1962

Geometry for the sixth grade "Figures in Space"

BOSTON UNIVERSITY

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Thesis

GEOMETRY FOR THE SIXTH GRADE

"Figures in Space"

An enrichment activity in non-metric geometry for sixth grade children academically talented in mathematics.

Submitted by

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The writer wishes to express her appreciation to Dr. J. Fred Weaver for his guidance and encouragement in the completion of this thesis.

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CHAPTER I

INTRODUCTION

Statement of the Problem

Research has brought about many changes in both the content and the method of teaching arithmetic in the elementary schools. It has been our concern that the program of mathematics as presented in the sixth grade does not adequately stimulate the talented or gifted child to the full use of his powers. We are encouraging these children to explore, achieve insight, reason, and deduce. It is necessary, therefore, to build into the curriculum subject matter which will insure these experiences.

The subject of geometry has not been included in the sixth grade course of study. Neither has geometry been considered as a program of enrichment for the gifted in arithmetic.

A study of current sixth grade textbooks shows that beyond a page or two devoted to perimeter, area, and perhaps volume, which are computational in nature, no geometrical information is included.

Perusal of seventh grade texts shows simply a review of material presented in the sixth grade. It is not until the eighth grade level that angles are measured and congruents are discussed.

It is the purpose of this paper to present a unit of study introducing noncomputational geometry to the sixth grade as a part of an enrichment program for children academically talented in arithmetic.

Justification

As to why a study of geometry is desirable at the elementary school level, a statement by Brune seems appropriate:

> Man has always needed geometric principles, however dimly he may have at first perceived them. Similarly children's lives cannot be devoid of geometry however unaware they may be of its formal aspects . . . geometry deserves a lifetime of interest. To study it in only the tenth grade hardly suffices. . . . Informal geometry in the elementary grades can counteract a serious deficiency.[1]

It has been only in the last few years that any attempt has been made to place some areas of geometry in the elementary grades, and these have been specific programs. It has been found that what was formerly assigned to eighth and tenth grades can well be assimilated and enjoyed by youngsters in the intermediate grades.

However, the approach to geometry today differs from the approach of years ago. It is the new approach which the writer wishes to consider in creating a new unit of study in geometry.

Scope

In order to introduce geometry to the talented sixth grade youngster, three units have been organized. The first unit describes space, points, lines, and planes in space. The second unit introduces the protractor in the study of angles. Unit three includes geometric figures, planes and solid.

Definite understandings and a vocabulary are stated with each unit. It is hoped that these understandings will establish a background for teachers, as well as the basis of the concepts to be learned by the

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[1]Irvin H. Brune, "Geometry in the Grades," The Arithmetic Teacher (May, 1961), 8:5:211.

children.

At the conclusion of each unit is a group of enrichment activities that may be used to stimulate the children to more productive and creative thinking.

This unit of study may be accomplished in the seventeen designated lessons or may take as long to complete as the enthusiasm and interest of the pupils indicate.

The basic interpretation of this material is dependent upon the concept of the School Mathematics Study Group Project on Elementary School Mathematics.

These ideas have been enlarged upon and extended, and new ideas have been introduced by the writer. Additional enrichment materials and activities have been developed and incorporated in the basic content.

CHAPTER II

REVIEW OF RELATED LITERATURE

There is very little material in our professional literature that specifically deals with nonmetric geometry in the elementary grades. There are two instructional programs which have introduced geometry in the primary grades, but a survey has found that there was a very limited amount of material available concerning these programs.

Hawley and Suppes are conducting experiments designed to teach geometric materials to primary grades. The authors of this material feel that abstract material can be handled much earlier in the grades than was thought possible a few years ago. This program has been designed for grades three and four at present. As Hawley states, "Since geometry is more typical of mathematics as a whole than arithmetic, it is desirable that this branch of mathematics be introduced early."[1]

One of the features of this program is that it is designed for the entire class but replaces none of the regular arithmetic. A child must read the instructions in order to carry out the steps in the constructions and in the other exercises. According to Lundberg, "The program has produced very satisfactory results."[2] It is felt that children of all ages should get ample opportunities to find out about things geometrical. Geometry can give satisfaction with things mathematical.

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[1]Newton S. Hawley and Patrick Suppes, "The Geometry Project," A mimeographed report, Stanford University, April 17, 1960.

[2]Hazel Lundberg, "Mathematics in the Elementary School," Educational Leadership (March, 1962), 368.

In the words of Brune, "Drawing, counting, and measuring lead pupils to observing, inferring, and generalizing."[1]

It is believed that through geometry and the handling of geometric figures, a consciousness of form begins to grow. An appreciation of numerous ideas develops which help the children to gain in understanding and to grow in geometric readiness.

Brune feels that the willingness and preparedness of mathematical maturity which geometry creates, help the child conquer new worlds in arithmetic.[2] He wrote:

> A proper study for all children is geometry--the geometry of form. Here pupils perceive, compare, measure and generalize. Here they sharpen intuition without plunging too far into abstractions. Above all, children see values in what they do. If we can encourage pupils to discover for themselves some principles in the science of space, then they will bring into their geometry classes a usable store of information about the Euclidean plane. They might also have a good start on three-space.[3]

The other group to introduce nonmetric geometry in the primary grades is the School Mathematics Study Group. The authors of this program are decidedly interested in the present-day approach to geometry content. The materials are developed from the viewpoint of structure, and beginning in grade four the child is introduced to the language and notations of sets, some of the properties of numbers, and topics from algebra and geometry. It is a mathematics program rather than an arithmetic program, for, according to Rutland and Hosier, "The content of the program places emphasis not only on arithmetic, but on geometric under-

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[1]Brune, op. cit., p. 211.

[2]Ibid.

[3]Ibid., p. 219.

standings and skills."[1]

Lundberg[2] reports that this program is also very successful. The responses of the children are excellent from the viewpoint of both interest and achievement. A noticeable result is the children's ability and willingness to attack new problems.

Far more students are reported to be interested in the new mathematics than were interested in the traditional program. This mathematics is taught as a complete program of arithmetic for all children in grades four, five, and six.

Materials such as those presented in these two programs, while differing slightly in certain aspects, aim to develop the concepts of mathematics through the structure of mathematics. We are finding that the study of geometry can be made meaningful and significant to children of the elementary grades, and can help them to grow in mathematical understandings and reasoning power with a deeper knowledge of space and the world about them.

As for the teacher's part in these programs, Rutland and Hosier believe,

> It is not necessary that an elementary teacher be an expert geometrician in order to help elementary-school children effectively develop some of these geometric ideas intuitively. On the other hand, no one can deny that the better the background and understanding one has, the better his teaching should be.[3]

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[1]Leon Rutland and Max Hosier, "Some Basic Geometric Ideas for the Elementary Teacher," The Arithmetic Teacher (November, 1961), 8:7:357.

[2]Lundberg, op. cit., p. 357.

[3]Rutland and Hosier, op. cit., p. 362.

While both of these programs have teacher-training workshops, there is very little other helpful material available. However, the National Council of Teachers of Mathematics,[1] in their Twenty-fourth Yearbook, presents the present-day approach to mathematics from kindergarten through grade twelve. Geometry may be found in their chapters on "Proof," "Relations and Function," and "Measurement and Approximation."

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[1]The National Council of Teachers of Mathematics, The Growth of Mathematical Ideas, Grades K-12, Twenty-fourth Yearbook, Washington, D. C., 1959.

CHAPTER III

INSTRUCTIONAL MATERIALS

Introduction

The basic content of this unit is so arranged as to teach the concepts of geometry required. It presupposes that there is no knowledge of geometry of any kind. The lessons are noncomputational in content, yet they introduce the pupil to physical geometry. The unit gives children the language and symbols of geometry and helps them to discover some of the basic principles for themselves.

The understandings presented at the beginning of each unit are a guide for the teacher. She may also use these as a check list with the pupils when the unit is finished.

The vocabulary which is at the end of each unit may be developed as the lessons progress. Some of the words may not appear to be new to the children, but they will, no doubt, be new in content.

It is hoped that the teacher will not feel restricted to the material presented, but will stop to explore areas of interest to the children or to explain concepts which may be difficult for the children to grasp.

OUTLINE FOR NONCOMPUTATIONAL GEOMETRY

UNIT ONE

UNDERSTANDINGS

1. A point is an exact location.
2. A point really has no size at all.
3. A point does not move.
4. Space is the set of all points everywhere.
5. A curve is a set of points.
6. A line segment is the most direct curve connecting two points.
7. The line segment ends are called end points.
8. Only one line may pass through two end points.
9. A line has no end points. It extends in either direction indefinitely.
10. A ray is a part of a line.
11. A ray starts at the end point and proceeds outward.
12. Planes and parts of planes may be thought of as sets of points all of which are on "flat" surfaces.
13. A plane contains more points and lines than can be counted.
14. Through two points in space there is an infinite number of planes.
15. Through a line in space there is an infinite number of planes.
16. Through three points not on a line there can be only one plane.
17. If two different lines in space intersect, their intersection is one point.
18. If a line and a plane intersect, the intersection is either a line or a point.
19. If two different planes intersect, their intersection is a line.

I. INTRODUCTION TO SPACE AND SETS OF POINTS

The four processes you have been using up to now are a part of arithmetic. When you think of arithmetic you immediately think of computation or problems in which you do these four processes. Let's think about a different kind of arithmetic. The arithmetic of space. We will talk about sets of points on a line, sets of points in space, curves and planes. We shall explore circles, angles, and polygons. This kind of mathematics is called geometry.

You have talked about sets before. A set is a group of things that have something in common. We could have a set of trees, a set of books, a set of the boys in this class, and many more. If we speak about a set of points we could name the points as sets of points,

A pencil is a set of points in space. Your desk is a set of points in space. The blackboard is a set of points in space.

Consider the air around us in the classroom. Is the air in the classroom empty? Not at all. There are more points in the space around us than you can count. If we had a magnifying glass we might be able to see some of the germs in the air, but not all. There would be far more than you could count. For instance, an atom has never been seen, but it occupies more points of space than can be counted.

Any object occupies more sets of points than you could ever count, and contains more points than can be counted. Is a ball empty?

Is a block of wood empty? Even a particle of dust occupies many points of space. Space is the set of all the exact locations everywhere.

SOME THINGS TO THINK ABOUT

1. Make a list of five things which occupy sets of points in space. Be ready to explain these.

2. How many points are there in the space of your own classroom?

   (more than could be counted)

3. How many points are there in a pound of candy?

   (more points than there are pieces of candy)

4. What is space?

II. POINTS AND CURVES

In geometry we let a dot or small pencil mark represent a point. The smaller the dot the better. Can you tell why?

(It covers less points and marks the spot more exactly.)

A point means an exact location in space so small that you cannot see it at all. Therefore any dot you make covers more points than can possibly be counted.

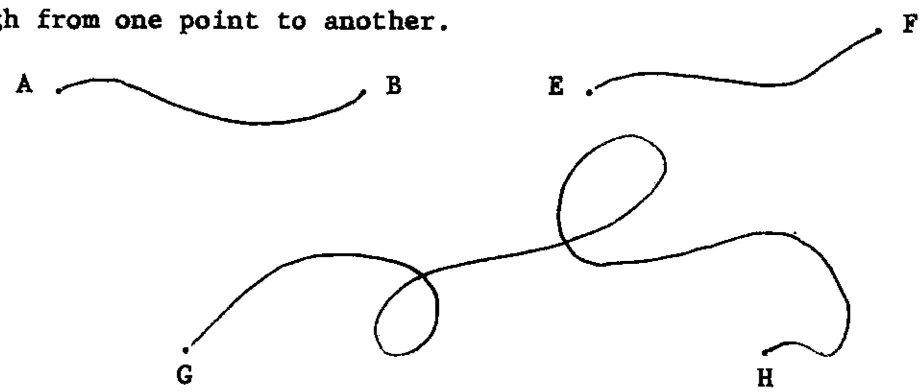
We use capital letters to identify dots or points.

. B

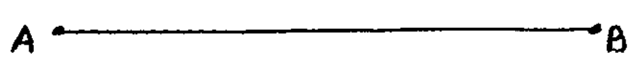
. A . C

Name this set of points. {ABC}

In going from one point to another you trace a curve. A curve is a set of points. It is all the different points your pencil goes through from one point to another.



There is a more direct way of going from A to B, or from E to F, or from G to H. This is called a line segment.



A line segment is also a set of points. It contains the two points named and all the other points between them. We write it $\overline{AB}$. Points A and B are ends of the line segment and are therefore called end points.

SOME THINGS TO THINK ABOUT

1. What is a point?

    (A definite location in space.)

2. Describe a set of four points in the classroom.

    (Four points of the walls, doors, desks, windows, etc.)

