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An investigation of the interrelationships existing between high school mathematics and physics courses

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AN INVESTIGATION OF THE INTERRELATIONSHIPS EXISTING BETWEEN HIGH SCHOOL MATHEMATICS AND PHYSICS COURSES

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THE PROBLEM AND THE PURPOSE OF THE THESIS

The Problem:

A problem all too often found in high school physics classes is that students have difficulty when it comes to problems involving mathematics. Of course if the student has not mastered the mathematics necessary for a given problem in physics one would expect him to have difficulty; however the situation frequently arises where the physics student has given evidence of having mastered the mathematics involved in a physics problem in work which the student has performed in the mathematics classroom at an earlier date but he has difficulty when faced with the mathematical problem in the context of the physics classroom. The problem here would appear to be a difficulty in applying mathematics to a situation not garbed in the language of the mathematics classroom.

Thus the problem can be seen to be twofold. First the expected difficulty which the physics student will have with a physics topic involving mathematics the student has not mastered, and second the difficulty which the physics student sometimes has in applying mathematics he has previously mastered to a situation in the physics classroom.

Purpose of the Thesis:

The purpose of this thesis is to attack both phases of the above problem. The first phase, that of the physics student's not having mastered the mathematics involved in a given physics topic, will be approached in this thesis by determining what mathematics is necessary for each topic generally taught in the high school physics course as evidenced by textbooks used in high school physics classrooms. Information of this nature will not guarantee that the first phase of the problem will be solved but it will at least provide a means by which the problem could be solved.

As some high schools offer physics in the junior year and some in the senior year we will limit the thesis topic somewhat by considering only the relationship between the junior and senior year mathematics courses and the physics course as generally taught in the high school. The junior year mathematics course is assumed here to be second year algebra and the senior year course is assumed to consist of trigonometry, solid geometry, some analytic geometry, and some elementary calculus.

After presentation of the connections existing between the mathematics and physics courses various ways in which the mathematics teacher and the physics teacher might cooperate in the teaching of their respective courses will be discussed. Cooperation between the two subject matter areas could insure that the mathematics necessary for a given physics topic would be presented before the physics topic and thus obviate the first phase of the problem to be attacked here. Cooperation between teachers in the two subject matter areas under discussion here would also tend to make the mathematics teacher more aware of the type of mathematical situation his students were likely to encounter and hence present him with the opportunity to prepare them for that type of situation while at the same time acquainting the physics teacher with the type of mathematical situation with which his students were accustomed, hence enabling him to assist in places where physics students encountered difficulty in applying mathematics to physics. Cooperation between the teachers could also enable students to see more clearly the importance of mathematics as a tool for physics. Thus cooperation between the teachers would provide an approach to the second phase of the problem, that of applying mathematics to new situations. A further approach to this phase of the problem will be made by indicating a few physics experiments of a mathematical nature which could be performed in the mathematics classroom, thus providing training in applying mathematics to the physical world; using mathematics to describe actual physical phenomena, an approach that might well give mathematics more meaning to the student.

Thus the purpose of this thesis is to attack the problem by finding out what mathematics is necessary for physics topics commonly taught in the high school; by suggesting ways in which mathematics teachers and physics teachers might cooperate, to the benefit of their students; and by suggesting specific ways in which the mathematics teacher could help his students to apply their mathematics to situations not of the type generally encountered in the mathematics classroom.

CHAPTER 2

SUMMARY OF PAST RESEARCH

That a problem exists in the physics student's ability to apply mathematics to physics problems is well verified by the research on the topic. Ralay[1] found that physics students' knowledge, or memory, of facts was greater than their understanding of the principles involved and concluded that the understanding of simple mathematical concepts was the largest single factor hindering progress in physics. Kilzer[2] constructed an inventory test and found that the students handled the elementary mathematics needed in high school physics very poorly. In another study Bryant[3] found that high school physics students were weak in interpreting formulas and equations and in applying variation, ratio, and proportion. Almost without exception the other studies cited in this chapter acknowledge this problem; this fact in itself indicating to some extent the seriousness of the problem. In addition to this problem students sometimes master the mechanical

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1. Lynn L. Ralya, "Investigation of Accomplishment in High School Physics by Means of Diagnostic Tests", Science Education, XXII, (November 1938), pp. 314-315.

2. L.R. Kilzer, "Mathematics Needed in High School Physics", School Science and Mathematics, XXIX, (April 1929), pp. 260-262.

3. Carroll W. Bryant, "Mathematics in Relation to Physics", The Mathematics Teacher, XXX, (December 1937), pp. 363-365.

mathematical processes without really understanding them and consequently encounter difficulty when approaching a new situation which involves mathematics which they do not understand but which they can manipulate mechanically if it is presented to them in the form in which they learned it.

The Need for Mathematics in Physics:

The difficulty encountered by physics students when dealing with mathematics is viewed as serious due to the need for mathematics in the study of physics. As indicated by Brown[1] mathematics can be used in conjunction with high school physics to:

1. Simplify the subject and make it more intelligible
2. Make it more directly applicable and useful
3. Enrich the concepts of physics
4. Show the interrelations of the various divisions of the subject matter.

To remove the mathematics from physics, i.e. to make it purely informational, would of course avoid the problem of an inadequate mastery of mathematics and would conceivably be suitable for some students; but this procedure, as stated by Breslich[2], does not provide the necessary background for those who wish to continue and who need an understanding of concepts and relationships which are largely of a mathematical nature. Mathematics is viewed as the tool of physics. Mathematics, as indicated

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1. Emmett H. Brown, Mathematics in Physics, Sixth Yearbook, National Council of Teachers of Mathematics, (1931), pp. 136-164.

2. E.R. Breslich, "Integration of Secondary School Mathematics and Science", School Science and Mathematics, XXXVI, (January 1936), pp. 58-67.

by Hall[1], is:

1. A way of communicating
2. A way of thinking about concepts
3. A way of problem solving

and consequently is of great value in the study of a science such as physics. Mathematics is viewed as the means used by the physicist to express the laws and relationships which he finds[2]. Thus to teach physics without mathematics would appear to be teaching a series of facts to be memorized without the understanding of the interrelationships existing between them, an understanding frequently obtainable through the use of mathematics. If it is possible to teach physics with this understanding, and it appears to be both very possible and in some cases accomplished, to do otherwise would seem not only pointless but actually a waste of time.

The Mathematical Content of High School Physics:

Quite a number of studies have been done to determine the mathematics contained in high school physics courses; these studies invariably used high school textbooks as their source of information. These studies generally deal with the computational abilities involved in solving problems and several lists have been compiled of the abilities needed, these lists being essentially in agreement with each other. The list reproduced below

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1. Arthur W. Hall, "Relations Between Science and Mathematics in Secondary Schools", Bulletin of the National Association of Secondary School Principals, XXXVII, (January 1953), pp. 92-95.

2. Carroll W. Bryant, op. cit.

is one made in a study of the mathematics required for physics, chemistry, engineering, and higher mathematics by Nickle[1] and is fairly representative.

Algebra:

1. Use of formulas, selecting the proper formula and making substitutions in it.
2. Solving linear equations
3. Use of ratio, proportion, and variation
4. Read and construct graphs
5. Make algebraic substitutions
6. Multiply and divide polynomials
7. Solve quadratic equations
8. Solve simultaneous equations
9. Square a quantity
10. Make application of parentheses
11. Form an equation in solving verbal problems
12. Use of exponents

Geometry:

1. Compute linear measure
2. Compute areas and volumes of common geometric figures
3. Apply pythagorean theorem

Trigonometry:

1. Make simple application of the sine, cosine, and tangent

In another study, Reagan[2] indicated that formulas, equations, ratio, proportion, and variation were needed early in physics. Hausdoerffer[3] states that the general nature of problem solving and conversion of units are important and that factoring and second degree equations are not used to any great extent. A somewhat

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1. George H. Nickle, "The Mathematics Most Used in the Sciences of Physics, Chemistry, Engineering, and Higher Mathematics", <u>The Mathematics Teacher</u>, XXXV, (February 1942), pp. 77-83.

2. G.W. Reagan, "The Mathematics Involved in Solving High School Physics Problems", <u>School Science and Mathematics</u>, XXV, (March 1925), pp. 292-299.

3. William H. Hausdoerffer, "The Mathematical Content of Two General College Physics Texts", <u>Science Education</u>, XXXVI, (October 1952), pp. 250-252.

different study was made by Carter[1]. He attempted to ascertain the importance of the understanding of mathematical concepts, as opposed to computational abilities, in the reading and understanding of high school physics texts using a list of thirty three mathematical concepts; examples of which are negative number, constant, parallel, equation, and variation. He found that recognition of mathematical concepts correlated highly with performance in physics, slightly higher than computational ability, and concluded that it was an important factor in comprehension of physics materials. Thus we find that not only is a certain degree of computational ability necessary but also the ability to recognize mathematical concepts when they appear, which recognition implies an understanding of these concepts. Curtis[2] concluded from a study of the mathematical terms used in secondary school physics textbooks that the mathematics used in these texts implies a knowledge of arithmetic, simple algebra and geometry, and a few basic facts of trigonometry. Making the assumption, which is probably valid, that the frequency of mathematical words in a given physics topic indicates whether more or less mathematics is needed in that topic than another (the more mathematical words found, the more mathematics is involved in the topic) she

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1. William Ray Carter, "A Study of Certain Mathematical Abilities in High School Physics", The Mathematics Teacher, XXV, (October 1932), pp. 313-330.

2. Charlotte Curtis, A Determination of Mathematical Terms in Secondary School Physics Textbooks, (Master's Thesis 1953, Boston University, Boston, Mass.)

concludes that of the topics generally taught in high school physics mechanics requires more mathematics than any other topic and atomic theory requires less than any other topic. Another study which was found useful for this thesis was done by Hannon[1] in which he made an analysis of the mathematical concepts necessary for a college physical science course. He breaks down the rather broad mathematical topic of formulas into formulas of various types and this itemization is employed in the list of mathematics topics used in this study (numbers ten to eighteen on the mathematics list presented in table one in chapter three).

Suggested Solutions of the Problem:

Accepting the assumption that it would be undesirable to eliminate the mathematics from physics for at very least the group which intends to go on in some field which will require an understanding of physics - an attempt was made previously to justify this assumption, whether that attempt was successful must be judged by the reader - we may now proceed to survey ways indicated in the literature by which the mathematics required for physics might be more effectively taught so that students would be better able to apply their mathematics to physics. As indicated earlier this process involves not only computational abilities but also an understanding of mathematical concepts.

