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REFERRAL AND INFORMATION ACQUISITION IN MARKETS AND ORGANIZATIONS

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ABSTRACT

This dissertation studies an economy where efficiency depends on the correct match between projects and two experts. A low-skill expert has low fixed cost and low productivity, so he is more efficient in handling low-potential or low-difficulty projects. And the opposite is true for a high-skill expert. The dissertation studies the effectiveness of markets and organizations in overcoming asymmetric information issues, experts' incentives to acquire project information at a cost, and how experts use the information to facilitate the correct match.

The first chapter studies a market where a referring expert privately knows a project's potential and may refer it at any price. Inspection benefits the referred expert. First, it allows him to find out the project's potential before accepting the referral offer. Second, it allows him to tailor production effort to the project's potential for maximum efficiency. In equilibrium, the referring expert pools projects into subsets and refers each subset at a different price. A higher price signals a subset of projects with higher potentials. The referred expert almost always inspects and then uses the information to make the acceptance decision. Each subset must be small enough to incentivize the referral at a price, but also large enough to incentivize inspection by the referred expert.

The second chapter studies contract design within an organization. A principal has to rely on the two experts to learn about projects' difficulties. If information cost is small, the principal can implement the first best by an optimal mechanism with the low-skill expert acting as a gatekeeper. The low-skill gatekeeper expert is incentivized to acquire information and report it truthfully. Subsequently, the principal efficiently assigns the project based on the report.

The third chapter studies a market where each of the two experts can exert a variable effort to acquire project information imperfectly. In the first best, experts coordinate their information acquisition efforts. In the market, either one or both experts acquire information. The two experts may fail to coordinate because one acquires information for efficient match but the other acquires information again to protect himself.

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Chapter 1

Referral and Inspection

1.1 Introduction

Market referral is a common business practice in many industries. For example, a small firm with low overhead cost can implement simple projects efficiently, but may lack the technology to exploit the full potential of complex projects. Therefore, the small firm may choose to refer complex projects to a specialized firm in the market for a fee. Conversely, a specialized firm with high overhead cost may find it worthwhile to refer simple projects to a small firm. Referrals are very commonly observed in the accounting, construction, consulting, and financial industries, etc.

Despite the prevalence of market referrals, recent research by Garicano and Santos (2004) and Grassi and Ma (2016) maintain that the referral market would unravel due to asymmetric information. In this chapter, I provide a model to demonstrate the referral market's effectiveness in matching projects to firms in the presence of hidden action, hidden information, and endogenous information acquisition. In contrast to the extant literature, I introduce information acquisition to my model; a firm may inspect a project before accepting a referral. The intuition is that a firm inspects not only to discern a project's profitability, but also to tailor its effort for maximum efficiency. I show that both forces play a key role in eliminating adverse selection and preventing the market from unraveling.

My model includes a set of projects and each project has a different productivity potential. There are two firms, a low-skill firm and a high-skill firm. Each firm can implement a project by incurring a fixed cost and exerting a variable effort that is tailored to the project potential. A project's return is a strictly concave single-peaked function in effort, and effort and project potential are complements in production. Each firm can achieve a higher return by implementing a project with higher potential, but the two firms have different comparative advantages. The low-skill firm has low productivity and low cost and the opposite is true for the high-skill firm. In the first best, the low-skill firm implements low-potential projects and the high-skill firm implements high-potential projects.

I study two types of markets which I call ascending referral and descending referral. In ascending referral, the low-skill firm has private information about a project and can decide whether to implement the project or refer it to the high-skill firm at a price. In descending referral, firms' roles are interchanged; the high-skill firm has private information about a project and can decide whether to implement or refer it to the low-skill firm. I first study ascending referral. If the low-skill firm decides to make a referral, then the high-skill firm can inspect the project potential at a cost before accepting or rejecting the offer. If the high-skill firm accepts, then it chooses effort to implement the project subject to the project's potential. The referral market is undermined by asymmetric information: (1) only the low-skill firm has private information about the project when the referral offer is made, and (2) inspection as well as project effort are the high-skill firm's hidden actions.

Inspection can benefit the high-skill firm through two channels. First, it enables the high-skill firm to avoid unprofitable projects. Second, inspection allows the high-skill firm to choose its effort optimally. I show that the high-skill firm must inspect every referred project in equilibrium. Otherwise, the low-skill firm would exploit the opportunity and refer low-potential projects to the high-skill firm at a high price, and the market would unravel. However, in equilibrium, the high-skill firm's incentive to inspect is not to reject unprofitable projects. The reason is that the low-skill firm's incentive to refer lemons is restrained if it expects the high-skill firm to inspect. Instead, the high-skill firm's inspection stems from the second channel: its incentive to tailor effort to implement projects with maximum efficiency.

