

2020-09

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G. Tsoukalas, B.H. Falk. 2020. "Token-Weighted Crowdsourcing." *Management Science*, Volume 66, Issue 9, pp. 3843 - 3859. <https://doi.org/10.1287/mnsc.2019.3515>  
<https://hdl.handle.net/2144/44074>

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# Token-Weighted Crowdsourcing

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Received: December 06, 2018

Revised: April 25, 2019; July 31, 2019

Accepted: August 26, 2019

Published Online in Articles in Advance:  
August 3, 2020

<https://doi.org/10.1287/mnsc.2019.3515>

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**Abstract.** Blockchain-based platforms often rely on token-weighted voting (“ $\tau$ -weighting”) to efficiently crowdsource information from their users for a wide range of applications, including content curation and on-chain governance. We examine the effectiveness of such decentralized platforms for harnessing the wisdom and effort of the crowd. We find that  $\tau$ -weighting generally discourages truthful voting and erodes the platform’s predictive power unless users are “strategic enough” to unravel the underlying aggregation mechanism. Platform accuracy decreases with the number of truthful users and the dispersion in their token holdings, and in many cases, platforms would be better off with a “flat”  $1/n$  mechanism. When, prior to voting, strategic users can exert effort to endogenously improve their signals, users with more tokens generally exert more effort—a feature often touted in marketing materials as a core advantage of  $\tau$ -weighting—however, this feature is not attributable to the mechanism itself, and more importantly, the ensuing equilibrium fails to achieve the first-best accuracy of a centralized platform. The optimality gap decreases as the distribution of tokens across users approaches a theoretical optimum, which we derive, but tends to increase with the dispersion in users’ token holdings.

**History:** Accepted by Gabriel Weintraub, revenue management and market analytics.



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**Funding:** This work was supported by the Ripple Research Fund at the Wharton School.

**Supplemental Material:** The e-companion is available at <https://doi.org/10.1287/mnsc.2019.3515>.

**Keywords:** blockchain • crowdsourcing • cryptocurrency • information aggregation • on-chain governance • strategic voting • tokenomics • token-curated registries

## 1. Introduction

Many blockchain-based platforms have implemented token-weighted voting ( $\tau$ -weighting) to incentivize efficient information crowdsourcing from their users in a decentralized way. These systems aggregate information through user votes, where the final action of the system is determined based on the weighted average of the users’ votes, and each user’s vote is weighted by his token holdings within the system. At their core, these systems work under the principle that users with more tokens have more skin in the game and are thus incentivized to provide higher-quality votes.

Though  $\tau$ -weighting has already been deployed on many live blockchain platforms, there is surprisingly little research attesting to its theoretical soundness. This paper examines some of its basic economics (*tokenomics*) to gauge its effectiveness. In particular, we focus on the following questions: Does  $\tau$ -weighting encourage or discourage truthful voting? How accurate is the resulting crowdsourced information? Does having skin in the game in the form of token

holdings adequately incentivize user effort to improve the platform’s predictive power?

The surge of blockchain-based platforms over the past several years has brought with it many new challenges. One of the most critical issues faced by these platforms is the need to crowdsource information from their users. This issue is at the core of many decentralized systems, regardless of how different they may otherwise be. Fittingly, there is a wide range of applications, ranging from governance issues such as how funds raised from an initial coin offering should be spent to evaluating the quality of code-upgrade proposals, to soliciting user feedback on service experience, aggregating product quality ratings, combating fake news, and many others.

One of the first applications of  $\tau$ -weighted voting was in the Slock.it decentralized autonomous organization (DAO) (Jentzsch 2016). The DAO was envisioned as a new type of user-managed investment fund, where users could deposit tokens into a joint fund, and the fund’s investment decisions would be made by the weighted votes of the users. Although the

Ethereum smart contract running the DAO was hacked, leading to its collapse, the DAO model inspired countless blockchain startups to adopt similar forms of on-chain governance, where token holders are allowed to direct the overall system through either informal (nonbinding) votes like CarbonVote (Lv and Ashu 2018) or binding resolutions (Warren and Bandeali 2018, Eufemio et al. 2018, Goodman 2014). CarbonVote has tallied more than five million Ether (the platform's currency) toward votes (Lv and Ashu 2019), and Tezos had more than 28,000 votes in for its first, binding hard-fork proposal (Kim 2019).

Token-weighted voting has also been used for content curation, whereby users can promote (upvote) content with their votes weighted proportionally to their token holdings—Steem (2018) and Sapien (Bhatia et al. 2018) are two notable examples. Steem identifies itself as a “blockchain-based rewards platform for publishers to monetize content” and has more than 60,000 daily active users (Team Steemit 2019). Sapien is a “social news platform that gives users control of their data, rewards content creators, and fights fake news.” In a different application, a prototype implementing a variant of  $\tau$ -weighted voting has been developed to allow users to vote on the trustworthiness of TLS and SSL certificates (Hentschker 2018). A general model of token-curated registries (TCRs) (Goldin 2017) has applied this principle to allow users to collectively curate a list of high-quality content. Civil (Iles 2018) has adopted the TCR model to combat fake news by identifying high-quality news sources, and AdChain (Goldin et al. 2017) uses a TCR to fight fraud in digital advertising.

Despite differences in their specific implementations, these systems all share a common underlying design and incentive mechanism: individual users are allowed to vote on the quality of content (e.g., a news source, a social media post, an investment plan, the quality of a product or a service), and the system aggregates these votes, weighting them according to each user's token holdings. The system then takes action based on this weighted average. Importantly, the system's reward is proportional to the quality of the decision. This reward is realized in the form of increased token value and is thus distributed among the users proportionally to their token holdings. The key design principle here is that users with large token holdings will be the most incentivized to increase the overall value of the platform.

These features are at the core of our model of  $\tau$ -weighted crowdsourcing. We consider a product or a service of unknown quality subjected to a vote on a platform. The platform commits (e.g., through a smart contract) to aggregate user votes according to a standard  $\tau$ -weighted mechanism. Platform value is driven by accuracy, which depends on the nature

of the incoming votes it receives from stakeholders. The more accurate the platform, the more valuable its tokens, and the more value it generates for its stakeholders. The voters are simply the token holders of the platform and are heterogeneous along two dimensions: their token holdings and the precision of their private signals. We first examine a simple voting game with exogenous signal precisions and consider two types of players: *strategic* and *truthful*. Truthful players simply vote their posterior beliefs after observing their own signal, while ignoring the presence of other voters on the platform. Strategic players, on the other hand, are fully rational—they report a vote that maximizes the expected value of their tokens, taking into account not only their own signal but other voters' strategies as well. Next, recognizing that strategic players may want to try to improve their information before voting, we extend the base model to account for endogenous information acquisition, adding an additional stage of effort provisioning; that is, before choosing a voting strategy, each player has the option to exert (costly) effort to improve the precision of his own signal. We characterize the equilibria (in linear strategies) of the crowdsourcing games with exogenous and endogenous information acquisition.

In brief, our results suggest that when it comes to crowdsourcing information,  $\tau$ -weighted aggregation generally (i) discourages truthful voting and (ii) reduces platform accuracy, unless players are sophisticated enough to endogenously unravel the underlying weighting mechanism. When we incorporate effort into our model to account for endogeneity in information acquisition, we show that  $\tau$ -weighting (iii) provides some desirable effort incentives of skin-in-the-game type for strategic players but (iv) nonetheless fails to achieve the first-best predictive power of a centralized platform.

Beyond these high-level insights, which provide answers to the questions raised earlier in the discussion, our analysis affords a series of more in-depth results on the nature and impact of strategic versus truthful voting, the effectiveness of centralized versus decentralized aggregation, and sensitivities to token dispersion (i.e., how tokens are distributed across users).

Intuitively, one might expect  $\tau$ -weighting to lead to suboptimal outcomes because it has the potential to exacerbate an inherent mismatch that may exist between the distribution of tokens and the distribution of information. For instance, a voter might have excellent information but low stake, in which case his vote will be underweighted in the aggregate, and vice versa. We find that strategic voters are able to fully overcome this mismatch by purposely misreporting their beliefs to the platform, effectively unraveling the platform's suboptimal weighting mechanism. However, this holds only when all voters are strategic.

First-best accuracy is elusive when even a single truthful voter is present, and the optimality gap increases in the number of truthful voters. In such cases, we show the platform is generally better off with an unweighted  $1/n$  aggregation mechanism. Taken together, these results suggest that if these decentralized platforms can effectively harness the wisdom of the crowd, it is despite of, not because of, their  $\tau$ -weighting mechanism.

To account for the fact that strategic voters may want to try to correct the aforementioned mismatch between the distribution of tokens and information, we then endogenize their information acquisition decisions in Section 5, where, before voting (stage 2), we give them the ability to improve the precision of their own private signal by exerting costly effort (stage 1). The extended two-stage model requires introducing two generic functions whose properties will be critical for the analysis: the information improvement function, which maps effort to signal accuracy, and the effort cost function, which maps effort to costs incurred.

We show that under certain technical/structural conditions on these functions, there is a unique effort-exerting equilibrium in which individual effort levels can be derived in closed form. Leveraging the expressions obtained, we show that effort increases with token holdings—a feature that platforms often tout in their marketing materials as a key advantage of  $\tau$ -weighted voting. We find the effect, however, not to be attributable to the token-weighting mechanism itself, which remains irrelevant when all users are strategic, even under endogenous information acquisition. Rather, it is simply that players with more tokens stand to benefit more from an increase in platform accuracy, because accuracy directly drives token value. In other words, the same effect would be observed if the platform adopted a flat  $1/n$  aggregation mechanism, as long as agents with more tokens stand to benefit more from an increase in platform accuracy. We also show that, *ceteris paribus*, an agent with higher precision will exert more effort than one with lower precision, implying some free-riding from less-informed voters.

Comparing the resulting equilibrium to that of a centralized platform that can coordinate user behavior, we find that decentralized equilibrium effort levels are strictly lower. As a result, the platform cannot achieve first-best effort provisioning in the decentralized setting, even in the best-case scenario in which all agents are strategic. We characterize the ensuing optimality gap and show that on average, it grows with the dispersion in players' token holdings, implying platforms may generally prefer that tokens are not too disproportionately held by platform users. The effect is stronger when all players have homogeneous precisions and is weaker, but still positive, otherwise. Finally, we derive the platform's optimal allocation of tokens, as a function of the distribution

of precisions across its users, and we show, through an appropriately chosen distance measure, that the platform would prefer dispersed distributions if these are close enough to the theoretical optimum and prefer a homogeneous distribution otherwise.