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1. Herbert Hannon, An Analysis of the Mathematical Concepts Necessary for a College Physical Science Course, (Western Michigan College, Kalamazoo, Michigan, 1954). (Mimeographed)

Breslich[1] enumerates several possibilities; the first, already discussed, is to eliminate the mathematics from physics and to make physics purely informational. The second possibility is to leave the courses (mathematics and physics) unchanged and to separate the students into groups according to their levels of mathematical competency, running the different groups through the physics course at different speeds; this procedure makes no attempt to correct any mathematical deficiencies and presupposes at least two and possibly several physics courses, which factor alone would make it impractical in many instances. A third suggestion was to administer a test in the mathematics needed in the physics course with those passing the test being allowed to take physics and those not passing being required to either do extra work in mathematics concurrently or to wait to take physics until they could pass the test. Other suggestions concerned various degrees of interdepartmental cooperation; an agreement could be made between the two departments as to which one would take care of mathematical deficiencies, the physics teacher could inform the mathematics teacher when certain skills would be needed and the mathematics teacher could provide them, further interdepartmental cooperation might involve transferring some experiments of a mathematical nature to the mathematics department. Finally Breslich states that the courses could be integrated, taught as one. In this manner principles common to both fields would be

1. E.R. Breslich, op. cit.

better understood, the mathematics would be more experimental and consequently more vivid to the student, time might be saved due to the avoidance of teaching the same mathematics twice (once in the mathematics classroom and once in the physics classroom), the student will see an application of the mathematics he is learning, and teachers would probably come to have a better understanding of courses other than their own. Wood[1] feels that it would be desirable to eliminate the pure theory from physics (demathematize it) and to transfer the mathematical part to mathematics. We have already discussed the demathematization of physics and to push a large quantity of physics into the mathematics course could of course detract from the amount of mathematics which would be taught; the desirability of doing that might be seriously questioned. Leuck[2] in a study of the mathematical troubles of elementary college physics students states:

> The great majority of skills required in physics problems is acquired by the pupils in grades five, six, seven, eight, and nine. If the mathematics of physics were emphasized in first year algebra, the pupil would not possess the necessary acquaintance with science itself so as to give meaning to the work. There is also no immediate motive for doing the work. Learning is most efficent when there is immediate use for the knowledge gained. Hence we conclude that teachers of physics must themselves teach the mathematics which their courses require. We should expect the high school teacher to provide

1. John W. Wood, "Physics", School Science and Mathematics, XXXVII, (June 1937), pp. 710-713.

2. William R. Leuck, The Arithmetic and Algebraic Disabilities of Students Pursuing First Year College Physics, (University of Iowa, Iowa City, Iowa, 1932).

> pupils with the general knowledge of his subject, but the specific applied mathematics of any course must be taught by the teacher of that course.

This passage seems to imply that there is a special brand of mathematics necessary for elememtary physics (whether high school or college) or more likely that the high school provides only the manipulative skills and not the understanding (in mathematics) necessary for, or the practice and training needed in, applying mathematics to other subjects. It is one of the purposes of this thesis to devise an arrangement whereby these skills might be provided through cooperation between the mathematics and physics departments.

Bryant[1] states that the solution of the student's mathematics difficulties in physics is to develop both subjects simultaneously, which, he states, will give life to the mathematics and through its application will tend to fix it in the mind of the student. Young[2] also proposes a simultaneous program but wishes to spread it out over a period of years. He feels that teaching the subjects together would make both more interesting and that mathematics would be seen as a necessary tool for physics. Spreading the physics out over a period of years (three or four) as he suggests might tend to destroy the relationships between the different parts of the subject matter in the mind of the student, but this

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1. Carroll W. Bryant, op. cit.

2. J.W.A. Young, The Teaching of Mathematics, (Longmans, Green, and Co., 1937), pp. 185-188.

difficulty could probably be avoided if care were exercised. Bain[1] states that bringing science into mathematics for explanatory purposes just makes the subject more difficult and suggests that the mathematics be developed apart from the science and then applied. Integrating mathematics and physics and using this technique will, he feels, assist the student in acquiring the technique of solving physics problems. The Groves (Ethel and Ewart)[2] experimented with correlating second year algebra and trigonometry with chemistry and physics with some rearrangement of topics in each course. Ratio and proportion were taught early in algebra and logarithms was taught early in trigonometry. Since the sine curve (alternating current) and sine law (parallelogram of forces) were needed early in physics but could not be taught meaningfully that early in trigonometry the physics topics effected were postponed until later in the course. The authors report that pupils felt they got more out of both subjects this way and were able to see the relation between the subjects. Also transfer was facilitated.

Research Relevant to This Topic:

It is apparent that much of the research done in the field of high school mathematics and physics courses is relevant to this topic to one degree or another. A great deal of work has been

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1. Sherwood C. Bain, "The Mathematical Problems of High School Physics", School Science and Mathematics, XXXXVI, (December 1946), pp. 846-854.

2. Ethel L. Grove and Ewart L. Grove, "An Experiment in the Integration of Mathematics and Science", School Science and Mathematics, LII, (June 1952), pp. 467-470.

done on determining what mathematics is needed in a high school physics course and the problem attacked in this thesis is generally recognized as significant. There is also substantial agreement that correlating or integrating high school mathematics and physics will enhance both subjects but no extensive work was found on actual ways of accomplishing this goal. This thesis attempts to go a step further by showing what mathematics is needed for each of the commonly taught physics topics (as indicated in textbooks) and by indicating several physics experiments which might be performed in a mathematics classroom (see table five in chapter three) as well as suggesting a possible arrangement of topics for a correlated or integrated mathematics and physics program.

CHAPTER 3

PROCEDURES USED AND GENERAL ORGANIZATION OF THE THESIS

In collecting data for the thesis the objective was to ascertain the mathematics needed for each topic generally presented in high school physics. In order to discover what topics were generally taught in the physics course the following textbooks were used, the topics in them being listed: Practical Physics by Black and Davis[1], Modern Physics by Dull[2], and Fundamental Physics by Semat[3], the latter being a college text which was used largely as a reference and to expand, in some cases, on the treatment found in the other texts. The same general procedure was used for the junior and senior year mathematics courses. For the purposes of this thesis a topic consists of a limited subject matter area such as Boyle's Law or temperature scales in physics and functional change or the pythagorean theorem in mathematics.

Since this thesis deals with the relationships between either junior or senior year high school mathematics courses and high school physics, those mathematical operations, skills, concepts, and understandings which are not generally taught in this part of the mathematics program are not considered in this thesis

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1. Black and Davis, New Practical Physics, New York, Macmillan Co., 1932.

2. Charles E. Dull, Modern Physics, New York, Henry Holt and Co., 1945.

3. Henry Semat, Fundamentals of Physics, New York, Rinehart and Co., 1951.

with the exception of some topics in plane geometry. However there is no intent to imply here that certain fundamental operations in mathematics not included in this thesis are not necessary for physics. The fundamental operations of arithmetic, fractions, and decimals are of great importance in physics and some students do apparently have difficulty with these operations. However the primary concern here is with the relationships between the upper level high school mathematics courses and physics and since the fundamental arithmetic operations are not taught as part of the upper level high school mathematics program, though they are essential to it, they are not included here.

Once having obtained a list of physics topics, the mathematics involved in each was ascertained from the previously named textbooks primarily from the text although some recourse was made to the problems also. Each physics topic was then examined in relation to the list of mathematics topics generally taught in the final two years of high school, note being made of the mathematics topics which appeared to be necessary or helpful for each physics topic in which mathematics was used. The list of mathematics topics was then revised, eliminating the topics taught in the final two years of high school which did not appear to be important for high school physics. Topics such as signed numbers, multiplying a polynomial by a monomial, solution of first degree equations, and the geometry included in the mathematics list, which is presented in table one at the end of this chapter, are included despite the fact that they have generally been taken up prior to the junior year in high school. Their inclusion is intended to give a more complete picture of the mathematics needed for each topic in physics and it is felt also that they are topics which could easily be taken up for review at any one of several points in the final two years without seeming out of place or having any detrimental effects. Indeed the elementary algebra topics listed above are generally reviewed at the beginning of the second year of algebra. Once both the mathematics and physics lists were drawn up they were cross-indexed. Each topic in mathematics and physics was assigned a number, the topics then being cross-indexed by means of a table; the table appears in table three and the list of physics topics in table two at the end of this chapter.

For each unit in physics, e.g. light, sound, heat, etc., a tally sheet was drawn up showing what mathematics topics were useful and the frequency with which the use of each occurred; although the same information can be gathered from a perusal of table three. These tally sheets were found helpful in formulating the discussion of relating mathematics and physics courses to be found in chapter four.

In some cases the mathematics involved in a given topic in physics did not appear evident from the description of the physics topic given and in these cases an explanation of the connection or relationship was given in what we shall term the connections index which will be found as table four at the end of this chapter.

Before proceeding further a comment or two on the mathematics list presented in table one appears to be in order. First, the order in which the topics are presented is in no way intended to imply that they are, should be, or could be, taught in that order; in many cases the order is quite arbitrary. The various subdivisions of the list are intended to facilitate the location of any given topic. The topics in the area of trigonometry are included under one heading despite the fact that many of those listed are taught in the junior year or earlier while others are not taught until the senior year. The mathematical topics are indicated in the form of statements designed to make further interpretation of the list unnecessary here.

In the physics list presented in table two the various physics topics are presented under the unit headings where they are generally found in textbooks; e.g. heat, sound, etc. In presenting the physics topics an attempt has been made to be quite brief; the general principle involved in the physics topic is stated and/or any key formulas are included on the assumption that this information will be sufficient to show what is meant by a given physics topic and that further information on the topic can either be deduced from the principles stated or obtained by reference to a physics text.

Table three contains a cross reference table showing what mathematics topics were found to be involved in each of the physics topics listed. The topics are listed by number and also by a key word or phrase to help avoid the inconvenience of checking back with the lists of topics. The mathematics topics are listed down the page at the side (from top to bottom) and the physics topics are listed across the top of the page. The numbers of the topics are also listed on the other side of the table for the mathematics topics and at the bottom of the page for the physics topics to facilitate the use of the table. To find what mathematics is involved in a given physics topic first locatethe physics topic and then read down the chart. Blank spaces indicate no significant relationship was found. The number 1 in a space indicates that an important relationship exists with the mathematics topic at the side and a number 2 in a space indicates that a significant relationship was found to exist with the mathematics topic indicated at the side of that row but that the relationship was not thought to be of primary importance. An asterisk with a number indicates that the connection is explained or illustrated in table four. If a topic on the mathematics list is not included under any of the physics topics it has been included on the mathematics list because it is involved in one of the experiments included in table five at the end of the chapter and it is indicated in the cross reference table of the experiments and the mathematics topics following the description of the experiments. In table three the mathematics topics listed under any one physics topic are not generally all necessary for an adequate understanding of the physics topic. Much of the mathematics can be avoided if a physics topic is to be covered in only a descriptive (informational) fashion and much more mathematics than indicated in the table could frequently be used if an extensive study of the topic is to be undertaken. The objective in compiling the table was to indicate the mathematics employed in the coverage found in the texts used in this study as a minimum. The mathematics topics needed in teaching a given physics topic will vary in some cases depending on the approach used in the physics course. Some mathematics topics which could be generally applied to much of physics, such as the use of the slide rule, are indicated only where they would be of particular significance, in the instance of the slide rule where extensive arithmetical work is called for.

