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# Econometric analysis of heterogeneity in financial markets using quantile regressions

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BOSTON UNIVERSITY  
GRADUATE SCHOOL OF ARTS AND SCIENCES

Dissertation

**ECONOMETRIC ANALYSIS OF HETEROGENEITY IN  
FINANCIAL MARKETS USING QUANTILE  
REGRESSIONS**

by

**YUAN TIAN**

B.A., Central University of Finance and Economics, 2011  
M.A., Boston University, 2014

Submitted in partial fulfillment of the  
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Approved by

First Reader

---

Zhongjun Qu, PhD  
Associate Professor of Economics

Second Reader

---

Ivan Fernandez-Val, PhD  
Professor of Economics

Third Reader

---

Pierre Perron, PhD  
Professor of Economics

*“To expect the unexpected shows a thoroughly modern intellect.” by Oscar Wilde*

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# ECONOMETRIC ANALYSIS OF HETEROGENEITY IN FINANCIAL MARKETS USING QUANTILE REGRESSIONS

YUAN TIAN

Boston University, Graduate School of Arts and Sciences, 2018

Major Professor: Zhongjun Qu, Associate Professor of Economics

## ABSTRACT

This dissertation studies *heterogeneity* in financial markets using quantile regressions.

The first chapter develops uniform confidence bands for the linear quantile regression estimator in a time series setting. The confidence bands are important for estimating the precision at different quantiles of the conditional distribution. The inference procedure is carried out through bootstrapping and allows for serially correlated error terms. An empirical application to the relationship between stock returns and investor sentiments suggests the method can be informative.

The second chapter analyzes the heterogeneity of firm characteristics on returns to capital. It develops a theoretical model under a utility maximization framework with imperfect insurance and credit markets constraints. From the model, the returns to capital are derived as a function of the parameters, which affects the production function of the firm and the entrepreneur utility form. Quantile regression is applied to analyze the field experiment data from the Sri Lanka Micro Enterprises Project (2005-2010). Empirical evidence shows that returns vary across different quantiles of firm profits. Further, the ability/risk aversion of entrepreneurs affect the returns

differently at different quantiles.

The third chapter examines capital account liberalization and its effect on price volatility in the Chinese housing market. The chapter assesses the extent to which: a) short-term capital flows and foreign direct investment may have impacted prices and volatility in the Chinese housing market; and b) whether 2006 Capital Account Regulations on foreign purchases of Chinese real estate were effective in reducing the level and volatility of prices in the housing market. The results show that hot money magnified the impacts of capital flows on housing prices during upward surges in housing prices. Quantile regression provides quantitative evidence that the more volatile the housing market was, the larger the impact short-term capital flows had on accentuating such volatility. Furthermore, the 2006 CAR continued to have a strong impact on reducing volatility in the Chinese housing market during the period under study.

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## List of Abbreviations

CAL	.....	Capital Account Liberalization
DMW	.....	De Mel, McKenzie and Woodruff (2008)
FDI	.....	Foreign Direct Investment
FE	.....	Fixed Effect
ICE	.....	Index of Consumer Expectation
<i>i.i.d.</i>	.....	Independent and Identically Distributed
IV	.....	Instrumental Variable
$L_q$ norm	.....	$\ X\ _q := (E X ^q)^{1/q}$ for a random variable $X$
OLS	.....	Ordinary Least Square
PDF	.....	Probability Distribution Function
QR	.....	Quantile Regression
$R^p$	.....	Mathematical Domain of Real Numbers
$S_T(\tau, \beta(\tau))$	.....	Full Sample <i>Subgradient</i>

## Chapter 1

# Uniform Quantile Regression Bands with an Application to Investor Sentiments

This chapter develops uniform confidence bands for the linear quantile regression (QR) estimator in a time series setting. The confidence bands are important for estimating the precision of the estimates at different quantiles of the conditional distribution. The inference procedure is carried out through bootstrapping and allows for serially correlated error terms. An empirical application to the relationship between stock returns and investor sentiments suggests the method can be informative.

### 1.1 Introduction

Quantile regression analysis offers a natural and flexible framework for the statistical analysis of conditional quantile functions. The appeal of quantile regression is in a complete characterization of the entire conditional distribution, including the impact on distributional features beyond the mean. This chapter explores the potential of quantile regression models as a tool for analyzing financial time series survey data.

Expectations of investors in financial markets can be measured using survey questions. Time series of expectations based on such surveys are in general different from expected returns calculated through financial modeling. Conditional quantile functions can potentially be used to measure the relationship between them, conveying more information than simply looking at the conditional mean function alone.

In this chapter, I derive uniform confidence bands for the linear quantile regres-

sion estimator. Compared with the traditional pointwise confidence interval, uniform confidence bands (defined in Section 1.3) are necessary for inference on a continuous range of quantiles. This property is necessary when comparing the differences in responses at different quantiles of the conditional distribution. For example, when analyzing the relationship between returns and sentiments in the financial market, uniform confidence bands allow for the comparison between bull and bear markets.

The main technical contribution of this chapter is to develop uniform confidence bands in a time series setting with serially correlated errors. I apply this method to weekly data of investor sentiments and S&P 500 stock returns. In particular, I estimate uniform confidence bands for the QR estimator of investor sentiments on the realized stock market returns six months ahead. This chapter is the first quantile regression study in the relationship between returns and sentiments at different quantiles in the financial markets.

The classical linear quantile regression model was first systematically proposed by Koenker and Bassett (1978). Koenker (2005) offers a comprehensive analysis of the classical quantile regression model and its applications. Recent contributions to conditional quantile regression estimation in the time series setting include Qu (2008), who worked on testing for structural changes in regression quantiles and derived the limiting distributions under the null hypothesis. Also, Härdle and Song (2010) established a strong uniform consistency rate for the QR estimator. Qu and Yoon (2015) derived inference theory for constructing uniform confidence bands in a nonparametric setting by applying local linear regressions to a grid of quantiles in an *i.i.d.* (independent and identically distributed) sample. Most existing QR studies focus on *i.i.d.* samples. This chapter allows for serially correlated errors.

From a methodological perspective, this chapter is related to Wu and Zhou (2014), who consider testing for structural changes in quantile regression models allowing for a

wide class of non-stationary regressors and serially correlated errors. From a technical perspective, it is closely related to Zhou (2013), who first proposed a simple and unified bootstrap testing procedure for inference in quantile regressions under general forms of smooth and abrupt changes in the temporal dynamics of the time series. From an empirical perspective, this chapter continues the study of Greenwood and Shleifer (2014), who compared survey data on expectations with expected returns. They showed evidence that the survey data is inconsistent with rational expectation representative investor models of returns. This chapter updates the dataset from Greenwood and Shleifer (2014). It estimates and compares QR coefficients of investor sentiments at different quantiles of realized stock returns.

The rest of the chapter is organized as follows. Section 1.2 introduces the notation and the assumptions. Section 1.3 develops the confidence bands and a bootstrapping procedure for constructing them. Section 1.4 examines the finite sample properties in the procedure to construct the bands. In section 1.5, the method is used to analyze data on U.S. investor sentiments and returns on the S&P 500 index. Section 1.6 concludes.

## 1.2 Quantile Regression

In this section, I briefly review the methodology of quantile regression and introduce assumptions needed to derive the bands.

### 1.2.1 Setup

Consider the parametric time series quantile regression model:

$$Q_{y_t|x_t}(\tau) = x_t'\beta(\tau), \quad (1.1)$$

where  $\tau \in (0, 1)$ ,  $\{y_t\}_{t=1,2,\dots,T}$  represents a time series of responses of size  $T$ , and  $\{x_t\}$  is a  $p \times 1$  vector of random variables. Throughout this chapter, I assume that for any quantile  $\tau \in [\omega, 1 - \omega]$ ,  $0 < \omega < 1/2$ , the conditional quantile function is always linear in  $x_t$  as given in the preceding display.

Let  $F_t(y, x_t)$  be the conditional distribution function of  $y_t$  given  $x_t$ :  $F_t(y, x_t) = P(y_t \leq y | x_t)$ . Then, the conditional quantile function is obtained from solving  $F_t(y, x_t) = \tau$ . The quantile regression estimator  $\hat{\beta}(\tau)$  can be obtained by solving

$$\hat{\beta}(\tau) = \arg \min_{\beta(\tau) \in \mathbb{R}^d} \sum_{t=1}^T \rho_\tau(y_t - x_t' \beta(\tau)), \quad (1.2)$$

where the check function  $\rho_\tau(u)$  equals

$$u \times \phi_\tau(u)$$

with  $\phi_\tau(u) = \tau - 1(u \leq 0)$  and  $1(\cdot)$  being the indicator function. The subgradient equals

$$S_T(\tau, \beta(\tau)) = T^{-1/2} \sum_{t=1}^T x_t \phi_\tau(y_t - x_t' \beta(\tau)) \quad (1.3)$$

### 1.2.2 Assumptions

To obtain the limiting distribution of the estimator, I impose the following assumptions on the model.

**Assumption 1.** The time series  $y_t$  and  $x_t$  are strictly stationary and ergodic such that

$$T^{-1/2} \max_{1 \leq t \leq T} \|x_t\| = o_p(1)$$

and  $E[\|x_t\|^2] < \infty$ , where  $\|\cdot\|$  is the Euclidean norm.

**Assumption 2.**  $F_t(\cdot)$  has continuous density  $f_t(\cdot) = f(\cdot|x_t)$  uniformly bounded away from 0 and  $\infty$  over quantiles  $\tau \in [\omega, 1 - \omega]$ . We have  $|f_t(y_1) - f_t(y_2)| \leq C|y_1 - y_2|$  for some constant  $C > 0$  and all  $y_1, y_2 \in R$ .

**Assumption 3.**  $E[\phi_\tau(y_t - x_t'\beta_0(\tau))|x_t] = 0$  almost surely for some unique  $\beta_0(\tau) \in B \subset R^p$ , where  $\phi_\tau(u) = 1(u < 0) - \tau$  and  $\beta_0(\tau)$  is an interior point of the compact set  $B$  for each  $\tau \in [\omega, 1 - \omega]$ .

**Assumption 4.** There exists a random variable  $A_t$  and a constant  $0 \leq k_1 < 1/2$  such that  $T^{-1} \sum_{t=1}^T \|x_t\| \leq T^{k_1} \times A_t$ . In addition,  $\sup E(A_t^{k_2}) < \infty$  for some  $k_2 > 2$ .

**Assumption 5.** There exist  $k_3 \geq k_4 > 1$ ,  $M < \infty$ , and  $V < \infty$  such that for all  $T > 1$ ,  $T^{-1} \sum_{t=1}^T [E(x_t'x_t)]^{k_3} \leq M$ ,  $E[T^{-1} \sum_{t=1}^T (x_t'x_t)^{k_4}] \leq V$ , and  $(k_3 - 1)/(k_4 - 1) > 1 + 2k_1$ . If  $E(x_t'x_t)^2 \leq W < \infty$  for  $\forall t$ , we can take  $k_3 = 2, k_4 = 3/2$ .

**Assumption 6.**  $T^{-1} \sum_{t=1}^T x_t x_t' = J_0 + o_p(1)$ , where  $J_0$  is a finite, symmetric and positive definite matrix.  $T^{-1} \sum_{t=1}^T f_t(x_t'\beta_0(\tau))x_t x_t' = H_0(\tau) + o_p(1)$  holds uniformly over  $\tau$  where  $H_0(\tau)$  is a  $p \times p$  symmetric and positive definite matrix for each  $\tau \in [\omega, 1 - \omega]$ .

I now discuss the assumptions. Assumption 1 imposes strict stationarity. Assumption 2 is standard in the quantile regression literature. Assumption 3 rules out mis-specification in the quantile regression model and provides an identification condition for the QR estimator of the true value  $\beta_0(\tau)$ .

Assumption 4 rules out trending regressors, under which the limiting distribution of the test statistics will be different. Assumption 5 is used to ensure the tightness of certain sequential weighted empirical processes. Assumption 6 provides general assumptions to facilitate the derivation of the asymptotic results.

Denote the error process  $e_t(\tau)$  in the quantile regression model by

$$e_t(\tau) = y_t - x_t' \beta_0(\tau).$$

I assume that the error term is generated as

$$e_t(\tau) = G(\tau, \mathcal{F}_t, \mathcal{G}_t),$$

where  $G(\cdot)$  is a function of  $\tau$  under two independent filtrations  $\mathcal{G}_t, \mathcal{F}_t$ .  $\mathcal{G}_t$  and  $\mathcal{F}_t$  are generated by  $(\dots, \eta_{t-1}, \eta_t)$  and  $(\dots, \varepsilon_{t-1}, \varepsilon_t)$ , where  $\{\eta_t\}$  and  $\{\varepsilon_t\}$  are independent. The filtration definition follows Definition 2.1 in Wu and Zhou (2014, 2016). Also, assume the regressors are generated as

$$x_t = H(\mathcal{F}_{t-1}, \mathcal{G}_t).$$

Note that  $\{e_t(\tau)\}$  and  $\{x_t\}$  are allowed to be dependent because they are generated by common filtrations.

The next assumption specifies that time series dependence is allowed and guarantees the smoothness of the error process  $e_t(\tau)$  with respect to  $\tau$ .

**Assumption S1.**  $G(\tau, \mathcal{F}_t, \mathcal{G}_t)$  satisfies, for some constant  $v > 1$ ,

$$\|G(i, \mathcal{F}_t, \mathcal{G}_t) - G(j, \mathcal{F}_t, \mathcal{G}_t)\|_v \leq C, \forall i, j \in (0, 1). \quad (1.4)$$

The time series dependence in the  $L_v$  norm satisfies

$$\|G(i, \mathcal{F}_t, \mathcal{G}_t) - G(i, \mathcal{F}_t^*, \mathcal{G}_t^*)\|_v \leq M\chi^{|l|}, \quad (1.5)$$

where  $M$  is a finite constant and  $\chi \in (0, 1)$ . In addition,  $\mathcal{F}_t^*, \mathcal{G}_t^*$  are defined by

$$\mathcal{F}_t^* = (\dots, \eta_{-1,t}, \eta'_{0,t}, \eta_{1,t}, \dots, \eta_{l,t}),$$

$$\mathcal{G}_t^* = (\dots, \varepsilon_{-1,t}, \varepsilon'_{0,t}, \varepsilon_{1,t}, \dots, \varepsilon_{l,t}),$$

where  $\eta'_{0,t}$  and  $\varepsilon'_{0,t}$  are independent copies of  $\eta_{0,t}$  and  $\varepsilon_{0,t}$ , respectively.

Assumption S1 requires the process  $e_t(\tau)$  be short-range dependent with exponentially decaying coefficients. It encompasses a broad class of serially dependent processes, such as invertible ARMA processes.

**Proposition 1.1** Under Assumptions 1-6,

$$\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) - H_0^{-1}(\tau)[\hat{S}_T(\tau, \hat{\beta}(\tau)) - S_T(\tau, \beta_0(\tau))] = o_p(1) \quad (1.6)$$

uniformly in  $\tau \in [\omega, 1 - \omega]$ , where  $\hat{\beta}(\tau)$  is the quantile regression estimator of  $\beta_0(\tau)$  and  $H_0(\tau)$  is defined in Assumption 6.

Serial correlation is common in financial time series. For example, let  $V_t$  denote the log of the monthly S&P 500 index. Let  $Z_t = V_t - V_{t-1}$ . Then the six-month return at time  $t$  can be expressed as

$$R_t = V_t - V_{t-6} = Z_t + Z_{t-1} + Z_{t-2} + Z_{t-3} + Z_{t-4} + Z_{t-5}.$$

At time  $t + 1$ , we have  $R_{t+1} = V_{t+1} - V_{t-5} = Z_{t+1} + Z_t + Z_{t-1} + Z_{t-2} + Z_{t-3} + Z_{t-4}$ . Clearly,  $R_{t+1}$  is correlated with  $R_t$ . Note that Proposition 1.1 does not depend on whether the error terms in the quantile regression are *i.i.d.* or serially correlated.

### 1.3 The Uniform Confidence Bands

The goal of this section is to construct the uniform confidence bands. First, the definition of uniform confidence bands is as follows:

**Uniform Confidence Bands** Let  $0 < \alpha < 1$  be the significance level. We want to construct  $L_\alpha(\tau)$  and  $U_\alpha(\tau)$  such that

$$\lim_{T \rightarrow \infty} \inf_{\tau \in [\omega, 1-\omega]} P(L_\alpha(\tau) \leq \beta_j(\tau) \leq U_\alpha(\tau)) \geq 1 - \alpha,$$

where  $\beta_j(\tau)$  is the  $j$ -th element of  $\beta(\tau)$ .

By Theorem 2.1 in Koenker (2005),  $\hat{S}_T(\tau, \hat{\beta}(\tau)) = o_p(1)$ , which holds whether serial correlation is present or not. Proposition 1 implies

$$\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) = -H_0^{-1}(\tau)S_T(\tau, \beta_0(\tau)) + o_p(1)$$

uniformly in  $\tau \in [\omega, 1 - \omega]$ . The process  $S_T(\tau, \beta_0(\tau))$  is conditionally pivotal. Define  $\xi_t(\tau) = \phi_\tau(y_t - x_t' \beta_0(\tau))x_t$ . Then  $S_T(\tau, \beta_0(\tau))$  can be expressed in terms of  $\xi_t(\tau)$  as

$$S_T(\tau, \beta_0(\tau)) = \frac{1}{\sqrt{T}} \sum_{t=1}^T \xi_t(\tau).$$

Define

$$\Sigma(\tau_1, \tau_2) = \lim_{T \rightarrow \infty} Cov(S_T(\tau_1, \beta_0(\tau_1)), S_T(\tau_2, \beta_0(\tau_2))) = \sum_{j=-\infty}^{\infty} Cov(\xi_0(\tau_1), \xi_j(\tau_2)).$$

Let  $\Sigma^2(\tau) = \Sigma(\tau, \tau)$ . The result below shows that  $\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau))$  converges to a zero mean Gaussian process whose covariance depends on  $\Sigma(\tau_1, \tau_2)$ .

**Theorem 1.1** Suppose Assumptions 1-6 and S1 hold, then

$$\sqrt{T}(\hat{\beta}(\tau) - \beta(\tau)) \Rightarrow H_0^{-1}(\tau)U(\tau) \tag{1.7}$$

uniformly over  $\tau \in [\omega, 1 - \omega]$ , where  $U(\tau)$  is a  $p$ -dimensional zero-mean Gaussian process with

$$Cov(U(\tau_1), U(\tau_2)) = \Sigma(\tau_1, \tau_2).$$

Theorem 1.1 is the main theoretical result of this chapter. It generalizes results in Qu (2008) and Wu and Zhou (2016). In particular, Qu (2008) establishes the weak convergence of  $\sqrt{T}(\hat{\beta}(\tau) - \beta(\tau))$  over  $\tau \in [\omega, 1 - \omega]$ , assuming  $e_t(\tau)$  follows a martingale difference sequence. Wu and Zhou (2016) allow for general time series dependence but their result is pointwise with respect to  $\tau$ . Theorem 1.1 is not only uniform in  $\tau$  but also allows for time series dependence in  $e_t(\tau)$ . These two features are necessary for the empirical application considered later in this chapter.

As we observe from Theorem 1.1, the key to accurately estimate the uniform confidence bands is to mimic the behaviors of the processes  $\{U(\tau)\}$  and  $\{H(\tau)\}$ . Below, I implement a bootstrapping procedure which avoids directly estimating the densities and long-run variance.

The bootstrapping procedure requires two tuning parameters. It takes the form of a moving block bootstrap (Lahiri, 2003). It is constructed to preserve the temporal dependence structure and mimic the pattern of the subgradients over time. I now discuss the estimation of  $H(\tau)$  and  $U(\tau)$  separately.

To estimate  $H(\tau)$ , define

$$\hat{H}(\tau) = \sum_{t=1}^T \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{T c_T}. \quad (1.8)$$

$\hat{H}(\tau)$  is viewed as a progressive local constant kernel estimation of the integrated conditional density (Wu and Zhou, 2014).  $c_T$  is the bandwidth parameter, chosen based on the minimum volatility method (Zhou, 2013). Under Assumptions 1-6 and S1, as  $c_T \log^2 T \rightarrow 0$ ,  $T c_T^3 / \log^2 T \rightarrow \infty$ , we have  $|\hat{H}(\tau) - H_0(\tau)| = o_p(1)$ , uniformly over  $\tau \in [\omega, 1 - \omega]$ , as proved by Lemma 4 in the appendix.

To estimate the distribution of  $U(\tau)$ , define

$$\Psi_m(\tau) = \sum_{t=1}^{T-m+1} (m(T-m+1))^{-1/2} (\omega_{t,m}(\tau) - \frac{m}{T} \omega_{1,T}(\tau)) R_t, \quad (1.9)$$

where

$$\omega_{j,m} = \sum_{r=j}^{j+m-1} \phi(\hat{e}_r(\tau))x_r, \quad (1.10)$$

and  $\{R_t\}_{t=1}^T$  are *i.i.d.* standard normal random variables that are independent of  $\{\epsilon_i\}_{i=-\infty}^{\infty}$ . The subgradient-based process  $\Psi_m(\tau)$  is used here to approximate the distribution of  $U(\tau)$  by taking repeated draws of  $\{R_t\}_{t=1}^T$ .

Below I outline a bootstrap procedure for constructing the uniform confidence bands for  $\beta_{0,j}(\tau)$ , the  $j$ -th element of  $\beta_0(\tau)$ , over  $\tau \in [\omega, 1 - \omega]$ . Let  $\hat{\beta}_j(\tau)$  be the QR estimator and  $\hat{s}_j(\tau)$  be a consistent estimator for the standard deviation of  $\sqrt{T}(\hat{\beta}_j(\tau) - \beta_{0,j}(\tau))$ .

- Step 1. Select  $c_t$  based on the minimum volatility (MV) method in section 4 of Zhou (2013), first advocated in Politis et al. (1999);
- Step 2. Generate  $B$  times (e.g.  $B = 5000$ ) a sequence  $\{\Psi_m(\tau)\}$  of size  $T$ , and get  $\hat{H}(\tau)$ ;
- Step 3. For  $b = 1, \dots, B$ , calculate

$$\tilde{E}_b(\tau) = \hat{H}^{-1}(\tau)\Psi_m(\tau),$$

and let  $\tilde{E}_{b,j}$  denote the  $j$ -th element in  $\tilde{E}_b(\tau)$ , which corresponds to  $\beta_{0,j}(\tau)$ . Calculate  $s_j(\tau)$  as the sample standard deviation of  $\tilde{E}_{b,j}$ ;

- Step 4. Compute

$$E_{b,j} = \sup_{\tau \in [\omega, 1-\omega]} |\tilde{E}_{b,j}(\tau)/s_j(\tau)|. \quad (1.11)$$

Let  $E_{(1,j)} \leq E_{(2,j)} \leq \dots \leq E_{(B,j)}$  be the order statistics of  $E_{b,j}$ . Denote the floor function  $\lfloor x \rfloor$  as the greatest integer that is less than or equal to  $x$ . Then  $E_{(\lfloor (1-\alpha)B \rfloor, j)}$  is the level  $(1-\alpha)$  percentile of the empirical distribution. Compute

the uniform confidence bands as

$$[\hat{\beta}_j(\tau) - T^{-1/2}s_j(\tau)E_{(\lfloor(1-\alpha)B\rfloor,j)}, \hat{\beta}_j(\tau) + T^{-1/2}s_j(\tau)E_{(\lfloor(1-\alpha)B\rfloor,j)}]. \quad (1.12)$$

for the parameter  $\beta_{0,j}(\tau)$  over  $\tau \in [\omega, 1 - \omega]$ .

**Corollary 1.1** Under Assumptions 1-6 and S1, the expression (1.12) provides the asymptotically valid level- $\alpha$  confidence bands for  $\beta_{0,j}(\tau)$  over  $\tau \in [\omega, 1 - \omega]$ .

In the implementation, the bandwidth  $c_T$  is chosen by the minimum volatility (MV) method as follows:

- Step i) Choose a suitable interval  $I = [h_1, h_{100}]$ , and divide it equally into 99 cells so that we have 100 points over  $I$ .
- Step ii) Define  $h_i = h_1 + (i - 1)(h_{100} - h_1)/99$  for  $1 \leq i \leq 100$ .
- Step iii) For each value  $h_i$  in the interval  $I$ , use it as the bandwidth to calculate

$$T^{-1/2} \sum_i \phi_\tau(\hat{e}_i(\tau))x_t \quad \text{and} \quad \hat{H}_{h_i}(\tau).$$

Let  $C(i)$  be the maximal value of  $|H_0^{-1}(\tau)U(\tau)|$  with  $H_0(\tau)$  replaced by  $\hat{H}_{h_i}(\tau)$  and  $U(\tau)$  replaced by  $T^{-1/2} \sum_i \phi_\tau(\hat{e}_i(\tau))x_t$ .

