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Rectification of panoramic photographs

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RECTIFICATION OF PANORAMIC PHOTOGRAPHS

by

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TABLE OF CONTENTS

ILLUSTRATIONS

INTRODUCTION

Panoramic photography is photography which has been made with an extremely wide angle of coverage. A panoramic camera is a specialized camera employing a revolving lens to take panoramic pictures.

The purpose of this thesis is to discuss some of the more important errors in this type of photography, and to discuss three possible methods of rectification of photographs taken with panoramic cameras.

Designing and developing a suitable panoramic camera is a current problem in the U.S. Air Force. At this date a suitable camera has not been adopted by the Air Force. It will be assumed, however, that an efficient panoramic camera can be designed, and the only problem this thesis will consider will be rectification.

Only rectification of panoramic photographs in the field of aerial photography, where the photographs can be used for photo-interpretation and mapping and charting purposes will be discussed. It is doubtful if the rectification of ground panoramic photographs would have any value, therefore it will not be considered.

PART I

HISTORICAL BACKGROUND OF PANORAMIC PHOTOGRAPHY

Panoramic cameras are cameras which are able to make photographs of extremely wide angles of coverage. Until recently photographic lenses which enable cameras to have over 90^{o} of coverage had not been considered favorable for aerial photography or for mapping and charting purposes. The extreme wide angle lenses in existence today have unusual distortion, illumination, and construction problems. Therefore, for better overall efficiency a normal angle camera or camera lens must be altered so that wider angles may be covered.

The panoramic camera method of increasing the angular coverage has been in common practice for a long time. This method was developed to take panoramic scenery photographs, and to take large group portraits. The lens of this type camera was rotated about its second nodal point. Rotating the lens about the second nodal point will produce no shift of the image.

The rotating lens was first used by Friedrich von Martens, of Paris, in 1845[1]. It had an angle of coverage of 150^{o} and used a cylindrically curved daguerreotype plate.

The first successful camera using this principle was developed in

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[1] Eder, J.M., History of Photography, p. 255.

1900 by KODAK[2]. This KODAK panoramic camera utilized flexible silver bromide film. The flexible film made it successful.

The first aerial panoramic photography was accomplished in Moscow, in 1903, by Thiele[3]. He used a combination of 7 cameras held aloft by a balloon. A single lens tilted downward was rotated to describe a circle.

Relatively fast lenses have been developed with angular fields up to 180°. For example, one of the better lenses of this type, German patent no. 620538-1932, patented in 1932 has distortion of 100 per cent at 165°. Another lens system, the Pleon Lens[4], was developed by Richter of Zeiss in Germany during World War II. The Pleon lens has a field of 180°, but only 130° can be utilized effectively for mapping and charting purposes. The distortion is designed into the lens, in this case negative distortion, to compress a 180° coverage into a finite image site. An advantage of this negative distortion is that negative distortion tends to make the illumination more uniform from center to edge, usually by the $\cos^4$ factor.

By knowing the exact amount of the negative distortion in the lens it is easily compensated for by designing a rectifying lens with the same

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[2]Eder, J.M., History of Photography, p. 256.
[3]Ibid., p. 396.
[4]Journal of the Optical Society of America, Vol. 38, No. 5, May 1948, pp. 421-431.

amount of distortion, but in a positive degree. The distortion is consequently cancelled during rectification and printing.

Long focal length lenses (24 inches and longer) are presently impractical for distortion free wide angular fields because of format size and because of the difficulty in correcting off axis aberrations.

The present trend is toward slit scanning devices. In this country Dr. U.K. Heidelauf, Dr. James G. Baker, Col. Richard W. Philbrick, and the Boston University Physical Research Laboratory have been the most instrumental in furthering research in this field.

Dr. Baker has suggested the concept of wielding the image across the line of flight by means of a rotating dove prism[5]. This camera was developed by the Perkin-Elmer Corporation.

Col. Richard Philbrick, USAF, suggested the principle of the Wieldable Strip Camera[6]. This camera, constructed at Boston University, is a modification of a standard Air Force camera, which is mounted so that it can rotate and at the same time the film is pulled past a slit.

The camera suggested by Dr. U.K. Heidelauf is under development by the Vectron Corporation[7]. This camera is enclosed in a streamlined

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[5]BU Optical Research Laboratory, Technical Report 59, Wieldable Strip Camera Experiment.
[6]Ibid.
[7]Heidelauf, U.K., Office Memorandum to Chief, Photographic Laboratory, Wright-Patterson Air Force Base, Ohio, Panoramic Camera and Film Rectifying System, 2 August 1949.

pod, approximately 12 feet long and 3 feet in diameter. The pod will be self sufficient, except for power. The pod enables it to be carried in a bomb bay or slung under a larger type of fighter aircraft. The camera is stabilized by electronic gyro-stabilizers to remove tip, tilt, etc. The focal length is 24 inches, but larger models can be built for 48 inches or 96 inches for detailed reconnaissance purposes. The scan is controlled by a revolving mirror and the film is pulled past a slit according to the speed of the aircraft.

Several other less well known systems have also been proposed.[8]

One of the advantages of an aerial panoramic system is that the extreme wide angle enables that fewer flight lines need be flown to cover a required area, thereby reducing the cost, and in time of hostile enemy action, exposure to hostile enemy fire.

The conventional horizon-to-horizon photography in the United States Air Force today is tri-Metrogon photography. This system incorporates 3 cameras, 1 vertical and 2 obliques. The cameras are rigidly fixed as to angular orientation to one another. The shutters are activated simultaneously. The horizon-to-horizon coverage provides vertical reference.

When the final charting procedure is started the oblique photos have to be rectified, or at least all angular measurements must be reduced to the horizontal. When a suitable rectification process is found for panor-

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[8]Boston University Optical Research Laboratory, Technical Report 59, Wieldable Strip Camera Experiment.

amic photographs one photograph will replace three tri-Metrogon photos. Thus a large amount of control planning, identification, transfer, and computation may be eliminated.

There are two types of image motion errors which affect aerial cameras. One type due to vibration, pitch, roll, etc., will be assumed to be compensated for by the stabilized mount. The second type, image motion error due to the relative movement of the object during the scanning cycle; this is at present one of the more serious problems. It can be compensated for only if the exact ground speed of the aircraft is known. Aircraft speed can be ascertained to within 5 per cent. At a speed of 600 miles per hour this 5 per cent error would be an error of 30 miles per hour in determining true ground speed. This means that the object would have a relative speed of 30 m.p.h. or 44 feet per second, to cause image motion. At an altitude of 40,000 feet the scale of the photography, for the 24 inch camera, is 1:20,000. Therefore, if the object moves 44 feet in one second the image will move .0022 feet = .0264 inches = .67 mm in one second.

Because of the slit scanning device this image motion error becomes an image displacement.

IMAGE DISPLACEMENT PER SECOND OF SCAN

	Flying Height				
Speed	10,000	20,000	30,000	40,000	50,000
200 mph	0.88mm	0.44mm	0.29mm	0.22mm	0.18mm
300 mph	1.34mm	0.67mm	0.45mm	0.33mm	0.27mm
600 mph	2.68mm	1.34mm	0.89mm	0.67mm	0.54mm

Successive photographs would probably have the same error as the velocity error usually remains constant over a period of time. This error would not affect reconnaissance operations, but would make plotting of a chart very difficult. A project is under way to refine ground speed determinations so as to substantially reduce this error. Inasmuch as this thesis is the rectification of panoramic negatives, it will be assumed that image motion errors are compensated for. This assumption will probably be realized in the near future by the United States Air Force Research and Development Command.

PART II

GENERAL DESIGN CONSIDERATIONS

For a 24 inch focal length panoramic camera the dimensions of the negative will be 9.5 inches (exposed only 9 inches wide) by 6.283 feet. The usable portion of the film, covering 120° (-60° to $+60^\circ$) will be 4.188 feet. The nadir point must be marked somehow on the negative so that it can be properly oriented in the rectifying printer.

The camera is intended to compensate for image motion. Consequently, it will not be discussed in rectification. To make matters easier the built in electronic gyro-stabilizer will eliminate all tip and tilt for practical charting operations. The only error to be compensated for will be errors in flying height. With the equipment of today, this problem is negligible.

Film shrinkage is a major factor in mapping operations. But this camera and printer are intended for charting operations primarily, and photo-reconnaissance secondarily. The compilation scales of aeronautical charts are so small that this error introduced by the small amount of film shrinkage will not affect the plotting accuracy. Also, this error caused by film shrinkage will be overshadowed by the errors of the radial line plot, the system normally used in this country for aeronautical chart compilation. The actual error is found by the following method: From the Kodak Handbook it is found that the shrinkage characteristics of Kodak Aerographic Film (Topographic Military) Type 1A is as follows:[1]

length	width	difference
.05 percent	.06 percent	.01 percent

The longest dimension is from the nadir to $\theta = 60^{\circ}$, which is 2.0944 feet. Consequently the maximum error due to film shrinkage will be at this point. The nadir point will be fixed in the printer therefore all other points on the film will shrink .0005 times their distance from the nadir, toward the nadir.

The shrinkage distance (image displacement) at $\theta = 60^{\circ}$ is equal to $.0005 \times 2.0944 = .00105$ feet. This displacement is magnified two times by the lens system during rectification, and is also lengthened by geometric projection by $1/\cos\theta$. Therefore the final displacement on the print, at $\theta = 60^{\circ}$, due to a .05 percent shrinkage will be:

Image displacement = $(.00105 \times 2)/\cos 60^{\circ} = .00420$ feet.

A displacement of .00420 feet at a scale of 1:20,000 (print scale) is an error of 84 feet on the ground. From an airplane in flight an object misplotted by 84 feet could never be detected. At the published chart scale of 1:250,000, 84 feet would plot as .000336 feet (.004032 inches), and at the chart scale of 1:500,000 this would be .000168 feet

[1]Eastman Kodak Company, Kodak Materials for Aerial Photography, p. 9.

(.002016 inches). Obviously this is too small for a navigator or pilot to reckon with.

The question now arises: How much error is introduced into the scale of the negative by the curvature of the earth, if the earth is assumed to be flat.

Referring to Figure 1. Scale in the X-direction is equal to the object distance divided by the focal length at the camera.

S_x = object distance/focal length

$S_x = a/f$; (assuming the earth to be a plane)

$S_x' = b/f$; (assuming the earth to be a sphere)

The error, expressed as a percent will be given by the following:

$$e = (1 - S_x/S'_x) \times 100 = (1 - a/b) \times 100$$

$$a = H/\cos\theta$$

$$b = (H + R)\cos\theta \pm \sqrt{(H + R)^2 \cos^2\theta - H(H + 2R)}$$

By neglecting some infinitesimals this formula can be practically represented by:

$$e = (H/2R)\tan^2\theta \times 100$$

It is anticipated that the usable portion of the negative will be up to 60°. The maximum flying height will be assumed to be 60,000 feet. At $\theta = 60^\circ$ and at a flying height of 60,000 feet the percent error in scale due to the curvature of the earth is computed from the foregoing formula to be:

.44 percent

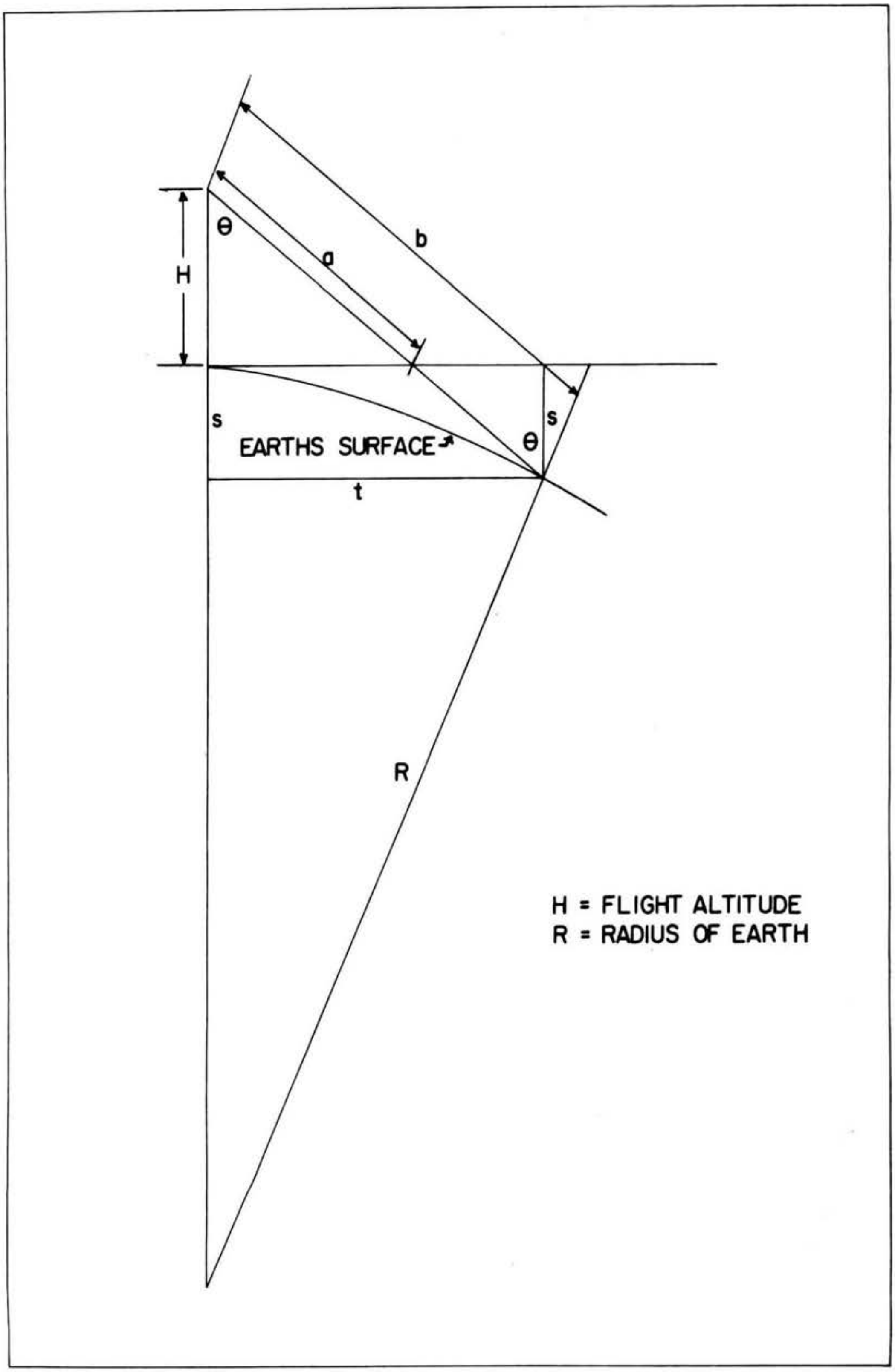

FIGURE I
ERROR DUE TO CURVATURE OF THE EARTH

Although not intended to be used, the printer will be able to print up to $\theta = 70^{o}$. At a flying height of 60,000 feet and at an angle of 70^{o} the percent error is found to be only .97.