To summarize, our results bring to light some of the more subtle pros and cons of  $\tau$ -weighted aggregation systems, and, in contrast to what their widespread use implies, we raise some questions about their effectiveness at harnessing information and user effort. Despite the ever-increasing popularity of  $\tau$ -weighted aggregation systems in practice, the academic literature has remained relatively silent on their theoretical soundness. This paper, which can be seen as a step to bridge this gap, strives to put the emerging topic of  $\tau$ -weighted aggregation on a firmer foundation and to provide guidance on the design of blockchain-based voting systems.

## 2. Related Literature

Though we are not aware of any other theoretical papers focusing specifically on the effectiveness of  $\tau$ -weighted crowdsourcing, our work is related to several literature streams.

### 2.1. Blockchain Systems and TCRs

Blockchain-based systems provide a natural platform for  $\tau$ -weighted voting, because each player's token holdings are usually publicly known (on the blockchain) and ballots can be cast, aggregated, and verified in a completely automated manner. In fact, numerous  $\tau$ -weighted voting mechanisms have been deployed on various blockchain platforms, as mentioned in the introduction. There is a growing practitioner literature on the topic of TCRs (Goldin 2017); however, previous studies focused on practical implementation (e.g., coding details) and did not formally model strategic agent behavior, which we do here. On the academic side, some studies have started to appear; for instance, Asgaonkar and Krishnamachari (2018) dive into some of the technical details of TCRs in a deterministic setting with known information. Many variants of TCRs have been proposed (Simon 2017, Lockyer 2018), and we expect these platforms to continue to grow. Abstracting away from the technicalities of these systems, our study sheds light on the effectiveness of the token-weighting mechanism that often sits at their core.

More broadly, the paper contributes to a rapidly growing literature discussing economic incentives in blockchain systems (Cong et al. 2018, Saleh 2018, Biais et al. 2019, Hinzen et al. 2019, Rosu and Saleh 2019) and studying the implications of the technology for a variety of areas such as auditing (Cao et al. 2018), corporate governance (Yermack 2017), crowdfunding (Chod and Lyandres 2018, Gan et al. 2020), finance (Biais et al. 2018), innovation (Catalini and Gans 2017),

and operations management and supply chains (Babich and Hilary 2020, Chod et al. 2020).

## 2.2. Crowdsourcing and Information Sharing in Networks

Our work is closely related to the literature on crowdsourcing, which studies the ability of firms to source information (Araman and Caldentey 2016, Papanastasiou et al. 2017), funds (Alaei et al. 2016, Strausz 2017, Babich et al. 2020, Belavina et al. 2020), or innovation (Terwiesch and Xu 2008, Bimpikis et al. 2015, Stouras et al. 2017) from users. We are not aware of any work in this literature that, like we do, examines the feasibility and effectiveness of crowdsourcing information and effort using a  $\tau$ -weighted mechanism.

More broadly, our work is related to the literature on information sharing in networks that studies information exchange and aggregation, either through direct communication (Acemoglu et al. 2014) or through observational learning (Acemoglu et al. 2011). In a related but different setting, Saghafian et al. (2018) examine information aggregation through (imperfect) sensors that can solicit information from other sensors. Unlike ours, these papers do not focus on the strategic incentives that agents have to (mis)report their information and exert effort when subjected to a  $\tau$ -weighted voting mechanism.

## 2.3. Weighted Voting and Shareholder Voting

Our work is broadly related to the literature on weighted voting mechanisms that have been studied in a variety of settings. For example, Banzhaf (1964) and a series of papers that build on that work (see, e.g., Snyder et al. 2005 and references therein) examine electoral weighted voting, focusing on the distinction between voting rights and voting power. Taylor and Zwicker (1993), Nordmann and Pham (1999), and Elkind et al. (2008) focus on coalition structures and the division of power in weighted voting games. Gifford (1979) and Tong and Kain (1991) study weighted voting in distributed computing systems, focusing on characterizing computational complexity.

Weighted voting is also relevant for the literature on shareholder voting, given that one share often entitles one vote, and shareholders hold different share amounts. Papers in this area tend to focus on issues specific to the context, for example, the mismatch of incentives between managers and shareholders (Shleifer and Vishny 1986), issues of vote trading (Christoffersen et al. 2007), and the differences between binding and nonbinding votes (Levit and Malenko 2011).

Our work departs from these settings in a number of ways. First, the input and/or output space studied

in these papers is generally constrained to be discrete and, in most cases, binary (yes/no). With these constraints, the complexity comes from the combinatorial nature of the problem. In contrast, the choice of continuous input and output spaces in our model deliberately draws from the established economics and finance literature on information aggregation and acquisition (Myatt and Wallace 2012, Colombo et al. 2014) and price informativeness (Vives 1988, Vives 2011, Ostrovsky 2012, Even et al. 2019) and is meant to capture situations that are best approximated by continuous states. For instance, our model is more suitable for situations in which agents are asked to estimate quality of content or products, as in the TCR examples mentioned previously, or investment amounts, as in the DAO example, and is less suitable to inform on yes/no issues of governance, such as electing board members, stock splits, and merger and acquisition decisions. Second, papers in these areas generally do not focus on user effort incentives, which is one of the main contributions of our work and one of the main areas of focus of  $\tau$ -weighted platforms in their marketing materials. Third, these papers do not specifically focus on the tokenomics of  $\tau$ -weighted crowdsourcing systems. As such, they do not address the specific questions we examine, including truthful versus strategic voting, centralized versus decentralized aggregation,  $\tau$ -weighting versus  $1/n$ -weighting, token dispersion, and the implications these have for token value and platform effectiveness.

## 3. Model

Consider a product (or a service) of unknown quality,  $q$ , subject to a vote on a platform with  $n$  token holders (“players”), indexed by  $i \in \{1, \dots, n\}$ . Players cannot directly observe  $q$  but have a common Gaussian prior belief

$$q \sim \mathcal{N}(\mu, \sigma_q^2),$$

where  $\mathbb{E}[q] = \mu$  is the mean quality, and  $\text{Var}[q] = \sigma_q^2$  captures quality dispersion, which can be a proxy for the novelty of the product.<sup>1</sup>

Players are tasked to vote on quality and are heterogeneous along two dimensions: their relative token holdings  $\tau_i \in (0, 1)$ , normalized so that  $\sum_{i=1}^n \tau_i = 1$ , and the signal they obtain about the product quality. More specifically, player  $i$  receives a private noisy signal

$$s_i = q + \epsilon_i, \quad (1)$$

where  $\epsilon_i$  is a normally distributed noise term with  $\mathbb{E}[\epsilon_i] = 0$  and  $\text{Var}[\epsilon_i] = \sigma_i^2$ , that is,  $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2), \forall i \in \{1, \dots, n\}$ . It follows that signals are independent and normally distributed with  $s_i \sim \mathcal{N}(\mu, \sigma_q^2 + \sigma_i^2)$ . In line with extant literature,  $\sigma_i, i \in \{1, \dots, n\}$ , are publicly visible,

but players cannot observe others' private signals. Similarly to how these platforms operate in practice, token holdings  $\tau_i, i \in \{1, \dots, n\}$ , are also publicly visible.<sup>2</sup>

After observing their private signal, players simultaneously submit their votes  $v_i = v_i(s_i)$ ; in line with extant literature (e.g., Vives 1988, Myatt and Wallace 2012), we restrict our attention to the class of linear strategies, represented by letter  $\mathbb{L}$ , that is,

$$v_i = \alpha_i s_i + (1 - \alpha_i)\mu, \quad (2)$$

where  $\alpha_i \in \mathbb{R}$  is the weight player  $i$  chooses to place on his signal.<sup>3</sup> We note that we purposely do *not* restrict  $\alpha_i$  to the interval  $[0, 1]$ , so assuming  $v_i$  to be a linear combination of  $s_i$  and  $\mu$  does not restrict the range of  $v_i$ . The linear model also facilitates comparisons with Bayesian posteriors that are also linear in the signal. In particular, conditional on  $s_i$ ,  $q$  is normally distributed with mean  $\mathbb{E}[q|s_i] = \beta_i s_i + (1 - \beta_i)\mu$ , where  $\beta_i = \sigma_q^2(\sigma_q^2 + \sigma_i^2)^{-1}$  is the weight player  $i$  places on his signal, and  $\text{Var}[q|s_i] = (\sigma_q^{-2} + \sigma_i^{-2})^{-1}$ .

In Section 5, we will allow users to try to improve their signals, and we defer the required modeling extension to that section.

The platform aggregates incoming votes weighted by token holdings, which forms the platform's quality estimate  $\hat{q}$ , that is,

$$\hat{q}(v_1, \dots, v_n) = \sum_{i=1}^n \tau_i v_i. \quad (3)$$

Consistent with how these platforms operate in practice, the platform commits to the aggregation mechanism in Equation (3), for example, through an auditable "smart contract." Throughout the paper we will compare this mechanism, which we refer to as  $\tau$ -weighted aggregation, or simply  $\tau$ -weighting, to a benchmark of an equally weighted (flat) mechanism, which we refer to as  $1/n$ -weighted aggregation, or simply  $1/n$ -weighting, that assumes all players are attributed a weight  $1/n$ , irrespective of their token holdings; formally  $\hat{q}_{1/n}(v_1, \dots, v_n) = \frac{1}{n} \sum_{i=1}^n v_i$ . For the sake of completeness, we will also consider the case of a platform optimizing the aggregation mechanism in Online Appendix A.3.

Players are rewarded through the value of their tokens, which in turn depends on the accuracy of the platform.<sup>4</sup> In particular, the closer the aggregate quality,  $\hat{q}$ , is to  $q$ , the more valuable the platform becomes and hence the more valuable the tokens become.

To model the platform's value, we work with a general class of differentiable and concave payoff functions taken over the error  $(q - \hat{q})$ , and we assume these to be well behaved if they satisfy the following properties.

**Assumption 1 (Well-Behaved Payoffs).** A payoff function  $\pi : \mathbb{R} \rightarrow \mathbb{R}$  is said to be well behaved if (1)  $\pi$  is symmetric about the origin, that is,  $\pi(x) = \pi(-x)$ , and (2)  $\pi$  is decreasing away from the origin, that is,  $\pi(x) \leq \pi(y)$ , whenever  $|x| \geq |y|$ .

The "well-behaved" assumption means that overestimating and underestimating the quality are equally bad and that the more accurate the estimate, that is, the lower the error  $(q - \hat{q})$ , the higher the payoff. As we shall see, these basic assumptions suffice to obtain meaningful results in Section 4, without having to restrict the payoff to a specific functional form. Many standard functional forms would satisfy these conditions, for instance,  $\pi(x) \propto -x^2$ , which would correspond to a quadratic utility, or  $\pi(x) \propto e^{-x^2}$ .