3. How many different paths could you travel in going from one end of the playground to the other?

    (More than could be counted.)

4. Suppose we say that D, E, and F are endpoints. Name all the different line segments you can.

    ($\overline{DE}$, $\overline{DF}$, $\overline{EF}$, $\overline{ED}$, $\overline{FD}$, $\overline{FE}$)

5. Using point E as an endpoint, how many line segments can you draw?

    (More than could be counted.)

6. How many line segments can you draw through points A . .B?

(Exactly one.)

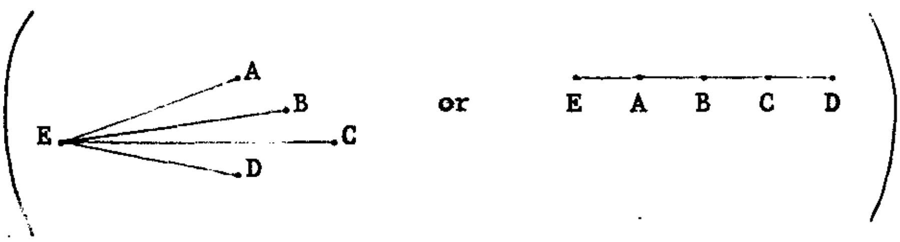
7. How many line segments can you draw? Name them.

D . (CD, DE, CE, FE, FC, FD)

. E

C . . F

8. Mark a point on your paper and call it E. Draw four line segments having E as an endpoint. Label the endpoints A, B, C, and D.



III. LINES AND RAYS

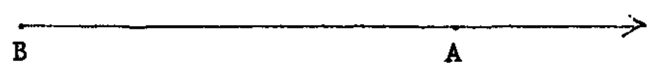
A line segment can be extended in either direction indefinitely.

Extend line segment $\overline{AB}$ in both directions. Label the new endpoints C and D. Is $\overline{AB}$ contained in $\overline{CD}$? (Yes) Extend the line once again in both directions. Label the new endpoints E and F. Is $\overline{AB}$ contained in $\overline{EF}$? (Yes) We could call the line $\overline{AB}$ or $\overline{AC}$ or by any of the points on the line. What other names can this line have?

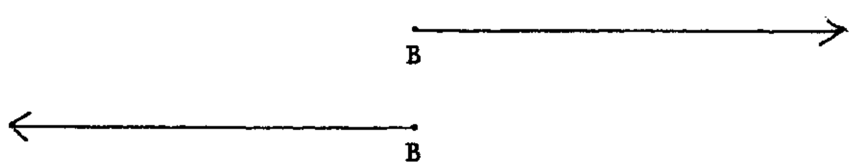
We could extend EF until there was no paper. If we think of the line as a piece of string, it could go on forever. A line has no endpoint. Arrows show this.

We can see that only one line can be drawn through points A and B. We call this line AB and write it $\overleftrightarrow{AB}$. This is always true of any two points.

Suppose that we have a line segment that we want to make longer--one that begins at an endpoint and continues. We show this



with an arrow. We call this a ray, and use the arrow symbol of $\overrightarrow{BA}$. A set of rays may be compared to the rays of a flashlight. The rays start with the flashlight and go on indefinitely. In the case of the sun, we may go on in either direction.



B is the endpoint for either direction.

REVIEWING

1. Here is a picture of $\overline{LR}$.

   Label points M, N, O, P, and Q on this line.

   Name all the line segments.

   Can you draw a complete picture of $\overleftrightarrow{LR}$? (No, it extends without end)

2. How many line segments are there which connect

   (a) two points (1)

   (b) three points (3)

   (c) four points (6)

   (d) five points (10)

   Make an illustration of each.

3. In how many ways could you name this ray?

   (only two: $\overrightarrow{DE}$ $\overrightarrow{DF}$)

4. Mark a point on your paper. Label it A. Draw five rays from this point.

5. Here are two points.

. S

T .

(a) How many lines can you draw which contain point T?

(More than can be counted.)

(b) How many lines can you draw that contain point S?

(More than can be counted.)

(c) How many lines can you draw that contain points T and S?

(One)

6. What is a line?

(A line segment continues without end.)

7. What is a line segment?

(Has two endpoints.)

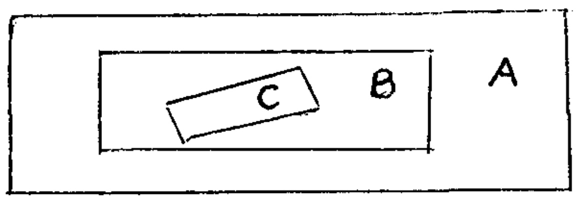
8. What is a ray?

(Has one endpoint.)

IV. PLANES AND LINES

The flat surfaces in your room help you think of planes. The wall is a plane. Your desk top is a plane. Name the other planes that you see around the room. (Floor, ceiling, chalkboard, etc.) How many points are there on the desk top? (More than can be counted.) These are all parts of a large plane. A part of a plane is a set of points in space. The cover of your book is part of a plane. It is a set of points in space.

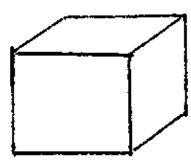
In this figure below, region A is the largest region, or part of a plane.



Region B is included as part of region A. Region C is part of region B and also part of region A. A whole plane contains larger and larger flat surfaces. A plane extends without end, just as a line extends without end.

By looking around the room you can see that if you count the planes above and below just your desks, there are more planes in space than can be counted. How many planes are there between your desk and the ceiling? (More than can be counted.)

How many lines can you draw in the following box? More than can be counted? Yes, any plane contains more lines than can be counted. Open your book and pretend that the binding of the book represents a line. What do the pages represent? (A part of a plane.) Each page passes



through the back or line of the book. You can see that there can be an infinite number of planes that could pass through this line.

Imagine a door swinging open and shut. How many times could it stop, or how many planes could there be? Could you ever count them? Through the two points of the door or the hinges there could pass more points than you could ever count. Can you think of other examples of this kind?

SOME THINGS TO THINK ABOUT

1. Name some objects which represent parts of a plane.

   (Any flat surface)

2. How can you have two points A and B and four planes passing through them?

3. Name as many instances as you can where two points contain many planes.

   (Cover of a box, revolving doors, etc.)

4. Two points in space, a line segment and a line are contained in

   (More planes than can be counted)

   ____ .

5. Suppose this page were part of plane E and I were to draw lines on this plane. How many lines could I draw?

   (More than could be counted)

6. Fold a piece of paper. What does the crease represent?

   (A line)

   What do the two parts show?

   (Parts of a plane which contain the line segment)

V. INTERSECTION OF LINES AND PLANES

When you are out riding in your car, have you ever come to an intersection? What is an intersection? Your dictionary will help you.

We have talked about a line being a set of points. Suppose two lines crossed. At how many points do these lines intersect? (One) Is this always true? (Yes) Draw $\overline{AF}$ and $\overline{MP}$ on your paper and make them intersect at point R. How many points are in the intersection of point R? (One) This is always true.

Could there be two lines that never intersect? (Yes, parallel) Just imagine the railroad tracks as lines. Do they ever intersect each other? Lines that never intersect are said to be parallel lines. Think of other times when two lines are parallel.

We know that if lines intersect they intersect at just one point. What about a line and a plane? Hold your pencil, a line, so that the point is on this page. This page represents a plane. Do a line and a plane intersect? At how many places? (One)

Now lay your pencil on the paper. Keep in mind that your pencil is still a line. Do a line and a plane intersect? At how many places? (More than you could count) The intersection of a line and a plane can be a line as well as a point.

Could a line and a plane intersect at just two points? No, it must be just one point or the whole line. If they are held parallel to each other they will not intersect at all. Look around the room and see if you can find places where lines or lines and planes intersect.

It is also possible for planes to intersect. Let us think about this. Could two planes ever intersect at just one point? Two points? No, two planes must intersect at many points. For instance, this book is lying on your desk. The intersection of this book and your desk is a line. Stand two books up together on your desk. Again the intersection of two planes is a line. What about the wall and the floor in your classroom? Look about for other examples.

If two planes intersect at no points (opposite walls) they are said to be parallel.

SOME THINGS TO THINK ABOUT

1. Make a list of as many examples as you can of

   (a) The intersection of two lines

   (b) The intersection of a line and a plane (One point and many points)

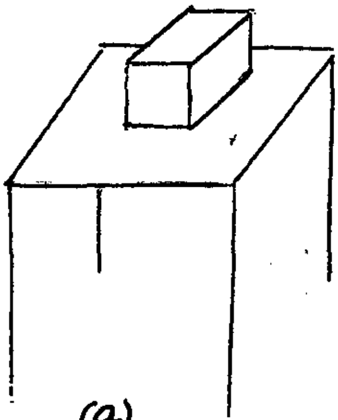
   (c) The intersection of two planes

2. Two plants are sitting on the table. In one case there is an intersection of two planes. In the other case there is the intersection of a plane and a line. Draw two diagrams to illustrate this.

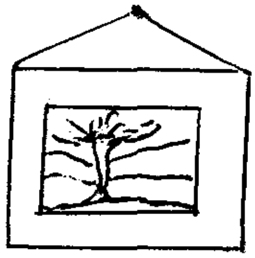
3. On a folded piece of paper what does the fold represent?

   (The fold is the intersection of two planes, being a line.)

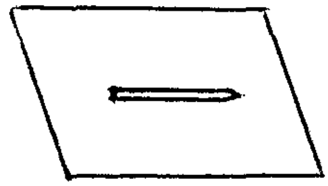
4. Tell about the intersection of each of these figures.



(a)



(b)



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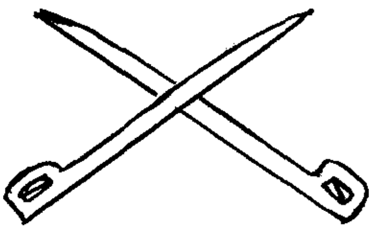
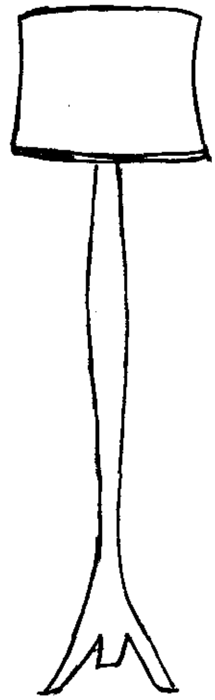
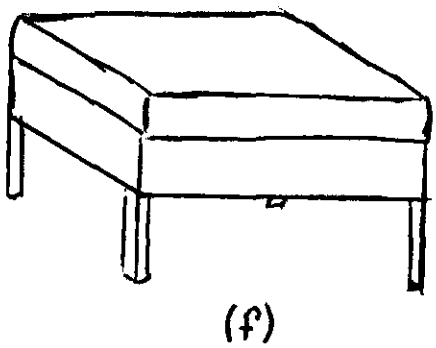


(d)



(e)



(f)

UNIT ONE

VOCABULARY

geometry

geometric

point

space

line

curve

plane

set

line segment

ray

endpoint

intersection

region

observation

parallel

union

UNIT ONE ACTIVITIES

1. How long ago was geometry invented? Perhaps you could make a report to the class on the History of Geometry.

2. Who wrote the first geometry book and what was it like?

3. Make a chart showing various kinds of intersections.

4. Construct devices to show the intersection of lines and planes.

5. Using a box, show intersection of lines and planes.

6. Make a mobile showing the intersection of two planes.

UNIT TWO

UNDERSTANDINGS

1. In tracing the path of a simple closed curve, one eventually gets back to the starting point.
2. The path of a simple closed curve never intersects itself.
3. All of the points of a closed curve lie in a plane.
4. A polygon is a union of line segments.
5. A circle is a closed curve with all its points the same distance from the center, and all its radii equal in length.
6. An angle is the union of two rays with a common endpoint and not on a line.
7. The endpoint of an angle is called a vertex.
8. Each ray is the side of an angle.
9. The standard instrument for measuring angles is a protractor, which is read in degrees.
10. The measurement of a right angle is ninety degrees.
11. An angle less than ninety degrees is an acute angle.
12. An angle more than ninety degrees and less than one hundred and eighty degrees is an obtuse angle.
13. A line segment may be bisected by finding its perpendicular.

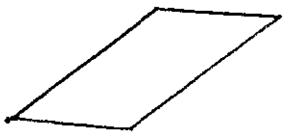
VI. SIMPLE CLOSED CURVES OR POLYGONS

In a previous lesson we called this set of points a curve.

A  B or C D is also a curve.

Now if we continue curve AB it is possible to return to the starting point and have a simple closed curve. Have you ever used stencils or patterns? You drew simple closed curves.

