Asset pricing implications of the mismatch between performance window and benchmark duration

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Asset Pricing Implications of Heterogeneous Investment Horizons

Idan Hodor *Monash University*

Fernando Zapatero *Boston University*

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Abstract

Short performance windows shrink fund managers' investment horizons well below value investors' long-term investment mandates. We unravel that the frictions tied to the asset management industry are responsible for the recent empirical findings showing that the risk premium, volatility, and Sharpe ratio on short-term dividend strips are higher than long-term dividend strips — findings that are at odds with the leading equilibrium asset pricing models. The interplay between fund managers' relative performance objective and short-term performance window is the primary equilibrium channel, which remains robust to various extensions. Our continuous-time setup admits closed-form expressions and is supported by additional empirical evidence.

1 Introduction

A growing body of literature studies the impact of professional asset managers on asset markets. Asset managers' contracts are expected to align their incentives with those of

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the investors to address possible agency problems, but the resulting investment strategy affects asset prices. This literature establishes several facts. First, mutual fund managers' performance is evaluated relative to a benchmark. In particular, Ma, Tang, and Gómez (2019) recently showed that 79% of the mutual fund managers in the US are evaluated relative to a prespecified benchmark with rolling windows of various lengths. The minimum rolling performance window is as short as one quarter, and a typical manager faces a one-year median for their minimum rolling performance window and a three-year average across all their windows.

An equally significant group of investors, in general, value investors like Warren Buffet, has a much longer investment horizon objective than mutual fund managers' performance window of one quarter, or one year, or even three years. This group does not suffer from the frictions of the asset management industry and typically cares about absolute performance rather than performance relative to a benchmark.

The heterogeneity of the investment horizons of these two groups of investors combined with the relative performance objectives of asset managers might be able to explain anomalies between short-term and long-term asset prices because the proportion of assets under management affected by these frictions is very high.

Despite its potential, the asset pricing implications of the heterogeneous investment horizons have yet to be explored. This paper proposes a dynamic asset pricing equilibrium model with a short-term asset manager with relative performance concerns, and a value, long-term investor. The model, thus, captures the heterogeneous investment horizons in order to study empirical regularities of short-term and long-term dividend strips recently established in the literature. Dividend strips grant the buyer dividends between two specified future dates. By looking at options data, the influential work of van Binsbergen, Brandt, and Koijen (2012) showed that short-term dividend strips have a higher risk premium, Sharpe ratio, and volatility than long-term dividend strips. van Binsbergen, Hueskes, Koijen, and Vrugt (2013) and, more recently, van Binsbergen and Koijen (2017) extend this evidence using dividend futures for the US, Europe, Japan, and the UK. These empirical findings are at odds with the leading equilibrium asset pricing models.

Our work's main contribution is to show that asset management frictions can explain the recent empirical evidence. That is, the heterogeneity in investment horizons between short-term fund managers with benchmarking incentives and long-term investors can explain these recent empirical regularities, thereby providing an original theoretical foundation for what has been considered a puzzle. While other explanations for dividends' strips empirical regularities exist, this paper is the first to show that frictions tied to the asset management industry can account for these empirical regularities.

The model features two types of investors: (i) asset managers who have short-term horizons because their performance is evaluated in the short-term — annually or even quarterly; additionally, their performance is evaluated relative to indexes; (ii) value investors who do not care about the indexes and have long term goals. This model shows that two potential mechanisms drive the equilibrium. The first mechanism is the heterogeneous investment horizons *individually*: the short-term performance window relative to the value investor's long-term goals. Alternatively, the *combined* effect of the heterogeneous investment horizons and the relative performance objectives of asset managers drives the equilibrium: the interaction between the short-term performance window and benchmarking incentives. We explicitly disentangle these two channels and show how the individual and combined effects determine equilibrium.

Significantly, the assets in the model mimic dividend strips' payout: the short-term asset pays dividends in the short-term, and the long-term asset pays dividends in the long-term, and their payouts do not overlap. Since the short-term asset manager cares only about the short-term investment horizon due to the short performance window, eventually, he must hold the short-term asset at the end of his performance window. Like a long-term dividend strip, the long-term asset does not pay off in the short term when the manager is at the end of their performance window. As a result, equilibrium adjusts assets' returns and market prices of risk so that, in the end, the short-term asset manager ends up holding the short-term asset while the long-term value investor ends up holding the long-term asset, regardless of the benchmark composition and the strength of the asset manager incentives to benchmark.

The equilibrium market price of the short-term risk is always higher than the equilibrium market price of the long-term risk. However, it does not immediately imply that the short-term risk premium is higher than the long-term risk premium. In fact, without benchmarking incentives, the short-term risk premium is lower than the long-term risk premium. This result becomes apparent when we realize that the quantity of short-term risk is higher for the long-term asset when there are no benchmarking motives, leading to a higher long-term risk premium.

In contrast, with benchmarking incentives, the price of the short-term drops, and the price of the long-term risk increases. Even though it is not immediately apparent, the short-term risk premium is higher than the long-term risk premium in this case; again, the reasoning lies in the changes to the quantity of risk. First, the short-term asset quantity of short-term risk remains unchanged while the long-term asset quantity of short-term risk drops, tilting the risk premium towards the short-term. Second, the short-term asset has no exposure to long-term shocks, so the quantity is zero, while the long-term asset has a negative quantity of risk; again, tilting the risk premium towards the short-term. Eventually, the two forces lead to a negative sloping risk premium.

The heterogeneity in investment horizons, through the individual effect, emphasizes the importance of non-fundamental short-term cash-flow news when market participants have different investment horizons. First, the individual effect introduces a negative fundamental long-term asset risk exposure, meaning long-term fundamental cash-flow news reduces the long-term asset price in equilibrium. Second, the individual effect implies that short-term news affects the long-term asset more than long-term news. For instance, a common shock to both short-term and long-term cash flow news would increase the long-term asset despite the negative long-term fundamental exposure. These effects run against a homogenous investment horizon economy in which fundamental news is the main driving force behind asset pricing fluctuations and correlates positively with asset returns.

When reflecting on the combined effects of benchmarking incentives on the assets' return exposures, we find that in most aspects, but not all, the heterogeneous investment horizons economy is distinct from an economy with homogenous investment horizons. In a heterogeneous horizon economy, when the long-term asset belongs to the benchmark, its fundamental exposure increases but remains negative, and its non-fundamental short-term risk exposure decreases but remains positive. This result reduces the long-term asset return's total volatility, aligning with Cuoco and Kaniel (2011) but reversing Basak and Pavlova (2013)s' findings when the investment horizons are homogeneous. When the short-term asset belongs to the benchmark, its fundamental return exposure and total volatility do not change.

When reflecting on the combined effects of the benchmarking incentives on the market prices of risk, we find that in a heterogeneous horizons economy, when the long-term asset belongs to the benchmark, the long-term (fundamental) market price of risk increases while the short-term (non-fundamental) market price of risk decreases. In contrast, in a homogenous horizon economy, benchmarking reduces the market price of benchmarked assets' fundamental risk while it does not affect non-fundamental market prices of risk. Adding the short-term asset to the benchmark entails another downforce on the short-term market price of risk, while it does not affect the long-term market price of risk, aligning with the findings of homogeneous investment horizon economies. However, regardless of the benchmark composition, the market price of the short-term risk is always higher than the long-term risk.[1]

The equilibrium dynamics of the combined effect work as follows. The asset manager's benchmark hedging requires exposure to the long-term risk; to satisfy this desire, the asset manager must hold the long-term asset since the short-term asset has no long-term risk exposure. However, in equilibrium, any mean-variance upside precisely offsets a benchmark hedge downside when the asset manager holds the long-term asset. As a result, the asset manager cannot attain exposure to the long-term risk and cannot satisfy the benchmark hedging desires by holding the benchmark asset in equilibrium.

Consequently, if the asset manager eventually holds the benchmark asset, it is not due to the benchmark hedging but rather the long-term asset risk-return trade-off. However, equilibrium ensures that the asset manager does not gain risk-return trade-off benefits from holding the benchmark asset and sets the short-term market price of risk higher than the long-term market price of risk. Eventually, this outcome leads the asset manager to hold the short-term asset in equilibrium, even when the benchmark hedge mandates otherwise.

The negative long-term risk exposure kills two birds with one stone since it also satisfies the value investor's risk aversion hedging desires. In equilibrium, the value investor is willing to forgo a better short-term risk-return trade-off to satisfy the long-term risk aversion hedging desires. Eventually, this outcome leads the value investor to hold the long-term asset in equilibrium.

Lastly, we provide novel empirical evidence to substantiate the importance of heterogeneous investment horizons. Our model predicts that (i) the asset manager buys the benchmark assets due to the benchmark hedging and sells short the short-term asset due to the risk aversion hedging, and (ii) as the size of the asset manager increases, the short-term asset price increases relative to the long-term asset price. Our empirical analysis verifies those

[1]Please refer to Basak and Pavlova (2013), and Buffa and Hodor (2018) for a comprehensive analysis of the equilibrium effects of homogenous investment horizons.

predictions. The empirical analysis abstracts away from features of the asset management industry. In particular, hedging against benchmark fluctuations means a different composition of long- and short-term assets for asset managers with different benchmark mandates. Still, the equilibrium predictions show up in various specifications of well-known benchmarks.

The remainder of the paper is organized as follows. Section 2 summarizes the related literature; Section 3 sets up the economy with benchmark duration mismatch; Section 4 discusses the equilibrium mechanism and addresses the three main puzzles of van Binsbergen et al. (2012); Section 5 introduces additional predictions accompanied by empirical support; Section 6 extends the main setup; and Section 7 concludes.

2 Related Literature

Our article builds upon the growing literature studying the effects of professional asset management on asset prices. The most relevant to our paper is a strand in that literature focusing on the asset pricing implications induced by an asset manager with a performance benchmark objective.

In this strand of literature, it is typical to embed the performance benchmark into the asset manager's objective function. Brennan (1993) and Gómez and Zapatero (2003) introduce a static setup with a mean-variance asset manager that maximizes the portfolio return relative to the return on a benchmark portfolio. Cuoco and Kaniel (2011) introduce a dynamic setup with a constant relative risk aversion asset manager that cares about the benchmark through performance-based fees. Basak and Pavlova (2013) and Basak and Pavlova (2016) introduce a dynamic setup with a reduced-form approach to incorporate the benchmark incentives into the manager's objective. They provide the salient features a reduced-form asset manager objective should satisfy and study the asset pricing implications of a single manager. Buffa and Hodor (2018) adopt their reduced-form asset manager objective and introduce a dynamic setup to study the asset pricing implications of multiple asset managers with different performance benchmarks.

Similarly, Basak, Pavlova, and Shapiro (2007) and, more recently, Sotes-Paladino and Zapatero (2019) embed the performance benchmark into the asset manager's objective function to study the asset manager's optimal behavior due to benchmarking and given asset prices. A common thread in all these papers is that there is no heterogeneous investment horizons, and performance windows align with long-term investment mandates. In contrast, this paper studies the asset pricing implications when there is heterogeneous investment horizons and fund managers' performance windows are strictly shorter than long-term investment mandates.

Our paper proposes another constructive addition to the literature by introducing an endogenous benchmark that depends on the equilibrium price rather than the terminal exogenous dividend, as the literature assumes. Despite the substantial complexity of considering an endogenous benchmark, our paper provides analytical closed-form solutions to the equilibrium quantities. We achieve this goal by considering Basak and Pavlova (2013)s' objective and letting the benchmarking importance parameter approach infinity.

A related strand of literature addresses the importance of benchmarks to align asset managers' incentives. Ou-Yang (2003), Cadenillas, Cvitanić, and Zapatero (2007), and Lioui and Poncet (2013) show that benchmarking is a part of an optimal contract given prices. Benchmarking is essential to align incentives even when considering the interplay between the equilibrium asset pricing and optimal contracting, as Cvitanić and Xing (2018) and Buffa, Vayanos, and Woolley (2019) show.

While there are other explanations for dividends' strips empirical regularities, our paper is the first that connects the frictions arising from asset managers' objectives to the empirical regularities of dividend strips. van Binsbergen and Koijen (2017) provides an extensive review of the different models that generate the dividend strips irregularities and classify their mechanisms into six broad categories: alternative models of preferences (Berrada, Detemple, and Rindisbacher (2013), Marfè (2014), Curatola (2015), Eisenbach and Schmalz (2016), Andries, Eisenbach, and Schmalz (2019), Andries (2021)), alternative models of technology (Gourio (2008), Nakamura, Steinsson, Barro, and Ursúa (2013), Belo, Collin-Dufresne, and Goldstein (2015), Lopez, Lopez-Salido, and Vazquez-Grande (2015), Hasler and RobertoMarfè (2016), Marfè (2013), Ai, Croce, Diercks, and Li (2018), Corhay, Kung, Schmid, and Nieuwerburgh (2020)), alternative models of beliefs (Croce, Lettau, and Ludvigson (2015)), heterogeneous agent models (Lustig and Nieuwerburgh (2006), Marfè (2017), Favilukis and Lin (2016),), asset pricing models with an exogenous stochastic discount factor (Lettau and Wachter (2007), Lettau and Wachter (2011), Lynch and Randall (2011)), market microstructure and tax effects (Boguth, Carlson, Fisher, and Simutin (2011), Schulz (2016)). In their review, van Binsbergen and Koijen (2017) argue that frictions introduced by asset managers may cause the dividend strips empirical irregularities.

