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An application of Berek's method to triplet design.

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Thesis

AN APPLICATION OF BEREK'S METHOD TO TRIPLET DESIGN

by

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LIST OF ILLUSTRATIONS

INTRODUCTION

The purpose of this project was a third order investigation, by Berek's method, of flat field photographic tripletswith an intended coverage of about 25° half-angular field. This design procedure is described in "Grundlagen der Praktischen Optik" by Dr. M. Berek.

The Berek procedure is essentially based on the Seidel Analysis derived in 1856.[1]

The scope of this project was limited to the one purpose of investigating Berek's design procedure applied to the design of a flat field photographic triplet. It is hoped that the procedure set forth in this thesis will serve two purposes:

1. A basis for comparison with other methods of 3rd order triplet design.
2. A useful design reference for those who wish to make use of this method of design.

Wide angle triplets have, in general, suffered from an excessive curvature of field in their third order approximations. And for that reason, the particular design problem chosen for this investigation was that of producing a somewhat flatter field than is usually found in photographic triplets.

A convenient measure of field curvature is the ratio of the curvature of the 3rd order image surface to the focal length. The usual ratio is between

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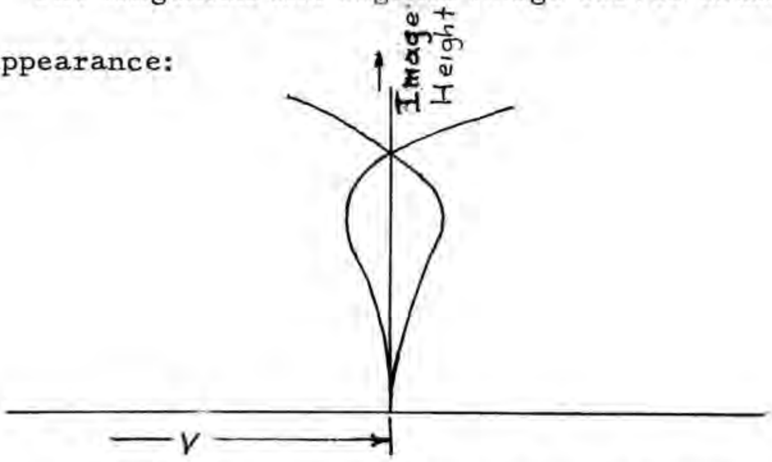
[1] The Encyclopaedia Britannica; p. 58, vol 1, 11th Ed.

2 and 3, but there appears to be a possibility of a higher ratio - that is, a flatter field. The effect of low ratios is that when astigmatism and curvature are corrected by higher order aberrations at a field angle near the margin of the field, the imagery at the lower angles is afflicted with significant astigmatism, which deteriorates image quality in the intermediate field. The tangential and sagittal image curves usually have the following appearance:



Now it is hoped that by systematic third-order exploration of triplets with a higher than usual curvature ratio, a basis can be provided for a satisfactorily corrected triplet with a considerably reduced zonal astigmatism. This curvature ratio is usually called "Petzval ratio". Our original aim was to obtain a Petzval ratio of about 5. This application of Berek's method permits the selection of glasses by numerical procedures. Most optical design procedures involve a half-random choice of glasses based mostly on that intangible called "experience". This is, then, somewhat of an advantage for Berek's method, since even expert designers may have to make several guesses. It will be seen later that it is possible to pick out of several dozen possible combinations - the one combination which best fits the problem.

Basically, the procedure is simple. First, establish the basic design conditions such as the following:

Focal Length

Desired Petzval Ratio

Over-all Thickness of the Lens.

Distortion ------ Corrected

Axial Color ---- Corrected

Oblique Color -- Corrected

Distortion, Axial, and Oblique Color are to be corrected to an acceptably small residual value.

Then we solve six basic equations, using only three assumptions:

1. Lens length-distance from first vertex to last vertex.
2. The sum of the powers of the individual elements.
3. The ratio of ray height on the second element to the height of the same ray on the 1st element.

The solution of the equations (simple ones at that) yields the following information:

1. Power of each element
2. Spacing between elements
3. ν-number ratios for the glasses in the elements.

The third item, ν-number ratios for the glasses in the elements, is the extra information provided by Berek's method, and, when properly applied, will almost automatically select the glasses for each element.

After the glasses have been selected, three equations are set up to correct astigmatism, coma, and spherical aberration by the three available bendings. Solution of these equations yields the radii for all elements. Since all calculations to this point conveniently assume zero thickness for each element, the next step is to assign reasonable thicknesses and readjust radii and spacing (by minor amounts). After this is done, the performance of the lens is checked by ray tracing, and then the work proceeds, utilizing the conventional ray-tracing methods.

Solution of the basic equations resulted in many possible arrangements. We chose the usual form (positive-negative-positive) for this design, but the others, negative-positive-positive and positive-positive-negative, could provide interesting material for further investigation. A telephoto triplet is one possibility, utilizing these unconventional arrangements.

To prevent overlooking any unusual but valid solutions, Dr. K. Pestrecov derived equations corresponding to extreme values of p_2 (power of the second element) since the second element has the most critical curvatures in many triplet designs. These equations were solved, but the solutions were not usable because of the extremely low ν-number ratios required. There exist no such stable glasses. Extreme solutions (minimum absolute values of p_2) have the additional disadvantage of excessive over-all lens thickness.

This thesis covers only the thin lens solution. Remaining steps of design are the assignment of thicknesses; the readjustment of radii and spacing; and finally finishing the design by conventional ray trace methods. However, it is believed that a reasonable amount of time spent on these further steps would produce a photographic triplet of satisfactory performance.

I am greatly indebted to Dr. Pestrecov for first proposing the problem and an approach; outlining for me a summary of Berek's method and then explaining the procedure to me in such a manner that I was able to proceed with the work in a reasonably efficient manner. His constant availability for questions, his enthusiasm for the work, and his frequent sound advice from his rich store of experience and perceptive abilities - all of these were of extreme value in the successful completion of the project.

II. NOTATIONS, DESIGN ASSUMPTIONS, AND REQUIREMENTS

A. Notation

The notations adopted in this design project are as follows:

f	focal length
p_1, p_2, p_3	powers of the individual elements
$p = 1/f = 1$	total power of the combination
$\Sigma p = p_1 + p_2 + p_3$	sum of powers of individual elements
h_1, h_2, h_3	heights of paraxial ray on the individual elements
d_1, d_2	separation between elements
$f = d_1 + d_2$	over-all thickness of the lens
P	Petzval sum
$P = \frac{1}{P}$	Petzval ratio. When $f = 1$, $P = 1/P$
u	Object distance from the surface vertex
v	Image distance from the surface vertex
n	Index of refraction for glass used in the elements.
$\frac{\nu_2}{\nu_1}, \frac{\nu_3}{\nu_1}$	ν-number ratios for element glasses.

Sign conventions:

a. All distances to the right of a surface are positive.

b. All distances to the left of a surface are negative.

c. All rays with a positive slope have positive angles and sines. (Here, standard analytic geometry notation is assumed.)

d. All rays with a negative slope have negative angles and sines.

e. All radii whose centers are to the right of a surface are positive.

f. All radii whose centers are to the left of a surface are negative.

A _______________ Seidel specific coefficient for spherical aberration

B _______________ Seidel specific coefficient for coma

C _______________ Seidel specific coefficient for astigmatism

D _______________ Seidel specific coefficient for distortion

B. Design Assumptions and Requirements

1. Assumptions

   a. Thin lens basic solution (element thickness = 0)

   b. $u_1 = -\infty$

   c. $f = 1.0$

   d. Distortion correction term is assumed to be zero. Discussion is on page 1.

   e. Oblique color correction term is to be zero for Cand F light.

   f. Axial color correction term is to be zero value for C and F light.

   g. Seidel specific coefficients for third order aberrations other than distortion are desired to be optimum values estimated from experience as follows:

      A ---------- -0.60 to -1.20

      B ---------- -.20

      C ---------- +.06

   h. A Petzval ratio of about 5.0 is set as a goal.

   i. Over-all lens thickness (ℓ) is to be kept to a minimum-consistent with good performance.

j. Design data to be obtained are:

- 1. From the solution of the basic equations:
  - (a) Powers of the individual elements: p_1 , p_2 , p_3 .
  - (b) Spacings between elements: d_1, d_2.
  - (c) ν-number ratios:
- 2. From the solution of the three bending equations
  - (a) Radii for each element.
- 3. Assigned on the basis of manufacturing requirements:
  - (a) Thickness for each element.

III. DETAILED PROCEDURE

A. Discussion of design variables listed in j. -1. above, and the basic equations for their determination.

I am indebted to Dr. K. Pestrecov for the following discussion of the values to be assigned to the distortion and other aberrations before solution of the basic equations.

> " It should be noted that setting the condition of a corrected distortion at this stage of the design is based on the assumption that the stop is in coincidence with the second element. Then the third order distortion will be nearly independent of the bending of the lens elements; and the designer is relieved from the necessity of dealing with a rather complicated general expression for distortion. The validity of this simplification has been questioned by some designers. It should be understood, however, that no claim is made here that the simplified condition should yield a system highly corrected for distortion. The expectation, confirmed by actual experience, is that the distortion will be kept within some reasonable bounds, and its ultimate correction, as the correction of all other higher order aberrations, can be effected in the final design stages. The convenience of the basic assumption seems to outweigh the objections based on a rigorous analysis of the situation.
>
> Essentially of the same nature is the question whether some residual values should be assigned to the expressions for the axial color and oblique color, or whether these expressions could be equated to zero; and likewise, the question arises whether the lens elements should be assigned some thicknesses in the beginning, or whether these thicknesses should be assigned after the thin lens solution has been completed. Again, the fact that we can expect to make some adjustments in the final design stages - regardless of which assumption is made - and the considerations of convenience make a powerful argument for the adoption of zero values for these quantities. "

The six basic equations follow: Berek, p 124

1. $p_1 + p_2 \frac{h_2}{h_1} + p_3 \frac{h_3}{h_1} = \frac{1}{f}$ (focal length)

2. $\frac{p_1}{n_1} + \frac{p_2}{n_2} + \frac{p_3}{n_3} = -P$ Petzval sum

3. $\frac{p_1}{\nu_1} + \left(\frac{h_2}{h_1}\right)^2 \frac{p_2}{\nu_2} + \left(\frac{h_3}{h_1}\right) = 0$ Axial Color

4. $\frac{d_2 \cdot \frac{h_3}{h_1} \cdot \frac{p_3}{\nu_3} - \frac{d_1 p_1}{\nu_1}}{d_2 - \frac{h_2}{h_1}\left(1 - \frac{h_3}{h_1}\right) f} \frac{h_3}{h_1} = 0$ Lateral Color

5. $p_1 d_1 - p_3 d_2 = 0$ Distortion

6. $d_1 + d_2 = \ell$ Over-all Lens Thickness.

