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MATHEMATICS CONTENT COURSES FOR PREPARING ELEMENTARY TEACHERS: CURRICULUM AND INSTRUCTION

by

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DEDICATION

For Matthew and Kennedy Callis

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MATHEMATICS CONTENT COURSES FOR

PREPARING ELEMENTARY TEACHERS:

CURRICULUM AND INSTRUCTION

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ABSTRACT

Mathematics content courses for prospective elementary teachers have the potential to increase future teachers' mathematical knowledge for teaching as well as model high quality instructional practices. This study investigated the instructional practices and curriculum usage of instructors of elementary mathematics-for-teaching courses. This mixed-methods study included a nationwide survey of instructors to identify the instructional practices and curriculum used in these courses. Additionally, this study compared the difference in reported use of instructional practices by survey participants' academic and professional background characteristics. Two case studies of instructors who used instructional materials developed by the Elementary Pre-service Teachers Mathematics Project (EMP) were also conducted to more deeply describe instructional practices and use of curriculum materials in these courses.

Results from the Instructional Practices and Curriculum Use (IPCU) survey (n = 458) indicate that college instructors of mathematics content courses for elementary teachers report using instructional practices supported by research and policy recommendations at higher levels than previous studies on general college STEM courses would suggest. In particular, survey participants reported using instructional practices

such as engaging students in mathematical practices, attending to mathematical knowledge for teaching, pursuing students' ideas, sharing mathematical authority with students, and supporting student-to-student interaction. Use of lecture, small groups, formative assessment, practices that lower cognitive demand, and efforts to achieve active participation varied substantially. The use of these instructional practices varied according to these characteristics, such as the subject and level of a participant's terminal degree, their appointment to a mathematics department versus a school of education, their experience teaching in PreK–12 schools, at statistically significant levels. This study suggests that the common perception of mathematics content courses for pre-service elementary teachers as remedial and dominated by lecture is not the norm.

Analysis of the case studies identified four ways that the participants used the EMP curriculum materials to create mathematically powerful experiences for their pre-service teachers. The case study instructors used the materials to (1) prompt pre-service teachers to examine and use mathematical relationships, (2) hold pre-service teachers responsible for engaging in rigorous mathematical work, (3) assess and make use of pre-service teachers' thinking, and (4) support pre-service teachers to use mathematical language. The elements of the curriculum that supported the case study instructors were identified at the overall programmatic level, the unit and lesson level, and at the individual problem level. This study demonstrates that curriculum materials can support instructors in using research-based instructional practices, but the design of the materials impacts how instructors are able to use the materials to create mathematically powerful experiences for their students.

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CHAPTER 1:

Section I: Introduction

Research has identified instructional practices in mathematics that result in learning that is long-lasting, generative, and supports the development of productive dispositions (Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007). Schoenfeld (2014) has codified much of this knowledge about best practices into five categories that he labels as the “five dimensions of mathematically powerful classrooms.” Classrooms that exhibit these five dimensions may be more likely to lead to robust student learning and productive disposition. These dimensions are 1) attending to high quality mathematical content, 2) selecting cognitively demanding problems and maintaining cognitive demand during implementation, 3) using strategies to support equitable participation, 4) allowing students to share in mathematical authority, and 5) using formative assessment to inform instruction. Classes that exhibit these five dimensions feature opportunities for students to develop their identities as doers of mathematics by pursuing their own ideas and conjectures.

It appears that much of the mathematics instruction in the United States fails to employ these practices, both at the K–12 and university levels. Instead, lecture is the norm (e.g., Walczyk, & Ramsey, 2003) and most of the mathematical work is done by the teacher, not the students (e.g. Hiebert et al., 2005). Pre-service elementary teachers (PSTs), as a subset of the American public, are similarly likely to experience instruction that focuses on procedures and facts instead emphasizing conceptual understanding or engagement in mathematical disciplinary practices. As a consequence, many PSTs have a rote understanding of elementary mathematics concepts (e.g. Newton, 2008). They have had few experiences engaging in mathematical practices around elementary mathematics content (Simon & Blume, 1996; Stylianides, 2007). Many PSTs hold a vision of mathematics teaching and learning that does not align with research-supported instructional practices (e.g. van Es & Conroy, 2009; Walkowiak, Lee, & Whitehead, 2015). Without intervention during their post-secondary education, elementary teachers may continue this cycle of instruction characterized by teaching-as-telling with a focus on executing procedures without conceptual understanding.

University-level mathematics content courses for elementary teachers that employ instructional practices that support the dimensions of mathematically powerful classrooms have the potential to influence pre-service teachers' future teaching in two important ways. First, these practices can serve as a model for future teachers for their own instruction with elementary students. Indeed, there is some research suggesting that when instructors use these practices in their content courses, PSTs develop more productive beliefs about teaching mathematics in elementary grades. For example, PSTs placed more importance on group work and discussion as opposed to direct explanation after completing a content course where the instructor shared mathematical authority with PSTs (Spielman & Lloyd, 2004). Second, since the goal of these dimensions is to assist learners in becoming more mathematically proficient (Schoenfeld, Floden, The Algebra Teaching Study, & Mathematics Assessment Project, 2014), mathematics content courses for elementary teachers that use these practices may assist PSTs in developing a deeper understanding of mathematics. Stronger knowledge of elementary mathematics among teachers has been linked to elementary students learning more mathematics (Hill, Rowan, & Ball, 2005). Therefore, using instructional practices that support mathematically powerful classrooms in mathematics content courses for elementary teachers can lead to greater elementary student learning by both improving future teachers' mathematical knowledge and by providing them with a vision of high quality mathematics instruction.

Little is known about the kind of instruction PSTs receive in these specific college-level mathematics content courses. One research study suggests that there are some instructors who actively engage PSTs in doing mathematics during class time (McCrory, Zhang, Francis, & Young, 2009), but the details are limited. There is evidence that some instructors primarily lecture to PSTs, reinforcing the notion of teaching as telling (Hart, Oesterle, & Swars, 2013; Masingila, Olanoff, & Kwaka, 2012). More robust detail on the kinds of practices instructors use in these courses can inform efforts to improve instruction. Identifying practices currently used and practices that are neglected, along with connections to instructors' backgrounds, can inform staffing decisions and professional development efforts for these courses. This knowledge can help inform professional development efforts targeted at instructors with different academic and professional backgrounds.

While we know very little about the type of instruction PSTs experience in these college-level mathematics content courses nationwide, we do know that not all instructors have PSTs actively doing mathematics during class time (Hart et al., 2013; Masingila et al., 2012; McCrory et al., 2009). Moreover, professional development, a potential lever to improve college instructors teaching practice, is rarely provided for these professors

(Masingila et al., 2012). Curriculum may be one way to improve PSTs' experiences in these courses.

In and of itself, curriculum is an important part of these courses. McCrory and colleagues (2009) found that courses using curriculum specifically designed for mathematics content courses for pre-service elementary teachers result in PSTs learning more mathematics for teaching when compared to courses that use textbooks designed for a general mathematics course or self-created materials. In addition, a curriculum can be educative (Davis & Krajcik, 2005) and may help college instructors improve their instruction. At the elementary level, there is evidence that teachers can learn from curriculum and as a result change their practice (e.g. Collopy, 2003; Remillard, 1991; Remillard & Bryans, 2004). However, the relationship among written curriculum, teachers' learning, and instructional change is extraordinarily complex. Ball and Cohen (1996) suggest that curriculum can influence instruction because of its close tie to the daily work of instructors and its ubiquitous presence (Ball & Cohen, 1996).

Most of the research on instructors' use of and learning from curriculum is at the elementary or middle grade levels. College instructors are different from elementary teachers; they have different academic backgrounds and pedagogical preparation and they work in different contexts (Mesa & Griffiths, 2012). Therefore, their use of textbooks is likely to be different. Furthermore, college level textbooks are designed differently from elementary textbooks. While it is not uncommon for Standards-based elementary textbooks to have teacher editions that include pedagogical advice, college-level instructor editions rarely do. Instead, they typically are identical to student textbooks with the addition of answers or worked solutions to problems. Elements that provide pedagogical advice may be misused or unnoticed by college instructors if they are not familiar with these textual features. Indeed, another issue that has been documented at the elementary level is the inappropriate use of curriculum features (Collopy, 2003).

While there is some research on college instructors' use of textbooks, these studies either quantify the percentage of mathematical ideas covered in a text that are addressed in class (Lo, Kim, & McCrory, 2008) or focus on first-year college mathematics courses, in general, rather than mathematics content courses for elementary teachers (Mesa & Griffiths, 2012). Whether or how curriculum can influence a college instructor's instructional practices remains an unanswered question. Indeed, if the link between textbooks and instructional practices can be sufficiently articulated and supported by empirical data, then strategic design of curriculum materials may be one way to help instructors provide powerful mathematical experiences for prospective teachers in college-level mathematics courses. To this end, this study aimed to answer two research questions:

1. What are the instructional practices and curriculum resources used in college-level mathematics content courses for pre-service elementary teachers? How do these differ by instructor characteristics, if at all?
2. How can a curriculum for college-level mathematics content courses for pre-service elementary teachers support instructors in creating mathematically powerful experiences for prospective teachers?

Section II: Definition of Terms

Powerful Instructional Practices

The phrase "powerful instructional practices" extends from Schoenfeld and colleagues' (2013, 2014) notion of mathematically powerful classrooms. Schoenfeld and colleagues started using the phrase "mathematically powerful classrooms" when developing the Teaching for Robust Understanding of Mathematics (TRUMath) Rubric. Powerful instructional practices are instructor actions that coincide with classroom episodes that score high on the TRUMath Rubric in one or more of the five dimensions the researchers identified as leading to mathematically powerful classrooms. The five dimensions are: mathematics; cognitive demand; access to mathematical content; agency, authority, and identity; and uses of assessment.

Mathematics. The *mathematics* dimension measures the extent to which the mathematics taught in the class portrays mathematics as a coherent discipline that can be figured out through sense-making, as opposed to a collection of isolated facts and procedures. The dimension also measures to what extent students are engaged in mathematical practices, such as reasoning and problem solving.

Cognitive demand. Related to the *mathematics* dimension, *cognitive demand* measures whether the tasks in which students are engaged consist of practicing procedures or doing mathematics and making connections among concepts. This dimension further measures *who* is engaged in high-level mathematical work, the instructor or the students. If the instructor is doing all of the work by presenting new mathematical ideas and explaining solution strategies that students then follow, the students have no opportunity to engage in productive struggle themselves. Such a class would rate low in the dimension of *cognitive demand*.

Access to mathematical content. Research has shown that, even within the same classroom, not all students have the same opportunities for participation and engagement with mathematical content (e. g. American Association of University Women & National Education Association, 1992). The dimension *access to mathematical content* measures the distribution of opportunity in the classroom. Namely, it measures whether all students have an opportunity to engage in rich mathematics or if only a few are actively engaged.

Agency, authority, and identity. This dimension measures the extent to which students are responsible for generating and validating the mathematical ideas studied in the classroom, and the degree to which the instructor recognizes the students as authors of ideas. Such classrooms are in contrast to those where the instructor or the textbook is the main source of ideas and of mathematical authority.

Uses of assessment. In many college math classes, most assessment activities are summative and are used to evaluate student progress, rather than to inform instruction. Conversely, formative assessment involves instructors making decisions based upon student responses. This dimension measures the extent to which instructors solicit, build upon, and address student ideas during instruction.

Mathematics Content Courses for Pre-Service Elementary Teachers/ Elementary Mathematics-For-Teaching (MFT) Courses

In this study, a mathematics content course for pre-service elementary teachers refers to a course designed to examine K–8 mathematics topics in depth. Topics usually involve a subset of the following: whole number operations, whole number algorithms, fractions, decimals, geometric measurement, geometric ideas, and occasionally statistical or algebraic ideas (Blair, Kirkman, & Maxwell, 2013). The emphasis of content courses is to assist future teachers in deeply understanding the content they will teach and building connections between these mathematical topics. Ideally, the methods of teaching elementary mathematics would be addressed in a separate course; however, the reality is that many institutions only have one course, where instructors must address both methods and pedagogy (Greenberg, Walsh, & McKee, 2014). Therefore, the survey used in this study captured data both from instructors of content-only courses and content-and-methods courses. These courses are also referred to as “elementary mathematics-for-teaching (MFT) courses.”

Section III: Justification

Improving the mathematical education of future elementary teachers is essential in order to improve the mathematical learning of K–12 students. Research has shown that teachers’ mathematical knowledge is related to student learning (Campbell et al., 2014; Hill et al., 2005). Whereas particular instructional practices are thought to support greater mathematical learning and productive disposition (Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007; National Research Council, 2001; Schoenfeld et al., 2014), U.S. teachers are unlikely to have experienced learning mathematics through such methods (Hiebert et al., 2005a). Mathematics content courses for pre-service teachers that focus on the deep ideas underlying elementary content and use powerful instructional practices can contribute to improving pre-service teachers' future work with their elementary students in two ways. First, these courses can deepen pre-service teachers' mathematical knowledge. Second, these courses can help pre-service teachers develop a vision for teaching mathematics through the use of powerful instructional practices.

Policy documents continue to call for more effective instruction in the mathematical preparation of elementary teachers (Conference Board of the Mathematical Sciences, 2012; National Research Council, 2012, 2015). In order to effectively improve instruction in such content courses, a better understanding of the kind of instruction currently occurring in these courses is necessary. The existing research on these courses is limited. Typically, research has focused on what types of courses are required or offered for pre-service teachers (Blair et al., 2013; Greenberg & Walsh, 2008; Lutzer, Rodi, Kirkman, & Maxwell, 2007) or who teaches these courses (Masingila et al., 2012). Some of these studies have also reported on instructional practices generally, such as the use of lecture (Masingila et al., 2012) or the percent of class time spent in different class structures such as individual, small group, or whole class formats (McCrory, Francis, Young, & Hall, 2008). Some studies have described general PST and instructor experiences (Hart et al., 2013) or broad factors that impact PST learning, such as use of class time and textbooks (McCrory, Francis, et al., 2008). There are few details about instructional practices in these studies. It is unknown, for instance, whether instructors tend to present mathematical ideas to PSTs or if they engage PSTs in tasks that help them make connections themselves. It is unknown if instructors solicit or encourage broad participation among the PSTs in their courses. A more comprehensive understanding of instructional practices in elementary mathematics for teaching courses can inform efforts to improve instruction in these courses.

One method for improving instruction could be through the use of curriculum materials. There is evidence that using textbooks that are specifically designed for mathematics content courses for elementary teachers results in PSTs learning more mathematics for teaching (McCrory et al., 2009). Yet there is very little research on how instructors use such textbooks. Studies on curriculum in mathematics content courses for elementary teachers have tended to focus on the mathematical topics addressed and the types of textbooks selected for such courses (Greenberg & Walsh, 2008; McCrory, 2006). One study (Lo et al., 2008) that did look at textbook use focused on factors that may have led instructors to add or omit different mathematical examples or topics, but it did not describe how instructional practices were or were not related to the instructors' use of the textbook. This focus is especially needed for mathematical content courses for elementary teachers because the textbook is often the instructor's primary resource (McCrory, Francis, et al., 2008). Instructors for these courses are often mathematicians without training in education or experience with elementary-aged children (Masingila et al., 2012; Walczyk, Ramsey, & Zha, 2007). Indeed, many mathematicians educating teachers admit that their knowledge of K–12 school contexts or pedagogical strategies is inadequate (Hart et al., 2013; Hodge, Gerberry, Moss, & Staples, 2010). Furthermore, mathematicians typically do not participate in professional development focused on improving instruction (Walczyk et al., 2007). Masingila and colleagues (2012) noted that professional development specific to elementary mathematics for teaching courses is rarely available. One study (Mesa & Griffiths, 2012) of mathematicians using curriculum in first-year college mathematics courses found that textbooks were an important tool for these instructors. Thus, since mathematicians' reliance on textbooks may be considerable, identifying how curriculum resources can support instructors in using powerful instructional practices may contribute to improved PST learning outcomes.

This study provides insight into the instructional practices used in mathematics content courses across the country. Policy documents have called for improvements in instruction in mathematics content courses for elementary teachers, but without knowing the current practices of instructors, it is difficult to plan for improvement. Furthermore, while policy documents call for these content courses to be taught by mathematicians (Conference Board of Mathematical Sciences, 2001), there is no empirical evidence – and indeed some evidence to the contrary (Chapin, Feldman, Salinas, & Callis, in review) – that instructors with advanced degrees in mathematics and appointed to mathematics departments are inherently better prepared to teach these courses. Information gathered from this study will better support decision-making on who should teach these courses. In addition, the results of this study may inform the development of curriculum materials that support college instructors in using practices that can both improve PSTs' mathematical understanding and serve as a model for their future work with elementary students.

In summary, this study contributes to the research on instruction in mathematics courses for pre-service teachers and the corresponding curriculum materials that support university instructors in their efforts to teach pre-service teachers. In addition, faculty members, curriculum developers, and curriculum selection committees may benefit from knowing how textbooks can support instructors in engaging in powerful practices. Research on curriculum use and instructional practices in elementary mathematics for teaching courses can contribute to improving the mathematical preparation of future elementary teachers.

This chapter has introduced the research questions, provided definitions for terms and provided a justification for the research. In Chapter 2, I will detail the research on the mathematical education of elementary teachers, with a particular focus on mathematics content courses specifically designed for this population. I will also detail the research on instructors' use of curriculum. In Chapter 3, I will outline the line of inquiry for this research, including the data collection and analysis methods. Chapters 4 and 5 will discuss the results, and Chapter 6 will summarize the overall findings from this study.

CHAPTER II: REVIEW OF THE LITERATURE

In mathematics education, curriculum materials have long been used to impact student outcomes, change instruction, and improve teachers' learning (Ball & Cohen, 1996). However, the relationship between curriculum materials and instruction is complex. This chapter first describes the theoretical and conceptual frameworks used in this study. Next, I describe the factors based on the research on K–12 curriculum that influence how teachers use and learn from curriculum. I then detail the research studies on the use of curriculum in higher education, both in mathematics classes generally and in mathematics content courses for elementary teachers specifically. Finally, I present the research on mathematics content courses for elementary teachers.

Section I: Theoretical Framework

This study has been informed by Remillard's (2005, 2011) research on curriculum. In her 2005 work, she noted that there are four potentially overlapping paradigms in research on curriculum and its use. The first is the *Following or Subverting* paradigm, which is oriented toward measuring fidelity with the assumption that an objective curriculum exists. The second paradigm is the *Drawing On* paradigm, which portrays teachers as active curriculum designers, using curriculum as one of many resources. The third paradigm is the *Interpreting* paradigm, which conceptualizes the use of curriculum through reading theory; it recognizes that teachers bring their beliefs, orientations, and knowledge to the act of reading curriculum. The *Interpreting* paradigm also notes that teachers "read" or interpret their students' reactions to curriculum and to instruction, which then impacts their further interpretation of curriculum. Lastly, the *Participating With* paradigm portrays curriculum as an artifact or tool and draws on sociocultural theory. This dissertation is rooted in the *Participating With* paradigm.

The *Participating With* paradigm does not portray written curricula as an exact representation of practice that should, or even can be, matched by a teacher when enacting curricula with students. Instead, researchers using this paradigm view curricula as tools. Like other tools created by humans, curricula are used to accomplish goals. The functional capacity is distributed between both the tool and the user of the tool; neither can be considered in isolation. For example, teachers may use problems in a mathematics textbook in a variety of different ways. They could use problems to introduce new ideas to students, or they may assign problems for students to practice ideas that have previously been introduced. Teachers may choose problems from a textbook to help them assess student thinking, to help students make real-world connections, or to differentiate instruction. The design of the problems likely influences teachers' choices, but teachers bring their own goals to their work. Both the content of the problems and the ways in which the teacher uses the problems impact students' instructional experience. Neither the curriculum nor the actions of the teacher can be considered in isolation. Of the four paradigms, the *Participating With* paradigm is the one that Remillard believed assists researchers in thinking about both the written curriculum and the user of the curriculum in tandem.

Brown (2012) used a pole vaulting analogy to illustrate how curriculum might be considered a tool. The task of pole vaulting cannot be separated from the pole or the vaulter; the two must be considered together to make sense of the activity. Brown wrote, "As with the pole vaulter and the pole, a teacher's ability to enact a curriculum unit cannot be understood solely in terms of individual instructional capacity, since the activity is characterized by the sharing of functional capacity across both the teacher and the curriculum materials" (p. 20–21).

Brown named three ways that teachers use curriculum materials: offloading, adapting, and improvising. These terms are not measures of fidelity, nor is one use of curriculum inherently better than another. Offloading, adapting, and improvising describe the ways in which teachers share the functional capacity with curriculum materials. Teachers may offload some work by using a curriculum supplied handout or using the example problems in a textbook. By offloading some of this work on to the curriculum material, teachers are able to use their time and focus in different ways. As students work on a handout, teachers may confer with students individually. Instead of creating their own examples, teachers can use their time to anticipate student difficulties. Brown's study found that teachers may also offload some tasks onto the curriculum materials for areas that are outside their expertise, such as a setup of a particular experiment in a science classroom. A second way teachers may use curriculum materials is to adapt them to better achieve their own goals. In Brown's study, one teacher adapted a science lesson so that the students had to develop their own procedures for an experiment. Supporting students to develop their own scientific procedures was one of her instructional goals. Teachers can also both adapt and offload simultaneously. For instance, a teacher might use a particular problem in a curriculum, but adapt the context to a topic students might find more interesting.

A third way teachers use curriculum is improvising. In the moment, teachers improvise in response to students' reaction to the enacted curriculum. Teachers might improvise by addressing unanticipated misconceptions that arise during a lesson or to take advantage of an interesting idea a student has during a lesson. Improvisation in particular shows how the functional capacity is shared between the instructor and the written curriculum materials. Curriculum materials cannot anticipate all possible student responses, so teachers will necessarily have to improvise at one time or another. However, the student responses to which teachers react are a function of the written curriculum materials. Improvisation also demonstrates the difference between the *Participating With* paradigm and the *Following and Subverting* paradigm. While some improvisations may be more productive than others, improvisations, like adaptations, are not seen as deviations from the curriculum. Instead, they are one of the ways in which teachers share the capacity to create instructional experiences for students with written curriculum.

Teachers use curriculum materials in different ways, but the design of the materials can suggest different uses. Brown (2012) noted that tools are designed "with the capability to cue activity through constraints and affordances," (p. 20). Different textual features of printed curriculum materials suggest different uses. For example, annotations in teachers' editions may detail common student difficulties or they may simply list answers. The former suggests that teachers use the problems as formative assessment to determine how their students are thinking about concepts. The latter suggests teachers use the problems as summative assessment, an evaluation tool. The curriculum studied by Grant, Kline, Crumbaugh, Kim, and Cengiz (2012) included sample classroom dialog as a textual feature. The elementary teachers in their study used this feature to anticipate student thinking, develop questions to spark discussion, and create an image of classroom instruction in their minds.

Textual features of grades K–12 textbooks include a range of potential resources: worked examples for students, extra examples for instructors to present, questions to ask for leading a class discussion, and information on common student thinking to inform instructors, to name a few. However, a teacher's use of such textual features in a curriculum is not automatic. As Cohen, Raudenbush, and Ball (2003) noted, resources have to be noticed and used in order to impact instruction and learning. Likewise, these resources may be used inappropriately. For example, Collopy (2003) noted that one of the two teachers in her study read the sample dialogs in the teacher editions aloud to the students, despite directions that explicitly noted that the sample dialogs were for informing the teacher, not for in-class use. Collopy conjectured that teachers' beliefs about mathematics, mathematics learning, and about themselves as mathematics teachers all contributed to the ways in which the curriculum materials were used.

Thus, through the *Participating With* paradigm, curriculum and teachers are seen as impacting each other. The teacher, given his or her existing goals, conceptions, and context, perceives different resources within the written curriculum. In turn, the content and structure of the curriculum suggest particular uses and can be educative for the teacher.

Previous research has not examined college mathematics professors use of curriculum through the *Participating With* paradigm. One study (Lo et al., 2008) on college instructors' use of curriculum used the lens of fidelity and attempted to explain deviations from the curriculum. However, it did not look at the impact that the curriculum had on the instructor. Another set of studies (Johnson, Caughman, Fredericks, & Gibson, 2013; Johnson & Larsen, 2012; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007) described mathematicians' challenges in enacting inquiry curriculum, but these studies did not address how these instructors used the written text. While there is research on K–12 teachers' use of curriculum materials, college instructors are different from elementary teachers. In addition to differences in content and pedagogical knowledge, college instructors may have different understandings of curriculum use and genre-specific features. Furthermore, college-level curriculum is typically designed differently from elementary and middle school level mathematics curriculum. For example, elementary Standards-based curricula often have teachers' guides with pedagogical advice that are separate from the student textbooks. This practice is uncommon in college-level curriculum, where instructor's editions are nearly identical to student editions, only enhanced with answers and worked solutions. Therefore, a college instructor would have different expectations of curriculum features and may not attend to such features as pedagogical advice even if it appeared in their college-level textbook. The interplay, then, between the way the instructor enacts the curriculum and the affordances and constraints present in the written or multimedia curricular resources may be very different among college-level instructors compared to the K–12 teachers who have been studied in the past. This research study identified some instances of this dynamic relationship between the written and enacted curriculum in college-level mathematics classes designed for elementary teachers.

Section II: Conceptual Framework

This research used a modified version of Brown's (2012) framework to conceptually frame the study. Brown proposed the Design Capacity for Enactment Framework (Figure 2.1) to explain how the enacted curriculum is a result of the interplay between a teacher and a written curriculum. This framework highlighted both the resources of the teacher and the resources of the curriculum. Teachers' resources included their goals and beliefs. Teachers' beliefs included beliefs about the subject matter, teaching and learning the subject matter, about curriculum, and beliefs about their own particular students. Teachers' resources also included teachers' subject matter knowledge and their pedagogical content knowledge.

Curriculum resources in Brown's model included "physical objects," "procedures," and "domain representations" (p. 26). Brown studied teachers' use of science curricula, so physical objects, such as laboratory equipment, were particularly important to him. In mathematics classes, while physical objects such as rulers, calculators, manipulative materials, and graph paper are used during instruction, they may not rise to the same level of importance or saliency as a component of curricula. In Brown's explanation, procedures referred to directions for teachers, scripts of enactment, instructions, or problems for students to solve. Domain representations referred to explanations or representation of concepts.

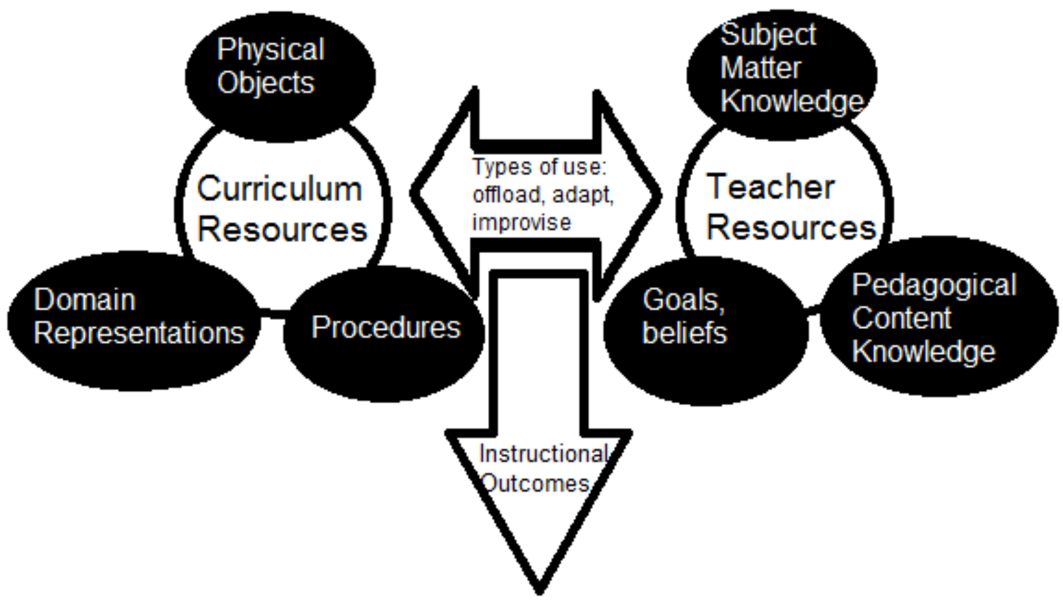

Figure 2.1. Brown's (2012) Design Capacity for Enactment Framework

Because Brown used this framework to describe the curriculum use of science teachers, further articulation of each of the curriculum resources in the context of mathematics courses for pre-service teachers is helpful. Domain representations referred to explanations of mathematical ideas, models, and representations. Because these courses concern mathematics *for teaching*, representations of elementary students' mathematical thinking would also be a domain representation. These could be found in the student editions of textbooks or in the Instructor's Guides. Some of these domain representations are on Power Point slides. Procedures include problems and activities that students engage with. They also include instructions to the instructor about how to facilitate the class. In this study, two of the curricula also had videos of the curriculum being enacted in other college courses, with commentary. These videos are also a component of the procedures. Physical objects relevant to this study include graph paper and three-dimensional solid blocks. Some of these physical objects were paper shapes that pre-service teachers cut out. The categorization of these different resources is less important than the specific resource being studied. For example, one problem (or procedure) resulted in pre-service teachers creating trapezoid cut outs (physical objects) in order to create a model that explained a mathematical formula (domain representation). Within the Instructor's Guide, there were sample questions to ask pre-service teachers (procedures) and different ways pre-service teachers might justify the formula (domain representations). This research study holistically analyzed the participants' use of this set of curriculum resources around this mathematical idea and problem and ones like it, rather than identify the category of the different resources. Similarly, this study investigated how instructors use curriculum resources and what elements supported them in their instruction. It did not label the curriculum usage as *offloading, adapting,* or *improvising*.

In this study, some adjustments to this model were used. First, "instructional outcomes," for specificity, were referred to as "instructional practices." This change clarified that it was the practices that were impacted, and not necessarily student learning outcomes, which involved other variables not present in the framework. Second, for clarity, "teacher" was changed to "instructor" to represent college faculty. Furthermore, mathematical knowledge for teaching teachers (MKTT, Olanoff, 2011; Superfine & Li, 2014; Superfine & Wenjuan Li, 2014) was added to Teacher Resources. Though it is not yet completely clear whether this knowledge is an extension, subset, or altogether different construct from subject matter knowledge (SMK) and pedagogical content knowledge (PCK), it is clear that instructors need and have knowledge of mathematics, pedagogy, and elementary contexts that goes beyond the knowledge of that needed by elementary teachers.

In addition, the framework was adapted to show the dynamic nature of instructor's use of curriculum. Curriculum materials are tools, and tools are used for enacting a practice. Just as the musician learns by listening to himself play sheet music, or a pole vaulter learns by reflecting on his performance, instructors learn by enacting a curriculum (Choppin, 2009, 2011; Remillard, 1999; Remillard & Bryans, 2004). Their knowledge, goals, and beliefs are changed or reinforced after noticing student responses to the enactment of curriculum. Therefore, after each iteration of enactment, what instructors notice or attend to in curriculum materials changes. While the published curriculum materials themselves do not change, the instructor's interpretation of them may. This phenomenon is made more explicit in Figure 2.2.

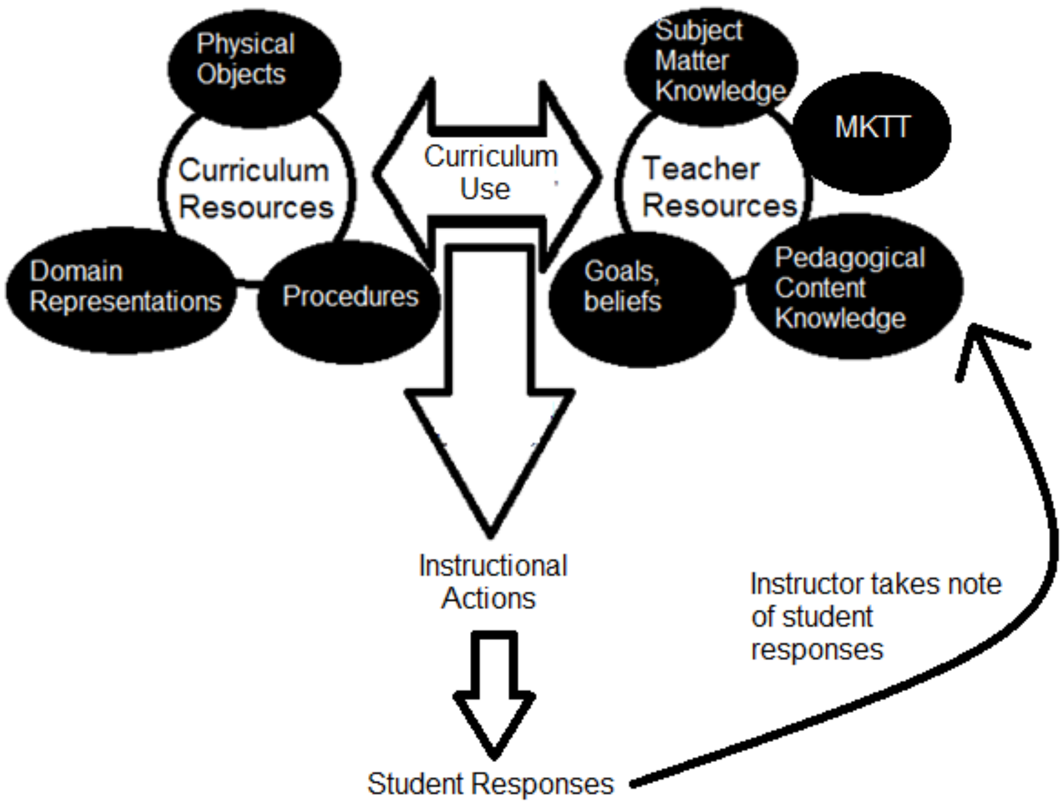

Figure 2.2. Dynamic Adapted Conceptual Framework

According to this dynamic framework, while curriculum resources have the potential to impact instructional practices, instructional practices also impact how instructors view curriculum materials. For example, instructors who establish norms where students explain their thinking have different opportunities to learn about their students and about the affordances of particular aspects of curriculum materials than instructors who do not use this instructional practice. This learning in turn impacts how instructors interpret, modify, or adapt curriculum materials in subsequent enactments.

Instructors of mathematics content courses for elementary teachers come with a wide range of backgrounds. Some have K–12 teaching experience while others do not; some have advanced degrees in mathematics while others have advanced degrees in education, psychology, or other fields (Masingila et al., 2012). Based on their past experiences, they come with different knowledge bases and different beliefs about what high-quality mathematics instruction looks like. Therefore, they may notice different aspects of curriculum materials in their planning and interpret them in different ways. Additionally, the curriculum materials themselves can also influence instructors' actions, beliefs, and knowledge. While curriculum materials can influence instructional practice, the instructor also has an existing repertoire of practices and works within a professional community that has existing norms and standards for teaching practices. Instructors with K–12 teaching experience may be more likely to have been socialized into teaching practices that actively engage students during class time, while mathematicians may have been socialized into teaching practices that focus on precise, well-articulated delivery of information (see also Schoenfeld, Thomas, & Barton, 2016). This study seeks to describe how curriculum materials might interact with these other factors to support powerful instructional practices.

Different instructional practices, in addition to the instructors' beliefs, knowledge, and goals, then impact what instructors notice from student responses to curricular enactment. Some instructors may attend to student affect, while others may attend to conceptual understanding, while still others may attend to students' developing abilities to engage in mathematical practices like justification. While goals and beliefs influence whether instructors notice these issues, these issues also require instructional practices that allow them to surface. For example, some questions posed to students may elicit evidence of conceptual understanding, but if instructors answer all false claims themselves, rather than giving the authority to the students, they may not have an opportunity to notice whether students improved their abilities to make sense of ideas. The next section, therefore, reviews the literature on instructors' learning from and use of curriculum materials.

Section III: Research on Instructors' Learning from and Use of Curriculum Materials

Several factors influence instructors' use of curriculum materials. Instructors' beliefs, their knowledge, and contextual factors impact they ways in which they use curriculum materials (Stein, Remillard, and Smith, 2007). The relationship between these factors and their use of curriculum differs according to whether they teach in K–12 schools or higher education institutions. This section first describes the research on K–12 teachers' use of curriculum materials and then describes the research on college instructors' use of curriculum materials.

One of the most important factors that impact curriculum usage is teachers' beliefs. Different beliefs are more salient at different moments; one belief does not govern teachers' entire practice. Researchers have found that teachers' beliefs about the nature of mathematics, about student learning, and about their own mathematical ability impact the ways in which they use curriculum materials. In turn, the opportunities they have for learning from enacting curriculum are impacted.

A study by Remillard (1991) demonstrated how different beliefs become salient and impact a teachers' use of curriculum materials from one instructional moment to the next. In this study, a third-grade teacher's beliefs about the nature of mathematics and the nature of knowing and learning mathematics led to two different actions. The teacher believed that knowing mathematics included knowing why a fact was true. Therefore, she would press her students for reasoning, even in instances where it was not called for in the curriculum script. This example of improvising (Brown, 2012) seemed aligned with a vision of high quality mathematics instruction supported by Remillard and the authors of the curriculum materials. On the other hand, the teacher also believed that there is one right answer in mathematics. This belief caused her to have selective hearing, only attending to correct student responses that could be found in the textbook. The teacher did not attend to students' incorrect answers or to correct answers that differed from the ones found in the text. This practice seemed less aligned with Remillard and the authors' vision of high quality mathematics instruction. In addition, Remillard commented that the failure to listen to and investigate students' responses limited the teacher's ability to learn about student thinking. Furthermore, because the teacher did not believe herself to be good at mathematics, she "abdicated authority for knowing" (Remillard, 1991, p. 3) to the textbook. That is, she did not further investigate the mathematics behind content that she did not herself understand.

Another study by Collopy (2003) demonstrated how teachers' beliefs in the nature of mathematics and of learning mathematics interacted with their beliefs about themselves as teachers and doers of mathematics. One of the elementary school teachers in this study believed that being “good” at mathematics meant being able to solve problems quickly and efficiently. She saw herself as a skilled teacher and doer of mathematics. Together, these beliefs led her to implement a Standards-based curriculum procedurally in order to help students solve problems in ways that fit her beliefs. She eventually abandoned the text all together. Conversely, another teacher in the study embraced the same text and learned from it. She did not have strong beliefs about mathematics. Instead, her actions were tied more closely to her belief in the necessity of making learning engaging. Because she believed that her own mathematical abilities were weak, she was willing to more thoroughly engage with and learn from the curriculum.

Interestingly, teachers’ beliefs about curriculum, in general, may supersede teachers’ beliefs about the nature of mathematics. Remillard and Bryan (2004) studied eight elementary teachers’ use of a Standards-based curriculum. The teachers who were suspicious of published curriculum materials, in general, were less likely to use the curricula to the same extent. This phenomenon was true even when a teacher’s beliefs about mathematics and learning mathematics aligned with those of the printed curriculum. The researchers described one teacher, Ms. Reston, and her use of curriculum materials to illustrate this idea. Ms. Reston’s beliefs about mathematics teaching aligned with the curriculum, but she distrusted published curriculum materials. She had developed her own repertoire that she felt supported her students in learning mathematics. Ms. Reston did not see that the curriculum had anything new to offer. She used the curriculum as a source of activities but did not use the instructor guides to organize the structure of her lessons. The researchers concluded that her limited use of curriculum materials resulted in fewer opportunities for the teacher to learn from the curriculum materials.

The way in which a teacher enacts curriculum also depends upon his or her knowledge. For example, in Remillard's (1991) study, the topics one teacher understood conceptually, she taught conceptually. Conversely, she taught procedurally when using representations with which she was uncomfortable. In her later work, Remillard and her colleagues conceptualized this kind of knowledge as "Curriculum-Embedded Mathematical Knowledge," and defined it as the knowledge needed to make sense of mathematical representations, problems, and other features found in K–12 curricula (Kim & Remillard, 2011).

Similar to the importance of teachers' curriculum embedded mathematical knowledge is their mathematical knowledge for teaching (MKT), both at a general and at a specific level, in terms of how they enact curriculum. Hill and Charalambous (2012) conducted a cross-case analysis of middle school teachers enacting a Standards-based curriculum. The researchers found that the teachers' score on Learning Mathematics for Teaching (LMT), an assessment to determine teachers' MKT, test items was related to the quality of their instruction. Teachers with high overall scores or high scores on items directly related to the observed lessons also had higher quality enactments as determined by the researchers. Specifically, these teachers were better able to make sense of and build upon student thinking. Their explanations were clearer and they used a higher level of mathematical language. The adaptations that these teachers made to the curriculum resulted in situations where students could make sense of mathematical ideas. In contrast, teachers with low content knowledge made more mathematical mistakes, provided unhelpful explanations or metaphors, or allowed students to engage in unproductive explorations. The one exception to this trend was a teacher who scored relatively high on mathematical knowledge for teaching, but who disagreed with the philosophy of the curriculum and held a procedural view of mathematics. This teacher used the tasks, but she taught the content procedurally rather than focusing on meaning-making or exploring multiple methods. Hill and Charalambous' (2012) findings are corroborated by other studies on middle school teachers. For example, Wilhelm's (2014) large quantitative study found that mathematical knowledge for teaching and having a vision of high-quality mathematics instruction were linked to maintaining the cognitive demand of instructional tasks. Thus, while knowledge is important, orientation toward curriculum materials and beliefs about mathematics are also important.

Choppin (2009, 2011) identified a broader kind of knowledge specific to a curriculum, one that extended over multiple lessons or units. He coined the term *curriculum-context knowledge*. This type of knowledge is about how particular components of a curriculum function to impact the mathematical thinking of a particular set of students. To illustrate this concept, Choppin described how a set of middle school teachers using a new edition of a Standards-based curriculum developed their knowledge through several adaptations of the lessons. For example, through listening to her students, one teacher developed her pedagogical content knowledge regarding the power of rectangles for helping her students understand the distributive property. Another teacher developed knowledge about a particular unit in the curriculum. During the first enactment, this teacher pressed students to develop the general form of an exponential equation in the first lesson of the unit. Witnessing the students' confusion, she adapted the lesson in future enactments to make the connection clearer, to no avail. Eventually, she realized that the materials were designed so that students would come to develop the general form after the third lesson of the unit. She began to see how the parts of the unit worked together to impact students' mathematical thinking. This ability to see how curriculum materials impact student thinking can influence how instructors use curriculum materials.

The way teachers enact curriculum materials can also impact their own opportunities for learning. Choppin (2009, 2011) found that teachers who made minimal adaptations to curriculum materials in their first enactments were better able to develop their curriculum-context knowledge by attending to students' responses. This practice enabled them to learn how the different components of the curriculum impacted student thinking. Those who adapted tasks in the first enactment were more likely to make subsequent adaptations that failed to support the teachers' stated mathematical goals for students. This failure occurred even though these teachers were able to identify the mathematics behind different components of the curriculum and the curriculum designers' intent. Furthermore, the teachers who heavily adapted the curriculum in the first iterations expressed frustration at not being able to see how the components impacted student thinking.

Similarly, Remillard and Bryans (2004) also found that opportunities to learn through enacting curriculum depended upon the extent to which teachers enacted the curriculum. The researchers categorized teachers as 1) using materials in an intermittent and narrow way, 2) adopting and adapting curriculum materials, or 3) thoroughly piloting curriculum materials. Teachers who use the curriculum in an intermittent and narrow way had few opportunities to learn through interacting with the curriculum. When teachers used materials in an intermittent and narrow way, they typically only used curriculum components that aligned with their existing practice. When teachers adopted and adapted, they adapted material to fit with their existing teaching practice and adopted suggested routines or teaching practices that aligned with their existing goals and beliefs. Typically these teachers did not attend to teachers' guides because they viewed curriculum as a source of activities. Teachers who were "thorough piloters" read the teachers' guides closely and tried to enact the lessons according to these guides. Those who adopted and adapted or thoroughly piloted had opportunities to expand their repertoire of instructional activities. Thorough piloters also had the opportunity to gain insight into student thinking. They explored mathematics while observing how students interacted with the tasks and activities. They also had the opportunity to learn about their role in orchestrating student learning both through reading the support documents as well as through their in-the-moment decision-making during instruction.

Institutional factors like collaboration and societal factors like accountability policies can influence teachers' enactment of curriculum materials and, subsequently, the opportunities they have for learning. For example, Choppin's (2009, 2011) studies of curriculum enactment included a school under pressure to raise standardized test scores. One teacher at this school often viewed the lessons in the curriculum through this lens, emphasizing those activities that would support skills that appeared on the standardized test. Another teacher in the study felt less pressure to raise standardized test scores. She co-planned with another instructor and developed her knowledge of how the components of the curriculum supported student thinking.

Most studies on K–12 teachers' use of curriculum relied on a small sample and mentioned contextual factors in passing. These research studies generally recognized the reality that contextual factors impact teachers' use of and learning from curriculum materials. However, few studies analyze at a detailed level how these contextual factors influence teachers' use of curriculum materials. Furthermore, all of these studies on curriculum enactment tended to be case studies of a few teachers. While they offered valuable conjectures and theory, the extent of their implications to other settings is unclear. More research is needed.

The research on K–12 teachers' use of and learning from curriculum materials offers some potential insight into factors that may affect how college-level instructors use and learn from curriculum materials. However, college mathematics instructors work in different contexts, have different knowledge bases, and hold different beliefs than K–12 teachers. Therefore, the way that these factors impact their enactment may vary significantly. The next section describes the limited research on college instructors' use of mathematics curricula.

Research on the Use of Instructional Materials at the College Level

The research on instructors' use of college-level mathematics curriculum materials is limited to eight studies which can be classified into three categories: (1) mathematicians' use of traditional curriculum materials for typical first-year mathematics courses, (2) mathematicians' use of inquiry curriculum materials, and (3) instructors' use of curriculum materials designed for mathematics content courses for elementary teachers. This study is focused on the third category, but the research in the first two categories may provide insight into the use of curriculum in content courses for pre-service teachers.

There is only one study on mathematicians' use of curriculum in first-year mathematics courses. Mesa and Griffiths (2012) found that the way instructors used textbooks at the post-secondary level depended upon the content of the courses and the instructors' perceptions of the students. The researchers interviewed 15 instructors of first-year, credit-bearing college mathematics courses from nine different institutions and supplemented their data with classroom observations and surveys of students. In the instructors' explanations of how they used textbooks, they compared their use of textbooks in first-year courses to their use of textbook in upper-level courses. All instructors used the textbooks heavily for planning for class, but there were two different patterns of how instructors interacted with the textbooks, depending upon their perception of their students. In particular, instructors distinguished between "math students" and "undergrad students" (Mesa & Griffiths, 2012, p. 98). "Math students," included honors students and students in upper-level mathematics courses. "Undergrad students" included students taking remedial courses, first-year students, and students taking mathematics courses as a requirement. Importantly, these terms do not delineate *courses*; they are descriptions of the type of students they believed they were predominantly teaching in a given course. For “undergrad students,” instructors identified skill goals by looking at problem sets. They then searched the text for examples and exercises that would help students develop such skills. Instructors of these students believed the students should read the textbook, but they did not expect them to do so. In contrast, instructors used the text differently in classes that enrolled “math students.” They read the text as a coherent whole. They considered how their own notation differed from the notation in the textbook and the impact such differences would have on students. They evaluated whether their own proofs or the textbook proofs were more appropriate. This phenomenon resulted in different learning experiences for these two different groups of students.

Mesa and Griffith (2012) found that the college instructors used the textbooks heavily to plan their lessons, but they viewed textbooks primarily as resources *for students*. The researchers claimed that since college instructors have significant subject matter expertise, the instructors did not expect to learn from textbooks. However, this conjecture conflicted with the researchers’ data. For instance, Mesa and Griffith found that instructors attended to the subtleties of different examples provided by the text. Thus, they developed some pedagogical content knowledge about the choice of different examples by using the text to prepare their lectures. Perhaps college mathematics instructors do not expect to learn *mathematics* from textbooks, but this expectation does not prevent them from learning in other ways. In fact, pedagogical content knowledge may be a fruitful area for learning through textbooks.

Instructors’ perceptions of textbooks likely have an impact on the use of curriculum materials for mathematics content courses for elementary teachers, but this is an area in need of more research. Do instructors view these materials as resources for their own learning, since their professional preparation typically does not include this content? Or does their orientation of thinking of curriculum materials as resources primarily for students extend to materials for courses for elementary teachers as well? Attention to this topic is important as publishers increasingly provide materials beyond traditional textbooks for mathematics courses for elementary teachers. For example, some curricula include mathematical and pedagogical information for instructors (e.g., Beckmann, 2014a; Chapin, Feldman, Callis, & Salinas, 2015). Others include videos of lesson enactments (e.g., Beckmann, 2014a; Chapin et al., 2015). Making sense of these resources requires instructors to see curriculum materials as more than a resource for students.

Another factor that may limit the extent to which mathematicians learn from traditional textbooks is how their use of the textbook over time changes (Mesa & Griffiths, 2012). When teaching a new course, instructors read the text closely. They used the text to create lecture notes, homework assignments, and assessments. Lecture notes in particular were an interesting artifact, one instructors used to communicate to themselves, to junior instructors, and to students. Over time, as the instructor became more familiar with teaching the course, the lecture notes often replaced the textbook as the primary resource for the instructor. Therefore, when mathematicians use traditional textbooks, there may be a short window of time for them to be influenced by the materials.

A few studies have examined college mathematics instructors' use of inquiry curriculum materials in linear algebra, differential equations, and abstract algebra. In these studies, the inquiry curricula have students "re-invent" or develop concepts based on their intuition and prior knowledge. The curricula used in these studies have students explore ideas through problems and discuss ideas as a whole class. These studies have identified two salient themes: instructors' knowledge and instructors' beliefs (Johnson et al., 2013; Johnson & Larsen, 2012; Speer, 2008; Speer & Wagner, 2009; Wagner et al., 2007). It is assumed that mathematicians have extensive content knowledge. However, content knowledge alone proved insufficient when enacting an inquiry-based curriculum (Speer & Wagner, 2009). In their study of an instructor using an inquiry curriculum in a linear algebra course, Speer and Wagner identified the importance of specialized content knowledge. Though the instructor had taught linear algebra traditionally in the past, he found enacting the inquiry curriculum to be challenging. Specifically, this instructor struggled to see the mathematics in students' ideas during whole class discussion. He also struggled to anticipate students' thinking. The instructor noted that he had not "thought enough about linear algebra *as a subject to be taught*" (Speer & Wagner, 2009, p. 553, italics original). The researchers conjectured that stronger specialized content knowledge might enable the instructor to conduct more productive discussions more easily.

When using inquiry curriculum in advanced mathematics courses, instructors need to be able to understand the mathematics in students' statements. In addition, instructors also have to be able to know what to do with student ideas to move a discussion in a productive direction. In their case studies, Speer and Wagner (2009) described one instructor who struggled with this aspect. They saw this as an instance of a lack of pedagogical content knowledge. The linear algebra instructor struggled with using student ideas to move a discussion forward, as did a differential equations instructor in another study (Wagner et al., 2007). In a third study focused on an instructor of abstract algebra, Johnson and Larsen (2012) also found that the instructor they studied occasionally struggled with implementation. Interestingly, this instructor succeeded in understanding the mathematical ideas and misconceptions expressed by students. However, she did not always understand why students were confused, nor did she always notice when her counterexample to one of their claims failed to convince them. Johnson and Larsen suggested that this was an instance where the instructor had insufficient knowledge of content and students.

Mathematicians do have pedagogical content knowledge. Iannone and Nardi (2005) noted that the instructors they interviewed in focus groups recognized common student challenges and the power of particular examples. They frequently gleaned this information from looking at student homework assignments. However, the diversity of student ideas that can arise in a class discussion in an inquiry course is much broader than when students respond to instructor-generated questions (Wagner et al., 2007). Questions generated by an instructor necessarily constrain what students can demonstrate they know or don't understand. Curriculum that uses whole class discussion can be a particularly challenging to enact because instructors must learn about student thinking as they simultaneously manage the discussion.

In addition to pedagogical and specialized content knowledge, curricular knowledge appears to be important for enacting inquiry curriculum at the college level.

At a macro planning level, one instructor struggled to manage the pacing of his course (Wagner et al., 2007). He was unable to identify which activities he could skip or when to move on in discussions. The researchers noted that the instructor was not "accustomed to attending to the relationship between classroom activities and subsequent ideas and ways of thinking that emerged from students" (Wagner et al., 2007, p. 260). This kind of knowledge is what Choppin (2009) referred to as curriculum-context knowledge – understanding how different features of a curriculum impact the way students think about different mathematical ideas.

Mathematicians' beliefs about the nature of learning and knowing mathematics can support them in negotiating the tension between covering many topics and using inquiry-based learning. Johnson and colleagues (2013) conducted interviews and collected written responses from three instructors using an inquiry-based curriculum for abstract algebra. The tension between coverage and inquiry-driven instructional practices emerged as a theme. The three instructors all maintained a commitment to the inquiry-based learning, though their reasoning for this commitment differed, depending upon their beliefs. The instructor most concerned about coverage recognized that students were understanding ideas more deeply than when he had taught the course more traditionally. He felt that understanding ideas more deeply was well worth the sacrifice of covering less content in the course. Another instructor explained that students do not necessarily learn an idea that an instructor explains to the class; students have to work through ideas to really understand them. Therefore, she felt that more material was actually learned in an inquiry-based course. She also believed that if students deeply understood the fundamentals, they would be able to individually make sense of topics that were not explicitly addressed in a particular course.

A third instructor did not believe the tension between coverage and inquiry was important. This instructor believed that studying abstract algebra was much more about the process. That is, developing mathematical practices that the study of abstract algebra highlighted was more important than the individual topics within the domain. The other two instructors were concerned with getting students to a particular mathematical goal within whole class discussion. However, the third instructor cautioned that such a commitment meant deprioritizing the pursuit of student ideas and allowing them to experience the reinvention of mathematics. In a written reflection on the curriculum, he wrote, “The mathematics major has the perversely unique quality of producing graduates who have not, at any point, been asked to engage in activities representative of mathematics research…. Shame on us for hiding the heart of our discipline from our disciples” (Johnson et al., 2013, p. 753). This quote is particularly interesting in the context of this study – could instructors of mathematics courses for elementary teachers similarly be concerned with engaging their students in the mathematics *as they would use it in their careers?* Would this be possible for instructors who have little knowledge about the mathematical work of elementary teachers? The instructors in Johnson and colleagues' (2013) study were motivated by their beliefs in what it means to understand abstract algebra. Would mathematicians have similar beliefs about what it means to understand fractions?

Johnson and colleagues’ (2013) findings that beliefs support powerful instructional practices at the collegiate level is also supported by Speer's (2008) study of a calculus instructor using problem-based learning. Speer interviewed and videotaped a doctoral student leading a discussion section in calculus focused on collaborative problem solving. Speer found that the instructor's actions could be explained by his beliefs in what it means to learn and understand mathematical ideas. However, the opposite was reported by Iannone and Nardi (2005). In their study of five mathematicians, Iannone and Nardi found that these instructors were aware that listening to lectures is a poor means of learning mathematics. However, they did not change their practice. Instead, they encouraged students to collaborate outside of class in designated spaces. These conflicting findings leave an outstanding question. What enables mathematicians to act on their beliefs about learning mathematics in their instruction? Can the use of different curriculum influence their instruction? Given that mathematicians have strong and complex beliefs about the nature of mathematics (Felbrich, Müller, & Blömeke, 2008; Mura, 1995), identifying how to support them in acting on their beliefs should be a priority.

In sum, the research on the use of inquiry materials in higher-level mathematics classes demonstrates the importance of instructors' beliefs and knowledge. Instructors' beliefs in the nature of knowing and learning mathematics seem a particularly promising lever for enacting powerful instructional practices. However, the impact of beliefs on their instruction may be tempered by professional norms, access to curriculum, or other factors. Furthermore, research on the effect of instructors' beliefs on practice is limited and tied to specific mathematics courses. It remains to be seen whether these findings extend to other domains, such as mathematical knowledge for teaching.

A number of studies report that textbooks are highly used in mathematics content courses for pre-service elementary teachers. A CBMS report found that between 90% and 100% of institutions with multiple sections of math content courses for elementary teachers had all sections use the same text (Lutzer et al., 2007). Textbooks can therefore serve as a touchstone for common conversations for instructors. On the other hand, with at least 14 different textbooks in the market, PSTs' experiences in these courses may vary considerably (McCrory, Siedel, & Stylianides, 2008). Perhaps even more importantly, the ways in which instructors use textbooks vary, though many of them have reported that the textbook influences their work.

Research suggests that the majority of instructors use textbooks specifically designed for mathematics content courses for elementary teachers, though understanding to what extent these texts influence their instruction is more complex. McCrory and her colleagues' research, the Mathematical Education of Elementary Teachers (ME.ET) study (2009), found that 65% of instructors in surveyed used a textbook *specifically* designed for such courses, and 38% used one of the top three rated by NCTQ: Beckmann's (2007) *Mathematics for Elementary Teachers,* Billstein, Libeskind, and Lott's (2003) *A Problem Solving Approach to Mathematics for Elementary School Teachers,* and Parker and Baldridge's *Elementary Mathematics for Teachers* (2004) and *Elementary Geometry for Teachers* (2008). This finding is similar to NCTQ's study, which found 38% of programs in their study used one of these three textbook, while 6% did not use a text (Greenberg & Walsh, 2008). The ME.ET study found a slightly higher percentage of instructors who used no text at all – 14% (McCrory, Francis, et al., 2008).

Instructors used these texts in varying ways. The ME.ET study (McCrory, Francis, et al., 2008) reported that approximately 45% of instructors used a single text as their main resource, while approximately another 40% used multiple textbooks or a mix of textbooks and other resources. Most of these instructors relied heavily on their textbooks. Thirty percent of instructors reported that over 80% of their weekly class time was based on their text, while over 50% of instructors reported that they based 60% or more of their weekly class time on the text. Furthermore, two-thirds of instructors reported following the textbook closely, at most changing the order of a few topics (McCrory, Francis, et al., 2008).

While the data from the ME.ET project provide a macro view of how instructors use textbooks, Lo, Kim, and McCrory's (2008) case study of two graduate teaching assistants provides more detail on how instructors use textbooks in their work. In particular, Lo and her colleagues found that the instructors used the texts heavily in their planning and course management. However, the material that the instructors modified, skipped, or added differed as a result of perceived time constraints, PSTs' reactions, and the instructor's own self-perceived professional role. For example, the instructors in Lo and colleagues' study used one text, Parker and Baldridge (2004), heavily in their planning and course management. They assigned readings and homework problems from the text. Though the instructors had an academic background in mathematics, neither of them had experience teaching mathematics in elementary school. Therefore, they used the texts to support their own learning.

While the textbook was an important resource for the instructors and PSTs, Lo's instructors covered less than 60% of the material involving fraction concepts and procedures in the text. This low level of coverage was in spite of the fact that they spent more time than the textbook suggested on these topics. Their changes included modifying, skipping, and adding content. One limitation of this study was that the researchers failed to ask the instructors why they made each decision to modify, skip, or add content. However, the descriptions of what the instructors modified, skipped, or added were provided and highlighted how written curriculum materials may have to be more explicit about the purposes of the examples, activities, and topics present in the text.

The instructors in Lo and colleagues' (2008) study made modifications to existing content in the textbook such as changing the context and numbers in examples and problems. The changes had mathematical implications. For example, when discussing improper fractions and mixed numbers, both instructors used different numerical examples, but only one instructor included the case of $\frac{a}{b}$ with $a = b$, as the text did, as an example of an improper fraction. Their motivation for their decisions to modify content differed as well. One instructor perceived her PSTs to be more engaged when she modified the material. The other instructor thought her PSTs would think she was unprepared if she followed the text too closely. These findings correspond to Remillard's (1999, p. 330) notion of teachers as "reading students" as they make curricular decisions. That is, teachers notice and interpret their students' reactions to instruction and modify their actions accordingly.