These are all simple closed curves.

T  (a)

C  (b)

but <u>not</u>

D 

(c)

In picture (c) we have lines intersecting. This is not a simple closed curve.

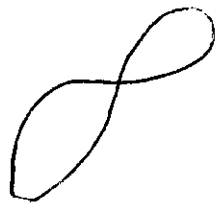
Picture (a) is a polygon. It is the joining of line segments together. Since it does not intersect, however, it is a simple closed curve. What polygon is made with three line segments? Four line segments? Draw them on the board. Can you give them their more common names? Perhaps you know the names for figures made with five, six, and eight line segments. We shall talk more about these later.

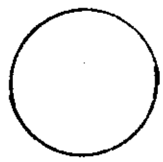
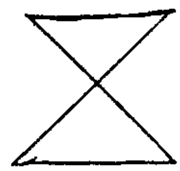
There is a family name for the four-sided polygon. It is called a quadrilateral. It may be in any shape you desire as long as it has just four sides which form a simple closed curve. Look in the dictionary for the exact meaning of quadrilateral.

Figure (b) is probably the way you always thought about a closed curve.

SOME THINGS TO THINK ABOUT

1. Which of these are simple closed curves?

(a)  (b)  (c) 

(d)  (e)  (f) 

(g)  (h)  (a, c, e, g)

2. Make a list of line segments in your classroom which form polygons.

   (a) Three-line segments or triangles

   (b) Four-line segments or quadrilaterals

3. Draw and name the polygons with five, six, seven, and eight line segments.

4. In each of these polygons at how many points do the sides intersect?

   (a) Triangle

   (b) Quadrilateral

   (c) Pentagon

5. Locate these points on your paper. Connect these points and name the line segments.

. G

F .

. H

E .

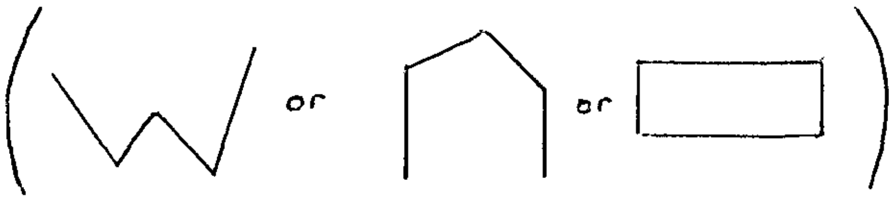
By intersecting two lines draw and name two more line segments.

6. Draw a set of points which is the union of four line segments.

Draw a closed curve which is the union of four line segments.

Are these two drawings different? (They could be.)

Are these two drawings the same? (They could be.)



7.

B
.

A . . C

E . . D

(a) Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, $\overline{AD}$, $\overline{CE}$, $\overline{AC}$.

(b) Locate point F at the intersection of $\overline{AD}$ and $\overline{EC}$.

(c) What polygon is represented by the figure EFD? (Triangle)

(d) What is the name of figure ACED? (Quadrilateral)

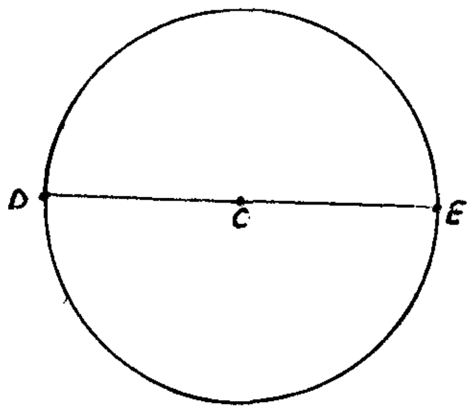
(e) What do you call the figure ABCDE? (Pentagon)

VII. CIRCLES

Another kind of simple closed curve is a circle. To draw a picture of a circle it is convenient to use a compass.

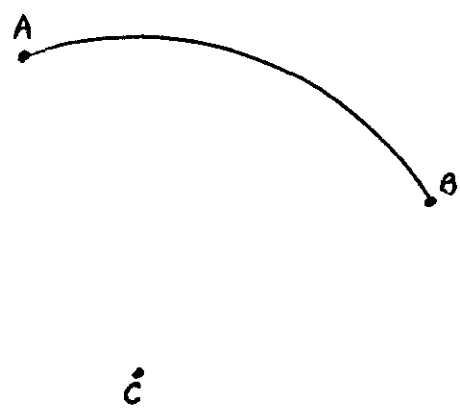
Hold the compass at the top between your thumb and index finger. Do not move the compass once you start. Lean slightly in the direction in which the pencil is moving. Make your lines light. Change the distance between the steel point and the pencil point and try it again. Remember that the steel point is very sharp. Perhaps it would be better to put a cardboard under your paper when using a compass.

Choose one of your circles and mark the center C. Take your ruler and measure from C to any point on the circle. Call this point P. Make points Q, R, and S the same way. Draw $\overline{CP}$, $\overline{CQ}$, $\overline{CR}$, $\overline{CS}$. A circle always has all its points the same distance from the center. Each of the lines drawn is a radius. Together they are called radii. How many radii are there for any circle? Yes, more than could be counted. When a compass is used to draw a circle, what is the relationship between the compass points and the circle? (The compass gives you true radii, and makes sure the points are the same distance from the center of the circle.)



Line segment $\overline{DE}$, which passes through the center point C of the circle, has a special name. It is called the diameter. However, since $\overline{CE}$ is a radius and $\overline{CD}$ is also a radius, and all radii are equal, what can you tell about the size of all diameters? They are always twice the radius and they are all equal.

Sometimes we want to draw a part of a circle. We call this an arc. Since an arc is part of a circle, what can you tell about every point on the arc? It is an equal distance from the center.

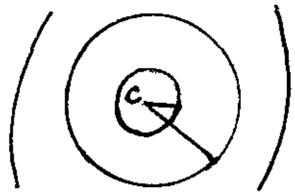


REVIEW

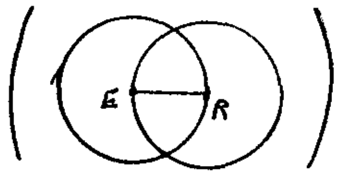
1. Draw a circle with point C as a center.

   Now draw another larger circle, still using point C as a center.

   Draw a radius for each circle.



2. Set your compass for two inches. Mark center E and draw a circle. Mark point R on the circle and draw $\overline{ER}$. Now make a second circle using R as a center and the same radius. What can you tell about these circles? (Have the same radii and intersect at two places.)



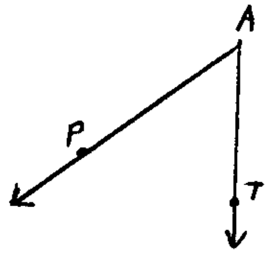
3. With a radius of one and a half inches, draw a circle whose center is M. Now draw its diameter. How large is the diameter? If you were asked to draw as many diameters as you could, would this be possible? (No, you could draw more than you could count.)

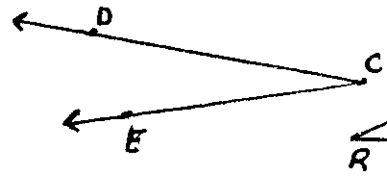
4. Draw $\overleftrightarrow{LM}$. On $\overleftrightarrow{LM}$ place your compass point and call it A. Make an arc which intersects $\overleftrightarrow{LM}$. Call this intersection point B. Label your arc.

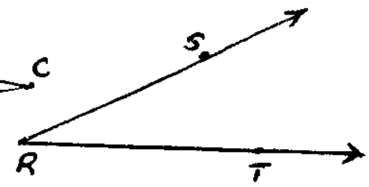
5. Draw line P. With your compass draw $\overline{AB}$ and $\overline{AD}$ on P. Make both of these line segments equal. Now continue to draw the arcs which intersect L and D and B using A as a center. Now make several statements which you have discovered about the figure.

(Some of these answers should be given: The length of $\overline{AB} = \overline{AD}$, A circle has been formed, $\overline{DB}$ is the diameter, $\overline{AB}$ is a radius, $\overline{AD}$ is a radius.)

VIII. ANGLES--PART A





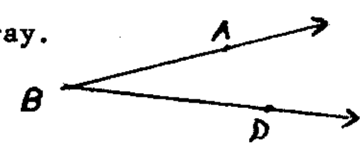


These new geometric figures are called angles. They are made up of the union of two rays which have the same endpoint and which do not lie on the same line. This endpoint is called the vertex of the angle.

Open the cover of your book. Hold it in many positions to make angles. The cover and the page form an angle. The point where these two lines join form a vertex. Look at the cover of your book. It has four angles. Identify the rays and vertex of each. They are all alike.

Locate more angles around the room.

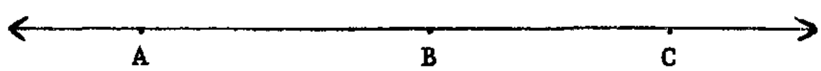
In order to read an angle in geometry we mark a point on each ray.



This is read $\angle$ ABD. Its rays are $\overrightarrow{BA}$ and $\overrightarrow{BD}$. Its vertex is B.

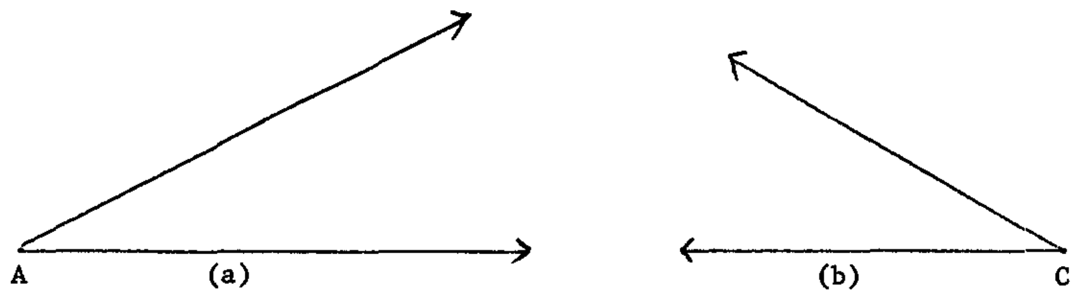
How would you read and write the three angles at the beginning of this lesson? Also give the rays and vertexes for each angle.

Since $\overleftrightarrow{AC}$ and $\overleftrightarrow{BA}$ fall on the same line $\overleftrightarrow{AC}$, they do not form an angle.



MEASURING ANGLES--PART B

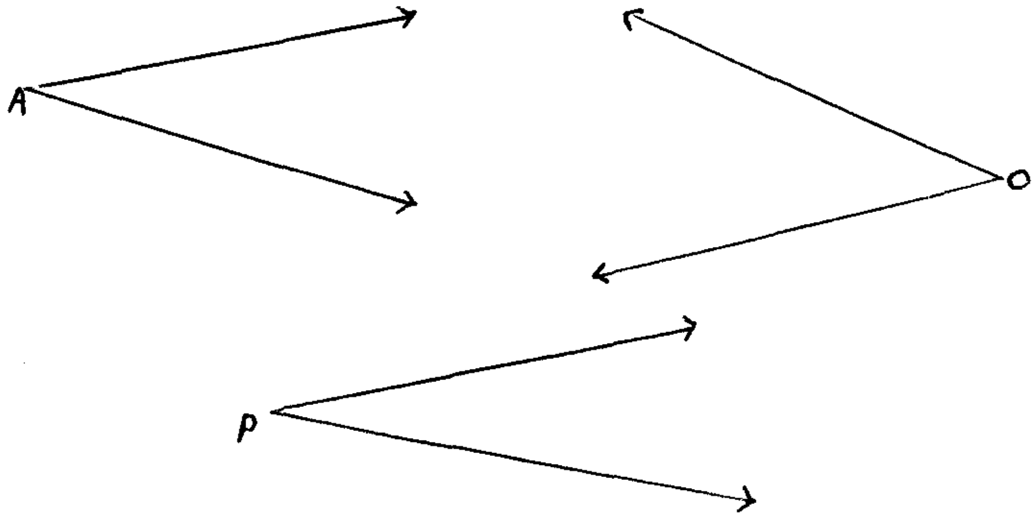
To measure an angle we need a protractor. Your teacher will give each of you a protractor to work with. Notice that the center of the protractor is cut out. This allows you to see the ray through the space. To measure an angle put the arrow of the protractor on the vertex of your angle, and the bottom of your protractor on one of the rays. Try this on (a).



The number that you see on the circular part of your protractor tells you the size of your angle. Look carefully at your protractor and you will see small numbers at either end. This allows you to measure any angle from two directions. Try this on (b). Angles are measured in degrees designated by $^{\circ}$ after the number, as 45°. Notice that only every tenth degree is numbered on your protractor. What is the largest number? What is the smallest number? Are the largest and smallest numbers printed on your protractor? Why?