The extreme error will be found at the largest usable angle, of $\theta = 60^{o}$, and at the maximum flying height. For all other combinations of altitude and angle of θ the error will be smaller. As this maximum value was computed to be .44 percent, it can be assumed that the effect of the curvature of the earth is negligible insofar as affecting the scale.

Actually, if the earth were flat and the scale at the nadir was 1:20,000 the scale at 60^{o} would be 1:40,000, but due to the curvature of the earth the scale is decreased to 1:40,176.

The camera can be flown at any altitude for reconnaissance photography, depending on the required detail. Charting photography should be flown at a specific altitude in order to obtain a certain scale. Chart compilation equipment and procedures are designed to operate at one optimum scale, although adjustments can be made so that compilation can efficiently be made at one of a few other scales.

The scale of a panoramic photograph will be considered to mean the scale at the nadir.

The scale of the negatives and photographs used in the present 6 inch tri-Metrogon system is 1:40,000. To obtain this scale a flying height of 20,000 feet is necessary. Twenty thousand feet is a dangerous altitude during hostile operations due to anti-aircraft fire and enemy interceptors. There are two obvious alternatives, either fly lower, or higher. The lower altitude system, which utilizes the Sonne Strip Camera, is in operation and under further development for reconnaissance, not charting photography. The other alternative, of flying higher is one of the advantages of the panoramic system.

In order to compile the charts at 1:40,000 from 1:20,000 negatives either the scale of the photography must be reduced, or the compilation scale must be increased. It would be very inefficient to go to the trouble and expense of obtaining a large print for subsequent reduction. The only other alternative would be to rectify the negatives and print them at a 1:1 ratio at the nadir. The overall scale of the print will then be 1:20,000. By use of radial line plotting the chart can be compiled at 1:20,000 or 1:40,000. By usual tri-Metrogon methods the compilation scale is 1:80,000. By compiling at twice the usual scale a much more accurate chart is obtained. After compilation the charts are published at 1:250,000, 1:500,000 and 1:1,000,000.

Also, by designing the rectifying printer for a 1:1 ratio any future increase in flying height capabilities can easily be absorbed; but if designed for a reduction of one half then it could not effectively be used for flying heights over 40,000 feet.

PART III

RECTIFICATION BY USE OF PANCRATIC LENSES AND A SLIT SCAN

The first design to be considered will be a 2 lens pancratic system with a slit scan.

The design of the pancratic lens involves the limitation imposed by several parameters. First, the magnification, and second, the focal length. The magnification must vary with the scanning angle to insure proper and uniform scale over the entire rectified photograph. The focal length must vary as a function of the scan angle to insure proper focus at all times. Figure 2 clearly illustrates the varying object-to-image distance which dictates the required varying focal length.

There are many pancratic lenses, but so far as could be ascertained, none have been designed that are limited by these 2 parameters simultaneously.

Magnification at $\theta = 0^{o} = M_o = -1$ (Given design consideration)

Magnification at any angle = M

From Figure 2:

$M = M_o/\cos\theta = -1/\cos\theta$.

Inasmuch as the focal length of the camera is 24 inches the center of curvature of the negative, in the object plane, will be constant and equal to 24 inches. This center of curvature will also be the axis of

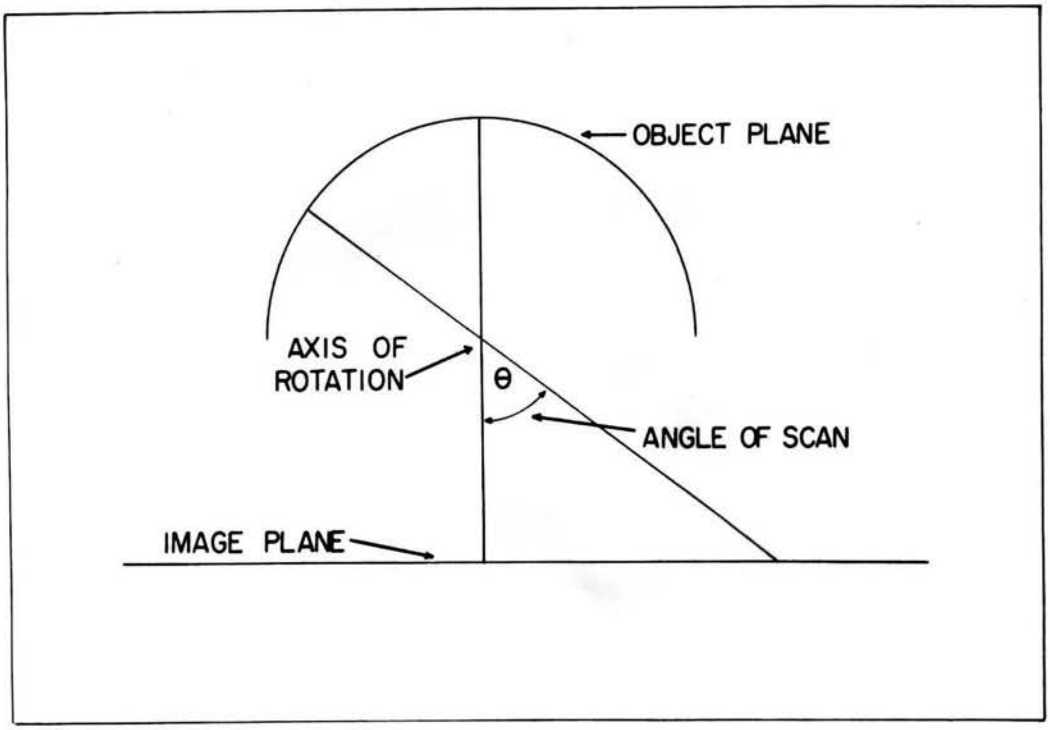

FIGURE 2
SCHEMATIC DIAGRAM OF PANCRATIC LENS – SLIT SCAN DEVICE

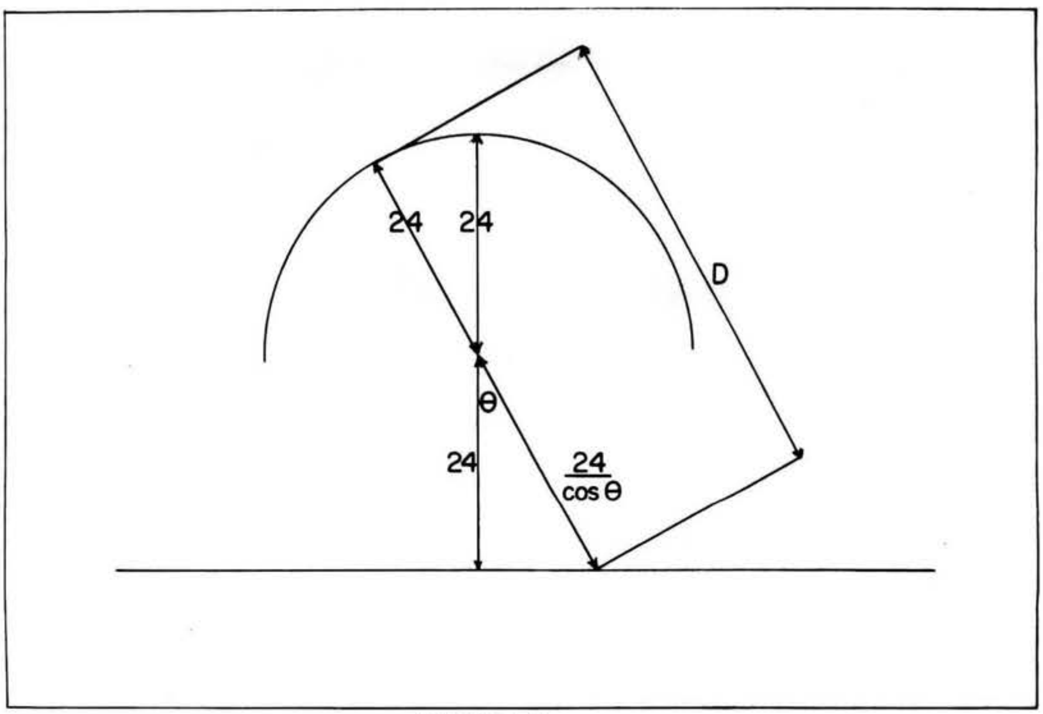

FIGURE 3
DESIGN DIMENSION RELATIONSHIPS

rotation at the scanning device. The distance from the axis at rotation to the image plane is equal to 24 inches divided by the cosine of the scanning angle, θ. This is clearly shown in Figure 3.

D = object to image distance

$D = 24 + (24)/\cos\theta$

at $\theta = 0^{o}$ $\quad D = 48$ inches

at $\theta = 60^{o}$ $\quad D = 72$ inches.

The optical relationship of any 2 lens system is shown in Figure 4. For the sake of simplicity both lenses are going to be considered to be of the same focal length; or $f_1 = f_2$. Magnification of a 2 lens system being equal to the product of the magnification of the 2 individual lens; the following results:

$M = M_1 M_2 = (f_1 X'_2)/(f'_2 X_1) = - X'_2/X'$.

One of the conditions imposed on this system is that $M = -1/\cos\theta$. Therefore:

$M = -X'_2/X_1 = -1/\cos\theta$

Solving for X'_2;

$X'_2 = X_1/\cos\theta$.

It is evident that the conditions at magnification will be satisfied as long as the above relationship is maintained.

The image-to-object distance, D, is equal to:

$-X_1 + X'_1 - X_2 + X'_2 - f_1 + f'_1 - f_2 + f'_2 = D = 24 + (24)/\cos\theta$

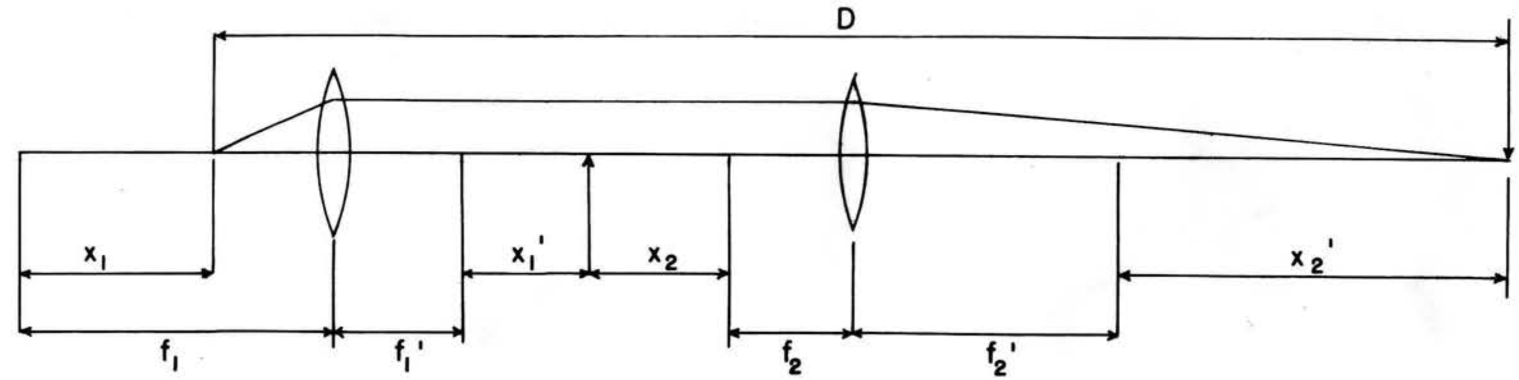

FIGURE 4
NEWTONIAN OPTICAL RELATIONSHIP OF 2 LENSES

letting $f_1 = f_2 = f$; $f = -f'$; and $X'_2 = (X_1)/\cos\theta$, this becomes:

$$-X_1 - (f^2)/(X_1) + (f^2 \cos\theta)/(X_1) + (X_1)/(\cos\theta) + 4f = 24 + (24)/\cos\theta.$$

Solving for X_1:

$$X_1 = (2f\cos\theta - 12\cos\theta - 12)/(\cos\theta - 1) \pm \sqrt{(2f\cos\theta - 12\cos\theta - 12)^2/(\cos\theta - 1)^2 + f^2\cos\theta}.$$

This formula gives the X_1 distance for any focal length at any scan angle. The X'_2 distance and the distance between the lens can easily be computed once the X_1 distance is used.

By substituting various focal lengths it was found that if a focal length of 24 inches was used the optimum movements were obtained, but due to the physical thicknesses of the lenses they could not be superimposed at $\theta = 0^\circ$. Therefore, for practical reasons a focal length of 22" was assumed to be best for the 2 panoratic projection lenses. Letting f = 22 inches the above formula for X_1 becomes:

$$X_1 = (32\cos\theta - 12)/(\cos\theta - 1) \pm \sqrt{(32\cos\theta - 12)^2/(\cos\theta - 1)^2 + 484\cos\theta}$$

Solving this equation for 10° increments of θ yields:

θ	X_1 (inches)	X'_2 (inches)
0°	0.0	0.0
10°	0.0	0.0
20°	0.5	1.0
30°	1.7	3.4
40°	3.3	6.6
50°	5.8	11.6
60°	9.5	19.0
70°	14.6	29.2
75°	17.3	34.6

Still assuming thin lens optics; $f_1 + X_1 = S_1$ and $f'_2 + X'_2 = S'_2$. The distance between the lenses, d, is equal to $D - (S_1 + S'_2)$.

θ	S_1 (inches)	S'_2 (inches)	d (inches)	D (inches)
0°	-22	+22	4.0	48.0
10°	-22	+22	4.2	48.2
20°	-21.5	+23	5.0	49.5
30°	-20.3	+25.4	6.0	51.7
40°	-18.7	+28.6	8.1	55.4
50°	-16.2	+33.6	11.5	61.3
60°	-12.5	+41.0	18.5	72.0
70°	-7.4	+51.2	35.6	94.2
75°	-4.7	+56.6	55.5	116.8

The above values are graphed on Figure 5 for a clear indication of the movements involved.

This indicates that proper magnification and proper focus can be maintained at all times throughout the scanning cycle.

One other lens dimension, the diameter was calculated to be 6 inches. This was done by finding the worst condition of vignetting. At $\theta = 60^\circ$ (maximum θ for rectification) the worst possible condition of vignetting occurs. The aperture stop is fixed at the axis of rotation. From simple geometry of similar triangles, as clearly shown in Figures 6 and 7, the lens diameters were computed.