A referral equilibrium, one in which the low-skill firm successfully refers projects to the high-skill firm, exists if and only if the inspection cost is small. An equilibrium is partial-pooling; the low-skill firm pools projects into convex subsets and refer each subset at a different price. A project with higher potential is referred at a higher price. Each equilibrium referred subset must be small enough to incentivize the referral, but must also be large enough to incentivize the high-skill firm's inspection. The high-skill firm always inspects and always accepts, although the firm can infer the project's conditional distribution from equilibrium prices. It is costly for the low-skill firm to make referrals because the firm has to pool different subsets of projects at different prices. Therefore, the low-skill firm refers fewer projects than the first best. However, the high-skill firm implements each referred project efficiently and the only inefficiency stems from the high-skill firm's inspection and insufficient referrals.

I then study descending referral in which the high-skill firm can refer a project to the low-skill firm. In contrast to ascending referral, there are some referrals in any equilibrium regardless of the magnitude of the inspection cost. The reason is that the low-skill firm can generate a positive surplus for projects with any potential because of its low fixed cost. On the other hand, the high-skill firm cannot generate a positive surplus for projects with very low potential. Therefore, the high-skill firm is willing to send these low-potential projects away with little or no compensation. As a result, there are two classes of equilibria. An equilibrium in the first class is similar to an equilibrium in ascending referral; the low-skill firm accepts a referral only after inspection. The second class has a minor difference; the low-skill firm accepts the lowest equilibrium price without inspection, and it accepts other higher equilibrium prices only after inspection. Without inspection, the low-skill firm's project effort on a referred project at the lowest equilibrium price is not efficient.

I also consider two extensions for ascending referral.[1] In the first extension, I allow inspection to be contractible. In equilibrium, the low-skill firm refers each project at a different price by paying for the inspection cost, so the high-skill firm must inspect. The high-skill firm still accepts referral only after inspection but the low-skill firm no longer leaves any rent to the high-skill firm. I find that all equilibria are separating and there is a unique equilibrium allocation. In equilibrium, low-potential projects are implemented by the low-skill firm and high-potential projects by the high-skill firm, but the equilibrium threshold is higher than the first-best threshold. In the second extension, I endogenize the low-skill firm's information on project potential. In the beginning of the game, the low-skill firm must decide whether to learn about a project's potential by incurring an inspection cost. I show that if the inspection cost is small, then the low-skill firm always inspects a project before implementing or referring it. In this sense, my main result is robust to treating all firms as only able to learn the project potential through inspection.

[1] The extensions also apply to descending referral.

In the final section, I discuss how my model can be reinterpreted to understand subcontracting in the construction industry. In the modified model, a contractor can complete a project by tailoring effort to reduce construction cost. The cost-minimizing effort for a contractor depends on the project difficulty. A more difficult project costs a contractor more to complete, but a completed project yields a fixed return regardless of the project's difficulty. Unlike the referral game, in a subcontracting game, a contractor cannot sell the ownership of a project. However, he can subcontract the project completion right by offering a fixed payment and the subcontractor can inspect the project before accepting the offer. The analysis and results are similar to the main model's. In equilibrium, a contractor pools projects into convex subsets and subcontracts each subset at a different payment. The subcontractor inspects a project at all but the highest equilibrium payment. He may or may not inspect at the highest equilibrium payment depending on the size of the corresponding subset.

The modified model's setup and results resonate with the practices in the construction industry. According to Carty (1995), practitioners in the construction industry use a lump-sum (fixed-price) contract "when the scope of work is clearly defined and understood by both parties to the contract." This supports the usage of a fixed price contract in my model since the "scope of work" in a subcontract is the completion of a project, which I assume to be a contractible event. Moreover, Kaplanogu and Arditi (2008) conduct a survey of the top 400 construction companies in the world and find that 78% of the contractors "evaluate the status of a project prior to committing" to an agreement. Those contractors who do not conduct "pre-project" inspection cite "the cost of the process and the lack of the appropriate company culture and resources" as their reason for not inspecting. This is consistent with the model's prediction that subcontractors who receive a project almost always inspect it and they do not evaluate the project before accepting if the inspection cost is too high. Secondly, according to Kaplanogu and Arditi (2008), the contractors cite the evaluation of "appropriateness of schedule" as one of the main reasons for them to carry out "pre-project" inspection. This matches my result that a contractor inspects to tailor its effort for maximum efficiency in an equilibrium. Lastly, subcontracts of various lump-sum values are observed in reality. This is in line with the chapter's analysis that there exist equilibria with multiple equilibrium subcontract payments as long as the inspection cost is small.

