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Boston University Center for Adaptive Systems
and
Department of Cognitive and Neural Systems
677 Beacon Street
Boston, MA 02215

A Quantitative Evaluation of the AVITEWRITE Model of Handwriting Learning

R. W. Paine ^{*}, S. Grossberg ^{}, A. W. A. Van Gemmert ^{***}**

^{*}RIKEN Brain Science Institute
Laboratory for Behavior and Dynamic Cognition
2-1 Hirosawa, Wako-shi
Saitama, 351-0198, Japan
Tel: 81-048-462-1111
Fax: 81-048-467-7248

^{**}Department of Cognitive and Neural Systems
and
Center for Adaptive Systems
Boston University
677 Beacon Street
Boston, MA 02215 USA
email: steve@bu.edu
<http://www.cns.bu.edu/Profiles/Grossberg>
phone: 617-353-7857
fax: 617-353-7755

^{***}Motor Control Laboratory
Department of Kinesiology
Arizona State University
Tempe, AZ 85287-0404
Email: VanGemert@asu.edu
Tel: 480-965-4718
Fax: 480-965-8108

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Corresponding author: S. Grossberg

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Abstract

Much sensory-motor behavior develops through imitation, as during the learning of handwriting by children. Such complex sequential acts are broken down into distinct motor control synergies, or muscle groups, whose activities overlap in time to generate continuous, curved movements that obey an inverse relation between curvature and speed. The Adaptive Vector Integration to Endpoint (AVITEWRITE) model of Grossberg & Paine (2000) proposed how such complex movements may be learned through attentive imitation. The model suggested how frontal, parietal and motor cortical mechanisms, such as difference vector encoding, under volitional control from the basal ganglia, interact with adaptively-timed, predictive cerebellar learning during movement imitation and predictive performance. Key psychophysical and neural data about learning to make curved movements were simulated, including a decrease in writing time as learning progresses; generation of unimodal, bell-shaped velocity profiles for each movement synergy; size scaling with isochrony, and speed scaling with preservation of the letter shape and the shapes of the velocity profiles; an inverse relation between curvature and tangential velocity; and a Two-Thirds Power Law relation between angular velocity and curvature. However, the model learned from letter trajectories of only one subject, and only qualitative kinematic comparisons were made with previously published human data. The present work describes a quantitative test of AVITEWRITE through direct comparison of a corpus of human handwriting data with the model's performance when it learns by tracing the human trajectories. The results show that model performance was variable across the subjects, with an average correlation between the model and human data of 89 +/-10 %. The present data from simulations using the AVITEWRITE model highlight some of its strengths while focusing attention on areas, such as novel shape learning in children, where all models of handwriting and the learning of other complex sensory-motor skills would benefit from further research.

Keywords: handwriting, sensory-motor learning, development, imitation, attention, vector coding, adaptive timing, motor cortex, parietal cortex, frontal cortex, cerebellum, basal ganglia, planning, working memory.

1. Introduction

How do children learn curvilinear movements by imitating written letters? How do varying, error-prone movements during learning become correct, efficient movements after repeated trials? The Adaptive VITEWRITE (AVITEWRITE) model of Grossberg & Paine (2000) contributes to answering these questions by modelling the perception/action cycle of handwriting. Although Grossberg & Paine (2000) demonstrated good qualitative performance of AVITEWRITE, the model learned from letter trajectories of only one subject, and only qualitative kinematic comparisons were made with previously published human data. The present work describes a quantitative test of AVITEWRITE through direct comparison of a corpus of human handwriting data with the model's performance when it learns by tracing the human trajectories.

AVITEWRITE describes how the complex sequences of movements involved in handwriting can be learned through the imitation of previously drawn curves. The model shows how initially segmented movements with multimodal velocity profiles during the early stages of learning, corresponding to early childhood, can become the smooth, continuous movements with the unimodal, bell-shaped velocity profiles observed in adult humans (Abend et al., 1982; Edelman & Flash, 1987; Morasso, 1981; Morasso et al., 1983) after multiple learning trials. Early, error-prone handwriting movements with many visually reactive, correctional components gradually improve over time and many learning trials, to become automatic, error-free movements. These learned movements can even be performed without visual feedback, as when a human signs his name with his eyes shut.

The AVITEWRITE model architecture is schematized in Figure 1. The model attempts to explain aspects of how visually reactive and planned movement commands can cooperate or compete to determine what movement will next occur. Because the planned commands are typically learned, the model proposes how new learning can occur even during reactive movements before becoming the basis for later planned movements that are read out of memory. At the start of movement, visual attention (1) focuses on the current hand position and moves to select a target position (2) on the curve being traced. A Difference Vector representation (3) of the distance and direction to the target is formed between an efference copy of the current hand position (*PPV*) and the new target position (*TPV*) (Andersen et al., 1995; Bullock, Cisek & Grossberg, 1998; Bullock & Grossberg, 1988; Georgopoulos et al., 1982; Mussa-Ivaldi, 1988). This Difference Vector activates the appropriate muscle synergy (4) to drive a reactive movement to that target. At the same time, a cerebellar adaptive timing system (5) learns the activation pattern of the muscle synergy involved in the movement.

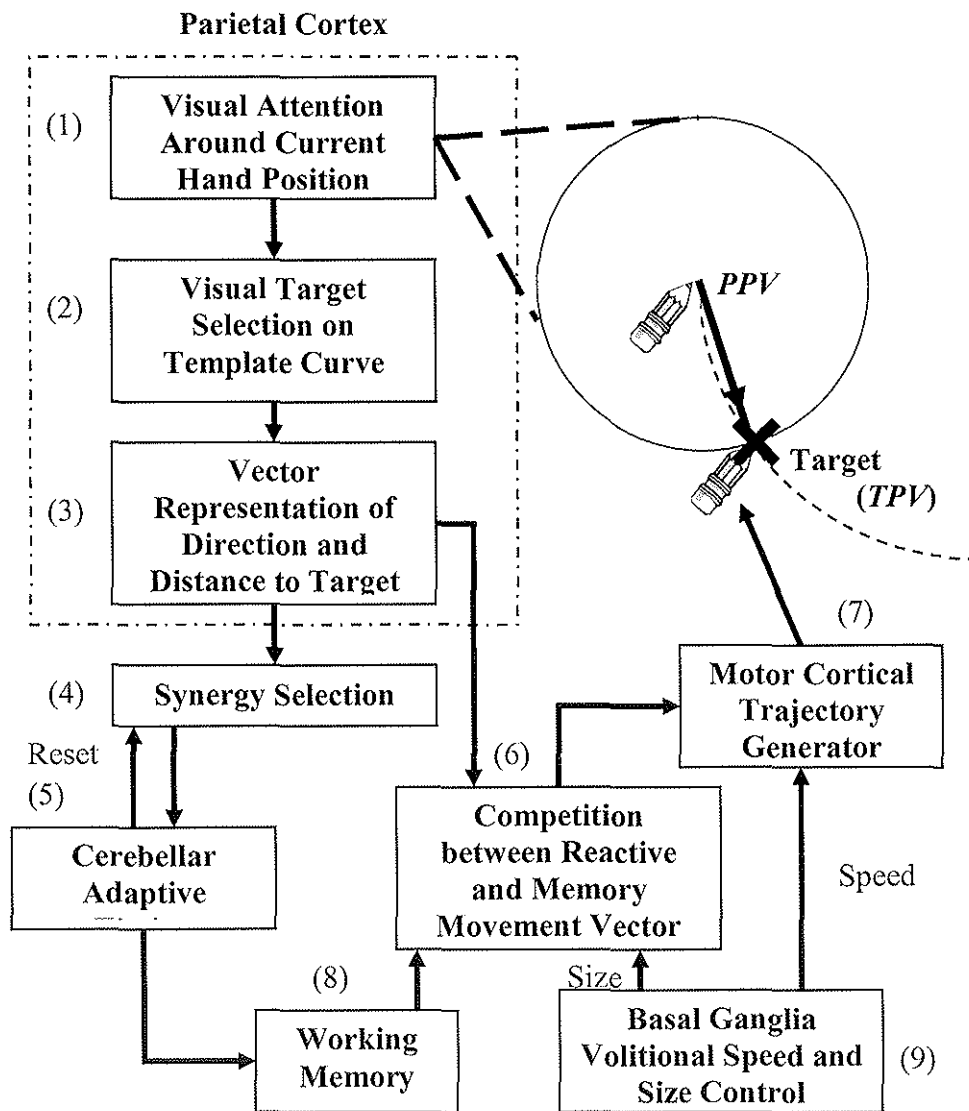


Figure 1. Conceptual diagram of the AVITEWRITE architecture. Numbers in parentheses indicate the order of discussion in the text. *PPV*: Present Position Vector; *TPV*: Target Position Vector

