$ and $\mathcal{I}_y$, let $\hat{F}_{x,d,\mathcal{I}_x}$ and $\hat{F}_{y,d,\mathcal{I}_y}$ be the associated empirical distributions calculated from $(X_i)_{i\in\mathcal{I}_x}$ and $(Y_i)_{i\in\mathcal{I}_y}$ respectively, then $\hat{\theta}_{x,\mathcal{I}_x} = Q(\hat{F}_{x,d,\mathcal{I}_x})$

and $\hat{\theta}_{y,\mathcal{I}_y} = Q(\hat{F}_{y,d,\mathcal{I}_y})$ represent the parameter estimators calculated from $(X_i)_{i\in\mathcal{I}_x}$ and $(Y_i)_{i\in\mathcal{I}_y}$ respectively. We can now form the time-warped self-normalized statistic as

$$T^{\theta}_{i,B_N}(\Delta) = \frac{B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}} - \Delta)^2}{B_N^{-2}\sum_{j=1}^{B_N} j^2\{(\hat{\theta}_{x,\mathcal{I}_{x,i,j}} - \hat{\theta}_{y,\mathcal{I}_{y,i,j}}) - (\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\}^2}, \tag{3.6}$$

which extends (3.5) from the mean case to more general quantities. Similarly, we write $T^{\theta}_{i,B_N} = T^{\theta}_{i,B_N}(0)$ for the null hypothesis of $\theta_x = \theta_y$. By substituting this generalized statistic (3.6) into the algorithm described in Section 3.4.1, we can similarly compute the global time-warped self-normalized statistic $T^{\theta}_{1,N}$ along with its subsample version $T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})$ for $1 \le i \le M_N$ where $\hat{\Delta}^{\theta} = \hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}}$. Let $\hat{q}_{T^{\theta},1-\alpha}$ be the $(1-\alpha)$-th quantile of $T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})$, $1 \le i \le M_N$, we can then reject the null hypothesis of $\theta_x = \theta_y$ at level α if $T^{\theta}_{1,N} > \hat{q}_{T^{\theta},1-\alpha}$. To provide the theoretical justification under this more general setting, we introduce the notion of influence function (Hampel et al., 1986) defined as

$$\mathrm{IF}_Q(z, F_d) = \lim_{\varepsilon\downarrow 0} \frac{Q\{(1-\varepsilon)F_d + \varepsilon\delta_z\} - Q(F_d)}{\varepsilon},$$

where $z \in \mathbb{R}^d$ and δ_z denotes the point mass at z. Then for any index set $\mathcal{I}$, we can follow Shao (2010) and apply the expansion

$$\hat{\theta}_{x,\mathcal{I}} = Q(\hat{F}_{x,d,\mathcal{I}}) = Q(F_{x,d}) + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \mathrm{IF}_Q(X_i, F_{x,d}) + R_{x,\mathcal{I}}$$

and

$$\hat{\theta}_{y,\mathcal{I}} = Q(F_{y,d}) + \frac{1}{|\mathcal{I}|}\sum_{i\in\mathcal{I}} \mathrm{IF}_Q(Y_i, F_{y,d}) + R_{y,\mathcal{I}},$$

where $\mathrm{E}\{\mathrm{IF}_Q(X_i, F_{x,d})\} = \mathrm{E}\{\mathrm{IF}_Q(Y_i, F_{y,d})\} = 0$ and $R_{\mathcal{I}} = (R_{x,\mathcal{I}}, R_{y,\mathcal{I}})^{\top}$ represents the remainder term. We make the following assumptions.

(C1) The process (X_i, Y_i) is strong mixing with

$$E\{|\mathrm{IF}_Q(X_i, F_{x,d})|^\iota\} < \infty, \quad E\{|\mathrm{IF}_Q(Y_i, F_{y,d})|^\iota\} < \infty,$$
$$\sum_{k=1}^{\infty} \{\alpha(k)\}^{1-2/\iota} < \infty$$

for some $\iota > 2$.

(C2) The long-run covariance matrix of the influence function process $\{\mathrm{IF}_Q(X_i, F_{x,d}), \mathrm{IF}_Q(Y_i, F_{y,d})\}^\top$ is positive definite.

(C3) The remainder term $|\mathcal{I}|R_{\mathcal{I}} = o_p(N^{1/2})$ uniformly over $\mathcal{I} = \{i \in \mathbb{Z} : \ Ns \le i \le Nt\}$ for $0 \le s \le t \le 1$ for all large N.

Assumptions (C1)–(C3) are standard in self-normalized inference; see for example Shao (2010) and the discussions therein. Intuitively, Assumptions (C1) and (C2) imply an invariance principle for the influence functions which generalizes the mean invariance principle (IP) in Section 3.3. Assumption (C3) is a negligibility condition on the remainder term, which is also mild; see for example the discussion in Shao (2010) and its verification for the class of smooth function models. We also remark that it is possible to replace or alleviate Assumptions (C1)–(C3) by, for example, the functional delta approach as in Volgushev and Shao (2014). Theorem 8 provides the asymptotic justification of the proposed warped self-normalized subsampling method for quantities beyond the mean.

Theorem 8. *Assume (C1)–(C3), $N_x/N \to p_x \in (0,1)$, $N_y/N \to p_y \in (0,1)$ and $l/N \to \ell \in [0,1)$ as $N \to \infty$. If $B_N \to \infty$ and $B_N/N \to 0$, then for any $\alpha \in (0,1)$ we have (i) under the null hypothesis of $\theta_x = \theta_y$,*

$$\mathrm{pr}(T_{1,N}^\theta > \hat{q}_{T^\theta, 1-\alpha}) \to \alpha;$$

and (ii) under the alternative when $N^{1/2}|\theta_x - \theta_y| \to \infty$ we have

$$\mathrm{pr}(T_{1,N}^\theta > \hat{q}_{T^\theta, 1-\alpha}) \to 1.$$

Although in practice one is often interested in testing the null hypothesis of $\theta_x = \theta_y$ or equivalently $\theta_x - \theta_y = 0$, it is possible to extend the proposed time-warped self-normalized test to handle the more general null hypothesis of $\theta_x - \theta_y = \Delta$ for some prespecified $\Delta \in \mathbb{R}$. For this, we use the global time-warped self-normalized statistic $T^{\theta}_{1,N}(\Delta)$ and follow the algorithm described in Section 3.4.1 to obtain its subsample version $T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})$ for $1 \leq i \leq M_N$ where $\hat{\Delta}^{\theta} = \hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}}$. Note that the subsample statistics $T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})$, $1 \leq i \leq M_N$, use the estimate $\hat{\Delta}^{\theta}$ in their construction, and as a result the associated subsampling distribution will not be affected by whether the null $\theta_x - \theta_y = \Delta$ is true or not. Corollary 1 provides the theoretical justification of the proposed time-warped self-normalized test for handling such a more general null hypothesis.

Corollary 1. *Assume (C1)–(C3), $N_x/N \to p_x \in (0,1)$, $N_y/N \to p_y \in (0,1)$ and $l/N \to \ell \in [0,1)$ as $N \to \infty$. If $B_N \to \infty$ and $B_N/N \to 0$, then for any $\alpha \in (0,1)$ we have (i) under the null hypothesis of $\theta_x - \theta_y = \Delta$,*

$$\mathrm{pr}(T^{\theta}_{1,N}(\Delta) > \hat{q}_{T^{\theta},1-\alpha}) \to \alpha;$$

and (ii) under the alternative when $N^{1/2}|(\theta_x - \theta_y) - \Delta| \to \infty$ we have

$$\mathrm{pr}(T^{\theta}_{1,N}(\Delta) > \hat{q}_{T^{\theta},1-\alpha}) \to 1.$$

The proposed warped self-normalized subsampling method provides a unified approach for two-sample testing of mean and other quantities when the two time series to be compared can depend on each other, be collected on staggered time periods, and have different lengths. In contrast, existing results are often developed for the setting when the two time series are either independent or collected on the same period, and may no longer work in the current more general setting; see for example the discussion in Section 3.2 and 3.3.

3.5 Implementation

3.5.1 Simulation Results

We shall here provide a simulation study to examine the finite-sample performance of the proposed warped self-normalized subsampling method, denoted by WSNS hereafter. We also make a comparison with the two-sample subsampling approach of Politis and Romano (2010), denoted by PR10 hereafter, and the self-normalized two-sample inference of Shao (2015), denoted by S15 hereafter. For this, let (ζ_i) and (ξ_i) be independent autoregressive processes generated as

$$\begin{pmatrix} \zeta_i \\ \xi_i \end{pmatrix} = \rho \begin{pmatrix} \zeta_{i-1} \\ \xi_{i-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1i} \\ \epsilon_{2i} \end{pmatrix},$$

where ϵ_{1i} and ϵ_{2i}, $i \in \mathbb{Z}$, are independent standard normal random variables. Additional simulation results for other innovation types are provided in Section 3.8. We consider the joint process

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ r & \sqrt{1-r^2} \end{pmatrix} \begin{pmatrix} \zeta_i \\ \xi_i \end{pmatrix},$$

where r controls the degree of dependence between the two time series (X_i) and (Y_i). The observations are taken as $X_k, \ldots, X_{k+N_x-1}$ and $Y_l, \ldots, Y_{l+N_y-1}$, for which we consider three scenarios.

- Scenario 1: $k = l = 1$, $N_x = N_y = 500s$, $s \in \{1, 2, 3\}$, which represents the situation when the two time series share the same observation period.

- Scenario 2: $k = 1$, $l = 100s + 1$, $N_x = N_y = 600s$, $s \in \{1, 2, 3\}$, which represents the situation when the two time series are recorded on shifted observation periods.

- Scenario 3: $k = 1$, $l = 250s + 1$, $N_x = 750s$, $N_y = 500s$, $s \in \{1, 2, 3\}$, which

represents the situation when the observation period of one time series is a subset of the other.

For all the three scenarios, the overlapping length between (X_i) and (Y_i) is set as $N_c = 500s$, $s \in \{1, 2, 3\}$. For each realization, we apply the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method of Shao (2015) to perform two-sample tests on the mean, median and variance of the two time series. Let $\rho = 0.3$, $B_N = 100$, and $r \in \{0, 0.4, 0.8\}$, the results based on 1000 realizations for each setting are summarized in Tables 3.1–3.3 for Scenarios 1–3 respectively. Additional simulation results for other choices of ρ and B_N are provided in Section 3.8. From the simulation results, we can observe the followings.

First, the PR10 method of Politis and Romano (2010) was developed for independent time series and is therefore expected to work when $r = 0$. When there exists dependence between the two time series to be compared, however, it is no longer guaranteed to work and it can be seen from Tables 3.1–3.3 that the PR10 method starts to exhibit a certain degree of size distortions when $r = 0.4$ and further deteriorates when $r = 0.8$. For example, if we consider the mean case with $r = 0.8$ in Table 3.1, then the empirical coverage probabilities of the PR10 method are distorted to 1.000, 1.000 and 1.000 at 90%, 95% and 99% nominal levels even when $N_c = 1500$. Note that the PR10 method requires estimating the normalizing sequence of the test statistic, which relates to the long-run variance of a dependent process and we use the banding estimate as described in Zhang (2021). As a comparison, the S15 method of Shao (2015) uses a self-normalizer to pivotalize the two-sample test statistic that avoids direct estimation of the normalizing sequence. It can be seen from Table 3.1 that the S15 method performs reasonably well under Scenario 1 even when the two time series depend on each other with $r = 0.4$ or $r = 0.8$. However, as discussed in Section 3.3, the S15 method described in Shao (2015) is not directly applicable to ad-

dress the situation when the two time series have different starting times unless they are independent. In particular, it can be seen from Table 3.2 that the S15 method performs reasonably well under Scenario 2 when $r = 0$ but exhibits size distortions similar to the PR10 method when $r = 0.4$ and $r = 0.8$. For example, if we consider the mean case with $r = 0.8$ in Table 3.2, then the empirical coverage probabilities of the S15 method are distorted to 0.989, 0.995 and 1.000 at 90%, 95% and 99% nominal levels even when $N_c = 1500$. In comparison, the empirical coverage probabilities of the proposed WSNS method for the same setting are 0.922, 0.964 and 0.993 which are reasonably close to their 90%, 95% and 99% nominal levels. The main reason is that, when the two time series exhibit joint dependence and are observed on staggered observation periods, the original self-normalizer as used in Shao (2015) may no longer be able to pivotalize the two-sample statistic; see also the discussion in Section 3.3. By incorporating an appropriate time-warping into the construction of a new time-warped self-normalizer as proposed in Section 3.4, our WSNS method can effectively handle time series exhibiting joint dependence with staggered observation periods. In addition, it provides a unified approach to handle the mean and other quantities such as the median and variance. It can be seen from Tables 3.1–3.3 that the proposed WSNS method seems to perform reasonably well as the empirical coverage probabilities are mostly close to their nominal levels for Scenarios 1–3 no matter if the two time series are independent ($r = 0$) or dependent ($r = 0.4$ and $r = 0.8$); see also the additional simulation results in the supplementary material.

3.5.2 Minimum Volatility Bandwidth Choice

The problem of selecting a bandwidth or block size has been widely studied in the subsampling literature; see for example Hall et al. (1995a), Hall et al. (1995b), Politis and White (2004), and references therein. In our data analysis in Section 3.5.3, we follow Politis et al. (1999) and consider a minimum volatility bandwidth choice which

Table 3.1: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.855	0.911	0.954	0.873	0.923	0.966	0.852	0.908	0.957
		PR10	0.852	0.915	0.966	0.863	0.912	0.976	0.867	0.928	0.975
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.878	0.928	0.976	0.874	0.922	0.976	0.866	0.924	0.973
		PR10	0.873	0.933	0.982	0.868	0.923	0.984	0.877	0.942	0.989
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.893	0.937	0.982	0.895	0.950	0.992	0.874	0.925	0.979
		PR10	0.884	0.934	0.985	0.879	0.945	0.991	0.882	0.942	0.986
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.861	0.911	0.966	0.857	0.921	0.977	0.846	0.914	0.965
		PR10	0.941	0.972	0.992	0.936	0.967	0.990	0.901	0.944	0.985
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.875	0.923	0.978	0.878	0.921	0.978	0.871	0.919	0.974
		PR10	0.965	0.980	0.997	0.927	0.973	0.995	0.908	0.960	0.992
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.889	0.938	0.983	0.894	0.944	0.981	0.871	0.927	0.972
		PR10	0.965	0.989	0.998	0.950	0.976	0.997	0.916	0.960	0.989
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.867	0.920	0.968	0.864	0.914	0.964	0.843	0.907	0.962
		PR10	0.995	0.997	1.000	0.986	0.997	0.999	0.976	0.993	1.000
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.883	0.936	0.980	0.882	0.931	0.978	0.869	0.925	0.971
		PR10	1.000	1.000	1.000	0.993	1.000	1.000	0.989	0.997	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.887	0.926	0.983	0.868	0.929	0.984	0.887	0.928	0.974
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.984	0.995	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.2: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.861	0.914	0.967	0.876	0.923	0.975	0.825	0.893	0.957
		PR10	0.858	0.929	0.974	0.866	0.918	0.976	0.846	0.919	0.973
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.895	0.937	0.979	0.884	0.938	0.988	0.863	0.918	0.967
		PR10	0.887	0.940	0.979	0.881	0.941	0.984	0.889	0.945	0.984
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.900	0.953	0.988	0.902	0.954	0.987	0.881	0.932	0.983
		PR10	0.903	0.942	0.982	0.892	0.950	0.982	0.888	0.941	0.993
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.885	0.929	0.970	0.862	0.922	0.975	0.840	0.908	0.965
		PR10	0.922	0.965	0.994	0.919	0.950	0.989	0.887	0.943	0.976
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.902	0.942	0.984	0.896	0.951	0.990	0.871	0.929	0.981
		PR10	0.936	0.973	0.994	0.930	0.973	0.992	0.918	0.958	0.992
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.909	0.954	0.991	0.908	0.961	0.994	0.876	0.937	0.985
		PR10	0.953	0.978	0.996	0.936	0.977	0.996	0.915	0.963	0.994
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.895	0.938	0.984	0.878	0.927	0.981	0.893	0.938	0.975
		PR10	0.992	0.999	1.000	0.971	0.987	0.998	0.962	0.986	0.997
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.915	0.960	0.994	0.899	0.948	0.986	0.897	0.949	0.982
		PR10	0.995	0.998	1.000	0.980	0.994	1.000	0.982	0.994	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.922	0.964	0.993	0.892	0.957	0.994	0.890	0.937	0.981
		PR10	0.991	0.998	1.000	0.980	0.994	1.000	0.980	0.996	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.3: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.851	0.906	0.970	0.869	0.923	0.971	0.856	0.907	0.959
		PR10	0.860	0.920	0.975	0.865	0.921	0.972	0.878	0.932	0.966
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.886	0.939	0.977	0.874	0.934	0.984	0.876	0.932	0.977
		PR10	0.893	0.933	0.980	0.881	0.935	0.982	0.884	0.939	0.988
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.894	0.945	0.992	0.903	0.961	0.991	0.866	0.933	0.986
		PR10	0.896	0.940	0.980	0.884	0.933	0.985	0.881	0.939	0.988
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.873	0.929	0.972	0.860	0.931	0.975	0.863	0.905	0.960
		PR10	0.933	0.970	0.994	0.919	0.964	0.993	0.896	0.939	0.982
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.882	0.943	0.986	0.872	0.927	0.988	0.884	0.938	0.972
		PR10	0.935	0.976	0.995	0.925	0.965	0.992	0.910	0.951	0.986
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.877	0.938	0.988	0.896	0.948	0.990	0.866	0.931	0.982
		PR10	0.947	0.979	0.992	0.932	0.969	0.997	0.913	0.961	0.994
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.895	0.942	0.980	0.870	0.920	0.971	0.879	0.925	0.965
		PR10	0.991	0.997	1.000	0.967	0.990	1.000	0.968	0.987	0.998
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.887	0.947	0.988	0.884	0.935	0.982	0.865	0.923	0.972
		PR10	0.989	0.998	1.000	0.980	0.996	1.000	0.967	0.986	0.998
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.895	0.952	0.990	0.877	0.936	0.989	0.869	0.927	0.973
		PR10	0.992	0.999	1.000	0.981	0.994	1.000	0.974	0.991	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

can be obtained as follows.

(i) For any bandwidth B in a suitable range, we take a small integer K to form a neighborhood $\mathcal{N}_K(B) = \{B - K, \ldots, B + K\}$.

(ii) For each bandwidth in $\mathcal{N}_K(B)$, we compute the associated subsampling cut-off threshold, denoted by $\hat{q}_{T,1-\alpha}(B - K), \ldots, \hat{q}_{T,1-\alpha}(B + K)$.

(iii) We compute the standard deviation of $\hat{q}_{T,1-\alpha}(B - K), \ldots, \hat{q}_{T,1-\alpha}(B + K)$, and choose the bandwidth that minimizes such a standard deviation.

As commented in Politis et al. (1999), the choice of K is generally not very important and a choice of $K = 2$ or $K = 3$ usually suffices. We use $K = 5$ in our data analysis in Section 3.5.3.

3.5.3 Application to a Precipitation Data

We shall here apply the proposed method to a precipitation data to compare the monthly precipitation series in the Northwest England and Wales (NwEW) region and the Northern Ireland (NI) region of the United Kingdom (UK). The data is available from the Met Office Hadley Centre website at `https://www.metoffice.gov.uk`, and time series plots are provided in Figure 3·1. For the NwEW region, the precipitation series begins in January 1873, while for the NI region it begins in January 1931. Therefore, one of the time series has an earlier starting time than the other and the two time series are of different lengths. This makes it desirable to use the proposed method for their two-sample tests, and we use all the available data through December 2021 in the analysis. The sample size for the NwEW series is 1788 and the sample size for the NI series is 1092. We consider testing if the NwEW region and the NI region share the same mean, median and variance for their history precipitation. The subsampling bandwidth is selected using the minimum volatility method described

Table 3.4: Two-sample test results of the monthly precipitation in the Northwest England and Wales (NwEW) region and Northern Ireland (NI) region of the United Kingdom (UK).

Quantity	$T^{\theta}_{1,N}$	p-value
Mean	3.70	0.501
Median	9.12	0.356
Variance	253.59	0.000

in the supplementary material, and the selected bandwidths are $B_N = 43$, 79 and 132 for the mean, median and variance respectively. The results are summarized in Table 3.4, from which we can see that, at the 5% significance level, the NwEW region and the NI region seem to possess the same mean and median in precipitation with p-values 0.501 and 0.356 respectively. However, the p-value for the variance test is 0.000 indicating that the NwEW region and the NI region have statistically different variations in their precipitation. This could possibly be related to the mountainous terrain in the NwEW region that makes its rainfall vary a lot across its different subareas. In particular, according to the fact sheet published by the Met Office (2013), the moist westerly winds in general tend to produce orographic rainfalls that lead to more precipitation over the mountains. On the other hand, as commented in the climatological memorandum published by the Meteorological Office (1983), the protected areas behind the mountains can be much drier than even the driest zones of NI. Therefore, our analysis seems to provide the statistical support that complements the climatological intuition.

3.6 Conclusion

Applications from various disciplines often desire the comparison of a given quantity from two time series in the form of a two-sample test. In practice, there are many circumstances that can cause the two time series to depend on each other and be

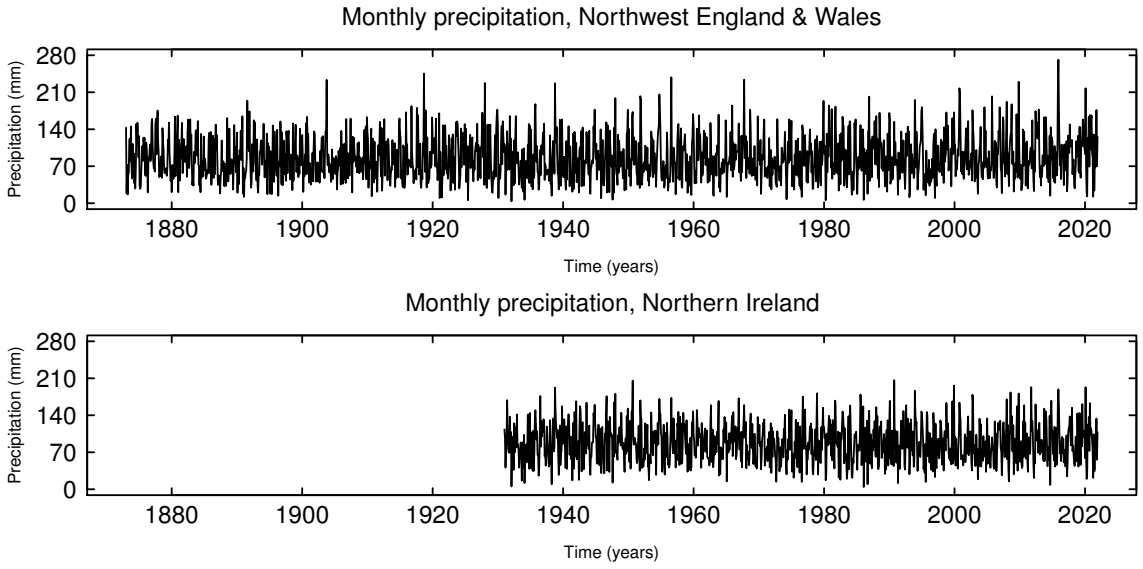

Figure 3·1: Time series plots for monthly precipitation records in the Northwest England and Wales (NwEW) region (top) and the Northern Ireland (NI) region (bottom) through December 2021.

recorded on different time periods. For example, climate science data recorded in different regions often exhibit joint dependence, and one region may get monitored earlier than the other. The joint dependence between the two time series together with their staggered observation periods make many existing methods not directly applicable, and it is desirable to propose a new method to handle such situations. We consider incorporating a suitable time domain warping into the construction of a new time-warped self-normalizer as proposed in Section 3.4, and the resulting method can effectively handle two-sample testing of time series that exhibit joint dependence with staggered observation periods. In addition, it provides a unified approach to handle the mean and other quantities such as the variance or quantiles which can be a convenient feature for practitioners. By applying the proposed method to a precipitation data, we found that the NwEW region and the NI region in UK share the same mean and median in their history precipitation but have statistically different variations. This provides the statistical support and complements the climatological

intuition that the mountainous subareas in NwEW tend to receive much more rainfalls due to orographic precipitation than the protected subarea that is often much drier than the NI.

3.7 Technical Proofs

3.7.1 Proof of Theorem 5

Proof. Recall that $N = N_x + N_y$, it suffices to show that, as $N \to \infty$,

$$\mathrm{E}[\{\hat{F}_{\mathrm{PR10}}(u) - \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy}u)\}^2] \to 0.$$

Note that $\mathrm{E}[\hat{F}_{\mathrm{PR10}}(u) - \mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\}] = 0$, we can decompose the above into

$$\begin{aligned} & \mathrm{E}[\{\hat{F}_{\mathrm{PR10}}(u) - \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy}u)\}^2] \\ = \; & \mathrm{E}([\hat{F}_{\mathrm{PR10}}(u) - \mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\}]^2) + \mathrm{E}([\mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\} - \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy}u)]^2) \\ = \; & \mathrm{var}\{\hat{F}_{\mathrm{PR10}}(u)\} + [\mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\} - \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy}u)]^2. \end{aligned}$$

We first deal with the term $[\mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\} - \mathrm{pr}(Z_{\mathrm{PR10}} \leq c_{xy}u)]^2$. For this, recall that

$$\hat{F}_{\mathrm{PR10}}(u) = q_x^{-1} q_y^{-1} \sum_{i=1}^{q_x} \sum_{j=1}^{q_y} \mathrm{I}(H_{i,j} \leq u).$$

where

$$H_{i,j} = (B_x^{-1} g_x + B_y^{-1} g_y)^{-1/2} (\bar{X}_{i,i+B_x-1} - \bar{Y}_{j,j+B_y-1}),$$

Define $\mathcal{S}_1(\eta) = \{(i,j) : \; |j-i| > \eta, \; 1 \leq i \leq q_x, \; 1 \leq j \leq q_y\}$, then by the m-dependence structure we have $\mathrm{cov}(\bar{X}_{i,i+B_x-1}, \bar{Y}_{i+k,i+k+B_y-1}) = 0$ if $|k| > m + \max(B_x, B_y)$. As a result, for any index pair $(i,j) \in \mathcal{S}_1\{m + \max(B_x, B_y)\}$, we have

$$\mathrm{cov}(\bar{X}_{i,i+B_x-1}, \bar{Y}_{j,j+B_y-1}) = 0$$

and as a result

$$\begin{aligned} & (B_x^{-1}g_x + B_y^{-1}g_y)^{-1}\mathrm{var}(\bar{X}_{i,i+B_x-1} - \bar{Y}_{j,j+B_y-1}) \\ = \; & (B_x^{-1}g_x + B_y^{-1}g_y)^{-1}\{\mathrm{var}(\bar{X}_{i,i+B_x-1}) + \mathrm{var}(\bar{Y}_{j,j+B_y-1}) \\ & \qquad -2\mathrm{cov}(\bar{X}_{i,i+B_x-1}, \bar{Y}_{j,j+B_y-1})\} \\ \to \; & 1. \end{aligned}$$

Under the null hypothesis of $\mu_x = \mu_y$, one has $\mathrm{E}(\bar{X}_{i,i+B_x-1} - \bar{Y}_{j,j+B_y-1}) = 0$ and thus $H_{i,j}$ follows a standard normal asymptotic distribution for any $(i,j) \in \mathcal{S}_1\{m + \max(B_x, B_y)\}$. On the other hand, since $N_x = N_y$, the variance of the global difference satisfies

$$(N_x^{-1}g_x + N_x^{-1}g_y - N_x^{-1}a_{xy})^{-1}\mathrm{var}(\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1}) \to 1,$$

and as a result

$$\begin{aligned} c_{xy}^{-1}Z_{\mathrm{PR10}} &= \left(\frac{g_x + g_y - a_{xy}}{g_x + g_y}\right)^{-1/2} N_x^{1/2}(g_x + g_y)^{-1/2}(\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1}) \\ &= (g_x + g_y - a_{xy})^{-1/2}N_x^{1/2}(\bar{X}_{k,k+N_x-1} - \bar{Y}_{l,l+N_y-1}) \end{aligned}$$

follows a standard normal asymptotic distribution. Let $|\mathcal{S}_1\{m+\max(B_x, B_y)\}|$ denote the cardinality of $\mathcal{S}_1\{m + \max(B_x, B_y)\}$, then $q_x^{-1}q_y^{-1}|\mathcal{S}_1\{m + \max(B_x, B_y)\}| \to 1$ as $N \to \infty$ and we can write

$$\begin{aligned} \hat{F}_{\mathrm{PR10}}(u) &= q_x^{-1}q_y^{-1}\sum_{i=1}^{q_x}\sum_{k=-i+1}^{-i+q_y} \mathrm{I}(H_{i,i+k} \le u) \\ &= q_x^{-1}q_y^{-1}\sum_{i=1}^{q_x}\sum_{k=\max\{-m-\max(B_x,B_y),-i+1\}}^{\min\{m+\max(B_x,B_y),-i+q_y\}} \mathrm{I}(H_{i,i+k} \le u) \\ &\quad + q_x^{-1}q_y^{-1}\sum_{i=1}^{q_x}\sum_{j\in\{1\le l\le q_y:\ |l-i|>m+\max(B_x,B_y)\}} \mathrm{I}(H_{i,j} \le u). \end{aligned}$$

Note that

$$q_x^{-1}q_y^{-1}\sum_{i=1}^{q_x}\sum_{k=\max\{-m-\max(B_x,B_y),-i+1\}}^{\min\{m+\max(B_x,B_y),-i+q_y\}} \mathrm{I}(H_{i,i+k} \le u) \le \frac{2q_x\{m + \max(B_x, B_y)\}}{q_xq_y} \to 0,$$

we have $[\mathrm{E}\{\hat{F}_{\mathrm{PR10}}(u)\} - \mathrm{pr}(c_{xy}^{-1} Z_{\mathrm{PR10}} \le u)]^2 \to 0$ as $N \to \infty$. It now suffices to show that $\mathrm{var}\{\hat{F}_{\mathrm{PR10}}(u)\} \to 0$. For this, we define $\Omega = \{(i,j,i',j') : \ 1 \le i, \ i' \le q_x, \ 1 \le j, \ j' \le q_y\}$ and $\mathcal{S}_2(\eta) = \{(i,j,i',j') : \ \min(|i-i'|, |i-j'|, |j-i'|, |j-j'|) > \eta, \ 1 \le i, \ i' \le q_x, \ 1 \le j, \ j' \le q_y\}$. Then by the m-dependence structure, for any quadruple (i,j,i',j') that satisfies $|i-i'| > m + B_x$, $|j-j'| > m + B_y$, $|i-j'| > m + \max(B_x, B_y)$ and $|j-i'| > m + \max(B_x, B_y)$, we have $\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\} = 0$. As a result, $\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\} = 0$ if $(i,j,i',j') \in \mathcal{S}_2\{m + \max(B_x, B_y)\}$. Note that $|\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\}| \le 1$, we have

$$
\begin{aligned}
& q_x^{-2} q_y^{-2} \sum_{(i,j,i',j') \in \Omega \setminus \mathcal{S}_2\{m + \max(B_x, B_y)\}} |\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\}| \\
\le \ & 2 q_x^{-2} q_y^{-2} \{m + \max(B_x, B_y)\}(q_x q_y^2 + q_x q_y^2 + q_y q_x^2 + q_y q_x^2) \\
\le \ & \frac{8\{m + \max(B_x, B_y)\}}{\min\{q_x, q_y\}} \to 0.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\mathrm{var}\{\hat{F}_{\mathrm{PR10}}(u)\} &= q_x^{-2} q_y^{-2} \sum_{i=1}^{q_x} \sum_{i'=1}^{q_x} \sum_{j=1}^{q_y} \sum_{j'=1}^{q_y} \mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\} \\
&\le q_x^{-2} q_y^{-2} \sum_{(i,j,i',j') \in \mathcal{S}_2\{m + \max(B_x, B_y)\}} |\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\}| \\
&\quad + q_x^{-2} q_y^{-2} \sum_{(i,j,i',j') \in \Omega \setminus \mathcal{S}_2\{m + \max(B_x, B_y)\}} |\mathrm{cov}\{\mathrm{I}(H_{i,j} \le u), \mathrm{I}(H_{i',j'} \le u)\}|
\end{aligned}
$$

which goes to zero as $N \to \infty$, entailing the desired result. □

3.7.2 Proof of Theorem 6

Proof. Given that $k = 1$, we can write

$$
D_{i,N}^{\star} = \bar{X}_{1, \lfloor i N_x / N \rfloor} - \bar{Y}_{l, l + \lfloor i N_y / N \rfloor - 1}, \quad i = 1, \ldots, N,
$$

and under the null hypothesis of $\mu_x = \mu_y$ we have

$$\begin{aligned}\frac{i}{\sqrt{N}} D^\star_{i,N} &= \left(\frac{i}{\lfloor iN_x/N \rfloor}, 0\right) N^{-1/2} \sum_{j=1}^{\lfloor iN_x/N \rfloor} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \\ &\quad - \left(0, \frac{i}{\lfloor iN_y/N \rfloor}\right) N^{-1/2} \left\{ \sum_{j=1}^{l-1+\lfloor iN_y/N \rfloor} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \right. \\ &\qquad \left. - \sum_{j=1}^{l-1} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \right\}.\end{aligned}$$

Since $p_x = p_y = 0.5$ when $N_x = N_y$, under assumption (IP) we have the weak convergence of

$$\frac{\lfloor Nt \rfloor}{\sqrt{N}} D^\star_{\lfloor Nt \rfloor, N} \Rightarrow V^\star_\ell(t; a, b, c)$$

on $t \in [0, 1]$. Note that we can write

$$\begin{aligned}N^{-2} \sum_{i=1}^N i^2 (D^\star_{i,N} - D^\star_{N,N})^2 &= \sum_{i=1}^N \frac{1}{N} \left(\frac{i}{\sqrt{N}} D^\star_{i,N} - \frac{i}{N} \frac{N}{\sqrt{N}} D^\star_{N,N} \right)^2 \\ &= \sum_{i=1}^N \int_{(i-1)/N}^{i/N} \left(\frac{\lfloor Nt \rfloor}{\sqrt{N}} D^\star_{\lfloor Nt \rfloor, N} - \frac{\lfloor Nt \rfloor}{N} \frac{N}{\sqrt{N}} D^\star_{N,N} \right)^2 dt \\ &= \int_0^1 \left(\frac{\lfloor Nt \rfloor}{\sqrt{N}} D^\star_{\lfloor Nt \rfloor, N} - \frac{\lfloor Nt \rfloor}{N} \frac{N}{\sqrt{N}} D^\star_{N,N} \right)^2 dt,\end{aligned}$$

then by assumption (IP) we can obtain the weak convergence of

$$N^{-2} \sum_{i=1}^N i^2 (D^\star_{i,N} - D^\star_{N,N})^2 \to_d \int_0^1 \left\{ V^\star_\ell(t; a, b, c) - t V^\star_\ell(1; a, b, c) \right\}^2 dt$$

on $t \in [0, 1]$, and this can be made jointly with the weak convergence of $\frac{\lfloor Nt \rfloor}{\sqrt{N}} D^\star_{\lfloor Nt \rfloor, N}$. The result then follows by an application of the continuous mapping theorem. □

3.7.3 Proof of Theorem 7

Proof. We first provide the proof for (i) that relates to the null. In particular, under the null hypothesis of $\mu_x = \mu_y$, we have

$$\begin{aligned}
&\frac{i}{\sqrt{N}}(\bar{X}_{\mathcal{I}_{x,1,i}} - \bar{Y}_{\mathcal{I}_{y,1,i}}) \\
= \; & \left(\frac{i}{\lfloor iN_x/N \rfloor}, 0\right) N^{-1/2} \left\{ \sum_{j=1}^{l-\lfloor i(l-1)/N \rfloor + \lfloor iN_x/N \rfloor - 1} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \right. \\
& \qquad \left. - \sum_{j=1}^{l-\lfloor i(l-1)/N \rfloor - 1} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \right\} \\
& - \left(0, \frac{i}{\lfloor iN_y/N \rfloor}\right) N^{-1/2} \left\{ \sum_{j=1}^{l+\lfloor iN_y/N \rfloor - 1} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} - \sum_{j=1}^{l-1} \begin{pmatrix} X_j - \mu_x \\ Y_j - \mu_y \end{pmatrix} \right\}.
\end{aligned}$$

Let

$$\begin{aligned}
U_{\ell,p_x,p_y}(t;a,b,c) \; = \; & \frac{a}{p_x}\{W_1(\ell - \ell t + p_x t) - W_1(\ell - \ell t)\} \\
& + \frac{c}{p_y}\{W_1(\ell + p_y t) - W_1(\ell)\} + \frac{b}{p_y}\{W_2(\ell + p_y t) - W_2(\ell)\},
\end{aligned}$$

then by Corollary 2.2 of Phillips and Durlauf (1986) the invariance principle holds and we can obtain the weak convergence

$$\frac{\lfloor Nt \rfloor}{\sqrt{N}}(\bar{X}_{\mathcal{I}_{x,1,\lfloor Nt \rfloor}} - \bar{Y}_{\mathcal{I}_{y,1,\lfloor Nt \rfloor}}) \Rightarrow U_{\ell,p_x,p_y}(t;a,b,c)$$

on $t \in [0,1]$. Throughout the proof of this theorem, we denote

$$A_{i,N} = N^{-2} \sum_{j=1}^{N} j^2 \{(\bar{X}_{\mathcal{I}_{x,i,j}} - \bar{Y}_{\mathcal{I}_{y,i,j}}) - (\bar{X}_{\mathcal{I}_{x,i,N}} - \bar{Y}_{\mathcal{I}_{y,i,N}})\}^2,$$

and we can write

$$\begin{aligned} A_{1,N} &= \sum_{j=1}^{N} \frac{1}{N} \left\{ \frac{j}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,j}} - \bar{Y}_{\mathcal{I}_{y,1,j}}) - \frac{j}{N} \frac{N}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}}) \right\}^2 \\ &= \sum_{j=1}^{N} \int_{(j-1)/N}^{j/N} \left\{ \frac{\lfloor Nt \rfloor}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,\lfloor Nt \rfloor}} - \bar{Y}_{\mathcal{I}_{y,1,\lfloor Nt \rfloor}}) - \frac{\lfloor Nt \rfloor}{N} \frac{N}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}}) \right\}^2 dt \\ &= \int_{0}^{1} \left\{ \frac{\lfloor Nt \rfloor}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,\lfloor Nt \rfloor}} - \bar{Y}_{\mathcal{I}_{y,1,\lfloor Nt \rfloor}}) - \frac{\lfloor Nt \rfloor}{N} \frac{N}{\sqrt{N}} (\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}}) \right\}^2 dt. \end{aligned}$$