3 An Economy with Heterogeneous Horizons

This section lays out a simple and tractable model to study the mismatch between investors with short-term performance windows and long-term investment mandates. We consider a standard pure-exchange finite horizon economy. Time t is continuous and goes from zero to T. Uncertainty is driven by two independent Brownian motions, (Z_{1t}, Z_{2t}). This model has two investment horizons: a short-term investment horizon, $\frac{T}{N}$, and a long-term investment horizon, T, whereby $N \geq 2$ is a finite number. For the primary analysis, we set $N = 2$.[2]

There are two dividend payout dates: short-term and long-term. The supply of dividends at the short-term is denoted by $D_{\mathcal{S}\frac{T}{2}}$ and at the long-term by $D_{\mathcal{L}T}$, where $\mathcal{S}$ and $\mathcal{L}$ stand for short- and long-term dividends, respectively. These short-term and long-term terminal dividends are determined by the dynamics of

$$dD_{\mathcal{S}t} = D_{\mathcal{S}t} \left(\mu dt + \sigma dZ_{1t} \right), \tag{1}$$

$$dD_{\mathcal{L}t} = D_{\mathcal{L}t} \left(\mu dt + \sigma dZ_{2t} \right), \tag{2}$$

where μ and σ are positive constants. We refer to these processes as news about the short- and long-term dividends. Notice that the short- and long-term dividends are independent and have the same distributional properties for any given $t < T/2$. Therefore, any price difference arises due to the equilibrium mechanism that prices short-term (Z_{1t}) and long-term (Z_{2t}) risks differently.

Potential differences in the distributional properties of the short- and long-term cash-flows may have adverse effects on our results. However, Belo et al. (2015) report that short-term dividends have higher volatility than long-term dividends — making our results stronger.

There are two risky assets. The first asset, $S_{\mathcal{S}t}$, represents a claim on the short-term dividend, and the second asset, $S_{\mathcal{L}t}$, represents a claim on the long-term dividend. We

[2] A longer performance window, $N > 2$, affects the levels of the equilibrium quantities, but as long as there are two distinct investment horizons, the equilibrium mechanism remains unchanged.

assume that both assets are in unit supply and follow

$$dS_{\mathcal{S}t} = S_{\mathcal{S}t}\left(\mu_{\mathcal{S}t}dt + \sigma_{\mathcal{S}1t}dZ_{1t} + \sigma_{\mathcal{S}2t}dZ_{2t}\right), \tag{3}$$

$$dS_{\mathcal{L}t} = S_{\mathcal{L}t}\left(\mu_{\mathcal{L}t}dt + \sigma_{\mathcal{L}1t}dZ_{1t} + \sigma_{\mathcal{L}2t}dZ_{2t}\right). \tag{4}$$

Prices, (instantaneous) expected returns, and (instantaneous) volatilities are endogenous and determined in equilibrium. To capture the short- and long-term assets' interdependencies, our analysis focuses on $t \leq T/2$, meaning the economy resets to $t = 0$ when time reaches $T/2$. In addition to the risky assets, investors can trade with a riskless bond. We assume the bond is in zero net supply and denote it by B_t, and for $t \leq T/2$, the bond pays a continuous, exogenous riskless interest payment r, which we set to zero to simplify the analysis.

3.1 Investors

Two types of investors populate the economy: long-term value and short-term asset managers. The long-term value investor, subject to *absolute* wealth concerns, evaluates his portfolio at the long-horizon date and has standard constant relative risk aversion preferences over the terminal value of his portfolio ($W_T^{\mathcal{V}}$):

$$E\left[\frac{\left(W_T^{\mathcal{V}}\right)^{1-R}}{1-R}\right]. \tag{5}$$

The short-term asset manager, subject to *relative* wealth concerns, evaluates the terminal value of his portfolio ($W_{T/2}^{\mathcal{I}}$) relative to a benchmark ($S_{\mathcal{L}T/2}$) at the short-horizon date:

$$E\left[\left(S_{\mathcal{L}T/2}\right)\frac{\left(W_{T/2}^{\mathcal{I}}\right)^{1-R}}{1-R}\right], \tag{6}$$

where $\mathcal{V}$ and $\mathcal{I}$ stand for the value and asset managers, respectively.

The benchmark is the long-term asset, and Section 6 discusses an extension in which the benchmark combines the short-term and long-term assets. Though, such a benchmark does not affect the main equilibrium implications. The economy further assumes that investors have the same risk aversion parameter, $R > 1$. Still, the economic setup allows for different risk aversion parameters while still providing closed-form precise expressions to all the equilibrium quantities. Qualitatively, the equilibrium outcomes remain the same when the asset manager is less risk-averse than the value investor.

The asset manager utility function captures the two essential frictions in the asset management industry: short-term performance window and relative performance objective, commonly referred to as benchmarking. First, the model captures the short-term performance window by setting the time the asset manager collects rewards shorter than the value investor ($T/2 < T$). Second, the model captures the relative performance objective by interacting the asset manager utility over end-of-period wealth with the benchmark. This interaction term captures the critical aspect of benchmarking laid out by Basak and Pavlova (2013), implying that the asset manager strives to post higher returns when the benchmark is high than when it is low. It captures asset managers' relative wealth concerns in a reduced form, allowing a highly tractable equilibrium outcome.

The asset manager utility function (6) introduces two variations to the established utility function of Basak and Pavlova (2013), introduced to study the equilibrium effects of asset managers on asset prices. We divide their utility function by the constant b and obtain the following equivalent preference representation

$$E\left[\left(\frac{1}{b}+I\right)\log\left(W\right)\right], \tag{7}$$

where b represents the benchmark's importance, and I represents the exogenous benchmark news in their model.

In our first modification of their utility function, we let $b \to \infty$ and assume that benchmarking is extremely important. It substantially simplifies the subsequent analysis and allows for analytical, closed-form solutions. Our second modification introduces a risk aversion parameter strictly bigger than one ($R > 1$) because myopic investors with $R = 1$ do not care about long-term shocks. Therefore, the financial markets with short- and long-term investment mandates are not dynamically complete. This assumption is innocuous because the model subsumes the myopic, $R = 1$ case. We can set $R = 1 + \epsilon$ with arbitrarily small $\epsilon > 0$ and investigate the myopic case with $R \to 1$. The institutional performance objective (6) captures the critical aspect of benchmarking laid out by Basak and Pavlova (2013): the asset manager strives to post higher returns when the benchmark is high than when it is low.

One constructive and essential deviation from the literature is that, in our model, the asset manager's benchmark is endogenous and determined in equilibrium. It includes the long-horizon asset price ($S_{\mathcal{L}T/2}$) and not its dividend news, as typically assumed in the literature.

We denote the total asset market at $t = 0$ by S_m and assume that at $t = 0$, the asset manager is endowed with λ shares of the total asset market, while the value investor with the residual $1 - \lambda$. Starting with these initial endowments, each investor dynamically chooses a portfolio π_{it}^k, where π_i^k represents the fraction of wealth investor k invests in security i, where $k = \mathcal{I},\ \mathcal{V}$ and $i = \mathcal{S},\ \mathcal{L}$. The wealth processes of the two investors then follow the dynamics

$$\frac{dW_t^{\mathcal{V}}}{W_t^{\mathcal{V}}} = \pi_t^{\mathcal{V}\prime} \begin{pmatrix} \mu_{\mathcal{S}t} \\ \mu_{\mathcal{L}t} \end{pmatrix} dt + \pi_t^{\mathcal{V}\prime} \Sigma_t \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix}, \quad \frac{dW_t^{\mathcal{I}}}{W_t^{\mathcal{I}}} = \pi_t^{\mathcal{I}\prime} \begin{pmatrix} \mu_{\mathcal{S}t} \\ \mu_{\mathcal{L}t} \end{pmatrix} dt + \pi_t^{\mathcal{I}\prime} \Sigma_t \begin{pmatrix} dZ_{1t} \\ dZ_{2t} \end{pmatrix}, \tag{8}$$

where we denote the vector of portfolio weights of investor k by π_t^k, and the matrix of return volatilities by Σ_t, such that

$$\pi_t^k \equiv \begin{bmatrix} \pi_{\mathcal{S}t}^k \\ \pi_{\mathcal{L}t}^k \end{bmatrix}, \quad \Sigma_t \equiv \begin{bmatrix} \sigma_{\mathcal{S}1t} & \sigma_{\mathcal{S}2t} \\ \sigma_{\mathcal{L}1t} & \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{9}$$

4 Equilibrium with Heterogeneous Horizons

This section unravels the heterogeneous investment horizons equilibrium mechanism. Our model introduces two frictions: (i) the heterogeneous investment horizons and (ii) the relative performance objective of asset managers. Accordingly, three potential equilibrium channels drive the equilibrium asset pricing quantities. There are two *individual* effects due to the heterogeneous investment horizons and the relative performance objective separately. One *combined* effect comes from the interaction of the heterogeneous investment horizons and the relative performance objective.

However, notice that the individual relative performance objective effect cannot drive our equilibrium mechanism and explain the dividend strips irregularities because, without heterogeneous investment horizons, dividend payouts occur in a single time. Ultimately, we remain with one individual effect and the combined effect. Throughout the analysis, we distinguish between these two potential channels and eventually show that both frictions are responsible for the three empirical dividend strips irregularities.

We define the equilibrium in a standard way: equilibrium prices and portfolio holdings are such that (i) both the asset manager and the value investor choose their optimal portfolios for given prices, and (ii) stocks, the bond, and consumption-good markets clear.

The separation between the short-term and the long-term implies that the equilibrium quantities are set up to ensure that, in the end, the asset manager holds the short-term asset while the value investor holds the long-term asset.

In the first step towards unraveling the equilibrium, we present investors' optimal risk exposures. It is a partial equilibrium result because investors choose their risk exposures (and portfolios) given prices. Still, it allows us to separate the different shock propagation channels and analyze how the heterogeneous investment horizons mechanism affects prices.

Lemma 1 (**Risk Exposure**). *The value and asset managers' risk exposures are given by*

$$\Sigma_t' \pi_t^{\mathcal{V}} = \theta_t + \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix}, \tag{10}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} = \theta_t + \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{11}$$

The Lemma shows that investors' wealth has three separate shock propagation channels that affect them. The first channel is the myopic mean-variance channel (θ_t). It states that investors buy and sell assets to correlate their wealth with the market prices of risk; in other words, risk exposure is proportional to excess return per unit of risk. We have yet to identify which asset has a better risk-return trade-off; it will be revealed explicitly in equilibrium.

The second channel is the risk aversion intertemporal hedge channel. Since the risk aversion parameter (R) is strictly greater than one, the risk aversion hedge states that wealth is less valuable when the investment opportunities are better because it is easier to increase wealth through investments. When the value investor is risk-averse ($R > 1$), a positive long-term cash-flow shock lowers the value of wealth, while the asset manager wealth is less valuable after a positive short-term cash-flow shock. The intertemporal risk aversion hedge goes back to Merton (1971).

The third remaining channel is an intertemporal benchmark hedge channel. It is entirely driven by the asset manager's benchmarking motives, as this hedge would disappear without a benchmark. The benchmark hedge states that the asset manager strives to do well when the benchmark does well. To achieve that goal, Lemma 1 reveals that the asset manager optimally correlates his risk exposure with the benchmark's return and simply buys the benchmark to hedge against benchmark fluctuations. Since the benchmark is the long-term asset, the asset manager's wealth is more valuable after a shock increases the long-term asset value.

Notice that the benchmark hedge is endogenous and has a short-term risk exposure entirely driven by the equilibrium mechanism because the short-term and long-term dividend news processes are independent ($D_{\mathcal{L}t} \perp D_{\mathcal{S}t}$), and the short-term asset does not belong to the benchmark (6). This outcome is unique to our economy and does not exist in a homogeneous investment horizon economy in which the benchmark hedge is exogenous as specified in the model setup.