PSTs' knowledge of content and pedagogy may be affected when instructors modify or skip content. Both instructors in Lo and colleagues' (2008) study skipped about half (48% or 54%) of the content that connected the mathematics to the teaching of fractions. For example, both instructors skipped a discussion about how the division interpretation of fractions (i.e., $\frac{a}{b} = a \div b$) can help elementary students understand the connections between the different forms of answers for division problems (mixed numbers, improper fractions, remainders, and decimals). Furthermore, while the material in the textbook connected rules, models, or properties between whole numbers and fractions, the instructors skipped a relatively high percentage of this content, 21% and 39%. In addition, one instructor was much more likely than the other (24% compared to 9%) to skip examples or exercises that demonstrated or provided an opportunity for PSTs to practice an idea. Both instructors reported feeling the pressure of time constraints and a desire to maintain pace with other instructors teaching different sections of the same course (Lo et al., 2008).

Another way that these instructors adapted the textbook was that they added additional material (Lo et al., 2008). For example, one instructor saw herself as primarily a mathematics teacher educator, as opposed to a mathematics instructor. This instructor reported that she prioritized helping PSTs make connections to teaching children mathematics. Though she skipped 54% of the content that made these connections, she added problems taken from elementary curricula. The second instructor saw herself primarily as a mathematics instructor, providing "a bridge between the mathematics that mathematicians do and the mathematics" (Lo et al., 2008, p. 1–6) studied in the course. She added examples that were more mathematically complex, both in terms of the needed calculation strategy and in the necessity of coordinating concepts. The researchers suggested that the instructors' professional identities, as a mathematics professor or as a mathematics teacher educator, influenced their enactment of curriculum. However, as evidenced by the material that the instructors chose to skip, it is unclear if professional identity fully explained these instructional decisions.

Lo and colleague's (2008) study also found that instructors' perception of their students influenced their use of curriculum materials. The second instructor in Lo and colleagues' (2008) study lectured more in a section where she perceived the students to be minimally engaged. Both the instructors in Lo's study and the instructors in in Mesa and Griffiths' (2012) study altered examples in the textbook so that students would have a reason to come to class, rather than just read the book. This practice may be specific to higher education institutions, where attendance is voluntary. Instructors may perceive students as needing an incentive to attend class. Remillard (1999) noted that instructors interpret their students' responses when using curriculum and making instructional decisions.

Instructor's decisions to skip or modify content are not completely understood, in part due to the small number of studies. One conjecture involves "curriculum embedded mathematics knowledge" (Kim & Remillard, 2011, p. 2), or teachers' curricular noticing (Males, Dietiker, Earnest, & Amador, 2015). Perhaps instructors do not see the mathematical or pedagogical potential in particular examples or problems in the textbook, even though they may value the ideas behind these examples and problems. For example, one of the instructors in Lo and colleagues' (2008) study did not seem to notice the importance of the improper fraction example where the numerator and denominator were equal. If this hypothesis is true, then educative curriculum materials that draw instructors' attention to the choices made in the curriculum may be even more important.

In addition to individual instructor factors, contextual factors have a significant and complex impact on the way that instructors use curriculum materials. Contextual factors and the use of curriculum materials also influence the amount of student-centered instruction employed in mathematics content courses for elementary teachers. Jeppsen (2010) studied 21 instructors of elementary mathematics-for-teaching courses at four community colleges. Two of the institutions achieved a high level of student-centered instruction, one through extensive use of the curriculum and the other through a high level of collaboration among instructors. The first institution achieved a high level of student-centered instruction through extensively using a textbook and the accompanying explorations manual. This college used this textbook as a source of in-class activities, unlike the other institutions. Jeppsen (2010) argued that contextual factors worked to support extensive use of the textbook. Specifically, the department chose the textbook because it aligned with its philosophy and goals for its pre-service teachers. In addition, alignment with the philosophy of the textbook was one of the criteria used to recruit and hire instructors. In their first year of teaching the course, new instructors were mentored by an experienced instructor. After this first year, there was little collaboration among instructors, but there was a high level of consistency because of the extensive use of the textbook. Thus, a departmental commitment to a particular teaching philosophy, maintained through strategic recruitment and supported by a purposefully chosen textbook, resulted in a high level of student-centered instruction. This case confirms Remillard and Bryan's (2004) findings that orientation toward curriculum impact its use. However, in this case, it is orientation toward a particular curriculum, not toward curriculum materials in general. At this community college, the department chose both the curriculum and the instructors to align with its pedagogical philosophy.

The second institution in Jeppsen's (2010) study achieved a high level of student-centered instruction in a different manner, specifically, through extensive instructor collaboration. Though both colleges used the same text, the second institution used it primarily to sequence topics and as a source of homework problems. For in-class activities, the instructors used a collaboratively-developed course packet. After each enactment, the instructors continually and collectively refined the materials. This high level of collaboration was further supported by the close proximity of instructors' offices. Therefore, it seems that a high level of student-centered instruction could be possible with a high level of collaboration around instructor-created curriculum materials. Jeppsen was sure to note, however, that the nature of the collaboration may be important. At a third institution, the collaboration consisted mostly of experienced instructors sharing resources, in contrast to the democratic, generative collaboration at the second institution. This third college had a lower level of student-centered instruction. Therefore, using a curriculum that supports student-centered instruction and recruiting instructors whose philosophies align with those of the text may be a more productive method for institutions where such extensive collaboration is difficult to achieve. Jeppsen's study is only a case study of four colleges. The detail about the instructional practices is limited to identifying the extent of student-centered instruction. More research is needed that more deeply examines instructional practices at a detailed level and their connection to curriculum materials.

Contextual factors can also inhibit extensive use of a curriculum and, possibly, student-centered instructional strategies. In Jeppsen's (2010) study, institutional factors impacted instructors' perception of a curriculum. The fourth college, which had the lowest rate of student-centered instruction, had a transfer agreement with a nearby four-year institution. This agreement required the use of a particular textbook for the course. Thus, the instructors saw little need to collaborate since the curriculum was mandated. They also felt little connection to the text, since the perception was that the textbook was forced upon them. Therefore, they often drew upon outside resources. Jeppsen concluded that the low level of collaboration and the lack of fidelity to the text were related to high levels of teacher-centered instruction.

Perceptions of a textbook have been identified as a factor in the enactment of curriculum in K–12 education. However, this factor has typically been seen as a characteristic that individual teachers act upon. In higher education, it may be a characteristic of a department, impacted by contextual factors such as credit transfer policies or instructor recruitment strategies. Jeppsen's (2010) study brings important attention to the fact that college instructors, like teachers, are influenced by the organization within which they work.

Jeppsen's study identified factors that influence curriculum use and instruction in mathematics content courses for elementary teachers. However, it used a macro-level lens. That is, it did not provide detail about what instructional strategies were used and how those strategies were connected to feature of a curriculum. Likewise, Lo and colleagues' (2008) study addressed only content coverage, not specific instructional practices. Research about how college instructors use materials to enact powerful instructional practices is still needed.

Conclusion

The research on college-level instructors’ use of curriculum is still in its infancy. The studies that do exist demonstrate that the issues that impact K–12 instructors’ enactment of curriculum – knowledge, goals, beliefs, professional identity, perceptions of students, contextual factors – also impact instructors at the post-secondary level. There are certainly differences in the way these factors present themselves. For example, while common content knowledge may be a barrier for some teachers, mathematicians typically do not struggle with common content knowledge. Instead, they may struggle with pedagogical content knowledge. Elementary teachers’ beliefs about the nature of mathematics may impact their use of curriculum, but these beliefs may be overridden by beliefs about engaging students. In contrast, mathematicians’ beliefs about instructional practices that engage students does not seem to reliably lead to more student-centered instruction. However, mathematicians’ beliefs about the nature of mathematics do seem to support them in persisting with inquiry curriculum materials that promote mathematically powerful instruction. Thus, college-level instructors are likely to use curriculum materials to support powerful instructional practices in ways that are different from the ways K–12 teachers use curriculum materials.

Mathematics content courses for elementary teachers are different from other college-level mathematics courses. The student populations and the goals of such courses (both learning about mathematical ideas and how to use these ideas to teach elementary students) are different. The instructors of these courses come from a variety of backgrounds, some traditional mathematicians with no K–12 teaching experience, but others with a background in education. Therefore, while the research on mathematicians' use of curriculum materials in other mathematics courses is informative, the use of curriculum materials in mathematics content course for elementary teachers is likely to be different.

Lastly, the research on curriculum use and instructional practices in elementary mathematics-for-teaching courses is limited. Lo and colleagues (2008) documented mathematical alignment to the written curriculum, but their study did not attend to instructional practices. Jeppsen's (2010) study addressed instruction broadly, indicating the percent of time spent on student- versus teacher- centered instruction. More detail about how instructors of these courses use curriculum materials to create mathematically powerful experiences for their studies is needed.

This section has summarized the research on the use of curriculum materials at both the K–12 and college level. The next section will provide background on the mathematical educational of elementary teachers, with a focus on the research on content courses for this population.

Section VI: Research and Recommendations on the Mathematical Education of Elementary Teachers

Research has shown that teachers' mathematical knowledge and their dispositions toward teaching mathematics influences their instruction and student outcomes (e.g. Campbell et al., 2014; Hill & Charalambous, 2012; Hill et al., 2005; Wilhelm, 2014). Furthermore, policy documents that call for a richer mathematics education for K–12 students typically include calls for improvements in teacher education (e. g., Gardner et al., 1983; National Research Council, 1989; Science and Mathematics Teacher Imperative (SMTI) and The Leadership Collaborative (TLC) Working Group on Common Core State Standards, 2011). However, there is still little research on effective strategies for preparing elementary teachers to teach mathematics (National Research Council, 2010). Though the research may be limited, the mathematical preparation of teachers should be informed by the research, scholarship, and professional consensus that does exist. In this section, I highlight the recommendations by professional and policy organizations, the research upon which such recommendations are based, and the research on mathematics content courses for elementary teachers.

Policy Recommendations on the Mathematical Education of Elementary Teachers

Policy organizations, such as the National Council for Teacher Quality (NCTQ), and professional organizations, such as the Conference Board of Mathematical Sciences (CBMS), have put forth recommendations on the mathematical education of teachers

(CBMS, 2012; Greenberg & Walsh, 2008). NCTQ, an advocacy organization, published their recommendations alongside their studies of the existing state of elementary teachers' pre-service education, one of which is referred to as *No Common Denominator* (Greenberg & Walsh, 2008). CBMS, an umbrella organization composed of professional organizations of mathematicians and mathematics educators, has published two volumes documenting their recommendations, *The Mathematical Education of Teachers (MET I)* (2001) and *The Mathematical Education of Teachers II (MET II)* (2012). The Association of Mathematics Teacher Educators (2017) has reiterated the recommendations of the MET II.

A primary recommendation of these policy documents is that elementary teachers complete coursework that specifically engages them in developing a deep understanding of the mathematics they will teach. While unopposed to additional mathematics coursework, both organizations contend that such courses – for example, Calculus or College Algebra – are not appropriate substitutes for courses specifically designed for elementary teachers. The National Council of Teachers of Mathematics (NCTM) (2005) and NCTQ (2008) both advocate for pre-service elementary teachers to take three courses to develop deep conceptual understanding of the mathematics they will teach. CBMS (2012) recommends 12 credit hours of content courses designed specifically for elementary mathematics teachers.

Despite these recommendations, multiple studies have found that both states and higher education institutions are failing to uniformly provide adequate mathematical preparation for future elementary teachers. In 2014, only 19 states required a subject matter test where each subject matter section had to be passed independently in order for candidates to earn elementary teaching certification (Greenberg et al., 2014). Two studies report that about one-fifth of institutions preparing elementary teachers do not require or do not offer mathematics courses specifically for elementary teachers (Greenberg & Walsh, 2008; Masingila et al., 2012). A third survey by the Conference Board of Mathematical Sciences (Lutzer et al., 2007) found that between 4 and 16 percent of institutions require *no* mathematics coursework of *any* kind for future elementary teachers. At many institutions, elementary teachers meet mathematical requirements by taking general courses. For example, Lutzer and colleagues (2007) found that, among institutions with certification requirements differentiated by grade level, between 20% and 40% reported that PSTs were likely to meet their mathematics requirement by taking college algebra. In contrast, between 28% and 70% of these institutions reported that elementary teacher candidates were likely to meet their mathematics requirement through a multi-semester course focusing on elementary mathematics for teaching. Similarly, Masingila and her colleagues (2012) found that over 70% of the surveyed institutions that offer courses in elementary mathematics-for-teaching required pre-service elementary teachers to take two or fewer courses, well below the *MET II* recommendations. Certainly, the fact that many institutions do not require elementary mathematics-for-teaching courses at all is problematic. The limited number of semester hours dedicated to this coursework at other institutions is also problematic. The *MET II* makes clear that the extent of the content elementary teachers must learn, both in breadth and depth, is sizable. Moreover, the recommendations also indicate the necessity of sufficient time for

PSTs to engage in mathematical practices.

Beyond credit hours, there are two significant components of the recommendations for content courses. First, rather than focus on advanced mathematical topics or review K–12 school mathematics, the course content “should examine the mathematics they [pre-service teachers] will teach in depth, from a teacher’s perspective” (CBMS 2012, p. 17; c.f. AMTE 2017). That is, the courses should address mathematical knowledge for teaching, a concept further discussed in the next section (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008). Second, the recommendations advocate that these courses provide both time and the pedagogical structure for pre-service teachers to engage in mathematical practices.

> All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing. Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students. .... A worthy goal of mathematics instruction for any undergraduate is to develop not only knowledge of content but also the ability to work in ways characteristic of the discipline. (CBMS, 2012, p. 19).

Additionally, there are recommendations based on research about quality mathematics teaching in higher education, generally. Recommendations and research point to several key principles in learning in higher education (National Research Council, 2015). Students' prior knowledge must be taken into account. Students must be actively involved in making sense of new concepts and integrating these concepts into their existing understanding. Developing metacognition by reflecting on their learning can help students learn more effectively. Interacting with others also supports students' learning. Bain's (2004) qualitative study investigating the practices and characteristics of excellent college instructors across disciplines found similar results. Thus, in addition to focusing on mathematical knowledge for teaching and engaging in mathematical practices, instruction should also emphasize the use of formative assessment, sense-making, reflection, and student-to-student interaction in mathematics content courses for elementary teachers.

Research on Mathematical Knowledge for Teaching

Teachers who have mathematical knowledge for teaching (MKT) have a deep knowledge of school mathematics. They understand how mathematical ideas build across grade levels. They understand the pedagogical consequences of the choices of different examples, contexts, representations, and strategies. Ball and her colleagues (2005, 2008) developed a conceptual framework of MT and identified and described six domains of mathematical knowledge for teaching, divided into two groups, subject matter knowledge and pedagogical content knowledge. Within subject matter knowledge (SMK), there is common content knowledge (CCK), specialized content knowledge (SCK) and horizon content knowledge (HCK). Within pedagogical content knowledge (PCK), there is knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC). Developed from studying teachers and the ways in which they use mathematics in their work, the researchers built upon Shulman's (1986) emphasis on the role of content knowledge in teaching and his notion of pedagogical content knowledge. This section will describe the domains and their role in the work of teaching.

Subject matter knowledge is particularly mathematical in nature. Common content knowledge (CCK) is the mathematical knowledge that most educated adults have, for instance, the ability to solve a real-world problem that calls for multiplication. In contrast, specialized content knowledge (SCK) is mathematical knowledge that is particular to the work of teaching. All mathematical careers use mathematics in ways that are particular to their discipline, but for elementary teachers, the distinction is quite striking. For example, ordinary mathematically literate adults do not necessarily need to know or remember why the standard multiplication algorithm works. However, teachers use this knowledge in their work with elementary students. The algorithm uses many mathematical ideas: the fact that our numerals can be decomposed by place value, the distributive property of multiplication over addition, and the pattern in multiplying multiples of ten, to name a few. Teachers use their specialized content knowledge about multi-digit multiplication to plan activities that help students conceptually understand the algorithm. They use their SCK to analyze student errors for fundamental misconceptions. Even in classrooms with direct instruction, teachers use their knowledge to directly explain the concepts behind the algorithm to students. Horizon content knowledge

(HCK), a less-researched domain (Jakobsen, 2014), includes the ways in which topics are linked throughout the grades and to advanced mathematics. For example, teachers with deep HCK about the distributive property recognize its role in both the standard multiplication algorithm and in simplifying polynomials. They see connections between the representations that are used at lower grade levels, such as the area model or open arrays, and the representations that are used in secondary school, such as algebra tiles. They might also recognize some limitations of different models in future grade levels. For example, discussions about area models may need to be adjusted when working with negative terms or with variables with greater exponents. Some researchers looking to develop the notion of HCK have claimed that HCK is not just about topics, but about disciplinary practices – for example, that proof by contradiction is a valid method (Jakobsen, Thames, Ribeiro, & Delaney, 2012) while proof by a multitude of examples is not (Jakobsen, 2014).

Pedagogical content knowledge (PCK) is the other major category in Ball and colleagues' (2008) construct. While SMK emphasizes the mathematics, PCK emphasizes the *learning* of mathematics. PCK consists of three domains: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Knowledge of content and students (KCS) is a teacher's knowledge of how students come to understand the content – it might include common student intuitions, misconceptions, or over-generalizations. Knowledge of content and teaching (KCT) includes the pedagogical affordances and drawbacks of different representations. Teachers with KCT consider their use of manipulative materials and diagrams in furthering the mathematical objectives during instruction. In addition, teachers with strong KCT understand the consequences of choosing particular examples. Teachers with strong KCT know "the most powerful analogies, illustrations, examples, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9).

Ball's work did not further define knowledge of content and curriculum. However, other researchers have defined curricular knowledge. Ball and colleagues referred to Shulman's articulation of curricular knowledge. Shulman (1986) likened curricular knowledge to a doctor's knowledge of drugs and procedures, a knowledge of the tools of the trade and how to use them. He differentiated between vertical curricular knowledge and lateral curricular knowledge. Vertical curricular knowledge, knowledge of the topics in previous and following grade levels, might be the same as horizon content knowledge in some interpretations. Lateral curricular knowledge includes connections between *different* disciplines at the same grade level (Shulman, 1986). Such knowledge is particularly valuable in teaching mathematics, so that students can see how mathematics is useful across disciplines.

Where Shulman described a kind of mental inventory of the different curriculum resources available, Kim and Remillard (2011) described a kind of skill that relies on mathematical knowledge. Kim and Remillard defined *curriculum embedded content knowledge* (CECK) as the mathematical knowledge necessary to make sense of the representations in and the design of curriculum materials. The researchers identified four dimensions. First, teachers with a high level of CECK can identify the mathematical point of an activity or the key mathematical ideas in student work. Second, teachers with a high level of CECK can identify how the mathematical ideas within an activity are situated among other mathematical ideas, such as prerequisite and more advanced topics. Third, such teachers can identify task complexity and potential areas for student confusion. Fourth, they can connect different representations. Some of these dimensions seem to overlap with other domains of subject matter knowledge or pedagogical content knowledge. However, these researchers are concerned with this knowledge as it applies to understanding curriculum materials.

Choppin's (2009) contribution to curricular knowledge, curriculum-context knowledge, likewise addressed a deep understanding of particular aspects of a curriculum. His conceptualization of this knowledge was tied closely to the impact that curriculum had on learners. For example, teachers with a high level of curriculum-context knowledge would understand how a sequence of lessons would support students in understanding a particular idea. That is, these teachers understand both how an idea mathematically develops over a sequence of activities and how student understanding is impacted by this sequence of activities. Because this knowledge is dependent upon the background knowledge of a particular set of students, he refers to this type of teacher knowledge as curriculum-*context* knowledge.

These different domains of mathematical knowledge for teaching intersect in ways that may be difficult to parse out, as Ball and her colleagues noted (2008). As demonstrated in the following section, empirical research has focused on some of these domains and demonstrated that teachers' knowledge has an impact on student learning.

Research on the Impact of Teachers' Mathematical Knowledge for Teaching on Student Learning

A number of studies have investigated the effects of teachers' mathematical knowledge for teaching on student learning. Hill, Rowan, and Ball (2005), in a large-scale quantitative study, found that elementary students made greater academic gains in mathematics over the course of the school year if their teachers had higher levels of content knowledge for teaching mathematics (CKT-M,) compared to other teachers. CKT-M combines common and specialized content knowledge. Hill, Rowan, and Ball's study involved 115 elementary schools, 1190 first-graders, 1773 third graders, and 699 teachers in 15 different states. The teachers completed CKT-M test items from what would become the Learning Mathematics for Teaching (LMT) assessment from the University of Michigan. Students' gains on the Terra Nova correlated with their teachers' scores on the LMT assessment at a statistically significant level. Indeed, the standardized coefficients for teachers' scores on the LMT assessment was greater than coefficients for their years of experience, certification status, the amount of mathematics and mathematics education coursework they had taken, students' levels of absenteeism, and even greater than the amount of time spent on mathematics instruction. Teachers' MKT scores were nearly as strong a predictor of student achievement gains as a student's socioeconomic status, according to the standardized coefficients in the multiple regression model (Hill et al., 2005). These results suggest that improving teachers' mathematical knowledge for teaching may be a potential lever for closing the achievement gap (Ball et al., 2005).

A second study by Campbell and colleagues (2014) described the impact of teachers' MKT as it interacted with teachers' beliefs about mathematics teaching. Similar to Hill and colleagues' (2005) work, which focused on subject matter knowledge, Campbell and her colleague measured teachers' subject matter knowledge, but also measured their pedagogical content knowledge. Their knowledge assessment used a combination of released teacher-knowledge items and researcher created items aligned to state content standards for students and teachers. Their study also included a separate 40-item Likert-scale teacher survey to measure beliefs about mathematics teaching and learning and awareness of student dispositions. Factor analysis identified two constructs that contributed to the researchers' multiple regression model to predict student achievement gains in mathematics. The first was a belief in the importance of teacher modeling before student practice and in incremental mastery of procedural skills before engaging in applications. The second construct combined two beliefs. First, the construct measured teachers' perceived awareness of students' dispositions and abilities. Second, it measured a focus on teaching multiple approaches to solving problems and a preference for using problems with multiple solution strategies.

With a sample of 443 early career teachers (259 fourth- and fifth- grade teachers and 184 middle school teachers) and 17,303 upper elementary and middle school students, the study found a statistically significant effect of teachers' content knowledge when controlling for student and teacher-level variables. Combined content and pedagogical content knowledge for *middle school* teachers was a statistically significant predictor ($p < 0.001$) of student assessment scores. This finding was true *regardless* of whether the model controlled for teachers' beliefs, years of experience, or if they were teaching from advanced curriculum. However, the results differed for elementary teachers and their students. Elementary teachers' knowledge was a statistically significant predictor of student achievement gains *only* in the model that controlled for the above variables and for special education certification. In the model that included only student-level variables and teacher knowledge, teacher knowledge was not a statistically significant predictor of elementary student learning. Furthermore, whereas Ball and her colleagues (2005) claimed that the impact of mathematical knowledge for teaching rivaled the impact of socio-economic status, in Campbell et al.'s (2014) study, the poverty variable met a higher level of statistical significance than the teacher knowledge variable. This result does not imply that poverty is a stronger predictor of student learning than teacher knowledge, since the researchers did not provide standardized coefficients or comparable effect sizes, but it is cause for recognizing that improving teacher knowledge is only one, albeit very important, aspect of improving students' mathematical learning.

Additionally, the researchers found one other factor that further enhanced student learning. Teachers who were aware of their students' skills and dispositions, who reported using multiple methods to solve problems, and who used problems with multiple solutions had an even greater impact upon their students' learning. In other words, while an elementary teacher's content knowledge positively impacted his or her students' learning, if the teacher scored particularly high on the factor measuring awareness of students' skills and disposition and use of multiple methods and problems with multiple solutions, his or her students performed even better. This finding reiterates the need for mathematical courses for teachers to include problems with multiple solutions and attention to student thinking.

While many factors influence students' learning, teachers' mathematical knowledge is one factor that teacher preparation programs may be able to influence. Even more important is the fact that the teacher knowledge that is the most powerful in influencing elementary students' learning is deep knowledge of the content they will teach. Unlike earlier studies that used rough proxies for teacher knowledge, such as the number of college mathematics courses elementary teachers had completed, these later studies demonstrate the power of this unique professional knowledge of teachers, even given other factors.

Research on Mathematics Content Courses for Elementary Teachers

Institutions in the United States, including states, colleges, and universities, enjoy significant autonomy in their work preparing the next generation of teachers. Particularly at the institutional level, this autonomy results in different course offerings, different standards for the experience and background of instructors of teacher candidates, variations in the allocation of time in class and type of pedagogy used, and differences in both the printed and enacted curriculum.

It is assumed that specialized mathematics courses for elementary teachers can increase PSTs' mathematical knowledge for teaching. However, the results of many studies also show variation in outcomes. In this section, I will discuss the research on both the positive outcomes of such courses, as well as the hesitation researchers have expressed about the uniform ability of these types of courses to improve PSTs' mathematical knowledge. Second, I will discuss additional issues that might impact PSTs in acquiring mathematical knowledge for teaching.

Many policy documents (e.g., CBMS, 2012; Greenberg & Walsh, 2008) call for mathematics content courses for elementary pre-service teachers to be uniquely designed to the work of teachers, not just mathematics courses for the general population of students. One reason for this recommendation is that the mathematical knowledge that teachers need, as described previously, is different from the mathematics typically taught in college math courses. Second, empirical evidence demonstrates that specially designed courses can, in fact, have an impact on pre-service teachers' mathematical knowledge for teaching.

One study, Matthews and Seaman (2007), compared the mathematical knowledge for teaching of PSTs who had completed a general mathematics course ($n = 19$) with that of PSTs who had completed a course specifically designed for teachers ($n = 29$). When controlling for ability with cumulative GPA and ACT scores, the researchers found that which course PSTs took was a statistically significant predictor ($p < 0.05$) of their score on the mathematical knowledge for teaching assessment. The researchers used an assessment they had developed and previously tested on other teachers, the Mathematical Content Knowledge for Teaching test. The 20-item test primarily included common content knowledge items, although there were 2 items potentially measuring pedagogical content knowledge and 3 items potentially measuring specialized content knowledge. Pre-service teachers in the specially designed course outperformed their peers with comparable GPA and ACT scores. PSTs in the specially designed course also had better attitudes about mathematics than their peers. However, one limitation to the study was the time between the completion of the mathematics course and administration of the assessment, which differed by group. The time between the end of the course and administration of the assessment was between four to six months longer for the control group than for the group who took the specially designed course. Therefore, PSTs in the specialized course may have simply remembered more mathematics since their experience was more recent.

Three larger, quantitative studies supported Matthews and Seaman's (2007) conclusions that elementary math-for-teaching courses help PSTs develop their mathematical knowledge for teaching. Superfine, Li, and Martinez (2013), over the course of five years, determined that 213 PSTs enrolled in an elementary mathematics-for-teaching (MFT) course at their university improved their mathematical knowledge for teaching (MKT) at statistically significant levels, with moderate to large effect sizes. This study used the Learning Mathematics for Teaching (LMT) assessment (Ball et al., 2008.). A second, larger study examined the impact of three curriculum units on PSTs' specialized and common content knowledge (Chapin et al., in review). Enrolling over 400 PSTs per unit from 33 institutions, the study used researcher-created pre- and post-assessments that required PSTs to explain and justify their answers. Content courses using these materials saw statistically significant gains in their PSTs' mathematical knowledge for teaching, with large effect sizes. A third study spanning across 17 universities (McCrory et al., 2009) further supported the conclusion that PSTs learn a significant amount of mathematical knowledge for teaching in content courses designed specifically for teachers. On average, the 1706 PSTs in McCrory and colleagues' (2009) study gained over three-fourths of a full standard deviation on an assessment using Learning Mathematics for Teaching items (Ball, Bass, et al., 2008) items after completing an elementary mathematics-for-teaching course. However, PSTs' gains varied at the individual level and, more substantially, at the instructor level.

Newton (2008) found mixed success with the five sections of a elementary MFT course that she studied. In her study, 85 PSTs took a pre- and post- assessment on fractions composed mostly of straight computation problems. At the end of the course, PSTs made fewer mistakes stemming from conceptual misunderstanding, such as adding both the numerators and denominators when finding the sum of two fractions. However, mistakes stemming from errors in whole number arithmetic did not decrease. Furthermore, while PSTs' common content knowledge about fraction operations seemed to have improved, their flexibility in using non-standard procedures when it would simplify a computation did not increase, even though this was a focus of the course. At the end of the course, only 26% of PSTs could transfer their learning to a non-routine problem that called for them to consider the referent whole when adding fractions. In sum, while Newton's content courses remedied some of the incorrect ideas that PSTs had from their own K–12 education, PSTs' strategic competence (National Research Council, 2001) and problem-solving skills appeared still to be weak. Unfortunately, Newton's study did not provide details about the pedagogy or curriculum used in the courses studied.

Furthermore, even though elementary MFT courses may have an impact on PSTs mathematical knowledge, these courses may not be enough to make up for PSTs' previous 12 years of schooling. Luo, Lo, and Leu (2011) conducted a study comparing Taiwanese and U.S. PSTs near the end of their preparation. The 89 U.S. PSTs in the study had been required to take two college math courses, either general college math courses or courses specifically designed for PSTs. However, the 85 Taiwanese PSTs were required only to take a mathematics methods course. Both sets of teachers took a multiple-choice assessment measuring their fraction knowledge before beginning a unit on fractions in their methods class. Taiwanese PSTs vastly outperformed their U.S. counterparts, despite their additional mathematics coursework. Fourteen (14) of the 15 questions were answered correctly by 70% to 99.8% of the Taiwanese PSTs. In contrast, the percent of U.S. PSTs answering each of these questions correctly ranged from 33.7% to 84.3%. The authors suggested this gap between the two cohorts paralleled the gap between K–12 students in the two countries. While some studies, like McCrory and colleagues' (2009) and Newton's (2008), found that elementary MTF courses were effective at increasing PSTs' knowledge, Luo and her colleagues' work is a reminder that these courses are only one part of a much larger puzzle. Again, lacking the specific details about the content or pedagogy of the Taiwanese and U.S. courses, it is difficult to determine if the fault lies in the K–12 educational system or with the university-level preparation of teachers or both.

An even closer look at Newton's (2008) and Luo et al.'s (2011) studies, along with the work of Tobias and her colleagues (Tobias, 2013; Tobias, Roy, & Safi, 2015), demonstrates finer-grained differences between the impact of elementary MTF courses as well. While Luo and her colleagues (2011) found a wide range in U.S. PSTs' performance on each of the fraction questions (from 19% to 84%), Newton's PSTs entered their courses with relatively high performance rates. On average, PSTs answered 92% of addition problems, 85% of subtraction problems, 75% of multiplication problems, and 64% of division problems correctly. On the post-test, PSTs' correct responses increased, averaging 88% correct or higher for each operation. The difference between the quantitative results in the two studies may have been due to the difference between students' facility with whole numbers versus fraction computation. It might also have been due to the nature of the assessments. Newton's (2008) assessment primarily featured straight computation, while Luo et al.'s (2011) assessment featured word problems and diagrams. The type of knowledge assessed, and how it is assessed, greatly determines whether these content courses are deemed successful. In addition, few of the studies assessing PSTs' knowledge after completing MFT courses provide details about PSTs' prior mathematical knowledge or preparation.

Furthermore, a comparison of several studies suggests that pre-service teachers may be better at demonstrating their understanding by explaining ideas to other learners, rather than completing individual assessments. The assessment method may be the reason for such different results, rather than the effectiveness of the courses. In particular, Luo and colleagues (2011) found that U.S. pre-service teachers struggled interpreting linear models of fractions, such as number lines. Furthermore, they also had difficulty solving word problems that suggested linear models for fractions, such as those dealing with length or distance, compared to similar problems that suggested area models. However, the results of another, qualitative study contradicted this finding. Thirty-three PSTs (Tobias's 2013; 2015) in an elementary math content class did not experience such difficulty with linear models of fractions. In fact, PSTs in this study readily used linear models of fractions to solve problems. In addition, the subjects were able to use iteration in their solutions, though other studies had suggested PSTs struggle with this concept. (Behr, Khoury, Harel, Post, & Lesh, 1997). These differing findings may be due to the method of assessment. Luo and her colleagues (2011) assessed individual PSTs with a multiple-choice test, Newton (2008) used an individual written assessment, Behr and her colleagues (1997) used individual interviews, but Tobias used transcripts of class discussions. PSTs may be better at demonstrating their learning through experiences where they are trying to explain ideas to other learners, much like teaching, than in situations where they are being individually evaluated. Likewise, assessments that parallel the work of teaching, as opposed to assessments that recreate PSTs' past schooling experiences, may be more authentic evaluations of their knowledge. This conjecture may be pertinent not only for research, but for instructors of elementary MTF courses as they plan their assessments.

In summary, the results of research on the effectiveness of MFT courses varies. This may be due to the method or content of the assessments used in the studies. But it may also be the result of the natural and significant variations between subjects at different institutions across the United States. On the other hand, the variation in the courses themselves may have resulted in different outcomes for PSTs. Unfortunately, few of the studies on mathematics content courses for elementary teachers described the nature of the courses or curriculum materials that were used. Research on K–12 education has found that curriculum, instruction, and teachers' knowledge and beliefs all have an impact on student learning (Campbell et al., 2014; Hiebert & Grouws, 2007; Hill et al., 2005; Remillard et al., 2014; Senk & Thompson, 2003). It is reasonable to conjecture that such factors contribute to pre-service elementary teacher learning as well. In the next section, I describe relevant research results that also may contribute to variations in PSTs' learning.

Research on instructors. One source of variability in PST learning may be the academic and professional background of the instructors. McCrory and her colleagues (2009) found that 67% of the variance in PST gains from a pre- to a post- assessment using the Learning Mathematics for Teaching instrument (Ball et al., 2008) could be ascribed to the instructor. It may be that the academic and professional backgrounds of the instructors, which vary significantly according to several studies, could be a significant source of variability in PSTs' learning. The academic and professional background of instructors are important in a unique way: instructors must draw upon advanced knowledge of mathematics, knowledge of children's mathematical conceptions, knowledge of elementary curriculum, and knowledge of pre-service teachers' conceptions (Chauvot, 2009; Olanoff, 2011; Superfine & Li, 2014; Zazkis & Zazkis, 2011). These multiple knowledge bases may not necessarily be developed by obtaining a Ph.D. in mathematics or by teaching elementary school mathematics. Given that there is variation among mathematics education doctoral programs (Reys, Glasgow, Teuscher, &

Nevels, 2007), this pathway may also fail to address some of these knowledge bases.

Masingila and her colleagues' (2012) survey found that most of the responding institutions reported that their elementary MTF courses (88.3%) were offered through mathematics departments. A minority (7.6%) were offered by education departments, and even fewer (4.1%) were joint efforts between the two departments. However, Masingila and her colleagues' (2012) results may not provide the whole story. Only 42.8% of 1,926 contacted institutions responded. Furthermore, Masingila and her colleagues (2012) contacted *mathematics* departments in their work, a design choice that might result in the under-representation of institutions that provide these courses through education departments. Even with this caveat, it is reasonable to assume that the majority of mathematics courses for elementary teachers are offered through a department of mathematics.

Many institutions offer multiple sections of mathematics courses for elementary teachers (Lutzer et al., 2007). Lutzer and colleagues (2007) found in their study that between 69% and 90% of institutions with multiple sections, depending upon the institution type, appointed a course supervisor. This finding was corroborated by Masingila and colleagues (2012), who found that vast majority of course coordinators (92.6% of responding institutions) typically taught at least one of the sections. Therefore, it is worth considering the backgrounds of both course supervisors and other instructors.

The employment status of instructors may have an impact on undergraduate students across a range of academic disciplines. Some studies have found that undergraduate students' exposure to part-time faculty can decrease retention (e. g., Ehrenberg & Zhang, 2005; Jaeger & Hinz, 2008), though not all studies have found a connection between part-time faculty teaching and students' course success or learning outcomes (e.g., Bolge, 1995; Fike & Fike, 2007). One large scale study (Umbach, 2007) of 17,914 faculty members from 130 institutions found that part-time faculty used collaborative teaching techniques less frequently, spent less time preparing, challenged students less, and had fewer interactions with students outside of class time than their full-time peers. This same study found that non-tenure-track faculty were less likely to challenge students and had fewer interactions with students than tenure track faculty. However, these studies are across a variety of departments and may not be generalizable to education courses. Indeed, institutions may hire practicing K–12 teachers to teach mathematics content courses for elementary teachers. Practicing K–12 teachers may be more likely to use collaborative teaching techniques and may be able to more readily make connections to the K–12 classroom than full-time faculty. An important first step is to identify how common it is for institutions to hire part-time faculty and non-tenure track faculty to teach these courses. Then, research can further investigate the impact of these faculty populations on instruction in mathematics content courses for elementary teachers.

Studies identifying the characteristics of faculty teaching mathematics-for-teaching courses differ in their conclusions. Of particular interest is the background on course coordinators, the faculty members who would be most likely to provide coherence for other instructors. Lutzer and colleagues (2007) found that 90% to 100% of the coordinators were tenure track or were full-time faculty with a Ph.D. In contrast, Masingila and colleagues (2012) found that just over half of the course coordinators had a doctorate, nearly equally split between mathematics (24.3%) and mathematics education (28.7%), and most of the remaining supervisors held masters' degrees. The reason behind this discrepancy may be due to changes in the allocation of resources between 2005 and 2012, natural variation among samples, or some methodological difference between the studies.

While course supervisors are important, many institutions offer multiple sections of elementary MTF courses (Lutzer et al., 2007), so the characteristics of all faculty members teaching elementary MTF courses matters. In their survey, Blair and colleagues (2013) found that 62% of institutions offering elementary MTF courses reported that tenured or tenure-track professors generally taught this course, while 26% reported that other full-time faculty members generally taught the course – that is, all but 12% of institutions reported that full-time faculty were charged with teaching the course (Blair et al., 2013). In contrast, Masingila and colleagues (2012) found that over one-third of the responding institutions hired part-time faculty to some degree to teach these courses. Only between 20–40% of the institutions responding to Masingila's survey reported that the courses were taught exclusively by full-time, tenure track faculty, depending upon institution type. McCrory, Francis, Young, and Hall's (2008) much smaller, in-depth survey (n = 63) results aligned more closely to Masingila and her colleagues' (2012) findings. Given that previous research (Umbach, 2007) indicated that full-time, tenure-track faculty outperform part-time faculty and full-time, non-tenure track faculty on measures of instructional practice and interactions with students, the employment status of instructors may be an important factor in the quality of pre-service teachers' mathematical education. On the other hand, mathematics-for-teaching courses may be so specialized that the trend in other departments may not hold.

The academic backgrounds of instructors tend to be in either mathematics or mathematics education. Masingila and colleagues (2012) and McCrory and colleagues (2008) found that both coordinators and instructors were approximately equally likely to have advanced degrees in mathematics as in mathematics education. However, Masingila and colleagues (2012) found that fewer than half of the course supervisors at four-year colleges in their study held a doctorate in their discipline. Among two-year colleges, which are estimated to provide 40% of elementary teachers with some mathematics coursework, only 8% reported that their instructors typically held a doctorate in either discipline (Masingila et al., 2012).

Just as instructors' academic backgrounds vary, so does their teaching experience. Both Masingila and colleagues (2012) and McCrory and colleagues (2008) found that instructors have extensive tertiary teaching experience: 83.2% of the institutions in Masingila and colleagues' (2012) study reported that their course supervisors had college teaching experience, and instructors in McCrory's study had taught the mathematics content course for elementary teachers an average of 14 times.

However, instructors' experience in K–12 schools is generally lacking. McCrory and colleagues. (2008) found that less than half (31 of 63) of instructors were certified to teach K–12; even fewer (26) had actually taught in a K–12 school. Masingila's larger study (2012) found a similar lack of K–12 teaching experience, shown in Table 2.1.

Table 2.1
Percent of Institutions with Course Supervisors or Instructors with Various K–12 Teaching Experience (compiled from Masingila et al., 2012)

Characteristics of Course Supervisors and Instructors	Percent of Institutions
Course supervisors typically have secondary teaching experience (grades 7–12)	61.0%
Course supervisors typically have elementary teaching experience (grades K–6)	28.9%
Course supervisors typically do not have pre-college teaching experience	12.8%
Full-time, tenure-track professors other than course supervisors typically have secondary teaching experience. (Varies by institution type)	30–50%
Full-time, tenure-track professors other than course supervisors typically have elementary teaching experience. (Varies by institution type)	15–22%

Is the lack of elementary MFT course instructors with elementary teaching experience a cause for concern? According to the instructors themselves, not necessarily. Some instructors cited their college teaching experience, tutoring, or research as sufficient preparation for teaching elementary MFT courses (Welder, McCloskey, & Searle, 2013). Other instructors perceived their advanced mathematical knowledge to be more important than having elementary teaching experience when instructing pre-service elementary teachers (Zazkis & Zazkis, 2011). However, some researchers have noted that the mathematical knowledge used in content courses for elementary pre-service teachers goes beyond purely mathematical knowledge. For example, Superfine and Li (2014) analyzed a database of artifacts, including video, from five sections of math content courses for elementary teachers taught by four different instructors. They found that the instructors drew upon knowledge that was both different from mathematicians' and from elementary teachers' knowledge. For example, instructors knew how to use common *elementary students'* errors or elementary curriculum materials to help pre-service teachers identify their *own* mathematical misconceptions. Knowledge of common elementary student errors might come from working in elementary schools. Knowledge of PSTs' misconceptions, which are different from children's misconceptions (Newton, 2008; Superfine & Li, 2014), may come from working with PSTs. Such knowledge might also come from reading research (Chauvot, 2009) or working with educative curriculum materials specifically designed for such courses. However, it is unlikely that an instructor could develop this multi-faceted knowledge by reflecting on his or her own experience as a mathematical learner, a key method that many mathematicians use to develop their pedagogical content knowledge (Finn, 2010; Iannone & Nardi, 2005).

One other aspect of necessary knowledge that researchers and mathematicians themselves report may be lacking is the ability to make connections to the elementary classroom context. Mathematics professors teaching mathematics courses in which secondary pre-service teachers were enrolled were able to speak at length about how their courses helped PSTs develop deep conceptual knowledge and awareness of the nature of mathematics as a discipline (Hodge et al., 2010). However, the professors voiced their concerns that they were unaware of the responsibilities and expectations of contemporary U.S. high school mathematics teachers (Hodge et al., 2010). McCrory and her colleagues (2008) found that, while instructors of elementary content courses were highly aware of the 2000 NCTM Principles and Standards, few were "very familiar" (p. 6) with the content of certification tests, state curriculum guides, K–8 state assessments, or even the publication uniquely targeted toward this audience, the Conference Board of Mathematical Sciences' Mathematical Education of Teachers (*MET I,* 2001). Hart and her colleagues' (2013) found that some instructors of elementary MFT courses did not feel it was their role to make connections to the elementary classroom and, when pressed, could not explain the connection between course content and teaching elementary students.

In sum, there is a variety of academic and professional backgrounds instructors bring to mathematics content courses for pre-service elementary teachers. It must be recognized that the knowledge involved in such courses is complex and does not typically come from one standard preparation path. Moreover, there may be little professional development or support from the university for instructors of these courses (Masingila et al., 2012). As Ball and Cohen (1996) have suggested, curriculum materials could offer support for instructors because they are embedded into instructors' daily work. However, such materials must take into account the varied background and knowledge bases of this population of college instructors.

Research on pedagogy. Research on post-secondary mathematics instruction has corroborated the research at the K–12 level: instruction significantly impacts students' learning and beliefs (Boaler & Staples, 2008; Cox, 2015; Freeman et al., 2014; Hiebert & Grouws, 2007; Mesa, Burn, & White, 2015). While recommendations abound, there is little research on the pedagogy used in mathematics content courses for elementary teachers. However, what research does exist supports policy recommendations (CBMS, 2012). Specifically, two studies demonstrate that the recommended instructional practices increase PSTs' mathematical knowledge for teaching (McCrory et al., 2009; Superfine et al., 2013) and two studies indicate that these practices can encourage PSTs' productive beliefs and affect (Lubinski & Otto, 2004; Spielman & Lloyd, 2004). However, several studies suggest that these recommended practices are not occurring in all mathematics courses for future teachers (Hart et al., 2013; Masingila et al., 2012; McCrory, Francis, et al., 2008; Olanoff, 2011; Walczyk & Ramsey, 2003). While some studies have shown that professional development can support high quality instruction in mathematics courses (Bleiler, 2014; Walczyk et al., 2007), these opportunities appear to be rare (Blair et al., 2013; Masingila et al., 2012; Mathematical Education of Teachers' Project, 2006; McCrory, Francis, et al., 2008; Walczyk et al., 2007).

One study articulated course design principles that seemed promising for increasing pre-service teachers' mathematical knowledge for teaching. Superfine, Li, and Martinez (2013), over the course of five years and across ten sections of math content courses for elementary teachers, developed tasks for use with PSTs and demonstrated moderate to large effect sizes in PSTs' growth in mathematical knowledge as measured by the Learning Mathematics for Teaching assessment. Superfine and her colleagues (2013) ascribed their success to three design principles:

> (1) Mathematics content is organized to deepen PSTs' understanding of mathematics for teaching, (2) PST learning includes opportunities for engaging in the mathematical practices of explanation and representation, and (3) PSTs' learning of mathematics is grounded in the practices of teaching mathematics. (p. 45)

Under the first principle, the researchers identified three strands: understanding why algorithms work, understanding why rules work, and examining definitions. In implementing the second principle, the researchers had PSTs involved in the mathematical practices of explaining, justifying, creating representations, and evaluating explanations through classroom discussions facilitated by a knowledgeable instructor. The third principle situated PSTs' learning in authentic teaching tasks that drew upon mathematical knowledge, such as making an elementary math problem more or less difficult. While Superfine, Li, and Martinez did not suggest that these design principles would result in more learning than a course using different principles, their PSTs' gains in mathematical knowledge for teaching were sizable.

In contrast, McCrory and colleagues' (2009) findings did link specific instructional practices to higher gains in PSTs' growth in mathematical knowledge for teaching. Their results are corroborated by the recommendations of Superfine and colleagues (2013). McCrory and colleagues' (2009) study used regression to compare the relationship between self-reported instructional practices and PSTs' gains on measures of mathematical knowledge for teaching. These researchers collected data on 1,706 pre-service teachers and their instructors at 17 institutions across four states. Participating PSTs completed pre- and post- assessments using previously validated items measuring mathematical knowledge for teaching. While on the whole, mathematics content courses for pre-service elementary teachers increased PSTs' MKT by nearly a full standard deviation, particular instructional practices were associated with even higher gains. If their instructors reported frequently using instructional techniques that actively engaged the PSTs, then the PSTs in their classes learned more MKT than those whose instructors reported using such techniques less often. Active engagement was measured by the instructors' reported frequency in involving their pre-service teachers in each of the following: explaining the reasoning behind an idea; working on problems for which there was no immediate method or solution; analyzing similarities and differences among several representations, solutions, or methods; working on mathematical communication and/or representation; making conjectures and exploring possible methods to solve a mathematical problem; discussing different ways that they solve particular problems; writing about how to solve a problem in an assignment or test; doing problems that had more than one correct solution; as opposed to listening to the instructor explain terms definitions, or mathematical ideas, computational procedures, or methods (McCrory, 2009). From this list of instructional practices, it is clear that active engagement was not just a measure of the amount of time PSTs spent doing something active. Rather, this list demonstrated that courses that engaged PSTs in mathematical practices – conjecturing, explaining, representing, communicating, and solving novel problems – resulted in PSTs' developing more mathematical knowledge for teaching. These results support recommendations in the *MET II* (CBMS, 2012) about giving PSTs opportunities to engage in mathematical practices in mathematics-for-teaching courses. McCrory and colleagues' (2009) findings also align with Superfine and colleagues' (2013) articulation of their second design principle. In both cases, these activities may have contributed to PSTs' achievement gains.

Both teachers' knowledge and teachers' beliefs have been shown to have an impact on the mathematical instruction their students receive, so the impact of mathematics content courses on PSTs' beliefs is important as well (A. Philipp, 2007). Lubinski and Otto (2004) demonstrated how instructional practices in elementary MFT courses can impact PSTs' beliefs and attitudes about mathematics and themselves. The researchers reported on an elementary MFT course that used problems to deepen PSTs' conceptual understanding of elementary mathematics. The course focused on fewer topics but featured high expectations for the depth of understanding and level of reasoning. The course designers sought to challenge the view of mathematics as a solitary activity. They aimed to challenge PSTs' notion that there is only one solution for every problem and that the solution must be learned by watching the teacher. The course used class discussion and small-group problem solving. The instructor pressed for reasoning, built upon PSTs' ideas, and shared mathematical authority with the PSTs. Written surveys and interviews of 16 of the 20 students in the course indicated that the PSTs held more positive attitudes about mathematics as a result of the course. They became more patient problem solvers and were more comfortable with struggle. They felt that the class prepared them to teach elementary children. While the study did not report on PSTs' learning, other research has addressed the importance of beliefs, attitudes, and confidence about mathematics for learners' mathematical development and for teachers' instructional practice and future learning (Collopy, 2003; A. Philipp, 2007; J. T. Remillard & Bryans, 2004). Therefore, the impact of instructional practices in elementary MFT courses on PSTs' affect should also be considered.

A second study (Spielman & Lloyd, 2004) examined the impact of curriculum and instruction on PSTs' mathematical learning and beliefs. In Spielman and Lloyd's (2004) quasi-experimental study, two sections of a mathematics content course for pre-service elementary teachers taught by the same instructor were given two different treatments. The first section (n = 19) used the textbook Billstein, Libeskind, and Lott (2001) with the associated activity manual. PSTs had many opportunities to work in small groups, explain their solution strategies at the board, and ask questions. Thus, this section experienced, to some degree, instruction that involved PSTs engaging in mathematics themselves, as recommended by McCrory and colleagues (2009). However, following the perceived "philosophy of the textbook authors" (Spielman & Lloyd, 2004, p. 34), the instructor portrayed himself and the textbook as the mathematical authority. He encouraged PSTs to refer to the text, directly answered PSTs' questions, and introduced new topics through short lectures. In contrast, the second section (n = 34) used two NSF-funded middle school curricula, *Math in Context* (2001) and *Connected Mathematics Program* (1991–1997), instead of a textbook designed specifically for such courses. Following the perceived intended pedagogy of these texts, the PSTs were held as the mathematical authority. As in the course section using Billstein textbook (2001), PSTs spent time presenting and discussing homework solutions and working in small groups. However, the instructor did not introduce new ideas with a lecture, provide sample problems, nor answer PSTs' questions directly. When PSTs had questions, the instructor would ask questions that helped them re-examine their own thinking. When PSTs presented solution strategies, they asked each other questions, to repeat ideas, and to consider alternative solutions, rather than speaking through the instructor.

The results in terms of PSTs' achievement on a pre- and post- assessment demonstrated that neither the text nor the pedagogy resulted in differences in PSTs' mathematical knowledge for teaching, though they did result in differences in PSTs' beliefs. PSTs in the Billstein (2001) section were much more likely to see the instructor, as opposed to their peers, as the source of learning – 68.4% compared to 21.1%, respectively – when compared to PSTs in the middle school materials section, where only 8.8% identified the instructor as the source of learning and 82.4% identified their peers. Pre-service teachers in the middle school materials section increased the amount of time they thought should be spent on group work, as opposed to lecturing. They were also more likely to prioritize the exploration of new ideas over practicing for skills mastery than the section using the Billstein text. The textbook and pedagogy also impacted PSTs' beliefs about curriculum. At the beginning of the course, PSTs in both sections viewed examples, practice problems, and explanations as important for textbooks. While the PSTs in the section using the Billstein textbook (2001) did not change their views significantly, PSTs in the course using middle school curriculum began to see these features as less important. The textbooks and pedagogy used in such courses can influence how PSTs think about mathematics teaching in their future work with children.

In sum, evidence suggests that there are a set of instructional practices that improve PSTs' development of mathematical knowledge for teaching, such as actively engaging in doing mathematics during class time, explaining, comparing, and justifying solutions, and connecting the mathematics to the practice of teaching elementary school children. There are also practices that appear promising for impacting PSTs' beliefs about mathematics, teaching and learning mathematics, and their beliefs about themselves as mathematical learners: collaborative problem solving, class discussion, and sharing mathematical authority with PSTs. However, several research studies suggest that not all pre-service teachers experience this kind of instruction.

Instructors in mathematics content courses for elementary teachers may not necessarily be using research-proven strategies in their instruction. McCrory and colleagues (2009) found that instructors used a range of instructional practices and suggested there were some instructors who infrequently had PSTs engage in the listed practices. In another report (McCrory, Francis, et al., 2008), these researchers documented the ways in which instructors reported using class time. On average, instructors in their survey (n = 63) spent 32% of class time lecturing and 23% of the time having PSTs work in small groups. However, the researchers noted that the time spent in these two categories varied widely, even while other categories, like time spent on independent practice (10%) or homework (11%), were relatively consistent. Masingila and colleagues' (2012) results also indicate that a range of instructional practices maybe occurring in mathematics content courses for elementary teachers. Over 13% of the 825 responding institutions reported using a primarily lecture-based format in their math content courses for elementary teachers, while 71.2% of these institutions reported using a combination of lectures and activities.

Quantitative reports of time spent on “group work” or attention to “conceptual understanding” may, however, obscure the practices occurring in mathematics content courses for elementary teachers. Olanoff's (2011) qualitative study examined three experienced instructors of mathematics content courses. All of the instructors professed a commitment to conceptual understanding and allowed for group work. However, in lessons on multiplication and division of fractions, it was not clear that the instructors allowed students to make sense of conceptual ideas themselves. One instructor in the study did the conceptual mathematical work for the students, rather than allowing the students to engage in sense making. The other two instructors scaffolded the experience so highly it was unclear if pre-service teachers had an opportunity to make sense of the reasoning or representations themselves. One instructor did not hold pre-service teachers accountable for understanding representations; the assessments consisted of calculations.

A cross-case synthesis of two studies identified pedagogical methods and course content as a concern among both instructors of elementary MFT courses and their students (Hart et al., 2013). One study used interview data of PSTs who had taken an elementary MFT course at a U.S. institution with a poor success rate in these courses. The second study used interview data of instructors of elementary MFT courses at a Canadian institution. Pre-service teachers in the first study described uncaring classroom environments dominated by lecture. They reported having to “memorize” and “regurgitate” (Hart et al., 2013, p. 444) content that was presented too abstractly and seemed disconnected from teaching elementary school students. They reported feeling anxious, stressed, stupid, and alone. The instructors at the Canadian institution also addressed the content and pedagogy of the courses. Some instructors reported using activities. These instructors were motivated by the need to help PSTs make connections to the elementary school classroom. Other instructors, though concerned about their PSTs’ affect, lectured due to perceived time constrains or discomfort with other methods. Some instructors did not feel that making links to teaching elementary students, whether through activities or through explication of connections, was within their expertise or their role. When pressed, some instructors could not make specific connections between the course content and teaching mathematics in elementary school. This study identified instructors’ challenges and motivations for using more active learning strategies. It also identified how instructors’ knowledge of teaching in elementary school or their perception of their role could impact their pedagogy. Moreover, the study highlighted the very real impact poor instruction can have on PSTs’ affect, self-perception, and success in their future work.

There is no data to indicate whether the instructional practices in these case studies is common in mathematics content courses for elementary teachers. As Speer, Smith, and Harvath (2010) noted in their review of the literature, there is little research into instructional practices in college level mathematics courses in general. However, the few studies that do exist on university mathematics and science courses generally may provide some insight. Two large-scale surveys (Finn, 2010; Walczyk & Ramsey, 2003) provided information about instructional practices in university mathematics courses broadly.

Walczyk & Ramsey's (2003) large scale survey (*n* = 230) of mathematics and science instructors in Louisiana found a moderate to low use of student-centered instructional practices. With a response rate of 28%, the study indicated that lecture was a dominant form of instruction. Use of small group work and informal summative assessment were rare. The researchers found that instructors teaching pedagogical methods courses in science and mathematics were not likely to use more learner-centered instructional practices than other mathematics and science at a statistically significant level.

Finn's (2010) survey focused on mathematics courses with enrollment that included prospective secondary teachers. This study had an estimated response rate of 38% with 877 respondents. Within the sample, 47% scored high on instructional practices that suggested transmission of information to students. A little over one-third (36%) scored low on the scales that measured a focus on changing students' conceptual understanding. On average, survey respondents rated well-organized lectures as the most important feature of good instruction. Whole class discussion, student-to-student discussion, small group work, and student presentations were all ranked lower in importance. As a whole, instructors reported less frequently orienting students toward each other's mathematical thinking or having them interact with each other.

This prevalence of the use of lecture over student-to-student interaction and addressing students’ own ideas conflicts with the research recommendations for effective teaching in college level mathematics courses (see National Research Council, 2015, discussed earlier). Freeman and colleagues’ (Freeman et al., 2014) meta-study found students in STEM courses that use lecture were nearly twice as likely to fail (*n* = 67 studies). In contrast, this meta-study found that students in courses that used active learning, broadly defined, were more likely to perform better on examinations and concept inventories (*n* = 158 studies).

There is reason to believe that improving college instructors’ use of strategies that actively engage learners is possible. Walczyk, Ramsey, and Zha (2007) found that the instructors in their survey who had pedagogical training in graduate school were more likely to engage in instructional innovation. In another survey, faculty who attended trainings on learner-centered instruction were slightly more likely to enact such practices (Walczyk & Ramsey, 2003). Bleiler's (2014) case study demonstrated how mathematicians and mathematics teacher educators can learn from one another by co-teaching a course for pre-service teachers.

However, graduate school training in pedagogy, co-teaching, and formal professional development opportunities seem to be rare. According to one survey, 71% of mathematics and science faculty reported receiving no training in pedagogy in graduate school (Walczyk et al., 2007). Well over half of the institutions in Masingila's (2012) study reported no formal training or support for instructors of math content courses for elementary teachers, with a small minority (less than 12% at Ph.D. granting institutions) engaging in co-teaching. Collaboration of any kind between education and mathematics departments has reportedly been rare; only 6–8% of institutions reported extensive collaboration in planning or teaching courses for prospective elementary teachers or conducting research (Mathematical Education of Teachers' Project, 2006; McCrory, Francis, et al., 2008). Blair and colleagues (2013) similarly reported that few (less than 15%) of the institutions responding to their survey had mathematics and education faculty co-teach courses. Even when faculty had resources to improve their instruction provided by the university, they don't always use them (Walczyk et al., 2007). Therefore, resources that are embedded into the daily work of an instructor such as curriculum support materials need to be investigated as one powerful way to help instructors improve their practice.

Research on textbooks for content courses for elementary teachers. As described earlier, textbooks specifically written for elementary MFT courses are heavily, if variably, used by instructors. More importantly, courses that use such textbooks lead to PSTs' developing stronger mathematical knowledge for teaching than courses utilizing instructor-created materials or other texts, such as those designed for college algebra courses. Research on the content of such textbooks can inform understanding of these courses.

As discussed earlier, McCrory and her colleagues' (2009) study identified factors that led to PSTs' developing more MKT. The study involved 1709 PSTs at 17 institutions taking pre- and post-assessments composed of items from the Learning Mathematics for Teaching assessment. Compared with PSTs in courses using instructor-created materials or books designed for other courses, PSTs in courses using textbooks specifically for elementary MFT courses saw an additional 4.58-point gain, nearly one half of a standard deviation. Given this impact that textbooks had on PSTs' learning, McCrory and her colleagues (McCrory, Siedel, & Stylianides, 2008) looked more closely at the contents of the textbooks. In light of the *MET II* recommendations and the tensions reported by faculty, some of the researchers' findings are worth discussing here. In particular, the format of the textbook may contribute to instructors' feelings of an overwhelming amount of material to address. The presence or absence of attention to mathematical practices in the texts is notable given the *MET II* recommendations. In addition, the researchers' observations about the abstract presentation of ideas and limited connections to teaching in elementary school echoed the concerns of pre-service teachers in Hart and colleague's (2013) study.