Defining the platform's payoff as  $\pi(q - \hat{q})$ , we can also define each player's individual share of this payoff by  $\pi_i = \tau_i \cdot \pi$ , that is, consistent with how these platforms operate in practice, each player obtains a share of the payoff proportional to his relative share of the tokens. This feature drives several of our results, as we shall see in Section 3.

Our core model assumes all players are *strategic* (fully rational), represented by letter  $\mathbb{S}$ , and we seek to characterize the linear pure-strategy Bayesian Nash equilibria of the game. More specifically, each player  $i$  maximizes his expected payoff over his linear voting strategy  $v_i \in \mathbb{L}$ , conditional on the private signal he receives,  $s_i$ , given the platform aggregation mechanism in Equation (3), and the linear strategies played by others,  $v_{-i} \in \mathbb{L}$ ,

$$\max_{v_i \in \mathbb{L}} \mathbb{E} [\pi_i(q - \hat{q}(v_1, \dots, v_n)) | s_i]. \quad (4)$$

In Section 4, the platform composed of strategic players will be compared with a benchmark platform consisting of nonstrategic players following simple *truthful* voting strategies. Truthful voters, represented by letter  $\mathbb{T}$ , vote their true, albeit individual, Bayesian posteriors. That is, they update their beliefs based on their own private signal, setting weight  $\alpha_i = \beta_i = \sigma_q^2(\sigma_q^2 + \sigma_i^2)^{-1}$  and vote  $v_i = \beta_i s_i + (1 - \beta_i)\mu$ , without considering other players. Practically speaking, platforms may be composed of players exhibiting different degrees of strategic behavior (another interpretation is that players may have different behavioral biases), hence, it is meaningful to examine the implications that player heterogeneity along this dimension can have for platform accuracy. The two player types we consider can be thought of as a coarse representation of the spectrum.<sup>5</sup> In Section 4.3, we extend the model to the more realistic case in which both player types are simultaneously present on the platform.

### 4. Crowdsourcing Information

In this section, we examine equilibrium voting strategies and their implications for the platform’s predictive accuracy. We begin with the first-best variance obtainable in the centralized setting.

#### 4.1. First-Best Platform Variance

Before presenting the result, we introduce one intermediate technical lemma that will be useful throughout the analysis.

**Lemma 1** (Payoff-Variance Equivalence). *Suppose  $\pi : \mathbb{R} \rightarrow \mathbb{R}$  satisfies Assumption 1 and let  $N(0, \sigma^2)$  denote a normally distributed random variable with mean 0 and variance  $\sigma^2$ ; then*

- i.  $\mathbb{E}[\pi(N(0, \sigma^2))]$  is monotonically decreasing in  $\sigma^2$ ;
- ii.  $\mathbb{E}[\pi(N(0, \sigma^2))] \geq \mathbb{E}[\pi(N(x_0, \sigma^2))]$  for all  $x_0$ ;
- iii. if  $V(x_1, \dots, x_n) : \mathbb{R}^n \rightarrow \mathbb{R}^+$ ,  $S \subset \mathbb{R}^n$  is a set of constraints, and  $x^* \stackrel{\text{def}}{=} \arg \min_{x \in S} V(x)$ , then  $\mathbb{E}[\pi(N(0, V(x^*)))] = \max_{x \in S} \mathbb{E}[\pi(N(0, V(x)))]$ .

Proofs are provided in the online appendix. One useful implication of Lemma 1 is that since the payoff function is well behaved and the aggregate error ( $q - \hat{q}$ ) is normally distributed with mean 0, we can interchangeably talk about the platform/players maximizing expected payoff, or minimizing the variance of the aggregate error.

Consider an omniscient central planner that can directly observe all signals and set  $\hat{q}$  to maximize  $\mathbb{E}[\pi(q - \hat{q})|s_1, \dots, s_n]$ .<sup>6</sup> In light of Lemma 1, this can equivalently be written as a minimization problem over the variance,  $V^{fb} \stackrel{\text{def}}{=} \min \text{Var}[q - \hat{q}|s_1, \dots, s_n] = \text{Var}[q|s_1, \dots, s_n]$ . The latter is simply the Bayesian posterior variance in the standard multivariate Gaussian setting,<sup>7</sup> thus

$$V^{fb} = \left( \sigma_q^{-2} + \sum_i \sigma_i^{-2} \right)^{-1}. \tag{5}$$

As expected,  $V^{fb}$  increases in  $\sigma_q$  and  $\sigma_i$ , that is, the more imprecise the prior and signals, the worse the platform’s predictive power over  $q$ . Furthermore, the platform becomes perfectly accurate as the number of players  $n \rightarrow \infty$ . In other words, the centralized platform can fully harness the wisdom of the crowd at the limit. This, however, hinges on “reasonable”  $\sigma_i, \forall i$ , for example, independence of individual signals and of their variance from  $n$ . For an alternative setting in which aggregate information does not necessarily grow with the number of users, see Bergemann and Välimäki (1997).

Finally, as is standard in the literature, it will be convenient to formally define the platform’s first-best precision as the inverse of its first-best variance,  $\rho^{fb} = (V^{fb})^{-1}$ . Combining this definition with the fact that the platform’s estimate is unbiased (given all

signals are unbiased and the aggregation mechanism is linear), we can say that the centralized setting yields the first-best (highest possible) accuracy the platform can achieve, and we use the terms “precision” and “accuracy” interchangeably.

#### 4.2. Equilibrium with Strategic Players

Next, we examine player voting strategies and the resulting platform accuracies they can achieve, and then compare these to the first-best benchmark.

To build intuition, consider first the simple case of a platform consisting of a single strategic player, holding all tokens (i.e.,  $\tau_1 = 1$ ). In this case,  $\hat{q} = \tau_1 \cdot v_1 = v_1$  and the player’s optimal strategy, conditional on observing signal  $s_1$ , is simply to truthfully report his Bayesian posterior,  $v_1^* = \mathbb{E}[q|s_1] = \beta_1 s_1 + (1 - \beta_1)\mu$ , where  $\beta_1 = \sigma_q^2(\sigma_q^2 + \sigma_1^2)^{-1}$  (see Online Appendix A.2 for details). We emphasize two points. The first is the importance of Assumption 1. In Online Appendix A.1 we give some natural payoff functions (that are not well behaved, i.e., violate Assumption 1), where the player’s optimal strategy is *not* to vote his Bayesian posterior. One particularly illustrative case is that of a price-is-right type of payoff function.<sup>8</sup> Second, although truthfully voting one’s posterior is intuitive, it does not carry over to the general  $n$ -player game, which we describe next.

**Proposition 1** (Voting Equilibrium). *If the payoff function satisfies Assumption 1, and all players are strategic, then in equilibrium, (i) player  $i$  votes  $v_i^* = \alpha_i^* s_i + (1 - \alpha_i^*)\mu$ , where  $\alpha_i^*$  is given in Equation (6), and (ii) the platform achieves the first-best variance given by Equation (5).*

$$\alpha_i^* = \frac{\sigma_q^2}{\tau_i \sigma_i^2 \left( 1 + \sum_{j=1}^n \frac{\sigma_q^2}{\sigma_j^2} \right)}, \quad i \in \{1, \dots, n\}. \tag{6}$$

We outline three main takeaways from Proposition 1. First, strategic players are able to recover first-best optimality in token-weighted platforms, but to do so, they must be willing to adjust votes based on the presence of their peers on the platform and their own token holdings. The former is observable from the dependence of the optimal weight on  $\sigma_i, \forall i$ , and the latter from its inverse relationship to  $\tau_i$  (and its independence from  $\tau_j, j \neq i$ ).

Second, the aggregation mechanism is irrelevant when the platform is composed of strategic players, that is, first best can also be restored under  $1/n$ -weighted aggregation (this is best seen by going through the proof of Proposition 1, replacing  $\tau_i$  by  $1/n, \forall i$ ).

Third, the Bayesian weight  $\beta_i \neq \alpha_i^*$ . This implies that truthful voting is generally suboptimal under  $\tau$ -weighted aggregation. Put differently, strategic players’ votes will not reflect their true individual

posterior beliefs. The votes cast can be either higher (vote inflation) or lower (vote shading) compared with truthful votes. To illustrate, Figure 1 compares player 1’s equilibrium vote upon receipt of a positive ( $s_1 = 4 > \mu = 1$  in Figure 1(a)), or negative ( $s_1 = 1 < \mu = 4$  in Figure 1(b)) signal, in an  $n = 2$ -player platform.

This type of strategic behavior may not be harmless in practice. Consider, for instance, the implications for rating and review platforms. If individual ratings are visible (as opposed to just aggregate scores), customers who rely on such platforms for information may be exposed to a disproportionate amount of extreme ratings, either very good or very bad scores. For example, in Figure 1, strategic players with few tokens ( $\sim 0.2$ ) who receive a positive (negative) signal, inflate (deflate) their ratings well beyond their true beliefs. Without diving into a discussion on behavioral foundations, this type of outcome could possibly undermine the credibility of the platform as customers may fail to realize that this gaming of ratings is actually in their best interest, in terms of achieving maximal aggregate accuracy.

How realistic it is to assume all voters are strategic in practice, and are thus capable of restoring first best, is debatable. If some voters are in fact less sophisticated, for instance, if they have an inherent preference to report their true beliefs, two sets of questions arise. First, (Q1) can strategic players restore first best in the presence of truthful voters, and, if not, (Q2) how does the resulting optimality gap vary with the relative number of truthful voters? Second, (Q3) in the presence of truthful voters, should the platform prefer a  $\tau$ -weighted or an equally weighted mechanism? and, (Q4) relatedly, how is this preference affected by the distribution of tokens across voters?

We address the first set of questions in Section 4.3, and the second set in Section 4.4.