The remainder of the chapter is organized as follows. Section 1.1.1 provides a review of the literature. Section 1.2 presents the model setup and the first best. Section 1.3 studies an ascending referral market in which the low-skill firm may refer to the high-skill firm. Section 1.4 studies a descending referral market where firms' roles are interchanged. Section 1.5 discusses equilibrium selection. Section 1.6 considers two extensions of the main model. Section 1.7 presents a modified version of the model in order to understand subcontracting in the construction industry. Section 1.8 concludes. Finally, the Appendix contains proofs of results.

1.1.1 Literature Review

My model is closely related to two papers on referrals when firms differ in marginal productivities and fixed costs.[2] In Garicano and Santos (2004), an informed firm may refer a project by either a fixed-price or an output-sharing contract. The potential of a project is either high or low, and effort is required for production. In a market equilibrium, ascending referral may be inefficient and may unravel completely,

[2]The literature calls them agents or experts. I call them firms to emphasize the chapter's focus on market referral.

whereas descending referral is first-best.[3] In Grassi and Ma (2016), a referring firm may exert effort to acquire a continuous signal about a project's cost before referring it to another firm at a price. In an equilibrium, only one of the two firms acquires information about a project and successfully refers it to the other. In both papers, the referred firm cannot acquire information and a market cannot support two-way referral.[4] My chapter demonstrates that the referred firm's inspection can solve the asymmetric information problem and leads to richer results. First, market referral can be two-way; either firm can refer projects to the other. Second, in an equilibrium, projects with different potentials can be referred at different equilibrium prices. Third, inspection leads to efficient project implementation.

My chapter contributes to the adverse selection literature when a seller is privately informed in a market. Chan and Leland (1982) and Bester and Ritzberger (2001) assume that a buyer can become fully informed about product quality at a cost, whereas Martin (2017) assumes that the buyer is rational inattentive to product quality. My model adopts a simple information acquisition technology under a richer environment; unlike a buyer purchasing a product, a firm can implement a project with a variable effort after accepting a referral. I point out that a referred firm can use project information to make acceptance decision as well as tailor its production effort. This intuition allows me to solve an otherwise challenging problem.

Garicano (2000) and Garicano and Rossi-Hansberg (2004, 2006) propose a framework in which an agent carries out production by solving a problem. Each problem

[3] My chapter uses a framework with continuous project type and a more general project return function. Because of the former assumption, the first-best result in the descending referral market of Garicano and Santos (2004) does not extend to my model.

[4] Both papers consider how a non-market institution may allow two-way referral. However, my chapter shows that a market is sufficient to sustain two-way referral.

differs in difficulty but has the same binary production outcome; it is either solved or not solved. Agents differ in their knowledge in solving a problem. Using this framework, Fuchs and Garicano (2010) and Fuchs, Garicano, and Rayo (2015) study a referral problem in which agents' knowledge and the referred problem's difficulty are private information. My model uses a different framework; production return is a continuous function of production effort and project potential. My focus is on referral in the presence of private production effort, project information, and information acquisition when firms' productivities and costs are common knowledge.

Information acquisition by an agent before contract acceptance in a principal-agent model has been considered in the previous literature.[5] Crémer and Khalil (1992) and Terstiege (2016) assume that the agent either acquires information at a cost before signing a contract or it learns at no cost after signing it. Crémer et al. (1998a) assume that if the agent does not acquire information before signing, then he only finds out the information ex-post after production. My chapter uses this approach. In contrast to the principal-agent literature, my chapter assumes that the referring firm is privately informed and it can only use a simple price contract.[6] However, in equilibrium, the referring firm uses the simple contract to share rent such that the referred firm inspects. The referred firm's inspection is redundant but prevents markets from unraveling and maximizes production efficiency.

[5]Other papers allow information to be acquired at different stages of the extensive form. The literature shows that timing matters. Crémer and Khalil (1994) and Crémer et al. (1998b) assume that the agent can acquire information before a contract is offered. Lewis and Sappington (1997) and Iossa and Martimort (2015) assume that the agent can acquire information after agreeing to a contract. In my chapter, the referred firm must acquire information before signing a deal in order to deter lemons.

[6]In Mezzetti and Tsoulouhas (2000), the principal is privately informed and agent's information acquisition is observable. The latter assumption allows them to concentrate on multiple rounds of contract proposal.