McCrory and her colleagues (2008) analyzed 14 textbooks for elementary MFT courses that were on the market at that time. In examining the format of the textbook, they categorized books by "coverage" and by "presentation" (p. 8). Books were classified as having extensive or intensive coverage; books that included connections to history, curriculum, or additional puzzles were labeled as extensive, while books that concerned themselves primarily with "pure" (p. 10) mathematics were labeled as intensive. While the extra features in extensive books may be helpful for PSTs, McCrory and her colleagues noted that these connections, which usually appeared in sidebars, could also make it difficult for PSTs and their instructors to focus on key ideas. Moreover, there was little direction as to how to use these sidebars in most extensive textbooks. Books were also classified as encyclopedic or narrative in presentation. Encyclopedic texts treated mathematical topics with relatively equal weight, while narrative texts told a “story” (p.10) of mathematics. McCrory and her colleagues hypothesized that the narrative texts may help instructors to focus on relevant topics and make connections, though the instructors may not necessarily agree with the given narrative or may want to prioritize other topics. Moreover, the researchers noted that the extensive and encyclopedic texts are difficult to read (McCrory, Siedel, et al., 2008), even though it seemed clear that the textbook authors assume PSTs will learn from reading the texts closely (McCrory, 2006). Extensive, encyclopedic texts could contribute to instructors and PSTs feeling like there is so much content to address that they must resort to lecture and memorization, as reported by Hart and colleagues (2013).

One major contribution of McCrory and colleagues’ (2008) extensive work is the attention to mathematical practices, captured in their analysis on reasoning and proof and their investigation of the “mathematical stance” (p. 8) of the different texts. Their methods for analyzing these two areas were different. The analysis of reasoning and proof took a more macro view, identifying sections of the book or places in the index where the terms reasoning, proof, explanation, problem-solving, or other terms could be found. That is, they looked for evidence of explicit descriptions of reasoning and proof in the exposition of the texts. Their work on mathematical stance was more nuanced. By “mathematical stance,” they meant disciplinary practices: the nature and role of assumptions, definitions, precision, and mathematical reasoning. The researchers found that some texts were explicit about such practices, calling attention to them in the exposition. Other texts were more implicit. For example, the authors may have used definitions in a meaningful way without calling attention to the practice. In addition, there were books that neither implicitly nor explicitly included meta-mathematical ideas. The researchers made no judgment on whether implicit or explicit treatment of meta-mathematical ideas is more beneficial; there's little evidence to suggest that reading about such ideas is inherently superior to helping PSTs value mathematical practices through other means. However, they do caution against textbooks for which attention to meta-mathematical ideas is absent. Given that textbooks often set the boundaries for opportunities to learn, one wonders where PSTs will learn how to engage in mathematical practices, as recommended by the *MET II* (CBMS, 2012), if such ideas are completely absent from textbooks.

The notion of explicitness and implicitness in McCrory and her colleagues' (in revision, 2008) analysis extended from meta-mathematical ideas to other aspects of the textbooks, particularly aspects relevant to mathematical knowledge for teaching. Of particular interest was the common occurrence of a lack of explicit discussion of connections between representations or meanings. For example, few books explained why $a \div b$, the result of an operation on two numbers, is the same as $\frac{a}{b}$, typically viewed as a part of a whole, a number in and of itself. Connections among representations, such as area and equal group models for multiplication, were rarely made explicit. The difficulties or nuances of using such models as one moves across number systems – from whole numbers to fractions, or from whole number to integers – were not discussed in the texts analyzed. McCrory and her colleagues also noted that connections between the topics in the text and mathematical knowledge *for teaching* – that is, how such learning objectives for PSTs relate to the work of the teacher – were implicit. Given that Superfine, Li, and Martinez's (2013) study recommended that PSTs' opportunities to learn mathematics be rooted in the practices of teaching mathematics, one wonders where instructors will be able to find such opportunities if they are not present in the textbooks – particularly if the instructors themselves do not have elementary teaching experience. Indeed, McCrory and colleagues' (2008) finding in regards to textbooks is echoed by the PSTs' concerns about the disconnect between course content and teaching mathematics in elementary school in Hart and colleagues' (2013) study.

The fact that PSTs must learn not just elementary mathematics but elementary mathematics *for teaching* makes such explicit details in textbooks even more important. One example McCrory and her colleagues (in revision, 2008) cited is the definition of a fraction that different texts used. While some texts built upon the definition of a fraction used in early elementary grades as a part of a whole, other books defined a fraction in ways more typical of higher mathematics. For example, a text might define $\frac{3}{4}$ as the solution to the equation $4x = 3$. McCrory (2006) speculated that beginning with a definition that is less closely aligned to PSTs' own existing conceptions of fractions may not be the best way to help them examine and deepen their existing understanding. She further pondered whether such a formal definition as the one below would be helpful for PSTs' work with future elementary students:

> [N]umbers of the form a/b are solutions to equations of the form $bx = a$. This set, denoted Q, is the set of rational numbers and is defined as

> follows: $Q = \{a/b \mid a \text{ and } b \text{ are integers and } b \neq 0\}$ (Billstein, 2003, p. 266, as cited by McCrory, 2006, p. 25)

Indeed, this example resonates with the concerns of Hart and colleagues' (2013) PSTs that the mathematics in the course was "too high level" (p. 448), i.e., too abstract and unusable for teaching mathematics to elementary school children.

Much of previous research on mathematics content courses and curriculum for elementary pre-service teachers has often focused on mathematical topics, such as the NCTQ (Greenberg & Walsh, 2008) report. McCrory's research highlights some broader questions about curriculum: What opportunities do pre-service teachers have to learn about mathematical practices? What might they be inferring about mathematical definitions? Are there opportunities for them to see mathematics as a coherent discipline by making connections across representations and number systems? Are there opportunities for them to connect their learning to their future work as teachers? These questions are reminders that mathematics content courses for elementary teachers have a much more complex charge than helping college students master content. These courses are also charged with helping future teachers develop a productive disposition and powerful vision of both mathematics and the teaching and learning of mathematics.

In conclusion, pre-service teachers' MKT gains as a result of the textbook choice or pedagogical style may be mediated by other factors. Yet, textbook choices and enactment of the implied pedagogy associated with the text can impact PSTs in other ways, such as their views of teaching and learning mathematics, their views of curriculum, and their notion of mathematical authority. The choice of textbook, the instructor's interpretation of the text, and the instructor's pedagogical strategy are all important in the education of pre-service elementary teachers.

Conclusion

Both research (especially McCrory et al., 2009; Superfine et al., 2013) and policy recommendations (Association of Mathematics Teacher Educators, 2017; CBMS, 2012; Greenberg & Walsh, 2008) lead to three main priorities for the mathematical education of pre-service elementary teachers:

1. Pre-service teachers should study the deep ideas of the mathematics they will teach.
2. Pre-service teachers should be able to connect their knowledge with the act of teaching. Mathematical knowledge for teaching is inherently rooted in the tasks of teaching.
3. Pre-service teachers need time and opportunity to engage in mathematical practices.

There is research to indicate that some institutions are succeeding at meeting these three priorities (e.g., Superfine et al., 2013). However, research also indicates that pre-service teachers' opportunities to engage in these three areas vary widely from institution to institution, perhaps even from course to course. On the first, most basic priority, it is estimated that one-fifth of institutions provide no formal opportunities to study elementary mathematics at a deep level through a content course designed specifically for them (Greenberg & Walsh, 2008; Lutzer et al., 2007; Masingila et al., 2012). In fact, some institutions have no mathematical requirements for PSTs at all (Lutzer et al., 2007).

The second priority, drawing connections between mathematical knowledge developed in these courses and teaching elementary students (CBMS, 2012; Superfine & Li, 2014), is likely to be a challenge even for institutions that provide math courses specifically for elementary teachers. The majority of textbooks for these courses analyzed failed to make connections to children's thinking (Greenberg & Walsh, 2008). Most of these textbooks also failed to explicate how the mathematical content they cover is related to mathematical knowledge *for teaching* (McCrory, Siedel, & Stylianides, 2008). Instructors tend not to have elementary teaching experience (Masingila et al., 2012). How, then, are these connections between mathematics and teaching supposed to surface in these classes? According to some PSTs and instructors of elementary MFT courses, they do not.

Lastly, it is not clear that PSTs have either the time or the opportunity to engage in mathematical practices in elementary MFT courses. Most institutions provide less than the recommended credit hours (Masingila et al., 2012). These institutions may be sacrificing breadth or depth in their content coverage, or they may be sacrificing time for PSTs to engage in mathematical practices. Moreover, not all instructors use pedagogical techniques that would provide PSTs with frequent opportunities to engage in problem-solving, explaining, conjecturing, or representing mathematical ideas (McCrory et al., 2009). Indeed, lecture as a pedagogical technique in both content and methods courses still exists (Hart et al., 2013; Masingila et al., 2012; McCrory, Francis, et al., 2008;

Walczyk & Ramsey, 2003). Many textbooks do not explicitly or implicitly call attention to reasoning, proof, or other "metamathematical ideas," (McCrory, Siedel, et al., in revision, 2008, p. 45). Furthermore, the research on curriculum for elementary MFT courses only examines the opportunities PSTs have to *observe*, or read about, mathematical practices. There is no research on the opportunities in published textbooks for PSTs to engage in mathematical practices themselves.

Some research on the efficacy of mathematics content courses suggest that they are not a universal panacea for improving pre-service teachers' mathematical knowledge for teaching (Luo et al., 2011; Newton, 2008). However, there is a notable body of work that suggests that these courses can be powerful levers in improving PSTs' MKT. Moreover, this research points to ways we can make improvements in instruction and in curriculum. As we work to improve the mathematical education of teachers, however, we must bear in mind the widely varying background of instructors, as well as the variation in pre-service teachers' own mathematical knowledge.

CHAPTER III
METHOD AND PROCEDURES

The purpose of this study was twofold: (1) to identify different instructional practices and their frequency in mathematics content courses for pre-service elementary teachers as well as relationships with instructor background characteristics and (2) to investigate how curriculum and related instructor support materials can support instructors in creating mathematically powerful experiences for pre-service elementary teachers in mathematics content courses. In order to answer the research questions, the study employed a mixed methods approach, using both quantitative and qualitative data. In Section I of this chapter, Schoenfeld's TRUMath rubric is presented and the instruments used are discussed. In Section II, the sample and the procedures for data collection are described. In Section III, the methods used to analyze the data are described.

Section I: Description and Design of the Testing Instruments

To investigate the different instructional practices used in elementary mathematics-for-teaching courses and how curriculum and related instructor support materials can support powerful instructional practices in these courses, the Instructional Practices & Curriculum Usage (IPCU) survey and two case studies were used. The IPCU survey was designed to collect information about the (1) instructional practices used in mathematics courses for elementary teachers, (2) the curriculum materials used in these courses, (3) characteristics of the survey participants' academic and professional background, and (4) the context within which the participants taught elementary mathematics-for-teaching courses. The two case studies employed videotaped observations, interviews, and video stimulated recall (VSR) interviews. The observations were designed to collect instances of instructional practices that led to a mathematically powerful classroom episode. The interviews were designed to collect background information on the instructors, identify the instructors' goals for the course and for their students, and learn generally about their use of curriculum materials. The VSR interviews were designed to gather data about the instructors' thinking behind their instructional actions and any connections they saw between their actions and the curriculum materials. This study was informed by Schoenfeld and colleagues' (2013, 2014; 2014) *Five Dimensions of Mathematically Powerful Classrooms.* Schoenfeld and his colleagues synthesized decades of research on mathematics teaching into these five dimension and created an observational rubric, called the TRUMath (Teaching for Robust Understanding of Mathematics) rubric. Schoenfeld's TRUMath rubric was used to analyze the observational data from the case study participants. Furthermore, the five dimensions of mathematically powerful classrooms influenced the design of the IPCU survey. These dimensions were also used in the analyses of the survey. Schoenfeld (2014) believes that learning mathematics in environments that feature these five dimensions contributes to learners' understanding of mathematics as well as impacts their views of themselves as mathematics learners.

The TRUMath rubric is an observational instrument that provides data on five different components or dimensions of mathematics teaching: Mathematics; Cognitive Demand; Equitable Access to Mathematics Content; Agency, Authority, and Identity; and Uses of Assessment. Schoenfeld states that these components are both “necessary and sufficient” (Schoenfeld, 2013, p. 607) for students’ to develop a robust understanding of mathematics. In Schoenfeld and colleagues’ use of this rubric, video of mathematics classes is broken into episodes lasting no longer than five minutes. Episode breaks are determined by a change in class structure (e.g., moving from small group to whole class discussion) or by a change in mathematical focus. Each episode is scored using either 1, 2, or 3 within each dimension; no fractional scores are allowed. An overview of the five dimensions and general scoring are provided below.

Mathematics. This dimension measures the extent to which the mathematics taught in the class portrays mathematics as a coherent discipline that can be figured out through sense-making, as opposed to a collection of isolated facts and procedures. The dimension also measures to what extent students are engaged in mathematical practices, such as reasoning and problem solving. A score of 1 would indicate that the class is completely dominated by rote skills while a score of 3 would indicate that the class features a connected and coherent view of mathematics and opportunities to engage in mathematical practices. A score of 2 indicates that there are few opportunities to make connections; the mathematics is primarily skill-oriented.

Cognitive Demand. Cognitive demand is related to the quality of the mathematics. Classes where the tasks are focused on "routine worksheets with detailed step-by-step procedures, or sets of repetitive exercises" (Schoenfeld, Floden, The Algebra Teaching Study, & Mathematics Assessment Project, 2014, p. 12) receive the lowest score on the rubric. However, the dimension of Cognitive Demand also reflects the implementation of the task and the degree to which *students* are engaged in productive struggle as opposed to having the struggle scaffolded away by the teacher. Classes where the instructor provides support that gives students access to rich tasks and allows students to encounter and engage with complexity themselves receive high scores under the *cognitive demand* dimension. A score of 1 in *cognitive demand* would indicate that students are primarily applying known procedures to routine problems, while a score of 3 would indicate that there are opportunities for productive struggle. A score of 2 would indicate that there is potential for productive struggle, but that the teacher "scaffolds away" the challenges. As Schoenfeld and his colleagues (2014) note in their description of the rubric, an instructor may talk about mathematical connections but offer students no opportunity to engage with the connections themselves. This situation would result in a high score on the *mathematics* dimension and a low score on the *cognitive demand* dimension.

Access to Mathematical Content. This dimension measures the distribution of opportunity in the classroom. Namely, it measures whether all students have an opportunity to engage in rich mathematics or only a few are actively engaged. It is not uncommon for only a subset of the students in a class to be called upon or asked conceptual questions. It may be that girls, students of color, students with less academic status, or quieter students have less access to the higher level mathematics being studied in the classroom, or it may be that the instructor has structures, routines, or norms in place that allow all students access to the mathematics. A score of 1 would indicate a class where only some students get to participate and the instructor does not attempt to remedy the problem. A score of 3 indicates that there is broad participation among students. A score of 2 indicates that the instructor makes some attempt at broadening participation, though he or she is unsuccessful within a particular moment.

Agency, Authority, and Identity. This dimension measures the extent to which students are responsible for generating and validating the mathematical ideas studied in the classroom, and the degree to which the instructor recognizes the students as authors of ideas. In some classrooms, it is the instructor or the textbooks that provide the solution strategies or the proofs of ideas. In other classrooms, students make conjectures from patterns or develop their own solution strategies, though the instructor plays a role in drawing students' attention to particular aspects of strategies or claims. This dimension is related to the *mathematics* dimension by ensuring that students have opportunities to engage in mathematical practices themselves. However, the *agency, authority, and identity* dimension is slightly different; the goal is helping students see themselves as capable of framing and tackling mathematical questions. A score of 1 indicates that the students have opportunities to say very little, while a score of 3 indicates that students explain their reasoning and build upon one another's ideas. In between, a score of 2 indicates that students have an opportunity to explain their thinking, but that the instructor determines whether individual students' ideas are correct and does not build upon student ideas.

Uses of Assessment. In many college mathematics classes, assessment activities are summative and are used to evaluate student progress, rather than to inform instruction. Formative assessment, on the other hand, involves instructors making decisions based upon student responses. Ideally, rather than only using assessment to assign students a grade, instructors use assessment formatively to build upon student ideas or address students' preconceptions. This dimension measures the extent to which student ideas are solicited, built upon, and addressed in instruction. A score of 1 indicates that student thinking is not elicited; a score of 3 indicates that students' ideas are used in the course of instruction. A score of 2 indicates that student thinking is surfaced, but not necessarily built upon.

Each of these dimensions and corresponding scoring are summarized in Table 3.1.

Table 3.1

Summary TRUMath Rubric (Schoenfeld et al., 2014, p. 408)

	The Mathematics	**Cognitive Demand**	**Access to Mathematical Content**	**Agency, Authority, and Identity**	**Uses of Assessment**
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

The TRUMath rubric is a classroom observation rubric. As such, it was used as designed to choose classroom episodes for the video stimulated recall interviews. In additions, items on the IPCU survey were designed to align to the TRUMath rubric. The next section describes the IPCU survey, including the alignment of items to the TRUMath rubric.

Instructional Practices and Curriculum Usage (IPCU) Survey

To gather information about instructors' pedagogy and use of curriculum in mathematics content courses for elementary teachers, the researcher developed a 74-item survey, called the Instructional Practices and Curriculum Usage (IPCU) Survey. The IPCU contained questions in four domains: (1) instructional practices, (2) curriculum usage, (3) instructor background, and (4) teaching context. The IPCU survey used items from the Postsecondary Instructional Practices Survey ("PIPS" Walter, Henderson, Beach, & Williams, 2015) and researcher-created items.

The Postsecondary Instructional Practices Survey is a previously tested survey instrument designed to measure the instructional practices of college faculty across a range of disciplines(Walter et al., 2015). This research study uses two sets of questions from the PIPS instrument. The first is a set of items measuring the percent of weekly class time spent in different formats, i.e., small group, individual work, or listening to the instructor. This set of items is referred to as "Question 3" in the IPCU Survey. The second set of items used from PIPS is a set of 24 Likert statements measuring instructional practices. This set of items is referred to as "Question 4" in the IPCU

Survey. Researchers who developed these items established face, content, and construct validity on them (Walter et al., 2015). The researchers administered the survey to 827 instructors across 72 departments at four institutions. For the set of Likert statements, the researchers established reliability for a five-factor model. This five-factor model was supported with good fit statistics. Items from the PIPS instrument were because the results from this study's sample could be compared to the results from a previous study by Walter and colleagues (2015) on mathematics instructors' use of instructional practices in other college mathematics classes.

The five-factor model for Question 4, the item containing 24 Likert statements, also makes conceptual sense for this study. The five factors are: (1) student-student interaction ($\alpha = 0.825$); (2) content delivery ($\alpha = 0.644$); (3) formative assessment ($\alpha = 0.641$); (4) student-content engagement ($\alpha = 0.606$); and (5) summative assessment ($\alpha = 0.447$). The first factor, student-student interactions, measured the extent to which instructors reported that they had students interact with each other about content during class. The second factor, content delivery, measured whether faculty reported directly presenting material to students. The third factor, formative assessment, measured whether the participant reported using assessment to inform instruction and whether he or she provided students with feedback. This factor is related to the fifth dimension of mathematically powerful classrooms, *use of assessment*. The fourth factor in the five-factor model, student-content engagement, was a measure of whether students were actively engaging in the discipline during class time. This factor is related to the mathematical practices aspect of the *mathematics* dimension of mathematically powerful classrooms (Schoenfeld et al., 2014). The last factor is about evaluative assessment practices. To summarize, this set of Likert statements provides information about general instructional practices and can further illuminate instructional practices relevant to two of the five dimensions of mathematically powerful classrooms, *mathematics* and *uses of assessment*. Because survey respondents may attend differently to the first item in a series of Likert statements, a researcher-created buffer item was added to the beginning of the set of statements in Question 4 *(I emphasize important mathematics)*. Therefore, there are a total of 25 Likert statements in Question 4, but data from the first statement is not used.

The remaining questions on the IPCU Survey were researcher developed and fall into four categories: (1) instructional practices, (2) curriculum usage, (3) teaching context, and (4) instructor background. Items addressing instructional practices were designed to be aligned to the TRUMath Rubric. For example, Question 2 was a multiple-select question that asked about five potential topics addressed in the course: (1) review of calculation procedures; (2) reasons why algorithms or formula work; (3) the nature of justification, reasoning, or proof in mathematics; (4) common misconceptions of elementary students; and (5) representations used in elementary school curricula or classrooms. If participants chose only the first topic, review of calculation procedures, this would align with a score of 1 on the TRUMath Rubric in the dimension of *mathematics*. Selection of the second topic addresses the attention in the *Mathematics* dimension given to connections between concepts and procedures, a component of the score of 3 on the rubric. Selection of the third topic is related to the engagement in mathematical practices, another component of scoring a 3 on the rubric. Topics 4 and 5 are related to mathematical knowledge for teaching. Other questions have answer choices aligned to a 1, 2, or 3 on the TRUMath Rubric for a particular dimension. There are also questions that work as a set to assign the participant a 1, 2, or 3 on a particular dimension. These connections to the rubric are described in more detail in the Analysis section.

Among the questions addressing instructional practices, a variety of question types are used. There are sets of Likert statements, open response questions, multiple-choice question, and multiple select questions. Some items have participants respond to a hypothetical scenario. The category, question type, question number, and more details about the topic of the question can be found in Table 3.2.

Questions about curriculum usage focused on the name of the textbook used, the extent of usage, and the use of ancillary materials, such as instructor guides or video and multimedia resources. This section was branched, so that participants only answered questions relevant to them. Questions about the teaching context addressed the perceived selectivity of the survey participants' institution or program, the number of students in each course section, and the number of sections taught each semester. The section about the participant's background include questions about the subject and level of the terminal degree earned, PreK–12 teaching experience, full or part time teaching status, and the participant's departmental appointment.

Table 3.2

Outline of IPCU Survey

Question Number	Topic	Subtopic	Question Type
1	Teaching Context	Content course or combination methods and content course	Multiple choice
2	Instructional Practices	Mathematics Content	Multiple select
3	Instructional Practices	Use of weekly class time	Numeric
4	Instructional Practices	General Instructional Practices: 1. Student-to-student interaction 2. Lecture and direct instruction 3. Formative assessment 4. Student engagement in disciplinary practices 5. Summative assessment	25 Likert-type statements, including 1 buffer item
5	Instructional Practices	Cognitive demand	5 Likert-type statements
6	Instructional Practices	Cognitive demand: response to student difficulty	7 Likert-type statements
7	Instructional Practices	Authority, Agency, and Identity: faulty student ideas	Open response
8	Instructional Practices	Access: definition of full student participation	Open response
9	Instructional Practices	Access: student participation strategies	Open response
10	Instructional Practices	Access: full student participation achieved	Yes/no
11	Instructional Practices	Authority, Agency, and Identity: novel student ideas	Multiple choice
12	Instructional Practices	Authority, Agency, and Identity: faulty student ideas	Multiple choice
13	Curriculum Use	Use a textbook	Yes/no
13a	Curriculum Use	Number of textbooks used	Numeric
13b	Curriculum Use	Extent of use	Multiple choice
13c	Curriculum Use	Materials other than textbook	Open response
13d, e1	Curriculum Use	Identify textbooks used	Multiple choice
14	Curriculum Use	How the textbook is used	Multiple choice
14a	Curriculum Use	Instructor manual use	Multiple choice
14b	Curriculum Use	Helpfulness of instructor manual	Multiple choice
14c	Curriculum Use	How instructor manual is used	Open Response
15a	Curriculum Use	Activity manual use	Multiple choice
15b	Curriculum Use	Activity manual use	Multiple choice

15c	Curriculum Use	Activity manual use	Open Response
16a	Curriculum Use	Multimedia use	Multiple choice
16c	Curriculum Use	Multimedia helpfulness	Multiple choice
16d	Curriculum Use	Multimedia helpfulness	Open Response
17a	Curriculum Use	Use of other resources	Multiple choice
17b	Curriculum Use	Use of other resources	Open Response
18	Teaching context	Number of sections per year	Numeric
19	Teaching context	Number of students per section	Numeric
20	Teaching context	Perceived selectivity of institution	Multiple choice
21a	Instructor background	Level of terminal degree	Multiple choice
21b	Instructor background	Subject of terminal degree	Multiple choice
21c	Instructor background	Departmental appointment	Multiple choice
21d	Instructor background	Full or part time status	Multiple choice
21e	Instructor background	Experience teaching PreK–12	Multiple select
23	Instructional Practices	Authority, Agency, Identity and Cognitive Demand: responding to students' incomplete ideas	Open Response

The IPCU Survey was designed by the researcher in conjunction with four mathematics teacher educators and one methodologist from Boston University. Content validity was established by three mathematics teacher educators and one mathematician. Feedback was sought from instructors of mathematics content courses for elementary teachers outside of Boston University. Four instructors from three different institutions participated in cognitive interviews about particular questions to ensure the intent of the questions was clear and that the language elicited data on the intended practice. In addition, several survey items were triangulated against directly observed practice. Specifically, the case study participants completed the survey and their responses were compared against their observed instructional practice.

To measure internal consistency reliability, factor analysis was conducted on questions using Likert-type scale measures. In addition, questions on the survey were triangulated with each other. Reliability is discussed in more detail in the Analysis section.

Use of TRUMath Rubric to Select Episodes for Video Stimulated Recall Interviews

The case studies used video stimulated recall (VSR) interviews (Lyle, 2003) to understand participants' thinking behind instructional moves that led to mathematically powerful moments, as well as how the participants saw the curriculum materials supporting them in these moments. To identify episodes for discussion in the VSR interviews, the TRUMath Rubric was used. First, the videos were segmented by class structure (whole class, small group, and individual student interactions). Then, I further segmented the video into episodes typically lasting between 45 seconds and 10 minutes, though a few were slightly longer. (While the TRUMath Scoring Guide recommends episodes of 5 minutes in length, to maintain coherence in this study, it was sometimes necessary to have episodes as long as 10 minutes.) These episodes were delineated by the instructor's move to another small group or individual student, discussion of a different problem, or discussion of a different solution strategy.

I then coded each episode using the TRUMath rubric (Schoenfeld et al., 2014) to assign a score of 1, 2, or 3 to each of the five dimensions of mathematically powerful classrooms. From all of the episodes that scored 3 on one or more of the dimensions, I chose episodes for which one or more of the five dimensions were particularly salient and for which there were a number of instructor actions. For example, if students were having a mathematically rich, cognitively demanding discussion in their small groups but the instructor did little other than say, “Interesting,” such an episode would not be likely to be chosen because there are few visible instructor actions, and therefore there would be little fodder for a VSR interview. Next, I chose episodes so that there were a variety of class formats, where possible: whole class discussion, interactions with individual students, interactions with small groups, student presentations, and teacher directions or exposition. Last, I attempted to choose episodes so that a variety of students were represented: those struggling with fundamental concepts as well as those quick to solve problems; those who were particularly eager to participate as well as those who spoke up more rarely. The TRUMath rubric scores on these selected episodes were verified by another researcher who had previously used the rubric in a different context. This researcher and the interrater reliability coder reached a consensus 100% of the time after discussion. There was originally disagreement on 28% of scores which led the researcher and interrater reliability coder to discuss the rubric and scoring guides provided by Schoenfeld and colleagues (2014) to delineate between the five dimensions. For the first case study, this process led to a selection of seven episodes totaling just under 30 minutes. Five of these episodes featured the participant using EMP and two of these episodes featured the participant using Billstein, for contrast. For the second case study, this process led to a selection of 11 episodes totaling just under 27 minutes. Ten of these episodes featured the participant using EMP and one of the episodes featured the participant engaged in an activity before using the Beckmann materials. As the study is about EMP materials, the non-EMP episodes were used to spark discussions about the participants’ typical practice and their perceptions of the differences between the two materials. The Billstein and Beckmann materials themselves were not studied.

Next, I listed the instructor actions occurring in each of the episodes. This list guided me in pressing for further details about their thinking in each of the episodes the case study participants and I watched together. After asking the case study participants about their initial thoughts about a particular episode, I would name the particular actions I saw in the episode and ask them to comment on why they took these actions. Typically, I would then ask them whether there was anything in the curriculum materials that supported them in their actions.

Interview Protocols

Case study participants were each interviewed ten times: once before any observations occurred; before and after each of the four class observation; and once following the conclusion of all observations in a video stimulated recall (VSR) interview. During these interviews, a set protocol was followed in order to determine participants' thinking regarding their instructional decisions. In the interview before the observations, I asked the participant about their goals for the course and for their students and their use of curriculum materials generally. In the interviews directly before each observation, I asked the participants about their goals for the course session and what they intended to do during the course session. After each session, I asked the participant whether they felt the particular class session was typical of their course. During the VSR interview, I first asked if the participant had any general thoughts about the episodes. After viewing a particular episode, I listed the instructional actions I saw and asked the participant about her thinking behind these actions. I then asked the participant whether the curriculum materials supported her in these actions. The interviews were conversational in nature and followed a semi-structured protocol (Patton, 2002). The full interview protocols can be found in Appendix A.

Confidentiality and Informed Consent.

Case study participants' data were recorded using pseudonyms and did not include any identifying information. Confidentiality was guaranteed for the survey respondents by using the Qualtrics option that automatically anonymizes survey responses. However, it was still possible to determine whether individuals completed the survey. The software allows the researcher to send reminder emails while maintaining confidentiality about the content of survey responses. At the end of this study, I downloaded the anonymized data and deleted the data from Qualtrics. All data was stored in a password-protected file on a password-protected computer that was only accessible to the researcher.

Case study participants were given an informed consent form. The researcher reviewed the form with the case study participants before any interviews or observations were conducted. During the interviews no participant was identified by name, and the identities of participants appearing on video tapes are not disclosed in this document. Survey participants indicated their informed consent by clicking on the first page of the survey.

Section II: The Sample and Procedures for Data Collection

The sample for this study was composed of two groups. The first group of participants in this study was a national volunteer sample of instructors of mathematics content courses for teachers. Five hundred and seven (507) instructors responded to the survey and reported teaching content courses or courses that combined content and methods. The second group consisted of two instructors of mathematics content courses for elementary teachers, the foci of the case studies.

Administration of the Instructional Practices and Curriculum Usage Survey

The first group of participants in this study was a national sample of instructors of mathematics content courses for teachers. In September of 2016 the IPCU survey was sent to over 1800 mathematics departments, the email lists of four professional societies, and professional contacts.

Instructors were sent an anonymous survey about their instructional practices and curriculum choices. The survey was sent to organizations and department heads until over 500 respondents replied, in November 2016. It was also sent to previous field testers of the Elementary Mathematics Teacher Project curriculum materials and the researcher's personal network. The survey link was shared on social media websites. Because of the snowballing nature of the sampling method, it was not possible to determine the exact response rate, since not all who received the invitation to take the survey were responsible for teaching such courses. Sometimes the survey was passed along to colleagues. Furthermore, some organizations sent the survey out to their membership themselves. Surveys were sent to AMTE (Association of Mathematics Teacher Educators) members, AMTE's Service, Teaching and Research (STaR) network, and the Mathematical Association of America's Research in Undergraduate Mathematics Education Special Interest Group. Due to the policies of professional organizations, reminder emails were not sent out to these networks. Department heads whose survey links had not yet been used were sent one reminder email. The researcher investigated and corrected all email addresses that bounced back or were outdated. The researcher completed paperwork for those institutions that required their own IRB approval. One of these institutions did not get approval before the close of the survey.

Of the 507 participants taking the survey, 402 completed more than 95% of the survey. In addition, 56 respondents completed between 13% and 74% of the survey, with a mean of 17.5% and a median of 15%. The remaining 49 respondents completed 10% or less of the survey, giving no usable data. Partial response were used in the analysis where there were sufficient data to report. Details about the completion rates are displayed in Table 3.3.

Table 3.3

Survey Completion Rates

Characteristics of Respondents	Number (Percent)
Instructors of mathematics content courses or combination courses responding to the survey	507 (100%)
Respondents completing 10% or less of the survey. Data is not reported on these respondents.	49 (10%)
Respondents completing 95% or more of the survey	402 (79%)
Respondents completing between 13% and 83% of the survey	56 (11%)
Total respondents included in the analysis	458 (90%)
Content Courses	423
Combination Courses	35

The sample consisted of instructors from a wide range of academic and professional backgrounds. Four hundred two (402) of the respondents answered questions about their academic and professional backgrounds. The vast majority (82%) of these respondents were full-time faculty members, as shown in Table 3.4.

Table 3.4

Employment Status of Survey Participants

Employment Status	Number of Respondents	Percent
Full time faculty member	330	82%
Adjunct/part-time faculty member	44	11%
Graduate student	20	5%
Other	8	2%
Total responding to this question	402	100%

Participants were asked to indicate their terminal degree (doctorate, masters, or other) as well as their field of study. The majority of participants responding to this question (70%) either held doctorates or were currently working toward their doctorates. Respondents whose terminal degree was in mathematics education were represented at a higher rate than those with advanced degrees in mathematics (57% compared to 32%). A few participants indicated that their terminal degree was in another field, such as physics, educational leadership, educational technology, elementary education, and counseling. Twenty-nine percent of respondents (29%) held masters degrees and a few participants held bachelor's degrees as their terminal degree. The number of participants with specific terminal degrees is shown in Table 3.5.

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Table 3.5

Terminal Degree of Survey Participants (n = 402)

Subject	Bachelors		Masters		Doctorate/ Working on Doctorate	
	number	percent	number	percent	number	percent
Mathematics	4	1%	51	13%	75	19%
Mathematics Education	0	0%	46	11%	184	46%
Other	1	0%	19	5%	22	5%
Total	5	1%	116	29%	281	70%

About one-fourth of the participants responding to this set of questions (28%) had not taught in a PreK–12 classroom. Twenty percent (20%) of this sample had experience teaching elementary grades. There were 38 (9%) respondents who had taught in a total of three different grade bands (e.g., grades PreK–5, 6–8, 9–12). Because participants were directed to check all of the grade bands that they had experience teaching, the categories do not total to 100%. These data are presented in Table 3.6.

Table 3.6

Survey Participants' Experiences Teaching Grades PreK–12

Experience	Number of Respondents (Percent)
No experience teaching grades PreK–12	114 (28%)
Experience teaching grades PreK–5	82 (20%)
Experience teaching grades 6–8	165 (41%)
Experience teaching grades 9–12	225 (56%)
Total Responding	402 (100%)

The majority of respondents (75%) were appointed to mathematics departments. About one-sixth of the sample was appointed to a school or department of education (17%), and a few participants held a joint appointment (6%) with mathematics and education. Finally a small number of participants were appointed to another department (2%). These data are summarized in Table 3.7.

Table 3.7

Departmental Appointment of Survey Participants

Departmental Appointment	Number of Respondents	Percent
Mathematics department	300	75%
School/department of education	68	17%
Joint appointment	25	6%
Other	9	2%
Total	402	100%

Participants were asked to indicate the selectivity of their program based on their views of the caliber of students in their courses for pre-service teachers. The majority of respondents described their program as “moderately selective” or “not selective,” as shown in Table 3.8.

Table 3.8

Response to the Question, "Thinking about the caliber of your students in these classes, how would you describe this institution or program?"

Response	Number of Respondents (Percent)
Highly selective	29 (7%)
Moderately selective	208 (52%)
Not selective or open enrollment	163 (41%)
Total responding	400 (100%)

Finally, the 402 participants who completed more than 95% of the survey were collectively responsible for over 1,115 sections of content courses for pre-service elementary teachers each year. According to self-reported class sizes, these respondents taught over 27,200 future elementary teachers each year.

Case Studies Sample

Both case study participants used the Elementary Mathematics Project (EMP) curriculum (Chapin, 2011) in their classes on geometric measurement. The EMP curriculum has the potential to support powerful practices and has had a proven impact on pre-service teachers' mathematical knowledge for teaching (Chapin, Feldman, Salinas, & Callis, in review). Problems and activities for pre-service teachers are the focus of the written materials in the EMP curriculum.

This was a convenience sample based on faculty members' willingness to participate. It was also a purposeful sample. One instructor had both extensive mathematical knowledge and mathematical knowledge for teaching. She had completed coursework for a doctorate in mathematics and held a doctorate in mathematics education. She had experience working in public schools. She taught elementary mathematics methods courses for over twenty years. However, she was new to teaching mathematics content courses for elementary teachers and had not used the EMP curriculum materials before. Therefore, I refer to her as the novice EMP user. The other instructor had a doctorate in Curriculum & Instruction in Mathematics and Science Education. She had been teaching the three content courses at her institution for over six years and therefore had a strong knowledge about and good relationships with her pre-service teachers. She had a long history of using the different revisions of the EMP curriculum materials as a pilot instructor. Therefore, I refer to her as the veteran EMP user.

Both instructors taught at colleges in the Northeastern United States. The novice EMP user taught at a public university with just over 9,000 students. The university accepted 75% of applicants in 2015. The veteran EMP user taught at a small liberal arts college with under 3,000 students. The college accepted 88% of applicants in 2015.

(“U.S. News & World Report Best Colleges,” 2016)

In addition to the EMP materials, the novice EMP user used a popular textbook for elementary mathematics content courses: Billstein et al.’s *A Problem Solving Approach to Mathematics for Elementary School Teachers.* The experienced EMP user likewise used an additional text, a widely used, activity-based textbook: Beckmann’s *Mathematics for Elementary Teachers with Activities.*

Conduct of Case Study Observations and Individual Interviews

In the spring and fall of 2016 I videotaped the case study participants teaching four lessons in their mathematics courses for elementary teachers. Each lesson was videotaped using an average of two cameras in order to capture the instructor’s interactions with the whole class, small groups, and individuals. For each participant, three EMP consecutive lessons were taped as well as one non-EMP lesson. These instructors were given a gift card to Amazon to thank them for their time.

After videotaping each class session, I recorded field notes summarizing my observations. Classroom handouts or artifacts from each lesson were collected. All videotapes were transcribed and then each transcript was segmented by class structure (whole class, small group, and individual work) and then further segmented into episodes that typically lasted 10 minutes or less. Natural breaks between discussions of different problems were also used to designate episodes. Each episode was then rated a 1, 2, or 3 using the TRUMath rubric for each of the five dimensions. The episodes that contained instances of powerful instructional practices, as indicated by a score of 3 in one or more of the dimensions, were used in the video stimulated recall interviews with participants.

In order to gain insight into each participant's views of instruction, they were interviewed several times. First, each participant participated in a background interview. Second, an interview was conducted prior to each observation. These interviews were approximately 20 minutes in length. Third, short post-observational interviews were conducted when possible. Questions used in the background, pre-observation, and post-observation interviews are found in Appendix A. Finally, once all observations were completed, video stimulated recall (VSR) (Lyle, 2003) interviews were conducted. The video-stimulated recall interviews were audio-recorded and followed the VSR interview protocol found in Appendix A. The goals of the VSR interviews were to (1) determine if the instructional practices or actions captured on video were typical of the participant's practice, or if not, why, (2) explore the participating instructor's thought processes behind specific instructional practices captured on video, (3) inquire about the connections the participants saw between the curriculum and their actions in these mathematically powerful moments. These VSR interviews lasted about 2 hours.

Positionality

The researcher was on the research team that developed materials. The researcher taught courses using the materials and participated in writing the lessons and developing the instructor support materials. Both case study participants were aware that the researcher was involved in the development of the EMP materials. This may have influenced their responses. However, the researcher attempted to make it clear that the

EMP materials were currently being adjusted and refined, so both positive, negative, and neutral feedback was welcome. Both participants provided suggestions for improvements or limitations of different features, so it appeared that this collegial atmosphere was created. While being an author of the curriculum materials studied supported the researcher in understanding the intent of different elements within the materials, it also may result in a limitation to interpreting the results.

Section III: Analysis of Research Questions

This study investigated the instructional practices used in mathematics content courses for teachers and the ways in which curriculum can support powerful instructional practices.

Research Question 1

What are the instructional practices and curriculum resources used in mathematics content courses for pre-service elementary teachers? How do these differ by instructor characteristics, if at all?

Data from the Instructional Practices and Curriculum Use (IPCU) Survey were used to answer Research Question 1. There were four categories of questions on the IPCU Survey: (1) instructional practices, (2) curriculum usage, (3) teaching context, and (4) instructor background. To analyze the data about instructional practices, I used descriptive statistics and exploratory and confirmatory factor analysis. I compared the results of different subgroups with inferential statistics using the data from questions on instructor background and teaching context. To analyze the data on curriculum usage, I used descriptive statistics.

Within the section on instructional practices, there were questions designed to address general instructional practices and questions aligned to the five dimensions of mathematically powerful classrooms and the TRUMath rubric (Schoenfeld et al., 2014). Question 3 looks at a general instructional practice, the use of class time. Participants indicated the percent of weekly class time the class spent in different formats – working in small groups, listening to the instructor, having students work individually, or "other." I reported the mean, standard deviation, and five number summary of the percent of class time spent in these different formats. I compare the mean percent of class time among these different subgroups of participants using ANOVA:

- Participants with advanced degrees in mathematics compared to participants with advanced degrees in mathematics education or other fields.
- Participants with PreK–12 teaching experience compared to participants without PreK–12 teaching experience.
- Participants with doctoral degrees compared to other participants.
- Participants appointed to mathematics departments compared to participants appointed to non-mathematics departments.
- Participants who perceive their programs as highly or moderately selective versus not selective.

Question 4 is a set of Likert-type scale statements about instructional practices. I conducted a confirmatory factor analysis on this set of statements using the five-factor model described in Section I. The five factors were: (a) student-student interaction, (b) content delivery, (c) formative assessment, (d) student-content engagement, and (e) summative assessment. Each participant was assigned a score for each factor, equivalent to the proportion of possible points for each factor. That is, each response within a Likert statement was coded as a 0, 1, 2, 3, or 4, with reverse coding as appropriate. Then, the sum of a participant's response to the items within a factor was calculated. This sum was then divided by the maximum possible points for that factor and multiplied by 100. I used this scoring method, an unweighted aggregate score, instead of a factor score because it allows me to make comparisons to results from other researchers who have used this instrument. I reported the mean, standard deviation, and five number summary for each of the five factors and for the entire sample. I also reported these descriptive statistics for each of the subgroups listed in the previous paragraph. I then use ANOVA to compare the means of the different subgroups to determine if there is any statistically significant difference in instructional practices among the subgroups.

In addition to Questions 3 and 4, there are nine items that measure instructional practice. Two of these (Q7 and Q22) are open-response items that are not analyzed as part of this study. The analysis of the remaining seven items was conducted in alignment with the TRUMath rubric in order to measure the extent to which the five dimensions of mathematically powerful classrooms were exhibited in the participant's self-reported practice.

Question 2 measured the *mathematics* dimension of the TRUMath rubric. As noted in Section I, the *mathematics* dimension has two sub-dimensions: (1) the quality of the mathematical topics and connections addressed and (2) the opportunity for engaging in the disciplinary practices of mathematics, such as reasoning, justifying, and generalizing. Classes that exhibit these two sub-dimensions score a 3 on the rubric, while classes focused primarily on rote procedures score a 1. Accordingly, Question 2 asked whether participants' course include a (a) review of calculation procedures, (b) the reasons why algorithms or formula work, and (c) the nature of reasoning, justification, or proof in mathematics. In addition, Question 2 asked whether two aspects of mathematical knowledge for teaching were addressed in their courses: (d) common misconceptions of elementary students and (e) representations used in elementary school classes.

The percent of the entire sample that addressed each of the five topics is reported. I also reported the percent of the entire sample that addressed *only* calculation procedures, equivalent to a score of 1 on the TRUMath rubric. These percentages are also reported for the subgroups listed on page 126. A Chi-squared test was used to determine if the proportion of any subgroup was statistically more likely to address any of these five mathematical topics.

Questions 5 and 6 in the instructional practices section of the IPCU survey were designed to measure two key aspects of cognitive demand: (1) maintaining the cognitive demand when setting up tasks (Q5) and (2) maintaining cognitive demand in face of pre-service teachers' difficulties (Q6). Participants responded to a series of Likert statements for these two questions. Statements in these two questions were grouped together and a factor analysis was run to determine if there were, in fact, two different constructs. One of the statements loaded on multiple factors, so that statement was dropped *(I ask guiding questions that direct students to the main mathematical idea)*. Two factors emerged across the two questions. The first factor included statements that suggested student independence. The second factor included statements that suggested lowering cognitive demand by the instructor taking over the mathematical work, for instance, by explaining the steps to solve a problem. Therefore, participants were assigned two scores in the area of cognitive demand: a score indicating the extent of their attempts to support student independence and a score indicating the extent to which they reported using practices that may lower cognitive demand. An ANOVA with the aggregate scores on each factor as dependent variables and membership in the subgroups above as independent variables was run to determine if there was a difference in the mean scores in cognitive demand for the different subgroups.

The dimension *access to mathematical content* was measured by two dichotomous questions (Q9 and Q10) and one short response question (Q8). First, I identified whether the participants defined full participation as active participation as opposed to passive participation, Q8. Active participation involved interacting with others, while passive participation was limited to listening, taking notes, and completing problems. Defining full participation as passive participation would be equivalent to scoring a 1 on the TRUMath rubric. I used a test for difference of proportion to determine if there were any statistically significant differences between those who defined full participation as active participation and those who defined participation as passive participation in regards to instructor characteristics. Among the participants who defined participation actively, I determined the percent who felt that all of their students actively participated most days (Q10). These participants would have scored a 3 on the TRUMath rubric. I used a test for difference of proportion to determine if there were any statistically significant differences between those who felt all of their students participated most days and those who did not in regards to instructor characteristics. This test was limited to those who defined participation as active participation. Among participants who defined full participation as active participation but felt that all of their students did not fully participate most days, I determined the percent who indicated that they had strategies for ensuring broad, active participation (Q9). These participants would have scored a 2 on the TRUMath rubric, since they made attempts to broaden active participation. This subgroup was too small to conduct statistical tests. The relationship between the answers to these three survey questions and the TRUMath rubric scores is shown in Figure 3.1.

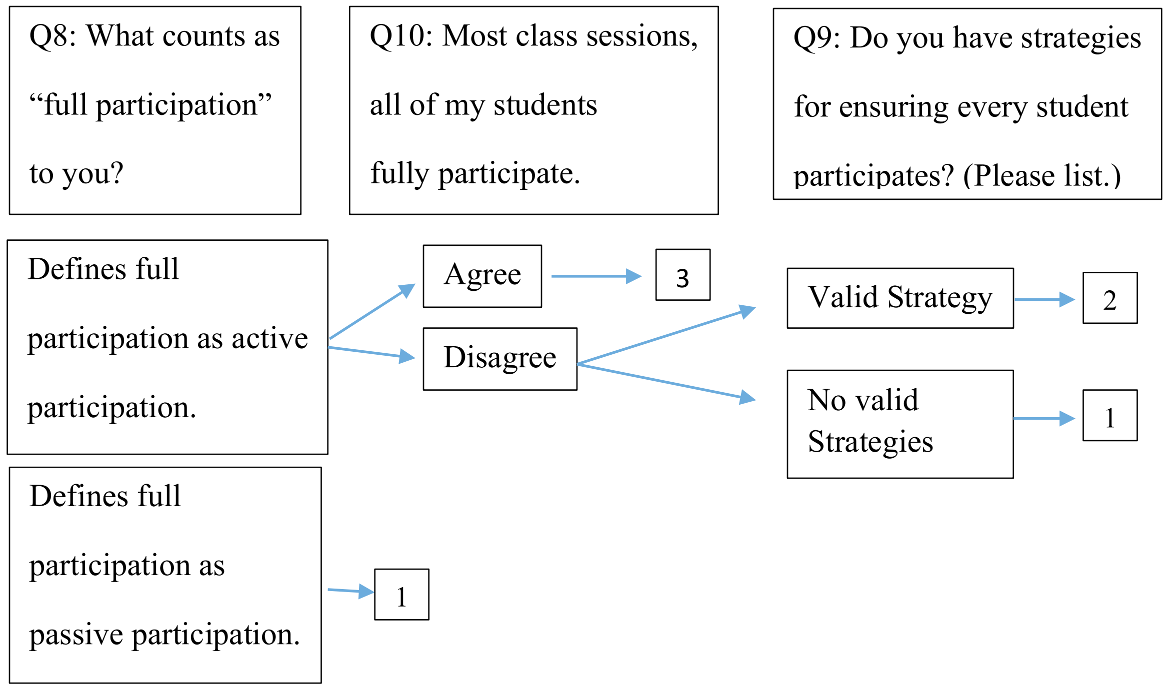


Figure 3.1. Equivalence of responses on IPCU Survey and the TRUMath Rubric

The dimension *agency, authority, and identity* was measured using two multiple choice questions (Q11 and Q12). These questions required choosing one of the multiple-choice answers. Each multiple-choice option was assigned a score of 1, 2, or 3, which aligned with the TRUMath rubric. There was an *other* box for those survey participants who did not feel comfortable choosing one of the multiple choice answers; these responses were not included in the analysis. About 21% of participants chose *other* for Q11. This percent was comparable for Q12. I compared the percent of each subgroup of participants answering in each of the three categories and tested for a difference of proportion.

Reliability for these two questions was determined by calculating the Chi squared test of independence. Since the two questions measured related elements of the same construct, they should not be independent.

The *uses of assessment* dimension measures the extent to which formative assessment impacts instruction. This study used data from Question 4 to address this dimension. Specifically, participants' scores on the formative assessment factor were analyzed. An ANOVA was conducted to determine if the mean scores among the different subgroups differed.

To analyze the questions addressing curriculum usage, I used descriptive statistics. Specifically, I identified the most commonly used published textbook materials. I reported the frequencies on the extent of the use of textbooks (Q13d, Q13e1, and Q13b), the ways in which the textbooks were used (Q14), and the use of ancillary materials such as multimedia resources and instructor guides (Q14a and Q16a).

Reliability of Scoring Methods and Instruments

To establish the reliability of the scoring methods and the participants' responses to survey items, I used interrater reliability, factor analysis, and triangulation among survey items. First, Question 8 asked participants to define full participation. These responses were coded as "active" or "passive." A code book can be found in Appendix C. Two other mathematics education researchers each coded 10% of the responses to Q8. After clarification of the code book, 100% agreement was reached. Next, the reported use of class time and scores calculated from responses on the Likert-type scale items in Question 4 were triangulated. Specifically, I calculated a correlation coefficient between the percent of time spent listening to the instructor with the aggregate score on the content delivery factor and between the percent of time spent in small groups with the student-student interaction factor. The percent of time spent listening to the instructor was correlated with the content delivery factor at a statistically significant level, $r = 0.53$, $p < 0.0001$. The percent of time spent working in small groups was correlated with the student-student interaction factor at a statistically significant level, $r = 0.62$, $p < 0.0001$. Similarly, I triangulated the student-content engagement factor from Question 4 with Question 2. Specifically, I compared the mean aggregate score on student-content engagement factor for participants who indicated they addressed the nature of reasoning, justification, or proof in mathematics within their courses. The student-content engagement factor measures the extent to which students engage in disciplinary practices during class time, so responses on these items should be related to the responses on Question 2. Participants who indicated that they addressed proof and reasoning in Q2 on average scored higher on the student-content engagement score at a statistically significant level [$F(1, 431) = 4.7958$, $p = 0.0291$]. Finally, factor analysis was conducted on questions that asked instructors to use a Likert-type scale to respond to a set of statements. In factor analysis, if the identified factors have high loadings (above 0.7), then the items share some of the variation; they are tapping into the same construct. A goodness of fit index was also calculated. These results are provided in Chapter 4.

Research Question 2

How can a curriculum for mathematics content courses for pre-service elementary teachers support instructors in creating mathematically powerful experiences for prospective teachers?

To answer the second research question, I developed case studies of two instructors teaching mathematics content courses for elementary teachers. For each participant, I analyzed videos of three class sessions where the participant used EMP curriculum materials and one class session where the participant used a different written curriculum. In the first case study, the participant enacted the EMP lessons on prisms and the two lessons on surface area. In the second case study, the participant enacted the EMP lessons on area concepts and area formula of parallelograms, triangles, and trapezoids. Both participants used the EMP materials for other lessons as well, but these are not the subject of the study. I interviewed these participants about their instructional practices and their use of curriculum in general. I conducted stimulated recall interviews to learn about participants' thinking behind their observed instructional practice and how the curriculum materials may have supported them. The episodes chosen for the stimulated recall interviews were selected using the TRUMath rubric to identify instances when instructional practices promoted a mathematically powerful learning environment.

The clips that were selected for use in the stimulated recall interview were also coded for interrater reliability. A researcher with experience using the TRUMath rubric in other contexts coded the episodes for each of the five dimensions. Following discussion, 100% agreement between all codes was achieved.

The VSR interviews were audio recorded and transcribed. The transcripts were sorted into two components: general thoughts, which were at the beginning or end of the interview, and the responses that followed the viewing of each video episode. I segmented the VSR interview transcript by video episode to which it was aligned.

Within these episodes, I coded individual sentences or groups of sentences within the participants' responses for one of the five dimensions of mathematically powerful classrooms. While the analysis of the video conducted by the researcher and the interrater reliability coder helped to identify the strengths of a particular episode in terms of these dimensions, coding the transcript illuminated the participants' motivations in terms of these dimensions, which may not have been transparent from the video. I attempted to assign only one code to each section of a response. Within some of these dimensions, additional sub-codes emerged. For instance, within the *mathematics* dimension, there were these sub-codes: mathematical connections, mathematical knowledge for teaching, and mathematical practices. Second, to align with Brown's (2012) framework, I coded sentences or groups of sentences within the participant's response with the following: (1) curriculum resources, (2) instructor resources: (a) goals, (b) beliefs, (c) knowledge. Examples of these different codes can be found in Appendix D and Appendix E.

From these codes, I then organized the data into a matrix like that in Table 3.9 for each of the video episodes discussed during the VSR interviews. These matrices allowed me to look across episodes and across participants to identify commonalities.

Table 3.9

Matrix for Cross Case Synthesis

<table>
<tr><td colspan="2">Episode Number:</td><td colspan="3">Summary of the Episode:</td></tr>
<tr><td colspan="2">Curriculum Resources:</td><td colspan="3">Instructor Resources:</td></tr>
<tr><td>Summary</td><td>Evidence (quotes)</td><td>Category (goals, beliefs, knowledge)</td><td>Summary</td><td>Evidence (quotes)</td></tr>
<tr><td colspan="5">Instructional Actions</td></tr>
</table>

<table>
<tr><td></td><td>Dimensions</td><td>Evidence</td><td>Source</td></tr>
<tr><td>Researcher Identified</td><td></td><td></td><td></td></tr>
<tr><td>Participant Identified</td><td></td><td></td><td></td></tr>
</table>

There were two components to my analysis in using these matrices. First, I consider *what is it that the instructor accomplishes using the curriculum materials?* To answer this question, I considered, *what do I as a researcher perceive the instructor accomplishing?* I answered this question based on the instructor actions I listed, the codes for the five dimensions of mathematically powerful classrooms, and my notes on the evidence from the videos for assigning these codes. I also considered, *what does the instructor perceive she is accomplishing?* I answered this based on the quotes from the VSR interviews, which were coded according to the five dimensions and according to Instructors Resources. While I as the researcher may have found a particular aspect of the video episode salient, the instructors' perceptions of their own practice may have differed from mine. Thus, the answer to *what is it the instructor accomplishes?* was jointly constructed by the researcher and the instructor. From considering these questions and organizing the matrices, I identified four themes, discussed in more detail in Chapter 4. These themes answer the question, *What does the curriculum support instructors to do?* My next question was *how* the curriculum supported the case study participants; that is, *what element of the curriculum did the instructors feel supported them?* In the VSR interviews, I asked the case study participants whether they felt anything in the curriculum supported them. I coded their responses in the VSR interviews as "curriculum resources" and placed the quotes in the matrices in the cell labeled "curriculum resources." In some cases, the participants were specific in their explanation of particular features of the resources that helped them accomplish their goals or act on their beliefs. In other cases, their responses were more vague, and I had to further investigate what some of their statements might mean by looking more closely at the curriculum materials. I looked at both the problems used during class and at the instructor support materials. In some cases, I compared the EMP curriculum materials to the alternative curriculum materials and the way in which the instructors enacted the alternative materials in order to understand their responses at a more detailed level. For each element that one of the participants identified, I looked to see if there was evidence that this element may have supported the other participant as well by examining the videos of enactment and the relevant lesson materials.

The notion of “support” was largely defined by the case study participants. After the episodes during the VSR interviews, I asked them, “Was there anything in the curriculum materials that helped you or supported you in doing this?” The participants identified particular features of the program, units, lessons, and problems. For example, they indicated that the design of the materials using class discussion throughout the program and the fact that problems within a lesson progressed from specific to general. The participants did not identify the Instructor’s Guides or videos of enactment themselves as supportive, though the instructors did use these materials. As described in Chapter 2, this study is based on Brown’s (2012) notion of curriculum as a tool, an object that enables someone to do something, or to do something better. A wrench can help someone to turn a bolt; a ratchet can help someone to turn a bolt tighter or more easily or efficiently. To understand how a wrench helps someone, one could explain the physics, or one could ask a person using a wrench. The two explanations have different purposes and affordances. This study attempted to do both, by jointly identifying how the curriculum supported the instructors to create mathematically powerful experiences for their pre-service teachers according to the instructor, with insight from the researcher’s experience as one of the curriculum authors and an experienced user.

This chapter has explained the data collection and analysis processes used in this study. Chapter 4 describes the findings from the IPCU survey to answer to the first research question. Chapter 5 describes the findings from the case studies to answer the second research question.

CHAPTER 4: ANALYSIS OF SURVEY DATA

This study sought to answer the questions, *What are the instructional practices and curriculum resources used in mathematics content courses for pre-service elementary teachers? How do these differ by instructor characteristics, if at all?* To this end, I conducted a survey of instructors of mathematics content courses for elementary teachers. I solicited responses from instructors via individual emails to department heads, emails to instructors I knew personally, and emails sent out on behalf of professional organizations. The survey contained questions on instructional practices and on curriculum use, as well as background questions. The instructional practices section included items from the Postsecondary Instructional Practices Survey (Walter et al., 2015) as well as items I designed to measure the five dimensions of mathematically powerful classrooms (Schoenfeld, 2014). To analyze the data, I use descriptive and inferential statistics and confirmatory factor analysis with multi-group comparisons.

In this chapter, I first describe the survey participants. Then, I describe the reported instructional practices of the survey participants, both generally and according to each of the five dimensions of mathematically powerful classrooms. I report how different subsets of the respondents, such as those with and without K–12 teaching experience, compared in their instructional practices. I then report on the curriculum materials used by the participants.

Section I: Survey Respondents

In the fall of 2016, a link to an online survey was sent to mathematics departments in the United States and to members of four professional organizations by email. The link was also posted on websites and social media and sent to professional contacts. Four hundred and two (402) college instructors who had taught mathematics content courses for pre-service teachers completed 95% or more of the survey. In addition, another 56 survey participants completed between 13% and 83% of the survey. These participants were included in the analysis on the items for which they provided data.

The participants taught in a variety of contexts. The majority of the 458 participants (92%) taught courses that focused primarily on mathematics content for pre-service elementary teachers. Eight percent (8%) taught courses that combined mathematics content and pedagogy for elementary school. Most of the participants (66%) were appointed to the mathematics department. Fewer (15%) were appointed to schools of education. The remainder held a joint appointment (5%), were appointed to another department (2%), or did not indicate their departmental appointment (12%). When asked to consider the caliber of the students in their classes, participants most frequently described their institutions as moderately selective (45%), while others reported their institutions as highly selective (6%) or not selective (36%). The remainder (13%) did not respond to the question. Four hundred (400) participants provided their average class size. Participants' class sizes ranged substantially. The mean class size was 23.6 students. Just over 26% of these 400 participants indicated that their typical class size in these courses was 30 or more students, while another 25% reported that their typical classes had 18 or fewer students. Four participants indicated they had 50 or more students in a typical section. On average, participants taught 2.8 sections of mathematics content courses for elementary teachers each year. The distribution of the number of courses each participant taught yearly is displayed in Table 4.1.