From a brief survey of table three it can be seen that the physics unit mechanics - force and motion is the only physics unit to involve much of the mathematics on the list below functional relationships, which end with topic 41. The unit found which required the least mathematics at the elementary level among those listed was atomic physics, while heat was found to be the only unit to involve no mathematics below functional relationships on the mathematics list. Some of the topics in functional relationships appear more than virtually any other topic but in most instances these topics are not absolutely essential to the understanding of the physics principles though they are generally helpful. The topics which appear to be most generally important are ratio, proportion, variation, functional variation, formulas of various types, and the ability to translate verbal relationships into the language of algebra.

TABLE 1

LIST OF MATHEMATICS TOPICS FOUND IN THE HIGH SCHOOL PHYSICS TEXTS USED IN THIS STUDY

Review of First Year Algebra Fundamentals:

1. Concept of similar terms; ability to determine what terms are similar and facility in combining similar terms in an algebraic expression.

2. Signed numbers; an understanding of and ability to work with signed numbers.

3. Multiplication of a polynomial by a monomial; the ability to do so.

4. Multiplication of a polynomial by a polynomial; ability to do so and also the concept of a square of a polynomial.

5. Powers of monomials; ability to raise a monomial to a power and the concept of a power of a letter or a number whether part of a monomial or not.

6. Root of a monomial; concept of the principal root of a monomial.

7. Skill in expressing verbal relationships in terms of algebraic symbols and in setting up algebraic expressions.

8. First degree equations and formulas; concept of what constitutes an equation or formula and skill in solving equations and formulas by use of the division, multiplication, subtraction, and addition axioms; transposition.

9. Formulas; concept of what constitutes a formula and facility in solving formulas for one unknown given values for the other unknowns.

10. Concept of what constitutes a formula and facility in solving formulas of the form $y=kx^2$ given values for all but one unknown.

11. Concept of what constitutes a formula and facility in solving

formulas of the form $y=\frac{k}{x^2}$ given values for all but one unknown.

12. Concept of what constitutes a formula and facility in solving formulas of the form $z=kxy$ given values for all but one unknown.

13. Concept of what constitutes a formula and facility in solving formulas of the form $z=k\frac{x}{y}$ given values for all but one unknown.

14. Concept of what constitutes a formula and facility in solving formulas of the form $z=kxy^2$ given values for all but one unknown.

15. Concept of what constitutes a formula and facility in solving formulas of the form $z=k\frac{x}{y^2}$ given values for all but one unknown.

16. Concept of what constitutes a formula and facility in solving formulas of the form $w=k\frac{xy}{z}$ or $k\frac{x^2}{y}$ given values for all but one unknown.

17. Concept of what constitutes a formula and facility in solving formulas of the form $w=k\frac{xy}{z^2}$ given values for all but one unknown.

18. Concept of what constitutes a formula and facility in solving formulas of the form $w=k\frac{xy^2}{z}$ given values for all but one unknown.

19. Equations and formulas containing parentheses; skill in solving equations and formulas containing parentheses; involves the removing of the parentheses and the solving of the equation.

20. Literal equations and formulas; skill in solving literal equations and formulas, finding what one variable equals in terms of the other variable or variables.

21. Concept of dependence as expressed in formulas; an understanding of the concept that the value of one variable in an equation depends upon the value of the other variable or variables.

Special Products and Factoring:

22. Skill in factoring out a monomial from an algebraic expression; involves the concept of a monomial.

23. Quadratic trinomials; skill in recognizing and factoring a quadratic trinomial.

24. The difference of two squares; skill in recognizing and factoring the difference of two perfect squares.

25. Quadratic equations; concept of a quadratic equation and skill in recognizing them.

26. Quadratic equations; an understanding of why quadratic equations can be solved by factoring and skill in doing so.

27. Quadratic equations; understanding of the formula and skill in solving quadratic equations using it.

Fractions:

28. Complex fractions; understanding of what constitutes a complex fraction and skill in simplifying them.

29. Fractional equations; an understanding of what they are and skill in solving them.

30. Formulas and literal equations containing fractions; understanding of what they are and skill in solving them for one variable in terms of the others. We will consider a formula of the form $z = k\frac{x}{y}$ as fractional.

31. Reciprocal equations; an understanding of what they are, how to recognize them, and skill in solving them. The only reciprocal equations encountered in this study are of the type $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.

32. Ratio; concept of and ability to work with ratio.

33. Proportion; concept of and ability to work with proportion.

Functional Relationships:

34. Dependent variable, independent variable, and constant; concept of dependent variable, independent variable, and constant and of the fact that the same equation in different forms may have different dependent and independent variables, i.e. the independent variable may become the dependent variable.

35. Writing formulas from rules, the ability to do so.

36. The concept of function, functional notation, and functional change or variation. Skill in using functional notation and in perceiving the effect on the dependent variable of a given change in an independent variable.

37. The concept of and skill in working with direct variation, ratio of two variables as a constant.

38. Concept of and skill in working with inverse variation, product of two variables constant.

39. Concept of and skill in working with joint variation, one variable varies as the product of two or more other variables.

40. Ability to construct and interpret graphs of formulas and to construct and interpret graphs from data in tabular form.

41. Understanding of the difference between relative and absolute change, e.g. percentage change as opposed to change in units such as grams, or a given change related to two different frames of reference.

Systems of Linear Equations:

42. Concept of a linear equation as represented by a straight line.

43. Concept of and skill in solving simultaneous equations both by algebraic and graphical methods.

44. Finding the formula for a table of values relating two variables. Involves establishing a straight line relationship so that the function is of the type $y=ax+b$, then substituting values for x and y from the table and solving for a and b using simultaneous equations.

Powers and Roots:

45. Concept of scientific notation and its uses; e.g. 3.16×10^{-12}.

46. Concept of radicals, especially principal roots.

47. Skill in finding the square root of a number by some process.

48. Concept of radical or irrational equations and ability to solve them.

49. Understanding of the pythagorean theorem and how to use it in problems involving right triangles.

Logarithms and the Slide Rule:

50. Logarithms; concept of a logarithm and an understanding of how they can be used in certain arithmetical operations and why they can be so used.

51. Skill in interpolating using logarithms.

52. The slide rule; understanding of what it is and how and why it may be used to perform certain arithmetical operations,

also an awareness of its limitations.

Trigonometry:

53. Trigonometric functions; concept of the sine, cosine, and tangent; skill in determining their values from a table, and knowledge of the sine curve.

54. Ability to interpolate in finding values of trigonometric functions from the angles or in finding the size of an angle from the value of the trigonometric function.

55. Skill in solving right triangles.

56. Concept of logarithms of trigonometric functions and skill in using them.

57. Skill in using logarithms in the solution of right triangles.

58. Trigonometric functions of any angles (0 to 180°); ability to find the value of a trigonometric function for any such angle and the concept of a trigonometric function having a value for any such angle.

59. Understanding of and ability to use reduction formulas.

60. Skill in working with expressions containing sines and cosines.

61. Trigonometric equations; ability to solve simple equations containing trigonometric functions.

62. Law of cosines; an understanding of the law of cosines and skill in using it in the solution of problems.

63. Law of sines; an understanding of the law of sines and skill in using it in the solution of problems.

Solid Geometry:

64. Volumes; ability to calculate the volumes of simple solids.

Plane Geometry:

65. Concept of an area and a perimeter and skill in calculating areas and perimeters.

66. Concept of parallel lines and an understanding of their elementary properties.

67. Parallelogram; concept of a parallelogram and an understanding of its elementary properties.

68. The circle; concept of a circle and an understanding of its elementary properties.

69. Concept of an angle and skill in working with them.

70. Concept of a perpendicular or normal to a line or plane.

71. Triangles; concept of a triangle and of congruent and similar triangles.

Analytic Geometry:

72. Concept of the slope of a line.

73. The parabola; concept of a parabola and knowledge of its elementary properties.

Rates of Change:

74. Concept of rate of change and average rate of change in graphic (secant) and general form.

75. Concept of instantaneous rate of change.

76. Understanding of the delta process.

77. Understanding of and ability to perform differentiation of second degree (or lower) polynomial functions.

78. Understanding of and ability to perform antidifferentiation of second degree (or lower) polynomial functions.

TABLE 2

LIST OF PHYSICS TOPICS CONTAINING MATHEMATICS

Symbols Commonly Appearing on the List:

A - area
a - acceleration
d - distance
E - voltage (E.M.F.)
e - efficiency
F - force
I - amperes
m - mass
p - pressure
R - resistance
t - time
v - velocity
w - weight

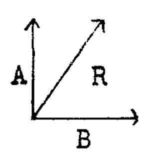
c,k, and g are constants

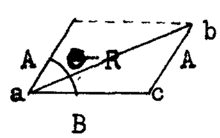
Mechanics - Force and Motion:

1. Units; the shifting of units within a system and between systems (construction and use of formulas).
$$\text{centimeters} = 2.54(\text{inches})$$

2. Hooke's Law; the displacement of a spring is directly proportional to the force exerted upon it within the limits of usefulness of the spring.
$$x = cF \qquad x = \text{displacement}$$

3. Vectors:

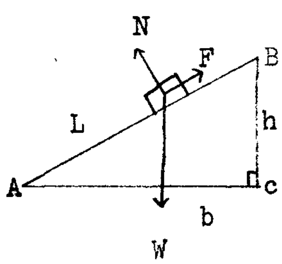
1. 

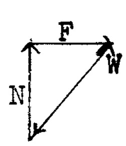
2. 

1. $R^2 = A^2 + B^2 + 2AB\cos\theta$

2. $\dfrac{R}{\sin(180-\theta)} = \dfrac{A}{\sin\angle bac}$

4. Equilibrium under the action of three forces:





$$\frac{F}{w} = \frac{BC}{AB} = \frac{h}{L}; \qquad F = w\frac{h}{L}$$

$$\frac{N}{w} = \frac{AC}{AB} = \frac{b}{L}; \qquad N = w\frac{b}{L}$$

$$F^2 + N^2 = w^2$$

5. Moments; in equilibrium the sum of all clockwise moments equals the sum of all counterclockwise moments. Levers.

6. Uniformly accelerated motion:

$$v = \frac{d}{t} \qquad a = \frac{v_2 - v_1}{t}$$

$$\bar{v} \text{ (average v)} = \frac{v_1 + v_2}{2}$$

derivation (from the above) of:

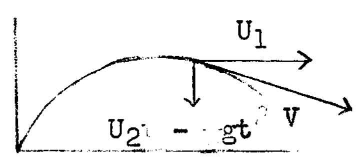
$$d = v_1 t + \tfrac{1}{2}at^2$$

$$v_2^2 = v_1^2 + 2ad; \quad \text{when } v_1 = 0, \ v_2 = \sqrt{2ad}$$

Application of formulas to freely falling bodies (substitution of g for a), concept of rate of change, application of calculus.