- Step iv) Define

$$D(s) = \frac{1}{2k} \left( \sum_{j=s-k}^{s+k} [C(j) - \frac{1}{2k+1} \sum_{j=s-k}^{s+k} C(j)]^2 \right)^{1/2}$$

for some  $k > 0$ ; let  $l$  be the minimizer of  $\{D(s)\}_{s=k+1}^{100-k}$ , then we select  $h_l$  as the optimal bandwidth  $c_T$  in the simulation.

## 1.4 Simulations

I perform a simulation study to examine the coverage properties of the proposed uniform confidence bands.

I consider the following data generating process:

$$y_t = 1 + x_t + (1 + x_t)u_t, \quad (1.13)$$

where  $x_t \sim \chi^2(3)/3$ . Denote  $Q_u(\tau)$  as the  $\tau$ -th quantile of  $u_t$ , then the conditional quantile functions of  $y_t$  can be written as

$$Q_{y_t|x_t}(\tau) = 1 + Q_u(\tau) + (1 + Q_u(\tau))x_t. \quad (1.14)$$

Therefore, to map with respect to the model specified in the previous section, we have

$$e_t(\tau) = y_t - 1 - (1 + Q_u(\tau))x_t - Q_u(\tau).$$

I consider two specifications for the process  $u_t$ . In Case I,  $u_t \sim i.i.d.N(0, 1)$ . In Case II,  $u_t = 0.5u_{t-1} + \epsilon_t$  and  $\epsilon_t \sim i.i.d.N(0, 1)$ . In both cases,  $x_t$  and  $u_t$  are mutually independent.

The simulation focuses on constructing uniform confidence bands for the coefficient of  $x_t$ , i.e.,  $(1+Q_u(\tau))$ . Below, I consider two sample sizes 250 and 500 and two nominal levels 5% and 10%. The quantile range is set to  $[0.1, 0.9]$ .

The coverage probability is obtained as follows. Step 1: Estimate the model to get the parameter estimates and the residuals. Step 2: Carry out bootstrap using the estimates from Step 1. Check whether the population coefficient  $(1 + Q_u(\tau))$  lies inside the uniform confidence bands for every quantile ( $\tau = 0.1, 0.2, \dots, 0.9$ ). Step 3: Repeat Steps 1 and 2 for 1000 times to get the coverage probability.

Table 1.1 reports the simulated coverage probabilities. In general, the coverage

**Table 1.1:** Simulated Coverage Probabilities under Different Error Structures

	$\alpha=5\%$		$\alpha=10\%$	
	n=250	n=500	n=250	n=500
Case I	0.925	0.941	0.883	0.899
Case II	0.931	0.946	0.885	0.901

probability is close to the significance level even when the sample size is small. As the sample size increases, the coverage gets even closer to the nominal level.

## 1.5 Application

Greenwood and Shleifer (2014) have provided evidence of a negative correlation between investor expectations and model-based expected returns. They analyzed time series of investor expectations of future stock market returns from six data sources. The six measures are found to be highly positively correlated with each other, as well as with past stock returns.

Here, I analyze the relationship between investor sentiments and the *conditional distribution* of stock returns. I look into the quantiles of the return distribution and apply the uniform confidence bands method to assess the significance of the estimates. The research questions are two folds:

- a) How do investor expectations about the stock market relate to the real S&P 500 stock market performance?
- b) Do such relationship vary across the distribution of realized returns of the stock market?

### 1.5.1 Data

The data are collected from several sources including the American Association of Individual Investors (AAII), the Michigan Survey of Consumers, the Survey of Consumer Finances and CRSP. After matching the S&P 500 index with investor senti-

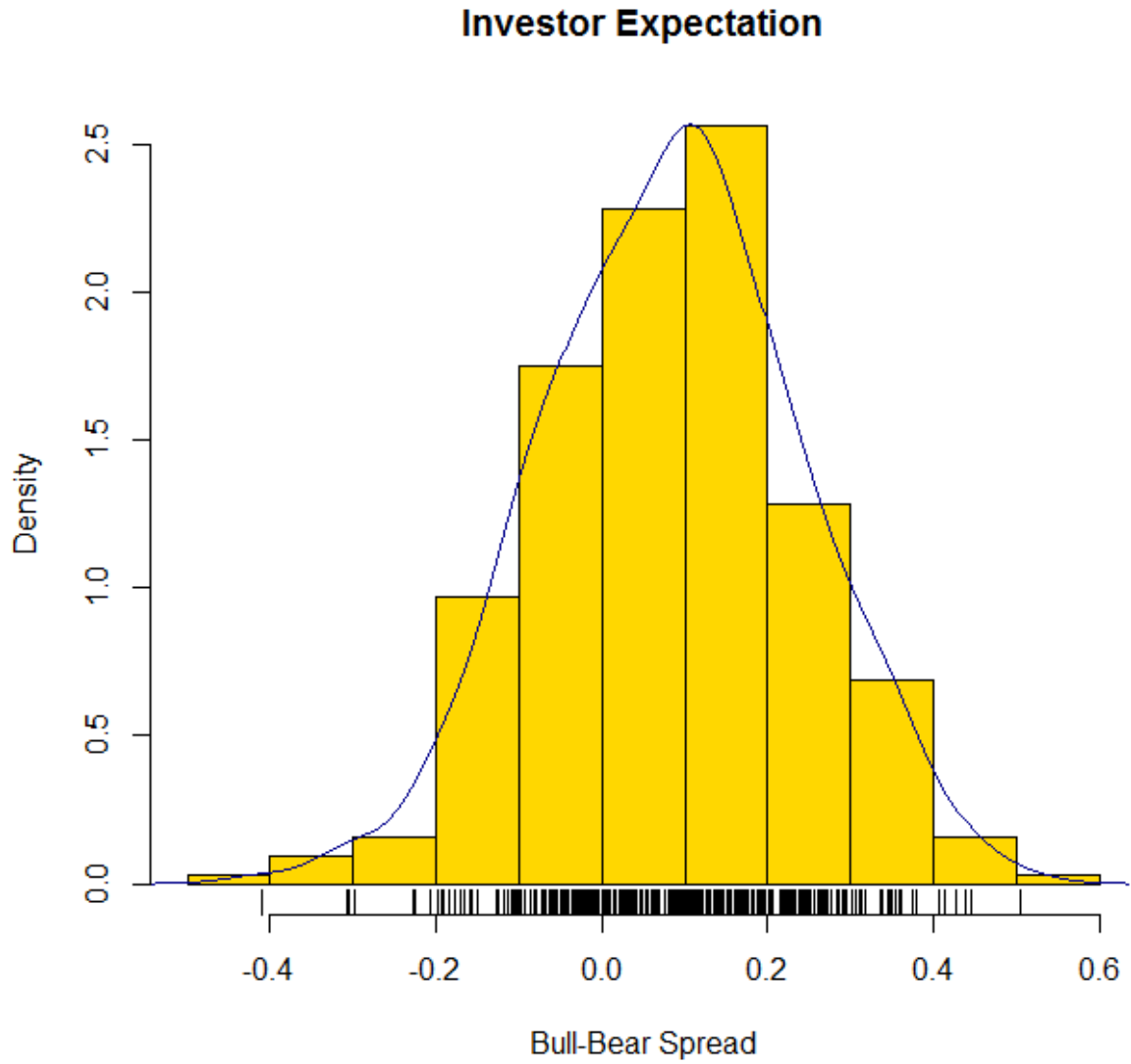
ments data, I compare the real stock returns with investor sentiments.

The investor expectation measure is computed as the bull-bear spread. More specifically, each investor reports his/her sentiment about the stock market before every Wednesday. AAI collects all responses and reports the spread by each Thursday. The bull-bear spread equals the difference between the percentage of investors who think the stock market will go up in the next six months and the percentage of investors who believe the stock market will go down in the next six months. I denote the spread at time  $t$  by  $E_t$ .

Figure 1.1 shows the density of the bull-bear spread. The spread has a mean of 0.08 and standard deviation of 0.19. In comparison, the distribution of the realized stock market returns is slightly skewed to the right with a fat left tail. Investor sentiments spread and realized returns appear to share some common features.

Historical data shows that the investor sentiments spread reached its minimum of -0.54 on Oct 19, 1990 and the second smallest value -0.51 on Mar 5, 2009. The maximum of the spread is 0.63, which happened on June 26, 2003. In that week, 81.5% of individual investors expected the stock market to go up six months from then. The second largest value is 0.62, which happened on January 6, 2000. These dates correspond to important economic events related to financial crises or economic booms. Consistent with Greenwood and Shleifer (2014), there is a negative correlation between investor expectation and model-based expected returns in the data set.

I now turn to the calculation of realized returns. Note that the S&P 500 index is collected as the daily closing value for each business day. I first calculate the average of the index levels over the week and match with the weekly spread at time  $t$ . The reason to take the average of the index during the week is that investor sentiments were reported during the week without specifying the exact date. I denote this value as  $P_t$ . Let  $k$  be the number of weeks in advance of time  $t$ . Then, the  $k$ -week ahead



**Figure 1.1:** Investor Sentiments: Bull-Bear Spread

return is computed as

$$R_{t+k} = \log P_{t+k} - \log P_t .$$

When reporting the results, I choose six months ahead stock returns for  $R_{t+k}$ . This is because the survey question asks each individual investor his/her expectation about the stock market six months forward in time. This way, I can match the expectations with the returns. This leads to the following quantile regression:

$$R_{t+k} = \alpha(\tau) + \beta(\tau)E_t + e_{t+k}(\tau) .$$

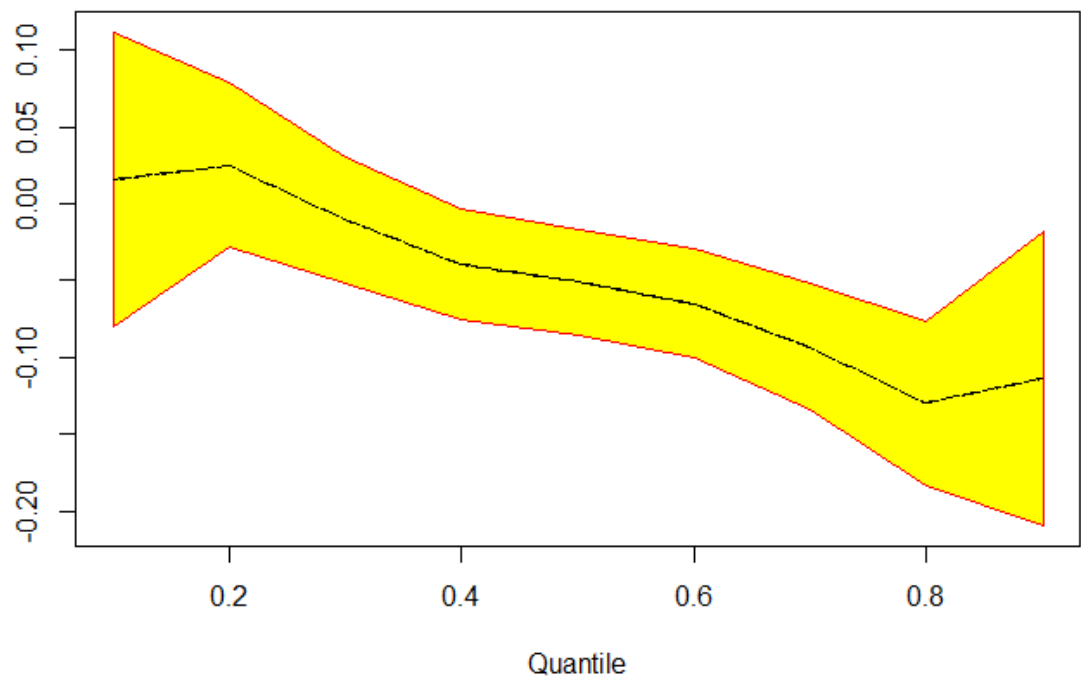
### 1.5.2 Parameter Estimates and the Uniform Confidence Bands

Figure 1.2 reports the point estimates for  $\beta(\tau)$  and the 95% uniform confidence bands. To facilitate comparison, in Table 1.2, I report at selected quantiles the point estimates, the intervals resulting from the 95% confidence bands, and the pointwise confidence intervals. The pointwise confidence intervals are computed following the bootstrapping procedure without taking the supremum over  $\tau \in [\omega, 1 - \omega]$ . U-length is the difference between the upper bound and the lower bound in the uniform confidence band. P-length is the length of the pointwise confidence interval.

The significance of  $\beta(\tau)$  is indicated through three notations: \* means significant at 90% confidence level; \*\* means significant at 95% confidence level;\*\*\* means significant at 99% confidence level.

**Table 1.2:** Uniform Confidence Bands for QR Estimates: Investor Sentiments

$\tau$	$\beta(\tau)$	95% Uniform CB	U-length	95% Confidence Interval	P-length
0.1	0.016	(-0.079,0.112)	0.191	(-0.066, 0.099)	0.165
0.2	0.025	(-0.030,0.079)	0.109	(-0.026, 0.075)	0.101
0.25	0.019	(-0.051,0.050)	0.101	(-0.014, 0.049)	0.064
0.5	-0.051	(-0.086,-0.017)**	0.068	(-0.076,-0.026)**	0.050
0.75	-0.112	(-0.156,-0.055)**	0.101	(-0.146, -0.078)***	0.068
0.8	-0.130	(-0.184,-0.076)***	0.108	(-0.165,-0.095)***	0.070
0.9	-0.114	(-0.209,-0.018)**	0.191	(-0.147, -0.080)***	0.067



**Figure 1.2:** Uniform QR Confidence Bands for Investor Sentiments

Three patterns emerge from the Figure and Table. First,  $\hat{\beta}(\tau)$  is downward sloping. The value is positive at lower quantiles and turns negative after crossing the median. Second, the estimates are insignificant at lower quantiles, but are all significant after crossing the median. Third, the bands are narrower in the middle and wider when approaching to the two tails.

How should the empirical results be interpreted? Consider two covariate values  $E_t$  and  $E_s$  with  $E_t < E_s$ . Intuitively,  $E_t$  represents a more pessimistic market expectation than  $E_s$ . Evaluating the conditional quantile function at these two covariate values and taking the difference, we obtain

$$\hat{Q}_{R|E_t}(\tau) - \hat{Q}_{R|E_s}(\tau) = \hat{\beta}(\tau)(E_t - E_s) .$$

The estimates then imply the following. At percentiles above the median, the future realized distributions corresponding to these two market events  $E_t$  and  $E_s$  are subtly different. If the market outcomes turn out to be non-favorable, then these outcomes are not systematically different when evaluated at the same quantile level. If the market outcomes are favorable, then the one with higher optimism actually has lower returns, when again the comparison is made at the same quantile level. This heterogeneity is a new finding. It appears hard to rationalize with existing theory built conditional on the market efficiency hypothesis.

To further examine the robustness of the above findings and also obtain an even clearer interpretation, below I conduct the same analysis but using a binary specification for the market expectation variable  $E_t$ .

### 1.5.3 Results with a Binary Regressor Specification

I generate a binary variable  $E_t^b$  that takes value 1 when the investor sentiment *spread*  $> 0$  and zero otherwise. That is, it takes value 1 when  $E_t > 0$ . Then, I

repeat the same regression as before with  $E_t$  replaced by  $E_t^b$ . The findings are reported in Table 1.3 and Figure 1.3. They have the same structure as Table 1.2 and Figure 1.2.

**Table 1.3:** Uniform Confidence Bands for QR Estimates in the Binary Case

$\tau$	$\beta(\tau)$	95% Uniform CB	U-length	95% Confidence Interval	P-length
0.1	0.022	(-0.007,0.051)	0.058	(-0.001, 0.045)	0.046
0.2	0.014	(-0.002,0.031)	0.033	(-0.002, 0.030)	0.032
0.25	0.003	(-0.010,0.015)	0.025	(-0.006, 0.013)	0.020
0.5	-0.013	(-0.024,-0.003)**	0.021	(-0.023, -0.004)**	0.019
0.75	-0.028	(-0.040, -0.015)**	0.025	(-0.038,-0.017)***	0.020
0.8	-0.034	(-0.051,-0.018)***	0.033	(-0.051, -0.018)***	0.033
0.9	-0.045	(-0.074,-0.015)***	0.058	(-0.057, -0.032)***	0.025

The results are consistent with those in the previous subsection. Again, the estimates are significant only for quantiles at and above the median. And they are all negative. Therefore, the finding is robust to the change in the regressor specification.

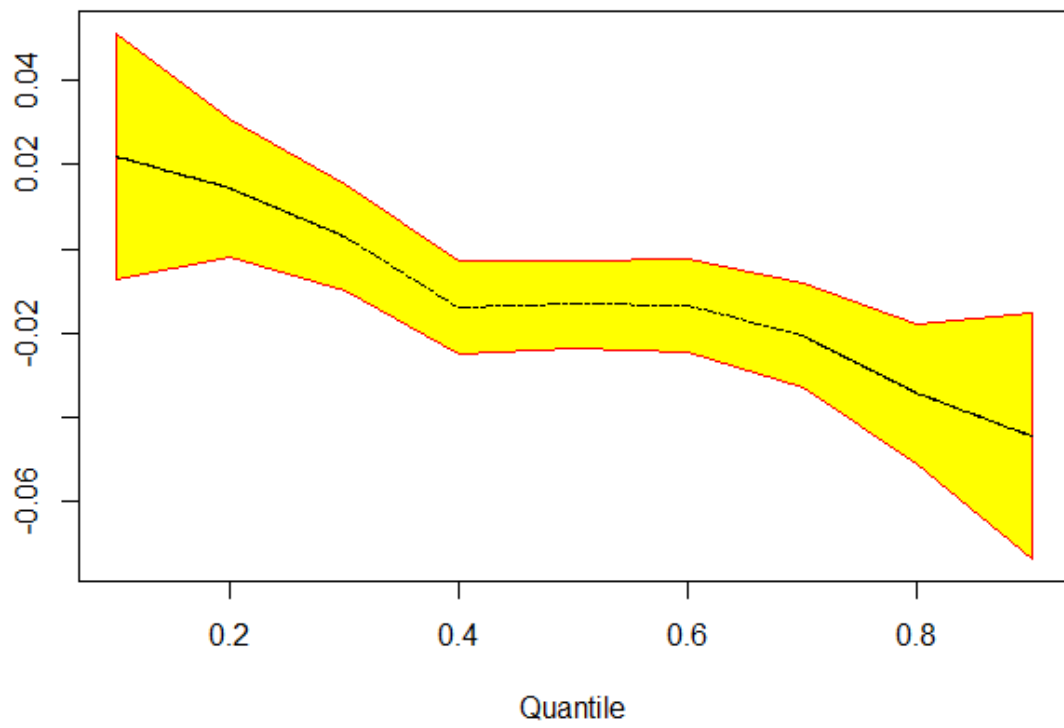
To interpret the results, we can consider two values for the regressor:  $E_t^b = 1$  and  $E_s^b = 0$ . Then:

$$\hat{Q}_{R|E_t^b}(\tau) - \hat{Q}_{R|E_s^b}(\tau) = \hat{\beta}(\tau) ,$$

which is negative and significant at median and upper quantiles only. If we treat  $E_t^b$  as bullish and  $E_s^b$  as bearish, then the result implies that a bullish expectation actually corresponds to a more bearish overall market outcome. The failure to predict accordingly is more apparent when the market realization is more favorable.

## 1.6 Conclusion

I have developed uniform confidence bands for the linear quantile regression estimator in a time series setting. The inference procedure allows for serially correlated errors. By applying this method, I analyze the future returns in S&P 500 stock market and sentiment survey data using a quantile regression model. This empirical application



**Figure 1-3:** Uniform QR Confidence Bands in the Binary Case

shows that a more optimistic investor sentiment actually corresponds to a lower right tail of the future stock return distribution.

## Appendix: Proofs

The following notation is used in this appendix. The Euclidean norm of a vector  $z$  is denoted by  $\|z\|$ . For matrices, the vector-induced norm is used, i.e., for matrix  $A$ ,  $\|A\| = \sup_{x \neq 0} \|Ax\|/\|x\|$ . The subscript 0 indicates the true value.  $D_{[0,1]}$  stands for the set of functions on  $[0, 1]$  that are right continuous and have left limits, equipped with the Skorohod metric.  $(D_{[0,1]})^m$  is a product space, which is equipped with the corresponding product Skorohod topology. The symbols “ $\Rightarrow$ ” and “ $\rightarrow^p$ ” signify weak convergence under the Skorohod topology and convergence in probability.  $O_p(\cdot)$  and  $o_p(\cdot)$  are the usual symbols for the order of stochastic magnitude.

Recall that  $S_T(\tau, b)$  is defined as

$$S_T(\tau, b) = T^{-1/2} \sum_{t=1}^T x_t \{1(y_t - x'_t b \leq 0) - \tau\}.$$

By re-centering  $1(y_t - x'_t b \leq 0)$  at its expectation conditional on  $x_t$ , instead of at  $\tau$ , we can define a related quantity  $S_T^d(\tau, b)$ :

$$S_T^d(\tau, b) = T^{-1/2} \sum_{t=1}^T x_t \{1(y_t - x'_t b \leq 0) - F_t(x'_t b)\},$$

where

$$F_t(x'_t b) = \Pr(y_t \leq x'_t b | x_t).$$

When serial correlation is present, the summands in  $S_T^d(\tau, b)$  have mean zero, but are in general serially correlated. Following Wu (2007) and Wu and Zhou (2016), we construct a martingale difference sequence with respect to a new filtration. Specifically, as in the main text, the error process  $e_t(\tau)$  is assumed to be generated as

$$e_t(\tau) = G(\tau, \mathcal{F}_t, \mathcal{G}_t).$$

Also, recall that the regressors are assumed to be generated by

$$x_t = H(\mathcal{F}_{t-1}, \mathcal{G}_t).$$

Using the above notation,  $S_T^d(\tau, b)$  can be further decomposed as

$$\begin{aligned} S_T^d(\tau, b) &= T^{-1/2} \sum_{t=1}^T x_t \{1(y_t - x_t' b \leq 0) - E(1(y_t - x_t' b \leq 0) | \mathcal{F}_{t-1}, \mathcal{G}_t)\} \\ &\quad + T^{-1/2} \sum_{t=1}^T x_t \{E(1(y_t - x_t' b \leq 0) | \mathcal{F}_{t-1}, \mathcal{G}_t) - F_t(x_t' b)\}. \\ &= M_T(\tau, b) + N_T(\tau, b) \end{aligned} \quad (1.15)$$

The following relationship will be used repeatedly in the subsequent analysis:

$$S_T(\tau, b) = M_T(\tau, b) + N_T(\tau, b) + T^{-1/2} \sum_{t=1}^T x_t \{F_t(x_t' b) - \tau\}. \quad (1.16)$$

We now establish some preliminary results. The next lemma provides the stochastic equicontinuity of the subgradient process. Along with the finite dimensional convergence (as already established in the literature), it implies the weak convergence of  $S_T(\tau, \beta_0(\tau))$  on  $D_{[0,1]}$  under the Skorohod topology.

**Lemma 1** *Let  $\Phi = [0, 1]$  be a parameter set with metric  $\rho(\tau_1, \tau_2) = |\tau_2 - \tau_1|$ . Under Assumptions 1-6 and S1, the process  $S_T(\tau, \beta_0(\tau))$  is stochastically equicontinuous under  $(\Phi, \rho)$ . That is, for any  $\epsilon > 0$ ,  $\eta > 0$ , there exists a  $\delta > 0$  such that for large  $n$ ,*

$$P \left( \sup_{[\delta]} \|S_T(\tau_1, \beta_0(\tau_1)) - S_T(\tau_2, \beta_0(\tau_2))\| > \eta \right) < \epsilon,$$

where  $[\delta] = \{\tau_1, \tau_2 \in \Phi : \rho(\tau_1, \tau_2) < \delta\}$ .