Then by the invariance principle (IP) we have the weak convergence

$$A_{1,N} \to_d A := \int_0^1 \{U_{\ell,p_x,p_y}(t;a,b,c) - tU_{\ell,p_x,p_y}(1;a,b,c)\}^2 dt,$$

which can be made jointly with

$$\sqrt{N}(\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}}) \to_d U_{\ell,p_x,p_y}(1;a,b,c).$$

By applying the continuous mapping theorem, we can derive that the global time-warped self-normalized two-sample statistic satisfies

$$T_{1,N} = \frac{N(\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}})^2}{A_{1,N}} \to_d T := \frac{U^2_{\ell,p_x,p_y}(1;a,b,c)}{A}. \tag{3.7}$$

Similarly, we can show that

$$T_{i,B_N}(0) \to_d T, \quad 1 \le i \le M_N.$$

Note that the proposed warped self-normalized subsampling method preserve the underlying dependence structure in each block, and the stationarity ensures that T_{i,B_N}, $1 \le i \le M_N$, all share the same limiting distribution. Let $F_{\mathrm{SNTS}}(u) = \mathrm{pr}(T \le u)$ be the distribution function of T defined in (3.7), $\tilde{F}_{\mathrm{SNTS}}(u)$ be the empirical distribution function of T_{i,B_N} given by

$$\tilde{F}_{\mathrm{SNTS}}(u) = M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}(T_{i,B_N} \le u),$$

and $\hat{F}_{\mathrm{SNTS}}(u)$ be the empirical distribution function of $T_{i,B_N}(\hat{\Delta})$ given by

$$\hat{F}_{\mathrm{SNTS}}(u) = M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\{T_{i,B_N}(\hat{\Delta}) \leq u\}.$$

We first show that, as $N \to \infty$,

$$\tilde{F}_{\mathrm{SNTS}}(u) \to_p F_{\mathrm{SNTS}}(u) \tag{3.8}$$

for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$, the set of continuity points of $F_{\mathrm{SNTS}}(u)$. For this, it suffices to show that

$$\mathrm{E}[\{\tilde{F}_{\mathrm{SNTS}}(u) - F_{\mathrm{SNTS}}(u)\}^2] \to 0$$

for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$ as $N \to \infty$. Note that $\mathrm{E}[\tilde{F}_{\mathrm{SNTS}}(u) - \mathrm{E}\{\tilde{F}_{\mathrm{SNTS}}(u)\}] = 0$, we can decompose the above into

$$\begin{aligned}
& \mathrm{E}[\{\tilde{F}_{\mathrm{SNTS}}(u) - F_{\mathrm{SNTS}}(u)\}^2] \\
= \; & \mathrm{E}([\tilde{F}_{\mathrm{SNTS}}(u) - \mathrm{E}\{\tilde{F}_{\mathrm{SNTS}}(u)\}]^2) + \mathrm{E}([\mathrm{E}\{\tilde{F}_{\mathrm{SNTS}}(u)\} - F_{\mathrm{SNTS}}(u)]^2) \\
= \; & \mathrm{var}\{\tilde{F}_{\mathrm{SNTS}}(u)\} + \{\mathrm{pr}(T_{i,B_N} \leq u) - \mathrm{pr}(T \leq u)\}^2.
\end{aligned}$$

Since $T_{i,B_N} \to_d T$, $1 \leq i \leq M_N$, we have $\{\mathrm{pr}(T_{i,B_N} \leq u) - \mathrm{pr}(T \leq u)\}^2 \to 0$ for $u \in \mathcal{C}(F_{\mathrm{SNTS}})$ as $N \to \infty$. It remains to show $\mathrm{var}\{\tilde{F}_{\mathrm{SNTS}}(u)\} \to 0$. Note that T_{i,B_N} can be viewed as a process indexed by i, the stationarity assumption on (X_i, Y_i) implies the stationarity of T_{i,B_N}, which leads to

$$\begin{aligned}
\mathrm{var}\{\tilde{F}_{\mathrm{SNTS}}(u)\} &= M_N^{-2} \sum_{i,j=1}^{M_N} \mathrm{cov}\{\mathrm{I}(T_{i,B_N} \leq u), \mathrm{I}(T_{j,B_N} \leq u)\} \\
&\leq M_N^{-2} \times 2 \times M_N \times \sum_{k=0}^{M_N-1} |\mathrm{cov}\{\mathrm{I}(T_{1,B_N} \leq u), \mathrm{I}(T_{k+1,B_N} \leq u)\}| \\
&= 2M_N^{-1} \sum_{k=0}^{B_N-1} |\mathrm{cov}\{\mathrm{I}(T_{1,B_N} \leq u), \mathrm{I}(T_{k+1,B_N} \leq u)\}| \\
&\quad + 2M_N^{-1} \sum_{k=B_N}^{M_N-1} |\mathrm{cov}\{\mathrm{I}(T_{1,B_N} \leq u), \mathrm{I}(T_{k+1,B_N} \leq u)\}|.
\end{aligned}$$

By Lemma 3.5 of Bai et al. (2016), we can obtain that

$$|\mathrm{cov}\{\mathrm{I}(T_{1,B_N} \le u), \mathrm{I}(T_{k+1,B_N} \le u)\}| \le \begin{cases} 1 & \text{if } k < B_N \\ \alpha(k - B_N + 1) & \text{if } k \ge B_N \end{cases}.$$

Note that $\lim_{N\to\infty} M_N/N > 0$ and $B_N/N \to 0$ as $N \to \infty$, we have

$$\begin{aligned} \mathrm{var}\{\tilde{F}_{\mathrm{SNTS}}(u)\} &\le 2M_N^{-1}B_N + 2M_N^{-1}\sum_{k=B_N}^{M_N-1}\alpha(k - B_N + 1) \\ &= 2M_N^{-1}B_N + 2M_N^{-1}\sum_{k=1}^{M_N-B_N}\alpha(k) \to 0, \end{aligned}$$

and (3.8) follows. We shall now provide the proof for

$$\hat{F}_{\mathrm{SNTS}}(u) \to_p F_{\mathrm{SNTS}}(u), \tag{3.9}$$

which differs from (3.8) in the sense that the estimator $\hat{\Delta}$ is used when calculating the test statistic for each subsample. For this, we write

$$\begin{aligned} \hat{F}_{\mathrm{SNTS}}(u) &= M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\{T_{i,B_N}(\hat{\Delta}) \le u\} \\ &= M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\left\{T_{i,B_N} \le u + \frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N\hat{\Delta}^2}{A_{i,B_N}}\right\}, \end{aligned}$$

and then by the above argument on $\tilde{F}_{\mathrm{SNTS}}(u)$ it suffices to show that the additional term

$$\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N\hat{\Delta}^2}{A_{i,B_N}}$$

is negligible in probability. To this end, for any $\epsilon > 0$, let

$$\Xi_N(\epsilon) = M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\left\{\left|\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N\hat{\Delta}^2}{A_{i,B_N}}\right| > \epsilon\right\},$$

then we have

$$\Xi_N(\epsilon) \le \Xi_{N,1}(\epsilon) + \Xi_{N,2}(\epsilon),$$

where

$$\Xi_{N,1}(\epsilon) = M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left\{ \left| \frac{2B_N \hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} \right| > \frac{\epsilon}{2} \right\}$$

and

$$\Xi_{N,2}(\epsilon) = M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left(\left| \frac{B_N \hat{\Delta}^2}{A_{i,B_N}} \right| > \frac{\epsilon}{2} \right).$$

Note that, as $N \to \infty$, $B_N/N \to 0$ and $\sqrt{N}\hat{\Delta} \to_d U_{\ell,p_x,p_y}(1; a, b, c)$, we have, for any $\delta > 0$,

$$\mathrm{pr}(|\sqrt{B_N}\hat{\Delta}| \leq \delta) \to 1.$$

As a result,

$$\mathrm{pr}\left[\Xi_{N,1}(\epsilon) \leq M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left\{ \left| \frac{2\sqrt{B_N}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} \right| > \frac{\epsilon}{2\delta} \right\} \right] \to 1$$

and

$$\mathrm{pr}\left\{ \Xi_{N,2}(\epsilon) \leq M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left(\left| \frac{1}{A_{i,B_N}} \right| > \frac{\epsilon}{2\delta^2} \right) \right\} \to 1.$$

Then by the joint convergence of $\sqrt{B_N}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}}) \to_d U_{\ell,p_x,p_y}(1; a, b, c)$ and $A_{i,B_N} \to_d A$, we have

$$\frac{\sqrt{B_N}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} \to_d \frac{U_{\ell,p_x,p_y}(1; a, b, c)}{A}.$$

Following a similar argument as that for (3.8) but replacing T_{i,B_N} by

$$|2A_{i,B_N}^{-1}\sqrt{B_N}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})|$$

we can obtain that

$$M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left\{ \left| \frac{2\sqrt{B_N}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} \right| > \frac{\epsilon}{2\delta} \right\} \to_p \mathrm{pr}\left\{ \left| \frac{2U_{\ell,p_x,p_y}(1; a, b, c)}{A} \right| > \frac{\epsilon}{2\delta} \right\}.$$

Similarly, we can show that

$$M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}\left(\left| \frac{1}{A_{i,B_N}} \right| > \frac{\epsilon}{2\delta^2} \right) \to_p \mathrm{pr}\left(\left| \frac{1}{A} \right| > \frac{\epsilon}{2\delta^2} \right).$$

For any given $\epsilon > 0$, we can choose $\delta > 0$ to be arbitrarily small so that the probabilities $\mathrm{pr}\{|2U_{\ell,p_x,p_y}(1;a,b,c)/A| > \epsilon/(2\delta)\}$ and $\mathrm{pr}\{|1/A| > \epsilon/(2\delta^2)\}$ are arbitrarily close to zero. Note that $\Xi_{N,1}(\epsilon)$ and $\Xi_{N,2}(\epsilon)$ are both nonnegative, we have

$$\Xi_{N,1}(\epsilon) + \Xi_{N,2}(\epsilon) \to_p 0$$

as $N \to \infty$ for any given $\epsilon > 0$. Since

$$\begin{aligned} & \mathrm{I}\left\{T_{i,B_N} \le u + \frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N(\hat{\Delta})^2}{A_{i,B_N}}\right\} \\ \le\ & \mathrm{I}\{T_{i,B_N} \le u + \epsilon\} + \mathrm{I}\left\{\left|\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N(\hat{\Delta})^2}{A_{i,B_N}}\right| > \epsilon\right\}, \end{aligned}$$

we have

$$\hat{F}_{\mathrm{SNTS}}(u) \le \tilde{F}_{\mathrm{SNTS}}(u+\epsilon) + \Xi_{N,1}(\epsilon) + \Xi_{N,2}(\epsilon).$$

This, together with (3.8), entail that $\mathrm{pr}\{\hat{F}_{\mathrm{SNTS}}(u) \le F_{\mathrm{SNTS}}(u+\epsilon)+\eta\} \to 1$ as $N \to \infty$ for any $\eta > 0$ and $u + \epsilon \in \mathcal{C}(F_{\mathrm{SNTS}})$. Since $\epsilon > 0$ can be chosen arbitrarily small, we have $\mathrm{pr}\{\hat{F}_{\mathrm{SNTS}}(u) \le F_{\mathrm{SNTS}}(u) + \eta'\} \to 1$ for any $\eta' > \eta > 0$. On the other hand, we can write

$$\begin{aligned} & \mathrm{I}\{T_{i,B_N} \le u - \epsilon\} \\ \le\ & \mathrm{I}\{T_{i,B_N} \le u - \epsilon\}\mathrm{I}\left\{\left|\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N(\hat{\Delta})^2}{A_{i,B_N}}\right| \le \epsilon\right\} \\ & \quad + \mathrm{I}\left\{\left|\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N(\hat{\Delta})^2}{A_{i,B_N}}\right| > \epsilon\right\} \\ \le\ & \mathrm{I}\{T_{i,B_N}(\hat{\Delta}) \le u\} + \mathrm{I}\left\{\left|\frac{2B_N\hat{\Delta}(\bar{X}_{\mathcal{I}_{x,i,B_N}} - \bar{Y}_{\mathcal{I}_{y,i,B_N}})}{A_{i,B_N}} - \frac{B_N(\hat{\Delta})^2}{A_{i,B_N}}\right| > \epsilon\right\}, \end{aligned}$$

and as a result,

$$\tilde{F}_{\mathrm{SNTS}}(u-\epsilon) \le \hat{F}_{\mathrm{SNTS}}(u) + \Xi_{N,1}(\epsilon) + \Xi_{N,2}(\epsilon).$$

Then, by a similar argument, we have $\mathrm{pr}\{\hat{F}_{\mathrm{SNTS}}(u) \ge F_{\mathrm{SNTS}}(u) - \eta'\} \to 1$ for any $\eta' > 0$, and thus $\mathrm{pr}\{|\hat{F}_{\mathrm{SNTS}}(u) - F_{\mathrm{SNTS}}(u)| \le \eta'\} \to 1$ as $N \to \infty$. Since $\eta' > 0$ can

be chosen arbitrarily small, (3.9) follows. Note that

$$|\hat{F}_{\mathrm{SNTS}}(u) - \mathrm{pr}(T_{1,N} \le u)| \le |\hat{F}_{\mathrm{SNTS}}(u) - F_{\mathrm{SNTS}}(u)| + |\mathrm{pr}(T_{1,N} \le u) - F_{\mathrm{SNTS}}(u)|$$

by the triangle inequality, combining (3.9) and the fact that $T_{1,N} \to_d T$ we have

$$|\hat{F}_{\mathrm{SNTS}}(u) - \mathrm{pr}(T_{1,N} \le u)| \to_p 0$$

for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$, and (i) follows. For (ii), by a similar argument as that for (3.7) we can show that the centered version satisfies

$$T_{1,N}(\mu_x - \mu_y) = \frac{N\{\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}} - (\mu_x - \mu_y)\}^2}{A_{1,N}} \to_d T,$$

where

$$\begin{aligned} & T_{1,N} - T_{1,N}(\mu_x - \mu_y) \\ = \; & \frac{N(\mu_x - \mu_y)^2 + 2N(\mu_x - \mu_y)\{\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}} - (\mu_x - \mu_y)\}}{A_{1,N}}. \end{aligned}$$

In addition, note that $T_{i,B_N}(\hat{\Delta})$ uses the estimate $\hat{\Delta}$ in its construction, and therefore its distribution remains at the same regardless of what the true value of $\mu_x - \mu_y$ is. As a result, by a similar argument as that in (i), we can show that

$$|\hat{F}_{\mathrm{SNTS}}(u) - \mathrm{pr}\{T_{1,N}(\mu_x - \mu_y) \le u\}| \to_p 0$$

continues to hold for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$. Therefore, it suffices to show that

$$\frac{N(\mu_x - \mu_y)^2 + 2N(\mu_x - \mu_y)\{\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}} - (\mu_x - \mu_y)\}}{A_{1,N}} \to_p \infty$$

when $N^{1/2}|\mu_x - \mu_y| \to \infty$. For this, note that by the invariance principle (IP) and a similar argument as that in (i), we can obtain the joint convergence

$$\left\{ \begin{array}{c} \sqrt{N}\{\bar{X}_{\mathcal{I}_{x,1,N}} - \bar{Y}_{\mathcal{I}_{y,1,N}} - (\mu_x - \mu_y)\} \\ A_{1,N}^{-1} \end{array} \right\} \to_d \left\{ \begin{array}{c} U_{\ell,p_x,p_y}(1; a, b, c) \\ A^{-1} \end{array} \right\},$$

and as a result we have

$$\frac{2N(\mu_x-\mu_y)\{\bar{X}_{\mathcal{I}_{x,1,N}}-\bar{Y}_{\mathcal{I}_{y,1,N}}-(\mu_x-\mu_y)\}}{A_{1,N}}=o_p\left\{\frac{N(\mu_x-\mu_y)^2}{A_{1,N}}\right\}$$

as $N^{1/2}|\mu_x-\mu_y|\to\infty$. Since $A_{1,N}^{-1}\to_d A^{-1}$ has a nondegenerate limiting distribution, we have $A_{1,N}^{-1}N(\mu_x-\mu_y)^2\to_p\infty$ and (ii) follows. □

3.7.4 Proof of Theorem 8

Proof. Note that

$$\begin{aligned}\hat{\theta}_{x,\mathcal{I}_{x,i,j}}-\hat{\theta}_{y,\mathcal{I}_{y,i,j}}=\theta_x-\theta_y \quad &+ \quad \frac{1}{|\mathcal{I}_{x,i,j}|}\sum_{k\in\mathcal{I}_{x,i,j}}\mathrm{IF}_Q(X_k,F_{x,d})+R_{x,\mathcal{I}_{x,i,j}}\\ &- \quad \frac{1}{|\mathcal{I}_{y,i,j}|}\sum_{l\in\mathcal{I}_{y,i,j}}\mathrm{IF}_Q(Y_l,F_{y,d})-R_{y,\mathcal{I}_{y,i,j}},\end{aligned}$$

we can write:

$$\begin{aligned}&\frac{j(\hat{\theta}_{x,\mathcal{I}_{x,i,j}}-\hat{\theta}_{y,\mathcal{I}_{y,i,j}})}{\sqrt{B_N}}\\ =\ &\frac{j(\theta_x-\theta_y)}{\sqrt{B_N}}+\frac{j}{\sqrt{B_N}|\mathcal{I}_{x,i,j}|}\sum_{k\in\mathcal{I}_{x,i,j}}\mathrm{IF}_Q(X_k,F_{x,d})+\frac{j}{\sqrt{B_N}|\mathcal{I}_{x,i,j}|}|\mathcal{I}_{x,i,j}|R_{x,\mathcal{I}_{x,i,j}}\\ &-\frac{j}{\sqrt{B_N}|\mathcal{I}_{y,i,j}|}\sum_{l\in\mathcal{I}_{y,i,j}}\mathrm{IF}_Q(Y_l,F_{y,d})-\frac{j}{\sqrt{B_N}|\mathcal{I}_{y,i,j}|}|\mathcal{I}_{y,i,j}|R_{y,\mathcal{I}_{y,i,j}},\end{aligned}$$

Since $|\mathcal{I}_{x,i,j}|=\lfloor jN_x/N\rfloor$, we have

$$\frac{j}{|\mathcal{I}_{x,i,j}|}=\frac{j\frac{N}{N_x}}{\left\lfloor j\frac{N}{N_x}\right\rfloor}\times\frac{N}{N_x}\le\frac{\left\lfloor j\frac{N}{N_x}\right\rfloor+1}{\left\lfloor j\frac{N}{N_x}\right\rfloor}\times\frac{N}{N_x}\le\frac{2N}{N_x}\to\frac{2}{p_x}$$

as $N\to+\infty$ for $\lfloor jN_x/N\rfloor\ge 1$ where $p_x\in(0,1)$. Similarly,

$$\frac{j}{|\mathcal{I}_{y,i,j}|}\le\frac{2N}{N_y}\to\frac{2}{p_y}$$

as $N \to +\infty$ for $\lfloor jN_y/N \rfloor \geq 1$ where $p_y \in (0,1)$. Therefore, under the null hypothesis of $\theta_x = \theta_y$,

$$\begin{aligned} & \left| \frac{j(\hat{\theta}_{x,\mathcal{I}_{x,i,j}} - \hat{\theta}_{y,\mathcal{I}_{y,i,j}})}{\sqrt{B_N}} - \left\{ \frac{j}{\sqrt{B_N}|\mathcal{I}_{x,i,j}|} \sum_{k \in \mathcal{I}_{x,i,j}} \mathrm{IF}_Q(X_k, F_{x,d}) \right. \right. \\ & \qquad \left. \left. - \frac{j}{\sqrt{B_N}|\mathcal{I}_{y,i,j}|} \sum_{l \in \mathcal{I}_{y,i,j}} \mathrm{IF}_Q(Y_l, F_{y,d}) \right\} \right| \\ = & \left| \frac{j}{\sqrt{B_N}|\mathcal{I}_{x,i,j}|} |\mathcal{I}_{x,i,j}| R_{x,\mathcal{I}_{x,i,j}} - \frac{j}{\sqrt{B_N}|\mathcal{I}_{y,i,j}|} |\mathcal{I}_{y,i,j}| R_{y,\mathcal{I}_{y,i,j}} \right| \\ \leq & \frac{2N}{N_x} \frac{1}{\sqrt{B_N}} |\mathcal{I}_{x,i,j}| |R_{x,\mathcal{I}_{x,i,j}}| + \frac{2N}{N_y} \frac{1}{\sqrt{B_N}} |\mathcal{I}_{y,i,j}| |R_{y,\mathcal{I}_{y,i,j}}| = o_p(1). \end{aligned}$$

Then, by applying the continuous mapping argument as in the proof of Theorem 7 on the linear combination of influence functions, we can obtain that

$$T^{\theta}_{1,N} \to_d T$$

and

$$T^{\theta}_{i,B_N}(0) \to_d T$$

for each $1 \leq i \leq M_N$. Let

$$\tilde{F}^{\theta}_{\mathrm{SNTS}}(u) = M_N^{-1} \sum_{i=1}^{M_N} \mathrm{I}(T^{\theta}_{i,B_N} \leq u),$$

then by Lemma 3.5 of Bai et al. (2016),

$$|\mathrm{cov}\{\mathrm{I}(T^{\theta}_{1,B_N} \leq u), \mathrm{I}(T^{\theta}_{k+1,B_N} \leq u)\}| \leq \begin{cases} 1 & \text{if } k < B_N \\ \alpha(k - B_N + 1) & \text{if } k \geq B_N \end{cases},$$

and by a similar argument as in the proof of Theorem 7 we can show that

$$\begin{aligned}\mathrm{var}\{\tilde{F}^{\theta}_{\mathrm{SNTS}}(u)\} &\leq 2M_N^{-1}\sum_{k=0}^{B_N-1}|\mathrm{cov}\{\mathrm{I}(T^{\theta}_{1,B_N}\leq u),\mathrm{I}(T^{\theta}_{k+1,B_N}\leq u)\}| \\ &\quad +2M_N^{-1}\sum_{k=B_N}^{M_N-1}|\mathrm{cov}\{\mathrm{I}(T^{\theta}_{1,B_N}\leq u),\mathrm{I}(T^{\theta}_{k+1,B_N}\leq u)\}| \\ &\leq 2M_N^{-1}B_N + 2M_N^{-1}\sum_{k=1}^{M_N-B_N}\alpha(k)\to 0,\end{aligned}$$

for any $u\in\mathcal{C}(F_{\mathrm{SNTS}})$. As a result, $\tilde{F}^{\theta}_{\mathrm{SNTS}}(u)\to_p F_{\mathrm{SNTS}}(u)$ for any $u\in\mathcal{C}(F_{\mathrm{SNTS}})$, and we shall now focus on

$$\hat{F}^{\theta}_{\mathrm{SNTS}}(u) = M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\{T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})\leq u\}.$$

For this, let $A^{\theta}_{i,B_N} = B_N^{-2}\sum_{j=1}^{B_N} j^2\{(\hat{\theta}_{x,\mathcal{I}_{x,i,j}}-\hat{\theta}_{y,\mathcal{I}_{y,i,j}})-(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}}-\hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\}^2$, then we can write

$$T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta}) = T^{\theta}_{i,B_N} - \frac{2B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}}-\hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\hat{\Delta}^{\theta}}{A^{\theta}_{i,B_N}} + \frac{B_N(\hat{\Delta}^{\theta})^2}{A^{\theta}_{i,B_N}}.$$

Similar to the proof of Theorem 7, we define the quantities

$$\Xi^{\theta}_N(\epsilon) = M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\left\{\left|\frac{2B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}}-\hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\hat{\Delta}^{\theta}}{A^{\theta}_{i,B_N}} - \frac{B_N(\hat{\Delta}^{\theta})^2}{A^{\theta}_{i,B_N}}\right|>\epsilon\right\},$$

$$\Xi^{\theta}_{N,1}(\epsilon) = M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\left\{\left|\frac{2B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}}-\hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\hat{\Delta}^{\theta}}{A^{\theta}_{i,B_N}}\right|>\frac{\epsilon}{2}\right\},$$

and

$$\Xi^{\theta}_{N,2}(\epsilon) = M_N^{-1}\sum_{i=1}^{M_N}\mathrm{I}\left\{\left|\frac{B_N(\hat{\Delta}^{\theta})^2}{A^{\theta}_{i,B_N}}\right|>\frac{\epsilon}{2}\right\},$$

then

$$\Xi^{\theta}_N(\epsilon)\leq\Xi^{\theta}_{N,1}(\epsilon)+\Xi^{\theta}_{N,2}(\epsilon).$$

Note that for any $\delta > 0$, by the influence function argument,

$$\mathrm{pr}(|\sqrt{B_N}\hat{\Delta}^{\theta}| \leq \delta) \to 1,$$

and thus we have

$$\mathrm{pr}\left[\Xi^{\theta}_{N,1}(\epsilon) \leq M_N^{-1}\sum_{i=1}^{M_N} \mathrm{I}\left\{\left|\frac{2\sqrt{B_N}(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})}{A^{\theta}_{i,B_N}}\right| > \frac{\epsilon}{2\delta}\right\}\right] \to 1$$

and

$$\mathrm{pr}\left\{\Xi^{\theta}_{N,2}(\epsilon) \leq M_N^{-1}\sum_{i=1}^{M_N} \mathrm{I}\left(\left|\frac{1}{A^{\theta}_{i,B_N}}\right| > \frac{\epsilon}{2\delta^2}\right)\right\} \to 1.$$

By applying the influence function argument again, we can obtain that

$$\frac{\sqrt{B_N}(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})}{A^{\theta}_{i,B_N}} \to_d \frac{U_{\ell,p_x,p_y}(1;a,b,c)}{A}$$

and

$$\frac{1}{A^{\theta}_{i,B_N}} \to_d \frac{1}{A}.$$

Then by bounding the covariance as in the argument of (3.8), we can show that

$$\begin{aligned} M_N^{-1}\sum_{i=1}^{M_N} \mathrm{I}\left\{\left|\frac{2\sqrt{B_N}(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})}{A^{\theta}_{i,B_N}}\right| > \frac{\epsilon}{2\delta}\right\} \\ \to_p \mathrm{pr}\left\{\left|\frac{2U_{\ell,p_x,p_y}(1;a,b,c)}{A}\right| > \frac{\epsilon}{2\delta}\right\} \end{aligned}$$

and

$$M_N^{-1}\sum_{i=1}^{M_N} \mathrm{I}\left\{\left|\frac{1}{A^{\theta}_{i,B_N}}\right| > \frac{\epsilon}{2\delta^2}\right\} \to_p \mathrm{pr}\left(\left|\frac{1}{A}\right| > \frac{\epsilon}{2\delta^2}\right).$$

Since δ can be chosen arbitrarily small, we can obtain that

$$\Xi^{\theta}_{N,1}(\epsilon) + \Xi^{\theta}_{N,2}(\epsilon) \to_p 0$$

holds for any $\epsilon > 0$. Note that

$$\begin{aligned} & \mathrm{I}\left\{T^{\theta}_{i,B_N} \le u + \frac{2B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\hat{\Delta}^{\theta}}{A^{\theta}_{i,B_N}} - \frac{B_N(\hat{\Delta}^{\theta})^2}{A^{\theta}_{i,B_N}}\right\} \\ \le \;& \mathrm{I}\{T^{\theta}_{i,B_N} \le u + \epsilon\} + \mathrm{I}\left\{\left|\frac{2B_N(\hat{\theta}_{x,\mathcal{I}_{x,i,B_N}} - \hat{\theta}_{y,\mathcal{I}_{y,i,B_N}})\hat{\Delta}^{\theta}}{A^{\theta}_{i,B_N}} - \frac{B_N(\hat{\Delta}^{\theta})^2}{A^{\theta}_{i,B_N}}\right| > \epsilon\right\}, \end{aligned}$$

we have

$$\hat{F}^{\theta}_{\mathrm{SNTS}}(u) \le \tilde{F}^{\theta}_{\mathrm{SNTS}}(u+\epsilon) + \Xi^{\theta}_{N,1}(\epsilon) + \Xi^{\theta}_{N,2}(\epsilon).$$

Since ϵ can be chosen arbitrarily small, we have for any $\eta' > 0, \mathrm{pr}\{\hat{F}^{\theta}_{\mathrm{SNTS}}(u) \le F_{\mathrm{SNTS}}(u) + \eta'\} \to 1$. Similarly, we can show that $\mathrm{pr}\{\hat{F}^{\theta}_{\mathrm{SNTS}}(u) \ge F_{\mathrm{SNTS}}(u) - \eta'\} \to 1$, (i) follows by applying the triangle inequality. For (ii), by a similar argument as that in (i) we can show that the centered version satisfies

$$T^{\theta}_{1,N}(\theta_x - \theta_y) \to_d T,$$

and the empirical distribution of time-warped self-normalized subsamples satisfies

$$|\hat{F}^{\theta}_{\mathrm{SNTS}}(u) - \mathrm{pr}\{T^{\theta}_{1,N}(\theta_x - \theta_y) \le u\}| \to_p 0$$

for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$. Note that we can write

$$\begin{aligned} & N(\hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}})^2 - N\{\hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}} - (\theta_x - \theta_y)\}^2 \\ = \;& N(\theta_x - \theta_y)^2 + 2N(\theta_x - \theta_y)\{\hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}} - (\theta_x - \theta_y)\}, \end{aligned}$$

by a similar argument as that in the proof of Theorem 7 we have

$$T^{\theta}_{1,N} - T^{\theta}_{1,N}(\theta_x - \theta_y) \to \infty$$

under the alternative when $N^{1/2}|\theta_x - \theta_y| \to \infty$, and (ii) follows. □

3.7.5 Proof of Corollary 1

Proof. Note that

$$T^{\theta}_{1,N}(\Delta) = \frac{N(\hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}} - \Delta)^2}{N^{-2}\sum_{j=1}^{N} j^2\{(\hat{\theta}_{x,\mathcal{I}_{x,1,j}} - \hat{\theta}_{y,\mathcal{I}_{y,1,j}}) - (\hat{\theta}_{x,\mathcal{I}_{x,1,N}} - \hat{\theta}_{y,\mathcal{I}_{y,1,N}})\}^2},$$

which, under the null hypothesis of $\theta_x - \theta_y = \Delta$, satisfies

$$T^{\theta}_{1,N}(\theta_x - \theta_y) \to_d T$$

by the proof of Theorem 8. In addition, due to the use of $\hat{\Delta}^{\theta}$ for centering, the time-warped self-normalized subsamples $T^{\theta}_{i,B_N}(\hat{\Delta}^{\theta})$, $i = 1, \ldots, M_N$, are invariant with respect to what the true value of $\theta_x - \theta_y$ is, and as a result

$$|\hat{F}^{\theta}_{\mathrm{SNTS}}(u) - \mathrm{pr}\{T^{\theta}_{1,N}(\theta_x - \theta_y) \leq u\}| \to_p 0$$

holds for any $u \in \mathcal{C}(F_{\mathrm{SNTS}})$ by a similar argument as in the proof of Theorem 8. Corollary 1 then follows. □

3.8 Additional Simulation Results

We in Tables 3.5–3.94 provide simulation results in Section 3.5.1 for Scenarios 1–3 with $\rho \in \{0.3, 0.6\}$, $B_N \in \{50, 75, 100, 125, 150\}$, and $r \in \{0, 0.4, 0.8\}$. The observations made in Section 3.5.1 about the simulation results continue to hold, and similar to the PR10 subsampling method of Politis and Romano (2010) the proposed WSNS method seems to be reasonably robust to different choices of the bandwidth.

Table 3.5: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.879	0.933	0.980	0.907	0.953	0.985	0.861	0.922	0.966
		PR10	0.872	0.927	0.982	0.880	0.933	0.989	0.875	0.945	0.985
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.900	0.944	0.990	0.886	0.949	0.990	0.869	0.917	0.983
		PR10	0.877	0.941	0.983	0.874	0.932	0.986	0.881	0.950	0.990
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.912	0.955	0.993	0.904	0.970	0.996	0.877	0.925	0.977
		PR10	0.884	0.937	0.988	0.886	0.947	0.993	0.886	0.943	0.992
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.888	0.943	0.983	0.896	0.952	0.988	0.872	0.923	0.969
		PR10	0.956	0.980	0.997	0.943	0.981	0.995	0.910	0.955	0.990
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.894	0.949	0.986	0.889	0.940	0.992	0.871	0.927	0.983
		PR10	0.963	0.982	0.999	0.938	0.974	0.998	0.913	0.964	0.996
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.909	0.953	0.993	0.907	0.956	0.994	0.874	0.926	0.973
		PR10	0.963	0.989	0.998	0.955	0.982	0.997	0.918	0.959	0.993
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.903	0.947	0.985	0.890	0.941	0.980	0.861	0.919	0.969
		PR10	0.998	1.000	1.000	0.994	0.997	0.999	0.984	0.996	1.000
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.889	0.950	0.989	0.888	0.947	0.988	0.868	0.931	0.979
		PR10	1.000	1.000	1.000	0.995	1.000	1.000	0.986	0.999	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.901	0.952	0.992	0.884	0.952	0.991	0.880	0.930	0.976
		PR10	1.000	1.000	1.000	0.997	0.999	1.000	0.985	0.997	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.6: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.917	0.957	0.992	0.927	0.958	0.994	0.885	0.936	0.980
		PR10	0.867	0.918	0.979	0.864	0.926	0.979	0.833	0.922	0.976
		S15	0.890	0.939	0.988	0.903	0.947	0.991	0.892	0.945	0.990
	1000	WSNS	0.931	0.970	0.995	0.933	0.970	0.996	0.893	0.953	0.987
		PR10	0.872	0.932	0.985	0.863	0.927	0.978	0.841	0.932	0.989
		S15	0.892	0.941	0.987	0.908	0.952	0.987	0.903	0.959	0.989
	1500	WSNS	0.941	0.978	1.000	0.934	0.978	0.996	0.893	0.934	0.985
		PR10	0.872	0.924	0.986	0.872	0.937	0.985	0.850	0.920	0.986
		S15	0.891	0.951	0.990	0.901	0.950	0.992	0.900	0.947	0.989
0.4	500	WSNS	0.919	0.964	0.995	0.903	0.953	0.994	0.882	0.941	0.985
		PR10	0.939	0.975	0.994	0.937	0.970	0.995	0.865	0.935	0.981
		S15	0.889	0.949	0.993	0.887	0.941	0.992	0.886	0.947	0.991
	1000	WSNS	0.937	0.967	0.997	0.926	0.972	0.995	0.900	0.940	0.986
		PR10	0.962	0.985	0.999	0.947	0.978	0.996	0.883	0.955	0.988
		S15	0.898	0.943	0.992	0.913	0.953	0.989	0.904	0.948	0.991
	1500	WSNS	0.936	0.974	0.997	0.930	0.974	0.999	0.885	0.933	0.984
		PR10	0.955	0.987	0.997	0.949	0.974	0.998	0.881	0.935	0.991
		S15	0.896	0.945	0.992	0.900	0.950	0.993	0.897	0.947	0.989
0.8	500	WSNS	0.920	0.967	0.991	0.913	0.955	0.989	0.872	0.934	0.985
		PR10	0.998	1.000	1.000	0.994	0.999	1.000	0.968	0.990	1.000
		S15	0.899	0.951	0.990	0.931	0.965	0.994	0.879	0.947	0.989
	1000	WSNS	0.937	0.971	0.996	0.911	0.961	0.994	0.894	0.929	0.982
		PR10	1.000	1.000	1.000	0.996	1.000	1.000	0.985	0.996	1.000
		S15	0.910	0.947	0.993	0.920	0.960	0.995	0.892	0.941	0.986
	1500	WSNS	0.935	0.970	0.998	0.913	0.958	0.993	0.871	0.915	0.968
		PR10	1.000	1.000	1.000	0.997	0.999	1.000	0.972	0.992	1.000
		S15	0.883	0.944	0.992	0.920	0.960	0.991	0.883	0.925	0.978