Significantly, the assets in the model mimic dividend strips' payout: the short-term asset pays dividends in the short-term, and the long-term asset pays dividends in the long-term, and their payouts do not overlap. Since the short-term asset manager cares only about the short-term investment horizon due to the short performance window, eventually, he must hold the short-term asset at the end of his performance window. Holding the long-term asset does not pay off in the short-term, similar to a long-term dividend strip. As a result, equilibrium (through the individual effect) adjusts assets' returns and market prices of risk so that, in the end, the short-term asset manager ends up holding the short-term asset while the long-term value investor ends up holding the long-term asset, regardless of the benchmark composition and the strength of the asset manager incentives to benchmark.

The benchmarking incentives (through the combined effect) introduce a benchmark hedging portfolio that correlates perfectly with the benchmark's return and forces equilibrium market prices of risk and asset returns to readjust so that the short-term asset manager would still hold the short-term asset while the long-term value investor would still hold the long-term asset.

To identify the individual effect, we solve for an equilibrium without the relative performance objective but with the heterogeneous investment horizons and denote the equilibrium quantities by the upper bar ($\bar{X}$) throughout the analysis. We then compare the asset pricing quantities arising from these two equilibriums and identify whether the individual channel or the combined channel drives the difference between the short-term and long-term equilibrium quantities.

To further our understanding of the equilibrium mechanism, next, we characterize the equilibrium return volatilities and market prices of risk, and plug them back into investors' risk exposures we characterized in Lemma 1. We start with the return volatilities.

Proposition 1 (**Volatility**). *The short-term asset has short-term risk exposure and no long-term risk exopsure,*

$$\sigma_{\mathcal{S}1t} = \bar{\sigma}_{\mathcal{S}1t} = \sigma, \quad \sigma_{\mathcal{S}2t} = \bar{\sigma}_{\mathcal{S}2t} = 0, \tag{12}$$

where $\bar{\sigma}_{\mathcal{S}1t}$ and $\bar{\sigma}_{\mathcal{S}2t}$ are the equilibrium risk exposures without benchmarking motives. The long-term asset has a positive exposure to the short-term risk and a negative exposure to the long-term risk,

$$\sigma_{\mathcal{L}1t} = \bar{\sigma}_{\mathcal{L}1t} - \frac{R}{2}\sigma > 0, \quad \sigma_{\mathcal{L}2t} = \bar{\sigma}_{\mathcal{L}2t} + \frac{R-1}{2}\sigma < 0, \tag{13}$$

where $\bar{\sigma}_{\mathcal{L}1t}$ and $\bar{\sigma}_{\mathcal{L}2t}$ are the equilibrium risk exposures without benchmarking motives, given by

$$\bar{\sigma}_{\mathcal{L}1t} = R\sigma > 0, \quad \bar{\sigma}_{\mathcal{L}2t} = -\left(R-1\right)\sigma < 0. \tag{14}$$

The individual effect of the heterogeneous investment horizons

(i) implies a negative fundamental exposure, $(\bar{\sigma}_{\mathcal{L}2t}) < 0$, and a positive non-fundamental exposure, $(\bar{\sigma}_{\mathcal{L}1t}) > 0$.

(ii) non-fundamental cash-flow news have a bigger impact on prices than fundamental cash-flow news, $(|\sigma_{\mathcal{L}1t}| > |\sigma_{\mathcal{L}2t}|)$ and $(|\bar{\sigma}_{\mathcal{L}1t}| > |\bar{\sigma}_{\mathcal{L}2t}|)$;

The combined effect of the heterogeneous investment horizons and the relative performance objective

(i) increases the (fundamental) long-term exposure $(\sigma_{\mathcal{L}2t})$ and decreases the (non-fundamental) short-term exposure $(\sigma_{\mathcal{L}1t})$; however, their sign remains the same.

(ii) reduces the long-term asset total volatility: $\sqrt{\bar{\sigma}^2_{\mathcal{L}1t} + \bar{\sigma}^2_{\mathcal{L}2t}} > \sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}}$;

In a traditional economy with homogenous investment horizons and without benchmarking incentives, positive fundamental cash-flow news increases the stock price. When comparing the asset pricing implications of fundamental and non-fundamental cash-flow news, the traditional economy illustrates that fundamental cash-flow news is the main driving force behind asset pricing fluctuations. These two outcomes persist in am economy with homogenous investment horizons and benchmarking incentives. Basak and Pavlova (2013) show that fundamental cash-flow news remains the main driving force behind asset pricing fluctuations. However, Proposition 1 reveals that the heterogeneous investment horizons (the individual effect) reverse both of these results. That is, (i) positive, long-term, fundamental cash-flow news ($D_{\mathcal{L}t}$) reduces the long-term asset price ($\sigma_{\mathcal{L}2t} < 0$), and (ii) the asset pricing implications of the non-fundamental short-term cash-flow news are more substantial than the fundamental long-term cash-flow news ($|\sigma_{\mathcal{L}1t}| > |\sigma_{\mathcal{L}2t}|$).

These two outcomes are *not* due to the interaction of the heterogeneous investment horizons and relative performance objective (the combined effect) since they persist in an economy without benchmarking incentives: (i) $\bar{\sigma}_{\mathcal{L}2t} < 0$, and (ii) $|\bar{\sigma}_{\mathcal{L}1t}| > |\bar{\sigma}_{\mathcal{L}2t}|$. It emphasizes the importance of non-fundamental short-term cash-flow news when market participants have different investment horizons.

An essential feature of the equilibrium arising from the interaction of the heterogeneous investment horizons and the relative performance objective (the combined effect) is that the total volatility of a benchmarked asset *decreases* in the presence of benchmarking aligning with Cuoco and Kaniel (2011) but reversing Basak and Pavlova (2013)s' findings when the investment horizons are homogeneous, as item (ii) in Proposition 1 reveals. This outcome arises because the long-term asset risk exposures decrease in absolute terms. The short-term risk exposure becomes less positive and decreases toward zero, while the long-term risk exposure becomes less negative and increases toward zero.

Including the long-term asset risk exposure in the asset manager benchmark hedging component (11) reveals a negative exposure to the long-term cash-flow news due to his benchmark hedge, $\sigma_{\mathcal{L}2t} < 0$. This negative long-term fundamental exposure turns out to be essential in equilibrium, as we soon uncover. The characterization of the market prices of risk completes the picture.

Proposition 2 (**Market Price of Risk**). *The short- and long-term market prices of risk are given by*

$$\theta_{1t} = \frac{R}{2}\sigma > 0, \qquad \theta_{2t} = \frac{R-1}{2}\sigma > 0, \tag{15}$$

where $\bar{\theta}_{1t} = R\sigma$ *and* $\bar{\theta}_{2t} = 0$ *are the equilibrium market prices of risk without benchmarking* *motives.*

The individual effect of the heterogeneous investment horizons implies that the short-term market price of risk is higher than the long-term market price of risk

$$\theta_{1t} > \theta_{2t}, \quad \bar{\theta}_{1t} > \bar{\theta}_{2t}. \tag{16}$$

The combined effect of the heterogeneous investment horizons and the relative performance objective

(i) increases the long-term market price of risk ($\theta_{2t} > \bar{\theta}_{2t}$)*;*

(ii) and decreases the short-term market price of risk ($\bar{\theta}_{1t} > \theta_{1t}$)*;*

Typically, in a homogenous horizon economy, benchmarking reduces the market price of benchmarked assets' fundamental risk while it does not affect non-fundamental market prices of risk. Economically, equilibrium depresses the market price of risk so that a retail investor without benchmarking motives finds the benchmarked asset unattractive due to its low risk-return trade-offs. The asset manager is willing to forgo the low risk-return trade-off because the benchmarked asset satisfies the benchmark hedging desires. Following the homogenous horizon economy's logic, we would expect the long-term market price of risk to decrease and to have no impact on the short-term market price of risk. However, the heterogeneous investment horizon equilibrium's workings are different, and the results from the homogenous case do not follow.

Interestingly, item (i) in Proposition 2 reveals that when the long-term asset belongs to the benchmark, the market price of the long-term risk increases. Further, item (ii) in Proposition 2 indicates that the market price of the short-term risk decreases even though the short-term asset does not belong to the benchmark. These two outcomes run against the theoretical findings with homogenous investment horizons, such as Basak and Pavlova (2013) and Buffa and Hodor (2018).[3]

However, despite the opposing price pressures reducing the short-term and increasing the long-term market prices of risk, the short-term market price risk always remains higher than the long-term market price of risk.

[3]Later, in Section 6, we show that when only the short-term asset belongs to the benchmark, the short-term market price of risk decreases while the long-term price of risk is unaffected, aligning with the findings of homogenous investment horizons economies.

So far, we have identified the market prices of risk and return volatilities. Next, we revisit investors' partial equilibrium risk exposures, given in Lemma (1). We plug the long-term asset return volatility and the market prices of risk, and characterize the equilibrium risk exposures.

$$\Sigma_t'\pi_t^{\mathcal{V}}/\sigma = \begin{bmatrix} R/2 \\ (R-1)/2 \end{bmatrix} + \begin{bmatrix} 0 \\ -(R-1) \end{bmatrix}, \tag{17}$$

$$\Sigma_t'\pi_t^{\mathcal{I}}/\sigma = \begin{bmatrix} R/2 \\ (R-1)/2 \end{bmatrix} + \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix} + \begin{bmatrix} R/2 \\ -(R-1)/2 \end{bmatrix}. \tag{18}$$

Interestingly, the positive long-term mean-variance risk exposure offsets the negative long-term benchmark hedge exposure entirely ($\theta_{2t} + \sigma_{\mathcal{L}2} = 0$), as the sum of red and blue in (18) illustrates.

Economically, the asset manager's benchmark hedging requires exposure to the long-term risk; to satisfy this desire, the asset manager must hold the long-term asset since the short-term asset has no long-term risk exposure (12). However, the equilibrium risk exposure (18) reveals that any mean-variance upside offsets a benchmark hedge downside when the asset manager holds the long-term asset. As a result, the asset manager cannot attain exposure to the long-term risk and cannot satisfy the benchmark hedging desires from holding the benchmark asset in equilibrium.

Consequently, if the asset manager eventually holds the benchmark asset, it is not due to the benchmark hedging but rather the long-term asset risk-return trade-off. However, equilibrium ensures that the asset manager does not gain risk-return trade-off benefits from holding the benchmark asset and sets the short-term market price of risk higher than the long-term market price of risk, $\theta_{1t} > \theta_{2t}$, as the difference between blue and green in (18) illustrates. Eventually, this leads the asset manager to hold the short-term asset in equilibrium, even when the benchmark hedge mandates otherwise.

The negative long-term risk exposure satisfies the value investor risk aversion hedging desires (Lemma 1). In equilibrium, the value investor is willing to forgo a better short-term risk-return trade-off in exchange for satisfying the long-term risk aversion hedging desires. Eventually, leading the value investor to hold the long-term asset in equilibrium.

It is now clear why the long-term asset fundamental news exposure is negative ($\sigma_{\mathcal{L}2t} < 0$).

It satisfies the conflicting desires of the value investor and the asset manager. With negative long-term asset fundamental news exposure, the asset manager gains no exposure to the long-term news ($\theta_{2t} + \sigma_{\mathcal{L}2t} = 0$) eliminating his desire to hold the long-term asset. At the same time, the value investor holds the long-term asset because the negative fundamental news exposure satisfies the risk aversion hedging desire, which guarantees that the value investor holds the long-term asset while the asset manager holds the short-term asset in equilibrium.

Notice that in an economy with heterogeneous investment horizons and without benchmarking incentives (the individual effect), the long-term mean-variance risk exposure becomes zero and does not offset the negative long-term risk aversion hedge exposure ($\bar{\theta}_{2t} + \bar{\sigma}_{\mathcal{L}2t} \neq 0$). In this economy, there is no intertemporal benchmark hedging demand (the third component in 10 vanishes). So, equilibrium only needs to ensure that the short-term asset has better risk return trade-offs than the long-term asset; hence, equilibrium sets the short-term market price of risk higher than the long-term market price of risk ($\bar{\theta}_{1t} > \bar{\theta}_{2t}$). The value investor is willing to forgo a better short-term risk-return trade-off because the long-term asset provides access to negative exposure to the long-term shock ($\bar{\sigma}_{\mathcal{L}2t} < 0$). Eventually, this leads to a similar outcome in which the asset manager holds the short-term asset while the value investor holds the long-term asset even without the combined effect.