Adaptive timing of strokes may be achieved by spectral timing in the cerebellum. Fiala et al. (1996) and others (Ito, 1984; Perrett, Ruiz, & Mauk, 1993) suggest that the cerebellum may be involved in the opening of a timed gate to express a learned motor gain. A Conditioned Stimulus (CS) arrives via parallel fibers at a population of cerebellar Purkinje cells, triggering a spectrum of phase-delayed depolarizations of the Purkinje cells. When a teaching signal is triggered by an Unconditioned Stimulus (US) in climbing fibers at some fixed Interstimulus Interval (ISI) after the CS, then Long Term Depression (LTD) of the Purkinje cells may occur at that time, leading

to disinhibition of the cerebellar nuclei at that time; hence the term "adaptive timing" (Fiala, Grossberg, & Bullock, 1996; Grossberg & Merrill, 1992, 1996).

The cerebellar adaptive timing system begins to cooperate or compete (6) with reactive visual attention for control of the motor cortical trajectory generator (7). A working memory (8) transiently stores learned motor commands to allow them to be executed at decreased speeds as the speed and size of trajectory generation are volitionally controlled through the basal ganglia (9). Reactive visual control takes over when planned read-out from memory causes mistakes, defined as deviation beyond the attentional radius around the curve. Both the movement trajectory and the memory are then corrected, allowing memory to take over control again. As successive, visually reactive movements are made to a series of attentionally chosen targets on the curve, a memory is formed of the muscle synergy activations needed to draw that curve. After tracing the curve multiple times, planned read-out from memory alone can yield error-free movements.

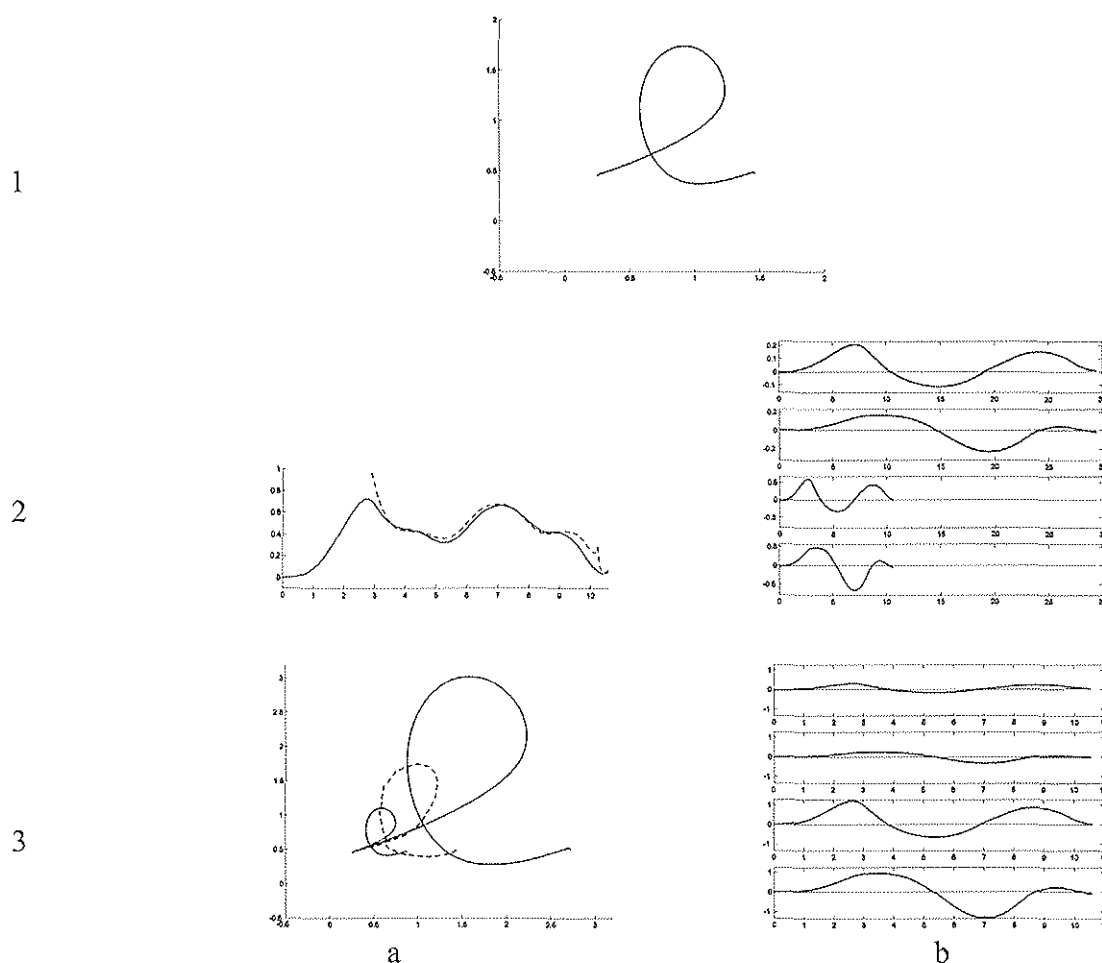


Figure 2: Example of AVITWRITE's writing and various psychophysical properties: (1) Letter *L* learned by tracing a human trajectory; (2a) Tangential velocity of the model's letter *L* (solid) compared to that predicted by the $2/3$ power law (dash); (2b) Velocity profiles after scaling the writing speed of the letter *L* with trajectory invariance; (3a) Size scaling of the letter *L*, halving and doubling the original (dash); (3b) Velocity profiles after size scaling, exhibiting isochrony. [Reproduced with permission from Grossberg & Paine (2000).]