It is very convenient to have two sets of numbers on one instrument. Examine the yardstick in your classroom. It will have two sets of numbers, also. Why is this useful?

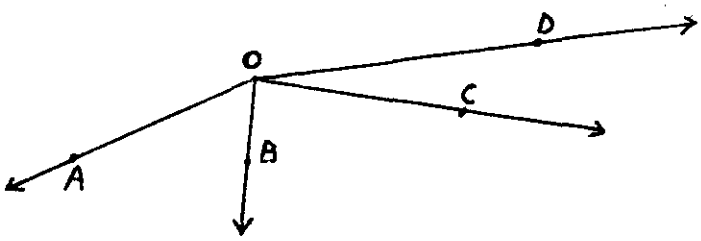
Here are some angles on which you may practice.



SOME THINGS TO THINK ABOUT

1. Draw ∠ COB. Will everyone's angle look just alike? (No) Why?

2. Name the six angles and the rays for each angle.

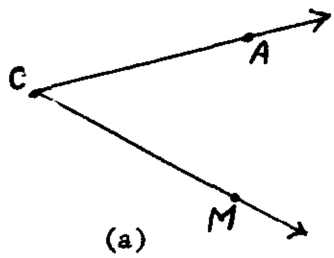


(As ∠ AOB etc.)

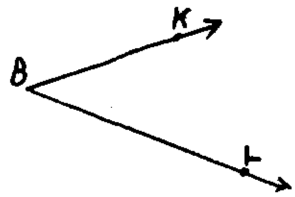
$\overrightarrow{OA}$

$\overrightarrow{OB}$

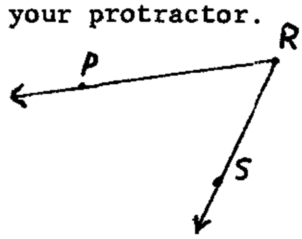
3. Give the measurements of these angles, using your protractor.



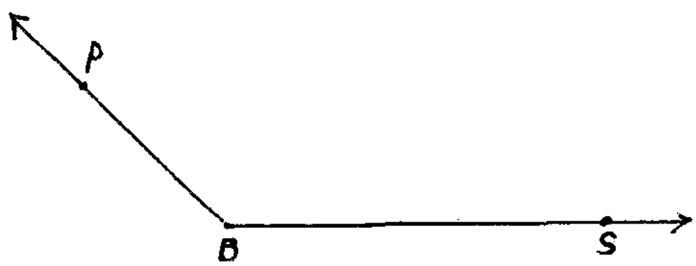
(a)



(b)



(c)

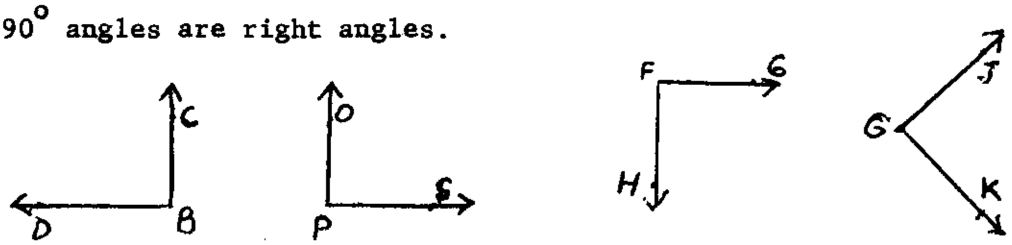


(d)

4. Would it make any significant difference to have an error of one degree in your measurement of an angle? Explain your answer.

IX. RIGHT ANGLES

We have mentioned that the corner of your book forms an angle. There are actually four angles on the cover of your book that are the same size. Take your protractor and measure them. All four angles are ________ degrees. How about the page in this book? Try the corners of a piece of paper. All these angles are alike. They all measure 90 degrees. Even your desk has 90-degree angles at the corners. All 90° angles are right angles.



(a) (b) (c) (d)

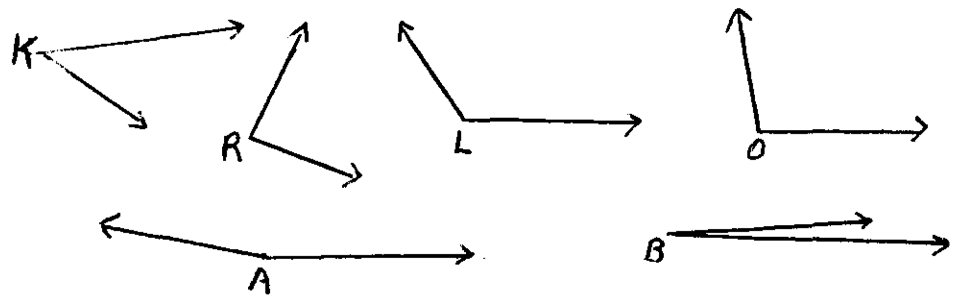
When one line segment or ray intersects another at a 90-degree angle, we call that line <u>perpendicular</u> to the other. $\overrightarrow{BC}$ is perpendicular to $\overrightarrow{BD}$. Also $\overrightarrow{PO}$ is perpendicular to $\overrightarrow{PS}$. Name the other perpendiculars in figures (c) and (d).

Hold your pencil up straight with the point on your desk. What can you say about the relationship of the pencil to the desk? Hold the yardstick on the floor the same way. Explain the relationship. What do you have to be careful about?

If we do not have a protractor and wish to make a right angle, we may do so with a compass. Draw $\overline{AB}$ on your paper one inch long. Now put your compass point on B and with $\overline{AB}$ as a radius draw a circle. Next, using A as a center of your circle and $\overline{AB}$ still as a radius, draw another circle. Label the points of intersection C and D. Now, draw $\overline{CD}$. If you have worked carefully, you have drawn $\overline{CD}$ perpendicular to $\overline{AB}$. Prove this with your protractor. You should have four right angles, all intersecting at O. $\overline{CD}$ has also divided $\overline{AB}$ in half at O. This is called bisecting a line segment. Will this happen every time? Since $\overline{CD}$ is perpendicular to $\overline{AB}$, as well as bisecting it, $\overline{CD}$ is called a <u>perpendicular</u> <u>bisector</u>.

REVIEW

1. Estimate the measurement of these angles and then check your answers with a protractor. Use 0° - 45° - 90° - 135° - 180° in your estimates.



2. List the right angles you can see around the room.

3. There is a shorter way to draw a perpendicular or bisect a given segment using only arcs. See if you can do it.

4. Draw $\overline{DE}$ six inches long. Place point F in the center of this segment. Construct ∠ EFB, ∠ BFA, ∠ AFC, and ∠ CFD so that their sum equals 180 degrees. Should everyone's drawing look alike?

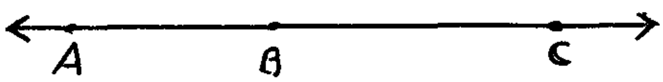
5. Using the drawing you have just made, list three ways in which two of these angles equal 180 degrees.

∠ DFC and ∠ CFE

∠ DFA and ∠ AFE

∠ DFB and ∠ BFE

6. Construct a line segment perpendicular to AC at point B.

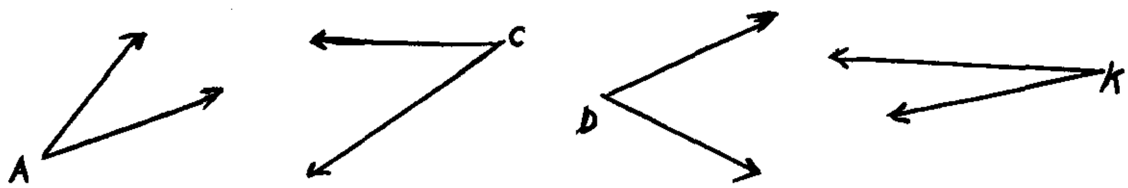


Can you make a statement about drawing a perpendicular to a line any place on that line?

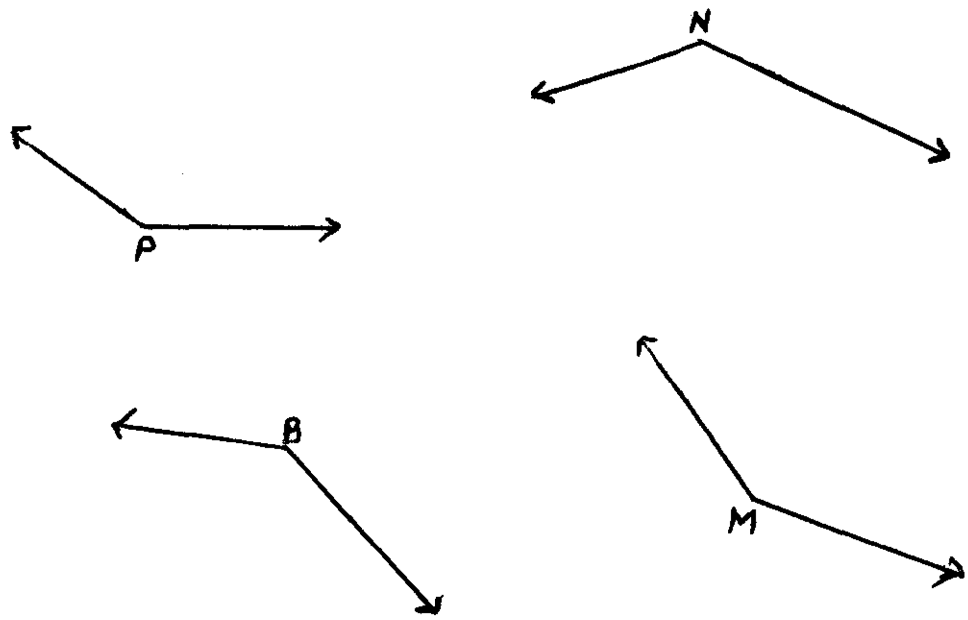
7. Using a compass, divide any segment into four equal parts.

X. OTHER KINDS OF ANGLES

We have been discussing right angles. There are names for other kinds of angles, also. Any angle less than ninety degrees is called an acute angle. If you look up the meaning of acute in your dictionary you will see why these are acute angles.



Another kind of angle is an obtuse angle. Again your dictionary will help you to better understand the meaning. If an angle is more than 90 degrees, but less than 180 degrees, it is called an obtuse angle.



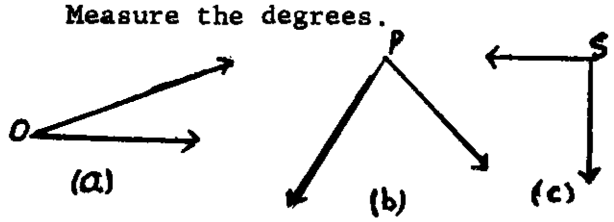
SOME THINGS TO THINK ABOUT

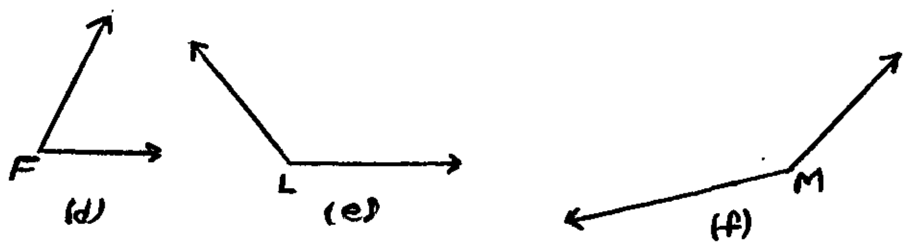
1. Name the angles.

   Estimate the degrees.

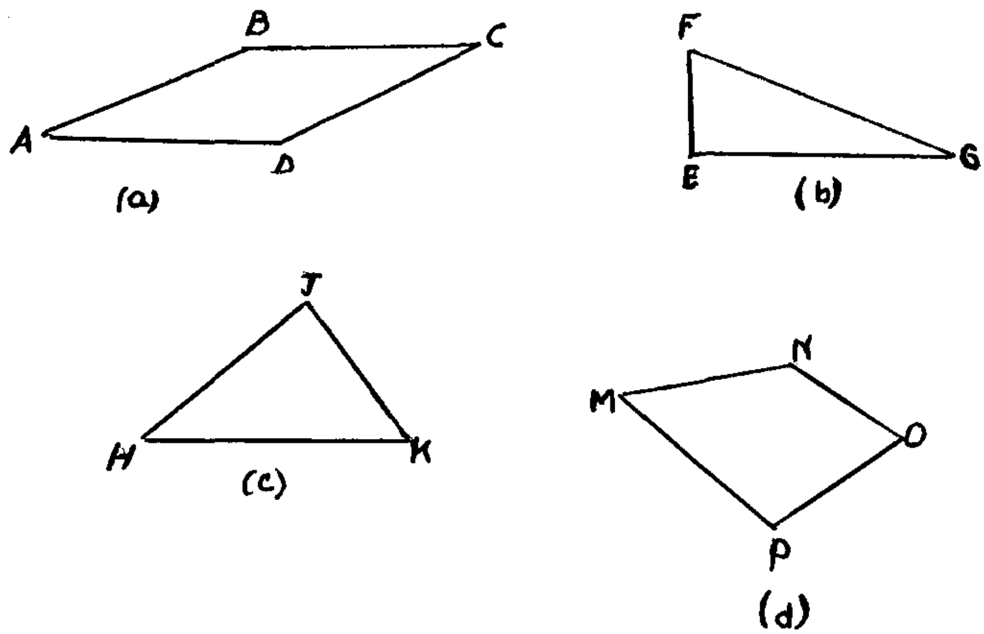
   Measure the degrees.