4.1 Equilibrium Dynamics of the Heterogeneous Horizons

We conclude this paragraph with a description of the equilibrium dynamics. When summing the three propagation channels affecting investors' risk exposures in (17) and (18), we find that overall, the value investor risk exposure correlates with the long-term asset, while the asset manager risk exposure correlates with the short-term asset.

$$\Sigma_t' \pi_t^{\mathcal{V}} = \theta_t + \begin{bmatrix} 0 \\ -(R-1)\,\sigma \end{bmatrix} = \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}, \tag{19}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} = \theta_t + \begin{bmatrix} -(R-1)\,\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} = \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix}. \tag{20}$$

Suppose that the short-term asset gets positive fundamental cash-flow news ($D_{\mathcal{S}t} \uparrow$). The mean-variance risk exposure states that the value and asset manager's wealth increases, as the first component in (19) and (20) is positive. The asset manager's benchmark hedge correlates the risk exposure with the long-term asset return, and since the long-term asset price increases after non-fundamental short-term news, the asset manager's wealth increases further due to the benchmark hedge. However, the asset manager's wealth decreases following short-term cash-flow news due to the risk aversion hedge, which reverses some of the positive impacts of the mean-variance and the benchmark hedge, but not all of it. Overall, the value and asset managers' wealth increase following short-term cash-flow news. However, the asset manager's wealth increases more than the value investor's wealth ($\sigma_{\mathcal{S}1t} > \sigma_{\mathcal{L}1t}$), implying that the short-term asset increases more than the long-term asset following short-term cash-flow news.

Alternatively, suppose that the long-term asset gets positive fundamental cash-flow news ($D_{\mathcal{L}t} \uparrow$). The mean-variance risk exposure states that both the value and asset managers' wealth increase, similar to the case with short-term cash-flow news. However, the asset manager's wealth decreases due to the negative benchmark hedge exposure to the long-term cash-flow news ($\sigma_{\mathcal{L}2t} < 0$). Equilibrium offsets these two counter effects entirely so that overall the asset manager has no long-term cash-flow news exposure. The value investor also has negative exposure to the long-term cash-flow news due to the risk aversion hedge. The negative exposure of the risk aversion hedge is more pronounced than the mean variance positive exposure so overall the value investor has negative exposure to the long-term asset. This result is intuitive because the value investor correlates his overall exposure to the long-term asset with negative long-term news exposure.

The following Proposition presents the short- and long-term security prices explicitly, in closed-form.

Proposition 3 (**Security Prices**). *The short- and long-term asset prices are given by*

$$S_{\mathcal{L}t} = \bar{S}_{\mathcal{L}t} \left(D_{\mathcal{L}t}\right)^{\frac{R-1}{2}} \left(D_{\mathcal{S}t}\right)^{-\frac{R}{2}} A_{\mathcal{L}t}, \qquad S_{\mathcal{S}t} = \bar{S}_{\mathcal{S}t} A_{\mathcal{S}t}, \tag{21}$$

where $\bar{S}_{\mathcal{L}t}$ and $\bar{S}_{\mathcal{S}t}$ are the equilibrium exposures without benchmarking incentives given by

$$\bar{S}_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right) \left(D_{\mathcal{L}t}\right)^{1-R} \left(D_{\mathcal{S}t}\right)^{R} \bar{A}_{\mathcal{L}t}, \qquad \bar{S}_{\mathcal{S}t} = D_{\mathcal{S}t} \bar{A}_{\mathcal{S}t}, \tag{22}$$

where $t \leq \frac{T}{2}$. The functions $\bar{A}_{\mathcal{L}t}$, $\bar{A}_{\mathcal{S}t}$, $A_{\mathcal{L}t}$, and $A_{\mathcal{S}t}$ are deterministic and positive functions

of time, defined in (74), (75), (76), and (77), respectively.
The heterogeneous investment horizons individual effect (22) implies that

- *(i) the short-term asset price increases relative to the long-term asset price as the asset manager size increases (λ).*
- *(ii) the long-term asset price is convex and increases in the short-term news, indicating that short-term news is more critical in good short-term states*
- *(iii) the long-term asset price is convex and decreases in the long-term news, indicating that long-term news is more critical in bad long-term states*

The combined effect of heterogeneous investment horizons and relative performance objective (21) reverses (ii) above and implies that

- *(iv) the long-term asset price becomes concave and increasing in the short-term news, indicating that short-term news is more critical in bad states*

Proposition 3 reveals several critical features of the heterogeneous investment horizons individual effect. Item (i) reveals that the long-term asset price drops as the asset manager size increases relative to the value investor ($\lambda \uparrow$), while the short-term asset price remains unchanged. In addition, items (ii) and (iii) reveal that, indeed, the long-term asset price falls following positive long-term news and increases following positive short-term news.

Perhaps more importantly, short-term dividend news is more critical for long-term prices in good short-term states; a one percent increase in short-term dividend news has a more substantial effect on the long-term price when prices are high than when they are low. Similarly, long-term dividend news is more critical for long-term prices in bad long-term states; a one percent increase in long-term dividend news has a more substantial effect on the long-term price when prices are high than when they are low.

Due to the interaction between heterogeneous investment horizons and relative performance objectives (the combined effect), short-term and long-term news become more critical for long-term prices in bad states. The convexity with respect to the long-term news implies that following positive long-term news, the decrease in the long-term price is more severe for low levels of long-term dividend news. Similarly, the concavity with respect to the short-term news implies that, following positive short-term news, the increase in the long-term price is more substantial for low levels of short-term dividend news. Figure 1 illustrates these results.

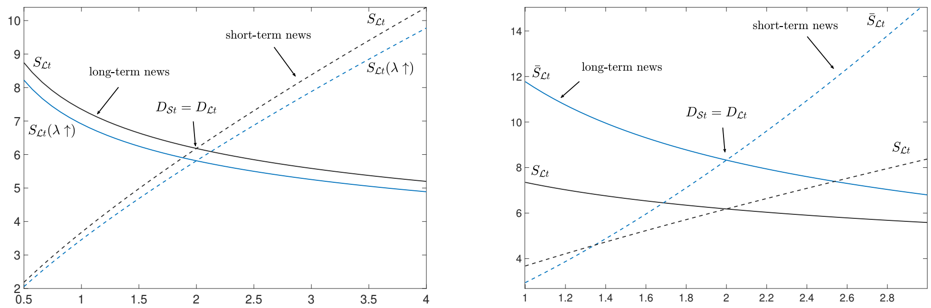

Figure 1. This figure plots asset prices as a function of news. The left figure shows that long-term assets are convex in long-term news and concave in short-term news. Further, as λ increases from 0.2 to 0.21, the long-term asset price drops from the black to the blue lines. The solid line plots the long-term price as a function of the long-term news while short-term news remains fixed ($D_{\mathcal{S}t} = 2$). The dashed line plots the long-term price as a function of the short-term news while long-term news remains fixed ($D_{\mathcal{L}t} = 2$). The right figure plots the changes in asset prices due to the combined effect. Without the combined effect ($\bar{S}_{\mathcal{L}t}$), the long-term assets shifts from concavity to convexity in the short-term news, as the black and blue dashed lines indicate. The rest of the parameters are $R = 1.5$, $\mu = 0.1$, $\sigma = 0.2$, $T = 3$, $t = 0.5$, $\lambda = 0.2$, $D_{\mathcal{L}0} = D_{\mathcal{S}0} = 1$.

Finally, we present the closed-form expression for the discount factor.

Proposition 4 (Discount Factor). *The equilibrium discount factor is given by*

$$\xi_{0,t} = \frac{(D_{\mathcal{L}t}/D_{\mathcal{L}0})^{\frac{1-R}{2}}}{(D_{\mathcal{S}t}/D_{\mathcal{S}0})^{\frac{R}{2}}} \frac{1}{\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right)\mathcal{E}_{0,t}\left(-\frac{R}{2}\right)}, \quad t \leq T/2, \tag{23}$$

where $\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right)\mathcal{E}_{0,t}\left(-\frac{R}{2}\right)$ *are deterministic functions (41), and (42).*

The discount factor is inversely related to both the short- and long-term dividend news. A feature similar to a traditional asset pricing model: an asset that pays off in states with low dividends gets a high value. Interestingly, the discount factor is more sensitive to short-term news than long-term news, implying that news about the short-term cash flow has a higher impact on prices. The differential sensitivity of the discount factor is another manifestation of the equilibrium requiring that the short-term asset risk-return trade-off is better than the long-term asset risk-return trade-off so that the asset manager would prefer to hold the short-term asset. Figure 2 illustrates this idea.

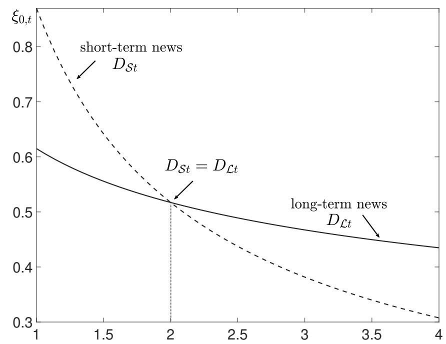

Figure 2. This figure plots the discount factor ($\xi_{0,t}$) as a function of news. In the solid line, we plot the discount factor as a function of the long-term news, while short-term news remains fixed ($D_{\mathcal{S}t} = 2$). In the dashed line, we plot the discount factor as a function of the short-term news, while long-term news remains fixed ($D_{\mathcal{L}t} = 2$). The rest of the parameters are as in Figure 1.

5 Implications of the Heterogeneous Horizons

So far, we have analyzed the equilibrium mechanism and its dynamics. This section establishes that the two frictions coming from the asset management industry (short performance window, benchmarking concerns) are responsible for the dividend strips' empirical regularities. These empirical regularities cannot be rationalized with workhorse asset pricing models. Dividend strips are a good candidate for teasing out the short-term equilibrium effects from the long-term ones because they inform us about the prices of risky cash-flows at different horizons.

An empirical analysis of short- and long-term dividend strips by van Binsbergen et al. (2012) showed that the short-term asset risk-premium and volatility are higher than the long-term asset volatility. A recent complementary analysis by van Binsbergen and Koijen (2017) verifies the original claim using a more comprehensive data set. Further, van Binsbergen et al. (2012)s' empirical findings go beyond risk premiums and volatilities. They empirically observe that, on average, the short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio. These three main findings are at odds with standard equilibrium asset pricing models, such as Campbell and Cochrane (1999)s' external habit formation model and Bansal and Yaron (2004)s' long-run risk model. Gabaix (2012)'s variable rare disaster model predicts the reverse for both volatility and Sharpe ratio and finds that risk premiums of the short- and long-term assets are identical.

The following Proposition verifies that this model generates these three empirical regularities.

Proposition 5 (**Risk Premium, Total Volatility, and Sharpe Ratio**).

(i) **Risk Premium:** *The short- and long-term risk premiums are given by*

$$\mu_{\mathcal{S}t} = \sigma^2 \frac{R}{2} > 0, \qquad \mu_{\mathcal{L}t} = \sigma^2 \frac{1}{2}\left(R - \frac{1}{2}\right) > 0. \tag{24}$$

The short-term asset risk premium is higher than the long-term asset risk premium,

$$\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}, \tag{25}$$

while the reverse is true without benchmarking incentives: $\bar{\mu}_{\mathcal{S}t} < \bar{\mu}_{\mathcal{L}t}$, where $\bar{\mu}_{\mathcal{S}t} = \sigma^2 R$ and $\bar{\mu}_{\mathcal{L}t} = \sigma^2 R^2$. The combined effect further implies

(a) A reduction in expected returns of both the benchmarked and non-benchmarked assets

$$\bar{\mu}_{\mathcal{S}t} > \mu_{\mathcal{S}t}, \quad \bar{\mu}_{\mathcal{L}t} > \mu_{\mathcal{L}t}. \tag{26}$$

The reduction in the long-term asset expected return is more pronounced

$$\bar{\mu}_{\mathcal{S}t} - \mu_{\mathcal{S}t} > \bar{\mu}_{\mathcal{L}t} - \mu_{\mathcal{L}t}. \tag{27}$$

(ii) **Total Volatility:** *The short-term asset volatility is higher than the long-term asset volatility if and only if investors' risk aversion is sufficiently low, such that*

$$\sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}} < \sqrt{\sigma^2_{\mathcal{S}1t} + \sigma^2_{\mathcal{S}2t}} \iff R < \bar{R}, \tag{28}$$

where $\bar{R} < 2$ is given in (94). The reverse is true without benchmarking incentives and for any risk aversion parameter $R > 1$.