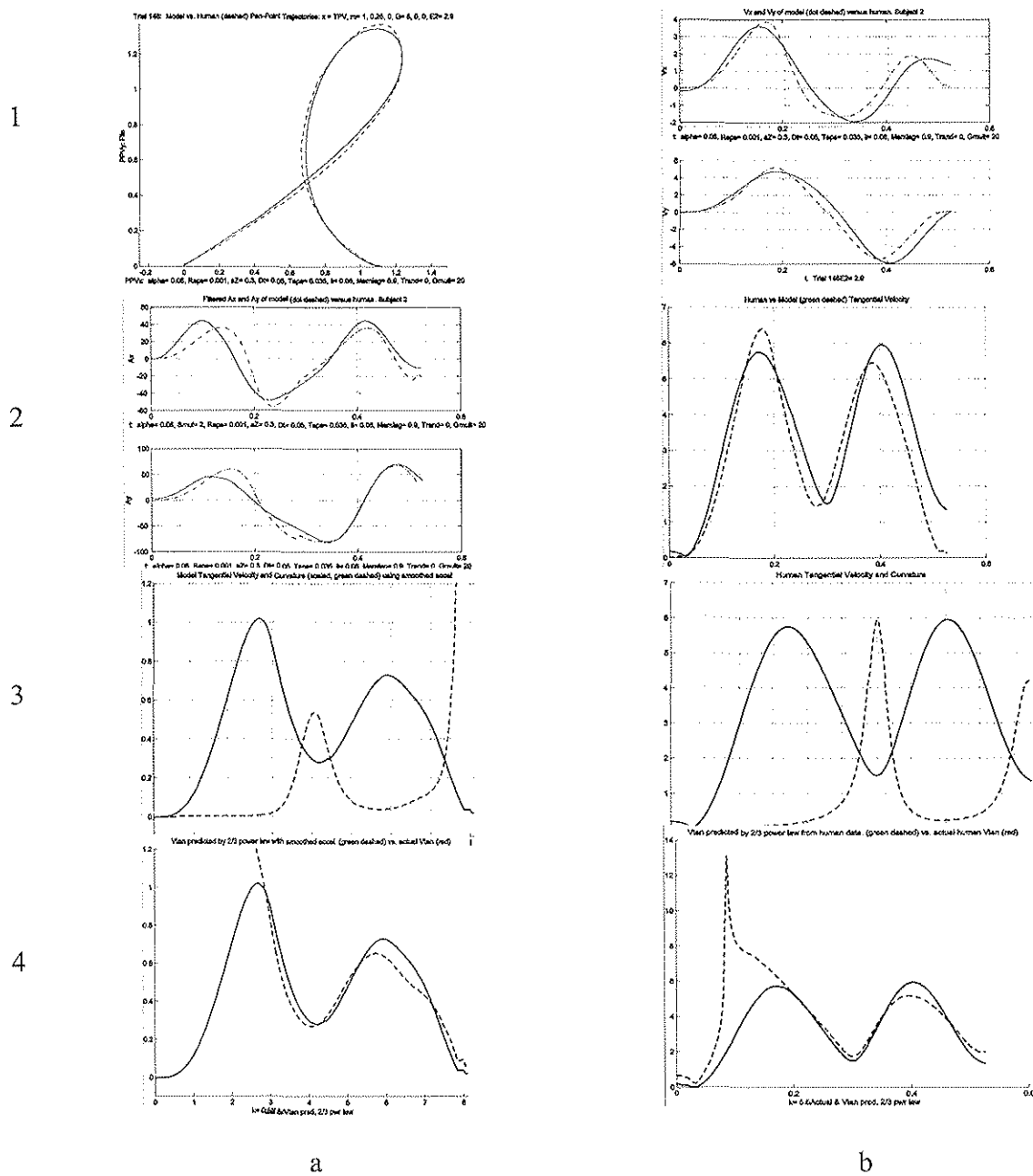


Figure 3: Good correlations for L subject 1. (1a) Human (dash) and model (solid) trajectory; (1b) Human (solid) and model (dash) velocity (x top); (2a) Human (solid) and model (dash) acceleration (x top); (2b) Human (solid) and model (dash) tangential velocity; (3a) Model tangential velocity (solid) and curvature (dash); (3b) Human tangential velocity (solid) and curvature (dash); (4a) Model 2/3 power law tangential velocity prediction (dash) vs. actual model tangential velocity (solid); (4b) Human 2/3 power law tangential velocity prediction (dash) vs. actual human tangential velocity (solid). (Note: Subjects 1-7 are labeled 2-8 in the small figure captions of all figures.)

Several properties of human handwriting movements emerge when AVITEWRITE learns to write a letter; see Figure 2. Size and speed can be volitionally varied (Figure 1, stage 9) after learning while preserving letter shape and the shapes of the velocity profiles (Plamondon et al., 1997; Schillings et al., 1996; van Galen & Weber, 1998; Wann & Nimmo-Smith, 1990; Wright, 1993). Isochrony, the tendency for humans to write letters of different sizes in the same amount of time, is also an emergent property of model interactions (Thomassen & Teulings, 1985; Wright, 1993). Speed can be varied during learning, and learning at slower speeds facilitates future learning at faster speeds (Alston & Taylor, 1987, p. 115; Burns, 1962, pp. 45-46; Freeman, 1914, pp. 83-84). Unimodal, bell-shaped velocity profiles for each movement synergy emerge as a letter is learned, and they closely resemble the velocity profiles of adult humans writing those letters (Abend et al., 1982; Edelman & Flash, 1987; Morasso, 1981; Morasso et al., 1983). An inverse relation between curvature and tangential velocity is observed in the model's performance (Lacquaniti et al., 1983). It also yields a Two-Thirds Power Law relation between angular velocity and curvature, as seen in human writing under certain conditions (Lacquaniti et al., 1983; Thomassen & Teulings, 1985; Wann et al., 1988). Finally, context effects become apparent when AVITEWRITE generates multiple connected letters, reminiscent of carryover coarticulation in speech (Hertrich & Ackermann, 1995; Ostry et al., 1996), and are similar to handwriting context effects reported by Greer & Green (1983) and Thomassen & Schomaker (1986).

2. Methods

Handwriting data were collected from seven adult subjects. The subjects were asked to write separate strings of the letters e, l, i, o, and t (eeee..., llll..., etc.) using cursive handwriting on ten separate trials. Data were collected using a Wacom 12x18 Intuos digital writing tablet with an X and Y pen-tip position sampling frequency of 206 Hz. The raw position and time data were collected and velocity, acceleration, and curvature were calculated. The raw data were then smoothed using a fourth-order low pass digital butterworth filter with a 7Hz cutoff frequency. The data from a subject's ten trials were averaged to create an average string of letters. The first letter of the string was selected as the single letter prototype for that subject. Each letter prototype was then scaled in size for input to the AVITEWRITE model. The size range was equivalent to the letter sizes of the corresponding letters learned in Grossberg & Paine (2000).

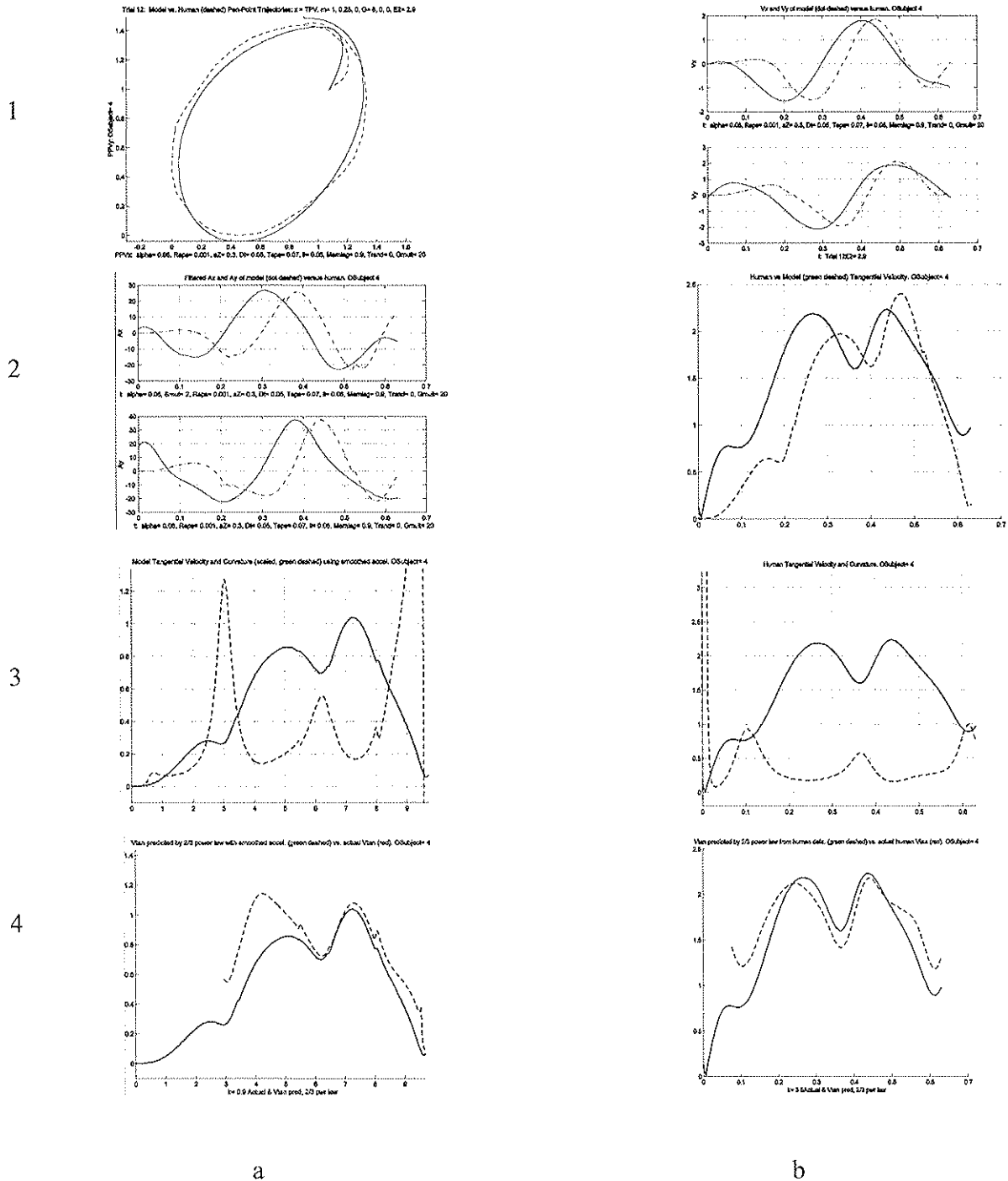


Figure 4: Poor correlations for O subject 3. (1a) Human (dash) and model (solid) trajectory; (1b) Human (solid) and model (dash) velocity (x top); (2a) Human (solid) and model (dash) acceleration (x top); (2b) Human (solid) and model (dash) tangential velocity; (3a) Model tangential velocity (solid) and curvature (dash); (3b) Human tangential velocity (solid) and curvature (dash); (4a) Model 2/3 power law tangential velocity prediction (dash) vs. actual model tangential velocity (solid); (4b) Human 2/3 power law tangential velocity prediction (dash) vs. actual human tangential velocity (solid).