There are 13 variables in these equations: $\frac{h_2}{h_1}, \frac{h_3}{h_1}, p_1, p_2, p_3, \nu_1, \nu_2, \nu_3, d_1, d_2, \ell, P, f,$

Berek ignores the glass indices when listing the variables, because they are not very powerful variables; and only a limited choice of glasses is available. The glass indices may be excluded by the convenient procedure of replacing the second equation by $p_1 + p_2 + p_3 = \sum p$, where $\sum p$ is a reasonably small value assigned on the basis of a study of a number of possible solutions, or on the basis of previous experience. Then glasses are selected whose indices will yield the proper Petzval sum when this sum is calculated with the powers obtained from the solution of the six simultaneous equations. They are selected from a family of glasses, whose ν-number ratios are reasonably close to the ratios calculated from the solution of basic equations.

Since the solution of six simultaneous equations can only yield answers for six unknowns, it is necessary to account for 7 of them as follows: Solution of these equations yields $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ rather than absolute values of ν_1, ν_2 and ν_3. Therefore we may use $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ as variables and set ν_1 as an arbitrary parameter. But $\frac{\nu_3}{\nu_1} = \frac{h_3}{h_1}$; and $\frac{h_3}{h_1}$ is determined by the powers and spacing of the elements and is therefore a dependent variable. This can be shown through the paraxial ray trace relationships. ℓ is chosen arbitrarily on the basis of experience and varied in discrete steps. Σ p is set as an arbitrary quantity based on experience or investigation; and it is also varied in discrete steps in combination with the length, ℓ. f is set at 1.0.

$\frac{h_2}{h_1}$ is arbitrarily set, within limits determined by solution of the cubic equation previously mentioned. This equation is derived from a part of the six basic equations and possibly could even be considered a seventh equation.

Now we have shown that $\nu_1, \frac{\nu_3}{\nu_1}, \ell, \Sigma p, \frac{h_2}{h_1}, \frac{h_3}{h_1}$, f are either arbitrarily set or are dependent variables deriving their values from one or more of the six independent variables $p_1, p_2, p_3, \frac{\nu_2}{\nu_1}, d_1$ and d_2.

Therefore we have six equations in six unknowns and may proceed to their solution in a straightforward manner.

It may be pointed out that ℓ, Σ p, $\frac{h_2}{h_1}$, f must be known before the equations can be solved; $\frac{h_3}{h_1}$ and $\frac{\nu_3}{\nu_1}$ are dependent variables appearing during the solution; and ν_1 is not chosen until after their solution. For convenience

Berek derives simpler expressions for direct determinations of the six independent variables. The forms actually used follow below: Berek, p 126

1. $p_2 = \dfrac{\frac{h_2}{h_1}\Sigma p - 1}{\frac{h_2}{h_1}\left(1 - \frac{h_2}{h_1}\right)}$

2. $d_1 = .5\ell\left(1 \pm \sqrt{1 - \frac{4}{\ell}\frac{1-\frac{h_2}{h_1}}{\Sigma p - p_2}}\right.$

3. $d_2 = \ell - d_1$

4. $p_1 = \dfrac{1 - \frac{h_2}{h_1}}{d_1}$

5. $p_3 = \dfrac{1 - \frac{h_2}{h_1}}{d_2}$

6. $h_3 = 1 - \dfrac{d_2}{\frac{h_2}{h_1}}$

7. $\dfrac{\nu_3}{\nu_1} = \dfrac{h_3}{h_1}$

8. $\dfrac{\nu_2}{\nu_1} = \dfrac{p_1\left(\frac{h_2}{h_1}\right)^2}{p_2\left(\frac{h_2}{h_1}\right) - 1}$

Berek gives Equation 8 in the following form to allow for residual axial color (ΔS_K) and focal lengths other than 1.0: $\nu_2 = \dfrac{\left(\frac{h_2}{h_1}\right)^2 p_1 \nu_1}{p_2\frac{h_2}{h_1} - \frac{\Delta S_K \nu_1}{F^2} - \frac{1}{F}}$

It will be noticed that the expression for d_1 yields two roots, thus giving two solutions for every combination of ℓ, Σp, and $\frac{h_2}{h_1}$. These solutions are the reverse of one another insofar as powers and spacings are concerned; but the $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ ratios are changed. This has the practical effect of doubling the ν-number ratio combinations in which we can try to fit our available glasses. Some values of $\frac{h_2}{h_1}$ (for a given Σp and ℓ) will cause the quantity under the square root symbol to become negative, yielding an imaginary root. When the known values of l and Σ p are substituted; and p_2 is substituted in terms of $\frac{h_2}{h_1}$ a cubic equation is obtained.

$$\left(\frac{h_2}{h_1}\right)^3 - \left(\frac{h_2}{h_1}\right)^2\left(2 - \Sigma p \ell\right) + \frac{h_2}{h_1} - .25\,\ell = 0$$

The solution of this cubic equation yields three real roots. These roots define the regions of $\frac{h_2}{h_1}$ for which real -- values for d_1 and d_2 may be obtained. It will be shown later that evaluation of the possible regions dictates an $\frac{h_2}{h_1}$ of approximately +0.70 to +0.80 to avoid unusual element arrangements and steeply curved surfaces.

B. A Detailed Design Outline for a Third Order Solution.

After investigation and rejection of minimum p_2 solutions, solution of the six basic equations was undertaken according to the following design procedure plan. This plan will be briefly stated to serve as a topical and chronological design outline; and then each step will be discussed with summarized data:

1. Choose Petzval sum limit (i.e., P = - 0.20).
2. Choose several Σ p's and lengths.
3. Set $\sqrt{1 - \frac{4}{\ell}\frac{1 - \frac{h_2}{h_4}}{\Sigma p - p_2}}$ =0, substituting for p_2 in terms of $\frac{h_2}{h_1}$ and Σ p.

4. Solve the resultant cubic equation $\left(\frac{h_2}{h_1}\right)^3-\left(\frac{h_2}{h_1}\right)^2\left(2-\Sigma p\ell\right)+\frac{h_2}{h_1}-.25\ell=0$ by synthetic division for each combination of ℓ and Σ p. There will be solutions with three real roots. Ignore the largest and smallest.
5. Solve the basic setup equations using the intermediate root or a value slightly higher. One value of $\frac{h_2}{h_1}$ is usually sufficient for each combination.
6. Check the resultant p_2, $\frac{\nu_2}{\nu_1}$, and $\frac{\nu_3}{\nu_1}$ for practicability. The problem is to keep p_2 low and at the same time have a low Petzval sum.
7. Choose from this collection by selection (or by interpolation and recalculation), a single combination of ℓ and Σ p.
8. List the obtained values of powers, spacings and ν-number ratios for the chosen combination.
9. Prepare a list of glasses of high index and high ν-number ratio for element 1. $\nu_1 \geqslant 50$ and $n_1 \geqslant 1.60$ are reasonable minimums.
10. Prepare a table listing column headings as follows: Glass Type, ν_2 calc., ν_2-Glasses, ν_3 calc., ν_3-Glasses. See Figure 5.
11. Calculate maximum and minimum ν_2 and ν_3 corresponding to maximum and minimum ν_1.

12. Prepare a separate list, in order of ascending or descending ν-numbers, of all available glasses in these ranges. n_2 will automatically be approximately 1.6 or higher. Choose n_3 greater than 1.6 to keep the Petzval sum down.
13. Now, from this list, enter in the appropriate spaces of the glass selection table in step 10, all glasses whose ν-numbers are within ±0.4 of the calculated values of ν_2 and ν_3.
14. From this completed table, choose combinations and compute Petzval sums to see if the Petzval sum is satisfactory. Trial and error is necessary until the desired sum is obtained.
15. We are now ready to determine the radii for correction of aberrations using the glass combination chosen in step 14. On the basis of experience choose values for A, B, C (Seidel specific coefficients for spherical aberration, coma, and astigmatism, respectively).
16. Derive the three bending equations using the powers, spacings and indices already determined.
17. Set up the three thin lens bending equations, substituting B and C as chosen, but do not substitute for A. The chosen A will serve as a comparison standard.
18. Solve the C equation for r_3, substituting reasonable values of r_2.
19. Solve the B equation, using the r_2 and r_3 determined in step 18.
20. Solve the A equation for A, using the r_1, r_2, r_3, thus far determined.

21. Repeat steps 18, 19, 20, for strategically chosen values of r_2 and plot the resulting values of A vs r_2.

22. Now from the resulting curve pick an r_2 that will give A sufficiently close to the assigned design value.

23. If no satisfactory solution is found, it will be necessary to go back to step 8 and repeat all following steps, using a different ℓ and Σ p combination until a satisfactory solution is obtained.

24. When a satisfactory solution is obtained, a check may be run by means of a Seidel aberration calculation sheet- which will be described later. This involves tracing a parallel paraxial D ray through the system to determine u and v for each surface. The obtained values should check with the values given by the equations.

25. After the check in step 24, axial color should be determined and plotted as follows:
    a. trace C and F and G' axial rays.
    b. plot v_6 against the wavelength of the light.
    c. calculate axial color as Δv_6 between the vertex of the curve and the worst deviated wavelength in the range to be used.

26. Now if the resultant Δv_6 (axial color) is considered not quite satisfactory, proceed to correct as follows:

   a. choose another glass with a slightly different ν-number for a single element of the lens, remembering that element # 2 will be stronger in effect than element # 1 or # 3.
   b. recalculate radii to maintain the element power constant and the ratio between radii constant (the shape factor K).
   c. redetermine axial color as in step 25.
   d. on the basis of this trial choose another glass for the same or different element.
   e. redetermine axial color as in step 25.
   f. repeat this procedure until the optimum compromise is found. Note: This is the end of the thin lens solution.

27. On the basis of the necessary edge thickness for good manufacturing practice, determine the thicknesses of the lens elements.
28. Recalculate radii for each element keeping element powers and principal plane separations constant between elements and maintaining a constant ratio between radii (K factor).
29. Trace a D axial ray for an infinitely distant object or obtain u's and v's for each surface.
30. Calculate a new Seidel sheet to determine third order aberrations for the new system. This will probably not be satisfactory.

31. Prepare a change table as follows:

    a. vary r_1 (the reasonable amount of variation is based on previous experience, it may be as high as 25% in some cases).

    b. recalculate r_1' (second surface of element 1).

    c. recalculate spacings to maintain constant separation between principal planes.

    d. trace a new D axial ray as in step 29.

    e. redetermine the Seidel aberrations as in step 30.

    f. repeat a, b, c, d, e for r_2 .

    g. repeat a, b, c, d, e for r_3 .

    h. tabulate these changes in each aberration against changes in the radii.

32. Now by inspection or by solution of simultaneous equations which can be set up from the change table, determine new radii for one or more elements.

33. Repeat steps 28, 29, 30.

34. a. If the resultant Seidel aberrations are close to the assigned design values, then the 3rd order design may be considered complete and the remainder of the design completed by geometrical ray tracing techniques.

    b. If the resultant Seidel aberration, are not sufficiently close to the pre-assigned design values, then probably one or more of the elements will have to be changed in an opposite direction (based on an analysis of the accumulated data); and steps 29, 30, 31 repeated. When a satisfactory answer is obtained, proceed as in 34a.