(iii) **Sharpe Ratio:** *The short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio*

$$\frac{\mu_{\mathcal{S}t}}{\sqrt{\sigma^2_{\mathcal{S}1t} + \sigma^2_{\mathcal{S}2t}}} > \frac{\mu_{\mathcal{L}t}}{\sqrt{\sigma^2_{\mathcal{L}1t} + \sigma^2_{\mathcal{L}2t}}}. \tag{29}$$

The equilibrium market price of the short-term risk is always higher than the equilibrium market price of the long-term risk (16). However, it does not immediately imply that the short-term risk premium is higher than the long-term risk premium. In fact, without benchmarking incentives, the short-term risk premium is lower than the long-term risk premium ($\bar{\mu}_{\mathcal{S}t} < \bar{\mu}_{\mathcal{L}t}$). This result becomes apparent when we realize that the quantity of short-term risk is higher for the long-term asset ($\bar{\sigma}_{\mathcal{L}1t} > \bar{\sigma}_{\mathcal{S}1t}$) when there are no benchmarking motives, leading to a higher long-term risk premium:

$$\bar{\theta}_{1t}\bar{\sigma}_{\mathcal{S}1t} + \underbrace{\bar{\theta}_{2t}}_{=0}\bar{\sigma}_{\mathcal{S}2t} < \bar{\theta}_{1t}\bar{\sigma}_{\mathcal{L}1t} + \underbrace{\bar{\theta}_{2t}}_{=0}\bar{\sigma}_{\mathcal{L}2t}. \tag{30}$$

In contrast, with benchmarking incentives, the price of the short-term drops ($\theta_{1t} < \bar{\theta}_{1t}$), and the price of the long-term risk increases ($\theta_{2t} > \bar{\theta}_{2t}$). In this case, the short-term risk premium is higher than the long-term risk premium; again, the reasoning lies in the changes to the quantity of risk. First, the short-term asset quantity of short-term risk remains unchanged ($\bar{\sigma}_{\mathcal{S}1t} = \sigma_{\mathcal{S}1t}$), while the long-term asset quantity of short-term risk drops ($\bar{\sigma}_{\mathcal{L}1t} > \sigma_{\mathcal{L}1t}$), tilting the risk premium towards the short-term. Second, the short-term asset has no exposure to long-term shocks, so the quantity is zero ($\sigma_{\mathcal{S}2t} = 0$), while the long-term asset has a negative quantity of risk ($\sigma_{\mathcal{L}2t} < 0$); again, tilting the risk premium towards the short-term. Eventually, the two forces lead to a negative sloping risk premium:

$$\mu_{\mathcal{S}t} = \underbrace{\theta_{1t}}_{>0}\underbrace{\sigma_{\mathcal{S}1t}}_{=\bar{\sigma}_{\mathcal{S}1t}} + \underbrace{\theta_{2t}}_{>0}\underbrace{\sigma_{\mathcal{S}2t}}_{=0} > \underbrace{\theta_{1t}}_{>0}\underbrace{\sigma_{\mathcal{L}1t}}_{<\bar{\sigma}_{\mathcal{L}1t}} + \underbrace{\theta_{2t}}_{>0}\underbrace{\sigma_{\mathcal{L}2t}}_{<0} = \mu_{\mathcal{L}t}. \tag{31}$$

The Proposition further shows that the total volatility (28) of the long-term asset ($S_{\mathcal{L}t}$) is, in fact, lower than the short-term asset ($S_{\mathcal{S}t}$) when risk aversion is not too high.

Significantly, the combined effect plays a crucial role in these two outcomes since both the risk-premium and volatility results reverse without the benchmarking incentives when only the individual effect exists.

When looking at the Sharpe ratio, it is not immediately apparent that the short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio because both the risk premium and the total volatility of the short-term asset are higher than the long-term counterparts. So, when dividing the risk premium by the total volatility, the Sharpe ratio of the short-term asset may turn out to be smaller. Even more so, it is not entirely clear whether the combined effect of heterogeneous investment horizons and relative performance objective drives this result or the heterogeneous investment horizons individually. Proposition 5 concludes that the combined effect drives the risk-premium and volatility results; however, the individual effect drives the Sharpe ratio result.

Lastly, Proposition 5 further reveals that the risk premium and the Sharpe ratio results do not depend on the model's parameters, while the total volatility result requires that investors' risk aversion parameter is not too high and roughly below two. This parameter restriction is in line with established asset pricing models such as the external habit formation of Campbell and Cochrane (1999) and others. When the model allows for a short-term benchmark, the risk premium also requires risk aversion roughly below two, as Section 6 reveals.

5.1 Additional Empirical Evidence

So far, we have analyzed the equilibrium mechanism and showed that the equilibrium volatility, risk premium, and Sharpe ratio support the recent empirical evidence. This section provides novel empirical evidence to further substantiate the heterogeneous investment horizons mechanism and support other model predictions. We start by introducing investors' portfolios.

By inverting the volatility matrix transpose $\left(\Sigma_t'\right)$, we convert investors' risk exposures to portfolio holdings, as the following equations illustrate:

$$\pi_t^{\mathcal{V}} = \left(\Sigma_t'\right)^{-1}\theta_t + \left(\Sigma_t'\right)^{-1}\begin{bmatrix} 0 \\ -\left(R-1\right)\sigma \end{bmatrix}, \tag{32}$$

$$\pi_t^{\mathcal{I}} = \left(\Sigma_t'\right)^{-1}\theta_t + \left(\Sigma_t'\right)^{-1}\begin{bmatrix} -\left(R-1\right)\sigma \\ 0 \end{bmatrix} + \left(\Sigma_t'\right)^{-1}\begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix}. \tag{33}$$

To stay consistent with the equilibrium mechanism laid out in the previous section, we think of the value investor's portfolio $\left(\pi_t^{\mathcal{V}}\right)$ as the sum of two separate portfolios: (i) the first is the mean-variance portfolio ($\phi_{\text{m.v.}}$), and (ii) the second is the short-term risk aversion hedging portfolio ($\phi^{\mathcal{V}}_{\text{r.a.}}$). Similarly, we think of the asset manager's portfolio as the sum of three separate portfolios: (i) the first is the mean-variance portfolio ($\phi_{\text{m.v.}}$), (ii) the second is the long-term risk aversion hedging portfolio ($\phi^{\mathcal{I}}_{\text{r.a.}}$), and (iii) the third is the benchmark hedging portfolio ($\phi^{\mathcal{I}}_{\text{b}}$). The following Proposition characterizes these portfolios.

Proposition 6 (**Portfolios**). *The asset manager benchmark hedging portfolio and long-term risk aversion hedging portfolio are given by*

$$\phi^{\mathcal{I}}_{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \phi^{\mathcal{I}}_{r.a.} = \begin{bmatrix} 1-R \\ 0 \end{bmatrix}, \tag{34}$$

while the value investor short-term risk aversion hedging portfolio is given by

$$\phi^{\mathcal{V}}_{r.a.} = \begin{bmatrix} -R \\ 2 \end{bmatrix}. \tag{35}$$

The mean-variance portfolio is given by

$$\phi_{m.v.} = \begin{bmatrix} R \\ -1 \end{bmatrix} \tag{36}$$

and $\phi^{\mathcal{I}}_{b} + \phi^{\mathcal{I}}_{r.a.} + \phi_{m.v.} = \pi^{\mathcal{I}}_{t}$ *and* $\phi^{\mathcal{V}}_{r.a.} + \phi_{m.v.} = \pi^{\mathcal{V}}_{t}$.

Proposition 6 reveals that the asset manager has a one-leg long portfolio and a one-leg short portfolio. Specifically, in the benchmark hedge portfolio, the asset manager buys the long-term asset, and in the short-term risk aversion hedge portfolio, sells the short-term asset. The mean-variance portfolio is a two-leg long-short portfolio that offsets the short position in the short-term asset and the long position in the long-term asset.

We expect the two simple one-leg portfolios to show up more pronouncedly in a monthly data since they are straightforward to replicate, relative to the mean-variance long-short portfolio, which requires interacting the two assets; it is more complex and expensive to replicate. Arguably, unconditional regression analysis with monthly observations will not fully capture the mean-variance offsetting effect. With this idea in mind, we provide empirical evidence supporting Proposition 6.

We estimate the asset manager's wealth using the total net assets (TNA) of mutual actively managed funds and we estimate the short-term and long-term asset prices using van Binsbergen et al. (2012) dividend prices data. They employ the European put-call parity to extract prices of dividends of the S&P 500 index between the quoted time and the options' maturity time. For instance, the dividend price in January 1996, for dividends paid out until June 1997 is $20. They derive a time series of monthly dividend prices for 6-month, 12-month, 18-month, and 24-month maturities. Accordingly, we define the short-term asset price as the 6-month dividend price and the long-term asset price as the difference between the 24-month and 18-month dividend prices. In other words, our short-term asset is a dividend strip paying dividends within zero to six months, while our long-term asset is a dividend strip paying dividends within one and a half and two years.[4] Then, we carry out an OLS regression of the total net assets (TNA) on the short-term and long-term dividend strip prices,

$$\text{TNA}_t = \alpha + \beta_s \times (S_{\mathcal{S}t}/\text{S\&P } 500_t) + \beta_l \times (S_{\mathcal{L}t}/\text{S\&P } 500_t) + \epsilon_t. \tag{37}$$

We hypothesize that an increase in the short-term asset price decreases the asset manager's wealth ($\beta_s < 0$) due to its risk aversion hedge, while an increase in the long-term asset price increases the asset manager's wealth ($\beta_l > 0$) due to its benchmark hedge.

We focus our regression analysis on four different groups of funds. The first is the broadest group, and it consists of all the mutual actively managed funds with a Morningstar category of US Equity. These funds invest in all the possible styles and market caps. The remaining three groups consist of mutual actively managed funds within the Russell 3000 family of prospectus benchmarks: (i) Russell 3000 ; (ii) Russell 3000 Growth; (iii) and the Russell 3000 Value.

Table 1 reports that the asset manager loads negatively on the short-term asset and positively on the long-term asset, as the two one-leg asset manager's portfolios predict. The result is robust to different specifications of asset managers' benchmarks, as the four different columns in Table 1 show. For instance, the first column in Table 1 states that one standard deviation increase in the short-term dividend strip price decreases the US Equity TNA by

[4] The TNA monthly data is from Morningstar and our sample period runs from 31/12/1998 until 30/10/2009.

0.72 standard deviations, while one standard deviation increase in the long-term dividend strip price increases the US Equity TNA by 0.93 standard deviations.

Table 1. TNA on long-term and short-term dividend strips' prices

Dependent Variable: $\mathbf{TNA}_t$	US Equity	Russell 3000	Russell 3000 Growth	Russell 3000 Value
constant	4.3071(***)	-0.06457	5.7401(***)	2.8293(***)
	(0.48127)	(0.372213)	(1.34844)	(0.59541)
$S_{\mathcal{L}t}$/S&P 500	0.9268(**)	0.81502(**)	0.3685	1.0897(***)
	(0.22124)	(0.199148)	(0.26738)	(0.28014)
$S_{\mathcal{S}t}$/S&P 500	-0.7180(**)	-0.10453	-0.6679(**)	-0.6940(*)
	(0.28694)	(0.253569)	(0.20143)	(0.26584)
R^2	0.1871	0.5232	0.1427	0.149

Notes: The table presents the OLS regression of dollar TNA on the long-term and short-term dividend strip prices. The first column consists of all the actively managed mutual funds with a US Equity Morningstar category. The remaining three columns reflect the actively managed funds with the indicated prospectus benchmark. Newey-West standard errors in parenthesis. To avoid non-stationarity issues, we divide the S&P 500 index dividend strip prices by the S&P 500 index level, similar to van Binsbergen et al. (2012). Variables are standardized. (***),(**),(*),(.) corresponds to 0.1%, 1%,5%,10% confidence levels, respectively.

One should not confuse an asset manager with a value index mandate and a value investor without a performance benchmark mandate. As long as the manager has a performance benchmark objective, the manager behaves as the model describes.

The empirical analysis in this section abstracts away from essential features of the asset management industry and, in particular, the duration of the benchmarks. That is, hedging against benchmark fluctuations means a different composition of long- and short-term assets for asset managers with different benchmark mandates. For instance, a manager with a growth mandate may view a dividend strip that pays in two years as a short-term asset and not a long-term asset, resulting in negative exposure to both the long-term and the short-term assets. Accordingly, to follow the equilibrium's mechanism, one would need a dividend strip that pays off much later than two years, which our data does not provide. Still, in line with the equilibrium's mechanism, Table 1, column 3 shows that the results are less pronounced for the long-term dividend strip for the Russell 3000 Growth managers.

Proposition 3 reveals that the short-term asset price increases relative to the long-term asset price as the asset manager size increases relative to the value investor. To support this hypothesis, we carry out the following OLS regression

$$\left(S_{\mathcal{S}t} - S_{\mathcal{L}t}\right)/\text{S\&P } 500_t = \alpha + \beta \times \lambda_t + \epsilon_t \tag{38}$$

and hypothesize that as the asset manager size increases relative to the value investor size ($\lambda_t \uparrow$), the short-term asset price increases relative to the long-term asset price ($\beta > 0$). We approximate the equity asset market fluctuations by fluctuations in the S&P 500 index and fluctuations in the asset manager's size by fluctuations in the total TNA of actively managed mutual funds with a particular benchmark. Using these approximations, we measure λ_t by comparing the cumulative log return in the asset manager's TNA relative to the cumulative log return in the S&P 500 index. For instance, if the asset manager's TNA increases by 2% while the S&P 500 index increases by 5% over the same period, the asset manager size and λ shrink. Intuitively, when the equity asset market grows faster than the asset manager size, then it is because the value investor size increases faster than the asset manager size, indicating a shrinking λ.