The aperture stop diameter determines the relative speed of the lens. It is desirable to operate as close to f/16 as possible. The diameter of the pupil is equal to the focal length divided by f/no. A pupil diameter of 1.375 inches is required. But the pupil is the image

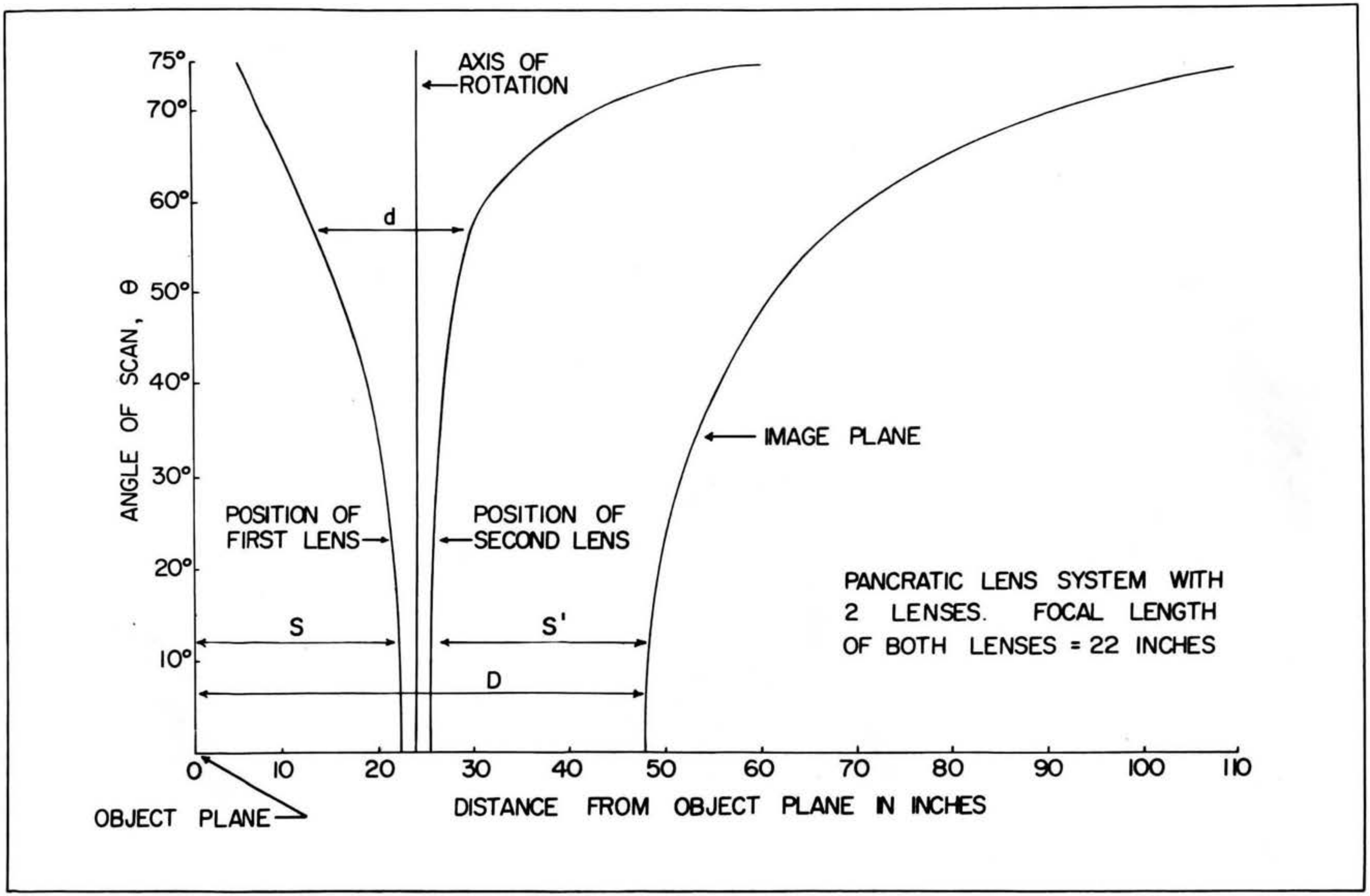

FIGURE 5

POSITION OF LENSES AND IMAGE PLANE AS A FUNCTION OF SCAN ANGLE

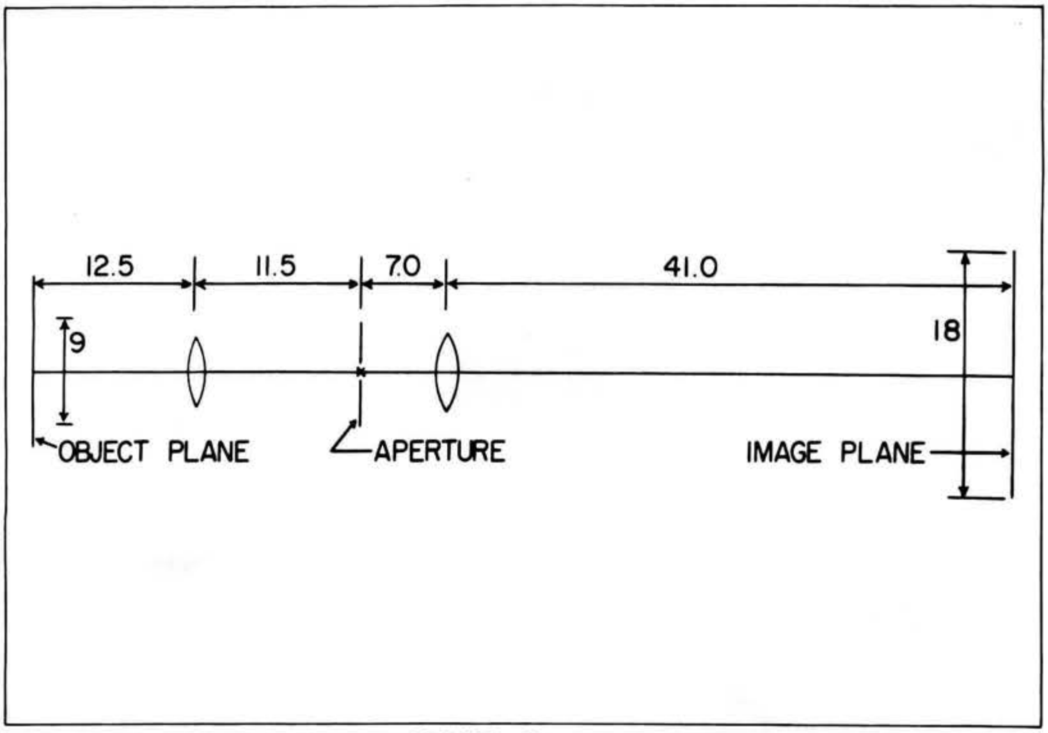

FIGURE 6
DETERMINATION OF LENS DIAMETER

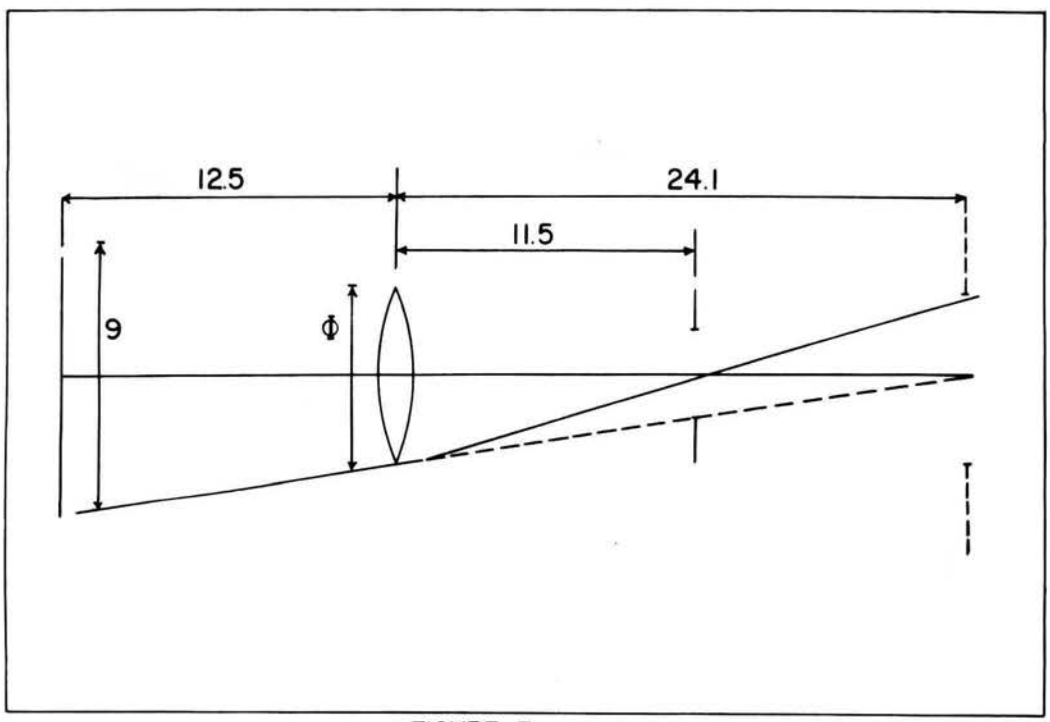

FIGURE 7
DETERMINATION OF LENS DIAMETER (cont'd)

at the aperture stop. During the scan the lenses move therefore with a constant aperture diameter the pupil size, and hence the f/no will change. The proper aperture stop is now computed so as to obtain an f/no of 16 at $\theta = 0^{o}$; see Figure 8. The physical diameter of the aperture for f/16 at $\theta = 0^{o}$ is 1.25 inches.

Calculating the equivalent f/no at each 10^{o} position we get the following values (see Figure 9).

θ	S	S'	M	f/no
0^{o}				16
10^{o}				16
20^{o}	2.5	2.82	1.125	17.25
30^{o}	3.7	4.45	1.20	14.6
40^{o}	5.3	6.98	1.32	15.25
50^{o}	7.8	12.1	1.55	14.95
60^{o}	11.5	24.1	2.09	14.05
70^{o}	16.6	67.6	4.06	14.75

Inasmuch as the density of each individual negative will vary within wide limits, and inasmuch as the equivalent f/no's vary only from 14.05 to 17.25 it will be assumed that the equivalent f/no remains constant, and at f/16 for all values of θ.

It was stated previously that under the given conditions perfect focus would be obtained. This would be true if the image plane was perpendicular to the optical axis. From Figure 2 this is obviously not true. The true condition, at the image plane, is shown in Figure 10. FR = focal range and e = width of image that is within focal range. Due to the design of the rectifier the condition of Schampflug cannot be