Completion of the design by geometrical ray tracing techniques was not included in this design project. Work was stopped short of the completion of a satisfactory 3rd order design with thick elements. Therefore no results are offered here. However, the outline procedure for the 3rd order thick element solution may be of interest to those desiring to go beyond the thin lens solution.

IV. DISCUSSION OF DESIGN OUTLINE AND SUMMARIZATION OF DATA

1. Choosing Petzval Sum Limit:

The choice of a Petzval sum and consequently the Petzval ratio, defined- $P_r = \frac{1}{\rho}$ is dictated by two opposing tendencies. A low Petzval sum (high Petzval ratio) will give a flatter image surface. This will result in: (1) less curvature of field and less zonal astigmatism, (2) steeper curves on the negative element. Now the first tendency is entirely desirable, but steeper curves on the second element give larger third-order aberration contributions. These larger aberration contributions are more difficult to balance out and often involve extreme bending. It is necessary therefore to resort to a compromise between these two tendencies. The usual compromise is a Petzval ratio between 2.0 and 3.0. In this design a Petzval ratio of 5.0 was set as a goal; and this ratio was kept throughout the design until the point was reached where element thicknesses were assigned. When reasonable element thicknesses were put into the design, the Petzval ratio was reduced from 5.0 to 4.2. This reduction may be explained as follows:

After thickness is assigned to a positive element (of the shape usually found in triplets -- i.e., double convex) and we trace a ray through it (keeping the same radii as before), we find that the power is reduced.

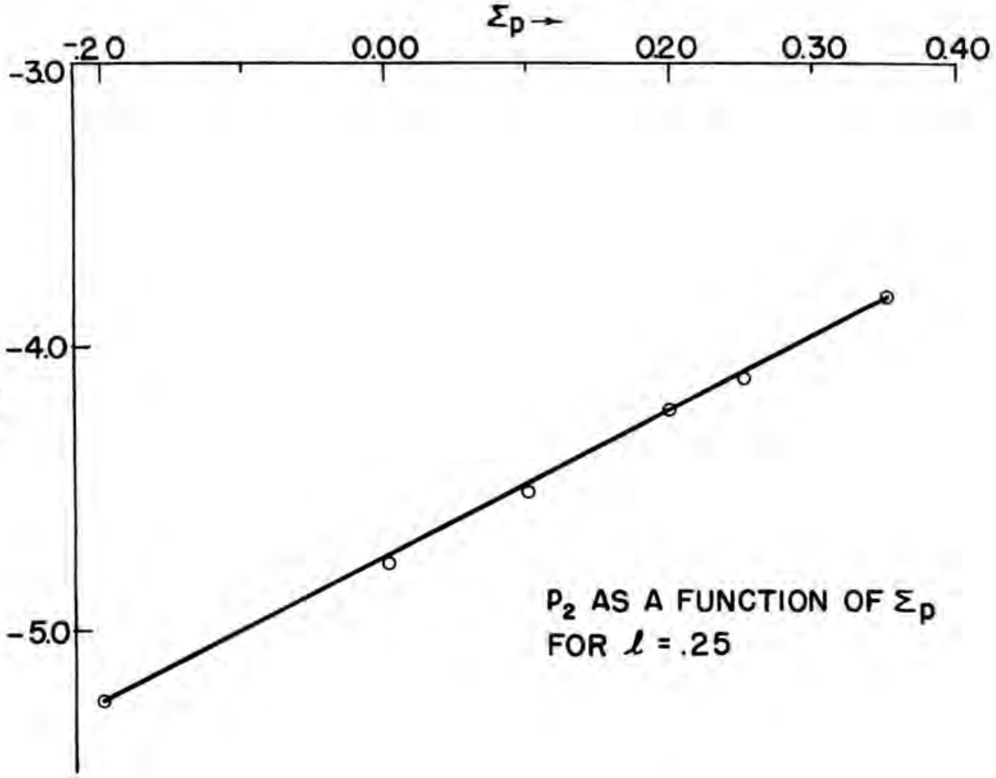
It may also be shown that adding thickness to a negative element which is double concave causes increased power, if we keep the radii constant. Since in this design procedure element powers and shape factors (ratio of radii) are kept constant, it will be seen that the radii of the positive elements become steeper and the radii of the negative elements become shallower. Since $P = \sum_{r_1}^{r_K} \frac{n-n'}{r}$, substitution of these new radii in this equation will reveal that the effect of all three elements is to increase the Petzval sum, or conversely, to reduce the Petzval ratio. Therefore, when setting out to design a triplet by this method, it is necessary to attain a Petzval ratio for the thin lens solution which is 20-25% greater than that desired for the final solution.

2. Choosing $\Sigma\rho$'s and overall lens thicknesses:

Overall lens thicknesses were chosen on the basis of experience for maximum compactness and minimum element diameters consistent with reasonable values for p_2. Solution of the basic equations for several different thicknesses revealed that p_2 increased as the thicknesses were decreased while $\Sigma\rho$ was kept constant. Overall lens thicknesses of 0.25, 0.30, and 0.35 were chosen and investigated. The criterion was that the absolute value of p_2 would not be allowed to exceed 4.0. This criterion was arbitrarily set-also on the basis of experience. Results will be discussed under step 6.

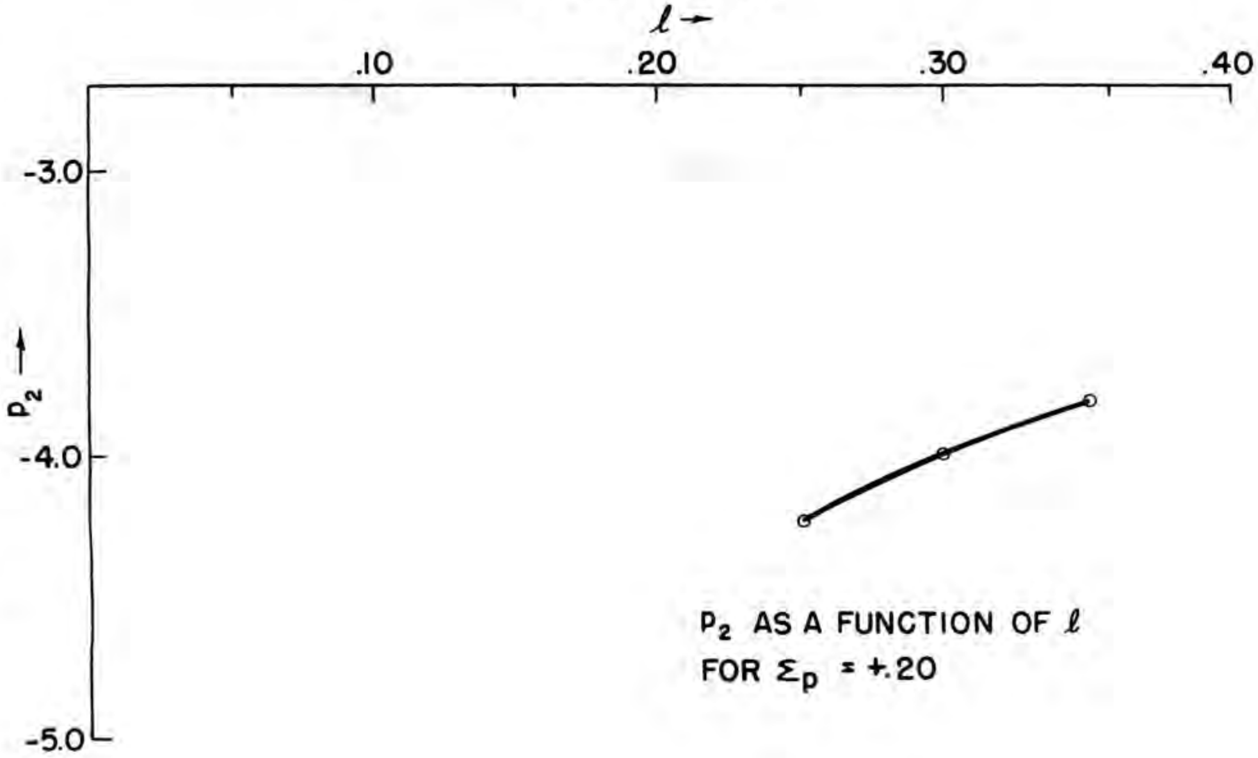
$\Sigma\rho$'s are chosen to bracket the solution of the equation: $\Sigma\rho = -nP$ where P was taken 0.20 and n(the effective index) was taken to be roughly 1.0-1.5. This would put $\Sigma\rho = 0.20$ to 0.30. To avoid the possibility of a bad guess, values of $\Sigma\rho = -0.20, 0.00, +0.10, +0.20, +0.25, +0.35$ were investigated to determine the relationship between $\Sigma\rho$ and p_2. The results are embodied in the graph in Figure 1a where p_2 is plotted vs $\Sigma\rho$ while ℓ remains constant at 0.25. Also in Figure 1b p_2 is plotted vs ℓ for $\Sigma\rho = +0.20$.