AVITEWRITE learned to draw the letters after multiple learning trials. The letters learned by AVITEWRITE were then compared to the original human templates from which AVITEWRITE learned. Model performance was evaluated by calculating the correlations between the model trajectory, velocity, and acceleration with the human data. Model velocity and acceleration were first scaled to the time and amplitude range of the human data. Further, the correlation between the model's tangential velocity and the tangential velocity predicted by the two-thirds power law was calculated. The correlation between the human tangential velocity and that predicted by the two-thirds power law was also calculated. Correlations were calculated using Equation 14 in the Appendix, based on Equation 8 from Edelman & Flash (1987).

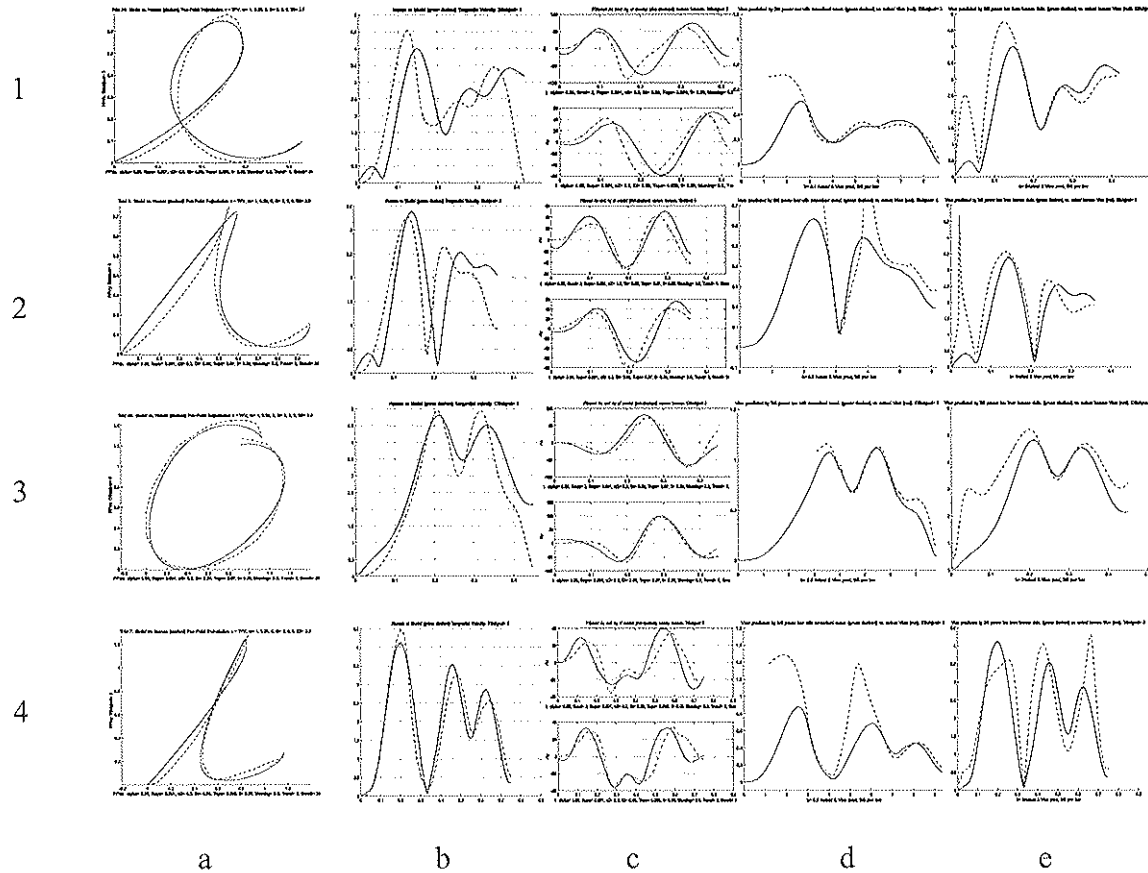


Figure 5: Simulations with best correlations: (a) trajectory of model vs. human (dash); (b) tangential velocity of model (dash) vs. human (solid); (c) x (top) and y (bottom) acceleration of model (dash) vs. human (solid); (d) tangential velocity predicted by the 2/3 power law (dash) compared to model tangential velocity (solid); (e) tangential velocity predicted by the 2/3 power law (dash) compared to human tangential velocity (solid). (1) E of subject 1; (2) I of subject 1; (3) O of subject 7; (4) T of subject 4. See Table 3 for correlations.

3. Results

The results of the simulations are shown in Figures 3-6 for the best and worst model results on individual letters. The correlations between the model and the human data, averaged over x and

y position, velocity, and acceleration over all letters for all subjects, are shown in Table 1. The correlations were calculated using Equation 14. Table 2 shows the average correlations between the tangential velocities predicted by the Two-Thirds Power Law and the model's and humans' tangential velocities. The correlations are further subdivided for the best (top five rows) and worst (bottom five rows) model results for individual letters in Table 3.

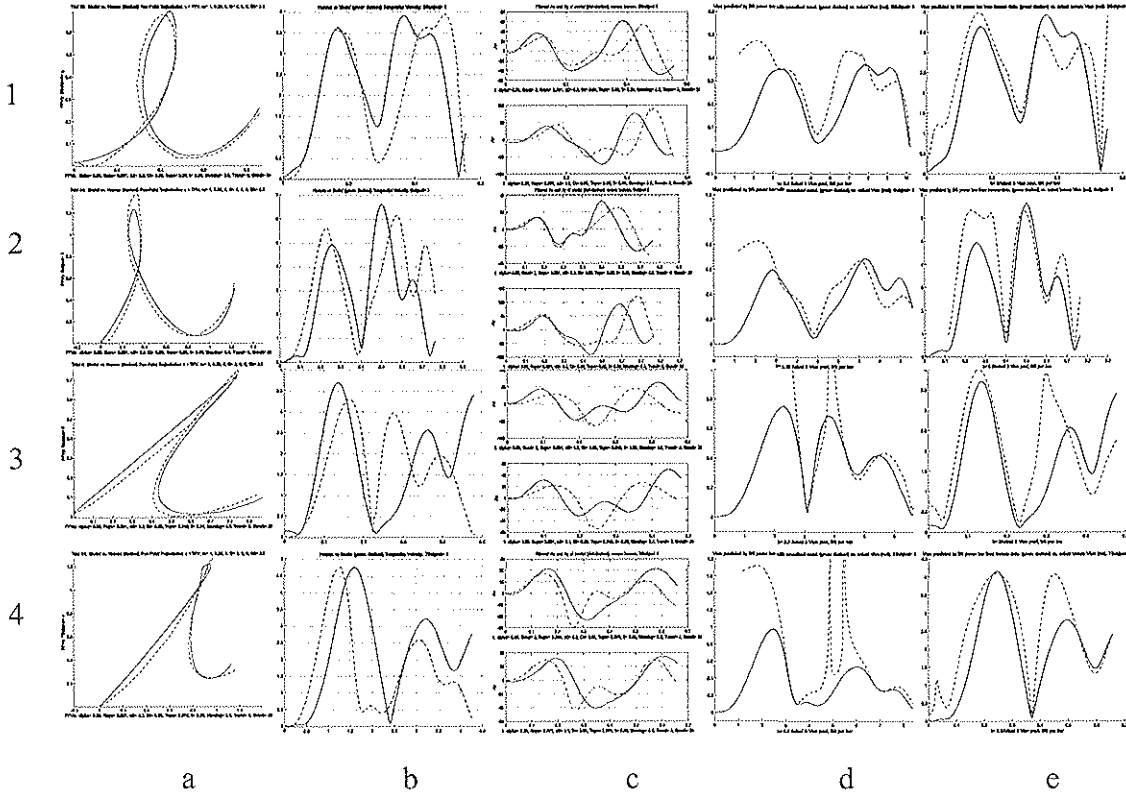


Figure 6: Simulations with worst correlations: (a) trajectory of model vs. human (dash); (b) tangential velocity of model (dash) vs. human (solid); (c) x (top) and y (bottom) acceleration of model (dash) vs. human (solid); (d) tangential velocity predicted by the 2/3 power law (dash) compared to model tangential velocity (solid); (e) tangential velocity predicted by the 2/3 power law (dash) compared to human tangential velocity (solid). (1) E of subject 2; (2) L of subject 2; (3) I of subject 5; (4) T of subject 3. See Table 3 for correlations