### 4.3. Equilibrium with Mixed Player Types

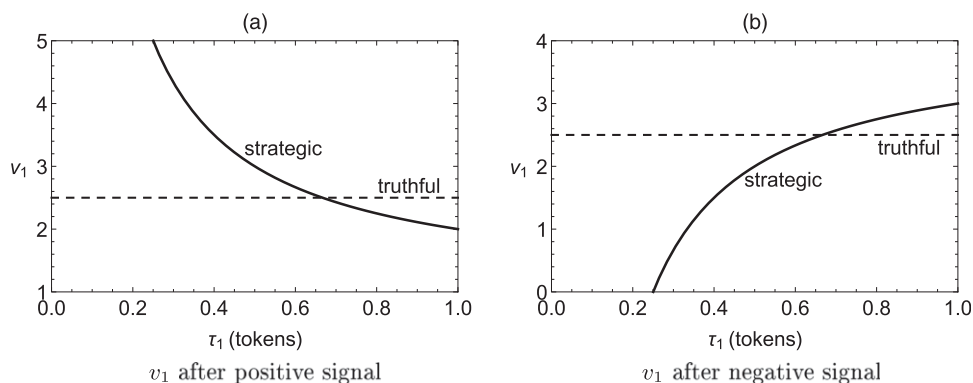
We extend the model from Section 3 to consider a platform containing a mixture of  $|\mathbb{T}|$  truthful and  $|\mathbb{S}|$  strategic players, such that  $|\mathbb{T}| + |\mathbb{S}| = n$  (notation  $|\cdot|$  represents set cardinality). As before, strategic players cannot observe other players’ signals or votes, but they can observe truthful voters’ identities.<sup>9</sup> Truthful players, by definition, consider only their own signals. Before stating the result, let weights  $\alpha_i^{\mathbb{S}} \stackrel{\text{def}}{=} \alpha_i^*$ ,  $i \in \mathbb{S}$ , from Equation (6), where the summation 1 to  $n$  in the denominator is now over the set of strategic players  $\mathbb{S}$ .

**Proposition 2** (Voting Equilibrium with Mixed Types). *In the presence of truthful voters on the platform, if the payoff function satisfies Assumption 1, then in equilibrium, (i) strategic player  $i$  votes  $v_i^{\mathbb{T},\mathbb{S}} = \alpha_i^{\mathbb{T},\mathbb{S}} s_i + (1 - \alpha_i^{\mathbb{T},\mathbb{S}}) \mu$ , where  $\alpha_i^{\mathbb{T},\mathbb{S}} = \alpha_i^{\mathbb{S}} (1 - \sum_{j \in \mathbb{T}} \tau_j \beta_j)$ ,  $i \in \mathbb{S}$ , and (ii) the corresponding platform variance  $V^{\mathbb{T},\mathbb{S}} = (1 - \sum_{j \in \mathbb{T}} \tau_j \beta_j)^2 V^{fb} + \sum_{j \in \mathbb{T}} (\tau_j \beta_j \sigma_j)^2 > V^{fb}$ .*

Proposition 2 shows that in this more realistic setting, there is still a unique equilibrium in which strategic players adjust their optimal signal weights from Equation (6) to try to correct for the presence of truthful voters. However, in contrast to the result in Proposition 1, the platform no longer achieves first-best predictive accuracy. Interestingly, even the presence of a single truthful voter ( $|\mathbb{T}| = 1$ ) prevents strategic players from restoring first best under  $\tau$ -weighted aggregation, and the more truthful players present, the worse the platform accuracy becomes. Proposition 3 formalizes the latter statement.

**Proposition 3** (Platform Accuracy with Mixed Types). *Consider a platform with a partition  $|\mathbb{S}|, |\mathbb{T}|$  of a set of  $n$  total players. For fixed  $n$ , if any number  $k \in \{1, \dots, |\mathbb{T}|\}$  of truthful players become strategic, then the platform’s accuracy increases.*

**Figure 1.** Player 1’s Vote  $v_1$  on Receiving Signal  $s_1$  vs. Relative Token Holdings  $\tau_1$  in Two-Player Platform



Note. Parameters  $\sigma_1 = \sigma_2 = \sigma_q = 1/2$ .

As a reminder, the results hold assuming the platform has precommitted to a token-weighted (or equally weighted) aggregation mechanism, for example, via a smart contract. For the sake of completeness, we relax this assumption in Online Appendix A.3, where we assume the platform can optimally set the weights it attributes to each player’s incoming vote. With this type of precise control, we show that the platform would indeed be able to recover first-best variance as long as it knows the identity of truthful voters. Note, however, that centralizing the aggregation mechanism in this way would go against the core philosophy these platforms were built on—that of decentralizing decisions—and so may be of limited applicability.

**4.4. Properties of the Optimality Gap**

Having shown that the presence of truthful players generates an indelible optimality gap in Section 4.3, answering questions (Q1) and (Q2), we now examine platform preference between  $\tau$ - and  $1/n$ -weighting (question (Q3)) and the effects of token dispersion (question (Q4)). To ease exposition, for the rest of this subsection we will assume the worst-case scenario of a platform consisting entirely of truthful players,  $|\mathbb{T}| = n$ . Our qualitative insights remain valid for  $1 < |\mathbb{T}| < n$  (for  $|\mathbb{T}| = 0$ , there is no optimality gap as per Proposition 1). Before proceeding with the analysis, we introduce some necessary notation and intermediate results.

**Platform Variance.** Let  $V_j^i, i \in \{\mathbb{T}, \mathbb{S}\}, j \in \{\tau, 1/n\}$ , represent the variance achieved by the platform under truthful ( $i = \mathbb{T}$ ) or strategic ( $i = \mathbb{S}$ ) voting and under  $\tau$ -weighting ( $j = \tau$ ) or  $1/n$ -weighting ( $j = 1/n$ ). Note that from Proposition 1,  $V_\tau^\mathbb{S} = V_{1/n}^\mathbb{S} = V^\mathbb{S} = V^{fb}$ , that is, strategic players achieve first best under both mechanisms. The corresponding values are given in Table 1.

Finally, define the optimality gap between truthful and strategic (first-best) voting as follows:

$$\mathcal{G}_j = V_j^\mathbb{T} - V^{fb} = V_j^\mathbb{T} - V^\mathbb{S}, \quad j \in \{\tau, 1/n\}.$$

**Token Dispersion.** Define  $d(x, y)$  as the Euclidean distance ( $L^2$  norm) between vectors  $x$  and  $y$ , that is,

$d(x, y) = \|x - y\|_2 = (\sum_{i=1}^n (x_i - y_i)^2)^{1/2}$ . Let the  $n$ -dimensional vectors  $\tau \stackrel{\text{def}}{=} \{\tau_1, \dots, \tau_n\}$ ,  $\mathbf{n}^{-1} \stackrel{\text{def}}{=} \{\frac{1}{n}, \dots, \frac{1}{n}\}$ , and  $\mathbf{1} \stackrel{\text{def}}{=} \{1, \dots, 1\}$ . Lastly, define the dispersion of vector  $\tau$  as the Euclidean ( $L^2$ ) norm from its average value, that is,

$$\text{disp}(\tau) = d(\tau, \bar{\tau}\mathbf{1}) = \|\tau - \bar{\tau}\mathbf{1}\|_2, \quad \text{where } \bar{\tau} = \frac{1}{n} \sum_{i=1}^n \tau_i = \frac{1}{n}.$$

With notation and definitions in place, we proceed with the analysis focusing first on the simple case in which all players have homogeneous signal precisions, before turning to the more general case with precision heterogeneity.

**4.4.1. Homogeneous Precisions ( $\sigma_i = \sigma, \forall i$ ).** Within this case of homogeneous precisions, we also distinguish between homogeneous and dispersed token holdings. The fully homogeneous case is relatively straightforward. Given there is no token dispersion in this case, there is no difference between  $\tau$ - and  $1/n$ -weighting. Simplifying the expressions in Table 1 we obtain

$$\mathcal{G}_\tau = \mathcal{G}_{1/n} = \frac{(n-1)^2 \sigma^4 \sigma_q^4}{n(\sigma^2 + \sigma_q^2)^2 (\sigma^2 + n\sigma_q^2)} \geq 0,$$

which is decreasing in  $n$ . Note that as  $n \rightarrow \infty, \mathcal{G}_\tau \rightarrow \frac{\sigma^4 \sigma_q^4}{(\sigma^2 + \sigma_q^2)^2} \geq 0$ , that is, though the optimality gap is decreasing in the number of players, it can persist at the limit as  $n$  tends to infinity, meaning that neither aggregation scheme can fully harness the wisdom of a truthful voting crowd, no matter how large.<sup>10</sup>

Next, we consider heterogeneity in token holdings while keeping all players’ signals equally precise.

**Proposition 4 (Dispersed Tokens).** *When players have heterogeneous token holdings and homogeneous signal variances,*

- i.  $\mathcal{G}_\tau - \mathcal{G}_{1/n} \geq 0$ , that is,  $1/n$ -weighting dominates token weighting;
- ii.  $\mathcal{G}_\tau - \mathcal{G}_{1/n}$  increases in  $\text{disp}(\tau)$ , that is,  $1/n$ -weighting dominance increases with token dispersion.

In line with intuition, when all players have the same information precision, there is nothing to be gained by weighing one player’s vote more or less

**Table 1.** Player Voting Strategies and Platform Variance

	Vote	Platform variance
Truthful $\tau$ -weighting	$v_i = \beta_i s_i + (1 - \beta_i)\mu$	$V_\tau^\mathbb{T} = \sum_{i=1}^n (\tau_i \beta_i \sigma_i)^2 + \sigma_q^2 (1 - \sum_{i=1}^n \tau_i \beta_i)^2$
Truthful $1/n$ -weighting	$v_i = \beta_i s_i + (1 - \beta_i)\mu$	$V_{1/n}^\mathbb{T} = \sum_{i=1}^n (\frac{1}{n} \beta_i \sigma_i)^2 + \sigma_q^2 (1 - \sum_{i=1}^n \frac{1}{n} \beta_i)^2$
Strategic	$v_i = \alpha_i^* s_i + (1 - \alpha_i^*)\mu$	$V^\mathbb{S} = V^{fb} = (\sigma_q^{-2} + \sum_{i=1}^n \sigma_i^{-2})^{-1}$

than any other’s. Thus, when the platform is composed of truthful, equally informed players, it is better to adopt an equally weighted mechanism and/or reduce token dispersion, if possible.

**4.4.2. Heterogeneous Precisions.** Here, we consider heterogeneity in both token holdings and signal variances, and we examine the impact of token dispersion on the optimality gap. In this most general case, the comparison between  $\tau$ -weighting and  $1/n$ -weighting is nontrivial and either aggregation mechanism could a priori outperform. Intuitively, which mechanism “wins” will depend on the mismatch that may exist between the distribution of tokens and the distribution of information. For instance, a voter might have excellent information but low stake, in which case his vote will be underweighted in the aggregate, and vice versa. As we shall see, this intuition is only partially true.

The set of  $\tau$ ’s for which  $\tau$ -weighting dominates is given by the sublevel set of  $V_\tau^T$ ,  $\mathcal{L} \stackrel{\text{def}}{=} \{\tau | V_\tau^T(\tau) \leq V_{1/n}^T \cap \mathbf{1}'\tau = 1\}$ . This set generally needs to be computed numerically, with the exception of low values of  $n$ . We focus here on the simplest case  $n = 2$ , which suffices to extract the main qualitative insights. The general case  $n > 2$  is analyzed in Online Appendix A.4 and confirms result robustness.