**Proof:** We have

$$\begin{aligned} S_T(\tau, \beta_0(\tau)) &= T^{-1/2} \sum_{t=1}^T x_t \{1(y_t - x_t' \beta_0(\tau) \leq 0) - E(1(y_t - x_t' \beta_0(\tau) \leq 0 | \mathcal{F}_{t-1}, \mathcal{G}_t))\} \\ &\quad + T^{-1/2} \sum_{t=1}^T x_t \{E(1(y_t - x_t' \beta_0(\tau) \leq 0 | \mathcal{F}_{t-1}, \mathcal{G}_t) - \tau)\}. \end{aligned}$$

The summands of the first term on the right hand side are martingale differences with respect to  $\{\mathcal{F}_{t-1}, \mathcal{G}_t\}$ . Hence, arguments in Theorem A1 of Bai (1996) are applicable. As a result, this term is stochastically equicontinuous with respect to  $\tau$ . The second term can be analyzed using the same argument as in Lemma 5 in Wu (2007) together with the monotonicity of the conditional quantile function. Therefore, this term is also stochastic equicontinuous. Combining the two results completes the proof. ■

**Lemma 2** *Let  $D$  be an arbitrary compact set in  $R^p$ . Under Assumptions 1-6 and S1, we have*

$$\sup_{\tau \in \mathcal{T}_\omega} \sup_{\xi \in D} \|S_T^d(\tau, \beta_0(\tau) + T^{-1/2}\xi) - S_T(\tau, \beta_0(\tau))\| = o_p(1),$$

where  $\mathcal{T}_\omega = [\omega_1, \omega_2]$  with  $0 < \omega_1 < \omega_2 < 1$ .

**Proof:** This proof proceeds along similar lines as that of Bai (1996, Theorem A3), with the further decomposition (1.15) to handle time series dependence.

Without loss of generality, we can assume the components of  $x_t$  are nonnegative. Otherwise, let  $x_{i,j}$  denote the  $j$ th component of  $x_t$  and we can write

$$x_{i,j} = x_{i,j}^+ - x_{i,j}^- \equiv x_{i,j}1(x_{i,j} \geq 0) - x_{i,j}1(x_{i,j} < 0).$$

Then  $x_{i,j}^+$  and  $x_{i,j}^-$  are nonnegative and they satisfy the assumptions in the main text, and the subsequent argument can be applied separately to  $x_{i,j}^+$  and  $x_{i,j}^-$ . Under the nonnegativity assumption,  $x_t1(y_t \leq x_t' \beta_0(\tau) + T^{-1/2}x_t' \xi)$  and  $F_t(x_t' \beta_0(\tau) + T^{-1/2}x_t' \xi)$  are nondecreasing in  $\tau$ .

As in Bai (1996), let  $\varepsilon_T = T^{-1/2-d}$ , where  $d \in (0, (\gamma - 1)/2)$  with  $\gamma > 1$ . Let  $N(\varepsilon_T) = [(\omega_2 - \omega_1)/\varepsilon_T] + 1$ .  $\mathcal{T}_\omega$  can then be partitioned into  $N(\varepsilon_T)$  intervals of equal length:  $\omega_1 = \tau_0 < \tau_1 < \dots < \tau_{N(\varepsilon_T)} = \omega_2$ . Suppose  $\tau \in [\tau_{j-1}, \tau_j]$ , then

$$\begin{aligned} & S_T^d(\tau, \beta_0(\tau) + T^{-1/2}\xi) - S_T(\tau, \beta_0(\tau)) \\ \leq & S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi) - S_T(\tau_{j-1}, \beta_0(\tau_{j-1})) + T^{-1/2} \sum_{t=1}^T x_t \{\tau_j - \tau_{j-1}\} \\ & + T^{-1/2} \sum_{t=1}^T x_t \{F_t(x'_t \beta_0(\tau_j) + T^{-1/2} x'_t \xi) - F_t(x'_t \beta_0(\tau_{j-1}) + T^{-1/2} x'_t \xi)\} \end{aligned}$$

and

$$\begin{aligned} & S_T^d(\tau, \beta_0(\tau) + T^{-1/2}\xi) - S_T(\tau, \beta_0(\tau)) \\ \geq & S_T^d(\tau_{j-1}, \beta_0(\tau_{j-1}) + T^{-1/2}\xi) - S_T(\tau_j, \beta_0(\tau_j)) + T^{-1/2} \sum_{t=1}^T x_t \{\tau_{j-1} - \tau_j\} \\ & + T^{-1/2} \sum_{t=1}^T x_t \{F_t(x'_t \beta_0(\tau_{j-1}) + T^{-1/2} x'_t \xi) - F_t(x'_t \beta_0(\tau_j) + T^{-1/2} x'_t \xi)\}. \end{aligned}$$

Hence,

$$\begin{aligned} & \|S_T^d(\tau, \beta_0(\tau) + T^{-1/2}\xi) - S_T(\tau, \beta_0(\tau))\| \\ \leq & \|S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi) - S_T(\tau_{j-1}, \beta_0(\tau_{j-1}))\| \\ & + \|S_T^d(\tau_{j-1}, \beta_0(\tau_{j-1}) + T^{-1/2}\xi) - S_T(\tau_j, \beta_0(\tau_j))\| \\ & + \|T^{-1/2} \sum_{t=1}^T x_t \{\tau_j - \tau_{j-1}\}\| \\ & + \left\| T^{-1/2} \sum_{t=1}^T x_t \{F_t(x'_t \beta_0(\tau_j) + T^{-1/2} x'_t \xi) - F_t(x'_t \beta_0(\tau_{j-1}) + T^{-1/2} x'_t \xi)\} \right\| \\ \equiv & (a) + (b) + (c) + (d). \end{aligned}$$

Therefore, to complete the proof, it is sufficient to show that (a), (b), (c) and (d) are  $o_p(1)$  uniformly in  $\tau \in \mathcal{T}_\omega$  and  $\xi \in D$ .

Term (c)'s supremum is less than or equal to

$$\|T^{-1-d} \sum_{t=1}^T x_t\| = o_p(1) \text{ because } d > 0.$$

For term (d), we follow the argument of Koul (1991, Lemma 2.1):

$$\begin{aligned} & \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \|(c)\| & (1.17) \\ & \leq \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \left\| T^{-1} \sum_{t=1}^T [f_t(x'_t b(\tau_j)) - f_t(x'_t b(\tau_{j-1}))] x_t x'_t \xi \right\| + o_p(1) \\ & \leq 2 \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \left\| T^{-1} \sum_{t=1}^T [f_t(x'_t b(\tau_j)) - f_t(x'_t \beta_0(\tau_j))] x_t x'_t \xi \right\| \\ & \quad + \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \left\| T^{-1} \sum_{t=1}^T [f_t(x'_t \beta_0(\tau_j)) - f_t(x'_t \beta_0(\tau_{j-1}))] x_t x'_t \xi \right\| + o_p(1), \end{aligned}$$

where  $b(\tau_k)$  ( $k = j - 1$  and  $j$ ) is some vector that lies between  $\beta_0(\tau_k)$  and  $\beta_0(\tau_k) + T^{-1/2}\xi$ , and the first inequality follows from the mean value theorem and  $\tau_j - \tau_{j-1} \leq n^{-1/2-d}$ . Now, (1.17) =  $o_p(1)$  if

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq t \leq T} \|f_t(x'_t b(\tau_j)) - f_t(x'_t \beta_0(\tau_j))\| = o_p(1) \quad (1.18)$$

and

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq t \leq T} \|f_t(x'_t \beta_0(\tau_j)) - f_t(x'_t \beta_0(\tau_{j-1}))\| = o_p(1). \quad (1.19)$$

(1.18) holds because  $f_t(s)$  are uniformly continuous in  $s$  for all  $t$  and

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq t \leq T} \|x'_t b(\tau_j) - x'_t \beta_0(\tau_j)\| = \max_{1 \leq t \leq T} \|x'_t\| O_p(T^{-1/2}) = o_p(1).$$

For (1.19),

$$x'_t \beta_0(\tau_j) - x'_t \beta_0(\tau_{j-1}) = \frac{\tau_j - \tau_{j-1}}{f_t(z_t)} = O_p(\tau_j - \tau_{j-1}) = O_p(T^{-1/2-d}), \quad (1.20)$$

where the first equality follows from the mean value theorem with  $x'_t\beta_0(\tau_{j-1}) \leq z_t \leq x'_t\beta_0(\tau_j)$ , and the second equality follows because  $f_t(\cdot)$  is bounded away from 0 for all  $t$ . (1.20) implies (1.19) because  $f_t(s)$  are uniformly continuous in  $s$  for all  $t$ .

We now turn to terms (a) and (b). We have

$$\begin{aligned} & \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} (\|a\| + \|b\|) \\ & \leq 2 \max_{1 \leq j \leq N(\varepsilon_T)} \|S_T(\tau_j, \beta_0(\tau_j)) - S_T(\tau_{j-1}, \beta_0(\tau_{j-1}))\| \\ & \quad + 2 \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \|S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi) - S_T(\tau_j, \beta_0(\tau_j))\|. \end{aligned}$$

The first term is  $o_p(1)$  by the stochastic equicontinuity of  $S_T(\tau_j, \beta_0(\tau))$  as proved in the previous lemma. For the second term, because  $D$  is compact, for any given  $\delta > 0$ ,  $D$  can always be partitioned into a finite number of subsets such that the diameter of each subset is less than or equal to  $\delta$ . Denote these subsets by  $D_1, D_2, \dots, D_{N(\delta)}$ . If  $\xi \in D_h$  ( $h \in \{1, 2, \dots, N(\delta)\}$ ), there exist two points,  $\xi_{h,1}$  and  $\xi_{h,2}$ , on the boundary of  $D_h$  satisfying  $x'_t\xi_{h,1} \leq x'_t\xi \leq x'_t\xi_{h,2}$ , leading to

$$\begin{aligned} & \max_{1 \leq j \leq N(\varepsilon_T)} \sup_{\xi \in D} \|S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi) - S_T(\tau_j, \beta_0(\tau_j))\| \\ & \leq \max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq h \leq N(\delta)} \max_{k=1,2} \left\| T^{-1/2} \sum_{t=1}^T x_t \{F_t(x'_t\beta_0(\tau_j) + T^{-1/2}x'_t\xi_{h,k}) - F_t(x'_t\beta_0(\tau_j))\} \right\| \\ & \quad + \max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq h \leq N(\delta)} \max_{k=1,2} \|S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi_{h,k}) - S_T(\tau_j, \beta_0(\tau_j))\|. \end{aligned}$$

The first term on the right side is of the same order as  $\xi_{h,1}$  and  $\xi_{h,2}$ , which can be made arbitrarily small by choosing a small  $\delta$ . To bound the second term, apply the decomposition (1.15):

$$\begin{aligned} & S_T^d(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi_{h,k}) - S_T(\tau_j, \beta_0(\tau_j)) \\ & = M_T(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi_{h,k}) - M_T(\tau_j, \beta_0(\tau_j)) \\ & \quad + N_T(\tau_j, \beta_0(\tau_j) + T^{-1/2}\xi_{h,k}) - N_T(\tau_j, \beta_0(\tau_j)). \end{aligned}$$

Therefore, it suffices to show that

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq h \leq N(\delta)} \max_{k=1,2} \left\| M_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - M_T(\tau_j, \beta_0(\tau_j)) \right\| = o_p(1) \quad (1.21)$$

and

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq h \leq N(\delta)} \max_{k=1,2} \left\| N_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - N_T(\tau_j, \beta_0(\tau_j)) \right\| = o_p(1). \quad (1.22)$$

For (1.21), because  $N(\delta)$  and  $k$  are finite, it is sufficient to show, for any  $\epsilon > 0$ ,  $1 \leq j \leq N(\varepsilon_T)$ ,  $1 \leq h \leq N(\delta)$  and  $k \in \{1, 2\}$ ,

$$N(\varepsilon_T) \Pr \left( \left\| M_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - M_T(\tau_j, \beta_0(\tau_j)) \right\| > \epsilon \right) \rightarrow 0. \quad (1.23)$$

Let

$$\begin{aligned} \zeta_t &= \left\{ \mathbf{1}(y_t \leq x'_t \beta_0(\tau_j) + T^{-1/2} x'_t \xi_{h,k}) - E \left( \mathbf{1}(y_t \leq x'_t \beta_0(\tau_j) + T^{-1/2} x'_t \xi_{h,k}) \mid \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right\} \\ &\quad - \left\{ \mathbf{1}(y_t \leq x'_t \beta_0(\tau_j)) - E \left( \mathbf{1}(y_t \leq x'_t \beta_0(\tau_j)) \mid \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right\}. \end{aligned}$$

Then

$$M_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - M_T(\tau_j, \beta_0(\tau_j)) = T^{-1/2} \sum_{t=1}^T x_t \zeta_t,$$

where  $x_t \zeta_t$  is a vector of martingale differences with respect to  $\{\mathcal{F}_{t-1}, \mathcal{G}_t\}$ . Apply Doob's and Rosenthal's inequalities; the left side of (1.23) is then bounded from below by

$$N(\varepsilon_T) \bar{M} \epsilon^{-2\gamma} \left\{ T^{-(\gamma-1)} E \left( T^{-1} \sum_{t=1}^T (x'_t x_t)^\gamma |\zeta_t|^{2\gamma} \right) + E \left( T^{-1} \sum_{t=1}^T x'_t x_t E \left( \zeta_t^2 \mid \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right)^\gamma \right\} \quad (1.24)$$

Note that, for  $\gamma > 1$ ,

$$\begin{aligned}
E(|\zeta_t|^{2\gamma} | x_t) &= E(\zeta_t^2 |\zeta_t|^{2\gamma-2} | x_t) \\
&\leq CE(E(\zeta_t^2 | x_t)) \\
&\leq T^{-1/2} C \|x'_t \xi_{h,k}\| \\
&\leq T^{-1/2} C \|x_t\| \|\xi_{h,k}\|,
\end{aligned}$$

where the first inequality follows because  $\zeta_t$  is always bounded and the third because of the mean value theorem. Therefore,

$$\begin{aligned}
E\left(T^{-1} \sum_{t=1}^T (x'_t x_t)^\gamma |\zeta_t|^{2\gamma}\right) &= E\left(T^{-1} \sum_{t=1}^T E((x'_t x_t)^\gamma \zeta_t^{2\gamma} | x_t)\right) \\
&\leq CT^{-1/2} E\left(T^{-1} \sum_{t=1}^T E((x'_t x_t)^\gamma \|x_t\|)\right) \\
&= O(T^{-1/2}),
\end{aligned}$$

which implies

$$T^{-(\gamma-1)} E\left(T^{-1} \sum_{t=1}^T (x'_t x_t)^\gamma |\zeta_t|^{2\gamma}\right) \leq O(T^{-\gamma+1/2}) = O(T^{-\gamma/2}).$$

For the second term in (1.24), it follows from similar arguments as above that

$$E(\zeta_t^2 | \mathcal{F}_{t-1}, \mathcal{G}_t) \leq T^{-1/2} C \|x_t\| \|\xi_{h,k}\|$$

and therefore

$$E\left(T^{-1} \sum_{t=1}^T x'_t x_t E(\zeta_t^2 | \mathcal{F}_{t-1}, \mathcal{G}_t)\right)^\gamma \leq CT^{-\gamma/2}.$$

Combining the above results, the order of (1.24) is

$$O(T^{1/2+d} \bar{M} \epsilon^{-2\gamma} T^{-\gamma/2}) = o(1),$$

which holds because  $d \in (0, (\gamma - 1)/2)$ . Notice that the above result does not depend on  $j$ ,  $h$  or  $k$ .

It only remains to show that

$$\max_{1 \leq j \leq N(\varepsilon_T)} \max_{1 \leq h \leq N(\delta)} \max_{k=1,2} \left\| N_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - N_T(\tau_j, \beta_0(\tau_j)) \right\| = o_p(1).$$

Again, because  $N(\delta)$  and  $k$  are finite, it suffices to show that

$$\max_{1 \leq j \leq N(\varepsilon_T)} \left\| N_T(\tau_j, \beta_0(\tau_j) + T^{-1/2} \xi_{h,k}) - N_T(\tau_j, \beta_0(\tau_j)) \right\| = o_p(1).$$

To this end, apply the same argument as in Wu and Zhou (2016, Lemma A2). Although their Lemma is for a fixed quantile, the  $o_p(1)$  order holds uniformly over  $\mathcal{T}_\omega$  under the stronger assumption S1 used in this chapter. This completes the proof. ■

**Lemma 3** *Under Assumptions 1-6 and S1, we have*

$$\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) = O_p(1) \tag{1.25}$$

*holds uniformly over  $\mathcal{T}_\omega = [\omega_1, \omega_2]$  with  $0 < \omega_1 < \omega_2 < 1$ .*

**Proof.** The proof is very similar to that of Lemma 2 in Qu (2008).

Because  $\left\| S_T(\tau, \hat{\beta}(\tau)) \right\| = o_p(1)$  (Theorem 2.1 in Koenker, 2005), (1.32) holds if for any  $\epsilon > 0$  there exists a  $K_0 > 0$ ,  $N_0 > 0$  and  $\eta > 0$ , such that if  $\left\| \sqrt{T}(\beta^*(\tau) - \beta_0(\tau)) \right\| > K_0$ ,

$$P \left\{ \inf_{\tau \in \mathcal{T}_\omega} \left\| S_T(\tau, \beta^*(\tau)) \right\| < \eta \right\} < \epsilon \quad \text{for all } T > N_0. \tag{1.26}$$

To show (1.26), rewrite  $\beta^*(\tau)$  as

$$\beta^*(\tau) = \beta_0(\tau) + T^{-1/2} t e,$$

where  $t = \|\beta^*(\tau) - \beta_0(\tau)\|$  and  $e = (\beta^*(\tau) - \beta_0(\tau)) / \|\beta^*(\tau) - \beta_0(\tau)\|$ . Notice that  $e$

satisfies  $e \in R^p$  and  $\|e\| = 1$ . Using the Cauchy-Schwarz inequality,

$$\begin{aligned} & P \left\{ \inf_{\tau \in \mathcal{T}_\omega} \|S_T(\tau, \beta^*(\tau))\| < \eta \right\} \\ &= P \left\{ \inf_{\tau \in \mathcal{T}_\omega} \|S_T(\tau, \beta_0(\tau) + T^{-1/2}te)\| * \|e\| < \eta \right\} \\ &\leq P \left\{ \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau) + T^{-1/2}te)| < \eta \right\}. \end{aligned}$$

Hence, to prove (1.26) it is sufficient to show that for any unit vector  $e$ , if  $t \geq K_0$  then,

$$P \left\{ \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau) + T^{-1/2}te)| < \eta \right\} < \epsilon \quad \text{for all } T > N_0. \quad (1.27)$$

Further, because the quantity  $e'S_n(\tau, \beta_0(\tau) + T^{-1/2}te)$  is a non-decreasing function of  $t$ , it is sufficient to show that, for any unit vector  $e$ , (1.27) holds when  $t$  is replaced by  $K_0$ , i.e.,

$$P \left\{ \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e)| < \eta \right\} < \epsilon \quad \text{for all } T > N_0. \quad (1.28)$$

We now study (1.28) in detail. The left-hand side is less than or equal to

$$\begin{aligned} & P \left\{ \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e)| < \eta, \right. \\ & \quad \left. \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau)) + e'H_0(\tau)K_0e| \geq 2\eta \right\} \quad (\text{I}) \\ & + P \left\{ \inf_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau)) + e'H_0(\tau)K_0e| < 2\eta \right\}. \quad (\text{II}) \end{aligned}$$

For (I), we apply the following inequalities: for arbitrary real functions of  $\tau$ , say  $A(\tau)$  and  $B(\tau)$ ,

$$\inf_{\tau \in \mathcal{T}_\omega} |B(\tau)| - \inf_{\tau \in \mathcal{T}_\omega} |A(\tau)| \leq \sup_{\tau \in \mathcal{T}_\omega} ||B(\tau)| - |A(\tau)|| \leq \sup_{\tau \in \mathcal{T}_\omega} |B(\tau) - A(\tau)|.$$

This implies that term (I) is less than or equal to

$$P \left\{ \sup_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau)) + e'H_0(\tau)K_0e - e'S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e)| \geq \eta \right\}. \quad (1.29)$$

Using the definition of  $S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e)$ , we have

$$\begin{aligned} & e'S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e) \\ = & e'S_T^d(\tau, \beta_0(\tau) + T^{-1/2}K_0e) + T^{-1/2} \sum_{t=1}^T e'x_t \{F_t(x'_t\beta_0(\tau) + T^{-1/2}K_0x'_te) - \tau\}. \end{aligned}$$

This implies

$$\begin{aligned} & e'S_T(\tau, \beta_0(\tau)) + e'H_0(\tau)K_0e - e'S_T(\tau, \beta_0(\tau) + T^{-1/2}K_0e) \\ = & e'S_T^d(\tau, \beta_0(\tau)) - e'S_T^d(\tau, \beta_0(\tau) + T^{-1/2}K_0e) \\ & + e'H_0(\tau)K_0e - T^{-1/2} \sum_{t=1}^T e'x_t \{F_t(x'_t\beta_0(\tau) + T^{-1/2}K_0x'_te) - \tau\}. \end{aligned}$$

By Lemma 2, the difference of the first two terms is  $o_p(1)$  uniformly in  $\tau \in \mathcal{T}_\omega$ . The difference of the remaining two terms is also  $o_p(1)$  by a Taylor expansion. Hence the term (1.29) can be made arbitrarily small for large  $T$ . So can term (I). Term (II) is less than or equal to

$$P \left\{ \sup_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau))| > -2\eta + K_0 \inf_{\tau \in \mathcal{T}_\omega} e'H_0(\tau)e \right\}.$$

Because  $e'S_T(\tau, \beta_0(\tau))$  is stochastically equicontinuous, for any  $\epsilon > 0$ , there always exists a  $K'$  such that when  $T$  is large enough,

$$P \left\{ \sup_{\tau \in \mathcal{T}_\omega} |e'S_T(\tau, \beta_0(\tau))| > K' \right\} < \epsilon. \quad (1.30)$$

Meanwhile, because  $\inf_{\tau \in \mathcal{T}_\omega} e'H_0(\tau)e$  is bounded away from zero, we can choose a  $K_0$  such that

$$K_0 \inf_{\tau \in \mathcal{T}_\omega} e'H_0(\tau)e > K' + 2\eta. \quad (1.31)$$

Combining (1.30) and (1.31), term (II) can be made arbitrarily small with an appropriate choice of  $K_0$ .

**Lemma 4** Under Assumptions 1-6 and S1, we have

$$|\hat{H}(\tau) - H_0(\tau)| = o_p(1) \quad (1.32)$$

holds uniformly over  $\mathcal{T}_\omega = [\omega_1, \omega_2]$  with  $0 < \omega_1 < \omega_2 < 1$ .

**Proof.** The proof is similar to that of Theorem 3.3 in Wu and Zhou (2016), with the same partition notations as Lemma 2 in this appendix. Recall that  $\mathcal{T}_\omega$  is partitioned into  $N(\varepsilon_T)$  intervals of equal length:  $\omega_1 = \tau_0 < \tau_1 < \dots < \tau_{N(\varepsilon_T)} = \omega_2$ . Suppose  $\tau \in [\tau_{j-1}, \tau_j]$ , then

$$\begin{aligned} & |\hat{H}(\tau) - H_0(\tau)| \\ = & \left| \sum_{t=1}^T \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} - \sum_{t=1}^T \frac{f_t(x_t' \beta_0(\tau))x_t x_t'}{T} \right| \\ = & \left| \sum_{t=1}^T \left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} - E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right) \right. \\ & + \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) - E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \right) \right) \\ & \left. + \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \right) - \frac{f_t(x_t' \beta_0(\tau))x_t x_t'}{T} \right) \right| \\ \leq & \left| \sum_{t=1}^T \left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} - E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right) \right| \\ & + \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) - E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \right) \right) \right| \\ & + \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau)/c_T)x_t x_t'}{Tc_T} \right) - \frac{f_t(x_t' \beta_0(\tau))x_t x_t'}{T} \right) \right|. \end{aligned}$$

By the property of conditional expectation, it is easy to see that

$$H_0(\tau) = \sum_{t=1}^T E\left( \frac{f_t(x_t' \beta_0(\tau))x_t x_t'}{T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right).$$

Hence, for  $1 \leq j \leq N(\varepsilon_T)$ , we have

$$\begin{aligned}
|\hat{H}(\tau_j) - H_0(\tau_j)| &\leq \left| \sum_{t=1}^T \left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} - E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right) \right| \\
&\quad + \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) - E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \right) \right) \right| \\
&\quad + \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \right) - E\left( \frac{f_t(x_t' \beta_0(\tau_j) | \mathcal{F}_{t-1}, \mathcal{G}_t) x_t x_t'}{T} \right) \right) \right| \\
&= H^M(\tau_j) + H^N(\tau_j) + H^O(\tau_j),
\end{aligned}$$

where

$$\begin{aligned}
H^M(\tau_j) &= \left| \sum_{t=1}^T \left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} - E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right) \right|, \\
H^N(\tau_j) &= \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) - E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \right) \right) \right|, \\
H^O(\tau_j) &= \left| \sum_{t=1}^T \left( E\left( \frac{\phi(\hat{e}_t(\tau_j)/c_T)x_t x_t'}{T c_T} \right) - E\left( \frac{f_t(x_t' \beta_0(\tau_j) | \mathcal{F}_{t-1}, \mathcal{G}_t) x_t x_t'}{T} \right) \right) \right|.
\end{aligned}$$

Therefore, to complete the proof, it is sufficient to show that  $H^M(\tau_j)$ ,  $H^N(\tau_j)$  and  $H^O(\tau_j)$  are  $o_p(1)$  uniformly in  $\tau_j \in \mathcal{T}_\omega$ .