Table 3.7: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.873	0.925	0.971	0.897	0.942	0.980	0.858	0.920	0.968
		PR10	0.862	0.921	0.978	0.870	0.922	0.984	0.879	0.934	0.979
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.889	0.937	0.983	0.888	0.937	0.987	0.868	0.933	0.979
		PR10	0.877	0.937	0.982	0.876	0.926	0.984	0.880	0.944	0.990
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.902	0.947	0.986	0.898	0.962	0.997	0.884	0.924	0.978
		PR10	0.881	0.936	0.988	0.886	0.946	0.992	0.883	0.943	0.987
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.882	0.935	0.977	0.880	0.938	0.982	0.874	0.922	0.970
		PR10	0.949	0.973	0.996	0.939	0.974	0.991	0.902	0.952	0.987
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.893	0.939	0.982	0.882	0.936	0.987	0.873	0.926	0.980
		PR10	0.963	0.981	0.997	0.938	0.973	0.998	0.913	0.960	0.994
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.907	0.945	0.987	0.901	0.953	0.993	0.883	0.935	0.974
		PR10	0.963	0.989	0.999	0.953	0.980	0.997	0.917	0.959	0.990
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.895	0.933	0.979	0.879	0.928	0.975	0.855	0.911	0.964
		PR10	0.997	0.998	1.000	0.992	0.998	0.999	0.981	0.995	0.999
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.887	0.949	0.981	0.887	0.943	0.985	0.879	0.932	0.975
		PR10	0.999	1.000	1.000	0.994	1.000	1.000	0.989	0.998	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.893	0.944	0.989	0.879	0.939	0.987	0.888	0.932	0.980
		PR10	1.000	1.000	1.000	0.997	1.000	1.000	0.986	0.998	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.8: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.914	0.946	0.974	0.909	0.952	0.986	0.879	0.932	0.974
		PR10	0.859	0.917	0.977	0.867	0.922	0.978	0.847	0.923	0.973
		S15	0.890	0.939	0.988	0.903	0.947	0.991	0.892	0.945	0.990
	1000	WSNS	0.913	0.963	0.988	0.918	0.959	0.991	0.896	0.946	0.986
		PR10	0.872	0.934	0.985	0.867	0.931	0.980	0.858	0.933	0.987
		S15	0.892	0.941	0.987	0.908	0.952	0.987	0.903	0.959	0.989
	1500	WSNS	0.923	0.967	0.994	0.917	0.966	0.997	0.898	0.945	0.987
		PR10	0.873	0.925	0.986	0.881	0.937	0.985	0.866	0.928	0.984
		S15	0.891	0.951	0.990	0.901	0.950	0.992	0.900	0.947	0.989
0.4	500	WSNS	0.905	0.945	0.981	0.887	0.936	0.985	0.879	0.937	0.983
		PR10	0.942	0.975	0.993	0.934	0.969	0.993	0.877	0.935	0.980
		S15	0.889	0.949	0.993	0.887	0.941	0.992	0.886	0.947	0.991
	1000	WSNS	0.923	0.956	0.995	0.918	0.962	0.991	0.905	0.943	0.987
		PR10	0.961	0.984	0.999	0.948	0.978	0.995	0.900	0.964	0.991
		S15	0.898	0.943	0.992	0.913	0.953	0.989	0.904	0.948	0.991
	1500	WSNS	0.924	0.967	0.995	0.917	0.966	0.995	0.893	0.929	0.985
		PR10	0.956	0.987	0.997	0.952	0.976	0.998	0.892	0.938	0.989
		S15	0.896	0.945	0.992	0.900	0.950	0.993	0.897	0.947	0.989
0.8	500	WSNS	0.905	0.948	0.984	0.894	0.950	0.979	0.871	0.935	0.975
		PR10	0.997	0.999	1.000	0.991	0.999	1.000	0.970	0.990	0.999
		S15	0.899	0.951	0.990	0.931	0.965	0.994	0.879	0.947	0.989
	1000	WSNS	0.926	0.963	0.997	0.894	0.953	0.992	0.893	0.932	0.979
		PR10	1.000	1.000	1.000	0.996	1.000	1.000	0.990	0.995	1.000
		S15	0.910	0.947	0.993	0.920	0.960	0.995	0.892	0.941	0.986
	1500	WSNS	0.919	0.964	0.996	0.904	0.949	0.990	0.879	0.920	0.975
		PR10	1.000	1.000	1.000	0.999	0.999	1.000	0.976	0.993	1.000
		S15	0.883	0.944	0.992	0.920	0.960	0.991	0.883	0.925	0.978

Table 3.9: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.855	0.911	0.954	0.873	0.923	0.966	0.852	0.908	0.957
		PR10	0.852	0.915	0.966	0.863	0.912	0.976	0.867	0.928	0.975
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.878	0.928	0.976	0.874	0.922	0.976	0.866	0.924	0.973
		PR10	0.873	0.933	0.982	0.868	0.923	0.984	0.877	0.942	0.989
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.893	0.937	0.982	0.895	0.950	0.992	0.874	0.925	0.979
		PR10	0.884	0.934	0.985	0.879	0.945	0.991	0.882	0.942	0.986
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.861	0.911	0.966	0.857	0.921	0.977	0.846	0.914	0.965
		PR10	0.941	0.972	0.992	0.936	0.967	0.990	0.901	0.944	0.985
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.875	0.923	0.978	0.878	0.921	0.978	0.871	0.919	0.974
		PR10	0.965	0.980	0.997	0.927	0.973	0.995	0.908	0.960	0.992
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.889	0.938	0.983	0.894	0.944	0.981	0.871	0.927	0.972
		PR10	0.965	0.989	0.998	0.950	0.976	0.997	0.916	0.960	0.989
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.867	0.920	0.968	0.864	0.914	0.964	0.843	0.907	0.962
		PR10	0.995	0.997	1.000	0.986	0.997	0.999	0.976	0.993	1.000
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.883	0.936	0.980	0.882	0.931	0.978	0.869	0.925	0.971
		PR10	1.000	1.000	1.000	0.993	1.000	1.000	0.989	0.997	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.887	0.926	0.983	0.868	0.929	0.984	0.887	0.928	0.974
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.984	0.995	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.10: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.883	0.931	0.960	0.890	0.932	0.972	0.859	0.912	0.964
		PR10	0.852	0.908	0.970	0.859	0.910	0.971	0.848	0.917	0.976
		S15	0.890	0.939	0.988	0.903	0.947	0.991	0.892	0.945	0.990
	1000	WSNS	0.898	0.949	0.982	0.899	0.943	0.987	0.892	0.943	0.983
		PR10	0.867	0.936	0.981	0.871	0.929	0.978	0.866	0.936	0.985
		S15	0.892	0.941	0.987	0.908	0.952	0.987	0.903	0.959	0.989
	1500	WSNS	0.908	0.951	0.989	0.908	0.955	0.992	0.890	0.936	0.981
		PR10	0.872	0.930	0.985	0.878	0.939	0.985	0.877	0.935	0.977
		S15	0.891	0.951	0.990	0.901	0.950	0.992	0.900	0.947	0.989
0.4	500	WSNS	0.880	0.927	0.968	0.863	0.920	0.974	0.859	0.919	0.971
		PR10	0.936	0.971	0.992	0.928	0.965	0.991	0.877	0.930	0.978
		S15	0.889	0.949	0.993	0.887	0.941	0.992	0.886	0.947	0.991
	1000	WSNS	0.908	0.940	0.990	0.891	0.942	0.986	0.900	0.935	0.978
		PR10	0.962	0.984	0.996	0.944	0.978	0.995	0.908	0.965	0.993
		S15	0.898	0.943	0.992	0.913	0.953	0.989	0.904	0.948	0.991
	1500	WSNS	0.903	0.953	0.990	0.904	0.952	0.987	0.880	0.927	0.980
		PR10	0.956	0.988	0.997	0.952	0.976	0.996	0.894	0.943	0.990
		S15	0.896	0.945	0.992	0.900	0.950	0.993	0.897	0.947	0.989
0.8	500	WSNS	0.877	0.931	0.972	0.876	0.928	0.970	0.849	0.911	0.963
		PR10	0.995	0.999	1.000	0.988	0.998	1.000	0.964	0.989	0.999
		S15	0.899	0.951	0.990	0.931	0.965	0.994	0.879	0.947	0.989
	1000	WSNS	0.912	0.950	0.990	0.879	0.938	0.984	0.883	0.925	0.975
		PR10	1.000	1.000	1.000	0.996	1.000	1.000	0.989	0.995	1.000
		S15	0.910	0.947	0.993	0.920	0.960	0.995	0.892	0.941	0.986
	1500	WSNS	0.899	0.947	0.984	0.888	0.941	0.986	0.871	0.918	0.969
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.977	0.993	0.999
		S15	0.883	0.944	0.992	0.920	0.960	0.991	0.883	0.925	0.978

Table 3.11: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.847	0.905	0.946	0.869	0.922	0.959	0.847	0.894	0.950
		PR10	0.842	0.905	0.964	0.856	0.906	0.969	0.855	0.915	0.975
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.874	0.922	0.969	0.865	0.925	0.975	0.868	0.917	0.971
		PR10	0.871	0.931	0.976	0.865	0.920	0.983	0.876	0.937	0.989
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.893	0.932	0.979	0.890	0.945	0.990	0.879	0.931	0.979
		PR10	0.878	0.935	0.987	0.878	0.942	0.989	0.881	0.941	0.986
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.847	0.913	0.952	0.854	0.919	0.969	0.842	0.897	0.958
		PR10	0.933	0.967	0.991	0.927	0.964	0.989	0.888	0.941	0.982
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.874	0.920	0.975	0.876	0.915	0.979	0.871	0.920	0.973
		PR10	0.959	0.976	0.997	0.929	0.971	0.994	0.906	0.958	0.990
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.886	0.937	0.978	0.894	0.941	0.981	0.871	0.933	0.974
		PR10	0.964	0.988	0.998	0.948	0.974	0.997	0.918	0.956	0.987
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.861	0.908	0.953	0.863	0.913	0.963	0.840	0.908	0.955
		PR10	0.997	0.997	0.999	0.985	0.995	0.998	0.974	0.990	0.997
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.869	0.931	0.971	0.878	0.931	0.975	0.869	0.924	0.972
		PR10	0.999	1.000	1.000	0.989	1.000	1.000	0.987	0.998	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.884	0.925	0.981	0.865	0.927	0.984	0.881	0.922	0.976
		PR10	1.000	1.000	1.000	0.996	0.999	1.000	0.983	0.995	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.12: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.871	0.921	0.958	0.877	0.933	0.966	0.852	0.906	0.958
		PR10	0.845	0.902	0.960	0.852	0.908	0.964	0.849	0.906	0.962
		S15	0.890	0.939	0.988	0.903	0.947	0.991	0.892	0.945	0.990
	1000	WSNS	0.891	0.941	0.981	0.881	0.941	0.981	0.890	0.938	0.983
		PR10	0.868	0.934	0.979	0.869	0.921	0.972	0.874	0.932	0.982
		S15	0.892	0.941	0.987	0.908	0.952	0.987	0.903	0.959	0.989
	1500	WSNS	0.904	0.943	0.985	0.900	0.950	0.989	0.893	0.938	0.981
		PR10	0.875	0.929	0.985	0.881	0.940	0.986	0.881	0.930	0.978
		S15	0.891	0.951	0.990	0.901	0.950	0.992	0.900	0.947	0.989
0.4	500	WSNS	0.871	0.923	0.962	0.857	0.911	0.967	0.861	0.903	0.959
		PR10	0.931	0.967	0.988	0.920	0.958	0.989	0.871	0.927	0.966
		S15	0.889	0.949	0.993	0.887	0.941	0.992	0.886	0.947	0.991
	1000	WSNS	0.897	0.938	0.979	0.878	0.943	0.983	0.899	0.939	0.974
		PR10	0.959	0.982	0.994	0.942	0.976	0.994	0.905	0.960	0.990
		S15	0.898	0.943	0.992	0.913	0.953	0.989	0.904	0.948	0.991
	1500	WSNS	0.897	0.943	0.991	0.900	0.948	0.988	0.883	0.930	0.980
		PR10	0.958	0.985	0.997	0.953	0.976	0.996	0.898	0.942	0.989
		S15	0.896	0.945	0.992	0.900	0.950	0.993	0.897	0.947	0.989
0.8	500	WSNS	0.872	0.920	0.964	0.866	0.921	0.964	0.832	0.896	0.962
		PR10	0.993	0.997	1.000	0.987	0.995	1.000	0.962	0.987	0.998
		S15	0.899	0.951	0.990	0.931	0.965	0.994	0.879	0.947	0.989
	1000	WSNS	0.901	0.951	0.986	0.878	0.939	0.980	0.883	0.928	0.975
		PR10	0.998	1.000	1.000	0.995	0.998	1.000	0.983	0.995	1.000
		S15	0.910	0.947	0.993	0.920	0.960	0.995	0.892	0.941	0.986
	1500	WSNS	0.898	0.945	0.984	0.890	0.940	0.984	0.869	0.919	0.971
		PR10	1.000	1.000	1.000	0.997	0.999	1.000	0.978	0.993	0.999
		S15	0.883	0.944	0.992	0.920	0.960	0.991	0.883	0.925	0.978

Table 3.13: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.830	0.889	0.947	0.849	0.908	0.956	0.826	0.881	0.940
		PR10	0.837	0.894	0.953	0.849	0.901	0.959	0.849	0.904	0.964
		S15	0.887	0.952	0.991	0.911	0.953	0.992	0.896	0.948	0.989
	1000	WSNS	0.862	0.911	0.962	0.858	0.916	0.969	0.860	0.909	0.967
		PR10	0.869	0.925	0.978	0.869	0.914	0.978	0.873	0.932	0.987
		S15	0.894	0.943	0.987	0.888	0.944	0.991	0.894	0.954	0.994
	1500	WSNS	0.886	0.925	0.976	0.885	0.932	0.987	0.877	0.923	0.975
		PR10	0.878	0.932	0.986	0.881	0.945	0.990	0.875	0.938	0.983
		S15	0.898	0.951	0.991	0.904	0.958	0.995	0.896	0.954	0.990
0.4	500	WSNS	0.841	0.887	0.947	0.847	0.904	0.961	0.833	0.882	0.943
		PR10	0.926	0.961	0.983	0.924	0.960	0.983	0.881	0.935	0.973
		S15	0.895	0.951	0.992	0.917	0.964	0.992	0.904	0.950	0.988
	1000	WSNS	0.868	0.914	0.963	0.867	0.913	0.972	0.859	0.917	0.968
		PR10	0.957	0.978	0.993	0.927	0.971	0.993	0.896	0.955	0.989
		S15	0.891	0.943	0.989	0.901	0.941	0.993	0.899	0.956	0.995
	1500	WSNS	0.882	0.928	0.975	0.882	0.936	0.979	0.863	0.924	0.971
		PR10	0.958	0.988	0.999	0.948	0.974	0.997	0.914	0.954	0.985
		S15	0.900	0.947	0.992	0.913	0.959	0.991	0.902	0.947	0.987
0.8	500	WSNS	0.845	0.895	0.945	0.855	0.905	0.953	0.831	0.890	0.943
		PR10	0.995	0.996	0.999	0.978	0.992	0.998	0.972	0.985	0.997
		S15	0.912	0.956	0.991	0.928	0.966	0.995	0.897	0.953	0.987
	1000	WSNS	0.861	0.922	0.972	0.877	0.922	0.969	0.863	0.926	0.968
		PR10	0.999	1.000	1.000	0.991	0.997	1.000	0.984	0.997	1.000
		S15	0.884	0.955	0.992	0.926	0.958	0.990	0.903	0.950	0.992
	1500	WSNS	0.869	0.923	0.980	0.857	0.926	0.981	0.877	0.924	0.971
		PR10	1.000	1.000	1.000	0.996	0.999	1.000	0.979	0.993	1.000
		S15	0.890	0.946	0.994	0.925	0.964	0.997	0.903	0.944	0.992

Table 3.14: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with standard normal innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.848	0.902	0.952	0.851	0.919	0.964	0.839	0.885	0.947
		PR10	0.834	0.894	0.959	0.848	0.898	0.962	0.849	0.906	0.962
		S15	0.890	0.939	0.988	0.903	0.947	0.991	0.892	0.945	0.990
	1000	WSNS	0.882	0.931	0.972	0.867	0.930	0.978	0.882	0.934	0.978
		PR10	0.865	0.931	0.976	0.868	0.918	0.972	0.867	0.933	0.980
		S15	0.892	0.941	0.987	0.908	0.952	0.987	0.903	0.959	0.989
	1500	WSNS	0.891	0.933	0.982	0.893	0.944	0.990	0.881	0.938	0.979
		PR10	0.877	0.927	0.984	0.875	0.938	0.985	0.880	0.931	0.977
		S15	0.891	0.951	0.990	0.901	0.950	0.992	0.900	0.947	0.989
0.4	500	WSNS	0.853	0.909	0.949	0.841	0.899	0.957	0.844	0.887	0.952
		PR10	0.926	0.958	0.985	0.912	0.950	0.990	0.864	0.922	0.959
		S15	0.889	0.949	0.993	0.887	0.941	0.992	0.886	0.947	0.991
	1000	WSNS	0.887	0.929	0.974	0.869	0.929	0.975	0.886	0.932	0.973
		PR10	0.958	0.980	0.995	0.937	0.975	0.994	0.905	0.956	0.991
		S15	0.898	0.943	0.992	0.913	0.953	0.989	0.904	0.948	0.991
	1500	WSNS	0.890	0.933	0.985	0.885	0.940	0.987	0.874	0.923	0.976
		PR10	0.957	0.985	0.998	0.953	0.978	0.996	0.898	0.939	0.987
		S15	0.896	0.945	0.992	0.900	0.950	0.993	0.897	0.947	0.989
0.8	500	WSNS	0.847	0.903	0.959	0.847	0.908	0.958	0.821	0.882	0.943
		PR10	0.991	0.995	0.997	0.985	0.995	0.999	0.958	0.981	0.994
		S15	0.899	0.951	0.990	0.931	0.965	0.994	0.879	0.947	0.989
	1000	WSNS	0.889	0.944	0.981	0.867	0.923	0.974	0.870	0.923	0.972
		PR10	0.999	1.000	1.000	0.994	0.998	1.000	0.984	0.994	1.000
		S15	0.910	0.947	0.993	0.920	0.960	0.995	0.892	0.941	0.986
	1500	WSNS	0.885	0.936	0.984	0.880	0.935	0.980	0.862	0.917	0.963
		PR10	1.000	1.000	1.000	0.997	0.999	1.000	0.977	0.994	0.999
		S15	0.883	0.944	0.992	0.920	0.960	0.991	0.883	0.925	0.978

Table 3.15: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.887	0.938	0.983	0.902	0.947	0.990	0.847	0.903	0.959
		PR10	0.869	0.933	0.981	0.881	0.933	0.981	0.854	0.923	0.982
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.903	0.952	0.989	0.908	0.960	0.996	0.858	0.919	0.977
		PR10	0.891	0.941	0.981	0.885	0.942	0.986	0.884	0.945	0.990
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.915	0.959	0.991	0.921	0.963	0.993	0.871	0.929	0.980
		PR10	0.898	0.943	0.983	0.896	0.943	0.985	0.892	0.945	0.995
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.901	0.942	0.990	0.891	0.941	0.989	0.870	0.925	0.971
		PR10	0.932	0.978	0.997	0.926	0.961	0.992	0.896	0.941	0.984
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.911	0.953	0.990	0.914	0.963	0.994	0.875	0.933	0.984
		PR10	0.943	0.975	0.998	0.940	0.972	0.994	0.915	0.962	0.996
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.927	0.964	0.993	0.928	0.973	0.993	0.878	0.937	0.985
		PR10	0.953	0.978	0.998	0.945	0.979	0.997	0.917	0.959	0.995
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.919	0.964	0.996	0.902	0.948	0.988	0.902	0.945	0.983
		PR10	0.995	0.999	1.000	0.973	0.993	0.999	0.970	0.991	1.000
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.934	0.978	0.997	0.915	0.958	0.993	0.901	0.951	0.988
		PR10	0.994	0.999	1.000	0.981	0.997	1.000	0.982	0.996	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.940	0.981	0.996	0.918	0.959	0.997	0.892	0.936	0.983
		PR10	0.994	0.998	1.000	0.980	0.992	1.000	0.981	0.998	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.16: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.933	0.968	0.994	0.925	0.972	0.996	0.891	0.935	0.980
		PR10	0.864	0.929	0.979	0.858	0.930	0.976	0.812	0.900	0.972
		S15	0.878	0.941	0.989	0.897	0.939	0.988	0.889	0.949	0.989
	1000	WSNS	0.946	0.980	0.997	0.940	0.972	0.995	0.889	0.939	0.988
		PR10	0.881	0.938	0.977	0.887	0.933	0.980	0.849	0.918	0.979
		S15	0.902	0.943	0.985	0.908	0.949	0.986	0.898	0.952	0.988
	1500	WSNS	0.955	0.977	0.996	0.948	0.982	0.998	0.898	0.942	0.990
		PR10	0.888	0.939	0.983	0.896	0.939	0.981	0.827	0.922	0.990
		S15	0.904	0.941	0.987	0.902	0.944	0.986	0.896	0.943	0.989
0.4	500	WSNS	0.944	0.974	0.997	0.922	0.959	0.993	0.879	0.940	0.981
		PR10	0.927	0.976	0.996	0.915	0.959	0.989	0.873	0.928	0.979
		S15	0.925	0.969	0.997	0.931	0.967	0.993	0.910	0.956	0.991
	1000	WSNS	0.952	0.985	0.997	0.951	0.974	0.996	0.905	0.953	0.992
		PR10	0.940	0.975	0.995	0.919	0.963	0.995	0.881	0.933	0.991
		S15	0.928	0.971	0.996	0.938	0.969	0.997	0.931	0.972	0.994
	1500	WSNS	0.958	0.990	1.000	0.947	0.983	0.998	0.910	0.957	0.988
		PR10	0.947	0.977	0.998	0.934	0.965	0.995	0.867	0.941	0.993
		S15	0.935	0.971	1.000	0.937	0.965	0.995	0.907	0.957	0.990
0.8	500	WSNS	0.961	0.987	0.999	0.936	0.976	0.998	0.906	0.955	0.991
		PR10	0.992	0.998	1.000	0.984	0.995	0.999	0.952	0.984	0.998
		S15	0.985	0.995	1.000	0.976	0.994	1.000	0.976	0.985	1.000
	1000	WSNS	0.971	0.994	0.999	0.943	0.973	0.999	0.924	0.971	0.995
		PR10	0.991	0.998	1.000	0.985	0.997	1.000	0.959	0.988	0.999
		S15	0.983	0.999	1.000	0.984	0.995	1.000	0.977	0.992	0.999
	1500	WSNS	0.974	0.994	0.999	0.954	0.983	0.998	0.927	0.966	0.994
		PR10	0.993	0.998	1.000	0.985	0.995	1.000	0.965	0.991	0.999
		S15	0.989	0.997	1.000	0.978	0.994	0.999	0.967	0.990	0.998

Table 3.17: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.876	0.934	0.974	0.891	0.942	0.983	0.846	0.904	0.960
		PR10	0.863	0.933	0.976	0.874	0.931	0.976	0.856	0.926	0.980
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.904	0.950	0.983	0.897	0.953	0.991	0.864	0.928	0.980
		PR10	0.889	0.941	0.979	0.887	0.942	0.984	0.889	0.946	0.989
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.909	0.960	0.990	0.916	0.961	0.991	0.887	0.937	0.984
		PR10	0.901	0.944	0.981	0.893	0.949	0.983	0.894	0.946	0.993
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.896	0.938	0.982	0.881	0.940	0.988	0.860	0.925	0.971
		PR10	0.927	0.976	0.995	0.925	0.957	0.991	0.894	0.943	0.981
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.905	0.945	0.986	0.910	0.955	0.995	0.884	0.939	0.986
		PR10	0.939	0.976	0.995	0.933	0.972	0.994	0.919	0.960	0.994
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.921	0.961	0.994	0.922	0.965	0.994	0.885	0.943	0.990
		PR10	0.954	0.978	0.997	0.945	0.979	0.996	0.918	0.959	0.994
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.913	0.956	0.991	0.894	0.940	0.986	0.901	0.940	0.981
		PR10	0.995	0.998	1.000	0.972	0.987	0.997	0.964	0.990	0.997
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.928	0.970	0.996	0.909	0.956	0.989	0.908	0.957	0.984
		PR10	0.993	0.998	1.000	0.982	0.996	1.000	0.982	0.996	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.934	0.974	0.993	0.913	0.959	0.996	0.898	0.945	0.985
		PR10	0.994	0.998	1.000	0.980	0.993	1.000	0.979	0.997	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.18: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.918	0.959	0.986	0.903	0.952	0.992	0.893	0.939	0.978
		PR10	0.858	0.923	0.977	0.857	0.928	0.973	0.823	0.901	0.965
		S15	0.878	0.941	0.989	0.897	0.939	0.988	0.889	0.949	0.989
	1000	WSNS	0.940	0.969	0.995	0.924	0.962	0.991	0.893	0.943	0.985
		PR10	0.884	0.934	0.976	0.887	0.935	0.979	0.862	0.928	0.979
		S15	0.902	0.943	0.985	0.908	0.949	0.986	0.898	0.952	0.988
	1500	WSNS	0.948	0.973	0.995	0.939	0.974	0.997	0.901	0.945	0.989
		PR10	0.891	0.941	0.984	0.899	0.944	0.981	0.850	0.932	0.988
		S15	0.904	0.941	0.987	0.902	0.944	0.986	0.896	0.943	0.989
0.4	500	WSNS	0.930	0.957	0.993	0.905	0.947	0.986	0.887	0.927	0.977
		PR10	0.922	0.975	0.993	0.917	0.957	0.987	0.878	0.933	0.975
		S15	0.925	0.969	0.997	0.931	0.967	0.993	0.910	0.956	0.991
	1000	WSNS	0.938	0.979	0.996	0.938	0.965	0.992	0.909	0.954	0.994
		PR10	0.932	0.976	0.994	0.924	0.964	0.994	0.894	0.939	0.988
		S15	0.928	0.971	0.996	0.938	0.969	0.997	0.931	0.972	0.994
	1500	WSNS	0.947	0.977	0.999	0.940	0.976	0.997	0.914	0.958	0.990
		PR10	0.947	0.976	0.997	0.936	0.968	0.994	0.875	0.947	0.990
		S15	0.935	0.971	1.000	0.937	0.965	0.995	0.907	0.957	0.990
0.8	500	WSNS	0.938	0.974	0.999	0.920	0.965	0.994	0.904	0.949	0.987
		PR10	0.989	0.998	1.000	0.981	0.995	0.999	0.957	0.983	0.999
		S15	0.985	0.995	1.000	0.976	0.994	1.000	0.976	0.985	1.000
	1000	WSNS	0.962	0.992	0.999	0.932	0.965	0.997	0.926	0.969	0.995
		PR10	0.992	0.999	1.000	0.986	0.997	1.000	0.964	0.988	0.998
		S15	0.983	0.999	1.000	0.984	0.995	1.000	0.977	0.992	0.999
	1500	WSNS	0.955	0.988	0.999	0.945	0.977	0.998	0.929	0.964	0.992
		PR10	0.994	0.998	1.000	0.987	0.994	1.000	0.973	0.992	0.999
		S15	0.989	0.997	1.000	0.978	0.994	0.999	0.967	0.990	0.998

Table 3.19: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.861	0.914	0.967	0.876	0.923	0.975	0.825	0.893	0.957
		PR10	0.858	0.929	0.974	0.866	0.918	0.976	0.846	0.919	0.973
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.895	0.937	0.979	0.884	0.938	0.988	0.863	0.918	0.967
		PR10	0.887	0.940	0.979	0.881	0.941	0.984	0.889	0.945	0.984
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.900	0.953	0.988	0.902	0.954	0.987	0.881	0.932	0.983
		PR10	0.903	0.942	0.982	0.892	0.950	0.982	0.888	0.941	0.993
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.885	0.929	0.970	0.862	0.922	0.975	0.840	0.908	0.965
		PR10	0.922	0.965	0.994	0.919	0.950	0.989	0.887	0.943	0.976
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.902	0.942	0.984	0.896	0.951	0.990	0.871	0.929	0.981
		PR10	0.936	0.973	0.994	0.930	0.973	0.992	0.918	0.958	0.992
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.909	0.954	0.991	0.908	0.961	0.994	0.876	0.937	0.985
		PR10	0.953	0.978	0.996	0.936	0.977	0.996	0.915	0.963	0.994
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.895	0.938	0.984	0.878	0.927	0.981	0.893	0.938	0.975
		PR10	0.992	0.999	1.000	0.971	0.987	0.998	0.962	0.986	0.997
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.915	0.960	0.994	0.899	0.948	0.986	0.897	0.949	0.982
		PR10	0.995	0.998	1.000	0.980	0.994	1.000	0.982	0.994	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.922	0.964	0.993	0.892	0.957	0.994	0.890	0.937	0.981
		PR10	0.991	0.998	1.000	0.980	0.994	1.000	0.980	0.996	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.20: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.891	0.943	0.976	0.888	0.934	0.982	0.872	0.927	0.975
		PR10	0.859	0.920	0.973	0.852	0.920	0.971	0.831	0.906	0.965
		S15	0.878	0.941	0.989	0.897	0.939	0.988	0.889	0.949	0.989
	1000	WSNS	0.921	0.953	0.990	0.909	0.949	0.985	0.886	0.933	0.982
		PR10	0.881	0.931	0.976	0.888	0.934	0.979	0.872	0.933	0.976
		S15	0.902	0.943	0.985	0.908	0.949	0.986	0.898	0.952	0.988
	1500	WSNS	0.931	0.963	0.993	0.925	0.964	0.994	0.895	0.939	0.988
		PR10	0.893	0.941	0.986	0.899	0.944	0.980	0.861	0.934	0.988
		S15	0.904	0.941	0.987	0.902	0.944	0.986	0.896	0.943	0.989
0.4	500	WSNS	0.910	0.946	0.982	0.881	0.927	0.976	0.859	0.914	0.971
		PR10	0.919	0.970	0.994	0.916	0.952	0.984	0.879	0.932	0.972
		S15	0.925	0.969	0.997	0.931	0.967	0.993	0.910	0.956	0.991
	1000	WSNS	0.918	0.964	0.995	0.924	0.958	0.987	0.894	0.946	0.987
		PR10	0.937	0.976	0.993	0.924	0.963	0.993	0.895	0.942	0.986
		S15	0.928	0.971	0.996	0.938	0.969	0.997	0.931	0.972	0.994
	1500	WSNS	0.932	0.970	0.996	0.927	0.961	0.997	0.905	0.953	0.988
		PR10	0.948	0.977	0.997	0.936	0.969	0.994	0.885	0.949	0.989
		S15	0.935	0.971	1.000	0.937	0.965	0.995	0.907	0.957	0.990
0.8	500	WSNS	0.927	0.957	0.993	0.906	0.949	0.990	0.895	0.943	0.979
		PR10	0.986	0.998	1.000	0.980	0.996	0.999	0.954	0.984	0.999
		S15	0.985	0.995	1.000	0.976	0.994	1.000	0.976	0.985	1.000
	1000	WSNS	0.950	0.983	0.999	0.917	0.960	0.995	0.913	0.968	0.993
		PR10	0.992	0.998	1.000	0.984	0.996	1.000	0.973	0.990	1.000
		S15	0.983	0.999	1.000	0.984	0.995	1.000	0.977	0.992	0.999
	1500	WSNS	0.947	0.981	0.999	0.931	0.968	0.996	0.916	0.959	0.993
		PR10	0.994	0.998	0.999	0.987	0.993	1.000	0.975	0.992	0.999
		S15	0.989	0.997	1.000	0.978	0.994	0.999	0.967	0.990	0.998

Table 3.21: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.853	0.910	0.958	0.866	0.917	0.970	0.816	0.893	0.948
		PR10	0.851	0.913	0.970	0.858	0.912	0.972	0.836	0.910	0.974
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.893	0.936	0.978	0.885	0.937	0.982	0.866	0.920	0.970
		PR10	0.882	0.934	0.974	0.878	0.933	0.983	0.886	0.943	0.982
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.904	0.946	0.987	0.905	0.953	0.989	0.882	0.934	0.979
		PR10	0.900	0.942	0.982	0.894	0.953	0.982	0.883	0.939	0.991
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.876	0.921	0.966	0.860	0.919	0.973	0.840	0.899	0.954
		PR10	0.915	0.957	0.991	0.905	0.948	0.984	0.881	0.933	0.975
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.895	0.943	0.979	0.890	0.947	0.985	0.877	0.932	0.981
		PR10	0.932	0.974	0.994	0.930	0.971	0.990	0.903	0.957	0.990
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.907	0.955	0.989	0.906	0.958	0.990	0.882	0.938	0.982
		PR10	0.953	0.977	0.996	0.937	0.979	0.996	0.913	0.960	0.993
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.892	0.936	0.980	0.871	0.931	0.973	0.888	0.930	0.975
		PR10	0.983	0.998	1.000	0.965	0.986	0.995	0.959	0.982	0.996
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.917	0.961	0.993	0.902	0.945	0.988	0.899	0.951	0.980
		PR10	0.996	0.998	1.000	0.981	0.994	1.000	0.979	0.992	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.924	0.961	0.992	0.896	0.953	0.995	0.893	0.936	0.986
		PR10	0.991	0.998	1.000	0.980	0.992	1.000	0.978	0.996	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.22: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.874	0.928	0.968	0.875	0.928	0.971	0.863	0.920	0.963
		PR10	0.853	0.909	0.970	0.845	0.917	0.967	0.822	0.895	0.960
		S15	0.878	0.941	0.989	0.897	0.939	0.988	0.889	0.949	0.989
	1000	WSNS	0.912	0.949	0.985	0.901	0.946	0.984	0.883	0.932	0.981
		PR10	0.879	0.933	0.973	0.883	0.936	0.978	0.871	0.931	0.976
		S15	0.902	0.943	0.985	0.908	0.949	0.986	0.898	0.952	0.988
	1500	WSNS	0.927	0.962	0.992	0.922	0.961	0.993	0.899	0.934	0.983
		PR10	0.896	0.939	0.984	0.901	0.944	0.979	0.864	0.932	0.987
		S15	0.904	0.941	0.987	0.902	0.944	0.986	0.896	0.943	0.989
0.4	500	WSNS	0.890	0.935	0.973	0.874	0.919	0.965	0.860	0.914	0.965
		PR10	0.915	0.964	0.993	0.910	0.951	0.985	0.871	0.929	0.965
		S15	0.925	0.969	0.997	0.931	0.967	0.993	0.910	0.956	0.991
	1000	WSNS	0.911	0.950	0.987	0.914	0.952	0.986	0.895	0.945	0.991
		PR10	0.937	0.976	0.993	0.921	0.963	0.992	0.896	0.944	0.988
		S15	0.928	0.971	0.996	0.938	0.969	0.997	0.931	0.972	0.994
	1500	WSNS	0.929	0.968	0.994	0.923	0.963	0.994	0.903	0.949	0.990
		PR10	0.944	0.977	0.996	0.937	0.970	0.993	0.892	0.951	0.988
		S15	0.935	0.971	1.000	0.937	0.965	0.995	0.907	0.957	0.990
0.8	500	WSNS	0.914	0.949	0.991	0.897	0.940	0.986	0.887	0.934	0.981
		PR10	0.986	0.997	1.000	0.977	0.993	0.999	0.955	0.981	0.999
		S15	0.985	0.995	1.000	0.976	0.994	1.000	0.976	0.985	1.000
	1000	WSNS	0.946	0.979	0.998	0.912	0.953	0.993	0.912	0.961	0.992
		PR10	0.992	0.998	1.000	0.984	0.995	1.000	0.974	0.990	0.999
		S15	0.983	0.999	1.000	0.984	0.995	1.000	0.977	0.992	0.999
	1500	WSNS	0.944	0.980	0.999	0.927	0.970	0.995	0.915	0.960	0.991
		PR10	0.996	0.998	1.000	0.988	0.993	0.999	0.976	0.993	0.999
		S15	0.989	0.997	1.000	0.978	0.994	0.999	0.967	0.990	0.998

Table 3.23: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.836	0.899	0.953	0.841	0.896	0.960	0.804	0.872	0.940
		PR10	0.847	0.907	0.965	0.856	0.907	0.967	0.829	0.899	0.967
		S15	0.888	0.946	0.989	0.906	0.948	0.990	0.886	0.941	0.986
	1000	WSNS	0.883	0.929	0.969	0.870	0.929	0.982	0.857	0.913	0.966
		PR10	0.882	0.933	0.975	0.878	0.937	0.982	0.882	0.939	0.980
		S15	0.905	0.948	0.989	0.901	0.955	0.991	0.901	0.952	0.987
	1500	WSNS	0.900	0.937	0.982	0.893	0.948	0.984	0.876	0.924	0.977
		PR10	0.897	0.940	0.978	0.895	0.952	0.983	0.885	0.936	0.990
		S15	0.901	0.940	0.989	0.907	0.958	0.991	0.887	0.948	0.992
0.4	500	WSNS	0.859	0.914	0.957	0.851	0.907	0.967	0.832	0.897	0.949
		PR10	0.912	0.952	0.987	0.901	0.944	0.983	0.870	0.923	0.972
		S15	0.927	0.972	0.996	0.928	0.972	0.996	0.907	0.946	0.988
	1000	WSNS	0.884	0.930	0.975	0.882	0.941	0.979	0.871	0.925	0.979
		PR10	0.930	0.973	0.994	0.933	0.967	0.990	0.905	0.956	0.991
		S15	0.935	0.968	0.996	0.937	0.971	0.995	0.924	0.971	0.994
	1500	WSNS	0.902	0.946	0.987	0.899	0.949	0.989	0.871	0.938	0.983
		PR10	0.951	0.973	0.996	0.938	0.977	0.996	0.910	0.956	0.992
		S15	0.942	0.973	0.999	0.939	0.973	0.995	0.896	0.960	0.993
0.8	500	WSNS	0.889	0.933	0.974	0.874	0.920	0.968	0.871	0.923	0.970
		PR10	0.980	0.997	1.000	0.966	0.982	0.997	0.951	0.977	0.996
		S15	0.980	0.996	1.000	0.966	0.986	0.998	0.958	0.981	0.997
	1000	WSNS	0.909	0.948	0.992	0.897	0.940	0.988	0.892	0.948	0.978
		PR10	0.995	0.999	1.000	0.976	0.992	1.000	0.980	0.991	0.999
		S15	0.981	0.999	1.000	0.968	0.990	1.000	0.970	0.989	0.999
	1500	WSNS	0.913	0.955	0.990	0.891	0.946	0.995	0.891	0.934	0.983
		PR10	0.991	0.999	1.000	0.980	0.993	1.000	0.977	0.996	1.000
		S15	0.989	0.995	1.000	0.970	0.988	0.999	0.973	0.991	0.999