FIGURE 1a



P_2 AS A FUNCTION OF Σ_p
FOR ℓ = .25

FIGURE 1b



P_2 AS A FUNCTION OF ℓ
FOR Σ_p = +.20

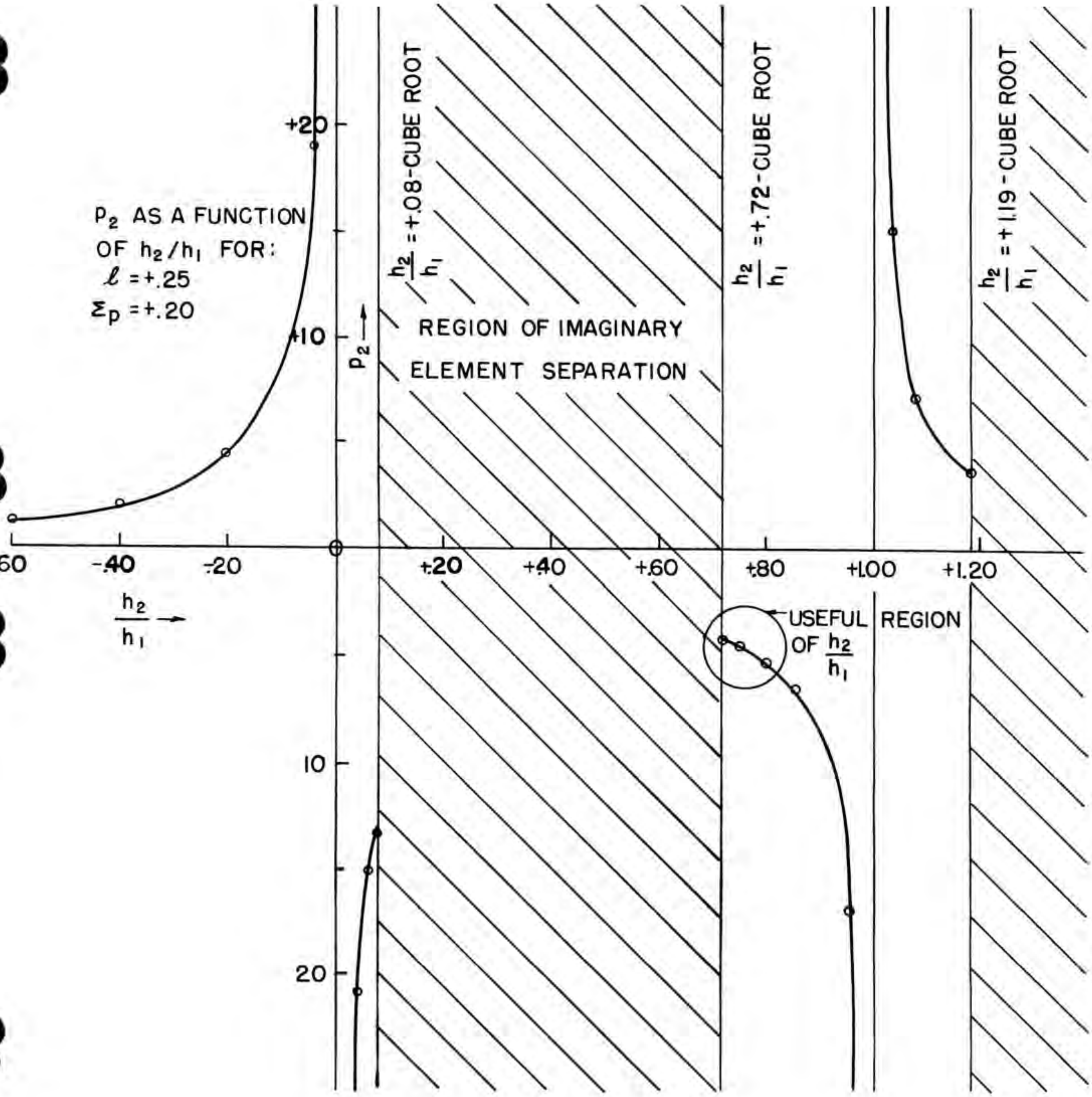
3. and 4. Transformation and solution of the length equation:

After preliminary calculations, it became evident that the minimum absolute value of p_2 for a given combination of Σp and ℓ would be obtained when the quantity under the radical was set equal to zero. To determine at what $\frac{h_2}{h_1}$ this would occur, it was necessary to substitute for P_2 in terms of $\frac{h_2}{h_1}$ and solve the resulting cubic equation by synthetic division for its three real roots in $\frac{h_2}{h_1}$. Regions in which the cubic remainder was negative gave real roots for the square root term and therefore real quantities for spacing, whereas regions where the cubic remainder was positive gave imaginary roots for the square root term. This meant an imaginary spacing and an unusable solution. Negative values of $\frac{h_2}{h_1}$ gave one positive and one negative spacing with an overall thickness greater than the assigned ℓ. This region was discarded as unpromising, but a further investigation might very well be worth while. Figure 2 shows p_2 plotted against $\frac{h_2}{h_1}$ from -0.60 to -1.20 and shows how p_2, considered to be the most critical design element, varies as $\frac{h_2}{h_1}$ is changed. Inspection of this curve quickly drives home the conclusion that, for conventional triplet arrangement the most advantageous $\frac{h_2}{h_1}$ is the intermediate cubic root or a value slightly higher. On the graph, the cubic roots are indicated by the vertical margins of the imaginary regions; and the useful region is circled.

5. Solution of the basic equations:

Having selected an ℓ, a Σp, and having solved the cubic equation for an approximate $\frac{h_2}{h_1}$, solution of the basic equations now proceeds in a straight-forward manner. It was found advantageous to utilize a tabular form

FIGURE 2


P₂ AS A FUNCTION
OF h₂/h₁ FOR:
ℓ = +.25
Σp = +.20
+20
+10
10
20
p₂ →
h₂/h₁ →
-.60
-.40
-.20
0
+.20
+.40
+.60
+.80
+1.00
+1.20
h₂/h₁ = +.08-CUBE ROOT
h₂/h₁ = +.72-CUBE ROOT
h₂/h₁ = +1.19-CUBE ROOT
REGION OF IMAGINARY
ELEMENT SEPARATION
USEFUL REGION
OF h₂/h₁

for the solution of the equations; and a sample computation sheet is included as Figure 3.

6. Checking p_2, ν_2, ν_3 for practicability:

In choosing a solution for further calculation, the desirable goals to keep p_2 low, keep the Petzval sum low, keep the overall thickness to a practical minimum and to obtain $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ ratios which will fit the available ranges of glasses. In this project, it was found that there was sufficient variety in the American glasses to provide a number of possible solution; and only American glasses were considered in the final selection.

As previously mentioned, under step 2, it was considered very desirable to keep p_2 under 4.0 absolute. An overall thickness equal to or less than 0.30 was desired. The Petzval sum was to be equal to or less than 0.20 absolute; and $\frac{\nu_2}{\nu_1}$ equal to or greater than 0.50 was considered advantageous. All solutions are summarized in the table in Figure 4.

In picking out likely solutions with minimum p_2 value the following relationships were considered: Solutions with $\ell = 0.35$ gave a low p_2, but they also gave lower $\frac{\nu_2}{\nu_1}$ ratios than were considered advantageous for the available glasses. This greater overall thickness might also be somewhat of a disadvantage for wide angle lenses-requiring larger element diameters than the smaller overall thicknesses and a consequent increase in manufacturing difficulty and expense.

The purpose of this stage of the investigation was to select glasses and check the Petzval sum against the design goal. Inspection of the table of solutions (figure 4) and figure 1a, b will show that as the overall lens thickness increases, the p_2 decreases and the $\frac{\nu_2}{\nu_1}$ ratio decreases.

FIGURE 3

SOLUTION OF THE BASIC EQUATIONS

$\ell = +0.30$
$\Sigma p = +0.25$

CUBIC ROOTS = $\frac{h_2}{h_1}$ = 0.10; +0.691; +1.15
REAL REGIONS = <0.10> +0.691 < 1.15
USEFUL REGION = 0.691 → <1.00; $p_2 \to -\infty$ AT 1.00

$\frac{h_2}{h_1}$	+0.691	+0.72	+0.75	+0.775	+0.80	+0.85	+0.90
$1/\frac{h_2}{h_1}$	+1.447	+1.389	+1.333	+1.290	+1.250	+1.176	+1.111
$1-\frac{h_2}{h_1}$	+0.309	+0.280	+0.250	+0.225	+0.200	+0.150	+0.100
$\Sigma p - 1/\frac{h_2}{h_1}$	+ 1.197	-1.139	-1.083	-1.040	-1.000	-0.926	-0.861
$p_2 = \frac{\Sigma p - 1/h_2/h_1}{1-h_2/h_1}$	-3.874	-4.068	-4.333	-4.624	-5.000	-6.177	-8.611
$\Sigma p - p_2$	+4.124	+4.318	+4.583	+4.874	+5.250	+6.427	+8.861
$d_1 = .5\ell(1-\sqrt{1-\frac{4}{\ell}\frac{1-h_2/h_1}{\Sigma p - p_2}}$	+0.155	+0.205	+0.228	+0.243	+0.255	+0.274	+0.288
$d_2 = \ell - d_1$	+0.145	+0.095	+0.072	+0.057	+0.045	+0.026	+0.012
$p_1 = \frac{1-h_2/h_1}{d_1}$	+1.995	+1.365	+1.095	+0.926	+0.784	+0.546	+0.347
$p_3 = \frac{1-h_2/h_1}{d_2}$	+2.130	+2.953	+3.489	+3.948	+4.466	+5.880	+8.264
$p_2 = 1/\frac{h_2}{h_1}$	-5.322	-5.456	-5.667	-5.914	-6.250	-7.353	-9.722
$\frac{V_2}{V_1} = \frac{p_2\ h_2/h_1}{p-1/h_2/h_1}$	+0.503	+0.537	+0.574	+0.606	+0.640	+0.714	+0.797
$\frac{h_3}{h_1} = \frac{V_3}{V_1} = 1-\frac{d_2}{h_2/h_1}$	+0.790 CHOSEN	+0.868	+0.904	+0.926	+0.944	+0.970	+0.987
$d_1 = .5\ell(1-\sqrt{1-\frac{4}{\ell}\frac{1-h_2/h_1}{\Sigma p - p_2}}$	+0.145	+0.095	+0.072	+0.057	+0.045	+0.026	+0.012
$d_2 = \ell - d_1$	+0.155	+0.205	+0.228	+0.243	+0.255	+0.275	+0.288
$p_1 = \frac{1-h_2/h_1}{d_1}$	+2.130	+2.953	+3.489	+3.948	+4.466	+5.880	+8.264
$p_3 = \frac{1-h_2/h_1}{d_2}$	+1.995	+1.365	+1.095	+0.926	+0.784	+0.546	+0.347
$\frac{V_2}{V_1}$ (AS ABOVE)	+0.503	+0.536	+0.574	+0.606	+0.640	+0.714	+0.797
$\frac{V_3}{V_1} = 1-\frac{d_2}{h_2/h_1}$	+0.776	+0.715	+0.696	+0.686	+0.681	+0.677	+0.680

FIGURE 4

SUMMARIZED BEST SOLUTIONS FOR EACH COMBINATION OF ℓ AND Σ_P

COMBI-NATION NUMBER	ℓ	Σ	$\frac{h_2}{h_1}$	p_1	p_2	p_3	d_1	d_2	$\frac{\nu_2}{\nu_1}$	$\frac{\nu_3}{\nu_1}$
1	0.25	+0.10	0.71	+2.29	-4.52	+2.32	0.13	0.12	+0.54	+0.83
2	0.25	+0.00	0.70	+2.38	-4.78	+2.40	0.13	0.12	+0.54	+0.82
3	0.25	-0.20	0.68	+2.46	-5.26	+2.60	0.13	0.12		
4	0.25	+0.20	0.72	+2.10	-4.26	+2.36	0.13	0.12	+0.54	+0.84
5	0.25	+0.25	0.73	+2.17	-4.12	+2.20	0.13	0.12	+0.55	+0.83
6	0.25	+0.35	0.74	+2.09	-3.84	+2.10	0.13	0.12	+0.55	+0.83
7	0.30	+0.20	0.69	+2.09	-4.00	+2.11	0.15	0.15	+0.50	+0.78
8	0.30	+0.25	0.69	+1.99	-3.87	+2.13	0.15	0.15	+0.50	+0.79 ← CHOSEN
9	0.30	+0.35	0.70	+1.95	-3.61	+2.01	0.15	0.15	+0.50	+0.79
10	0.35	+0.20	0.65	+1.98	-3.82	+2.04	0.18	0.17	+0.46	+0.73
11	0.35	+0.25	0.65	+1.97	-3.70	+1.98	0.18	0.17	+0.46	+0.73
12	0.35	+0.35	0.67	+1.90	-3.45	+1.90	0.18	0.17	+0.50	+0.74

Similarly, as the Σp increases, p_2 decreases but $\frac{\nu_2}{\nu_1}$ remains almost constant. The Petzval sum does not vary with changes in the overall lens thickness; but it tends to increase with an increase in Σp. This is easily seen from the modified equation $\Sigma p = -nP$. The $\frac{\nu_3}{\nu_1}$ ratios were not found to be critical-provided Eastman Kodak glasses were available for the third element. Other American glasses will work-but not as well.