Table 2 reports that as the asset manager size increases, the short-term asset price increases relative to the long-term asset price. The result is robust whether we measure asset manager size with the cumulative log return of (i) the TNA across all funds with the US Equity Morningstar category, (ii) the TNA of funds with the Russell 3000 prospectus benchmark, or (iii) the TNA of funds with the Russell 3000 Growth prospectus benchmark. For instance, the third column in Table 2 states that a 1% increase in the Russell 3000 Growth funds size increases the difference between the short- and long-term dividend strip prices by 1.64 standard deviations. The result attenuates when measuring the asset manager size with the Russell 3000 Value prospectus benchmark TNA, but also the R-squared vanishes.

Table 2. Short-term minus long-term dividend strips' prices on λ.
Dependent Variable: $(S_{\mathcal{S}t} - S_{\mathcal{L}t})$/S&P 500

constant	-0.8718	-0.1671	-1.7472(.)	1.0696(*)
	(1.13946)	(0.3067)	(1.00677)	(0.50250)
λ (US Equity)	1.2443	—	—	—
	(0.925)	—	—	—
λ (Russell 3000)	—	0.5031(.)	—	—
	—	(0.1708)	—	—
λ (Russell 3000 Growth)	—	—	1.6399(*)	—
	—	—	(0.68835)	—
λ (Russell 3000 Value)	—	—	—	-0.3215
	—	—	—	(0.42552)
R^2	0.03281	0.063	0.09824	0.004122

Notes: The table presents the OLS regression of the difference between the short-term and the long-term dividend strip prices on the size of the asset manager. We measure the relative size by comparing the cumulative log return in the asset manager's TNA relative to the cumulative log return in the S&P 500 index. The different specifications refer to the US actively managed mutual funds with various mandates are similar to Table 1. To avoid non-stationarity issues, we divide the dividend strip prices by the S&P 500 index level, similar to van Binsbergen et al. (2012), and to adjust for potential heteroskedasticity, we use robust standard errors in parenthesis. The dependent variable is standardized. (***),(**),(*),(.) corresponds to 0.1%, 1%,5%,10% confidence levels, respectively.

We have presented two novel and straightforward pieces of evidence supporting our equilibrium mechanism. It is now clear that the heterogeneous investment horizons mechanism is a critical determinant of asset pricing and provides an original theoretical foundation for the recent empirical evidence of van Binsbergen et al. (2012) and the subsequent literature.

6 Extensions - TBD

7 Conclusion

Empirical evidence points to a critical difference between the asset managers' short-term performance window and value investors' long-term investment mandates. The heterogeneity in their investment horizons combined with relative performance objectives of asset managers has potential explanatory power of asset price anomalies because the professional assets under management flowing through these frictions is enormous. Despite its economic significance, the asset pricing effects of the heterogeneity in investment horizons have yet to be explored. This paper proposes a dynamic asset pricing equilibrium model with an institutional and value investor that captures the combined effect of the heterogeneous investment horizons and relative performance objective to address short-term and long-term cash-flows empirical regularities.

van Binsbergen et al. (2012) found that short-term dividend strips have a higher risk premium, Sharpe ratio, and volatility than long-term dividend strips, all of which are at odds with the leading equilibrium asset pricing models. This paper is the first to show that frictions tied to asset managers' objectives are responsible for these empirical regularities, thereby providing a novel theoretical foundation for these recent empirical puzzles.

Our continuous-time setup admits precise closed-form expressions, and despite the economic setup's simplicity, we provide novel empirical evidence to support other predictions. Our model predicts that the asset manager buys the long-term asset due to the benchmark hedging and sells short the short-term asset due to the risk aversion hedging. Further, as the size of the asset manager increases, the short-term asset price increases relative to the long-term asset price. Our empirical analysis verifies those predictions. Methodologically, our setup introduces a benchmark that depends on endogenous prices rather than exogenous dividend news, currently assumed in the literature.

There are several avenues for future research. Theoretically, it would be interesting to extend the model to allow for variations in the risk premium and observe if the benchmark duration mismatch addresses other well-known empirical phenomena such as momentum and reversal. Empirically, data on short- and long-term dividend strips is scarce and explored by relatively few papers. It would be interesting to know whether periods, where the heterogeneous investment horizons are less pronounced mitigate van Binsbergen et al. (2012)s' findings.

A Proofs

In this section we show how to derive the equilibrium quantities. We introduce two generalizations to the short-term asset manager utility function in the main setup.

$$E\left[\left(S_{\mathcal{S}T/N}\right)^{\beta}\left(S_{\mathcal{L}T/N}\right)^{\alpha}\frac{\left(W_{T/N}^{\mathcal{I}}\right)^{1-R}}{1-R}\right]. \tag{39}$$

First, the short-term period is T/N for a general $N \geq 2$, and second, the parameters α and β respectively control the importance of the long-term and short-term assets to asset manager's incentives. We assume that $\alpha \geq 0$ and $\beta \geq 0$. When $\alpha = \beta = 0$, there is no benchmark and the equilibrium coincide with a traditional two consumption dates setup. When $\alpha = 1$ and $\beta = 0$, we obtain the main setup, and when $\beta > 0$, the short-term asset becomes important for benchmarking. We do not restrict $\alpha, \beta \leq 1$, and analyze cases in which the benchmark is extremely important for incentive purposes. Throughout the analysis, the long-term value investor utility function remains identical to the main setup (5). The proofs' order reflects the most convinient way to solve for equilibrium and does not reflect the order in which the propositons appear in the text.

Before we begin with the formal proofs, we solve the following expected values.

$$\mathcal{E}_{t_1,t_2}\left(1-R\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{1-R}\right] / \left(D_{\mathcal{L}t_1}\right)^{1-R} = e^{(1-R)\left(\mu-\frac{1}{2}\sigma^2 R\right)(t_2-t_1)}, \tag{40}$$

$$\mathcal{E}_{t_1,t_2}\left(\frac{(1-R)\,\alpha}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{\frac{(1-R)\alpha}{1+\alpha}} = e^{\frac{(1-R)\alpha}{1+\alpha}\left(\mu-\frac{\sigma^2}{2}\frac{1+\alpha R}{1+\alpha}\right)(t_2-t_1)}, \tag{41}$$

$$\mathcal{E}_{t_1,t_2}\left(\frac{\beta-R}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{\frac{\beta-R}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{\frac{\beta-R}{1+\alpha}} = e^{\frac{\beta-R}{1+\alpha}\left(\mu-\frac{\sigma^2}{2}\frac{1+\alpha-\beta+R}{1+\alpha}\right)(t_2-t_1)}, \tag{42}$$

$$\mathcal{E}_{t_1,t_2}\left(1+\frac{\beta-R}{1+\alpha}\right) \equiv E_{t_1}\left[\left(D_{\mathcal{L}t_2}\right)^{1+\frac{\beta-R}{1+\alpha}}\right] / \left(D_{\mathcal{L}t_1}\right)^{1+\frac{\beta-R}{1+\alpha}} = e^{\left(1+\frac{\beta-R}{1+\alpha}\right)\left(\mu-\frac{\sigma^2}{2}\frac{R-\beta}{1+\alpha}\right)(t_2-t_1)}. \tag{43}$$

We reference to these expected values throughout the proofs.

Proof of Proposition 4 (Discount Factor). The security market is dynamically complete. As such, there exists a unique state price density process, ξ, and the no arbitrage relations

$$\xi_t S_{\mathcal{L}t} = E_t\left[\xi_T D_{\mathcal{L}T}\right], \quad t \in [0,T], \tag{44}$$

$$\xi_t S_{\mathcal{S}t} = E_t\left[\xi_{T/N} D_{\mathcal{S}T/N}\right], \quad t \in [0,T/N], \tag{45}$$

are always satisfied. In our setup, we set $r = 0$ for $t \leq T/N$, and thus the state price density evolves according to

$$d\xi_t = -\xi_t \left(\theta_{1t} dZ_{1t} + \theta_{2t} dZ_{2t}\right), \quad t \leq T/N. \tag{46}$$

The processes θ_{1t} and θ_{2t} are the market prices of risk of the short-term and long-term shocks, respectively. Restating the dynamic budget constraints as

$$\xi_t W_t^{\mathcal{V}} = E_t \left[\xi_T W_T^{\mathcal{V}}\right], \quad t \in [0, T], \tag{47}$$

$$\xi_t W_t^{\mathcal{I}} = E_t \left[\xi_{T/N} W_{T/N}^{\mathcal{I}}\right], \quad t \in [0, T/N], \tag{48}$$

and maximizing each investor's objective function (5) and (39), subject to (47) and (48) at time $t = 0$, respectively, we obtain the first order conditions

$$\left(\frac{S_{\mathcal{S}T/N}^{\beta} S_{\mathcal{L}T/N}^{\alpha}}{y^{\mathcal{I}} \xi_{T/N}}\right)^{\frac{1}{R}} = W_{T/N}^{\mathcal{I}}, \tag{49}$$

$$\left(y^{\mathcal{V}} \xi_T\right)^{-\frac{1}{R}} = W_T^{\mathcal{V}}, \tag{50}$$

where $y^{\mathcal{S}}$ and $y^{\mathcal{L}}$ are the corresponding Lagrange multipliers. By utilizing the budget constraints at $t = 0$, (47), and (48), we find that the Lagrange multipliers satisfy

$$\left(\frac{1}{y^{\mathcal{I}}}\right)^{\frac{1}{R}} = \frac{\xi_0 \lambda S_m}{E\left[\left(S_{\mathcal{S}T/N}^{\beta} S_{\mathcal{L}T/N}^{\alpha}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]}, \tag{51}$$

$$\left(\frac{1}{y^{\mathcal{V}}}\right)^{\frac{1}{R}} = \frac{\xi_0 \left(1-\lambda\right) S_m}{E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]}. \tag{52}$$

We obtain the state price density at the long- and short-term by utilizing the market clearing conditions and observing that $D_{\mathcal{S}T/N} = S_{\mathcal{S}T/N}$ due to the no arbitrage condition. By doing so, we obtain

$$\xi_{T/N} = \left(\frac{\xi_0 \lambda S_m}{D_{\mathcal{S}T/N}}\right)^R \frac{D_{\mathcal{S}T/N}^{\beta} S_{\mathcal{L}T/N}^{\alpha}}{E\left[\left(D_{\mathcal{S}T/N}^{\beta} S_{\mathcal{L}T/N}^{\alpha}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]^R}, \tag{53}$$

$$\xi_T = \left(\frac{\xi_0 \left(1-\lambda\right) S_m}{D_{\mathcal{L}T} E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]}\right)^R. \tag{54}$$

Next, to pin down the benchmark, we derive the long-term security price at the short-term date, $S_{\mathcal{L}T/N}$.