Table 3.24: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with standard normal innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.857	0.912	0.958	0.848	0.908	0.962	0.847	0.896	0.954
		PR10	0.849	0.898	0.967	0.842	0.906	0.960	0.822	0.891	0.956
		S15	0.878	0.941	0.989	0.897	0.939	0.988	0.889	0.949	0.989
	1000	WSNS	0.896	0.939	0.979	0.894	0.930	0.979	0.876	0.930	0.976
		PR10	0.877	0.927	0.975	0.885	0.930	0.978	0.873	0.926	0.975
		S15	0.902	0.943	0.985	0.908	0.949	0.986	0.898	0.952	0.988
	1500	WSNS	0.916	0.957	0.991	0.921	0.955	0.991	0.890	0.934	0.980
		PR10	0.897	0.939	0.982	0.901	0.945	0.977	0.866	0.934	0.987
		S15	0.904	0.941	0.987	0.902	0.944	0.986	0.896	0.943	0.989
0.4	500	WSNS	0.880	0.920	0.967	0.860	0.912	0.954	0.848	0.902	0.951
		PR10	0.909	0.955	0.989	0.902	0.942	0.981	0.871	0.925	0.963
		S15	0.925	0.969	0.997	0.931	0.967	0.993	0.910	0.956	0.991
	1000	WSNS	0.903	0.944	0.987	0.900	0.943	0.983	0.881	0.940	0.987
		PR10	0.936	0.973	0.993	0.925	0.963	0.993	0.891	0.944	0.986
		S15	0.928	0.971	0.996	0.938	0.969	0.997	0.931	0.972	0.994
	1500	WSNS	0.917	0.964	0.992	0.918	0.955	0.992	0.901	0.943	0.987
		PR10	0.943	0.977	0.996	0.941	0.970	0.993	0.894	0.947	0.987
		S15	0.935	0.971	1.000	0.937	0.965	0.995	0.907	0.957	0.990
0.8	500	WSNS	0.906	0.941	0.981	0.880	0.930	0.979	0.875	0.926	0.971
		PR10	0.982	0.996	1.000	0.972	0.991	0.999	0.948	0.978	0.998
		S15	0.985	0.995	1.000	0.976	0.994	1.000	0.976	0.985	1.000
	1000	WSNS	0.923	0.975	0.998	0.905	0.945	0.990	0.900	0.958	0.989
		PR10	0.993	0.998	1.000	0.985	0.997	1.000	0.973	0.990	1.000
		S15	0.983	0.999	1.000	0.984	0.995	1.000	0.977	0.992	0.999
	1500	WSNS	0.935	0.970	0.999	0.919	0.964	0.992	0.915	0.953	0.991
		PR10	0.996	0.998	1.000	0.988	0.994	0.999	0.975	0.993	0.999
		S15	0.989	0.997	1.000	0.978	0.994	0.999	0.967	0.990	0.998

Table 3.25: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.884	0.935	0.985	0.898	0.941	0.984	0.866	0.919	0.970
		PR10	0.872	0.931	0.980	0.868	0.928	0.974	0.882	0.935	0.983
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.910	0.958	0.989	0.897	0.958	0.991	0.874	0.934	0.979
		PR10	0.892	0.936	0.983	0.876	0.938	0.986	0.880	0.944	0.993
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.919	0.959	0.996	0.920	0.968	0.997	0.869	0.929	0.983
		PR10	0.893	0.944	0.984	0.884	0.933	0.987	0.883	0.943	0.994
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.892	0.942	0.984	0.890	0.939	0.981	0.866	0.908	0.976
		PR10	0.939	0.976	0.998	0.921	0.968	0.996	0.901	0.944	0.988
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.903	0.947	0.989	0.891	0.945	0.995	0.874	0.935	0.976
		PR10	0.936	0.980	0.996	0.932	0.965	0.992	0.914	0.953	0.988
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.891	0.952	0.993	0.904	0.954	0.995	0.865	0.929	0.978
		PR10	0.951	0.980	0.994	0.933	0.974	0.996	0.913	0.966	0.994
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.900	0.947	0.984	0.871	0.918	0.978	0.874	0.923	0.969
		PR10	0.993	0.996	1.000	0.972	0.994	1.000	0.971	0.993	1.000
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.896	0.948	0.990	0.886	0.941	0.991	0.849	0.911	0.969
		PR10	0.992	0.998	1.000	0.980	0.993	1.000	0.968	0.989	0.997
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.900	0.956	0.991	0.880	0.943	0.991	0.854	0.917	0.968
		PR10	0.992	0.999	1.000	0.982	0.993	1.000	0.974	0.991	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

Table 3.26: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.932	0.972	0.995	0.921	0.967	0.991	0.884	0.936	0.986
		PR10	0.855	0.927	0.976	0.857	0.910	0.976	0.842	0.905	0.982
		S15	0.882	0.940	0.994	0.884	0.927	0.986	0.898	0.942	0.988
	1000	WSNS	0.946	0.980	0.997	0.945	0.982	0.997	0.905	0.952	0.988
		PR10	0.873	0.934	0.981	0.862	0.940	0.978	0.842	0.917	0.983
		S15	0.892	0.955	0.985	0.903	0.954	0.990	0.900	0.952	0.992
	1500	WSNS	0.952	0.990	1.000	0.942	0.986	0.998	0.889	0.941	0.989
		PR10	0.892	0.938	0.982	0.877	0.935	0.985	0.831	0.908	0.985
		S15	0.900	0.952	0.989	0.902	0.943	0.993	0.887	0.936	0.988
0.4	500	WSNS	0.931	0.977	0.995	0.921	0.964	0.993	0.895	0.938	0.981
		PR10	0.934	0.968	0.998	0.914	0.957	0.993	0.865	0.923	0.983
		S15	0.924	0.972	0.996	0.917	0.952	0.993	0.910	0.959	0.991
	1000	WSNS	0.947	0.982	0.996	0.950	0.977	0.994	0.894	0.944	0.987
		PR10	0.931	0.974	0.995	0.923	0.961	0.995	0.875	0.931	0.989
		S15	0.929	0.965	0.998	0.926	0.964	0.996	0.906	0.955	0.991
	1500	WSNS	0.947	0.985	0.999	0.933	0.972	1.000	0.902	0.950	0.986
		PR10	0.946	0.975	0.993	0.926	0.971	0.995	0.871	0.940	0.989
		S15	0.939	0.976	0.998	0.932	0.971	0.997	0.898	0.953	0.994
0.8	500	WSNS	0.945	0.976	0.998	0.906	0.964	0.994	0.892	0.939	0.980
		PR10	0.990	0.998	1.000	0.980	0.998	1.000	0.943	0.982	0.997
		S15	0.981	0.995	0.999	0.969	0.990	1.000	0.953	0.983	0.999
	1000	WSNS	0.954	0.980	0.998	0.916	0.961	0.992	0.888	0.936	0.987
		PR10	0.987	0.996	1.000	0.986	0.994	1.000	0.946	0.981	0.999
		S15	0.981	0.994	1.000	0.975	0.992	0.999	0.954	0.985	0.998
	1500	WSNS	0.957	0.985	0.999	0.919	0.963	0.997	0.902	0.939	0.979
		PR10	0.990	0.999	1.000	0.977	0.994	1.000	0.956	0.983	1.000
		S15	0.983	0.994	1.000	0.966	0.988	0.997	0.970	0.991	0.999

Table 3.27: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.870	0.925	0.973	0.877	0.932	0.977	0.864	0.912	0.970
		PR10	0.870	0.925	0.977	0.867	0.926	0.973	0.880	0.930	0.976
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.898	0.943	0.979	0.885	0.944	0.985	0.880	0.931	0.984
		PR10	0.893	0.932	0.982	0.884	0.940	0.983	0.887	0.943	0.992
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.900	0.952	0.991	0.903	0.962	0.993	0.863	0.931	0.986
		PR10	0.892	0.944	0.983	0.887	0.937	0.985	0.883	0.942	0.992
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.884	0.934	0.976	0.879	0.931	0.983	0.865	0.912	0.970
		PR10	0.937	0.974	0.997	0.925	0.962	0.994	0.896	0.941	0.985
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.891	0.947	0.986	0.880	0.935	0.993	0.880	0.936	0.974
		PR10	0.935	0.980	0.994	0.931	0.965	0.994	0.912	0.954	0.988
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.881	0.943	0.990	0.900	0.951	0.991	0.863	0.931	0.981
		PR10	0.950	0.979	0.993	0.935	0.974	0.997	0.916	0.960	0.994
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.903	0.949	0.986	0.866	0.923	0.979	0.885	0.933	0.965
		PR10	0.993	0.997	1.000	0.971	0.994	1.000	0.973	0.990	0.998
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.890	0.948	0.989	0.887	0.937	0.979	0.860	0.918	0.977
		PR10	0.990	0.998	1.000	0.980	0.995	1.000	0.968	0.987	0.998
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.895	0.954	0.993	0.878	0.939	0.991	0.867	0.923	0.974
		PR10	0.991	0.999	1.000	0.982	0.995	1.000	0.974	0.990	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

Table 3.28: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.905	0.950	0.988	0.894	0.950	0.987	0.873	0.929	0.980
		PR10	0.854	0.917	0.976	0.858	0.913	0.974	0.855	0.912	0.971
		S15	0.882	0.940	0.994	0.884	0.927	0.986	0.898	0.942	0.988
	1000	WSNS	0.920	0.964	0.995	0.928	0.969	0.994	0.900	0.947	0.987
		PR10	0.876	0.934	0.982	0.868	0.938	0.981	0.860	0.922	0.983
		S15	0.892	0.955	0.985	0.903	0.954	0.990	0.900	0.952	0.992
	1500	WSNS	0.931	0.977	0.999	0.928	0.971	0.997	0.895	0.934	0.987
		PR10	0.895	0.939	0.980	0.880	0.939	0.986	0.850	0.917	0.986
		S15	0.900	0.952	0.989	0.902	0.943	0.993	0.887	0.936	0.988
0.4	500	WSNS	0.911	0.952	0.989	0.900	0.946	0.988	0.886	0.936	0.981
		PR10	0.929	0.968	0.994	0.918	0.959	0.991	0.874	0.927	0.981
		S15	0.924	0.972	0.996	0.917	0.952	0.993	0.910	0.959	0.991
	1000	WSNS	0.928	0.969	0.992	0.923	0.966	0.991	0.890	0.937	0.984
		PR10	0.933	0.977	0.994	0.928	0.965	0.996	0.888	0.938	0.986
		S15	0.929	0.965	0.998	0.926	0.964	0.996	0.906	0.955	0.991
	1500	WSNS	0.932	0.978	0.998	0.916	0.961	0.997	0.905	0.944	0.986
		PR10	0.948	0.975	0.993	0.933	0.969	0.994	0.892	0.946	0.989
		S15	0.939	0.976	0.998	0.932	0.971	0.997	0.898	0.953	0.994
0.8	500	WSNS	0.938	0.974	0.995	0.893	0.948	0.989	0.886	0.937	0.976
		PR10	0.990	0.998	1.000	0.981	0.998	1.000	0.948	0.980	0.998
		S15	0.981	0.995	0.999	0.969	0.990	1.000	0.953	0.983	0.999
	1000	WSNS	0.937	0.974	0.997	0.901	0.956	0.993	0.889	0.938	0.986
		PR10	0.990	0.997	1.000	0.985	0.994	1.000	0.958	0.981	0.998
		S15	0.981	0.994	1.000	0.975	0.992	0.999	0.954	0.985	0.998
	1500	WSNS	0.946	0.978	0.999	0.913	0.955	0.996	0.904	0.943	0.974
		PR10	0.990	0.999	1.000	0.984	0.994	1.000	0.962	0.984	1.000
		S15	0.983	0.994	1.000	0.966	0.988	0.997	0.970	0.991	0.999

Table 3.29: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.851	0.906	0.970	0.869	0.923	0.971	0.856	0.907	0.959
		PR10	0.860	0.920	0.975	0.865	0.921	0.972	0.878	0.932	0.966
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.886	0.939	0.977	0.874	0.934	0.984	0.876	0.932	0.977
		PR10	0.893	0.933	0.980	0.881	0.935	0.982	0.884	0.939	0.988
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.894	0.945	0.992	0.903	0.961	0.991	0.866	0.933	0.986
		PR10	0.896	0.940	0.980	0.884	0.933	0.985	0.881	0.939	0.988
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.873	0.929	0.972	0.860	0.931	0.975	0.863	0.905	0.960
		PR10	0.933	0.970	0.994	0.919	0.964	0.993	0.896	0.939	0.982
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.882	0.943	0.986	0.872	0.927	0.988	0.884	0.938	0.972
		PR10	0.935	0.976	0.995	0.925	0.965	0.992	0.910	0.951	0.986
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.877	0.938	0.988	0.896	0.948	0.990	0.866	0.931	0.982
		PR10	0.947	0.979	0.992	0.932	0.969	0.997	0.913	0.961	0.994
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.895	0.942	0.980	0.870	0.920	0.971	0.879	0.925	0.965
		PR10	0.991	0.997	1.000	0.967	0.990	1.000	0.968	0.987	0.998
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.887	0.947	0.988	0.884	0.935	0.982	0.865	0.923	0.972
		PR10	0.989	0.998	1.000	0.980	0.996	1.000	0.967	0.986	0.998
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.895	0.952	0.990	0.877	0.936	0.989	0.869	0.927	0.973
		PR10	0.992	0.999	1.000	0.981	0.994	1.000	0.974	0.991	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

Table 3.30: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.878	0.931	0.975	0.873	0.927	0.975	0.866	0.922	0.970
		PR10	0.852	0.909	0.974	0.848	0.908	0.968	0.855	0.914	0.963
		S15	0.882	0.940	0.994	0.884	0.927	0.986	0.898	0.942	0.988
	1000	WSNS	0.908	0.955	0.991	0.917	0.958	0.991	0.891	0.944	0.985
		PR10	0.878	0.930	0.982	0.871	0.937	0.981	0.865	0.922	0.981
		S15	0.892	0.955	0.985	0.903	0.954	0.990	0.900	0.952	0.992
	1500	WSNS	0.917	0.969	0.996	0.918	0.964	0.993	0.894	0.937	0.984
		PR10	0.894	0.941	0.980	0.879	0.937	0.984	0.855	0.922	0.984
		S15	0.900	0.952	0.989	0.902	0.943	0.993	0.887	0.936	0.988
0.4	500	WSNS	0.896	0.944	0.984	0.893	0.931	0.983	0.876	0.932	0.972
		PR10	0.927	0.962	0.992	0.911	0.954	0.987	0.875	0.929	0.973
		S15	0.924	0.972	0.996	0.917	0.952	0.993	0.910	0.959	0.991
	1000	WSNS	0.913	0.961	0.992	0.913	0.958	0.986	0.894	0.942	0.984
		PR10	0.937	0.976	0.994	0.925	0.962	0.994	0.891	0.939	0.985
		S15	0.929	0.965	0.998	0.926	0.964	0.996	0.906	0.955	0.991
	1500	WSNS	0.918	0.965	0.996	0.907	0.955	0.990	0.900	0.947	0.984
		PR10	0.947	0.977	0.992	0.932	0.969	0.993	0.901	0.950	0.988
		S15	0.939	0.976	0.998	0.932	0.971	0.997	0.898	0.953	0.994
0.8	500	WSNS	0.932	0.965	0.991	0.889	0.941	0.985	0.876	0.933	0.982
		PR10	0.989	0.995	1.000	0.978	0.994	1.000	0.947	0.977	0.999
		S15	0.981	0.995	0.999	0.969	0.990	1.000	0.953	0.983	0.999
	1000	WSNS	0.926	0.971	0.997	0.897	0.951	0.991	0.893	0.942	0.987
		PR10	0.988	0.997	1.000	0.986	0.994	1.000	0.961	0.984	0.998
		S15	0.981	0.994	1.000	0.975	0.992	0.999	0.954	0.985	0.998
	1500	WSNS	0.936	0.975	0.998	0.911	0.952	0.994	0.903	0.941	0.977
		PR10	0.989	0.998	1.000	0.982	0.993	1.000	0.963	0.985	0.999
		S15	0.983	0.994	1.000	0.966	0.988	0.997	0.970	0.991	0.999

Table 3.31: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.850	0.898	0.961	0.855	0.915	0.962	0.846	0.897	0.952
		PR10	0.858	0.914	0.973	0.859	0.920	0.969	0.872	0.923	0.966
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.878	0.936	0.973	0.869	0.929	0.976	0.873	0.930	0.974
		PR10	0.886	0.934	0.981	0.879	0.938	0.980	0.882	0.936	0.986
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.884	0.937	0.987	0.893	0.955	0.989	0.867	0.926	0.981
		PR10	0.895	0.940	0.980	0.885	0.936	0.983	0.879	0.939	0.986
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.870	0.920	0.961	0.853	0.920	0.970	0.856	0.907	0.959
		PR10	0.934	0.965	0.994	0.922	0.962	0.993	0.890	0.932	0.981
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.879	0.933	0.980	0.868	0.922	0.983	0.883	0.931	0.969
		PR10	0.932	0.976	0.994	0.923	0.964	0.990	0.904	0.948	0.986
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.873	0.937	0.988	0.895	0.941	0.987	0.868	0.929	0.979
		PR10	0.945	0.976	0.992	0.931	0.971	0.997	0.914	0.956	0.993
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.894	0.943	0.974	0.864	0.914	0.968	0.878	0.919	0.962
		PR10	0.985	0.995	1.000	0.967	0.989	0.999	0.966	0.986	0.998
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.882	0.945	0.990	0.886	0.937	0.978	0.862	0.921	0.975
		PR10	0.992	0.999	1.000	0.982	0.994	1.000	0.965	0.987	0.998
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.894	0.949	0.994	0.877	0.935	0.989	0.874	0.928	0.975
		PR10	0.991	0.998	1.000	0.979	0.995	1.000	0.973	0.986	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

Table 3.32: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.861	0.914	0.968	0.858	0.916	0.969	0.860	0.909	0.961
		PR10	0.851	0.902	0.967	0.849	0.906	0.965	0.850	0.915	0.966
		S15	0.882	0.940	0.994	0.884	0.927	0.986	0.898	0.942	0.988
	1000	WSNS	0.891	0.941	0.987	0.906	0.947	0.985	0.885	0.938	0.982
		PR10	0.873	0.934	0.978	0.875	0.935	0.980	0.870	0.922	0.978
		S15	0.892	0.955	0.985	0.903	0.954	0.990	0.900	0.952	0.992
	1500	WSNS	0.909	0.959	0.992	0.906	0.957	0.989	0.889	0.934	0.981
		PR10	0.895	0.941	0.980	0.878	0.937	0.985	0.862	0.928	0.986
		S15	0.900	0.952	0.989	0.902	0.943	0.993	0.887	0.936	0.988
0.4	500	WSNS	0.885	0.936	0.977	0.875	0.923	0.973	0.871	0.921	0.966
		PR10	0.924	0.959	0.989	0.908	0.952	0.987	0.873	0.926	0.971
		S15	0.924	0.972	0.996	0.917	0.952	0.993	0.910	0.959	0.991
	1000	WSNS	0.900	0.951	0.987	0.905	0.949	0.983	0.884	0.942	0.984
		PR10	0.936	0.974	0.995	0.927	0.961	0.993	0.894	0.938	0.984
		S15	0.929	0.965	0.998	0.926	0.964	0.996	0.906	0.955	0.991
	1500	WSNS	0.909	0.955	0.993	0.897	0.948	0.989	0.894	0.943	0.984
		PR10	0.948	0.977	0.992	0.934	0.965	0.993	0.900	0.950	0.985
		S15	0.939	0.976	0.998	0.932	0.971	0.997	0.898	0.953	0.994
0.8	500	WSNS	0.926	0.961	0.987	0.881	0.934	0.982	0.871	0.928	0.977
		PR10	0.988	0.996	1.000	0.976	0.994	1.000	0.952	0.977	0.997
		S15	0.981	0.995	0.999	0.969	0.990	1.000	0.953	0.983	0.999
	1000	WSNS	0.915	0.964	0.996	0.887	0.954	0.988	0.894	0.938	0.983
		PR10	0.989	0.998	1.000	0.988	0.995	1.000	0.963	0.984	0.998
		S15	0.981	0.994	1.000	0.975	0.992	0.999	0.954	0.985	0.998
	1500	WSNS	0.930	0.968	0.996	0.912	0.953	0.994	0.904	0.941	0.982
		PR10	0.990	0.998	1.000	0.980	0.991	1.000	0.963	0.986	0.999
		S15	0.983	0.994	1.000	0.966	0.988	0.997	0.970	0.991	0.999

Table 3.33: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.835	0.888	0.954	0.846	0.902	0.958	0.837	0.892	0.942
		PR10	0.848	0.903	0.967	0.858	0.910	0.957	0.863	0.924	0.954
		S15	0.893	0.945	0.991	0.887	0.930	0.987	0.896	0.937	0.992
	1000	WSNS	0.872	0.928	0.965	0.856	0.924	0.974	0.872	0.926	0.973
		PR10	0.878	0.935	0.981	0.875	0.936	0.981	0.879	0.936	0.984
		S15	0.898	0.947	0.988	0.901	0.949	0.986	0.902	0.950	0.995
	1500	WSNS	0.879	0.934	0.986	0.883	0.951	0.988	0.866	0.925	0.980
		PR10	0.892	0.937	0.981	0.882	0.936	0.984	0.878	0.938	0.985
		S15	0.896	0.951	0.989	0.917	0.962	0.989	0.894	0.949	0.990
0.4	500	WSNS	0.863	0.913	0.958	0.844	0.904	0.961	0.853	0.901	0.951
		PR10	0.921	0.959	0.989	0.912	0.956	0.990	0.880	0.925	0.972
		S15	0.938	0.973	0.994	0.934	0.968	0.998	0.914	0.957	0.991
	1000	WSNS	0.870	0.929	0.974	0.863	0.915	0.978	0.873	0.928	0.974
		PR10	0.935	0.973	0.995	0.920	0.958	0.994	0.901	0.944	0.985
		S15	0.941	0.967	0.996	0.916	0.962	0.989	0.910	0.962	0.991
	1500	WSNS	0.872	0.934	0.985	0.893	0.940	0.985	0.867	0.925	0.979
		PR10	0.945	0.979	0.994	0.931	0.970	0.997	0.908	0.956	0.993
		S15	0.940	0.976	0.997	0.933	0.971	0.997	0.908	0.960	0.998
0.8	500	WSNS	0.891	0.938	0.980	0.856	0.905	0.965	0.875	0.915	0.966
		PR10	0.980	0.995	1.000	0.954	0.985	0.997	0.958	0.986	0.996
		S15	0.981	0.995	1.000	0.961	0.987	1.000	0.958	0.982	0.999
	1000	WSNS	0.885	0.938	0.985	0.872	0.931	0.980	0.868	0.916	0.972
		PR10	0.991	0.998	1.000	0.982	0.993	1.000	0.965	0.984	0.998
		S15	0.977	0.994	1.000	0.972	0.993	1.000	0.958	0.982	0.998
	1500	WSNS	0.893	0.946	0.989	0.874	0.931	0.989	0.880	0.927	0.975
		PR10	0.991	0.997	1.000	0.977	0.996	1.000	0.973	0.986	1.000
		S15	0.982	0.994	1.000	0.971	0.988	1.000	0.964	0.987	0.998

Table 3.34: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with standard normal innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.851	0.900	0.959	0.845	0.904	0.962	0.851	0.902	0.954
		PR10	0.845	0.897	0.957	0.843	0.905	0.956	0.850	0.910	0.954
		S15	0.882	0.940	0.994	0.884	0.927	0.986	0.898	0.942	0.988
	1000	WSNS	0.879	0.940	0.977	0.897	0.944	0.985	0.877	0.935	0.981
		PR10	0.875	0.927	0.982	0.867	0.936	0.978	0.869	0.917	0.981
		S15	0.892	0.955	0.985	0.903	0.954	0.990	0.900	0.952	0.992
	1500	WSNS	0.905	0.949	0.992	0.899	0.951	0.991	0.883	0.928	0.981
		PR10	0.894	0.940	0.979	0.878	0.936	0.983	0.862	0.930	0.986
		S15	0.900	0.952	0.989	0.902	0.943	0.993	0.887	0.936	0.988
0.4	500	WSNS	0.872	0.923	0.967	0.868	0.916	0.963	0.862	0.914	0.960
		PR10	0.919	0.953	0.989	0.902	0.947	0.982	0.868	0.927	0.966
		S15	0.924	0.972	0.996	0.917	0.952	0.993	0.910	0.959	0.991
	1000	WSNS	0.894	0.943	0.984	0.897	0.947	0.980	0.884	0.937	0.979
		PR10	0.938	0.974	0.994	0.927	0.961	0.994	0.894	0.941	0.984
		S15	0.929	0.965	0.998	0.926	0.964	0.996	0.906	0.955	0.991
	1500	WSNS	0.898	0.953	0.993	0.892	0.944	0.990	0.888	0.941	0.983
		PR10	0.945	0.977	0.992	0.932	0.967	0.993	0.905	0.948	0.986
		S15	0.939	0.976	0.998	0.932	0.971	0.997	0.898	0.953	0.994
0.8	500	WSNS	0.914	0.958	0.982	0.875	0.934	0.978	0.872	0.925	0.973
		PR10	0.981	0.994	1.000	0.976	0.994	1.000	0.944	0.974	0.995
		S15	0.981	0.995	0.999	0.969	0.990	1.000	0.953	0.983	0.999
	1000	WSNS	0.909	0.957	0.990	0.878	0.942	0.984	0.896	0.938	0.985
		PR10	0.987	0.997	1.000	0.987	0.994	1.000	0.963	0.986	0.997
		S15	0.981	0.994	1.000	0.975	0.992	0.999	0.954	0.985	0.998
	1500	WSNS	0.929	0.968	0.992	0.901	0.948	0.992	0.902	0.942	0.980
		PR10	0.990	0.996	1.000	0.982	0.993	1.000	0.961	0.986	0.999
		S15	0.983	0.994	1.000	0.966	0.988	0.997	0.970	0.991	0.999

Table 3.35: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.886	0.941	0.982	0.873	0.932	0.978	0.894	0.948	0.985
		PR10	0.875	0.929	0.976	0.878	0.934	0.976	0.852	0.953	0.993
		S15	0.894	0.953	0.991	0.889	0.943	0.983	0.911	0.958	0.994
	1000	WSNS	0.893	0.949	0.985	0.903	0.957	0.992	0.902	0.944	0.990
		PR10	0.874	0.930	0.985	0.886	0.938	0.986	0.851	0.938	0.997
		S15	0.892	0.946	0.988	0.906	0.957	0.989	0.918	0.948	0.994
	1500	WSNS	0.928	0.965	0.993	0.901	0.951	0.988	0.901	0.934	0.980
		PR10	0.891	0.946	0.987	0.897	0.956	0.988	0.843	0.924	0.994
		S15	0.915	0.959	0.990	0.905	0.951	0.985	0.906	0.951	0.987
0.4	500	WSNS	0.887	0.936	0.980	0.871	0.935	0.979	0.891	0.940	0.977
		PR10	0.948	0.975	0.992	0.947	0.981	0.994	0.891	0.972	0.996
		S15	0.890	0.947	0.989	0.904	0.954	0.990	0.902	0.952	0.992
	1000	WSNS	0.890	0.946	0.986	0.898	0.950	0.992	0.910	0.951	0.987
		PR10	0.955	0.982	0.997	0.949	0.986	0.998	0.894	0.966	1.000
		S15	0.896	0.943	0.988	0.907	0.959	0.991	0.916	0.958	0.993
	1500	WSNS	0.919	0.962	0.990	0.890	0.941	0.990	0.890	0.934	0.981
		PR10	0.965	0.988	0.999	0.955	0.986	0.999	0.874	0.948	0.997
		S15	0.918	0.957	0.991	0.909	0.950	0.989	0.910	0.954	0.988
0.8	500	WSNS	0.896	0.938	0.983	0.865	0.938	0.985	0.875	0.927	0.978
		PR10	0.995	0.998	1.000	0.991	0.996	1.000	0.992	0.996	1.000
		S15	0.898	0.952	0.989	0.927	0.966	0.996	0.901	0.949	0.994
	1000	WSNS	0.885	0.944	0.985	0.888	0.956	0.994	0.905	0.948	0.987
		PR10	0.999	1.000	1.000	0.996	0.999	1.000	0.996	1.000	1.000
		S15	0.883	0.938	0.989	0.931	0.975	0.997	0.921	0.961	0.994
	1500	WSNS	0.907	0.956	0.992	0.879	0.939	0.992	0.870	0.930	0.982
		PR10	1.000	1.000	1.000	0.995	1.000	1.000	0.990	1.000	1.000
		S15	0.908	0.951	0.988	0.914	0.960	0.995	0.895	0.953	0.989

Table 3.36: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.924	0.964	0.988	0.919	0.959	0.990	0.926	0.974	0.996
		PR10	0.880	0.926	0.972	0.860	0.922	0.974	0.816	0.919	0.985
		S15	0.900	0.945	0.987	0.897	0.943	0.988	0.903	0.961	0.995
	1000	WSNS	0.926	0.976	0.993	0.920	0.965	0.994	0.921	0.961	0.994
		PR10	0.863	0.921	0.983	0.867	0.935	0.978	0.805	0.916	0.991
		S15	0.892	0.943	0.989	0.881	0.943	0.987	0.906	0.950	0.993
	1500	WSNS	0.953	0.982	0.996	0.945	0.977	0.995	0.912	0.955	0.987
		PR10	0.886	0.935	0.986	0.901	0.949	0.986	0.800	0.905	0.990
		S15	0.915	0.958	0.992	0.912	0.959	0.990	0.910	0.948	0.987
0.4	500	WSNS	0.925	0.969	0.988	0.923	0.963	0.992	0.915	0.962	0.994
		PR10	0.941	0.969	0.991	0.936	0.974	0.996	0.862	0.942	0.990
		S15	0.901	0.947	0.987	0.901	0.959	0.993	0.902	0.958	0.991
	1000	WSNS	0.936	0.971	0.996	0.920	0.965	0.996	0.913	0.963	0.989
		PR10	0.946	0.980	0.997	0.939	0.973	0.996	0.847	0.937	0.998
		S15	0.892	0.948	0.989	0.899	0.946	0.990	0.902	0.954	0.989
	1500	WSNS	0.945	0.983	0.996	0.936	0.976	0.997	0.912	0.948	0.992
		PR10	0.965	0.987	0.998	0.960	0.983	0.998	0.854	0.932	0.994
		S15	0.908	0.949	0.990	0.902	0.961	0.995	0.912	0.949	0.990
0.8	500	WSNS	0.932	0.964	0.990	0.921	0.958	0.991	0.910	0.954	0.990
		PR10	0.996	0.997	1.000	0.992	0.998	0.999	0.978	0.998	1.000
		S15	0.907	0.948	0.991	0.928	0.963	0.993	0.895	0.948	0.991
	1000	WSNS	0.924	0.964	0.995	0.908	0.960	0.993	0.910	0.953	0.982
		PR10	0.999	0.999	1.000	0.998	1.000	1.000	0.986	0.999	1.000
		S15	0.875	0.935	0.991	0.917	0.961	0.995	0.893	0.950	0.991
	1500	WSNS	0.942	0.976	0.996	0.921	0.967	0.995	0.927	0.965	0.987
		PR10	1.000	1.000	1.000	0.997	1.000	1.000	0.981	1.000	1.000
		S15	0.904	0.944	0.989	0.928	0.967	0.992	0.921	0.963	0.989

Table 3.37: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.880	0.931	0.971	0.862	0.930	0.972	0.898	0.945	0.987
		PR10	0.869	0.922	0.972	0.872	0.929	0.977	0.862	0.946	0.989
		S15	0.894	0.953	0.991	0.889	0.943	0.983	0.911	0.958	0.994
	1000	WSNS	0.896	0.943	0.983	0.896	0.951	0.988	0.908	0.947	0.986
		PR10	0.869	0.927	0.982	0.887	0.937	0.983	0.857	0.940	0.994
		S15	0.892	0.946	0.988	0.906	0.957	0.989	0.918	0.948	0.994
	1500	WSNS	0.922	0.957	0.991	0.899	0.946	0.987	0.905	0.937	0.980
		PR10	0.889	0.944	0.985	0.898	0.955	0.986	0.855	0.932	0.992
		S15	0.915	0.959	0.990	0.905	0.951	0.985	0.906	0.951	0.987
0.4	500	WSNS	0.887	0.937	0.975	0.866	0.923	0.972	0.895	0.932	0.979
		PR10	0.940	0.972	0.991	0.940	0.972	0.995	0.892	0.966	0.993
		S15	0.890	0.947	0.989	0.904	0.954	0.990	0.902	0.952	0.992
	1000	WSNS	0.891	0.944	0.984	0.883	0.949	0.985	0.918	0.951	0.989
		PR10	0.954	0.980	0.998	0.946	0.983	0.998	0.901	0.963	0.999
		S15	0.896	0.943	0.988	0.907	0.959	0.991	0.916	0.958	0.993
	1500	WSNS	0.918	0.965	0.990	0.892	0.941	0.986	0.893	0.944	0.979
		PR10	0.965	0.987	0.999	0.956	0.987	0.999	0.883	0.957	0.996
		S15	0.918	0.957	0.991	0.909	0.950	0.989	0.910	0.954	0.988
0.8	500	WSNS	0.894	0.942	0.975	0.868	0.920	0.980	0.881	0.928	0.971
		PR10	0.995	0.997	1.000	0.986	0.996	0.999	0.990	0.996	0.999
		S15	0.898	0.952	0.989	0.927	0.966	0.996	0.901	0.949	0.994
	1000	WSNS	0.890	0.939	0.985	0.887	0.957	0.992	0.917	0.949	0.990
		PR10	0.999	1.000	1.000	0.995	0.998	1.000	0.996	1.000	1.000
		S15	0.883	0.938	0.989	0.931	0.975	0.997	0.921	0.961	0.994
	1500	WSNS	0.912	0.950	0.989	0.880	0.941	0.989	0.883	0.942	0.984
		PR10	1.000	1.000	1.000	0.995	1.000	1.000	0.987	1.000	1.000
		S15	0.908	0.951	0.988	0.914	0.960	0.995	0.895	0.953	0.989

Table 3.38: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.907	0.953	0.985	0.902	0.948	0.981	0.920	0.962	0.992
		PR10	0.878	0.922	0.970	0.863	0.918	0.972	0.834	0.919	0.981
		S15	0.900	0.945	0.987	0.897	0.943	0.988	0.903	0.961	0.995
	1000	WSNS	0.914	0.964	0.989	0.900	0.952	0.990	0.922	0.963	0.992
		PR10	0.864	0.917	0.981	0.874	0.930	0.981	0.826	0.927	0.988
		S15	0.892	0.943	0.989	0.881	0.943	0.987	0.906	0.950	0.993
	1500	WSNS	0.943	0.978	0.993	0.931	0.970	0.992	0.916	0.952	0.988
		PR10	0.888	0.936	0.988	0.901	0.949	0.987	0.824	0.918	0.990
		S15	0.915	0.958	0.992	0.912	0.959	0.990	0.910	0.948	0.987
0.4	500	WSNS	0.914	0.961	0.981	0.900	0.951	0.988	0.911	0.958	0.990
		PR10	0.938	0.969	0.990	0.933	0.974	0.994	0.878	0.950	0.988
		S15	0.901	0.947	0.987	0.901	0.959	0.993	0.902	0.958	0.991
	1000	WSNS	0.923	0.965	0.989	0.909	0.954	0.990	0.921	0.967	0.987
		PR10	0.944	0.979	0.997	0.941	0.972	0.997	0.875	0.940	0.997
		S15	0.892	0.948	0.989	0.899	0.946	0.990	0.902	0.954	0.989
	1500	WSNS	0.938	0.976	0.997	0.915	0.967	0.993	0.915	0.952	0.989
		PR10	0.964	0.988	0.999	0.963	0.981	0.999	0.873	0.945	0.994
		S15	0.908	0.949	0.990	0.902	0.961	0.995	0.912	0.949	0.990
0.8	500	WSNS	0.913	0.960	0.983	0.908	0.954	0.985	0.904	0.946	0.983
		PR10	0.993	0.997	1.000	0.989	0.997	0.999	0.986	0.998	1.000
		S15	0.907	0.948	0.991	0.928	0.963	0.993	0.895	0.948	0.991
	1000	WSNS	0.913	0.956	0.993	0.899	0.949	0.990	0.906	0.953	0.987
		PR10	0.999	0.999	1.000	0.998	1.000	1.000	0.992	0.999	1.000
		S15	0.875	0.935	0.991	0.917	0.961	0.995	0.893	0.950	0.991
	1500	WSNS	0.933	0.969	0.997	0.908	0.959	0.990	0.930	0.964	0.987
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.992	0.999	1.000
		S15	0.904	0.944	0.989	0.928	0.967	0.992	0.921	0.963	0.989

Table 3.39: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.864	0.918	0.961	0.850	0.911	0.961	0.886	0.935	0.977
		PR10	0.864	0.921	0.972	0.859	0.924	0.972	0.878	0.932	0.976
		S15	0.894	0.953	0.991	0.889	0.943	0.983	0.911	0.958	0.994
	1000	WSNS	0.884	0.926	0.979	0.883	0.945	0.986	0.907	0.942	0.983
		PR10	0.869	0.926	0.980	0.881	0.932	0.982	0.875	0.951	0.991
		S15	0.892	0.946	0.988	0.906	0.957	0.989	0.918	0.948	0.994
	1500	WSNS	0.910	0.956	0.987	0.894	0.942	0.981	0.894	0.935	0.981
		PR10	0.890	0.939	0.985	0.894	0.946	0.984	0.864	0.936	0.990
		S15	0.915	0.959	0.990	0.905	0.951	0.985	0.906	0.951	0.987
0.4	500	WSNS	0.863	0.917	0.965	0.847	0.907	0.962	0.879	0.922	0.967
		PR10	0.935	0.970	0.991	0.929	0.965	0.989	0.911	0.961	0.985
		S15	0.890	0.947	0.989	0.904	0.954	0.990	0.902	0.952	0.992
	1000	WSNS	0.880	0.931	0.977	0.873	0.925	0.982	0.909	0.952	0.984
		PR10	0.955	0.979	0.997	0.942	0.980	0.995	0.909	0.970	0.998
		S15	0.896	0.943	0.988	0.907	0.959	0.991	0.916	0.958	0.993
	1500	WSNS	0.908	0.955	0.987	0.877	0.932	0.983	0.886	0.934	0.978
		PR10	0.963	0.986	0.999	0.954	0.982	0.999	0.889	0.960	0.994
		S15	0.918	0.957	0.991	0.909	0.950	0.989	0.910	0.954	0.988
0.8	500	WSNS	0.876	0.922	0.967	0.845	0.913	0.970	0.866	0.919	0.957
		PR10	0.993	0.997	0.999	0.983	0.994	0.999	0.985	0.996	0.998
		S15	0.898	0.952	0.989	0.927	0.966	0.996	0.901	0.949	0.994
	1000	WSNS	0.874	0.926	0.983	0.874	0.939	0.988	0.913	0.946	0.982
		PR10	0.999	1.000	1.000	0.995	0.998	1.000	0.997	1.000	1.000
		S15	0.883	0.938	0.989	0.931	0.975	0.997	0.921	0.961	0.994
	1500	WSNS	0.901	0.943	0.987	0.870	0.932	0.985	0.879	0.928	0.982
		PR10	1.000	1.000	1.000	0.996	1.000	1.000	0.991	1.000	1.000
		S15	0.908	0.951	0.988	0.914	0.960	0.995	0.895	0.953	0.989