7. Relative velocity; a body moving in a stream of water has a velocity relative to the water and also has a velocity relative to the bank of the stream.

8. Projectiles:

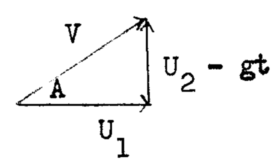


U_1 = initial horizontal velocity

U_2 = initial vertical velocity

x(horizontal position) $= U_1 t$

y(vertical position) $= U_2 t - \frac{1}{2}gt^2$



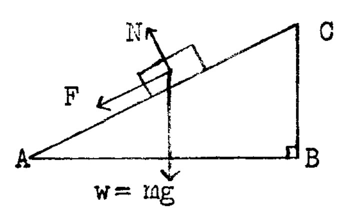
$$y = \frac{U_2}{U_1}x - \frac{1}{2}\frac{g}{U_1^2}(x^2)$$

To find the range of a projectile set y equal to 0, to find what angle yields the maximum range set U_1 equal to v(cosA) and U_2 equal to v(sinA).

9. Newton's second law: the unbalanced force acting upon a body produces an acceleration which is directly proportional to the unbalanced force.

$$F = ma$$

10. The inclined plane:



$$\sin A = \frac{F}{mg}; \quad F = mg(\sin A); \ ma = mg(\sin A)$$

$$a = g(\sin A)$$

11. The coefficient of friction:

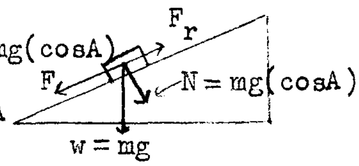
$$f(\text{coefficient of friction}) = \frac{\text{force needed to overcome friction}}{\text{weight}}$$

The force of friction is thus directly proportional to the weight of the body.

12. The inclined plane - friction:

f = coefficient of friction
force of friction(F_r) = $fN = fw(\cos A) = fmg(\cos A)$

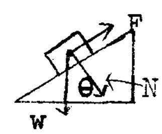
$F = ma = mg(\sin A) - fmg(\cos A)$



derivation of $f = \tan A_c$ where A_c is the angle at which the acceleration of the body is zero.

13. Variation of the efficiency of the inclined plane:

$F_r = fN$
$N = w(\cos\theta)$
$F_r = fw(\cos\theta)$



As θ increases, cosθ and N decrease, and hence F_r decreases and the efficiency increases

14. Impulse and momentum: the loss of momentum by one body equals the gain in momentum of the other body or bodies.

$$\text{Impulse} = Ft = mv_1 + mv_2$$

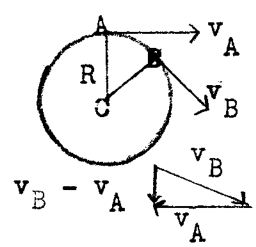
15. Circular motion:

$$\frac{v_B - v_A}{v_A} = \frac{AB}{AC} = \frac{AB}{R}$$

$$\frac{v_B - v_A}{v} = \frac{vt}{R}$$

arc $AB = vt$
$v_A = v(\text{speed})$

$$\frac{v_B - v_A}{t} = \frac{v^2}{R} = a$$



16. Centrifugal force:
a. Centrifugal force is directly proportional to the mass of the body.
b. Centrifugal force is inversely proportional to the radius of curvature.
c. Centrifugal force is directly proportional to the square of the velocity.

$$F = ma;\ \text{for circular motion}\quad a = \frac{v^2}{R};\ F = \frac{mv^2}{R}$$

17. Periodic motion; the pendulum: the period of vibration is directly proportional to the square root of the length of the pendulum.

$$T = c\sqrt{L} \qquad T = \pi\sqrt{\frac{L}{g}}$$

T = period
L = length

18. Periodic motion:

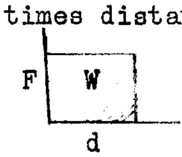
$$v = \frac{2r}{T^2};; \quad a = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

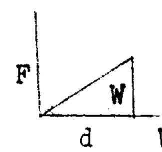
T = period
r = radius

19. Law of universal gravitation; there is an attractive force between two bodies which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers.

Mechanics - Work, Power, and Energy:

20. Work is done when a force moves through a distance. Work is the product of force times distance.

Graphical methods: 



Work done in stretching a spring
W = work

21. Conservation and transformation of energy:

$$mgh = \tfrac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

potential energy at a height h equals kinetic energy a distance h down from the previous point.

22. Potential and kinetic energy:

$$P.E. = mgh$$

$$v = \sqrt{2gh}; \quad h = \frac{v^2}{2g} \qquad K.E. = mg\frac{v^2}{2g} = \tfrac{1}{2}mv^2$$

23. Power; power equals work per unit time:

$$\text{power} = \frac{\text{work}}{t} = \frac{Fd}{t} = Fv; \quad \text{Horse power} = \frac{Fd}{550t}$$

when d is in feet

24. Efficiency of machines:

$$e = \frac{\text{output}}{\text{input}} = \frac{wh}{Fd}$$

h = height

25. Mechanical advantage of machines:

$$\text{Actual mechanical advantage} = \frac{\text{resisting force}}{\text{acting force}}$$

ideal mechanical advantage $= \frac{\text{distance effort moves}}{\text{distance resistance moves}}$

$e = \frac{\text{actual mechanical advantage}}{\text{ideal mechanical advantage}}$

Machines: lever, pulley, wheel and axle, inclined plane, wedge, screw

Mechanics of Fluids:

26. Fluids at rest:

$d = \frac{m}{V}$; $P = \frac{F}{A}$ or $\frac{W}{A}$

d = density
V = volume

pressure is directly proportional to the depth of the liquid
pressure is directly proportional to the density of the liquid
pressure is independent of the shape of the container or body of liquid.

$p = khd = hd$

27. Force of a liquid; the force on the wall of a dam is equal to the density of the liquid multiplied by the perpendicular area of the wall of the dam multiplied by the average depth.

28. Specific weight:

the specific weight of substance $x = \frac{\text{density of x}}{\text{density of water}}$

29. Pressure in a confined liquid:

Balanced columns of liquids; $p = h_1 d_1 = h_2 d_2$

d = density
h = height

$$\frac{h_1}{h_2} = \frac{d_1}{d_2}$$

30. The hydraulic press:

$F = pA$; $F = f\frac{A}{a}$

F = force exerted by large piston
f = force exerted by small piston
A = area of large piston
a = area of small piston

Mechanical advantage $= \frac{\text{distance effort moves}}{\text{distance resistance moves}} = \frac{\text{resistance}}{\text{effort}}$

31. Fluids in motion:

for steady flow in a pipe $\frac{v_1}{v_2} = \frac{A_2}{A_1}$

$F = (p_1 - p_2)A$

$Work = Fd = FL = (p_1 - p_2)AL = (p_1 - p_2)V = \frac{1}{2}mv^2$

$p_1 - p_2 = hdg$; $hdgV = \frac{1}{2}mv^2$; $v = \sqrt{2gh}$

A = cross section area
L = length
V = volume
v = velocity
d = density

32. Bernoulli's theorem:

$p_1 + \frac{1}{2}dv_1^2 = p_2 + \frac{1}{2}dv_2^2$ derived from work $= (p_1 - p_2)V$ and K.E. $= \frac{1}{2}mv^2$

V = volume
d = density

<u>Gas Laws and Kinetic theory</u>:

33. Boyle's Law; at constant temperature the volume of a gas is inversely proportional to the pressure exerted upon it.

$$p_1V_1 = p_2V_2$$

V = volume
p = pressure

34. Charle's Law; at a constant pressure the volume of a gas is directly proportional to the absolute temperature, at a constant volume the pressure exerted by a gas is directly proportional to the absolute temperature.

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \qquad \frac{p_1}{p_2} = \frac{T_1}{T_2}$$

T = absolute temperature
V = volume

35. Kinetic theory of gases:

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = K$$

T and V same as in 34.

36. Avagadro's hypothesis; equal volumes of gases at the same temperature and pressure have equal numbers of molecules.

for two gases at the same volume, temperature, and pressure:

$$p_1V_1 = \frac{1}{3}N_1m_1\bar{v}_1^2$$

$$p_2V_2 = \frac{1}{3}N_2m_2\bar{v}_2^2$$

$$p_1V_1 = p_2V_2; \quad N_1m_1\bar{v}_1^2 = N_2m_2\bar{v}_2^2$$

N = number of molecules
$\bar{v}$ = average velocity
V = volume

since the temperature and pressure are the same

K.E.$_1$ = K.E.$_2$ and $m_1\bar{v}_1^2 = m_2\bar{v}_2^2$

Heat:

37. Temperature scales:

Centigrade $T_c = \frac{5}{9}(T_f - 32)$

Fahrenheit $T_f = \frac{9}{5}T_c + 32$

Absolute $T_a = T_c + 273.2$

T_c = temperature centigrade
T_f = temperature fahrenheit
T_a = temperature absolute

38. Specific heat:

$$S_p = \frac{\text{heat capacity of x}}{\text{heat capacity of water}}$$

S_p = specific heat

39. Thermal expansion of solids and liquids:

$a = \frac{L - Li}{LiT}$; $b = \frac{V - Vi}{ViT}$

$L^3 = Li^3(1 + aT)^3$

$L^3 = Li^3(1 + 3aT + 3a^2T^2 + a^3T^3)$

$V = Vi(1 + bT)$, $b = 3a$ approximately

a = coefficient of linear expansion
b = coefficient of volume expension
Li = length at 0° C.
T = temperature cent.
Vi = volume at 0° C.
L = length
V = volume

40. Heat and work:

$H = ms(T_2 - T_1)$; $W = JH$

H = heat; s = specific heat
W = work; T = temperature centigr.
J = conversion factor joules/calorie

41. Laws of evaporation:
  a. The rate of evaporation increases with the temperature.
  b. The rate of evaporation increases with an increase in surface area.
  c. THe rate of evaporation depends on the nature of the liquid.
  d. The rate of evaporation decreases with increasing pressure.
  e. The rate of evaporation decreases with increasing humidity.
  f. The rate of evaporation increases with the rate of change. of the air in contact with the surface.

function, concept of dependence even though no exact relationship is given

42. Heat of vaporization and fusion:

$H = mH_v$ (water to steam)

$H = mH_f$ (ice to water)

H_f = heat of fusion

H_v = heat of vaporization

H = heat

43. Transfer of heat:

Temperature gradient $= \frac{T_1 - T_2}{d}t$

T_1 and T_2 = temperatures at different points on a conductor

The amount of heat transfer is directly proportional to the cross-section area of the conductor and the temperature gradient.