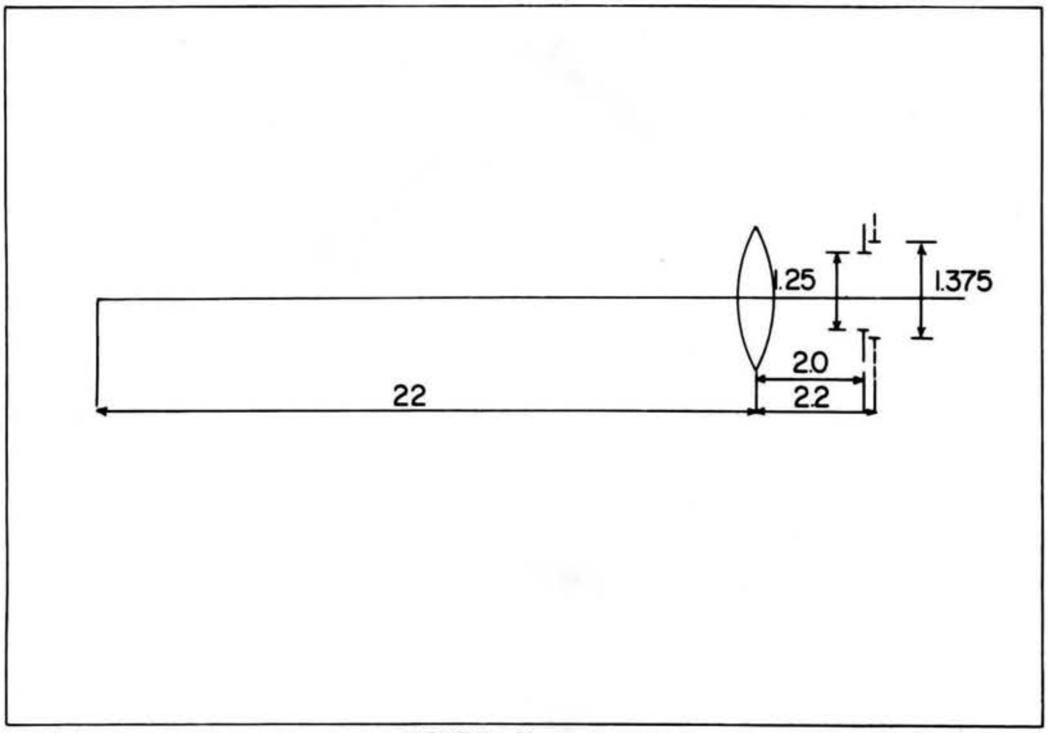

FIGURE 8
DETERMINATION OF APERTURE DIAMETER

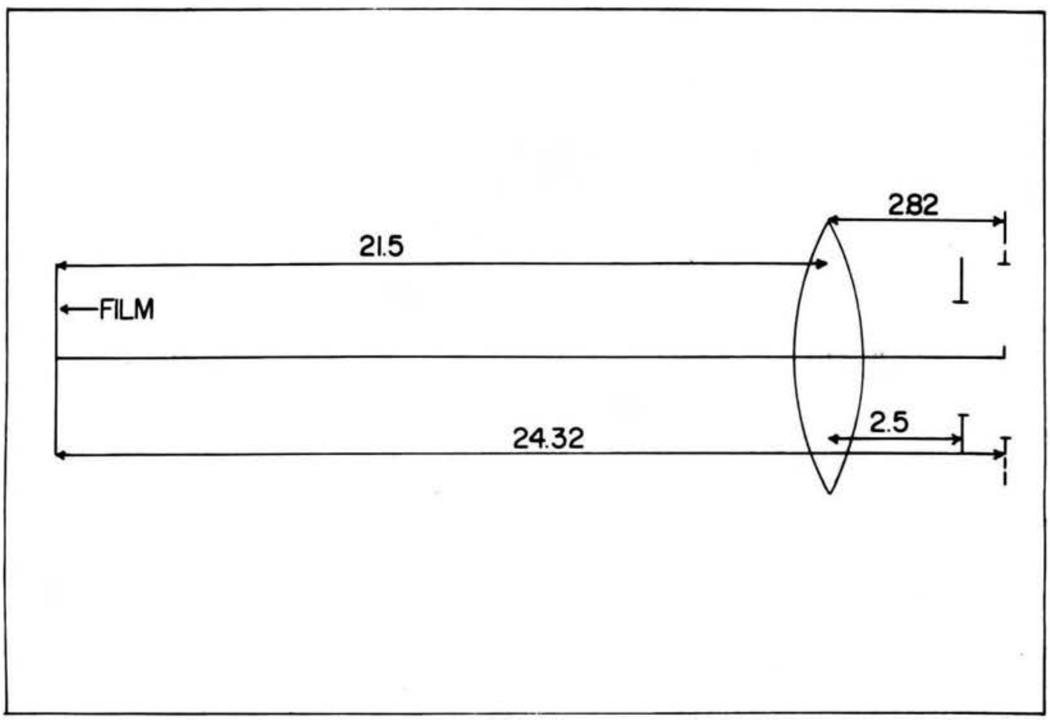

FIGURE 9
F/NO DETERMINATION

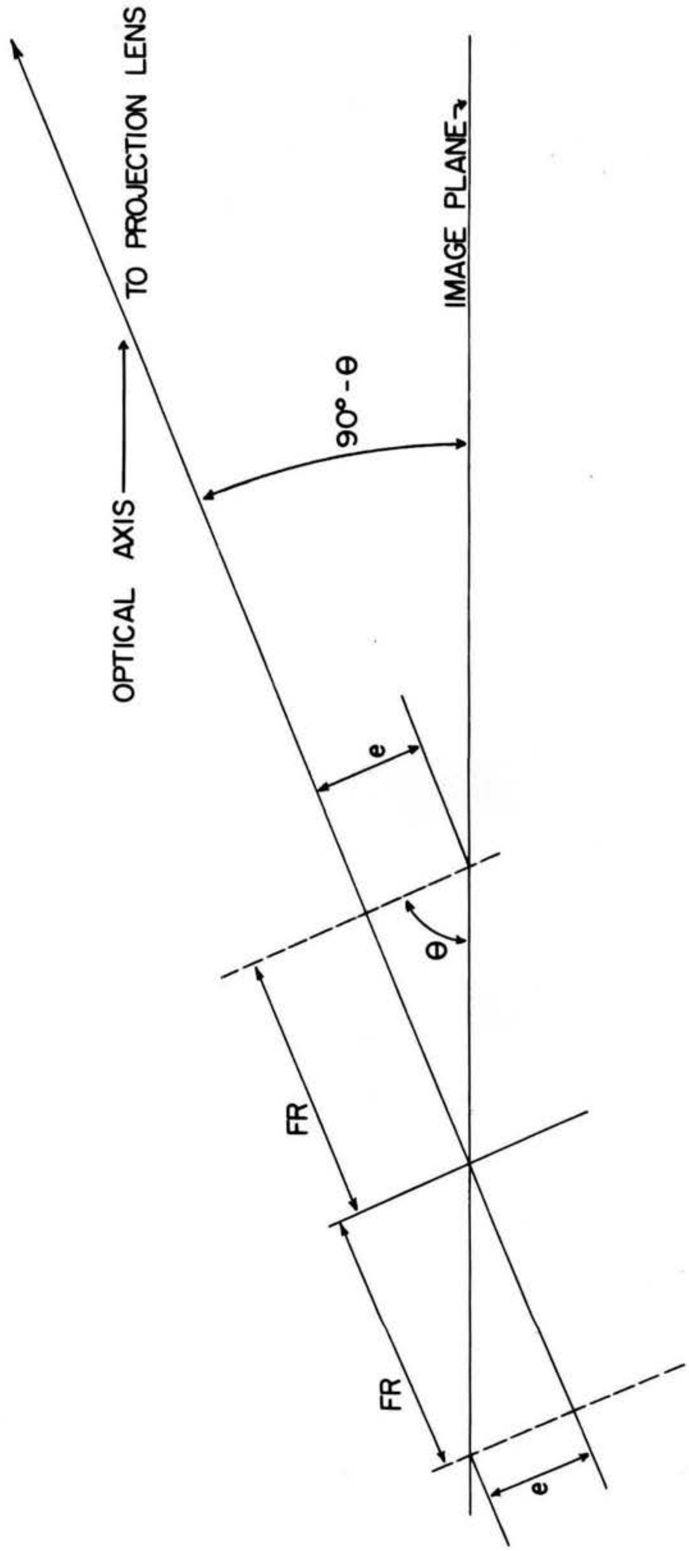

FIGURE 10
DETERMINATION OF SLIT WIDTH

met. Therefore, the slit width must be reduced to keep the light rays within the range e, of Figure 10.

In order to determine e, the focal range (FR) must be calculated and the effective exit f/no must be known. The effective exit f/no varies with the scanning angle. From Figure 10 it is obvious that the minimum e occurs at maximum θ; or when $\theta = 60^{o}$. The effective exit f/no is equal to:

(exit) f/no = (distance)/(aperture diameter x exit magnification)

Exit magnification is treated later on in this thesis.

at $\theta = 60^{o}$

(exit) f/no = (48)/(1.25) (1.46) = 26.3

The focal range (FR) = (4) $(f/no)^2$. Assuming a wave length of 5,000 Å

FR = (4) (.0005) $(26.3)^2 = \pm 1.38$ mm

At $\theta = 60^{o}$

e = (FR)/(tan θ) = (1.38)/ (1.732) = .796 mm

The image of the slit must not exceed 2e = 1.592 mm, in order to insure proper focus. At $\theta = 60^{o}$ the magnification of the slit is 2, therefore the slit width must be one half this value or .796 mm (.8 mm).

Since the method being used is a slit scanning method the area of the image near the edge of the scan will remain out of focus for only a very short time. Therefore, it is estimated that a slit width of 1 mm could effectively be used. The slit width could perhaps be opened as far as 1.5 mm or even to 2 mm under actual conditions.

This concludes the design of the optical considerations of the pancratic lens and the slit scan. Assuming that the rectifier will operate efficiently the problem now arises as to illumination requirements, slit scan velocities, and light source determinations.

There are 2 methods of obtaining the correct overall exposure. One method is to have the light source constant and vary the scanning speed. The other method is to have the scanning speed constant and to vary the light source luminance. The basic equation for exposure (E) is $E=It$. I=illuminance, and t=time of exposure. In the first method I is held constant and t is varied. In the second method t is held constant and I is varied. To determine E as a function of the scanning angle θ there are several factors to be considered.

The resultant illuminance on the image plane during the scan is found by the following method. Referring to figure 11;

I_1 = Required illuminance of $\theta=0^{\circ}$, (to be determined later in this thesis for a typical case)

I_2 = Resultant illuminance at a distance from the source

I_3 = Resultant illuminance at I_2, but in a plane not perpendicular to light rays, but parallel to plane at I_1.

It will be assumed that the light source luminance remains constant for values of θ. Illuminance is inversely proportional to the square of the

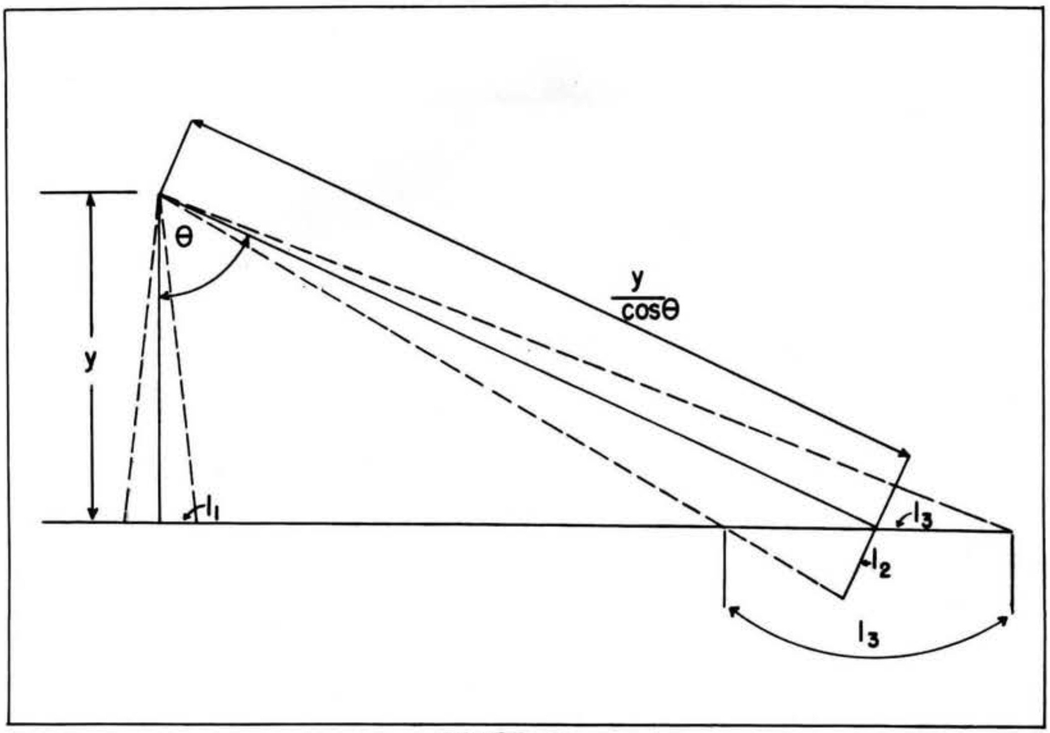

FIGURE 11
ILLUMINANCE ON IMAGE PLANE

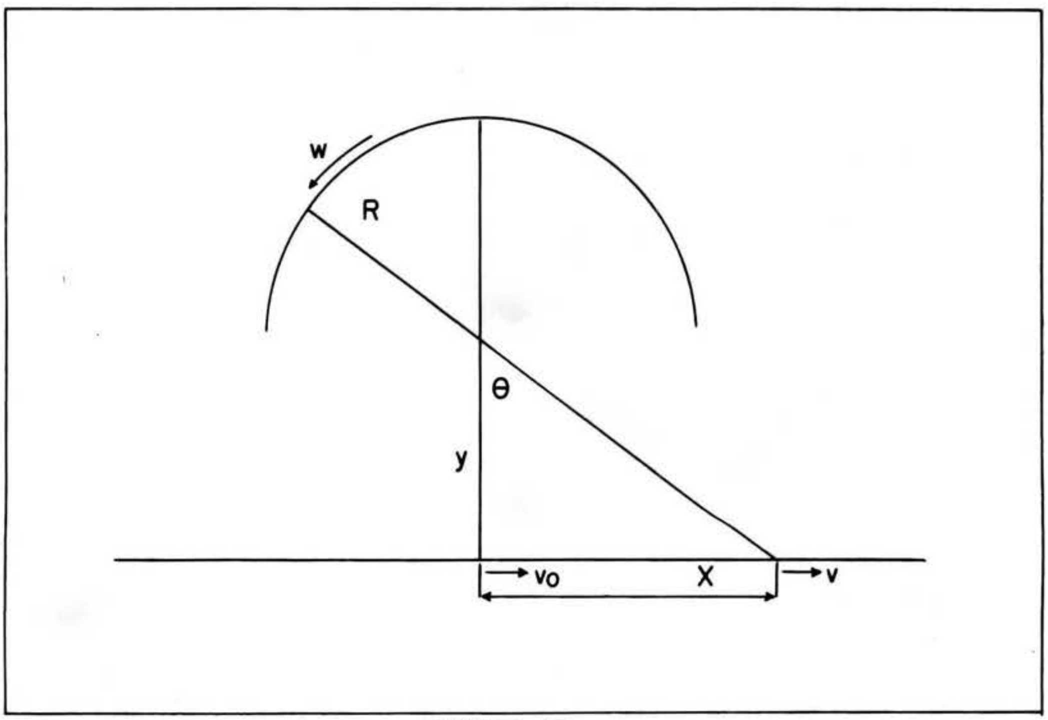

FIGURE 12
VELOCITY RELATIONSHIPS

distance. The illuminance at $I_1 = (K)/y^2$ and at $I_2 = (K)/y^2/\cos^2\theta$. I_2 is then equal to $I_1 \cos^2\theta$. But the light does not fall on an area perpendicular to the light rays (Area of I_2), but on the area of I_3. $I_3 = I_2 \cos\theta$. Therefore the resultant illuminance falling on area of I_3 is equal to $I_1 \cos^3\theta$.

This means that in order to effect an even, overall illuminance the luminance of the aperture must be increased by a factor of $1/\cos^3\theta$, thereby obtaining the correct exposure. As will be shown later, it seems more practical to hold the light source luminance constant and to vary the scanning speed so as to increase the exposure time by this $1/\cos^3\theta$ factor.

θ	$1/\cos^3\theta$
0°	1.0000
10°	1.0500
20°	1.2020
30°	1.5420
40°	2.2300
50°	3.7650
60°	8.0000
70°	25.15
75°	57.3

If it is desired to maintain a constant scanning speed and vary the light source intensity; this can be readily accomplished by driving the scanning device by a small motor mounted on the scanning mechanism, and gearing the motor to the negative holder frame. The intensity of the light could be varied by use of resistors wound so as to vary the actinic output of the light source by a factor $1/\cos^3\theta$. This method does not utilize maximum light efficiency, and it is felt that the following method is much more practical.

The better method would be to operate the light source at maximum output and vary the speed at the scanning device. The speed of the driving motor can be controlled by controlling the current to the field of the motor or the scan speed can be varied by a mechanical linkage with cams.

Assuming the variable speed method is best the scan velocity as a function of the scan angle must be computed.

If the image of the slit was the same width for all values of θ (or X) the scanning velocity, v, would have to be reduced by a factor of $\cos^3\theta$. But, since the width of the slit is magnified in the direction by a factor equal to $1/\cos^2\theta$ the velocity in the direction, v, need be reduced only by a $\cos\theta$ factor to correct for proper illumination. The other $\cos^2\theta$ factor is hence made up by an increase in the time it takes the widening slit image to pass a given point.

Refering to figure 12,

v_o = velocity of image at $\theta=0^\circ$

v = instantaneous velocity of scan image

w = instantaneous angular velocity of scan

R = y = radius of scan

X = position of image on image plane

$$x = y \tan \theta \qquad\qquad w = d\theta/dt$$

$$x = \int_{t_1}^{t_2} v\,dt \qquad\qquad v = v_o \cos \theta$$

$$\text{but, } v = (R/\cos^2\theta) \times (d\theta/dt) = v_o \cos \theta$$

$$dt = (R\,d\theta)/(v_o \cos^3\theta)$$

$$d\theta/dt = w = (v_o/R) \cos^3\theta$$

and finally:

$$w = w_o \cos^3\theta$$

$$t = (R/v_o)\int_{\theta_1}^{\theta_2} \sec^3\theta\, d\theta$$

As the scan moves from $-\theta$ to $+\theta$ the time of scan becomes,

$$t = (2R/v_o)\int_{0}^{\theta} \sec^3\theta\, d\theta$$

It is now necessary to determine typical values for exposure time and scan velocity. The first source investigated was a standard, low pressure, mercury fluorescent tube. Using a slit width of 1/4 inch and converting to an f/16 system the exposure time was found to be 4.23 seconds for each individual point. The required a v_o of .0093 inches/second. Obviously too slow to be of any practical value.