Consider a platform with two players, and assume player 1 is better informed than player 2, that is,  $\sigma_1 = 1 < \sigma_2 = \sigma_q = 1.5$ .

Figure 2(a) shows the optimality gaps under each mechanism as a function of player 1’s token holdings  $\tau_1$  and illustrates the dominance set  $\mathcal{L}$ , which is represented by a simple interval  $\tau_1 \in [1/2, \tau^c]$ , with  $\tau^c = \frac{3}{2} - \frac{2\sigma_1^2}{\sigma_1^2 + \sigma_2^2}$  and  $\tau_2 = 1 - \tau_1$ . We highlight two main takeaways. First, the basic intuition that  $\tau$ -weighting dominates when the better-informed player has more tokens does not always hold. If this player has too many tokens ( $\tau_1 \geq \tau^c \sim 0.9$ ),  $1/n$ -weighting outperforms. Second,  $\mathcal{G}_\tau$  has a minimum (reached at  $\tau_1 \sim 0.7$ ) that represents the “best possible” allocation of tokens, and at this point  $\tau$ -weighting is understandably

better. However, as the platform does not control token allocation, it is more meaningful to consider the entire range of possible realizations of  $\tau_1$  in the interval  $[0, 1]$ . Doing so reveals that  $\tau$ -weighting dominates only about 40% of the time. More importantly, when it does dominate, it tends to do so by much a lower margin than in cases where it lags behind (e.g., compare  $\mathcal{G}_\tau$  to  $\mathcal{G}_{1/n}$  in Figure 2(a), as  $\tau_1 \rightarrow 0.7$ , and then as  $\tau_1 \rightarrow 0$ ), suggesting that  $1/n$ -weighting might be the better mechanism on average.

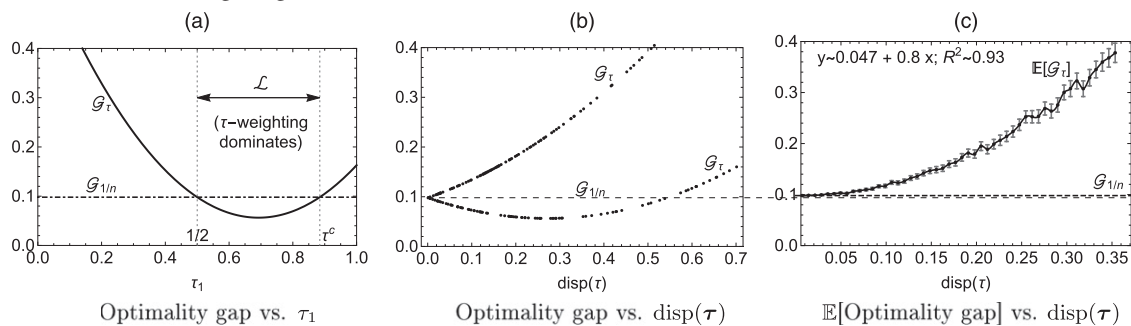
Figure 2(b) by and large confirms this result but nuances it further by examining how the optimality gap changes with  $\text{disp}(\tau)$ . To construct the values of  $\text{disp}(\tau)$ , we simulate independent realizations of vector  $\tau = \{\tau_1, \tau_2\}$  drawn from a uniform  $[0, 1]$  (normalized so that  $\tau_1 + \tau_2 = 1$ ). The results show that the relationship between  $\mathcal{G}_\tau$  and  $\text{disp}(\tau)$  is not trivial and is represented by two branches (reflecting the fact that  $\text{disp}(\tau)$  is symmetric under permutation of the players, i.e.,  $\text{disp}(\tau_1, \tau_2) = \text{disp}(\tau_2, \tau_1)$ ). The average value of  $\mathcal{G}_\tau \sim 0.15 > \mathcal{G}_{1/n} \sim 0.10$ , and hence the optimality gap under  $1/n$ -weighting, is  $\sim 1/3$  lower on average. Finally, Figure 2(c) digs one step deeper by showing how the expectation over  $\mathcal{G}_\tau$ ,  $\mathbb{E}[\mathcal{G}_\tau]$ , increases with  $\text{disp}(\tau)$  (this is roughly equivalent to averaging the two branches from Figure 2(b)).<sup>11</sup>

These results, as well as the extended analysis in Online Appendix A.4, support the following conclusion.

**Numerical Result 1.** *If players have heterogeneous token holdings and signal variances, then on average, as token dispersion increases, (i)  $\mathcal{G}_\tau$  increases and (ii)  $1/n$ -weighting increasingly outperforms  $\tau$ -weighting.*

To summarize, the results suggest that when all players follow truthful voting strategies,  $1/n$ -weighting outperforms  $\tau$ -weighting, on average (across a uniform spectrum of possible token holding realizations). We make the following caveat: the entire analysis in Section 4 has so far ignored possible endogeneity in acquiring information. Presumably, strategic players may have an incentive to exert effort to improve their information, and this could plausibly

**Figure 2.** When Does  $\tau$ -Weighting Dominate?



affect these insights. Section 5 analyzes these endogenous information acquisition effects.

### 5. Crowdsourcing Effort

Having analyzed player voting behavior in Section 4, we now turn our attention to an important feature that these crowdsourcing platforms were designed around: the incentives that players have to endogenously *improve* the platform’s accuracy. Marketing materials often claim token weighting provides proper effort-exerting incentives. For instance, in the case of TCRs,

[t]oken-curated registries are decentrally-curated lists with intrinsic economic incentives for token holders to curate the list’s contents judiciously. (Goldin 2017)

This section examines this claim in depth. To ease exposition, we focus on effort-exerting incentives when all players are strategic. As a reminder, in this case we showed that first-best variance is fully restored in the exogenous setting of Section 4. As we shall see, even in this best-case scenario, first-best platform variance can no longer be restored in the endogenous setting. The main implications of the analysis extend to the more complicated situations in which the platform is composed of a mixture of player types (and would be more pronounced in these cases).

For the rest of the section, it will be more convenient to work with signal precisions as opposed to signal variances,  $\rho_i \stackrel{\text{def}}{=} \frac{1}{\sigma_i^2}, i \in \{1, \dots, n\}$ .

#### 5.1. Model with Endogenous Information Acquisition

We begin by expanding the model in Section 3 to account for endogenous information acquisition. To this end, we introduce an additional stage to the game: before observing his signal and choosing his vote, player  $i$  has the option to exert effort,  $u_i$ , to improve the precision of his signal from  $\rho_i$  to  $\rho_i g(u_i)$ , at a cost of  $c(u_i)$ . We refer to  $g(\cdot)$  and  $c(\cdot)$  as the precision improvement and effort cost functions, respectively. This new setting is a two-stage game similar in spirit to that of Colombo et al. (2014), that is, in stage 1, players simultaneously choose how much effort to exert, and in stage 2, players simultaneously choose their votes after observing their private signals.

Formally, this extension renders the private signal in Equation (1) a function of one’s effort,

$$s_i(u_i) = q + \epsilon_i(u_i), \tag{7}$$

where  $\epsilon_i(u_i)$  is normally distributed, with  $\mathbb{E}[\epsilon_i(u_i)] = 0$  and  $\text{Var}[\epsilon_i(u_i)] = \frac{1}{\rho_i g(u_i)}$ . To ensure that functions  $c(\cdot)$  and  $g(\cdot)$  have desirable properties, for example, effort

should lead to improved information, we make the following basic assumptions.

**Assumption 2.** *Precision improvement  $g(u)$  and effort cost  $c(u)$  are continuous differentiable functions with  $g(u) \geq 0, c(u) \geq 0, \forall u \geq 0$  (positive),  $g'(u) \geq 0, c'(u) \geq 0, \forall u \geq 0$  (increasing), and  $g(0) = 1, c(0) = 0$  (boundary conditions).*

The boundary conditions are set such that if zero effort is exerted in stage 1, the outcome of the game in stage 2 is identical to that of Section 4 (with strategic agents).

As before, votes are linear in the signal, which means that players’ voting strategy is determined by the weight,  $\alpha_i$ , they place on their signal, but votes now also depend on effort levels since they affect the signal,  $v_i = \alpha_i s_i(u_i) + (1 - \alpha_i)\mu$ . Therefore, the platform’s aggregate quality estimate  $\hat{q} = \sum_{i=1}^n \tau_i v_i$  now also depends on effort. We use the notation  $\hat{q}(\mathbf{u}, \mathbf{v})$  with  $\mathbf{u} = \{u_1, \dots, u_n\}$  and  $\mathbf{v} = \{v_1, \dots, v_n\}$  to make the dependence explicit when necessary.

Up until this stage, we have kept the platform payoff function as general as possible. We now specialize it to a standard form satisfying Assumption 1. In particular, the platform’s payoff is quadratic in the aggregate error  $q - \hat{q}$ , that is,  $\pi = k_1 - k_2(q - \hat{q}(\mathbf{u}, \mathbf{v}))^2$ , where  $k_1 \geq 0, k_2 > 0$  are constants.<sup>12</sup> Specializing the payoff enables us to keep functions  $g(\cdot)$  and  $c(\cdot)$  as general as possible and shift the focus of the analysis to understanding their effects on player effort incentives.

In Section 4, player  $i$  maximized his payoff over his vote  $v_i$ . Here, player  $i$  maximizes over both effort level and vote:

$$\max_{u_i \geq 0} \left\{ -c(u_i) + \tau_i \max_{v_i \in \mathbb{L}} \mathbb{E}[k_1 - k_2(q - \hat{q}(\mathbf{u}, \mathbf{v}))^2 | s_i] \right\}, \tag{8}$$

where the inner maximization corresponds to the second stage in which players vote after observing their signals, and the outer maximization corresponds to the first stage in which players choose their effort levels.

Note that although the payoff function is quadratic in the error, the problem is not quadratic in all decisions, which potentially makes the optimization nontrivial. To proceed, we first use existing results from Section 4 to solve the second stage. This allow us to express the players’ second-stage optimal voting strategies as a function of effort levels, which reduces the two-stage optimization problem, Equation (8), to a simpler single-stage problem optimizing over effort levels.