H = heat transfer

$$H = KA \frac{T_1 - T_2}{d} t$$

t = time
A = cross-section area

44. Heat radiation:

for a black body $R = kT^4$
for a non-black body $R = ekT^4$

R = radiation
e = emissivity of the body
T = temperature centig.

45. Conversion units: gasoline engine:

Horse power $= \frac{D^2N}{2.5}$

N = number of cylinders
D = diameter of cylinder in inches

Sound:

46. Speed of sound; at 0° centigrade the speed of sound is approximately 1090 feet per second, the speed increases 2 feet per second for each degree increase in temperature.

$$v = 1090 + 2T$$

T = temperature centigrade

47. Velocity, wave length, and vibration:

$$v = Ln$$

n = frequency (vibrations per second)
L = wave length

48. Sound ranging; to determine the position of an enemy gun by noting the times at which the report of a shot is heard at various points. See experiment seven in table five.

49. Loudness: the loudness of a sound is inversely proportional to the square of the distance from the source.

$$L = \frac{k}{d^2}$$

L = loudness

50. Music; laws of strings:
   a. lengths - vibration is inversely proportional to the length of the string.
   b. diameters - vibration is inversely proportional to the diameter of the string.
   c. tension - the rate of vibration is directly proportional to the square root of the tension.

Light:

51. Light intensity; the intensity of light is inversely proportional to the square of the distance from the source.

$$\text{Foot candles} = \frac{\text{C.P. (candle power)}}{d^2}$$

52. Image formed by allowing rays from a light source to pass through a small aperture (image inverted).

$$\frac{\text{size of object}}{\text{size of image}} = \frac{\text{object distance}}{\text{image distance}}$$

53. Reflection; the angle of incidence equals the angle of reflection when a light ray is reflected from a reflecting surface.

54. Lense formula for a concave mirror:

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$$

F = focal length $\left(\frac{\text{radius of curvature}}{2}\right)$
D_o = object distance
D_i = image distance

$$\frac{\text{size of object}}{\text{size of image}} = \frac{\text{object distance}}{\text{image distance}}$$

55. Index of refraction;

$$I = \frac{\text{speed in air}}{\text{speed in substance x}} = \frac{\sin A}{\sin B}$$

I = index of refraction
A = angle of incidence
B = angle of refraction

56. Refraction - lenses:

$$\frac{1}{D_o} + \frac{1}{D_i} = \frac{1}{F}$$

F = focal length $\left(\frac{\text{radius of curvature}}{2}\right)$
D_o = object distance
D_i = image distance

D_i and F are negative for concave lenses

$$\frac{\text{size of object}}{\text{size of image}} = \frac{\text{object distance}}{\text{image distance}}$$

57. Optical instruments:
a. microscope

f = focal length of eyepiece lense
L = length of tube
F = focal length of objective lense

magnification of the objective lense $= \frac{L}{F}$

magnification of the eyepiece $= \frac{25}{f}$

all lengths in centimeters

total magnification $= \frac{25L}{fF}$

b. telescope

magnification $= \frac{F}{f}$

58. Color:

$$v = Ln$$

v = velocity
n = frequency of vibration
L = wave length

Electricity and Magnetism:

59. Magnetism; the force between two magnetic poles is directly proportional to the product of the strengths of the poles and inversely proportional to the square of the distance between them.

60. Alternating current; the sine curve.

61. Current electricity; ohms law:

$$I = \frac{E}{R}$$

62. Resistance in a wire:
   a. The resistance of a wire is inversely proportional to the square of the diameter, i.e. the cross-section area.
   b. The resistance of a wire is directly proportional to the length of the wire.
   c. The resistance of a wire increases with increasing temperature in metallic conductors.

$$R = \frac{kL}{d^2}$$

L = length in feet
d = diameter in mils
k = resistance of 1 foot of wire of diameter 1 mil

63. Voltaic cells; resistance of a cell:

$$I = \frac{E}{R_i + R_e}$$

R_i = internal resistance

R_e = external resistance

64. Series grouping of cells:

$$I = \frac{nE}{R_e + nR_i}$$

cells assumed to be identical

n = number of cells
R_i = internal resistance

R_e = external resistance

65. Parallel grouping of cells:

$$I = \frac{E}{R_e + \frac{R_i}{n}}$$

cells assumed to be identical

n = number of cells
R_i = internal resistance

R_e = external resistance

66. Series wiring:

$$R = R_1 + R_2 + R_3 + \ldots\ldots \qquad E = E_1 + E_2 + E_3 + \ldots\ldots$$

$$I_1 = I_2 = I_3 = \ldots\ldots$$

67. Parallel wiring:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots$$

$$E_1 = E_2 = E_3 = \ldots\ldots \qquad I = I_1 + I_2 + I_3 + \ldots\ldots$$

68. Effects of electric current; laws of electrolysis:
   a. The amount of metal deposited by an electric current is directly proportional to the length of time the current flows.
   b. The amount of metal deposited during electrolysis is directly proportional to the strength of the current in amperes.
   c. The amount of an element deposited in a given time is directly proportional to the electro-chemical equivalent of the element.

69. Heating effects of electric current (Joule's laws):
   a. The heat produced by an electric current is directly proportional to the resistance of the conductor in ohms.
   b. The amount of heat produced by an electric current is directly proportional to the square of the current in amperes.
   c. The amount of heat produced by an electric current is directly proportional to the time the current flows.

$H = kRI^2t$; $H = .24RI^2t$ — H = heat

70. Fall of potential; the fall of potential along a wire of uniform resistance is directly proportional to the length of the wire.

L = length

$E = E_1 - kL$

71. Electric power:

$$W = EI = \frac{E^2}{R}$$

W = watts

72. Voltage transformer:

$$\frac{E_1}{E_2} = \frac{\text{number of turns of wire in the primary coil}}{\text{number of turns of wire in the secondary coil}}$$

Atomic Physics:

73. Einstein's law of relativity:

$E = mc^2$

c = velocity of light in centimeters per second
E = energy

74. Scientific notation:

e.g. 6.32×10^{-21}

75. Charges on atomic particles; positive, negative, or neutral.

TABLE 3

CROSS REFERENCE TABLE OF PHYSICS AND MATHEMATICS TOPICS

The cross reference table found on page 41 has mathematics topics listed vertically at the left hand side of the page and physics topics listed horizontally across the top. The numbers of the topics correspond to the numbers of topics in tables one and two.

To read from the table the mathematics topics found to be useful for a given physics topic locate the physics topic at the top of the page and read down the column headed by that physics topic. The spaces in that column which are left blank indicate that no significant relationship was found to exist with the mathematics topic listed at the left in the row in which the space is located. A number one in a space indicates that an important relationship was found to exist and a number two in a space indicates that a significant relationship was found to exist but that it was not considered to be of primary importance.

To locate the physics topics which make use of a given mathematics topic read across the page in the row in which the mathematics topic is listed, noting the physics topics at the top of any column in which a number has been placed.

An asterisk after any number in the table indicates that the connection between the mathematics and physics topics, indicated by the number, is illustrated or explained in table four.

. kx/y						1									2							2	1	1	1		1		2								1	2				
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1. Dependence		2		2	2	2							2	2	2	2	2	2	2		2	2	2	2	2		2	2	2	2		2	2	2	2	2		2	2		2	2
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2. Ratio		1	1	1							1												1	2			1		1			2		2								
3. Proportion		1	1	1					1																		2	1	2	2		2										
4. Variable	2	1				2		2	2	2	2	2	2		2	2	2	2	2	2	2	2	2	2	2	2		2		2				2	2	2		2	2		2	2
5. Rules	2	1				1		2	1		2	2	2	2			2	2	2	2	2			1	2		2	2		2				2				2			2	
6. Function	2	2				1		2	2	2	2	2	2	2	2	2	2	1			2	2	2	2	2	2	2	2		2	2	2	2	2	2	2		2	2	1	2	2
7. Direct var	1	1				1		1	1	1	2	2	1	1	1	1	1	1	1	1	2	2	2		2	2		2		2		1		1		2		2	2	2	2	2
8. Inverse															1	1		2	1			2	2		1			2					1	1						2		
9. Joint															2	1			2		2	1			1		2	2						2								
40. Graphs						2																																				
41. Relative		2					2																																			
42. Linear																																										
. Simultaneous																																										
44. Table																																										
45. Scientific													2																													
46. Principal						2											1																									
47. Square root			2			2	2						2		2	2	2	2		2	2									2												
48. Radical						2		2							2	2	1	2		2	2									1												
49. Pythagorean			1	1			2	1		2																																
50. Logarithms			2				2																																			
51. Interpolate			2				2																																			
52. Slide Rule				2																												2	2	2								
53. Trig funct			1	1				1		2		1																														
54. Tr interpola			2	2				2		2		2																														
55. Rt triangle			1	1	2*			2		2		1																														
56. Log trig fn			2																																							
57. Log rt tri			2																																							
58. Any angle			2																																							
59. Reduction			2																																							
60. Trig expres			2					2					2																													
61. Trig equa								1		1		1	2																													
62. Cosine law			1																																							
63. Sine law			1																																							
64. Volumes																									2		2															
65. Area, perim																																	2									
66. Parallel			2																																							
67. Parallelog			1																																							
68. Circle																		2																								
69. Angle			2	2																																						
70. Normal			2	2																																						
71. Triangle			2	1																																						
72. Slope													2																													
73. Parabola								1																																		
74. Change						2																																				
75. Instantaneou						2																																				
76. Delta proc						2																																				
77. Differenti						2																																				
78. Antidiffer						2																																				
Phrase Topics	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	4

	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
13					1				1		1	1	1										
14																	1		1		1		
15										1													
16																							
17							1																
18																							
19																							
20		2		2	2	2			1	2	2	2	2			2			2		2		
21					2				2	2	2	2	2		2	2	2	2			2		
22																							
23																							
24																							
25																							
26																							
27																							
28													2										
29			2																				
30					2					2	2	2	2						2				
31		1		1											1								
32			1		2																		
33		2		2																1			
34						2	2			2		2	2	2		2							
35						2	2			2	2	2	2	2	2	2	2	2	2		2		
36							2		2	2	2	2	2	2	2	2	2	2	2		2		
37		2		2	2		1		2	2	2	2	2	2		2	2	2	2	2	2		
38					2	2	1		2	2	2	2	2							2			
39							1									2	2						
40																							
41																							
42																							
43																							
44																							
45																					2	1	
46																							
47										2							2						
48																							
49																							
50																							
51																							
52																							
53			1					2															
54			2																				
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61			2																				
62																							
63																							
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65																							
66																							
67																							
68																							
69																							
70	1		1																				
71			1																				
72																							
73																							
74																							
75																							
76																							
77																							
78																							