Table 4.1

Number of Course Sections Taught Each Year by Survey Participants

Number of course sections taught each year	Number of Participants (n = 401)	Percent
Fewer than 2	86	21.4%
2–3	194	48.4%
4–5	80	20.0%
6–7	37	9.2%
8–10	4	1.0%

The participants also had a wide variety of academic and professional backgrounds. The majority (72%) were full-time faculty members. Fewer were adjunct or part-time faculty members (10%) or graduate students (4%). Some indicated “other,” which included at least one recent retiree. The majority of the participants (61%) held doctoral degrees as their terminal degree, or they were working toward a doctoral degree. A quarter held a Master’s degree as their terminal degree and 1% held a bachelor’s as their terminal degree. Just over half of the participants held their terminal degree in mathematics education. However, those whose terminal degree was in mathematics were also well represented (28%). The majority of participants (63%) also had PreK–12 teaching experience, and some (18%) had elementary teaching experience. Twelve percent (12%) did not respond to questions about their academic and professional background or employment status. Further details about the academic and professional background of the participants can be found in Chapter 3.

Section II: General Instructional Practices

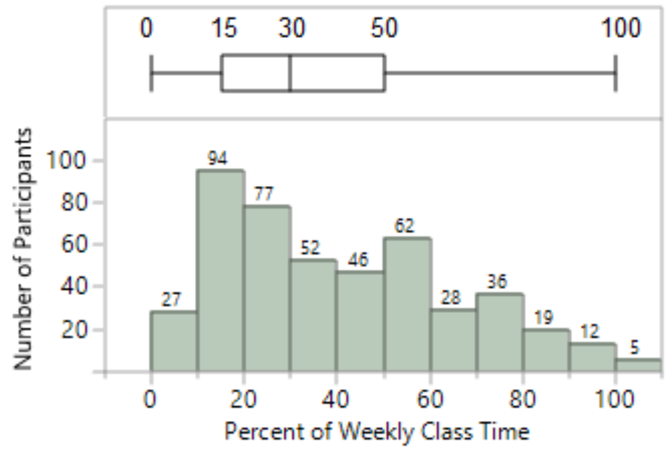
The survey measured general instructional practices in two ways. First, respondents reported what percent of weekly class time was spent in different formats, such as listening to the instructor or working in small groups. Second, participants responded to a series of Likert-type scale statements from the Postsecondary Instructional Practices Survey (PIPS, Walter, Henderson, Beach, & Williams, 2015).

Use of Class Time

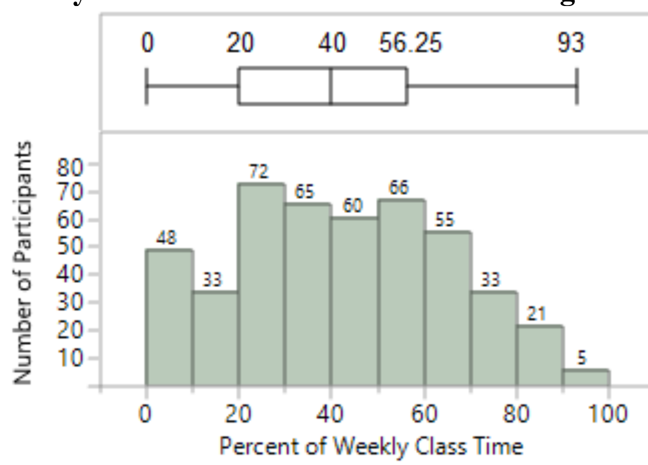
The question measuring allocation of weekly class time listed three formats: the instructor talking to the whole class, students working individually, and students working in small groups. In addition, participants could write in other uses of time. Whole class discussions and student presentations were frequently written under other uses of time. To a lesser extent, participants reported "other" such as "assessment" or "using technology." Some participants reported activities that were impossible to classify as individual, small group, or whole class work. For example, "working with manipulatives" could be completed in any of these formats. Therefore, I did not report on these other uses of class time.

On average, the most frequently reported use of class time was having students work in small groups or having them listen to the instructor. The mean percent of weekly class time spent listening to the instructor was 37%. The mean percent of weekly class time spent working in small groups was 39%. However, use of these class structures varied widely. A quarter (25%) of respondents indicated that students spent more than half of class time listening to the instructor. On the other hand, more than 25% of participants indicated that students spent over half of class time working in small groups. Thus, the range of class structures that pre-service teachers experience varied widely. Figure 4.1 shows the distribution of weekly class time in three different class formats. Table 4.2 provides the means, standard deviations, and five number summaries of the percent of time participants spent in the different formats.

Percent of Weekly Class Time Spent Listening to the Instructor Talking



Percent of Weekly Class Time with Students Working in Small Groups



Percent of Weekly Class Time Spent with Students Working Individually

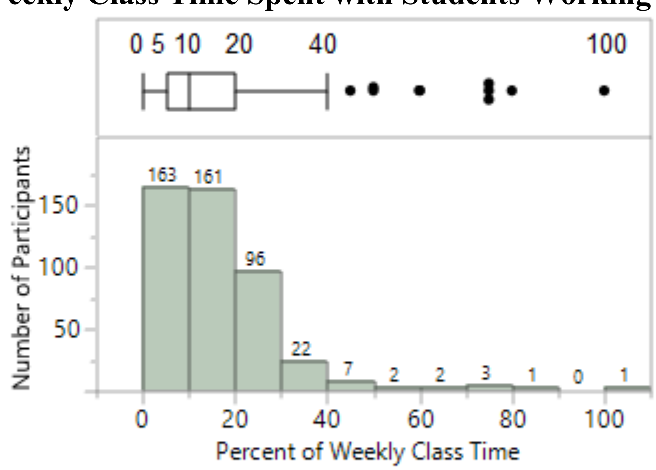


Figure 4.1. Distribution of weekly class time in three formats

Table 4.2

Percent of Weekly Class Time Spent in Different Formats

Class Format	Mean (Standard Deviation)
Instructor Talking	36.86 (24.5)
Small Group Work	38.8 (22.9)
Individual Work	12.7 (12.5)

There was a statistically significant difference between the uses of class time by instructors of different background characteristics, as shown in Table 4.3. First, participants whose terminal degree was in mathematics were more likely to spend more class time using the instructor talking format than those whose terminal degree was in mathematics education [$F(2, 399) = 10.6862$, $p < 0.0001$]. Relatedly, those with their terminal degree in mathematics were also less likely to spend class time with students working in small groups [$F(2, 399) = 8.1447$, $p = 0.0003$]. Additionally, those with their terminal degree in mathematics varied in their use of these class formats at a much higher rate than those with their terminal degree in mathematics education, who were more alike in their use of class formats. This difference is shown in the box and whisker plots in Figure 4.2. There was not a statistically significant difference in the percent of weekly class time spent in individual work [$F(2, 300) = 0.2296$, $p = 0.7194$].

Table 4.3

Mean Percent of Weekly Class Time Spent in Different Formats, by Subject of Terminal Degree

Class Format	Subject of Terminal Degree		
	Mathematics *(n = 130)*	Mathematics Education *(n = 230)*	Other Subject *(n = 42)*
Instructor Talking **(p < 0.0001)**	44.0 (28.2)	32.4 (21.5)	42.2 (22.7)
Small Group Work **(p = 0.0003)**	34.2 (25.0)	43.0 (21.1)	33.2 (21.6)
Individual Work (p = 0.7194)	12.4 (14.7)	11.8 (10.6)	13.4 (9.9)

Note. In all tables comparing subgroups, the mean is displayed followed by the standard deviation in parentheses.

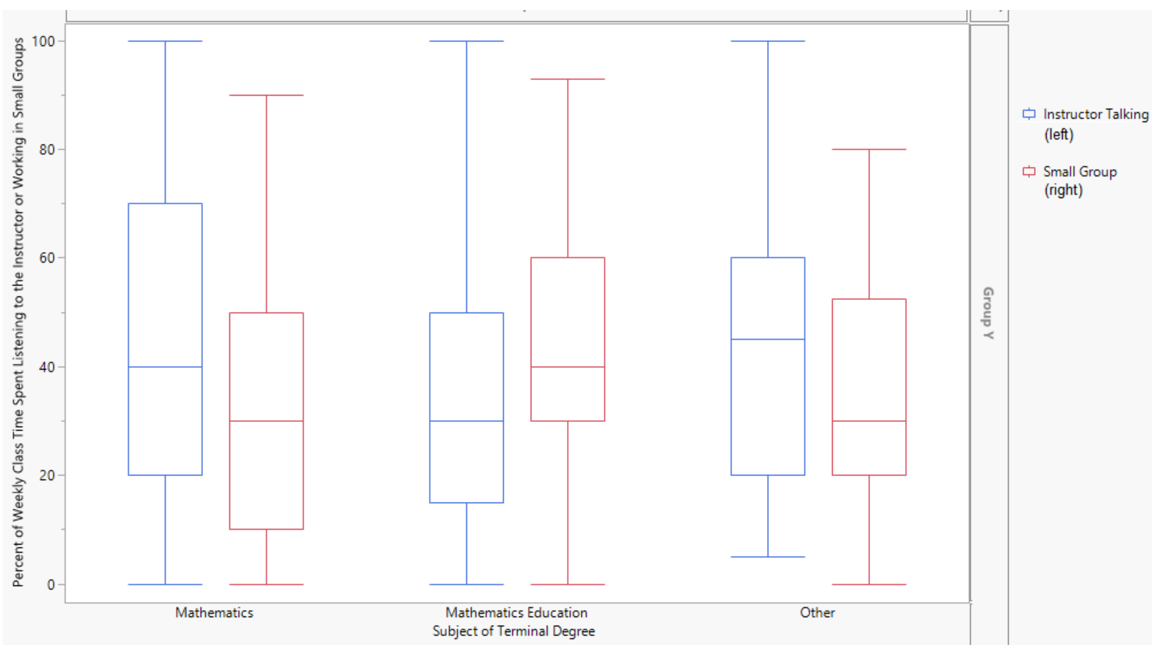


Figure 4.2. Distribution of percent of class time spent in small group or instructor talking format, by subject of terminal degree

There were also statistically significant differences in the use of class time between participants who were appointed exclusively to mathematics departments, as opposed to schools of education, joint appointments, or other departments. Specifically, those appointed to mathematics departments generally spent more time having students listening to the instructor, as shown in Table 4.4 [$F(1, 400) = 10.4953$, $p = 0.0013$]. In addition, while the difference between the percent of time spent in small groups is not statistically significant, it is substantial. (A one-tailed test found there to be a statistically significant difference with $p = 0.0254$.) However, there is a strong relationship between the subject of participants' terminal degree and their departmental appointment. Approximately 40% of the respondents appointed to mathematics departments had terminal degrees in mathematics, whereas less than 12% of respondents appointed to other departments had a terminal degree in mathematics, as shown in Table 4.5. Including participants who had a joint appointment in the same group as participants appointed to mathematics departments did not change the results.

Table 4.4

Mean Percent of Weekly Class Time Spent in Different Formats, by Departmental Appointment

Class Format	Appointed to a math department *n = 300*	Joint appointment or appointment to another department. *n = 102*
Instructor Talking **(p = 0.0013)**	39.5 (25.8)	30.47 (19.2)
Small Group Work (p = 0.0507)	37.8 (24.1)	43.0 (18.6)
Individual Work (p = 0.3401)	12.5 (13.0)	11.2 (8.3)

Table 4.5

Percent of Participants Appointed to Different Departments with Their Terminal Degree in Mathematics, Mathematics Education, or Other Discipline

	Subject of Terminal Degree			
Departmental Appointment	Mathematics	Mathematics Education	Other Discipline	Total
Mathematics	118	160	22	300
Department	39.3%	53.3%	7.3%	100%
Other Appointments	12	70	20	102
	11.8%	68.7%	19.6%	100%
School of	4	51	13	68
Education	5.9%	75.0%	19.1%	100%
Joint	5	16	4	25
Appointment	20.0%	64.0%	16.0%	100%
Other	3	3	3	9
	33.3%	33.3%	33.3%	100%

The level of a respondent's terminal degree impacted the percent of class time spent in these different formats as well. Compared to participants with master's or bachelor's degree, participants with their doctorate or working toward their doctorate on average spent more of the class time in the small groups format [$F(1, 400) = 19.9737$, $p < 0.0001$]. Participants with their doctorate or working toward their doctorate also typically spent less class time having students listen to the instructor [$F(1, 400) = 18.5824$, $p < 0.0001$] or work individually [$F(1, 400) = 14.6487$, $p = 0.0002$]. These differences were statistically significant, as shown in Table 4.6. It is worth noting that, while the differences between those with doctorates and those without was different for participants whose terminal degree was in mathematics as opposed to another discipline, the direction of the difference was the same. In other words, holding a doctorate in one's discipline impacted the use of class time, regardless of the discipline.

Table 4.6

Mean Percent of Weekly Class Time Spent in Different Formats, by Level of Terminal Degree

Class Format	Doctorate/Working on Doctorate *n = 281*	Bachelors, Masters, or Masters and Additional Coursework *n = 121*
Instructor Talking **(*p* <0.0001)**	33.8 (23.4)	45.1 (25.6)
Small Group Work **(*p* < 0.0001)**	42.4 (22.5)	31.5 (22.0)
Individual Work **(*p* = 0.0002)**	10.7 (8.9)	15.6 (16.8)

There was no statistically significant difference in the use of class time among those with PreK–12 teaching experience and those without PreK–12 teaching experience, as shown in Table 4.7. [For instructor talking, $F(1, 400) = 1.0266$, $p = 0.3116$. For small group work, $F(1, 400) = 0.2684$, $p = 0.6047$. For individual work, $F(1, 400) = 2.7061$, $p = 0.1007$.] Those with PreK–5 teaching experience generally spent less time having students listen to the instructor at a statistically significant level [$F(1, 400) = 8.1403$, $p = 0.0046$], but there was not a statistically significant difference in their other use of class formats, as shown in Table 4.8. [For small group work, $F(1, 400) = 2.0778$, $p = 0.1502$. For individual work, $F(1, 400) = 0.1443$, $p = 0.7043$.]

Table 4.7

Mean Percent of Weekly Class Time Spent in Different Formats, by K–12 Teaching Experience

Class Format	PreK–12 Teaching Experience *n = 288*	No PreK–12 Teaching Experience *n = 114*
Instructor Talking ($p = 0.3116$)	36.4 (23.5)	39.2 (27.2)
Small Group Work ($p = 0.6047$)	38.8 (21.6)	40.1 (25.8)
Individual Work ($p = 0.1007$)	12.8 (12.1)	10.6 (11.6)

Table 4.8

Mean Percent of Weekly Class Time Spent in Different Formats, by Elementary Teaching Experience

Class Format	PreK–5 Teaching Experience *n = 82*	No PreK–5 Teaching Experience *n = 320*
Instructor Talking **(p = 0.0046)**	30.4 (21.1)	39.0 (25.1)
Small Group Work (p = 0.1502)	42.4 (19.6)	38.3 (23.6)
Individual Work (p = 0.7043)	12.6 (11.0)	12.1 (12.3)

Participants' perception of their students and institutions' selectivity also influenced the use of class time. Specifically, respondents who indicated that they would consider their institution moderately or very selective when thinking about the caliber of their students in their mathematics courses for elementary teachers spent less class time having students listen to the instructor [$F(1, 398) = 5.2436$, $p = 0.0225$]. While there was a difference in the percent of class time these two groups had their students work in small groups, this difference was not statistically significant [$F(1, 398) = 2.9112$, $p = 0.0887$]. (A one-tailed test found this difference to be statistically significant level with $p = 0.0444$.) No statistically significant difference was found between the amount of time instructors in the two different groups spent having students work individually [$F(1, 398) = 2.3601$, $p = 0.1253$]. These differences are shown in Table 4.9.

Table 4.9

Mean Percent of Weekly Class Time Spent in Different Formats, by Selectivity

Class Format	Very Selective or Moderately Selective *n = 237*	Not Selective *n = 163*
Instructor Talking **(p = 0.0225)**	34.9 (23.1)	40.6 (26.3)
Small Group Work (p = 0.0887)	40.8 (22.2)	36.9 (23.7)
Individual Work (p = 0.1253)	11.4 (8.9)	13.3 (15.5)

The question measuring allocation of class time did not provide “whole class discussion” or “student presentations” as options. However, 157 participants, just over one-third of the sample, indicated “other” and described using whole class discussions or student presentations during their class time. Among these participants, their use of whole class discussion or student presentation class formats range substantially, from 2% to 70% of weekly class time, with a median of 20%. Compared to the sample at large, this subset of participants were more likely to have earned or have been working on their doctorate (81% compared to 70%), held their terminal degree in mathematics education (66% compared to 57%), and viewed their institutions as moderately or very selective, based on the caliber of their students in these courses (66% compared to 59%). They were slightly less likely to have been appointed exclusively to a mathematics departments (71% compared to 75%). There did not seem to be a substantial difference between this subgroup and the sample at large in regards to PreK–12 or elementary education experience, less than a 2% difference. Relatedly, within this subset, the mean percent of class time spent in whole class discussion or on student presentations was higher among participants who had earned or were working toward their doctorates or who held their terminal degree in mathematics education. However, the design of the question limited additional analysis. Further research on instructional practices in higher education should include student presentations and whole class discussion as options for use of class time.

In sum, the experiences pre-service elementary teachers are exposed to range significantly, and there appears to be a connection to their instructors' backgrounds. Pre-service elementary teachers whose instructors hold their terminal degrees in mathematics or are appointed to mathematics departments spent more listening to the instructor. Pre-service teachers whose instructors hold doctoral degrees spent more time working in small groups and less time listening to the instructor, compared to their counterparts. Instructors who perceived their programs as less selective, based on the caliber of their students, spent more time having students listening to the instructor talk. In addition, the results of this survey suggest that the use of whole class discussion and student presentations may also be impacted by these instructor characteristics. These results are summarized in Table 4.10.

Table 4.10

Instructors and Class Formats: Which instructors are likely to spend more time in various class formats?

Class Format	Instructor Characteristics
Instructor Talking	Terminal degree is in mathematics Appointed to mathematics department Do not hold/not working toward doctoral degree Do not have PreK–5 teaching experience Perceive their program to be not selective based on the caliber of their students
Small Group Work	Terminal degree is in a subject other than mathematics Hold a doctorate or are working toward a doctorate
Individual Work	Do not hold/not working towards doctoral degree

Use of Instructional Practices

To measure general instructional practices, participants responded to 24 Likert-type scale items taken from the Postsecondary Instructional Practices Survey (Walter et al., 2015). This instrument uses a five factor model. The five factors are (1) student-student interactions, (2) content delivery, (3) formative assessment, (4) student-content engagement, and (5) summative assessment. These factors are more thoroughly described in Chapter 3.

Analysis of the data set in this study led to adjustments in the scoring method recommended by Walter and colleagues (2015). First, item 07, “I frequently ask students to respond to questions during class time,” was removed. Over 70% of respondents indicated it was very descriptive of their teaching, leaving little variability for analysis. Second, the fifth factor, summative assessment, did not have good fit statistics. Further content analysis of the individual items indicated that the items were measuring very different constructs within assessment. Therefore, I report item-level statistics only for the items Walter and colleagues assigned to this factor. Each of the retained items, the associated factor, and the standardized factor scores for each retained item can be found in Table 4.11.

Table 4.11
Four Retained Factors and Aligned Items

Factor and Associated Items	Standardized Factor Loading
(1) Student-student interactions	
10) I structure class so that students explore or discuss their understanding of new concepts before formal instruction.	0.7464000
12) I structure class so that students regularly talk with one another about course concepts.	0.7463626
13) I structure class so that students constructively criticize one another's ideas.	0.7455466
14) I structure class so that students discuss the difficulties they have with this subject with other students.	0.6441289
15) I require students to work together in small groups.	0.6307594
19) I require student to make connections between related ideas or concepts when completing assignments.	0.3743998
(2) Content delivery	
01) I guide students through major topics as they listen and take notes.	0.7270199
03) My syllabus contains the specific topics that will be covered in every class session.	0.2477095
05) I structure my course with the assumption that most of the students have little useful knowledge of the topics.	0.3413041
11) My class sessions are structured to give students a good set of notes.	0.9217962
(3) Formative assessment	
04) I provide students with immediate feedback on their work during class (e.g., student response systems, short quizzes).	0.3842267
06) I use student assessment results to guide the direction of my instruction during the semesters.	0.8522292
08) I use student questions and comments to determine the focus and direction of classroom discussion.	0.4884412
18) I give students frequent assignments worth a small portion of their grade.	0.2041171
20) I provide feedback on student assignments without assigning a formal grade.	0.3782486
(4) Student-content engagement	
02) I design activities that connect course content to my students' lives and future work.	0.4903475
09) I have students use a variety of means (models, drawings, graphs, symbols, simulations, etc.) to represent phenomena.	0.4761862
16) I structure problems so that students consider multiple approaches to finding a solution.	0.5543609
17) I provide time for students to reflect about the processes they used to solve problems.	0.5303648

I conducted confirmatory factor analyses (CFA) to determine whether the categorization of the items as recommended by Walter and colleagues (2015) was legitimate for this data set. I determined the goodness of fit of these models by using the root mean square error of approximation (RMSEA), the Tucker-Lewis non-normed fit index (NNFI), the Bentler comparative fit index (CFI), and the standardized root mean square residual (SRMR). I also conducted a Chi-squared test to determine whether the data matched the model. A root mean square error of approximation less than 0.06 indicates a good fit. For the Tucker-Lewis NNFI and the Bentler CFI, above 0.95 is ideal. A SRMR of less than 0.08 is considered indicative of a good model fit. For the Chi-squared test, I failed to reject the null hypothesis, and thus felt the data fit the model, for p values greater than 0.05. These statistics for each of the four factors are shown in Table 4.12.

Table 4.12
Fit Statistics for the Four Factors

Factor	Student-Student Interaction	Content Delivery	Student-Content Engagement	Formative Assessment
RMSEA	0.0254121	0.02576532	0.05033849	0.06894667
NNFI	0.9956325	0.9946921	0.9465772	0.955754
CFI	0.9976707	0.9982307	0.9786309	0.9852513
SRMR	0.01916428	0.02134879	0.02916923	0.02321025
Chi-squared	10.2318 $df = 8$ $p = 0.2491323$	2.57224 $df = 2$ $p = 0.2763409$	8.378689 $df = 4$ $p = 0.07865082$	6.107148 $df = 2$ $p = 0.04718996$

Most of the fit statistics indicate that this model is a good fit. There are a few exceptions: the Tucker-Lewis NNFI for the third factor and the Chi-squared test and RMSEA for the fourth factor. However, even these exceptions are very close to the recommended values.

I calculated the aggregate score for each of the participants on each of the four factors. Each of the responses was assigned a value of 0 (not at all descriptive of my teaching) through 4 (very descriptive of my teaching). These values were then totaled for each participant on the items relating to a particular factor. This sum was divided by the maximum total possible points and multiplied by 100. The aggregate score is equivalent to the percent of possible points earned on the set of items assigned to a given factor. A score of 100 would indicate that the participant reported all of the practices within a given factor were "very descriptive" of the participant's teaching. A score of 0 would indicate that the participant reported that all of the practices within a given factor were "not at all" descriptive of the participant's teaching. This next section describes the results for the entire sample, displayed in Table 4.13. Then, I compare each of the subgroups previously listed.

Table 4.13
Scores on the Four Instructional Factors

Factor	*n*	Mean Score	Standard Deviation
1) Student-student interaction	433	72.83	19.37
2) Content delivery	432	51.37	22.04
3) Formative assessment	433	61.28	17.06
4) Student-content engagement	433	81.57	15.40

In the sample as a whole, engaging students in working together and engaging them in the practices of the discipline, factors 1 and 4, both have high means. Within the items aligned to factor 1, the average was highest among more general instructional practices: students talking to each other (item 12, mean 3.42), requiring students to work in small groups (item 15, mean 3.34), and making connections among concepts in assignments (item 19, mean 3.24). Participants generally scored lower on more specific practices, having students explore ideas before formal instruction (item 10, mean 2.62), having students criticize each other's ideas (item 13, mean 2.36) and prompting students to discuss difficulties (item 14, mean 2.51). This variability among the mean scores for items aligned to student-content engagement was not present; the mean on these items ranged from 3.0 to 3.6.

The mean scores on student-student interaction and student-content engagement are aligned with policy recommendations (Bain, 2004; Conference Board of the Mathematical Sciences, 2012; National Research Council, 2015; Schoenfeld, 2014). In contrast, the mean score on formative assessment is not aligned with research on effective instruction (Bransford & National Research Council, 2000). More specifically, on item 6, the use of student assessment results to guide instruction, 47% of respondents indicated that this practice was only somewhat, minimally, or not at all descriptive of their teaching; 15% said it was minimally or not at all descriptive. On item 8, on whether student questions guide instruction, 33% indicated it was only somewhat, minimally, or not at all descriptive of their teaching.

As a whole, participants scored an average of about 50 on content delivery.

Walter and colleagues (2015) described this factor as a measure of more "traditional teaching practices," as practices that situate the instructor as the deliverer of content to students. In their study, the researchers found STEM faculty to be more likely to indicate that the content delivery factor was more descriptive of their instruction. However, this was not the case for instructors in this sample, as content delivery had the lowest factor score of the four retained factors. Compared to the other factors, the distribution on this score was much more symmetrical and the standard deviation for this factor was greater than the other factors. At the item level, one of the interesting results was that one-third (34%) of participants indicated that item 05 was mostly or very descriptive of their teaching, that they structure their course with the assumption that most of the students have little useful knowledge of the topics.

The following section investigates the differences in these factors by previously discussed subgroups:

- Participants with advanced degrees in mathematics compared to participants with advanced degrees in mathematics education or other fields.
- Participants appointed to mathematics departments compared to participants appointed to non-mathematics departments.
- Participants with doctoral degrees compared to other participants.
- Participants with PreK–12 teaching experience compared to participants without PreK–12 teaching experience.
- Participants who perceive their programs as highly or moderately selective versus not selective.

Participants with advanced degrees in mathematics as a group scored lower on the student-student interaction [$F(1, 397) = 25.8660$, $p < 0.0001$], use of formative assessment [$F(1, 397) = 5.9338$, $p = 0.0153$], and student-content engagement [$F(1, 397) = 16.4223$, $p < 0.0001$] at statistically significant levels. There was no meaningful difference on the content delivery factor [$F(1, 396) = 0.7696$, $p = 0.3809$]. These results are displayed in Table 4.14.

Table 4.14

Difference in Mean Score, by Subject of Terminal Degree

	Subject of Terminal Degree					
	Mathematics			Mathematics education, education, or another discipline		
Factor	*n*	mean	SD	*n*	mean	SD
1) Student-student interaction **($p < 0.0001$)**	129	65.76	22.30	270	76.01	17.09
2) Content delivery ($p = 0.3809$)	129	52.23	1.94	269	50.16	1.34
3) Formative assessment **($p = 0.0153$)**	129	58.02	17.59	270	62.43	16.54
4) Student-content engagement **($p < 0.0001$)**	129	77.08	1.32	270	83.63	0.91

Participants appointed exclusively to mathematics departments, as opposed to joint appointments or appointed to another department, also scored lower on student-student interaction [$F(1, 397) = 19.5846$, $p < 0.0001$] and student-content engagement [$F(1, 397) = 7.8599$, $p = 0.0053$] on average, at statistically significant levels. There was not a meaningful difference in the content delivery [$F(1, 396) = 0.3288$, $p = 0.5667$] or formative assessment factors [$F(1, 397) = 1.0646$, $p = 0.3028$]. These results did not change when participants with joint appointments were grouped with participants appointed exclusively to mathematics departments. These results are displayed in Table 4.15.

Table 4.15

Difference in Mean Score, by Departmental Appointment

	Departmental Appointment					
	Mathematics			Other Department, or Joint Appointment		
Factor	*n*	mean	SD	*n*	mean	SD
1) Student-student interaction **($p < 0.0001$)**	297	70.26	20.33	102	79.94	14.76
2) Content delivery ($p = 0.5667$)	296	51.20	22.43	102	49.75	20.72
3) Formative assessment ($p = 0.3028$)	297	60.49	16.79	102	62.50	17.57
4) Student-content engagement **($p = 0.0053$)**	297	80.26	16.07	102	85.17	12.59

Participants who held doctoral degrees or who were working toward a doctoral degree scored higher on the student interaction factor [$F(1, 397) = 13.2441$, $p = 0.0003$] and lower on the content delivery factor [$F(1, 396) = 42.1525$, $p < 0.0001$] at statistically significant levels. There was no major difference between these two groups on the formative assessment factor [$F(1, 397) = 0.0039$, $p = 0.9504$]. The difference between the two groups in terms of student-content engagement was not statistically significant at the $p = 0.05$ level [$F(1, 397) = 3.3403$, $p = 0.0684$]. (A one-tailed test found statistical significance with $p = 0.0342$.) These results are displayed in Table 4.16.

Table 4.16

Difference in Mean Score, by Level of Terminal Degree

	Terminal Degree					
	Doctorate or working toward a doctorate			Bachelors, Masters, or Masters with advanced coursework.		
Factor	*n*	mean	SD	*n*	mean	SD
1) Student-student interaction **($p = 0.0003$)**	279	75.03	18.69	120	67.39	20.39
2) Content delivery **($p < 0.0001$)**	278	46.36	20.45	120	61.20	22.00
3) Formative assessment ($p = 0.9504$)	279	60.97	16.90	120	61.08	17.26
4) Student-content engagement ($p = 0.0684$)	279	82.44	15.15	120	79.38	15.81

Participants with PreK–12 teaching experience scored higher on uses of formative assessment [$F(1, 397) = 11.7993$, $p = 0.0007$] and student-content engagement [$F(1, 397) = 18.2949$, $p < 0.0001$] at statistically significant levels. The differences between the two groups on the student-student interaction factor [$F(1, 397) = 3.5865$, $p = 0.0590$] and the content delivery factors [$F(1, 396) = 2.5114$, $p = 0.1138$] were not statistically significant. (A one-tailed test found a statistically significant difference on the student-student interaction factor with $p = 0.0295$.) These results are reported in Table 4.17.

Table 4.17

Difference in Mean Score, by PreK–12 Teaching Experience

	PreK–12 Teaching Experience			No PreK–12 Teaching Experience		
Factor	*n*	mean	SD	*n*	mean	SD
1) Student-student interaction ($p = 0.0590$)	286	73.89	18.21	113	69.80	22.29
2) Content delivery ($p = 0.1138$)	285	51.93	21.91	113	48.06	22.04
3) Formative assessment **($p = 0.0007$)**	286	62.81	16.77	113	56.42	16.76
4) Student-content engagement **($p < 0.0001$)**	286	83.54	13.65	113	76.38	18.20

In addition to formative assessment and student-content engagement, participants with *elementary* teaching experience (PreK–5, $n = 81$) also scored higher on the student-student interaction factor at a statistically significant level [$F(1, 397) = 9.5545$, $p = 0.0021$]. On average, participants with elementary teaching experience scored 78.65

on the student-student interaction factor, compared to 71.23 for other participants.

Participants who viewed their institution as highly or moderately selective, based upon the caliber of their students in mathematics courses for elementary teachers, scored higher on student-student interaction [$F(1, 396) = 11.4129$, $p = 0.0008$] and student-content engagement [$F(1, 396) = 7.1552$, $p = 0.0078$] at statistically significant levels. There was no meaningful difference on the formative assessment [$F(1, 396) = 0.2228$, $p = 0.6372$] or content delivery factors [$F(1, 395) = 0.8146$, $p = 0.3673$]. These results are shown in Table 4.18.

Table 4.18

Difference in Mean Score, by Perception of Institutional Selectivity and Students

	Perception of Institution based on Caliber of Students					
	Moderately or Very Selective			Not Selective		
Factor	*n*	mean	SD	*n*	mean	SD
1) Student-student interaction **(*p* = 0.0008)**	236	75.39	17.22	162	68.75	21.90
2) Content delivery (*p* = 0.3673)	236	50.03	21.95	161	52.06	22.11
3) Formative assessment (*p* = 0.6372)	236	61.31	16.93	162	60.49	17.16
4) Student-content engagement **(*p* = 0.0078)**	236	83.18	14.21	162	79.01	16.73

In sum, instructors' characteristics do influence their reported instructional practices. The participants who used instructional practices that encourage student-

student interaction most frequently (1) held their terminal degree in a subject other than mathematics; (2) were appointed to departments other than mathematics departments or held joint appointments; (3) held a doctorate or were working toward a doctorate; (4) had elementary teaching experience; or (5) viewed their institution as selective. Those who did not hold a doctorate or who were not working toward a doctorate were more likely to engage in practices that position themselves as the deliverer of content. Participants who reported using formative assessment most frequently tended to hold a terminal degree in a subject other than mathematics or had PreK–12 teaching experience. Participants who used instructional practices to encourage student-content engagement (1) held their terminal degree in a subject other than mathematics; (2) were appointed to a department other than mathematics or held a joint appointment; (3) had PreK–12 teaching experience; or (4) viewed their institutions as selective. These results are summarized in Table 4.19.

Table 4.19

Instructor Characteristics and Instructional Practices: Which instructors are likely to more frequently engage in the following instructional practices?

Factor	Instructor Characteristics
Student-student interaction	• Hold a terminal degree in mathematics education or a subject other than mathematics • Have a joint appointment or are appointed to a department other than mathematics • Hold a doctorate or are working toward a doctorate • Have PreK–5 teaching experience • View their institution as selective, based on the caliber of their students.
Content delivery	• Hold a bachelor's or master's degree as their terminal degree
Formative assessment	• Hold a terminal degree in a subject other than mathematics • Have PreK–12 teaching experience
Student-content engagement	• Hold a terminal degree in mathematics education or a subject other than mathematics • Have a joint appointment or are appointed to a department other than mathematics • Have PreK–12 teaching experience • View their institution as selective, based on the caliber of their students.

Summative Assessment

In addition to these five factors, the survey measured different components of summative assessment. These items are listed below.

- 21) My test questions focus on important facts and definitions from the course.
- 22) My test questions require students to apply course concepts to unfamiliar situations.
- 23) My test questions contain well-defined problems with one correct solution.
- 24) I adjust student scores (e.g. curve) when necessary to reflect a proper distribution of grades.

With the exception of item 24, about grading on a curve, the responses to these items were approximately normally distributed. More than half of the participants responded that they did not grade on a curve at all, while an additional 24% indicated the statement was minimally descriptive of their teaching. The proportions of participants answering the three other statements in various ways are displayed in Table 4.20.

Table 4.20

Features of Participants' Summative Assessments

	How Descriptive Participants Indicate the Feature is of Their Instruction				
Feature	Not at all	Minimally	Somewhat	Mostly	Very
Important facts and definitions	11%	29%	29%	17%	14%
Apply course concepts to unfamiliar situations	6%	14%	34%	31%	14%
Well defined problems with one correct solution	8%	21%	38%	23%	10%

Section III: Five Dimensions of Mathematically Powerful Classrooms

In the following sections, I report the survey results in regards to instructional practices using Schoenfeld's five dimensions of mathematically powerful classrooms: (1) mathematics, (2) cognitive demand, (3) access to mathematical content, (4) agency, authority, and identify, and (5) assessment.

Mathematics

The *mathematics* dimension measures the extent to which students are engaged in conceptual understanding of mathematics, as opposed to only practicing rote procedures. It also measures the opportunities students have to engage in mathematical practices. Question 2 in this survey asked participants which of the following mathematical topics they addressed in their mathematics course for elementary teachers: (1) review of calculation procedures; (2) the mathematical reasons why formulas or algorithms work; (3) the nature of justification, reasoning, or proof in mathematics; (4) common misconceptions of elementary students; and (5) representations used in elementary school classrooms or curricula. Thus, this question measured the attention to both procedures and concepts, to mathematical practices, and to the application of these ideas to the work of elementary school teaching.

The majority of participants in this study were addressing mathematical concepts, as shown in Table 4.21. The majority were also addressing the nature of reasoning in mathematics. The majority of participants (94.97%) were also making connections to the work of elementary school teachers: 83.62% addressed both elementary students' misconceptions and representations commonly found in the curricula, while an additional 6.33% addressed misconceptions and an additional 5.02% addressed representations. Fifty-four percent (54%) of participants indicated that they addressed all five topics. Only 0.22% indicated that they only reviewed calculation procedures.

Table 4.21

Topics Addressed in Mathematics Content Courses for Elementary Teachers

Topic	Percent of Participants Addressing this Topic *n* = 458
Review of calculation procedures	77.51%
The mathematical reasons why formula or algorithms work	94.32%
The nature of justification, reasoning, or proof in mathematics	81.88%
Common misconceptions and mistakes of elementary students	89.96%
Representations used in elementary school classrooms or curricula	88.65%

There were statistically significant differences in the topics addressed between subgroups with different instructor characteristics. However, even though these differences were statistically significant, they were not always practically significant. A Pearson Chi-squared test of proportions was calculated to compare each pair of groups.

Instructors whose terminal degree was in mathematics, as opposed to mathematics education or another discipline, were more likely to review calculation procedures, $\chi^2(1, N = 402) = 7.059, p = 0.0079$. Those with their terminal degree in mathematics were less likely to address common misconceptions of elementary students, $\chi^2(1, N = 402) = 10.080, p = 0.0015$. Likewise, they were less likely to address representations used in elementary school classrooms, $\chi^2(1, N = 402) = 8.161, p = 0.0043$. However, in both groups, the vast majority, over 80%, addressed connections to elementary school. These results are displayed in Table 4.22.

Table 4.22

Topics Addressed, by Subject of Instructors' Terminal Degree

	Subject of Terminal Degree	
	Mathematics	Mathematics Education or Other Discipline
Topic	$n = 130$	$n = 272$
Review of calculation procedures**	85%	74%
The mathematical reasons why formula or algorithms work	96%	93%
The nature of justification, reasoning, or proof in mathematics	81%	83%
Common misconceptions and mistakes of elementary students**	84%	94%
Representations used in elementary school classrooms or curricula**	82%	92%

Note. $^*p < 0.05$, $^{**}p < 0.01$

Departmental appointment did not impact whether participants addressed these topics. There were no significant differences in the percent of participants addressing each of the five topics between those appointed exclusively to mathematics departments and those with joint appointments or appointed to a department in a different discipline: (1) review of calculation procedures, $\chi^2(1, N = 402) = 1.147$, $p = 0.2842$, (2) reasons why formula or algorithms work, $\chi^2(1, N = 402) = 1.141$, $p = 0.2855$, (3) nature of reasoning in mathematics, $\chi^2(1, N = 402) = 0.367$, $p = 0.5448$, (4) misconceptions of elementary students, $\chi^2(1, N = 402) = 1.071$, $p = 0.3007$, and (5) representations used in elementary classrooms, $\chi^2(1, N = 402) = 0.773$, $p = 0.3794$. Grouping participants with a joint appointment with participants appointed exclusively to mathematics departments did not change these results. These results are displayed in Table 4.23.

Table 4.23

Topics Addressed, by Departmental Appointment

	Departmental Appointment	
Topic	Mathematics $n = 300$	Joint or Other $n = 102$
Review of calculation procedures	79%	74%
The mathematical reasons why formula or algorithms work	95%	92%
The nature of justification, reasoning, or proof in mathematics	82%	84%
Common misconceptions and mistakes of elementary students	90%	93%
Representations used in elementary school classrooms or curricula	88%	91%

Having a doctorate as a terminal degree impacted the likelihood a participant would address some of these topics. Specifically, those with doctorates were less likely to review calculation procedures at a statistically significant level, $\chi^2(1, N = 402) = 8.759$, $p = 0.0031$. In the sample, those with doctorates were also more likely to address the nature of justification, reasoning, and proof, though this difference was not statistically significant, $\chi^2(1, N = 402) = 3.573$, $p = 0.0587$. (A one-tailed test would show the difference to be significant with $p = 0.0421$.) These results are displayed in Table 4.24.

Table 4.24

Topics Addressed, by Level of Terminal Degree

	Level of Terminal Degree	
Topic	Doctorate or Working toward Doctorate $n = 281$	Bachelor's, Master's, or Master's with Additional Coursework $n = 121$
Review of calculation procedures**	73%	87%
The mathematical reasons why formula or algorithms work	95%	92%
The nature of justification, reasoning, or proof in mathematics	85%	77%
Common misconceptions and mistakes of elementary students	91%	88%
Representations used in elementary school classrooms or curricula	90%	85%

Note. **$p < 0.01$

Participants' PreK–12 teaching experience did not seem to have a major impact on their likelihood of addressing these five topics. Those with PreK–12 experience were more likely to address common misconceptions of elementary school students at a statistically significant level, $\chi^2(1, N = 402) = 3.904$, $p = 0.0482$. However, there was no difference in the other four topics: (1) review of calculation procedures, $\chi^2(1, N = 402) = 1.013$, $p = 0.3142$, (2) reasons why formula or algorithms work, $\chi^2(1, N = 402) = 0.052$, $p = 0.8200$, (3) the nature of reasoning and proof, $\chi^2(1, N = 402) = 0.384$, $p = 0.5357$, (4) representations in elementary school, $\chi^2(1, N = 402) = 2.213$, $p = 0.1368$. These results are shown in Table 4.25. Participants with PreK–5 teaching experience ($n = 45$) were not more likely to address connections in elementary school at a statistically significant level.

Table 4.25

Topics Addressed, by PreK–12 Teaching Experience

Topic	PreK–12 Teaching Experience $n = 288$	No PreK–12 Teaching Experience $n = 114$
Review of calculation procedures	76%	81%
The mathematical reasons why formula or algorithms work	94%	94%
The nature of justification, reasoning, or proof in mathematics	82%	84%
Common misconceptions and mistakes of elementary students *	92%	86%
Representations used in elementary school classrooms or curricula	90%	85%

Note. $^*p < 0.05$

Participants' perception of their institution and the caliber of their students impacted their likelihood of reviewing calculation procedures, $\chi^2(1, N = 400) = 4.860$,

$p = 0.0275$. It did not impact whether they addressed the other four topics: (1) reasons why formula work, $\chi^2(1, N = 400) = 2.173$, $p = 0.1404$, (2) the nature of reasoning and proof, $\chi^2(1, N = 400) = 0.667$, $p = 0.4140$, (3) elementary students' misconceptions, $\chi^2(1, N = 400) = 1.488$, $p = 0.2225$, and (4) representations used in elementary school classrooms, $\chi^2(1, N = 400) = 2.254$, $p = 0.1332$. These results are displayed in Table 4.26.

Table 4.26

Topics Addressed, by Perception of Institution and Students

Topic	Moderately or Very Selective	Not Selective
Review of calculation procedures*	73%	83%
The mathematical reasons why formula or algorithms work	93%	96%
The nature of justification, reasoning, or proof in mathematics	85%	80%
Common misconceptions and mistakes of elementary students	92%	88%
Representations used in elementary school classrooms or curricula	91%	86%

Note. $^*p < 0.05$

In sum, most participants reported attending to conceptual understanding in their courses for elementary teachers. Most participants also made connections to the work of elementary school teachers. A sizable majority addressed the nature of justification in mathematics. While fewer participants reported reviewing procedures, the majority did review calculation procedures. Hardly any participants (0.22%) reported focusing on the review of procedures to the exclusion of other mathematical concepts.

Although there were some statistically significant differences between these subgroups, these differences may not be practically significant. Participants who perceived their institutions as less selective, whose terminal degree was in mathematics, or who did not hold or were not working toward their doctorate were more likely to review calculation procedures than their counterparts. Participants with PreK–12 teaching experience or whose terminal degree was in mathematics education or a discipline other than mathematics were more likely to address connections to elementary school teaching. However, the majority of participants reported addressing the conceptual ideas in mathematics, mathematical practices, and connections to elementary school, across all subgroups, regardless of instructor characteristics.

Cognitive Demand

Cognitive demand is related to the *mathematics* dimension in that cognitively demanding tasks are necessarily those that support conceptual understanding and engage learners in mathematical practices. However, studies at the secondary level have shown that U.S. teachers can lower the cognitive demand of rich tasks by doing the mathematical work for students (Hiebert et al., 2005b). In this study, *cognitive demand* was measured by two sets of Likert-type statements, (Questions 5 and 6). One set of questions asked participants about their selection and launch of tasks. The second set of questions asked participants about practices that maintained or lowered cognitive demand.

Exploratory factor analysis and confirmatory factor analysis indicated that there were two correlated factors across these two questions that underlie participants' responses: (1) practices that support students in doing the cognitively demanding mathematical work and (2) practices that may reduce cognitive demand for students by having the instructor do the mathematical work. One item, *I ask guiding questions that direct students to the main mathematical idea*, was removed because it loaded nearly equally on both factors and seemed to have been ambiguous. The model fit statistics are displayed in Table 4.27. These show a reasonable fit. The factor loadings are displayed in Table 4.28. I assigned each participant an aggregate score for each factor, following the process outlined for the four factors discussed under General Instructional Practices. These factor scores could be interpreted as the percent of total possible points. A score of 100 would indicate that the participant felt all the practices within a factor were very descriptive of his or her teaching. A score of 0 would indicate that the participant felt all practices within a factor were not at all descriptive of his or her teaching. This section will describe the results generally and then compare the different subgroups.

Table 4.27

Model Fit Statistics for Two Factors of Cognitive Demand

Model Fit Statistics	Support Cognitive Demand	Lower Cognitive Demand
Model Chi-square	14.23572	7.018228
Degrees of freedom	8	4
Probability	0.07582459	0.1349281
RMSEA	0.04323449	0.04258919
Tucker-Lewis NNFI	0.982306	0.9934686
Bentler CFI	0.9905632	0.9973875
SRMR	0.02306578	0.01865828

Table 4.28

Standardized Factor Loadings of Associated Items on Cognitive Demand

Item	Support Cognitive Demand	Lower Cognitive Demand
Q5) I facilitate opportunities for students to figure out ideas for themselves.	0.8341	
Q5) I expect students to attempt problems they may not know how to solve.	0.7726	
Q5) I use problems to introduce new ideas.	0.6232	
Q6) I see how far they can get without my help, either individually or as group.	0.4603	
Q6) I listen and offer encouragement, but not direction on what to do.	0.4677	
Q6) I ask questions that get students to articulate their thinking.	0.4692	
Q5) I explain mathematical ideas to students before having them solve problems.		0.7886
Q5) I demonstrate how to solve problems before having students attempt problems.		0.8325
Q6) I clearly explain the steps for how to solve the problem.		0.8626
Q6) I ask questions that lead them through the steps for solving the problem.		0.4359
Q6) I clearly explain the mathematics in the problem without telling them the solution.		0.3931

As a whole, participants scored high on practices that would encourage students to engage in cognitively demanding tasks. Participants scored moderate on practices that would lower cognitive demand. While participants seem to be mostly in agreement on practices that support a high cognitive demand, the spread in scores on practices that lowered cognitive demand was much wider, as shown in Figure 4.3. The mean, standard deviation, and five number summary for the two factors are in Table 4.29.

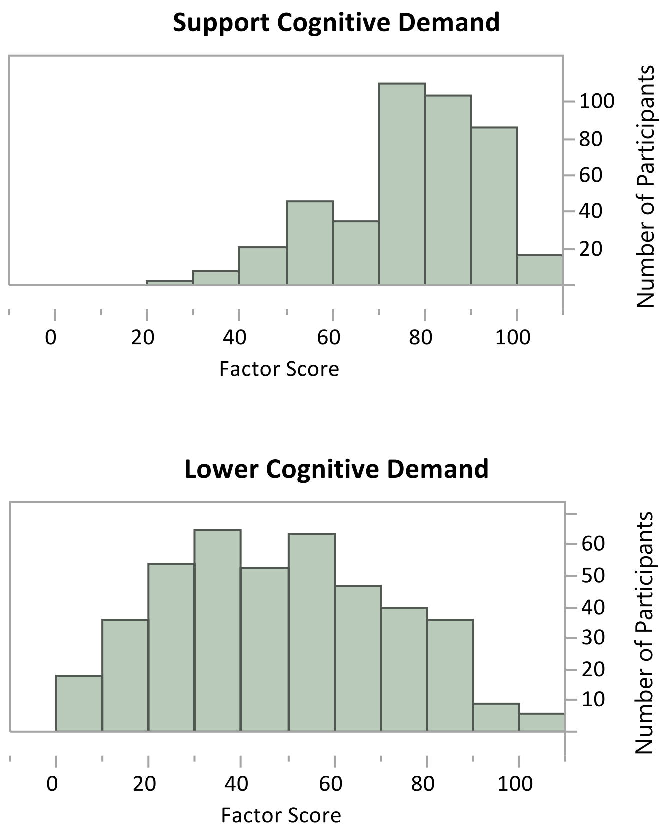


Figure 4.3. Distribution of scores on cognitive demand factors

Table 4.29

Descriptive Statistics for Cognitive Demand Factor Scores

Factor	Five Number Summary	Mean	Standard Deviation
Support Cognitive Demand	{25, 66.7, 79.2, 87.5, 100}	76.82	15.72
Lower Cognitive Demand	{0, 25, 45, 65, 100}	45.79	23.43

Instructor characteristics mattered significantly in regards to practices involving cognitive demand. As a group, participants with their terminal degree in mathematics scored lower on practices that support cognitive demand [$F(1, 398) = 18.8971$, $p < 0.0001$] and higher on practices that may lower cognitive demand [$F(1, 397) = 24.1935$, $p < 0.0001$]. These differences were statistically significant. Relatedly, participants appointed exclusively to mathematics departments scored lower in practices that supported cognitive demand [$F(1, 398) = 9.6057$, $p = 0.0021$] and higher in practices that lowered cognitive demand [$F(1, 397) = 15.0274$, $p < 0.0001$], at statistically significant levels. Participants who held doctorates or who were working toward their doctorates scored higher on supporting cognitive demand [$F(1, 398) = 14.8628$, $p < 0.0001$] and lower on lowering cognitive demand [$F(1, 397) = 52.4236$, $p < 0.0001$], at statistically significant levels. This trend was true in the sample both for those whose terminal degree was in mathematics and those whose terminal degree was in another discipline, though the difference was more pronounced among the latter. Participants' PreK–12 teaching experience and perceived selectivity of their institution only impacted their scores on supporting cognitive demand at a statistically significant level, not on lowering cognitive demand. Participants with PreK–12 teaching experience were more likely to describe high cognitive demand practices as more descriptive of their teaching [$F(1, 398) = 7.0902$, $p = 0.0081$], as were participants who perceived their institutions to be very or moderately selective [$F(1, 397) = 4.8172$, $p = 0.0288$]. There was no substantial difference between participants with PreK–12 teaching experience and those without in regards to practices that lower cognitive demand [$F(1, 397) = 0.9191$, $p = 0.3383$]. Likewise, there was no appreciable difference between participants who viewed their institution as moderately or very selective and those who viewed it as not selective on practices lowering cognitive demand [$F(1, 396) = 0.9486$, $p < 0.3307$] These results are displayed in Table 4.30.

Table 4.30

Mean and Standard Deviation of Scores on Cognitive Demand Factors, by Instructor Characteristic

Instructor Characteristic	Support Cognitive Demand	Lower Cognitive Demand
Subject of terminal degree	***	***
Mathematics	72.1 (16.9)	53.4 (23.1)
Mathematics education or other	79.2 (14.6)	41.5 (22.3)
Departmental appointment	**	***
Exclusively mathematics	75.5 (16.1)	47.9 (22.9)
Joint appointment, school of education, or other	81.0 (14.1)	37.8 (22.7)
Level of terminal degree	***	***
Doctorate or working toward doctorate	78.9 (15.0)	40.1 (22.6)
Master's or Bachelor's	72.4 (16.6)	57.4 (20.0)
Teaching Experience	**	
PreK–12 teaching experience	78.2 (15.3)	44.7 (23.1)
No PreK–12 teaching experience	73.6 (16.5)	47.1 (23.5)
Perception of students and institution	*	
Selective	78.4 (15.1)	44.5 (23.9)
Not selective	74.8 (16.5)	46.8 (22.2)

Note. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.0001$

In sum, while it appears that instructors of mathematics content courses are engaging in practices to support a high cognitive demand, they are also engaging in practices that lessen the cognitive demand. Conceptually, this aligns with a score of 2 on the TRUMath Rubric. Certain populations of instructors are more likely to engage in some practices than others. More advanced degrees and connections to the education sector, either through their academic or professional training or through a departmental appointment, appears to support instructors in engaging in practices to maintain cognitive demand.

Access to Mathematical Content

The dimension *access to mathematical content* measures the extent to which the teacher achieves active participation of all students in the classroom. Active participation in this study was defined as students actively interacting with others. Active participation goes beyond asking questions and listening to also include contributing one's own ideas.

In this study, the IPCU survey measured three aspects of this dimension. First, participants answered a short, open-ended question about their definition of full participation. Their responses were coded as active participation and passive participation. The coding guidelines can be found in Appendix C. Second, participants indicated whether they felt they achieved full participation among all of their students on most days. Third, participants listed strategies that they had for ensuring participation.

Of the 399 participants who answered one or more of the questions regarding participation, 65% defined full participation as active participation. Active participation included working with others, answering questions, and contributing to discussions. Participants whose responses were limited to attending class, listening, paying attention, taking notes, completing problems, having a good attitude, coming prepared to class, or staying awake, were coded as defining full participation as passive participation. These participants represented 22% of those who answered the set of participation questions. Five percent (5%) the participants provided qualifiers that indicated they believed that a student who only “actively listened,” thought about problems individually, or asked questions when confused would be considered fully participating. These responses were more in alignment with the characteristics of passive participation listed above and so were coded accordingly. Some participants did not provide enough information (e.g., “engaged listening or participating”). Some left the question blank, though they answered other questions about participation. Some indicated that they did not care about participation. These participants, representing 8% of those answering questions about participation, were also coded as defining full participation as passive. Altogether, 35% of participants answering questions about participation defined full participation as passive participation. Even among these participants, 39% disagreed with the statement, *Most class sessions, all of my students fully participate.* The coding guidelines can be found in Appendix C.

Participants whose terminal degree was in mathematics, as opposed to mathematics education or another discipline, were less likely to define full participation as active participation at a statistically significant level, according to a Pearson Chi-squared test of proportions, $\chi^2(1, N = 396) = 8.657$, $p = 0.0033$. Only 55% of those with their terminal degree in mathematics defined full participation as active participation, compared to 70% of participants with their terminal degree in other disciplines. There were no statistically significant differences among the other instructor characteristics investigated in this study on this dimension.

Among participants who defined full participation as active participation, 172, or 66% indicated that on most days, all of their students actively participated. In other words, 43% of 399 participants responding to questions about participation would have received a 3 on the TRUMath rubric. I conducted a Pearson Chi-squared tests of independence to determine whether there was a relationship between groups with different instructor characteristics and the participants' belief that they achieved full participation among all of their students on most days. I considered only the subset of 259 participants who defined full participation as active participation in this test. Participants appointed to mathematics departments, as opposed to joint appointments or other departments, were less likely to indicate they achieved full participation from all of their students most days at a statistically significant level, $\chi^2(1, N = 257) = 4.685, p = 0.0304$. Sixty-two percent (62%) of participants appointed to a mathematics department in this subset reported achieving full participation from all of their students most days, compared to 76% of other participants. Participants with PreK–12 experience were more likely to report achieving full participation among all of their students most days at a statistically significant level $\chi^2(1, N = 257) = 7.830, p = 0.0051$. Seventy-one percent (71%) of participants with PreK–12 teaching experience in this subset reported achieving full participation most days while only 52% of participants without PreK–12 teaching experience believed they achieved full participation most days. The level and discipline of a participant's terminal degree and the perceived selectivity of their institution did not impact whether participants felt they achieved full participation among all of their students on most days: (1) level of terminal degree, $\chi^2(1, N = 257) = 2.076, p = 0.1496$, (2) discipline of terminal degree, $\chi^2(1, N = 257) = 2.466, p = 0.1163$, (3) perceived selectivity, $\chi^2(1, N = 257) = 0.245, p = 0.6204$.

The TRUMath rubric also measures whether instructors makes an effort to achieve broad active participation. Instructors who make attempts to broaden participation, even though they do not achieve participation by all students, would receive a score of 2 on the rubric. Consequently, this survey asked participants whether they had strategies to ensure all students full participated. I analyzed the subgroup of 87 participants who defined full participation as active participation but did not feel that all of their students fully participated most days. Fifty-four (54) of these participants, or 62% of this subset, listed strategies that they used for encouraging participation. This represents 14% of the entire group of 399 participants who answered questions about participation. Because this subset of the sample was small, I did not conduct a Chi-squared test of proportions to determine if instructor characteristics influenced the participants' likelihood of having strategies. These statistics can be found in Table 4.31.

Table 4.31
Associated TRUMath Scores of Participants Answering Questions about Participation (n = 399)

Aligned TRUMath Score	Description of participants	*n*	percent
3	define full participation as active participation and feel most days all of their students fully participate	172	43%
2	define full participation as active participation, do not feel most days all of their students fully participate, but have strategies for broadening participation	54	14%
1	define full participation as active participation, do not feel most days all of their students fully participate, and do not have strategies for broadening participation	33	8%
1	define full participation as passive participation	140	35%

Authority, Agency, and Identity

The dimension *agency, authority, and identity* measures the extent to which students' ideas are addressed in class, as well as the extent to which students are the ones who determine whether ideas are mathematically valid. In the survey, this dimension was measured by two questions, Q11 and Q12, which examined how participants planned to address students' novel, incorrect, or incomplete ideas. The answer choices were aligned to a score of 1, 2, or 3 to correspond to the rubric. A score of 1 indicated that students' ideas were not pursued. A score of 3 indicated that student ideas were pursued and students were the ones who determine mathematical validity. A score of 2 indicated that student ideas were pursued, but the instructor played a more substantial role in determining the validity. The two survey questions and the scores assigned to the answer choices are reproduced in Figure 4.4. Approximately 20% of respondents selected “other.” As explained in Chapter 3, these responses were not analyzed.

Q11) When a student brings up a novel idea, I generally:

- Indicate whether or not the idea is correct **(1)**
- Ask other students what they think about the idea **(3)**
- Explain how this idea fits in with other mathematics we have been covering **(2)**
- Other (please explain) **(not analyzed)**

Q12) In the next question, we ask you to respond to a scenario that might occur in your class.

Some students brought up an idea in your class that was related to the topic you were addressing, but outside of what you planned to discuss. For example,

> Your class is beginning to study prime and composite numbers. You have just asked students for definitions and written their definitions on the board. Mary adds: "Prime numbers are mostly small." Sam agrees: "Composite numbers are mostly large numbers."

What is the first thing you would do in response?

- Nothing, since this idea is outside of the topics for the course **(1)**
- Provide some counterexamples for the class to consider **(2)**
- Ask Mary to further explain her thinking **(3)**
- Correct the students **(1)**
- Ask other students to comment on Mary’s idea **(3)**
- Other (please explain): **(not analyzed)**

Figure 4.4. Questions measuring the dimension *authority, agency, and identity* and the scores assigned to the answer choices

The vast majority of all participants selected answer choices that were assigned a score of 3, indicating that they intended to investigate the students' ideas and to engage other students in determining their validity. Nearly all participants indicated they would address the idea in some way. The results are displayed in Table 4.32.