1.2 Model

1.2.1 Projects, Firms, and Information Structure

There is a set of projects with total mass set at 1. Each project is indexed by a productivity potential q, which is a random variable distributed on a support $Q \equiv [0, \bar{q}]$, with a continuous distribution function and a density function $G(\cdot)$ and $g(\cdot)$, respectively.[7]

There are two risk-neutral firms, a low-skill firm and a high-skill firm. Each of them can implement a project. The low-skill firm can incur a fixed cost f_l and exert a variable effort e on a project q for a return of $y_l(e, q)$, and the high-skill firm can incur a fixed cost f_h and exert an effort e on a project q for a return of $y_h(e, q)$. Let the return functions be $y_i : \mathbb{R}_+ \times Q \to \mathbb{R}$, $i = l, h$. Given an effort e and a project q, a firm's return can be interpreted as project q's output net of firm's effort disutility. I make the following assumptions on the return functions. First, the return functions are strictly increasing in project potential q; $\frac{\partial y_l(e,q)}{\partial q} > 0$ and $\frac{\partial y_h(e,q)}{\partial q} > 0$. Second, the return functions $y_l(e, q)$ and $y_h(e, q)$ are twice-differentiable and strictly concave in e. Also, $\max_e y_l(e, 0) = \max_e y_h(e, 0) = 0$ and $y_l(0, q) = y_h(0, q) = 0$ for $\forall q \in Q$. Given a project with positive potential q, $y_l(e, q)$ and $y_h(e, q)$ each achieves an interior maximum at a positive number at a positive effort e. Finally, effort e and project potential q are complementary in production. That is, the cross derivatives of the return functions are positive; $\frac{\partial^2 y_l(e,q)}{\partial e \partial q} > 0$ and $\frac{\partial^2 y_h(e,q)}{\partial e \partial q} > 0$.

The two firms have different comparative advantages. The high-skill firm has to incur a higher fixed cost than the low-skill firm in order to implement a project. I also normalize the low-skill firm's fixed cost to be zero, so $f_h > f_l = 0$. However, the

[7] Throughout the chapter, I use project q and project with potential q interchangeably.

high-skill firm is more productive than the low-skill firm. First, given a project q and an effort e, the high-skill firm generates a higher return than the low-skill firm, which means $y_h(e,q) > y_l(e,q)$. Second, the high-skill firm has a weakly higher marginal return of effort than the low-skill firm; $\frac{\partial y_h(e,q)}{\partial e} \geq \frac{\partial y_l(e,q)}{\partial e}$. Third, a project with a higher potential enhances the high-skill firm's return more than the low-skill firm's; $\frac{\partial y_h(e,q)}{\partial q} > \frac{\partial y_l(e,q)}{\partial q}$.[8] Finally, I assume that $\max_e y_h(e,\bar{q}) - f_h > \max_e y_l(e,\bar{q})$. This assumption makes sure that the high-skill firm generates a higher surplus than the low-skill firm for some projects, when each firm exerts surplus-maximizing effort on the project.

Two information structures are included in the model. In the first, a firm is privately informed about a project's potential. In the second, a firm can inspect and learn about a project's potential at a cost c.

1.2.2 First Best

An allocation is an assignment of a project q to one of the firms and a production effort by the assigned firm. The first-best allocation is one that maximizes project q's expected surplus. In the first best, a project q is assigned to the firm who can generate the highest surplus. The assigned firm chooses an effort to maximize project q's surplus. If the low-skill firm is the assigned firm, it chooses effort $\hat{e}_l(q) \equiv \text{argmax}_e y_l(e,q)$. Alternatively, the high-skill firm chooses effort $\hat{e}_h(q) \equiv \text{argmax}_e \{y_h(e,q) - f_h\}$. I call $\hat{e}_l(q)$ and $\hat{e}_h(q)$ the low-skill and the high-skill firms' efficient efforts, respectively. They are one-to-one functions of project potential q, because each return function

[8] Here is an example of the return functions that satisfy the assumptions I just introduced: $y_l(e,q) = eq - \phi(e)$ and $y_h(e,q) = req - \phi(e)$, with $r > 1$, $\phi(0) = 0$, $\phi' > 0$ and $\phi'' > 0$. A firm's return function consists of two separable functions. The first is a multiplicative production function and the second is a convex disutility function. This separable functional form is the one Garicano and Santos (2004) adopt.

is strictly concave and has an interior maximum in project potential q. Lemma 1 characterizes firms' efficient effort functions (proof in the Appendix).