Another light source investigated was a standard 500 Watt projection lamp. The dimensions of the filament were 10.9 mm x 11.5 mm. At a distance of 915 cm (30 feet) an adequate exposure was found to be in the neighborhood of 1/2 second. The equivalent f/no of the lamp is, (using 1. 1 cm for the diameter of the filament):

$$f/no. = 915/1.1 = 832.$$

To obtain the time for the same exposure through on f/16 system, (at a 1:1 relationship the effective f/no = 32):

$$t_{16}/t_{832} = (f/32)^2/(f/832)^2; \; t_{16} = .00074 \text{ seconds.}$$

Assuming 50 percent loss in transmission, the exposure time then becomes .00148 seconds. From this v_o is found to be 675 mm/second or 26.6 inches/second. The time of scan from -60° to $+60^\circ$ becomes 3.6 seconds. This time of 3.6 seconds is a maximum value to be used under ideal conditions. Any changes in exposure time would increase this 3.6 seconds.

The next step in this design is to develop a condensing system to tie the light source to the projection elements. For maximum efficiency it is necessary for the condensing system to image the filament at the aperture of the projection lens system; and to magnify it so that it just fills the area of the aperture stop. Due to the movement of the projection lenses during the scanning cycle the condensing system can only be designed for one value of θ. A condensing system, for this case, was designed for the conditions of $\theta = 0^\circ$ and the error at other values of θ was computed. For proper magnification a condensing system of focal length of 7" was found to be necessary. See figures 13 and 14. This can easily be accomplished by using 2 identical plano-convex lenses of

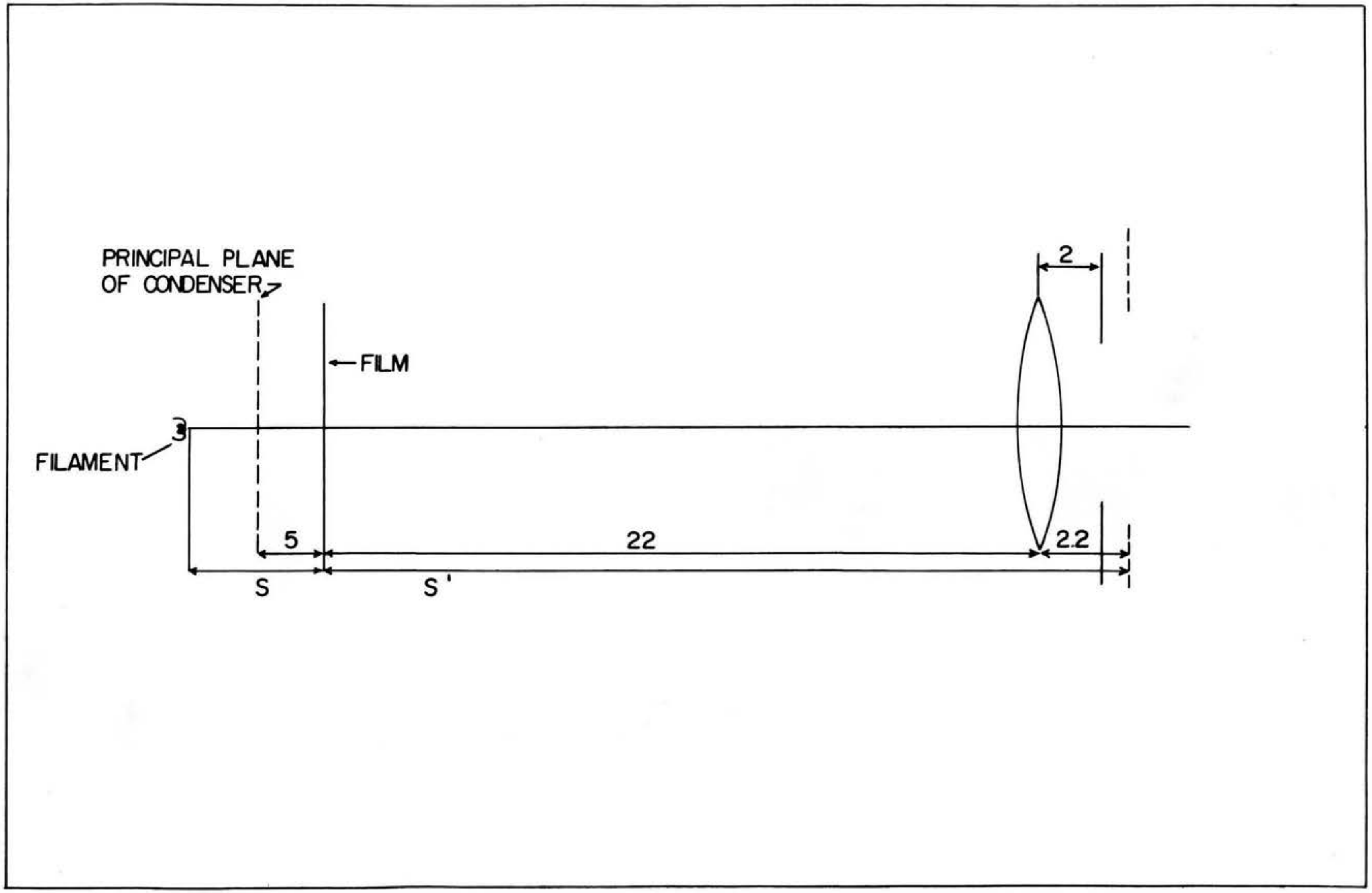

FIGURE 13
SCHEMATIC DIAGRAM OF CONDENSING SYSTEM

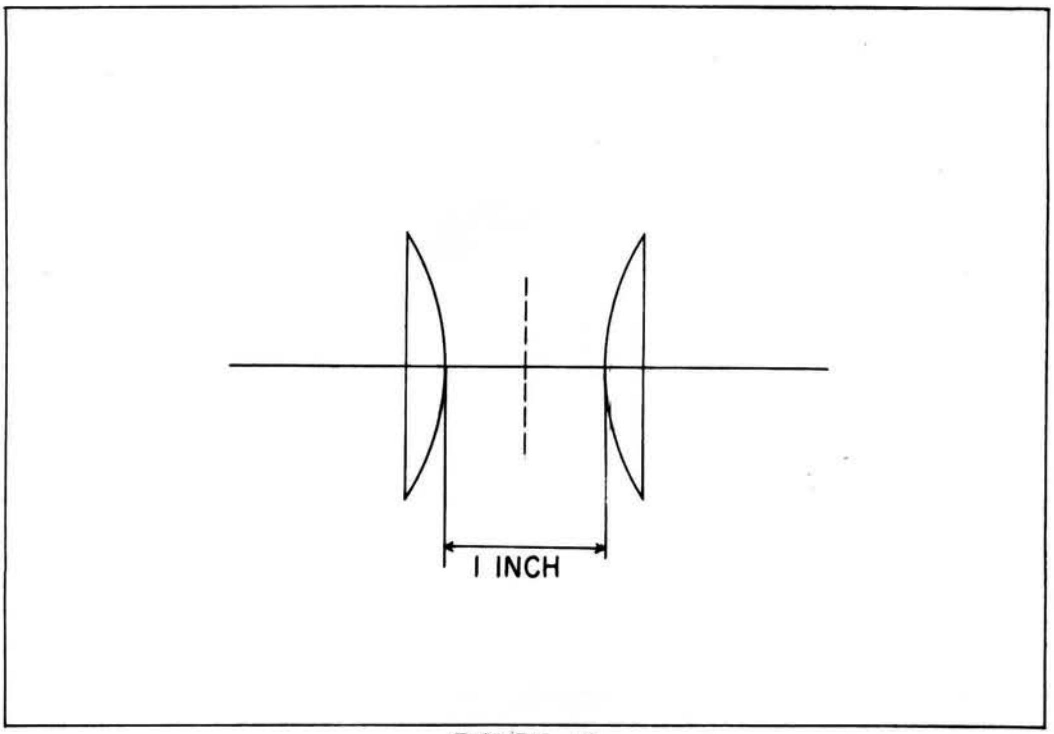

FIGURE 14
CONDENSING SYSTEM

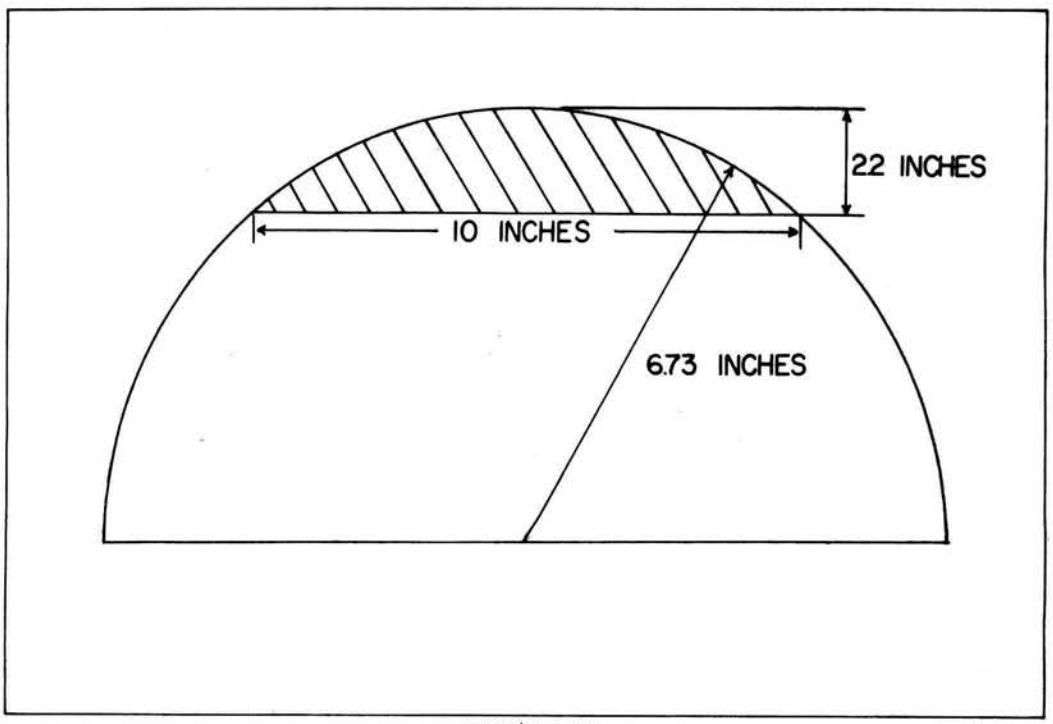

FIGURE 15
LONGITUDINAL SECTION OF CONDENSING LENS

focal length 13.47'' and spaced 1'' apart. Assuming an index of refraction of 1.5 the radius of curvature of the lenses would be 6.73'', see figure 15.

In order to keep the filament imaged onto the aperture either the condensers or the filament must move as a function of θ. It will be more practical to move the filament, see figure 16. This movement is given by:

$$S_1 = \left[154\, S'_2 + 7d\,(22 - S'_2)\right] / \left[(7-d)(22 - S'_2) - 22\, S'_2\right]$$

S'_2 is the distance between the first projection lens and the aperture, and is a function of θ. Therefore, this equation determines S_1 as a function of θ. This movement is graphed on figure 17.

To find the magnification of filament when imaged onto the aperture the following relationship was used:

$$M = \Pi\, S'/S = (S'_1 \times S'_2)/(S_1 \times S_2)$$

The diameter of the aperture was previously found to be 1.25 inches. The diameter of the filament was found to be .433 inches (100 Watt lamp). The following table indicates the size of the filament image at the various angles of θ as compared to the aperture.

θ	aperture	Diameter of filament image
0°	1.25 inches	1.260 inches
10°	1.25 inches	1.260 inches
20°	1.25 inches	1.230 inches
30°	1.25 inches	1.170 inches
40°	1.25 inches	1.105 inches
50°	1.25 inches	1.042 inches
60°	1.25 inches	1.026 inches
70°	1.25 inches	1.110 inches