7. Choosing ℓ and Σp:

Discarding the $\ell = .35$ solutions on the basis discussed in step 6, four combinations were chosen from the six solutions available with the other two overall lens thicknesses:

1. $\Sigma p = +.20$; $\ell = .25$
2. $\Sigma p = +.35$; $\ell = .25$
3. $\Sigma p = +.20$; $\ell = .30$
4. $\Sigma p = +.25$; $\ell = .30$

All of these solutions had $\frac{\nu_2}{\nu_1}$ ratios greater than .5 and p_2 values in the usable range. Had none of these four been satisfactory, the remaining two solutions would have been investigated. Each of the four was investigated alone and in turn.

8. Listing powers, spacing and ν-number ratios:

This information is conveniently entered at the top of the table described in step 10. A sample is shown in Figure 5.

9. Preparing a list of glasses:

In the outline, lower limits of index and dispersion for the glass to be used in element one were given as 1.50 and 50.0 respectively. This

FIGURE 5 - GLASS SELECTION TABLE

$\ell = .30$

$\Sigma p = .25$

$\frac{\nu_2}{\nu_1} = 0.50308$

$\frac{\nu_3}{\nu_1} = \begin{matrix} 0.79003 \\ 0.77582 \end{matrix}$

$P_1 = +1.9947$
$P_2 = -3.8744$
$P_3 = +2.1297$

$P_1 = +2.1297$
$P_2 = -3.8744$
$P_3 = +1.9947$

TYPE	ν_1	ν_2 CALC.	ν_2 - GLASSES	ν_3 CALC	ν_3 - GLASSES
1) EDBC-2 1.657	50.9	25.61	NONE	40.21 *39.49*	
2) EK210	51.1	25.71	NONE	40.37 *39.64*	
3) DBC-51 1.617	53.7	27.02	NONE	42.42 *41.66*	
4) EDBC-1 1.617	53.9	27.12	NONE	42.58 *41.82*	
5) DBC-2 1.617	54.9	27.62	EDF4 - 27.8 (1.75060)	43.37 *42.59*	
6) DBC-50 1.617	55.1	27.72	DITTO	43.53 *42.75*	
7) C8380 1.638	55.5	27.92	DITTO	43.85 *43.06*	C8490-44.2 (1.61300)
8) EK110 1.697	56.1	28.22	DITTO	44.32 *43.52*	
9) DBC-8 1.623	56.8	28.57	NONE	44.87 *44.07*	
10) C8360 1.623	56.8	28.57	NONE	44.87 *44.07*	
11) DBC-3 1.611	57.2	28.78	NONE	45.19 *44.38*	
12) DBC-1 1.611	58.8	29.58	EDF3 - 29.3 (1.72000)	46.45 *45.62*	EK 310 - 46.4 (1.7450) *
13) C8440 1.612	59.5	29.93	NONE	47.01 *46.16*	
14) C8400 1.620	60.3	33.33	NONE	47.64 *46.78*	

was based on a fairly simple logic as follows:

If $\frac{\nu_2}{\nu_1}$ is approximately .5 to .55; then, considering that EDF-4 has an dispersion of 27.8, it can be shown by substitution in the equation $\frac{\nu_2}{\nu_1} = \frac{27.8}{\nu_1} = .5 - .55$, that a ν_1 of 50-55 is required to comply with this ratio. EDF-4 has perhaps the lowest ν-number of any standard American glass; and other low ν-number glasses with slightly higher ν-numbers will take even higher ν-numbers in the first element.

All American glasses of low ν-number have high indices of refraction (1.65-1.75); and an examination of the Petzval sum equation, $\frac{p_1}{n_1} + \frac{p_2}{n_2} + \frac{p_3}{n_3} = -P$, will show that since p_2 must oppose the effects of p_1 and p_3, it is desirable either that n_2 should be small in comparison with n_1 and n_3 or that p_2 should be large. But we have said that glasses of low ν-number (which cannot be avoided for element 2) have high indices and p_2 is severely restricted on account of higher order aberrations. Therefore, we are left with the choice of making p_1 and p_3 smaller in comparison with p_2 (i.e. decrease Σp) or we can make n_1 and n_3 as large as possible. If we decrease Σp, the solution of the basic equations will give larger negative powers for p_2, which is not permissible. There, the only quantities which can be varied to real advantage are the indices of the first and third elements; and there should be a real effort to use glasses with the highest possible indices provided that their dispersions are suitable and their stabilities are satisfactory.

Glasses used in this project were taken from Bausch & Lomb, Corning and Eastman Kodak glass listing. The Eastman high index glasses seem particularly adapted to this design procedure.

10. Preparation of the glass selection table:

The outline is self-explanatory, except that it may be pointed out that a new glass selection table is required for each basic solution investigated. Figure 5 serves to illustrate Steps 8, 10, 12, 13, and 14.

11. Calculation of maximum and minimum ν_2 and ν_3 for the combination:

The $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ ratios for the particular combination are used to calculate these values corresponding to the maximum and minimum values of for the glasses listed as possibilities for element 1.

12. Preparing a list of glasses for elements 2 and 3:

Self explanatory except that if several combinations are to be tried, it may be convenient to prepare a master list for each element. Comprehensive ranges for these lists will be approximately as follows: ν_1 = 50-65; ν_2 = 25-33; and ν_3 = 40-53. This will eliminate a great deal of confusion in repeatedly switching back and forth from one glass catalog to another and will tend to prevent overlooking desirable glasses.

13. The Entry of correct glasses for the elements:

See Figure 5 for example. The tolerance of ± 0.4 in dispersion ratio was chosen on the basis of experience.

14. Choosing combinations and computing Petzval sum:

It is not too tedious to compute the Petzval sum for all complete combinations meeting the criterion in Step 13, but a little practice will soon enable one to choose the best two or three combinations by visual inspection.

As explained in Step 9, the thing to look for is a comparatively low index for element 2 combined with a comparatively high index for elements 1 & 3. It is for this reason that the Eastman Kodak glasses of 1.70-1.80 with ν-numbers in the 40-50 range are particularly good for this type of triplet. Because their resistance to weathering may be questioned, it is perhaps best to restrict their use to element 3 which will not normally be exposed to weather. A more stable glass of somewhat lower index will perhaps be better for the front element. The starred choice in Figure 5 was chosen on just this basis. If there is no worry about exposure to the elements, a very nice combination can be obtained from two E. K. glasses for the first and third elements and a conventional B. & L. or Corning glass for the second element. The high index glasses have the additional advantage that the curvature required for a given surface power is less for a high index glass than for a low index glass ($P = \frac{n'-n}{r}$) where P is the "surface power". This is usually very important in the correction of higher order aberrations.

15. Determination of radii for correction of spherical aberration, coma and astigmatism:

Correction of these aberrations is done by bending. There are three elements which we can bend; and there are three aberrations to correct. Expressions for each of these aberrations can be derived; and then by simultaneous or trial and error methods, they can be solved for the first radius of each element. It was found that simultaneous solution was unreasonably complicated and the trial and error method actually used will be described a little later. Desired values for Seidel specific coefficients were given in the design requirements as follows:

$A = -0.60 - 1.20$

$B = -0.20$

$C = +0.06$

These must be substituted in transformation equations to yield actual Seidel aberrations. I am indebted to Dr. K. Pestrecov for the following discussion of Seidel transformations:

> "The Berek specific coefficient A is a measure of the image spread caused by the third-order spherical aberration. The B and C specific coefficients are, respectively, measures of the image spread caused by the third-order coma and astigmatism, if the center of the entrance pupil is in coincidence with the vertex of the first surface. For any other position of the entrance pupil, the A, B, and C coefficients should be multiplied by a factor, whose value is determined by the position of the entrance pupil and of the object plane. Then these new quantities are summed up in a certain manner (indicated below), and the sums so obtained become the measures of the image spreads caused by third-order aberrations. These sums are known under the name of Seidel."
>
> "If the entrance pupil factor is denoted K, the Seidel sums acquire the forms tabulated below. For the reason of uniformity, the specific coefficient A (which, as was stated previously, is, by itself a measure of spherical aberration) is denoted S_1 in the tabulation. K is defined as follows: $K = \frac{U \times U_e}{U - U_e}$
>
> u_e = distance from first vertex to entrance pupil.
>
> u = object distance from first vertex, when $U = -\infty$, $K = u_e$
>
> Seidel sum for spherical aberration $S_1 = A$
>
> Seidel sum for coma $S_2 = -KA + B$
>
> Seidel sum for tangential curvature $S_3 = 3K^2A - 6KB + 3C + P$
>
> Seidel sum for sagittal curvature $S_4 = K^2A - 2KB + C + P$
>
> Seidel sum for distortion $S_5 = -K^3A + 3K^2B - K(3C + P) + D$

"In our analysis of the triplet design, the assumption was made that the diaphragm stop coincides with the second element. Hence, the center of the entrance pupil was not at the first vertex and K is not zero. It should be pointed out that there has been no particular need to take account of K until now - when the aberrations are evaluated. Furthermore K (related to stop position) may be adjusted to minimize one or more aberrations - without changing any of the other characteristics of the system. It will be noted, from an inspection of the equations that coma is especially susceptible to changes in stop position. Then a question may be asked why the Seidel sums in their general form were not used in the course of this investigation. The reason is that the general Seidel sums are more difficult to work with than the specific coefficients; and at the first stage of third order design (only this stage is covered in the thesis) the effort should be concentrated on obtaining the specific coefficients of reasonably small values which would be consistent with the indications of previous experience. Then it is reasonable to expect that the general Seidel sums will also be of sufficiently small values, which would provide a satisfactory basis for the subsequent stages of the third-order and the trigonometric design. On the other hand, the early reduction of the Seidel sums to some predetermined values may lead to a satisfactory final design in some exceptional cases only, when the correlation between the third-order and the actual performance of the system has been well established. Otherwise, the effort spent on obtaining some "reasonable" Seidel sums may prove to be rather useless. It may be of interest to note that the transformation of the Seidel sums into conventional aberrations is relatively simple. As derived by myself, the transformations listed below give entirely sufficient approximations for an object at infinity:

Longitudinal spherical aberration -- Sph. = $(0.125/f_n^2)S_1^{f}$

Tangential coma -- Coma = $(0.375/f_n^2)\, S_2^{ftan}$

Distance of tangential focus from the
paraxial image plane -- Tan. = $0.5S_3 f\tan^2$

Distance of sagittal focus from
the paraxial image plane -- Sag. = $0.5S_4 f\tan^2$

Linear distortion -- Dist. = $0.5S_5 f\tan^3$

In these formulas: f = the equivalent focal length, f_n = the f-number of the cone limited by the lower and upper rays under the consideration, α = the field angle of the beam (in the object space). With the entrance pupil at the first vertex, K = 0, and the Seidel sums become:

$$S_1 = A,\ S_2 = B,\ S_3 = 3C + P,\ S_4 = C + P,\ S_5 = D''$$

16. Derivation of the bending equations:

Powers, spacings and indices already determined are as follows:

$p_1 = \underline{+}\,1.99$

$p_2 = -3.87$

$p_3 = +2.13$

$d_1 = +0.154$

$d_2 = +0.146$

$n_1 = +1.611$ (all indices are for D light in these equations)

$n_2 = +1.72$

$n_3 = +1.745$

In the derivation of these equations, the quantities listed above are substituted in expressions for the contribution of each element; and then the expression is solved for an equation in terms of the first radius of one or more elements. Then the resulting three equations are substituted in a combined equation which will yield the Seidel specific coefficient for the whole lens--for that particular aberration. Each aberration is treated in order below, with the equations for the individual elements listed first, then the combining equation-with the explicit expressions for the data given above following the appropriate general equation:

<u>Spherical Aberration</u>

a. For the individual element: Berek, p 91

$$A_i = [-]\left\{\left(\frac{n_i}{n_i-1}\right)^2 p_i^3 + \left(\frac{3n_i+1}{n_i-1}\right) p_i^2 \left(\frac{1}{v_i}\right) + \left(\frac{3n_i+2}{n_i}\right) p_i \left(\frac{1}{v_i}\right)^2 - \frac{1}{r_i}\left[\left(\frac{2n_i+1}{n_i-1}\right) p_i^2 + \left(\frac{4n_i+1}{n_i}\right) p_i \left(\frac{1}{v_i}\right)\right] + \frac{1}{r_i^2}\left(\frac{n_i+2}{n_i}\right) p_i \right\}$$

the derived equations for the individual elements:

$$A_1 = -55.17 + \frac{27.49}{r_1} + \frac{4.47}{r_1^2}$$

$$A_2 = +95.56 + \frac{21.82}{r_2^{\cancel{2}}} + \frac{8.38}{r_2^2}$$

$$A_3 = -26.79 + \frac{15.76}{r_3} - \frac{4.57}{r_3^2}$$

b. For the whole lens:

$$A = A_1 + \left(\frac{h_2}{h_1}\right)^4 A_2 + \left(\frac{h_3}{h_1}\right)^4 A_3 \quad \text{Berek, p 122}$$

the derived equation for the whole lens:

$$A = -43.83 + \frac{27.49}{r_1} - \frac{4.47}{r_1^2} + \frac{4.98}{r_2} + \frac{1.91}{r_2^2} + \frac{6.14}{r_3} - \frac{1.78}{r_3^2}$$

Coma

a. For the individual element:

$$B_i = \left(\frac{n_i}{n_i - 1}\right) P_i^2 + \frac{2n_i + 1}{n_i}(P_i)\frac{1}{v_i} - \frac{1}{r_i}\left(\frac{n_i + 1}{n_i}\right) P_i \quad \text{Berek, p91}$$

the derived equations for the individual elements:

$$B_1 = 10.49 - \frac{3.23}{r_1}$$

$$B_2 = 6.99 + \frac{6.13}{r_2}$$

$$B_3 = 5.89 - \frac{3.35}{r_3}$$

b. For the whole lens: Berek, Page 122

$$B = B_1 + \left(\frac{h_2}{h_1}\right)^2 B_2 + \left(\frac{h_3}{h_1}\right)^2 B_3 + d_1\left(\frac{h_2}{h_1}\right)^3 A_2 + \left(\frac{h_3}{h_1}\right)^3 \left[\frac{d_1 \frac{h_3}{h_1} + d_2}{\frac{h_2}{h_1}}\right] A_3$$

the derived equation for the whole lens:

$$B = 17.28 - \frac{3.23}{r_1} + \frac{4.04}{r_2} + \frac{0.92}{r_3} + \frac{0.43}{r_2^2} - \frac{0.87}{r_3^2}$$

Astigmatism

a. For the whole lens: Berek, Page 122

$$C = -\Sigma p + 2d_1 \frac{h_2}{h_1} B_2 + 2\left(\frac{h_3}{h_1}\right)\left[\frac{d_1 \frac{h_3}{h_1} + d_2}{\frac{h_2}{h_1}}\right] B_3 + d_1\left(\frac{h_2}{h_1}\right)^2 A_2 + \left(\frac{h_3}{h_1}\right)^2 \left[\frac{d_1 \frac{h_3}{h_1} + d_2}{\frac{h_2}{h_1}}\right]^2 A_3$$

the derived equation for the whole lens:

$$C = -3.44 + \frac{1.56}{r_2} - \frac{0.57}{r_3} + \frac{0.096}{r_2^2} - \frac{0.43}{r_3^2}$$

17. Setting up equations and substitution for B and C:

An attempt was made to solve the bending equations simultaneously, but this was quickly abandoned when it became evident that terms up to tenth power would be involved - to say nothing of square root terms liberally sprinkled throughout. Then a synthetic division method was tried with considerably more success.

Numerical values given under design requirements were substitued for B and C; but no substitution was made for A. The reason will appear under step 18.

18, 19, 20 and 21. Solution of the equations:

Since the expression for C has only two unknowns in it, we may immediately obtain numerical values from it by assuming a numerical value for one of the radii and solving for the other. Since the central element has almost twice as much power as the other two elements, it was considered probably that it would have the steepest curves of any of the elements: and therefore it was deemed best to solve the equation for r_3 and substitute reasonable values of r_2. These values were varied on both sides of the equi-concave radii, determined from solution of the paraxial lens makers equation: $P = n-1 \left(\frac{1}{r_1} - \frac{1}{r_{1'}}\right) = (n-1)\left(\frac{2}{r_1}\right)$ for equal radii. The values of r_2 and r_3 thus obtained were substituted in the equation for B; and the resulting equation in r_1 was solved. This then gave three radii from two equations and was obviously not a unique solution. Now if a definite value of A was inserted in the first equation, the substitution of these radii simply would not check out, since there would be no real relationship between the arbitrary value of A and the radii determined from the solution of two other equations. Therefore, the actual procedure was to substitute the three radii and solve for A. When this had been repeated for several strategically chosen values of r_2, a graph of A vs r_2 was prepared; and from this graph, an r_2 giving a satisfactory value of A was selected. This radius and the associated r_1 and r_3 then became the first radii of their respective elements. It was then a simple matter to determine the second radius of each element by the lens makers equation. The solution of these equations for several values of r_2 is facilitated by the use of tabular entries similar to that used in the solution of the basic equations. See Figure 3. The plot of A vs r_2 is shown as Figure 6. In this particular lens design the selection of r_2 was influenced also by r_1 which turned out, surprisingly, to have steeper curvature than r_2 or r_2'. Therefore a compromise combination of radii was adapted. The spherical aberration at r_2,= $-.42$ ($A = -1.06$) was considered to be satisfactory.

23. Repetition of procedure after unsatisfactory solution:

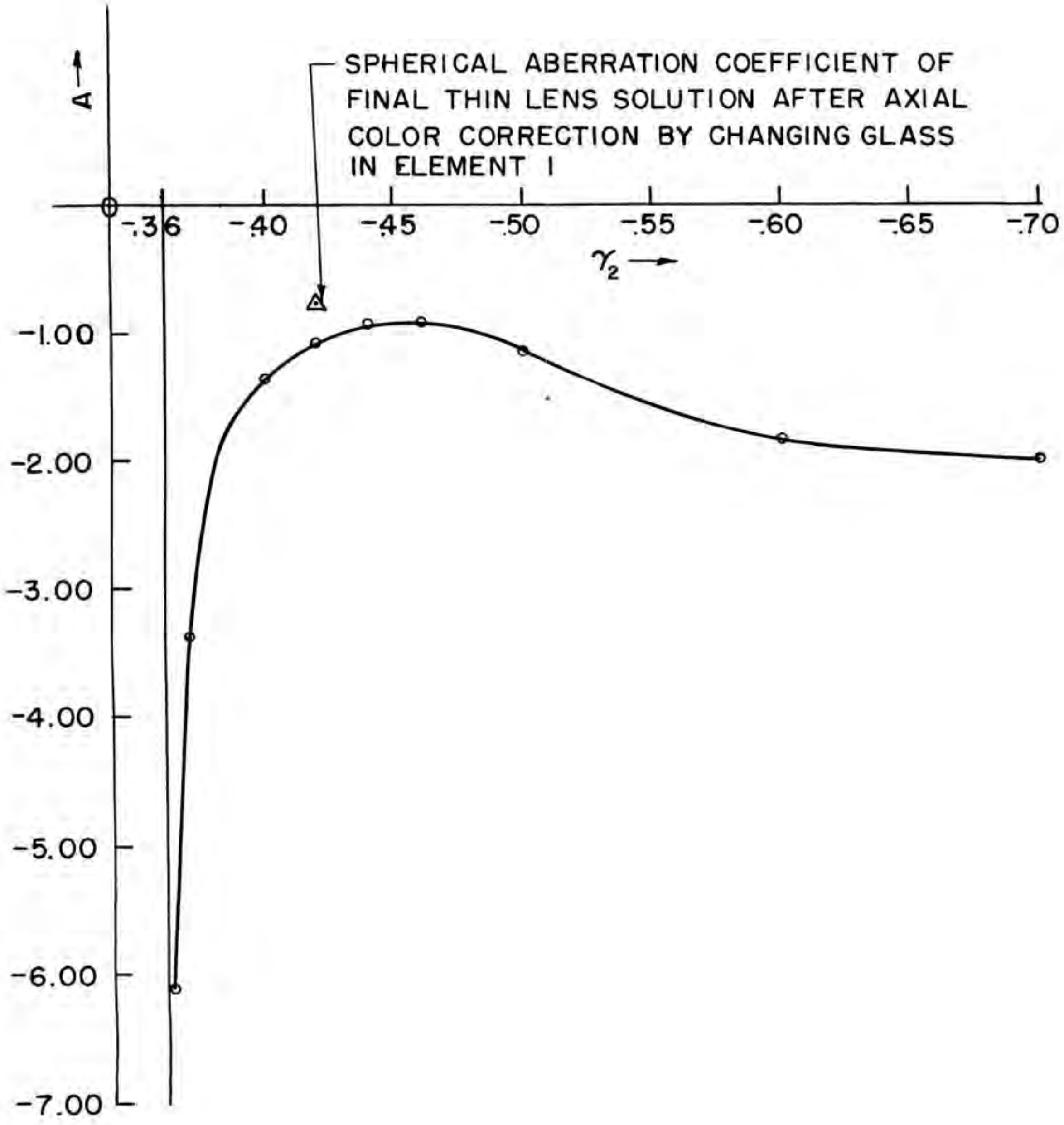
    Repetition of steps 8 thru 22 was not necessary.