Table 4.32

Responses on Questions Addressing Authority, Agency, and Identity

		Response		
Question	*n*	1	2	3
Q11)	316	1.6%	19.9%	78.5%
Q12)	317	1.6%	9.2%	89.6%

There were differences in the ways different subgroups answered these two questions. A Pearson's Chi-squared test was conducted to determine whether the differences were statistically significant. The differences only reached statistical significance on Q11, a general question about responding to students' novel ideas. The differences did not reach statistical significance on Q12, a question about responding to a specific student misconception. Specifically, participants with their terminal degree in mathematics, as opposed to mathematics education or another discipline, were less likely to share mathematical authority with students, as measured by Q11, $\chi^2(2, N = 314) = 16.430, p = 0.0003$. Likewise, participants appointed exclusively to mathematics departments were less likely to share mathematical authority with their students in comparison to their counterparts appointed to schools of education, other departments, or jointly appointed, $\chi^2(2, N = 314) = 6.738, p = 0.0344$. Additionally, participants who held doctoral degrees or who were working on doctoral degrees were more likely to share mathematical authority with students, as measured by Q11, $\chi^2(2, N = 314) = 16.144, p = 0.0003$. Participants' PreK–12 teaching experience and perceptions of their institution did not have an impact on their answers to Q11 at a statistically significant level, $\chi^2(2, N = 314) = 4.573, p = 0.11016$ and $\chi^2(2, N = 314) = 4.781, p = 0.0916$, respectively. These results are shown in Table 4.33.

Table 4.33
Impact of Instructor Characteristics on the Dimension Authority, Agency, and Identity, as Measured by Q11

		Response to Q11		
Instructor characteristics	*p*	1	2	3
Subject of terminal degree				
Mathematics	**0.0003**	3%	32%	65%
Mathematics education or other		1%	14%	85%
Departmental appointment				
Exclusively mathematics	**0.0344**	2%	23%	75%
Joint appointment, school of education, or other		0%	11%	89%
Level of terminal degree				
Doctorate or working toward doctorate	**0.0003**	0%	15%	84%
Master's or Bachelor's		4%	30%	66%
Teaching Experience				
PreK–12 teaching experience	0.1016	2%	17%	82%
No PreK–12 teaching experience		1%	27%	72%
Perception of students and institution				
Moderately or Very Selective	0.0916	2%	16%	79%
Not selective		1%	26%	73%

Participants' responses on Q12 followed these same trends, though the differences between subgroups on these questions did not reach statistical significance, as shown in Table 4.34: the subject of a participant's terminal degree, $\chi^2(2, N = 314) = 3.456$, $p = 0.1777$; a participant's departmental appointment, $\chi^2(2, N = 314) = 2.791$, $p = 0.2477$; the level of a participant's terminal degree, $\chi^2(2, N = 314) = 5.282$, $p = 0.0713$; whether a participant had taught in PreK–12 schools, $\chi^2(2, N = 314) = 0.318$, $p = 0.8529$; or the participant's perception of the institution as moderately or very selective, $\chi^2(2, N = 314) = 3.834$, $p = 0.1471$.

Table 4.34
Impact of Instructor Characteristics on the Dimension Authority, Agency, and Identity, as Measured by Q12

		Response to Q12		
Instructor characteristics	*p*	1	2	3
Subject of terminal degree				
Mathematics	0.1777	3%	12%	85%
Mathematics education or other		1%	8%	91%
Departmental appointment				
Exclusively mathematics	0.2477	2%	11%	88%
Joint appointment, school of education, or other		1%	4%	94%
Level of terminal degree				
Doctorate or working toward doctorate	0.0713	2%	7%	91%
Master's or Bachelor's		1%	15%	84%
Teaching Experience				
PreK–12 teaching experience	0.8529	1%	10%	89%
No PreK–12 teaching experience		2%	9%	89%
Perception of students and institution				
Selective	0.1471	1%	9%	90%
Not selective		3%	9%	87%

Section IV: Use of Curriculum

This study also examined the use of curriculum materials in mathematics content courses for elementary teachers. This survey collected data about the use of different textbooks and ancillary materials. In this section, I report descriptive statistics about participants' use of textbooks and ancillary materials, according to the survey results.

The majority of respondents used a textbook in their courses. Of the 405 participants who responded to questions about their use of curriculum, over 87% indicated they used a textbook for some part of their course. Most of these respondents, over 75%, indicated that they used only one textbook, though some used multiple textbooks. Participants who did not use textbooks used materials they had created themselves, materials that other instructors within their department had collaboratively developed over several years, or resources from a variety of sources, such as journal articles and websites. Of those who used textbooks, 79.5% used them for most or almost every topic in the course. Less than 2% reported that the textbooks were listed on the syllabus but used very little in the course by the students or instructor. Of the 349 participants who reported using textbooks, 84% indicated that they used the textbook as a reference for students. However, very few (5%) of the participants who reported using textbooks indicated that they used the textbook *only* as a reference for students. Eighty-three percent (83%) of participants used the textbook as a source of homework problems. Fewer participants, 71%, used the textbook in their planning, either for the daily lessons or for the overall course. Even fewer, 61%, used the textbook as a source of in-class activities. Nearly every instructor used the textbook in multiple ways.

The most popular textbooks among those who use primarily one textbook are displayed in Table 4.35. Only five participants indicated that they used textbooks that were not designed specifically for prospective mathematics teachers, e.g. College Algebra textbooks or textbooks designed for Liberal Arts Mathematics courses. Seven participants indicated that they used mathematics textbooks designed for K–8 students.

Table 4.35

Most Popular Textbooks Used by Participants

Textbook title	Number of Participants (Percent)
Beckmann's *Mathematics for Elementary Teachers*	76 (22%)
Billstein, Libeskind & Lott's *A Problem Solving Approach to Mathematics for Elementary Teachers*	48 (14%)
Musser, Burger, & Peterson's *Mathematics for Elementary Teachers: A Contemporary Approach*	31 (9%)
Sowder, Sowder, & Nickerson's *Reconceptualizing Mathematics*	26 (8%)
Van de Walle's *Elementary and Middle School Mathematics: Teaching Developmentally*	24 (7%)
Long & DeTemple's *Mathematical Reasoning for Elementary Teachers*	23 (7%)
Bassarear's *Mathematics for Elementary School Teachers*	22 (6%)
Bennett's *Mathematics for Elementary Teachers: A Conceptual Approach*	20 (6%)
Total number of participants listing the textbooks used	346

The two most popular textbooks, Beckmann's *Mathematics for Elementary*

Teachers and Billstein, Libeskind & Lott's *A Problem Solving Approach to Mathematics for Elementary, Teachers*, have associated activity or exploration manuals. In the former, the activity manual is bound in the same book in later editions. In the latter textbook, the exploration manual is an additional publication. Nearly every participant using the Beckmann textbook used the activity manual, 92%. Only 28% of participants using the Billstein textbook used the exploration manual; 26% were unaware that such an addition was available.

The use of instructor's guides varied. Of the 350 participants answering questions about the instructor's guides, 34% used the instructor's guides. Another 13% indicated that they were unaware of instructor's guides or such documents did not exist for the textbook they used. Of those who used the instructor's guides, 76% found them moderately, very, or extremely helpful. The participants' written responses indicated that some of them used the instructor's guides primarily for finding the answers to specific problems or for additional assessment questions. Some participants used the instructor's guides for pedagogical tips, for understanding the author's intent with particular questions or activities, for quality mathematical explanations, and for connections to the K–12 classroom. There were also a few comments that indicated that participants would like more guidance from instructor's guides, such as guidance for teaching particular topics, insight into how students will respond to different activities, or ways to adjust activities.

Of the 348 participants providing information about their use of online and multimedia resources, 40% indicated that they used the online or multimedia materials;

nearly 20% were unaware of any online materials. These materials included online homework, Power Point slide shows, virtual manipulatives, dynamic graphing activities, test banks, electronic versions of the textbooks, as well videos of elementary students or elementary classrooms. Participants who used these materials generally reported that they were helpful.

Participants in this survey used curriculum materials in a range of ways. The majority of participants used one textbook for their planning and as a source of activities for their students. Instructors were less likely to use ancillary materials, such as exploration manuals separate from the main textbook, instructor's guides, and multimedia resources. However, those who did use instructor's guides or multimedia resources generally found them helpful. Some participants even volunteered suggestions for material that they would like to see in such resources.

Conclusion

This chapter has reported on the results of a nationwide survey on the instructional practices and curriculum usage of instructors of mathematics content courses for elementary teachers. I identified relationships between instructor characteristics and their use of different instructional practices. Chapter 6 situates these findings in the context of other research on mathematics content courses for elementary teachers and college level mathematics courses generally. Chapter 5 looks more closely at the instructional practices and curriculum usage of two instructors of elementary mathematics content courses in two case studies.

CHAPTER 5: ANALYSIS OF CASE STUDIES

The goal of this study was to identify the ways in which curriculum materials can support instructors of mathematics content courses for pre-service elementary teachers to create mathematically powerful experiences for prospective teachers. Specifically, the study sought to answer the question, *How can a curriculum for mathematics content courses for pre-service elementary teachers support instructors in creating mathematically powerful experiences for prospective teachers?* I conducted a case study of two instructors using curriculum materials from the Geometric Measurement Unit of the Elementary Mathematics Pre-Service Teachers Project (Chapin, 2015). These materials were used with undergraduate students enrolled in mathematics content courses for pre-service elementary teachers. This chapter describes the curriculum and introduces the two participating instructors. I then explain the four themes that were found through this analysis and the ways in which the instructors reported that the curriculum materials supported them in planning and implementing instruction.

Section I: The Case Study Participants and the Curricula

This section provides background for this study. First I describe the two case study instructors and the institutions within which they worked. Then I describe the EMP curriculum studied, as well as the two alternative curriculum also used by the case study instructors.

Case Study Participants

Two instructors volunteered to participate in this study. Dr. H. was a professor at a public university. Dr. C. was a professor at a small liberal arts college. The following sections provide detail about the instructors' backgrounds and their reported typical instructional practice.

Participant 1: Dr. H, Public University

Dr. H. taught mathematics content courses for pre-service elementary teachers at a mid-sized state university in the Northeastern United States. The course had traditionally been taught by instructors in the mathematics department. Dr. H. had taught many of the mathematics-related courses in the school of education, but this was her first time teaching the content course for the mathematics department. The majority of students had taken a related course on number, operations, and algebra the previous semester. This course had been taught by an instructor in the mathematics department. Nearly all of the students in the course were education majors. The course included freshmen, sophomores, juniors, and seniors. This was the first time Dr. H. had used the EMP materials. She had taught five of the lessons in the geometric measurement unit before I observed her class. Dr. H. also used the textbook *A Problem Solving Approach to Mathematics for Elementary Teachers* by Billstein, Libeskind, and Lott for content not addressed by the EMP materials. She referred to this textbook as "Billstein." I observed her teaching one class session using the Billstein text, after the EMP unit had been taught.

Dr. H. earned her PhD in mathematics education. She completed all but her dissertation for a PhD in mathematics prior to her studies in mathematics education. She recounted the moment when she realized the depth of elementary mathematics in a seminar where a professor asked her class to think about drawings to represent $\frac{3}{4} \div \frac{1}{2}$. Her dissertation had focused on the mathematics in children's literature. Prior to teaching at this university, Dr. H. had taught middle school, served as a content specialist for a public school district, and taught at a nearby university.

Dr. H.'s primary goal for the course was to help students conceptually understand enough mathematics to pass the state teacher certification test. While she did not "teach to the test," she used the objectives of the assessment as a guide. Secondarily, she wanted her students to be less afraid of mathematics. She wanted them to believe in their own ability to reason through mathematical situations.

According to Dr. H., her typical class session differed depending on the materials she used. When using the EMP materials, she attempted to use them as designed, following the Instructor's Guide and the videos of enactment closely. She launched the lesson by reviewing material from the first class session. During this time, she asked general questions of her students and followed up with specific questions based on their comments. Students then worked in small groups on the problems in the lesson. She would bring the class back together for the whole class discussion questions. She used teacher discourse moves to facilitate these discussions. In contrast, when Dr. H. used the Billstein materials, she would present information. Students would use guided note handouts during this time. They would then work on problems from the textbook in small groups. Students would present their solution methods to the class. Dr. H. would then try to “generalize” or summarize key ideas from the problems.

I observed Dr. H. in three class sessions using the EMP materials, followed by one class session using the Billstein textbook. In my observations, Dr. H.’s reported typical class session followed these formats, with two exceptions. In the class session I observed in which she used the Billstein materials, she did not use direct instruction to present new information to the whole class. Instead, she answered questions regarding an assessment that had been passed back, then directed students to work in their groups on guided notes and problems from the Billstein materials. She also did not summarize at the end of the class session. These differences may have been due to perceived time constraints, as this lesson was one during the last few weeks of the semester.

Dr. H. believed that the configuration of the room may have impacted her students’ willingness to engage in small group and whole class discussion. Her class had been assigned to work in a small room with desks attached to chairs. For the three EMP lessons I observed, she swapped rooms with another instructor so that students could sit with their small group at tables. As a result, the small groups in the EMP lessons typically included the same students from lesson to lesson. In our initial interview, Dr. H. felt that students seemed reluctant to talk in small groups or in whole class discussion. She found that the lessons took less time than the Instructor’s Guides recommended because students said little. However, in my observations, students talked naturally in their small groups. Though there were several students who were more likely to talk in the whole class discussion than others, Dr. H. used teacher discourse moves to invite more voices into the conversation. During the lesson using the Billstein materials, which occurred in the smaller classroom, students also quickly set to work in their small groups. They asked questions of each other's presentations. Students seemed to actively participate in small group and whole class discussion in all four of the lessons I observed, regardless of the classroom or curriculum materials. There are two potential reasons for the difference between what I observed and what Dr. H. initially reported. First, the change of classroom may have helped students more actively engage in discourse. Once these norms were established, the materials or setting had less of an impact. Second, students may have simply needed time to adjust to a different pedagogy. Five class sessions with the EMP materials helped them become comfortable in this type learning environment. Regardless, students actively participated in all four class sessions observed.

Participant 2: Dr. C., Small Liberal Arts College

Dr. C taught mathematics content courses for pre-service elementary teachers at a small, private college in the Northeastern United States. The course observed was the last in a three-course sequence required by the college. The content of the course focused on geometry, measurement, data, and analysis. Dr. C had been teaching this sequence of courses for six years and reported knowing the students who took this course well. In addition to teaching these particular students in the other mathematics-for-elementary-teachers courses, she had taught some of them in other mathematics courses, such as statistics. She reported having strong relationships with her students. Dr. C had been a field tester for the EMP materials and had taught the EMP units on fractions, number theory, and geometric measurement for three or more years. Therefore, she was familiar with the lessons, though the lessons had been revised and altered during this time period. Dr. C. also used the textbook, *Mathematics for Elementary Teachers with Activities* by Beckmann, for content not addressed by the EMP materials.

Dr. C earned her bachelor's degree in elementary education and literature. Much of her elementary teaching career focused on teaching sixth grade science, so she pursued a master's degree in science education. She then continued her studies to complete a doctorate of education in curriculum and instruction with a focus on mathematics and science education. Her dissertation examined the use of remote coaching and videos of science teachers' instructional practice in teachers' professional development. Dr. C's first and only full-time academic appointment was at this college. Prior to teaching at this college, Dr. C. had taught upper elementary students and held a position in an educational consulting firm where she was in charge of teacher professional development.

Dr. C.'s goals for the course were two-fold. First, because the course also counted toward the college's general education quantitative literacy requirement, she wanted reasoning about numbers in context, rather than applying formulas, to be the emphasis. Second, she wanted her students to develop a strong understanding of the mathematics they would teach from an advanced perspective to fulfill state certification requirements. In addition to her content goals, Dr. C. also hoped that she could help students become "less anxious about math and teaching math" by the end of the three-course sequence.

Dr. C. described her typical class session as using four formats. First, she collected homework, reviewed difficult homework questions, and administered quizzes. While one quiz was administered in the four days observed, the other three class sessions began immediately with what Dr. C. referred to as "their real content work," the main part of the lesson. Dr. C. reported that this part of her lesson typically began with her identifying the topic or goal and soliciting ideas from the students as a whole class. She would then direct them to work on packets of problems that the students had been given at the beginning of the semester, or to pages in the Beckmann Activity Manual. After working on these problems, they would summarize the main ideas in a whole class discussion.

Dr. C. reported that students worked in small, self-selected groups when working on the problems. She indicated that her stronger students had distributed themselves evenly throughout the classroom. She expressed frustration at the configuration of the room, long, heavy tables that took up most of the space and prompted students to work in pairs rather than in groups of three or four as she would have preferred. She further reported that she circulated among the groups to listen to their thinking, but she would refrain from "providing any feedback" until the final part of the lesson where they debriefed as a whole class, typically one or two problems at a time. During this final phase which she referred to as the whole class discussion, she would transcribe student ideas on the board and synthesize ideas. She mentioned that she had been attempting to push students to write down their ideas verbally before sharing out as a whole class so that more of the students' "voice" was expressed in the whole class discussion. I observed that students would sometimes raise their hands but more frequently would volunteer statements without raising their hands. In the class sessions I observed, Dr. C. did not call on students who did not raise their hands. When I asked at a later interview whether she ever had students present at the board, she said she did not, though she did ask three students to present their strategies at a subsequent class session. It was more common for students to describe their thinking verbally from their seats.

The components of Dr. C's "typical class session" were evident in the observed class sessions. There were two ways in which her practice during the observed lessons deviated from her described practice. First, as Dr. C. visited small groups, she did provide feedback. Sometimes, this would take the form of listening to a student's idea and asking clarifying questions or agreeing with or questioning a conclusion. Other times, she would direct a student's attention to a particular feature of a problem, such as the length of the base of a quadrilateral, and prompt them to talk about it. Second, because the groups were fluid rather than assigned, there were a number of instances where students worked individually, rather than in pairs or small groups. Of the episodes observed where Dr. C. used the EMP materials, 29% involved her talking with an individual and 38% involved her talking with a small group. In general, Dr. C.'s description of her typical class session aligned with the class sessions observed in this study.

Elementary Pre-Service Teachers Mathematics Project (EMP) Curriculum

The Elementary Pre-Service Teachers Mathematics Project (EMP) is a NSF-funded research project (Phase I, 2009–2011, DUE 0837349; Phase II, 2013–2015, TUES 1323156; Phase III, 2016–2021, DUE 1625784) located at Boston University. It was designed to improve the mathematics preparation of future elementary teachers through the development of curricula and corresponding educative instructor support materials.

By the end of Phase II, three instructional units had been developed and tested nationally. These units proved effective in increasing pre-service teachers' mathematical knowledge for teaching (Chapin et al., in review). This section briefly describes the materials and explains the design principles that were used in writing them. It then identifies some differences between the EMP materials and the alternative curriculum that the case study instructors used in addition to EMP. Broader access to the materials can be requested by going to the website at www.elementarymathproject.com.

The EMP units include both student and instructor materials. Each unit includes six to ten individual lesson handouts for students with corresponding instructor guides. The EMP lessons are designed so that users engage in problem solving and discussion during class time. The materials do not support lecturing or presentation of content to students. Each lesson focuses on a key mathematical idea, such as surface area or volume. The authors identified "high-leverage content," the particular topics and practices that are vital for beginning teachers to understand and be able to teach (TeachingWorks, 2015). Undergraduate students engage in recurring cycles of collaborative problem solving, small group and whole class discussions, and presentations that deepen their conceptual understanding of the mathematical ideas they will teach. Discussion questions support the development of pre-service teachers' understanding of mathematical practices such as generalization and justification.

Materials specifically for the instructor are on a web-based platform. For each lesson, these include an instructor guide, a Power Point presentation, homework files, and answer keys. The instructor guides provide information on each lesson, such as the learning goals, the problems or tasks, the mathematical representations, pacing, strategies for teaching the content, information about pre-service teachers as learners, connections to the K–5 classroom, and suggestions for facilitating the whole class discussion. In addition, supplemental documents provide in depth suggestions for facilitating discussion generally, establishing classroom norms, working with groups of students, and understanding the mathematics. An unusual feature of the curriculum is that every lesson has accompanying video clips for instructors. These video clips show the lessons being enacted as designed. Examples of the lessons and instructor manuals can be found in Appendix G and Appendix H.

The lessons were developed using a sociocultural framework (Lave & Wenger, 1991). Sociocultural theory portrays learners as taking increasingly larger participatory roles in a community of practice. The EMP lessons were designed to engage pre-service teachers in two communities: a community of mathematical learners and a community of beginning teachers. As members of a community of mathematical learners, pre-service teachers take on increasingly greater responsibility for explaining and justifying mathematical ideas. As members of a community of beginning teachers, pre-service teachers learn to orient themselves to the thinking of others and consider the mathematical ideas relevant to the work of teaching.

Two design principles underlie the EMP lessons. The first principle emphasizes the structure of the presentation and enactment of the lessons by asking pre-service teachers to engage in cycles of collaborative problem solving and whole class discussion. In small groups, pre-service teachers collaborate on a set of one to four problems designed to draw their attention to one to two important mathematical idea. Each set of problems is followed by one to two whole class discussion questions that refine, extend, or generalize the idea raised in the preceding problems. For example, Figure 5.1 shows the one cycle in the Area Concepts lesson. Problems 1 and 2 are designed to have students explore different methods for finding area: tiling with square units, surrounding with a rectangle, and decomposing into rectangles and triangles. This concept supports their understanding of area as additive and subtractive, a fourth grade Common Core Standard, and these strategies are used in later lessons within the unit. Problems 3 and 4 spur discussions about the definition of area as the amount of space within a two-dimensional shape and the role of square units in measuring area. Following these problems, there are questions designed to be used in whole class discussion. The different strategies and topics addressed in the earlier problems provide fodder for this discussion. Each lesson involves one or two of these cycles.

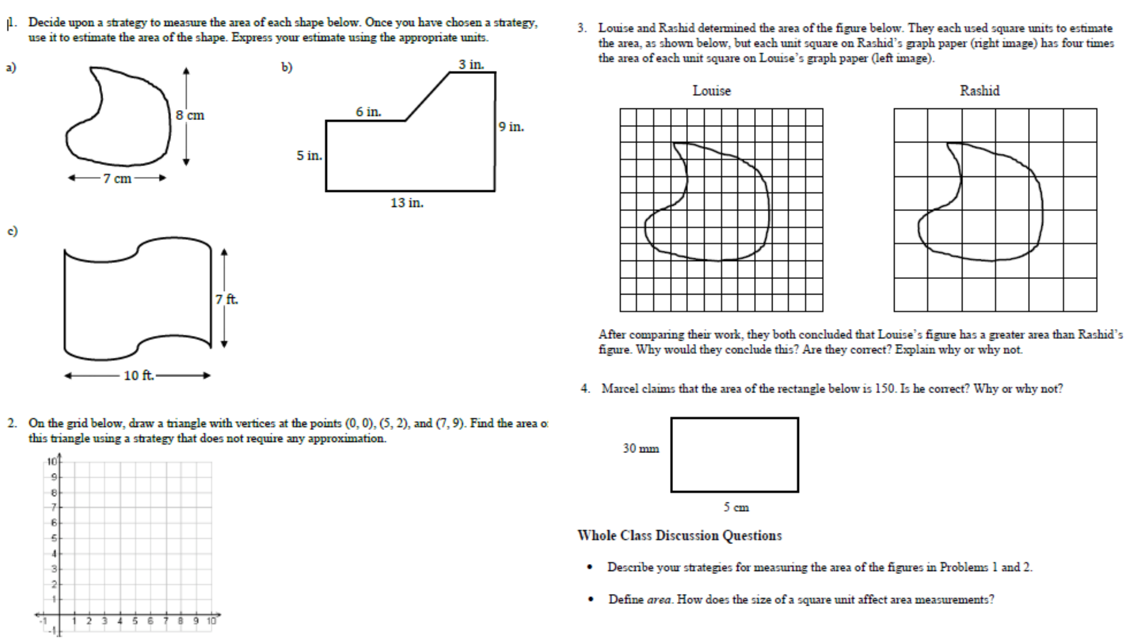


Figure 5.1. The first cycle in the EMP Area Concepts lesson

These cycles are related to Simon's (1994) learning cycles. In his description of learning cycles, Simon explained how mathematical problem situations spur discussions among pre-service teachers about mathematical concepts. Once these concepts are identified, pre-service teachers then apply these concepts to new situations, which lead to additional identification of mathematical concepts. In the cycle in Figure 5.1, one concept pre-service teachers are likely to identify is the role of square units in quantifying the area of a figure. This concept is further used in later problems within the lesson, which leads to additional refinement of ideas.

The second design principle is that problems and discussion questions focus on deepening PSTs' understanding of content knowledge for teaching mathematics, which consists of both common and specialized content knowledge. For example, pre-service teachers may know how to compute the area of common shapes, and they may recall the convention of writing "$units^2$" when answering problems in mathematics classes. However, many may not have had the opportunity to think about concepts regarding area. For example, previous research has suggested that many pre-service teachers have not had experiences that would give meaning to the notion of "square units," the idea that a square is iterated to tile or cover a space (Browning et al., 2014; Ghosh Hajra, McNeal, & Bowers, 2016). The problems within EMP lessons are designed to focus pre-service teachers' attention on these deeper, foundational ideas.

The EMP curriculum authors (of which the researcher is one) describe four problem types within the EMP lessons: (1) problems that connect mathematical concepts and ideas; (2) problems that highlight why a formula, rule, or algorithm works; (3)

problems that prompt pre-service teachers to make sense of mathematical structure; and (4) problems that prompt pre-service teachers to identifying the mathematics in elementary students' thinking. In Figure 5.1, Problem 3 is an example of the first problem type. It prompts pre-service teachers to recognize the difference between the area of a figure as a quantity and the numerical value assigned to the area. Figure 5.2 is an example of the second problem type. Pre-service teachers make rectangles and parallelograms from two identical copies of triangles to justify why the area formula for a triangle is $A = \frac{1}{2}bh$. One problem that focused on mathematical structure has pre-service teachers identify the number of faces, edges, and vertices on a prism. They write a formula for finding the number of such features given a prism with a *n*-sided base. Through this discussion, they are able to see individual features of prisms, but also how those features connect. For instance, they explain how each vertex on one base is connected to a vertex on the opposite base by one edge. Problem 4 in Figure 5.1 about Marcel's misconception is an example of the fourth problem type. This problem orients pre-service teachers toward understanding how the mathematical concepts they are studying would be used in the work of teaching elementary school children. These problem types are not necessarily disjoint; one problem may have characteristics of two or more problem types.

2. Cut out 2 copies of each of the following triangles from the end of this packet:
    - right triangle
    - isosceles triangle
    - scalene triangle
    - equilateral triangle

    Your goal is to explain to others why the formula for the area of a triangle ($A = ½bh$) makes sense. Your reasoning must work for any type of triangle. Consider using some of the strategies for determining area that you learned in prior lessons.

Figure 5.2. EMP Problem helping pre-service teachers explain why formulae work

Both participants in the case study also used another textbook to teach content not addressed by the EMP materials. Dr. H. used Billstein, Libeskind, and Lott's (2016) *A Problem Solving Approach to Mathematics for Elementary School Teachers*. Described in more detail in Chapter 2, this textbook has been referred to by the publisher as "the number one book on the market for the traditional approach" (personal correspondence, Greenhut, 2015). The relatively large market share of this textbook is verified by other researchers and the survey used in this study (Greenberg & Walsh, 2008). A typical section of a chapter includes exposition about a mathematical idea with worked-out examples, followed immediately by one to two practice problems that are similar in structure to the example. At the end of each section are problems about the topic. Dr. H. indicated that she used the text primarily as a source of problems for homework and classwork and as a reference for students.

Dr. C. used Beckmann's (2014) *Mathematics for Elementary Teachers with Activities*, which the publisher referred to as "the number one book on the market for the inquiry approach" (personal correspondence from the publisher, Greenhut, 2015). The relatively large market share of this textbook is verified by other researchers and the survey used in this study (Greenberg & Walsh, 2008). The textbook contains two sections, separated by a divider: (1) a section with explication of mathematical ideas and collections of problems and (2) a section with "activities" that highlight particular mathematical ideas. Some of these "activities" involve manipulative materials while others are mathematical problems. As an example of the former, one activity prompts students to build geometric solids with toothpicks and clay. In contrast, another activity prompts students to identify the three-dimensional solid formed by pictures of two-dimensional nets. Dr. C. used the activities during class time and assigned some problems from the first section for homework. The students also used the text as a reference during class time.

There are a number of differences between the EMP curriculum and these two alternative texts. Two of the differences concern the physical materials: (1) the nature of the materials for students and (2) the number of the topics addressed. First, the EMP student materials or handouts consist of 10 or fewer problems and questions for them to complete and discuss, whereas both of the alternative texts consist of explanatory text including worked out examples of solution methods. At the end of each section in the alternative texts, there are problems that could be used for homework. In the Billstein textbook there are 436 of these problems in the chapter on geometric measurement. In the Beckmann textbook, there are 201 of these problems in the two chapters on geometric measurement. In EMP, there are 71 homework problems. Second, the number of topics addressed are quite different. EMP focuses on “high-leverage content.” Therefore, there are topics addressed by the alternative texts that are not addressed in EMP. Some examples of these topics are temperature, unit conversion, the surface area of a sphere, and the Pythagorean theorem.

In addition, there were two major differences that influenced the participants’ use of the curriculum materials: (1) the perceived cohesion of a problem set and (2) the prominence of instructor support materials. First, the case study participants seemed to use the alternative texts as a collection of problems or activities from which they selected a few for use in class or for homework. In fact, the Beckmann Instructor Resource Manual supports this view (Beckmann, 2014a). This orientation is different from the way the participants viewed the EMP lessons, which were seen as cohesive wholes, though they did skip individual problems when pressed for time. Second, the prominence of the instructor support materials was different. At the lesson, unit, and overall program level, the EMP instructor support documents were the first documents listed on the webpage, as shown in Appendix F. In contrast, instructor support materials for the alternative texts were not sent by the publisher with exam copies of the textbook. While videos of enactment and tips for instructors do exist for the Beckmann textbook, they have to be accessed separately, either online or by requesting a DVD. The design of the instructor support materials for the alternative texts made them challenging to use as well. Dr. C. explained that she had stopped using the answer guide for the Beckmann activities because it contained written descriptions, rather than diagrams, and therefore it made her work less efficient. Indeed, both case study participants used the EMP instructor support materials in their planning but did not report using instructor support materials for the alternative texts. The case study participants did not explicitly indicate that the form of the curriculum materials led them to different instructional actions. However, recognizing these differences can add insight to the ways in which the participants indicated that the curriculum supported them in creating mathematically powerful experiences for their pre-service teachers.

Treatment of Topics within the Three Curricula

The two case study participants chose to use the EMP materials to teach a number of topics in geometric measurement, while they used their alternative text to address other topics. In this section, I describe how EMP developed the topics taught in the three observed lessons in each of the case studies. I then describe how the alternative text developed those same topics for comparison.

In the first case study, Dr. H. used the EMP materials to teach several topics within geometric measurement. In this case study, I observed her using three EMP lessons: Prisms, Surface Area I, and Surface Area II. This set of lessons was designed to take three class sessions. The Prism lesson consisted of three learning cycles. In the first learning cycle, students discussed different vocabulary related to prisms while referring to three-dimensional blocks on their tables: base, edge, face, lateral face, prism, and vertex. They then discussed a problem where they evaluated hypothetical elementary student statements about prisms. These statements were designed to help pre-service teachers use language more precisely and distinguish between prisms, pyramids, and cylinders. The whole class discussion question asked pre-service teachers to define a prism. In the second cycle, pre-service teachers investigated the number of faces, edges, and vertices of different prisms. They were asked to write a formula to determine the number of each feature, given the number of sides of the base of the prism. In the whole class discussion question, they justified these formulae. This cycle was intended to help pre-service teachers make sense of the structure of prisms by drawing their attention to the relationships between sides of the base and the number of edges, faces and vertices.

In the third cycle, pre-service teachers investigated the nets of cubes by folding and unfolding examples and non-examples. They identified which faces in a net would be on opposite sides on a cube. During the whole class discussion question, they articulated the features of nets that allow or don't allow them to be folded into cubes. This learning cycle supported pre-service teachers in visualizing two-dimensional representations of solids, a skill further developed in the subsequent lessons.

The objective of the two lessons on surface area was to help students make sense of a surface area formula that would work for all right prisms and right cylinders. The Surface Area I lesson consisted of one learning cycle. It began by prompting pre-service teachers to make sense of one formula for the surface area of a rectangular prism ($SA = 2lw + 2wh + 2lh$) by asking them to connect each term in the formula to a picture of a rectangular prism. Next, students were asked to explain whether or not this formula could be applied to other prisms or to cylinders. The first problem in this lesson connected to a formula pre-service teachers learned in their secondary schooling but then examined the fact that it could not be used to find the surface area for other solids, motivating the need for a more general formula.

The subsequent problems in the lesson supported pre-service teachers in thinking about surface area more broadly. Pre-service teachers built two triangular prisms, two rectangular prisms, and two cylinders with open bases from 8 ½ by 11 inch sheets of paper. They drew and labeled the nets of these six solids and then found the surface area of each as if they had bases. The whole class discussion question prompted the pre-service teachers to describe how these solids were similar and different, identifying how the lateral surface of all of these prisms and cylinders formed a rectangle.

The Surface Area II lesson continued the Surface Area I lesson to develop a formula that could be used to find the surface area of all right prisms and cylinders. It consisted of two learning cycles. In the first cycle, students drew and labeled the nets of three different solids, a triangular prism, a rectangular prism, and a cylinder. They articulated the connections between the dimensions of the nets and the dimensions of the solids. They computed the surface area for the three figures and generated a formula for the surface area of any right prism or cylinder, $SA = ph + 2B$, where B is the area of the base, p is the perimeter of the base, and h is the height of the prism or cylinder. They justified the formula in the whole class discussion question. In the second cycle, the pre-service teachers applied the formula by finding the lateral surface area and the total surface area of a rectangular prism in three different ways, depending upon which faces were designated as the bases.

Dr. H. used *A Problem Solving Approach to Mathematics for Elementary School Teachers* by Billstein, Libeskind, and Lott in addition to EMP. She did not use the text for teaching surface area; however, examining Billstein et al.'s development of surface area concepts can inform interpretations of how the EMP curriculum impacted her instruction. The Billstein textbook had one six-page section on surface area. It demonstrated first how to find the surface area of a rectangular prism with an example problem. Then, it demonstrated how to find the surface area of a cube. It explained the reasoning behind the formula $SA = 6s^2$, where s is the side length of one face of the cube. The text then presented the more general formula $SA = ph + 2B$. The text did not directly explain why this formula is true, but it provided an example of a general pentagonal prism and its net from which one could infer the reasoning behind the formula. This paragraph and diagram was followed by two example problems, one of a triangular prism and one of a rectangular prism. The nets of these latter two prisms were not provided.

The Billstein textbook then spent less than one page on a cylinder. It introduced a cylinder as the result of increasing the number of sides on a base of a prism infinitely. A general explanation of the surface area formula of a cylinder, $SA = 2\pi r^2 + 2\pi rh,$ was provided with a diagram. The diagram showed how the lateral surface of a cylinder can be cut to form a rectangle with the two circular bases attached.

The textbook then spent one and a half pages on the surface area of right regular pyramids. The example problems were limited to square-based pyramids. This section began by explaining that the lateral faces of a right regular pyramid are congruent isosceles triangles. The text explained the formula $SA = \frac{1}{2}pl + B,$ where p is the perimeter of the base, l is the slant height, and B is the area of the base. Slant height was defined but the height of the pyramid was not mentioned in the exposition, in the general diagram of a square based pyramid, or in the first example problem. The second example problem was contextual and the heights of the two pyramids was mentioned, but these quantities were not used in the solution. There was no comment in the solution on the difference between the height of a pyramid and the slant height.

The next section addressed the surface area of a cone. It introduced a cone as the result of increasing the number of sides of the base of a regular pyramid an infinite number of times. A diagram demonstrated this process. The text used this connection to explain the formula for the surface area of a cone. Slant height was defined, but it was not contrasted with the height of the cone. There was one example problem. This problem provided the height, but the number was not used in the solution. No commentary was made on this fact.

The section on cones was followed by one paragraph on the surface area of a sphere. It defined a great circle of a sphere and provided a diagram. The surface area formula for a sphere was provided. There was no explanation for this formula, except to say that it was typically justified with calculus.

The exposition and example problems were followed by 67 problems that could have been used for homework. Many of these problems had the answers in the back of the book. These problems primarily involved multi-step applications of formula to specific contexts. There were some variations. For example, one problem directed students to find the surface area of a frustum instead of a cone. Five problems involved visualization rather than calculation, including one question from the National Assessment of Educational Progress. Thirteen problems seemed to motivate general ideas, such as the impact of doubling one dimension on the surface area of different solids. Five of these composed a section called “Connecting Mathematics to the Classroom.” These problems featured hypothetical elementary students making various claims. In general, however, the problems about general ideas or requiring visualization were not separated from the application problems. The problems were typically independent of one another. Only one problem referred to an idea used in an earlier problem.

The pedagogy suggested by this textbook is direct instruction that includes conceptual explanations for some formula and examples (c.f. Spielman & Lloyd, 2004). It appeared that the surface area of the four different solids (prisms, pyramids, cones, and spheres) were meant to be taught within one class session, since the content was located within one six-page section of the textbook. Given that the problems were primarily application problems and were not organized in a particular way, the intention seemed to be that the instructor would select a subset of these problems for homework or classwork. Indeed, Dr. H.'s description of her typical class session when using this textbook aligned with this interpretation.

In the second case study, Dr. C. used the EMP materials to teach several topics within geometric measurement. In this case study, I observed her using three EMP lessons: Area Concepts, Parallelograms & Triangles, and Trapezoids. This set of lessons was designed to take three class sessions. The Area Concepts lesson consisted of two learning cycles. The first learning cycle is displayed in Figure 5.1 and discussed on page 212. The full lesson can be found in Appendix G. In this cycle, the first problem directed students to find the area of irregular shapes. The second problem directed them to find the area of an obtuse triangle on a coordinate grid. Students typically used one of two strategies (1) decomposing a figure into familiar shapes and adding the areas of these smaller shapes or (2) surrounding the figure with a rectangle, determining the area of the rectangle, and subtracting the area outside of the given figure. These problems were designed to help pre-service attend to the fact that area is additive and subtractive. Then, in two problems, students were presented with errors made by elementary students. Both were designed to focus on the importance of the size of the units used to record the area of figures. The whole class discussion questions asked pre-service teachers to describe their methods for finding the area of the figures in the first two problems, to define area, and to discuss the nature of square units. Dr. C. decided not to enact the second learning cycle in the Area Concepts lesson because of time. The second learning cycle consisted of one problem. The problem provided three identical rectangles on grid paper and prompted students to tile each rectangle in one of three ways: with 1 by 1 square tiles, 1 by 4 rectangular tiles, and 3 by 2 rectangular tiles. This problem was designed to help pre-service teachers to (1) see a rectangle as an array of units and (2) connect the linear dimensions of a rectangle with the number of tiles that align the edges of the rectangle. The whole class discussion question asked students to justify the area formula for a rectangle.

According to the EMP Instructor Guide for the lesson, the objective of the Parallelogram & Triangles lesson was to support pre-service teachers in justifying the area formulas for parallelograms and triangles. There were two learning cycles in this lesson, the first one focused on parallelograms and the second focused on triangles. The problem in the first learning cycle prompted pre-service teachers to draw five different parallelograms with a base of four units and a height of three units on grid paper. Then, the problem directed students to use these parallelograms to justify the area formula of these parallelograms. The whole class discussion question prompted pre-service teachers to generalize their argument for any parallelogram. Two methods were described in the

Instructor's Guide: (1) decomposing the parallelogram into triangles and rectangles and recomposing these shapes to create a rectangle, and (2) surrounding the parallelogram with a rectangle and subtracting the exterior area between the parallelogram and the rectangle. The Instructor Guide stressed that if students choose to make sense of method 1, instructors should push for students to justify how they know the figure they form is a rectangle, without any gaps or overlaps, drawing on the properties of parallelograms. In the second learning cycle, students were given two identical copies of right triangles, isosceles triangles, scalene triangles, and equilateral triangles. They used these triangles to make sense of and justify the area formula of the triangles. The whole class discussion question directed students to generalize their argument for all triangles. In the Instructor's Guide, an example of forming a parallelogram out of two triangles was provided, but the authors also indicated that other methods often are developed by students.

The objective of the Trapezoid lesson was for students to derive a formula for the area of a trapezoid. Unlike the Parallelograms & Triangles lesson, the formula was not provided to the students. The lesson is composed of one learning cycle. The first problem provided students with three identical pairs of trapezoids: (1) a pair of right trapezoids, (2) a pair of isosceles trapezoids, and (3) a pair of scalene trapezoids. The problem directed students to use these trapezoids to derive a formula for the area of a trapezoid. The second problem prompted students to use additional methods to find the area of a trapezoid with specific measurements. The problem then asked the students to label this trapezoid with the variables b_1, b_2, and h to derive a general formula. The whole class discussion question asked participants to describe some ways to find the area of a trapezoid that had its dimensions labeled with variables.

Dr. C. used *Mathematics for Elementary Teachers with Activities* by Beckmann in addition to EMP. She did not use this text for teaching area concepts or area formulae. However, examining Beckmann's development of these concepts can provide context for how the EMP materials may have contributed to her ability to create mathematically powerful experiences for her pre-service teachers. The Beckmann textbook was divided into two separate sections, exposition and activities. Within the exposition were prompts that directed the student to activities relevant to that topic in the separate section of the book. In most cases, the exposition after a class activity prompt provided an explanation of the content in the class activity. Most of these explanations were directly parallel to the problems in the activities, even using identical figures. Like the Billstein textbook, the Beckmann exposition contained worked examples, which were typically closely aligned with the preceding activity.

The chapter on the area of shapes consisted of sections that addressed the following topics: (1) area of rectangles, (2) moving and additivity principles of area, (3) area of triangles, (4) areas of parallelograms and trapezoids, (5) shearing and Cavalieri's principle, (6) area of circles, (7) area of irregular non-polygonal shapes, (8) contrasting area and perimeter, and (9) the Pythagorean theorem. The first four topics are directly related to the content taught with the EMP materials in the observed lessons as well as the topic of area of irregular non-polygonal figures. Within these five sections of the chapter, there were 12 pages of exposition and 11 activities, four of which were designated as "central" (p. xiv) activities with an icon. The Beckmann (2014a) Instructor Resource

Manual indicates that there is probably not enough time to complete all of the activities, so instructors may use this icon to help them choose which activities to use.

The “central” activities included: (1) Using the Moving and Additivity Principle, (2) Explaining Why the Area Formula for Triangles is Valid, (3) Do Side Lengths Determine the Area of a Parallelogram, and (4) Determining the Area of an Irregular Shape. Students were prompted to do the first central activity at the end of the section explaining the moving and additivity principles. The first problem in the activity directed students to use four different strategies to find the area of a rectilinear figure: (1) a subdividing strategy, (2) a takeaway strategy, (3) a move and reattach strategy, and (4) a combine two copies and talk half strategy. The second problem asked students to find the area of a square inscribed in a larger square in several ways. The second central activity followed the exposition and activities about different methods for finding the area of triangles, identifying the base and height, and computing with the formula $A = \frac{1}{2}bh$. In the second central activity, students were asked to justify the area formula of a triangle using a right triangle and an acute triangle labeled with variables. The problems asked students to develop justifications that most naturally fit with different representations of the formula: $\frac{1}{2}(bh)$, $\left(\frac{1}{2}b\right)h$, and $b\left(\frac{1}{2}h\right)$. They were then asked to correct a faulty justification of the area formula for an obtuse triangle. The third central activity introduced the section on parallelograms. The first problem presented a rectangle and two other parallelograms with the same side lengths on a grid. It asked students whether the area of a parallelogram depended on the side lengths of a parallelogram. The fourth central activity introduced the section on approximating the area of irregular non-polygonal shapes. This activity asked students how they might use 1 by 1 inch plastic squares, graph paper, a postage scale, string, and modeling dough to find the area of an irregular shape.

There were similarities and differences between the conceptual development of these ideas in the EMP curriculum and in the Beckmann textbook. For example, both used the conservation of area to justify different area formulae. (EMP referred to "composing and recomposing" and Beckmann referred to "moving and additivity principles.") Both also used the duplicate-and-take-half strategy. The Beckmann textbook introduced triangles before parallelograms and justified both using rectangles. As a result, the constructions required more steps than those suggested in the EMP Instructor's Guide.

There were also similarities and differences in the suggested pedagogy. Like EMP, Beckmann suggested in the Instructor's Resource Manual (Beckmann, 2014a) and the preface of the textbook that the class activities be used with collaborative problem solving in small groups followed by whole class discussion. However, the text is more agnostic about whether students should discover ideas on their own or read about ideas first. Beckmann (2014a) indicated that she placed several of the prompts for activities before the exposition because she personally preferred to have students explore ideas first before assigning the related reading. However, she also indicated that it was acceptable to have students read about the concepts first and then engage in the activities. Some of the activity prompts came at the end of the exposition of a particular section as well.

In addition to explicit instructions in the instructor support materials, the student-facing materials suggested slightly different pedagogies. In the Beckmann textbook, the explanations in the exposition often paralleled the activities closely, as if revealing the correct or ideal answer. This structure was different from the EMP materials, where there were no student-facing answers, solutions, or explanations. In the EMP materials, it appeared that students were expected to generate the solution strategies and main ideas. In the Beckmann materials, it appeared that there were ideal solution strategies the author believed should be used. The two curricula were both designed to support collaborative problem solving and discussion about mathematical concepts, but the design of the materials implied slightly different pedagogical philosophies.

Section II: Major Themes

The purpose of this research project was to answer the question, *How can a curriculum for mathematics content courses for pre-service elementary teachers support instructors in creating mathematically powerful experiences?* Through cross-case synthesis of two case studies, I identified four themes. The EMP curriculum materials supported the participants to do the following:

1. Prompt pre-service teachers to examine and use mathematical relationships;
2. Hold pre-service teachers responsible for engaging in rigorous mathematical work;
3. Assess and make use of pre-service teachers' thinking;
4. Support pre-service teachers to use mathematical language.

In this section, I explain each of these themes in more detail and I present a classroom episode that illustrates the theme. I describe the curricular elements that the participants perceived supported them in regards to that theme. There often were multiple themes represented by a given episode. Likewise, the same curricular feature often supported a participant in multiple ways. Finally, I summarize how curricular features supported both participants.

Prompting Pre-service Teachers to Examine and Use Mathematical Relationships

Mathematics is a discipline rich in relationships. There are hierarchical relationships, such as those used to classify quadrilaterals or solids. There are relationships between symbols and the measure of attributes they represent, such as the terms in geometric formula and different features of a figure. There are relationships among different representations of the same mathematical object, such as solid figures, nets, and two-dimensional projections. There are also relationships among figures that might not often be classified together but involve decomposition. For example, a parallelogram can be decomposed into two identical triangles, and the lateral surfaces of both cylinders and prisms are formed by rectangles. Students' understanding of such relationships can support conceptual understanding (Hiebert & Carpenter, 1992). Attending to such relationships can help learners see mathematics as a coherent discipline, rather than a collection of isolated facts and procedures. This theme aligns with the first of the five dimensions of mathematically powerful classrooms, *mathematics*, which focuses on mathematical connections between concepts and procedures, on the portrayal of mathematics as a coherent discipline, and on mathematical practices, such as attending to structure.

In every episode in which the participants used EMP materials, the focus of the class was examining and using mathematical relationships. In this section, I provide some examples of how the pre-service teachers were prompted to examine and use mathematical relationships in the observed classes. After each example, I describe the features of the curriculum materials that supported the participants in focusing pre-service teachers' attention on such relationships.

During the teaching of lessons on surface area, Dr. H. and her students articulated the relationships among the nets of cylinders and prisms. In particular, Dr. H. wanted her students to see that (1) the lateral faces of prisms were rectangles of the same height; (2) when unfolded, these lateral faces formed a larger rectangle, which they called the lateral surface rectangle; (3) cylinders were also formed from lateral surface rectangles; (4) the length of these lateral surface rectangles was the perimeter or circumference of the solid's base; and (5) these features could be used to create a surface area formula that would work for all prisms and cylinders, according to interviews and class observation data. To accomplish these goals, Dr. H. had her students construct open-based prisms and cylinders with 8 ½ by 11 inch paper in the Surface Area I lesson. In the Surface Area II lesson, she directed her students to draw and explain their nets of a triangular prism, a rectangular prism, and a cylinder on the board. She asked the class how the three nets were similar and directed them to talk in small groups. As she circulated around the classroom, she asked students what similarities they saw among the nets. Students explained to her that the lateral surface of all four solids is a large rectangle composed of smaller rectangles. They explained how the dimensions of each individual rectangular face are connected to the dimension of the base. Then, in the whole class discussion, students discussed these ideas in detail. Dr. H. also demonstrated to the whole class how the perimeter or the circumference of the base was equivalent to the length of the lateral surface rectangle. She explained how the surface area formula for a cylinder could be understood as finding the sum of the area of this lateral surface rectangle and the area of the circular bases.

These mathematical relationships among the nets of prisms and cylinders and the connection to a general surface area formula for these solids were explicit in the Instructor's Guides for the lessons on surface area:

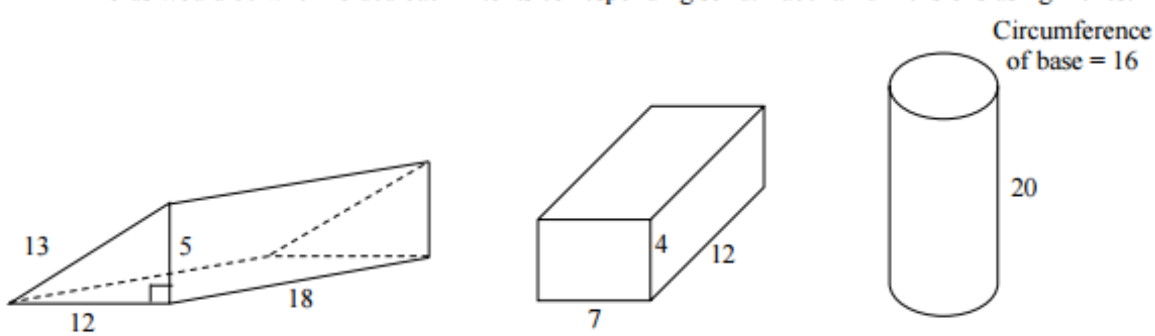
> The learning goal for this lesson focuses on developing participants' [pre-service teachers'] understanding of the relationship between the dimensions of a prism or cylinder and the measurements on its corresponding net…. Additionally, this work supports initial intuitions about a generalizable formula for the surface area of prisms and cylinders.
>
> Surface Area I Instructor's Guide (Chapin et al., 2017)

> Participants … deepen their understanding that the surface area of prisms and cylinders have two general components: the lateral surface and the two bases. By establishing that the lateral surfaces can be combined to form a lateral surface rectangle in the two dimensional net, participants

> will make the connection that the lateral surface component can be calculated by multiplying the perimeter of the base of the solid by the height of the solid.
>
> Surface Area II Instructor's Guide (Chapin et al., 2017)

The problems and questions in the EMP lessons supported pre-service teachers in identifying these relationships among the nets and the solids. Consider, for example, the problem that this discussion focused on, shown in Figure 5.3. While drawing nets is not an uncommon task in curriculum designed for mathematics content courses for elementary teachers, follow-up questions prompting pre-service teachers to explicitly articulate connections between the dimensions of the solid and the net appear to be less common. Certainly, they were not found in Dr. H.'s alternative text.

1. a) Sketch the net of each solid below (including the bases). Use dashed lines to show where the folds would be when folded back into its corresponding solid. Label all dimensions using inches.



b) How are the dimensions of each net related to the dimensions of the corresponding solid?

Figure 5.3. EMP problem prompting pre-service teachers to articulate the relationship between the dimensions of a solid and the dimensions of its net.

When asked how the curriculum supported her to prompt pre-service teachers to examine such mathematical relationships, Dr. H. cited the nature of the curriculum materials as using “discovery learning,” where questions were posed to students that required them to “discover” and then make sense of mathematical relationships. Indeed, Dr. H. saw the discovery nature of the materials as supporting her across many of the themes. Students made sense of ideas themselves, with members of their small group, or as a whole class, as opposed to listening to the instructor or reading a textbook. Comparisons between her use of the alternative text, Billstein, can further illuminate this idea of “discovery learning.” Specifically, Dr. H. indicated that she did not draw students’ attention to relationships among representations or solution strategies when using her alternative text because “Billstein doesn’t do that.” Unlike her alternative text, the questions themselves in EMP are focused on such relationships. For example, one whole class discussion question in EMP asks: *What is the relationship between the dimensions of a prism or cylinder with open bases and the dimensions of a rectangular piece of paper that can be folded to construct it?* This question appeared in the materials after students had built six different open-based solids from the same sized sheet of paper. Students investigated the relationship in small groups before considering this whole class discussion question. In contrast, the problems in Billstein were each independent from one another. The majority of problems involved applications of formula or definitions, rather than focusing on relationships. An example from the section on surface area in Billstein is reproduced below.

> A soup can has a $2\frac{5}{8}$ in. diameter and is 4 in. tall. What is the area of the paper that will be used to make the label for the can if the paper covers the entire lateral surface area?
>
> (Billstein, Libeskind, & Lott, 2016, p. 870)

The problems Dr. H. chose to use from the Billstein text focused on applying definitions or procedures. In contrast, the problems and questions in the EMP lesson focused on noticing and articulating relationships among mathematical ideas and representations, one of the explicit design principles articulated by the authors (Chapin et al., in review). This focus on identifying and making sense of relationships among mathematical objects, as opposed to applying ideas explicated in the text, supported Dr. H. in prompting pre-service teachers to examine and use mathematical relationships in her course.

In addition, the focus of the materials on a few fundamental ideas may have supported Dr. H. in prioritizing mathematical relationships as well. When asked whether she addressed similar relationships in her other courses, Dr. H. admitted that she typically did not. She indicated that she found herself under significant pressure with time constraints, the amount of material she was expected to cover, and the ability level of her students. When mathematical relationships were addressed in her other courses, she typically directly explained them to students. Therefore, the focus of the EMP materials on central ideas may have helped Dr. H. feel like she had the time to spend on attending to the relationships that the EMP material addressed. Dr. H. noted, in fact, that the EMP curriculum doesn't cover "everything." Other researchers have found that many of the curriculum materials for mathematics content courses for elementary teachers are "encyclopedic," covering a large volume of material (McCrory, 2006). The focus of the materials on a few fundamental ideas and the nature of the problems as centering on general relationships as opposed to applications of facts and definitions, an element of the "discovery" nature of the materials, supported Dr. H. in prioritizing the study of mathematical relationships with her pre-service teachers.

The EMP materials lend themselves to making sense of mathematical relationships both within particular lessons as well as across lessons. Dr. C. likewise focused on drawing her students' attention to mathematical relationships explicit in the learning goals of individual lessons. For example, the students justified the area formula for a triangle by explaining the relationship between a triangle and a parallelogram. In addition, she worked to help her pre-service teachers use relationships that had been studied in earlier lessons. For example, in her class on the area of a parallelogram, the first problem directed students to draw five different parallelograms with the same base and height on grid paper. These parallelograms would then later be decomposed as a step in the justification of the area formula for a parallelogram. Most of the students initially could not progress beyond drawing a rectangle. A few students were able to only draw one or two parallelograms. Seeing this stumbling block, Dr. C. brought the class together to review the features of a general parallelogram. She explicitly addressed how these features are similar to and different from the features of a rectangle. She then demonstrated a strategy for generating additional parallelograms with the same base and height by translating the top base to the right. This dynamic view of a rectangle was not explicitly part of the EMP materials, but proved to be successful in supporting students in generating additional parallelogram. Dr. C. stated,

> "I wanted to reinforce those ideas that a rectangle is a parallelogram. They're special cases of parallelograms because they have additional features that not all parallelograms have. So using that as a starting point because it's a clear way of seeing the base and the height and the constraining features that we were trying to have in place."

For Dr. C., the EMP materials supported her to continually address relationships that had been explored in earlier lessons in order to deepen her pre-service teachers' understanding and develop their problem solving abilities. In this particular episode, Dr. C. found that specific problem features supported her in prompting her pre-service teachers to continually examine these relationships. Specifically, the fact that the problem asked pre-service teachers to draw their own parallelograms, rather than provide parallelograms for them to decompose, required her students to think more carefully about the properties of parallelograms and their relationship to rectangles. Indeed, this was an explicit design choice by the authors, who recognized that pre-service teachers often have difficulty drawing more than one non-rectangular parallelogram:

> "In previous enactments, we have found that many participants are very unfamiliar with how to draw parallelograms. Many are able to draw a rectangle and one non-rectangular parallelogram with a base of 4 units and

> a height of 3 units, but they struggle to recognize that additional parallelograms are possible. In these cases, ask participants to consider the characteristics of parallelograms…."
>
> Parallelograms and Triangles Instructor Guide, Chapin et al., 2017

This episode is an example of a learning cycle at work. Simon (1994) explained that, in a learning cycle, students first explore mathematical situations and then identify a concept through discussion and negotiation of meaning. They then apply this concept to a new situation, which then leads to further development of a mathematical concept. This process is depicted in Figure 5.4. In Dr. C.'s course, prior to this class session, the pre-service teachers had explored situations involving quadrilaterals. They had identified the hierarchical relationships between quadrilaterals by discussing their properties. During the episode presented here, the pre-service teachers were prompted to apply their knowledge of the properties of quadrilaterals to construct five different parallelograms. This led them to further examine their own understanding of the relationship between rectangles and parallelograms, the meaning of the word parallel, and the difference between height and side length. While the EMP authors indicate that there are one to three learning cycles within a lesson (Chapin et al., in review), there are also learning cycles across a set of lessons. Dr. C. found the learning cycles that occur over multiple lessons supported her in prompting her pre-service teachers to continually examine important mathematical relationships. There was some evidence of cross-lesson learning cycles supporting Dr. H.'s work, as well. The lesson on Prisms, Surface Area I, and Surface Area II all engaged pre-service teachers in attending to features of solids, visualizing solids from nets, and drawing nets of solids or projections of solids. A continual theme of connecting features and dimensions of these different representations appeared in each of the three lessons observed, though it was not discussed to the same degree in the VSR interview.

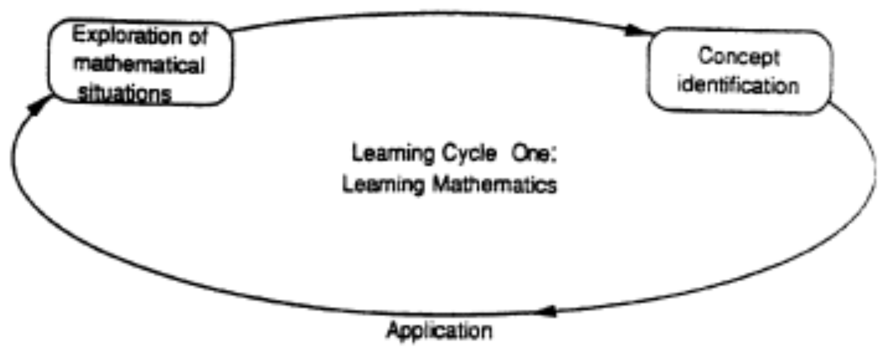


Figure 5.4. Simon's (1994) learning cycle

In both case studies, when the participants used EMP materials, the focus of their class was on examining mathematical relationships. They examined the relationship between geometric measurement formula and the features of shapes or solids. They articulated the relationships between different representations of mathematical objects. They identified and used mathematical relationships among different shapes and solids. The EMP materials supported the case study participants in prompting their pre-service teachers to examine these relationships. First, the content of the materials prioritized mathematical relationships, as can be seen in the Instructor's Guides, in the authors' articulation of their design principles, and in the problems and questions themselves. Whereas other curriculum materials may focus on the application of facts and ideas to specific contexts, the EMP problems are designed to highlight general mathematical relationships and they explicitly prompt students to articulate these relationships. The focus of the materials on a few fundamental ideas also seemed to help participants to prioritize the study of these relationships. In addition, the design principle of using learning cycles seemed to support the participants in revisiting key mathematical relationships to deepen their pre-service teachers' understanding and mathematical proficiency. However, while the curriculum authors indicate that each lesson contains one to three learning cycles, this study demonstrates that implicit learning cycles *across* lessons also supported the case study instructors in their work with pre-service teachers. These design features helped participants create mathematically powerful experiences for their students by focusing on making sense of important mathematical relationships. These relationships involved the structure of geometric figures, the connections between terms in measurement formula and components of geometric figures, and the properties and relationships between shapes. The study of these relationships portrayed a coherent view of mathematics as described in the TRUMath rubric (Schoenfeld, Floden, The Algebra Teaching Study, et al., 2014).