The supremum of  $H^O(\tau_j)$  is less than or equal to  $O(c_T^2) = o_p(1)$  (equation 53 in Wu and Zhou, 2016). By Doob's inequality, Jensen's inequality and Theorem 3.3 in Wu and Zhou (2016), we have that

$$\left\| \max_{1 \leq j \leq N(\varepsilon_T)} |H^M(\tau_j)| \right\| \leq C \left( \sum_{t=1}^T \left\| \frac{\phi(e_t/c_T)x_t x_t'}{T c_T} \right\|^2 \right)^{1/2} = O(T^{-1/2} c_T^{-1/2}) = o_p(1).$$

By the triangle inequality and Doob's inequality,

$$\begin{aligned}
& \left\| \max_{1 \leq j \leq N(\varepsilon_T)} |H^N(\tau_j)| \right\| \\
& \leq \sum_{l=0}^{\infty} \left\| \max_{1 \leq j \leq N(\varepsilon_T)} \left| \sum_{t=1}^j \mathcal{P}_{t-l} E \left( \frac{\phi(\hat{e}_t(\tau_j)/c_T) x_t x'_t}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right| \right\| \\
& \leq C \sum_{l=0}^{\infty} \left\| \sum_{t=1}^j \mathcal{P}_{t-l} E \left( \frac{\phi(\hat{e}_t(\tau_j)/c_T) x_t x'_t}{T c_T} \middle| \mathcal{F}_{t-1}, \mathcal{G}_t \right) \right\|.
\end{aligned}$$

where  $\mathcal{P}_i(\cdot) = E(\cdot | \mathcal{F}_{i-1}, \mathcal{G}_i) - E(\cdot | \mathcal{F}_{i-2}, \mathcal{G}_{i-1})$ . Note that for  $l \leq 0$ ,

$$\begin{aligned}
& \mathcal{P}_{t-l}(\phi(\hat{e}_t(\tau)/c_T) x_t x'_t) \\
& = \mathcal{P}_{t-l}(E(\phi(\hat{e}_t(\tau)/c_T) | \mathcal{F}_{t-1}, \mathcal{G}_t)) x_t x'_t \\
& = c_T \mathcal{P}_{t-l} \left( \int \phi(y) f_t(c_T y | \mathcal{F}_{t-1}, \mathcal{G}_t) dy x_t x'_t \right).
\end{aligned}$$

As in Lemma 1 in Wu (2007) and using the triangle inequality, we have that  $\|\mathcal{P}_{t-l} \phi(\hat{e}_t(\tau)/c_T) x_t x'_t / c_T\| = O(\chi^l)$ . Combining the equations above, we have that  $\|\max_{1 \leq j \leq N(\varepsilon_T)} |H^N(\tau_j)|\| = O_p(T^{-1/2})$ . The order of  $|\hat{H}(\tau) - H_0(\tau)|$  is  $O_p(T^{-1/2} c_T^{-1/2} + T^{-1} c_T^{-3} \log^2 T + c_T^2) = o_p(1)$ .

This lemma is different from Theorem 3.3 in Wu and Zhou (2016) which holds for a fixed quantile. The  $o_p(1)$  order holds uniformly over  $\mathcal{T}_\omega$  under the stronger assumptions S1. This completes the proof. ■

**Proof of Proposition 1.1:** We have already proved that  $Z_T(\tau) = \sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) = O_p(1)$  uniformly in  $\tau \in \mathcal{T}_\omega$ . Thus, it is sufficient to establish the result for  $Z_T(\tau)$  satisfying  $\|Z_T(\tau)\| \leq K$ , where  $K$  is some finite constant. Let  $\delta$  be a generic  $p$ -vector satisfying  $\|\delta\| \leq K$ . Then, following Gutenbrunner and Jurečková (1992),

we have

$$\begin{aligned}
& S_T(\tau, \beta_0(\tau) + T^{-1/2}\delta) \\
= & S_T^d(\tau, \beta_0(\tau) + T^{-1/2}\delta) - S_T^d(\tau, \beta_0(\tau)) \quad (e) \\
& + T^{-1/2} \sum_{t=1}^T x_t \{F_t(x'_t\beta_0(\tau) + T^{-1/2}x'_t\delta) - \tau\} \quad (f) \\
& + S_T(\tau, \beta_0(\tau)).
\end{aligned}$$

For term (e), Lemma 2 implies that it is  $o_p(1)$  uniformly in  $\|\delta\| \leq K$  and  $\tau \in \mathcal{T}_\omega$ . For term (f), the mean value theorem implies

$$(f) = T^{-1/2} \sum_{t=1}^T f_t(x'_t\beta_0(\tau) + T^{-1/2}x'_t\delta^*)x_t x'_t$$

with  $\|\delta^*\| \leq \|\delta\|$ . Because  $\max_{1 \leq t \leq T} T^{-1/2} \|x_t\| = o_p(1)$ , we have

$$f_t(x'_t\beta_0(\tau) + T^{-1/2}x'_t\delta^*) = f_t(x'_t\beta_0(\tau)) + o_p(1)$$

uniformly. Hence,

$$(f) = T^{-1/2} \sum_{t=1}^T f_t(x'_t\beta_0(\tau))x_t x'_t + o_p(1)$$

uniformly in  $\|\delta\| \leq K$  and  $\tau \in \mathcal{T}_\omega$ . Combining the above results, we have

$$S_T(\tau, \beta_0(\tau) + T^{-1/2}\delta) = S_T(\tau, \beta_0(\tau)) + H_0(\tau)\delta + o_p(1),$$

which holds uniformly in  $\|\delta\| \leq K$  and  $\tau \in \mathcal{T}_\omega$ . The result follows because  $S_T(\tau, \beta_0(\tau) + T^{-1/2}\delta) = o_p(1)$  when  $\delta = Z_T(\tau)$ . ■

**Proof of Theorem 1.1.** By Proposition 1.1,

$$S_T(\tau, \hat{\beta}(\tau)) = S_T(\tau, \beta_0(\tau)) + H_0(\tau)\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) + o_p(1).$$

Because

$$S_T(\tau, \hat{\beta}(\tau)) = o_p(1),$$

we obtain

$$\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) = -H_0(\tau)^{-1}S_T(\tau, \beta_0(\tau)).$$

Because  $S_T(\tau, \beta_0(\tau))$  is stochastically equicontinuous by Lemma 1, we have

$$-S_T(\tau, \beta_0(\tau)) \Rightarrow U(\tau),$$

where  $U(\tau)$  is a zero mean  $p$ -dimensional continuous Gaussian process. Therefore,

$$\sqrt{T}(\hat{\beta}(\tau) - \beta_0(\tau)) \Rightarrow H_0(\tau)^{-1}U(\tau).$$

This completes the proof. ■

**Proof of Corollary 1.1.** Denote the level  $(1 - \alpha)$  uniform confidence bands for  $\beta_j(\tau)$  as  $C_\alpha$ , then

$$\begin{aligned} & P(\beta_j(\tau) \notin C_\alpha \text{ for some } \tau \in [\omega, 1 - \omega]) \tag{1.33} \\ &= P\left(\beta_j(\tau) \notin [\hat{\beta}_j(\tau) - T^{-1/2}s_{0,j}(\tau)E_{(\lfloor(1-\alpha)B\rfloor,j)}, \hat{\beta}_j(\tau) + T^{-1/2}s_{0,j}(\tau)E_{(\lfloor(1-\alpha)B\rfloor,j)}]\right) \\ &= P\left(\frac{T^{1/2}|\beta_j(\tau) - \hat{\beta}_j(\tau)|}{s_{0,j}(\tau)} > E_{(\lfloor(1-\alpha)B\rfloor,j)} \text{ for some } \tau \in [\omega, 1 - \omega]\right) \\ &= P\left(\sup_{\tau \in [\omega, 1 - \omega]} \frac{T^{1/2}|\beta_j(\tau) - \hat{\beta}_j(\tau)|}{s_{0,j}(\tau)} > E_{(\lfloor(1-\alpha)B\rfloor,j)}\right). \end{aligned}$$

Note that  $\hat{H}^{-1}(\tau)$  converges in probability uniformly to  $H^{-1}(\tau)$  and  $\Psi_m(\tau)$  converges weakly to the same zero mean Gaussian process as does  $U(\tau)$ . As a result,  $s_{0,j}(\tau)$  converges to the standard deviation of  $[H_0(\tau)^{-1}U(\tau)]_j$ , where  $[\cdot]_j$  stands for the  $j$ -th element of the vector inside the brackets. Denote it by  $\sigma_{0,j}(\tau)$ . Also,  $E_{(\lfloor(1-\alpha)B\rfloor,j)}$

converges to  $(1 - \alpha)$  percentile of

$$\sup_{\tau \in [\omega, 1-\omega]} \left| \frac{[H_0(\tau)^{-1}U(\tau)]_j}{\sigma_{0,j}(\tau)} \right|.$$

Denote this quantile by

$$F_{(\lfloor(1-\alpha)B\rfloor, j)}.$$

Combing these two results, the last line of (1.33) is equal to

$$P \left( \sup_{\tau \in [\omega, 1-\omega]} \frac{T^{1/2} |\beta_j(\tau) - \hat{\beta}_j(\tau)|}{\sigma_{0,j}(\tau)} > F_{(\lfloor(1-\alpha)B\rfloor, j)} \right) + o(1),$$

which converges to  $\alpha$  by Proposition 1.1.

## Chapter 2

# Heterogeneity Effect of Development Funding on Micro-enterprises: Evidence from a Field Experiment in Sri Lanka

This chapter analyzes the heterogeneity of firm characteristics on returns to capital. It develops a theoretical model under a utility maximization framework with imperfect insurance and credit markets constraints. From the model, the returns to capital are derived as a function of the parameters, which affects the production function of the firm and the entrepreneur utility form. Quantile regression is applied to analyze the field experiment data from the Sri Lanka Micro Enterprises Project (2005-2010). Empirical evidence shows that returns vary across different quantiles of firm profits. Further, the ability/risk aversion of entrepreneurs affect the returns differently at different quantiles.

### 2.1 Introduction

As a potential source of growth and employment, micro-enterprises are essential components of development funding. Micro-finance institutions and nongovernmental organizations (NGOs) have recently become the most common source of household enterprises financing with more than 70 million clients worldwide. A question for policymakers in development funding channels is how these micro-enterprises make investment decisions and what characteristics of entrepreneurs affect their profits. Broad

field experiments (De Mel, McKenzie, and Woodruff, 2008 (henceforth DMW); Banerjee and Duflo, 2006) show evidence that the return to capital among micro-enterprises is much higher than market interest rate. However, little empirical evidence has been found on the choice of optimal scales and timing of interventions as well as the evaluation of riskiness from the perspective of policymakers and NGOs in the developing countries.

The previous literature on micro-enterprises in developing countries can be categorized as having three perspectives: i) the form of production function; ii) the estimation of the returns to capital; iii) the implications for the functioning of markets. Akerberg, Benkard, Berry, and Pakes (2007) pointed out the absence of plausible instruments with substantial inter-firm variation, while the approach in DMW advances the related literature on instrumental variables estimates of production functions by giving randomly selected firms lump sums of cash or physical materials or capital. They use these random grants as instruments for capital in the production function. Banerjee and Duflo (2009) showed that with perfectly functioning capital markets, all firms should have the same risk-adjusted return to capital. Estimating the extent and sectors in which this prediction does not hold true may then inform us of the extent of capital market imperfections in the broader economy.

The theoretical literature on the poverty trap postulates that entrepreneurs might remain inefficiently small for some period of time, but would be able to grow by reinvesting profits. How to maintain a longer-range future for micro-enterprises is critical for policy designers given the rapid urbanization of developing countries. Tracking the growth pattern of small-scale entrepreneurs provides quantitative analysis on the best timing and amount for development funding to support. Therefore by investigating the growth path of returns to capital generated by the random investment, we find that the returns to development funding vary across micro-enterprises in the Sri

Lanka field experiment.

There is a broad literature on heterogeneous return to human capital (Tazeen Fasih et al., 2012). However, due to the complexity of capital markets, there are few papers concentrating on the heterogeneity of returns to physical capital. In contrast with the previous literature, we investigate how entrepreneurs make investment decisions and operate micro-enterprises in developing countries. I concentrate on the heterogeneity of micro-enterprises, such as the initial capital endowment, the entrepreneur's ability and risk awareness.

This chapter measures the effects caused by heterogeneity and evaluates the investment intervention. How nongovernmental organizations (NGOs) and private/state-run financial institutions can best design cost-effective interventions for micro-enterprises is relevant to improving access to credit and offering more investment opportunities in developing countries. Therefore this chapter explores the optimal scale of capital stock for interventions in order to determine the peak effectiveness of investment funding policy.

To measure the treatment effects and test its consistency, I apply quantile regression and find differences in treatment effects at different quantiles of the distribution of the variable of interest. As one step further based on DMW (2008), I look into the quantiles and sub-samples of the field experiments. The results are different from DMW regarding the actual returns to capital. Quantile difference is not negligible and the differences between quantiles remain significant.

From the perspective of micro-enterprises characteristics, there is a long list of distorting factors affecting the growth of micro-enterprises, including government policies, credit market failures, intro-family inefficiencies, learning externalities, and even behavioral factors. It is hard to conclude that a particular distortion has resulted in a significant loss in productivity even if the *prima facie* evidence suggest it is

the strongest (Banerjee and Duflo, 2004). It becomes an essential issue about how micro-enterprises can use development funding efficiently and what kind of policy microfinance institutions should pursue.

From an econometric standpoint, the panel data embeds different firm profits in different time periods. Different firms benefit from a certain amount of increase in capital stock differently due to the heterogeneity of firms and entrepreneur characteristics (Karlan and Zinman, 2010). To separate these heterogeneity effects can help: 1) sharpen the comparison between long-run and short-run effects of a capital shock for a single average firm from the panel data set; 2) clarify the character of firms which benefit most from such a capital shock or obtain higher profit increment from the inventions. Therefore this chapter introduces quantile methodology in analyzing the heterogeneity effects in development funding.

Evaluating the micro-enterprises before issuing funding is essential for micro-finance organizations. It provides better assistance to entrepreneurs in order to set up longer-term improvements. The education level, the intellectual ability and the genders of the entrepreneurs affect the management of the micro-enterprises. Risk aversion of entrepreneurs and uncertainty of projects are main factors characterizing the riskiness of enterprises. How these characteristics affect the returns to capital offers specific characteristics of the desirable quality of entrepreneurs for policymakers. The characteristics corresponding to higher return rate are attractive to lenders in practice. This chapter also shows evidence of nonlinear effects of risk aversion and indicates uncertainty in the imperfect financial markets.

As is pointed by Karlan and Morduch (2009), one way in which access to funding may promote economic development is by providing some poor individuals the opportunity to set up their own businesses. Evaluating the micro-enterprises before issuing funding is essential for micro-finance organizations and better helps entrepreneurs set

up longer-term improvements, as well as providing feedback to micro-finance policies applied to imperfect markets.

### **2.1.1 Microfinance in Sri Lanka**

The micro-finance movement in Sri Lanka dates as far back as 1906 with the establishment of Thrift and Credit Co-operative Societies (TCCSs) under the Co-operative Societies Ordinance introduced by the British colonial administration. Following the tsunami which struck Sri Lanka in 2004, there was an influx of foreign aid to the country, of which a substantial amount was channeled to the micro-finance sector. While many donors worked through established microfinance institutions, some funded the establishment of multi-sectoral livelihood programs which included micro-finance components. These were largely unsustainable in the long-term. They had some detrimental effects on the sector in the short term. These effects came through their mix of grants and subsidized loans and might result in damages to the established credit intuition.

There is a recently emerging trend: the entry of commercial banks and registered finance companies and other large corporate entities into the microfinance business. For many commercial banks and finance companies in the emerging markets, micro-finance is more a Corporate Social Responsibility (CSR) or image building activity. The absence of a cohesive regulatory and supervisory system for the micro-finance sector is one of the barriers to the future growth of the sector. The methods and standards of supervision vary widely. The absence of one single regulatory and supervisory authority has resulted in missing uniform standards and inefficient development paths.

In Sri Lanka, Micro-Finance Institutions (MFIs) are required to obtain a license and are expected to meet certain capital requirements depending on their scales of operation under the MFI Act proposed by the Central Bank of Sri Lanka (CBSL).

Micro-finance is currently classified as a money lending business. It is restricted from obtaining offshore equity investment into such business. This has a negative impact on the larger, better performing, unregulated MFIs which cannot scale up operations through offshore equity capital.

The attempt to introduce a regulatory and supervisory system for the micro-finance sector has been going on for a number of years. There are concerns over some provisions in the MFI Act released by CBSL, which has been withheld for restructuring and amendment. This chapter is written from the perspective of regulating and supervising microfinance development given the current state of regulation and supervision in Sri Lanka.

This chapter is structured as follows. Section 2.2 outlines the theoretical model of the household/micro-enterprise problem in the setting of an imperfect credit market. The model serves as the foundation of the econometric regression and endogenously generates the heterogeneity. Section 2.3 applies quantile regression to the DMW data set and discusses the returns to capital in terms of different quantiles of the sample. The empirical econometric model, based on the structure of the theoretical model, studies the effect of entrepreneur characteristics towards the capital shock on micro-enterprise profits. Section 2.4 connects the empirical results with policy analysis, especially focusing on the current situation in developing countries such as Sri Lanka. Section 5 concludes and provides an outlook of potential future work.

## 2.2 Theoretical Model

### 2.2.1 Model Setup

The baseline theoretical model is a one-period household model. Household  $H$  has a constant relative risk aversion (CRRA) utility function:

$$U(C, \bar{L} - L) = (C^{1-\gamma} - 1)/(1 - \gamma) + (\bar{L} - L)^\theta \quad (2.1)$$

$H$  has  $\bar{L}$  hours to allocate between operating the micro-enterprise and leisure. For simplicity, assume there is no outside labor market and the household does not hire employees.  $H$  has an initial asset holding  $A$  and can borrow  $B$  from the formal credit market. There is a borrowing limit  $\bar{B}$  for the household. The utility function is separable in consumption and leisure.  $\gamma$  measures the relative risk aversion coefficient of the household. There is a production function of a single technology in the traditional Cobb-Douglas form:

$$f(K, L, \theta) = \theta K^\alpha L^\beta \quad (2.2)$$

$K$  is the capital amount that household invests in his small business.  $\theta$  represents the technology shock and  $\alpha + \beta$  measures the returns to scale level of the micro-enterprise. The household also faces a production risk  $\epsilon$  and receives  $\epsilon f(K, L, \theta)$  from production.  $\epsilon$  is a random variable with mean 1 and standard deviation  $\eta$ . In addition, we make the standard assumption that there are a fixed number of households in this economy. In addition, each micro-enterprise needs one household to operate and manage it. The market rate of return to capital is  $r$ .

### 2.2.2 Household Problem

The household problem is to optimally choose the amount of capital stock  $K$ , financed through both formal credit market and internal household capital market, the labor  $L$  devoted to the micro-enterprise. The maximization problem is

$$\max E[U(C, \bar{L} - L)], \text{ s.t. } C = \epsilon f(K, L, \theta) - rB, K = A + B, B \leq \bar{B}, L \leq \bar{L} \quad (2.3)$$

$A$  is given as the household internal net asset holding.  $B$  is the amount of borrowing from the formal credit market, paid back at the market rate  $r$ . The borrowing constraint is exogenously determined by the credit market. The first order conditions can be derived from the method of Lagrange multipliers.

$$f_K(K, L, \theta) = \frac{1}{1 + Cov(U_C, \epsilon)/E[U_C]} \left( r + \frac{\lambda}{E[U_C]} \right) \quad (2.4)$$

$$f_L(K, L, \theta) = \frac{\sigma(\bar{L} - L)^{\sigma-1} + \mu}{E[\epsilon \times U_C]} \quad (2.5)$$

Define  $\lambda > 0$  as the Lagrange multiplier. It represents the tightness of the overall credit market constraints. It is the shadow cost of capital from the credit market borrowing. Based on the first order conditions above,  $\lambda$  depends on the household initial asset  $A$  and the production function form. Note that  $E(\epsilon) = 1$  means that the production function is a one-to-one mapping with all risks hedged.

**(I) Perfect Insurance Market with a Missing Credit Market** Under the assumption of a perfect insurance market with a missing credit market, production risks can be fully insured by insurance,  $\epsilon = 1$ . The model indicates that the returns to capital are implicitly a nonlinear function of household's initial wealth, labor supply and the productivity properties of the micro-enterprise.

Consider the simple case with a binding borrowing constraint. The household will use all they can borrow to invest in the capital. The tightness of the overall credit market constraint is strictly positive as there is a strict restriction for the household to borrow in the credit market. The higher the credit market tightness measure, the more restrictive the credit market.

Assume there is an interior solution for the optimal labor supply. The household allocates labor and leisure when the marginal benefit from working equals to the marginal cost of forgoing leisure. In this case the equilibrium can be solved explicitly. It can be shown that the capital investment is a function of the initial wealth, given the constraint in the credit market.

The characteristics of the entrepreneur, such as risk aversion and labor-leisure

elasticity, affect the household (as an entrepreneur) when making labor decision. This also relates to the tightness measure of the credit market. Such underlying heterogeneity is apparent in the labor-leisure equality. It has a nonlinear effect on the household decision of how much to work.

**(II) Perfect Credit Market with Perfect Insurance Market** When the borrowing constraint is not binding, we have a perfect credit market with the Lagrange multiplier zero. Assume the household decides the labor supply based on the capital investment and that the capital-labor ratio is constant in equilibrium. This assumption implies that the household effort depends on the scale of the firm.

Large firms have more capital endowment. They also require more labor inputs. A linear labor supply in terms of capital implies, as in the ideal case with Cobb-Douglas production function, that labor and capital have an equal impact on production. In a constant returns to scale production function, the log of the capital return is a linear function of log of capital. Without this parameter condition on the returns to scale, such a linear relationship no longer holds.

The capital/labor ratio is independent of the production technology. In a reduced log form, heterogeneity of household characteristics, such as entrepreneur ability, initial wealth and risk aversion, affect the labor and capital investment so as to the returns. We can see that even in a perfect insurance and credit market, heterogeneity of households influences the capital accumulation in a nonlinear way. The underlying heterogeneity includes the household initial wealth, the entrepreneur ability (education, risk aversion and gender) as well as technology shocks.

With increasing returns to scale in the production function, a high level of technology does not contribute more to the capital compared with a low level of technology initially. This can be explained by the higher cost of equipment and facility investment.

With constant returns to scale (CRS), entrepreneur characteristics and technology have no further effect on the capital accumulation as well as the returns to capital. A CRS production function would generate a constant treatment effect to the capital shock.

With decreasing returns to scale, heterogeneity of advantageous characteristics has a positive effect on the capital accumulation. The market rate of return has a negative effect on capital.

**(III) Perfect Credit Market with Missing Insurance Market** In a missing insurance market with perfect credit market, the riskiness random variable  $\epsilon$  and the risk aversion coefficient  $\gamma$  affect the equilibrium conditions. The exact form of the first order condition requires further information about the moments of the  $\epsilon$  distribution.  $X$  is a linear transformation of the riskiness random variable and  $n$  relates to the risk aversion of the entrepreneur.

$$f_K(K, L, \theta) = \frac{E(U_C)}{E(\epsilon_C)} r = rE(X^n)/E(X^{n+1}) \quad (2.6)$$

Suppose  $X$  has a normal distribution with mean  $f - rK + rA > 0$ , the moment generating function of  $X$  is an exponential function of the mean and variance. Therefore the effect of riskiness on returns to capital is approximately an exponential function under a normal distribution assumption. Regressing on a linear component of riskiness underestimates the heterogeneity effect, leading to insignificant results. Suppose  $X$  has an exponential distribution, based on the moment generating function, the effect of riskiness on returns to capital can be estimated by the multiplication of riskiness and the inverse of household risk aversion. In both cases, the covariance between marginal utility and production risk is more relevant in explaining the imperfectness of insurance market.