Lemma 1 *Let* $\hat{e}_l(q) \equiv argmax_e y_l(e,q)$ *and* $\hat{e}_h(q) \equiv argmax_e\{y_h(e,q) - f_h\}$*. Then, for* $\forall q \in Q$*,*

(i) $\hat{e}_h(q) \geq \hat{e}_l(q)$*, and*

(ii) $\frac{d\hat{e}_l(q)}{dq} > 0$ *and* $\frac{d\hat{e}_h(q)}{dq} > 0$.

The high-skill firm's efficient effort to implement a project with positive potential is higher than the low-skill firm's, because the high-skill firm has a higher marginal return of effort and the return functions are strictly concave in effort. Furthermore, each firms' efficient effort is an increasing function of project potential. This is due to two assumptions. First, effort and project potential are complementary in production. Second, return functions are strictly concave in effort. To save on notations, define the low-skill firm's efficient surplus of project q to be $S_l(q) \equiv y_l(\hat{e}_l(q), q)$, and the high-skill firm's efficient surplus of project q to be $S_h(q) \equiv y_h(\hat{e}_h(q), q) - f_h$. With the new notations, the assumption $\max_e y_h(e, \bar{q}) - f_h > \max_e y_l(e, \bar{q})$ translates into $S_h(\bar{q}) > S_l(\bar{q})$. Lemma 2 characterizes firms' efficient surplus functions (proof in the Appendix).

Lemma 2 *Let* $S_l(q) \equiv \max_e y_l(e,q)$ *and* $S_h(q) \equiv \max_e y_h(e,q) - f_h$*. Then* $\frac{dS_h(q)}{dq} > \frac{dS_l(q)}{dq} > 0$ *for* $q \in Q$*. There exists a unique* $q^{fb} \in Q$ *such that* $S_l(q) > S_h(q)$ *for* $q < q^{fb}$*,* $S_l(q) < S_h(q)$ *for* $q > q^{fb}$*, and* $S_l(q^{fb}) = S_h(q^{fb})$*. Also, there exists a unique* q^* *such that* $S_h(q^*) = 0$.

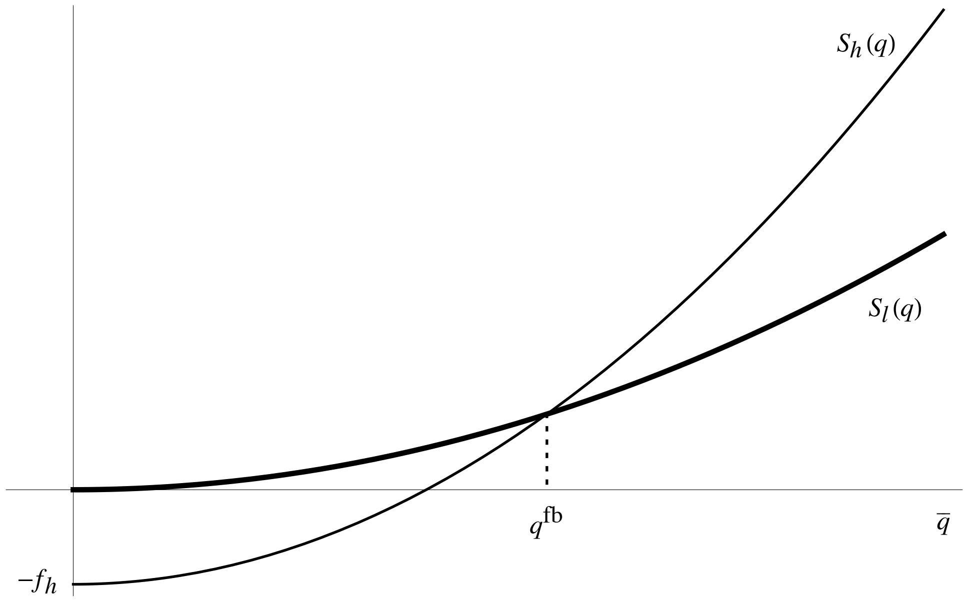

Figure 1·1: First-best allocation

Lemma 2 is summarized in Figure 1.1. The solid line denotes the high-skill firm's efficient surplus $S_h(q)$ and the bold line denotes the low-skill firm's efficient surplus $S_l(q)$.[9] As project potential q increases, $S_h(q)$ increases faster than $S_l(q)$, because (i) the high-skill firm has a higher marginal return of project potential, (ii) the high-skill firm's efficient effort is higher than the low-skill firm's by Lemma 1, and (iii) the cross derivative of a return function is positive. However, the high-skill firm's fixed cost is higher than the low-skill firm's. Hence, the low-skill firm is more efficient for projects with potential lower than q^{fb}, whereas the high-skill firm is more efficient for projects with potential higher than q^{fb}. To sum up, the first-best allocation is: (i) $q < q^{fb}$ is assigned to the low-skill firm, $q \geq q^{fb}$ is assigned to the high-skill firm, and (ii) the assigned firm exerts efficient effort on the assigned project.