24. Determination of Seidel specific coefficients by means of independent calculations;

Utilization of another means of computing the specific coefficients will serve as a check on the solution of the bending equations. If the specific coefficients, as figured by the Seidel calculation sheet, check with the assigned B and C and also check with the A obtained when the determined radii were substituted in Berek's expression for spherical aberration (A),

FIGURE 6

SPHERICAL ABERRATION (A) AS A FUNCTION OF THE FIRST SURFACE RADIUS OF THE SECOND ELEMENT



then the solution may be assumed to be correct. A sample sheet, showing the calculation of specific coefficients for the experimental design is included as Figure 7. However, before this calculation can be made, it will be necessary to calculate u and v for each surface. This may be done by a paraxial ray trace with D light or some of the simpler paraxial equations. Since these elements have zero thickness, it will be noticed that for each element, the u for the second surface is the same as the v for the first surface. The sheet is considered to be self-explanatory.

25. Determination of axial color:

The procedure given in the outline is self explanatory, but again it should be pointed out that the chosen combination of glasses gives axial color correction for C and F light. The reason for this is simply that the dispersions which were used in the selection of the glasses were figured as $\frac{n_F - n_C}{n_D}$ (ν-number = $\frac{n_D}{n_F - n_C}$). If some other wavelengths had been used in the calculation of the dispersion, the axial (and the lateral color) would have been corrected for those wavelengths. No exact check was made on lateral color during the stages of design covered by this paper. But the usual approximate criterion $\Delta V_6 = \Delta f$ gave the indication that the lateral color was satisfactory. The axial color curve is given for the basic solution in Figure 8. ΔV_6 may be measured from the curve as approximately .008 (for f = 1) $\lambda = G \to D$ inclusive. An inspection of the curve reveals that the focal point is changing rapidly between the wavelengths for F and G' light. This would not be too good for color photography. However, it would be satisfactory for black and white photography with a very light yellow filter - ideally, a filter with a cut off around the F wavelength. This would prevent the out of

FIGURE 7 - SEIDEL DATA CALCULATION SHEET

Lens:

Form:

EFL = **1.0** BF =

Date:

TERM	SURFACE								
	1	2	3	4	5	6			
R	.310	-126.15	- .420	+ .333	+3.432	- .390			
t	0	0	.155	0	.145	0			
n	1	1.617	1	1.72	1	1.745			
n'	1.617	1	1.72	1.	1.745	1			
u		+.813	+.346	+1.467	-1.157	-2.698			
v	+.813	.501	+1.467	-1.012	-2.698	+ .790			
n/R	3.224	-.013	-2.381	+5.160	.291	-4.480			
n/u	0	1.990	2.887	+1.172	-.864	-.647			
$n/R - n/u = Q$	3.225	-2.003	-5.268	+3.988	+1.155	-3.833			
$1/n$	1	.618	1.000	.581	1	.573			
$1/n'$	.618	1	.581	1	.573	1			
$1/nu$	0	.761	2.887	.396	-.864	-.212			
$1/n'v$	.761	1.995	+.396	-.988	-.212	1.266			
$1/nu - 1/n'v = \Delta$	-.761	-1.234	2.490	+1.384	-.651	-1.478			
u_k/v_{k-1}	1	1	.691	1	+1.143	1			
$(u_k/v_{k-1})h_{k-1} = h$	1	1	.691	.691	.790	.790			
h^2	1	1	.477	.477	.624	.624			
h^2Q	3.225	2.003	-2.515	-1.904	.721	-2.392			
$1/h^2Q = \varepsilon$	.310	-.499	-.397	+.525	1.387	-.418			
t/n	0	0	.155	0	.145	0			
$h_{k-1} \times h_k$	1	1	.691	.477	.546	.624			
$(t/n)/h_{k-1}h_k$	0	0	.224	0	.266	0			
$\Sigma = \delta$	0	0	.224	.224	.490	.490			
$\varepsilon + \delta = \tau$	+.310	-.499	-.173	+.749	+1.877	+.072			
h^4Q^2	+10.400	4.011	6.326	3.626	.520	5.724			
$h^4Q^2\Delta = A$	-7.915	-4.948	+15.755	+5.018	-.339	-8.461	A = -0.889		
$\tau A = B$	-2.454	+2.471	-2.732	+3.761	-.636	-.609	B = -0.200		
$\tau B = C$	-.761	-1.234	+.474	+2.818	-1.193	-.044	C = +0.060		
							Pr= 4.964		
$(1/n' - 1/n)/R$ P	-1.231	-.003	+.997	1.256	-.124	-1.096	P = -0.201		
$\tau(C+P) = D$	-.618	+.618	-.255	3.053	-2.473	-.082	D = +0.243		

NOTE: THIS SHEET IS FOR THE FINAL THIN LENS SOLUTION AFTER AXIAL COLOR HAS BEEN CORRECTED of focus blue light from fuzzing up the image.

26. Correction (swinging) of the axial color curve:

If, as for the color photography case mentioned above, the basic axial color curve is not satisfactory, we can attempt a correction by changing the glass in one or more elements, keeping the element powers constant and keeping the ratio between first and second radii constant. Since these characteristics of the lens remain constant, the third-order aberrations remain practically unaffected. That is, spherical aberration, coma, and astigmatism and distortion, to a high degree of approximation, depend only on the ratio between radii, powers and spacing-none of which change. Unless the index n_D of the substituted glass happens to be the same as the one it replaces, the Petzval sum will change slightly, but this is usually negligible.

Four color curves were obtained-three besides the basic combination. Curves for the basic combination of glasses and for the combination used in the fourth trial are shown in Figures 8 and 9. A different glass was tried in each element, but the change was too great when element 2 was changed and not enough when element 3 was changed. The effect of a glass change on axial color seems to be related to the power of the element and the change in dispersion of the glass. The results of the determinations are shown here in tabular form:

Element changed	Old Glass	New Glass	Element Power	$\Delta\nu$ $\nu_{new} - \nu_{old}$	Axial Color (Δv_6) C-G' inclusive
no change	---	---	1.99	0	0.0038
1	DBC-1	DBC-2	1.99	+3.9	0.0021(Chosen

FIGURE 8

AXIAL COLOR ABERRATION CURVE
FIRST TRIAL

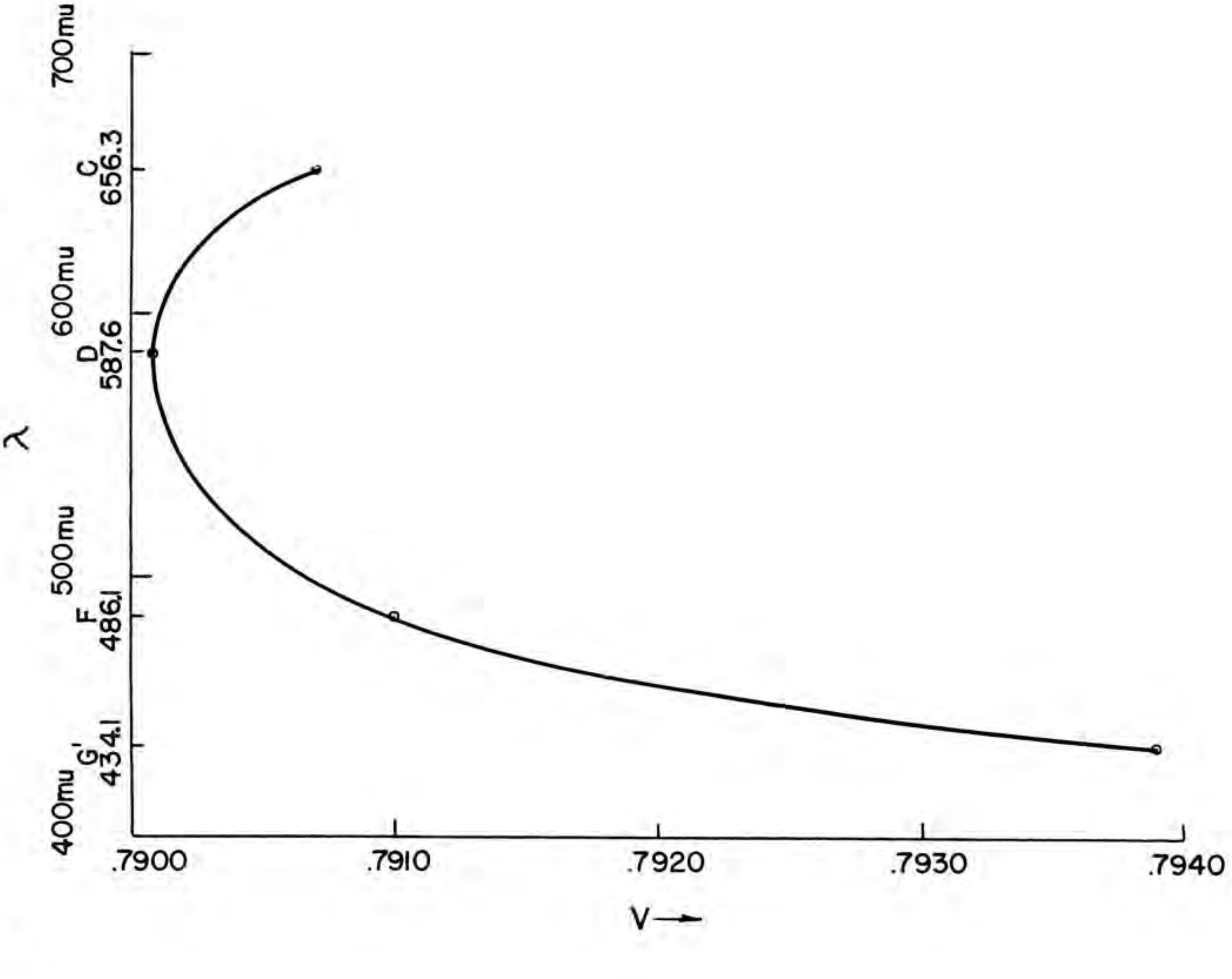
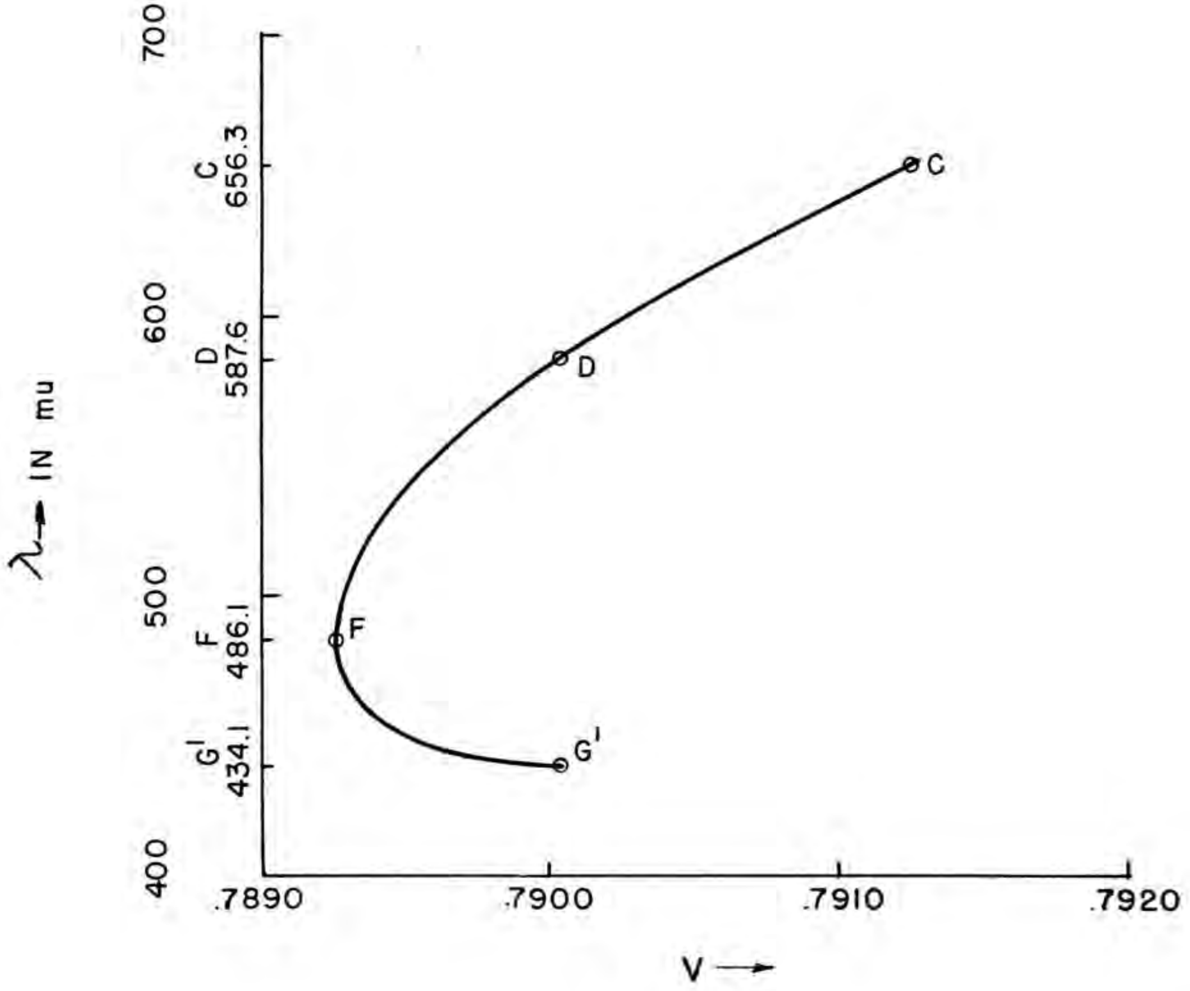