## 2.3 Empirical Analysis

Based on the theoretical analysis, the dependent variables in the econometric model are determined. The model aims at the individual firm level. It does not take the group effects into account. It avoids peer effects and learning benefits for simplicity.

First, it suggests the heterogeneity of returns among households might come from the inconstant returns to scale in the production function or the tightness of the credit market. Constant return to capital is based on the assumption of a constant returns to scale production function and a perfect credit market. OLS IV coefficient estimates the average returns to scales while quantile regression tells the median, quarter and even 10th percentile estimators of returns. Quantile regression offers the distribution of returns in terms of profit quantiles and tests the return to scale of the sample.

Second, the regression is in log forms and linear to initial wealth. Heterogeneity effects are in terms of different quantiles of initial profits. Take the log of profits and regress on the IV treatment variable as well as the characteristics of firms. The initial wealth of the entrepreneur is proportional to the initial capital endowment. The baseline model takes the initial profit of the firm as the capital endowment.

Third, the underlying heterogeneity includes household labor supply, household ability to make decision, household risk aversion and labor-leisure evaluation. There is no explicit formula for the effects in the model while it suggests these parameters affect the profit simultaneously. The quantile regression model includes the number of waged workers in the firm, the ability, risk aversion and education, gender of the entrepreneur.

Last but not least, it suggests a test of imperfect insurance market by adding exponential terms of riskiness and interacting measures of the inverse of risk aversion with uncertainty of business returns. I also add the intersection of treatment instrument and the entrepreneur characteristics. This separates the heterogeneity effects

between the treatment group and the control group. By comparing the differences in the corresponding coefficients, we can learn the propagation of capital investment from different characteristics of micro-enterprises.

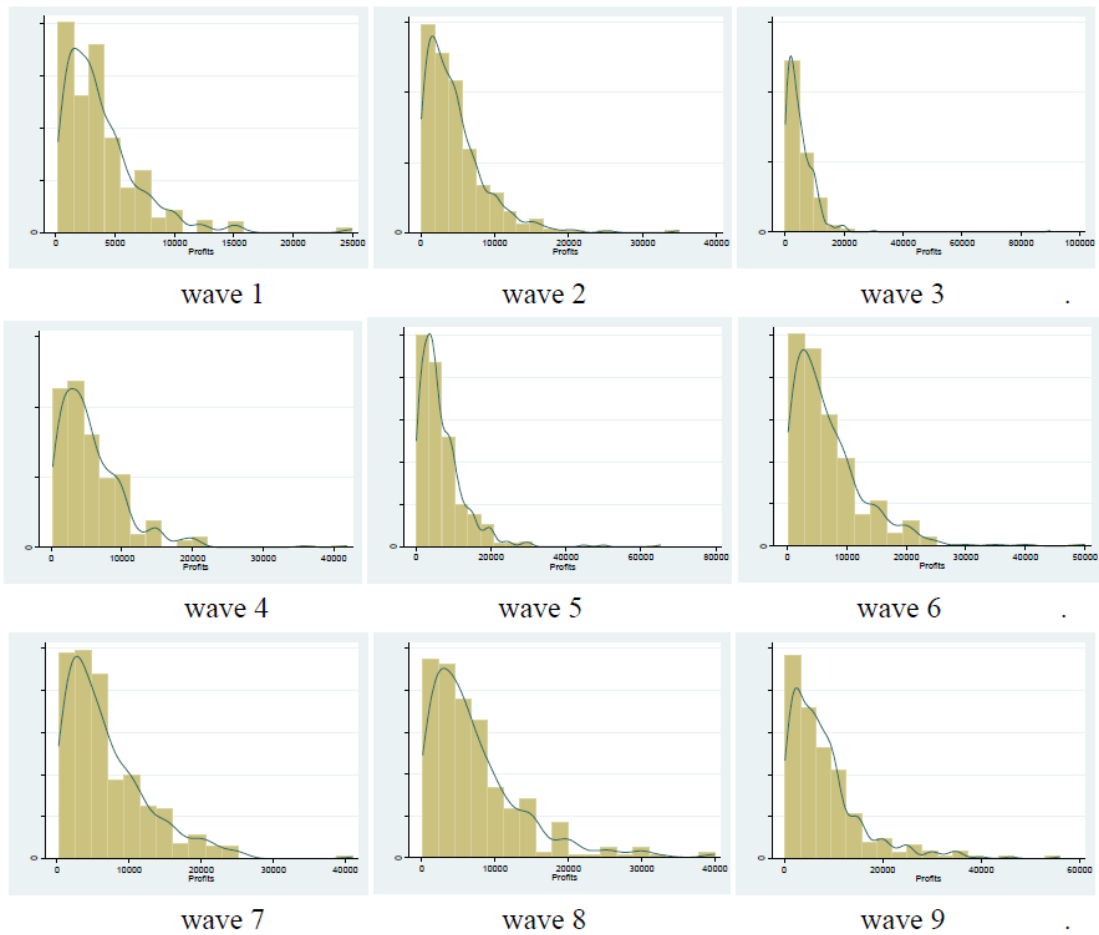
The empirical analysis is based on the unique data set collected from the DMW field experiment. The field experiment is carried out in Sri Lanka. The analysis of the Sri Lanka Micro-enterprise Project 2005-2007 aims to help development funding in the emerging markets.

### **2.3.1 Data**

The data are based on the random field experiment carried in Sri Lanka. They were originally collected from Sri Lanka Micro-enterprise Project (2005-2010) by De Mel, McKenzie and Woodruff. This field experiment was originally designed to study the returns to capital in micro-enterprises and the process of recovery of micro-enterprises from the tsunami in December 2004, carried out in three Western and Southern districts of Sri Lanka: Kalutara, Galle and Matara. The baseline survey was carried out in April 2005, with micro-enterprises re-interviewed quarterly.

The door-to-door screening survey of households was used to identify enterprises with invested capital of 100,000 Sri Lanka Rupees (LKR) (about 100 USD) or less, excluding investments in land and buildings. There are 618 enterprises in retail trade and manufacturing operated by owners aged from 22 to 65, and with no paid employees. The micro-enterprises include common self-employment activities such as running small grocery stores, selling tea, food preparation, sewing clothes, making lace products, and coir production. They cover a variety of small-scale activities in many developing countries.

To rule out the effect of the 2004 tsunami, this paper focuses only on the 408 enterprises located away from the boundary. The selected enterprises are either in industries less affected by the weather or based on the geographic location. The lack



**Figure 2-1:** Distribution of the Sample Profit during the Experiments

of record keeping suggests that reported profits may be subject to a range of different types of measurement errors. However, though the lack of record keeping is a general phenomenon among small businesses in developing countries, there are still ways to determine what can be done to elicit reasonable information on profits from small firm owners.

From the sample profit distribution of nine waves in the experiment starting from April, 2005 to April, 2007, we can postulate the effects of capital investment shocks on profits are different for different scales of micro-enterprises in terms of profits. Wave 1 is the baseline sample and the histograms include all the available observations. The

treatment effects are hard to detect from the figure.

There are unbalanced growth rates across different sizes of the firms. In addition, the sample is not normally distributed in profits. Rather than the OLS regression, it is better to choose quantile regression in analyzing the returns of capital. Tracking the capital accumulation process across different firms in terms of profit quantiles helps establish a cost effective policy for development funding.

The median owner in the sample is 41 years old and has 10 years of education. The randomized sample is almost equally divided between male and female owners. The coefficient of relative risk aversion comes from a lottery B game collected in the second wave of the survey. Respondents were asked whether they would choose a certain payoff of 40 LKR (about two hours of mean reported earnings) or a gamble with payoffs of 10 or 100 LKR. The CRRA is calculated using the switch over point from the certain payoff to the gamble. The data were randomized so any differences between the treatment and control groups are purely due to chance.

From the comparison between the density of profits and corresponding capital stock, we see the benefits from investment are not a monotone linear function of the capital stock. Initially, there appear to be increasing profits driven by capital increment while those effects diminish as firm size enlarges. From the theoretical model, there is an optimal proportion of incremental investment to the capital stock when the enterprises are on its best growth path. Therefore we apply the quantile regression method to obtain the heterogeneity effects in different quantiles of profits and generate the distribution of effects on returns to capital.

### **2.3.2 Quantile Regression**

Quantile regression is applied in order to investigate the relationship between capital stock shocks and a set of predictors, such as: the number of waged workers, entrepreneurial ability, risk aversion, education, and gender. Quantile regression fo-

cuses on answering the question: how does the capital investment shock affect micro-enterprises profits at different quantiles?

This section discusses the distribution of the heterogeneity effects of the predictors across firms. The response variable is distributed based on the profits of the micro-enterprises in the pre-treatment baseline survey. Quantile regression specifies changes in the quantiles of the response variable. It also provides evidence of the heterogeneity of treatment effects separately from the treatment effects of the returns from the scale of the production function form.

Based on the theoretical model, the predictors reflect the heterogeneity of micro-enterprises characteristics. Variables and explanations in details are listed as follows:

1) Evertreat: A dummy variable representing the random capital shock in the first wave of the survey. As the property of randomization in the field experiment, it is used as an instrument variable (IV) of capital investment shock. The coefficient of this IV measures the treatment effect of returns to capital.

2) Number of workers: The number of paid wage workers in the household in the pre-treatment baseline survey. It reflects the labor supply of the households and is negatively correlated with wages. The elasticity of labor supply is also considered in the extension of the model.

3) Ability of the entrepreneur: Maximum digit span measurement follows Djankov et al. (2005) and represents the score of the entrepreneur on a forward digit span recall test. Households were shown a three digit number. The card showing the number was then taken away. Ten seconds later, respondents were asked to repeat the number as written on the card. Those responding correctly were shown a four digit number, and so forth up to 11 digits. The mean digit span recalled was 5.9 digits.

4) Relative Risk Aversion: This is estimated by lottery B and is collected from

experiments played with real money with each firm owner. Firm owners were given the choice between 40 LKR (1 US dollar= 128.55 Sri Lanka rupees) for certain, or a gamble with  $x$  percent chance of 10 rupees and  $(100-x)$  percent chance of 100 rupees. A 10-sided dice was used to vary odds of the higher payment from 10 percent up to 90 percent. The probability threshold at which an individual switches from the safe payment to the risky gamble provides a measure of risk aversion. The midpoint of the implied CRRA interval is used as the measure of risk aversion.

5) Years of education of entrepreneur: This is the most standard characteristics of the enterprise owner (Paulson and Townsend 2004). The average years of education are 9.0 years with a standard deviation of 3.1 years.

6) Gender: The gender of the entrepreneur. The proportion of female entrepreneurs is 0.491 in the baseline data set.

In order to estimate the heterogeneity effects in quantiles, we write the regression model of the following form:

$$y_{it} = \alpha_i + \beta_i S_{it} + z_{it} D_{it} + \gamma_t + \mu_{it} \quad (2.7)$$

$y_{it}$  is the real profit in log of enterprise  $i$  at wave  $t$ .  $S_{it}$  is a vector gathering other determinants of firm profits. These determinants include the number of waged workers and characteristics of entrepreneurs, such as gender, age, years of education, and the relative risk aversion coefficient.  $D_{it}$  is the treatment (capital shock) status of enterprise  $i$  at wave  $t$ .  $D_{it} = 1$  indicates that the enterprise is receiving a random increase in capital stock in the field experiment.

Note that Table 2.1 is based on the baseline survey. Real profits and total capital are in Sri Lanka rupees.

$\alpha_i$  is the fixed effect at the micro-enterprise level. It accounts for the unobserved micro-enterprise characteristics.  $\beta_i$  is a vector of the corresponding effects, represent-

**Table 2.1:** Micro-enterprises Statistics Summary

Baseline Survey (Round 1) & Total Data Set							
Characteristic	obs.	25%	50%	75%	mean	obs.	median
Real profits	391	1500	3000	5000	3850	3308	4063
Total capital	408	36041	81500	16041	146805	3216	89787
# Workers	408	0	1	1	0.699	3672	1
Age	408	32	41	50	41.833	3672	41
Digit span	377	5	6	7	5.598	3393	6
Lottery B CRRA	403	-1.48	0.065	1.59	0.143	3672	0.065
Gender	387	0	0	1	0.491	3483	0
Education	408	8	10	11	9.053	3672	10

ing firm-specific heterogeneity effects of the observed entrepreneur characteristics. It measures the development quality of the micro-enterprises.  $z_{it}$  is the treatment effect which can be interpreted as the returns to capital.  $\gamma_t$  is the fixed effect at the wave level.  $\mu_{it}$  is the error term.

This model can be interpreted as a production function of micro-enterprises. The production technology is represented by firm-specific characteristics, which are assumed to remain constant between waves. Since the capital shock is carried out as a random capital/cash prize in this field experiment, these parameters are uncorrelated with the treatment status.

First difference methodology is not applicable to this particular data set. Because the level of profits for these micro-enterprises are comparatively small and the progress of development is small as well. Therefore we use a fixed effect quantile regression model to see the different effects across distributions. The fixed effects include the firm level and wave level.

A common concern when estimating any type of production function is that there can be feedback effects on the choice of inputs. The chosen independent variables gather determinants of profits that present between-wave variation while the gender of the entrepreneur, the age of the entrepreneur, education level, and the hourly wage rate paid to the household stays unchanged in such a short term period. To allow for heterogeneity and avoid other noise, we focus on enterprises at the first 3 waves in

the field experiment.

### 2.3.3 Results

The results of the quantile regression are shown in Table 2.2 and Figure 2.2. The quantile regression coefficients are changing for different quantiles in Table 2.2. Compared with OLS results, quantile regression suggests the treatment effects vary differently at different quantiles. The returns to capital at the 0.25 and 0.75 quantiles are significantly positive while the median and both tails are not significant from zero.

For micro-enterprises, the number of workers hired has a negative impact on the real profits regardless of the quantiles. The results from quantile regression show that there is a significant negative impact of the entrepreneur ability for the 0.75 quantile. Female entrepreneur leadership affects micro-enterprise returns consistently and entrepreneurial risk awareness has a positive impact on their growth.

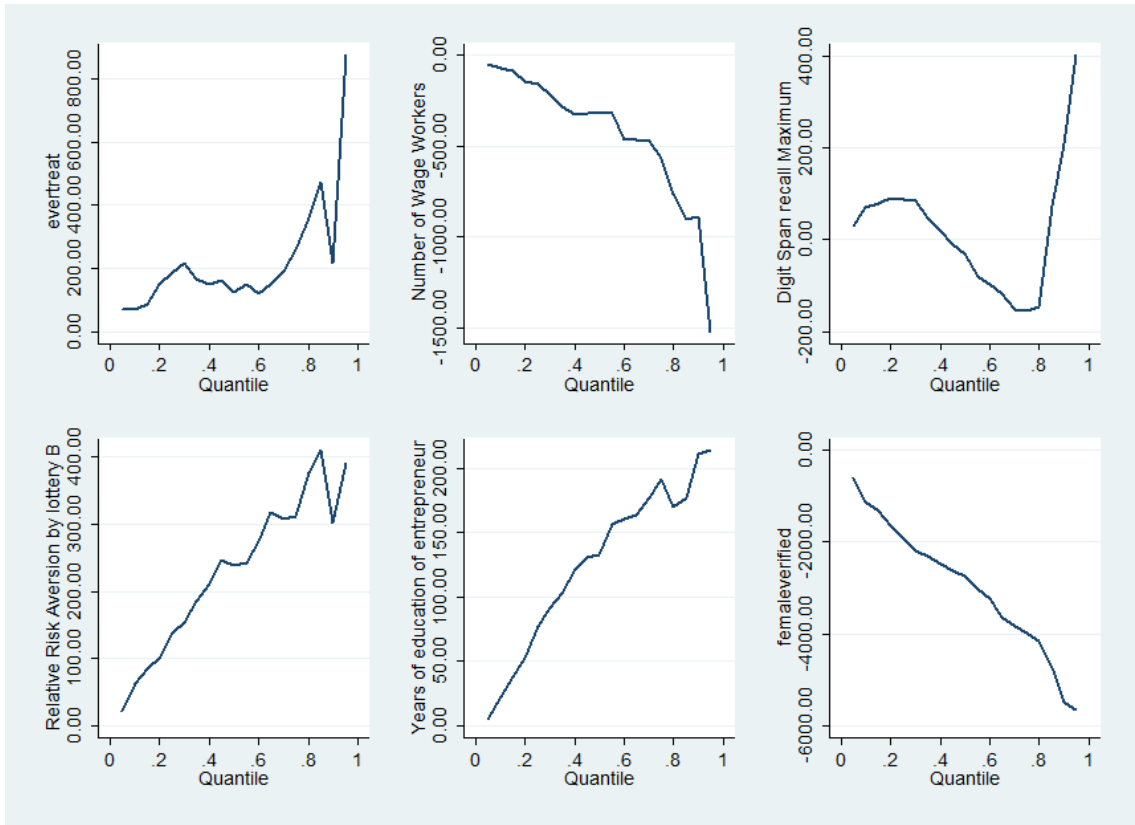
**Table 2.2:** Quantile Regression on Micro-enterprises Real Profits

Quantile	0.10	0.25	0.50	0.75	0.90
Treatment	70.72	187.57***	125.12	259.46**	217.85
# Workers	-69.65**	-159.39***	-317.58***	-567.71***	-889.10***
Ability	69.80***	86.13***	-30.61	-153.99*	209.40
Risk	62.53***	138.31***	238.24***	310.84***	301.14***
Education	22.65	77.12***	133.29***	191.81***	211.56***
Gender	-1130.7***	-1953.6***	-2747.1***	-3993.1***	-5496.1***
Constant	1160.7***	2088.9***	4736.6***	8508.8***	11595.8***

In Table 2.2, capital and profits are measured in Sri Lankan rupees, deflated by the Sri Lankan CPI to reflect March 2005 price levels. Profits are measured monthly. All regressions include enterprise and wave fixed effects. Standard errors, clustered at the enterprise level, are shown in parentheses. Sample is trimmed for the top 0.5% of changes in profits. The confidence intervals are denoted \* for 90%, \*\* 95% , and \*\*\* 99% confidence interval, respectively.

The horizontal axis in Figure 2.2 represents different quantiles of firm profits in the pre-treatment baseline survey. The vertical axis represents the effects of predictors on

the capital accumulation. In the top left of the figure, the treatment of capital stock shocks has the expected positive coefficients on the firm profits, and the effects are roughly proportional to the size of the treatment for the 0.8 quantile of the sample.



**Figure 2.2:** Quantile Regression

At the median, the effect of the shocks on capital stock is approximately 120-130% of the treatment amount. The number on the vertical line represents the treatment effect in percentage of the increment in capital flows. For firms with higher capital stock, especially at the 25% of the sample, capital shocks have significant effects on increasing the firm profit.

In the top middle of the figure, the QR coefficient of numbers of wage workers is always negative and decreases for larger enterprises. From the theoretical point of view, the slope of the curve equals to the marginal revenue minus the wage rate. This

non-convex curve shows that large firms pay higher wages and heterogeneity in wages dominates the impact on profits and capital stocks. There might exist an overpaying problem for large firms since for the quantile above 0.8, the marginal cost of hiring one more worker is more than ten times the treatment amount (10,000 LKR, which is equivalent to less than 100 USD).

One of the possible explanations is from the regulatory cost in developing countries such as training of wage workers. Regulations, especially those aimed at controlling prices and entry into markets that would otherwise be competitive, can limit growth and significantly reduce economic welfare (Hahn and Guasch, 2000).

Another explanation is that the type of workers matters in their contribution to the firm profit. Permanent workers contribute more to long-term profits while short-term workers have high job liquidity in the sense that they can readily and easily change jobs. Micro-enterprises have limitations in hiring permanent workers.

The entrepreneur ability shows different tracks of effects in digit span recall and years of education. The maximum digit span recalled has a U-shape curve across quantiles while from the quantile regression table summary, the negative coefficients for 0.5 and 0.75 quantiles are not significant from zero at the 95 confidence level.

Risk aversion effect supports the statement that more risk averse entrepreneurs are more reliable therefore can have more access to development funding. There is another impact on risk aversion that more risk averse entrepreneurs are less likely to make risky investments which might result in a potential loss in profits and hinder the growth of the firm. Based on the bottom left graph, entrepreneurial risk awareness affects the credibility of the entrepreneur and helps micro-enterprises grow healthily.

The coefficient of gender is consistent with prior literature. Male entrepreneurs have a relative advantage in operating and managing the firms, especially for large firms. In contrast with insignificant average impact of treatment for female-owned

enterprises in DMW, we can see female owners have a significant negative impact on the profits for the higher quantiles. In developing countries such as Sri Lanka, the gender difference is still a critical issue on the growth of micro-enterprises.

There are two possible explanations of the non-linearity in the returns to capital. One is the exact form of the production function. For firms with increasing returns to scale, the capital shocks take into effect above some threshold as shown in the quantile regression. It is hard for small firms to obtain large machines to improve the technology in a short period of capital accumulation. On the other hand, heterogeneity in entrepreneurs and technology can also affect profits as capital stock increases. Regulations on capital flows in developing countries will affect firms at different levels of capital stocks differently.

To separate these two causes of non-convexity in production function, I also consider the intersection between the treatment dummy of capital shocks and the predictors. Recall that the quantile regression is applied to six predictors: treatment amounts, number of wage workers, entrepreneur ability measured by maximum digit span recall, relative risk aversion by lottery B, years of education of the entrepreneur, and gender. The intersection results are discussed later with Figure 2.6.

Table 2.3 reports the two important effects: treatment effect (measure of returns to capital) and coefficient of risk aversion (risk awareness in the financial market). Both the treatment amount and risk aversion have significant positive effects on the real profits. However, the effects are not the same at different quantiles.

As a consequence of different scales of capital in micro-enterprises, there is a local peak in the effects of capital shocks at the 0.8 quantile, implying that for micro-enterprises in this range of profit level, the returns to capital is higher when they are at the 80% quantile of the log profit distribution.

The total number of observations is 3027 and the standard errors are obtained

**Table 2.3:** Quantile Coefficients with Standard Errors through Bootstrapping

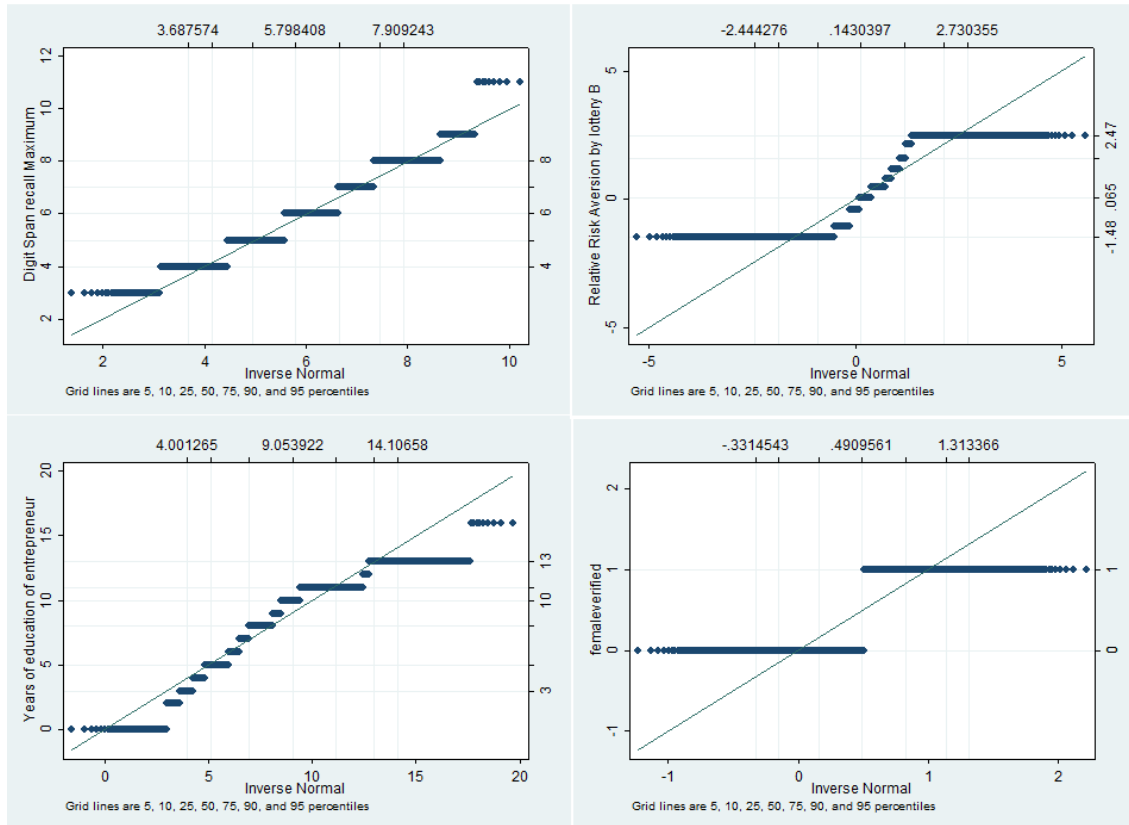
Quantile	0.10	0.25	0.50	0.75	0.90
Treatment	70.7234*	187.5738***	125.1211*	259.4602*	217.8542
B. S. E.	52.0415	91.1204	109.5926	195.7647	286.4098
Risk	62.5314***	138.3196***	238.2475***	310.8404***	301.14***
B. S. E.	23.6137	27.4102	69.4261	71.8958	185.8184
Pseudo R2	0.0457	0.0666	0.0824	0.0862	0.0889

from bootstrapping to fit the base model. The effects of number of wage workers are decreasing as the micro-enterprises grow. The coefficient of the entrepreneur ability reaches a local maximum around the 0.2 quantile. This is different from the effect of education level. For education level and risk aversion, the increasing trend indicates they play an important role in the future extension of the micro-enterprises. Therefore for a sustainable growth pattern for micro-enterprises, entrepreneurs should improve their risk awareness and education.