Note that some correlations exceed 100%, as in the “104%” correlation of y position in letter *i* of subject 1. This is an artifact of the correlation index used by Edelman & Flash (1987), who also reported correlations greater than 100% in some instances (cf., their Figures 3-6). Their equation is used here to allow direct comparison between their results and the present model.

Position	Velocity	Acceleration	Total
96 +/- 6	83 +/- 14	77 +/- 20	89 +/- 10

Table 1: Average correlation (%) with standard deviation between AVITEWRITE and human data across all letters *e*, *l*, *i*, *o*, *t*.

Note that some correlations exceed 100%, as in the "104%" correlation of y position in letter i of subject 1 in Table 3. This is an artifact of the correlation index used by Edelman & Flash (1987), who also reported correlations greater than 100% in some instances (cf., their Figures 3-6). Their equation is used here to allow direct comparison between their results and the present model.

AVITEWRITE	Human
83 +/- 10	86 +/-7

Table 2: Correlation (%) with standard deviation between actual tangential velocity and two-thirds power law prediction.

4. Discussion

This work further quantifies the performance of the AVITEWRITE handwriting learning model by comparing model performance to that of a group of human subjects. The model learned by tracing the trajectories for 5 average letters from 7 human subjects, for a total of 35 letters. Each average letter was generated from 10 writing samples per subject.

Letter/ Subject	X Position	Y Position	Tangential Velocity	X Acceleration	Y Acceleration	Model 2/3 Power Law	Human 2/3 Power Law
E/1	89	103	86	95	103	85	77
L/1	98	99	95	91	97	95	89
I/1	98	104	90	90	102	56	85
O/7	101	101	96	93	96	98	89
T/4	100	100	87	82	84	81	89
E/2	95	91	70	45	29	74	86
L/2	98	93	59	67	30	79	85
I/5	82	93	36	37	46	65	74
O/3	73	82	86	37	43	86	91
T/3	94	100	47	89	96	88	93

Table 3: Correlations (%) for simulations shown in Figures 3-6: Best (top five rows) and worst (bottom five rows) results.

The results show that model performance was variable across the subjects, with a maximum total correlation of 100% and a minimum of 63%. As seen in Table 1, the average correlation was 89% with a standard deviation of 10%. Analysis of the worst case simulation results, seen in Figures 4 and 6, indicates that the main differences between the human data and model output are a variable stretching or compression of parts of the model velocity and acceleration profiles relative to the human profiles.

One should note that the available human data were not from individual letters, but from connected letters (eeee, llll, etc.). The first letter of each subject’s sequence was selected for each of the ten trials per letter, after smoothing and averaging as described in the Methods. However, the human letter sequences did not generally have zero initial and final velocity and acceleration. Hence, there is a problem with non-zero starting and stopping velocities in the human data against which the model was compared, in contrast with the zero velocity and acceleration initial conditions and equifinality observed in the AVITEWRITE model output, as seen in Figures 3 and 4 (1b, 2b). Finally, note that the 2/3 Power Law prediction of tangential velocity has singularities at points where the curvature is zero (Equations 15 and 17), which occurs at points of zero acceleration at the start of each movement. These singularities were removed from the figures for clarity, and are the cause of the poor match between model output and 2/3 power law prediction at the extremes of the figures, as seen in Figure 3 (4a). Note that a similar singularity occurs in the human data of Figure 3(4b).

Letter	Average Attentional Radius (r_a) +/- standard deviation	Average Number of Trials for Model to Learn +/- standard deviation
e	0.034 +/- 0.002	33 +/- 27
l	0.044 +/- 0.005	104 +/- 158 (range: 14-447)
i	0.054 +/- 0.010	20 +/- 24 (range: 4-74)
o	0.069 +/- 0.005	21 +/- 12
t	0.09 +/- 0.05	12 +/- 10

Table 4: Model parameters for five letters across seven subjects. Note: attentional radius (r_a) held constant during learning for a given subject’s letter.

The only model parameter which was varied across letters and subjects was the attentional radius, as seen in Table 4. AVITEWRITE makes essential use of visual spatial attention to determine where the hand will move to imitate a curve. Attention was modeled algorithmically since it was not the main focus of Grossberg and Paine (2000). The model assumes, for simplicity, that attention may be focused within a circular region around the present fixation point. In the model, visual spatial attention is initially focused around the current hand position on a template curve (Figure 1, Box 1). If subsequent movement deviates from the attentional radius around the curve due to memory inaccuracy, then a new target is chosen on the curve. Decreasing the attentional radius increases the position correlation between the model and the template curve being traced at the cost of more learning trials for convergence to error-free performance. An excessively small attentional radius may prevent convergence in a reasonable period of time, just as an excessively large attentional radius will yield a poor trajectory, which converges quickly. Experimental data suggest that superior frontal, inferior parietal, and superior temporal cortex are part of a network for voluntary attentional control (Hopfinger et al., 2000), which is critical for directing unpracticed movements (Richer et al., 1999, p. 1427). Jueptner et al. (1997a, 1997b) reported that the prefrontal cortex was activated in a finger movement-sequence learning task during new learning but not during automatic performance after learning. Further, the left dorsal prefrontal cortex was reactivated when subjects paid attention to the performance of a previously learned movement sequence (Jueptner et al., 1997b, p. 1313). Although no data are available that characterize a precise mechanism for modulating attention during movement learning,

AVITEWRITE assumes that attention can be voluntarily controlled to achieve a desired level of accuracy, or else to complete learning in a limited time at the expense of accuracy.

Few published handwriting models attempt to measure their results through quantitative comparisons with a corpus of human data. One prior model which does so is the Edelman & Flash (1987) minimum snap model. For this reason, we compare AVITEWRITE to the minimum snap model here. A more extensive review of handwriting models and discussion with regard to AVITEWRITE can be found in Grossberg & Paine (2000).