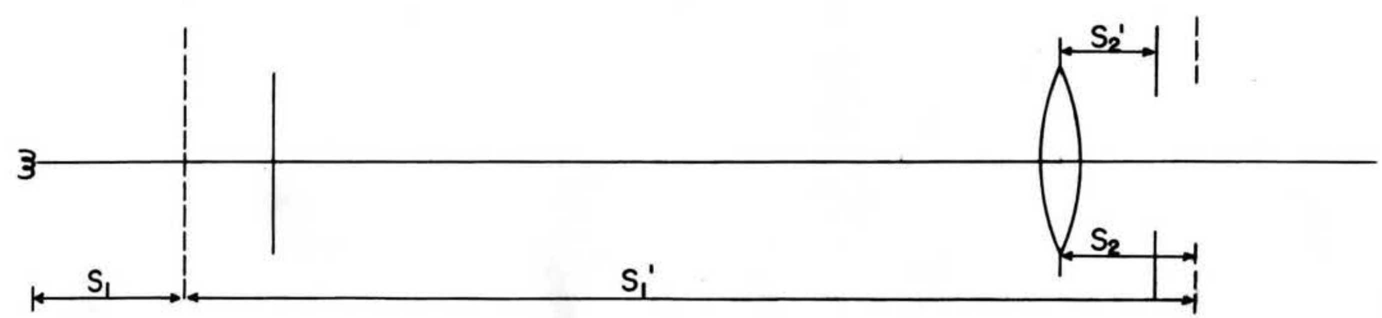

FIGURE 16

FILAMENT MOVEMENT AS A FUNCTION OF θ

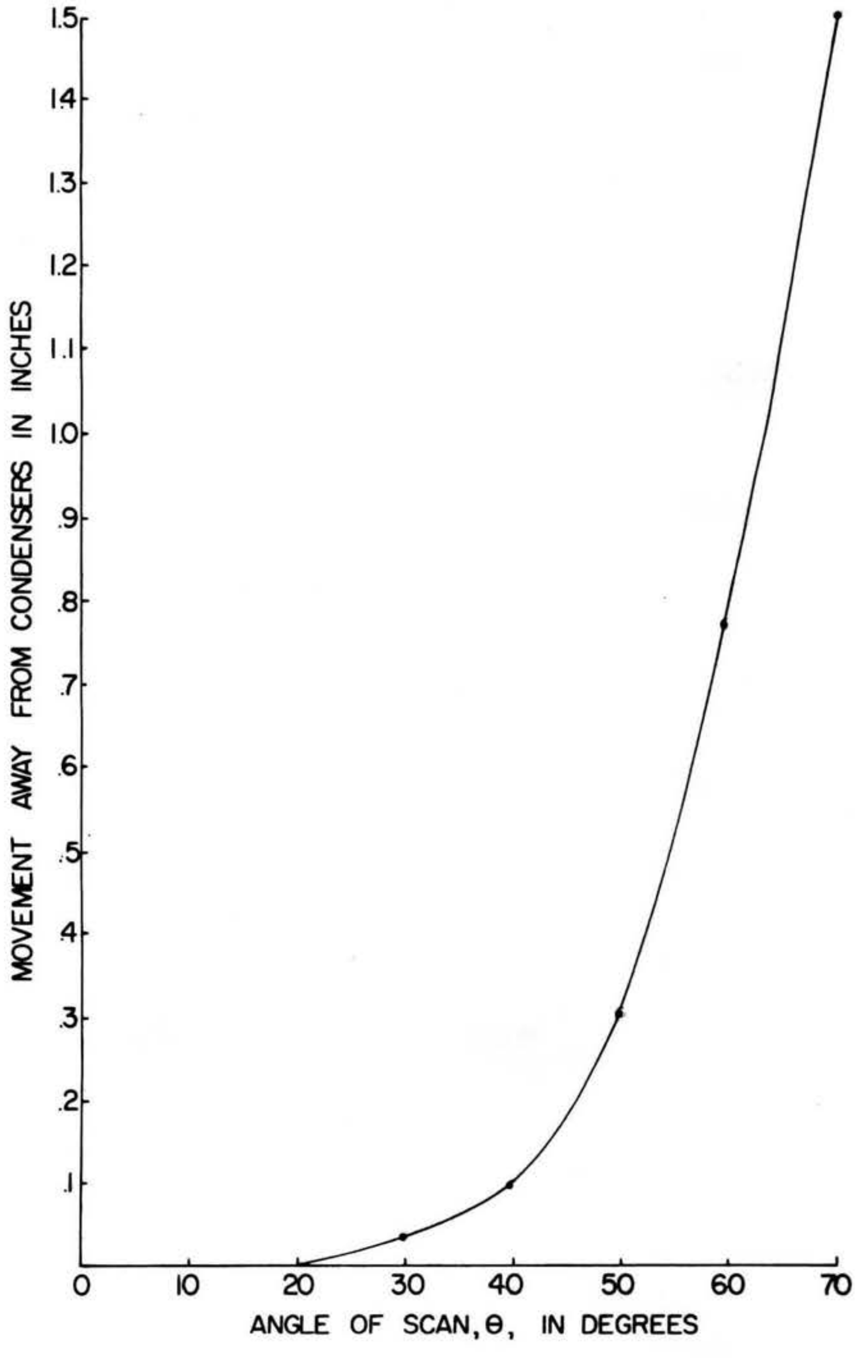

FIGURE 17

FILAMENT MOVEMENT AS A FUNCTION OF ANGLE OF SCAN

The image of the filament never exceeds the aperture size. But at $\theta = 60^{\circ}$ the image of the filament reaches its smallest diameter. The shape of the filament is approximately a square, while the aperture is circular. Because of this square shape of the filament the aperture is almost completely filled, see figure 18. It will therefore be assumed that the aperture is filled at all angles of θ up to 70°.

There is another factor to be considered, and this is overexposure (higher density) of the negative as the angle θ is increased. This is due to haze, scattered light, and/or other atmospheric conditions prevailing during the camera exposure. Although this relationship cannot be derived mathematically it has been found in general practice that an aerial camera acts as if light enters the camera as a function $1/\cos\theta$. This means that the general density of the negative increases from $\theta = 0^{\circ}$ to higher values of θ by this $1/\cos\theta$ factor. This increase in density also causes a loss of illuminance on the image plane of the rectifier by this $1/\cos\theta$ factor.

This assumed $1/\cos\theta$ factor of increased negative density is, for practical purposes, equally balanced out by the magnification of the aperture onto the image space.

Magnification of the aperture into the image space is equal to Z'/Z', see figure 19. Areal magnification is equal to the square of the magnification. The areal magnification at $\theta = 0^{\circ} = M_{o} = 1.21$.

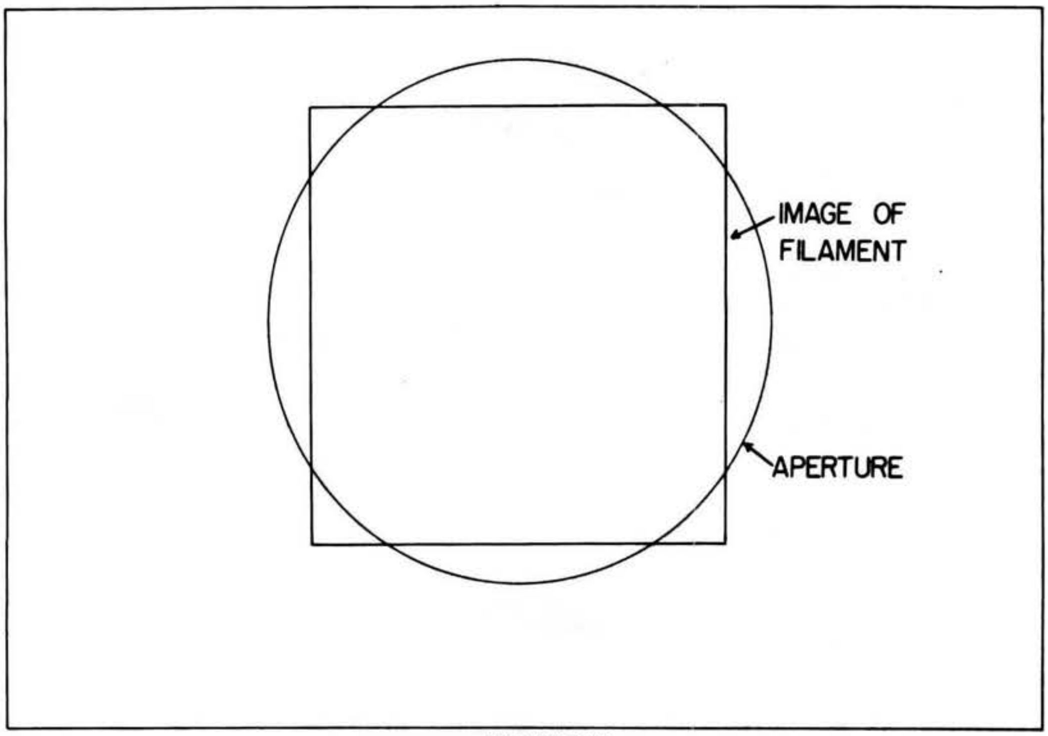

FIGURE 18
IMAGE OF FILAMENT ON APERTURE

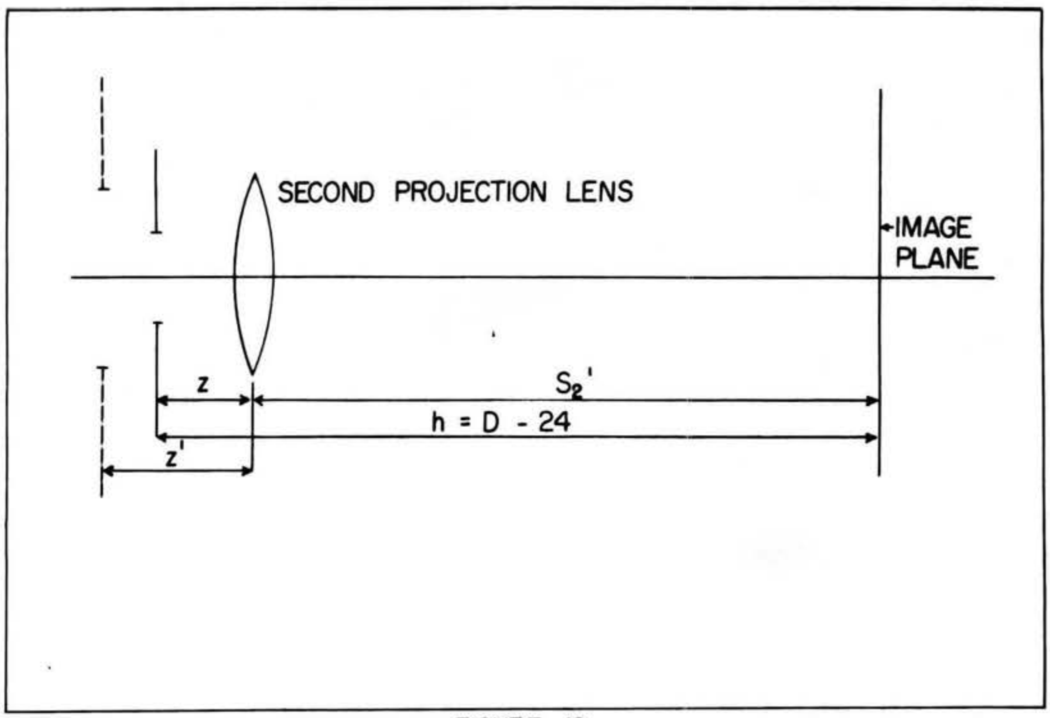

FIGURE 19
MAGNIFICATION OF APERTURE ONTO IMAGE PLANE

The following table compares the actual areal magnification with the necessary $1/\cos\theta$ factor to be compensated for.

θ	M	M^2	$M^2_o/\cos\theta$
0°	1.10	$1.21 = M_o$	1.21
10°	1.11	1.23	1.23
20°	1.13	1.27	1.28
30°	1.11	1.23	1.39
40°	1.14	1.30	1.58
50°	1.20	1.44	1.87
60°	1.46	2.14	2.42
70°	7.33	53.60	3.53

The usable area of the photograph is only from -60° to $+60^\circ$, therefore the erratic behavior between 60° and 70° will have no effect on the results. The final print will be overexposed at the margin beyond the usable portion.

The very close relationship between M^2 and the emperically determined $M^2_o/\cos\theta$ allows us to assume that this will compensate for the increased negative density.

This concludes the design of a pancratic lens and slit scanning rectifying printer. Before investigating another type of rectifying system it might be well to mention that even though this design has only two lenses (projection) in actual practice a more complicated system would be employed. This 2 lens system only indicates the feasability of this type of system. So far as is known a variable focal length lens system has not been developed which ties focal length <u>and</u>

magnification (scale) as specific parameters. There are many variable focal length lenses, and there are many variable magnification lenses, but they are independent of each other.

Also throughout the design simple lens formulas were used, both Newtonian and Gaussian. If this design were to be carried any further the thin lens formulae would have to be replaced by thick lens formulae, and be followed by formal optical design.

Some liberty was exercised in assumptions. This paper is not intended to be a design of a printer that is complete in every detail but to prove that a particular type of design is practical, simple to operate, and would be an asset in the mapping and charting field.

PART IV

MECHANICAL RECTIFICATION SYSTEM

The second method of rectification to be considered will utilize a slit scan, but the film will move past the slit as it is stationary. The printing paper will also move accordingly.

Two lens systems can be considered: A) a pancratic system, such as used in PART III; B) a fixed focal length system.

A. Pancratic System

Here again the Scheimpflug condition can be neglected if the slit width is taken sufficiently small so that the image will remain in the focal range. The slit width was determined to be 1 mm to comply with the above conditions.

In this design the optical relationships and dimensions are the same as in the pancratic lens-slit scan device of Part III. The difference is that in this case the lens does not rotate, the optical axis is held fixed, instead of the negative; and the negative and printing papers are moved.

To maintain the necessary angular relationship between the optical axis and the image plane the image plane must also rotate. The angle of rotation being equal to θ, the angle of scan.

From the nadir point of the negative, the distance is a function of the scan angle, θ.