Figure 2.3 gives a general summary of the distribution of each predictor: digit span (entrepreneur memory ability), relative risk aversion by lottery B, entrepreneur education level, and female verified (entrepreneur gender). The data set is diversified and representative to a wide range of heterogeneity.

Digit span recall maximum has a step figure with respect to normal distribution which matches the analysis of entrepreneur ability. It offers the feasibility of the basic test for the significant effects on profit quantile regression. The relative risk aversion by lottery B varies from -1.48 to 2.47. It follows a monotone pattern with extreme cases at the two boundaries. Years of education of the entrepreneur have an unbalanced step pattern different from the digit span recall ability. There is an upper ceiling for education while there is a possibility that the abilities of this group of entrepreneurs with the same education years differ in firm management. The gender of the entrepreneur is randomized with equal weights.

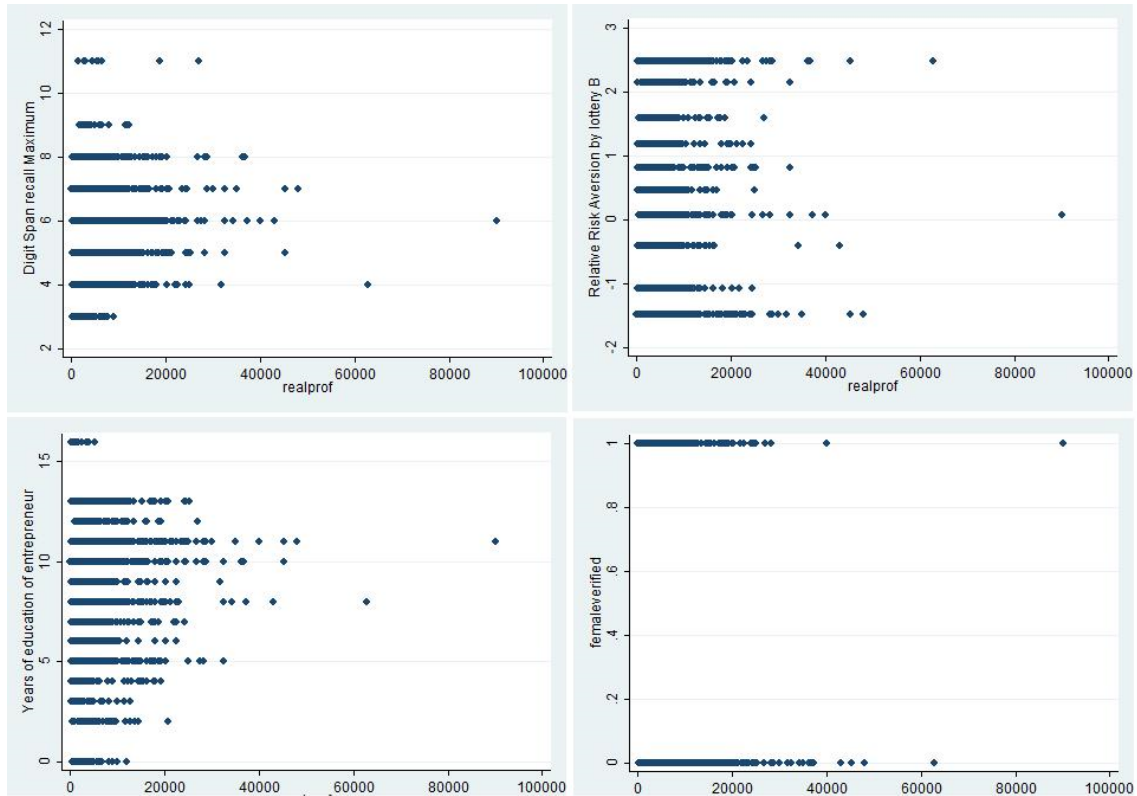
Figure 2.4 shows plots of characteristics in terms of quantiles of real profits at firm level. We can see most firms' profits are allocated between 0 and 40,000 LKR



**Figure 2-3:** Data in Quantiles

(approximately 300 USD). Intuitively, the expected result is a positive correlation between entrepreneur characteristics and real profits. However, from the graph, it's hard to draw this conclusion for micro-enterprises in Sri Lanka. Therefore we conduct a more precise quantile regression to explain the relation between returns to capital and entrepreneur characteristics.

Entrepreneurial ability is normally distributed with respect to the quantiles of real profits. The risk aversion variable can be viewed as randomly uniformly distributed in the range of real profits. The education level of entrepreneurs follows a positive correlation with profits. The larger the years of education, the higher the probability that the entrepreneur would get a high profit. Gender effects are hard to see from the plot but it shows that the sample is balanced in gender. As discussed above, it is

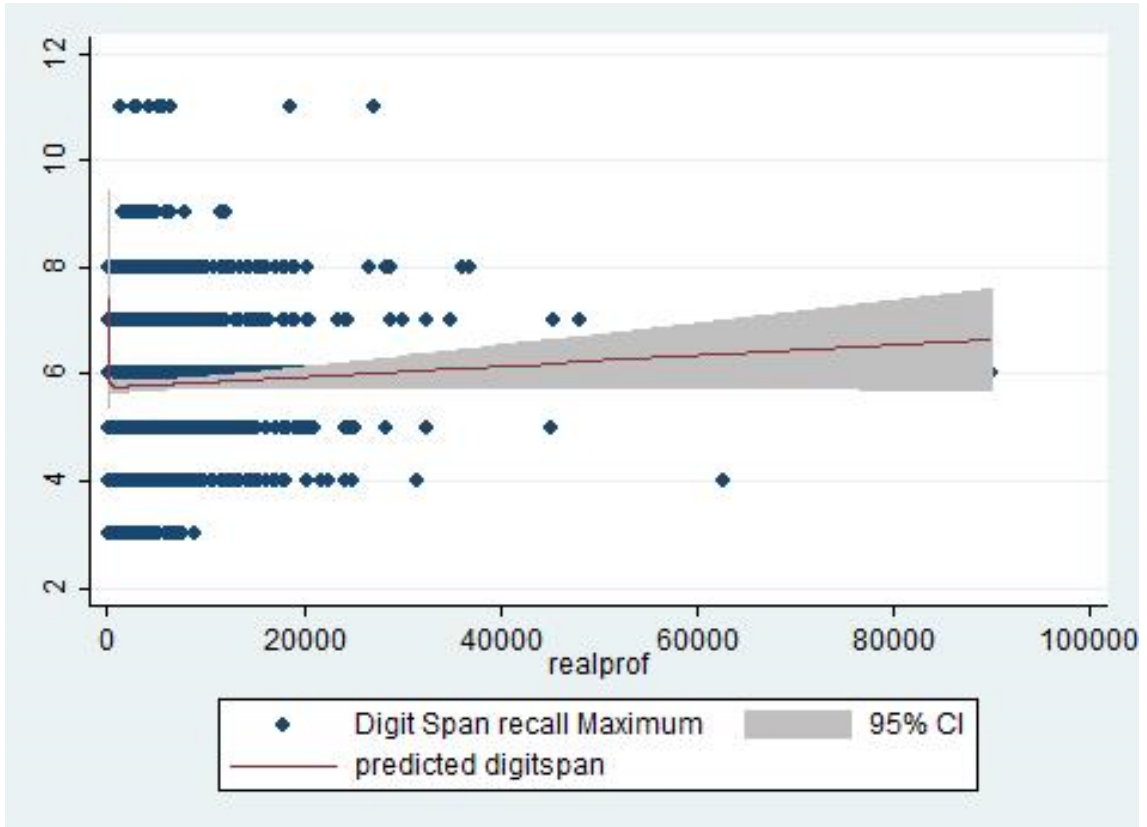


**Figure 2.4:** Distribution of Real Profits

hard to tell the relationship simply from two dimensional plots of the sample.

The sample was fitted with an OLS regression to see the trend. Figure 5 shows the OLS predicted effects of digit span with a 95% confidence interval. It has a slightly positive slope with the horizontal line representing the distribution of profits, taking the full sample into account.

The best choice to rule out outlier bias is to conduct quantile regression with respect to different small samples. This serves as a sensitivity test of quantile regression. The sample properties provide more reliable results for the analysis and prove feasibility of quantile regression. They also shed light on the heterogeneity tests by adding intersections of treatment dummy and entrepreneur characteristics.



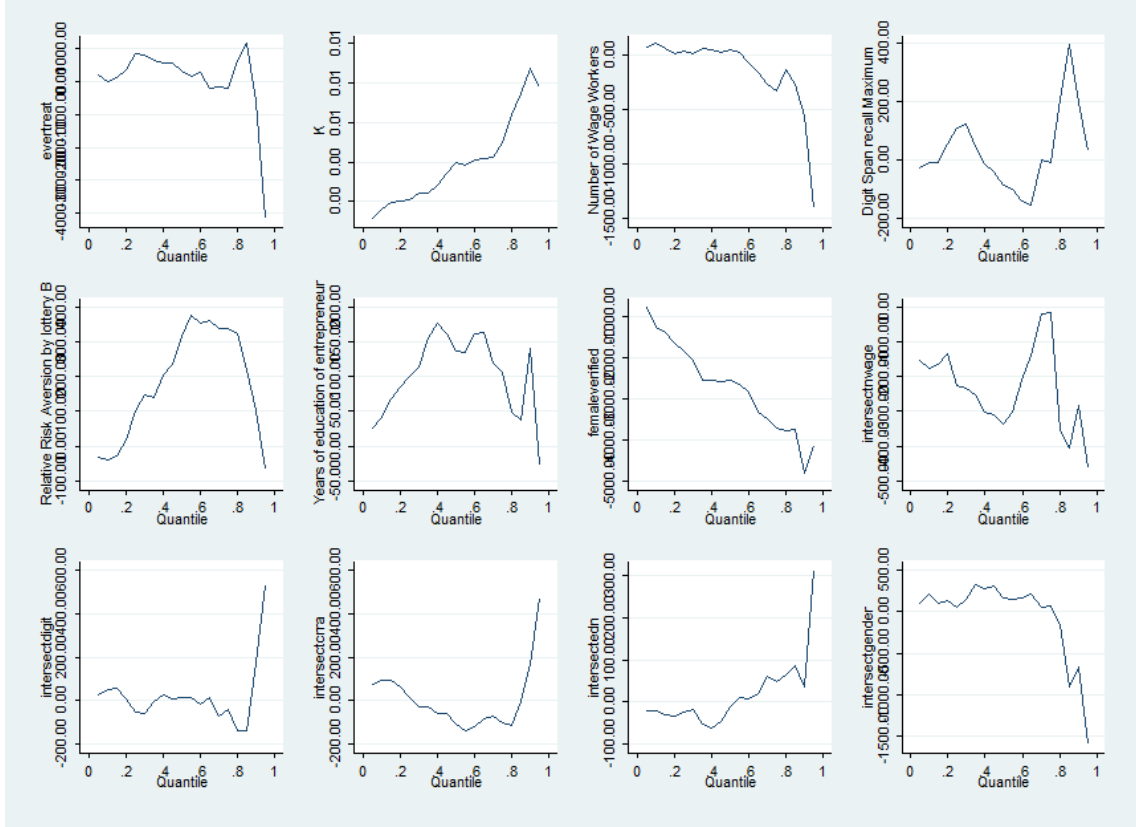
**Figure 2-5:** OLS Predicted Effects of Digit Span with 95% Confidence Interval

**Tests of Perfectness of Markets** As suggested by the theoretical model, we add five intersection elements to the quantile regression. We include the inventory stock in the independent regression.

$$y_{it} = \alpha_i + \beta_i S_{it} + z_{it} D_{it} + \theta_{it} S_{it} \times D_{it} + \mu_{it} \quad (2.8)$$

As shown in the model, the real profits are affected by capital shocks, initial wealth, labor hired, and entrepreneur ability such as risk aversion and education level. Figure 6 shows the results. We can see the path of the number of wage workers, digit span remembered and gender does not change much while patterns of both CRRA and education changes from monotone to non-monotone. One interesting result is that the pattern of treatment effects changes as a mirror reflection when adding the

initial capital inventory. The capital shocks affect the returns for certain quantiles significantly.



**Figure 2-6:** Quantile Regression with Intersections

Here the interpretation of the coefficients cannot simply represent the pure returns to capital. Though we use the instrumental variable method, the treatment effect includes returns to capital, labor, as well as intermediate inputs. The quantile regression is implemented as the effects on the conditional distribution at different quantiles of the firm real profits. The sharp change for the 0.8-1 quantile in profits is due to the small sample size in that part. The analysis would rule out these as outliers and focus mainly on the 0.2-0.8 quantile.

The initial wealth (initial inventory stock) follows a monotonic linear effect similar to the results of theoretical model for one period household in the imperfect credit

market. Number of wage workers becomes a insignificant factor compared to previous QR. There is no big change for the ability of digit span remembered. We can see for different quantile, ability acts differently while the intersection parts are not significant from zero.

In general, there is a positive effect of entrepreneurial risk aversion on the real profits of the micro-enterprises. Years of education can be a substitute to the digit span recall ability. They are significant, indicating a combined indicator can represent the overall entrepreneurial ability and has a constant effect on the growth of micro-enterprises across different quantiles.

Under the treatment of random capital shock, the combination of gender and gender-intersection-with-treatment is no longer significant. Without the treatment of grant, gender difference is highly significant for the real profits of micro-enterprises, especially for higher quantiles of profits.

Intersection indicators represent the difference of that indicator effects on treatment groups and control groups. For the 0.2 to 0.8 quantiles, four of them are not significant from zero: entrepreneur ability, risk aversion, gender and education level. However, we can see a non-monotonic pattern for the coefficients of number of wage workers. This is an interesting test to give more information about the labor market development of the small firms.

There is a gap of insignificance for the middle group of 0.5-0.8 quantiles while the lower and higher quantiles are both significantly negative from zero. The underlying heterogeneity and mechanism requires more theoretical analysis as well as further strict assumptions on the model.

## 2.4 Policy Analysis

There is a long tradition of informal savings and credit in Sri Lanka. According to the Central Bank, the volume of deposits in the financial system amounts to 1,700 billion in June 2007. A significant saving culture and a large proportion of the population access to financial services are strengths in Sri Lanka. Sri Lanka also has strong financial sector market infrastructure as well as specialized micro-finance training emerging.

These are encouraging steps towards formalizing the study of micro-finance and introducing international standards and best practices through the involvement of internationally recognized institutions such as Frankfurt School of Finance and Management. While the standard and quality of training may differ, the recognition of the need to such specialized training differs for the sector.

### 2.4.1 Macro Level Support

A long term vision and policy for micro-finance in Sri Lanka is in need. As shown in the previous section, micro-finance institutions can develop a long term contract with micro-enterprises and do additional survey/training to entrepreneurs. Such bundling of development help in funding makes the business cycle in a positive sustainable way.

The lack of a regulatory and supervisory framework for micro-finance is a major barrier to transformation and scaling-up of many MFIs. The NGO-MFIs operate in a grey zone as they are essentially unregulated and unsupervised. To balance NGOs and MFIs, government's macro regulations play an important role in the future combining with the market power.

### **2.4.2 Meso Level Support**

Local commercial funding institutions are reluctant to get involved in micro-finance due to their perception that it is a high risk activity. A risk-awareness training program combined with the promotion of micro-finance benefits would eliminate such worries and have a long-run reward to the entrepreneurship. Though there are a large number of off-shore micro-finance funding agencies available and interested in well-performing MFIs, the restrictive legal environment and the long process of obtaining approval from the Controller of Exchange serve as deterrent factors for many potential off-shore founders.

From the theoretical model, we see the capital market constraints influence the growth of micro-enterprises. Knowledge transformation and information change within sectors offer more learning activities and exposure to regional/international good practices to MFIs.

As shown in the empirical evidence, the credit market in Sri Lanka is imperfect and lacks credit information sharing. As membership of the Credit Information Bureau of Sri Lanka is mandatory for licensed commercial banks, voluntary participation of MFIs is unlikely as there are costs involved which most MFIs are unwilling to incur. How to handle the over-indebtedness and the probability of a high portfolio at risk for MFIs is crucial for development funding in the long run.

### **2.4.3 Micro Level Support**

The quality and skill levels of MFI staffs are important factors in the development funding. Similar to the field experiment conducted in Sri Lanka Micro-enterprise Survey, the large amount of subsidized funds hides the real sustainability picture of the MFIs as measured by financial self sustainability. In order to make institutions financially self-sustainable, the returns to financial investments are important in eval-

uating the project and designing the contract before funding. How to measure the load recovery rate and avoid providing a misleading perception of the portfolio is indeed the most urgent issue to solve.

In the long run, building up a healthy organizational culture with transparency and standardization is essential. A stronger focus on cost-efficiency and sustainability fits the market better and it is good to separate the micro-finance business from community development activities. Improving delivery technologies and reducing transaction costs is urgent for the sustainability of development funding in the very near future.

#### **2.4.4 Welfare Analysis**

In development economics, household welfare has the same importance as the growth rate of the economy. There is always a trade-off between consumption and leisure. The credit constraint, as an indicator of credit market openness, represents the development level in the credit market. In a perfect insurance market with missing credit market, it affects the capital stock decision directly in a linear way. The constraint affects consumption indirectly with an increasing marginal effect. So the household welfare is affected materially at the margin, especially for firms with higher initial capital wealth.

Welfare has influence on the confidence level of entrepreneurial investment decision making. The goal is to see how the positive cycle of development is established and how such effects passes through the characteristics of entrepreneurs. This would be a future research of interest. Cases are even more complicated for theoretical models with labor markets and imperfect insurance/credit markets.

In summary, credit market imperfections do affect the developing pattern of small firms. Welfare changes are even larger than capital accumulation. The amplification of such credit constraints forces more attention to be paid to the overall development

of markets, not only technology development in the production sector but also the corresponding financial markets to match the country's development level.

## **2.5 Conclusion**

This paper aims at opening the door to connect economic theory to micro-finance industry. By analyzing the heterogeneity of effects to returns, it offers a new insight in implementing and evaluating development funding contracts. Empirical evidence shows the characteristics of entrepreneurs influence the returns to capital in quantiles of micro-enterprise real profits.

The previous literature in implementing the returns to capital by instrumental variable of treatment, in deed, is translated in this paper as the general returns to development funding. These include returns to capital, labor as well as intermediate inputs. Therefore separating these effects are still of great interest for further research study.

### **2.5.1 Summary**

This chapter uses quantile regression to study how heterogeneity of entrepreneur characteristics affects the returns to capital shocks. Using the field experiment carried out by De Mel, McKenzie, and Woodruff (2008), it examines the heterogeneity in the treatment impact on micro-enterprise profits and first applies quantile regressions to illustrate the effects of capital shocks for different micro-enterprises. It also identifies the distributional characteristics of individual-specific coefficients.

The treatment effects of capital shock on micro-enterprises are nonlinear at different quantiles of real profits, suggesting firms with different capital levels have different profit growth rate. The effects of common parameters and treatment are identified in randomized coefficient panel data model. Results show that risk aversion of entrepreneur and the uncertainty of the projects have significant impacts on returns for

firms in the lower quantile of profits in the sample.

The quantile regression analyses show significant heterogeneity patterns of entrepreneurs' characteristics on real profits. The non-convex curve for coefficients of number of paid workers shows that large firms pay higher marginal wage on hiring and heterogeneity in wages dominates the impact on profits and capital stocks. Male entrepreneurs have a relative advantage in operating and managing the firms, especially for large firms.

This chapter provides two main explanations of the non-linearity in the returns to capital shocks: the exact form of the production function (increasing returns to scale) and the heterogeneity in entrepreneur characteristics. It separates these two causes of non-convexity in production functions by considering the intersection between the treatment dummy of capital shocks and the predictors.

With a single household model, I have analyzed the heterogeneity effects empirically. In sum, this chapter provides insights to micro-finance institutions on evaluation of micro-enterprise project and policy makers on development funding.

### **2.5.2 Future Research**

The broad role of micro-enterprises in developing countries is still under debate. One perspective argues that they may be highly productive firms held back by credit constraints or other frictions, while another view is that informal enterprises serve as a low-returns safety net for individuals excluded from the formal sector (Porta and Shleifer, 2008). In order to measure the market development level between credit and insurance markets, the future task is to seek a new Development Market Index (DMI) to setup a closer connection between theoretical model of micro-enterprises production and econometric nonlinear regression.

Contrary to the conclusion that high returns are closely associated with missing credit markets than missing insurance markets in DMW, this paper suggests both

credit and insurance markets imperfections affect the returns. One direction of further research is to define the development market measure to weigh the insurance market and credit market.

The data employed in this study are quarterly survey data from April 2005 to April 2007. The time series is not of sufficient length to include the seasonal lags (usually four lags for quarterly data) or by year (data only covers 2 years in total). Though there are nine waves available, the quantile regression only accounts for the first three waves as the treatment shocks happen at the first two waves. To quantitatively estimate the dynamics of the micro-enterprise development, new tools in quantiles for time-series- cross-section data are in need to control stationary co-variates and (non-)linear time trend. This is another promising direction of further research.

From the comparisons between the densities of profits and the corresponding capital stock, we see the benefits from investment are not a monotone linear function of capital stock. There might be first an increasing in profits driven by capital increment while effects diminish as firm size enlarges. A further research goal includes comparing the short-run and long-run return to capital to evaluate the projects and make policy suggestions.

## Chapter 3

# Housing Price Volatility and the Capital Account in China

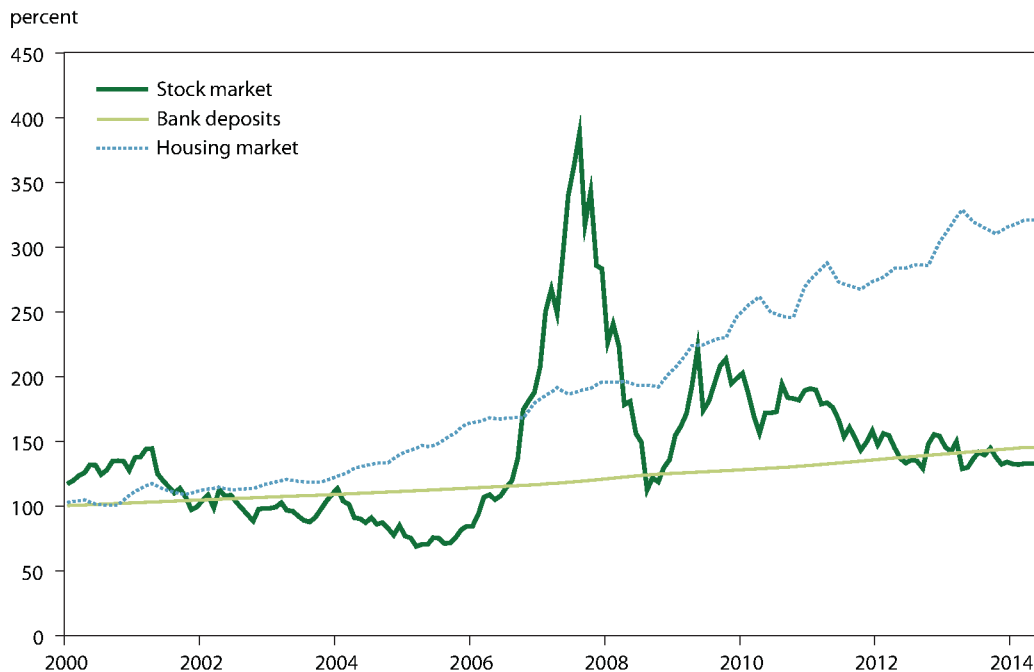
China has experienced significant surges in its housing market since 2005. This chapter examines the extent to which a) short-term capital flows and FDI may have further impacted the prices and volatility in the Chinese housing market and b) whether 2006 Capital Account Regulations (CARs) on foreign purchases of Chinese real estate were effective in reducing the level and volatility of prices in the housing market.