**Lemma 2** (Problem Reduction).

*i. At the second stage of the game, given a fixed vector  $\mathbf{u}$ , player  $i$ ’s optimal vote is  $v_i^*(\mathbf{u}) = \alpha_i^*(\mathbf{u})s_i + (1 - \alpha_i^*(\mathbf{u}))\mu$ ,*

where  $\alpha_i^*(\mathbf{u})$  and the platform's resulting variance are given by

$$\alpha_i^*(\mathbf{u}) = \frac{\rho_i g(u_i)}{\tau_i \left( \rho_q + \sum_{j=1}^n \rho_j g(u_j) \right)},$$

and 
$$V(\mathbf{u}) = \left( \rho_q + \sum_{i=1}^n \rho_i g(u_i) \right)^{-1}. \quad (9)$$

ii. At the first stage of the game, player  $i$ 's profit maximization problem, Equation (8), is equivalent to the following cost-minimization problem over effort level:

$$\min_{u_i \geq 0} \mathcal{C}_i(\mathbf{u}) = \tau_i V(\mathbf{u}) + c(u_i). \quad (10)$$

For convenience, we define the platform's precision  $\rho(\mathbf{u}) \stackrel{\text{def}}{=} V(\mathbf{u})^{-1} = \rho_q + \sum_{i=1}^n \rho_i g(u_i)$ .

## 5.2. Equilibrium

Before stating the equilibrium, we discuss the conditions under which one exists. From Lemma 2, it suffices to focus on Problem (10). When  $\mathcal{C}_i(\mathbf{u})$  is continuous and convex with respect to  $u_i$  for  $u_i \geq 0$ , there exists at least one equilibrium (Rosen 1965). Given that  $\mathcal{C}_i(\cdot)$  is the sum of two functions, it is convex if both functions are convex. While it is generally common to assume convex cost of information acquisition  $c(\cdot)$  (see, e.g., Vives 2011), convexity of  $\tau_i V(\mathbf{u})$  depends on the properties of the information improvement function  $g(u_i)$ . We have the following result.

**Lemma 3** (Convexity of  $V(\cdot)$ ).  $V(\mathbf{u})$  is convex in  $u_i$  iff

$$\frac{\rho_i}{\rho(\mathbf{u})} \geq \frac{1}{2} \frac{g''(u_i)}{g'(u_i)^2}. \quad (11)$$

A sufficient condition is  $g(\cdot)$  linear or concave.

Lemma 3 has important implications. First, note that  $\rho_i \geq 0$  and  $\rho(\mathbf{u}) \geq 0$  by construction, and thus the left-hand side is positive. It follows that convexity of  $V(\cdot)$  trivially holds when  $g(\cdot)$  is linear as  $g''(\cdot) = 0$  in this case. For instance,  $g(u_i) = 1 + u_i$ , which also satisfies Assumption 2 ( $g > 0, g' > 0$  and  $g(0) = 1$ ). Convexity of  $V(\cdot)$  also trivially holds when  $g(\cdot)$  is concave as  $g''(\cdot) \leq 0$  in this case.

Up until now, we have been working with general functions  $g(\cdot)$  and  $c(\cdot)$ , and our results, although also general, have not addressed the equilibrium outcome. To make headway, we need to impose some additional structural requirements on  $c(\cdot)$  and  $g(\cdot)$ . To see why, consider the first-order conditions of (10)  $\tau_i V' + c' = 0, \forall i$ . Using the expression for  $V'$  (see Equation (33) in the online appendix), these

are equivalent to the following system of nonlinear equations:

$$\begin{aligned} \tau_1 \rho_1 &= \left( \frac{\partial c(u_1)}{\partial u_1} \middle/ \frac{\partial g(u_1)}{\partial u_1} \right) \rho(u_1, \dots, u_n)^2 \\ &\vdots \\ \tau_n \rho_n &= \left( \frac{\partial c(u_n)}{\partial u_n} \middle/ \frac{\partial g(u_n)}{\partial u_n} \right) \rho(u_1, \dots, u_n)^2. \end{aligned} \quad (12)$$

While such nonlinear systems are generally intractable, the structure of these equations motivates the following additional assumption, which will facilitate the computation of the equilibrium.

**Assumption 3.** Let  $k > 0$  be a constant and suppose  $c$  and  $g$  satisfy the following equation:

$$c' / g' = k \cdot g^2. \quad (13)$$

In online Appendix A.5, we show that a broad class of linear and concave forms of  $g(\cdot)$  (which by Lemma 3 suffice for equilibrium existence), and corresponding functions  $c$ , satisfy this requirement. For instance, consider  $g(u_i) = 1 + u_i$ , thus  $g'(u_i) = 1$ . Condition (13) (with  $k = 1$ ) implies  $c'(u_i) = (1 + u_i)^2$ , and thus  $c(u_i) = (1/3)(1 + u_i)^3$ , which is convex, positive increasing for  $u_i \geq 0$ .

Leveraging Equation (13) and some additional properties, we obtain that the system in Equation (12) can have not only a unique solution but one that is also characterizable in closed form.

**Theorem 1** (Equilibrium). Let vectors  $\boldsymbol{\tau} = \{\tau_1, \dots, \tau_n\}$ ,  $\boldsymbol{\rho} = \{\rho_1, \dots, \rho_n\}$ . Suppose  $c(\cdot)$  and  $g(\cdot)$  satisfy Assumptions 2 and 3,  $c(\cdot)$  is convex, and  $g(\cdot)$  is either linear or a concave bijective (invertible) function. Then, the effort-exerting game has a unique equilibrium, characterized by votes  $v_i^*(\mathbf{u}^*) = \alpha_i^*(\mathbf{u}^*) s_i + (1 - \alpha_i^*(\mathbf{u}^*)) \mu$ , where the signal weights  $\alpha_i^*(\mathbf{u}^*)$  are given in Equation (9), and effort levels  $u_i^*$  are given by

$$\begin{aligned} u_i^*(\boldsymbol{\tau}, \boldsymbol{\rho}) &= g^{-1} \left( f_i^*(\boldsymbol{\tau}, \boldsymbol{\rho}) \right), \text{ with} \\ f_i^*(\boldsymbol{\tau}, \boldsymbol{\rho}) &= \frac{\left( \rho_q^2 + \frac{4}{k^{1/2}} \sum_{j=1}^n \left( \tau_j \rho_j^3 \right)^{1/2} \right)^{1/2} - \rho_q}{2 \frac{1}{(\rho_i \tau_i)^{1/2}} \sum_{j=1}^n \left( \tau_j \rho_j^3 \right)^{1/2}}. \end{aligned} \quad (14)$$

Having derived the equilibrium, we now examine how equilibrium effort levels depend on token holdings and information precision.

**Proposition 5** (Comparative Statics). Suppose the conditions of Theorem 1 are satisfied; then equilibrium effort levels (i) monotonically increase in token holdings and (ii) are nonmonotonic in information precisions. Further, consider

two hypothetical players  $i$  and  $i'$  and suppose they have the same precision; then (iii)  $\tau_i > \tau_{i'} \Rightarrow u_i^* > u_{i'}^*$ . Suppose instead the two players have the same token holdings; then (iv)  $\rho_i > \rho_{i'} \Rightarrow u_i^* > u_{i'}^*$ .

Proposition 5(i) states that the more tokens a player has, the more effort he provides in equilibrium—a desirable feature often touted in marketing materials. Relatedly, part (iii) can be interpreted as an equivalent statement on fairness—when comparing two equally informed players, the one with more tokens always exerts more effort in equilibrium. However, as we shall show in Section 5.3, these desirable features are not attributable to the  $\tau$ -weighted aggregation mechanism itself.

Proposition 5(ii) highlights that there is a more subtle (nonmonotonic) relationship between the amount of effort a player exerts and his information precision. Part (iv) shows that when comparing two players with equal token holdings, the one with better information precision exerts more effort in equilibrium. In other words, there is some amount of free-riding from less-informed players, which is not necessarily a desirable feature.

To illustrate these results, Figure 3 shows equilibrium effort levels in a  $n = 2$ -player game assuming a linear information improvement function  $g(x) = 1 + \eta x$  with  $\eta > 0$ . Figure 3(a) (parameters  $\eta = \rho_1 = \rho_2 = \rho_q = 1$ ) highlights the results of Proposition 5 parts (i) and (iii). That is, player 1's effort increases with his share of the tokens  $\tau_1$ , and player 1 exerts more effort than player 2 when  $\tau_1 > 1/2$ . Figure 3(b) (parameters  $\eta = \tau_1 = \tau_2 = 1/2, \rho_2 = \rho_q = 1$ ) shows the more subtle relationship between effort levels and information precision described in Proposition 5 parts (ii) and (iv). The first vertical line marks the point where player 1's precision and effort exceed player 2's, and the second vertical line highlights the nonmonotonicity.

For the rest of the analysis, we assume all results are subject to the conditions of Theorem 1, and we will work with the linear form of  $g(\cdot)$  described above and in Online Appendix A.5.1.

### 5.3. Centralized Solution and Optimality Gap

Having derived player equilibrium effort levels, we now compare the resulting platform accuracy they obtain to that of a centralized setting.

In the centralized case, we assume the platform is maximizing the sum of all expected player payoffs. Noting that  $\sum_{i=1}^n \tau_i V(\mathbf{u}) = V(\mathbf{u}) \sum_{i=1}^n \tau_i = V(\mathbf{u})$ , and using the equivalence between maximizing payoffs and minimizing variance from Lemma 1, the centralized problem can be written as a minimization problem over the platform variance and effort costs given by

$$\min_{\mathbf{u} \geq 0} V(\mathbf{u}) + \sum_{i=1}^n c(u_i). \tag{15}$$

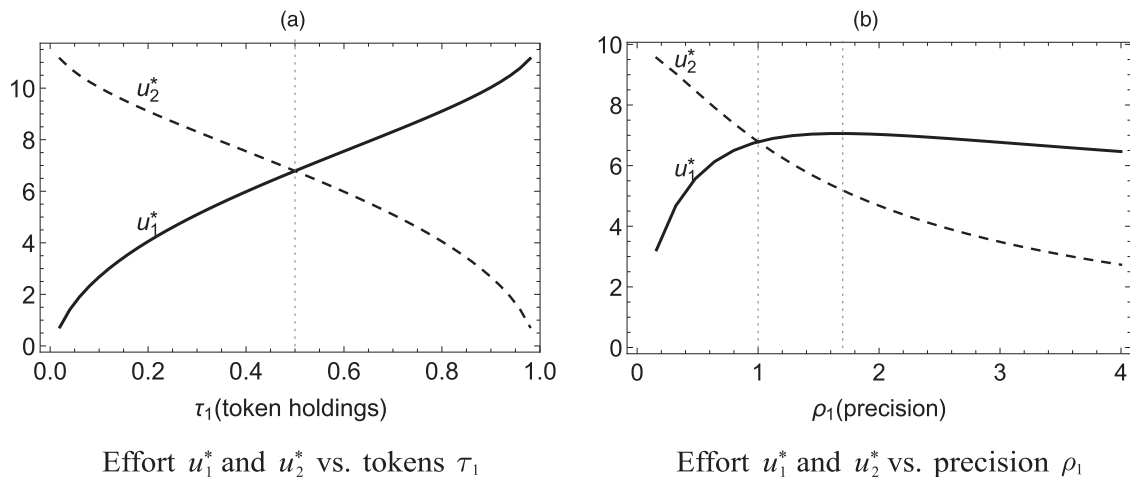
Convexity of  $V(\cdot)$  (see Lemma 3) and  $c(\cdot)$  suffice to guarantee a unique solution. Denote by  $\mathbf{u}^{fb}$  the vector representing this solution and let  $V^{fb} \stackrel{\text{def}}{=} V(\mathbf{u}^{fb})$ . Let  $V^{eq} \stackrel{\text{def}}{=} V(\mathbf{u}^*)$ , with  $\mathbf{u}^* = \{u_1^*(\boldsymbol{\tau}, \boldsymbol{\rho}), \dots, u_n^*(\boldsymbol{\tau}, \boldsymbol{\rho})\}$  given by Equation (14), be the equilibrium platform variance of the decentralized game, and define the optimality gap between the two as

$$\mathcal{G} = V^{eq} - V^{fb}. \tag{16}$$

**Proposition 6.** *Suppose the assumptions from Theorem 1 hold, and let  $\mathbf{1} = \{1, \dots, 1\}$  be an  $n$ -dimensional vector of ones; then*