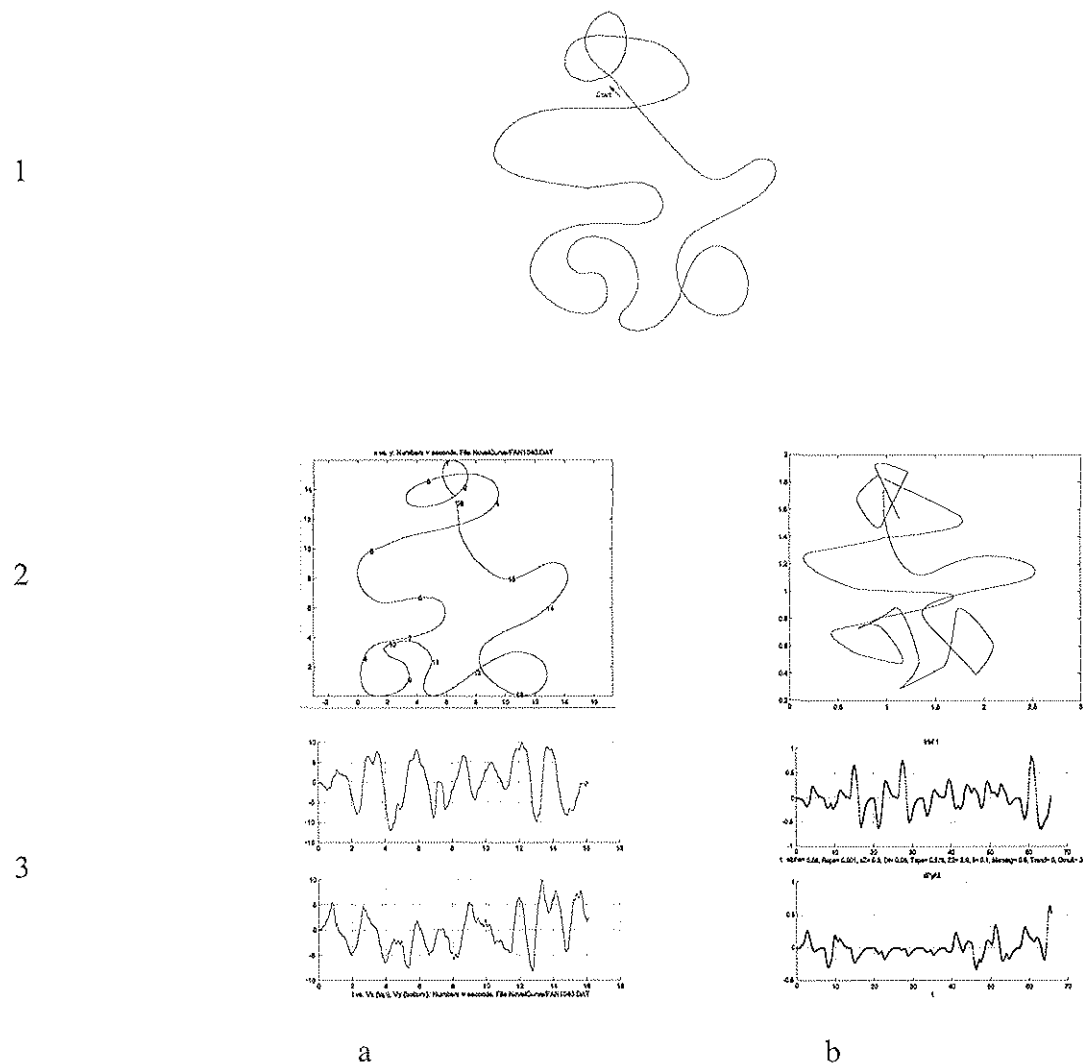


Figure 7: Novel curve learning: (1) Template curve presented to human and model; (2a) Human trajectory on copying trial 1; (2b) AVITEWRITE model trajectory on tracing trial 1; (3a) Human x (top) and y (bottom) velocity profiles for the trajectory in 2a; (3b) AVITEWRITE x (top) and y (bottom) velocity profiles for the trajectory in 2b.

Edelman & Flash (1987) presented a bottom-up model of trajectory formation based on dynamic minimization of the square of the third (jerk) or fourth (snap) derivative of hand position. The

version which minimizes snap yielded better correlation with human experimental data. The model assumes that all letters are formed by a concatenation of shape primitives, such as "cup", similar to a letter U, and "oval", like a letter O. Further, the model generates each stroke primitive by use of a viapoint, an intermediate target prior to the end of the stroke. The model output is compared to human experimental data, and strong correlations are reported between model-generated position, velocity, and acceleration traces and the human counterparts. The inverse relation between movement velocity and curvature seen in human writing is demonstrated by the model. The use of numerical estimations of the degree of fit to the data is emphasized and contrasted with the purely subjective fit estimates in some models.

One general problem with this hypothesis is that no known brain mechanisms can minimize a quantity across an entire movement trajectory before it occurs. Otherwise expressed, this model assumes that a kind of global information is available which seems to be biologically unrealizable. Even if this general concern could be alleviated, were humans to generate handwriting movements by minimizing the snap, it is unclear how they could minimize the fourth, or even the third, derivative of hand position across an entire movement trajectory. Golgi tendon organs measure muscle tension (Gordon & Ghez, 1991). Further, Matthews (1972) showed that muscle receptors exist that are sensitive both to the length of the muscle and to the velocity of stretching. Thus, the first derivative of hand position is probably available to higher motor control centers. However, evidence supporting neural computation of higher derivatives of hand position is lacking. This raises the concern that jerk or snap minimization may not be used in any form during human trajectory planning. Finally, the minimum snap model makes use of viapoints which are explicitly chosen at the curvature maxima. In contrast, AVITEWRITE uses local information to learn a written trajectory, notably an automatic, attention-based target selection algorithm.

Parameters	X Position	Y Position	X Velocity	Y Velocity	X Acceler- ation	Y Acceler- ation	Average Correl- ation
$\Delta t = 0.06$ $r_a = 0.07$	73	82	68	76	37	43	63
$\Delta t = 0.055$ $r_a = 0.075$	85	90	80	86	55	62	76

Table 5: Correlations (%) between model performance and human data for Subject 3's letter O when spectral density (Δt) is varied as well as the attentional radius (r_a).

Edelman & Flash computed the correlations of the minimum snap model to four curves (hook, cup, gamma, and oval) generated by three subjects, with ten curve samples per subject. Quantitatively, the minimum snap model yields better fits to the kinematic data than AVITEWRITE, with a mean correlation of 97.0%, +/- 0.3%. However, it should be noted that the minimum snap model required the extraction of a different set of parameters from each curve in order to regenerate that curve. Such an approach tacitly assumes that a different subject generates each curve, which was not true in the experiments. The AVITEWRITE model also achieves higher correlations if parameters are varied in this way (e.g., an improvement from 63% to 76% as seen in Table 5 and Figure 8). In the AVITEWRITE simulations that are reported here, only the attentional radius was varied since varying levels of attention are known to affect

task performance (Hopfinger et al., 2000; Richer et al., 1999; Jueptner et al., 1997a, 1997b). Other system parameters, such as those involved in the neuronal response dynamics and synaptic modification of Equations 1-3 in the Appendix, were held constant based on the assumption that they would not vary significantly among different humans or different letters. The volitional speed command and the corresponding Purkinje cell spectral activation density (Δt) (Equations 2, 8, and 9) were held constant for this analysis, although Grossberg & Paine (2000) did show that improved performance may be achieved if learning begins at a slow speed and gradually increases across trials via increases in the volitional speed command (GO signal) and the spectral density. The “spectrum” refers to the phase-delayed pattern of Purkinje cell activation hypothesized to occur in response to a Conditioned Stimulus (CS) that arrives via parallel fibers at the Purkinje cell population. This pattern of Purkinje cell activity plays an important role in the hypothesized mechanism of adaptive timing used for movement learning in the AVITEWRITE model (Fiala, Grossberg, & Bullock, 1996; Grossberg & Merrill, 1992, 1996; Grossberg & Paine, 2000). The “spectral activation density” refers to the time delay between Purkinje cell activations. As seen in Figure 4 (1a), the position correlation was relatively low (with a value of $c_x = 73\%$ and $c_y = 82\%$) for the letter “O” at the given GO signal and spectral density (Equations 2, 8, and 9) for the attentional radius of 0.07, even though the same GO and spectral density yielded a much better result for the letter T for this same subject, with position $c_x = 94\%$ and $c_y = 100\%$ and an attentional radius of 0.075. Higher correlations could have been achieved if different parameters had been used. For example, a modest improvement in simulating subject 3’s letter “O” can be achieved simply by increasing the spectral density and attentional radius slightly, yielding a 13% improvement in overall correlation from 63% to 76% (Table 5, Figure 8), with position $c_x = 85\%$ and $c_y = 90\%$.

This result points to a weakness of the current AVITEWRITE model; that is, the current lack of teaching signal normalization. For the letter “i” shown in Figure 5 (2a), distant targets are initially chosen, yielding large Difference Vectors, due to the low curvature of the initial portion of the letter. (See Grossberg & Paine, 2000 for details of the target selection algorithm.) The initial, shallow curve of the “i” upstroke is approximated by AVITEWRITE as a nearly straight line for the attentional radius of 0.07 used. As seen in Equations 3-7, the larger the Difference Vector, or DV, the larger the memory trace, and the resulting speed, other things being equal. Other things may not be equal, however, since the DV is gated by a volitional GO signal that releases the movement and controls overall movement speed (Bullock and Grossberg, 1988). Such a GO signal is controlled by the basal ganglia in the brain (Horak & Anderson, 1984a, 1984b; Turner et al., 1998). The present model simulations focus more on trajectory learning and performance by cortical and cerebellar circuits than on basal ganglia dynamics. For accurate learning and performance of letters of multiple sizes, the present simulations make clear that interactions of these brain regions with the basal ganglia are also needed. Related modeling work (e.g., Brown, Bullock, and Grossberg, 2003) has begun to clarify how these interactions work and will provide a foundation for future elaboration of the AVITEWRITE model.