Holding Pre-service Teachers Responsible for Engaging in Rigorous Mathematical Work

Rigorous mathematical work is actively engaging in sense-making and mathematical practices, such as authentic problem solving, justification, and explanation. Rigorous mathematical work goes beyond applying known procedures to find answers to textbook problems (Gojak, 2013). Other researchers have referred to rigorous mathematical work as simply "doing mathematics" (Boston & Wolf, 2006; Henningsen & Stein, 1997; Smith & Stein, 2011; Stein & Lane, 1996). This theme is strongly related to the dimension of *cognitive demand* (Schoenfeld, Floden, The Algebra Teaching Study, et al., 2014) because it is focused on whether the students are engaged in rigorous and meaningful mathematical work. This theme is also related to the *agency, authority, and identity* dimension because students are held responsible for developing their own solution strategies and critiquing each other's thinking.

In both case studies, pre-service teachers engaged in rigorous mathematical work: explaining solutions, critiquing other's solutions, developing their own strategies, and constructing mathematical objects. Furthermore, there were instances where both participants refused to do the mathematical work for the students. In these instances, the participants did not clarify a definition, explain a mistake, or funnel students toward an efficient strategy. Instead, they insisted that students be the ones doing the mathematical work, rather than doing the mathematical work for the students. They were able to accomplish this challenge even while negotiating time constraints and the pressure to "cover" more material. Every observed lesson featured multiple instances of this theme when the participants used EMP materials. In this section, I provide a few examples of this theme from the observed class sessions for each participant. I then describe how each participant felt the curriculum materials supported them to hold pre-service teachers responsible for engaging in rigorous mathematical work.

In Dr. H.'s observed lessons, she frequently pushed her pre-service teachers to explain their thinking, created opportunities for them to comment on each other's mistakes, and invited additional volunteers to share their thoughts. Her belief that students must struggle with and make sense of mathematical concepts themselves motivated these instructional actions. "They have to be the center of the learning," she said. "I can't open their brains and pour it in."

One episode in particular demonstrates how she refrained from doing the mathematical work for students. During a lesson on surface area, the students drew nets on the board. They explained to the class how they determined the dimensions of different parts of the nets. One student explained her net of a cylinder. In regards to the base, her explanation was vague; it was not clear if she meant the length of the lateral face was equivalent to the diameter or the circumference of the base. Without indicating whether or not the student was correct, Dr. H. pressed her for further explanations, including asking her to draw the dimension in two different ways. These diagrams showed that the student was thinking about the diameter, not the circumference. Dr. H. then called on another student, who explained why the dimension would be the circumference, not the diameter. The first student then revised her thinking. Dr. H. then called for one other person to articulate the connection between the lateral surface area and the dimension of the circle. In this episode, Dr. H. made a series of instructional decisions. She had students draw their nets on the board. Rather than review the solutions herself or ask the students to compare their answers to those on the board, Dr. H. asked the students to present their thinking. She asked several follow up questions of the first student to gain clarity. Rather than correcting the student herself, Dr. H. called on other students to share their own thoughts. With each instructional action, she ensured that students were doing the mathematical thinking, rather than doing the thinking for them. This episode was also an example of the first theme: students examined the relationship between the different features of a cylinder and its net. In addition, Dr. H. held the students responsible for engaging in the mathematical work of sense-making, explanation, and critique themselves.

Dr. H. cited two elements of the curriculum materials that supported her in these instances where she held pre-service teachers responsible for engaging in rigorous mathematical work. First, she again cited the discovery nature of the materials. That is, the problems within the materials were designed to support pre-service teachers in making sense of mathematical ideas themselves. They were cognitively demanding problems, prompting pre-service teachers to connect concepts and procedures or engage in "doing mathematics." Her understanding of the nature of the materials was informed by her reported close reading of the Instructor Guides and watching the videos of enactment on the website. Second, the materials were designed to be used with classroom discourse. Dr. H. reported that this design element fit well with her existing practice of having students work in small groups and then present their solutions.

The researcher confirmed that Dr. H.'s typical instructional practice included small group problem solving and student presentations by observing a class session where students discussed problems from the alternative textbook. However, the content of the conversations during this class session differed substantially from those where she used EMP materials. When the instructor worked with a small group in which a pre-service teacher was struggling on an EMP problem, other group members contributed to helping the pre-service teacher who was struggling. Both Dr. H. and other students were mathematical resources. In contrast, when pre-service teachers struggled with problems from the Billstein textbook, the instructor acted as the primary mathematical resource. Furthermore, the apparent goals of the two conversations were different. The goal of the conversations about EMP problems was to identify and articulate mathematical relationships. Typically, the small groups discussed relationships between geometric solids and nets, relationships among nets of different solids, or the relationship between measurement formula and features of a solid. In contrast, the goal of the conversations about problems from the alternative textbook was to solve a particular problem. In these instances, Dr. H., as she said, “tells too much.” The transcripts from two episodes illustrating this difference are presented below. In this instance of using the EMP materials, she directly told a student an idea. However, she shared the responsibility for explaining the idea with another student, and the focus was on understanding a relationship rather than finding a numerical answer.

Small Group Discussing an EMP Problem with Dr. H.

In the following episode, a student drew a net of a cylinder with the circular bases at the vertices of the lateral surface rectangle, as shown in Figure 5.5, so it was not clear if she understood the relationship between the length of the lateral surface rectangle and the circumference of the base. Dr. H. addressed this ambiguity in her second comment. Both the instructor and another student helped the student make sense of the connection *between the dimensions of the three dimensional cylinder and its net.*

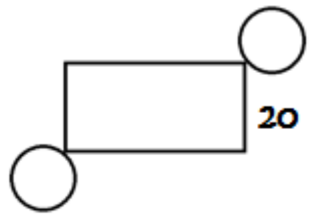


Figure 5.5. Student 2's drawing of a net of a cylinder

Dr. H.: Okay where are your dimensions?

Student 1: The dimensions are the same.

Student 2: Wait how are the dimensions the same?

Dr. H.: So you have a dimension of 20 here, [pointing to one of the edges of the rectangle] does

this circle go on this edge [points along the long edge] or it goes on this edge [points along the edge labeled 20]. I'm not sure. You have this on the comer. [Student 2 starts erasing.] You see this rectangle here you only have this dimension 20 [pointing to the side of the rectangle labeled 20.] What would this dimension be? [pointing to the perpendicular sides of the rectangle.]

Student 2: [Answers, but it is unclear. Student continues to erase.]

Dr. H.: Why?

Student 2: [indiscernible.]

Dr. H.: So what's this length around there? [Runs her finger along the edge of the circle in the picture of the cylinder.] [Instructor pauses, then points to where the paper says "Circumference = 16."]

Student 2: 16

Dr. H.: So where does that 16 come in the picture, where would you put 16?

Student 1: Can I help her?

Dr. H.: Give her a hint.

Student 1: [Pointing with his pencil on Student 2's paper.] So this is one side, so it's 20 and if you unravel it so this is all 16 [running his pencil along the circle in the picture of the cylinder] so if you unravel this and lay it flat [demonstrates with his hands] and that makes this side in the 20 so in this side it would be 16.

Dr. H.: [Gets blank paper.] This is the dimensions not quite right, but if this is 20, [folds the

paper into a tube] this is 20 [pointing her finger along side the height] which is 16 [running her finger along the circle at the top] then when I open it up one, what is this? [Points along the top edge of the paper.]

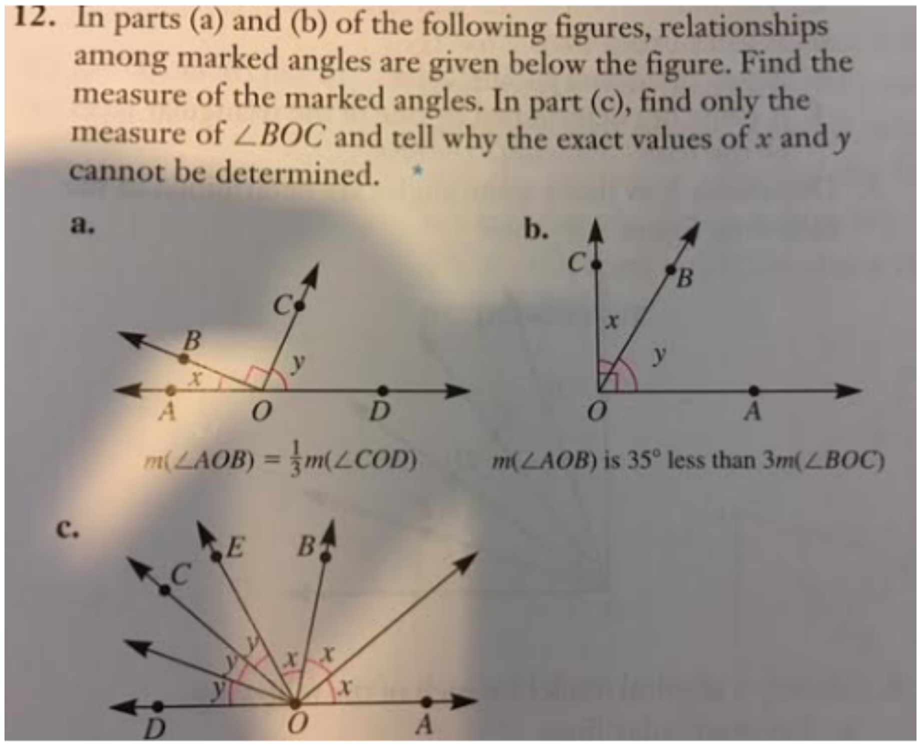
Student 2: 16.

Dr. H.: Yes. Okay?

Student 2: Yes.

Small Group Discussing a Problem from the Billstein Textbook with Dr. H.

In this episode, a small group was struggling with part (a) of the problem shown in Figure 5.6.



12. In parts (a) and (b) of the following figures, relationships among marked angles are given below the figure. Find the measure of the marked angles. In part (c), find only the measure of $\angle BOC$ and tell why the exact values of x and y cannot be determined.

a. $m(\angle AOB) = \frac{1}{3}m(\angle COD)$

b. $m(\angle AOB)$ is 35° less than $3m(\angle BOC)$

c.

Figure 5.6. A problem from the alternative textbook used in Dr. H.'s class. From Billstein et al., 2016, p. 638

Student 1: I don't understand how you get the sides.

Dr. H.: How do you get the what?

Student 1: How do you get the sides?

Dr. H.: Because they give you a relationship between them, don't they.

Student 1: Yeah.

Dr. H.: So what does it say.

Student 4: M angle A O B is equal to one-third m angle C O D.

Dr. H.: So if this [indicating angle AOB] is equal 1/3 of that [indicating angle COD], right? This little one is equal to one third of that one. I find that, how much is this? This is one unit, how many units would that be?

Student 1: *[indiscernible.]*.

Dr. H.: Three. So one unit plus three units is…

Student: Four.

Dr. H: Four units. This would be what, 4 X, right?

Student 1: Mm-hmm.

Dr. H.: I mean 3 X because 3 X plus X is 4 X and which 4 X has to equal what?

Student 1: *[indiscernible] [00:15:17]*

Dr. H.: Ninety. Solve for x. You see that?

Student 1: Divide ninety by four? *[00:15:23]*

Dr. H.: Divide ninety by four. That will give you x. And then you have to find out what y is.

The difference in the conversations about EMP problems versus Billstein problems could have been due to perceived time constrains, since the episode featuring the alternative text was one of the last few days of class. It could also have been due to the norms the class had of using these materials. Specifically, EMP problems were solved during class, while the students in the observed lesson had attempted the Billstein problems at home and then discussed during class time. It could, however, be the nature of the curriculum materials that led to such this difference in Dr. H.'s instruction. Specifically, EMP problems focused on understanding general concepts, while the Billstein problems focused on applying specific facts, such as the measure of a straight angle, to find numerical answers to particular situations. While there may have been curricular effects, it was likely that several factors contributed to the difference in her instruction.

For Dr. H., the EMP materials fit with the structure of her class, which included small group problem solving and student presentations. However, the discovery nature of the EMP lessons supported Dr. H. in ensuring that her pre-service teachers were the ones doing the mathematical work of explanation, critique, and sense-making. That is, the focus of the problems on supporting pre-service teachers in making sense of general mathematical ideas, rather than finding numerical answers to specific contexts, allowed Dr. H. to act on her belief that pre-service teachers must be “the center of the learning.” She insisted on and supported them in engaging in the mathematical work of sense-making and articulating ideas with precise mathematical language. She found opportunities for other students to be mathematical resources for their peers, rather than being the sole mathematical authority in the room.

Like Dr. H., Dr. C. held her students responsible for engaging in rigorous mathematics. In Dr. C.’s observed lessons, this theme manifested itself in two ways. First, Dr. C supported her pre-service teachers to pursue their own problem solving strategies, even if those strategies were less efficient, more complex, or had more potential roadblocks than the solution methods she anticipated students would use. Second, she maintained the high cognitive demand of the mathematical tasks, even in the face of student struggle. Her instructional decisions were connected to her overall goal to develop her pre-service teachers’ abilities to be patient problem solvers comfortable with “messy” situations. There were several instances when this occurred. For example, in the Trapezoid Area Formulas lesson, students were tasked with developing a formula for the area of a trapezoid. One student constructed an irregular hexagon from two scalene trapezoids, as shown in Figure 5.7. At first, the student intended to decompose the hexagon into triangles. She made a connection to the strategy that they had used to find the sum of the measures of the interior angles of a polygon in a previous lesson. Dr. C. first clarified the difference between finding angle measures and finding the area of a figure. The student then suggested enclosing the hexagon in a rectangle. She pointed to the triangles formed in the four corners of the rectangle and said, "these would all be the same." Dr. C. drew the student's attention to the fact that these triangles would not, in fact, be the same size. Dr. C. then encouraged the student to pursue her method. Even though this would be a complex way to derive an area formula for a trapezoid, Dr. C. acted on her goal for this particular student: to move her away from wanting to solve problems quickly with a procedure toward following her own ideas. The researcher identified two other instances in the episodes selected for the VSR interviews where Dr. C. similarly encouraged students to pursue their own complex strategies, offering support when needed. Dr. C. indicated that encouraging students to pursue their own creative, if inefficient, strategies was a common practice in her instruction.

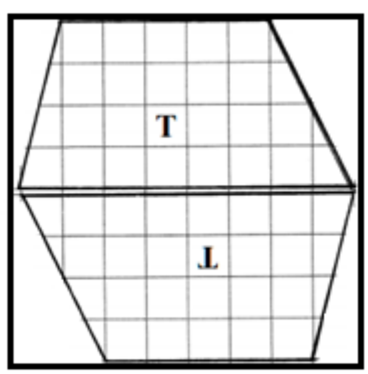


Figure 5.7. A construction for finding the area formula of a trapezoid in Dr. C.'s class

In addition, Dr. C. maintained a high level of cognitive demand during the observed class sessions. She accomplished this in two ways. First, she provided a significant amount of time for students to sovle the problem on their own, even when they were struggling. For example, in the episode discussed in the earlier section, she provided 18 minutes for the students to attempt to draw five parallelograms with the same height and base. Second, she insisted that her pre-service teachers complete the problems even when they struggled. For example, after she suggested a strategy to generate parallelograms from a “base rectangle,” she insisted that the students “really try” to draw five parallelograms. Dr. C. did not draw example parallelograms for them to copy on grid paper. She also did not reduce the number of parallelograms to be drawn. This decision was significant; in order to draw five unique, nonrectangular parallelograms with a base of four units using the grid lines, it was necessary to draw a parallelogram with a height outside of the shape, as shown in Figure 5.8. Thus, maintaining the number of parallelograms students had to draw made it more likely that they had to wrestle with the case where the height was outside of the figure. Dr. C. believed that physically drawing these parallelograms would help her students think about the properties of rectangles and parallelograms.

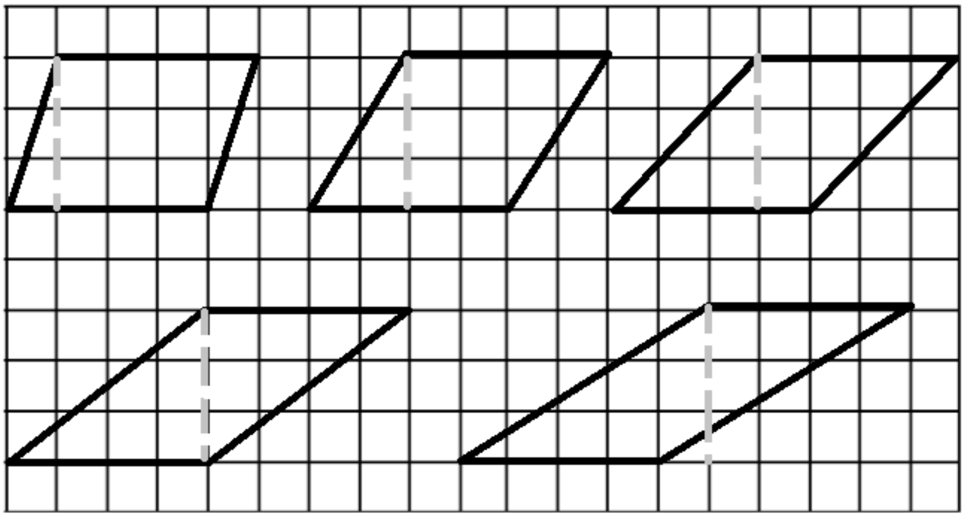
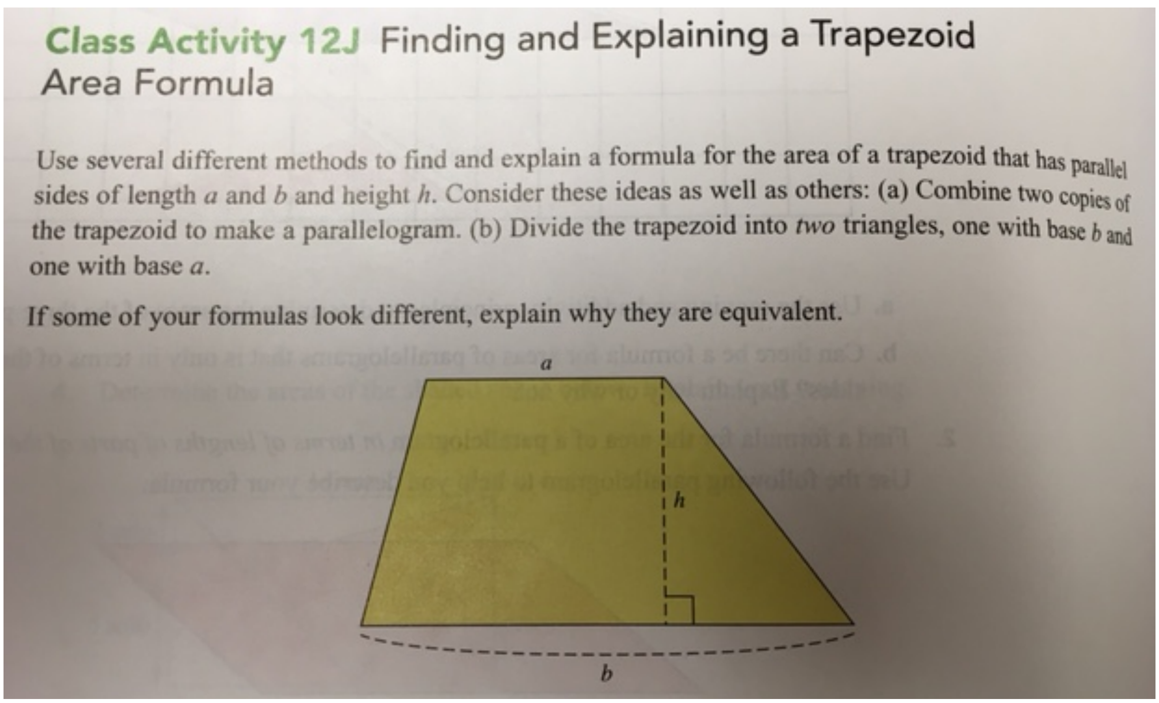


Figure 5.8. Parallelograms and their heights

There were two aspects of the EMP curriculum materials that supported Dr. C. in holding her pre-service teachers responsible for engaging in rigorous mathematical work. The first aspect supported her in encouraging her students to develop their own problem solving strategies. Specifically, Dr. C. felt the nature of the problems in the EMP lessons allowed “divergent thinking.” That is, there were multiple solution strategies. For instance, the problem in the episode above was designed to help pre-service teachers derive an area formula for trapezoids. Pre-service teachers were provided with identical pairs of three different types of trapezoids: right, isosceles, and scalene. This design allowed individual pre-service teachers to create many different kinds of construction (rectangles, parallelograms, and hexagons) in support of developing a formula. Because the trapezoids were paper cut outs, as opposed to printed in a textbook, students were also able to further decompose and recompose their constructions into more familiar shapes. In contrast, other curricula provide an example of a generalizable trapezoid (i.e. a scalene trapezoid,) and direct the learner to decompose it in one particular way (see Billstein et al., 2016, p. 836). Dr. C.'s alternative curriculum by Beckmann did suggest multiple constructions for justifying the area formula for a trapezoid, but it directed students on how to create these constructions, as shown in Figure 5.9. Therefore, problems that support "divergent thinking" are not just problems with multiple solution methods. These problems are open ended, leaving room for students to devise their own strategies, rather than directing them to use particular strategies.



Class Activity 12J Finding and Explaining a Trapezoid Area Formula

Use several different methods to find and explain a formula for the area of a trapezoid that has parallel sides of length a and b and height h. Consider these ideas as well as others: (a) Combine two copies of the trapezoid to make a parallelogram. (b) Divide the trapezoid into *two* triangles, one with base b and one with base a.

If some of your formulas look different, explain why they are equivalent.

Figure 5.9. Activity in Dr. C.'s alternative textbook for deriving the area formula for a trapezoid. From Beckmann, 2014b, p. CA–282

Dr. C. found many opportunities for pre-service teachers to develop their own solution methods, rather than being funneled into one solution path, in the EMP lessons. For instance, in one problem about finding the area of an irregular shape, some students surrounded the shape in a rectangle. Some students decomposed it into right triangles and rectangles. Other students decomposed the shape into triangles from a central point in an attempt to make connections to an earlier lesson on the sum of the measures of the interior angles of a polygon. Reproductions of their work are shown in Figure 5.10.

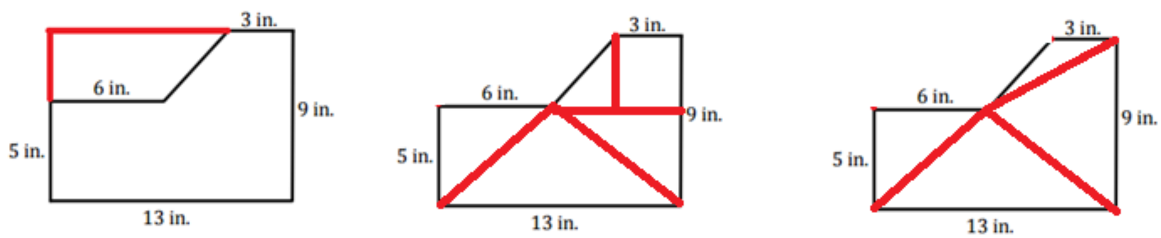


Figure 5.10. Students' solutions to find the area of an irregular shape

A second aspect of the curriculum supported Dr. C. to insist that pre-service teachers complete problems: focus. When asked whether there was anything in the curriculum that supported her in insisting that students generate the parallelograms themselves, she noted, “I think it’s communicated as important because the curriculum doesn’t try to teach everything. It tries to teach a few things really well. And I think that being able to see a pared down version of exactly what we really need to develop as foundational helps me to make decisions about what to pursue and what to cut.” This notion of focus was further articulated in her comparison between the Beckmann materials and the EMP materials, particularly when discussing how she managed to address her pre-service teachers’ misconceptions or incomplete ideas, as discussed in more detail later in this chapter. This idea of focus has been raised by other researchers, who have referred to published textbooks for these courses as “encyclopedic” (McCrory, 2006).

Both instructors held pre-service teachers responsible for engaging in rigorous mathematical work: articulating definitions and explanations, critiquing mistakes, developing problem solving strategies, and generating examples using mathematical properties. Dr. H. found the curriculum supported her work in this area through the overall "discovery" design, where problems led students to make sense of ideas themselves and through discussion. Dr. C. found support in the characteristics of specific problems, the fact that they supported divergent thinking and directed students to create their own examples. She also noted that the focus of the curriculum on foundational ideas allowed her to prioritize holding students responsible for engaging in mathematical work. The design of the problems, the focus on fundamental content, and the overall pedagogical philosophy of the materials supported the participants in creating mathematically powerful experiences for their pre-service teachers. Specifically, pre-service teachers engaged in cognitively demanding tasks, developed their identities as mathematical thinkers through devising their own problem solving strategies, and shared the authority with the instructor for explaining mathematical ideas to each other and critiquing each other's mistakes.

Assessing and Using Pre-service Teachers' Thinking

The third theme that emerged from the data was that participants found that the EMP materials assisted them in assessing and using pre-service teachers' thinking. Research indicates that assessment, when used formatively and integrated into daily instruction, impacts opportunities for student learning (Schoenfeld et al., 2014). *Uses of Assessment,* the fifth dimension of mathematically powerful classrooms, occurs in classrooms where instructors elicit and address student thinking during class time. In addition to making sense of what students currently think, instructors in mathematically powerful classrooms "build on productive beginnings and address emerging understandings," (Schoenfeld et al., 2014) That is, they use student thinking in their instruction, in real time.

Both case study participants cited the importance of understanding what their students were thinking when they were asked about different instructional actions. Both participants valued learning how their students' thought about problems for their own personal development. Moreover, both instructors productively engaged with pre-service teachers' mistakes or misconceptions. In this section, I describe instances where the participants sought to understand their students' thinking. I provide examples of the ways in which the participants used pre-service teachers' mistakes or misconceptions. Then, I describe the features of the curriculum that each participant felt supported them in these endeavors.

For Dr. H., assessing student thinking was closely tied to her push for precise language. She would frequently press students to continue explaining and ask for students to comment on a definition or explanation offered by other students. Dr. H. would often take students literally with their explanations or definitions. For example, when a pre-service teacher said that the lateral surface is "faces you can physically see" Dr. H. held up a rectangular prism and asked the class how many faces they could see. She then reoriented the prism so that they could see additional faces. This move resulted in the students further refining their definition of lateral surface. When asked about her tendency to take students literally or otherwise insist on more precise language, she indicated that she needed to hear students give a precise explanation in order to assess out how deeply they understood a concept. She said,

> "You have to be able to explain to me what the options are and how you're thinking. I don't know if you know unless you're precise in explaining to me what you know and then I can evaluate it as a teacher…What's area or what's the volume? Every time, you say 'volume,' it doesn't matter what it is, I get length times width times height. Well, that works sometimes, but not the volume of everything is that."

In addition, there were multiple instances where Dr. H. productively engaged with pre-service teachers' mistakes or misconceptions. In the observed class sessions, there were two instances where a student's mistake became an object of the whole class discussion, an opportunity for her students to further clarify their thinking about the concepts they were currently studying. In one case, a student had drawn a net of a triangular prism on the board with one dimension labeled incorrectly. Without indicating whether there was an error, Dr. H. asked the student, "how did you get five?" The student explained, but then indicated that she was unsure. Another student then explained how to get the right answer by showing the relationship between the base and the lateral surfaces, a key objective of the lesson. This same practice occurred in an episode described in an earlier section, where a student claimed the length of the lateral surface rectangle of a cylinder was the diameter, rather than the circumference, in her presentation. Dr. H.'s decisions to use mistakes in this way, as an object of discussion, was motivated by several factors. First, she wanted to create a safe environment for students to see themselves as mathematically capable and to understand mistakes as part of the learning process. Second, she saw mistakes in and of themselves as valuable. She cited mathematics education researchers in her explanation. Mistakes were opportunities to more closely examine mathematical ideas.

For Dr. H., there were two elements of the EMP materials that supported her in assessing student thinking and making productive use of mistakes. First, the materials were designed to be used with classroom discourse. Misconceptions could surface and be discussed in classrooms that employed class discussion. However, the amount of information that can be gained from listening to students talk about mathematics depends upon the problems they are discussing. In the two examples of mistakes provided, Dr. H. was able to determine that her students were still developing their abilities to mentally unfold prisms and cylinders. This topic can be fodder for a rich discussion, with many contributors. In contrast, the problem from her alternative textbook in Figure 5.6 on page 247, in which students were prompted to solve for particular angle measures, did not seem to illuminate how students were thinking about angles. Thus, the design of problems influences the richness of class discussion. In order for mistakes to be worthy objects of discussion, they must be related to important mathematical relationships.

Second, the EMP activities, she said, "lend themselves nicely to probing and having students probe their [own] thinking." There are explicit prompts within the EMP problems for students to explain and justify their thinking, as shown in the problem in Figure 5.3 on page 234. In addition, many of the problems themselves are designed for pre-service teachers to examine their own prior knowledge. For example, the first problem in the Surface Area I lesson has participants examine a formula for the surface area of a rectangular prism. Students are then asked whether the formula would work for other prisms, and why not. Similarly, the lesson on Volume, which was not part of this study but was used by the case study participants, contained problems that helped students make sense of the formula for volume of a rectangular prism. They then examined why the formula $V = lwh$ would not work for triangular prisms, but what conceptual underpinnings of the formula could be applied. These problems are shown in Figure 5.11. This problem set has been abridged for space.

2. You have learned that the formula for volume of a rectangular prism is *length* × *width* × *height*. But why does this formula work? Use interlocking cubes to build the following rectangular prisms:
   - 4 × 5 × 1
   - 4 × 5 × 2
   - 4 × 5 × 3

   a) Make sketches of your rectangular prisms.

   b) Explain why the formula gives the volume of the rectangular prism.

3. Zoe, Theresa, and Rafi are fifth graders who were asked to find the volume of a rectangular prism. Their conversation is shown below:

   Zoe: *I counted the number of cubic units in the bottom layer of the prism. Then I added up all of the prism's layers together.*

   Theresa: *I computed the volume by multiplying length, width, and height. We learned this method.*

   Rafi: *I found the area of the base of the prism, and then multiplied that area by the prism's height. I thought about stacking the base up and up but multiplied rather than added.*

   a) How are all three methods similar?

   c) Choose one of the methods above to find the volume of this triangular prism. Explain your choice of method.

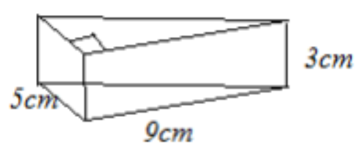


Figure 5.11. EMP problem prompting pre-service teachers to reexamine their prior knowledge about volume formulae

Dr. C. also believed in the importance of assessing student thinking for effective instruction. She used several instructional moves to assess what students were thinking. As she approached small groups or individuals, she would ask broad, general questions, "Tell me what you're doing." She would repeat back what students had said to make sure she understood. Rather than funneling students toward a particular strategy, she tried to hold back her comments so that she could fully understand how a student was thinking about a particular problem. For example, in one particular instance a student was explaining to Dr. C. her method of duplicating a trapezoid to form a parallelogram to determine its area. Dr. C.'s initial instinct was to prompt the student to recall the area formula for a parallelogram from the earlier lesson. When the student did not take up this idea, Dr. C. created space for the student to explain how she would further decompose the parallelogram formed by the trapezoids to make a rectangle. As a result, the conversation then focused on how the length of the base of the parallelogram was maintained throughout this recomposition. This topic may have been unexamined if Dr. C. had not listened to the student's explanation. Dr. C. indicated that these instructional actions, listening to students, checking to make sure she understood, and holding back her suggestions until she fully understood a student's thinking, were an important part of her practice. "I know in my mind where I want my students to be," she said, "but if I try to impose on them how I think the pathway is, it's not going to be as effective as if I see where they are first."

Like Dr. H., Dr. C. also productively engaged with pre-service teachers' misconceptions. However, the nature of these misconceptions differed from the mistakes that surfaced in Dr. H.'s class. In Dr. C.'s class, the misconceptions recurred throughout multiple lessons and were, in her words, foundational concepts. For instance, although the class had completed the Quadrilaterals lesson, students continued to confuse the names and classifications of shapes. They struggled to use the properties to make constructions. One student insisted squares were not rectangles. Another student drew a trapezoid and then a chevron when asked to draw a parallelogram. Few students were initially able to construct a non-rectangular parallelogram on grid paper. Dr. C. used a range of strategies to address these misconceptions. To address the classification of squares and rectangles, she invited other students to comment. As students worked on constructing parallelograms, she poised questions to small groups and individuals. These questions prompted other group members to comment on why a particular shape that had been drawn was not a parallelogram. The questions also directed students' attention to the similarities and differences between parallelograms and rectangles. Asking questions, inviting other students to comment, and directly addressing misconception were Dr. C.'s typical strategies for addressing these misconceptions.

One recurring misconception was particularly salient throughout the three observed lessons: the difference between height and side length. This misconception first surfaced in the Area Concepts lesson when pre-service teachers struggled to find the height for an obtuse triangle on a coordinate grid. One pre-service teachers drew right angle symbols in the corner of an obtuse angle, as shown in the Figure 5.12. Another drew a median from the obtuse angle to the longest side. This student defined the height as a line segment from the peak of a triangle to "the midpoint of the base."

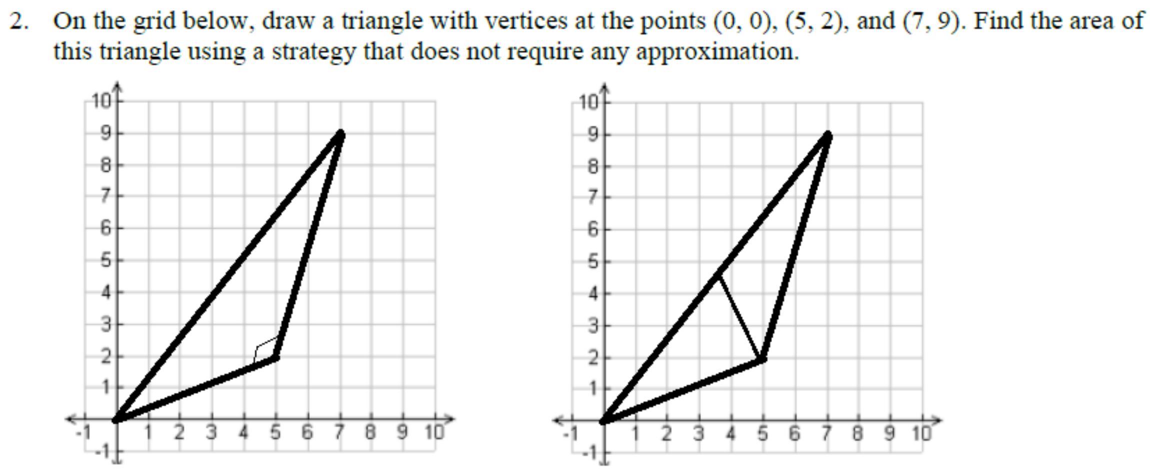


Figure 5.12. Sample student diagrams from Dr. C.'s class

The confusion about height resurfaced when students were drawing parallelograms on grid paper, as described earlier. As she visited small groups, Dr. C. noticed that some students were labeling the slanted side of their parallelogram with a 3, rather than the height. Other students could not progress beyond a rectangle because they believed that the side length and the height had to be the same segment. These students saw no other way to make an additional parallelogram with a vertical line segment measuring three units. In one struggling group, Dr. C. asked “What does height mean to you?” The students made an up and down motion in the air with their pencils and said, “Up and down.” These students may have been fixated on the typical orientation of a line segment representing height, rather than the fact that the segment must be perpendicular to the base. During the Trapezoids lesson, this confusion surfaced yet again. Dr. C. asked one pair about their labels. “You labeled this as B2. Where is your second base?” One student indicated a slanted side of the trapezoid, but then said, “That might be one. I’m not sure. More likely it’s the height.” Students had to wrestle with the idea of height repeatedly, making sense of it in the context of each different shape studied.

Dr. C. addressed this recurring misconception as she circulated among small groups. From students' diagrams, she could determine whether there were confusions about height. As students explained why they were struggling with a problem, additional misunderstandings about height surfaced. Her follow up questions clarified the nature of the confusion, such as a fixation on orientation, a belief that height had to be a side of the shape or within the shape's interior, or a misremembered theorem from high school geometry courses. She then addressed the misconception. She reminded students that a height must be perpendicular to a base. Once, she also demonstrated for a small group how to draw a height on the exterior of a shape. Dr. C. indicated that these instances were typical of her practice. She said, "When we get into the root of a misconception or something that's naively constructed, you can rebuilt it in a different way than if you just say, 'no, that's wrong, let's try it this way.'"

Dr. C. found that the EMP materials were instrumental in helping her to identify and address pre-service teachers' misconceptions about fundamental ideas, such as the nature of height. The fact that the lessons allowed foundational ideas to resurface over multiple lessons made it possible to "really get deep" with students' ideas. She compared this design to problems within the Beckmann materials, which she felt were not transparent about the assumed knowledge that pre-service teachers were expected to bring to the course.

> "You come to Beckmann with this understanding that your students are coming with a pretty solid foundation and can jump in from there and that you don't need

> to examine that prerequisite stuff because it's just a given. And it's not a given…I think they [EMP materials] called to the surface some prerequisite stuff that students need to know in order to be successful."

Dr. C. gave the example of a problem from Beckmann to create a net of a cone that would hold half a liter of water. Such a problem would require an awareness of the difference between height and slant height, knowledge of formulae for circumference and the volume of a cone, and the ability to convert between measures of liquid volume and cubic units. This prerequisite knowledge is not transparent to the instructor. While students are capable of solving such a problem with support, Dr. C. felt that it came at a cost to spending time reexamining more foundational ideas. "Is it worth the time and effort involved in getting them to that point or is our time better spent on the key pieces that can support strategic thinking and the examination of prior knowledge?"

These opportunities to revisit foundational ideas over multiple lessons is not one of the explicit design principles articulated by the EMP authors. However, it is clear that this design is implicit in the materials. The authors indicate that they chose "high-leverage content," which Dr. C. might refer to as "foundational ideas." The authors refer to learning cycles, which earlier analysis showed occurred over multiple lessons, not just within one lesson. These learning cycle allow students to revisit ideas in new contexts. Dr. C.'s comparison to the volume of a cone problem also highlights additional features of the EMP materials. The problems are designed to have a low threshold and a high ceiling, whereas the cone problem required several prerequisite ideas. When there are several prerequisite ideas, it can be difficult to identify the source of students' misunderstanding. Finally, the instructor support materials also aimed to make transparent the prerequisite ideas and potential areas where students may struggle. These features coalesced into one overarching element that was salient to the participant: foundational ideas and related student misconceptions were illuminated over multiple lessons, giving her multiple opportunities to help students develop their conceptual understanding.

Thus, there are several aspects of the curriculum that supported the participants in assessing and then productively engaging with her pre-service teachers' mistakes: (1) the focus on foundational ideas, (2) the way the materials allowed misconceptions to repeatedly surface over multiple lessons, (3) the nature of the activities as prompting students to question their prior knowledge and current thinking, and (4) the design of the materials as supportive of using classroom discourse. These materials supported the case study instructors in assessing students and then responding to their thinking, a key dimension of mathematically powerful classrooms.

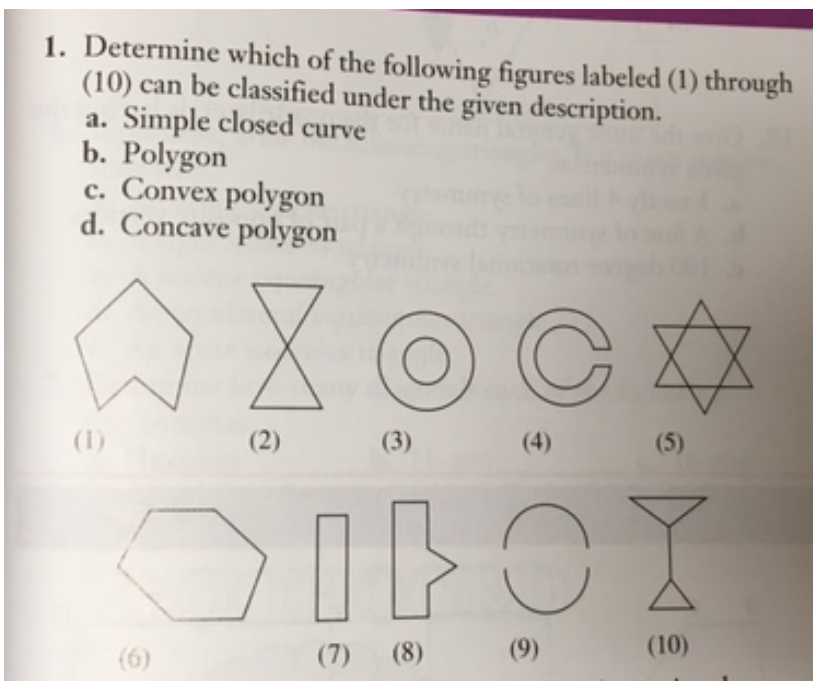
Supporting Pre-service Teachers to Use Mathematical Language.

The fourth theme that emerges from the data was the way the EMP materials helped the participants support pre-service teachers to use mathematical language. There are several different ways in which mathematical language was a focus in the two case studies: (1) the use of mathematical terminology, (2) precision in articulating ideas; (3) engagement in mathematical argumentation, and (4) the use of variables to represent relationships and ideas. This theme is related to the *mathematics* dimension of mathematically powerful classrooms. Instruction related to this theme resulted in students being engaged in the mathematical practices of justification, representation, and communication.

In both of the case studies, all four aspects of this theme were observed. However, the prominence of the instructional actions each participant took in these four aspects differed. In this section, I describe some instances where this theme was exhibited for the different instructors. I then explain how the curriculum materials supported them in their instruction.

Supporting pre-service teachers' use of mathematical terminology was a feature in both case studies, but especially prominent in Dr. H.'s class. Dr. H. frequently launched her class sessions with a review that included students generating definitions. For example, she asked the students to identify the features of different prisms using specific vocabulary such as edge, face, and vertex projected using Power Point. Often, Dr. H.'s focus on terminology was closely intertwined with her focus on precision. When Dr. H. asked her class to define a cylinder, one student suggested, "Two circles connected by a rectangle." Dr. H. asked for other students to comment on the definition. When no one volunteered, Dr. H. then drew a net of a cylinder on the board. This action led the student to add "three-dimensional" to her definition. Even though Dr. H. knew that students could identify cylinders during this lesson, she pushed her students to precisely define key mathematical terms. Dr. H. prioritized precision because of her own belief in its importance in the discipline of mathematics. In addition, she emphasized precision because of the nature of the course – her students were planning on being elementary teachers. As she said, "I'm so fussy about the language and I want them to be precise in the mathematics because they're going to be teachers…..if they are going to be teachers and they say that to a student, the student is going to take you literally….So, like I know what you mean, and you know what you mean. And I know you know it, but you have to be able to explain it."

Dr. H. did not believe that there was anything particular about the curriculum materials that prompted her to push students to use mathematical terms and precise language. However, this same level of a push for precision did not occur in her class session where she used Billstein materials. Indeed, such an opportunity could have surfaced when students discussed the problem about polygons and curves displayed in Figure 5.13.



1. Determine which of the following figures labeled (1) through (10) can be classified under the given description.
   a. Simple closed curve
   b. Polygon
   c. Convex polygon
   d. Concave polygon

(1) (2) (3) (4) (5)

(6) (7) (8) (9) (10)

Figure 5.13. A problem Dr. H. used from her alternative text.

(Billstein et al., 2016, p. 651)

The EMP lessons that Dr. H. enacted included questions that focus on terminology. The first question in the Prisms lesson directed students to "Examine the prisms at your table. Then review the following vocabulary: base, edge, face, lateral face, prism, and vertex." The following whole class discussion question prompted students to define a prism. Thus, language was an important component of the EMP materials. While not recognizing the emphasis on language in the EMP materials, Dr. H. definitely enhanced the lessons in this area. She began her lessons by prompting students to articulate what they had studied in the previous lesson. As they talk, she asked follow up questions about the mathematical terminology that they used. She pressed for precision by asking other students to comment on suggested definitions or by taking students literally in order to force them to refine their statements. The focus on mathematical terminology in the EMP lessons was a starting point, but her insistence on precision and focus on terminology enhanced this experience for her students.

Dr. C.'s observed class sessions likewise exhibited the use of mathematical terminology. During whole class discussion, she would transcribe her pre-service teachers' suggestions on the board, often adding more academic language such as "compose" or "decompose." "A lot of times my students will have good ideas, but won't be able to articulate them with language that's clear. So I do try to do a lot of modeling very specific language vocabulary when the students are presenting ideas…. I think if I surround them in the academic language, but also the language of the discipline, it helps to raise everybody's ability." This focus on modeling academic language occurred in individual interactions, but was especially prominent during whole class discussion.

Both case studies similarly had students engaging in mathematical argumentation. In Dr. C.'s VSR interview, this practice was an explicit topic of discussion. For example, in the whole class discussion, Dr. C. prompted her students to justify why the area formula for a triangle is half the product of the base and the height. She transcribed their descriptions of their constructions on the board. When one student explained generally how the strategy of doubling a triangle and finding the area of the parallelogram would work, she prompted him to continue his argument with unfinished sentences, shown in bold in the transcript below.

> **Dr. C.:** Okay. So, why does your formula work for all triangles? Is that two hands raised? You're jumping out of your seat.
>
> **Student:** Yeah.
>
> **Dr. C.:** Go for it, [student name].
>
> **Student**: So, any triangle you have you can make an identical triangle and they will have the same height and the same base and it will make a parallelogram
>
> **Dr. C.**: And joining those two together --
>
> **Student**: Yes.
>
> **Dr. C**.: -- you can form a parallelogram, awesome. **And that parallelogram has an area of?**
>
> **Student**: Two times of base times height [sic].
>
> **Dr. C.**: Base times height. **And so, each copy of the triangle would be?**
>
> **Student:** Exactly one half.
>
> **Dr. C.:** Exactly one half of that base times height. Okay.

Dr. C. explained that in all of her mathematics courses she pushed students to "make claims but then also have the evidence to support them. So I do try to model a lot of that. Don't just stop after you make the statement that you think is true, but bring us along the reasoning that you went through in order to get there." While in her other courses she explicitly introduced frameworks for logical reasoning, in this course she aimed to help students develop such skills in a more "organic" way.

According to Dr. C., the EMP materials supported her in this endeavor through the structure of the lessons. Specifically, the lessons first prompted students to consider many specific cases before justifying a general idea. In this instance, students were directed to work with three different pairs of triangles before justifying the area formula for a triangle. Likewise, students worked with three different pairs of trapezoids before generalizing the trapezoid formula. They drew and decomposed five different parallelograms before justifying the parallelogram formula. Dr. C. believed that this structure gave her students sufficient experiences to move beyond the particular cases in order to begin to justify general mathematical ideas.

Another way that both case study participants used mathematical language was they focused on using symbols to represent relationships. This was not surprising given that one of the objectives of the lessons was to make sense of geometric formulae. For Dr. C.'s class, however, supporting students in being comfortable using variables to represent relationships that they could explain orally was a goal that became particularly prominent during the Trapezoid lesson. In this lesson, students could explain a *process* for decomposing and recomposing a trapezoid to find the area, but they struggled to represent their processes with variables and operation symbols. Dr. C. worked with student both individually and as a whole class on this goal. In one episode, she brought the class back together to direct them to use variables in their explanations of how they found the area of their trapezoids. She used this opportunity to bring attention to the features of a trapezoid, particularly the length of the two bases. Likewise, after listening to student explanations when conferring individually, she pressed them to use variables to generalize their explanation. “They shun algebra,” she said of her students’ reluctance to use variables to represent their thinking. Attaching variables to the dimensions of the trapezoids and their constructions was important, she felt, for them to conceptually understand the trapezoid area formula. Furthermore, using algebraic notation was a way to “express something in a general way,” a key skill she wanted her students to develop. She was not satisfied with students’ oral descriptions of their construction; she wanted them to be able to use the conventions of the discipline by attaching meaning to variables and use variables and operation symbols to express general relationships.

When asked how the curriculum materials supported her in episodes where she pressed students to use variables, Dr. C. noted that symbolic notation was explicitly in the problems and questions in the lesson. In the Trapezoid lesson, the variables were introduced in the whole class discussion question, shown in Figure 5.14. This question appeared after students had used trapezoid cut-outs on grid paper to generate area formulae and articulate them verbally.

Whole Class Discussion Question

- Describe some ways to find the area of the trapezoid below.

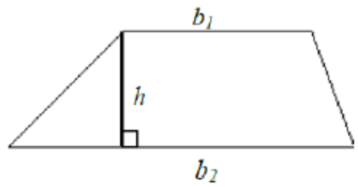


Figure 5.14. The use of symbolic notation in EMP

It is not unusual for college curricula to use variables. However, the way in which curricula expect students to use algebra may differ. For example, Billstein et al. (2016) provided a justification for the trapezoid area formula by demonstrating how to decompose a trapezoid labeled with variables. However, in the example problem and many of the homework problems, the dimensions were provided. Therefore, students were computing with a formula, but they were not using variables to represent general relationships. In fact, one could argue that variables were treated as parameters, not varying quantities (R. A. Philipp, 1992), because they were quickly transformed into constants and computed. The students did not have to consider how the quantities vary. Billstein also treated variables as unknowns (R. A. Philipp, 1992, p. 1992), as demonstrated in the problem in Figure 5.6 on page 247.

In contrast, both EMP and the Beckmann textbook aimed to have *students* be the ones using variables to represent general relationships. Both curricula provided the area formula for triangles and trapezoids and prompted the students to explain why the formula made sense. Both curricula also asked the students to derive a formula for the area of trapezoid, which proved to be the more challenging task. However, there may have been more scaffolding in the EMP materials than the Beckmann materials. Both EMP and Beckmann had students examine specific examples of shapes on grid paper before justifying different area formula. However, in EMP, students used the shapes on grid paper to derive and generalize their formula. The specific quantities were always present for them to return to, if necessary, as they made their arguments. During the Trapezoid lesson, several students in Dr. C.'s class used these grid lines to reason about their constructions. In the Beckmann textbook, the figures in the activities about generalizing the formula no longer had grid lines. Students therefore must have internalized several conceptions about area to be successful in the Beckmann problems. This study does not provide data to compare the two methods. In considering what aspects of curriculum to help instructors in develop their pre-service teachers' abilities to use algebra to represent general relationships, these types of supports are worth further investigation.

The EMP curriculum supported instructors in their goals to help pre-service teachers use mathematical language. It supported them in developing pre-service teachers' use of mathematical terminology, in pushing pre-service teachers to use precise language, in engaging pre-service teachers in mathematical argumentation, and in supporting pre-service teachers to use symbolic notation to represent general relationships and ideas. There were several elements that helped them in these objectives. First, several problems focused specifically on developing mathematical terminology. Second, the structure of the lessons, moving from specific examples in problems to generalized concepts in whole class discussion questions, provided support for instructors in developing pre-service teachers' skills in constructing mathematical arguments and generalizations. Third, the use of variables in the EMP problems to represent general relationships, as opposed to unknowns or rules for computing, provided opportunities for instructors to encourage students to use variables and operation symbols to represent their thinking. These curricular elements supported instructors in creating mathematically powerful experience for their pre-service teachers by providing opportunities to engage in the mathematical practices of justification, representation, and communication.

Section III: Elements of the EMP Curriculum that Support Instructors to Provide Mathematically Powerful Experiences

The participants in this case study used the EMP materials to create mathematically powerful experiences for their pre-service elementary teachers in a number of ways. First, they focused on helping pre-service teachers make sense of mathematical relationships in geometric measurement. Second, they held pre-service teachers responsible for rigorous mathematical work, whether it be explaining concepts or creating their own solution strategies. Third, they assessed student thinking and addressed pre-service teachers' misconceptions in their instruction. Fourth, they supported pre-service teachers in using the language of the discipline by using mathematical terminology, pushing for precision, engaging pre-service teachers in argumentation, and prompting them to use variables to represent relationships and ideas. Within each of the previous sections, I explained how the participants were supported by the curriculum materials in each of these endeavors. In this section, I summarize the elements that supported these instructors. These elements fell into three categories: (1) overall philosophy of the curriculum materials, (2) design of lessons and units, and (3) design of individual problems. These categories are not necessarily distinct. For instance, the overall philosophy of a curriculum certainly informs the design of individual problems.

For Dr. H., the overall philosophy of the curriculum materials was one of the more salient elements. There were three components of her understanding of the overall philosophy and design: (1) the focus on understanding concepts, (2) the expectation that students make sense of mathematical concepts themselves through problems, and (3) the belief that people learn through social interaction. Dr. H. frequently compared the conceptual nature of the EMP materials with more procedurally focused materials such as her alternative curriculum by Billstein, Lubinski, and Lott. Dr. H. used the term "discovery learning," to refer to the idea that the sequence of problems within the EMP units helped pre-service teachers make sense of ideas themselves. She noted how important it was that the materials use class discussion. These three overarching principles were evident in both the problems and the instructor support materials, especially the Instructor's Guides and the videos of enactment.

The focus on understanding concepts was evident throughout all of the EMP materials. The objective of the Surface Area I lesson, for instance, was to develop pre-service teachers' "understanding of the relationship between the dimensions of a prism or cylinder and the measurements on its corresponding net" (Instructor's Guide). The lessons were designed to support pre-service teachers in making sense of these concepts *themselves*, rather than be told the concepts by an instructor or a textbook. The lessons used a variety of strategies to help pre-service teachers make sense of ideas themselves. One method was to have students build multiple examples of mathematical objects themselves and then use repeated reasoning to identify general relationships. For example, pre-service teachers built several open-based prisms and cylinders from 8 ½ by 11 inch pieces of paper. They then drew nets of the prisms and cylinders with bases and labeled the dimensions. This activity led them to notice the relationship between the lateral surface rectangle and the height of the prism and the dimensions of the base of the prism. Additionally, the EMP materials positioned pre-service teachers as responsible for articulating the ideas they "discovered." Following the prism activity, students were prompted to articulate the relationship generally by the whole class discussion question.

The third component of the overall philosophy behind the EMP materials was the belief that learning occurs through social interaction. Dr. H. indicated that she believed the fact that EMP materials were designed to be used with whole class discussion was important. Particularly, she found this design element important for eliciting and using student thinking. She felt that the materials fit naturally within her practice of using small group discussions about problems and student presentations. There was evidence that the EMP materials enhanced her practice in this area. The social interactions within these class observations extended beyond students talking or presenting. Specifically, there were three ways that her use of class discussion differed when using EMP materials. First, conversations focused on mathematical relationships. Second, she made productive use of pre-service teachers' mistakes. Third, she shared mathematical authority with students when they struggled. These differences seemed to be driven by the nature of the problems and activities. Indeed, the overall philosophy of using classroom discourse must inform the design of particular problems. Problems must provide content to talk about.

Dr. H., as a "thorough piloter" (J. T. Remillard & Bryans, 2004), had read the Instructor's Guides and watched the videos of enactment. These informed her vision of the use of the materials. She found three components of the overall design to be supportive in focusing on mathematical relationships, holding pre-service teachers responsible for engaging in rigorous mathematical work, and eliciting and using student thinking. These three components were (1) the focus on understanding concepts, (2) the expectation that students make sense of mathematical concepts themselves through problems, and (3) the belief that people learn through social interaction.

At the lesson and unit level, three main elements supported mathematically powerful moments. First, the materials focused on foundational ideas. Both case study participants noted that the EMP materials didn't try to address "everything," but instead tried to address "a few things really well." For Dr. H., it seemed that the focus allowed her to feel like she had more time to focus on mathematical relationships. As she said, EMP "takes the time to build the understanding." For Dr. C., the focus supported her in holding pre-service teachers' responsible for engaging in rigorous mathematical work, rather than doing the mathematics for them. The focus on *foundational* ideas was particularly valued by Dr. C., who expressed frustration with other curriculum materials that either did not prioritize fundamental concepts or were not transparent about prerequisite ideas. She felt better able to assess pre-service teachers' thinking, respond to their confusion, and have them deeply explore and apply foundational ideas with the EMP materials.

The notion of *foundational* concepts adds further nuance to the notion of "high-leverage content." High-leverage content is defined as "the particular topics, practices, and texts that are foundational to the K–12 curriculum and vital for beginning teachers to be able to teach skillfully…. Examples…include place value, number concepts and operations, fractions, and representing and explaining mathematical ideas and relationships" (TeachingWorks, 2015). Within these topics, there could be many objectives. The topic of fractions includes fractions on a number line, decimal representation of fractions, comparing fractions, and operations on fractions. *Foundational* ideas within this domain include partitioning, piece size, and iteration of same-sized pieces. They are the ideas upon which other topics are built. In Dr. C.'s class sessions on geometric measurement in this study, the foundational ideas were the properties of quadrilaterals, the decomposition of shapes and the conservation of area, and representing measurement relationships with variables. A prerequisite idea that surfaced in her class sessions was the height of two-dimensional figures. Dr. C. felt that the focus of the EMP materials allowed her to engage students in deeply exploring foundational ideas and helped her to repeatedly identify and address the prerequisite ideas with which her students struggled.

The second aspect of the lesson and unit design that supported the case study instructors was the notion of learning cycles. In a learning cycle, there are three stages: exploration, concept identification, and application (Simon, 1994). The application stage triggers a new learning cycle, where an idea is further explored or a new idea is explored. This study suggests that the EMP lessons have learning cycles not only within a lesson, but across lessons. For example, within the Quadrilaterals lesson, students identified the relationships between categories of shapes based upon their properties. They then had to apply those properties and relationships to construct parallelograms in the Parallelograms & Triangles lessons and to create new figures out of congruent trapezoids in the Trapezoids lesson. This application led to opportunities to more fully explore the properties and the categorization of quadrilaterals. Dr. C. identified this aspect of the lessons, allowing ideas to resurface over time, as supportive for her to assess and address pre-service teachers' thinking. While not explicitly identified by Dr. H., the power of learning cycles was also present in Dr. H.'s class as students reexamined the structure of prisms and cylinders in different ways over multiple lessons.

The third element of the EMP lessons and units that supported the case study instructors was the organization of the problems within a lesson. Specifically, problem sequences started with problems that involved multiple specific examples. Students used repeated reasoning with these specific examples to identify general concepts. Dr. C. reported that this structure helped her support students in mathematical argumentation. For example, in the Parallelograms & Triangles lesson, students drew and decomposed five different parallelograms before justifying the area formula for a parallelogram in a general way. They created constructions with three different kinds of triangles before generalizing the area formula for triangles. In the Trapezoids lessons, participants created constructions with three different pairs of trapezoids. They used these constructions to articulate methods for finding the area of trapezoids before being prompted to justify a general formula. Students referred back to their specific constructions when explaining the area formulae generally.

At the problem level, there seemed to be six different aspects that supported the instructors. The first four were referred to by the case study instructors while the last two were inferred from observation and comparison with the alternative textbooks. First, the problems focused on mathematical relationships and explicitly prompted pre-service teachers to articulate these relationships. Second, the problems supported "divergent thinking." Third, the problems lent themselves to probing student thinking and students probing their own thinking. Fourth, the problems used algebra as a way to represent general relationships. Fifth, there were problems that specifically focused on mathematical terminology. Sixth, problems had a low threshold and a high ceiling, which made necessary prerequisite knowledge more transparent.