We do so by utilizing the no-arbitrage condition, given in (44), at $t = T/N$.

$$\xi_{T/N} S_{\mathcal{L}T/N} = E_{T/N}\left[\xi_T D_{\mathcal{L}T}\right]. \tag{55}$$

Plugging $\xi_{T/N}$ and ξ_T from (53) and (54) leads to an equation for $S_{\mathcal{L}T/N}$ given by

$$\frac{S^{1+\alpha}_{\mathcal{L}T/N}}{\left(D_{\mathcal{S}T/N}\right)^{R-\beta}} \left(\frac{\lambda S_m \xi_0}{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{T/N}\right)^{1-\frac{1}{R}}\right]} \right)^R = \left(D_{\mathcal{L}T/N}\right)^{1-R} \mathcal{E}_{T/N,T}\left(1-R\right) \left(\frac{\left(1-\lambda\right) S_m \xi_0}{E\left[\left(\xi_T\right)^{1-\frac{1}{R}}\right]} \right)^R. \tag{56}$$

To ease notation we define $\bar{\xi}_{\mathcal{S}}$ and $\bar{\xi}_{\mathcal{L}}$ by

$$\bar{\xi}_{\mathcal{S}} \equiv \xi_0 \left(\frac{\lambda S_m}{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]} \right)^R, \quad \bar{\xi}_{\mathcal{L}} \equiv \xi_0 \left(\frac{\left(1-\lambda\right) S_m}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]} \right)^R, \tag{57}$$

where $\xi_{s,t} \equiv \frac{\xi_t}{\xi_s}$. We obtain that $S_{\mathcal{L}T/N}$ is given by

$$S_{\mathcal{L}T/N} = \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{R-\beta}{1+\alpha}} \left(\mathcal{E}_{T/N,T}\left(1-R\right) \frac{\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}} \right)^{\frac{1}{1+\alpha}}, \tag{58}$$

where $\mathcal{E}_{T/N,T}\left(1-R\right)$ is given in (40), evaluated at $(t_1, t_2) = (\frac{T}{N}, T)$. Plugging $S_{\mathcal{L}T/N}$ back to $\xi_{T/N}$ we find

$$\xi_{T/N} = \left(D_{\mathcal{L}T/N}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\beta-R}{1+\alpha}} \left(\mathcal{E}_{T/N,T}\left(1-R\right)\bar{\xi}_{\mathcal{L}}\right)^{\frac{\alpha}{1+\alpha}} \left(\bar{\xi}_{\mathcal{S}}\right)^{\frac{1}{1+\alpha}}. \tag{59}$$

Further, because $\xi_{T/N}$ is a martingale it is given by the relationship $\xi_t = E_t\left[\xi_{T/N}\right]$, which leads to

$$\xi_t = \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \left(D_{\mathcal{S}t}\right)^{\frac{\beta-R}{1+\alpha}} \left(\mathcal{E}_{T/N,T}\left(1-R\right)\bar{\xi}_{\mathcal{L}}\right)^{\frac{\alpha}{1+\alpha}} \left(\bar{\xi}_{\mathcal{S}}\right)^{\frac{1}{1+\alpha}} \mathcal{E}_{t,T/N}\left(\frac{\left(1-R\right)\alpha}{1+\alpha}\right) \mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right), \tag{60}$$

for $t \le T/N$, where both $\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)$ and $\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)$ are deterministic functions of time given in (41) and (42), respectively, and evaluated at $(t_1, t_2) = (t, \frac{T}{N})$. Dividing ξ_t by ξ_0 (60), we obtain

$$\xi_{0,t} = \frac{\left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \left(D_{\mathcal{S}t}\right)^{\frac{\beta-R}{1+\alpha}}}{\left(D_{\mathcal{L}0}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}}} \frac{1}{\mathcal{E}_{0,t}\left(\frac{(1-R)\alpha}{1+\alpha}\right) \mathcal{E}_{0,t}\left(\frac{\beta-R}{1+\alpha}\right)}, \quad t \le T/N. \tag{61}$$

□

Proof of Proposition 2 (Market Price of Risk). We take Itô's Lemma on $\xi_{0,t}$ (61) and obtain

$$\theta_{1t} = \frac{R-\beta}{1+\alpha}\sigma, \qquad \theta_{2t} = \frac{\alpha\left(R-1\right)}{1+\alpha}\sigma, \tag{62}$$

Further, we find that $\theta_{1t} > \theta_{2t}$ if, and only if

$$R(1-\alpha) > \beta - \alpha, \tag{63}$$

which is always satisfied because $R(1-\alpha) > (1-\alpha) > \beta - \alpha$ since $R > 1$ and $\alpha, \beta \leq 1$. We set $\alpha = 1$ and $\beta = 0$ and obtain (15). We obtain the no benchmark case by setting $\alpha = \beta = 0$. □

Proof of Proposition 3 (Security Prices). To find the long-term security price, we again utilize the no-arbitrage condition

$$S_{\mathcal{L}t} = \frac{E_t\left[\xi_{T/N} S_{\mathcal{L}T/N}\right]}{\xi_t}, \quad t < T/N, \tag{64}$$

and find that

$$S_{\mathcal{L}t} = (D_{\mathcal{L}t})^{\frac{1-R}{1+\alpha}} (D_{\mathcal{S}t})^{\frac{R-\beta}{1+\alpha}} \left(\frac{\mathcal{E}_{T/N,T}(1-R)\,\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}}\right)^{\frac{1}{1+\alpha}} \frac{\mathcal{E}_{t,T/N}(1-R)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \tag{65}$$

By plugging $\bar{\xi}_{\mathcal{S}}$ and $\bar{\xi}_{\mathcal{L}}$, given (57), we find that

$$S_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right)^{\frac{R}{1+\alpha}} \left(\frac{E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{\frac{R}{1+\alpha}} (D_{\mathcal{L}t})^{\frac{1-R}{1+\alpha}} (D_{\mathcal{S}t})^{\frac{R-\beta}{1+\alpha}} \frac{\left(\mathcal{E}_{T/N,T}(1-R)\right)^{\frac{1}{1+\alpha}} \mathcal{E}_{t,T/N}(1-R)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}, \tag{66}$$

for $t \leq T/N$. To characterize $E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]$, we evaluate the above equation at $t = T/N$, raise it to the power of $\frac{\alpha}{R}$, multiply both sides by $D^{\frac{\beta}{R}}_{\mathcal{S}T/N}\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}$, and take expectations. By doing so, we get

$$\begin{aligned} &E\left[\left(D^{\beta}_{\mathcal{S}T/N} S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}} \left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right] \\ &= \left(\frac{1-\lambda}{\lambda}\right)^{\alpha} \left(\frac{1}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{\alpha} \left(E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}} \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right]\right)^{1+\alpha} \left(\mathcal{E}_{T/N,T}(1-R)\right)^{\frac{\alpha}{R}}. \end{aligned} \tag{67}$$

By plugging this identity back to $S_{\mathcal{L}t}$ (66) and rearranging, we find

$$S_{\mathcal{L}t} = \left(\frac{1-\lambda}{\lambda}\right)^{R} \left(\frac{E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}} \left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}} \left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right]}{E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]}\right)^{R} (D_{\mathcal{L}t})^{\frac{1-R}{1+\alpha}} (D_{\mathcal{S}t})^{\frac{R-\beta}{1+\alpha}} \frac{\mathcal{E}_{T/N,T}(1-R)\,\mathcal{E}_{t,T/N}(1-R)}{\mathcal{E}_{t,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}, \tag{68}$$

We are left to evaluate the two unconditional expected values in this last formulation of $S_{\mathcal{L}t}$. By evaluating the first expected value, we obtain

$$E\left[\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\left(D_{\mathcal{L}T/N}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}}\left(D_{\mathcal{S}T/N}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\right]=\frac{\left(D_{\mathcal{L}0}\right)^{\frac{1-R}{R}\frac{\alpha}{1+\alpha}}\left(D_{\mathcal{S}0}\right)^{\frac{\alpha+\frac{\beta}{R}}{1+\alpha}}\left(\mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\right)^{\frac{1}{R}}\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\left(\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)\right)^{\frac{R-1}{R}}}, \tag{69}$$

where $\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}$ is obtained by evaluating (61) at $t=T/N$ and by raising the expression to the power of $1-\frac{1}{R}$. The function $\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)$ is given in (43), evaluated at $(t_1,t_2)=(0,\frac{T}{N})$. We continue with the evaluation of the second expected value. By dividing ξ_T (54) by ξ_0 (60), we obtain

$$\frac{\xi_T}{\xi_0}=\left(\frac{\bar{\xi}_{\mathcal{L}}}{\bar{\xi}_{\mathcal{S}}}\right)^{\frac{1}{1+\alpha}}\frac{\left(D_{\mathcal{L}T}\right)^{-R}}{\left(D_{\mathcal{L}0}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}}\left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{\alpha}{1+\alpha}}\mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \tag{70}$$

By plugging $\bar{\xi}_{\mathcal{L}}$, $\bar{\xi}_{\mathcal{S}}$ from (57), by raising the expression to the $1-\frac{1}{R}$ power of both sides, taking expectations, and rearranging we obtain

$$E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]=\left(\frac{1-\lambda}{\lambda}\right)^{\frac{(R-1)}{\alpha+R}}\left(E\left[\left(D^{\beta}_{\mathcal{S}T/N}S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}}\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]\right)^{\frac{(R-1)}{\alpha+R}}$$
$$\left(\frac{\left(D_{\mathcal{L}0}\right)^{1-R}\mathcal{E}_{0,T}\left(1-R\right)}{\left[\left(D_{\mathcal{L}0}\right)^{\frac{(1-R)\alpha}{1+\alpha}}\left(D_{\mathcal{S}0}\right)^{\frac{\beta-R}{1+\alpha}}\left(\mathcal{E}_{T/N,T}\left(1-R\right)\right)^{\frac{\alpha}{1+\alpha}}\mathcal{E}_{0,T/N}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)\right]^{\frac{R-1}{R}}}\right)^{\frac{1+\alpha}{\alpha+R}}.$$

By plugging $E\left[\left(D^{\beta}_{\mathcal{S}T/N}S^{\alpha}_{\mathcal{L}T/N}\right)^{\frac{1}{R}}\left(\xi_{0,T/N}\right)^{1-\frac{1}{R}}\right]$, given in (67), using (69), and rearranging, we obtain

$$E\left[\left(\xi_{0,T}\right)^{1-\frac{1}{R}}\right]=\left(\frac{1-\lambda}{\lambda}\right)^{\frac{R-1}{R}}\left(D_{\mathcal{L}0}\right)^{\frac{1-R}{R}}\left(D_{\mathcal{S}0}\right)^{\frac{R-1}{R}}\left(\frac{\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{0,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}\right)^{\frac{R-1}{R}}\left(\mathcal{E}_{0,T}\left(1-R\right)\right)^{\frac{1}{R}}. \tag{71}$$

By plugging these two expected values back to $S_{\mathcal{L}t}$ (68), we obtain

$$S_{\mathcal{L}t}=\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{D_{\mathcal{L}t}}{D_{\mathcal{L}0}}\right)^{\frac{1-R}{1+\alpha}}\left(\frac{D_{\mathcal{S}t}}{D_{\mathcal{S}0}}\right)^{\frac{R-\beta}{1+\alpha}}\left(D_{\mathcal{S}0}\right)\frac{\mathcal{E}_{0,t}\left(\frac{(1-R)\alpha}{1+\alpha}\right)\mathcal{E}_{0,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{0,t}\left(1-R\right)\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \tag{72}$$

Similarly, the short-term security price is given by

$$S_{\mathcal{S}t}=\left(D_{\mathcal{S}t}\right)\frac{\mathcal{E}_{t,T/N}\left(\frac{1+\alpha+\beta-R}{1+\alpha}\right)}{\mathcal{E}_{t,T/N}\left(\frac{\beta-R}{1+\alpha}\right)}. \tag{73}$$

We get $\bar{S}_{\mathcal{S}t}$ and $\bar{S}_{\mathcal{L}t}$ by setting $\alpha = \beta = 0$, and $S_{\mathcal{S}t}$ and $S_{\mathcal{L}t}$ by setting $\alpha = 1$ and $\beta = 0$. Furhter, we define $\bar{A}_{\mathcal{L}t}$ and $\bar{A}_{\mathcal{S}t}$ to decompose the deteminstic parts affecting prices, as follows.

$$\bar{A}_{\mathcal{L}t} \equiv \left(\frac{1}{D_{\mathcal{L}0}}\right)^{1-R} \left(\frac{1}{D_{\mathcal{S}0}}\right)^{R} D_{\mathcal{S}0} \frac{\mathcal{E}_{0,t}\left(0\right) \mathcal{E}_{0,T/N}\left(1-R\right)}{\mathcal{E}_{0,t}\left(1-R\right) \mathcal{E}_{t,T/N}\left(-R\right)}, \tag{74}$$

$$\bar{A}_{\mathcal{S}t} \equiv \frac{\mathcal{E}_{t,T/N}\left(1-R\right)}{\mathcal{E}_{t,T/N}\left(-R\right)}. \tag{75}$$

Similarly, we define

$$A_{\mathcal{L}t} \equiv \frac{1}{\bar{A}_{\mathcal{L}t}} \left(\frac{1}{D_{\mathcal{L}0}}\right)^{\frac{1-R}{2}} \left(\frac{1}{D_{\mathcal{S}0}}\right)^{\frac{R}{2}} D_{\mathcal{S}0} \frac{\mathcal{E}_{0,t}\left(\frac{1-R}{2}\right) \mathcal{E}_{0,T/N}\left(1-\frac{R}{2}\right)}{\mathcal{E}_{0,t}\left(1-R\right) \mathcal{E}_{t,T/N}\left(-\frac{R}{2}\right)}, \tag{76}$$

$$A_{\mathcal{S}t} \equiv \frac{1}{\bar{A}_{\mathcal{S}t}} \frac{\mathcal{E}_{t,T/N}\left(1-\frac{R}{2}\right)}{\mathcal{E}_{t,T/N}\left(-\frac{R}{2}\right)}. \tag{77}$$