This chapter shows that short-term capital flows from abroad had a modest impact on the Chinese housing market's price levels. Hot money has had a more significant impact on increasing market price volatility. Moreover, the quantile regression results illustrate that hot money magnified the impacts of capital flows on housing prices during upward surges in the housing prices. In terms of market volatility, QR results show that the more volatile housing market is associated with larger impacts which the short-term capital flows had on accentuating such volatility. Furthermore, the 2006 CAR policy continued to have a strong impact on reducing volatility in the Chinese housing market during the period under study.

### 3.1 Introduction

China did not establish a market-determined real estate market until 1998 when it abolished the state-planned housing distribution system. Since then, the housing prices, as well as the price volatility, have been increasing at an alarming rate.

By 2015, housing sales in China accounted for roughly 15 percent of gross domestic product. This statistic does not account for the significant indirect effect on GDP through other sectors such as banking and construction. This is reflected in the fact that real estate investment has provided a compounded rate of returns of 10.1 percent a year over the past decade. From 2000 to 2014, returns on property investments have been attractive for Chinese residents. One of the reasons for this is the limited investment options available in China's less-developed financial markets.



Source: CEIC Data.

**Figure 3.1:** Time series of Housing, Stock Market Returns and Bank Deposits

Different from the stock market, the housing boom in China keeps increasing at an increasing pace (Figure 3.1). There were several attempts by the Chinese central bank (PBOC) and the government to keep prices affordable. Beijing imposed a variety of restrictions between 2005 and 2014 on the capital markets. Property market investment slowed to a five-year low of 10.5% in 2014 from a year earlier - the slowest pace since the first half of 2009. In November 2014, the country's central bank

unexpectedly cut interest rates to 2.75% for the first time since 2012 in an attempt to revive the economy.

Besides the reforms in the housing market, China embarked upon opening its capital account and loosening its exchange rate policies so as to rapidly internationalize the currency. This chapter looks into examining the extent to which measures pertaining to the capital account are at all associated with the Chinese housing market's prices and its price volatility.

The 2005-2013 property surges in China have been attributed to many factors. Low interest rates and increasing bank lending have been prevalent since 2003. Secretary Wen forged a cheap credit policy to trigger the construction and purchase of property while making competing debt investments less appealing. Local government relied on land sales for income (accounting for up to 50% of revenue). Limited access to foreign investments for Chinese citizens increased the appeal of domestic investments, such as property. Chinese citizens also faced cultural pressures encouraging home ownership.

Responding to the 2008 global financial crisis, the spending from the Chinese economic stimulus program also found its way into real estate, contributing to the bubble. Historical patterns show that the first rate cuts (September 2008 and June 2012) generated an increase in housing sales that lasted one to one and a half years. Home prices also started to rebound in one to two quarters after the first interest rate reduction.

Quantile regression is useful under this situation as the housing bubble particularly occurs at the higher quantiles of the price distribution. Quantile regression can be generalized in research analyzing the policy effectiveness on prices in quantiles. This chapter uses quantile regression from the perspective of housing price volatility in terms of capital account regulations.

## 3.2 Literature Review

Over the past few years, a number of economists both inside and outside of China have examined the determinants of Chinese housing prices. By and large, the empirical linear modeling results indicate that the changes in monetary policy and mortgage rates have had simultaneously the largest and negative effect on changes in house prices, while changes in all the other factors such as broader macroeconomic influences have positive but less significant effects. Empirical nonlinear modeling reveals that the product of mortgage rates and value-added industrial outputs, the product of real effective exchange rates and stock index, the product of export growth and short-term capital flows, and product of the house price index and producer price index dominate in the determination of house prices.

### 3.2.1 Academic Research Methodology

**(I) Theoretical literature on Chinese regime for capital flows and asset bubbles** China used to maintain tight restrictions on its capital account but has been gradually opening the capital account in a sequential manner for a decade. There has been a shift in official views which have become more sympathetic to capital controls (Ostry et al. 2011; IMF 2011). However, there are no international rules to constrain, discipline, or indeed legitimize restrictions that countries put in their capital account.

There has long been a debate on capital account liberalization and volatility controls (Subramanian, Jeanne, and Williamson, 2012). The pros and cons of prudential capital controls to curb the boom-bust cycle in capital flows have been discussed before (Williamson 2005), but economists now understand better the theoretical case for such policies with a new literature on the welfare economics of prudential capital controls (Korinek 2011). This literature essentially transposes to international capital

flows. The closed-economy analysis of the macro-prudential policies aims at curbing the boom-bust cycle in credit and asset prices.

In calibrated dynamic welfare optimizing models, capital account controls can be represented by imposing taxes with endogenous binding constraints (Bianchi 2011). Free capital mobility has been found to have little impact on economic development. Because of its importance in the global economy, China presents the most complicated example for research interests. The capital flow regime in China has unique characteristics in its pursuit of developing the financial asset markets and growing housing markets.

With large amounts of foreign direct investment and tax incentives, the Chinese capital account is hardly properly described as completely closed. Most of Chinese foreign assets are accumulated as international reserves. There are other forms of restrictions in capital inflows as well as outflows controls. However, full control over capital flows implies full control over its macroeconomic doppelganger, the trade balance, and hence the real exchange rate (Jeanne 2011). IMF (2012) came up with an up-to-date and operational framework for policy advice on liberalizing capital flows and on the management of capital outflows. It proposed an integrated approach and made systemically important implications on emerging market economies (EMEs) that extensively control capital flows (China and India).

The appropriate degree of liberalization for a country depends on its specific circumstances, notably on whether it has reached certain thresholds with respect to financial development. In China, further liberalization would be beneficial based on the implementation of the authorities' liberalization plans and more rapid progress on supporting reforms, particularly in the financial sector. However, controlling for market volatility during reform changes would avoid unstable situations.

New evidence (Mody and Murshid 2013) has been shown consistent with the

standard economic theory under volatility regimes of cross-border private capital flows. With volatility below a certain threshold, the inflow of foreign capitals has promoted growth. However, during periods of volatile growth, more flows have been associated with slower growth. Volatility levels and changes reflect an interaction between domestic production and institutional structures with global factors.

**(II) Links between capital account opening, short-term capital flows and the housing market** The literature on capital flows is not unified under a single theoretical framework in the macroeconomic analysis of capital controls. There is heterogeneity across countries over time when the capital account opening policy are implemented differently. In most of the empirical literature, there are no distinctions between capital controls on outflows and those on inflows. These exercises suffer from the same problem as the classification of exchange rate arrangements by IMF. There are also temporary versus permanent issues in the applications (Reinhart and Smith 2002).

Furthermore, China has never fully liberalized like other countries that actually went down the path of financial and capital account liberalization. While it had decided to reintroduce controls, institutions have developed with varying degrees of openness to the international capital markets. With substantial capital and exchange controls, separating the effects of capital controls is a more difficult task for China (Reinhart and Reinhart 1999). There are also reserve requirements and policies applied in China as an alternative to capital controls.

For the links between capital account opening and capital flows, Magud, Reinhart, and Rogoff (2011) explicitly defined the measures for capital controls. They standardized the results of over 30 empirical studies by introducing two indices of capital controls: Capital Control Effectiveness Index (CCE Index), and Weighted Capital Control Effectiveness Index (WCCE Index). They pointed out that current

empirical studies are in need of a common methodology. For example, Chile and Malaysia are over-weighted in the case study. Therefore, they modeled the effects of imposing capital controls on short-term flows using a portfolio balance approach. There also exist country-specific characteristics for capital controls to be effective. The equivalence in effects of price- vs. quantity-capital controls is conditional on the level of short-term capital flows.

There are also comparative quantitative studies on Chinese capital markets with developed economies. Financial capital and fixed capital tend to flow in opposite directions between poor and rich countries. Wang, et al. (2012) introduced frictions into a standard two-country neoclassical growth model to explain the pattern of two-way capital flows between emerging economies (such as China) and the developed world (such as the United States). They showed how underdeveloped credit markets in China can lead to an abnormally high rate of returns to fixed capital but an excessively low rate of returns to financial capital, compared with U.S.

### **3.2.2 Development of the Capital Market in China**

China has been gradually and systematically opening its capital account for some time. In the late 1970s, gradual liberalization in the foreign trade regime began with direct investment inflows and some opening up of credit markets. In 1994, to restructure the foreign exchange control system in China, substantial reforms resulted in the establishment of a nationwide foreign exchange inter bank market. In December 1996, China made the Renminbi (RMB) convertible for current account transactions, removing both quantitative and regulatory restrictions on the use of foreign exchange.

Since December 11, 2001, China has become a member of the World Trade Organization (WTO), which has also been seen as a catalyst for capital account liberalization and currency convertibility. In 2006, the current account and capital account surpluses in China were US\$ 170 billion and US\$ 60 billion, respectively. The growth

rate of net exports was more than 70 percent, after reaching a peak growth rate of 220 percent in 2005. Although the RMB exchange rate has appreciated by about 6 percent against the US dollar since July 2005, the appreciation pressure on the RMB is unabated.

In 2006, there were a few policies introduced to the financial and housing markets in China. They include implementing QDII schemes, phasing out the compulsory requirement on surrendering of foreign exchange by domestic entities, and imposing restrictions on property purchase by foreign entities.

The major changes in the capital account measures in China can be summarized as follows:

- Originally Chinese enterprises and commercial banks were allowed to keep a certain proportion of foreign exchange earnings from current account transactions in foreign exchange bank accounts. Currently, there are no such limits.
- Residents are allowed to convert RMB to foreign currency up to \$50,000 per year. They are now free to open foreign exchange accounts.
- Overseas investment by the Chinese firms is now much less restricted than in the past. Residents are allowed to buy foreign equities via qualified domestic institutional investors (QDII). The QDII scheme was introduced in June 2006, which allowed qualified domestic banks to conduct overseas wealth management for their clients and qualified securities brokers (such as fund management companies and securities companies) to make overseas portfolio investment. By the end of October 2007, a total of \$27 billion of funds had been invested outside the mainland under the QDII scheme.
- Non-residents are allowed to open RMB accounts in China and buy A shares via the qualified foreign institutional investors (QFII) scheme. The QFII scheme is

a transitional institutional arrangement opening the capital market to foreign capital. By the end of October 2007, a total of 52 foreign institutions had obtained QFII status from the China Securities Regulatory Commission (CSRC), of which 49 had been granted an aggregate investment quota of some \$10 billion by the SAFE. At this moment, the total quota of QFIIs is \$30 billion.

- Restrictions on the issuance of bonds abroad by the domestic institutions have been loosened. By the end of 2006, a total of 27 domestic institutions (including the Ministry of Finance) had been allowed to issue 141 international bonds in major capital markets such as Japan, the United States, Europe, Singapore and Hong Kong SAR of China, raising \$30.8 billion.
- International development institutions have been allowed to issue RMB bonds domestically. In October 2005, the International Finance Corporation (IFC) was permitted to issue RMB 1.13 billion and the Asian Development Bank (ADB) RMB 1 billion bonds domestically.
- By the end of October 2007, the China Development Bank, the Export-Import Bank of China and Bank of China had issued RMB 5 billion, RMB 2 billion and RMB 3 billion Yuan bonds in Hong Kong SAR, respectively.
- Non-residents are allowed to buy houses in China as long as they have been in China for work or study for more than one year. The "extra-national treatment" previously granted to foreign banks which had allowed them to borrow abroad with fewer restrictions than domestic banks has been abolished.
- A new foreign exchange settlement system has been established. Under this system, capital inflows are under stricter scrutiny. It is assumed that all inflows should be based on real transactions. Inflows of foreign exchange originating from "foreign investment" must be paid to designated recipients and are not

allowed to enter into and stay in RMB accounts of enterprises in order to benefit from an appreciation of the RMB.

According to the State Administration of Foreign Exchange, among 43 items of transactions under the capital account, 8 items are completely liberalized; 11 items are under loose restriction; 18 items are under moderate restriction. Convertibility is strictly prohibited only for 6 items. Calculations based on the International Monetary Fund (IMF) formula show that so far 80% of China's capital account has been liberalized.

The Ministry of Housing and Urban-Rural Development (MOHURD) and the State Administration of Foreign Exchange (SAFE) jointly issued the Circular on Further Standardizing the Administration of Property Purchases by Overseas Institutions and Foreigners (restrictions on the purchase of properties by foreign entities with a presence in China and foreign individuals). Circular 186 was issued to implement the wider macroeconomic policies set out in the Circular on Firmly Restraining Rapid Growth of Real Property Prices in Certain Cities (Guo Fa No. 10 of 2010) issued by the State Council on 17 April 2010 and to strengthen the implementation of Opinions on Regulating Foreign Investment in Real Estate Market Access and Administration of Foreign Investment in Real Estate (Jian Zhu Fang No. 171 of 2006) jointly issued by six ministries on 11 July 2006 (Circular 171).

Circular 171 states that foreign individuals and branches and representative offices established by foreign entities are only permitted to purchase properties in China for self-use. It requires that a foreign entity that purchases property for self-use purposes must, in addition to other documents required by laws and regulations, present the business license or registration certificate of its branch or representative office and a letter of undertaking certifying that the purchase of commercial property is genuinely for self-use purposes. Foreign investors must first establish a wholly foreign-owned

enterprise (with taxable presence) in China in order to do so.

The Beijing Municipal Government issued a Circular on Regulating the Purchase of Commodity Housing by Foreign Entities and Foreign Individuals on 29 January 2007. It provides that such properties bought by foreign entities and foreign individuals must not be leased or sold after purchasing without permission. Although the Beijing Circular should not apply to places outside Beijing, it does seem designed to put off foreigners purchasing property in Beijing. Having been the first municipality to have relaxed the one property rule applicable to foreigners subsequent to the issuance of the Circular 171 and then reintroducing the rule later, it appears that Beijing seems to have executed more policy flip-flops than most, perhaps due to the higher reported percentage of foreign buyers.

Although the Chinese government has also issued certain restrictive policies applicable to domestic investors (for example, the loan-to-value percentage for such investors is reduced if the investors have bought more than one property in China) and the percentage down payment increased, these restrictions do not restrict purchases in absolute terms, only the number of qualified buyers. A wealthy cash buyer in China can simply carry on buying in the market despite the new restrictions; when one compares with his/her foreign counterpart, the double standard is obvious.

Hence, the motivation behind applying these restrictive policies to foreign investors is somewhat questionable, given the relatively modest share of foreign investors in the market taken as a whole. Despite these policies, prices in many major cities in China such as Guangzhou have continued to rise in recent months, suggesting the policy of reining in real estate prices is not able to fully achieve its aims.

Whilst there is a genuine and understandable concern about foreign investment being a front for speculation on the RMB, the difficulty in extracting money from the real estate market for foreigners due to foreign exchange controls makes it difficult

to imagine that this is an easy target for hot money, which tends to move rapidly and freely from place to place. Therefore, one view about the real motivation is that the Chinese authorities are, by implicitly suggesting that foreign speculators and *hot money* are somehow to blame for high prices, once again trying to distract attention away from the more fundamental issues which are causing price surges in the market, and putting the dream of owning a property even further out of reach of most ordinary people in China.

Despite the new policy of *Difficult in Easy out*, capital inflows have continued to flow in rapidly through the current account as well as the capital account. The destinations of these capital inflows are China's money markets and capital markets, especially the real estate markets and stock markets. So far there is not hard data on how much speculative capital has flown into what markets. Some argued that at least a third of China's current account surplus in 2006 is disguised capital inflows through over-invoicing exports and under-invoicing imports.

To channel the large scale of speculative capital flows across borders into China is not that difficult. For example, there are numerous Taiwanese companies with a certain amount of registered capital. However, capital is allowed to be injected gradually over the years. To utilize these loopholes, hedge funds can put money into those firm accounts as injected capital legally with a price. Then the money will leave the account of these corporations and enter into their subsidiary accounts. Now it is entirely legal for these subsidiaries to put the money into their accounts held by securities companies on behalf of themselves to buy shares.

Under these circumstances, probably only two kinds of capital will be eager to utilize opportunities provided by loosening the control over capital outflows to flow out of China. The unwinding of speculative capital and money need to be bleached. Legitimate outbound investment and remittance of investment incomes already have

channels in place to move beyond the borders without undue difficulty. However, even legitimate outbound investment should be carefully monitored and supervised. Without careful monitoring and supervising, the results of the disorderly outflows of Chinese capital will fare much worse than Japan two decades ago.

Many emerging markets with closed capital accounts suffer from underlying fragility. Such fragility might come from a large fiscal deficit, flexible exchange rates, a large amount of corporate and government debt denominated in dollars, a banking sector with significant nonperforming loans, or an opaque corporate non-financial sector. All these factors trigger financial instability and make capital account convertibility riskier.

### **3.3 Determinants of Housing Price**

The determinants of housing prices in China are discussed below by key category/factor, as well as the data employed to represent these factors, along with economic rationales and citations.

a) Monetary policy: M2 and its growth rate. The sharp increase in property prices in 2009 was a consequence of the extremely loose monetary policy adopted by the Chinese government to counter the global financial crisis, such as easily-attainable bank loans and discounts on interest rates. Research shows a positive impact on the housing price index (Vrontis and Thrassou, 2013; Zhang et al 2011; Verry 2007; Rigobon and Sack, 2004);

b) Inflation: CPI. The current growing demand in the housing market is due to certain companies and investor fear of inflation. Li (2013) and Cochrane (2011) found evidence of inflation push in the Chinese housing market;

c) International trade: net export growth rate. Rapid increases in international trade volume contributed much to the booming of Chinese economy and hence may

provide a momentum for house price changes (IMF, 2013; Zhang et al, 2011; Wolf Jr. 2011; Bergsten et al 2008);

d) Exchange rate fluctuation can be represented by the real exchange rate of Yuan to US Dollar. Chinese exchange rate policy played a critical role in its international trade and FDI booms. China's foreign exchange policy improved its competitiveness in attracting FDI flows as well as creating favorable conditions for maximizing exports. The 12 Month RMB/dollar Non-deliverable Forward contracts (NDF) measures the effects of RMB appreciation expectations. (IMF 2013; Li, 2014; Devlin et al, 2006; Xing, 2006; Korenberg et al. 1988; Billings and Zhu, 1994);

e) Land price: real estate investment growth rate. Local governments, dependent on land financing, have strong incentives and abilities to generate significant revenue from real estate investment. Soaring land price also push up the housing prices (Chen et al, 2011; Zhang et al, 2011);

f) Capital flows: hot money. The speculative capital flow, due to RMB appreciation expectation, is one of the main factors that helped accelerate the bubble. Based on Guo and Huang (2010), hot money is calculated as (change in foreign exchange reserves) minus (trade and service balance) minus (foreign direct investment). There is a growing literature domestically and internationally(Liang, 2014; Guo and Huang, 2010; Martin, 2008; Li, 2007);

g) Price volatility such as volatility of housing index matters since Granger causality test indicated that the price fluctuations are a Granger-causality rationale for changes in monetary policy, economic growth and price level. Housing price volatility represents the fluctuation of the housing market. (Chen et al, 2014; Meidani, 2011; McCarthy and Steindel, 2009; Goodhart and Hofmann, 2008);

h) Economic growth: GDP growth rate. Variance decomposition analysis showed the house price fluctuations from the impacts of monetary policy, the lagged effects

last about three months. Research shows that the Chinese economy can develop smoothly and orderly through the automatic adjustment of the market and the government's macroeconomic control (IMF, 2013; Kaiwa and Lamberte, 2010; Wang, 2009);

i) Unemployment rate. The dynamic VAR model indicates that house price increases are highly related to the Chinese economy: money supply increase, higher price level, slower economic growth, and a higher unemployment rate in the long run. (WESP, 2013; IMF, 2013)

The research on capital account liberalization in China has been of interest for a long time. A variety of methods have been applied to solve the research question from different perspectives. Bayoumi and Ohnsorge (2013) analyzed the inflows and outflows in China and provided global implications of capital account liberalization with Chinese macroeconomic data. They concluded that China needs to be more cautious on the capital account liberalization. Gallagher et al (2014) examined the research question of *How Long will the China Capital Account Take to Get Fully Liberalized*. They suggested to reform and regulate first. The open capital accounts leave emerging markets susceptible to the pro-cyclical nature of global finance.

Gruben, McLeod (2002) focused on Capital Account Liberalization and Inflation. Romer (1993) and Quinn-Toyoda (1996) modeled with cross-country data and discovered that widespread capital account liberalization in the early 1990s, which contributed to the worldwide deflation. Huang et al (2011) studied the order of liberalization of the Chinese capital account by  $RGDP_{it} = \alpha_0 + \alpha_1 CACI_{it} + \alpha_2 X_{it} + \epsilon_{it}$  where CACI represents the Capital Account Control Index. Both Jeanne (2012) and He et al (2012) analyzed the capital account policies and the real exchange rate in China. They provided a simple small open economy model with capital controls, focusing on real exchange intervention. They found that public accumulation of foreign assets

might affect the real exchange rate even in the countries with a fairly open capital account.

Luo and Jiang (2005) developed an international asset pricing models under property rights regime with price distortion, moral hazard and monetary overhang. They pointed out that property rights reform should be given the first priority. The main impediments toward capital account liberalization are monetary overhang and moral hazard in the stock market. Wei and Zhang (2012) researched impacts of the related factors on the capital account liberalization strategies from the perspective of financial stability based on Chinese financial institution balance sheets. They predicted that the relative low saving rate and deterioration of external debt will exacerbate financial risks.

Yang and Leatham (2011) investigated the impacts of currency convertibility on the informational linkage between official and swap market exchange rates with the Unit Root Test and Error Correction Model. They suggested that a complete currency convertibility is needed for more informed RMA exchange rate.

The research on the Chinese housing market dates back to 1995. Zhang, Hua, and Zhao (2011) examined monetary policy impacts on housing prices in China during 2000-2010. They employed a nonlinear auto-regressive moving average model with exogenous inputs. Hui and Yue (2006) investigated whether there is a housing price bubble in Beijing, Shanghai, and Hong Kong in 2003. They applied Granger causality tests, generalized impulse response analysis in the reduced form of housing price determinants. Lu and Miao (2011) analyzed the dynamics of housing prices in 35 urban cities between 1995 and 2005. Yu (2011) detected and measured the characteristics of housing bubbles in major cities in China between 1999 and 2010. The research decomposed bubble characters and implemented panel unit root test with a VAR model. Wu et al (2012) looked back to the 2000-2010 time period. They proposed

measures of Chinese land and housing prices, evaluated conditions in major Chinese housing markets, and conducted comparative analyses of time series data.

From the perspective of econometric methodology, Zietz et al (2008) found that results from OLS differ across studies, not only in terms of the size of OLS coefficients and statistical significance but sometimes in direction of effect. To examine this issue, they applied quantile regression, with and without accounting for spatial autocorrelation, to identify the coefficients of a large set of diverse variables across different quantiles. This chapter addresses short-term capital flows on the Chinese housing price index using quantile regression.

### **3.4 Baseline Model**

Based on the related literature mentioned in the previous sections, the baseline model is a reduced form of housing price equation. The dependent variables include the price determinants mentioned above and the measure of hot money that was created. Moreover, the 2006 event dummy and hot money are included to examine the extent to which a) short-term capital flows may further impact the price/volatility swing in the Chinese housing market and b) whether 2006 Capital Account Regulation (CAR) in China were effective in stemming the bubble in terms of price/volatility.

#### **3.4.1 Data**

Monthly data of Chinese housing market were collected. The macro fundamentals (Dec 1998 - Oct 2013) begin immediately after the 1997 Asian crises. The data also cover the 2007 global financial crises. These variables are chosen based on the literature and research on the housing market in China. In contrast with previous research which used annual data, we chose the monthly frequency and calculate the volatility of housing price index based on a 12 months window.