The problems and activities in the EMP lessons used in the observed classroom sessions focused on mathematical relationships. There were explicit prompts following these questions that direct pre-service teachers to articulate these relationships. These problems stood in contrast to problems in alternative texts calling for pre-service teachers to apply mathematical facts and procedures to specific situations. As a result, the class sessions employing EMP materials focused on these mathematical relationships.

Dr. C. often brought up her priorities of supporting creative problem solving. She noted that the EMP problems supported "divergent thinking." Divergent thinking in mathematics includes the ability to come up with multiple different solution strategies

(Hasan Unal & Ibrahim Demir, 2009). The problems she identified had multiple solutions and were open-ended. That is, they did not direct pre-service teachers to a particular solution path. Some of these problems prompted students to create multiple examples or constructions. These types of problems supported Dr. C. in holding pre-service teachers responsible for engaging in rigorous mathematical work, especially developing their own solution strategies.

Dr. H. noted that the problems lent themselves to probing student thinking and students probing their own thinking. Many of these problems prompted students to reexamine their prior knowledge. Many problems included follow up questions asking students to explain and justify their thinking. These problems supported Dr. H. in assessing and using student thinking in her instruction.

In both case studies, the participants pushed pre-service teachers to use algebra to represent general relationships. This practice was more evident in Dr. C.'s class, likely because the objectives of the observed lessons included more geometric formulae. Dr. C. indicated that the EMP problems explicitly prompted her to introduce algebraic notation. In general, the EMP problems tended to use algebra to represent general relationships, as opposed to representing specific values which students were expected to find. This feature of the EMP problems supported the case study participants in this aspect of mathematical communication. Similarly, it seemed that the fact that there were problems that explicitly prompted pre-service teacher to discuss and define vocabulary supported Dr. H. in pushing her students to use mathematical terminology.

Finally, the EMP problems had a low threshold and a high ceiling. This feature stood in contrast to problems that required pre-service teachers to coordinate and apply multiple facts and procedures to find an answer. This feature of the problems may have assisted the instructors in identifying what prerequisite knowledge pre-service teachers held or did not hold. Their mistakes or unproductive solution paths often revealed their misconceptions. The case study participants did not explicitly mention this feature. However, Dr. C.'s comparison to a problem in another text suggests that problems that required the coordination of multiple facts and procedures detracted from examining students' prior knowledge. Relatedly, Dr. H. responded to students' struggles with Billstein problems by walking them through a process. This instructional practice stood in contrast to the way she dealt with students' difficulties when enacting EMP materials. It is possible that Dr. H. was unable to identify what concept pre-service teachers were struggling with when they were unable to attempt a problem in the Billstein text. Further research should examine the impact of problems with a low threshold and high ceiling have on instructors' ability to assess and use pre-service teachers' thinking.

Elements of the EMP curriculum at the problem, lesson, unit, and programmatic levels supported the case study participants in creating mathematically powerful experiences for their pre-service teachers in their mathematics content courses. Specifically, the participants found themselves better able to focus on mathematical relationships, hold their students responsible for engaging in rigorous mathematical work, assess and address students' thinking, and support pre-service teachers in using mathematical language. The participants found the three overall principles of the materials supportive: (1) the focus on understanding concepts, (2) the expectation that students make sense of mathematical concepts themselves through problems, and (3) the belief that people learn through social interaction. Additionally, the focus of the materials on *foundational* ideas in these units supported the instructors to emphasize mathematical relationships and hold pre-service teachers responsible for engaging in rigorous mathematical work. The learning cycles helped the instructors assess and address pre-service teachers' thinking. The structure of the lessons from specific examples to general ideas helped them in engaging pre-service teachers in mathematical argumentation. Problems that focused on making sense of and articulating general mathematical ideas supported them, as did problems that prompted pre-service teachers to examine their own prior knowledge and current thinking. Other problem-level features supported the instructors as well. Some of these features include the low threshold and high ceiling of the problems, problems that explicitly focused on mathematical terminology, and problems that prompted pre-service teachers to use algebra to express general relationships that they could articulate verbally and demonstrated visually.

This chapter has described the two case studies of instructors who used EMP in their mathematics content courses for pre-service teachers. These observed class sessions exhibited the dimensions of mathematically powerful classrooms. This chapter has identified four themes in the ways in which the curriculum materials supported the case study participants. It has also identified the features of the curriculum that supported the participants. In the next chapter, I discuss the relevance of these findings and situate them in the previous research on mathematics content courses for pre-service teachers.

CHAPTER 6: DISCUSSION

The purpose of this study was twofold: (1) first, to identify the instructional practices and curriculum used in college-level mathematics content courses for elementary teachers and (2) second, to determine how curriculum could support instructors of these courses in creating more mathematically powerful experiences for their pre-service teachers. As part of this study, a nationwide survey of college instructors of mathematics content courses for elementary teachers was conducted. Additionally, case studies of two instructors of these courses were conducted. These instructors were observed and videotaped teaching three lessons of the Elementary Pre-service Teachers Mathematics Project (EMP) curriculum and one lesson using their alternative curriculum. They participated in video stimulated recall (VSR) interviews. Themes were identified across the two cases to identify the ways in which these two participants felt the curriculum supported them in creating mathematically powerful experiences for their students.

The data were analyzed quantitatively and qualitatively. The data from the survey were analyzed using descriptive and inferential statistics, factor analysis, and multi-group comparisons. The videotape from the case study was analyzed using Schoenfeld and colleagues' (2014) five dimensions of mathematically powerful classrooms and the TRUMath rubric. Class episodes for discussion in video-stimulated recall (VSR) interviews were chosen from those episodes that scored 3 on one or more of the five dimensions. In the VSR interviews, case study participants were asked to comment on their instructional actions and whether they felt the curriculum supported them in these actions. The transcripts from these interviews were coded according to the five dimensions of mathematically powerful classrooms. The transcripts were also coded by Instructor Resources or Curriculum Resources from Brown's (2012) Pedagogical Design Capacity framework. I next identified four themes in the ways the instructors were supported by the curriculum. Finally, I identified the aspects of the curriculum they felt supported them in these four areas.

Section I of this chapter summarizes the study's key findings. Section II lists the limitations of this study. Section III makes recommendations for future research.

Section I: Study Findings

Previous research on mathematics content courses for pre-service teachers focused on the availability of such courses, the content of the courses, and the qualifications of the instructors who teach the courses (Blair et al., 2013; Greenberg & Walsh, 2008; Masingila et al., 2012). There have also been studies that suggested that the particular content, curriculum, and pedagogy led to pre-service teachers developing more mathematical knowledge for teaching and more productive beliefs (Chapin et al., in review; McCrory et al., 2009; Spielman & Lloyd, 2004; Superfine et al., 2013). Previous to this study, there has been no research on the type of instruction occurring in these courses across the country. Similarly, there was only one study that looked at the relationship between curriculum and instruction in mathematics content courses for pre-service elementary teachers (Jeppsen, 2010). Jeppsen used a macro-level lens, quantifying student- versus teacher-centered instruction. The present study adds to this work by more richly describing the instructional actions at a more detailed level and identifying the elements of the curriculum that supported the participating instructors in their practice.

Schoenfeld and colleagues' (2014) five dimensions of mathematically powerful classrooms were used in several aspects of this study. The five dimensions of mathematically powerful classrooms included: *mathematics; cognitive demand; access to mathematical content; authority, agency, and identity;* and *use of assessment*. The first dimension, *mathematics*, measured the extent to which the mathematics discussed in class was conceptually based. It measured whether mathematics was portrayed as a coherent body of knowledge that could be figured out or a set of isolated skills and procedures to be memorized. It also measured students' opportunities to engage in mathematical practices. The second dimension, *cognitive demand*, measured the extent to which students were responsible for engaging in mathematics, as opposed to the instructor doing the mathematical work for the students. The third dimension, *access to mathematical content*, measured whether the instructor achieved meaningful participation from all students. The fourth dimension, *authority, agency, and identity,* measured the extent to which student ideas were elicited and pursued. It measured the extent to which students were responsible for determining whether a mathematical idea was legitimate. The fifth dimension, *use of assessment,* measured the extent to which the teacher elicited student thinking, addressed misconceptions, and built on student ideas. The survey questions about instructional practice were designed around each of the five dimensions. The video of classroom instruction and the VSR interviews were analyzed according to the five dimensions.

This study determined that there are many ways in which mathematics courses for elementary teachers are, in fact, exhibiting high levels of these five dimensions, according to survey responses. The study also identified the ways in which curriculum materials can support instructors of these courses in creating experiences for their students that exhibit high levels of these five dimensions.

Question 1

What are the instructional practices and curriculum resources used in mathematics content courses for pre-service elementary teachers? How do these differ by instructor characteristics, if at all?

There is a typical image of college-level mathematics and science courses that serves as a backdrop for policy recommendations and conversations about improvement in higher education instruction, including mathematics content courses for prospective teachers (National Research Council, 2015). This image is dominated by an instructor lecturing to students. The prototypical instructor expects students to make sense of ideas themselves and uses assessment for evaluative purposes. There are a number of studies that support this image (e.g. Iannone & Nardi, 2005; Walczyk & Ramsey, 2003; Walter et al., 2015). In addition, previous research studies have suggested that this type of instruction occurs in mathematics courses that enroll prospective teachers (e.g. Finn, 2010; Hart et al., 2013; Hart & Swars, 2009). Case study research also describes instances where instructors did not focus on the conceptual understanding of procedures or did not make connections to teaching mathematics in elementary school (e.g. Hart et al., 2013; Olanoff, 2011). Other studies have suggested that these courses are remedial in nature (Greenberg & Walsh, 2008).

The results of the IPCU survey demonstrated that this image of the typical college level mathematics course may not the norm for mathematics content courses for elementary teachers. More than three-quarters of the survey participants indicated that their students spent 50% or less class time listening to the instructor. The participants reported having their students work in small groups regularly every week; more than half of the survey participants reported that students worked in small groups 40% or more of the time. The average student-student interaction score, which measures the extent to which instructors indicate they use practices that encourage students to interact with each other, was high, 72.83. The mean student-content engagement score, which measures the extent to which instructors indicate students engage in mathematical practices, was also high, 81.57. These results are in contrast to Walter and colleagues (2015), who found that STEM faculty were more likely to score low on these factors, as shown in Table 6.1. The majority of participants (65%) in this study also had high expectations for active student participation. A sizable number of participants (43%) both had high expectations and felt like *all* of their students met these expectations most days. Perhaps mathematics education at the tertiary level is changing, or perhaps instructors of mathematics content courses for elementary teachers prioritize using more student-centered instructional practices because of the nature of the course and the future careers of these students36.

Table 6.1

Mean and Standard Deviation of Scores on Five Factors in the Postsecondary Instructional Practices Survey

Factor	Instructors of Mathematics Content Courses of Elementary Teachers in This Study			Mathematics Instructors in Walter et al.'s (2015) Study		
	n	Mean Score	Standard Deviation	*n*	Mean Score	Standard Deviation
1) Student-student interaction	433	72.83	19.37	88	49.01	27.90
2) Content delivery	432	51.37	22.04	86	69.11	24.41
3) Formative assessment	433	61.28	17.06	87	59.60	21.43
4) Student-content engagement	433	81.57	15.40	87	59.36	19.00

Policy documents have also raised concerns that mathematics content courses for elementary teachers are treated as remedial mathematics courses (Greenberg & Walsh, 2008). The results from the IPCU survey indicate that this is primarily not the case. Only 0.22% of participants focused on the review of calculation procedures to the exclusion of connections to teaching mathematics in elementary school and the fundamental mathematical ideas underlying these procedures. By and large participants reported using instructional strategies that support a high cognitive demand, where students struggle to make sense of ideas themselves, even in the face of difficulty. The majority of participants also reported positioning their students as mathematical authorities by pursuing students' novel, incomplete, or incorrect ideas and inviting other students to comment on the legitimacy of those ideas. Hart and colleagues' (2013) study brought important attention to the idea that these courses may lack connections to elementary school. The IPCU survey, however, suggested this is not the case in the majority of courses. Over 90% of respondents indicated that they made connections to teaching mathematics in elementary school. In fact, more than 80% of the instructors in each subgroup studied reported making these connections, even those whose terminal degree was not related to education, even those with no experience teaching PreK–12 school children.

However, some of the findings also indicate that many instructors are not using research-based instructional practices that are known to be effective (Bain, 2004; National Research Council, 2015) or are not modeling high quality instruction for their prospective teachers (Association of Mathematics Teacher Educators, 2017). In particular, there was a high percentage of participants who did not prioritize the use of formative assessment. Forty-seven percent (47%) of participants indicated that using assessment to guide their instruction was only somewhat, minimally, or not at all descriptive of their teaching. One-third of participants indicated that using student questions or comments to inform class discussion was only somewhat, minimally, or not at all descriptive of their practice. A set of participants also reported using practices that lower the cognitive demand. Specifically, 26% of participants indicated that demonstrating how to solve problems before students attempted problems was "mostly" or "very" descriptive of their teaching. Relatedly, 19% indicated that it was "mostly" or "very" descriptive of their teaching to "clearly explain the steps for how to solve a problem" when students had difficulty. Additionally, not all instructors held their students responsible for actively participating. Thirty-five percent (35%) of participants described full participation as passive participation, or did not expect *all* students to actively participate, or did not think student participation was important. Regardless of their definition of full participation, a number of instructors felt they were unable to get all students to participate most days. Among those who defined participation passively ($n = 137$), 39% indicated that not all of their students participated on most days. Among those who defined participation actively ($n = 259$), 34% indicated that not all of their students participated on most days.

Many instructors in the sample reported that their instruction was aligned with recommendations for the mathematical education of elementary teachers (Association of Mathematics Teacher Educators, 2017; Conference Board of the Mathematical Sciences, 2012) and research on effective instruction (Bain, 2004; National Research Council, 2015; Schoenfeld, Floden, The Algebra Teaching Study, et al., 2014). Their courses focused on important mathematics. They focused on conceptual ideas and made connections to teaching mathematics to elementary students. They offered pre-service teachers opportunities to engage in mathematical practices, according to the student-content engagement factor. They provided opportunities for students to interact with each other, as indicated by their use of small groups and the high scores on the student-student interaction factor. They engaged in instructional practices that supported cognitive demand. The instructors positioned their students as mathematical authorities by pursuing students' novel or faulty ideas and inviting other students to comment on the legitimacy of these ideas. However, maintaining cognitive demand, broadening access by holding students accountable for actively engaging during class time, and using assessment to inform instruction were less aligned with policy recommendations.

The use of instructional practices differed by instructor characteristics. The results are summarized in Table 6.2. The most consistent predictor of instructional practices was the subject of a participant's terminal degree. Compared to participants whose terminal degree was in mathematics, participants whose terminal degree was in mathematics education or another discipline spent less time having students listen to the instructor and more time having students work in small groups. They were more likely to address the reasons behind procedures and make connections to elementary school, and less likely to review computation procedures. They engaged pre-service teachers in mathematical practices as measured by the student-content engagement score. They scored higher on instructional practices that support cognitive demand and lower on practices that lower cognitive demand. They were more likely to expect all of their students to actively participate. They reported using formative assessment. While both groups of instructors were likely to pursue student's novel, incomplete, or incorrect ideas, instructors with their terminal degree in mathematics education or a discipline other than mathematics were more likely to invite other students to comment on student's novel ideas.

Table 6.2

Instructor Characteristics and Instructional Practices

	Subject of Terminal Degree		Departmental Appointment		Level of Terminal Degree		PreK–12 Teaching Experience		Perceived Selectivity	
	Mathematics	Mathematics Education or Other Discipline	Mathematics Department	School of Education or Other Department	Doctorate or Pursuing Doctorate	Masters or Bachelors	PreK–12 Teaching Experience	No reK–12 Teaching Experience	Very or Moderately Selective	Not Selective
General Instructional Practices										
Listening to instructor	■		■			■				■
Working individually						■				
Small Groups		■		░	■					
Student-student interaction score		■		■	■		░		■	
Five Dimensions of Mathematically Powerful Classrooms										
Mathematics										
Content										
Review of procedures	■					■				■
Reasons behind procedures		■								
Connections to elementary school		■					■			
Practices										
Justification, reasoning, proof					░		■			
Student-content engagement score		■		■	░				■	
Cognitive Demand										
Supporting CD		■		■	■		■		■	
Lowering CD	■		■			■				
Access to Mathematical Content										
Define participation actively		■								
Achieve active participation among all students				■			■			
Authority, Agency & Identity										
Inviting students to comment on other student's novel ideas		■		■	■					
Assessment										
Formative assessment score		■					■			

Note. Black indicates a statistically significant difference between the two subgroups according to a two-tailed t-test. The group that scored higher or spent a greater percent of time is indicated in black. Gray indicates there was a statistically significant difference according to a one-tailed t-test.

Since there was such a strong relationship between the subject of participants' terminal degrees and their departmental appointment, it followed that most of these difference occurred among participants appointed to mathematics department and participants appointed to schools of education, participants with a joint appointment, or appointed to other departments. However, there were no statistically significant differences between these two groups in terms of the mathematical content of the course. There was a statistically significant difference between the percent of instructors in each group who felt they achieved active participation among all of their students most days. Participants with joint appointments, appointed to schools of education, or appointed to departments other than mathematics were more likely to feel that they achieved broad, active participation most days. Research has shown that when all students are expected to contribute to the learning of the class, stronger communities develop and more mathematical learning can occur (Boaler & Staples, 2008; Schoenfeld, Floden, The Algebra Teaching Study, et al., 2014).

The level of one's terminal degree also influenced many instructional practices. Participants who held a doctorate or were currently pursuing their doctorate were more likely than those whose terminal degree was a master's or bachelors to engage in instructional practices aligned with recommendations. Participants with a doctoral degree or working toward their doctorate spent less time having students listen to the instructor and more time having students work in small groups. They scored higher on student-student interaction. They were less likely to report reviewing computational procedures. They were more likely to engage in instructional practices supporting cognitive demand

and less likely to engage in practices lowering cognitive demand. While both groups were likely to pursue students' novel or faulty ideas, participants with their doctorate or working toward their doctorate were more likely to invite students' to comment on each other's novel ideas.

Having PreK–12 teaching experience also influenced participants' instructional practices. Those with PreK–12 teaching experience were more likely to address common elementary student misconceptions or mistakes in their course, compared to those participants without PreK–12 teaching experience. Participants with PreK–12 teaching experience also scored more highly on student-content engagement, supporting cognitive demand, and using formative assessment. The difference between the two groups on addressing representations used in elementary school was not statistically significant.

Instructors' perceived selectivity of their institution, based on the caliber of their students in mathematics courses for elementary teachers, influenced instructors' practices as well. Students in institutions that the instructor perceives as selective may be receiving a mathematical experience that is both mathematically more demanding and more engaging. Participants who reported that their institutions were not selective spent more time having students listen to the instructor and scored lower on student-student interaction. They also were more likely to review procedures and were less likely to engage in mathematical practices as measured by the student-content engagement factor. These instructors were also less likely to use instructional practices that supported the high cognitive demand of tasks. This difference is concerning and warrants further research. It suggests that the gulf between prospective teachers who appear well prepared and those who do not may be widened through their mathematical experiences rather than narrowed. It also prompts one to wonder whether the two different groups of students are exposed to different models for teaching mathematics.

While there were differences among groups, it is important not to overstate these differences. First, across all groups, participants scored high on many of the practices, such as student-content engagement and student-student interaction. Second, the variation from instructor to instructor *within* these groups may be more important than the differences between different groups. For example, the box and whisker plots in Figure 6.1 highlight this variability. While participants with their doctorate in mathematics education clearly spent less time having students listen to the instructor as a group, there are participants in each subgroup that spent little to no time with students listening to the instructor. There are participants in each subgroup who spent 40% of the time or more with students listening to the instructor. Likewise, Figure 6.2 shows the distribution of student-student interaction scores. While participants with a doctorate or pursuing a doctorate in mathematics education still scored higher on this factor, the differences between the distribution of scores among those with a masters in mathematics education and those with a doctorate in mathematics is not overwhelming. This spread exists in other measures as well.

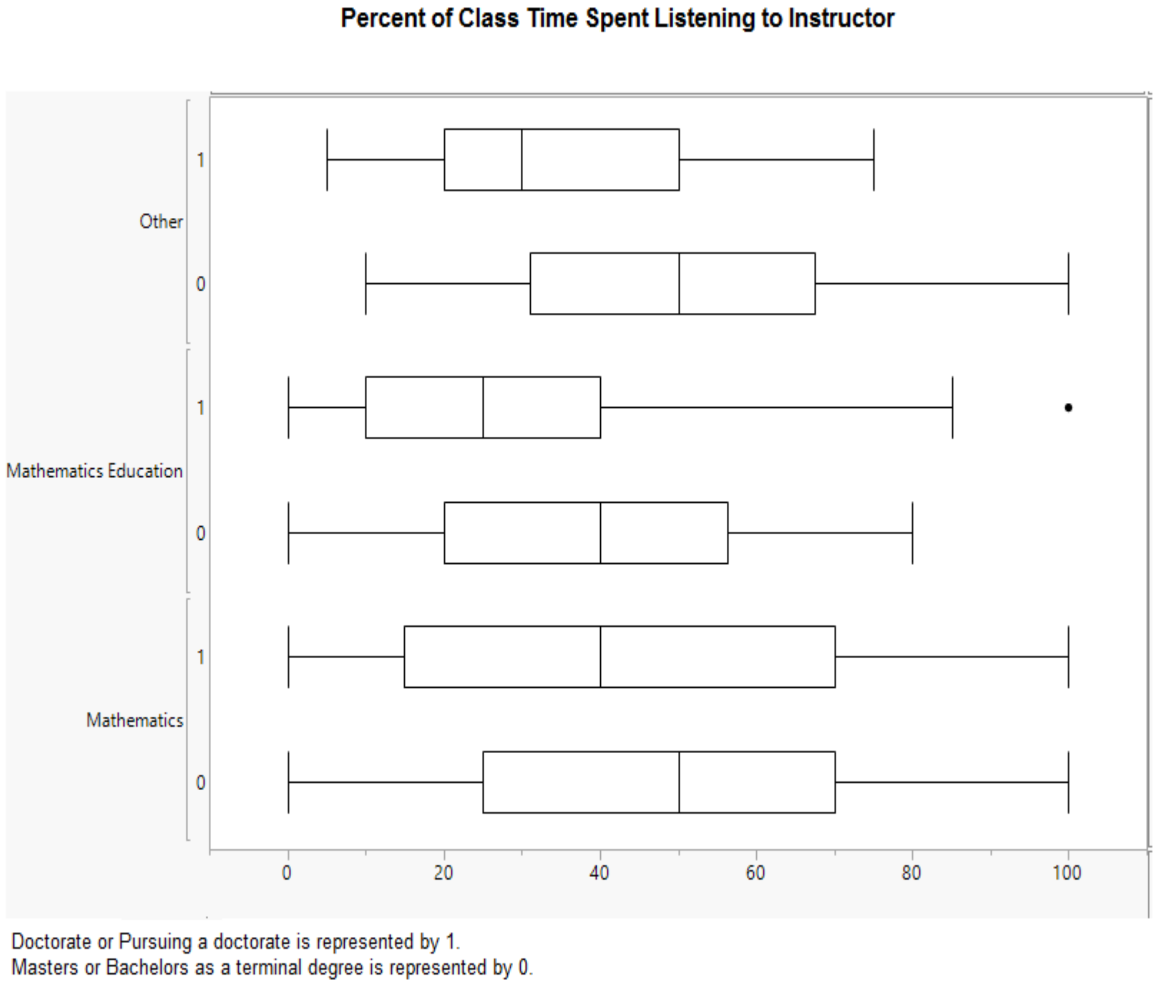


Figure 6.1. Percent of class time spent listening to instructor, by level and subject of terminal degree

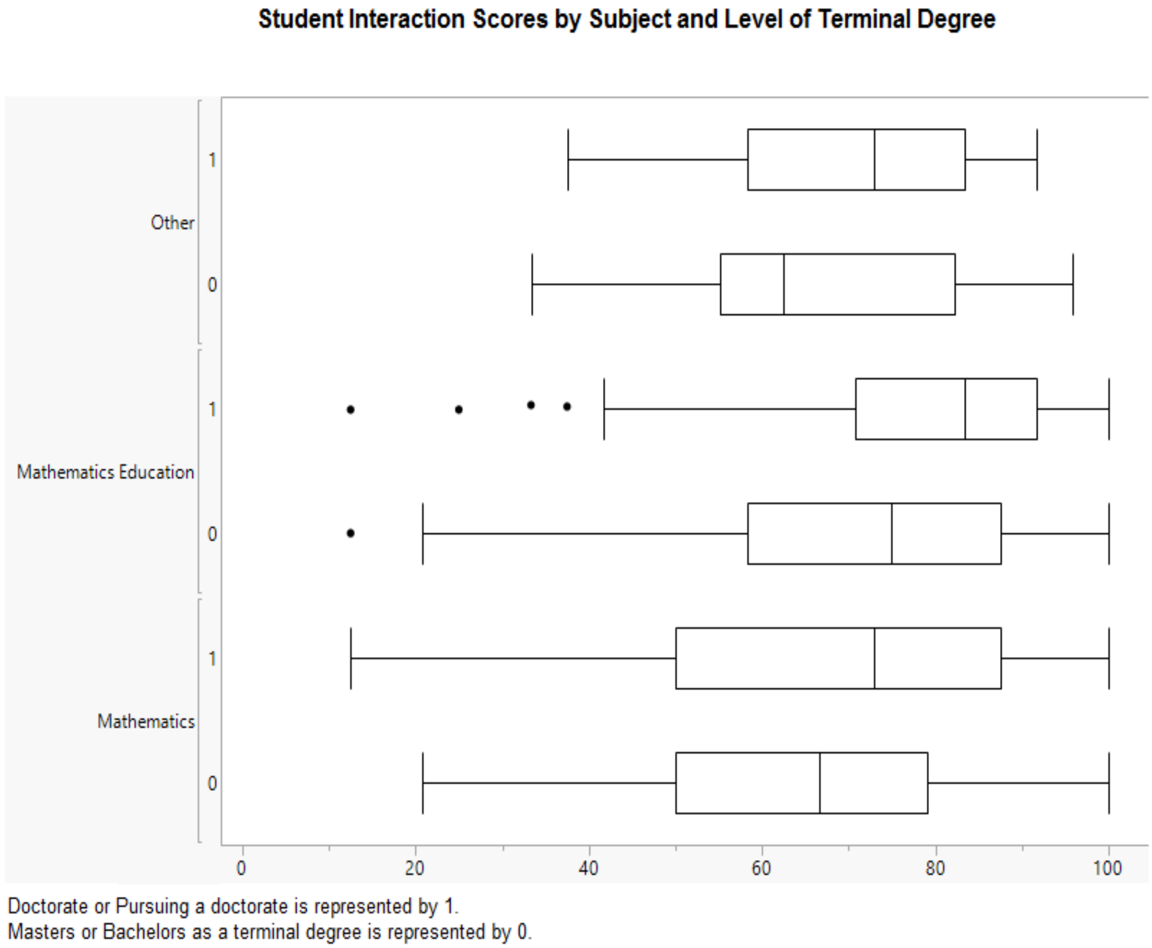


Figure 6.2. Student interaction scores by subject and level of terminal degree.

Some additional details about the instrument can inform interpretations of these results when looking at comparisons between instructors of different characteristics. First, this study used aggregate scores, rather than factor scores, for the items using Likert statements. This was for two purposes: (1) aggregate scores can be more intuitively understood (DiStefano, Zhu, & Mindrila, 2009; Walter et al., 2015) and (2) aggregate scores allowed me to compare my results to the results of Walter and colleagues (2015) on instructors of college mathematics courses. The correlation between the factor scores and the aggregate scores on student-student interaction, student-content engagement, and lowering cognitive demand, as shown in Table 6.3, support using aggregate scores. The correlation between the factor scores and the aggregate scores for these three factors are all above 0.95. However, the correlation between the factor scores and the aggregate scores for content delivery, formative assessment, and supporting cognitive demand are lower than 0.95, suggesting there is some measurement error.

Table 6.3.

Correlation Between Factor Scores and Aggregate Scores

Factor	Correlation
Student-Student Interaction	0.9884529
Content Delivery	0.8730681
Formative Assessment	0.8451788
Student-Content Engagement	0.9889478
Supporting Cognitive Demand	0.9294196
Lowering Cognitive Demand	0.9654208

The content delivery factor, which measures more traditional teaching practices, was a four-item factor primarily driven by two items with high factor loadings. This may be related to the lack of alignment between factor scores and aggregate scores. The supporting cognitive demand dimension may have been subject to a ceiling impact; there as little variability and most of the participants indicated that many of practices were descriptive of their teaching. Another hypothesis is that there as a second, overlapping factor within this factor, since the items about responding to student difficulty had lower factor loadings than the more general items. Further research could investigate these relationships.

The formative assessment factor also had a correlation less than 0.95 between the factor scores and the aggregate scores. There were few statistically significant differences between different subgroups on the mean score for this factor, and this sample did not substantially differ from Walter and colleagues (2015) mathematics instructors on this factor. Using factor scores may have found additional differences. An item-level content analysis provides more detail about the nature of this factor. Specifically, there seem to be several constructs within this factor: (1) the use of formative assessment to guide instruction (items 06 and 08), as opposed to a form of communication with students; (2) the frequency of formative assessment (items 04 and 18); the role of formative assessment in grades (items 18 and 20). Item level comparisons showed that there were statistically significant differences between various subgroups on some of these items and virtually no difference on other items. In future research, I will investigate this in more depth. Even with these issues, it is clear that there is significant variability in pre-service teachers experiences in mathematics content courses for elementary teachers, some of which is related to instructor characteristics.

The variability pre-service teachers experience in their college-level mathematics courses extends to curriculum as well. Although 36% of the sample used one of the two most popular textbooks on the market, 18 other textbooks were chosen as a primary textbook. This figure does not include the participants who indicated that they drew from multiple textbooks. This result is aligned with results found by earlier studies (Greenberg & Walsh, 2008; McCrory et al., 2009). Additionally, over 15% of survey respondents indicated that they did not use textbooks specifically designed for such courses. Using published curriculum specifically designed for these courses is associated with pre-service teachers learning more mathematical knowledge for teaching, so this fact is notable (McCrory et al., 2009). However, Jeppsen (2010) found that collaboratively developed materials can lead to more student-centered instruction, depending upon the nature of the collaboration. Whether it be instructional practices or the curriculum used, there is a substantial amount of variation in the mathematical education of pre-service elementary teachers.

Question 2

How can a curriculum for mathematics content courses for pre-service elementary teachers support instructors in creating mathematically powerful experiences for prospective teachers?

The research on mathematics curriculum use in higher education is scarce. Studies on the use of inquiry curriculum in higher mathematics classes have found that instructors' beliefs in the nature of mathematics and knowing mathematics can support their commitment to using the curriculum (Johnson et al., 2013; Johnson & Larsen, 2012; Speer & Wagner, 2009; Wagner et al., 2007). These studies have also found that instructors' pedagogical content knowledge can make implementing inquiry curriculum challenging. However, these small-sample case studies were conducted in higher level mathematics courses which typically enroll “math students” (Mesa & Griffiths, 2012, p. 96). As Mesa and Griffiths’ larger study found, instructors’ perception of their students influence their use of more traditional textbooks. Since pre-service elementary teachers are a special population, this study fills a gap in the literature on how instructors of mathematics content courses for elementary teachers use curriculum. There are only two other existing studies that look at the instruction and the curriculum use in such courses. The first considered primarily what content was skipped or modified, but did not address instructional practices (Lo et al., 2008). The second, Jeppsen's (2010) study of four community colleges, suggests there is a relationship between the way a department portrays a curriculum, the resulting commitment instructors feel towards a curriculum, and the degree of student-centered instructional practices. However, Jeppsen’s study used a macro-level lens, whereas this study described instruction in detail and identified the elements of a curriculum that instructors felt supported them in creating mathematically powerful experiences for their pre-service teachers.

The class sessions observed in this case study demonstrated several mathematically powerful instances (Schoenfeld, 2014). When asked about their instructional actions in these instances, the participating instructors explained how the curriculum supported them in these moments. In particular, the curriculum supported the participants in four ways that were aligned to Schoenfeld and colleagues’ (2014) dimensions of mathematically powerful teaching. In the *mathematics* dimension, the participants were supported in two ways. First, the EMP curriculum materials helped the instructors to focus on mathematical relationships. Second, the curriculum materials helped the instructors to support pre-service teachers in using mathematical language, which included attention to the mathematical practices of communication, justification, and representation. In regards to the *cognitive demand* and the *agency, authority, and identity* dimensions, the participants felt supported in holding pre-service teachers responsible for engaging in rigorous mathematical work. This mathematical work included explaining ideas, devising and pursuing their own problem solving strategies, critiquing each other's reasoning, and completing cognitively demanding problems. The fourth way in which the participants felt supported was in assessing and using pre-service teachers' thinking, aspects of the dimension, *use of assessment*. The last dimension of mathematically powerful classrooms, *access to mathematical content*, measured the extent to which an instructor achieved broad, meaningful participation. This study did not identify ways in which the instructors felt the curriculum materials supported them in this dimension, but this may have been due to the limitations of the study design.

The participants in this study focused their instruction on mathematical relationships. These relationships were various. One prominent relationship in both case studies was the relationship between terms in geometric formula and features of geometric objects. Over several lessons, Dr. H.'s students examined the relationships among the components of geometric solids. They articulated the connections between the solids and their nets. Students in Dr. C.'s class revisited the relationships among different quadrilaterals. They used relationships based on the decomposition of different two-dimensional figures to justify area formulae. The portrayal of mathematics as a domain of relationships, rather than consisting of isolated facts and procedures, is a key element of the *mathematics* dimension of mathematically powerful classrooms. These relationships were one way that conceptual understanding of important mathematical ideas, another aspect of the *mathematics* dimension, was developed by the curriculum.

In both case studies, participants supported their students in using mathematical language. This practice occurred in four ways. First, both participants supported their students in using mathematical terminology. There were questions with the EMP lessons focused on vocabulary. Second, Dr. H. in particular pushed for precision in articulating ideas. There were specific prompts for students to articulate ideas within the lessons. Both participants engaged students in mathematical argumentation. Dr. C. noted that the structure of the problems, from specific cases to general, supported her in accomplishing this goal. Both participants supported pre-service teachers in using variables to represent relationships and ideas, which was a focus of the geometric measurement lessons. These aspects of using mathematical language are related to the mathematical practices of communication, representation (National Council of Teachers of Mathematics, 2000), and justification (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Engaging in these mathematical practices is related to the *mathematics* dimension of mathematically powerful classrooms.

Both participants in this study also held their students responsible for engaging in rigorous mathematics. Dr. H. insisted that her students explain ideas, use precise language, and correct each other’s mistakes. Dr. C. insisted that her students devise and pursue their own solution strategies and complete cognitively demanding problems. In these instances, the participants did not "scaffold away the challenge" (Schoenfeld, Floden, & The Algebra Teaching Study and Mathematics Assessment Project, 2014); they did not do the mathematical work for the pre-service teachers. Thus, these instances were illustrative of the *cognitive demand* dimension of mathematically powerful classrooms. Additionally, these instances were examples of the dimension *authority, agency, and identity.* In these moments, students were the ones who were responsible for determining whether an idea was mathematically legitimate. Students had agency to develop and pursue their own solution strategies, even if these strategies were inefficient or unexpected. It appeared that the expectation that students make sense of ideas themselves, as an overarching design principle, as well as the focus on foundational ideas supported the participants in their endeavors. Additionally, the fact that many problems supported "divergent thinking" – had multiple solution methods and points of entry – supported Dr. C. in insisting that pre-service teachers develop and pursue their own solution strategies.

Both participants assessed and used pre-service teachers' thinking in their instruction. Dr. H. explained that when she insisted on precision, used teacher discourse moves, or took her students' explanations literally, she was assessing their understanding. Dr. C. explained that when she approached a group or individual and asked a broad, general question, she was assessing their understanding. She attempted to withhold her own comments in the beginning of an interaction to learn more about how her students were thinking about an idea. Both participants used mistakes or confusion in their instruction. Dr. H. used mistakes or misconceptions as opportunities to engage the rest of the class in articulating a mathematical relationship. Dr. C. directly addressed the whole class or individual students about confusion that would impede their progress on a task. Both of the instructors cited the curriculum as supportive in helping them to assess student thinking. Specifically, Dr. H. indicated that the fact that the curriculum was designed to be used with class discussion and prompted pre-service teachers to examine their own thinking supported her to assess and make use of pre-service teachers' thinking. Dr. C. felt that the fact that fundamental ideas could be revisited supported her in having multiple opportunities to assess and address pre-service teachers' confusion. That is, the learning cycles supported her in assessing and making use of pre-service teachers' thinking.

The instructors did not identify the ways in which the curriculum materials supported them in their instruction with regards to the dimension of *access to mathematical content*. While the instructors did employ practices that broadened participation, they did not indicate that the curriculum supported them in these instructional actions. Studying the connection between curriculum materials and this dimension could be an area for further research.

The aspects of the curriculum that supported the instructors fell into three categories: (1) overall philosophy of the curriculum materials, (2) design of the lessons and units, and (3) design of individual problems. Dr. H., as a new user, was better able to identify the elements that supported her at the overall philosophy level. Dr. C., as a veteran user, was better able to identify the aspects that supported her at the unit, lesson, and individual problem level. Additionally, examination of the materials themselves provided further detail on the elements that the instructors identified. Contrasting their instructional actions when using EMP compared to their actions when using an alternative curriculum also illuminated the different aspects that were supportive. Combining the participants' insights, examples from the EMP curriculum, and counterexamples from the alternative curricula all provided a fuller picture of how the EMP curriculum materials supported instructors in creating mathematically powerful experiences for pre-service teachers in content courses. The elements that this study identified as supportive are shown in Figure 6.3.

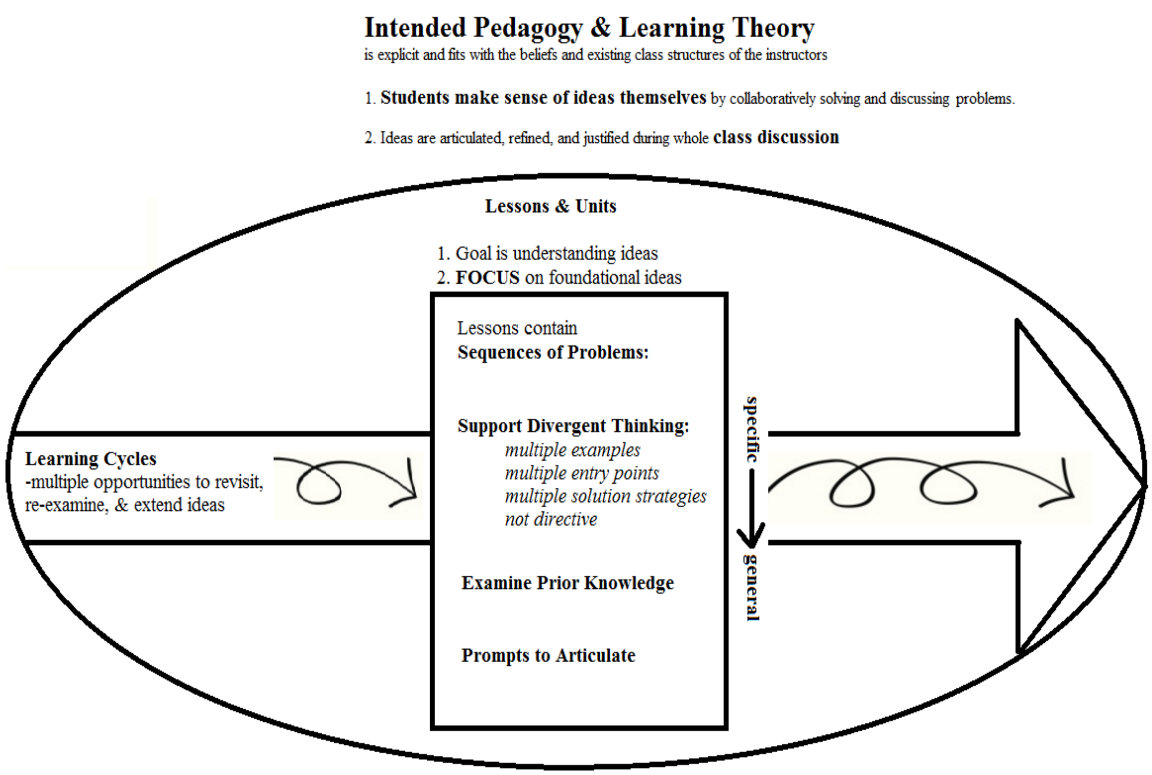


Figure 6.3. The elements of the EMP curriculum that supported the case study instructors in creating mathematically powerful experiences for their pre-service teacher

In Figure 6.3, the *intended pedagogy and learning theory,* social constructivism and the resulting use of class discussion, is the context within which these other elements are situated. It surrounds the other elements and features. At the lesson and unit level, two principles about the content were identified as supportive by the participants: the goal of understanding ideas and the focus on foundational ideas. These principles are a larger grain size than the aspects of individual problems, but as discussed earlier, individual problems, sets of problems, and sets of lessons can embody these principles. Thus, these principles about content surround the lessons and units. The rectangle at the center and the arrow speak to the structure of the lessons and units. The rectangle in the center represents individual lessons. Within a lesson, the problems moved from specific to general, as indicated along the right side of the rectangle. Features of individual problems within a lesson are listed within this rectangle. The arrow represents learning cycles that occur across these lessons and within lessons. Ideas are revisited from different perspectives over multiple lessons within a unit.

These elements can be understood at the programmatic level, the lesson and unit level, and at the level of individual problems, but they are all related. At the programmatic level, the overall philosophy of the materials was identified as supportive. The EMP materials were designed using sociocultural theory, the idea that learning is increased engagement in a social practice. One key practice in mathematics is sense-making. Therefore, the EMP materials were designed to help pre-service teachers make sense of fundamental ideas themselves and by talking with their peers. The lessons contained a series of problems to be solved in small groups, which led pre-service teachers to making sense of a mathematical concept. Students were then explicitly prompted to articulate that concept as they solved problems and discussed the whole class discussion questions. These problems led pre-service teachers to examine mathematical ideas through their engagement in the sociocultural practices of the mathematics community such as problem-solving, repeated reasoning, and critiquing arguments. Dr. H. referred to this design as "discovery learning," and "using class discussion." Her further explanations of her beliefs and of her experience with the materials demonstrated that she valued how the EMP lessons helped pre-service teachers make sense of mathematical relationships themselves. This design feature supported her especially in focusing on mathematical relationships and holding pre-service teachers responsible for engaging in rigorous mathematics.

This element must be understood within the context of this study. First, the intended pedagogy and learning theory of EMP was compatible with the case study participants' beliefs about learning mathematics and their existing class structures of small group problem solving and whole class discussion or student presentations. Second, the EMP authors made efforts to explicitly convey the intended pedagogy through instructor resources, both written materials and videos of enactment. Both instructors used these materials and thus their image of the intended pedagogy may have been impacted by them. This explicitness of intended pedagogy is in contrast with the alternative curriculum materials. The Billstein textbook contains exposition and worked out sample problems, suggesting direct instruction (Spielman & Lloyd, 2004). In the Instructor Resources Manual for the Beckmann textbook, the author provides general suggestions for facilitating class discussion, but she is more agnostic about whether students should read the exposition of mathematical ideas before or after the Class Activities. She suggests beginning class with a short "lecture" (Beckmann, 2014a, p. 5), but does not indicate whether such lectures should explain concepts or whether students should make sense of concepts through the activities. Therefore, it is not just that EMP is designed with the expectation that students make sense of ideas themselves through problem-solving and whole class discussion; it is that this intended pedagogy is explicitly conveyed to instructors and fits with their existing beliefs and class structures.

The elements at the lesson and unit level that the case study participants found supportive were two-fold: (1) the content of the lessons and units and (2) the structure of the lessons and units. Regarding the content of the lessons and units, there were two aspects. First, the goals of the lessons and units were about understanding ideas. While many curricula might purport to have understanding ideas as an objective, a look at the problems can indicate what "understanding" means to the curriculum authors. In the case of one alternative text, Billstein, the majority of the problems were about applying procedures that had been explicated in the exposition, as has been discussed earlier and can be seen in Appendix I. Dr. H. did not feel that Billstein was designed to have pre-service teachers make sense of mathematical ideas themselves.

The second aspect of the content of the lessons and units was the focus on foundational ideas. There were two dimensions to the notion of focus. First, the number of topics in the EMP materials units was fewer than the number of topics found in other curriculum for these courses. Dr. C. indicated that it helped her to see a "pared-down version" of the most important topics in order to find time to insist that her pre-service teachers engage in rigorous mathematical work. Likewise, Dr. H. indicated that the fact that EMP "takes the time to build understanding," which helped her to prioritize mathematical relationships. She often felt rushed in other courses by the number of topics she was expected to cover. Second, the lessons within a unit were built upon a few overarching foundational ideas. For example, the three lessons about prisms and surface area were built upon the idea of the structure of a prism. Defining prisms, computing surface area, recognizing the limitations and affordances of different formulae, drawing and labeling the dimensions of nets, and finding the number of vertices, edges, and faces on a prism were all worthwhile mathematical goals in their own right. However, these topics were in the service of helping pre-service teachers visualize solids, attend to individual features, and see the structure of prisms. Similarly, strategies for finding the area of irregular shapes and the conceptual underpinnings for the area formulae of parallelograms, triangles, and trapezoids are important components of mathematical knowledge for teaching. However, the problems in these EMP lessons were unified by more foundational ideas: the conservation of area and the properties of quadrilaterals. This focus on foundational ideas also allowed misconceptions about prerequisite knowledge, such as the height of a figure, to surface and be addressed. Thus, the focus helped the instructors to prioritize addressing mathematical relationships, hold pre-service teachers responsible for engaging in rigorous mathematics, and assess and use pre-service teachers' thinking.

There were two components of the structure of the units and lessons that the participants found supportive: (1) cycles of enactment and (2) the sequence of problems from multiple, varied, specific examples to more abstract and general ideas. The cycles of enactment were designed using Simon's work (1994) on learning cycles. In a learning cycle, there are three phases: the exploration phase, the concept identification phase, and the application phase. The application phase causes the learner to reexamine the concept in a new light, leading to a new cycle. The EMP authors indicate that there are one to three cycles in each lesson, but this study found that there are learning cycles across lessons as well. For example, the Quadrilaterals lesson, which was taught before the observed class sessions, had students first explore the properties and classifications of quadrilaterals. This cycle concluded with diagrams showing the relationships among different quadrilaterals based on their properties, the concept identification stage. Drawing the parallelograms on grid in the Parallelograms lesson was an example of the application stage. Students had to apply the properties of parallelograms and their knowledge of the relationships between rectangles and parallelograms to draw these figures. This led to a deeper examination of the relationship between parallelograms and rectangles and their properties. Dr. C. found that these cross-lesson learning cycles helped her to focus on mathematical relationships and to assess and use pre-service teachers' thinking. The learning cycles were a supportive structure both within individual lessons and across lessons within a unit.

A structure of the organization of problems within the lesson supported the participants as well. Dr. C. noted that there were multiple, varied specific examples that pre-service teachers worked with before using repeated reasoning to come to general conclusions. For example, students drew and decomposed five different parallelograms with a height of three units and a base of four units before justifying the area formula generally. The students noticed a pattern in their recompositions and discussed their figures specifically when justifying an area formula for all parallelograms. This element helped Dr. C. to support her pre-service teachers to engage in mathematical argumentation and use variables as they justified the different area formulae. While Dr. H. did not explicitly mention this element as supportive, it was present in her instruction. During the surface area lessons, students considered the prisms they had built from 8 ½ by 11 inch paper and the nets they had drawn of different prisms. They used repeated reasoning about these specific examples to generalize that the lateral surfaces of prisms were constructed by a rectangle whose dimensions were related to the height of the prism and the dimensions of the base. Therefore, this structure of using repeated reasoning about specific examples to identify general mathematical relationships seemed to have supported Dr. H. as well.

Aspects of individual problems themselves also supported the case study instructors. In particular, Dr. C. found that the fact that the problems supported "divergent thinking," helped her to insist that her pre-service teachers engage in the rigorous mathematical work of developing and pursuing their own solution strategies. The problems that supported divergent thinking had multiple solution strategies, such as the solutions to finding the area of an irregular figure in Figure 5.10 on page 255. These problems also had multiple examples. In some cases, the pre-service teachers developed the examples themselves, as in the case of the parallelogram problem described in the earlier paragraph. In other cases, pre-service teachers were provided with a variety of examples that led to different constructions, such as the problem in Figure 6.4. These problems also had multiple entry points. For example, the problem in Figure 6.4 asked pre-service teachers to derive an area formula for a trapezoid using paper cutouts.

1. Cut out 2 copies of each of the following trapezoids from the end of this packet:
    - Right trapezoid
    - Isosceles trapezoid
    - Trapezoid

    Use the 2 copies of these trapezoids to derive a formula for the area of any trapezoid and explain why this formula makes sense.

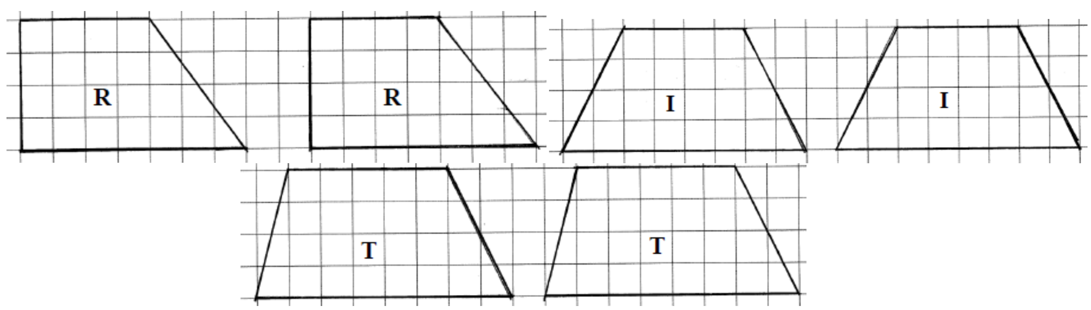


Figure 6.4. EMP problem supporting divergent thinking

When Dr. C. enacted the problem, two students set out to create models that supported a formula for the area of a trapezoid that they already knew. Other students looked for ways to construct rectangles because they were comfortable with finding the area of rectangles. These students used scissors to further decompose the trapezoids. The majority of students tried a variety of constructions, including hexagons and parallelograms, before they set down a solution path. Thus, there were entry points for those looking to make sense of their prior knowledge, for those who were able to identify their preferred strategy immediately, and for those who needed time to explore different options. Dr. C. contrasted the EMP problems with those in her alternative textbook that required the coordination of multiple. The EMP problems that supported “divergent thinking,” did not require as much prior knowledge to begin. Additionally, these EMP problems did not direct pre-service teachers to a particular strategy, in contrast to the Beckmann problem in Figure 5.9 on page 254. Problems that supported divergent thinking through multiple, varied examples, multiple entry points, multiple solution strategies, and that were not directive supported Dr. C. in holding her pre-service teachers responsible for engaging in the rigorous mathematical work of developing and pursuing their own solution strategies.

There were other problem-level features that supported the instructors as well. Several of the problems prompted pre-service teachers to examine their own prior knowledge, which supported Dr. H. in assessing and using her students’ thinking in her instruction. The problems were focused on mathematical relationships with explicit prompts to articulate these relationships. Thus, participants found themselves supported in focusing their course on mathematical relationships and had opportunities to encourage students to use mathematical language. Additionally, there were problems that focused explicitly on mathematical terminology, which also supported participants’ efforts to encourage pre-service teachers to use mathematical language. Problems and questions that focused on connecting variables to the features of geometric solids supported the participants in pushing pre-service teachers to use variables to expression general relationships. Problems that introduced variables after students had experiences with numeric examples may also have supported the instructors in this endeavor, though more research is needed in this area. The low threshold and high ceiling of the problems may also have supported the participants in identifying pre-service teachers' confusion about prerequisite knowledge.

The results of this study both corroborate and expand upon the design principles articulated by the EMP authors, particularly in regard to individual problems. The EMP authors identified four problem types that they used in the development of the materials: (1) problems that connect mathematical concepts and ideas; (2) problems focused on mathematical structure; (3) problems that support understanding why a rule, algorithm, or formula works, and (4) problems about elementary student thinking. There is overlap between these four problem types and the characteristics of problems that were identified in this study. Problems that focus on mathematical relationships is a broad category that includes the first two problem types. This study provides additional articulation of the third problem type. Some of the problems asked students to make sense of familiar formulae that were provided to them, the area formulae for parallelograms, triangles, and rectangles. In addition, students were asked to *generate* their own expressions that represented the constructions that they created in the Trapezoid lesson. Using variables to represent their own thinking proved to be a harder task than justifying an existing formula. While Dr. C. was aware of the fourth problem type, she did not find them particularly valuable in her instruction. However, she did value problems that supported "divergent thinking," described earlier. This element can be seen in several of the problems, but it was not explicitly a design principle stated by the EMP authors.

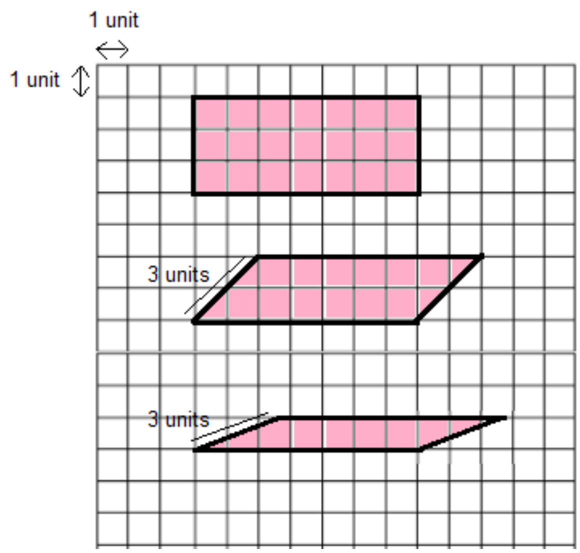
The EMP authors indicated that the lessons are designed to build on pre-service teachers' prior knowledge (Chapin et al., in review). Dr. H. reported that she valued that the materials lent themselves to students probing their own thinking. Several of the problems prompted pre-service teachers to examine their own prior knowledge. Furthermore, the explicit prompts for articulation and justification led to additional opportunities for pre-service teachers and their instructors to reflect on their thinking. Research indicates that addressing learners' prior knowledge is important (Bransford & National Research Council, 2000). This practice is particularly important for pre-service elementary teachers, who come with twelve or more years of mathematical learning. They have many conceptions about mathematics, some useful, some limited, and some incorrect (Browning et al., 2014). Furthermore, standards for mathematics teacher preparation stress the importance of helping prospective teachers develop the ability to build on their students' prior knowledge and modeling high quality mathematics instruction (Association of Mathematics Teacher Educators, 2017). Therefore, addressing pre-service teachers' prior knowledge in mathematics content courses is doubly important.

The EMP authors indicated in their description of the design that they build on pre-service teachers' prior knowledge, but this element is not called out as a particular design principle or problem type (Chapin et al., in review). Perhaps it should be. This study found that 35% of survey participants ($n = 432$) indicated that the statement "I structure my course with the assumption that most of the students have little knowledge of the topics" was mostly or very descriptive of their teaching. Educative curriculum materials like EMP may support instructors in learning about and using pre-service teachers' mathematical thinking.

Note that often I have drawn on the two alternative curricula to better illustrate the elements that the case study participants found supportive in EMP. I do this only for illustrative purposes; it can be easier to understand an object more thoroughly when examining counterexamples. Indeed, the case study participants likewise used their alternative curriculums during their VSR interviews to explain to me the EMP elements they found supportive. However, I do not make the claim that these elements are unique to EMP. In fact, in the Billstein section on Surface Area, 11 of the 66 problems were about general relationships or concepts, as opposed to applying procedures. For example, one problem read, "Reba says that if she takes a block of modeling clay and makes different shapes with it, the surface area always remains the same regardless of the shape she makes. How do you respond?" (Billstein et al., 2016, pp. 873, #14) These problems were not separated from the other problems in any way. Similarly, one activity in Beckmann could be an example of prompting pre-service teachers to examine their prior knowledge or conceptions. The activity used multiple examples to help pre-service teachers recognize that the area of a parallelogram is not determined by the length of its sides. This is reproduced in Figure 6.5 on the following page. These elements in the alternative curricula may have helped the instructors create mathematically powerful experiences with their pre-service teachers as well, but the impact of the other two curricula on instructors' practice is outside the scope of this study.

Class Activity 12H Do Side Lengths Determine the Area of a Parallelogram?

1. The three parallelograms below (the first of which is also a rectangle) all have two sides that are 3 units long and two sides that are 7 units long.



a. Use the moving and additivity principles to determine the areas of the three parallelograms.

b. Can there be a formula for areas of parallelograms that is only in terms of the lengths of the sides? Explain why or why not.

2. Find a formula for the area of a parallelogram *in terms of lengths of parts of the parallelogram.* Use the following parallelogram to help you describe your formula.

Figure 6.5. An activity from Dr. C.'s alternative text that focuses on a common misconception by providing multiple examples. (Beckmann, 2014b, p. CA–281)

While these elements might be present in other curricula individually, they were not all present. For example, both of the alternative curricula did not have the same degree of focus as EMP. Both Billstein and Beckmann contain many more topics that EMP; they are both “extensive” as opposed to “intensive” (McCrory, Siedel, et al., 2008). In addition, it is difficult to tell if Beckmann uses learning cycles in the same way as EMP. Both Billstein and Beckmann are portrayed as a resource from which instructors choose different activities and problems, so learning cycles may not appear during enactment. Beckmann explicitly described the materials in this way in the Instructor Guide:

> It is unlikely that you will have time to do all the Class Activities in class in the sections you cover, so most likely, you will have to pick and choose. I have labeled the activities that I view as most central and important with the “core” symbol to help you. Even so, please don’t feel that you must use all these core activities. It’s also fine not to do all the problems in a Class Activity. (Beckmann, 2014a, p. 4)

In contrast to the two alternative curricula, EMP was not portrayed as a resource from which instructors should “pick and choose” activities or problems. The two case study instructors also did not perceive or use EMP in this way. They intended to use lessons and units as a cohesive whole. The elements identified in this study must be understood within this context. It may be that these elements are not supportive in isolation from one another. Dr. C., for instance, explained how the focus of the lessons and units, one feature, allowed her to feel like she had time to insist that pre-service teachers engage in the rigorous mathematical work of developing their own examples and pursuing their own solution strategies, features of individual problems. Without the focus on the materials, problems that support divergent thinking may not provide support to instructors in the same way.

Additionally, the fact that the case study participants perceived and used EMP as a cohesive whole adds to the research on instructors' use of curricula. Past research indicates that instructors' perception of curricula influence how they use it (e.g., J. T. Remillard & Bryans, 2004). In regards to mathematics content courses for pre-service teachers, Jeppsen (2010) found that a departmental level of commitment to a textbook could impact usage of the textbook and the extent of student-centered instruction. This study suggests that the portrayal and use of a curriculum as a cohesive whole, as opposed to a collection of optional problems and activities, may also be related to the way college mathematics instructors use curriculum materials to create mathematically powerful experiences for their students.