	Mathemat Topics	1	2	3	4	5	6	7	8	9	10	11	12
Atomic Physics	75. Charges		1										
	74. Notation												
	73. Relativity							2		1			
Electricity and Magnetism	72. Transformer							2					
	71. El Power					2		2		1			2
	70. Potential							2	1				
	69. Heating					2		2		1			
	68. Electrolysis							2		1			1
	67. Par wiring							2		1			
	66. Ser wiring							2		1			1
	65. Parallel							2		1			
	64. Series							2		1			
	63. Cell							2		1			
	62. Resistance					2		2		1			
	61. Ohm's law							2		1			
	60. Alternating												
	59. Magnetism							2		1			
Light	58. Color							2		1			1
	57. Optical							2	2	1			
	56. Lenses		1					2		1			
	55. Refraction							2		1			
	54. Mirror		1					2		1			
	53. Incidence												
	[illegible]. Image							1					
	49.							1		2			
Sound	48. Ranging												
	47. Wave							2		1			1
	46. Speed							1	1				
Heat	45. Heat engine					2		2		1			
	44. Radiation					2		2		1	1		
	43. Transfer							2		1			
	42. Fusion							2	1				
	41. Evaporation							2					
	40. Heat work							2	1	2			2
	39. Therm expan		2	2	2			1	1	2			
	38. Spec heat							2		1			
	37. Temperature		1					2	1				
Gas Laws and Kinetic Theory	36. Avagadro					2		2		2	1		
	35. Gases							2	2	1			
	34. Charles'							2		1			
	33. Boyle's							2		1			1
Mechanics of Fluids	32. Bernoulli					2		2		2			
	31. Fl in motion			2		2		2		1			2
	30. Hydr press							2		1			
	29. Liquid pres							2		1			
	28. Sp weight							2		1			
	27. Liquid force							2		1			1
	26. Fl at rest							2		1			2
Mechanics — Work, Power, and Energy	25. Mech advant							2		1			
	24. Efficiency							2		2			
	23. Power							2		1			
	22. Pot and kin					2		2		1			
	21. Conservation					2		2		1			
	20. Work							2		1			1

Mechanics - Force and Motion

	1. Units	2. Hooke's	3. Vectors	4. Equi 3 force	5. Moments	6. Unif motion	7. Rel velocity	8. Projectiles	9. 2^{nd} law	10. Incl plane	11. Coeff of fri	12. Incl pl fric	13. Varia of c f	14. Momentum	15. Circ motion	16. Cent force	17. Pendulum	18. Per motion	19. Gravitation
1. Similar					2														
2. Signed																			
3. Poly X mono																			
4. Poly X poly																			
5. Powers			2	2		2		2		2					2	2		2	2
6. Root																			
7. Symbols	1	1	2	1	2	2		2	1	2	2	2	2	1	2	1	1	2	1
8. First degree	1	1			1	1		2	1		1	1		1					2
9. Formulas	1	1	2	1	1	1		1	1	2	1	1	2	2	1	1	1	2	2
10. kx^2						1		1									1*		
11. k/x^2																			
12. kxy						2		2		1		2	1	1					

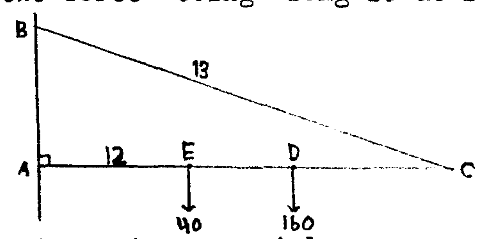
TABLE 4

INDEX OF THE CONNECTIONS EXISTING BETWEEN CERTAIN MATHEMATICS AND PHYSICS TOPICS

Relationships which did not appear to be clear from the presentation of the mathematics and physics topics are explained below. The connection to be explained is indicated first by the number of the physics topic followed by the number of the mathematics topic. These connections are those which are followed by an asterisk in table 3.

5. Moments, 55. Right triangle:

Used in finding the force acting along BC at B.



8. Projectiles, 22. Factoring out a momomial:

In finding the range of a projectile:

$$y = \frac{U_2}{U_1}x - \frac{1}{2}\frac{g}{U_1}x^2$$

Set y equal to 0 and solve for x. In the process of solving, x is factored out.

11. Coefficient of friction, 20. Literal equations:

$$f = \frac{F}{w}$$

Solving the formula for w or F involves topic 20.

17. Pendulum, 10. kx^2:

$T = \pi\sqrt{\frac{L}{g}}$; square both sides, $k = \frac{g}{\pi^2}$

33. Boyle's law, 16. $k\frac{xy}{z}$

$$P_1V_1 = P_2V_2 \qquad \text{i.e.} \quad P_1 = \frac{P_2V_2}{V_1}$$

TABLE 5: EXPERIMENTS

The following ten experiments of activities are of the type which could well be carried out in the mathematics classroom. Each involves at least one of the topics on the mathematics list and each makes use of a fairly simple, elementary physics principle.

At the end of this table there is a cross-reference chart showing what mathematics was used in each experiment. A blank space in the chart indicates that the mathematics topic listed at the side of the chart in that row was not used in the experiment whose number appears at the top of that column. A number one in a space indicates that the mathematics topic indicated is quite important in the experiment and a number two in a space indicates that the mathematics topic is useful but not of primary importance.

1. Hooke's Law: Using a helical spring and various weights which can be attached to it measure the displacement for several different weights keeping a table of values. Derive the relationship by use of proportion or by first establishing a linear relationship between the variables and then finding the values of the constants by use of simultaneous equations.

2. Moments: Using a set of weights which can be attached to a bar or yardstick suspended or supported at the center so that it is horizontal when no weights are attached, attach different weights on either side of the center in such a manner that the bar remains horizontal. Record the weights and the distance of each from the center and allow the students to

derive the relationship. After the relationship has been derived apply it predictively.

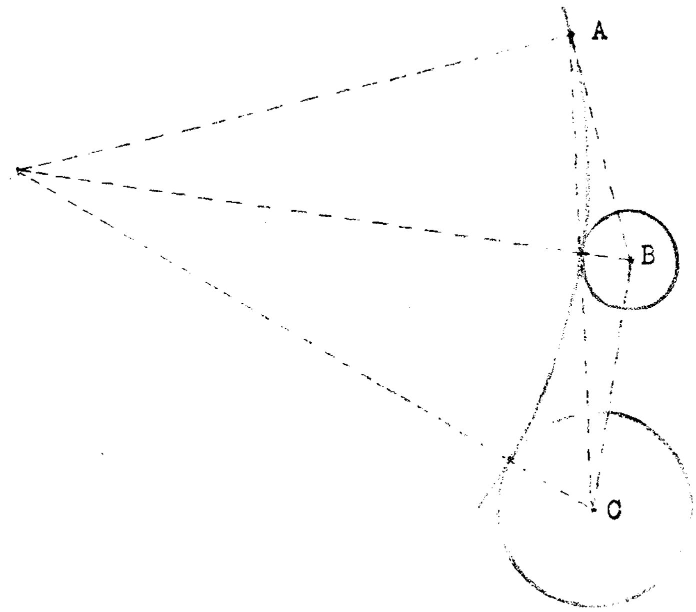
3. Trajectory, Projectiles: Use a stick with strings attached at equal intervals with some small object attached to the ends of the strings to make them clearly visible, the lengths of the strings being proportional to the squares 1,4,9,16,25, etc. proceeding from smaller to larger. At whatever angle the stick is held the points at the end of the strings form a parabola. This parabola can be fitted to a stream of water coming out of a tube to illustrate that the path of the water through the air is a parabola.

4. Pendulum: Use a string marked off at equal intervals with a weight attached at the end. Use this apparatus as a pendulum with the weight as the bob. Measure the period for different lengths of the pendulum and, using the values obtained, establish the relationship. If it is desired to go a step further the constant can be calculated.

5. Soap Bubble Experiment: Use of a wire loop with a loop of string attached to the wire loop by means of three other strings. Dip the wire loop in a solution such as a soap solution so that a film will be formed and puncture the film in the string loop while leaving the rest of the film intact. The loop will form a circle as the surface tension of the remaining film will cause it to contract to the smallest possible area so that the loop will possess the largest area for a figure of that perimeter.

6. Temperature Scales: Derivation of the formula relating fahrenheit and centigrade temperature scales using proportion.

7. Sound Ranging: (an exercise) Determination of the location of a gun by obtaining the times at which the report of a shot was heard at various points. In the diagram the report was heard at point B one second after it was heard at point A, and one second before it was heard at point C. Therefore, using 1120 feet per second as the velocity of sound, the sound of the report was on a circle of radius 1120 feet with center B and on a circle of radius 2240 feet with center C when it was heard at point A. The gun is located at the center of the circle whose arc passes through A and is tangent to the circles whose centers are B and C. Use of trigonometry to find the position of the gun using triangle ABC.



8. Bunsen Photometer: Use light sources of known intensities and a paper screen with a grease spot in the middle. When the grease spot is equally illuminated on both sides it disappears. Using light sources of known intensities establish the relationship between the distance from the screen and the intensity and then use that relationship to determine the intensities of unknown light sources.

9. Optical Bench: Use of an optical bench with light source, screen, and various lenses. Tabulate object distances and image distances with several examples for each lense and derive the lense formula from the data.

10. Current electricity: Test the fall in potential along different lengths of a wire, the more nearly uniform the wire the better, and derive the mathematical relationship between the two variables.

CROSS REFERENCE TABLE FOR EXPERIMENTS

	Experiment 1	2	3	4	5	6	7	8	9	10
1. Similar		2								
2. Signed		1								
7. Symbols	1	2	2	1		2		1	1	1
8. First degree	2	1				1				2
9. Formulas	2	2		2		2		1		2
10. kx^2			2	1						
11. k/x^2								1		
19. Parentheses						1				
20. Literal			1		2	1				2
21. Dependence	2		2	2		2		1	2	2
31. Reciprocal									1	
33. Proportion	1					1				
34. Variable	2		1	1		2		2		1
37. Direct var	1		1	1						1
38. Inverse		2						1		
39. Joint var		2								
40. Graphs	1					2				1
41. Relative	2									
42. Linear	1					2				1
43. Simultaneous	2									
44. Table	1									
65. Area					1					
68. Circle							1			
73. Parabola			1							
See note 1	1	1	1	1		1		1	1	1

Note: Experiment 7 also includes some trigonometry beyond that included in the list.

Note 1: The ability to perceive a relationship between the variables.

CHAPTER 4

RELATING MATHEMATICS AND PHYSICS COURSES

As mentioned previously one of the objectives of this study is to set up in general outline a plan whereby physics and mathematics courses taught in high school might be more closely related. Before discussing a specific order in which the subject matter topics might be arranged it would be advisable to comment on the merits of the two principal methods whereby the two courses could be more closely related: integration and correlation.