We are interested in how the housing price is affected by capital flows and policy changes. The housing prices are controlled by the macroeconomic fundamentals. Therefore, the baseline model is as follows:

$$Y_t = \alpha + \beta X_t + \delta D_t + \epsilon_t \quad (3.1)$$

**Table 3.1:** List of Variables and Resources

Notation Details and Source	Explanation
HI 199801 - 201310; NBS	Housing Index
HV 12-month Housing Index	Housing Volatility
NDF12M Bloomberg	NDF 12-month
NDF1M Bloomberg	NDF 1-month
MSCI QDII; QFII; MSCI GIMI; Bloomberg	MSCI China Indexes
CNY Monthly; Bloomberg	Yuan/Dollar exchange rate
USH US mortgage rate(PMMS); Bloomberg	US Housing
NEXGR Total Exports - Imports; NBS	Net Export Growth Rate
REIGR NBS	Real Estate growth rate
M2GR Monetary policy; NBS	M2 growth rate
CPI NBS	Consumption Price Index

NBS: National Bureau of Statistics

$Y_t$  represents the housing market price index HI or the volatility of housing prices HV.  $X_t$  is a vector including macroeconomic fundamentals of the Chinese economy, such as CPI (inflation), M2GR (monetary policy), CNY (foreign currency exchange market), NEXGR (net exports), REIGR (real estate investment growth rate).  $F_t$  measures capital flows in the short term, which is also called hot money. The deriva-

tion of hot money follows the equation:

$$HotMoney = \Delta(FXReserves) - (Trade/ServiceBalance) - FDI \quad (3.2)$$

$D_t$  is an indicator variable representing the policy changes in the capital market, where  $t$  represents the year when the events/policies taken place. 2006 is the year when policy changes took place in the capital market. In 2006, QFII schemes were introduced to attract more foreign investors which opened the capital account in China to a certain level. 2006 was also a year when more flexibility was released to the foreign exchange market as well as broaden the investment choices of domestic investors.

The first goal is to test whether the results remain the same during this time period compared with the previous literature, for which our analyses overlap in time periods. We find that the results have the same sign as prior research. Then we introduce two additional measurements in the Chinese housing market. One is hot money in the capital and financial market. The other is the 2006 policy dummy on the housing regulations in China.

Capital flows are difficult to measure, especially for developing countries like China. Based on macroeconomic literature, we calculate the hot money by the difference between foreign exchange reserves change and trade-service balance, then deduct the FDI part. This is a general formula widely applied in the capital market literature. As discussed in the previous section, the 2006 regulation policy on the housing market is of research interest for the housing volatility. Therefore we generate a dummy variable representing this policy with 1 after the policy is issued and zero before 2006.

The housing index (HI) ranges in values from 50 to 170 with the median larger than mean. The housing volatility (HV) is calculated based on HI and ranges from 0 to 50. The MSCI Emerging Markets Index for China is a free float-adjusted market

capitalization index that is designed to measure equity market performance in the global emerging markets.

**Table 3.2:** Summary Statistics

	max	median	min	mean	standard error
HI	161.27	106.694	53.624	101.695	35.511
HV	43.553	4.296	0.009	7.774	9.250
MSCI	102.98	40.56	13.73	42.872	19.903
CNY	8.28	7.856	6.054	7.460	0.850
USH	8.62	5.91	3.32	5.742	1.259
USHMF	8.27	5.53	3.4	5.544	1.133
NEX	$4.16 \times 10^9$	$6.34 \times 10^8$	$2.129 \times 10^7$	$1.01 \times 10^9$	$9.417 \times 10^8$
NEXGR	50.8	23.4	-29	19.432	16.840
M2	1209587	367326.5	117638.1	479188.3	322597.455
M2GR	29.7	16	12	16.744	3.813
REI	86013.38	7736.42	184.4	15529.3	17925.02
REIGR	50.2	24.6	1	25.371	7.849
CPI	108.7	101.9	97.8	102.074	2.387
FXR	$4.87 \times 10^{11}$	$2.08 \times 10^{11}$	$8.57 \times 10^9$	$2.27 \times 10^{11}$	$1.719 \times 10^{11}$
f	$3.56 \times 10^{11}$	$6.83 \times 10^{10}$	$-1.8 \times 10^{11}$	$9.04 \times 10^{10}$	$1.360 \times 10^{11}$
FDI	$2.8 \times 10^{11}$	$1.04 \times 10^{11}$	$3.84 \times 10^{10}$	$1.24 \times 10^{11}$	$8.241 \times 10^{10}$
D2006sep	1	1	0	0.508	0.500

Note that CNY is the currency exchange rate of US dollars in terms of the Yuan. USH is the mortgage rate in the US housing market and USHMF is the 30-year loan interest rate in the US housing market. Net Export (NEX) measures the international trade market in China and NEXGR is the growth rate based on NEX. M2 measures the monetary policy in China and M2GR is the growth rate. Real Estate Investment (REI) measures the land market in China and REIGR is the growth rate based on REI.

### 3.4.2 Results

We find the same results in monetary policy (significant positive), inflation (significant positive) and net exports (insignificant positive), compared with the prior literature on the determinants of the Chinese housing market. However, the land price push is not well represented by the real estate investment growth rate. The real estate investment growth rate has a significant negative impact on the housing price index.

In the model with capital flows  $F$  and 2006 reform dummy, there is a positive and statistically significant relationship between  $F$  and the HI, as well as the volatility variable. In addition, we also find a negative relationship between the 2006 dummy and the volatility. In general, our analysis adds to the existing explanations of the housing bubble and the discussions about the efficiency of capital account regulation.

Here are the three main findings from the results:

i) Add hot money  $F$  only into the baseline model of HI (housing index): positive impact, significant at 5% confidence interval;

ii) Add both hot money  $F$  and 2006 reform dummy into the baseline model of HI (housing index): positive impact, significant at 10% confidence interval, insignificant negative impact of the 2006 reform dummy;

iii) Add both hot money  $F$  and 2006 reform dummy into the baseline model of HV (housing volatility): positive impact, significant at 5% confidence interval,  $R^2$  improved from 0.2005 to 0.2882.

In addition, adding hot money improves the adjusted goodness of fit of the baseline model. Hot money has a significant positive impact on the housing price. As more and more hot money flows into the economy, they would enter the housing market causing surges in prices. We find quantitative evidence of short-term capital flows that increases the housing prices in China.

On the other hand, the policy change in September 2006 influences the housing market in a different way. Though there is an insignificant negative impact on the level of the housing prices, the major effect of such a policy is the volatility control of the housing market. We find a significant negative impact of the 2006 event dummy on the housing volatility. The policy stabilizes the Chinese housing index by lowering the volatility of the housing index.

In sum, capital flows have a significant impact on the Chinese housing market.

Capital flow liberalization in China might cause the bursting of housing bubbles. Regulations and policy changes stabilize the housing market.

**Table 3.3:** Quantile Regression: Housing Index and Housing Volatility

(A)HI	[1]Baseline	[2]	[3]
f		$2.1 \times 10^{-11}^{**}$ [ $1.01 \times 10^{-11}$ ]	$2.25 \times 10^{-11}^*$ [ $1.2 \times 10^{-11}$ ]
d2006			-1.13 [5.13]
M2GR	1.28*** [0.23]	0.71* [0.36]	0.69* [0.36]
CPI	3.04*** [0.41]	2.29*** [0.56]	2.3*** [0.57]
NEXGR	0.16*** [0.83]	0.058 [0.65]	0.058 [0.65]
CNY	-34.8*** [1.16]	-36.4*** [1.48]	-36.9*** [3.16]
constant	32.5 [47.1]	127.6* [67.6]	132* [70.8]
N	167	167	167
R-squared	0.899	0.904	0.904
(B)HV	[1]Baseline	[2]	[3]
f		$-1.34 \times 10^{-12}$ [ $7.82 \times 10^{-12}$ ]	$1.77 \times 10^{-11}^{**}$ [ $8.9 \times 10^{-12}$ ]
d2006			-15.1*** [3.81]
M2GR	0.64*** [0.18]	0.71** [0.28]	0.50* [0.27]
CPI	1.92*** [0.31]	1.69*** [0.43]	1.8*** [0.42]
NEXGR	0.11 [0.68]	0.071 [0.65]	0.069 [0.04]
CNY	3.22*** [0.89]	2.49*** [1.15]	-5.7** [2.35]
constant	-223*** [36.4]	-197.2*** [52.6]	-137.6*** [52.56]
N	167	167	167
R-squared	0.208	0.218	0.288

Note: Standard errors are listed in the brackets; [1] refers to the baseline regression model in the literature; [2] refers to the regression model with hot money; [3] refers to the regression model adding both hot money and 2006 event dummy.

The short-run capital flows and regulations play an important role on house prices in China. As we can see from Table 3.3, there is a positive impact of hot money on the level of housing prices. This result explains the increasing prices in the housing market. Though the government proposed policy changes for the prices in 2006, the effect of such regulations does not significantly impact the overall house price level. We do find significant impacts in the second regression regarding house volatility.

To further explore the effects of capital flows, we introduce quantile regression to assess the different impacts at different quantiles of the distribution of housing prices and price volatility. Even though OLS coefficients provide the mean impact of hot money, it is not adequate to learn about the change patterns of hot money for the

housing market. As the housing markets are fluctuating all the time, the effects are not constant and vary across time. Therefore, in the next section, we apply quantile regression to examine the changing pattern of effects of hot money on the housing market.

### 3.5 Quantile Regression

This section explores the potential of quantile regression models as a tool for analyzing capital flows and regulatory policies for the housing market in China. The higher quantile of housing prices is more relevant to determine whether short-term capital flows have a stronger effect on the housing index.

#### 3.5.1 Measurements

There is a significant difference in the quantile results between housing price index regression and volatility regression. The push from short-term capital flows has different effects on the housing market performances regarding the mean and the variance. Therefore the results are discussed separately here.

(I) For the housing price index at the market price level

The positive effect of hot money on the housing price index only occurs at the upper quantiles of the distribution of the housing prices. This indicates the OLS results could not represent the general case. In fact, only when the overall housing price index is high, it is the case that hot money have a positive impact on the price index. The 2006 policy dummy effect is not constant across the distribution of housing prices. We found significant negative impacts at the tails of the distribution; however, the results for the median are positive based on the quantile regression results.

Monetary policy and inflation show strong positive impact at the lower quantiles but lose significance at the higher quantiles. In general, quantile regression describes the effect of interest regarding the distribution of the price index. It describes the

performance of capital flows and economic policies at a specific level of the housing market. The lower quantiles represent the housing market in a recession while the higher quantiles indicate there might be a price bubble in the housing market.

(II) For the monthly housing volatility

Sharp changes at the higher quantiles in all dependent variables coefficients. Higher housing volatility makes the market unstable, therefore, much more easily affected by capital flows and related policies as well as macroeconomic fundamentals. The effect of hot money is significantly positive at the upper quantiles of housing price volatility. It remains the same for under 0.6 quantiles. The 2006 Sep event has a significant negative impact on housing price volatility, especially for the upper quantiles. The effects of hot money are significantly positive at the higher quantiles, reaching the maximum at around the 80th percentiles. For the dummy 2006 variable, the negative impact at the tails means that the policy has the effect of dragging the price away from the extreme cases.

The factor driving the housing prices surge includes hot money, which is significant for the upper distribution of the housing index. This result comes from the quantile regression. The policy is not significantly well represented by the dummy variable.

**Table 3.4:** Determinants of Housing Index

HI	0.1	0.25	0.5	0.75	0.9
F	$-1.37 \times 10^{-12*}$	$-3.35 \times 10^{-12}$	$-2.54 \times 10^{-12}$	$1.82 \times 10^{-11**}$	$1.77 \times 10^{-11*}$
d2006sep	-4.37	-0.32	16.78**	5.08	-1.55
M2GR	1.06***	1.19***	0.79 ***	-0.34	-0.22
CPI	1.74 ***	2.03***	2.26***	0.43	-0.16
NEXGR	0.035	0.068**	0.092 *	0.08	0.027
CNY	-45.23	-42.05***	-30.22*	-30.17***	-28.51***
Constant	237.96 ***	181.9 ***	73.48 ***	294.64***	352.23 ***
Pseudo R2	0.770	0.782	0.747	0.663	0.659

Note: the bootstrapping standard error is calculated in 20 times.

Based on our unique data set of the China housing market at the national level, we can also conclude that herding formation is stronger in increasing markets than

in declining markets. When the markets are turning turbulent, in the high quantile regression, there is a significant influence from short-term capital flows. The results support the asymmetry of herding behavior in increasing and declining markets.

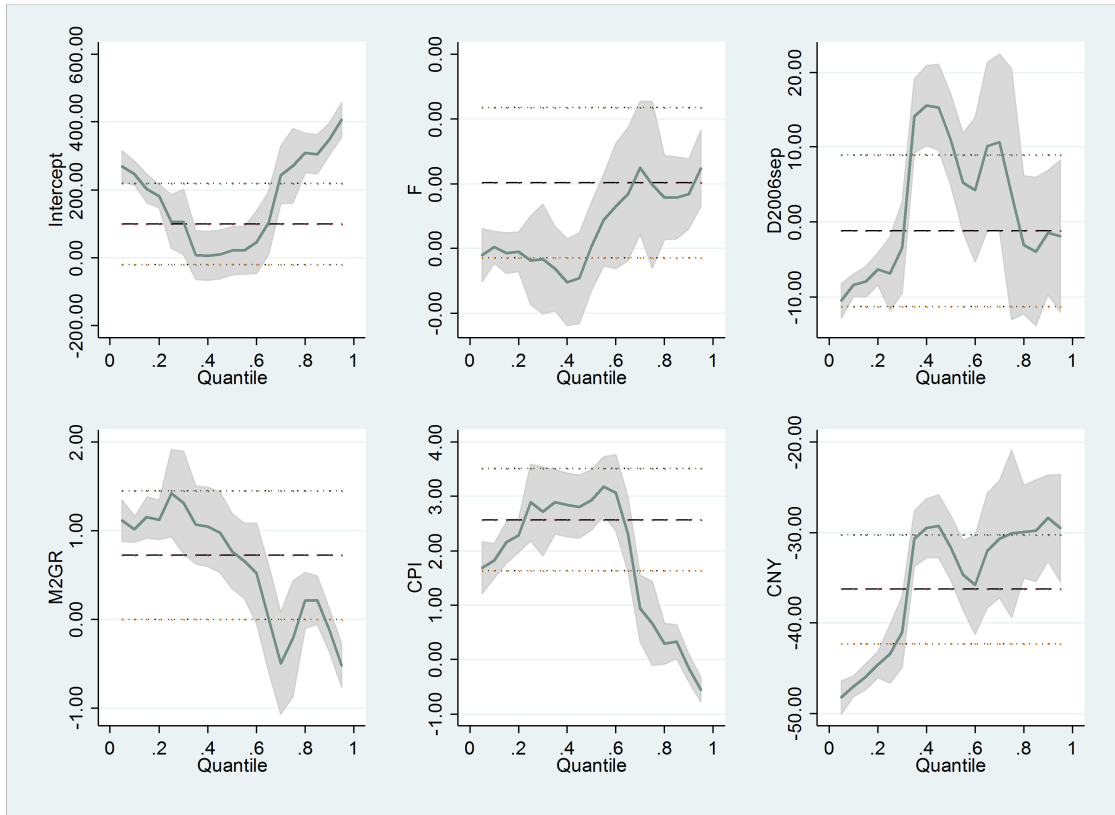
That investors in Chinese residential housing markets tend to herd is evident in the quantile regression when the housing market is increasing. Our quantile regression provides empirical evidence on investigating the abnormally increasing prices of the housing market in China. The policy analysis has not been captured in quantile regression before. This chapter applies QR to the Chinese housing price index with a focus of short-term capital flows.

### 3.5.2 Results

Figure 3.2 and 3.3 present the coefficients across different quantiles of the housing price index and volatility. We can see the differences in coefficients clearly from the graph, especially for the hot money  $F$  and the regulation policy dummy  $D2006sep$ . The results in Figure 3.2 show that the regulation policy is more effective for the median price level at its peak at the 40 percentile of the distribution of housing prices.

Figure 3.3 indicates that the effects at the right tail, which means at a higher volatility level of housing prices, are much larger. For example, hot money creates more volatility when the housing market is volatile. On the other hand, the policy becomes more effective in stabilizing the housing market at the 0.8 quantiles. The volatility changes are more relevant in the extreme situations.

The advantage of quantile regression is that it helps illuminate the effectiveness of regulation policy and indicates when would be the optimal time to issue a policy. For the housing market, policies like 2006 CAR would stabilize the housing market more efficiently during the volatile period. The effect of cooling down the hot housing market is larger during the extreme housing price periods.

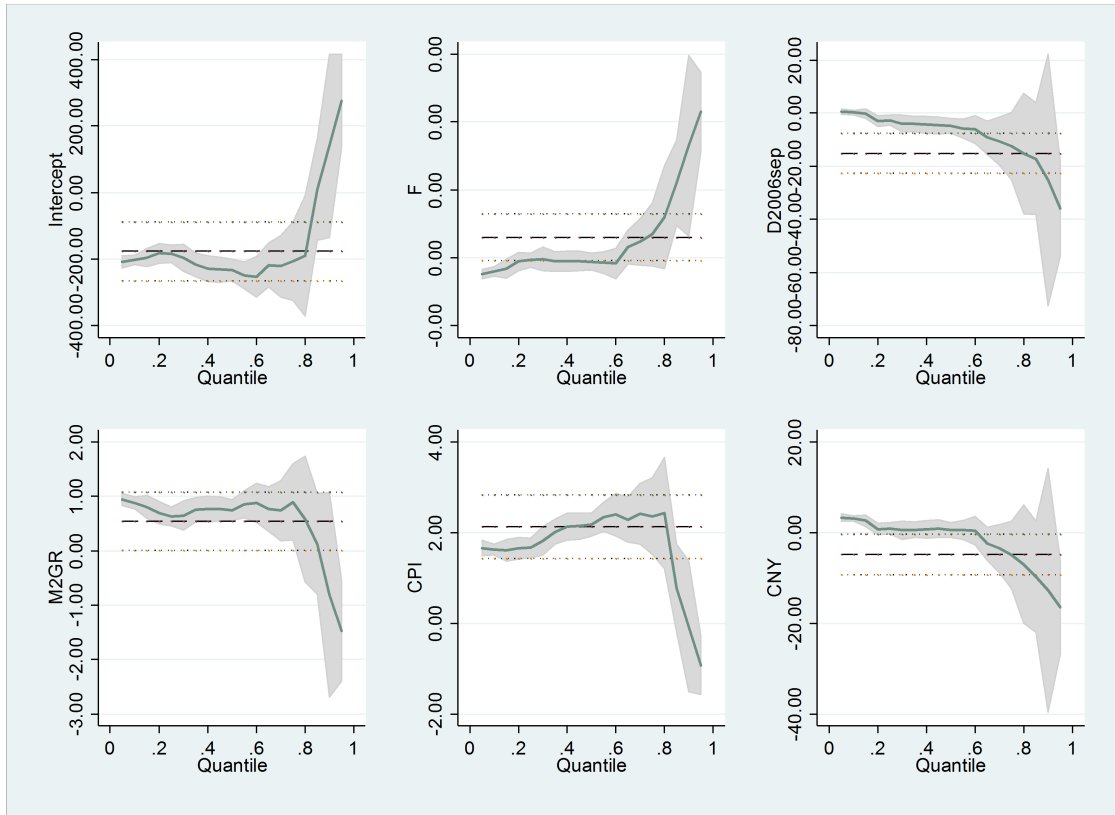


**Figure 3-2:** QR: Housing Price Index

In sum, short-term capital flows affect the housing market and create housing bubbles. Hot money not only increases the level of the housing price but also makes the market more unstable, especially when the fundamental economy is volatile. Regulatory policies on the housing market are effective in stabilizing the market, i.e., reducing volatility but have no significant impact on reducing the price level.

### 3.6 Conclusion

This chapter examines the extent to which external financial flows also have an impact on Chinese housing prices and their volatility. It finds that short-term capital flows from abroad had a modest impact on price level increases in the Chinese housing market, and a significant impact on increasing price volatility in the housing market.



**Figure 3-3:** QR: Housing Price Volatility

The quantile regression analysis shows that short-term capital flows magnify the impacts on capital flows on prices during upward surges in the housing price.

Our analysis shows that Chinese capital account regulations on foreign real estate purchases in 2006 had an impact on increasing the housing price and its volatility. Those policies did not appear to have an impact on reducing overall housing price levels, but there is a strong impact on reducing price volatility in the Chinese housing market. The effects of different price factors on different quantiles of the housing price index are mixed.

Although the Chinese housing bubble has appeared to ebb to some degree, this chapter empirically provides quantitative analysis of China's ongoing discussions regarding capital account liberalization. Quantile regression helps to evaluate the im-

pact of capital flows on housing prices. The contribution is to distinguish the effects of hot money at different states of the housing price cycle.

One issue this chapter does not cover is measuring the time period for a policy to take effect. The dynamics of when the policy works are beyond the scope of the research question. This chapter assumes the policy effect is constant over the time period it is implemented. However, one policy might be effective in a certain period of time or change afterward. Quantile regression and its applications require careful interpretations.

# Appendix

## Survey Data Details

Survey data has an intuitive advantage of direct measurement from questionnaires but also suffers from asymmetric information with noise. As technology develops and research facility improves, more and more survey data and field experiments are able to be carried out efficiently and effectively. In addition, an increasing number research analysis on survey data awaits econometric tools and quantitative skills in discovering these questions.

The first chapter in this dissertation is related to investor sentiment data and this appendix offers more details about the sentiments. The survey question has been asked every week to individual investors. This is shown through an email link with the question on investor sentiments. It has three options and the results are collected every Wednesday and released every Thursday. I also list a comparison of different sentiment data and discuss details of concerns regarding the survey data.

One of the concerns in the survey data: Who are these potential investors taking the survey and reporting their sentiments (public/inner information accessible)?

The Index for Consumer Expectations is from the monthly Survey of Consumers. This is an ongoing nationally representative survey. It is based on approximately 500 telephone interviews with adult men and women living in households in the coterminous United States. The sample is designed to maximize the study of change by incorporating a rotating panel sample design in an ongoing monthly survey program. For each monthly sample, an independent cross-section sample of households is drawn.

The bull-bear spread is taken from individual investors. It is assumed that investors armed with effective investment education materials and a bit of dedication could outperform the popular market averages. Over thirty years later, 150,000 members of American Association of Individual Investors (AAII) report investment returns that are consistently higher than those of the stock market as a whole. Further AAI Shadow Stock Portfolio (a real-money portfolio used to teach members about investing) has lower risk scores and better returns than the S&P 500 for the last 10 years. These members of AAI includes individual investors who are part-time financial investors, CFOs and institution investors, or market researchers.

The survey questions are cited from the University of Michigan Survey Research Center on Economic Behavior Program and Survey of Consumer Attitudes and Behavior in January 2014.

The survey was undertaken to measure changes in consumer attitudes and expectations, to understand why such changes occur, and to evaluate how they relate to consumer decisions to save, borrow, or make discretionary purchases. The data regularly include the Index of Consumer Sentiment, the Index of Current Economic Conditions, and the Index of Consumer Expectations.

Here is a selection of questions related to the expectations and sentiments.

1) What do you think the stock market will be in the next six months? (Go up, stay neutral, go down)

2) Looking ahead, do you think that a year from now you (and your family living there) will be better off financially, or worse off, or just about the same as now?

3) For business conditions in the country as a whole, do you think that during the next twelve months we'll have good times financially, or bad times, or what?

4) Looking ahead, which would you say is more likely, that in the country as a whole we will have continuous good times during the next five years or so, or that we

will have periods of widespread unemployment or depression, or what?

Based on the responses, the weighted average and spread difference are calculated to represent the index for expectation and investor sentiments. The concerns of survey data come from both textual side and non-textual side, which are beyond the econometric scope of this chapter. However, a large sample size can overcome some disadvantages of the survey and represent the whole population.

Sentiment questions are given with a time indicator such as six months ahead. The stock returns used in the baseline model are six months ahead based on the sentiment questions. Investors respond based on their own understanding of the future.

Different frequencies of the data (weekly, monthly or quarterly) at different starting times might introduce additional noise to the empirical analysis. The survey of investor sentiments is used as a direct measure of expected returns and match with returns at a weekly frequency. The strong negative correlation between investor expectations and macroeconomic uncertainty shows that investors take macroeconomic fundamentals and risks in forming investment expectations.

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# CURRICULUM VITAE