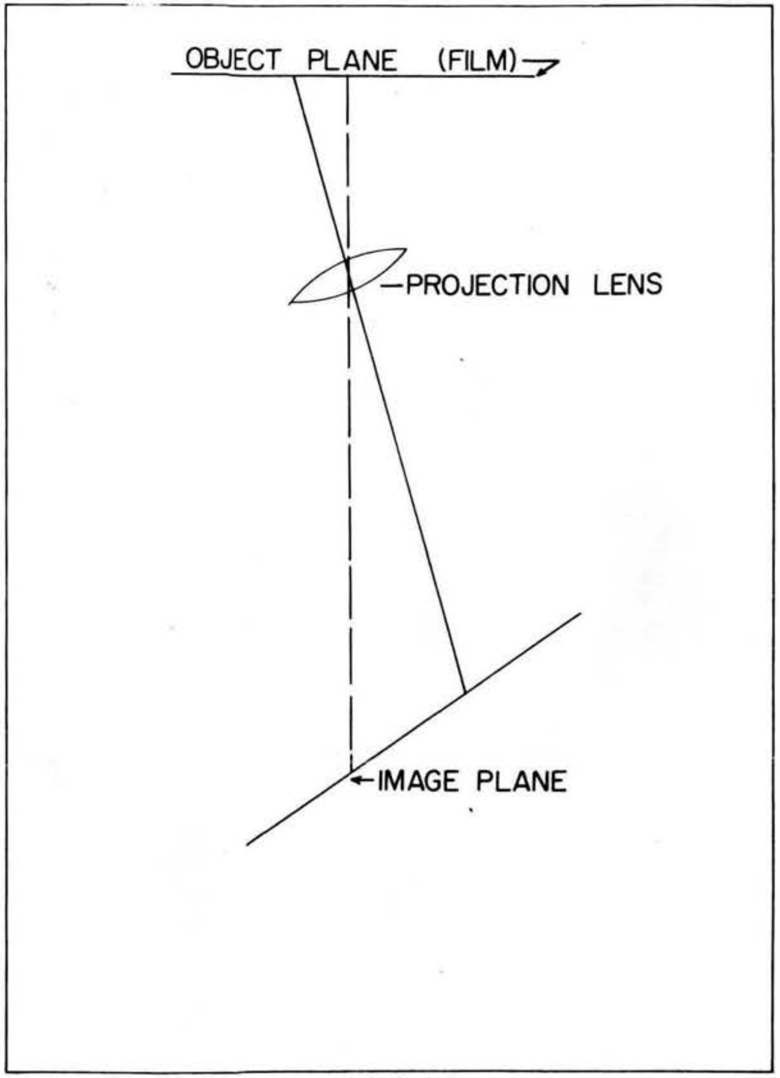

FIGURE 20

SCHEMATIC DIAGRAM OF
MECHANICAL RECTIFICATION METHOD

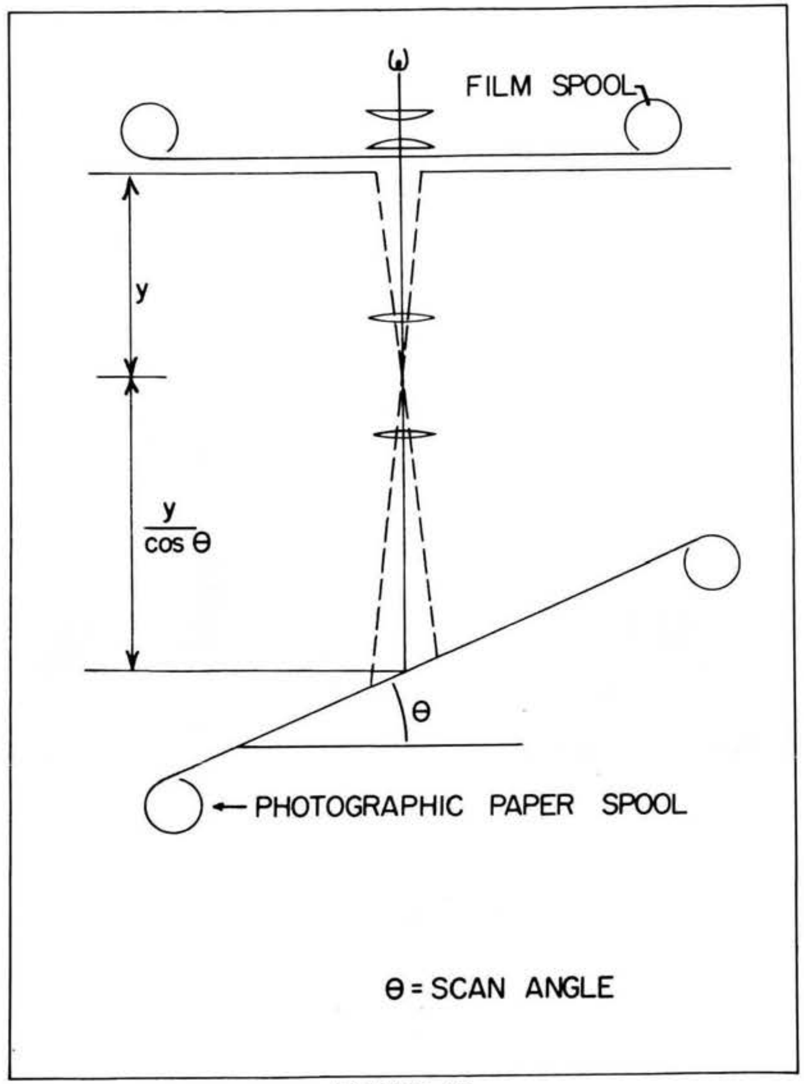

FIGURE 21

MECHANICAL RECTIFIER,
USING PANCRATIC - SLIT SCAN SYSTEM

s = distance along negative, from nadir

r = radius of scan = focal length of camera

θ = angle of scan in radians

$s = r\theta = 24\theta$ (inches) = 2θ (feet)

From Part III it was found that for proper exposure $w = w_o \cos^3\theta$

$w = d\theta/dt$ = angular velocity of scan

w_o = velocity at nadir point ($\theta = 0^o$)

From this it is obvious that:

$V = ds/dt = (d\theta/dt)\ r \cos^3\theta = rw_o\cos^3\theta = rw = V_o\cos^3\theta$

and $V_o = rw_o$

V_o = linear velocity of negative at nadir ($\theta = 0^o$)

V = linear velocity of negative at any value of θ.

A V_o of 26.6 inches/second was computed as a typical value for this type of design.

r = 24 inches = 2 feet

θ		$\cos^3\theta$	Velocity of Negative		X-Distance	
degrees	radians					
0^o	0.00000	1.0000	$1.0000V_o$	26.6''/sec	00.00''	0.0000'
10^o	0.17453	0.9525	$0.9525V_o$	25.3	4.19''	0.3485'
20^o	0.34907	0.8310	$0.8310V_o$	22.1	8.38''	0.697'
30^o	0.52360	0.6480	$0.6480V_o$	17.2	12.57''	1.046'
40^o	0.69813	0.4480	$0.4480V_o$	11.9	16.77''	1.397'
50^o	0.87267	0.2655	$0.2655V_o$	7.1	20.93''	1.745'
60^o	1.04720	0.1250	$0.1250V_o$	3.3	25.15''	2.092'

The velocity of the printing paper is equal to the velocity of the image of Part III.

v = velocity of printing paper = $V/\cos^2\theta$

Velocity of negative and paper as a function of θ

θ	s	V	v
0°	00.00 inches	26.6 in/sec	26.6 in/sec
10°	4.19	25.3	26.1
20°	8.38	22.1	25.05
30°	12.57	17.22	22.95
40°	16.77	11.90	20.30
50°	20.93	7.06	17.04
60°	25.15	3.32	13.28

One other variable is the overall distance (D) from the negative to the printing paper. From Part III

θ	D
0°	48.0 inches
10°	48.2
20°	49.5
30°	51.7
40°	55.4
50°	61.3
60°	72.0

The other variable is the tilt of the easel (image plane). This is simply equal to the angle θ itself.

The relationship of the two lens with each other and with respect to the object plane is the same as computed in Part III. The same aperture, light source, and condensing system will also be required. To keep the image within the proper focal range the width of the slit will have to be 1mm (or less), as computed in Part III.

One of the serious design limitations involves synchronization of the paper and negative speeds with the lens and filament movements, and with the rotation of the easel.

The most serious drawback in continuous contact printing equipment in use today is proper control and correlation of paper and negative velocity. And yet the problem in continous contact printing is easier as the negative and the paper move at the same velocity. The main problem is the prevention of slippage. One method of eliminating slippage would be by the use of sprockets, similar to those used in motion picture photography, and with the same degree of precision.

A suggested method for consideration would be to start the printing process at the nadir and print one side, then come back to the nadir and print the other side. The negative speed could be controlled by the use of the proper electrical circuits applying current to the field of the driving motor of the take-up spools, or the metering rollers.

The filament, easel tilt, object plane-image plane distance, and lens movements are functions of θ directly, therefore their control is relatively simple, and can be accomplished by simple mechanical linkage or cams.

The speed of the printing paper must increase by a function of $1/\cos^2$ over the speed of the negative. One way this can be done is by use of a seperate driving motor for the printing paper, in which the speed is synchronized to the negative position. Another method could be a mechanical linkage from the same motor that drives the negative. There are undoubtedly many other ways this could be done.

B. Fixed Focal Length System, Figure 22

For simplicities sake let us assume the projection lens to be of focal length equal to 12 inches. Magnification then becomes:

$$M = M_o/\cos\theta = -1/\cos\theta = s'/s$$

From this relationship, s is found to be related to s' by:

$$s = s'\cos\theta$$

Relating s to θ:

$$s = -12\cos\theta - 12$$

Relating s' to θ:

$$s' = (12\cos\theta + 12)/\cos\theta$$

The object to image distance becomes the sum of the absolute values of s and s'.

$$D = |s| + |s'|$$

θ	s	s'	D
0^o	-24.00 inches	24.00 inches	48.00 inches
10^o	-23.82 inches	24.20 inches	48.02 inches
20^o	-23.28 inches	24.80 inches	48.08 inches
30^o	-22.40 inches	25.85 inches	48.25 inches
40^o	-21.18 inches	27.66 inches	48.84 inches
50^o	-19.72 inches	30.78 inches	50.50 inches
60^o	-18.00 inches	36.00 inches	54.00 inches

The above results are graphed on Figure 23.

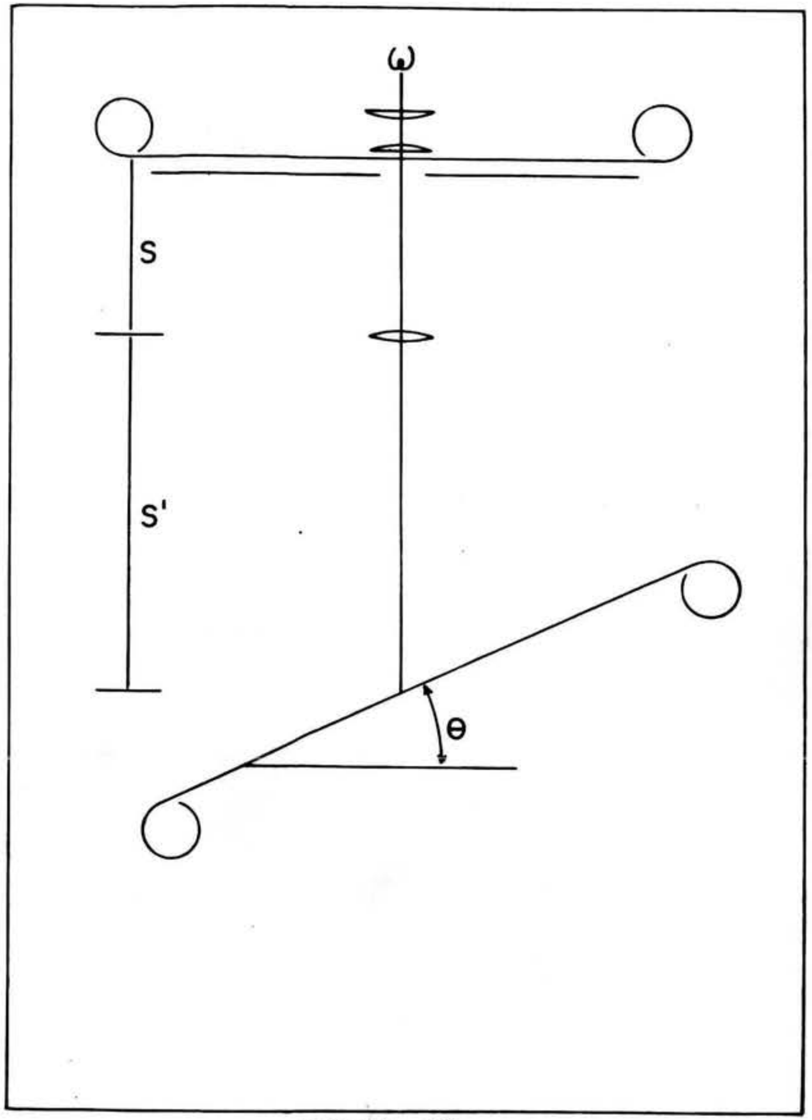

FIGURE 22
MECHANICAL RECTIFIER,
USING FIXED LENS SYSTEM

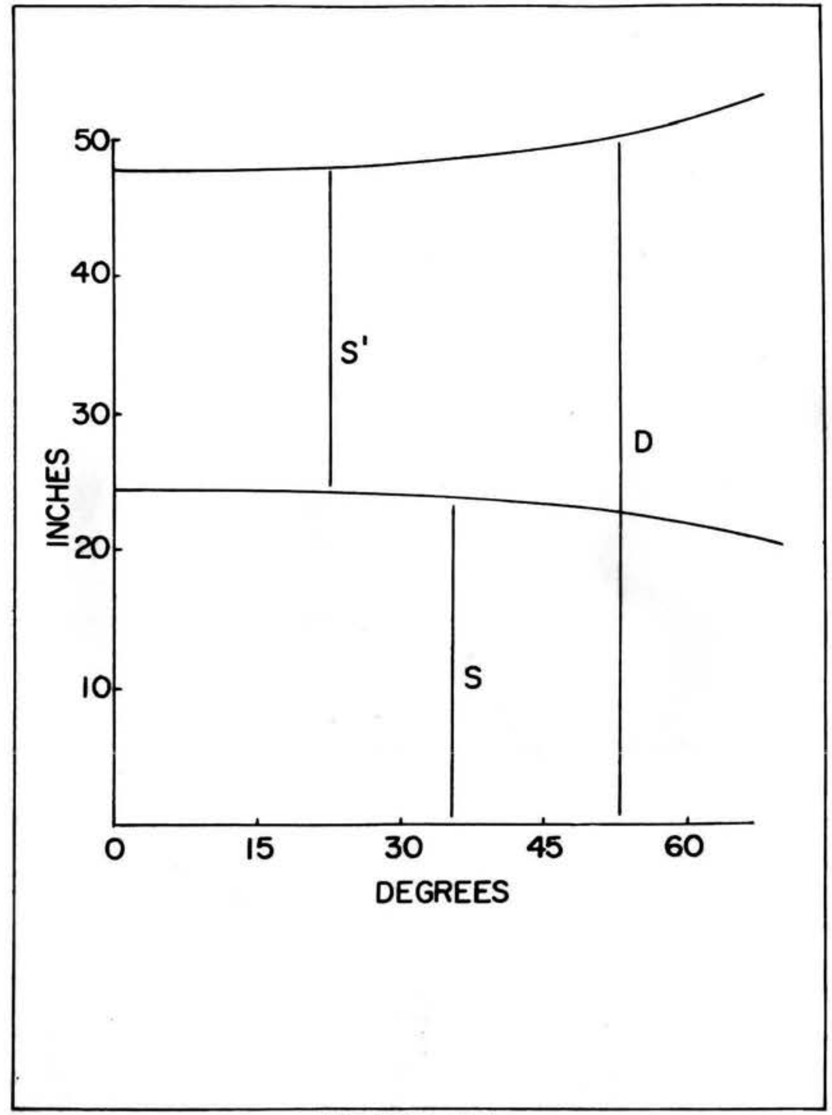

FIGURE 23
S, S' AND D AS A FUNCTION OF θ
FIXED LENS SYSTEM

The other variables which can readily be computed are:

(a) Aperture diameter and position
(b) Condensing System
(c) Filament distance
(d) Slit Width
(e) Negative Velocity
(f) Paper Velocity

PART V

MATHEMATICAL SYSTEM

A third method of rectification considered here is similar to ordinary photographic enlarging, but in this case both the object plane and image plane will have to be curved in one coordinate.

This problem was intensly investigated by Dr. Georg Joos and Mr. John Watson. Their results are published in Boston University, Optical Research Laboratory Technical Note No. 53, entitled, Projection Printer for rectification to the normal and to the 45° Inclination for Whirling Dervish negatives. In view of the complexity of the mathematics involved only a short description of this method will be given in this paper.

Two image surfaces can be computed by a series of integral differential equations, one to comply with the conditions of longitudinal magnification, and one to comply with the conditions of transverse magnification. These two image planes cannot be made to coincide mathematically. Therefore, at best, only an approximate solution can be found, and then only by trial and error.

By computation Dr. Joos found that a cylindrical object surface and an image surface resembling a hyperbola would be a satisfactory approximation. The focal length of the projection lens was found to be more satisfactory if made slightly longer than the focal length of the camera. The radius of the negative surface was found to be best if equal to the focal length of the camera.