Finally, for future references, define $S_l^{-1}(x)$ as the inverse function of $S_l(q)$ such

[9] $S_l(q)$ and $S_h(q)$ are not necessarily convex functions as depicted.

that $S_l^{-1}(S_l(q)) = q$ and $S_h^{-1}(x)$ as the inverse function of $S_h(q)$ such that $S_h^{-1}(S_h(q)) = q$. They are well-defined functions because the efficient surplus functions are strictly increasing.

1.3 Ascending Referral

I consider two types of markets which I call ascending referral and descending referral. In ascending referral, the low-skill firm can implement a project or refer it to the high-skill firm. The extensive form of ascending referral is:

Stage 1: The low-skill firm has a project with potential q determined by distribution G. The firm has private information about the project potential q. It decides between implementing the project itself and referring the project to the high-skill firm. The low-skill firm implements the project by choosing an effort e and incurring a fixed cost f_l, whereas it refers the project by choosing a price p.

Stage 2: The high-skill firm decides whether or not to inspect the referred project at an inspection cost c. The firm then decides whether or not to accept the referral. If it accepts, it pays p to the low-skill firm and implements the project by choosing an effort e and incurring a fixed cost f_h. If the high-skill firm rejects, the game ends.

Descending referral has the roles of the firms interchanged. I study both markets but the analysis of the two markets are similar. I present the analysis of ascending referral in detail in this section and indicate how the two markets differ in the next section.

The extensive form has the following features. Project potentials' distribution, return functions, fixed costs and inspection cost are firms' common knowledge. The low-skill firm is privately informed about the project potential, but the high-skill firm may privately inspect and learn about the project potential at an inspection cost before accepting or rejecting the referral. Last but not least, a firm's project effort and return are its hidden action and hidden information, respectively. In Section 1.6, I extend the model so that the high-skill firm's inspection is contractible. I also endogenize the low-skill firm's information.

In this environment the only contractible event is a transfer of project ownership, thus the low-skill firm refers a project with a fixed-price contract. Why is project return not contractible? Project return depends on both project potential and implementation effort, and it is generated in a complex way. In order to enforce a return-sharing contract, the referring firm needs to constantly monitor the referred firm's production process. This may be very costly. In addition, an outstanding work performance on a project may lead to future business opportunities through reputation effect, and reputation is typically not contractible.

Note that the game ends if the high-skill firm rejects a referral in stage 2. One can consider an alternative game in which the low-skill firm must implement a rejected project with a delay cost $\epsilon > 0$. I discuss how the alternative game is equivalent to the above game in Section 1.3.5.

1.3.1 Strategies and Beliefs

I define firms' strategies and the high-skill firm's belief in this section. First, the low-skill firm's strategy is defined by (i) a production effort if it decides to work on a project, and (ii) a referral price if it decides to refer the project to the high-skill firm. The low-skill firm's strategy is a function of project potential. Second, the high-skill firm's strategy is defined by (i) a decision whether to inspect a referred project, (ii) a decision whether to accept the referral, and (iii) a production effort if it accepts the referral. The high-skill firm's inspection decision is a function of referral price. Its acceptance decision and production effort are functions of referral price and inspection information.

I only consider referral prices in between zero and $S_h(\bar{q})$, which is without loss of generality. The low-skill firm can obtain a surplus weakly higher than zero with any project, so it would not offer a referral price lower than or equal to zero. The high-skill firm always rejects a price higher than $S_h(\bar{q})$ because it cannot obtain a surplus higher than $S_h(\bar{q})$ with any project. Upon receiving a price offer p in stage 2, the high-skill firm forms a belief about the project potential. The high-skill firm's belief about a project with potential q is a density function $\mu : (0, S_h(\bar{q})] \times Q \to \mathbb{R}_+$, so $\mu(p, q)$ is the probability density that the high-skill firm assigns to project q when the referral price is p, with $\int_Q \mu(p, x)dx = 1$. I assume that the low-skill firm prefers referring a project to implementing it when it is indifferent. Also, I assume that the high-skill firm prefers accepting a referral when it is indifferent.