□

Proof of Lemma 1 (Risk Exposure). We utilize the no arbitrage conditions (47) and (48), the market clearing conditions in the consumption good, and write

$$\xi_t W_t^{\mathcal{V}} = E_t\left[\xi_T D_{\mathcal{L}T}\right], \quad t \in [0, T], \tag{78}$$

$$\xi_t W_t^{\mathcal{I}} = E_t\left[\xi_{T/N} D_{\mathcal{S}T/N}\right], \quad t \in [0, T/N]. \tag{79}$$

By plugging ξ_T (54) and $\xi_{T/N}$ (59) we find that

$$\xi_t W_t^{\mathcal{V}} \propto \left(D_{\mathcal{L}t}\right)^{1-R}, \tag{80}$$

$$\xi_t W_t^{\mathcal{I}} \propto \left(D_{\mathcal{S}t}\right)^{1+\frac{\beta-R}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} = \left(D_{\mathcal{S}t}\right)^{1-R} \left(D_{\mathcal{S}t}\right)^{R\frac{\alpha+\frac{\beta}{R}}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}} \propto \left(D_{\mathcal{S}t}\right)^{1-R} \left(S_{\mathcal{L}t}\right)^{\alpha} \left(S_{\mathcal{S}t}\right)^{\beta}. \tag{81}$$

In the third term of (81), we separate the benchmarking from the non-benchmarking components by setting $\alpha = \beta = 0$, which leads to a non benchmarking component that equals $\left(D_{\mathcal{S}t}\right)^{1-R}$ and a benchmakring component that equals $\left(D_{\mathcal{S}t}\right)^{R\frac{\alpha+\frac{\beta}{R}}{1+\alpha}} \left(D_{\mathcal{L}t}\right)^{\frac{(1-R)\alpha}{1+\alpha}}$. We obtain the last term of (81) by observing that the benchmarking component equals $\left(S_{\mathcal{L}t}\right)^{\alpha} \left(S_{\mathcal{S}t}\right)^{\beta}$, as (72) and (73) reveals. We finish by applying Itô's Lemma to both sides of the above equations, (80), (81) and find that

$$\Sigma_t' \pi_t^{\mathcal{V}} - \theta_t = \begin{bmatrix} 0 \\ -\left(R-1\right)\sigma \end{bmatrix}, \tag{82}$$

$$\Sigma_t' \pi_t^{\mathcal{I}} - \theta_t = \begin{bmatrix} -\left(R-1\right)\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta, \tag{83}$$

which leads to the desired result when setting $\alpha = 1$ and $\beta = 0$. □

Proof of Proposition 1 (Volatility). We obtain the volatility coefficients by taking Itô's Lemma of $S_{\mathcal{L}t}$ (72) and $S_{\mathcal{S}t}$ (73) as follows.

$$\begin{bmatrix}\sigma_{\mathcal{S}1t}\\ \sigma_{\mathcal{S}2t}\end{bmatrix} = \begin{bmatrix}\sigma\\ 0\end{bmatrix}, \qquad \begin{bmatrix}\sigma_{\mathcal{L}1t}\\ \sigma_{\mathcal{L}2t}\end{bmatrix} = \sigma \begin{bmatrix}\frac{R-\beta}{1+\alpha}\\ \frac{1-R}{1+\alpha}\end{bmatrix}. \tag{84}$$

We set $\alpha = 1$ and $\beta = 0$ to get the results for the benchmark case and $\alpha = \beta = 0$ for the no benchmark case. □

Proof of Proposition 5 (Risk Premium, Total Volatility, and Sharpe Ratio). We start the proof by showing that the risk premium of the short-term asset is higher than the long-term asset. To find the risk premiums we use the identity

$$\begin{bmatrix}\mu_{\mathcal{S}t}\\ \mu_{\mathcal{L}t}\end{bmatrix} = \begin{bmatrix}\sigma_{\mathcal{S}1t} & \sigma_{\mathcal{S}2t}\\ \sigma_{\mathcal{L}1t} & \sigma_{\mathcal{L}2t}\end{bmatrix}\begin{bmatrix}\theta_{1t}\\ \theta_{2t}\end{bmatrix} = \sigma^2 \begin{bmatrix}1 & 0\\ \frac{R-\beta}{1+\alpha} & -\frac{R-1}{1+\alpha}\end{bmatrix}\begin{bmatrix}\frac{R-\beta}{1+\alpha}\\ \frac{R-1}{1+\alpha}\alpha\end{bmatrix} = \sigma^2 \begin{bmatrix}\frac{R-\beta}{1+\alpha}\\ \left(\frac{R-\beta}{1+\alpha}\right)^2 - \left(\frac{R-1}{1+\alpha}\right)^2\alpha\end{bmatrix}. \tag{85}$$

We find that $\mu_{\mathcal{S}t} > \mu_{\mathcal{L}t}$ if and only if

$$\frac{R-\beta}{1+\alpha} > \left(\frac{R-\beta}{1+\alpha}\right)^2 - \left(\frac{R-1}{1+\alpha}\right)^2\alpha. \tag{86}$$

Let us define $X \equiv \frac{R-1}{R-\beta}$, divide both sides by $\left(\frac{R-\beta}{1+\alpha}\right)^2$, and rewrite the inequality such that

$$X^2\alpha > 1 - \frac{1+\alpha}{R-\beta} = \frac{R-\beta-1-\alpha}{R-\beta} = \frac{R-1}{R-\beta} - \frac{\beta+\alpha}{R-\beta} = X - \frac{\beta+\alpha}{R-\beta}. \tag{87}$$

Transferring the X to the left hand side leads to

$$X\left(X\alpha - 1\right) = \frac{R-1}{R-\beta}\left(X\alpha - 1\right) > -\frac{\beta+\alpha}{R-\beta}. \tag{88}$$

Rearranging, we finally obtain

$$\left(R-1\right)X\alpha > \left(R-1\right) - \left(\beta+\alpha\right). \tag{89}$$

It is clear from (89) that if $\alpha = \beta = 0$ the inequality is not satisfied, implying that the short-term asset expected return is lower than the long-term asset expected return without benchmarking when $R > 1$. We left to show that when there is benchmarking, $0 < \alpha + \beta \leq 1$, for any $0 \leq \alpha \leq 1$ there exists a threshold $0 \leq \underline{\beta} < 1 - \alpha$, such that for any $\beta \in [\underline{\beta}, 1-\alpha]$ the inequality (89) is satisfied. Towards that goal, let $R = 1 + \epsilon < 2$ for $0 < \epsilon < 1$. Then, for any given $\alpha \in [0,1]$, we set $\underline{\beta} = \max\{\epsilon - \alpha, 0\}$. It is apparent that

$\underline{\beta} < 1 - \alpha$ because $\epsilon < 1$, so $[\underline{\beta}, 1 - \alpha]$ is non empty. Eventually, we obtain

$$(R-1)\,X\alpha > 0 \geq \epsilon - \left(\underline{\beta} + \alpha\right) \geq \epsilon - (\beta + \alpha) = (R-1) - (\beta + \alpha)\,, \tag{90}$$

where the right inequality holds because $\beta \geq \underline{\beta}$. We conclude that with benchmarking, the the short-term asset expected return is higher than the long-term asset expected return.

Next, we show that the total volatility of the short-term asset is higher than the long-term asset. Following the definition of total volatility (28) and the volatility coefficients (84) from the proof Proposition 1, we find that the total volatility of the short-term asset is higher than the long-term asset, if, and only if,

$$1 > \left(\frac{R-\beta}{1+\alpha}\right)^2 + \left(\frac{R-1}{1+\alpha}\right)^2. \tag{91}$$

An algebraic manipulation leads to

$$2R^2 - 2R\,(1+\beta) + \left[\left(1+\beta^2\right) - (1+\alpha)^2\right] < 0. \tag{92}$$

We solve for the roots of R and find

$$R = \frac{2\,(1+\beta) \pm \sqrt{4\,(1+\beta)^2 - 4 \times 2\left[\left(1+\beta^2\right) - (1+\alpha)^2\right]}}{4}. \tag{93}$$

Another algebraic manipulation finally leads to

$$R = \frac{(1+\beta) \pm \sqrt{\left[1 - \beta\left(\sqrt{2}-1\right)\right]\left[1 + \beta\left(\sqrt{2}+1\right)\right] + 2\alpha\,(2+\alpha)}}{2}. \tag{94}$$

This quadratic equation has two solutions. We define $\bar{R}$ as the upper solution. It is immediately observable that when $\alpha = \beta = 0$, the upper solution equals 1, implying that the short-term asset volatility is lower than the long-term asset volatility for $R > 1$ without benchmarking. When there is benchmarking, $1 \geq \alpha + \beta > 0$, we left to show that $1 < \bar{R} < 2$. Following (94), we find that $\bar{R} > 1$ if, and only if

$$\alpha\,(2+\alpha) > \beta\,(\beta - 2)\,, \tag{95}$$

which is satisfied for any $\beta, \alpha \in (0, 1]$, since the right hand side is negative and the left hand side is positive. Further, we find that $\bar{R} < 2$ if, and only if

$$\alpha\,(2+\alpha) < (2-\beta)^2\,. \tag{96}$$

Adding and subtracting 1 from the left hand side, and rearranging leads to

$$-1 < (1-\alpha-\beta)(3+\alpha-\beta), \tag{97}$$

which is satisfied for $\alpha+\beta \leq 1$.

Lastly, we find that the short-term asset Sharpe ratio is higher than the long-term asset Sharpe ratio if, and only if,

$$\frac{R-\beta}{1+\alpha} > \frac{\left(\frac{R-\beta}{1+\alpha}\right)^2 - \left(\frac{R-1}{1+\alpha}\right)^2 \alpha}{\sqrt{\left(\frac{R-\beta}{1+\alpha}\right)^2 + \left(\frac{R-1}{1+\alpha}\right)^2}}. \tag{98}$$

Let us define $X \equiv \frac{R-1}{R-\beta}$ and rewrite the inequality above in terms of X, leading to

$$\sqrt{1+X^2} > 1 - X^2\alpha. \tag{99}$$

This inequality is always satisfied because $X > 0$. Notice that the inequality holds for $\alpha = \beta = 0$ as well.

□

Proof of Proposition 6 (Portfolios). It is easy to verify from (85) that

$$\left(\Sigma_t^{'}\right) = \begin{bmatrix} 1 & \frac{R-\beta}{1+\alpha} \\ 0 & -\frac{R-1}{1+\alpha} \end{bmatrix} \sigma, \qquad \left(\Sigma_t^{'}\right)^{-1} = \begin{bmatrix} 1 & \frac{R-\beta}{R-1} \\ 0 & -\frac{1+\alpha}{R-1} \end{bmatrix} \frac{1}{\sigma}, \tag{100}$$

where Σ_t is defined in (9). Multiplying the risk exposures, given in (82) and (83), by the inverse of $\left(\Sigma_t^{'}\right)$, we obtain

$$\pi_t^{\mathcal{V}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} 0 \\ -(R-1)\sigma \end{bmatrix}, \tag{101}$$

$$\pi_t^{\mathcal{I}} = \left(\Sigma_t^{'}\right)^{-1} \theta_t + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} -(R-1)\sigma \\ 0 \end{bmatrix} + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta. \tag{102}$$

We define the portfolios of the individual equilibrium channels, which also appears in the text above Proposition 6) as

$$\phi_{\text{m.v.}} \equiv \left(\Sigma_t^{'}\right)^{-1} \theta_t, \quad \phi_{\text{r.a.}}^{\mathcal{V}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} 0 \\ -(R-1)\sigma \end{bmatrix}, \tag{103}$$

$$\phi_{\text{r.a.}}^{\mathcal{I}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} -(R-1)\sigma \\ 0 \end{bmatrix}, \quad \phi_{\text{b}}^{\mathcal{I}} \equiv \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{L}1t} \\ \sigma_{\mathcal{L}2t} \end{bmatrix} \alpha + \left(\Sigma_t^{'}\right)^{-1} \begin{bmatrix} \sigma_{\mathcal{S}1t} \\ \sigma_{\mathcal{S}2t} \end{bmatrix} \beta. \tag{104}$$

By taking the inner products we obtain

$$\phi_{\text{m.v.}} = \begin{bmatrix} R-\beta \\ -\alpha \end{bmatrix}, \quad \phi^{\mathcal{V}}_{\text{r.a.}} = \begin{bmatrix} -(R-\beta) \\ 1+\alpha \end{bmatrix}, \quad \phi^{\mathcal{I}}_{\text{r.a.}} = \begin{bmatrix} -(R-1) \\ 0 \end{bmatrix}, \quad \phi^{\mathcal{I}}_{\text{b}} = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{105}$$

leading to our desired results in (34), (35) and (36) when $\beta = 0$ and $\alpha = 1$. □

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