	Name	Estimate	Measure
(a)	_____	_______	_______
(b)	_____	_______	_______
(c)	_____	_______	_______
(d)	_____	_______	_______
(e)	_____	_______	_______
(f)	_____	_______	_______



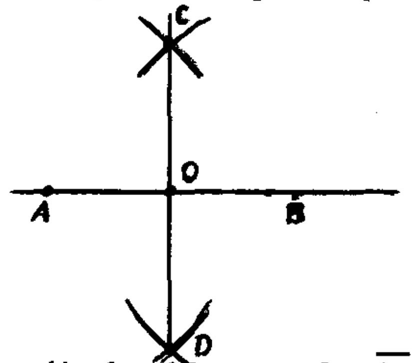


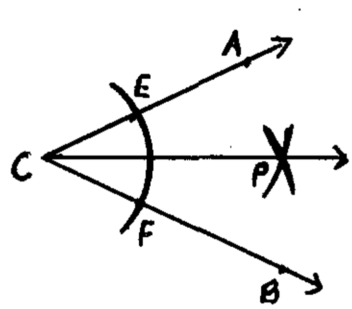
2. Name the angles of these figures.



XI. BISECTING AN ANGLE

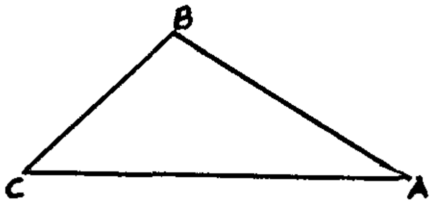
You have already learned how to bisect a line segment using a compass. To review let's bisect $\overline{AB}$. Using A as a center and AB as a radius, draw arcs over and under $\overline{AB}$. Now, using B as a center, draw the intersecting arcs. Label the intersecting points C and D, and draw $\overline{CD}$. You have just found the perpendicular bisector for $\overline{AB}$ at O.



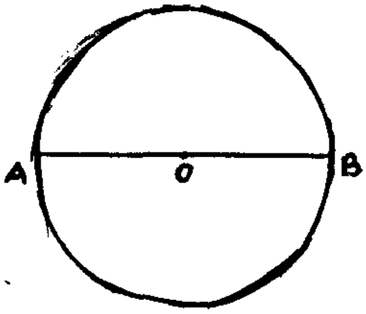


Sometimes it is necessary for us to bisect an angle with a compass. Let us see how to do this. Using C as a center of a circle and any convenient radius, draw an arc intersecting $\overrightarrow{CA}$ and $\overrightarrow{CB}$ at points E and F. Now, using point F as a center and any convenient radius, draw an arc within the angle. Then use point E as a center and the same radius, and intersect the arc you have just drawn at P. Draw $\overrightarrow{CP}$. This ray bisects $\angle$ ACB. You can prove this with your protractor.

REVIEW

1. 

Bisect C, B, and A. Draw all bisectors long enough so that they intersect.

2. 

Using bisectors, divide this circle into fourths and then eighths.

3. Draw a circle with a radius of one and a half inches. Mark a point on the circle and call it B. Using the same radius and starting at B, intersect the circle and continue doing this until you divide the circle into six equal pieces. Now draw six bisectors so that the circle will be divided into twelve pieces.

To think about:

Using any radius, can you always divide a circle into six equal parts?

4. How many ways can you prove that two angles are equal?

UNIT TWO

VOCABULARY

angles

closed curve

polygon

quadrilateral

diameter

circle

arc

radius

radii

vertex

vertexes

compass

right angle

obtuse

acute

perpendicular

bisect

bisector

UNIT TWO ACTIVITIES

1. Draw and color a design using just the compass.
2. Look into the history of the measurement of angles. Report to the class.
3. Using one large circle and dividing it into six equal parts, see how many interesting designs you can create.
4. Make a report on surveyors' instruments.
5. How are degrees and angles used in aviation?
6. Do any businesses use degrees and angles? How?
7. There is a way to draw a circle with straight lines. See if you can find out how to do this.
8. An archaeologist is able to tell from a fragment of a plate containing an arc the exact size the plate had been. Can you figure out how he does this?

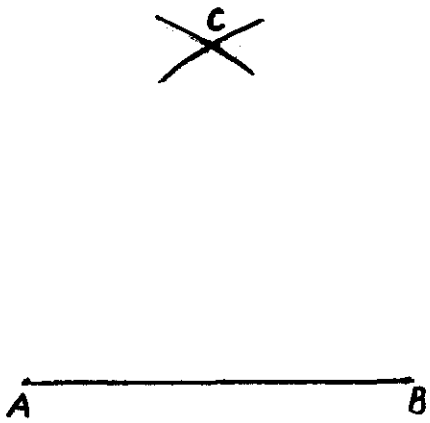
UNIT III

UNDERSTANDINGS

1. An equilateral triangle has congruent sides and congruent angles.
2. When triangles are exactly alike, they are called congruent.
3. When sides or lines are alike, they are called congruent.
4. When angles are alike, they are called congruent.
5. The sum of the measure of the three angles of any triangle is one hundred eighty degrees.
6. An isosceles triangle has two congruent sides and two congruent angles.
7. A scalene triangle has no congruent sides and no congruent angles.
8. A square is a figure with all sides congruent and all angles right angles.
9. A rectangle has opposite sides congruent and parallel and all angles right angles.
10. The surfaces of geometric solids are simple closed curves.
11. The intersection of the faces of any solid form an edge.
12. The endpoint of an edge is called a vertex.

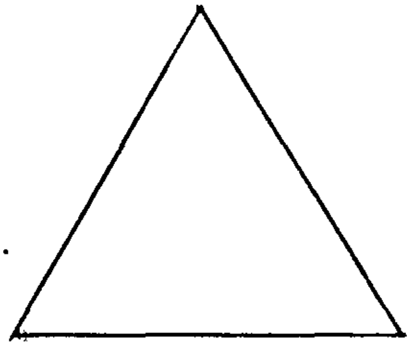
XII. EQUILATERAL TRIANGLES

We know that a triangle is a union of three line segments. Below, $\overline{AB}$ is two inches long. With A as a center and a radius of $\overline{AB}$ or two inches, make an arc above $\overline{AB}$. Again with B as a center and the same radius, intersect the arc you have just drawn at C. Now draw $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$. You now have a triangle with congruent sides called an equilateral triangle. Take your protractor and measure each angle.



Are the angles congruent? They should be. An equilateral triangle not only has congruent sides, but congruent angles as well. How much did each angle measure? What was the sum of the three angles? Take your compass and make a triangle the same way using a different measure. Measure the angles. Do you still get 60 degrees for each angle? Is the sum of the three angles of an equilateral triangle equal to 180 degrees? Does the size of the triangle make a difference in the measurement of the angles? (No)

Use a piece of tracing paper and copy this triangle from the book. Cut it out and place it on top of this one. Would you say that these triangles were exactly alike? Are they the same size and shape? When two triangles are the same size and shape we say that they are congruent. Are the sides the same length? Then we say that the sides are congruent. Are the angles the same size? We say that the angles are congruent. In this case, since you have traced it, you know that these two triangles are congruent.



The leather triangles on the desk blotter would be congruent. How would you know this? Draw equilateral triangles in the upper corners of the blackboard. Make sure that they are congruent. How can you do this? Remember, they are always congruent if they are exactly alike.

Draw three congruent equilateral triangles on this page. Label the vertices of the first triangle ABC, the second triangle DEF, and the third triangle GHI. Here is an interesting sentence you may write after you have drawn your three congruent triangles.

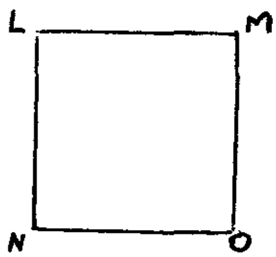
$$\triangle ABC \cong \triangle DEF \cong \triangle GHI$$

Can you guess what $\cong$ means in geometry?

SOME THINGS TO THINK ABOUT

1. Name things that you have in the classroom or at home of which you could make equilateral triangles. It might be fun to try this at home.

2. Draw equilateral triangles using $\overline{LM}$, $\overline{MO}$, $\overline{ON}$, and $\overline{NL}$ as bases.



3. Make an equilateral triangle in the center of an equilateral triangle.

4. Draw three equilateral triangles with bases of two, three, and four inches.

   Are these triangles congruent?

   Are the sides congruent?

   Are the angles congruent?

XIII. ISOSCELES TRIANGLES

Another kind of triangle has two sides congruent and two angles equal or congruent. This is called an isosceles triangle. Draw $\overline{AB}$ one and a half inches long. Using your compass construct the other two sides two inches long, intersecting at N.

Is $\overline{AN}$ congruent to $\overline{BN}$?

Is $\angle$ BAN congruent to $\angle$ ABN?

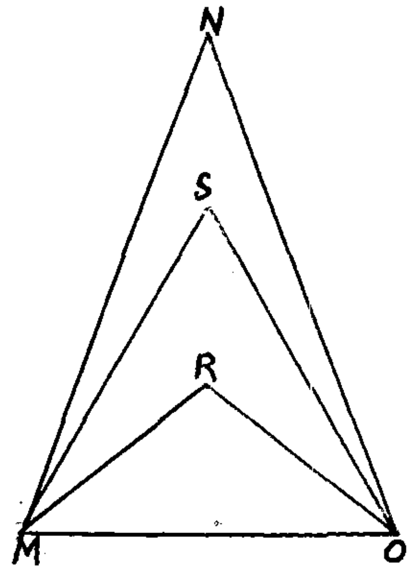
If this is true, you have drawn an isosceles triangle.

Use your protractor and measure the angles of your triangle. Add their measures together. Does their sum equal 180 degrees? Now make an isosceles triangle with a base of two inches and sides one and a half inch. Are two sides equal? Are two angles equal? Does the sum of the angles equal 180 degrees?

Would you say that the sum of the measures of the angles of an isosceles triangle always equals 180 degrees? Yes, this is always true. There was another triangle of which this was also true. Which one was that? Perhaps it is true of all triangles.

SOME THINGS TO THINK ABOUT

1. Look around you at school and at home and find examples of triangles you have studied.



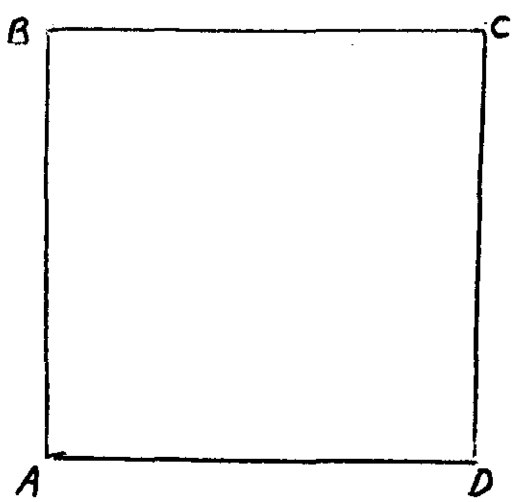
2. How many triangles are represented?

   What kind of triangles are they?

   How do you know?

   Name the opposite sides and angles which are congruent.

   Are ∠ OMN, ∠ OMS, and ∠ OMR congruent? How can you tell?

3. 

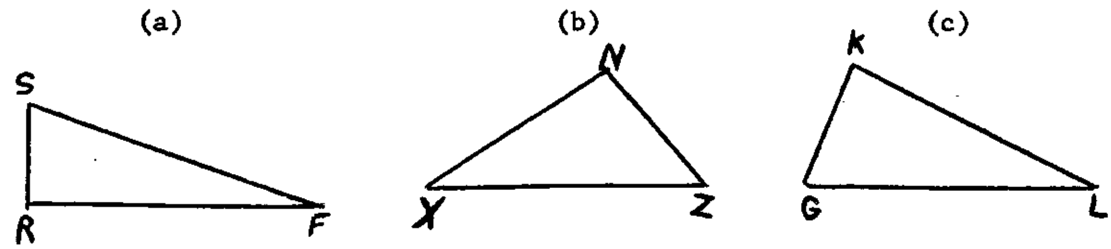
   Bisect C and A. Extend the lines until they intersect at O.