Integration:

If the mathematics and physics courses were integrated they would be taught as one course in the year in which physics was taken; such a course could be taught by one teacher with a sufficient grounding in both subjects or by two teachers cooperatively, the latter would require that both teachers be in the classroom at the same time. In an integrated course the subject matter content would be so arranged that any topic in mathematics necessary for a given topic in physics would be taught either before that physics topic or concurrently with it. This procedure would have the advantage of providing a use for that mathematics as soon as it is taught, thus tending to fix it in the mind of the student more thoroughly than would be possible otherwise.

Since the mathematics would be taught just prior to the physics topic there would be no need for review of the mathematics topic as might be the case if the student had not been exposed to that topic for some time prior to the need for it in physics and thus an economy in time might be effected. Approaching the subject in this manner would also have the advantage of giving the student training in applying the mathematics; i.e. applying it to the real world about him and using mathematics to describe actual phenomena instead of confining mathematics to the classroom. The student through working with concrete materials and through expressing physical relationships in mathematical terms will come to understand both the physical relationships and the mathematics better than if each were presented without any emphasis upon the interrelationship between the two. However these interrelationships must be emphasized if the student is to appreciate fully their significance. It seems to me that the best way in which to present these interrelationships would be to present the subjects together as one insofar as possible; that is to teach the mathematics along with the physics while at the same time of course pointing out that mathematics is significant and useful for other reasons than its value as a tool for the physicist.

Theoretically it would seem that the most effective manner in which to combine the mathematics and physics courses would be to have one teacher teaching a course we shall call math-physics (or if you prefer, physics-math). The course would combine the time generally spent on mathematics and physics and where appropriate would emphasize the mathematics with the physics, providing training in generalizing or applying the more abstract mathematics to the more concrete physical situation and in expressing the more concrete relationships in mathematical terms. There would of course be topics in each subject area where there was no significant interrelationship, at least at the high school level, and in these cases there would be no point in attempting to artificially create an interrelationship between the two; they could be taught separately in these instances. However there are disadvantages to an arrangement of this nature. There should be some sort of safeguard to insure that one subject is not emphasized to the detriment of the other; any one teacher might be better versed in one subject than in another and might consequently tend to slant the math-physics course toward one subject or another causing complaints that Johnny was not getting enough mathematics or physics as the case may be. To provide a safeguard in terms of the effective teaching time spent on each subject would be virtually impossible due to the frequent occasions when it would be most difficult to determine how much time had been spent on the mathematics and how much time on the physics when working on a topic where the mathematics and physics were closely related. A safeguard might be provided in terms of the subject matter content to be covered in mathematics and physics but there could well be a difficulty inherent here in that classes of different ability levels would not be able to handle the same amount of material and a teacher might still emphasize one subject more than another, i.e. teach it more thoroughly.

Having two teachers in the classroom cooperating, one a physics teacher and one a mathematics teacher, might obviate any bias on the part of either one but generally speaking any plan of this sort would be out of the question due to the expense in terms of money and personnel of having two teachers teach the same class at the same time.

Although it is felt here that teaching a math-physics course with a qualified instructor who would be virtually without bias toward either subject would be the most effective way in which to attack the problem there might be a practical difficulty in finding properly qualified teachers who would have a sufficient grounding in both subjects to carry through such a program effectively. It is true that many high schools, especially smaller ones, have one teacher for both subjects now and in these instances an integrated math-physics program might well be tried and if results appeared favorable such a program could be put on a permanent basis. A program of integration might prove more difficult in a larger high school with more complete departmentalization but is probably not beyond the realm of possibility.

Correlation:

The second possibility for relating mathematics and physics courses would be to correlate the two; that is to teach the courses separately but through cooperation between the different teachers the subjects could be arranged in such a sequence that the mathematics needed for a given topic in physics would be taught in the mathematics course before or concurrently with the physics topic for which it was needed was presented in the physics course. A further degree of cooperation between the mathematics and physics teachers would be evidenced if some of the simpler physics experiments, a few of which are presented in table five in chapter three, were undertaken in the mathematics classroom to illustrate the mathematical principles involved. The experiments indicated in table five with an attached chart showing the mathematics involved in each are not in any way intended to represent a complete collection. They are included largely to give an indication of the type of activity which might be profitably engaged in in the mathematics classroom. This sort of activity would also be a help in training students to apply their mathematics to situations outside the mathematics classroom. The physics teacher could cooperate by emphasizing mathematical concepts as they come up in physics, e.g. direct variation and functional change, and in this manner with the mathematics teacher using physical applications to illustrate mathematical principles and the physics teacher showing where and how mathematical principles are used in physics, the relationships between the two subjects could be brought to the attention of the students thereby enriching their understanding of both subjects. Using a correlation of separate mathematics and physics courses avoids to a large extent any detrimental effects of a bias towards his subject on the part of either teacher, for the physics would be supplementary to the mathematics course and the mathematics, beyond what was absolutely necessary, would be supplementary to the physics course. There would also not be as great a problem in finding qualified instructors. However it would be necessary for both the mathematics and physics teachers to maintain a close degree of cooperation between the two departments and to continually emphasize the interrelationships between the two subjects if such a program were to be most effective. The advantages of a correlated mathematics and physics course would be the same as those of integrating the two courses although to a lesser degree while the disadvantages attending the integrated course would be almost entirely absent. A disadvantage attending the correlated courses would be the additional work necessary on the part of the cooperating teachers, however this would also be true in an integrated program regardless of whether it would be taught by one or two teachers.. Whether or not the results produced by such additional work would be worth the effort is a question which cannot be answered conclusively here; however the articles which have been written, and which are mentioned in chapter two, in which a program with some degree of cooperation between the two departments has been attempted have indicated that student response in terms of both interest and understanding was quite good. These experiments have generally been fairly limited in scope but at least should present an indication of what could be expected from more complete cooperation between those responsible for mathematics and physics courses.

Topical Arrangements for a Cooperative Program:

The physics course is taught in either the junior or the senior year, the year varying among different schools. Therefore we will consider an arrangement of topics for a physics course taught in the junior year and related to the junior year mathematics course (second year algebra) and also for a physics course taught in the senior year and related to the senior year mathematics course (trigonometry, solid geometry, analytic geometry, and some calculus). In each case a possible order of topics will be discussed. The mathematics and physics topics will be listed and then discussed in terms of first an integrated and then a correlated course.

In actually teaching in a program where mathematics and physics are related the question of timing becomes quite important; for the purpose of even attempting such a program is to both insure that the student will have the requisite mathematics for a given topic in physics and to assist in his being able to apply that mathematics when the occasion does arise. In order that the student will have the requisite mathematics for a given topic in physics and will have it in readily usable form it would be best if that mathematics were taught immediately preceeding the physics topic so that the student would find a use for that mathematics as soon as he had originally mastered it and while it was still fresh in his mind. The writer feels that to teach the mathematics concurrently with the physics could well result in making both subjects more difficult and consequently it is felt that that procedure is best avoided. The immediate application of the mathematics would reinforce the mathematical knowledge gained and enable the physics topic to be mastered with greater facility than otherwise.

In order to accomplish the above objective it appears necessary to do a bit of juggling with the traditional order of both the physics and mathematics courses, particularly if the arrangement is to be effected in the junior year. We shall now consider a possible arrangement of topics in the junior year.

When Physics is Taught in the Junior Year:

Of the mathematics found necessary for physics in this study, a very large part is generally taught in the junior year mathematics course. This fact both enhances the potential of a closer relationship between the two subjects and at the same time tends to make an evolving of a workable program more difficult due to the many interrelationships to be considered. It would seem advisable in an integrated math-physics program to teach both the mathematics and the physics without an appreciable break in either, that is to give the student some mathematics and some physics, although not necessarily equal amounts, every day or almost every day. The purpose here is to avoid breaks in continuity of presentation for the subject matter and also to insure some variety of subject matter for the student.

A possible list of topics in mathematics and physics is presented below. These lists are followed by a chart indicating the week in which each topic might be started in a 36 week year. Each topic is listed in the week in which it could be started. Obviously any such arrangement presented here must be approximate due to the varying ability levels of different classes and to the varying degrees of emphasis which would be placed on any one topic by different teachers. After their presentation the lists and chart below will be discussed in relation to an integrated and a correlated arrangement of the mathematics and physics courses in the junior year.

Physics:
- Atomic Physics
- Heat
- Sound
- Gas Laws and Kinetic Theory
- Light
- Electricity and Magnetism
- Mechanics - Force and Motion
- Mechanics - Work, Power, and Energy
- Mechanics of Fluids

Mathematics:
- Fundamental Operations
- First Degree Equations and Problems
- Fractions - Including finding an arithmetical lowest common denominator, ratio, and proportion; not including work involving factoring.
- Functional Relationships, Variation, Graphs
- Slide Rule (Multiplication, division, powers, square roots)
- Reciprocal Equations
- Special Products and Factoring
- Fractions - Those involving factoring and complex fractions.
- Quadratic Functions and Equations
- Numerical Trigonometry - Including the sine and cosine laws.
- Logarithms
- Powers and Roots - Topics not brought in before such as fairly complex work involving radicals.
- Systems of Linear Equations
- Other Topics - Such as series, probability, analytic geometry, etc.

WEEK	PHYSICS TOPICS	MATHEMATICS TOPICS
1.	Atomic Physics	Fundamental Operations
2.		
3.	Heat	
4.		First Deg. Equa., Prob
5.		
6.		Fractions; Ratio, Propor.
7.		
8.	Sound	
9.		Functional Relationships
10.		
11.	Gas Laws and Kinetic Th.	
12.		Slide Rule
13.		
14.	Light	Reciprocal Equations
15.		
16.		Special Products, Factor.
17.		
18.	Electricity and Magnetism	
19.		Fractions; Factoring
20.		
21.		Quadratic Functions, Equ.
22.		
23.	Mechanics - Force, Motion	
24.		Numerical Trigonometry
25.		
26.		
27.		Logarithms
28.		
29.		Powers and Roots
30.		
31.	Mechanics - Work, Pow., E.	Systems of Linear Equat.
32.		
33.		Other Topics
34.	Mechanics of Fluids	
35.		
36.		

An integrated math-physics program has two principal advantages over a correlated program. First the interrelationship between the two subject matter areas is more easily seen and appreciated by the student and second the integrated course would appear to be far more flexible as to timing. There is no set time that must be devoted to either mathematics or physics each day so that it is possible to arrange the timing whereas in a correlated arrangement it is more difficult, though still possible, to do so. The discussion below is from the point of view of an integrated program.