- i. *the optimal effort levels of the centralized platform are given by  $u_i^{fb}(\boldsymbol{\rho}) = u_i^*(\mathbf{1}, \boldsymbol{\rho}), \forall i \in \{1, \dots, n\}$ ;*
- ii. *the effort levels of the centralized platform exceed those of the decentralized one,  $\mathbf{u}^{fb} > \mathbf{u}^*$ ;*
- iii. *the optimality gap is positive,  $\mathcal{G} > 0$ .*

**Figure 3.** Equilibrium Effort Levels in Two-Player Game with a Linear Information Improvement Function



Part (i) of Proposition 6 shows that the effort levels of the centralized platform have a simple connection to the equilibrium effort levels of the decentralized platform in Equation (14). Mathematically, they can be obtained by setting  $\tau_i = 1, \forall i$ . This implies the distribution of token holdings is irrelevant from the centralized platform’s perspective; intuitively, when it comes to assigning effort levels to players, only the vector of information accuracy,  $\rho$ , matters. Part (ii) follows from Proposition 5(i), which implies  $u_i^*(\mathbf{1}, \rho) > u_i^*(\tau, \rho), \forall n > 1$ , as long as  $\tau_i \neq 1, \forall i$ , that is, no single player has all the tokens. Part (iii) follows from part (ii) and the fact that  $V(u)$  is decreasing in  $u$  (see the proof of Proposition 6).

Unlike our results in Section 4, where strategic players can fully restore first best, here the optimality gap is strictly positive. This is because in contrast to voting, providing effort is costly, and each player stands to benefit differently depending on his token holdings, which implies that player  $i$ ’s individual profit incentives are no longer aligned with the platform’s.

Having shown that first-best accuracy can no longer be restored in the game with effort, even in the best-case scenario in which all agents are strategic, we now examine how the resulting optimality gap is affected by (i) the aggregation mechanism, comparing  $\tau$ - and  $1/n$ -weighted schemes, and (ii) the dispersion in token holdings.

**Irrelevance of Aggregation Mechanism.** Recall that in Section 4, we showed the aggregation mechanism to be irrelevant if all players are strategic. This result continues to hold in the broader game with effort. To see why, consider the voting game in Equation (8) and note that for a fixed vector  $u$ , player  $i$ ’s optimization problem in the second stage parallels the original voting game in Equation (4) analyzed in Section 4 (the two are in fact identical when  $u = 0$ ). When all players are strategic, they adjust their votes in the second stage to undo the platform’s aggregation mechanism and recover the first-best accuracy achievable for a given vector  $u$ . In other words, although the platform’s choice of aggregation mechanism influences how individual players vote, it does not influence the final voting outcome, once all votes are aggregated. This also means that the optimality gap is not affected by the choice of the aggregation mechanism (formal statements are provided in the proof of Lemma 2).

**Relevance of Token Dispersion.** It is important to emphasize that the previous result does *not* imply that the distribution of tokens is irrelevant. In fact, Theorem 1 shows the opposite: each player’s equilibrium effort level depends on the entire vector of token holdings  $\tau$ . This is because the distribution of

tokens directly affects each player’s expected payoff in the first stage of the game, because of the proportionality of returns. Recall that individual payoffs are defined as  $\tau_i \pi$ , that is, players with more tokens stand to benefit more from an increase in platform value and, in turn, are incentivized to exert more effort. Ironically, this runs contrary to some of the claims made by  $\tau$ -weighted platform operators, regarding the role played by the aggregation mechanism in providing adequate effort-exerting incentives. The fact that this effect is tied to the proportionality of returns means that it would persist even if platforms were to adopt an equally weighted  $1/n$  mechanism. This suggests it is particularly meaningful to explore question (ii) regarding the impact of token dispersion, in some depth. We do so next.

#### 5.4. The Impact of Token Dispersion

In this section, we examine the impact of token dispersion on the optimality gap. Before proceeding, we summarize the equilibrium player actions and derive the platform variances in the decentralized and centralized (first-best) settings in Table 2. Within the decentralized setting, it will be useful to distinguish between a special homogeneous case in which all players hold  $1/n$  tokens and the general heterogeneous case in which token holdings are dispersed. Abusing notation for simplicity, let  $\mathbf{n}^{-1} \stackrel{\text{def}}{=} \{1/n, \dots, 1/n\}$  and denote by  $V_{1/n}^{eq} \stackrel{\text{def}}{=} V^{eq}(\tau \rightarrow \mathbf{n}^{-1})$  the platform variance in the purely homogeneous case. This type of distinction is not necessary in the centralized case given token holdings are irrelevant in that setting. We emphasize that this notation is fundamentally different from the one adopted in Section 4, where the subscript  $1/n$  represented the platform choosing an equally weighted aggregation mechanism. Here, subscript  $1/n$  represents the special case of homogeneous holdings under  $\tau$ -weighted aggregation. Without loss of generality, we display all expressions taking the limit  $\sigma_q \rightarrow \infty$  to ease exposition.

We proceed by first examining the simple case in which all players have the same precision, and then we analyze the more complicated heterogeneous case.

##### 5.4.1. Optimality Gap with Homogeneous Precisions.

Similarly to Equation (16), let  $\mathcal{G}_{1/n} \stackrel{\text{def}}{=} V_{1/n}^{eq} - V^{fb}$  denote the optimality gap between the equilibrium platform variance when all players hold  $1/n$  tokens, and first best.

**Proposition 7.** *Under the conditions of Theorem 1, when all players strategically choose effort levels and votes and have homogeneous precisions, that is,  $\rho_i = \rho, \forall i \in \{0, \dots, n\}$ , we have  $\mathcal{G} \geq \mathcal{G}_{1/n} > 0$ , that is, the platform would prefer an even allocation of tokens.*

**Table 2.** Equilibrium Player Actions and Platform Variance as  $\sigma_q \rightarrow \infty$

Setting	Equilibrium votes and effort	Platform variance
Decentralized, dispersed holdings	$v_i(\alpha_i^*(u_i^*(\tau, \rho))); u_i^*(\tau, \rho)$	$V^{eq} = k^{1/4} \left( \sum_{i=1}^n \rho_i^{3/2} \sqrt{\tau_i} \right)^{-1/2}$
Decentralized, $1/n$ holdings	$v_i(\alpha_i^*(n^{-1}, u_i^*(n^{-1}, \rho))); u_i^*(n^{-1}, \rho)$	$V_{1/n}^{eq} = k^{1/4} \left( (1/\sqrt{n}) \sum_{i=1}^n \rho_i^{3/2} \right)^{-1/2}$
Centralized (first best)	$v_i(\alpha_i^*(1, u_i^*(1, \rho))); u_i^*(1, \rho)$	$V^{fb} = k^{1/4} \left( \sum_{i=1}^n \rho_i^{3/2} \right)^{-1/2}$

It is interesting to note that a homogeneous allocation is preferred although players with more tokens exert more effort in equilibrium (see Proposition 5). This is (in part) due to the convexity of  $V^{eq}$  with respect to  $\tau$  (see the proof of Proposition 7 for a step-by-step derivation).

We now examine the more general case with heterogeneous precisions.

**5.4.2. Optimality Gap with Heterogeneous Precisions.**

The set of  $\tau$ 's for which a dispersed allocation dominates is given by  $\{\tau | V^{eq}(\tau) \leq V_{1/n}^{eq} \cap \mathbf{1}'\tau = 1\}$ . We examine dominance numerically, considering a platform with  $n = 20$  players, described by a fixed vector  $\sigma$  (split evenly across the interval  $[0, 1]$ , meaning the average signal precision is 0.5). As before, we draw realizations of vector  $\tau$  from a uniform in  $[0, 1]$  (normalized so that the components sum to 1). For each of those draws, we record the corresponding value of  $\mathcal{G}$  and  $\text{disp}(\tau)$ .

Figure 4(a) shows the optimality gap  $\mathcal{G}$  as a function of  $\text{disp}(\tau)$ . The single point represented by an empty rectangle at the  $x$ -axis origin is the optimality gap resulting from an even allocation of tokens,  $\mathcal{G}_{1/n}$ . By definition,  $\text{disp}(n^{-1}) = 0$  (there is no dispersion when all players have the same number of tokens). From this point, we trace the horizontal dashed line. Points below this horizontal line, in black, represent dominance of dispersed token allocations, over the  $1/n$  allocation. Points above this line, in gray, represent the reverse. There is a positive, albeit low, correlation ( $\sim 10\%$ ) between  $\mathcal{G}$  and  $\text{disp}(\tau)$ , suggesting that, on average, the optimality gap slightly increases with  $\text{disp}(\tau)$ , though the effect is weak.

To bring to light the intricacies of their relationship, we run a more extensive simulation: Figure 4(b) shows that the probability that dispersed tokens are preferred,  $\Pr(\mathcal{G} < \mathcal{G}_{1/n})$ , tends to decrease with  $\text{disp}(\tau)$  (going from  $\sim 50\%$  to  $\sim 20\%$ ). Figure 4(c) expands the analysis beyond probabilities to take into account magnitudes and shows that the average  $\mathbb{E}[\mathcal{G}]$  tends to increase with  $\text{disp}(\tau)$ .<sup>13</sup> These results support the following conclusion.