FIGURE 9

AXIAL COLOR ABERRATION CURVE



2	EDF-3	EDF-2	-3.87	+1.6	0.0029
1	DBC-1	DBC-3	2.13	-1.6	0.0025

Examination of the curves will reveal that with a Wratten 12 filter such as used in aerial photography, it would be possible to reduce axial color to approximately 0.0010.

Since the design did not go beyond the thin lens stage, it was thought worthwhile to made use of the Seidel transformation equations and determine the actual third-order spherical aberration, coma, astigmatism (tangential and sagittal curvatures) and distortion. The equations were discussed under step 15. The results of these calculations follow:

(a) Seidel Sums:

$S_1 = -0.8891$	Spherical Aberration
$S_2 = -0.0003$	Coma
$S_3 = +0.1129$	Tangential Curvature
$S_4 = -0.0967$	Sagittal Curvature
$S_5 = +0.2076$	Distortion

(b) Actual 3rd order aberrations:

It was considered of interest to compare the actual 3rd order aberrations obtained first by the correct equations and then by the simplified equations which result from the assumption that $K = 0$ (see step 15 discussion). The comparison follows in tabular form:

$f = 1.0 \quad f_n = 5.0$

Aberrations

$K = 0.22418$	$K = 0.0$

	K = 0.22418	K = 0.0
Axial spherical aberration	-0.00444	-0.005
Tangential coma	-0.0000012	-0.0008
Distance of tangential focus from the paraxial image plane	-0.00405	-0.00072
Distance of sagittal focus from the paraxial image plane	-0.00347	-0.005
Linear distortion	0.0020	0.0023

It is hoped that when higher order aberrations are added in by means of ray tracing techniques, these third order sums would be balanced enough to obtain optimum lens characteristics.

CONCLUSIONS

1. Berek's method for the third order design of a photographic triplet was applied and resulted in a satisfactory thin lens solution corrected for:

   1. Spherical Aberration
   2. Coma
   3. Astigmatism
   4. Distortion
   5. Axial Color
   6. Lateral Color

2. A Petzval ratio of 4.96 was obtained for the thin lens solution. A design goal for a Petzval ratio of 5.0 had been set at the beginning of the project (A Petzval ratio of 4.96 corresponds to a Petzval sum of .2015 for f=1).

3. From a D axial trace, the paraxial focal length was determined to be 1.00002.

4. Axial color was first corrected for C and F light. This correction was suitable for black and white photography with a light yellow filter cutting off around the F wavelength (486.1 mu). Then it was corrected for C to G' wavelengths inclusive by substituting DBC-2 for DBC-1 in the first element. This change would be more suitable for color photography.

5. The glass selection procedure, used in the course of this investigation appears to be the most precise and systematic glass selection

procedure in use for third order triplet design. Once a list of available glasses has been prepared and reasonable dispersion tolerances assigned, one can quickly and unerringly pick out the best possible combination from the available list.

6. Design criteria assigned at the beginning of the project were all met for the thin lens solution. Preliminary calculations, with lens thicknesses inserted in the design, indicate however that the Petzval ratio will drop about 20% when the thicknesses are inserted. Therefore, it would be well to set a Petzval ratio about 20-25% higher for the thin solution than that desired in the finished lens.

7. It was concluded from a fairly extensive survey of all possible values of $\frac{h_2}{h_1}$ (an arbitrarily assigned parameter), that it is best to use a value of $\frac{h_2}{h_1}$ in the region from +0.60 to +0.90. And in this region, it is best to use the lowest value of $\frac{h_2}{h_1}$ that will give a real answer when the basic expression is solved for the spacing between the first and second element. The desirable byproduct of this procedure is that the curvatures of the elements are kept to a practical minimum for a given combination of Σp and ℓ.

8. When Σp increases, Petzval sum tends to increase (and Petzval ratio decreases) ; and p_2 decreases. These two effects are, respectively, undesirable and desirable. Therefore a compromise must be made based on the individual circumstances.

9. When ℓ increases, p_2 decreases and Petzval sum remains constant. However, element diameters also increase if a sizable field angle is to be covered; and if the lens is too long it may be unwieldy. Here too, a practical compromise must be made.

10. It would be desirable to continue this investigation through the third order thick lens solution, and finally, to the finish of the design by conventional ray trace methods. From such a study we might be able to prepare a "cook book" design procedure for the design of a photographic triplet, covering all stages from start to finish. This "primer of triplet design " should be useful to those who are just beginning the study of optical design to help them gain a perspective picture of the whole pattern of design procedure. Having such a detailed procedure would enable the beginner to design a simple triplet by himself from start to finish; and it is certainly true that, in lens design, one learns by doing.

Even experienced designers might find it useful as a memory refresher; and perhaps it might stimulate new attempts to consolidate optical design practice in a logical and easily understood manner.

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VII. ABSTRACT

This thesis describes the investigation of Berek's method of third order lens design as applied to the design of a flat field, photographic triplet. The fields of photographic triplets are usually flattened artificially by adjustment of the astigmatic image surfaces. This process results in degraded image definition in the zonal area of the film plane. The degree of degradation depends primarily upon the original curvature of the Petzval surface with which the astigmatic surfaces are intimately related. If the Petzval surface is steeply curved, then more degradation must be expected than if the Petzval surface is comparatively flat. Most photographic triplets have a Petzval surface whose radius is between two and three times as long as the focal length. The ratio of these radii (called Petzval ratio) chosen for this project was 5.0, which is attained in the final thin lens solution. Use of the Berek basic equations is analyzed in detail. It is shown that with thirteen quantities involved in six equations, it is necessary to account for seven of them either as arbitrarily set values or as dependent variables, leaving only six true variables for the solution of the basic equations. This accounting is made; and each of the thirteen quantities is identified as an arbitrarily set quantity, a dependent variable, or a true variable. The six basic equations include expressions for focal length, Petzval sum, oblique (lateral) and axial color correction, distortion, and overall lens thickness. The distortion expression is based on the assumption that the diaphragm stop coincides with the second element in the thin lens system.

Oblique and lateral color are to be set equal to zero. These two assumptions are justified on the basis of convenience and the lack of any valid method of assigning advantageous residuals.

The solution of the basic equations yields individual element powers, spacing, and the number ratio for the first and second elements.

A detailed outline of design procedure leading to a third order thin lens solution is presented with detailed discussion of the alternative possibilities of each step and the particular plan which was followed. Summarized data, including graphs and tables, accompany the discussion of the individual design steps. Bending equations are set up for the correction of spherical aberration, coma, and astigmatism. The equations are solved by an empirical method since their simultaneous algebraic solution is almost hopelessly complicated. When a satisfactory solution is found, the radii of the individual elements are determined. These radii, combined with the individual element powers, spacings and $\frac{\nu_2}{\nu_1}$ and $\frac{\nu_3}{\nu_1}$ ratios resulting from the solution of the basic equations, completely define the thin lens system. However, this system has its axial color corrected for C and F wavelengths of light, which is satisfactory for black and white photography with a light yellow filter, but which may not be entirely satisfactory for color photography. A further axial color correction procedure is employed to extend the useful region of wavelengths from F to G'. These steps are discussed and graphs showing the color correction obtained are included. Axial color correction changes at this point are feasible since bendings, powers, and spacings do not change; and monochromatic aberrations are therefore not affected.

Further design steps leading to the completion of a third order thick element design are included in the procedure outline, but are not discussed.

It is concluded that the Berek design procedure has one advantage over most triplet design procedures in that it has a systematic glass selection procedure which will select the glasses for the individual elements with a minimum of guess work on the part of the designer. Other than that, the design method seems to be straightforward, relatively simple and workable. The critical element of the system is the second or negative element; and as the Petzval radius is increased, the curvatures of the element surfaces get steeper and steeper. Intelligent choice of glasses can help some here; and it is pointed out that if more than one glass combination is obtained as a possible solution, it is usually best to take the combination which has the highest indices for number one and number two elements. This results in a lower Petzval sum and in lower surface aberration contributions. In the particular design described in this paper, the first radius of the first element turned out to be the shortest surface radius of the lens; and the rear surface of the element is almost plane. This is also true of the sample triplet described in Berek, p. 130, and of many other triplets. This in spite of the fact that the power of the second element is almost twice that of the first element. This merely points up the desirability of high index glasses, since the radii are longer with high index glasses; and higher order aberrations (not discussed) usually become worse as the radii become shorter.

Equations for Seidel sum calculations from Berek's Seidel specific coefficients are given; and the transformation equations for transforming Seidel sums to conventional aberrations are given as derived by Dr. K. Pestrecov. The final thin lens solution is evaluated by these equations and the summarized data included.