Table 3.40: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.881	0.936	0.974	0.875	0.934	0.974	0.906	0.946	0.987
		PR10	0.871	0.917	0.969	0.864	0.911	0.969	0.854	0.907	0.975
		S15	0.900	0.945	0.987	0.897	0.943	0.988	0.903	0.961	0.995
	1000	WSNS	0.898	0.947	0.984	0.878	0.937	0.982	0.909	0.950	0.988
		PR10	0.861	0.917	0.979	0.873	0.926	0.975	0.844	0.936	0.989
		S15	0.892	0.943	0.989	0.881	0.943	0.987	0.906	0.950	0.993
	1500	WSNS	0.926	0.966	0.990	0.921	0.963	0.991	0.911	0.947	0.987
		PR10	0.889	0.939	0.987	0.899	0.948	0.985	0.838	0.922	0.988
		S15	0.915	0.958	0.992	0.912	0.959	0.990	0.910	0.948	0.987
0.4	500	WSNS	0.890	0.946	0.973	0.877	0.933	0.977	0.892	0.937	0.983
		PR10	0.935	0.967	0.991	0.930	0.965	0.992	0.880	0.937	0.981
		S15	0.901	0.947	0.987	0.901	0.959	0.993	0.902	0.958	0.991
	1000	WSNS	0.907	0.949	0.985	0.892	0.941	0.982	0.904	0.956	0.987
		PR10	0.944	0.977	0.996	0.932	0.972	0.996	0.882	0.951	0.994
		S15	0.892	0.948	0.989	0.899	0.946	0.990	0.902	0.954	0.989
	1500	WSNS	0.917	0.964	0.990	0.893	0.957	0.990	0.907	0.951	0.986
		PR10	0.966	0.985	0.999	0.959	0.981	0.998	0.880	0.949	0.993
		S15	0.908	0.949	0.990	0.902	0.961	0.995	0.912	0.949	0.990
0.8	500	WSNS	0.888	0.942	0.978	0.887	0.935	0.978	0.889	0.928	0.972
		PR10	0.991	0.997	1.000	0.988	0.995	0.999	0.985	0.997	1.000
		S15	0.907	0.948	0.991	0.928	0.963	0.993	0.895	0.948	0.991
	1000	WSNS	0.891	0.934	0.984	0.877	0.938	0.980	0.897	0.939	0.981
		PR10	0.999	0.999	1.000	0.996	0.999	1.000	0.993	1.000	1.000
		S15	0.875	0.935	0.991	0.917	0.961	0.995	0.893	0.950	0.991
	1500	WSNS	0.915	0.956	0.991	0.892	0.952	0.984	0.923	0.964	0.988
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.991	0.999	1.000
		S15	0.904	0.944	0.989	0.928	0.967	0.992	0.921	0.963	0.989

Table 3.41: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.846	0.908	0.958	0.840	0.896	0.960	0.882	0.926	0.972
		PR10	0.860	0.916	0.966	0.859	0.912	0.967	0.867	0.925	0.971
		S15	0.894	0.953	0.991	0.889	0.943	0.983	0.911	0.958	0.994
	1000	WSNS	0.879	0.924	0.975	0.886	0.942	0.978	0.902	0.941	0.984
		PR10	0.865	0.921	0.979	0.880	0.926	0.981	0.872	0.950	0.986
		S15	0.892	0.946	0.988	0.906	0.957	0.989	0.918	0.948	0.994
	1500	WSNS	0.910	0.954	0.984	0.890	0.937	0.973	0.899	0.933	0.980
		PR10	0.893	0.934	0.984	0.889	0.947	0.984	0.865	0.940	0.989
		S15	0.915	0.959	0.990	0.905	0.951	0.985	0.906	0.951	0.987
0.4	500	WSNS	0.859	0.911	0.958	0.839	0.901	0.958	0.874	0.911	0.963
		PR10	0.931	0.964	0.989	0.921	0.961	0.988	0.901	0.947	0.980
		S15	0.890	0.947	0.989	0.904	0.954	0.990	0.902	0.952	0.992
	1000	WSNS	0.871	0.928	0.974	0.866	0.922	0.976	0.912	0.953	0.984
		PR10	0.951	0.978	0.996	0.938	0.975	0.994	0.916	0.965	0.993
		S15	0.896	0.943	0.988	0.907	0.959	0.991	0.916	0.958	0.993
	1500	WSNS	0.902	0.957	0.987	0.872	0.933	0.981	0.890	0.936	0.976
		PR10	0.963	0.986	0.999	0.955	0.983	0.999	0.893	0.962	0.993
		S15	0.918	0.957	0.991	0.909	0.950	0.989	0.910	0.954	0.988
0.8	500	WSNS	0.866	0.912	0.961	0.842	0.907	0.966	0.860	0.912	0.953
		PR10	0.988	0.996	0.999	0.982	0.993	1.000	0.981	0.994	0.998
		S15	0.898	0.952	0.989	0.927	0.966	0.996	0.901	0.949	0.994
	1000	WSNS	0.864	0.917	0.976	0.875	0.937	0.985	0.912	0.939	0.981
		PR10	0.997	0.999	1.000	0.996	0.998	1.000	0.995	0.999	1.000
		S15	0.883	0.938	0.989	0.931	0.975	0.997	0.921	0.961	0.994
	1500	WSNS	0.897	0.941	0.986	0.869	0.928	0.982	0.879	0.934	0.982
		PR10	1.000	1.000	1.000	0.993	0.999	1.000	0.992	1.000	1.000
		S15	0.908	0.951	0.988	0.914	0.960	0.995	0.895	0.953	0.989

Table 3.42: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.865	0.921	0.970	0.864	0.919	0.968	0.887	0.937	0.980
		PR10	0.862	0.915	0.966	0.855	0.905	0.963	0.840	0.897	0.970
		S15	0.900	0.945	0.987	0.897	0.943	0.988	0.903	0.961	0.995
	1000	WSNS	0.891	0.937	0.981	0.870	0.937	0.978	0.911	0.953	0.985
		PR10	0.859	0.913	0.979	0.872	0.923	0.974	0.850	0.928	0.985
		S15	0.892	0.943	0.989	0.881	0.943	0.987	0.906	0.950	0.993
	1500	WSNS	0.923	0.959	0.986	0.912	0.956	0.990	0.907	0.946	0.982
		PR10	0.893	0.938	0.985	0.899	0.949	0.985	0.848	0.925	0.985
		S15	0.915	0.958	0.992	0.912	0.959	0.990	0.910	0.948	0.987
0.4	500	WSNS	0.875	0.926	0.969	0.867	0.918	0.966	0.882	0.925	0.975
		PR10	0.932	0.963	0.990	0.927	0.962	0.991	0.877	0.925	0.977
		S15	0.901	0.947	0.987	0.901	0.959	0.993	0.902	0.958	0.991
	1000	WSNS	0.891	0.941	0.976	0.883	0.934	0.980	0.900	0.953	0.985
		PR10	0.942	0.977	0.996	0.932	0.974	0.994	0.882	0.951	0.992
		S15	0.892	0.948	0.989	0.899	0.946	0.990	0.902	0.954	0.989
	1500	WSNS	0.914	0.958	0.987	0.891	0.949	0.990	0.907	0.949	0.983
		PR10	0.964	0.987	0.999	0.962	0.984	0.998	0.886	0.949	0.993
		S15	0.908	0.949	0.990	0.902	0.961	0.995	0.912	0.949	0.990
0.8	500	WSNS	0.874	0.924	0.969	0.879	0.925	0.968	0.874	0.924	0.965
		PR10	0.990	0.997	0.999	0.981	0.995	0.997	0.985	0.996	0.999
		S15	0.907	0.948	0.991	0.928	0.963	0.993	0.895	0.948	0.991
	1000	WSNS	0.871	0.932	0.983	0.883	0.934	0.981	0.897	0.939	0.977
		PR10	0.998	0.999	1.000	0.994	0.999	1.000	0.994	0.998	1.000
		S15	0.875	0.935	0.991	0.917	0.961	0.995	0.893	0.950	0.991
	1500	WSNS	0.913	0.954	0.990	0.891	0.946	0.985	0.925	0.958	0.988
		PR10	1.000	1.000	1.000	0.998	0.999	1.000	0.993	0.999	1.000
		S15	0.904	0.944	0.989	0.928	0.967	0.992	0.921	0.963	0.989

Table 3.43: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.829	0.896	0.940	0.836	0.877	0.947	0.869	0.919	0.963
		PR10	0.846	0.905	0.962	0.850	0.906	0.964	0.853	0.912	0.958
		S15	0.894	0.953	0.991	0.889	0.943	0.983	0.911	0.958	0.994
	1000	WSNS	0.861	0.922	0.967	0.877	0.936	0.979	0.896	0.938	0.980
		PR10	0.861	0.916	0.974	0.880	0.925	0.981	0.868	0.945	0.985
		S15	0.892	0.946	0.988	0.906	0.957	0.989	0.918	0.948	0.994
	1500	WSNS	0.904	0.948	0.984	0.881	0.930	0.971	0.893	0.929	0.974
		PR10	0.899	0.935	0.984	0.891	0.946	0.984	0.865	0.946	0.987
		S15	0.915	0.959	0.990	0.905	0.951	0.985	0.906	0.951	0.987
0.4	500	WSNS	0.834	0.894	0.947	0.830	0.881	0.942	0.860	0.906	0.957
		PR10	0.922	0.959	0.984	0.916	0.956	0.987	0.895	0.937	0.977
		S15	0.890	0.947	0.989	0.904	0.954	0.990	0.902	0.952	0.992
	1000	WSNS	0.867	0.919	0.969	0.857	0.918	0.974	0.908	0.947	0.980
		PR10	0.952	0.975	0.995	0.937	0.971	0.994	0.909	0.963	0.993
		S15	0.896	0.943	0.988	0.907	0.959	0.991	0.916	0.958	0.993
	1500	WSNS	0.895	0.949	0.981	0.864	0.930	0.974	0.883	0.938	0.972
		PR10	0.960	0.982	0.999	0.954	0.981	0.998	0.894	0.964	0.992
		S15	0.918	0.957	0.991	0.909	0.950	0.989	0.910	0.954	0.988
0.8	500	WSNS	0.849	0.895	0.953	0.821	0.892	0.967	0.847	0.893	0.941
		PR10	0.990	0.994	0.999	0.973	0.992	0.998	0.975	0.984	0.997
		S15	0.898	0.952	0.989	0.927	0.966	0.996	0.901	0.949	0.994
	1000	WSNS	0.856	0.911	0.970	0.865	0.931	0.983	0.902	0.935	0.976
		PR10	0.995	0.999	1.000	0.993	0.998	1.000	0.992	1.000	1.000
		S15	0.883	0.938	0.989	0.931	0.975	0.997	0.921	0.961	0.994
	1500	WSNS	0.890	0.935	0.980	0.863	0.921	0.981	0.869	0.930	0.975
		PR10	1.000	1.000	1.000	0.994	1.000	1.000	0.993	1.000	1.000
		S15	0.908	0.951	0.988	0.914	0.960	0.995	0.895	0.953	0.989

Table 3.44: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with t_5 innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.854	0.905	0.951	0.853	0.905	0.952	0.874	0.925	0.969
		PR10	0.850	0.907	0.950	0.844	0.897	0.953	0.838	0.897	0.958
		S15	0.900	0.945	0.987	0.897	0.943	0.988	0.903	0.961	0.995
	1000	WSNS	0.884	0.928	0.977	0.864	0.923	0.978	0.899	0.949	0.983
		PR10	0.856	0.910	0.976	0.870	0.921	0.973	0.855	0.933	0.980
		S15	0.892	0.943	0.989	0.881	0.943	0.987	0.906	0.950	0.993
	1500	WSNS	0.914	0.953	0.985	0.896	0.951	0.984	0.901	0.940	0.980
		PR10	0.892	0.942	0.984	0.898	0.948	0.987	0.855	0.940	0.981
		S15	0.915	0.958	0.992	0.912	0.959	0.990	0.910	0.948	0.987
0.4	500	WSNS	0.857	0.912	0.957	0.853	0.908	0.964	0.871	0.919	0.970
		PR10	0.920	0.955	0.986	0.915	0.961	0.986	0.870	0.920	0.972
		S15	0.901	0.947	0.987	0.901	0.959	0.993	0.902	0.958	0.991
	1000	WSNS	0.878	0.933	0.973	0.867	0.926	0.976	0.891	0.944	0.981
		PR10	0.945	0.975	0.995	0.935	0.972	0.995	0.884	0.948	0.985
		S15	0.892	0.948	0.989	0.899	0.946	0.990	0.902	0.954	0.989
	1500	WSNS	0.904	0.955	0.984	0.883	0.934	0.984	0.901	0.941	0.980
		PR10	0.963	0.983	0.998	0.957	0.981	0.998	0.894	0.953	0.992
		S15	0.908	0.949	0.990	0.902	0.961	0.995	0.912	0.949	0.990
0.8	500	WSNS	0.860	0.910	0.959	0.865	0.907	0.959	0.853	0.913	0.961
		PR10	0.991	0.997	0.999	0.979	0.990	0.998	0.980	0.995	0.999
		S15	0.907	0.948	0.991	0.928	0.963	0.993	0.895	0.948	0.991
	1000	WSNS	0.860	0.921	0.971	0.862	0.928	0.975	0.888	0.936	0.978
		PR10	0.998	0.999	1.000	0.994	0.999	1.000	0.993	0.998	1.000
		S15	0.875	0.935	0.991	0.917	0.961	0.995	0.893	0.950	0.991
	1500	WSNS	0.903	0.942	0.983	0.886	0.938	0.977	0.913	0.954	0.986
		PR10	1.000	1.000	1.000	0.997	0.999	1.000	0.993	0.999	1.000
		S15	0.904	0.944	0.989	0.928	0.967	0.992	0.921	0.963	0.989

Table 3.45: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.893	0.946	0.986	0.899	0.953	0.986	0.894	0.942	0.980
		PR10	0.886	0.936	0.974	0.885	0.934	0.981	0.840	0.930	0.983
		S15	0.900	0.939	0.979	0.907	0.947	0.987	0.897	0.954	0.988
	1000	WSNS	0.905	0.947	0.985	0.908	0.951	0.993	0.887	0.937	0.986
		PR10	0.880	0.942	0.983	0.887	0.942	0.985	0.836	0.929	0.994
		S15	0.897	0.941	0.992	0.890	0.942	0.990	0.895	0.955	0.988
	1500	WSNS	0.924	0.961	0.993	0.931	0.971	0.995	0.886	0.944	0.983
		PR10	0.894	0.940	0.988	0.902	0.948	0.989	0.832	0.930	0.997
		S15	0.917	0.960	0.992	0.911	0.964	0.993	0.891	0.945	0.989
0.4	500	WSNS	0.913	0.951	0.988	0.908	0.952	0.983	0.897	0.943	0.983
		PR10	0.938	0.968	0.993	0.931	0.971	0.993	0.874	0.950	0.991
		S15	0.937	0.969	0.994	0.940	0.971	0.990	0.918	0.962	0.993
	1000	WSNS	0.920	0.957	0.990	0.921	0.960	0.993	0.893	0.945	0.980
		PR10	0.943	0.982	0.997	0.948	0.974	0.997	0.883	0.954	0.996
		S15	0.945	0.972	0.996	0.937	0.968	0.994	0.913	0.961	0.993
	1500	WSNS	0.926	0.964	0.993	0.905	0.966	0.999	0.889	0.940	0.981
		PR10	0.947	0.980	0.996	0.943	0.971	0.998	0.854	0.945	0.998
		S15	0.941	0.975	0.999	0.938	0.968	0.997	0.906	0.951	0.992
0.8	500	WSNS	0.932	0.966	0.994	0.923	0.970	0.995	0.908	0.956	0.994
		PR10	0.992	0.998	1.000	0.982	0.994	0.999	0.970	0.991	0.999
		S15	0.984	0.997	1.000	0.976	0.992	1.000	0.975	0.991	1.000
	1000	WSNS	0.936	0.969	0.998	0.918	0.960	0.995	0.924	0.960	0.992
		PR10	0.998	0.999	1.000	0.985	0.995	0.999	0.973	0.994	1.000
		S15	0.986	0.996	1.000	0.970	0.989	0.999	0.978	0.993	0.999
	1500	WSNS	0.943	0.971	0.995	0.912	0.962	0.995	0.892	0.958	0.991
		PR10	0.992	0.999	1.000	0.986	0.996	0.999	0.968	0.994	1.000
		S15	0.982	0.996	1.000	0.977	0.992	0.999	0.973	0.988	0.998

Table 3.46: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.933	0.975	0.994	0.933	0.970	0.996	0.921	0.964	0.991
		PR10	0.875	0.923	0.977	0.871	0.920	0.978	0.820	0.912	0.981
		S15	0.891	0.939	0.987	0.892	0.941	0.988	0.910	0.949	0.986
	1000	WSNS	0.940	0.974	0.998	0.935	0.977	0.996	0.918	0.958	0.989
		PR10	0.872	0.936	0.981	0.873	0.934	0.988	0.801	0.905	0.991
		S15	0.897	0.939	0.992	0.895	0.946	0.991	0.901	0.940	0.988
	1500	WSNS	0.953	0.980	1.000	0.956	0.987	0.998	0.907	0.952	0.994
		PR10	0.884	0.937	0.984	0.891	0.938	0.983	0.796	0.906	0.996
		S15	0.916	0.958	0.994	0.903	0.958	0.993	0.901	0.953	0.990
0.4	500	WSNS	0.947	0.976	0.995	0.939	0.974	0.998	0.921	0.960	0.991
		PR10	0.931	0.966	0.991	0.925	0.961	0.990	0.834	0.916	0.987
		S15	0.938	0.973	0.997	0.930	0.966	0.993	0.903	0.958	0.988
	1000	WSNS	0.950	0.983	0.997	0.942	0.983	0.999	0.909	0.959	0.992
		PR10	0.940	0.981	0.998	0.954	0.976	0.998	0.845	0.928	0.991
		S15	0.938	0.975	0.998	0.945	0.980	0.997	0.919	0.959	0.992
	1500	WSNS	0.962	0.984	0.999	0.952	0.977	0.999	0.914	0.960	0.994
		PR10	0.941	0.972	0.996	0.929	0.968	0.997	0.840	0.927	0.996
		S15	0.942	0.979	0.999	0.937	0.973	0.996	0.914	0.965	0.995
0.8	500	WSNS	0.972	0.986	0.998	0.954	0.979	0.997	0.925	0.975	0.997
		PR10	0.990	0.997	1.000	0.980	0.996	0.998	0.943	0.984	0.998
		S15	0.984	0.994	1.000	0.975	0.992	0.999	0.966	0.992	1.000
	1000	WSNS	0.975	0.992	1.000	0.947	0.976	0.999	0.930	0.963	0.995
		PR10	0.997	0.999	1.000	0.990	0.998	1.000	0.957	0.989	1.000
		S15	0.983	0.994	1.000	0.972	0.993	1.000	0.966	0.991	0.998
	1500	WSNS	0.969	0.990	0.998	0.962	0.981	0.999	0.944	0.975	0.996
		PR10	0.991	0.999	1.000	0.989	0.997	1.000	0.965	0.992	1.000
		S15	0.986	0.995	1.000	0.981	0.994	1.000	0.982	0.995	1.000

Table 3.47: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.895	0.937	0.979	0.888	0.943	0.983	0.894	0.940	0.983
		PR10	0.878	0.931	0.972	0.879	0.932	0.976	0.848	0.939	0.981
		S15	0.900	0.939	0.979	0.907	0.947	0.987	0.897	0.954	0.988
	1000	WSNS	0.907	0.944	0.986	0.902	0.944	0.988	0.898	0.945	0.985
		PR10	0.879	0.944	0.981	0.886	0.937	0.981	0.849	0.930	0.992
		S15	0.897	0.941	0.992	0.890	0.942	0.990	0.895	0.955	0.988
	1500	WSNS	0.921	0.962	0.990	0.932	0.969	0.995	0.894	0.953	0.985
		PR10	0.895	0.939	0.988	0.902	0.948	0.986	0.846	0.934	0.995
		S15	0.917	0.960	0.992	0.911	0.964	0.993	0.891	0.945	0.989
0.4	500	WSNS	0.908	0.950	0.980	0.903	0.947	0.976	0.895	0.942	0.978
		PR10	0.933	0.963	0.992	0.934	0.966	0.991	0.880	0.952	0.985
		S15	0.937	0.969	0.994	0.940	0.971	0.990	0.918	0.962	0.993
	1000	WSNS	0.912	0.949	0.989	0.919	0.955	0.991	0.899	0.949	0.983
		PR10	0.946	0.981	0.997	0.944	0.969	0.996	0.888	0.954	0.992
		S15	0.945	0.972	0.996	0.937	0.968	0.994	0.913	0.961	0.993
	1500	WSNS	0.921	0.964	0.991	0.906	0.960	0.997	0.890	0.944	0.984
		PR10	0.948	0.976	0.996	0.943	0.972	0.998	0.869	0.947	0.996
		S15	0.941	0.975	0.999	0.938	0.968	0.997	0.906	0.951	0.992
0.8	500	WSNS	0.920	0.961	0.991	0.919	0.966	0.996	0.912	0.957	0.989
		PR10	0.990	0.997	1.000	0.980	0.995	0.999	0.971	0.990	0.998
		S15	0.984	0.997	1.000	0.976	0.992	1.000	0.975	0.991	1.000
	1000	WSNS	0.932	0.965	0.998	0.912	0.958	0.994	0.922	0.963	0.994
		PR10	0.997	0.999	1.000	0.983	0.996	0.999	0.980	0.993	1.000
		S15	0.986	0.996	1.000	0.970	0.989	0.999	0.978	0.993	0.999
	1500	WSNS	0.936	0.967	0.992	0.915	0.958	0.992	0.897	0.949	0.988
		PR10	0.992	0.999	1.000	0.988	0.997	0.999	0.971	0.993	1.000
		S15	0.982	0.996	1.000	0.977	0.992	0.999	0.973	0.988	0.998

Table 3.48: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.911	0.961	0.990	0.904	0.956	0.991	0.915	0.962	0.987
		PR10	0.866	0.918	0.971	0.871	0.920	0.978	0.842	0.923	0.973
		S15	0.891	0.939	0.987	0.892	0.941	0.988	0.910	0.949	0.986
	1000	WSNS	0.930	0.966	0.996	0.924	0.971	0.994	0.923	0.959	0.989
		PR10	0.872	0.936	0.980	0.877	0.938	0.987	0.824	0.911	0.986
		S15	0.897	0.939	0.992	0.895	0.946	0.991	0.901	0.940	0.988
	1500	WSNS	0.943	0.974	0.998	0.941	0.981	0.997	0.912	0.958	0.993
		PR10	0.887	0.937	0.983	0.898	0.943	0.984	0.826	0.925	0.994
		S15	0.916	0.958	0.994	0.903	0.958	0.993	0.901	0.953	0.990
0.4	500	WSNS	0.924	0.965	0.992	0.927	0.969	0.988	0.919	0.958	0.989
		PR10	0.929	0.963	0.991	0.922	0.960	0.991	0.852	0.928	0.981
		S15	0.938	0.973	0.997	0.930	0.966	0.993	0.903	0.958	0.988
	1000	WSNS	0.940	0.978	0.997	0.927	0.975	0.995	0.914	0.957	0.992
		PR10	0.943	0.981	0.998	0.955	0.980	0.996	0.873	0.933	0.991
		S15	0.938	0.975	0.998	0.945	0.980	0.997	0.919	0.959	0.992
	1500	WSNS	0.953	0.975	0.997	0.938	0.975	0.998	0.921	0.963	0.994
		PR10	0.943	0.972	0.993	0.929	0.968	0.996	0.863	0.943	0.996
		S15	0.942	0.979	0.999	0.937	0.973	0.996	0.914	0.965	0.995
0.8	500	WSNS	0.957	0.981	0.996	0.937	0.972	0.998	0.924	0.966	0.995
		PR10	0.989	0.997	1.000	0.984	0.996	0.998	0.955	0.985	0.995
		S15	0.984	0.994	1.000	0.975	0.992	0.999	0.966	0.992	1.000
	1000	WSNS	0.966	0.989	0.999	0.933	0.975	0.997	0.927	0.961	0.991
		PR10	0.998	0.999	1.000	0.990	0.997	0.998	0.967	0.992	1.000
		S15	0.983	0.994	1.000	0.972	0.993	1.000	0.966	0.991	0.998
	1500	WSNS	0.960	0.982	0.999	0.953	0.977	0.997	0.942	0.979	0.997
		PR10	0.991	0.999	1.000	0.992	0.997	1.000	0.974	0.994	1.000
		S15	0.986	0.995	1.000	0.981	0.994	1.000	0.982	0.995	1.000

Table 3.49: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.872	0.929	0.968	0.861	0.924	0.972	0.889	0.928	0.968
		PR10	0.872	0.924	0.970	0.861	0.922	0.974	0.855	0.925	0.975
		S15	0.900	0.939	0.979	0.907	0.947	0.987	0.897	0.954	0.988
	1000	WSNS	0.894	0.935	0.978	0.889	0.930	0.984	0.894	0.938	0.984
		PR10	0.878	0.937	0.982	0.879	0.939	0.981	0.856	0.934	0.989
		S15	0.897	0.941	0.992	0.890	0.942	0.990	0.895	0.955	0.988
	1500	WSNS	0.908	0.952	0.989	0.917	0.961	0.992	0.884	0.943	0.986
		PR10	0.894	0.939	0.984	0.899	0.944	0.986	0.848	0.937	0.993
		S15	0.917	0.960	0.992	0.911	0.964	0.993	0.891	0.945	0.989
0.4	500	WSNS	0.887	0.926	0.976	0.887	0.934	0.969	0.882	0.922	0.975
		PR10	0.933	0.962	0.990	0.930	0.963	0.989	0.888	0.941	0.980
		S15	0.937	0.969	0.994	0.940	0.971	0.990	0.918	0.962	0.993
	1000	WSNS	0.897	0.942	0.981	0.906	0.952	0.990	0.894	0.945	0.982
		PR10	0.946	0.979	0.998	0.937	0.967	0.994	0.890	0.952	0.992
		S15	0.945	0.972	0.996	0.937	0.968	0.994	0.913	0.961	0.993
	1500	WSNS	0.909	0.955	0.989	0.891	0.948	0.994	0.883	0.943	0.984
		PR10	0.950	0.972	0.996	0.943	0.970	0.996	0.875	0.948	0.994
		S15	0.941	0.975	0.999	0.938	0.968	0.997	0.906	0.951	0.992
0.8	500	WSNS	0.907	0.952	0.987	0.904	0.957	0.988	0.906	0.949	0.985
		PR10	0.984	0.998	0.999	0.976	0.995	0.999	0.971	0.986	0.998
		S15	0.984	0.997	1.000	0.976	0.992	1.000	0.975	0.991	1.000
	1000	WSNS	0.914	0.958	0.993	0.901	0.948	0.988	0.917	0.957	0.992
		PR10	0.996	0.999	1.000	0.975	0.994	0.999	0.978	0.995	1.000
		S15	0.986	0.996	1.000	0.970	0.989	0.999	0.978	0.993	0.999
	1500	WSNS	0.925	0.960	0.991	0.902	0.954	0.989	0.891	0.950	0.987
		PR10	0.992	0.999	1.000	0.986	0.996	0.999	0.973	0.993	1.000
		S15	0.982	0.996	1.000	0.977	0.992	0.999	0.973	0.988	0.998

Table 3.50: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.890	0.933	0.979	0.881	0.941	0.979	0.897	0.951	0.985
		PR10	0.859	0.917	0.969	0.868	0.921	0.975	0.850	0.916	0.966
		S15	0.891	0.939	0.987	0.892	0.941	0.988	0.910	0.949	0.986
	1000	WSNS	0.922	0.954	0.988	0.914	0.960	0.990	0.920	0.954	0.985
		PR10	0.875	0.936	0.978	0.870	0.932	0.986	0.837	0.917	0.985
		S15	0.897	0.939	0.992	0.895	0.946	0.991	0.901	0.940	0.988
	1500	WSNS	0.933	0.964	0.994	0.922	0.974	0.995	0.904	0.954	0.991
		PR10	0.891	0.935	0.983	0.893	0.946	0.984	0.836	0.932	0.992
		S15	0.916	0.958	0.994	0.903	0.958	0.993	0.901	0.953	0.990
0.4	500	WSNS	0.905	0.951	0.986	0.896	0.959	0.982	0.898	0.938	0.980
		PR10	0.922	0.961	0.991	0.921	0.958	0.989	0.860	0.924	0.975
		S15	0.938	0.973	0.997	0.930	0.966	0.993	0.903	0.958	0.988
	1000	WSNS	0.929	0.963	0.993	0.910	0.958	0.995	0.904	0.946	0.988
		PR10	0.940	0.979	0.998	0.952	0.978	0.997	0.881	0.941	0.987
		S15	0.938	0.975	0.998	0.945	0.980	0.997	0.919	0.959	0.992
	1500	WSNS	0.934	0.970	0.996	0.927	0.969	0.996	0.907	0.957	0.990
		PR10	0.945	0.973	0.994	0.935	0.967	0.996	0.869	0.947	0.999
		S15	0.942	0.979	0.999	0.937	0.973	0.996	0.914	0.965	0.995
0.8	500	WSNS	0.935	0.971	0.994	0.920	0.962	0.990	0.911	0.956	0.988
		PR10	0.988	0.997	0.999	0.982	0.994	0.998	0.952	0.984	0.994
		S15	0.984	0.994	1.000	0.975	0.992	0.999	0.966	0.992	1.000
	1000	WSNS	0.946	0.981	0.999	0.928	0.967	0.992	0.920	0.953	0.988
		PR10	0.996	0.999	1.000	0.988	0.997	0.998	0.974	0.993	1.000
		S15	0.983	0.994	1.000	0.972	0.993	1.000	0.966	0.991	0.998
	1500	WSNS	0.948	0.977	0.997	0.935	0.973	0.995	0.931	0.973	0.995
		PR10	0.991	0.999	1.000	0.991	0.997	1.000	0.979	0.995	1.000
		S15	0.986	0.995	1.000	0.981	0.994	1.000	0.982	0.995	1.000

Table 3.51: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.864	0.915	0.959	0.864	0.912	0.967	0.879	0.925	0.963
		PR10	0.863	0.918	0.968	0.862	0.921	0.969	0.862	0.921	0.969
		S15	0.900	0.939	0.979	0.907	0.947	0.987	0.897	0.954	0.988
	1000	WSNS	0.887	0.935	0.970	0.889	0.937	0.976	0.894	0.945	0.982
		PR10	0.874	0.933	0.981	0.879	0.934	0.982	0.858	0.941	0.987
		S15	0.897	0.941	0.992	0.890	0.942	0.990	0.895	0.955	0.988
	1500	WSNS	0.906	0.951	0.987	0.916	0.960	0.991	0.888	0.948	0.985
		PR10	0.895	0.938	0.982	0.895	0.943	0.985	0.853	0.937	0.991
		S15	0.917	0.960	0.992	0.911	0.964	0.993	0.891	0.945	0.989
0.4	500	WSNS	0.888	0.922	0.968	0.884	0.929	0.959	0.881	0.922	0.964
		PR10	0.928	0.955	0.989	0.924	0.962	0.983	0.889	0.935	0.977
		S15	0.937	0.969	0.994	0.940	0.971	0.990	0.918	0.962	0.993
	1000	WSNS	0.889	0.936	0.981	0.905	0.950	0.982	0.894	0.944	0.983
		PR10	0.942	0.974	0.997	0.941	0.968	0.993	0.896	0.955	0.988
		S15	0.945	0.972	0.996	0.937	0.968	0.994	0.913	0.961	0.993
	1500	WSNS	0.910	0.957	0.986	0.895	0.949	0.994	0.884	0.944	0.984
		PR10	0.948	0.973	0.996	0.942	0.968	0.998	0.879	0.952	0.991
		S15	0.941	0.975	0.999	0.938	0.968	0.997	0.906	0.951	0.992
0.8	500	WSNS	0.908	0.949	0.981	0.906	0.952	0.981	0.903	0.947	0.979
		PR10	0.987	0.996	0.999	0.972	0.994	0.999	0.967	0.985	0.999
		S15	0.984	0.997	1.000	0.976	0.992	1.000	0.975	0.991	1.000
	1000	WSNS	0.918	0.954	0.990	0.905	0.947	0.988	0.925	0.956	0.993
		PR10	0.995	0.999	1.000	0.975	0.993	0.999	0.978	0.994	1.000
		S15	0.986	0.996	1.000	0.970	0.989	0.999	0.978	0.993	0.999
	1500	WSNS	0.923	0.959	0.992	0.897	0.952	0.990	0.892	0.950	0.990
		PR10	0.992	0.998	1.000	0.985	0.997	0.999	0.976	0.991	1.000
		S15	0.982	0.996	1.000	0.977	0.992	0.999	0.973	0.988	0.998

Table 3.52: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.881	0.922	0.978	0.877	0.926	0.970	0.888	0.942	0.980
		PR10	0.853	0.911	0.968	0.864	0.910	0.967	0.856	0.914	0.958
		S15	0.891	0.939	0.987	0.892	0.941	0.988	0.910	0.949	0.986
	1000	WSNS	0.915	0.949	0.985	0.908	0.951	0.985	0.915	0.951	0.985
		PR10	0.870	0.933	0.977	0.868	0.934	0.987	0.842	0.932	0.981
		S15	0.897	0.939	0.992	0.895	0.946	0.991	0.901	0.940	0.988
	1500	WSNS	0.931	0.962	0.993	0.915	0.970	0.994	0.904	0.952	0.994
		PR10	0.889	0.932	0.981	0.896	0.947	0.981	0.846	0.930	0.990
		S15	0.916	0.958	0.994	0.903	0.958	0.993	0.901	0.953	0.990
0.4	500	WSNS	0.905	0.939	0.980	0.889	0.943	0.973	0.896	0.933	0.974
		PR10	0.920	0.955	0.990	0.916	0.953	0.985	0.867	0.918	0.967
		S15	0.938	0.973	0.997	0.930	0.966	0.993	0.903	0.958	0.988
	1000	WSNS	0.921	0.957	0.991	0.905	0.951	0.992	0.903	0.945	0.986
		PR10	0.935	0.977	0.996	0.950	0.977	0.997	0.885	0.950	0.988
		S15	0.938	0.975	0.998	0.945	0.980	0.997	0.919	0.959	0.992
	1500	WSNS	0.930	0.968	0.992	0.919	0.960	0.994	0.911	0.958	0.992
		PR10	0.941	0.972	0.994	0.935	0.966	0.995	0.885	0.948	0.997
		S15	0.942	0.979	0.999	0.937	0.973	0.996	0.914	0.965	0.995
0.8	500	WSNS	0.931	0.963	0.989	0.918	0.952	0.986	0.903	0.944	0.982
		PR10	0.985	0.996	0.999	0.977	0.995	0.998	0.954	0.980	0.994
		S15	0.984	0.994	1.000	0.975	0.992	0.999	0.966	0.992	1.000
	1000	WSNS	0.944	0.977	0.997	0.923	0.959	0.990	0.922	0.954	0.986
		PR10	0.996	0.999	1.000	0.989	0.997	0.999	0.976	0.995	1.000
		S15	0.983	0.994	1.000	0.972	0.993	1.000	0.966	0.991	0.998
	1500	WSNS	0.941	0.973	0.994	0.933	0.971	0.995	0.928	0.971	0.994
		PR10	0.991	0.999	1.000	0.991	0.997	1.000	0.982	0.995	1.000
		S15	0.986	0.995	1.000	0.981	0.994	1.000	0.982	0.995	1.000

Table 3.53: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.848	0.906	0.952	0.851	0.906	0.962	0.867	0.908	0.959
		PR10	0.855	0.916	0.965	0.848	0.913	0.972	0.852	0.909	0.958
		S15	0.900	0.939	0.979	0.907	0.947	0.987	0.897	0.954	0.988
	1000	WSNS	0.887	0.927	0.965	0.882	0.932	0.973	0.887	0.937	0.976
		PR10	0.874	0.931	0.978	0.880	0.932	0.979	0.860	0.938	0.983
		S15	0.897	0.941	0.992	0.890	0.942	0.990	0.895	0.955	0.988
	1500	WSNS	0.897	0.948	0.986	0.907	0.956	0.987	0.883	0.940	0.983
		PR10	0.893	0.937	0.983	0.897	0.944	0.987	0.852	0.941	0.991
		S15	0.917	0.960	0.992	0.911	0.964	0.993	0.891	0.945	0.989
0.4	500	WSNS	0.880	0.913	0.962	0.862	0.922	0.959	0.863	0.915	0.958
		PR10	0.924	0.959	0.984	0.916	0.958	0.978	0.882	0.929	0.969
		S15	0.937	0.969	0.994	0.940	0.971	0.990	0.918	0.962	0.993
	1000	WSNS	0.886	0.933	0.978	0.897	0.941	0.977	0.879	0.942	0.978
		PR10	0.935	0.975	0.995	0.936	0.967	0.991	0.896	0.954	0.988
		S15	0.945	0.972	0.996	0.937	0.968	0.994	0.913	0.961	0.993
	1500	WSNS	0.908	0.952	0.986	0.887	0.942	0.990	0.880	0.935	0.979
		PR10	0.945	0.972	0.995	0.943	0.967	0.997	0.883	0.951	0.990
		S15	0.941	0.975	0.999	0.938	0.968	0.997	0.906	0.951	0.992
0.8	500	WSNS	0.891	0.936	0.978	0.896	0.946	0.984	0.898	0.937	0.973
		PR10	0.987	0.996	0.999	0.972	0.994	0.999	0.961	0.984	0.997
		S15	0.984	0.997	1.000	0.976	0.992	1.000	0.975	0.991	1.000
	1000	WSNS	0.906	0.949	0.989	0.901	0.946	0.984	0.918	0.955	0.988
		PR10	0.993	0.999	1.000	0.976	0.993	0.999	0.979	0.991	1.000
		S15	0.986	0.996	1.000	0.970	0.989	0.999	0.978	0.993	0.999
	1500	WSNS	0.921	0.953	0.988	0.891	0.949	0.990	0.890	0.946	0.985
		PR10	0.991	0.998	1.000	0.986	0.997	0.999	0.974	0.992	1.000
		S15	0.982	0.996	1.000	0.977	0.992	0.999	0.973	0.988	0.998