The EMP curriculum supported the case study participants in creating mathematically powerful experiences for their pre-service teachers. In particular, it helped them to focus on mathematical relationships, support pre-service teachers in using mathematical language, hold pre-service teachers responsible for engaging in rigorous mathematics, and assess and make use of pre-service teachers thinking. These themes are related to four of the dimensions of mathematically powerful classrooms: *mathematics; cognitive demand; authority, agency, and identity; and uses of assessment*. More research is needed to determine if curriculum materials can also support instructors in *access to mathematical content* by helping them to broaden participation. The overall philosophy of the materials, the lesson and unit level design, and aspects of individual problems supported the participants in these endeavors.

Section II: Limitations of the Study

Any research study has limitations due to its design. In particular, there are factors that limit the generalizability of the results to the greater population. The conclusions of this study must be evaluated in the context of the sample and the measurements used.

1. Research indicates that surveys are a fair proxy for actual instructional practice, but they do miss detail and only provide a partial picture (Stecher et al., 2002). Surveys can measure an instructor's *perception* of his or her own instructional practice, but they do not measure what actually occurred in the classroom. Thus, the conclusions from the survey must be interpreted with the understanding that participants' reported practice may differ from their actual practice.
2. Survey research provides breadth, but is less effective at providing depth. Therefore, while we know that the survey respondents reported using particular instructional practices, there is less information about what their practices looked like in the context of their classes. The results of the survey should be considered in light of this knowledge. Further analysis of the open response questions may provide more information.

3. The survey was voluntary. Some populations may have been underrepresented. In particular, less than 11% of respondents were part-time or adjunct faculty. In comparison, Masingila and colleagues (2012) found that more than a third of institutions responding to their survey employed part-time faculty to teach all or some of their mathematics content courses for pre-service teachers. This may have affected the results from the survey.
4. When asking about the allocation of class time among different formats, the survey did not provide an option for "Whole Class Discussion/Student Presentation." Many participants wrote this in the "other" section. It may be that instructors use this class format at a higher rate than what is reported in this study. Future survey research should include "Whole Class Discussion/Student Presentation" as an explicit category.
5. The aggregate scores of content delivery, formative assessment, and supporting cognitive demand factors may have measurement error, which may influence the comparisons among the different groups of instructors. Further research should investigate if using factor scores identifies the same differences in the reported use of instructional practices within these three factors among different groups of instructors.
6. The formative assessment factor may have contained additional factors, such as using assessment to inform instruction, frequency of formative assessment, and using formative assessment to communicate with students. Therefore, there may be additional differences between various subgroups of instructors when looking

at these factors individually that did not appear in this study.

7. Case studies, by their nature, are not generalizable to the population at large. In particular, this study used a convenience sample. While the two participants differed on a number of characteristics, they both had experience working in K–12 schools. Their doctoral degrees were related to education. Instructors who come from a more traditional university mathematics background without connections to K–12 education may interact with curriculum materials in a different way. This study may not include the ways in which curriculum can support instructors from more traditional mathematics backgrounds without K–12 teaching experience.
8. Stimulated recall interviews have limitations. Some have suggested that they may not be able to measure tacit knowledge and that participants may offer post-hoc reasoning, though others have found that they generate important insights into in-the-moment decision making (Lyle, 2003). Therefore, the aspects of curriculum that participants felt were supportive may not be as supportive as they indicated after the fact. It may also be that there were ways in which the curriculum was supportive that participants were unable to articulate.
9. Only four class sessions were observed. There may have been too few observations to make generalizations about the instructional practices used by the instructors.
10. The session in which the case study instructors used alternative curriculum followed the class session in which they used EMP materials. Their use of EMP

materials may have impacted their instructional practices in these episodes. The instructional practices in these episodes may not be typical of their practice when using the alternative curriculum.

11. The last class observation with the first case study instructor was in April, toward the end of the semester. Her instructional practice when using her alternative curriculum may have been affected by the pressure of the end of the semester and therefore not be typical of her practice.
12. The researcher interviewed the case study instructors before and after each observation. These interviews could have impacted their instruction. For example, questions about what the instructors intended to do during class may have helped them to think in more detail about their instruction. Therefore, the observed class sessions may not have captured their typical practice.
13. The case study participants knew that the researcher had been involved in the development of the EMP curriculum. This may have impacted their responses.

Section III: Further Research

This study identified a number of trends. Based on the findings of the study and the limitations, the following recommendations for future research are made.

1. There were two open-response questions that were not coded in this study. Analyzing the responses to these questions may provide additional insight into how instructors address pre-service teachers' incomplete or faulty ideas.

2. This study identified whether instructor characteristics influenced the use of different instructional practices, but it did not measure the size of the impact these characteristics had. It also did not measure whether or how these instructor characteristics interacted. Future research could include the use of linear modeling techniques to assess the impact. Specifically, I will use multivariable regression with categorical predictors for the instructor characteristics and use a logit transformation of the percentages of the different outcome variables for use of class time, student-student interaction, student-content engagement, use of formative assessment, and supporting and lowering cognitive demand.
3. The sample for this study did not include a significant number of part-time faculty members who were teaching mathematics content courses to pre-service elementary teachers. Further research should seek out increased participation from adjunct or part-time faculty to determine if there is a difference in part-time faculty members' instructional practices in mathematics content courses for elementary teachers to better inform such decisions.
4. The case studies suggested that different curriculum materials led to differences in instructional practices. Further research should investigate the use of other curriculum materials used by the same instructor to more thoroughly describe this connection. Further research could also address whether a curriculum can change an instructors practice by conducting observations before and after the use of a particular curriculum.

5. The case study instructors both had experience in K–12 education. They shared similar beliefs about the nature of learning mathematics that were informed by these experiences. A future study could be conducted that included participants who come from more traditional university mathematics teaching backgrounds.
6. The case studies involved only four observed class sessions. Further research should include more observations. More observations could identify additional ways in which instructors use curriculum materials to create powerful mathematical experiences for their students. Alternatively, observations could be spaced throughout the semester. A more longitudinal examination of an instructor's practice could determine whether the curriculum materials support the instructor in developing a classroom community or increase students' engagement with mathematics over time. It could also identify ways in which the curriculum supports instructors in the *access* dimension of mathematically powerful classrooms.
7. One case study instructor indicated that her students' opinion on her uses of pedagogies, direct instruction versus the EMP model, changed over time. A future study should include measures of student perceptions.

APPENDICES

Appendix A: Interview Protocol for Instructors Participating in the Case Studies

Background Interview (60 minutes)

1. What are your goals for this course? (Follow up with specific questions about mathematical and nonmathematical goals if necessary.
2. What are your goals for your students?
3. Can you walk me through a typical class session? What do you tend to do? What do your students tend to do?
4. Please explain to me how you go about planning for a lesson.
5. Do you ever use the curriculum materials in your planning? Please describe how to me.
6. Do you ever use ________ (list each ancillary material) when you prepare? How so?

Pre-Observation Interview (20–40 minutes)

1. Can you tell me about your goals for this class session?
   If goals are affective, social, or pedagogical, also inquire about mathematical content goals.
2. What is your plan for today?
3. Is there anything else I should know prior to observing this class (such as previous topics discussed or recent challenges)?

Post-Observation Interview (10–20 minutes)

1. What are you initial thoughts about how class went?
2. Was this class atypical in any way?

Stimulated Recall Interview (90 – 120 minutes)

There were a few moments in class that I wanted to discuss with you. (Play relevant video).

1. Please tell me a little about how or why you decided to do this (describe instructional action).
2. Is there anything in the curriculum materials that contributed to your decision?

I've noticed these elements in your teaching/these elements of your class. (Provide a written list of the structures or elements of the class with qualitative detail.)

3. If you had to map these elements to the curriculum you use, or something else in your experience, how would you do that?

Appendix B: Instructional Practices and Curriculum Use Survey

Thank you for participating in this survey on instructional practices and curriculum use in college-level mathematics content courses for preparing elementary teachers. The survey asks multiple choice and short answer questions and should take 20–40 minutes to complete.

There are no known risks to you if you decide to participate in this survey. There is no direct benefit to you for participating in this study. The alternative would be not participating in the study. We will not share any information that identifies you with anyone. The survey is anonymous.

We will do our best to keep your information confidential. All data is stored in a password protected electronic format. To help protect your confidentiality, the surveys will not contain information that will personally identify you. The results of this study will be used for scholarly purposes only and may be shared with Boston University representatives.

We hope you will take the time to complete this questionnaire; however, if you agree to complete the survey you are not required to answer all the questions or complete it. Your participation is voluntary and there is no penalty if you do not participate. If you have any questions or concerns about completing the questionnaire, about being in this study, or to receive a summary of our findings you may contact us at 339-224-4812 or at lkcallis@bu.edu. We hope to publish the results in a national journal.

By clicking the ">>" button, you indicate that you teach a mathematics course to prepare elementary teachers and that you consent to participate in the survey.

The following survey is about **mathematics content courses for elementary teachers** – that is, courses in which the focus in on pre- or in-service teachers learning mathematics relevant to teaching K–8 mathematics. This survey is not about courses that primarily focus on methods of teaching mathematics, or courses about pedagogy.

Q1 Have you ever taught mathematics content courses for elementary teachers, as described above?

- Yes
- I have taught courses that combine content and pedagogy.
- No

[If *No* is selected, participants are directed out of the survey with the message below.]

We appreciate your time. This survey focuses on mathematics content courses for elementary teachers. If you know instructors who teach these courses, we would appreciate it if you could forward thislink to them: [link to survey].

Q2 Please answer the following questions about your instruction in these content courses for elementary teachers. If you teach multiple sections, please respond about your typical practice in these courses or about the course you teach most frequently or have been teaching the longest. Think about the course section you are currently teaching, or have most recently taught.

Which of the following topics, if any, have you addressed in your classes? Please select all that apply.

- Reviewing calculation procedures
- The mathematical reasons why formulas or algorithms work
- The nature of justification, reasoning, or proof in mathematics
- Common misconceptions and mistakes of elementary students
- Representations used in elementary school classrooms or curricula
- None of these

Q3 Please indicate what proportion of class time during a typical week is spent in the following activities. The sum of these answers should equal 100%.

The instructor talking to the whole class. ___ %

Students working individually. ___ %

Students working in small groups. ___ %

Students doing something else. (Please specify:) ________ ___ %

Students doing something else. (Please specify:) ________ ___ %

Students doing something else. (Please specify:) ________ ___ %

Q4 Each of the 37 teaching practice items on the next two pages is a statement that may represent your current teaching practice. As you proceed through the survey, please consider the statements as they apply to your practice when teaching mathematics content courses for elementary teachers. Think about the most recent time you taught such a course.

Please read each statement, and then indicate the degree to which the statement is descriptive of your teaching in the math content courses you teach for elementary teachers. There are no "right" or "wrong" answers. The purpose of the survey is to understand how you teach, not to evaluate your teaching. [Participants select whether each statement is very, mostly, somewhat, minimally, or not at all descriptive of their teaching. The choices were displayed in drop down menus on mobile devises and in matrix form for computers. The statements were randomized. The numbers shown here were not displayed; they are provided for the reader only. The first statement was not part of the analysis. It is a buffer item.]

0. I emphasize important mathematics.
1. I guide students through major topics as they listen and take notes.
2. I design activities that connect course content to my students' lives and future work.
3. My syllabus contains the specific topics that will be covered in every class session.
4. I provide students with immediate feedback on their work during class (e.g., student response systems, short quizzes, etc.).
5. I structure my course with the assumption that most of the students have little knowledge of the topics.
6. I use student assessment results to guide the direction of my instruction during the semester.
7. I frequently ask students to respond to questions during class time.
8. I use student questions and comments to determine the focus and direction of classroom discussion.
9. I have students use a variety of means (models, drawings, graphs, symbols, simulations, etc.) to represent phenomena.

10. I structure class so that students explore or discuss their understanding of new concepts before formal instruction.
11. My class sessions are structured to give students a good set of notes.
12. I structure class so that students regularly talk with one another about course concepts.
13. I structure class so that students constructively criticize one another's ideas.
14. I structure class so that students discuss the difficulties they have with this subject with other students.
15. I require students to work together in small groups.
16. I structure problems so that students consider multiple approaches to finding a solution.
17. I provide time for students to reflect about the processes they use to solve problems.
18. I give students frequent assignments worth a small portion of their grade.
19. I require students to make connections between related ideas or concepts when completing assignments.
20. I provide feedback on student assignments without assigning a formal grade.
21. My test questions focus on important facts and definitions from the course.
22. My test questions require students to apply course concepts to unfamiliar situations.
23. My test questions contain well-defined problems with one correct solution.

24. I adjust student scores (e.g., curve) when necessary to reflect a proper distribution of grades.

Q5 Please read each statement, and then indicate the degree to which the statement is descriptive of your teaching in the math content courses you teach for elementary teachers. [Participants select whether each statement is very, mostly, somewhat, minimally, or not at all descriptive of their teaching. The choices were displayed in drop down menus on mobile devises and in matrix form for computers.]

- I explain mathematical ideas to students before having them attempt problems.
- I demonstrate how to solve problems before having students attempt problems.
- I facilitate opportunities for students to figure out ideas for themselves.
- I expect students to attempt problems they may not know how to solve.
- I use problems to introduce new ideas.

Q6 When students work on problems, they sometimes get stuck and seem unable to progress. In this situation, which would you say is most descriptive of your teaching? . [Participants select whether each statement is very, mostly, somewhat, minimally, or not at all descriptive of their teaching. The choices were displayed in drop down menus on mobile devises and in matrix form for computers.]

- I see how far they can get without my help, either individually or as a group.
- I listen and offer encouragement, but no direction on what to do.

- I clearly explain the steps for solving the problem.
- I ask questions that lead them through the steps for solving the problem.
- I ask questions that get students to articulate their thinking.
- I clearly explain the mathematics in the problem without telling them the solution steps.
- I ask guiding questions that direct students to the main mathematical idea.

Q7 When a student poses a faulty conjecture, procedure, or idea, what do you generally plan to do?

Q8 What counts as "full participation" to you?

Q9 Do you have strategies for ensuring that every student participates during class?

- o No.
- o Yes. (Please list two strategies:) ______________________________

Q10 Do you agree or disagree? Most class sessions, all of my students fully participate.

- o Agree
- o Disagree

Q10a Explain: (optional) ________________________

Q11 When a student brings up a novel idea, I generally:

- o indicate whether or not the idea is correct.
- o ask other students what they think about the idea.
- o explain how this idea fits in with other mathematics we have been covering.
- o other (Please explain:)________________________________

Q12 In the next question, we ask you to respond to a scenario that might occur in your class.

Some students brought up an idea in your class that was related to the topic you were addressing, but outside of what you planned to discuss. For example,

Your class is beginning to study prime and composite numbers. You have just asked students for definitions and written their definitions on the board. Mary adds: "Prime numbers are mostly small." Sam agrees: "Composite numbers are mostly large numbers."

What is the first thing you would do in response?

- o Nothing, since this idea is outside the topics for the course
- o Provide some counterexamples for the class to consider
- o Ask Mary to further explain her thinking
- o Correct the students
- o Ask other students to comment on Mary's idea
- o Other (please explain): _____________________

In the following question about textbooks, "use" refers to any kind of use by students or by you as the instructor. (E.g., as a reference for students, or a source of homework problems, for planning, or for any other purpose.) “Textbook” refers to publicly available texts. Textbooks may be online or print, free or for purchase.

Q13 Do you or your students use a textbook for any part of your course?

- o Yes
- o No

Q13a How many different textbooks do you use? ______

Q13b [If *Yes* is selected for Q13:] This course uses the textbooks in some way for:

- o Almost every topic covered in the course
- o Most topics covered in the course
- o A few of the topics covered in the course
- o The books are listed on the syllabus, but neither I nor my students tend to use them.

13c If you do not use a textbook, what materials do you use for your course?

____

Q13d What is the name of the primary textbook you use in class? [Drop down menu]

Bassarear's Mathematics for Elementary School Teachers
Beckmann's Mathematics for Elementary Teachers
Bennett's Mathematics for Elementary Teachers: A Conceptual Approach
Billstein's A Problem Solving Approach to Mathematics for Elementary Teachers
Burger & Starbird's The Heart of Mathematics
Chapin & Johnson's Math Matters: Understanding the Math You Teach
Darken's Fundamental Mathematics for Elementary and Middle School Teachers
Elementary Pre-Service Teachers Mathematics Program (EMP)
Heddens & Speer's Today's Mathematics
Jensen's Arithmetic for Teachers
Jones, Lopez & Price's A Mathematical Foundation for Elementary Teachers
Kaplan's Math on Call
Kutz, Lubell, & Burns Foundations of Mathematics
Long & DeTemple's Mathematical Reasoning for Elementary Teachers
Masingila, Lester, & Raymond's Mathematics for Elementary Teachers via Problem Solving
Miller, Heeren & Hornsby's Mathematical Ideas
Musser, Burger, & Peterson's Mathematics for Elementary Teachers: A Contemporary Approach
National Council of Teachers of Mathematics Essential Understanding Series
O'Daffer's Mathematics for Elementary School Teachers
Parker & Baldridge's Elementary Mathematics for Teachers
Parker & Baldrige's Elementary Geometry for Teachers
Sharhangi's Elements of Geometry for Teachers
Sonnabend's Mathematics for Teachers: An Interactive Approach for Grades K–8
Sowder, Sowder, & Nickerson's Reconceptualizing Mathematics
Van de Walle's Elementary and Middle School Mathematics: Teaching Developmentally
Wheeler & Wheeler Modern Mathematics for Elementary Educators
Wu's Understanding Numbers in Elementary School Mathematics
I use multiple textbooks equally.
I use textbooks designed for K–12 students.
Other

Q13e1 If other, or if you use multiple textbooks equally, please list the name and author of the textbook(s) here: ____________________________

Q14 How do you use your primary textbook(s)? (Choose all that apply.)

- As a reference for students
- As a source of homework problems
- As a source of activities during class
- For planning lectures
- For planning overall daily instruction
- For planning the overall course
- Other (please explain:) _______________

Q14a Do you use the instructor manual associated with this text?

- Yes.
- No.
- There is none/I'm unaware of an instructor manual.

Q14b If you do use the instructor manual, how helpful do you find this resource?

- Unhelpful
- Slightly helpful
- Moderately helpful
- Very helpful
- Extremely helpful

Q14c Please explain: ____________________-

Q15a [Questions 15a, b, and c were displayed for participants who selected Beckmann's Mathematics for Elementary Teachers with Activities, Billstein's A Problem Solving Approach to Mathematics for Elementary Teachers, or Masingila, Lester, & Raymond's Mathematics for Elementary Teachers with Problem Solving.]

The textbook you selected has an Activity or Exploration Manual, an additional text that contains problems and activities. These manuals may be physically bound to the back of the original text or they may be separate books.

Do you use the Activity or Exploration Manual associated with this text?

- o Yes.
- o No, I choose not to use it.
- o No, I was unaware of an Activity or Exploration Manual.

Q15b How do you use the Activity Manual? (Select one or both.)

- in class
- out of class

Q15c Please briefly explain how you use the Activity Manual: ____________

Q16a Do you use any of the online or multimedia resources associated with this text?

- Yes. (Please specify:) ___________________
- No, I choose not to use them.
- No, I was unaware of online or multimedia resources.

Q16c [If *Yes* is selected to 16a] How helpful do you find these online or multimedia resources?

  - Unhelpful
  - Slightly helpful
  - Moderately helpful
  - Very helpful
  - Extremely helpful

Q16d Please explain: _________________

Q17a Do you use other resources besides this textbook

- Yes, frequently
- Yes, occasionally
- Rarely
- Not at all

Q17b Please explain what other resources you use outside of your primary textbook:

____

This last section is about you and your students.

Q18 How many sections of mathematics content courses for elementary teachers do you teach each academic year, on average? (Enter a number.)

Q19 About how many students are in one section? (Enter a number.)

Q20 *Thinking about the caliber of your students in these classes*, how would you describe this institution or program?

- o highly selective
- o moderately selective
- o not selective or open enrollment

Q21a Please tell us more about your background. What is your highest degree?

- o Doctorate
- o Masters
- o Other (please explain:) _______________

Q21b What subject is your highest degree in?

- Mathematics
- Mathematics Education
- Other (please explain:) _______________

Q21c Are you appointed to a mathematics department or a school of education?

- Mathematics department
- School/department of education
- Joint appointment
- Other

Q21d Please indicate if you are a full-time faculty member or full-time post-doc, a part-time or adjunct instructor, or a graduate student.

- full time
- adjunct/part-time
- graduate student
- Other (please explain:) _____________

Q21e Have you ever taught in a PreK–12 school? Please select all that apply.

- No
- Yes, Elementary (grades PreK–5)
- Yes, Middle (grades 6–8)
- Yes, Secondary (grades 9–12)

Q22 Thank you for taking the time to tell us about your practice. We very much appreciate your input and wonder if you would be willing to answer one more question by responding to a short vignette that could hypothetically happen in your class. Would you be willing to take a few moments to answer one or more questions?

- Yes
- No

[If *Yes* is selected for Q22] Thank you so much for your additional time. For the following question, please first read the vignette. Then, please explain the next actions you would take in class. You may also wish to comment on the mathematics involved, but please be sure to explain what you as the instructor would do in class.

Your class is working on comparing fractions without using common denominators or decimals. In particular, they are trying to determine which is greater, $\frac{14}{19}$ or $\frac{13}{20}$.

In explaining her reasoning to the whole class, one student says "$\frac{14}{19}$ is greater than $\frac{13}{20}$ because it is 4.5 pieces over a half, and $\frac{13}{20}$ is only 3 pieces over a half."

How do you respond? Please explain the next actions you would take in class.

Appendix C: Coding Guidelines
for Active Versus Passive Participation for Survey Question 8

Score 1 if the response indicates the participant defines "full participation" as "active participation." Score 0 otherwise. Active participation could include:

- Sharing Ideas
- Responding to others
- Contributing to/engaging in/being a part of class discussion
- Helping others in small groups
- Discussing problems in small groups
- Working on problems with a partner
- Answering **and** asking questions
- Participating in group activities

There must be some indication that there is **interaction** occurring, not just individuals working alone.

The following are not sufficient to be coded as active participation; there must be something else in the list:

- Asking questions
- Listening
- Turning off cell phones
- Being awake and present

- Persevering
- Trying all problems
- Completing classwork or homework
- Writing answers on the board
- Making sense of mathematics
- Attendance

Also, if there are any hedges that would indicate a student would be considered "fully participating" if they were only listening or thinking, these should be marked as 0. Hedges include "whichever form," "as they are comfortable," "not necessarily speaking." Use of the word "or" frequently indicates a hedge.

Anything that is too vague to be labeled "active participation" is also scored 0. (For example, "active engagement.")

<u>Examples of responses coded "active participation"</u>

- Students should be both sharing their ideas and questions in class and listening and responding to the ideas and questions of others
- Persevere in group activities, share your reasoning, carefully attend to reasoning of others.
- Listening, asking questions (of peers and instructor), contributing ideas to the class

Examples of responses coded “passive participation”

- All students are engaged and working; they persevere.
- Participating in class in whichever form works for the student including listening, being prepared, engaging with ideas, etc.
- Active involvement in class.

Appendix D: Examples of Instructor Resources and Curriculum Resources Codes

In this section, I provide some example quotes from the VSR interviews to illustrate the coding of instructor and curriculum resources. Many of the quotes have been abridged, for space.

Instructor Resources

Beliefs: [Dr. H. in regards to the value of mistakes in learning.] "Sometimes I'll say to the student, 'Thank you for making that mistake because now I wanted to talk about that, that's good, oh, it's like I planned you made that,' … in making the mistake then you can correct the thinking or, you know, have them correct each other's thinking."

[Dr. C. in regards to how instructors must address misconceptions, in response to a clip where a student's description did not match her model.] "Putting a cap on something doesn't keep it from emerging later. But trying to get down to the root of why were you thinking this way, is it really the way that you were thinking or are you assigning words to it that are not the same as what you are really doing."

Knowledge: [Dr. H. in regards to her own advanced mathematical knowledge that helps her develop counterexamples and identify the legitimacy of alternative methods her students develop. She brings up an episode where a student got the right answer by estimating with the diagram.] "You know, obviously, I have to think about the legitimacy of some [alternative methods students devise]. There is one [instance] I think on day five that the student got the right answer but I don't know how they got it and I said well if you do it like that you're not going to get the right answer all the time, so you have to know enough math, obviously, as an elementary teacher to know when their methods are legitimate."

[Dr. C. in regards to her knowledge of her students' content knowledge.] "The other thing is they shun algebra. Any variability – they hate that. Like they don't want to find a way to notate it in such a way that we can reason through it."

Goals: [Dr. H. in regards to her goal of creating a safe environment where students feel mathematically capable and her instructional move of asking for additional volunteers instead of just calling on the first volunteer.] "And the rest of the class feels that they are not good enough. They're not good enough because 'oh, they [other students] always are called on, they always know how, I never know, so I never get called on, I never share because I have nothing important to say or what I'm going to say is wrong.'… The idea is to share ideas and then to have them sort out for themselves what ideas or what is right and what is not."

Goals: [Dr. C. in regards to a goal for a student to pursue her own creative ideas and experience messy problems.] "So that's why I let her go with the whole strategy of enclosing in a rectangle and looking at the missing pieces and what not. Just to support the idea that, yeah, it could be a valid strategy… I think our students have become so ingrained with textbook problems that make things so easy that they come to expect that everything is simplistic."

Curriculum Resources

Overall design: [Dr. C. in regards to her decision to insist students draw their own parallelograms, even with the pressure of time.] "I know it's important and I think it's communicated as important because the curriculum doesn't try to teach everything. It tries to teach a few things really well. And I think that being able to see a pared down version of exactly what, like, we really need to develop as foundational helps me to make decisions about what to pursue and what to cut."

Lesson Structure: [Dr. C. in regards to how the structure of moving from specific examples to general ideas helped her to support students in mathematical argumentation.] "There's a lot of taking the specific and making it more general which is, it fits with my teaching style anyway, but I think that those activities really support this idea that we look at things really in a specific concrete way and then try to generalize out from there."

Activities & Problems: [Dr. H. in regards to her use of teacher discourse moves in a whole class discussion about making sense of the formula for the surface area of a rectangular prism.] "EMP really helps you do that well, the activities that they have lend itself – lend themselves nicely to probing and having students probe their thinking. They are having the students really work with hands on and just questioning all the time."

Appendix E: Examples of Codes about the Five Dimensions of Mathematically Powerful Classrooms

The VSR interviews were coded for the five dimensions of mathematically powerful classrooms. These codes were used in combination with the codes and notes on the related video clips of classroom episodes. Below are some examples of quotes that were coded according to the five dimensions.

The mathematics:

(Sub-code: Relationships) "I wanted to reinforce those ideas that a rectangle is a parallelogram. They're special cases of parallelograms because they have additional features that not all parallelograms have."

(Sub-code: Practices: Representations) "So with my end goal of wanting to be able to come back together as a class and talk about things in a common way, we needed some common notation or some common ways of talking about the trapezoids."

Cognitive demand

"That's how patient problem solving, like – they're not always going to look so neat and clean. Sometimes they generate things that are not neat and clean, you know… And I think she can solve that one quickly, but I think that being able to see things from multiple perspective would benefit her."

Access to mathematical content: “I don’t want someone to give an answer, right or wrong, and have everybody starting thinking of, if I say that it’s right or wrong. I want them to continue thinking about it.”

Authority, agency, and identity: “I don’t want to be the fount of all knowledge that I pour into them.” “How you treat the student’s thinking, whether you treat it with respect or not is really important.”

Use of assessment: “I’m so glad I let her talk because what she was saying was not what I thought she was going to say… And it was powerful to be able to stop and listen because that led me to a different discussion than I would have had if we were just base 1 base 2 times height, end of story, right?”

Appendix F: Prominence of EMP Instructor Support Materials

The results of this study found that 66% of the 350 survey participants answering questions about instructor guides did not use them. Thirteen percent (13%) of these 350 participants indicated that either the curriculum they used did not have instructor guides or that they were unaware of instructor guides.

In contrast, the EMP materials put instructor support resources at the forefront of their web-based materials. The EMP landing page, after log in, features three documents that provide information about the pedagogy used with these lessons, as shown in Figure A.1. Within each of the lesson pages, the Instructor's Guide is the first document listed. Below the materials are three to five short videos of instructors enacting the lessons with written commentary, as shown in Figure A.2.

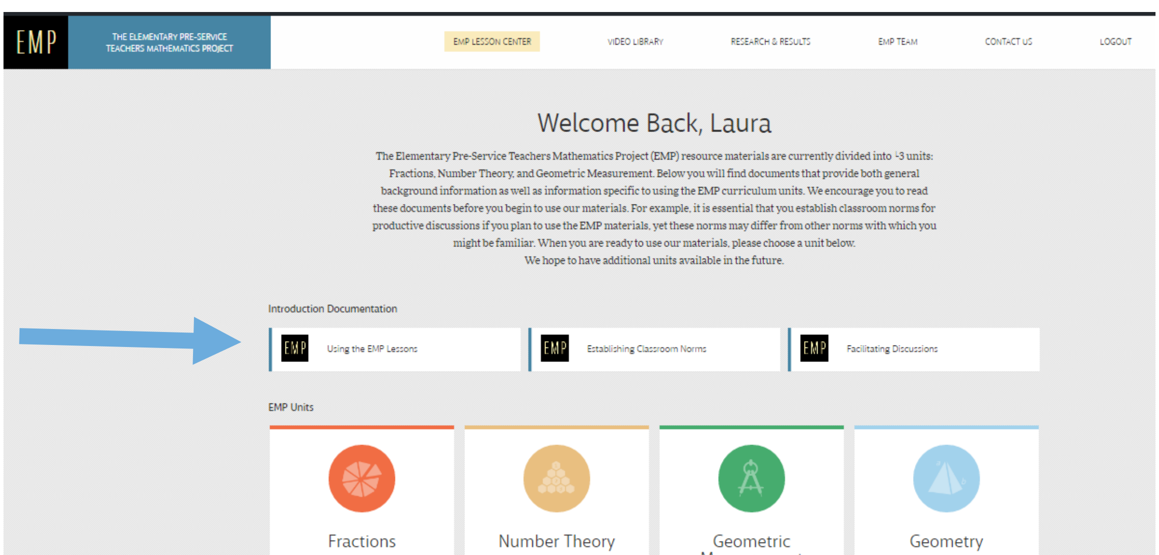


Figure A.1. EMP landing page

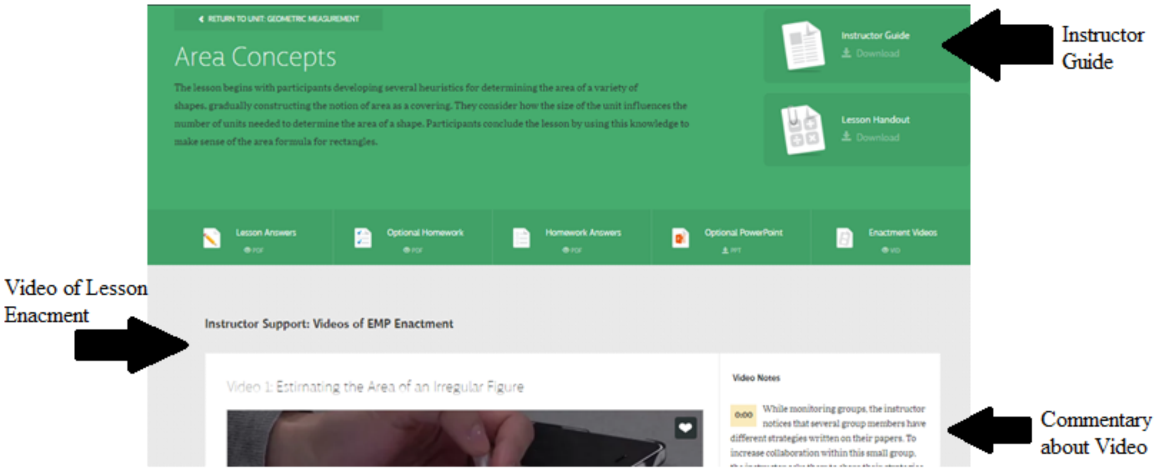


Figure A.2. Lesson level instructor support materials for EMP

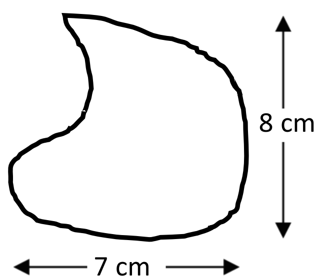
Appendix G: EMP Sample Lesson

Area Concepts

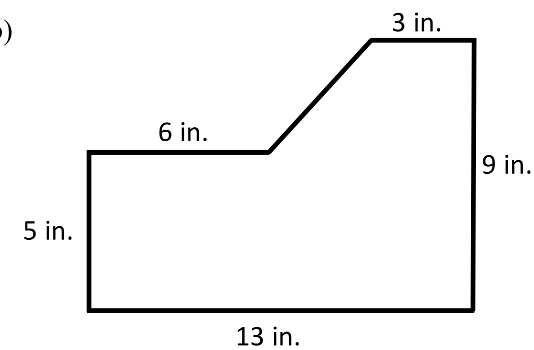
In this lesson, you will examine the notion of *area* and identify several heuristics for determining area. Work on the following sets of problems with your partner or group. Be sure to share your ideas. Work to make sense of the ideas together.

1. Decide upon a strategy to measure the area of each shape below. Once you have chosen a strategy, use it to estimate the area of the shape. Express your estimate using the appropriate units.

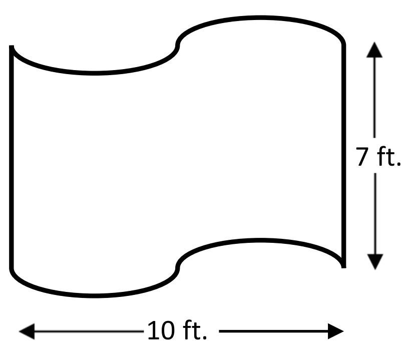
a)



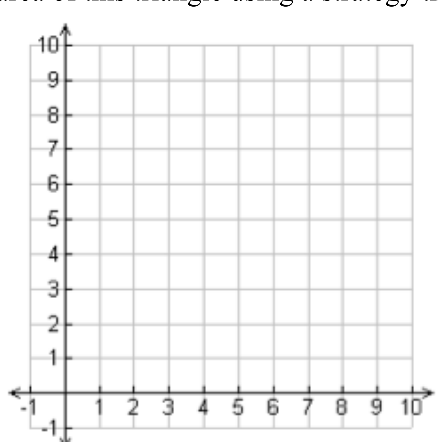
b)



c)

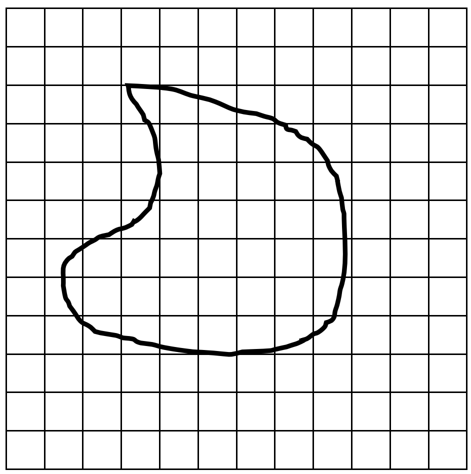


2. On the grid below, draw a triangle with vertices at the points (0, 0), (5, 2), and (7, 9). Find the area of this triangle using a strategy that does not require any approximation.

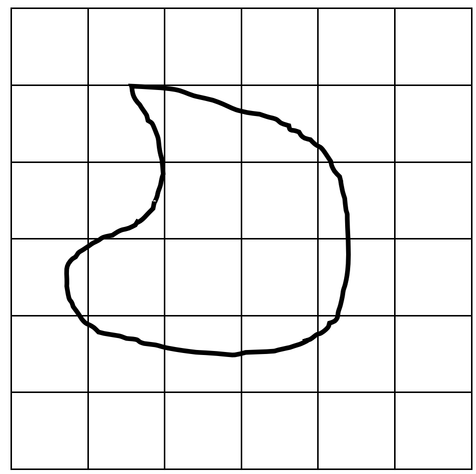


3. Louise and Rashid determined the area of the figure below. They each used square units to estimate the area, as shown below, but each unit square on Rashid's graph paper (right image) has four times the area of each unit square on Louise's graph paper (left image).

Louise

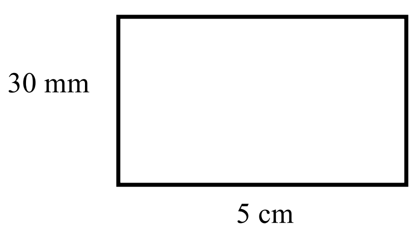


Rashid



After comparing their work, they both concluded that Louise's figure has a greater area than Rashid's figure. Why would they conclude this? Are they correct? Explain why or why not.

4. Marcel claims that the area of the rectangle below is 150. Is he correct? Why or why not?

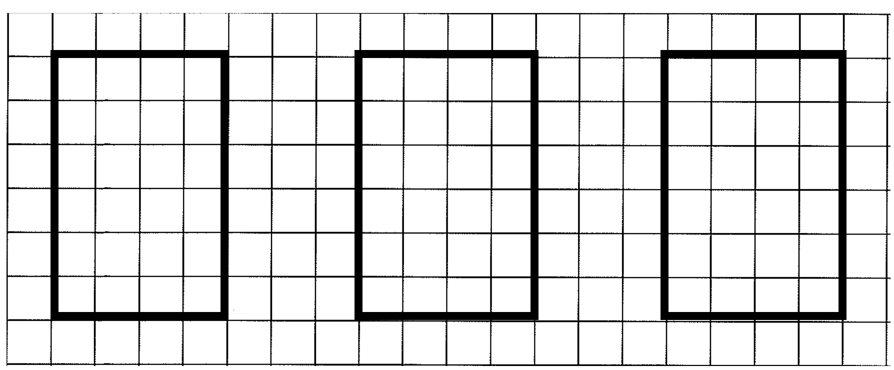


Whole Class Discussion Questions

- Describe your strategies for measuring the area of the figures in Problems 1 and 2.

- Define *area*. How does the size of a square unit affect area measurements?

5. The three rectangles below are identical. Tiling a rectangle involves drawing copies of the same shape on the rectangle until the surface of the rectangle is fully covered.



a) Tile the rectangle on the left with rectangular tiles that are 1 by 1. How many of these tiles cover the rectangle?

b) Tile the rectangle in the center with rectangular tiles that are 1 by 4. How many of these tiles cover the rectangle?

c) Tile the rectangle on the right with rectangular tiles that are 3 by 2. How many of these tiles cover the rectangle?

d) Claudia claims that these rectangles have three different area measures since they are covered by a different number of tiles (e.g., the area of the rightmost rectangle is 4 since it is covered by 4 tiles). Do you agree with her reasoning? Explain.

Whole Class Discussion Question

- Why does the formula *area* = *length* x *width* work to find the area of any rectangle?

Area Concepts and Formulas Homework: Summarize & Connect

1. Define the term *area*. Include a discussion of the benefit of using the square unit as the preferred unit of area measure.
2. Name and describe several different general strategies for finding the area of a figure.
3. Why is the area of a rectangle computed as length x width?

Appendix H: EMP Sample Instructor's Guide

Lesson Plan Overview

Timing	Content Emphases	Suggested Implementation
Problems 1–4 (20 min) Whole Class Discussion Questions (15 min)	**Problems 1–2** have participants develop general strategies for approximating the area of irregular shapes. The strategies are tiling with square units, decomposing into known shapes, decomposing and recomposing into known shapes, and surrounding with a rectangle. Participants will revisit these strategies throughout the Geometric Measurement unit. **Problems 3–4** help participants develop the concept of area as a covering of two-dimensional space. The size of the square unit is important and will affect how we record the area. **The first DQ** has participants articulate the general strategies for finding area explored in Problems 1–2. **The second DQ** provides participants an opportunity to develop a definition for area using the work completed in Problems 3–4 (see Video 2).	**Launch:** Ask participants to share their definitions of "area"; make a list and save as a reference for whole class discussion. Direct the class to work on P1–3 in their groups. **P1–2:** Circulate to observe the strategies different groups use. Take note of groups that use tiling with square units, decomposing, and recomposing on Problem 1 and surrounding on Problem 2. Resist suggesting strategies and do not comment on whether their strategies are correct or incorrect at this time (see Video 1) **P3–4:** These problems do not take much time, but they can help participants begin to develop a definition for area based on a covering of square units. **DQ1:** Ask different groups to present the four strategies for Problems 1–2. Keep a running list on the board of the strategies. Then facilitate a short discussion about when each strategy would be useful and why it is valid. **DQ2:** Ask the class to refine its earlier definition of area. Focus discussion on Problems 3–4. Ask questions such as "what does it mean to say the area of the figure is 150?" to focus their attention on the meaning of the numerical value of area.
Problem 5 (10 min) Whole Class Discussion Question (5 min)	Problem 5 has participants explore the relationship between the dimensions of a rectangle and its area measure. Tiling a rectangle with different sized tiles may change its dimensions, but the number of 1x1 square units remains the same. The DQ uses Problem 5 to press participants to explain why *area = length x width* for rectangles. Length refers to the number of 1x1 square units in one row, while width refers to the number of rows that cover the rectangle; their product refers to the number of 1x1 square units that cover the entire rectangle.	**P5:** Illustrate the process of tiling the leftmost rectangle with 1x1 squares before asking participants to work in groups. Mention that the other rectangles may require different sized tiles. Direct the class to begin working on the DQ if they finish Problem 5 early. **DQ:** Ask participants to explain the formula $A=lw$ using their work in Problem 5. Ask questions such as "what does each dimension of the three rectangles represent?" and "what do we get when we multiply these dimensions?" Many participants need to share their justifications and respond to others' thinking during this discussion.

Teaching the Lesson

Learning Goals

Participants learn that area is a measure that involves covering a defined surface using 1x1 square units. They develop general strategies for finding areas of figures, including decomposing into known shapes, decomposing and recomposing into known shapes, surrounding with a rectangle and subtracting the external area, and tiling with square units. They consider how the size of the unit influences the number of units needed to determine the area of a figure or shape.

Materials

- *Area Concepts* lesson
- *Area Concepts Homework*
- Document camera or overhead projector and transparency of irregular figures in Problems 1–2

Timing

50 Minutes	All problems and summary.
80 Minutes	Add in-depth discussion of Problems 1–2 and Problem 5.

Launching the Lesson

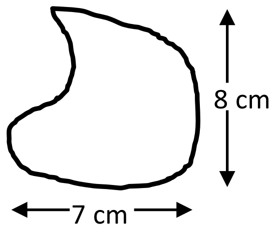
To launch the lesson, elicit definitions of area from different participants and make a list on the board. We have found that participants often have imprecise notions of area and define it as "how much space a shape takes up." Tell participants that they will refine their definitions in this lesson as well as develop specific strategies for finding the area of figures. Do not provide the definition of area during the launch. This should come out of their work and discussions. Direct participants to work on Problems 1–4 in groups. Remind them that if they finish these problems early, they should discuss the two Whole Class Discussion Questions following Problem 4 with their group mates.

Problems 1–4

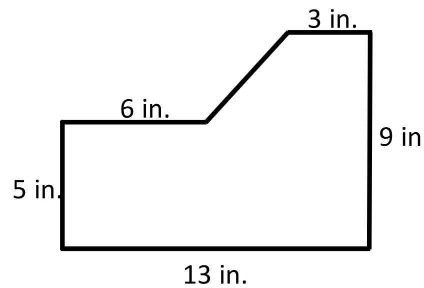
Emphasize to participants that they need to develop their own strategies for finding the areas of the irregular figures in Problem 1, and that they will be presenting their strategies to the class shortly. They may use a different strategy for each figure or the same strategy for all three. Also clarify that it is okay for their answer to Problem 1a to be an approximate area, but they should strive to find exact area measures for 1b and 1c.

1. *Decide upon a strategy to measure the area of each shape below. Once you have chosen a strategy, use it to estimate the area of the shape. Express your estimate using the appropriate units. .*

b)



b)



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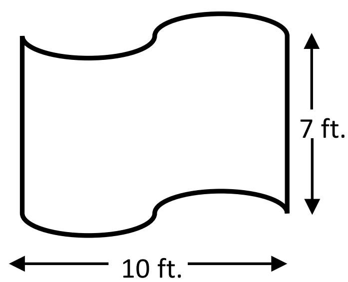
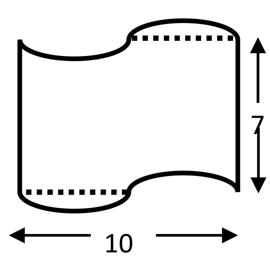
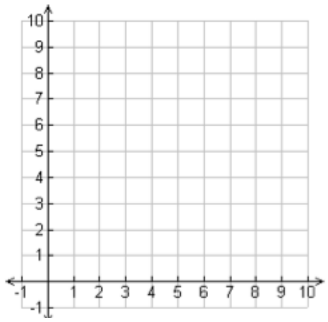


Many participants are unclear on how to determine the area of irregular figures. Thus, we have found that Problem 1 (especially 1a) takes a bit longer than one might expect. Do not suggest strategies, as it is important that participants struggle to generate their own methods, but do tell them that groups will be presenting their strategies to the class and encourage them to develop a plan fairly quickly. See Video 1 (http://elementarymathproject.com/emp_lesson/area-concepts/) for an example of the instructor eliciting various strategies from a small group. Note that in this video, the group is working with a different irregularly-shaped figure that was used in a previous version of this lesson. As participants work, check in with different groups to see which strategies they are using; you can use this time to make decisions about which groups you will ask to present. For 1a, participants may tile the figure with a variety of known figures, such as rectangles, triangles, and circles, or they may cover the figure with congruent square units. For 1b, decomposing the figure into known figures such as rectangles, triangles, and/or trapezoids is a productive strategy. For 1c, a third strategy of decomposing and recomposing works; the figure can be cut at the top and bottom as shown, reflected vertically, and translated across to fit in – recomposing the shape into one 7 by 10 rectangle.



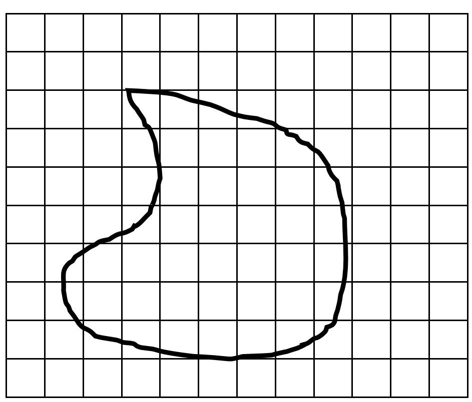
2. On the grid below, draw a triangle with vertices at the points (0, 0), (5, 2), and (7, 9). Find the area of this triangle using a strategy that does not require any approximation.



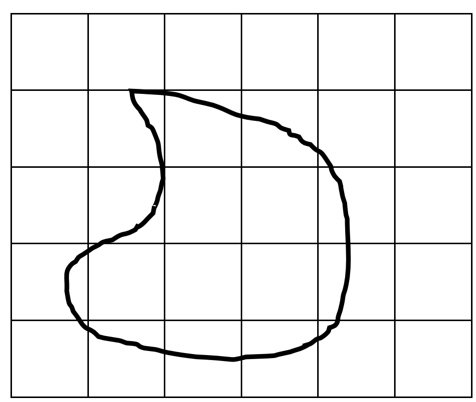
Problem 2 highlights the strategy of enclosing a figure inside a rectangle and subtracting the areas outside of the figure but inside of the rectangle. Do not provide this strategy to groups. Some may begin to count the number of square units inside the triangle; if so, remind groups to find a strategy that does not result in an approximate area value. Sometimes, participants may need a simple reminder that it is often helpful to alter the figure in some way that creates more recognizable shapes whose areas are obtainable. Once they identify several general strategies for finding area, groups are ready to tackle Problems 3–4.

3. Louise and Rashid determined the area of the figure below. They each used square units to estimate the area, as shown below, but each unit square on Rashid's graph paper (right image) has four times the area of each unit square on Louise's graph paper (left image).

Louise



Rashid



Problem 3 does not take long but helps participants develop the concept of area as a covering of two-dimensional space. It helps participants understand that the size of the unit affects how we report the area but this does not mean that the physical area is any different. Participants initially identify an inverse relationship between the area of a square unit and the number of square units needed to cover a figure – the greater the area of the square unit, the less square units are needed to cover the figure. However, some participants may not explain how they know that the area for each of Louise and Rashid's work is the same. Articulating this idea will help them define area in the Discussion Questions.

4. *Marcel claims that the area of the rectangle below is 150. Is he correct? Why or why not?*

Participants may have difficulty with Problem 4. Marcel is incorrect because the length and width of the rectangle are recorded using different units. Marcel must convert the units of one dimension to be the same as the other dimension. Those participants who calculate the area as 150 have not recognized that the labels on the dimensions are different. If you observe this error, consider asking participants to explain their answer –"What does an area of 150 mean? Is the area of the rectangle 150 mm^2 or 150 cm^2?" This may also be a good opportunity to ask participants to define area — a covering of a surface that is quantified using a number of *square* units. This idea is developed further during whole class discussion.

Whole Class Discussion Questions

- *Describe your strategies for measuring the area of the figures in Problems 1 and 2.*

The goal of this discussion question is to articulate several general strategies for finding the area of shapes. Ask participants to present their strategies to the rest of class using a document camera or overhead projector. Purposefully select individuals that will describe the following strategies for Problems 1–2:

- Draw a grid or array of equal-size squares over the figure and count/approximate the number of squares that cover the figure.
- Decompose the figure into pieces (e.g., rectangles and triangles), find the areas of each of the pieces, and then add the areas of the pieces together.
- Decompose the figure into pieces and then recompose them all into a known figure (e.g., rectangle).
- Enclose the figure in a rectangle and subtract the area between the rectangle and the figure from the rectangle's area.

Include a brief discussion about when one might use the different strategies and the need for young children to have many opportunities to cover a space with square tiles prior to developing more efficient methods.

- *Define area. How does the size of a square unit affect area measurements?*

Refer to Problems 3–4 when asking the class to refine their initial area definitions from the lesson launch. Ask questions such as "How can Problems 3 and 4 inform the way we define area? How can we refine our earlier definitions using the work we have done so far?" Area is a measure of how much surface or 2D space is covered. The unit of measure for area is the square unit since squares cover a surface efficiently with no holes, gaps, or overlaps. Thus, the area of a shape or figure is the number of equal-size square units that cover the shape or figure.

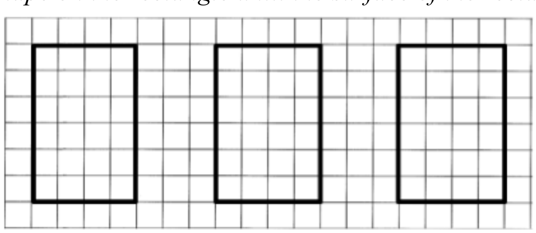
Push participants to also articulate the inverse relationship between the size of a square unit and the number of square units needed to cover a figure. It is important for participants to understand that the area of a shape does not change, although the number representing its area does change depending on the size of the square unit used. It may also come up that using small square units to measure the area of a shape will provide a closer approximation of the area of an irregular figure than using large square units. The size of the squares in Problem 3 on the left are ¼ the size of the squares on the right. Using these smaller squares to measure the area of the drawing of the head gives a closer approximation of the area of the figure. Refer to Video 2 (http://elementarymathproject.com/emp_lesson/area-concepts/) for an example of how such a discussion can unfold. Note that the discussion in this video concerns a different irregularly-shaped figure that was used in a previous version of this lesson.

If participants struggle to make sense of the area concept, refer to Problem 4 and ask participants to explain what an area of 150 represents? This should focus their attention on the importance of square units when determining area. Then ask them to explain why the units of both dimensions of the rectangle must be the same. Have several different participants explain their thinking around this question. Do not worry if some struggle to see the relationship between the linear dimensions and the square dimensions of area at this point. Problem 5 will address this in more depth.

Problem 5

Read Problem 5 as a class. Then illustrate tiling the leftmost rectangle with 1x1 square units. Mention that although square units are most efficient, the rectangles do not have to be tiled with square units, as parts (b) and (c) demonstrate. Then direct groups to work together on Problem 5 and the subsequent whole class discussion question.

5. The three rectangles below are identical. Tiling a rectangle involves drawing copies of the same shape on the rectangle until the surface of the rectangle is fully covered.



- e) *Tile the rectangle on the left with rectangular tiles that are 1 by 1. How many of these tiles cover the rectangle?*
- f) *Tile the rectangle in the center with rectangular tiles that are 1 by 4. How many of these tiles cover the rectangle?*
- g) *Tile the rectangle on the right with rectangular tiles that are 3 by 2. How many of these tiles cover the rectangle?*
- h) *Claudia claims that these rectangles have three different area measures since they are covered by a different number of tiles (e.g., the area of the rightmost rectangle is 4 since it is covered by 4 tiles). Do you agree with her reasoning? Explain.*

The goal of Problem 5 is to help participants understand the relationship between the linear dimensions of a rectangle and its area measure. Each dimension's linear measure represents the number of 1-unit partitions that can be made along the dimension, and their product represents the number of 1x1 square units that will cover the rectangle. The left rectangle represents the traditional tiling of twenty-four 1 by 1 square units. In comparison, the center rectangle uses 1 by 4 rectangular units to tile the figure; this tiling results in only six rectangular tiles. The rightmost rectangle is tiled by 3 by 2 rectangular units, resulting in four tiles. If you notice groups struggling to answer 5d, ask them the following:

- "We can say that the center rectangle has dimensions of 6 by 1 yet the same rectangle on the left has dimensions 6 by 4? "What does the 6 mean? What does the 1 mean?"
- "Why does the rightmost rectangle have dimensions 2 by 2 when the same rectangle on the left has dimensions 6 by 4? "What does the 2 mean? What does the other 2 mean?"

Whole Class Discussion Question

- *Why does the formula area = length x width work to find the area of any rectangle?*

Participants generally have a superficial understanding of area formulas. Some participants may initially say that multiplying the length and width ensures that you find all of the square units in the rectangle. Press for further explanation on how they know that this is true. If they are unsure how to explain it, you might ask them how the dimensions of the rectangle inform you about how many square units can fit in the shape. Refer to Problem 5, in which only the dimensions of the leftmost rectangle identified precisely how many square units can fit along each dimension. This can help them explain why the dimensions of the center and rightmost rectangles could not be multiplied to find their area.

One way to explain the area formula of a rectangle is that the *length* of the rectangle tells us how many square units can fit on that dimension and the *width* tells us how many iterations of the row we can have. Alternatively, participants may say that the *width* tells us how many square units can fit on that dimension of the rectangle and the *length* tells us how many of those columns we have. This relates to our definition of area because when we talk about how many square units fit along one dimension, we are implying that we are lining up square units without overlapping and without leaving any gaps. Then when we repeat this dimension of square units along the other dimension, we are completely covering the 2D space of the rectangle. Furthermore, arranging the square tiles in this array format allows us to easily calculate the total number of square units.

Summarize the lesson by articulating the key mathematical ideas that participants have learned.

- *Area* is found by using a unit of measurement that can be iterated in a shape to completely cover the 2D space of the shape with no gaps or overlaps.
- The size of the unit for area affects the numerical value for the area but the area itself is the same.
- $A = lw$ works to find the area of all rectangles because the length tells you how many square units fit on the length dimension and the width tells you how many of those rows we can iterate in that shape to completely fill it.

Area Concepts Homework: Summarize & Connect

Assign the following three problems to participants for homework to solidify their understanding of the key ideas of the lesson. If you wish to assign additional homework questions, they can be found in the *Area Concepts Homework* document.

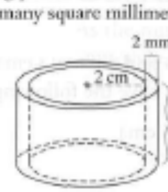
1. *Define the term area. Include a discussion of the benefit of using the square unit as the preferred unit of area measure.*

2. *Name and describe several different general strategies for finding the area of a figure.*

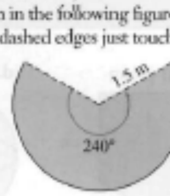
3. *Why is the area of a rectangle computed as length × width?*

Appendix I: Sample Problems From Billstein, Libeskind, and Lott (2016)

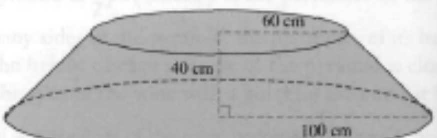
4. The napkin ring pictured in the following figure is to be re-silvered. How many square millimeters must be covered? 2688π mm²



5. Assume the radius of Earth is 6370 km and Earth is a sphere. What is its surface area? $162{,}307{,}600\pi$ km²
6. Two cubes have sides of length 4 cm and 6 cm, respectively. What is the ratio of their surface areas? 4:9
7. The base of a right pyramid is a regular hexagon with sides of length 12 m. The altitude of the pyramid is 9 m. Find the total surface area of the pyramid. $(108\sqrt{21} + 216\sqrt{3})$ m²
8. A soup can has a $2\frac{5}{8}$ in. diameter and is 4 in. tall. What is the area of the paper that will be used to make the label for the can if the paper covers the entire lateral surface area? *
9. The top of a right rectangular box has an area of 88 cm². The sides have areas 32 cm² and 44 cm². What are the dimensions of the box? $\ell = 11$ cm, $w = 8$ cm, $h = 4$ cm
10. How does the lateral surface area of a right circular cone change if
    a. the slant height is tripled but the radius of the base remains the same? The lateral surface area is multiplied by 3.
    b. the radius of the base is tripled but the slant height remains the same? The lateral surface area is multiplied by 3.
    c. the slant height and the radius of the base are multiplied by 3? The lateral surface area is multiplied by 9.
11. Find the surface area of a right square pyramid if the area of the base is 100 cm² and the height of the pyramid is 20 cm. $(100 + 100\sqrt{17})$ cm²
12. The sector shown in the following figure is rolled into a cone shape so that the dashed edges just touch. Find the following.



    a. The lateral surface area of the cone. 1.5π m²
    b. The total surface area of the cone (Include the base.). 2.5π m²
13. As seen in exercise 12, a sector of a circle can be used to construct a right circular cone. The length of the arc of the sector becomes the circumference of the circular base of the cone.
    a. If the length of the arc is 6π, what is the radius of the base of the cone that can be constructed? 3 units
    b. In part (a), the radius of the sector is 5 units; what is the slant height of the cone that can be constructed? 5 units
    c. Using the information in parts (a) and (b), what is the height of the cone that can be constructed? 4 units
    d. Using the information in parts (a)–(c), what is the angle measure for the original sector? 216°
14. If the cardboard tube of a toilet paper roll has diameter of 2.5 in. and is 4 in. tall, what is the lateral surface area of the outside of the cardboard roll? 10π in.²
15. If two right circular cones are similar with radii of the bases in the ratio 1:2, what is the ratio of their surface areas? 1:4
16. Water covers approximately 70% of Earth's surface. Assume Earth is a sphere with diameter about 13,000 km. What amount of Earth's surface is covered with water? *
17. If two cubes have total surface areas of 64 in.² and 36 in.², what is the ratio of the lengths of their edges? 4:3
18. The total surface area of a cube is 10,648 cm². What is the length of each of the following items?
    a. One of the edges *
    b. A diagonal that is not a diagonal of a face *
19. Find the total surface area of the following stand, which was cut from a right circular cone: $(6400\pi\sqrt{2} + 13{,}600\pi)$ cm²



20. Find the numerical difference in the surface area of a square pyramid, with height of 10 cm and base side of 10 cm, and the inscribed right circular cone whose height is 10 cm, and whose base is inscribed in the square base of the pyramid. $100 + 100\sqrt{5} - (25\pi\sqrt{5} + 25\pi) \approx 69.44$ cm²
21. A honeycomb is the basis of some window treatments. In one such treatment, the ends are open and the structure is a hexagonal prism without bases. What is the surface area (inside and outside) of one 4-cm (length of one side of the hexagon) cell if the window treatment is 48 cm wide? 2304 cm²
22. A building had 54 rectangular plate glass windows that were all 4 ft by 8 ft by $\frac{1}{4}$ in. What is the total surface area of the glass including edges? 3483 ft²

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