   How many triangles are formed?

   Measure all angles and sides. What kind of triangles did you draw? What is the sum of the angles of each?

XIV. SCALENE TRIANGLES

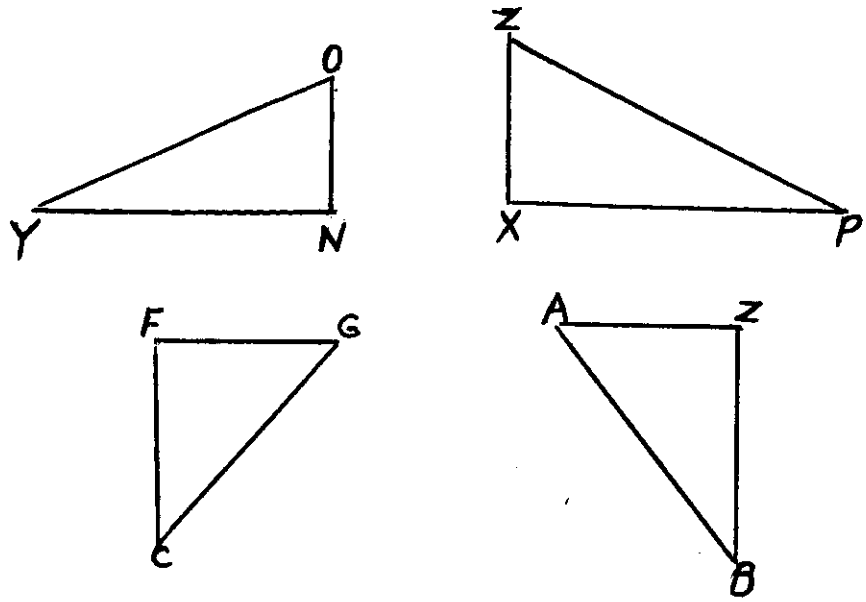
Triangles that have no sides congruent are called scalene triangles.



Use a protractor to prove that no angles are congruent either. What is the sum of the measures of the angles of each triangle? Even though all angles have a different measure, does their sum equal 180 degrees? What would you say about the largest and the longest side? Yes, they are opposite. This is always true.

To read these triangles and all triangles use the sign and the letters. Those above would be read △ SRF, △ XZY, △ GKL.

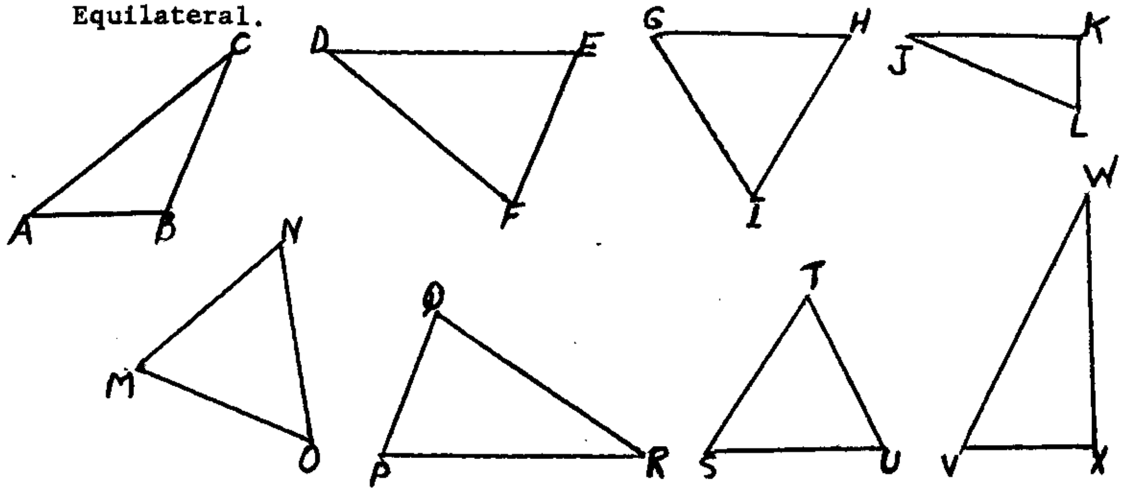
When we talk about the bottom of the triangle, we always use the word <u>base</u>. The bases of the triangles above are RF, XZ, and GL.



Study the figures above. They are all scalene triangles, yes. But they are something else, also. They have one special thing in common. One special thing is alike in all of them. For this reason they have a special name. See if you can tell what it is.

REVIEW

1. Divide these triangles into three sets--Scalene, Isosceles, and Equilateral.



2. When might it be helpful to know that the sum of the measures of all angles of any triangle equals 180 degrees?

3. With your compass draw a scalene triangle whose base is three inches, and whose sides are two and a half, and one and a half.

4. Follow directions to make $\triangle$ UVW.

   Draw base $\overline{UV}$ two and one quarter inches long.

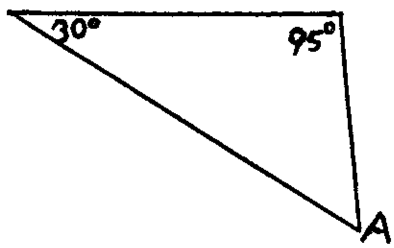
   At U make the $\angle$ 30°.

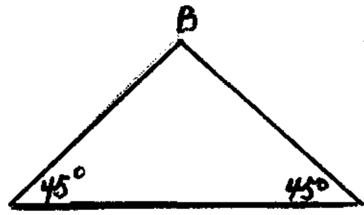
   At V make an $\angle$ 70°.

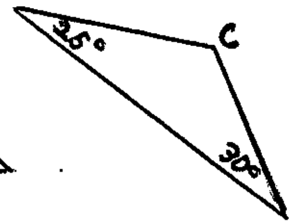
   How long is $\overline{UW}$, $\overline{VW}$?

   How many degrees is $\angle$ W?

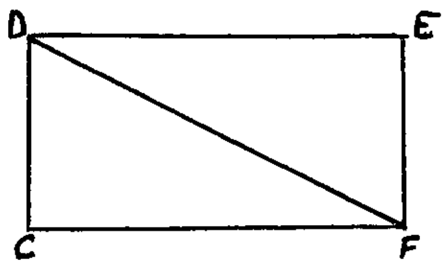
5. What is the measurement of the missing angle? How can you prove you are correct?







6.

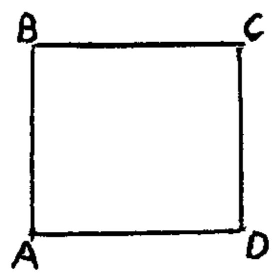


What is the sum of the measure of the angles of the △ DEF?

What is the sum of the measure of the angles of the △ DFC?

What is the sum of the measure of the angles of the rectangle DEFG?

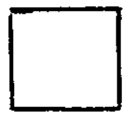
7. Give the sum of the measure of the angles of the square ABCD.



8. Try drawing different four-sided figures. These are called quadrilaterals, by the way. Find the sum of the measures of the angles of each quadrilateral. You may come to an interesting conclusion.

XV. GEOMETRIC FIGURES IN A PLANE

We have been studying triangles and circles. You have heard about a square whose sides are congruent. What about its angles? Are they congruent? Yes, they are always congruent.



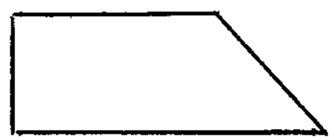
A figure with two sides longer than the other two and four right angles is called a rectangle.

Its opposite sides are congruent and parallel. Here is another figure. This one may be new to you. It is called a parallelogram.

Can you tell why? What about its angles? Would you say that opposite angles were congruent?

Here are some more:

Trapezoid



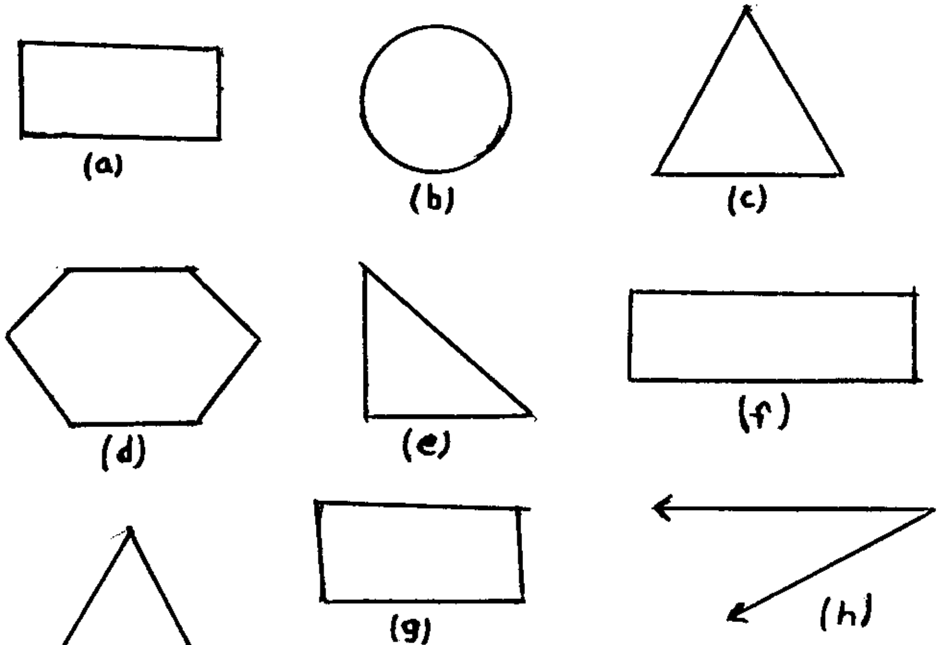
Rhombus

All these four-sided figures are called quadrilaterals. Geometric figures are named according to the number of sides. We have a pentagon with __________ sides. There is a hexagon with __________ sides. The octogon has __________ sides. Are there more?

What is the sum of the measures of the angles of each of these quadrilateral figures? Did you add them together? Can you figure out a quicker method?

REVIEW

1. Which pairs are congruent?



(a,g)

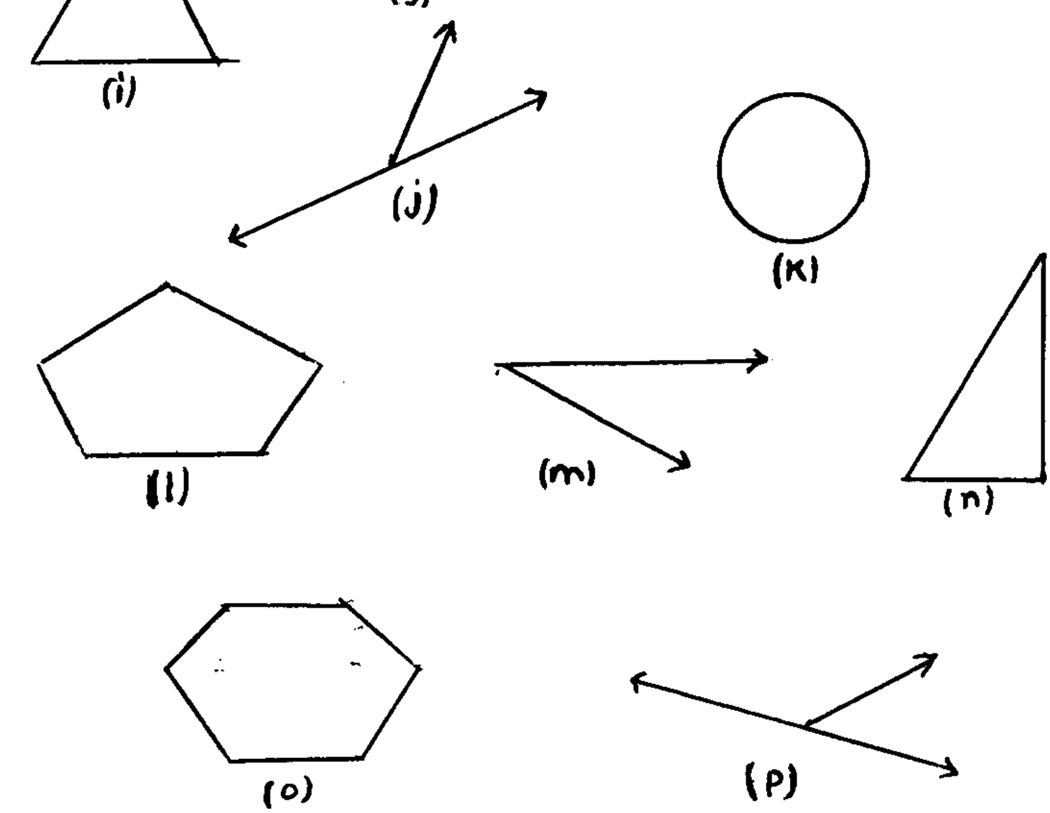
(c,i)