Table 3.54: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with t_5 innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.866	0.912	0.966	0.858	0.915	0.953	0.874	0.928	0.970
		PR10	0.851	0.906	0.962	0.852	0.910	0.961	0.854	0.909	0.955
		S15	0.891	0.939	0.987	0.892	0.941	0.988	0.910	0.949	0.986
	1000	WSNS	0.908	0.941	0.978	0.898	0.940	0.982	0.900	0.950	0.983
		PR10	0.870	0.924	0.978	0.865	0.932	0.986	0.847	0.929	0.980
		S15	0.897	0.939	0.992	0.895	0.946	0.991	0.901	0.940	0.988
	1500	WSNS	0.922	0.957	0.988	0.907	0.962	0.994	0.897	0.946	0.990
		PR10	0.891	0.932	0.981	0.890	0.945	0.980	0.848	0.929	0.992
		S15	0.916	0.958	0.994	0.903	0.958	0.993	0.901	0.953	0.990
0.4	500	WSNS	0.889	0.930	0.972	0.876	0.927	0.973	0.878	0.927	0.971
		PR10	0.911	0.956	0.987	0.909	0.950	0.982	0.867	0.911	0.959
		S15	0.938	0.973	0.997	0.930	0.966	0.993	0.903	0.958	0.988
	1000	WSNS	0.905	0.950	0.985	0.901	0.946	0.987	0.895	0.942	0.988
		PR10	0.933	0.976	0.995	0.949	0.977	0.997	0.884	0.948	0.985
		S15	0.938	0.975	0.998	0.945	0.980	0.997	0.919	0.959	0.992
	1500	WSNS	0.922	0.962	0.989	0.911	0.954	0.991	0.905	0.953	0.987
		PR10	0.939	0.972	0.995	0.939	0.967	0.994	0.887	0.946	0.996
		S15	0.942	0.979	0.999	0.937	0.973	0.996	0.914	0.965	0.995
0.8	500	WSNS	0.914	0.951	0.983	0.899	0.944	0.984	0.895	0.937	0.978
		PR10	0.987	0.995	0.999	0.976	0.994	0.997	0.953	0.976	0.994
		S15	0.984	0.994	1.000	0.975	0.992	0.999	0.966	0.992	1.000
	1000	WSNS	0.927	0.972	0.991	0.917	0.962	0.988	0.915	0.943	0.985
		PR10	0.994	0.999	1.000	0.990	0.996	1.000	0.976	0.993	1.000
		S15	0.983	0.994	1.000	0.972	0.993	1.000	0.966	0.991	0.998
	1500	WSNS	0.938	0.967	0.992	0.925	0.967	0.993	0.927	0.966	0.994
		PR10	0.989	0.999	1.000	0.990	0.997	1.000	0.982	0.995	1.000
		S15	0.986	0.995	1.000	0.981	0.994	1.000	0.982	0.995	1.000

Table 3.55: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.902	0.947	0.980	0.911	0.949	0.984	0.895	0.946	0.982
		PR10	0.883	0.924	0.979	0.882	0.935	0.984	0.833	0.945	0.993
		S15	0.886	0.942	0.982	0.915	0.963	0.993	0.907	0.949	0.987
	1000	WSNS	0.906	0.947	0.987	0.893	0.955	0.990	0.883	0.940	0.988
		PR10	0.878	0.939	0.986	0.875	0.933	0.989	0.831	0.918	0.994
		S15	0.914	0.955	0.986	0.898	0.952	0.995	0.906	0.951	0.993
	1500	WSNS	0.923	0.976	0.995	0.907	0.970	0.995	0.893	0.938	0.978
		PR10	0.898	0.954	0.988	0.878	0.942	0.985	0.843	0.940	0.998
		S15	0.911	0.955	0.990	0.906	0.951	0.992	0.909	0.964	0.991
0.4	500	WSNS	0.907	0.946	0.979	0.890	0.939	0.986	0.885	0.933	0.981
		PR10	0.932	0.971	0.993	0.946	0.976	0.999	0.872	0.955	0.993
		S15	0.926	0.969	0.994	0.932	0.971	0.998	0.915	0.955	0.994
	1000	WSNS	0.888	0.951	0.989	0.884	0.952	0.995	0.863	0.924	0.982
		PR10	0.946	0.980	0.996	0.934	0.969	0.997	0.859	0.943	0.995
		S15	0.940	0.975	0.996	0.927	0.969	0.993	0.911	0.959	0.997
	1500	WSNS	0.908	0.959	0.993	0.898	0.962	0.996	0.877	0.928	0.982
		PR10	0.950	0.978	0.998	0.935	0.978	0.995	0.879	0.952	1.000
		S15	0.942	0.976	0.994	0.933	0.974	0.996	0.925	0.964	0.996
0.8	500	WSNS	0.897	0.951	0.990	0.886	0.935	0.987	0.862	0.919	0.973
		PR10	0.985	0.997	1.000	0.977	0.995	0.999	0.969	0.993	0.999
		S15	0.970	0.992	1.000	0.958	0.982	0.999	0.972	0.991	0.999
	1000	WSNS	0.889	0.950	0.986	0.876	0.941	0.991	0.852	0.919	0.974
		PR10	0.995	0.999	1.000	0.978	0.996	1.000	0.966	0.991	1.000
		S15	0.977	0.996	1.000	0.960	0.986	0.999	0.976	0.990	0.998
	1500	WSNS	0.881	0.940	0.991	0.892	0.946	0.987	0.859	0.917	0.973
		PR10	0.991	0.999	1.000	0.982	0.996	1.000	0.972	0.991	1.000
		S15	0.983	0.993	0.999	0.963	0.986	0.999	0.979	0.993	1.000

Table 3.56: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.938	0.967	0.989	0.932	0.970	0.996	0.925	0.959	0.992
		PR10	0.872	0.916	0.974	0.877	0.930	0.977	0.796	0.912	0.989
		S15	0.881	0.936	0.986	0.903	0.948	0.990	0.904	0.947	0.984
	1000	WSNS	0.942	0.973	0.992	0.944	0.975	0.998	0.913	0.953	0.992
		PR10	0.866	0.930	0.984	0.867	0.934	0.983	0.799	0.892	0.984
		S15	0.907	0.949	0.988	0.903	0.951	0.990	0.900	0.949	0.989
	1500	WSNS	0.971	0.990	1.000	0.944	0.987	0.999	0.911	0.951	0.987
		PR10	0.893	0.948	0.986	0.873	0.931	0.991	0.808	0.911	0.995
		S15	0.909	0.954	0.991	0.895	0.954	0.986	0.900	0.961	0.994
0.4	500	WSNS	0.933	0.963	0.995	0.937	0.971	0.995	0.903	0.939	0.990
		PR10	0.926	0.963	0.993	0.921	0.960	0.994	0.826	0.932	0.987
		S15	0.917	0.968	0.995	0.933	0.967	0.995	0.910	0.958	0.992
	1000	WSNS	0.938	0.971	0.996	0.938	0.972	0.996	0.912	0.947	0.992
		PR10	0.938	0.974	0.995	0.941	0.972	0.995	0.824	0.926	0.993
		S15	0.938	0.971	1.000	0.939	0.973	0.994	0.920	0.957	0.991
	1500	WSNS	0.948	0.982	1.000	0.937	0.979	0.999	0.912	0.951	0.989
		PR10	0.951	0.976	0.998	0.935	0.973	0.999	0.838	0.932	1.000
		S15	0.945	0.973	0.998	0.925	0.971	0.996	0.914	0.957	0.996
0.8	500	WSNS	0.942	0.980	0.997	0.916	0.959	0.991	0.905	0.950	0.984
		PR10	0.981	0.995	1.000	0.977	0.992	1.000	0.947	0.982	0.999
		S15	0.965	0.994	1.000	0.959	0.986	1.000	0.966	0.987	0.999
	1000	WSNS	0.951	0.981	0.998	0.919	0.970	0.997	0.891	0.937	0.985
		PR10	0.992	0.999	1.000	0.981	0.993	1.000	0.947	0.990	1.000
		S15	0.982	0.996	1.000	0.981	0.991	0.998	0.972	0.986	0.999
	1500	WSNS	0.946	0.982	0.999	0.926	0.971	0.995	0.911	0.951	0.989
		PR10	0.987	0.998	1.000	0.978	0.995	1.000	0.957	0.988	1.000
		S15	0.984	0.994	0.999	0.971	0.987	0.998	0.976	0.992	0.999

Table 3.57: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.883	0.936	0.974	0.896	0.943	0.980	0.900	0.931	0.979
		PR10	0.879	0.919	0.977	0.884	0.941	0.981	0.852	0.946	0.988
		S15	0.886	0.942	0.982	0.915	0.963	0.993	0.907	0.949	0.987
	1000	WSNS	0.899	0.943	0.983	0.892	0.948	0.987	0.876	0.927	0.987
		PR10	0.881	0.939	0.985	0.872	0.932	0.987	0.840	0.929	0.990
		S15	0.914	0.955	0.986	0.898	0.952	0.995	0.906	0.951	0.993
	1500	WSNS	0.910	0.965	0.995	0.902	0.965	0.993	0.895	0.939	0.980
		PR10	0.902	0.952	0.986	0.884	0.941	0.983	0.857	0.944	0.996
		S15	0.911	0.955	0.990	0.906	0.951	0.992	0.909	0.964	0.991
0.4	500	WSNS	0.900	0.942	0.977	0.884	0.934	0.982	0.880	0.929	0.983
		PR10	0.930	0.959	0.991	0.942	0.977	0.998	0.885	0.957	0.992
		S15	0.926	0.969	0.994	0.932	0.971	0.998	0.915	0.955	0.994
	1000	WSNS	0.885	0.939	0.987	0.884	0.946	0.990	0.871	0.926	0.981
		PR10	0.946	0.979	0.996	0.935	0.971	0.996	0.868	0.945	0.994
		S15	0.940	0.975	0.996	0.927	0.969	0.993	0.911	0.959	0.997
	1500	WSNS	0.903	0.948	0.993	0.888	0.957	0.996	0.882	0.932	0.986
		PR10	0.952	0.975	0.998	0.937	0.978	0.995	0.893	0.959	0.995
		S15	0.942	0.976	0.994	0.933	0.974	0.996	0.925	0.964	0.996
0.8	500	WSNS	0.888	0.945	0.990	0.886	0.940	0.983	0.872	0.925	0.972
		PR10	0.984	0.995	1.000	0.975	0.994	0.996	0.969	0.992	0.999
		S15	0.970	0.992	1.000	0.958	0.982	0.999	0.972	0.991	0.999
	1000	WSNS	0.890	0.960	0.986	0.883	0.942	0.987	0.862	0.929	0.976
		PR10	0.993	0.999	1.000	0.980	0.997	1.000	0.970	0.992	1.000
		S15	0.977	0.996	1.000	0.960	0.986	0.999	0.976	0.990	0.998
	1500	WSNS	0.882	0.941	0.991	0.891	0.948	0.987	0.868	0.918	0.975
		PR10	0.991	0.999	1.000	0.980	0.994	1.000	0.975	0.992	1.000
		S15	0.983	0.993	0.999	0.963	0.986	0.999	0.979	0.993	1.000

Table 3.58: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.921	0.949	0.980	0.914	0.949	0.986	0.911	0.953	0.985
		PR10	0.868	0.914	0.969	0.877	0.932	0.975	0.823	0.934	0.981
		S15	0.881	0.936	0.986	0.903	0.948	0.990	0.904	0.947	0.984
	1000	WSNS	0.929	0.963	0.989	0.926	0.971	0.991	0.912	0.957	0.993
		PR10	0.871	0.930	0.985	0.873	0.934	0.986	0.827	0.908	0.987
		S15	0.907	0.949	0.988	0.903	0.951	0.990	0.900	0.949	0.989
	1500	WSNS	0.946	0.983	0.998	0.934	0.974	1.000	0.911	0.949	0.985
		PR10	0.897	0.947	0.988	0.877	0.939	0.990	0.832	0.920	0.993
		S15	0.909	0.954	0.991	0.895	0.954	0.986	0.900	0.961	0.994
0.4	500	WSNS	0.919	0.952	0.987	0.916	0.957	0.990	0.888	0.934	0.984
		PR10	0.924	0.961	0.987	0.924	0.959	0.994	0.851	0.945	0.989
		S15	0.917	0.968	0.995	0.933	0.967	0.995	0.910	0.958	0.992
	1000	WSNS	0.926	0.968	0.995	0.920	0.959	0.991	0.903	0.945	0.992
		PR10	0.943	0.975	0.995	0.941	0.974	0.994	0.848	0.939	0.991
		S15	0.938	0.971	1.000	0.939	0.973	0.994	0.920	0.957	0.991
	1500	WSNS	0.941	0.970	0.998	0.917	0.968	0.996	0.910	0.946	0.991
		PR10	0.953	0.976	0.997	0.940	0.975	0.999	0.864	0.942	0.999
		S15	0.945	0.973	0.998	0.925	0.971	0.996	0.914	0.957	0.996
0.8	500	WSNS	0.938	0.973	0.996	0.906	0.951	0.986	0.901	0.948	0.985
		PR10	0.980	0.993	1.000	0.977	0.990	1.000	0.964	0.986	0.999
		S15	0.965	0.994	1.000	0.959	0.986	1.000	0.966	0.987	0.999
	1000	WSNS	0.941	0.978	0.998	0.913	0.970	0.992	0.897	0.940	0.987
		PR10	0.992	0.999	1.000	0.985	0.995	1.000	0.959	0.993	1.000
		S15	0.982	0.996	1.000	0.981	0.991	0.998	0.972	0.986	0.999
	1500	WSNS	0.934	0.976	0.995	0.913	0.965	0.993	0.915	0.950	0.990
		PR10	0.989	0.998	1.000	0.982	0.996	1.000	0.969	0.991	0.999
		S15	0.984	0.994	0.999	0.971	0.987	0.998	0.976	0.992	0.999

Table 3.59: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.876	0.924	0.971	0.892	0.931	0.978	0.884	0.931	0.978
		PR10	0.871	0.916	0.971	0.877	0.930	0.978	0.852	0.940	0.985
		S15	0.886	0.942	0.982	0.915	0.963	0.993	0.907	0.949	0.987
	1000	WSNS	0.890	0.939	0.979	0.888	0.940	0.987	0.873	0.925	0.984
		PR10	0.881	0.934	0.984	0.870	0.933	0.985	0.843	0.927	0.988
		S15	0.914	0.955	0.986	0.898	0.952	0.995	0.906	0.951	0.993
	1500	WSNS	0.901	0.958	0.995	0.896	0.957	0.990	0.893	0.934	0.982
		PR10	0.899	0.952	0.984	0.884	0.942	0.982	0.862	0.943	0.992
		S15	0.911	0.955	0.990	0.906	0.951	0.992	0.909	0.964	0.991
0.4	500	WSNS	0.888	0.931	0.969	0.879	0.921	0.970	0.877	0.927	0.971
		PR10	0.924	0.954	0.987	0.939	0.974	0.995	0.887	0.956	0.986
		S15	0.926	0.969	0.994	0.932	0.971	0.998	0.915	0.955	0.994
	1000	WSNS	0.881	0.938	0.981	0.874	0.930	0.991	0.867	0.919	0.982
		PR10	0.951	0.979	0.995	0.931	0.971	0.995	0.874	0.947	0.990
		S15	0.940	0.975	0.996	0.927	0.969	0.993	0.911	0.959	0.997
	1500	WSNS	0.901	0.951	0.990	0.889	0.947	0.994	0.883	0.934	0.987
		PR10	0.951	0.974	0.997	0.938	0.977	0.995	0.897	0.960	0.995
		S15	0.942	0.976	0.994	0.933	0.974	0.996	0.925	0.964	0.996
0.8	500	WSNS	0.884	0.948	0.987	0.877	0.936	0.984	0.868	0.919	0.973
		PR10	0.981	0.995	1.000	0.970	0.993	0.999	0.970	0.989	0.999
		S15	0.970	0.992	1.000	0.958	0.982	0.999	0.972	0.991	0.999
	1000	WSNS	0.893	0.953	0.991	0.887	0.939	0.985	0.871	0.932	0.974
		PR10	0.994	0.999	1.000	0.980	0.997	1.000	0.975	0.994	1.000
		S15	0.977	0.996	1.000	0.960	0.986	0.999	0.976	0.990	0.998
	1500	WSNS	0.886	0.946	0.989	0.892	0.941	0.987	0.869	0.927	0.978
		PR10	0.989	0.999	1.000	0.981	0.993	1.000	0.979	0.991	1.000
		S15	0.983	0.993	0.999	0.963	0.986	0.999	0.979	0.993	1.000

Table 3.60: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.899	0.940	0.974	0.897	0.945	0.979	0.904	0.951	0.985
		PR10	0.867	0.908	0.967	0.878	0.927	0.970	0.834	0.929	0.980
		S15	0.881	0.936	0.986	0.903	0.948	0.990	0.904	0.947	0.984
	1000	WSNS	0.916	0.957	0.986	0.909	0.960	0.991	0.907	0.944	0.987
		PR10	0.873	0.927	0.986	0.867	0.932	0.982	0.838	0.913	0.981
		S15	0.907	0.949	0.988	0.903	0.951	0.990	0.900	0.949	0.989
	1500	WSNS	0.933	0.977	0.997	0.917	0.964	0.999	0.908	0.945	0.983
		PR10	0.897	0.946	0.987	0.876	0.937	0.988	0.851	0.927	0.992
		S15	0.909	0.954	0.991	0.895	0.954	0.986	0.900	0.961	0.994
0.4	500	WSNS	0.907	0.941	0.974	0.902	0.949	0.984	0.886	0.925	0.976
		PR10	0.923	0.956	0.985	0.923	0.960	0.992	0.864	0.944	0.986
		S15	0.917	0.968	0.995	0.933	0.967	0.995	0.910	0.958	0.992
	1000	WSNS	0.921	0.957	0.992	0.908	0.951	0.985	0.894	0.946	0.989
		PR10	0.938	0.973	0.993	0.938	0.975	0.993	0.857	0.946	0.989
		S15	0.938	0.971	1.000	0.939	0.973	0.994	0.920	0.957	0.991
	1500	WSNS	0.929	0.959	0.995	0.905	0.961	0.996	0.902	0.943	0.991
		PR10	0.951	0.977	0.995	0.942	0.975	0.998	0.872	0.944	0.998
		S15	0.945	0.973	0.998	0.925	0.971	0.996	0.914	0.957	0.996
0.8	500	WSNS	0.929	0.965	0.993	0.900	0.947	0.986	0.901	0.949	0.976
		PR10	0.978	0.993	1.000	0.975	0.991	1.000	0.965	0.990	0.999
		S15	0.965	0.994	1.000	0.959	0.986	1.000	0.966	0.987	0.999
	1000	WSNS	0.935	0.972	0.997	0.910	0.961	0.993	0.894	0.940	0.983
		PR10	0.992	0.998	1.000	0.985	0.995	1.000	0.964	0.993	1.000
		S15	0.982	0.996	1.000	0.981	0.991	0.998	0.972	0.986	0.999
	1500	WSNS	0.918	0.976	0.996	0.901	0.956	0.992	0.916	0.950	0.989
		PR10	0.988	0.998	1.000	0.983	0.996	1.000	0.973	0.993	0.999
		S15	0.984	0.994	0.999	0.971	0.987	0.998	0.976	0.992	0.999

Table 3.61: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.868	0.918	0.961	0.877	0.925	0.973	0.867	0.928	0.965
		PR10	0.868	0.913	0.966	0.876	0.924	0.975	0.857	0.927	0.981
		S15	0.886	0.942	0.982	0.915	0.963	0.993	0.907	0.949	0.987
	1000	WSNS	0.883	0.936	0.972	0.884	0.935	0.979	0.876	0.923	0.980
		PR10	0.880	0.931	0.982	0.870	0.926	0.986	0.848	0.937	0.982
		S15	0.914	0.955	0.986	0.898	0.952	0.995	0.906	0.951	0.993
	1500	WSNS	0.899	0.955	0.991	0.899	0.956	0.987	0.887	0.931	0.982
		PR10	0.900	0.947	0.985	0.885	0.941	0.980	0.862	0.946	0.991
		S15	0.911	0.955	0.990	0.906	0.951	0.992	0.909	0.964	0.991
0.4	500	WSNS	0.886	0.929	0.962	0.863	0.909	0.963	0.865	0.925	0.975
		PR10	0.923	0.955	0.987	0.928	0.966	0.997	0.898	0.949	0.984
		S15	0.926	0.969	0.994	0.932	0.971	0.998	0.915	0.955	0.994
	1000	WSNS	0.874	0.932	0.986	0.865	0.926	0.986	0.862	0.918	0.981
		PR10	0.942	0.976	0.995	0.930	0.968	0.995	0.884	0.956	0.991
		S15	0.940	0.975	0.996	0.927	0.969	0.993	0.911	0.959	0.997
	1500	WSNS	0.901	0.948	0.986	0.885	0.946	0.993	0.885	0.933	0.985
		PR10	0.947	0.976	0.996	0.935	0.973	0.994	0.903	0.963	0.995
		S15	0.942	0.976	0.994	0.933	0.974	0.996	0.925	0.964	0.996
0.8	500	WSNS	0.887	0.943	0.982	0.884	0.929	0.985	0.869	0.925	0.971
		PR10	0.980	0.996	1.000	0.972	0.989	0.997	0.974	0.989	0.998
		S15	0.970	0.992	1.000	0.958	0.982	0.999	0.972	0.991	0.999
	1000	WSNS	0.892	0.947	0.988	0.880	0.944	0.984	0.876	0.932	0.977
		PR10	0.994	0.998	1.000	0.979	0.997	1.000	0.974	0.996	1.000
		S15	0.977	0.996	1.000	0.960	0.986	0.999	0.976	0.990	0.998
	1500	WSNS	0.885	0.942	0.989	0.892	0.950	0.986	0.872	0.933	0.978
		PR10	0.988	0.998	1.000	0.980	0.993	1.000	0.978	0.993	1.000
		S15	0.983	0.993	0.999	0.963	0.986	0.999	0.979	0.993	1.000

Table 3.62: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.892	0.935	0.965	0.882	0.930	0.974	0.902	0.936	0.975
		PR10	0.859	0.903	0.955	0.868	0.924	0.966	0.845	0.926	0.975
		S15	0.881	0.936	0.986	0.903	0.948	0.990	0.904	0.947	0.984
	1000	WSNS	0.902	0.952	0.981	0.903	0.953	0.990	0.904	0.939	0.989
		PR10	0.872	0.929	0.983	0.873	0.931	0.982	0.848	0.922	0.980
		S15	0.907	0.949	0.988	0.903	0.951	0.990	0.900	0.949	0.989
	1500	WSNS	0.920	0.969	0.996	0.899	0.958	0.999	0.900	0.946	0.986
		PR10	0.897	0.947	0.986	0.873	0.941	0.987	0.856	0.928	0.991
		S15	0.909	0.954	0.991	0.895	0.954	0.986	0.900	0.961	0.994
0.4	500	WSNS	0.895	0.943	0.972	0.892	0.944	0.984	0.871	0.928	0.973
		PR10	0.915	0.948	0.984	0.913	0.956	0.993	0.875	0.943	0.977
		S15	0.917	0.968	0.995	0.933	0.967	0.995	0.910	0.958	0.992
	1000	WSNS	0.909	0.956	0.990	0.900	0.944	0.984	0.889	0.939	0.987
		PR10	0.939	0.977	0.994	0.938	0.975	0.993	0.867	0.949	0.989
		S15	0.938	0.971	1.000	0.939	0.973	0.994	0.920	0.957	0.991
	1500	WSNS	0.925	0.956	0.994	0.892	0.954	0.994	0.902	0.941	0.992
		PR10	0.945	0.977	0.995	0.941	0.972	0.998	0.879	0.952	0.996
		S15	0.945	0.973	0.998	0.925	0.971	0.996	0.914	0.957	0.996
0.8	500	WSNS	0.925	0.967	0.993	0.887	0.938	0.982	0.896	0.945	0.977
		PR10	0.977	0.991	1.000	0.971	0.988	0.998	0.969	0.989	0.998
		S15	0.965	0.994	1.000	0.959	0.986	1.000	0.966	0.987	0.999
	1000	WSNS	0.928	0.972	0.997	0.903	0.955	0.993	0.892	0.939	0.985
		PR10	0.992	0.998	1.000	0.983	0.996	1.000	0.964	0.995	0.999
		S15	0.982	0.996	1.000	0.981	0.991	0.998	0.972	0.986	0.999
	1500	WSNS	0.909	0.970	0.993	0.888	0.953	0.992	0.920	0.953	0.988
		PR10	0.989	0.997	1.000	0.981	0.996	1.000	0.974	0.994	1.000
		S15	0.984	0.994	0.999	0.971	0.987	0.998	0.976	0.992	0.999

Table 3.63: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.858	0.904	0.959	0.866	0.914	0.956	0.862	0.914	0.965
		PR10	0.859	0.905	0.956	0.870	0.922	0.967	0.855	0.927	0.970
		S15	0.886	0.942	0.982	0.915	0.963	0.993	0.907	0.949	0.987
	1000	WSNS	0.882	0.929	0.970	0.880	0.927	0.983	0.870	0.924	0.977
		PR10	0.875	0.930	0.982	0.867	0.925	0.982	0.850	0.932	0.981
		S15	0.914	0.955	0.986	0.898	0.952	0.995	0.906	0.951	0.993
	1500	WSNS	0.897	0.953	0.991	0.894	0.949	0.987	0.885	0.935	0.983
		PR10	0.902	0.943	0.983	0.884	0.937	0.982	0.871	0.944	0.990
		S15	0.911	0.955	0.990	0.906	0.951	0.992	0.909	0.964	0.991
0.4	500	WSNS	0.882	0.919	0.964	0.854	0.903	0.961	0.859	0.918	0.971
		PR10	0.914	0.950	0.982	0.931	0.971	0.996	0.889	0.946	0.980
		S15	0.926	0.969	0.994	0.932	0.971	0.998	0.915	0.955	0.994
	1000	WSNS	0.870	0.929	0.980	0.859	0.924	0.987	0.863	0.917	0.977
		PR10	0.938	0.974	0.996	0.925	0.965	0.994	0.885	0.952	0.989
		S15	0.940	0.975	0.996	0.927	0.969	0.993	0.911	0.959	0.997
	1500	WSNS	0.898	0.942	0.985	0.885	0.941	0.991	0.878	0.937	0.987
		PR10	0.946	0.973	0.996	0.933	0.972	0.993	0.903	0.962	0.995
		S15	0.942	0.976	0.994	0.933	0.974	0.996	0.925	0.964	0.996
0.8	500	WSNS	0.884	0.941	0.986	0.863	0.925	0.975	0.872	0.924	0.974
		PR10	0.975	0.993	1.000	0.971	0.990	0.998	0.970	0.990	0.998
		S15	0.970	0.992	1.000	0.958	0.982	0.999	0.972	0.991	0.999
	1000	WSNS	0.885	0.945	0.989	0.883	0.937	0.980	0.876	0.929	0.976
		PR10	0.994	0.997	1.000	0.973	0.997	1.000	0.975	0.995	1.000
		S15	0.977	0.996	1.000	0.960	0.986	0.999	0.976	0.990	0.998
	1500	WSNS	0.875	0.942	0.989	0.895	0.945	0.986	0.873	0.936	0.975
		PR10	0.988	0.998	1.000	0.978	0.993	1.000	0.979	0.992	1.000
		S15	0.983	0.993	0.999	0.963	0.986	0.999	0.979	0.993	1.000

Table 3.64: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with t_5 innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.878	0.922	0.964	0.871	0.918	0.968	0.888	0.929	0.971
		PR10	0.854	0.900	0.952	0.864	0.918	0.960	0.852	0.919	0.969
		S15	0.881	0.936	0.986	0.903	0.948	0.990	0.904	0.947	0.984
	1000	WSNS	0.899	0.948	0.981	0.892	0.946	0.987	0.892	0.933	0.987
		PR10	0.867	0.925	0.980	0.872	0.932	0.979	0.854	0.926	0.977
		S15	0.907	0.949	0.988	0.903	0.951	0.990	0.900	0.949	0.989
	1500	WSNS	0.918	0.970	0.991	0.903	0.950	0.992	0.898	0.943	0.982
		PR10	0.898	0.945	0.982	0.871	0.931	0.986	0.859	0.936	0.989
		S15	0.909	0.954	0.991	0.895	0.954	0.986	0.900	0.961	0.994
0.4	500	WSNS	0.888	0.936	0.970	0.894	0.928	0.973	0.859	0.920	0.961
		PR10	0.904	0.947	0.982	0.914	0.950	0.984	0.870	0.928	0.975
		S15	0.917	0.968	0.995	0.933	0.967	0.995	0.910	0.958	0.992
	1000	WSNS	0.898	0.950	0.987	0.891	0.939	0.983	0.885	0.937	0.980
		PR10	0.936	0.974	0.994	0.935	0.972	0.993	0.879	0.947	0.987
		S15	0.938	0.971	1.000	0.939	0.973	0.994	0.920	0.957	0.991
	1500	WSNS	0.919	0.952	0.993	0.891	0.952	0.990	0.894	0.941	0.986
		PR10	0.945	0.974	0.995	0.942	0.970	0.998	0.883	0.952	0.996
		S15	0.945	0.973	0.998	0.925	0.971	0.996	0.914	0.957	0.996
0.8	500	WSNS	0.911	0.958	0.990	0.876	0.932	0.977	0.906	0.942	0.979
		PR10	0.975	0.991	1.000	0.966	0.987	0.998	0.966	0.986	0.998
		S15	0.965	0.994	1.000	0.959	0.986	1.000	0.966	0.987	0.999
	1000	WSNS	0.919	0.973	0.994	0.903	0.954	0.993	0.893	0.940	0.978
		PR10	0.991	0.998	1.000	0.983	0.998	1.000	0.966	0.992	0.999
		S15	0.982	0.996	1.000	0.981	0.991	0.998	0.972	0.986	0.999
	1500	WSNS	0.906	0.963	0.990	0.892	0.956	0.991	0.921	0.951	0.988
		PR10	0.988	0.997	1.000	0.982	0.993	0.999	0.974	0.994	1.000
		S15	0.984	0.994	0.999	0.971	0.987	0.998	0.976	0.992	0.999

Table 3.65: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.877	0.938	0.983	0.888	0.938	0.985	0.864	0.921	0.968
		PR10	0.879	0.940	0.978	0.867	0.929	0.979	0.896	0.937	0.983
		S15	0.898	0.947	0.994	0.895	0.935	0.988	0.919	0.956	0.993
	1000	WSNS	0.898	0.954	0.990	0.910	0.957	0.996	0.836	0.890	0.963
		PR10	0.891	0.936	0.987	0.888	0.944	0.986	0.890	0.934	0.986
		S15	0.897	0.947	0.993	0.903	0.948	0.990	0.884	0.936	0.988
	1500	WSNS	0.911	0.960	0.993	0.928	0.967	0.999	0.856	0.912	0.972
		PR10	0.910	0.948	0.993	0.891	0.948	0.987	0.900	0.947	0.983
		S15	0.906	0.955	0.989	0.911	0.956	0.997	0.902	0.946	0.981
0.4	500	WSNS	0.892	0.942	0.985	0.884	0.942	0.989	0.873	0.922	0.967
		PR10	0.955	0.977	0.999	0.916	0.969	0.991	0.913	0.948	0.988
		S15	0.904	0.945	0.990	0.899	0.942	0.994	0.911	0.953	0.986
	1000	WSNS	0.908	0.954	0.992	0.911	0.960	0.996	0.851	0.905	0.969
		PR10	0.958	0.985	0.999	0.938	0.977	0.995	0.921	0.957	0.991
		S15	0.895	0.955	0.990	0.914	0.956	0.995	0.894	0.940	0.987
	1500	WSNS	0.909	0.957	0.994	0.916	0.961	0.991	0.873	0.927	0.974
		PR10	0.972	0.997	1.000	0.943	0.976	0.997	0.922	0.959	0.990
		S15	0.902	0.947	0.990	0.912	0.953	0.988	0.908	0.949	0.986
0.8	500	WSNS	0.891	0.945	0.987	0.883	0.937	0.987	0.861	0.912	0.962
		PR10	1.000	1.000	1.000	0.995	0.998	1.000	0.966	0.991	1.000
		S15	0.905	0.949	0.997	0.913	0.962	0.991	0.906	0.947	0.989
	1000	WSNS	0.906	0.958	0.995	0.909	0.954	0.986	0.865	0.923	0.974
		PR10	1.000	1.000	1.000	0.995	0.999	1.000	0.979	0.991	1.000
		S15	0.896	0.953	0.995	0.933	0.963	0.993	0.904	0.953	0.990
	1500	WSNS	0.907	0.953	0.990	0.885	0.937	0.984	0.873	0.921	0.978
		PR10	1.000	1.000	1.000	0.996	0.999	1.000	0.982	0.992	0.999
		S15	0.903	0.943	0.994	0.908	0.948	0.988	0.903	0.947	0.992

Table 3.66: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.924	0.955	0.992	0.915	0.952	0.994	0.881	0.932	0.980
		PR10	0.865	0.921	0.977	0.871	0.926	0.976	0.839	0.902	0.976
		S15	0.892	0.942	0.991	0.892	0.943	0.987	0.904	0.948	0.993
	1000	WSNS	0.939	0.973	0.996	0.943	0.974	0.994	0.876	0.928	0.981
		PR10	0.877	0.931	0.985	0.882	0.940	0.986	0.846	0.927	0.980
		S15	0.893	0.946	0.993	0.897	0.955	0.984	0.892	0.948	0.988
	1500	WSNS	0.947	0.980	0.998	0.938	0.983	0.997	0.880	0.934	0.980
		PR10	0.887	0.941	0.992	0.885	0.944	0.989	0.849	0.927	0.975
		S15	0.906	0.950	0.988	0.914	0.949	0.987	0.900	0.946	0.989
0.4	500	WSNS	0.919	0.960	0.991	0.908	0.954	0.989	0.873	0.926	0.983
		PR10	0.943	0.978	0.997	0.931	0.973	0.996	0.856	0.915	0.985
		S15	0.894	0.934	0.988	0.894	0.950	0.985	0.883	0.936	0.992
	1000	WSNS	0.942	0.976	0.995	0.916	0.966	0.996	0.898	0.944	0.979
		PR10	0.954	0.982	0.998	0.938	0.979	0.999	0.868	0.944	0.994
		S15	0.898	0.947	0.991	0.893	0.947	0.988	0.911	0.958	0.991
	1500	WSNS	0.945	0.985	0.999	0.944	0.974	0.996	0.885	0.938	0.988
		PR10	0.960	0.992	1.000	0.952	0.982	0.999	0.883	0.939	0.989
		S15	0.902	0.953	0.992	0.906	0.964	0.991	0.901	0.956	0.996
0.8	500	WSNS	0.917	0.965	0.991	0.894	0.944	0.985	0.886	0.932	0.975
		PR10	1.000	1.000	1.000	0.994	1.000	1.000	0.962	0.986	0.997
		S15	0.887	0.936	0.990	0.909	0.957	0.991	0.900	0.948	0.991
	1000	WSNS	0.939	0.979	0.998	0.896	0.966	0.994	0.885	0.941	0.984
		PR10	1.000	1.000	1.000	0.997	1.000	1.000	0.975	0.996	0.999
		S15	0.903	0.949	0.990	0.908	0.963	0.994	0.896	0.950	0.990
	1500	WSNS	0.943	0.976	0.998	0.904	0.957	0.993	0.882	0.935	0.985
		PR10	1.000	1.000	1.000	0.998	1.000	1.000	0.977	0.996	1.000
		S15	0.901	0.955	0.994	0.910	0.957	0.995	0.897	0.948	0.993