A tuple of strategies and belief is a perfect Bayesian equilibrium of the ascending referral game if the strategies are mutual best responses given the belief and the belief is obtained from the strategies by Bayes' rule whenever possible. I restrict my attention to pure strategy equilibria. I also impose the following assumption to facilitate my analysis.

Assumption A

$$\textit{For all } \hat{q} > q^{fb}, \quad c \leq \frac{1}{G(\hat{q})} \int_0^{\hat{q}} S_h(q) dG(q) - \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \left[y_h(e,q) - f_h \right] dG(q). \tag{1.1}$$

To understand Assumption A, consider a project q from the subset $(0, \hat{q})$, with $\hat{q} > q^{fb}$. What is the high-skill firm's expected surplus of implementing the project? The first term on the right-hand side of the inequality is the high-skill firm's expected surplus of implementing the project q with full information. The high-skill firm implements the project by choosing its efficient effort according to project potential. The second term on the right-hand side is the high-skill firm's expected surplus of implementing the project only knowing that the project q is from the subset $(0, \hat{q})$. The high-skill firm implements the project by choosing an average effort. The difference in surplus on the right-hand side is always positive. Assumption A says that, if the high-skill firm has a project from the subset $(0, \hat{q})$, with $\hat{q} > q^{fb}$, then the inspection cost is smaller than the difference in surplus on the right-hand side. The high-skill firm wants to inspect project q before implementing it. I discuss the implication of relaxing Assumption A at the end of this section. I rewrite Assumption A for simplicity.

Assumption A $\quad c \leq c_A \equiv \min_{\hat{q} > q^{fb}} \left\{ \frac{1}{G(\hat{q})} \int_0^{\hat{q}} S_h(q) dG(q) - \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \left[y_h(e,q) - f_h \right] dG(q) \right\}$.

In the rest of the section, I construct equilibria with the following characteristics. The low-skill firm works on some projects and refers others. Different subsets of project may be referred at different prices. Although the high-skill firm can infer a referred project's conditional distribution through an equilibrium referral price, it always inspects the referred project and always accepts it. As will be shown, they are the only equilibria where projects are referred. I first present some necessary conditions of an equilibrium, then I show that they are sufficient as well.

1.3.2 The High-skill Firm's Equilibrium Information Sets

In this section, I study the high-skill firm's equilibrium information sets. I make some observations before proceeding. The low-skill firm always prefers implementing a project rather than losing it due to a rejected referral. This has the following implications. First, in an equilibrium, no referral is rejected. Also, the low-skill firm who refers a project at price p obtains an equilibrium payoff p. Finally, the high-skill firm accepts referrals at equilibrium prices and rejects referrals at off-equilibrium prices.

A no-referral equilibrium always exists. In it, the high-skill firm rejects any price offer because the firm believes that the referred project's potential is too low to justify the price. In the rest of the section, I focus on equilibria in which the low-skill firm successfully refers projects to the high-skill firm. I call them referral equilibria. To facilitate the analysis, define the high-skill firm's equilibrium information set to be a referral set.

Definition 1 (Referral Set) *A referral set $R(p)$ is the set of projects which the low-skill firm refers at equilibrium price p.*

In an equilibrium, the low-skill firm may refer different projects at different prices, so there may be many referral sets. Any two referral sets must be disjoint because I concentrate on pure strategy equilibria. In an equilibrium, the high-skill firm infers that a project at price p comes from the referral set $R(p)$. Whether or not the high-skill firm inspects the project depends on the precision of the project potential information conveyed by the low-skill firm's strategy through the referral price. If the information conveyed by the referral price is too precise, the high-skill firm does not inspect the referred project. However, the low-skill firm may exploit a lack of inspection by referring lemons to the high-skill firm.

There may be many prices that are never offered by the low-skill firm. For example, in an equilibrium, the low-skill firm may refer all projects at price p_1. Perfect Bayesian equilibrium does not impose a restriction on the high-skill firm's belief when the firm observes prices different from p_1. Multiple equilibrium allocations can be supported by various off-equilibrium beliefs. In this section, I examine all possible equilibrium allocations by using the following belief restriction.

Definition 2 (Pessimistic Belief) *An equilibrium is said to satisfy pessimistic belief if the high-skill firm believes that a referred project at any off-equilibrium price has the lowest potential.*

Consider an equilibrium and its allocation. The equilibrium may not satisfy pessimistic belief, but if I replace the corresponding off-equilibrium belief by pessimistic belief, then the said allocation remains to be an equilibrium allocation.[10] The reason is that the high-skill firm rejects any off-equilibrium price with pessimistic belief. In

[10]Although I concentrate on referral equilibria in this section, I can easily construct a no-referral equilibrium using pessimistic belief.

this section, I only consider equilibria satisfying pessimistic belief, which is without loss of generality. In the next section, I introduce another belief restriction as a refinement.