**Numerical Result 2.** *When all players strategically choose effort levels and votes, and have heterogeneous precisions, equilibrium effort levels decrease and the optimality gap increases, on average, with token dispersion.*

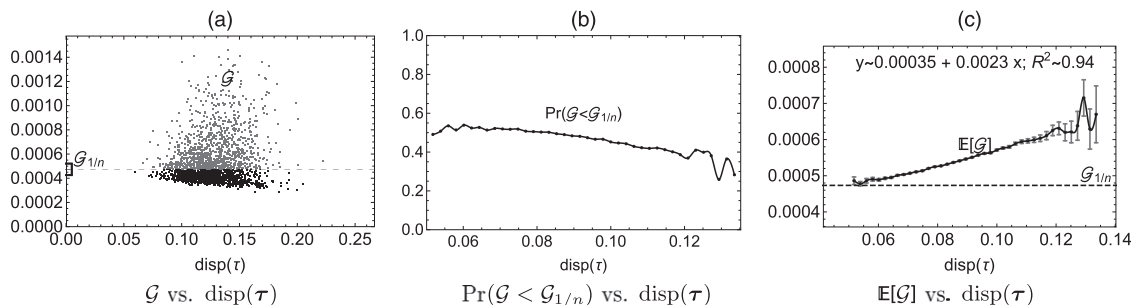
**5.5. Optimal Token Allocation**

Though the previous analysis suggests it is preferable on average to limit token dispersion across players, it does not account for the distribution of precisions that can be used to better inform the platform on the preferred allocation of tokens. Figure 4(b) shows that between 20% and 50% of the time, the platform would be better off with dispersed tokens. It is therefore of interest to examine these instances further.

Consider the equilibrium platform variance  $V^{eq}(\tau)$ , where we are now being explicit about the dependence on the vector of tokens ( $\tau$ ). The allocation of tokens that minimizes the platform's variance with strategic players is given by  $\tau^* \stackrel{\text{def}}{=} \{\arg \min_{\tau} V^{eq}(\tau), \text{subject to } \mathbf{1}'\tau = 1\}$ . For the linear effort model and taking the  $\lim_{\sigma_q \rightarrow \infty} V^{eq}(\tau)$  to simplify expressions, the optimal allocation is obtainable in closed form:

$$\tau_i^*(\rho) = \left( 1 + \frac{1}{\rho_i^2} \sum_{j \neq i} \rho_j^3 \right)^{-1} \tag{17}$$

**Figure 4.** Optimality Gap  $\mathcal{G}$  with Heterogeneous Precisions



As can be intuitively expected, player  $i$ 's optimal token holdings are a function of all players' precisions: they increase in his own precision and decrease in others' precisions.

Next, consider the Euclidean distance  $d(\tau, \tau^*)$ , which measures how far apart a random vector  $\tau$  is from the optimal allocation  $\tau^*$ . Using the same simulation parameters as in Section 5.4, Figure 5 shows that the optimality gap tends to increase with this distance. Furthermore, there is a positive correlation of 85% between the two, suggesting a strong effect. This leads to our last numerical result.

**Numerical Result 3.** *When all players strategically choose effort levels and votes and have heterogeneous precisions, (i) the optimality gap increases, on average, with  $d(\tau, \tau^*)$ ; (ii) there exists a critical distance above which an even allocation dominates all dispersed allocations and a critical distance below which all dispersed allocations dominate an even allocation.*

Note that the numerical results are generally robust when changing: the number of players  $n$ , the fixed vector  $\rho$ , and the metric chosen to compare vectors (e.g., similar results are obtainable if using the geometric angle between vectors, as opposed to the Euclidean distance).

## 6. Conclusion and Limitations

Many blockchain-based platforms have deployed voting mechanisms that use some form of token-weighted aggregation. These platforms rely on the argument that token weighting incentivizes users to provide higher-quality votes, which in turn improves the overall accuracy of the platform. Our results show that this intuition is at best only partially correct. In many cases, the platform could achieve equal or sometimes better accuracy (higher overall payoffs) by pursuing a different aggregation strategy and/or by limiting token dispersion across users.

Like all stylized models, some of our assumptions place limitations on the scope of the paper. For instance, our model assumes a continuous voting and outcome space in  $\mathbb{R}$ . In some cases, votes are constrained to be binary, that is, users can vote content up

or down, or can like or dislike, and in others, votes are constrained to be in discrete ranges, for example, Uber ratings following a completed trip allow users to provide only discrete feedback levels, one to five stars. Constraining the vote space would be an interesting direction to explore.

Second, some of our results rely on players knowing the token holdings, signal precisions, and/or the strategy types of other players. Relaxing these types of assumptions is challenging but could lead to interesting insights, such as whether platforms should purposely obfuscate information or disseminate it as much as possible.

Third, though our model considers endogenous information acquisition, it does not consider endogenous token acquisition, for instance, through an initial coin offering (ICO), or through secondary trading of tokens over time. Presumably, if signal precisions remain stable (e.g., users are constrained to vote on similar topics over time), users may have an incentive to work toward realizing the optimal token allocation derived in Section 5.5. Conversely, when signal precisions vary (e.g., users are asked to vote on different topics over time), and/or when trading tokens is subject to frictions, this type of endogeneity may be less relevant.

As mentioned in the introduction, token-weighted voting has been adopted for many different applications, and our model paints these systems with a broad enough brush to capture some of the common economic trade-offs they all share. As such, this paper should be viewed as a first step in expanding our theoretical understanding of these systems. These limitations could represent interesting directions for future work in this space.

## Endnotes

<sup>1</sup> For instance, when  $\sigma_q \rightarrow 0$ , there is no uncertainty about product quality. Conversely, when  $\sigma_q \rightarrow \infty$ , any value of  $q$  on the real line is equally likely; for example, this could represent an innovative product that has just hit the market. Formally,  $\sigma_q \rightarrow \infty$  represents the case of a uniform improper prior.

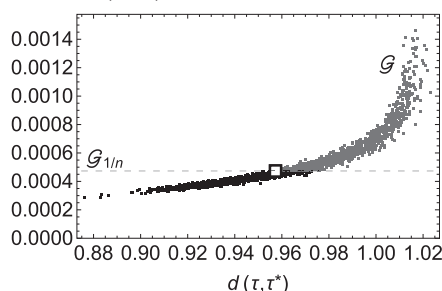
<sup>2</sup> Relaxing these basic assumptions generally has nontrivial consequences and could thus be an interesting direction for future research.

<sup>3</sup> It is known that when all other players are playing linear strategies, the focal agent's best response is linear (see, e.g., Myatt and Wallace 2012, p. 347). This assumption is often adopted in the literature to ensure tractability.

<sup>4</sup> Whereas token value may be driven by additional factors, such as the secondary market liquidity of the tokens, the quality of the underlying blockchain sustaining the tokens, the presence of speculators in the market, and so forth, our primary interest in this paper is to assess whether these platforms can effectively crowdsource information solely based on accuracy incentives. We therefore consider accuracy as the primary driver of token value and, by implication, platform value.

<sup>5</sup> Note that an additional, even less sophisticated, truthful player type that simply votes his raw signal,  $v_i = s_i$ , instead of his Bayesian

**Figure 5.**  $\mathcal{G}$  vs.  $d(\tau, \tau^*)$



posterior is subsumed in the current model by taking the limiting case of a diffuse prior,  $\sigma_q \rightarrow \infty$ , which implies  $\beta_i \rightarrow 1$ , and hence  $v_i \rightarrow s_i$ .

<sup>6</sup> Alternatively, we can consider a central planner that observes all signals and sets player votes  $v_i$  instead of directly setting  $\hat{q}$ , and the same results will hold.

<sup>7</sup> To see this, first note that the optimal  $\hat{q}^*$  solves  $\mathbb{E}[q - \hat{q}^* | s_1, \dots, s_n] = 0$ , which gives  $\hat{q}^* = \mathbb{E}[q | s_1, \dots, s_n]$ . As a result,  $\text{Var}[q - \hat{q}^* | s_1, \dots, s_n] = \text{Var}[q - \mathbb{E}[q | s_1, \dots, s_n] | s_1, \dots, s_n] = \text{Var}[q | s_1, \dots, s_n]$ . Since  $q$  and  $\{s_i\}_{i=1}^n$  are normally distributed, one can use the standard result concerning multivariate normals  $\text{Var}[q | s_1, \dots, s_n] = \sigma_q^2 - \Sigma_{qs} \Sigma_{ss}^{-1} \Sigma_{sq}^*$ , where  $\Sigma_{qs}$ ,  $\Sigma_{ss}^*$  and  $\Sigma_{sq}^*$  are the covariance matrices. A straightforward calculation gives Equation (5).

<sup>8</sup> In the popular U.S. game show *The Price is Right*, which has been running since 1972, contestants (voters in our framework) compete to guess the price of an item but face the prospect of elimination if their guess ends up being above the true price. We show that simple Bayesian posterior voting is generally suboptimal when it comes to dealing with this type of asymmetry in player payoffs.

<sup>9</sup> This assumption, made for tractability, eliminates the need to define additional belief sets but begs the question of how player types could be revealed in practice—a potentially interesting direction for future work. See Chandrasekhar et al. (2015) for a related experimental study.

<sup>10</sup> To see why, consider that as  $n \rightarrow \infty$ , the optimal weight to place on the prior  $1 - \alpha^* = 1 - n/(n + \frac{\sigma_q^2}{\sigma_s^2}) \rightarrow 0$ , whereas truthful voters maintain a fixed weight of  $1 - \beta > 0$ , which by definition is independent of  $n$ .

<sup>11</sup> To be more precise, for Figure 2(c), we ran a simulation drawing 50,000 independent realizations of  $\tau$ . To calculate  $\mathbb{E}[\mathcal{G}_\tau]$ , we discretized  $\text{disp}(\tau)$  into 50 bins of equal length, recording the mean  $\mathcal{G}_\tau$  within each bin. To preserve statistical significance, we discarded bins with fewer than 50 total values.

<sup>12</sup> The constants  $k_1, k_2$  are added for additional flexibility and to keep the value positive (if  $k_1$  is large enough), but alternatively,  $k_1$  could be set to zero and  $k_2$  set to one, and  $\pi$  could be interpreted as relative utility, rather than value.

<sup>13</sup> To be more precise, in both cases, we ran a simulation drawing 1-mm independent realizations of  $\tau$  from a standard (normalized) uniform distribution. We then discretized  $\text{disp}(\tau)$  into 50 bins of equal length, recording  $\Pr(\mathcal{G} < \mathcal{G}_{1/n})$  and  $\mathbb{E}[\mathcal{G}]$  within each bin. To preserve statistical significance, we discarded bins with fewer than 50 total values.

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