(j,p)

(h,m)

(d,o)

(b,k)



2. Match the definition with the symbol.

______ $\triangle$ (e) a. ray

______ $\cong$ (h) b. line

______ $\overrightarrow{CD}$ (a) c. segment

______ $\angle$ (d) d. angle

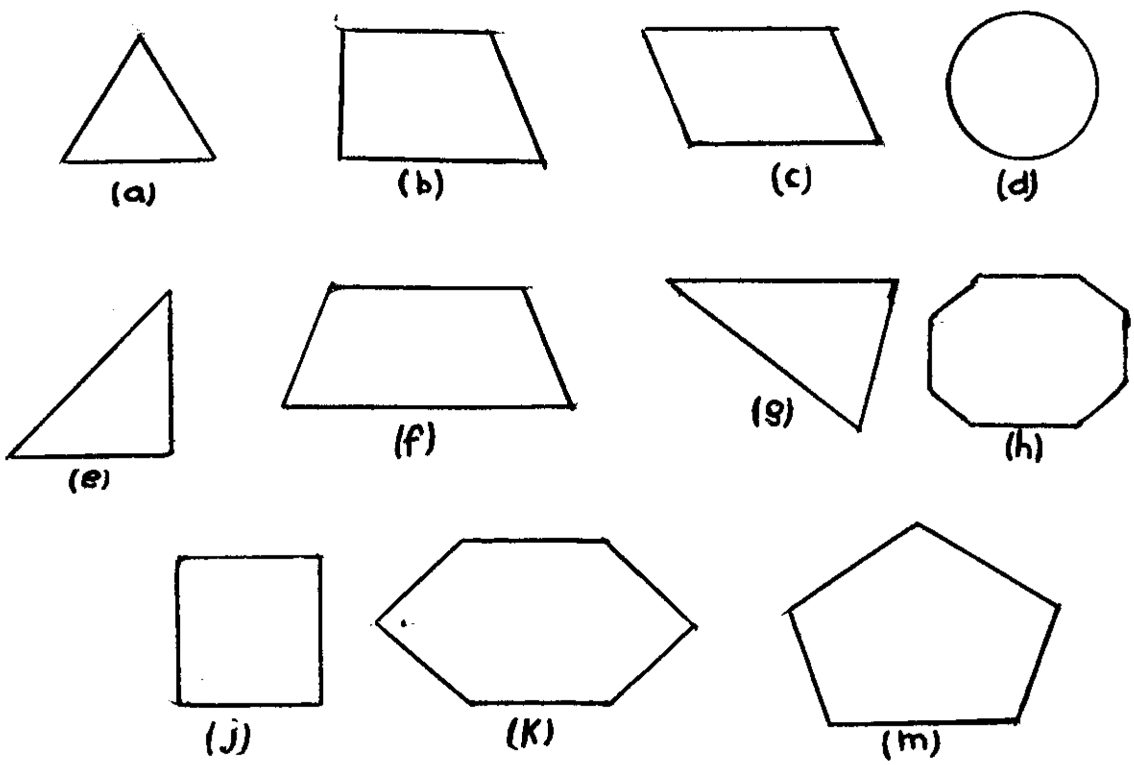
______ $\overline{AB}$ (c) e. triangle

______ $a > b$ (b) f. a is greater than b

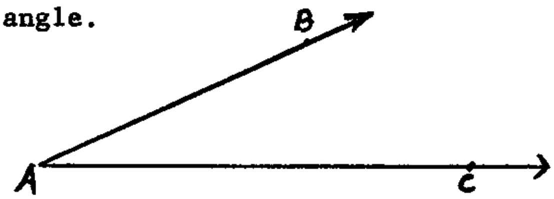
______ $\overleftrightarrow{EF}$ (h) g. a is less than b

______ $a < b$ (g) h. congruent

3. Name these geometric figures.

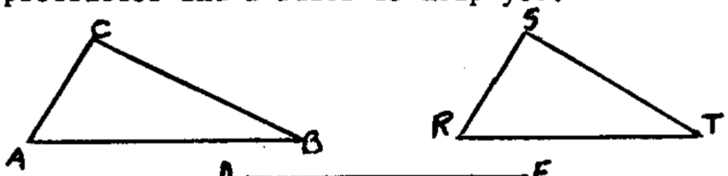


4. Bisect this angle.

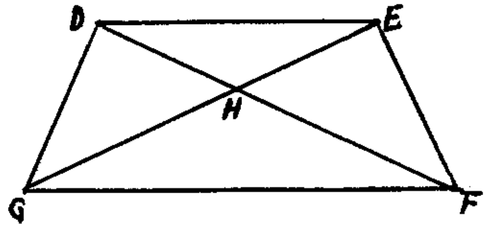


5. Tell which triangles, angles, and sides are congruent. You may use a protractor and a ruler to help you.

(a)



(b)



6. Compare the two angles and tell which angle is greater or smaller.

(a)

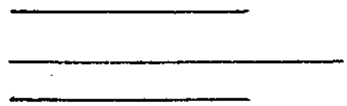
(b)

(c)

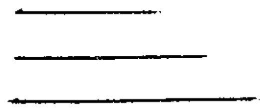
(d)

7. Construct a triangle using the line segments given. Name the triangles.

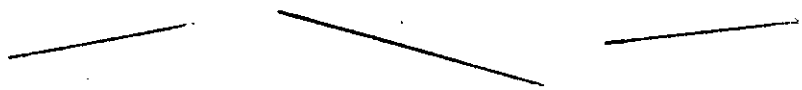
(a)



(b)



(c)

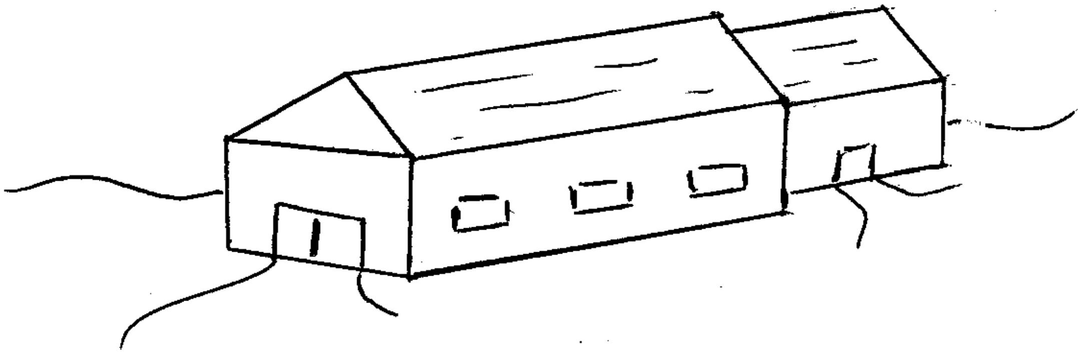


(d)

8. Construct a triangle whose base is two and one half inches and whose sides are one and a half by two inches.

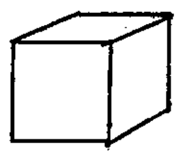
9. Draw the perpendicular bisector for the line segment $\overline{AB}$ at E.

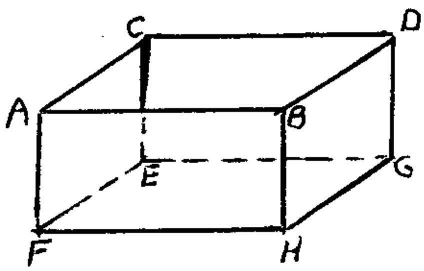
XVI. GEOMETRIC SOLIDS--PART A



We have been talking about geometric figures which are regions on a plane. There is another set of geometric figures which have interiors. We call these solid regions. The outside plane regions of these solids are called faces. The intersection of two faces of a solid is called an edge. The endpoint of an edge is called a vertex. Two or more endpoints are called vertices.

One rectangular solid is called a cube. On a cube all the edges, faces, and angles are congruent.

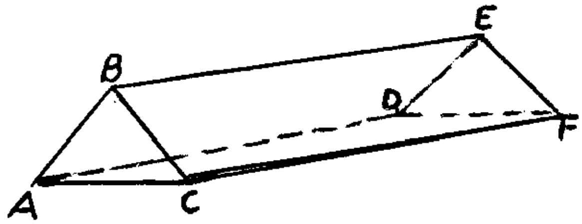




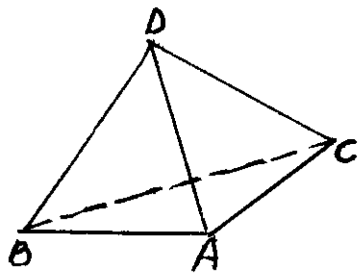
What plane geometric figures are formed by the faces of this figure on the left? (Four rectangular regions and two square regions.)

This is a rectangular prism. How many edges? How many vertices?

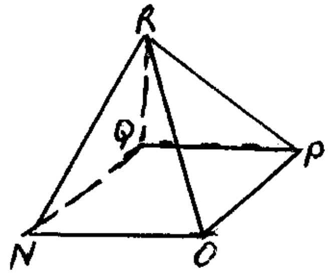
Here is a triangular prism. Name the plane geometric figures. How many edges? How many vertices?



Here are two kinds of pyramids.



Triangular Pyramid
(Tetrahedron)

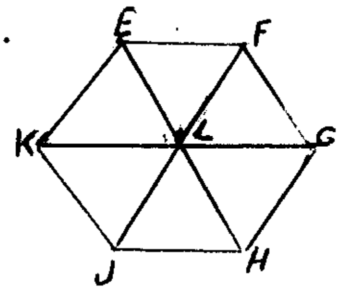


Square Pyramid

Which of these figures is like the pyramids of Egypt? How do you know? Can you name the faces? What about the base? Edges? Vertices?

Look around the classroom and see how many kinds of geometric solids there are. The picture at the beginning of the lesson has many geometric figures, both plane and solid. Can you name them?

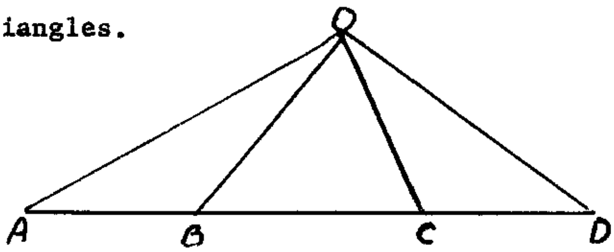
SOME THINGS TO THINK ABOUT

1. 

Name the different geometric figures you see in the hexagon at the left.

(1 hexagon
6 parallelograms
6 trapezoids
6 triangles)

2. Name the triangles.



3. Make a chart for yourself like this.

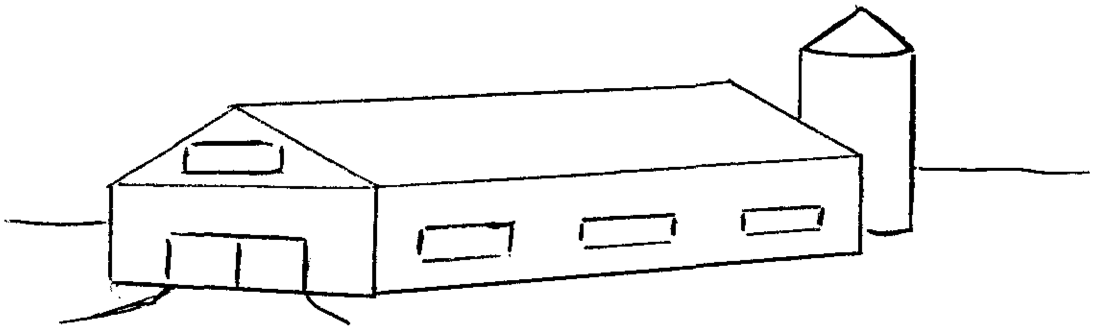
CUBES	RECTANGULAR SOLIDS	PRISM (2)	PYRAMIDS (2)

As you see examples of these figures around you, list them. Keep this chart for several days. Then compare it with your classmates.