In summary, although an exact solution is impossible, the approximate solution determined at the Boston University Optical Research Laboratory was found to have no distortion greater than 14 per cent as long as the rectification does not exceed 63° from the axis.

It is obvious from figure 24 that the illumination problem would be exceedingly difficult.

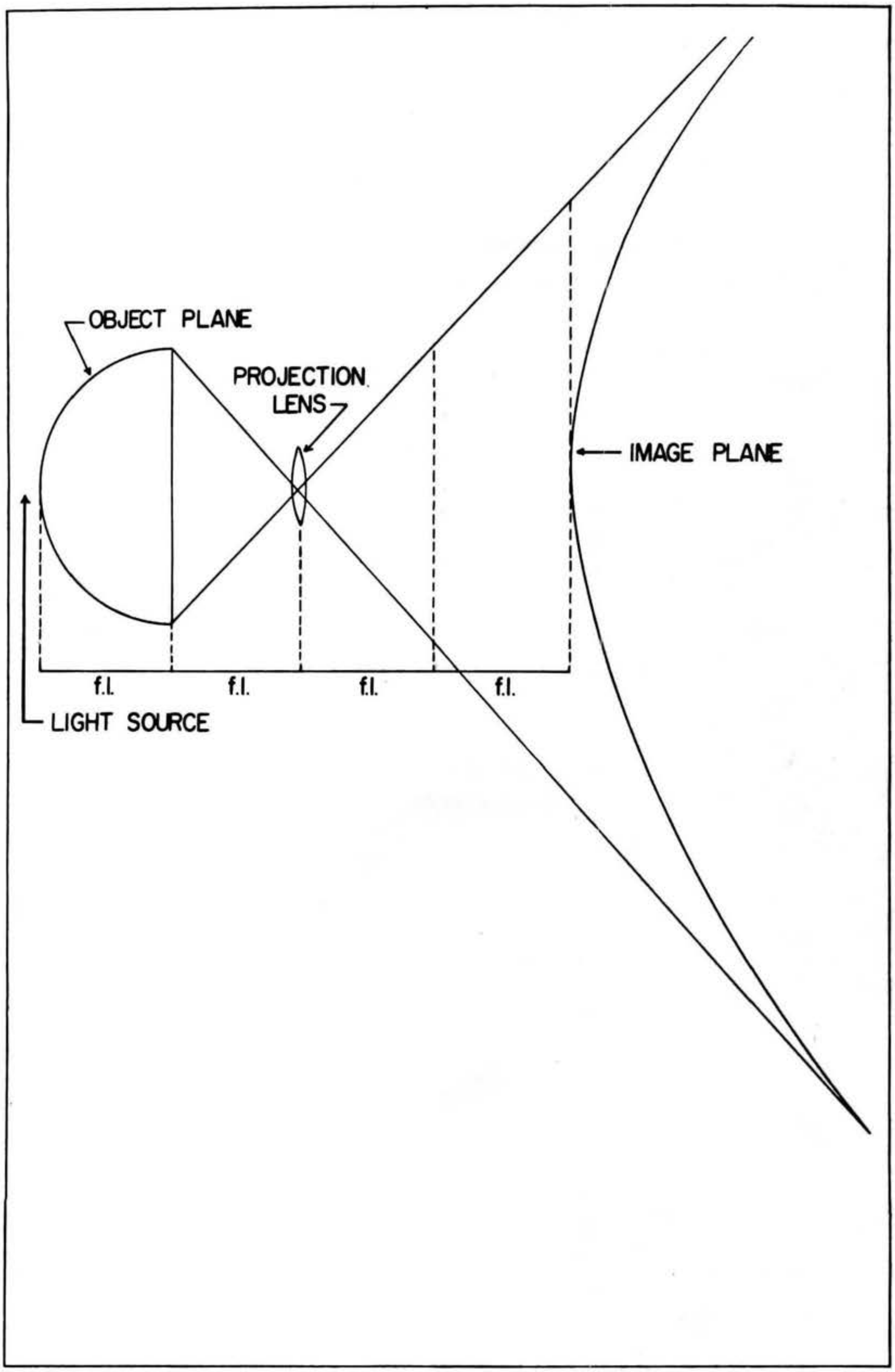

FIGURE 24

SPECIAL SOLUTION TO MATHEMATICAL RECTIFICATION METHOD

PART VI

CONCLUSIONS AND RECOMMENDATIONS

Panoramic photography is almost as old as photography itself. The design of a suitable rectifier depends on the development of an aerial panoramic camera that will produce a perfect negative, or a negative in which the errors can accurately be determined.

It is felt that the E-2 Panoramic Camera, currently under construction will comply with those requirements, except for image motion errors due to inability to compute the exact ground speed of the aircraft and errors resulting from the inability to determine the true plumb point due to lateral and forward accelerations. At present, ground speed can be determined to do within 5%. Current research indicates that this figure can be reduced to 1%. When the ground speed can be determined to within 1% the resulting errors can be considered negligible. Errors resulting from deflection from the vertical are currently being reduced by refinements of stabilized mounts.

The advantages of a panoramic camera over the present tri-metragon system were discussed in Part I, but bears repeating. The tri-metragon system utilizes three cameras, whereas the panoramic system uses one. This eliminates difficulties which arise from improper orientation of the three tri-metragon cameras. But more important, compilation techniques are simplified and reduced to a minimum, thereby necessitating less man-power.

One radial line plot made directly from one panoramic photograph would replace the present three radial line plots made from the corresponding tri-metragon photographs; one simple vertical and two complicated obliques. A panoramic system would not require the rectoblique plotter, nor the oblique sketchmaster as the rectified panoramic photograph is considered to be a vertical. This reduction in the number of steps in processing would lead to a reduction in man-power requirements.

Of the three systems discussed the pancratic lens-slit scan and the mechanical system will give the most accurate results. Previous experience with continuous printing equipment has shown that moving negatives and paper at predetermined speeds during processing is a difficult problem which is still not solved for field equipment. The mechanical system discussed herein is further complicated by demanding that the film and photographic paper must move at different speeds and that their individual speeds constantly change.

In view of the above comments it is felt that the pancratic lens-slit scan system would be the most effective method of rectifying panoramic negatives made by use of a slit scan panoramic camera of the design suggested by Dr. Heidelauf.

If a shorter focal length could be used, as in peace time or civilian precise mapping a split vertical system would be more efficient. This system has been thoroughly investigated by the United States Geological Survey. The Twinplex Plotter was developed along this principle. But this system must have focal lengths of 6 inches or less or the format size of the negative becomes excessively large. For instance, a 24 inch focal length camera with angular coverage of 90° (Total field) would need a negative of 4 feet by 4 feet.

APPENDIX

PROPOSAL OF DOCTOR CLAUS ASCHENBRENNER

Dr Claus Aschenbrenner of Boston University and the Photo Reconnaissance Laboratory, Wright/Patterson AFB, Ohio has suggested a method of rectification shown schematically in Figure 25. It operates as follows:

(1) Negative and paper fixed (or moving together) with respect to mirror 1.

(2) Mirrors 3 and 4 must move laterally to compensate for displacement ($f' \tan x - fx$).

(3) Move lens along axis for adjustment of scale.

(4) Move mirror 2, lens, and mirror 3 as a unit, back and forth for adjustment of sharpness, or use pancratic lens and leave distances of mirrors 1, 2 and 3, 4 constant.

This system could stand more investigation, but after a preliminary investigation it seems to be a modification of the mechanical system discussed in Part IV of this thesis.

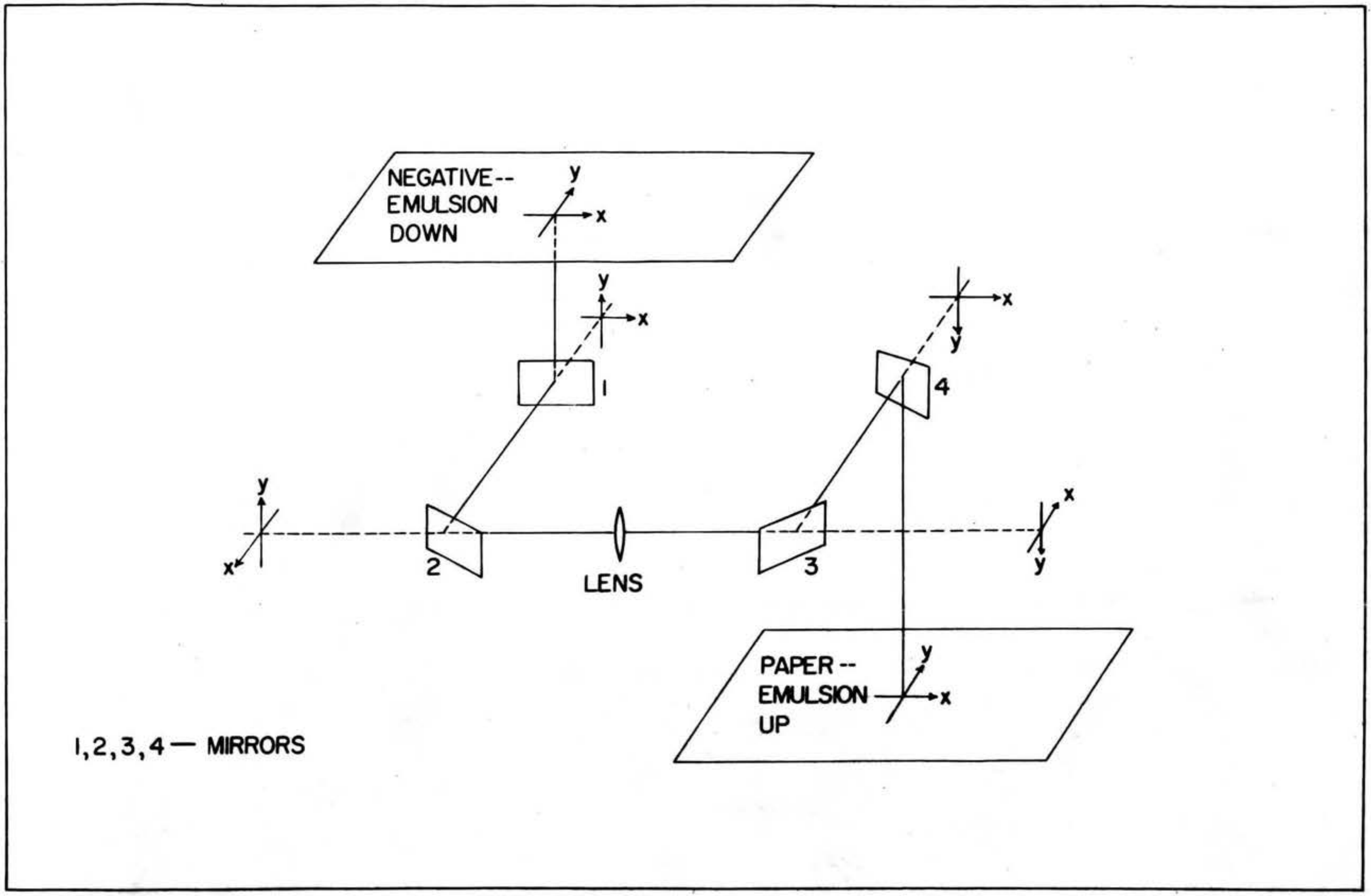

FIGURE 25
DR. ASCHENBRENNER'S PROPOSAL

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ABSTRACT

Panoramic photography is quite old, but is becoming increasingly important in the field of aerial photography. Due to increased efficiency of aircraft warning facilities and accuracy of anti-aircraft missles the problem of obtaining photographs for aerial reconnaissance and mapping and charting becomes extremely hazardous. The obvious solution is to fly higher and further away and to take oblique photographs. This doesn't seem to be difficult, at first glance, but the object to camera distance becomes extremely long, thereby reducing the scale of the photograph. Decreasing the scale of a photograph decreases its usability. The only way to keep a proper scale then, is to increase the focal length. Increasing the focal length increases the format size and the overall camera dimensions, and more important weight. Weight and size are at a premium in aircraft and to combat this a folded optical system is coupled to a slit scan device which utilizes moving film. This reduces weight and size, but introduces other technical problems, which are presently being solved by the U. S. Air Force.

Assuming that the panoramic camera is built and is operational suitable. A method of rectification must be devised in order to effectively utilize the photography. Three methods of rectification are discussed. First, a pancratic lens coupled to a slit scanning device. Second, a mechanical rectifier, utilizing moving film and

paper. Third, a mathematical Solution.

The pancratic lens-slit scan system tends to recreate the conditions in existence at the time of exposure. The Scheimpflug condition is neglected by selecting a slit width so small that the image remains within the focal range.

During the exposure the images were in perfect focus from nadir to horizon, as the distances involved are considered to be infinity. But in rectifier the object to image distance varies from 4 feet at the nadir to 6 feet at $\theta = 60^{\circ}$. Therefore a lens system had to be designed to keep proper focus for all object to image distances and the proper magnification, (to insure the correct scale). These two variables are a function of the angle of scan (θ). The basic design for this pancratic lens starts with selecting 2 lenses of 22 inch focal length and computing the movements necessary to accomplish the required magnifications and focal distances.

It was found that this could be easily accomplished, but the final lens system would have to be designed by a competent lens designer and would be much more complicated, due to a aberration corrections.

For proper illumination the filament of the light source has to move as a function of θ in order to keep imaged onto the aperture.

The effective f/no very nicely stayed approximately the same, 14.75 to 17.25, throughout all the lens movements.

For overall illumination the speed of the scan has to vary to compensate for loss due to the inverse square law, due to tilting of the image plane, and due to increased density of negative toward the horizon.

The second system, the mechanical system, consists of either a pancratic or a fixed lens system, with the negative moving past a slit and the image being projected down on a table onto moving paper.

The moving parts that have to be calibrated are: the negative, printing paper, light source filament, 1st projection lens, 2nd projection lens, rotation of lens system, image to object distance, and tilt of easel. Most of these relationships are easy to compute and control, but the paper speed when determined will be difficult to control. As has been observed in the field, continuous printing devices do not operate efficiently in operational organizations.

The third system, the mathematical solution, uses surfaces which were derived by trial and error after extensive mathematical research. This system can be proved, mathematically, not to be perfect. In fact the distortion has not been reduced less than 14 per cent. This is too much for reconnaissance and charting purposes.

In view of the limitations of the various rectifiers it is felt that the pancratic lens-slit scan system would be the most practical rectifier for field use.

If accepted for field use this pancratic lens system could eventually replace the current tri-metragon method of charting. The panoramic rectification system would eliminate many operations and specialized techniques and save money by reducing man-power requirements per chart.