Table 3.67: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.871	0.924	0.973	0.876	0.927	0.977	0.870	0.918	0.966
		PR10	0.869	0.931	0.974	0.870	0.923	0.979	0.892	0.933	0.983
		S15	0.898	0.947	0.994	0.895	0.935	0.988	0.919	0.956	0.993
	1000	WSNS	0.892	0.951	0.987	0.906	0.954	0.993	0.844	0.898	0.970
		PR10	0.890	0.933	0.983	0.888	0.942	0.988	0.888	0.931	0.981
		S15	0.897	0.947	0.993	0.903	0.948	0.990	0.884	0.936	0.988
	1500	WSNS	0.905	0.956	0.987	0.920	0.962	0.998	0.871	0.916	0.971
		PR10	0.903	0.949	0.992	0.891	0.950	0.989	0.899	0.948	0.983
		S15	0.906	0.955	0.989	0.911	0.956	0.997	0.902	0.946	0.981
0.4	500	WSNS	0.880	0.929	0.978	0.879	0.925	0.979	0.874	0.918	0.967
		PR10	0.949	0.975	1.000	0.916	0.962	0.991	0.913	0.945	0.985
		S15	0.904	0.945	0.990	0.899	0.942	0.994	0.911	0.953	0.986
	1000	WSNS	0.900	0.951	0.985	0.906	0.959	0.992	0.863	0.916	0.964
		PR10	0.959	0.984	0.996	0.940	0.973	0.995	0.918	0.956	0.987
		S15	0.895	0.955	0.990	0.914	0.956	0.995	0.894	0.940	0.987
	1500	WSNS	0.904	0.952	0.991	0.904	0.959	0.990	0.882	0.935	0.979
		PR10	0.968	0.995	1.000	0.946	0.979	0.997	0.917	0.956	0.990
		S15	0.902	0.947	0.990	0.912	0.953	0.988	0.908	0.949	0.986
0.8	500	WSNS	0.880	0.938	0.982	0.874	0.925	0.975	0.865	0.916	0.962
		PR10	1.000	1.000	1.000	0.994	0.996	1.000	0.962	0.989	0.999
		S15	0.905	0.949	0.997	0.913	0.962	0.991	0.906	0.947	0.989
	1000	WSNS	0.903	0.951	0.992	0.900	0.948	0.984	0.877	0.938	0.979
		PR10	1.000	1.000	1.000	0.995	0.999	1.000	0.978	0.989	0.999
		S15	0.896	0.953	0.995	0.933	0.963	0.993	0.904	0.953	0.990
	1500	WSNS	0.905	0.949	0.993	0.879	0.931	0.984	0.883	0.934	0.979
		PR10	1.000	1.000	1.000	0.996	0.999	1.000	0.982	0.992	1.000
		S15	0.903	0.943	0.994	0.908	0.948	0.988	0.903	0.947	0.992

Table 3.68: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.892	0.941	0.979	0.899	0.944	0.982	0.880	0.932	0.978
		PR10	0.856	0.916	0.971	0.873	0.923	0.967	0.842	0.903	0.977
		S15	0.892	0.942	0.991	0.892	0.943	0.987	0.904	0.948	0.993
	1000	WSNS	0.923	0.966	0.994	0.925	0.969	0.992	0.885	0.931	0.983
		PR10	0.882	0.935	0.984	0.880	0.945	0.987	0.860	0.930	0.977
		S15	0.893	0.946	0.993	0.897	0.955	0.984	0.892	0.948	0.988
	1500	WSNS	0.938	0.971	0.995	0.933	0.969	0.993	0.887	0.937	0.985
		PR10	0.888	0.941	0.993	0.889	0.946	0.988	0.869	0.930	0.979
		S15	0.906	0.950	0.988	0.914	0.949	0.987	0.900	0.946	0.989
0.4	500	WSNS	0.899	0.938	0.980	0.883	0.936	0.981	0.864	0.925	0.981
		PR10	0.939	0.974	0.996	0.933	0.971	0.993	0.859	0.923	0.984
		S15	0.894	0.934	0.988	0.894	0.950	0.985	0.883	0.936	0.992
	1000	WSNS	0.928	0.971	0.994	0.903	0.953	0.993	0.899	0.952	0.977
		PR10	0.955	0.984	0.998	0.940	0.980	0.999	0.886	0.948	0.993
		S15	0.898	0.947	0.991	0.893	0.947	0.988	0.911	0.958	0.991
	1500	WSNS	0.936	0.972	0.995	0.930	0.968	0.994	0.891	0.944	0.986
		PR10	0.964	0.994	1.000	0.953	0.984	0.999	0.895	0.942	0.989
		S15	0.902	0.953	0.992	0.906	0.964	0.991	0.901	0.956	0.996
0.8	500	WSNS	0.894	0.943	0.986	0.880	0.930	0.981	0.891	0.929	0.974
		PR10	1.000	1.000	1.000	0.993	0.998	1.000	0.967	0.983	0.997
		S15	0.887	0.936	0.990	0.909	0.957	0.991	0.900	0.948	0.991
	1000	WSNS	0.924	0.975	0.991	0.882	0.950	0.990	0.890	0.945	0.983
		PR10	1.000	1.000	1.000	0.998	1.000	1.000	0.979	0.996	0.999
		S15	0.903	0.949	0.990	0.908	0.963	0.994	0.896	0.950	0.990
	1500	WSNS	0.935	0.969	0.994	0.896	0.948	0.991	0.890	0.945	0.989
		PR10	1.000	1.000	1.000	0.998	1.000	1.000	0.981	0.997	1.000
		S15	0.901	0.955	0.994	0.910	0.957	0.995	0.897	0.948	0.993

Table 3.69: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.853	0.900	0.957	0.859	0.907	0.954	0.869	0.907	0.960
		PR10	0.858	0.919	0.970	0.861	0.913	0.970	0.888	0.931	0.978
		S15	0.898	0.947	0.994	0.895	0.935	0.988	0.919	0.956	0.993
	1000	WSNS	0.874	0.938	0.984	0.891	0.941	0.986	0.841	0.886	0.958
		PR10	0.882	0.933	0.984	0.884	0.941	0.985	0.884	0.931	0.981
		S15	0.897	0.947	0.993	0.903	0.948	0.990	0.884	0.936	0.988
	1500	WSNS	0.891	0.949	0.985	0.905	0.951	0.996	0.869	0.916	0.968
		PR10	0.897	0.944	0.992	0.895	0.951	0.988	0.899	0.940	0.984
		S15	0.906	0.955	0.989	0.911	0.956	0.997	0.902	0.946	0.981
0.4	500	WSNS	0.863	0.908	0.966	0.860	0.911	0.961	0.867	0.914	0.956
		PR10	0.941	0.972	0.998	0.905	0.949	0.985	0.914	0.945	0.977
		S15	0.904	0.945	0.990	0.899	0.942	0.994	0.911	0.953	0.986
	1000	WSNS	0.892	0.940	0.982	0.886	0.946	0.987	0.857	0.911	0.961
		PR10	0.958	0.983	0.997	0.938	0.968	0.994	0.916	0.955	0.989
		S15	0.895	0.955	0.990	0.914	0.956	0.995	0.894	0.940	0.987
	1500	WSNS	0.895	0.940	0.992	0.893	0.947	0.984	0.877	0.925	0.967
		PR10	0.965	0.992	1.000	0.944	0.977	0.997	0.915	0.955	0.991
		S15	0.902	0.947	0.990	0.912	0.953	0.988	0.908	0.949	0.986
0.8	500	WSNS	0.865	0.915	0.963	0.852	0.913	0.966	0.857	0.910	0.957
		PR10	1.000	1.000	1.000	0.988	0.998	1.000	0.962	0.980	0.997
		S15	0.905	0.949	0.997	0.913	0.962	0.991	0.906	0.947	0.989
	1000	WSNS	0.889	0.943	0.984	0.878	0.941	0.981	0.876	0.929	0.978
		PR10	0.999	1.000	1.000	0.993	0.998	1.000	0.980	0.989	0.997
		S15	0.896	0.953	0.995	0.933	0.963	0.993	0.904	0.953	0.990
	1500	WSNS	0.894	0.943	0.988	0.871	0.917	0.982	0.876	0.934	0.979
		PR10	1.000	1.000	1.000	0.995	0.999	1.000	0.984	0.991	0.999
		S15	0.903	0.943	0.994	0.908	0.948	0.988	0.903	0.947	0.992

Table 3.70: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.859	0.920	0.963	0.866	0.918	0.965	0.868	0.921	0.971
		PR10	0.853	0.903	0.965	0.858	0.915	0.964	0.842	0.907	0.969
		S15	0.892	0.942	0.991	0.892	0.943	0.987	0.904	0.948	0.993
	1000	WSNS	0.896	0.950	0.987	0.903	0.953	0.982	0.877	0.920	0.979
		PR10	0.876	0.932	0.981	0.878	0.941	0.983	0.865	0.933	0.981
		S15	0.893	0.946	0.993	0.897	0.955	0.984	0.892	0.948	0.988
	1500	WSNS	0.923	0.963	0.991	0.913	0.956	0.989	0.881	0.935	0.977
		PR10	0.887	0.940	0.990	0.883	0.943	0.989	0.873	0.933	0.978
		S15	0.906	0.950	0.988	0.914	0.949	0.987	0.900	0.946	0.989
0.4	500	WSNS	0.874	0.919	0.963	0.869	0.914	0.964	0.858	0.917	0.969
		PR10	0.934	0.967	0.994	0.925	0.967	0.992	0.865	0.922	0.979
		S15	0.894	0.934	0.988	0.894	0.950	0.985	0.883	0.936	0.992
	1000	WSNS	0.906	0.952	0.990	0.891	0.944	0.984	0.892	0.941	0.979
		PR10	0.950	0.982	0.998	0.946	0.976	0.997	0.899	0.960	0.993
		S15	0.898	0.947	0.991	0.893	0.947	0.988	0.911	0.958	0.991
	1500	WSNS	0.921	0.963	0.994	0.908	0.962	0.991	0.892	0.940	0.987
		PR10	0.963	0.990	1.000	0.954	0.982	1.000	0.897	0.948	0.989
		S15	0.902	0.953	0.992	0.906	0.964	0.991	0.901	0.956	0.996
0.8	500	WSNS	0.867	0.917	0.970	0.865	0.914	0.964	0.869	0.922	0.968
		PR10	0.999	1.000	1.000	0.989	0.999	1.000	0.967	0.983	0.999
		S15	0.887	0.936	0.990	0.909	0.957	0.991	0.900	0.948	0.991
	1000	WSNS	0.912	0.960	0.990	0.860	0.935	0.984	0.888	0.935	0.982
		PR10	0.999	1.000	1.000	0.997	1.000	1.000	0.983	0.996	0.999
		S15	0.903	0.949	0.990	0.908	0.963	0.994	0.896	0.950	0.990
	1500	WSNS	0.923	0.961	0.993	0.887	0.938	0.982	0.886	0.939	0.983
		PR10	1.000	1.000	1.000	0.997	1.000	1.000	0.983	0.996	1.000
		S15	0.901	0.955	0.994	0.910	0.957	0.995	0.897	0.948	0.993

Table 3.71: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.843	0.897	0.945	0.846	0.896	0.955	0.858	0.907	0.953
		PR10	0.843	0.914	0.968	0.850	0.909	0.963	0.885	0.924	0.969
		S15	0.898	0.947	0.994	0.895	0.935	0.988	0.919	0.956	0.993
	1000	WSNS	0.872	0.928	0.979	0.888	0.935	0.981	0.847	0.904	0.959
		PR10	0.879	0.926	0.983	0.886	0.938	0.986	0.879	0.930	0.978
		S15	0.897	0.947	0.993	0.903	0.948	0.990	0.884	0.936	0.988
	1500	WSNS	0.891	0.947	0.985	0.900	0.947	0.996	0.876	0.917	0.966
		PR10	0.892	0.944	0.992	0.892	0.948	0.987	0.895	0.938	0.982
		S15	0.906	0.955	0.989	0.911	0.956	0.997	0.902	0.946	0.981
0.4	500	WSNS	0.850	0.904	0.949	0.842	0.897	0.949	0.860	0.920	0.949
		PR10	0.938	0.966	0.995	0.897	0.944	0.985	0.906	0.941	0.974
		S15	0.904	0.945	0.990	0.899	0.942	0.994	0.911	0.953	0.986
	1000	WSNS	0.878	0.930	0.980	0.882	0.940	0.987	0.856	0.909	0.966
		PR10	0.956	0.981	0.996	0.937	0.968	0.993	0.918	0.952	0.988
		S15	0.895	0.955	0.990	0.914	0.956	0.995	0.894	0.940	0.987
	1500	WSNS	0.895	0.935	0.986	0.891	0.949	0.986	0.882	0.924	0.974
		PR10	0.965	0.993	1.000	0.941	0.976	0.997	0.915	0.956	0.988
		S15	0.902	0.947	0.990	0.912	0.953	0.988	0.908	0.949	0.986
0.8	500	WSNS	0.852	0.905	0.952	0.843	0.906	0.963	0.859	0.900	0.958
		PR10	0.996	1.000	1.000	0.985	0.995	1.000	0.959	0.980	0.995
		S15	0.905	0.949	0.997	0.913	0.962	0.991	0.906	0.947	0.989
	1000	WSNS	0.884	0.941	0.978	0.879	0.930	0.976	0.882	0.923	0.978
		PR10	0.998	1.000	1.000	0.991	0.998	1.000	0.979	0.987	0.996
		S15	0.896	0.953	0.995	0.933	0.963	0.993	0.904	0.953	0.990
	1500	WSNS	0.894	0.939	0.983	0.866	0.915	0.979	0.875	0.931	0.979
		PR10	1.000	1.000	1.000	0.994	0.998	1.000	0.978	0.992	0.999
		S15	0.903	0.943	0.994	0.908	0.948	0.988	0.903	0.947	0.992

Table 3.72: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.850	0.904	0.948	0.854	0.903	0.958	0.862	0.912	0.971
		PR10	0.844	0.903	0.959	0.850	0.914	0.959	0.843	0.894	0.959
		S15	0.892	0.942	0.991	0.892	0.943	0.987	0.904	0.948	0.993
	1000	WSNS	0.890	0.944	0.982	0.886	0.947	0.983	0.880	0.918	0.973
		PR10	0.873	0.926	0.981	0.877	0.941	0.981	0.870	0.932	0.974
		S15	0.893	0.946	0.993	0.897	0.955	0.984	0.892	0.948	0.988
	1500	WSNS	0.916	0.956	0.988	0.910	0.950	0.988	0.881	0.932	0.976
		PR10	0.886	0.940	0.991	0.887	0.944	0.989	0.877	0.932	0.973
		S15	0.906	0.950	0.988	0.914	0.949	0.987	0.900	0.946	0.989
0.4	500	WSNS	0.863	0.909	0.949	0.856	0.907	0.950	0.854	0.912	0.964
		PR10	0.927	0.965	0.991	0.925	0.960	0.990	0.865	0.915	0.975
		S15	0.894	0.934	0.988	0.894	0.950	0.985	0.883	0.936	0.992
	1000	WSNS	0.897	0.947	0.984	0.881	0.936	0.979	0.893	0.943	0.984
		PR10	0.954	0.982	0.998	0.941	0.977	0.996	0.900	0.958	0.992
		S15	0.898	0.947	0.991	0.893	0.947	0.988	0.911	0.958	0.991
	1500	WSNS	0.917	0.954	0.991	0.900	0.960	0.989	0.898	0.939	0.982
		PR10	0.965	0.988	1.000	0.952	0.981	1.000	0.898	0.951	0.988
		S15	0.902	0.953	0.992	0.906	0.964	0.991	0.901	0.956	0.996
0.8	500	WSNS	0.856	0.907	0.959	0.863	0.911	0.956	0.869	0.917	0.960
		PR10	0.997	1.000	1.000	0.988	0.998	1.000	0.959	0.976	0.994
		S15	0.887	0.936	0.990	0.909	0.957	0.991	0.900	0.948	0.991
	1000	WSNS	0.909	0.947	0.988	0.855	0.932	0.982	0.891	0.936	0.978
		PR10	0.998	1.000	1.000	0.997	1.000	1.000	0.983	0.995	0.999
		S15	0.903	0.949	0.990	0.908	0.963	0.994	0.896	0.950	0.990
	1500	WSNS	0.920	0.953	0.992	0.881	0.932	0.982	0.888	0.938	0.984
		PR10	1.000	1.000	1.000	0.998	1.000	1.000	0.983	0.996	1.000
		S15	0.901	0.955	0.994	0.910	0.957	0.995	0.897	0.948	0.993

Table 3.73: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.824	0.876	0.942	0.829	0.882	0.941	0.847	0.898	0.944
		PR10	0.830	0.900	0.958	0.847	0.906	0.956	0.867	0.924	0.966
		S15	0.898	0.947	0.994	0.895	0.935	0.988	0.919	0.956	0.993
	1000	WSNS	0.865	0.913	0.972	0.874	0.935	0.970	0.840	0.901	0.956
		PR10	0.875	0.925	0.980	0.883	0.937	0.982	0.874	0.925	0.978
		S15	0.897	0.947	0.993	0.903	0.948	0.990	0.884	0.936	0.988
	1500	WSNS	0.885	0.936	0.981	0.894	0.944	0.991	0.872	0.911	0.963
		PR10	0.890	0.942	0.992	0.897	0.951	0.985	0.892	0.936	0.981
		S15	0.906	0.955	0.989	0.911	0.956	0.997	0.902	0.946	0.981
0.4	500	WSNS	0.830	0.883	0.939	0.828	0.891	0.939	0.847	0.909	0.952
		PR10	0.926	0.960	0.987	0.891	0.943	0.980	0.897	0.932	0.968
		S15	0.904	0.945	0.990	0.899	0.942	0.994	0.911	0.953	0.986
	1000	WSNS	0.875	0.922	0.971	0.868	0.932	0.982	0.847	0.902	0.961
		PR10	0.954	0.978	0.996	0.936	0.971	0.993	0.912	0.952	0.983
		S15	0.895	0.955	0.990	0.914	0.956	0.995	0.894	0.940	0.987
	1500	WSNS	0.883	0.930	0.984	0.886	0.937	0.984	0.878	0.922	0.969
		PR10	0.965	0.990	0.999	0.942	0.977	0.996	0.911	0.955	0.987
		S15	0.902	0.947	0.990	0.912	0.953	0.988	0.908	0.949	0.986
0.8	500	WSNS	0.843	0.893	0.940	0.830	0.887	0.952	0.850	0.893	0.946
		PR10	0.993	0.998	1.000	0.981	0.995	1.000	0.956	0.974	0.991
		S15	0.905	0.949	0.997	0.913	0.962	0.991	0.906	0.947	0.989
	1000	WSNS	0.873	0.928	0.976	0.875	0.928	0.972	0.879	0.917	0.973
		PR10	0.998	0.999	1.000	0.990	0.999	1.000	0.976	0.985	0.994
		S15	0.896	0.953	0.995	0.933	0.963	0.993	0.904	0.953	0.990
	1500	WSNS	0.883	0.934	0.978	0.861	0.909	0.975	0.872	0.923	0.977
		PR10	1.000	1.000	1.000	0.993	0.999	1.000	0.976	0.991	0.999
		S15	0.903	0.943	0.994	0.908	0.948	0.988	0.903	0.947	0.992

Table 3.74: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 1 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.830	0.888	0.940	0.835	0.886	0.945	0.844	0.896	0.953
		PR10	0.831	0.892	0.953	0.840	0.909	0.952	0.836	0.894	0.952
		S15	0.892	0.942	0.991	0.892	0.943	0.987	0.904	0.948	0.993
	1000	WSNS	0.873	0.929	0.978	0.875	0.935	0.981	0.868	0.917	0.966
		PR10	0.866	0.925	0.978	0.877	0.935	0.982	0.871	0.933	0.971
		S15	0.893	0.946	0.993	0.897	0.955	0.984	0.892	0.948	0.988
	1500	WSNS	0.899	0.953	0.986	0.904	0.940	0.986	0.877	0.929	0.978
		PR10	0.885	0.939	0.991	0.890	0.946	0.986	0.880	0.932	0.974
		S15	0.906	0.950	0.988	0.914	0.949	0.987	0.900	0.946	0.989
0.4	500	WSNS	0.842	0.892	0.940	0.842	0.891	0.949	0.839	0.891	0.951
		PR10	0.923	0.954	0.988	0.913	0.959	0.987	0.855	0.909	0.962
		S15	0.894	0.934	0.988	0.894	0.950	0.985	0.883	0.936	0.992
	1000	WSNS	0.887	0.936	0.978	0.880	0.924	0.970	0.881	0.931	0.977
		PR10	0.951	0.978	0.998	0.941	0.973	0.995	0.902	0.952	0.991
		S15	0.898	0.947	0.991	0.893	0.947	0.988	0.911	0.958	0.991
	1500	WSNS	0.911	0.950	0.987	0.891	0.955	0.985	0.885	0.932	0.981
		PR10	0.963	0.988	1.000	0.954	0.980	0.998	0.901	0.952	0.990
		S15	0.902	0.953	0.992	0.906	0.964	0.991	0.901	0.956	0.996
0.8	500	WSNS	0.844	0.891	0.944	0.847	0.890	0.945	0.850	0.904	0.947
		PR10	0.993	0.997	1.000	0.988	0.997	1.000	0.954	0.974	0.993
		S15	0.887	0.936	0.990	0.909	0.957	0.991	0.900	0.948	0.991
	1000	WSNS	0.897	0.939	0.982	0.849	0.921	0.980	0.878	0.930	0.976
		PR10	0.997	1.000	1.000	0.997	0.999	1.000	0.984	0.994	0.999
		S15	0.903	0.949	0.990	0.908	0.963	0.994	0.896	0.950	0.990
	1500	WSNS	0.906	0.954	0.986	0.872	0.919	0.975	0.883	0.932	0.984
		PR10	1.000	1.000	1.000	0.999	1.000	1.000	0.984	0.997	1.000
		S15	0.901	0.955	0.994	0.910	0.957	0.995	0.897	0.948	0.993

Table 3.75: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.898	0.950	0.987	0.915	0.952	0.990	0.856	0.918	0.969
		PR10	0.872	0.924	0.984	0.883	0.927	0.976	0.890	0.945	0.987
		S15	0.887	0.941	0.988	0.894	0.949	0.985	0.910	0.957	0.989
	1000	WSNS	0.915	0.953	0.993	0.930	0.971	0.996	0.845	0.906	0.964
		PR10	0.891	0.945	0.992	0.898	0.945	0.990	0.887	0.932	0.982
		S15	0.906	0.945	0.991	0.912	0.956	0.989	0.886	0.928	0.986
	1500	WSNS	0.907	0.961	0.992	0.924	0.969	0.992	0.852	0.906	0.976
		PR10	0.880	0.938	0.990	0.885	0.932	0.979	0.896	0.940	0.987
		S15	0.886	0.954	0.987	0.903	0.958	0.992	0.892	0.941	0.987
0.4	500	WSNS	0.909	0.962	0.988	0.906	0.957	0.988	0.861	0.912	0.964
		PR10	0.936	0.974	0.994	0.915	0.951	0.988	0.900	0.951	0.987
		S15	0.931	0.968	0.994	0.926	0.962	0.989	0.909	0.958	0.993
	1000	WSNS	0.920	0.964	0.998	0.931	0.972	0.996	0.864	0.918	0.974
		PR10	0.952	0.986	1.000	0.940	0.976	0.993	0.909	0.961	0.992
		S15	0.939	0.973	0.999	0.938	0.969	0.995	0.916	0.960	0.992
	1500	WSNS	0.915	0.963	0.992	0.922	0.959	0.995	0.870	0.924	0.974
		PR10	0.945	0.981	0.996	0.916	0.961	0.992	0.915	0.958	0.993
		S15	0.933	0.973	0.993	0.922	0.963	0.996	0.923	0.958	0.993
0.8	500	WSNS	0.934	0.968	0.992	0.907	0.953	0.990	0.874	0.923	0.982
		PR10	0.987	0.995	0.999	0.973	0.991	0.999	0.954	0.982	0.999
		S15	0.981	0.994	0.999	0.958	0.977	0.997	0.961	0.982	1.000
	1000	WSNS	0.936	0.979	0.999	0.917	0.964	0.994	0.878	0.929	0.976
		PR10	0.997	1.000	1.000	0.982	0.996	1.000	0.968	0.989	1.000
		S15	0.987	0.999	1.000	0.979	0.991	0.998	0.948	0.986	1.000
	1500	WSNS	0.928	0.969	0.994	0.905	0.967	0.994	0.879	0.938	0.981
		PR10	0.995	0.999	1.000	0.983	0.993	0.999	0.957	0.986	0.999
		S15	0.980	0.989	1.000	0.969	0.990	1.000	0.946	0.977	0.997

Table 3.76: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.940	0.973	0.994	0.927	0.969	0.995	0.901	0.939	0.976
		PR10	0.869	0.925	0.980	0.871	0.930	0.979	0.857	0.918	0.977
		S15	0.889	0.947	0.992	0.890	0.950	0.986	0.906	0.953	0.986
	1000	WSNS	0.954	0.980	0.998	0.946	0.979	0.997	0.888	0.933	0.985
		PR10	0.876	0.939	0.987	0.875	0.948	0.985	0.838	0.915	0.985
		S15	0.909	0.945	0.987	0.894	0.947	0.994	0.887	0.943	0.993
	1500	WSNS	0.941	0.981	0.996	0.942	0.975	0.998	0.867	0.931	0.988
		PR10	0.874	0.929	0.983	0.859	0.929	0.977	0.850	0.918	0.983
		S15	0.892	0.949	0.987	0.893	0.950	0.986	0.896	0.956	0.994
0.4	500	WSNS	0.949	0.981	0.994	0.941	0.977	0.995	0.901	0.946	0.985
		PR10	0.937	0.970	0.994	0.925	0.966	0.991	0.875	0.932	0.986
		S15	0.934	0.976	0.996	0.928	0.968	0.991	0.918	0.966	0.996
	1000	WSNS	0.966	0.987	0.999	0.950	0.983	1.000	0.886	0.936	0.986
		PR10	0.944	0.986	0.999	0.928	0.972	0.998	0.870	0.937	0.990
		S15	0.946	0.972	1.000	0.935	0.974	0.997	0.913	0.959	0.994
	1500	WSNS	0.956	0.976	0.999	0.949	0.976	0.996	0.898	0.944	0.986
		PR10	0.938	0.976	0.996	0.916	0.958	0.994	0.880	0.941	0.989
		S15	0.933	0.969	0.993	0.933	0.967	0.994	0.928	0.960	0.994
0.8	500	WSNS	0.966	0.984	0.998	0.943	0.974	0.996	0.903	0.948	0.989
		PR10	0.987	0.993	1.000	0.980	0.992	0.999	0.957	0.981	0.998
		S15	0.984	0.994	0.998	0.985	0.994	0.999	0.959	0.987	0.999
	1000	WSNS	0.973	0.993	1.000	0.946	0.980	1.000	0.913	0.954	0.990
		PR10	0.996	1.000	1.000	0.984	0.997	1.000	0.949	0.978	0.996
		S15	0.989	0.998	1.000	0.971	0.993	1.000	0.967	0.986	0.998
	1500	WSNS	0.966	0.982	1.000	0.941	0.979	0.999	0.908	0.945	0.988
		PR10	0.990	1.000	1.000	0.981	0.993	1.000	0.960	0.984	0.998
		S15	0.975	0.989	1.000	0.973	0.988	0.998	0.961	0.991	0.999

Table 3.77: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.887	0.933	0.977	0.893	0.947	0.978	0.867	0.921	0.967
		PR10	0.871	0.923	0.981	0.877	0.927	0.973	0.883	0.936	0.983
		S15	0.887	0.941	0.988	0.894	0.949	0.985	0.910	0.957	0.989
	1000	WSNS	0.911	0.955	0.991	0.921	0.966	0.993	0.858	0.914	0.967
		PR10	0.892	0.940	0.995	0.899	0.944	0.992	0.891	0.931	0.983
		S15	0.906	0.945	0.991	0.912	0.956	0.989	0.886	0.928	0.986
	1500	WSNS	0.909	0.949	0.990	0.913	0.962	0.991	0.863	0.920	0.979
		PR10	0.879	0.935	0.991	0.887	0.937	0.981	0.893	0.939	0.989
		S15	0.886	0.954	0.987	0.903	0.958	0.992	0.892	0.941	0.987
0.4	500	WSNS	0.898	0.948	0.985	0.895	0.948	0.982	0.867	0.919	0.966
		PR10	0.933	0.970	0.991	0.910	0.953	0.982	0.899	0.946	0.984
		S15	0.931	0.968	0.994	0.926	0.962	0.989	0.909	0.958	0.993
	1000	WSNS	0.917	0.959	0.997	0.923	0.965	0.990	0.872	0.927	0.978
		PR10	0.947	0.987	1.000	0.939	0.979	0.994	0.911	0.960	0.991
		S15	0.939	0.973	0.999	0.938	0.969	0.995	0.916	0.960	0.992
	1500	WSNS	0.910	0.957	0.990	0.909	0.952	0.993	0.879	0.935	0.980
		PR10	0.945	0.980	0.996	0.919	0.961	0.994	0.913	0.957	0.993
		S15	0.933	0.973	0.993	0.922	0.963	0.996	0.923	0.958	0.993
0.8	500	WSNS	0.917	0.964	0.991	0.906	0.942	0.981	0.877	0.923	0.973
		PR10	0.986	0.996	0.999	0.973	0.988	0.999	0.952	0.981	0.999
		S15	0.981	0.994	0.999	0.958	0.977	0.997	0.961	0.982	1.000
	1000	WSNS	0.935	0.978	0.997	0.908	0.958	0.993	0.881	0.935	0.984
		PR10	0.996	1.000	1.000	0.984	0.993	1.000	0.965	0.987	1.000
		S15	0.987	0.999	1.000	0.979	0.991	0.998	0.948	0.986	1.000
	1500	WSNS	0.923	0.965	0.987	0.897	0.964	0.992	0.890	0.941	0.983
		PR10	0.995	0.999	1.000	0.983	0.994	0.999	0.959	0.986	0.999
		S15	0.980	0.989	1.000	0.969	0.990	1.000	0.946	0.977	0.997

Table 3.78: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.917	0.958	0.988	0.912	0.946	0.985	0.895	0.934	0.978
		PR10	0.867	0.923	0.975	0.862	0.923	0.977	0.869	0.918	0.971
		S15	0.889	0.947	0.992	0.890	0.950	0.986	0.906	0.953	0.986
	1000	WSNS	0.942	0.975	0.995	0.929	0.972	0.997	0.892	0.942	0.986
		PR10	0.881	0.938	0.990	0.878	0.951	0.986	0.858	0.921	0.985
		S15	0.909	0.945	0.987	0.894	0.947	0.994	0.887	0.943	0.993
	1500	WSNS	0.928	0.971	0.996	0.923	0.960	0.994	0.875	0.946	0.986
		PR10	0.879	0.930	0.985	0.866	0.933	0.983	0.867	0.927	0.986
		S15	0.892	0.949	0.987	0.893	0.950	0.986	0.896	0.956	0.994
0.4	500	WSNS	0.929	0.964	0.989	0.926	0.961	0.990	0.906	0.946	0.980
		PR10	0.936	0.966	0.989	0.923	0.962	0.989	0.880	0.938	0.985
		S15	0.934	0.976	0.996	0.928	0.968	0.991	0.918	0.966	0.996
	1000	WSNS	0.952	0.979	0.999	0.933	0.975	0.998	0.893	0.950	0.987
		PR10	0.943	0.986	0.999	0.932	0.975	0.997	0.877	0.948	0.989
		S15	0.946	0.972	1.000	0.935	0.974	0.997	0.913	0.959	0.994
	1500	WSNS	0.945	0.966	0.998	0.936	0.968	0.996	0.901	0.945	0.986
		PR10	0.940	0.976	0.996	0.922	0.960	0.994	0.897	0.949	0.992
		S15	0.933	0.969	0.993	0.933	0.967	0.994	0.928	0.960	0.994
0.8	500	WSNS	0.955	0.982	0.994	0.930	0.967	0.990	0.901	0.944	0.982
		PR10	0.985	0.993	1.000	0.976	0.991	0.999	0.959	0.983	0.998
		S15	0.984	0.994	0.998	0.985	0.994	0.999	0.959	0.987	0.999
	1000	WSNS	0.965	0.989	0.999	0.933	0.977	0.999	0.914	0.954	0.990
		PR10	0.995	1.000	1.000	0.986	0.998	1.000	0.957	0.983	0.996
		S15	0.989	0.998	1.000	0.971	0.993	1.000	0.967	0.986	0.998
	1500	WSNS	0.957	0.980	0.998	0.933	0.970	0.997	0.913	0.951	0.990
		PR10	0.990	1.000	1.000	0.982	0.994	1.000	0.966	0.986	0.999
		S15	0.975	0.989	1.000	0.973	0.988	0.998	0.961	0.991	0.999

Table 3.79: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.868	0.924	0.969	0.874	0.923	0.971	0.851	0.905	0.961
		PR10	0.863	0.917	0.974	0.875	0.926	0.969	0.883	0.933	0.975
		S15	0.887	0.941	0.988	0.894	0.949	0.985	0.910	0.957	0.989
	1000	WSNS	0.884	0.945	0.983	0.901	0.952	0.990	0.849	0.912	0.966
		PR10	0.891	0.938	0.991	0.900	0.946	0.990	0.893	0.926	0.981
		S15	0.906	0.945	0.991	0.912	0.956	0.989	0.886	0.928	0.986
	1500	WSNS	0.893	0.943	0.986	0.896	0.953	0.988	0.863	0.920	0.977
		PR10	0.878	0.937	0.991	0.885	0.935	0.980	0.888	0.934	0.987
		S15	0.886	0.954	0.987	0.903	0.958	0.992	0.892	0.941	0.987
0.4	500	WSNS	0.877	0.931	0.972	0.874	0.929	0.974	0.853	0.908	0.960
		PR10	0.932	0.960	0.987	0.904	0.950	0.981	0.890	0.940	0.984
		S15	0.931	0.968	0.994	0.926	0.962	0.989	0.909	0.958	0.993
	1000	WSNS	0.898	0.948	0.990	0.911	0.952	0.986	0.870	0.920	0.978
		PR10	0.945	0.984	1.000	0.935	0.979	0.994	0.908	0.956	0.987
		S15	0.939	0.973	0.999	0.938	0.969	0.995	0.916	0.960	0.992
	1500	WSNS	0.899	0.950	0.984	0.890	0.945	0.990	0.874	0.933	0.977
		PR10	0.945	0.979	0.995	0.916	0.960	0.994	0.912	0.954	0.989
		S15	0.933	0.973	0.993	0.922	0.963	0.996	0.923	0.958	0.993
0.8	500	WSNS	0.905	0.953	0.986	0.887	0.925	0.975	0.864	0.918	0.968
		PR10	0.986	0.993	0.998	0.967	0.983	0.998	0.952	0.976	0.998
		S15	0.981	0.994	0.999	0.958	0.977	0.997	0.961	0.982	1.000
	1000	WSNS	0.916	0.967	0.995	0.892	0.945	0.989	0.877	0.929	0.980
		PR10	0.997	1.000	1.000	0.982	0.995	1.000	0.963	0.989	1.000
		S15	0.987	0.999	1.000	0.979	0.991	0.998	0.948	0.986	1.000
	1500	WSNS	0.913	0.956	0.984	0.885	0.948	0.991	0.886	0.937	0.981
		PR10	0.994	0.999	1.000	0.982	0.994	0.999	0.955	0.983	0.999
		S15	0.980	0.989	1.000	0.969	0.990	1.000	0.946	0.977	0.997

Table 3.80: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.896	0.941	0.978	0.889	0.934	0.975	0.880	0.930	0.968
		PR10	0.862	0.919	0.973	0.860	0.919	0.971	0.865	0.923	0.971
		S15	0.889	0.947	0.992	0.890	0.950	0.986	0.906	0.953	0.986
	1000	WSNS	0.911	0.962	0.989	0.912	0.954	0.991	0.889	0.939	0.981
		PR10	0.885	0.936	0.988	0.879	0.950	0.986	0.865	0.923	0.983
		S15	0.909	0.945	0.987	0.894	0.947	0.994	0.887	0.943	0.993
	1500	WSNS	0.918	0.955	0.991	0.903	0.952	0.991	0.871	0.937	0.981
		PR10	0.880	0.932	0.983	0.862	0.933	0.977	0.875	0.933	0.983
		S15	0.892	0.949	0.987	0.893	0.950	0.986	0.896	0.956	0.994
0.4	500	WSNS	0.904	0.949	0.983	0.904	0.946	0.982	0.895	0.930	0.972
		PR10	0.932	0.962	0.987	0.921	0.955	0.988	0.878	0.934	0.986
		S15	0.934	0.976	0.996	0.928	0.968	0.991	0.918	0.966	0.996
	1000	WSNS	0.924	0.967	0.992	0.914	0.963	0.994	0.894	0.938	0.982
		PR10	0.942	0.984	0.999	0.934	0.975	0.997	0.881	0.950	0.988
		S15	0.946	0.972	1.000	0.935	0.974	0.997	0.913	0.959	0.994
	1500	WSNS	0.930	0.960	0.993	0.921	0.960	0.992	0.898	0.938	0.985
		PR10	0.937	0.974	0.995	0.921	0.960	0.995	0.906	0.955	0.991
		S15	0.933	0.969	0.993	0.933	0.967	0.994	0.928	0.960	0.994
0.8	500	WSNS	0.933	0.969	0.991	0.927	0.959	0.986	0.888	0.937	0.979
		PR10	0.983	0.992	0.999	0.979	0.990	0.999	0.956	0.983	0.995
		S15	0.984	0.994	0.998	0.985	0.994	0.999	0.959	0.987	0.999
	1000	WSNS	0.942	0.984	0.998	0.926	0.968	0.997	0.903	0.949	0.989
		PR10	0.995	1.000	1.000	0.983	0.997	1.000	0.955	0.984	0.997
		S15	0.989	0.998	1.000	0.971	0.993	1.000	0.967	0.986	0.998
	1500	WSNS	0.942	0.968	0.995	0.920	0.959	0.993	0.901	0.946	0.986
		PR10	0.990	1.000	1.000	0.981	0.996	1.000	0.969	0.984	0.999
		S15	0.975	0.989	1.000	0.973	0.988	0.998	0.961	0.991	0.999