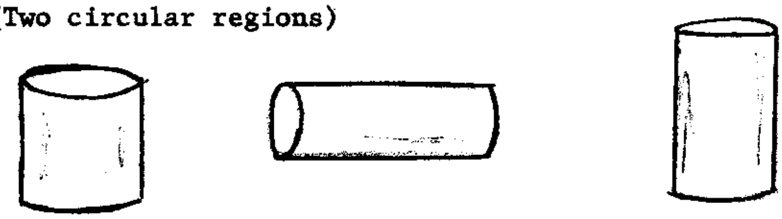
4. Draw a pentagonal pyramid.

5. Draw a hexagonal prism.

XVII. GEOMETRIC SOLIDS--PART B



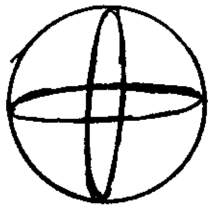
A trip to any supermarket will find you surrounded with more geometric solids than you would care to count. Just the canned goods department gives us cylinders by the basketful. Some are tall, and some are short, but they are all cylinders. What are the bases of a cylinder? (Two circular regions)



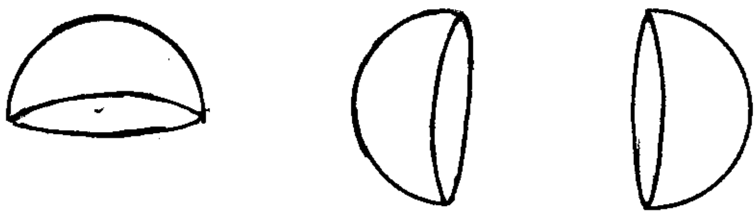
Now suppose you could spread a cylinder out. What geometric figure is formed?

Some cylinders look like this. Where have you seen this kind?

A cone has just one base that is a circular region. You are apt to be familiar with this one at a very early age. Why?



A sphere is also a geometric shape. You may call it a ball or a globe. It can even be the earth. In geography we talk about half the earth and call it the hemisphere. We live in the western hemisphere, or the northern hemisphere. What are the other two hemispheres called? These are made up of more circles than you could count.



Can you see why?

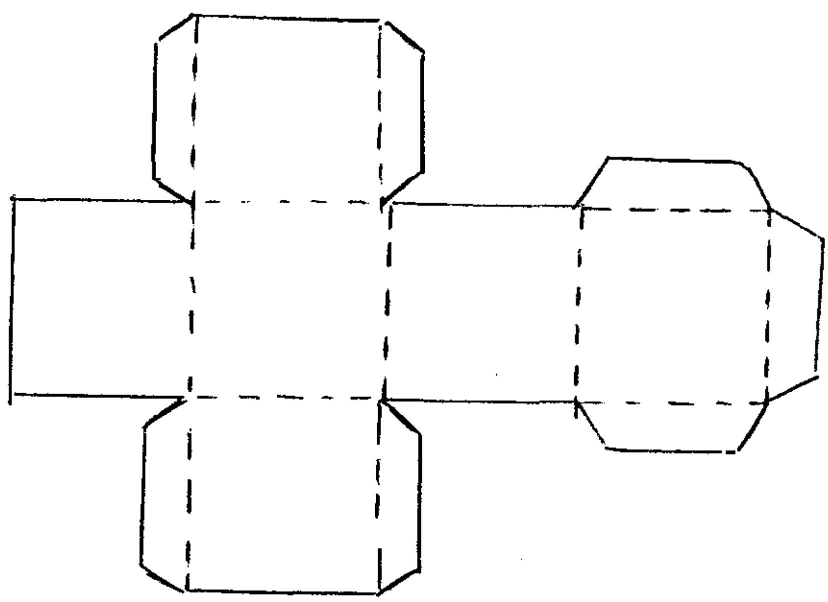
Perhaps you would be interested in making some models of the geometric solid figures you have studied. There are some patterns at the end of this unit which might be fun to use.

SOME THINGS TO DO

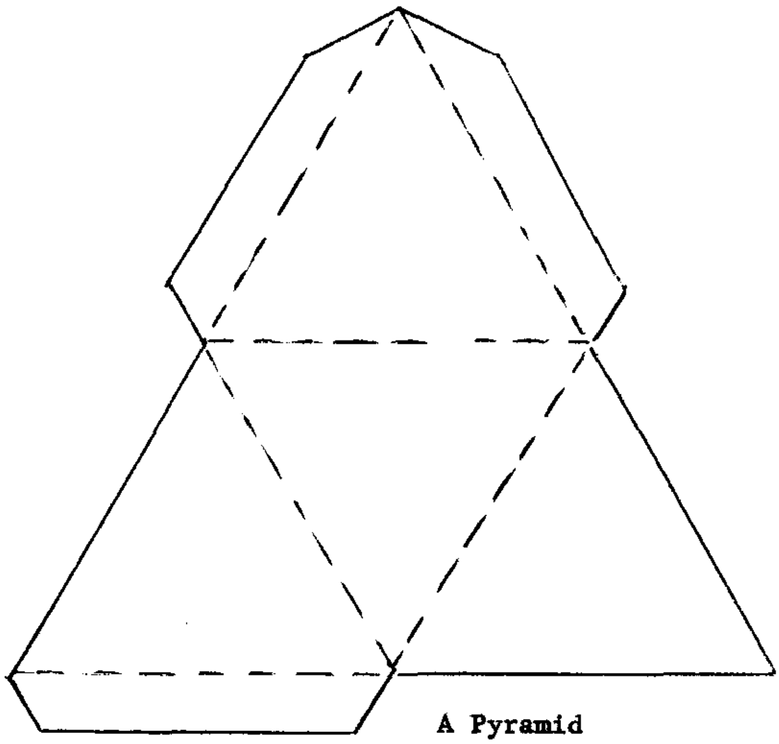
1. Here is another page for your chart.

2. If you would like to make models of some geometric solid figures, here are a few patterns to go by. A mobile to hang in your room would be fun.

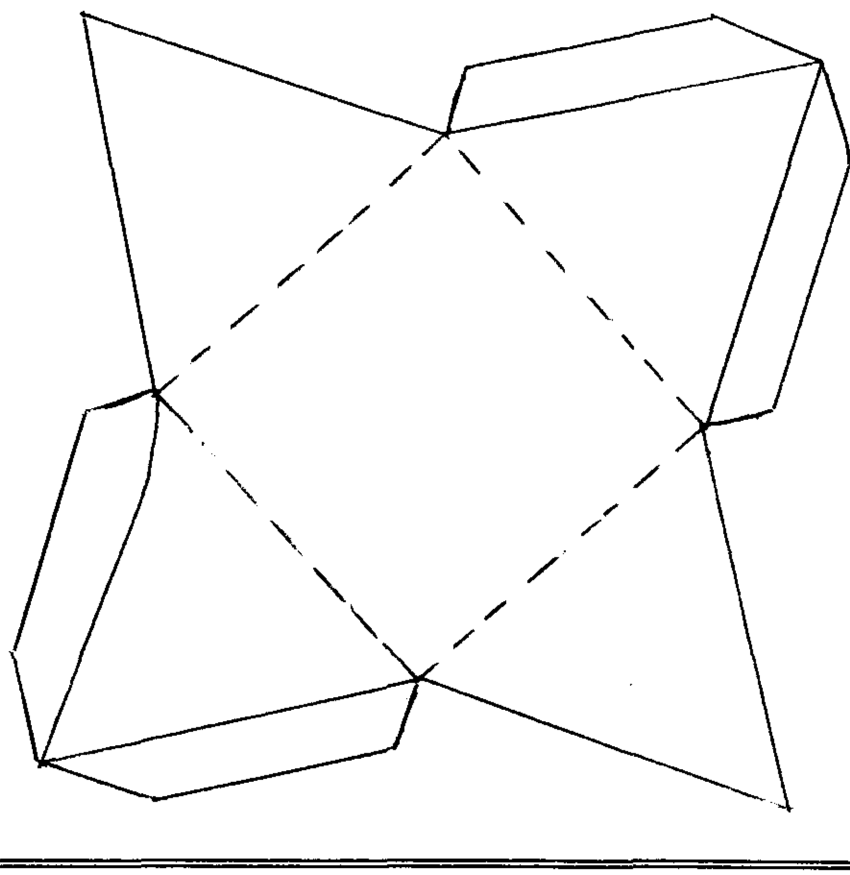
C Cube



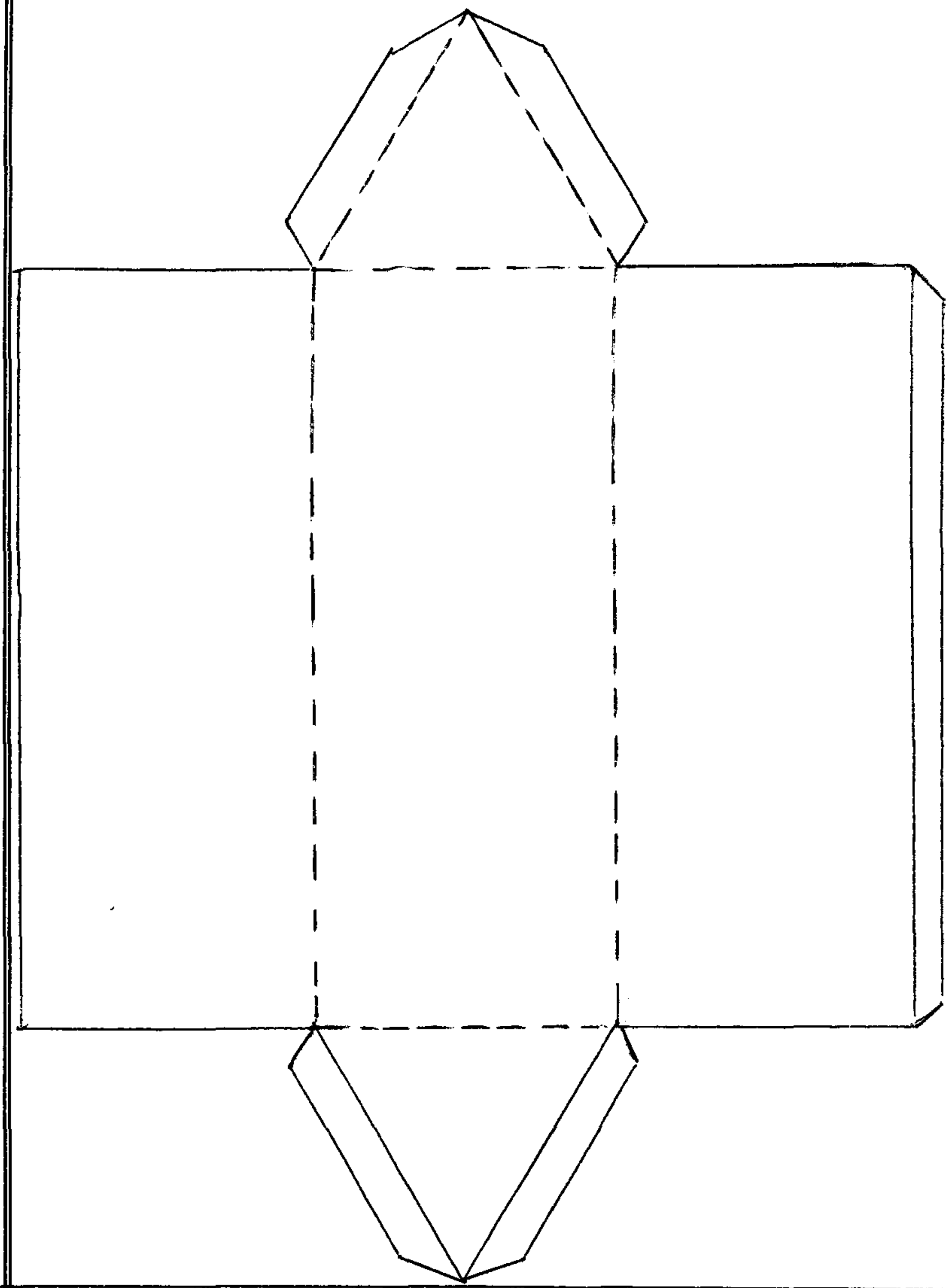
A Tetrahedron



A Pyramid



A Prism



UNIT THREE

VOCABULARY

triangle

congruent

equilateral

isosceles

scalene

base

square

parallelogram

hexagon

trapezoid

pentagon

cylinder

pyramid

cone

sphere

hemisphere

prism

cube

tetrahedron

face

edge

UNIT THREE ACTIVITIES

1. Here is a real puzzler:

   Draw an equilateral triangle with a six-inch base. Your compass is not big enough to do this in the manner you have learned. Find another way to do this, but use your compass.

2. Look up the history of triangles. Who used them first? Why?

3. Make a chart of all the geometric shapes studied in this unit.

4. Find out about the history and construction of pyramids.

5. Construct mobiles of paper geometric solids.

6. Create interesting designs using plane geometric shapes.

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