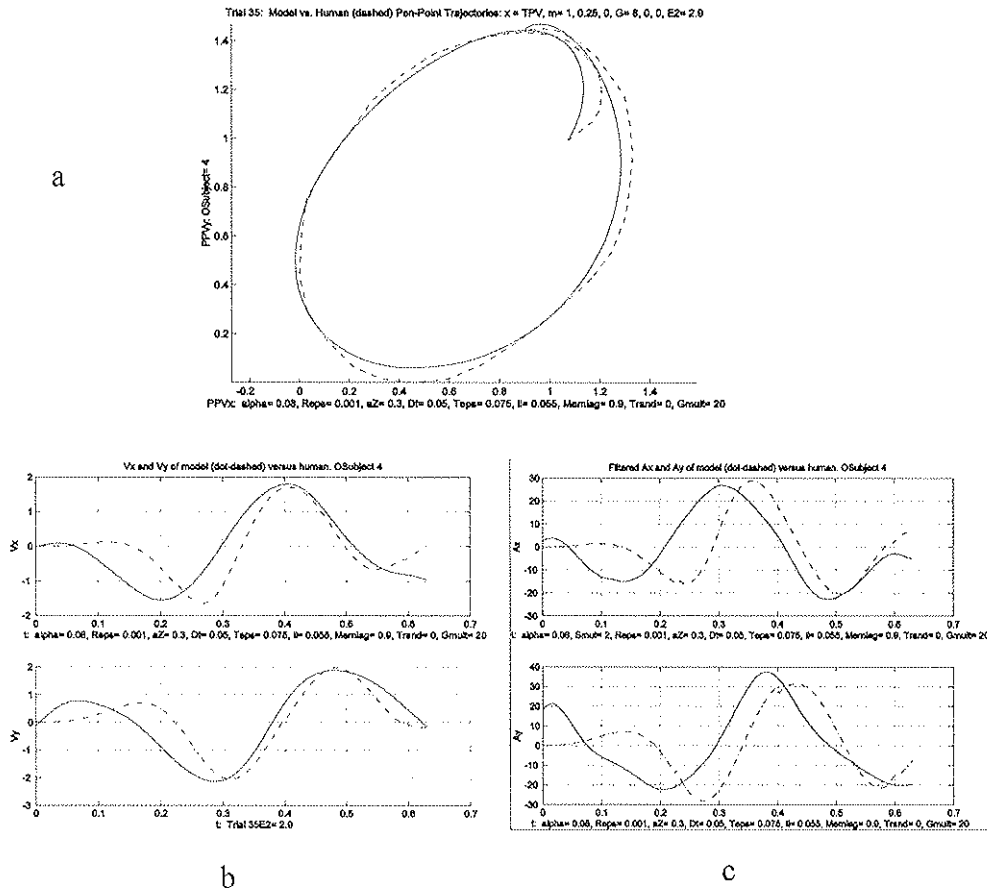


Figure 8: Results of using a different spectral density ($\Delta t = 0.055$) and attentional radius ($r_a = 0.075$) when AVITEWRITE learns the letter O from subject 3. (a) Trajectory of model (solid) and human (dash); (b) X(top) and Y (bottom) velocity of model (dash) and human (solid); (c) X (top) and Y (bottom) acceleration of model (dash) and human (solid). Compare with results when the same spectral density was used for all letters in Table 3 and Figure 4.

Further evaluation of the AVITEWRITE model would also be facilitated if there existed more studies of handwriting learning in children. Many handwriting studies have been done with children in order to improve the teaching of handwriting (Freeman, 1914; Burns, 1962; Hendricks, 1976; Furner, 1983). These studies reveal the progression of movement proficiency over years of practice. The fact that handwriting performance can improve over years of practice suggests that it is the result of cumulative learning from many individual writing trials. Unfortunately, few scientific studies of either adults or children address short-term changes in handwriting performance due to learning on individual movement trials. Preliminary attempts to learn a novel shape (Figure 7) were begun as part of this work. However, only adult subjects were available for the experiments. These adults, with years of writing experience, were able to copy the novel shape with smooth, continuous velocity profiles on the first trial, whereas AVITEWRITE begins learning each new shape in a naive state, and initially generates more segmented velocity profiles and straight line curve segments. These segmented velocity profiles

reflect a more discrete, multiple-stroke-driven strategy on early movement trials in AVITEWRITE compared to adult humans (Figure 7, see also Figure 23 of Grossberg & Paine, 2000). Edelman & Flash (1987), among others (Morasso, 1986; Wing, 1980), propose that this problem may be overcome by learning a discrete set of motor primitives, which are then concatenated to generate arbitrary shapes. Although the AVITEWRITE model does not explicitly describe motor primitives, concatenation of learned letters with coarticulatory context effects (Figure 9) was demonstrated in Grossberg & Paine (2000). The problem remains open of what motor primitives, including whole letter shapes, may be learned to generate a complex motor repertoire and how they may be rapidly assembled to generate an arbitrary, novel shape.

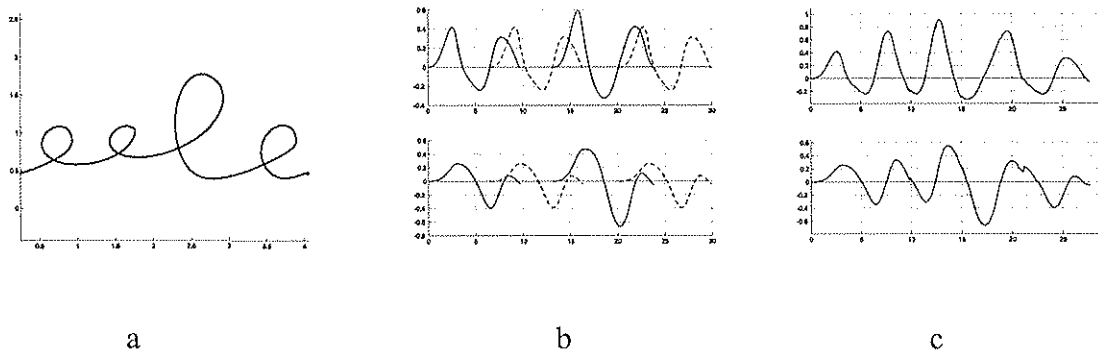


Figure 9: Simulation of coarticulatory context effect by varying the overlap between adjacent letters. As seen also in human data from Thomassen & Schomaker (1986), the second *e* in *eele* is smallest due to overlap with the upstroke of the following *l*. (a) Model trajectory; (b) Individual velocity profiles of the letters staggered through time (x top, y bottom); (c) The velocity profile of the connected letters *eele* generated by AVITEWRITE. Reproduced with permission from Grossberg & Paine (2000).

5. Conclusion

The AVITEWRITE model describes aspects of how the cerebral cortex, cerebellum, and basal ganglia may interact during complex learned handwriting movements. There is both cooperation and competition between reactive vision-based imitation and planned memory read-out. The model suggests that there is an automatic shift in the balance of movement control between cortical and cerebellar processes during the course of learning. AVITEWRITE shows how challenging psychophysical properties of planar hand movements may emerge from this cortico-cerebellar-basal ganglia interaction.

The present data from simulations using the AVITEWRITE model have highlighted some of its strengths while focusing attention on areas where all models of handwriting and the learning of other complex sensory-motor skills would benefit from further research. A key area concerns how to generalize prior sensory-motor learning to facilitate the learning of novel curves. Further evaluation of all handwriting models would also be facilitated by the availability of experiments that study novel curve learning in younger subjects, who may not yet have developed putative motor primitives or the skill for concatenating them for arbitrary novel curves.