Table 3.81: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.860	0.913	0.964	0.866	0.919	0.966	0.846	0.902	0.952
		PR10	0.858	0.910	0.963	0.859	0.920	0.962	0.869	0.927	0.971
		S15	0.887	0.941	0.988	0.894	0.949	0.985	0.910	0.957	0.989
	1000	WSNS	0.881	0.943	0.982	0.895	0.946	0.986	0.853	0.912	0.964
		PR10	0.886	0.938	0.989	0.889	0.945	0.992	0.886	0.921	0.983
		S15	0.906	0.945	0.991	0.912	0.956	0.989	0.886	0.928	0.986
	1500	WSNS	0.885	0.943	0.984	0.889	0.950	0.987	0.867	0.920	0.979
		PR10	0.879	0.936	0.988	0.885	0.935	0.977	0.890	0.936	0.984
		S15	0.886	0.954	0.987	0.903	0.958	0.992	0.892	0.941	0.987
0.4	500	WSNS	0.864	0.932	0.969	0.866	0.924	0.966	0.853	0.904	0.958
		PR10	0.924	0.955	0.984	0.902	0.945	0.976	0.885	0.935	0.981
		S15	0.931	0.968	0.994	0.926	0.962	0.989	0.909	0.958	0.993
	1000	WSNS	0.891	0.950	0.989	0.904	0.955	0.987	0.871	0.922	0.972
		PR10	0.944	0.988	1.000	0.935	0.978	0.994	0.905	0.953	0.987
		S15	0.939	0.973	0.999	0.938	0.969	0.995	0.916	0.960	0.992
	1500	WSNS	0.897	0.948	0.980	0.886	0.948	0.990	0.868	0.932	0.977
		PR10	0.942	0.978	0.996	0.918	0.960	0.993	0.912	0.957	0.990
		S15	0.933	0.973	0.993	0.922	0.963	0.996	0.923	0.958	0.993
0.8	500	WSNS	0.896	0.945	0.982	0.881	0.928	0.969	0.868	0.920	0.966
		PR10	0.982	0.992	0.998	0.961	0.982	0.995	0.946	0.972	0.996
		S15	0.981	0.994	0.999	0.958	0.977	0.997	0.961	0.982	1.000
	1000	WSNS	0.911	0.968	0.994	0.896	0.947	0.988	0.879	0.931	0.975
		PR10	0.996	1.000	1.000	0.983	0.994	1.000	0.962	0.986	1.000
		S15	0.987	0.999	1.000	0.979	0.991	0.998	0.948	0.986	1.000
	1500	WSNS	0.910	0.954	0.984	0.883	0.942	0.992	0.888	0.938	0.984
		PR10	0.993	0.999	1.000	0.979	0.995	0.999	0.954	0.983	0.998
		S15	0.980	0.989	1.000	0.969	0.990	1.000	0.946	0.977	0.997

Table 3.82: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.871	0.936	0.971	0.880	0.918	0.970	0.868	0.919	0.963
		PR10	0.853	0.911	0.967	0.863	0.915	0.965	0.865	0.920	0.968
		S15	0.889	0.947	0.992	0.890	0.950	0.986	0.906	0.953	0.986
	1000	WSNS	0.897	0.964	0.985	0.904	0.947	0.989	0.879	0.929	0.977
		PR10	0.877	0.936	0.986	0.878	0.951	0.984	0.868	0.919	0.981
		S15	0.909	0.945	0.987	0.894	0.947	0.994	0.887	0.943	0.993
	1500	WSNS	0.912	0.953	0.989	0.896	0.945	0.990	0.871	0.933	0.984
		PR10	0.881	0.931	0.984	0.864	0.934	0.979	0.877	0.934	0.982
		S15	0.892	0.949	0.987	0.893	0.950	0.986	0.896	0.956	0.994
0.4	500	WSNS	0.892	0.942	0.972	0.887	0.933	0.973	0.885	0.929	0.971
		PR10	0.927	0.958	0.984	0.920	0.954	0.982	0.878	0.930	0.977
		S15	0.934	0.976	0.996	0.928	0.968	0.991	0.918	0.966	0.996
	1000	WSNS	0.916	0.967	0.992	0.905	0.958	0.993	0.891	0.940	0.983
		PR10	0.941	0.985	0.999	0.933	0.973	0.997	0.886	0.950	0.986
		S15	0.946	0.972	1.000	0.935	0.974	0.997	0.913	0.959	0.994
	1500	WSNS	0.924	0.958	0.988	0.913	0.958	0.992	0.895	0.943	0.983
		PR10	0.937	0.975	0.995	0.921	0.958	0.994	0.911	0.955	0.992
		S15	0.933	0.969	0.993	0.933	0.967	0.994	0.928	0.960	0.994
0.8	500	WSNS	0.924	0.960	0.987	0.918	0.951	0.985	0.884	0.929	0.976
		PR10	0.980	0.990	0.999	0.975	0.989	0.997	0.957	0.980	0.994
		S15	0.984	0.994	0.998	0.985	0.994	0.999	0.959	0.987	0.999
	1000	WSNS	0.936	0.980	0.996	0.917	0.962	0.995	0.901	0.944	0.984
		PR10	0.997	1.000	1.000	0.984	0.997	1.000	0.954	0.986	0.997
		S15	0.989	0.998	1.000	0.971	0.993	1.000	0.967	0.986	0.998
	1500	WSNS	0.934	0.967	0.991	0.917	0.956	0.991	0.906	0.947	0.987
		PR10	0.991	1.000	1.000	0.981	0.996	1.000	0.970	0.985	0.998
		S15	0.975	0.989	1.000	0.973	0.988	0.998	0.961	0.991	0.999

Table 3.83: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.839	0.902	0.951	0.853	0.900	0.964	0.835	0.897	0.939
		PR10	0.851	0.900	0.964	0.868	0.918	0.965	0.860	0.922	0.971
		S15	0.887	0.941	0.988	0.894	0.949	0.985	0.910	0.957	0.989
	1000	WSNS	0.872	0.931	0.978	0.891	0.939	0.981	0.843	0.907	0.959
		PR10	0.882	0.937	0.986	0.889	0.942	0.991	0.884	0.919	0.978
		S15	0.906	0.945	0.991	0.912	0.956	0.989	0.886	0.928	0.986
	1500	WSNS	0.879	0.934	0.983	0.883	0.941	0.981	0.862	0.914	0.974
		PR10	0.876	0.934	0.990	0.884	0.936	0.978	0.884	0.933	0.983
		S15	0.886	0.954	0.987	0.903	0.958	0.992	0.892	0.941	0.987
0.4	500	WSNS	0.853	0.914	0.962	0.849	0.908	0.958	0.840	0.891	0.952
		PR10	0.921	0.954	0.983	0.903	0.936	0.979	0.876	0.919	0.980
		S15	0.931	0.968	0.994	0.926	0.962	0.989	0.909	0.958	0.993
	1000	WSNS	0.889	0.942	0.985	0.897	0.939	0.982	0.859	0.914	0.967
		PR10	0.942	0.980	0.999	0.934	0.979	0.994	0.900	0.955	0.986
		S15	0.939	0.973	0.999	0.938	0.969	0.995	0.916	0.960	0.992
	1500	WSNS	0.884	0.944	0.979	0.874	0.942	0.984	0.863	0.932	0.972
		PR10	0.940	0.979	0.995	0.915	0.960	0.994	0.910	0.954	0.989
		S15	0.933	0.973	0.993	0.922	0.963	0.996	0.923	0.958	0.993
0.8	500	WSNS	0.887	0.941	0.981	0.868	0.917	0.970	0.853	0.905	0.959
		PR10	0.979	0.991	0.999	0.962	0.982	0.996	0.937	0.968	0.994
		S15	0.981	0.994	0.999	0.958	0.977	0.997	0.961	0.982	1.000
	1000	WSNS	0.900	0.957	0.994	0.889	0.943	0.985	0.874	0.927	0.972
		PR10	0.996	1.000	1.000	0.983	0.994	0.999	0.956	0.986	1.000
		S15	0.987	0.999	1.000	0.979	0.991	0.998	0.948	0.986	1.000
	1500	WSNS	0.892	0.944	0.981	0.885	0.939	0.990	0.885	0.927	0.981
		PR10	0.994	0.999	1.000	0.982	0.994	0.998	0.954	0.981	1.000
		S15	0.980	0.989	1.000	0.969	0.990	1.000	0.946	0.977	0.997

Table 3.84: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 2 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.853	0.916	0.957	0.861	0.902	0.957	0.859	0.904	0.955
		PR10	0.848	0.898	0.965	0.853	0.911	0.960	0.862	0.920	0.957
		S15	0.889	0.947	0.992	0.890	0.950	0.986	0.906	0.953	0.986
	1000	WSNS	0.896	0.954	0.983	0.893	0.941	0.980	0.875	0.917	0.974
		PR10	0.882	0.936	0.985	0.876	0.946	0.984	0.865	0.916	0.979
		S15	0.909	0.945	0.987	0.894	0.947	0.994	0.887	0.943	0.993
	1500	WSNS	0.899	0.947	0.989	0.885	0.940	0.987	0.871	0.928	0.984
		PR10	0.879	0.933	0.982	0.864	0.930	0.979	0.879	0.937	0.984
		S15	0.892	0.949	0.987	0.893	0.950	0.986	0.896	0.956	0.994
0.4	500	WSNS	0.869	0.930	0.966	0.871	0.916	0.971	0.872	0.919	0.957
		PR10	0.919	0.954	0.984	0.916	0.953	0.977	0.880	0.922	0.971
		S15	0.934	0.976	0.996	0.928	0.968	0.991	0.918	0.966	0.996
	1000	WSNS	0.900	0.953	0.991	0.900	0.949	0.989	0.874	0.938	0.979
		PR10	0.941	0.984	0.998	0.927	0.973	0.998	0.881	0.952	0.986
		S15	0.946	0.972	1.000	0.935	0.974	0.997	0.913	0.959	0.994
	1500	WSNS	0.911	0.954	0.989	0.902	0.952	0.989	0.894	0.938	0.982
		PR10	0.934	0.974	0.996	0.921	0.957	0.994	0.912	0.955	0.992
		S15	0.933	0.969	0.993	0.933	0.967	0.994	0.928	0.960	0.994
0.8	500	WSNS	0.914	0.948	0.983	0.899	0.944	0.979	0.876	0.917	0.972
		PR10	0.977	0.988	0.999	0.972	0.986	0.997	0.945	0.981	0.994
		S15	0.984	0.994	0.998	0.985	0.994	0.999	0.959	0.987	0.999
	1000	WSNS	0.925	0.970	0.996	0.906	0.960	0.995	0.885	0.941	0.983
		PR10	0.997	1.000	1.000	0.984	0.997	1.000	0.954	0.986	0.996
		S15	0.989	0.998	1.000	0.971	0.993	1.000	0.967	0.986	0.998
	1500	WSNS	0.922	0.965	0.986	0.904	0.949	0.989	0.899	0.941	0.985
		PR10	0.990	1.000	1.000	0.981	0.996	1.000	0.967	0.985	0.998
		S15	0.975	0.989	1.000	0.973	0.988	0.998	0.961	0.991	0.999

Table 3.85: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.883	0.946	0.991	0.896	0.944	0.995	0.846	0.902	0.968
		PR10	0.882	0.942	0.983	0.864	0.925	0.982	0.887	0.941	0.986
		S15	0.898	0.954	0.988	0.897	0.950	0.990	0.899	0.947	0.986
	1000	WSNS	0.911	0.962	0.993	0.922	0.964	0.993	0.854	0.914	0.971
		PR10	0.908	0.953	0.990	0.882	0.933	0.987	0.891	0.939	0.988
		S15	0.908	0.952	0.995	0.909	0.956	0.989	0.897	0.949	0.989
	1500	WSNS	0.918	0.961	0.993	0.918	0.967	0.996	0.862	0.928	0.982
		PR10	0.901	0.952	0.985	0.882	0.938	0.978	0.903	0.948	0.991
		S15	0.905	0.954	0.989	0.902	0.957	0.990	0.906	0.947	0.987
0.4	500	WSNS	0.886	0.950	0.989	0.887	0.944	0.988	0.845	0.909	0.966
		PR10	0.947	0.975	0.995	0.914	0.958	0.988	0.909	0.955	0.988
		S15	0.934	0.967	0.996	0.916	0.965	0.996	0.903	0.945	0.993
	1000	WSNS	0.913	0.958	0.986	0.916	0.962	0.992	0.854	0.918	0.974
		PR10	0.959	0.985	0.998	0.919	0.962	0.994	0.906	0.957	0.986
		S15	0.939	0.970	0.997	0.930	0.967	0.993	0.901	0.945	0.984
	1500	WSNS	0.911	0.961	0.992	0.912	0.958	0.992	0.868	0.928	0.982
		PR10	0.951	0.978	0.995	0.922	0.969	0.988	0.924	0.961	0.992
		S15	0.944	0.974	0.996	0.937	0.967	0.994	0.922	0.965	0.992
0.8	500	WSNS	0.896	0.949	0.987	0.887	0.945	0.991	0.847	0.904	0.963
		PR10	0.989	0.997	1.000	0.977	0.994	0.999	0.953	0.988	0.996
		S15	0.980	0.995	0.999	0.963	0.984	0.998	0.947	0.981	0.999
	1000	WSNS	0.899	0.942	0.991	0.887	0.949	0.991	0.850	0.908	0.973
		PR10	0.994	1.000	1.000	0.985	0.999	1.000	0.963	0.986	0.997
		S15	0.980	0.991	0.999	0.969	0.989	1.000	0.940	0.967	0.997
	1500	WSNS	0.893	0.954	0.989	0.901	0.953	0.989	0.855	0.923	0.975
		PR10	0.988	0.998	1.000	0.984	0.995	0.999	0.957	0.983	1.000
		S15	0.986	0.995	0.999	0.968	0.992	1.000	0.948	0.978	0.997

Table 3.86: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 50$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.942	0.983	0.997	0.946	0.979	0.997	0.882	0.924	0.975
		PR10	0.866	0.928	0.983	0.863	0.928	0.978	0.836	0.909	0.975
		S15	0.898	0.948	0.988	0.885	0.949	0.991	0.892	0.948	0.993
	1000	WSNS	0.951	0.980	0.999	0.949	0.984	0.998	0.889	0.941	0.984
		PR10	0.897	0.947	0.989	0.889	0.936	0.982	0.842	0.905	0.976
		S15	0.912	0.957	0.990	0.907	0.955	0.986	0.895	0.952	0.993
	1500	WSNS	0.956	0.985	0.998	0.946	0.982	0.996	0.890	0.946	0.986
		PR10	0.891	0.946	0.984	0.880	0.933	0.981	0.855	0.924	0.986
		S15	0.906	0.955	0.992	0.905	0.959	0.989	0.899	0.947	0.987
0.4	500	WSNS	0.949	0.984	0.998	0.940	0.970	0.995	0.886	0.933	0.977
		PR10	0.935	0.972	0.991	0.928	0.969	0.988	0.871	0.927	0.982
		S15	0.927	0.964	0.992	0.920	0.971	0.993	0.913	0.957	0.991
	1000	WSNS	0.947	0.975	0.996	0.955	0.979	0.994	0.893	0.939	0.984
		PR10	0.949	0.984	0.997	0.942	0.976	0.995	0.861	0.918	0.983
		S15	0.939	0.965	0.994	0.935	0.971	0.995	0.900	0.948	0.992
	1500	WSNS	0.960	0.983	0.996	0.941	0.978	0.998	0.903	0.948	0.993
		PR10	0.945	0.978	0.995	0.935	0.964	0.993	0.877	0.942	0.993
		S15	0.945	0.973	0.995	0.933	0.974	0.996	0.919	0.963	0.994
0.8	500	WSNS	0.949	0.983	0.994	0.917	0.962	0.995	0.873	0.926	0.980
		PR10	0.988	0.996	1.000	0.983	0.995	1.000	0.939	0.976	0.994
		S15	0.981	0.994	0.999	0.972	0.990	1.000	0.953	0.985	0.996
	1000	WSNS	0.940	0.976	0.999	0.930	0.971	0.998	0.882	0.937	0.983
		PR10	0.993	1.000	1.000	0.996	0.999	1.000	0.934	0.980	0.999
		S15	0.981	0.993	0.999	0.974	0.995	0.999	0.946	0.979	0.998
	1500	WSNS	0.948	0.981	0.996	0.919	0.972	0.995	0.899	0.946	0.990
		PR10	0.989	0.998	1.000	0.983	0.996	1.000	0.956	0.990	1.000
		S15	0.983	0.996	1.000	0.974	0.989	0.999	0.966	0.992	1.000

Table 3.87: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.870	0.929	0.980	0.882	0.929	0.990	0.852	0.909	0.964
		PR10	0.870	0.934	0.982	0.874	0.928	0.983	0.884	0.925	0.981
		S15	0.898	0.954	0.988	0.897	0.950	0.990	0.899	0.947	0.986
	1000	WSNS	0.906	0.950	0.987	0.912	0.954	0.989	0.858	0.910	0.968
		PR10	0.906	0.952	0.989	0.886	0.939	0.986	0.890	0.937	0.986
		S15	0.908	0.952	0.995	0.909	0.956	0.989	0.897	0.949	0.989
	1500	WSNS	0.913	0.952	0.992	0.903	0.965	0.997	0.874	0.931	0.981
		PR10	0.899	0.951	0.984	0.892	0.943	0.981	0.904	0.946	0.990
		S15	0.905	0.954	0.989	0.902	0.957	0.990	0.906	0.947	0.987
0.4	500	WSNS	0.874	0.937	0.981	0.873	0.935	0.986	0.839	0.906	0.964
		PR10	0.940	0.977	0.994	0.916	0.959	0.990	0.905	0.952	0.985
		S15	0.934	0.967	0.996	0.916	0.965	0.996	0.903	0.945	0.993
	1000	WSNS	0.903	0.949	0.983	0.904	0.953	0.986	0.858	0.915	0.975
		PR10	0.959	0.984	0.998	0.926	0.968	0.993	0.908	0.954	0.985
		S15	0.939	0.970	0.997	0.930	0.967	0.993	0.901	0.945	0.984
	1500	WSNS	0.906	0.956	0.988	0.903	0.949	0.991	0.874	0.937	0.980
		PR10	0.952	0.978	0.995	0.926	0.970	0.990	0.923	0.961	0.992
		S15	0.944	0.974	0.996	0.937	0.967	0.994	0.922	0.965	0.992
0.8	500	WSNS	0.893	0.948	0.988	0.872	0.935	0.986	0.849	0.901	0.960
		PR10	0.988	0.996	1.000	0.976	0.994	0.999	0.955	0.981	0.996
		S15	0.980	0.995	0.999	0.963	0.984	0.998	0.947	0.981	0.999
	1000	WSNS	0.901	0.941	0.991	0.890	0.945	0.993	0.861	0.914	0.973
		PR10	0.993	1.000	1.000	0.987	1.000	1.000	0.963	0.985	0.998
		S15	0.980	0.991	0.999	0.969	0.989	1.000	0.940	0.967	0.997
	1500	WSNS	0.897	0.951	0.989	0.903	0.949	0.985	0.860	0.923	0.978
		PR10	0.989	0.998	1.000	0.986	0.994	1.000	0.957	0.981	0.998
		S15	0.986	0.995	0.999	0.968	0.992	1.000	0.948	0.978	0.997

Table 3.88: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 75$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.916	0.964	0.996	0.921	0.970	0.991	0.874	0.924	0.971
		PR10	0.868	0.929	0.981	0.866	0.927	0.982	0.851	0.908	0.977
		S15	0.898	0.948	0.988	0.885	0.949	0.991	0.892	0.948	0.993
	1000	WSNS	0.937	0.969	0.997	0.925	0.969	0.996	0.894	0.941	0.984
		PR10	0.897	0.951	0.987	0.892	0.938	0.985	0.855	0.921	0.978
		S15	0.912	0.957	0.990	0.907	0.955	0.986	0.895	0.952	0.993
	1500	WSNS	0.942	0.974	0.997	0.935	0.975	0.995	0.897	0.945	0.985
		PR10	0.899	0.944	0.983	0.886	0.937	0.983	0.880	0.931	0.989
		S15	0.906	0.955	0.992	0.905	0.959	0.989	0.899	0.947	0.987
0.4	500	WSNS	0.929	0.968	0.993	0.910	0.958	0.991	0.878	0.929	0.971
		PR10	0.934	0.970	0.990	0.924	0.969	0.990	0.885	0.933	0.980
		S15	0.927	0.964	0.992	0.920	0.971	0.993	0.913	0.957	0.991
	1000	WSNS	0.932	0.965	0.994	0.937	0.972	0.995	0.889	0.944	0.982
		PR10	0.950	0.984	0.996	0.950	0.975	0.995	0.880	0.923	0.985
		S15	0.939	0.965	0.994	0.935	0.971	0.995	0.900	0.948	0.992
	1500	WSNS	0.949	0.979	0.995	0.928	0.972	0.997	0.905	0.950	0.989
		PR10	0.950	0.977	0.995	0.938	0.968	0.996	0.898	0.952	0.993
		S15	0.945	0.973	0.995	0.933	0.974	0.996	0.919	0.963	0.994
0.8	500	WSNS	0.938	0.976	0.993	0.907	0.964	0.990	0.872	0.921	0.979
		PR10	0.987	0.996	1.000	0.983	0.994	1.000	0.951	0.979	0.994
		S15	0.981	0.994	0.999	0.972	0.990	1.000	0.953	0.985	0.996
	1000	WSNS	0.930	0.966	0.998	0.921	0.961	0.998	0.886	0.939	0.987
		PR10	0.993	0.999	1.000	0.995	0.999	1.000	0.945	0.982	1.000
		S15	0.981	0.993	0.999	0.974	0.995	0.999	0.946	0.979	0.998
	1500	WSNS	0.942	0.974	0.996	0.912	0.965	0.995	0.904	0.952	0.989
		PR10	0.990	0.997	1.000	0.984	0.996	1.000	0.971	0.993	1.000
		S15	0.983	0.996	1.000	0.974	0.989	0.999	0.966	0.992	1.000

Table 3.89: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.856	0.918	0.976	0.870	0.918	0.979	0.843	0.894	0.958
		PR10	0.862	0.929	0.979	0.871	0.924	0.979	0.872	0.923	0.978
		S15	0.898	0.954	0.988	0.897	0.950	0.990	0.899	0.947	0.986
	1000	WSNS	0.898	0.945	0.988	0.904	0.948	0.982	0.858	0.909	0.966
		PR10	0.900	0.952	0.989	0.888	0.941	0.985	0.887	0.931	0.985
		S15	0.908	0.952	0.995	0.909	0.956	0.989	0.897	0.949	0.989
	1500	WSNS	0.903	0.951	0.989	0.903	0.955	0.994	0.879	0.928	0.981
		PR10	0.899	0.951	0.980	0.896	0.943	0.982	0.902	0.946	0.987
		S15	0.905	0.954	0.989	0.902	0.957	0.990	0.906	0.947	0.987
0.4	500	WSNS	0.864	0.921	0.972	0.862	0.918	0.975	0.834	0.895	0.956
		PR10	0.937	0.975	0.990	0.915	0.955	0.984	0.898	0.951	0.980
		S15	0.934	0.967	0.996	0.916	0.965	0.996	0.903	0.945	0.993
	1000	WSNS	0.902	0.945	0.980	0.904	0.943	0.982	0.861	0.918	0.972
		PR10	0.957	0.983	0.996	0.922	0.964	0.993	0.905	0.954	0.982
		S15	0.939	0.970	0.997	0.930	0.967	0.993	0.901	0.945	0.984
	1500	WSNS	0.900	0.947	0.987	0.896	0.946	0.989	0.875	0.938	0.984
		PR10	0.948	0.976	0.995	0.922	0.969	0.990	0.923	0.959	0.989
		S15	0.944	0.974	0.996	0.937	0.967	0.994	0.922	0.965	0.992
0.8	500	WSNS	0.893	0.945	0.986	0.872	0.933	0.984	0.851	0.901	0.954
		PR10	0.986	0.993	0.999	0.976	0.993	0.999	0.953	0.974	0.994
		S15	0.980	0.995	0.999	0.963	0.984	0.998	0.947	0.981	0.999
	1000	WSNS	0.897	0.944	0.987	0.890	0.943	0.984	0.866	0.915	0.971
		PR10	0.994	0.998	1.000	0.985	0.999	1.000	0.961	0.982	0.998
		S15	0.980	0.991	0.999	0.969	0.989	1.000	0.940	0.967	0.997
	1500	WSNS	0.893	0.950	0.991	0.902	0.944	0.984	0.864	0.924	0.979
		PR10	0.988	0.996	1.000	0.987	0.993	1.000	0.959	0.979	0.999
		S15	0.986	0.995	0.999	0.968	0.992	1.000	0.948	0.978	0.997

Table 3.90: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 100$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.894	0.943	0.988	0.894	0.957	0.986	0.861	0.921	0.966
		PR10	0.861	0.925	0.978	0.857	0.933	0.983	0.854	0.904	0.976
		S15	0.898	0.948	0.988	0.885	0.949	0.991	0.892	0.948	0.993
	1000	WSNS	0.923	0.959	0.993	0.916	0.960	0.990	0.895	0.944	0.980
		PR10	0.900	0.951	0.986	0.888	0.941	0.984	0.864	0.922	0.975
		S15	0.912	0.957	0.990	0.907	0.955	0.986	0.895	0.952	0.993
	1500	WSNS	0.928	0.963	0.993	0.918	0.964	0.992	0.886	0.943	0.985
		PR10	0.894	0.945	0.983	0.887	0.936	0.983	0.886	0.936	0.986
		S15	0.906	0.955	0.992	0.905	0.959	0.989	0.899	0.947	0.987
0.4	500	WSNS	0.898	0.953	0.991	0.901	0.942	0.987	0.877	0.917	0.963
		PR10	0.932	0.969	0.989	0.923	0.971	0.989	0.885	0.928	0.979
		S15	0.927	0.964	0.992	0.920	0.971	0.993	0.913	0.957	0.991
	1000	WSNS	0.924	0.960	0.993	0.928	0.965	0.988	0.887	0.941	0.980
		PR10	0.949	0.982	0.994	0.948	0.976	0.993	0.884	0.929	0.979
		S15	0.939	0.965	0.994	0.935	0.971	0.995	0.900	0.948	0.992
	1500	WSNS	0.935	0.974	0.992	0.919	0.960	0.996	0.905	0.945	0.989
		PR10	0.950	0.977	0.995	0.938	0.969	0.994	0.901	0.960	0.991
		S15	0.945	0.973	0.995	0.933	0.974	0.996	0.919	0.963	0.994
0.8	500	WSNS	0.930	0.969	0.987	0.889	0.949	0.991	0.877	0.925	0.975
		PR10	0.986	0.994	0.999	0.984	0.994	0.999	0.954	0.979	0.992
		S15	0.981	0.994	0.999	0.972	0.990	1.000	0.953	0.985	0.996
	1000	WSNS	0.928	0.965	0.997	0.911	0.955	0.994	0.881	0.943	0.983
		PR10	0.992	0.999	1.000	0.997	0.999	1.000	0.950	0.982	0.998
		S15	0.981	0.993	0.999	0.974	0.995	0.999	0.946	0.979	0.998
	1500	WSNS	0.933	0.965	0.996	0.895	0.953	0.993	0.907	0.953	0.988
		PR10	0.990	0.997	1.000	0.985	0.996	1.000	0.972	0.995	1.000
		S15	0.983	0.996	1.000	0.974	0.989	0.999	0.966	0.992	1.000

Table 3.91: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.842	0.903	0.964	0.862	0.914	0.964	0.838	0.892	0.947
		PR10	0.858	0.925	0.976	0.867	0.920	0.977	0.867	0.915	0.970
		S15	0.898	0.954	0.988	0.897	0.950	0.990	0.899	0.947	0.986
	1000	WSNS	0.893	0.939	0.987	0.896	0.939	0.983	0.860	0.906	0.965
		PR10	0.901	0.953	0.987	0.891	0.937	0.988	0.881	0.928	0.980
		S15	0.908	0.952	0.995	0.909	0.956	0.989	0.897	0.949	0.989
	1500	WSNS	0.900	0.942	0.987	0.889	0.948	0.990	0.883	0.922	0.982
		PR10	0.898	0.947	0.980	0.895	0.943	0.983	0.902	0.948	0.986
		S15	0.905	0.954	0.989	0.902	0.957	0.990	0.906	0.947	0.987
0.4	500	WSNS	0.851	0.916	0.968	0.855	0.913	0.963	0.831	0.893	0.952
		PR10	0.928	0.972	0.988	0.915	0.953	0.984	0.891	0.945	0.970
		S15	0.934	0.967	0.996	0.916	0.965	0.996	0.903	0.945	0.993
	1000	WSNS	0.894	0.938	0.982	0.895	0.935	0.979	0.860	0.915	0.968
		PR10	0.955	0.982	0.995	0.922	0.964	0.993	0.902	0.949	0.981
		S15	0.939	0.970	0.997	0.930	0.967	0.993	0.901	0.945	0.984
	1500	WSNS	0.903	0.944	0.989	0.888	0.939	0.985	0.877	0.936	0.978
		PR10	0.944	0.974	0.995	0.920	0.968	0.990	0.925	0.959	0.989
		S15	0.944	0.974	0.996	0.937	0.967	0.994	0.922	0.965	0.992
0.8	500	WSNS	0.890	0.944	0.979	0.864	0.923	0.981	0.847	0.903	0.959
		PR10	0.986	0.992	0.998	0.971	0.989	0.998	0.947	0.975	0.994
		S15	0.980	0.995	0.999	0.963	0.984	0.998	0.947	0.981	0.999
	1000	WSNS	0.894	0.941	0.986	0.884	0.938	0.986	0.861	0.915	0.974
		PR10	0.993	0.998	1.000	0.984	0.997	1.000	0.958	0.979	0.998
		S15	0.980	0.991	0.999	0.969	0.989	1.000	0.940	0.967	0.997
	1500	WSNS	0.896	0.949	0.988	0.902	0.944	0.983	0.866	0.926	0.980
		PR10	0.988	0.996	1.000	0.988	0.993	1.000	0.957	0.979	0.998
		S15	0.986	0.995	0.999	0.968	0.992	1.000	0.948	0.978	0.997

Table 3.92: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 125$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.888	0.941	0.979	0.879	0.938	0.979	0.848	0.911	0.960
		PR10	0.858	0.920	0.976	0.859	0.927	0.974	0.854	0.904	0.973
		S15	0.898	0.948	0.988	0.885	0.949	0.991	0.892	0.948	0.993
	1000	WSNS	0.907	0.955	0.987	0.903	0.949	0.990	0.890	0.939	0.984
		PR10	0.898	0.946	0.987	0.885	0.938	0.983	0.864	0.920	0.973
		S15	0.912	0.957	0.990	0.907	0.955	0.986	0.895	0.952	0.993
	1500	WSNS	0.926	0.961	0.991	0.912	0.956	0.990	0.893	0.942	0.984
		PR10	0.894	0.946	0.979	0.885	0.933	0.981	0.886	0.936	0.985
		S15	0.906	0.955	0.992	0.905	0.959	0.989	0.899	0.947	0.987
0.4	500	WSNS	0.886	0.939	0.982	0.885	0.930	0.978	0.862	0.918	0.957
		PR10	0.927	0.966	0.987	0.920	0.964	0.989	0.880	0.923	0.973
		S15	0.927	0.964	0.992	0.920	0.971	0.993	0.913	0.957	0.991
	1000	WSNS	0.916	0.950	0.989	0.919	0.960	0.988	0.890	0.934	0.973
		PR10	0.949	0.980	0.993	0.944	0.972	0.992	0.887	0.925	0.981
		S15	0.939	0.965	0.994	0.935	0.971	0.995	0.900	0.948	0.992
	1500	WSNS	0.931	0.973	0.991	0.913	0.955	0.995	0.902	0.945	0.989
		PR10	0.950	0.977	0.995	0.938	0.969	0.994	0.903	0.963	0.993
		S15	0.945	0.973	0.995	0.933	0.974	0.996	0.919	0.963	0.994
0.8	500	WSNS	0.928	0.958	0.987	0.884	0.946	0.986	0.867	0.914	0.971
		PR10	0.985	0.994	1.000	0.982	0.994	0.999	0.953	0.978	0.992
		S15	0.981	0.994	0.999	0.972	0.990	1.000	0.953	0.985	0.996
	1000	WSNS	0.920	0.956	0.995	0.905	0.948	0.996	0.880	0.939	0.984
		PR10	0.992	0.999	1.000	0.994	0.999	1.000	0.949	0.980	0.999
		S15	0.981	0.993	0.999	0.974	0.995	0.999	0.946	0.979	0.998
	1500	WSNS	0.925	0.964	0.995	0.893	0.952	0.994	0.905	0.953	0.989
		PR10	0.990	0.997	1.000	0.986	0.997	1.000	0.973	0.995	1.000
		S15	0.983	0.996	1.000	0.974	0.989	0.999	0.966	0.992	1.000

Table 3.93: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.3$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.836	0.893	0.959	0.844	0.908	0.957	0.829	0.886	0.941
		PR10	0.843	0.913	0.974	0.864	0.915	0.968	0.863	0.903	0.964
		S15	0.898	0.954	0.988	0.897	0.950	0.990	0.899	0.947	0.986
	1000	WSNS	0.889	0.936	0.983	0.886	0.933	0.979	0.859	0.906	0.965
		PR10	0.894	0.951	0.985	0.891	0.935	0.985	0.878	0.929	0.979
		S15	0.908	0.952	0.995	0.909	0.956	0.989	0.897	0.949	0.989
	1500	WSNS	0.895	0.939	0.982	0.886	0.945	0.986	0.878	0.926	0.981
		PR10	0.896	0.942	0.980	0.893	0.942	0.980	0.900	0.943	0.986
		S15	0.905	0.954	0.989	0.902	0.957	0.990	0.906	0.947	0.987
0.4	500	WSNS	0.841	0.910	0.964	0.837	0.902	0.957	0.820	0.889	0.940
		PR10	0.918	0.968	0.982	0.904	0.955	0.979	0.888	0.935	0.963
		S15	0.934	0.967	0.996	0.916	0.965	0.996	0.903	0.945	0.993
	1000	WSNS	0.886	0.940	0.976	0.884	0.931	0.976	0.860	0.919	0.962
		PR10	0.950	0.978	0.994	0.923	0.960	0.993	0.901	0.951	0.981
		S15	0.939	0.970	0.997	0.930	0.967	0.993	0.901	0.945	0.984
	1500	WSNS	0.897	0.944	0.984	0.887	0.934	0.985	0.869	0.934	0.978
		PR10	0.945	0.975	0.995	0.922	0.968	0.988	0.921	0.956	0.986
		S15	0.944	0.974	0.996	0.937	0.967	0.994	0.922	0.965	0.992
0.8	500	WSNS	0.881	0.950	0.979	0.863	0.922	0.979	0.849	0.897	0.955
		PR10	0.982	0.991	0.998	0.971	0.990	0.999	0.946	0.971	0.991
		S15	0.980	0.995	0.999	0.963	0.984	0.998	0.947	0.981	0.999
	1000	WSNS	0.896	0.941	0.983	0.877	0.936	0.980	0.866	0.914	0.967
		PR10	0.993	0.998	1.000	0.979	0.995	1.000	0.958	0.979	0.999
		S15	0.980	0.991	0.999	0.969	0.989	1.000	0.940	0.967	0.997
	1500	WSNS	0.894	0.951	0.988	0.898	0.940	0.981	0.865	0.928	0.980
		PR10	0.988	0.996	1.000	0.984	0.993	1.000	0.959	0.979	0.997
		S15	0.986	0.995	0.999	0.968	0.992	1.000	0.948	0.978	0.997

Table 3.94: Empirical coverage probabilities of the proposed WSNS method, the PR10 method of Politis and Romano (2010) and the S15 method described in Shao (2015) for Scenario 3 with Uniform$(-2, 2)$ innovation, $\rho = 0.6$ and $B_N = 150$.

			Mean			Median			Variance		
r	N_c	Method	90%	95%	99%	90%	95%	99%	90%	95%	99%
0	500	WSNS	0.874	0.921	0.978	0.867	0.935	0.972	0.845	0.898	0.954
		PR10	0.854	0.906	0.971	0.858	0.919	0.973	0.852	0.898	0.969
		S15	0.898	0.948	0.988	0.885	0.949	0.991	0.892	0.948	0.993
	1000	WSNS	0.898	0.948	0.987	0.898	0.951	0.987	0.884	0.933	0.977
		PR10	0.894	0.948	0.983	0.883	0.938	0.979	0.865	0.918	0.968
		S15	0.912	0.957	0.990	0.907	0.955	0.986	0.895	0.952	0.993
	1500	WSNS	0.916	0.959	0.990	0.904	0.947	0.990	0.886	0.940	0.982
		PR10	0.894	0.944	0.978	0.886	0.934	0.978	0.887	0.935	0.985
		S15	0.906	0.955	0.992	0.905	0.959	0.989	0.899	0.947	0.987
0.4	500	WSNS	0.881	0.928	0.982	0.871	0.924	0.974	0.848	0.910	0.955
		PR10	0.920	0.961	0.984	0.915	0.962	0.988	0.881	0.922	0.970
		S15	0.927	0.964	0.992	0.920	0.971	0.993	0.913	0.957	0.991
	1000	WSNS	0.905	0.948	0.986	0.909	0.955	0.981	0.882	0.933	0.977
		PR10	0.944	0.977	0.992	0.940	0.969	0.991	0.889	0.925	0.977
		S15	0.939	0.965	0.994	0.935	0.971	0.995	0.900	0.948	0.992
	1500	WSNS	0.923	0.966	0.990	0.906	0.949	0.989	0.898	0.946	0.987
		PR10	0.950	0.978	0.995	0.940	0.968	0.994	0.905	0.963	0.991
		S15	0.945	0.973	0.995	0.933	0.974	0.996	0.919	0.963	0.994
0.8	500	WSNS	0.914	0.950	0.982	0.875	0.936	0.978	0.854	0.908	0.966
		PR10	0.984	0.994	0.998	0.980	0.992	0.996	0.949	0.975	0.990
		S15	0.981	0.994	0.999	0.972	0.990	1.000	0.953	0.985	0.996
	1000	WSNS	0.915	0.954	0.990	0.892	0.951	0.988	0.877	0.928	0.983
		PR10	0.993	0.998	1.000	0.991	0.998	1.000	0.950	0.977	0.997
		S15	0.981	0.993	0.999	0.974	0.995	0.999	0.946	0.979	0.998
	1500	WSNS	0.922	0.963	0.995	0.894	0.947	0.992	0.907	0.944	0.992
		PR10	0.990	0.998	1.000	0.981	0.997	1.000	0.972	0.995	1.000
		S15	0.983	0.996	1.000	0.974	0.989	0.999	0.966	0.992	1.000

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