The first two topics on the mathematics list are a review of topics which in general are covered in first year algebra as are several other topics included on the list; e.g. special products and factoring. While the work covered in first year algebra is being reviewed it would seem to be best to cover a physics unit which is largely descriptive in nature at the elementary level. Of the physics units listed that of atomic physics seems to be most conducive to descriptive treatment and the little mathematics needed, such as scientific notation (using powers of ten to express very large or very small numbers) could easily be introduced while reviewing fundamental operations in the mathematics course as can the problem of expressing verbal relationships in terms of mathematical symbols though this topic could be avoided altogether in this physics unit if so desired. The formula of the type $y = kx^2$ could be introduced while working on first degree equations and whatever needed from powers and roots could be introduced there also. If after atomic physics some elementary properties of matter are considered, first degree equations could be utilized in the conversion of units from one system to the other and also within a given system, e.g. the metric system.

The second physics topic listed is heat. Much of the mathematics involved here is computational and the types of formulas needed for the physics and not previously taught in the mathematics course could be brought up as need for them arises in the physics course. A simple experiment such as number six in table five relating centigrade and fahrenheit temperature scales would provide practice in working with proportion and would not necessarily have to be introduced early in the unit if proportion had not yet been covered in mathematics. If the mathematics had progressed sufficiently far, variation could well be brought in while studying heat. Sound, next on the physics list, gives ample opportunity for working with variation and proportion.

In the unit gas laws and kinetic theory, the gas laws particularly, a working knowledge of proportion and variation is virtually essential, giving ample opportunity for application of this phase of mathematics. The extensive calculations which are needed in working out problems involving the gas laws - given simultaneous changes in two of the variables (temperature, volume, and pressure) find the change in the third - provide a good opportunity to show the usefulness of the slide rule in performing extensive calculations and consequently the slide rule is listed after functional variation in the mathematics list. If it is desired to give students some understanding of why the slide rule works, a short introduction to logarithms would probably suffice and the actual mechanics of logarithms could be postponed until later in the course, unless of course it appeared that sufficient time was available to teach logarithms before the slide rule and still insure that any needed mathematics topic would preceed the physics topic for which it was needed.

The next physics unit listed is light and in mathematics the topic reciprocal equations is indicated, that topic being used in the mirror and lense equations in light. The trigonometry which could be used in the index of refraction can be easily avoided; numerical trigonometry would probably tend to be more meaningful if introduced with mechanics where it is more widely applicable. After taking up reciprocal equations in mathematics all the mathematics needed for light will have been covered and special products and factoring which has little application in elementary physics could be taken up. Next the work in fractions involving factoring and complex fractions, which had not been taught previously, could be covered. Complex fractions, or formulas involving them, are needed in the physics unit electricity and magnetism in conjunction with the voltaic cell.

Mechanics - force and motion, the next physics unit, makes use of both some elementary trigonometry, the bulk of which would be treated in numerical trigonometry, and, to a limited extent, quadratic equations. In the mathematics list the unit logarithms is listed after numerical trigonometry but if possible it would be helpful to introduce logarithms previous to the trigonometry so that the logarithms might be used to assist in the calculations in trigonometry involved in solving right triangles. The next physics unit is mechanics - work, power, and energy, but this unit, for which the previous one is prerequisite, involves no mathematics beyond that involved in the previous physics unit and the mathematics topics, or parts of mathematics topics, ignored or slighted previously could be taken up here. The last physics unit, mechanics of fluids, involves no mathematics not already taken up.

In general an attempt should be made to provide the generally applicable mathematics such as ratio, proportion, and functional relationships as early as possible and these topics could be emphasized where applicable throughout the physics course if so desired. The mathematics needed at only one or a few particular points in the physics course should be presented at the appropriate point, or reviewed if previously taught. There are several mathematics units, such as logarithms and systems of linear equations, which could be presented most any time and may be postponed until toward the end of the year or inserted earlier in the year if there appears to be sufficient leeway in the timing.

Although the above description is intended for an integrated mathematics and physics program the same general list of topics could be applied to a program in which the courses were correlated. The timing would probably not be as good in relating mathematics topics to physics topics due to a lesser degree of flexibility and the relationship of mathematics as a tool of physics probably would not be as well emphasized in a correlated program; however the relationship between the two courses would be given far greater emphasis in a correlated program than in the currently accepted set up where the relationship is not generally recognized.

When Physics is Taught in the Senior Year:

The senior year mathematics course is assumed to be composed of a little less than half a year of trigonometry, some solid geometry, the rest of the time being divided between analytic geometry and elementary calculus. Since the greater bulk of the high school physics course does not draw upon these areas in mathematics it does not appear that there is as much to be gained from integration or correlation as would be the case if the physics course were taught in the junior year. There is however a definite interrelationship in several places. The index of refraction of light can be expressed in terms of the sine and cosine and consequently the physics unit of light could be taught early in the physics course to show an application of these functions. The only other physics units to make notable use of the mathematics taught in the senior year are those of mechanics - force and motion and electricity. The sine curve for alternating current is the usage of trigonometry in electricity and thus electricity could also be introduced fairly early in the physics course, assuming here that the mathematics course is arranged in the order in which it is indicated above. The trigonometry involved in mechanics is largely connected with vectors and inclined plane problems and this unit could be started just after the sine law and cosine law had been taught in trigonometry. Later in the mathematics course the connection between projectiles and the parabola could be brought in when the parabola is studied in analytic geometry (see also experiment three in table five), and uniformly accelerated motion could be brought in when teaching elementary aspects of the calculus. The rest of the physics could be fitted in where convenient. There does not appear to be a sufficiently strong relationship between the two courses if the physics course is taught in the senior year to justify an integrated program although it is felt here that there would be definite benefits to be derived from the correlation of the two subjects.

In Which Year Should Physics be Taught:

Actually it is beyond the scope of this thesis to propose in which year the physics course should be taught; however since high schools do vary in this aspect we shall comment briefly on the question in the light of the results of this study. There would appear, from the standpoint of this thesis, to be substantial advantages to be derived from teaching physics in the junior year due to the close interrelationship with the mathematics course taught in that year, and further advantages to be derived from integrating or at least correlating to some degree the mathematics and physics courses. Presenting physics in the junior year would, by implication, relegate the chemistry course to the senior year, There would however be possible advantages to teaching the chemistry course in the junior year. There is a great deal of use made of ratio and proportion in chemistry and possibly other mathematical topics are widely used also, a possible topic for further research, which might indicate a possibility of correlating the junior year chemistry course with the mathematics of that year. Also the work on the chemical electric cell and chemical effects of electric current presented in physics might be better understood with a chemistry background, although these topics could probably be as well understood in a senior chemistry course with a physics background, i.e. a class with a grounding in the physical principles involved. There is also the fact that in the outline of topics proposed for the junior year mathematics and physics program a few physics topics are covered descriptively due to the students not having had the requisite mathematics. However it is felt here that this is not a serious difficulty. However we shall rest on this point, that on the basis of this thesis there would appear to be substantial advantages to teaching the physics course in the junior year although there are other factors which should be considered.

CHAPTER 5

SUMMARY OF RESULTS AND CONCLUSIONS

General Review and Conclusions:

The conclusions stated at various points in the thesis will be briefly reviewed here.

First there appears to be a very significant interrelationship between mathematics and physics at the high school level with the role of mathematics being that of a tool of physics. The concrete relationships of physics are stated in mathematical terms and due to these relationships being so stated the physicist is enabled to work with them with much greater facility than otherwise possible.

High school students apparently have difficulty when faced with physics problems of a mathematical nature due to a lack of the necessary mathematical knowledge or an inability to apply the mathematical knowledge they already possess. This problem is seen as serious due to the need for mathematics in physics.

Tables one, two, and three in chapter three are the results of a study of the mathematics involved in high school physics; being respectively a list of mathematics topics needed for high school physics, a list of physics topics involving mathematics which are commonly taught in high school physics, and a cross indexed chart showing what mathematics was found to be needed for each physics topic listed and what physics topics made use of a given mathematics topic. The mathematics topics which appear to be most generally significant are the ability to translate verbal expressions into algebraic symbols, formulas of various types, ratio, proportion, variation, and functional relationships. The physics unit listed which involved the least mathematics was that of atomic physics and that which involved the most mathematics, both in quantity and in the spread of topics, was mechanics - force and motion.

It was felt that both causes of student's difficulties with the mathematics involved in high school physics could be successfully attacked by means of cooperation between teachers of the two subject matter areas, the principal means of cooperation being correlation or integration although some degree of cooperation would be evidenced by the transferral of some physics experiments of a mathematical nature, such as those listed in table five, to the mathematics course or by having the physics teacher advise the mathematics teacher as to when various mathematics topics would be needed in the physics course, and the mathematics teacher seeing that his classes were provided with these topics in time for them to be used in the physics course. It was felt that integration would be advantageous because it would: give life to the mathematics by making it more experimental in nature; tend to fix it in the mind of the student by providing an application for the mathematics and by showing the student one way in which mathematics is applied would give him a better appreciation of the value of mathematics; integration would also tend to help the student understand both fields better through the study of principles common to both; and would aid the student in developing techniques of applying mathematics by making applications in the classroom. A correlated course would have the same advantages although to a lesser degree and would possess the disadvantage of being less flexible so far as what to teach in a given day (how much physics and how much mathematics) than an integrated course while the integrated course has the possible disadvantage of a bias on the part of the teacher toward one subject or the other and a resulting lack of attention to the subject not favored. Either arrangement would assist in solving the general problem, this being the principal advantage in so far as the physics teacher is concerned in addition to the possible time saved by not being forced to review mathematics with a physics class before teaching them physics.

The junior year mathematics course contains the bulk of the mathematics (of that used in the last two years of high school) used in the physics course and as a result it is felt that the greatest benefit would be derived from cooperation between the two departments if the physics course were presented in the junior year. Presentation of the physics course in the senior year would probably necessitate the reviewing of several mathematics topics in the physics course. However there may be substantial advantages to teaching the chemistry course in the junior year of which I am unaware. There is still much of value to be derived from a correlation of the senior year mathematics course with the physics course if taught in the senior year although the relationship between the two courses does not seem to me to be sufficiently strong to warrant integration.

Suggestions for Further Research and Limitations of the Thesis:

The research on this thesis was confined to investigating the mathematics contained in a few high school physics textbooks. Further study on the mathematics involved in the physics laboratory would probably reveal a very significant relationship there, especially in the area of graphs. Some indication of the possibilities inherent here can be had by reference to appendix five. It might be an interesting study to attempt to ascertain whether the physics as actually presented in the classroom conformed to the mathematical content of physics texts. It would seem to me that a study made of the mathematics involved in high school chemistry would assist greatly in ascertaining whether chemistry and physics should be taught in the junior and senior years respectively or whether the order might be profitably reversed. Finally further work might be done on the work started in this thesis; this work might well take the form of a day-by-day plan for a correlated mathematics and physics program.

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