Appendix: Model Equations

For a complete description of the AVITEWRITE model implementation, please see Grossberg & Paine (2000) and Paine (2002). At the beginning of movement learning, a visual target position (TPV) is chosen in a predefined forward direction on the curve to be learned such that the line from the current hand position, PPV , to TPV never exceeds an attentional threshold distance, or radius, from the curve being traced (the template curve). The difference vector to the target, DV_{vis} , is integrated toward the value of $TPV - PPV$, as in Equation (1):

Visual Difference Vector

$$\frac{dDV_{vis}}{dt} = [-\mu_1(DV_{vis}) + \mu_2(TPV - PPV)(1 - H(RH(tube) - \varepsilon))]. \quad (1)$$

In (1), R is the learned cerebellar output. $H(tube)$ equals 1 if the PPV is within the attentional radius of the template curve being traced, and it equals zero otherwise. Function $H(RH(tube) - \varepsilon)$ equals one if PPV is within the attentional radius of the template curve and the cerebellar output, R , is above some threshold value, ε . Otherwise, $H(RH(tube) - \varepsilon)$ equals zero and the visual difference vector, DV_{vis} , decays to zero. In (1), $\mu_1 = 1$; $\mu_2 = 0.25$; and $\varepsilon = 0.001$. Thus, if memory is available and movement is sufficiently accurate, then memory directs the movement. If the memory signal is too small or an error is made by deviating from the attentional radius around the template curve, then vision controls the movement direction.

Cerebellar learning is simulated as follows. A spectrum of Purkinje cell (PC) responses is created using Equation (2):

Cerebellar Spectral Components

$$g_i = \gamma((t - (i - 1) \cdot \Delta t)^2)(B - (t - (i - 1) \cdot \Delta t)^{2.9}). \quad (2)$$

In (2), $\Delta t = 0.06$: the time between the start of adjacent Purkinje cell spectral components. Term g_i models activation of Purkinje cell i at time t . $\gamma = 0.0136$ and $B = 25$.

The i^{th} synaptic weight z_i between the parallel fibers and the Purkinje cells is modified based on the climbing fiber inputs as described in Equation (3):

Cerebellar Synaptic Weights

$$\frac{dz_i}{dt} = \alpha_2 g_i (-z_i + \alpha(TPV - PPV)) \cdot H(TPV - PPV). \quad (3)$$

Each synaptic weight is modified only if its spectral component g_i is active and visual target information is available. Visual target information is defined by TPV . Climbing fiber activity is assumed to be proportional to the size of the difference between the target position, TPV , and the present position, PPV , with synaptic weights increasing in proportion to the value of $TPV - PPV$.

in Equation (3). $H(TPV - PPV)$ equals 1 if $(TPV - PPV) > 0$, and it equals 0 otherwise. Parameters $\alpha_z = 0.3$ and $\alpha = 0.08$ in (3).

The gated spectral activity $h_i = g_i z_i$. Each term h_i provides a local view in time of the learned information. The sum of these terms provides a population response to the effects of learning due to the climbing fiber teaching signals. This population response of the Purkinje cells is assumed to form the adaptively timed cerebellar output, R , as in Equation (4):

$$R = \sum_i h_i. \quad (4)$$

The cerebellar output, R , is generated at a fixed rate in response to a given density of Purkinje Cell spectral components g_i through time. The output rate of R can be altered by changing spectral density.

A cortical Working Memory buffer is hypothesized to allow performance of learned movements at variable speeds while preserving movement and velocity profile shape. R is temporarily stored in a working memory buffer, simulated as a discretely sampled set of values from the continuous cerebellar output:

$$WM(t) = R(t_i) \text{ for } t_i < t < t_{i+1}. \quad (5)$$

In (5), t_i is the i^{th} time that DV_{gate} , which is defined in (11) below, becomes zero from a positive value. At time $t = 0$, $WM(0) = R(0)$. This working memory output, WM , is combined with the visual difference vector, DV_{vis} , and scaled by a volitionally-controlled size-scaling GRO signal, S , to form the size-scaled, memory-enhanced difference vector, DV_s :

$$DV_s = S \cdot (WM + DV_{vis}). \quad (6)$$

In (6), $S = 0.3$

The outflow PPC command is determined by multiplying DV_s with a volitionally-controlled speed-scaling GO signal G , and then integrating this product:

Present Position Vector

$$\frac{dPPV(t)}{dt} = DV_s \cdot G(t). \quad (7)$$

Thus $DV_s \cdot G$ computes an outflow representation of commanded movement speed. The speed-controlling GO signal G is defined as follows:

GO Signal

$$\frac{dG}{dt} = \gamma_1(-G + J), \quad (8)$$

where parameters $\gamma_1 = 8$ and $J = 20$.

Readout of the Working Memory buffer's discrete movement commands is controlled by a memory-modulated target position vector (TPV_m):

Memory-Modulated Target

$$TPV_m(i+1) = TPV_m(i) + DV_s. \quad (10)$$

TPV_m tracks the cumulative DV_s through time. The PPV is subtracted from the TPV_m to form a

Gating Difference Vector

$$DV_{gate} = TPV_m - PPV. \quad (11)$$

The difference vector DV_{gate} controls readout from the WM buffer. The next cerebellar command that has been stored in Working Memory is read from the WM buffer when DV_{gate} is less than or equal to zero; that is, when the current TPV_m has been reached or surpassed. By altering the size of the GO signal, the rate at which TPV_m is reached by the outflow PPV can be controlled. Thus, Working Memory readout is controlled by the speed of the movement, which is determined by PPV . This gating rule ensures that the shapes of the movement and its velocity profile are preserved as performance speed is changed by a different choice of the volitional GO signal.

The movement velocity profiles generated by the model represent outflow movement commands, not the actual performance of the arm/hand system. There is filtering of the movement signal downstream of the central command by the peripheral muscle apparatus (Contreras-Vidal et al., 1997). An assumption of low-pass filtering in the command pathway is commonly made in muscle models (Barto et al., 1999, p.567). Therefore, the acceleration profile (12) generated by the present model is filtered using a first order differential equation (13):

Acceleration Profile

$$A(t) = \frac{\frac{dPPV(t)}{dt} - \frac{dPPV(t-D)}{dt}}{D} \quad (12)$$

This acceleration is time-averaged to compute the

Muscle-Filtered Acceleration Profile

$$\frac{dA_f}{dt} = (-A_f(t) + A(t)). \quad (13)$$

The step size in (12) is $D = 0.05$.

Correlation Equation (Edelman & Flash, 1987)

Correlations

$$c(a,b) = \max_{0 \leq r \leq R} \frac{\sum_{i=0}^{n-r} (a_i - \bar{a})(b_{i+r} - \bar{b})}{(n-r) \sqrt{\frac{1}{n} \sum_{i=0}^n (a_i - \bar{a})^2} \cdot \sqrt{\frac{1}{n} \sum_{i=0}^n (b_i - \bar{b})^2}} \quad (14)$$

were calculated for x and y position, velocity, acceleration, tangential velocity, and Two-Thirds Power Law tangential velocity predictions. Equation (14) defines the correlation for two sequences $a(t) = \{a_0, a_1, \dots, a_n\}$ and $b(t) = \{b_0, b_1, \dots, b_n\}$. \bar{a} and \bar{b} are the sequence means. R is the maximum permitted index shift between the two vectors and is equal to $0.1n$. Equation (14) yields correlations from -1 to 1. Similar curves yield positive correlation values, although the curves are not necessarily identical.

Curvature

There is an inverse relation between tangential velocity and curvature in Figure 3 (3a, b). The peaks in curvature near the ends of the simulated trajectories (Figure 3a) are the result of the x and y velocities (V_x, V_y) getting very small, with V_x and $V_y \ll 1$. As seen in Equation (15),

$$C = \frac{(V_x \cdot A_y) - (V_y \cdot A_x)}{(V_x^2 + V_y^2)^{1.5}} \quad (15)$$

curvature C approaches infinity as the sum of V_x^2 and V_y^2 approaches zero.

Two-Thirds Power Law

The Two-Thirds Power Law states that the angular velocity is proportional to the curvature raised to the two-thirds power (Lacquaniti et al., 1983):

$$A = kC^{\frac{2}{3}}, \quad (16)$$

where A = angular velocity, C = curvature, and k is a proportionality constant. Equivalently,

$$V_{tan} = kr^{\frac{1}{3}}, \quad (17)$$

where V_{tan} = tangential velocity, r = radius of curvature ($1/C$), and k is a proportionality constant.

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