Lemma 3 *Let p be an equilibrium price and $R(p)$ be the corresponding referral set. For any $q \in R(p)$, $S_l(q) \leq p$.*

Here is the proof. The low-skill firm can always implement a project with potential q to get its efficient surplus $S_l(q)$, because the firm knows the project potential. Therefore, in an equilibrium, the low-skill firm refers project q at price p only if the price is larger than its efficient surplus $S_l(q)$.

Lemma 4 *Let $u_l(q)$ be an equilibrium payoff of the low-skill firm with project q. Then $u_l(q)$ is a non-decreasing function of q.*

Lemma 4 says that the low-skill firm's equilibrium payoff is non-decreasing in project potential (proof in the Appendix). This monotonicity is driven by two factors. First, the low-skill firm always has the option of working on its project to get efficient surplus $S_l(q)$, which is an increasing function of project potential q. Second, consider an equilibrium. If the high-skill firm's equilibrium strategy calls for it to accept a project q' at a price p, then the equilibrium strategy also calls for it to accept a project $q'' > q'$. This is true whether or not the high-skill firm inspects the referred project. In an equilibrium, this guarantees that the low-skill firm's payoff of referring a project at a price p is non-decreasing in project potential. Two implications follow from Lemma 4. The first is a monotone property; if the low-skill firm refers projects

q_1 and q_2 at prices p_1 and p_2 in an equilibrium, respectively, then $q_1 < q_2$ implies $p_1 < p_2$. The second implication is stated in Corollary 1.

Corollary 1 *An equilibrium referral set is convex.*

Here is the proof. Consider an equilibrium in which the low-skill firm refers two projects q' and q'' at a price p, with $q' < q''$. The firm's equilibrium payoff with projects q' or q'' is p. By Lemma 4, the low-skill firm's equilibrium payoff with any project q satisfying $q' < q < q''$ is p as well. Hence, project q belongs to projects q' and q'''s referral set. Corollary 1 implies that a referral set must be of the form $R(p) = (q^a, q^b)$, where q^a is the infimum of the set, q^b is the supremum of the set, and p is the corresponding equilibrium price. Note that q^a and q^b are functions of price p. For convenience, I write a referral set as an open interval.[11]

Lemma 5 *If the high-skill firm accepts an equilibrium referral price p without inspection, then inf $R(p) = 0$.*

Lemma 5 is due to standard adverse selection as in Akerlof (1970). The proof is in the Appendix. If the high-skill firm accepts a price p without inspection, then the low-skill firm is free to refer any project at price p, including the project with the lowest potential. Lemma 5 implies that there is at most one equilibrium price at which the high-skill firm accepts without inspection. If there existed two such equilibrium prices, the low-skill firm would never refer at the lower price. Moreover, the equilibrium price at which the high-skill firm accepts without inspection must be

[11] Whether I write a referral set as an open or a closed set is insignificant because each point of a set has zero measure.

the lowest equilibrium price. To see that, suppose there exist multiple equilibrium prices and the high-skill firm accepts p without inspection. Compared to other referral sets, $R(p)$ contains projects with the lowest potentials because of Lemma 5 and the fact that referral sets are disjoint and convex. By Lemma 4, the low-skill firm's equilibrium payoff is non-decreasing in project potential. Therefore, p must be the lowest equilibrium price.

1.3.3 The High-skill Firm's Equilibrium Strategies and Beliefs

Let p be an equilibrium price and $R(p) = (q^a, q^b)$ be the corresponding referral set. Let $\tilde{q}$ be the project potential satisfying $S_h(\tilde{q}) = p$. Recall that in an equilibrium referral is never rejected, so price p must be lower than the high-skill firm's efficient surplus of some projects within the referral set. That is, $p < S_h(q^b)$ and $\tilde{q} < q^b$. At price p, the high-skill firm believes that the project comes from the interval (q^a, q^b), hence its belief density is $\mu(p, q) = \frac{g(q)}{G(q^b)-G(q^a)}$ for $q \in (q^a, q^b)$ and $\mu(p, q) = 0$ otherwise.

In a continuation equilibrium in stage 2, does the high-skill firm inspect a referred project at price p? Inspection benefits the high-skill firm in two aspects. First, the high-skill firm has the option to reject a lemon after inspection. If $q^a < \tilde{q}$, then the high-skill firm rejects any project with potential $q < \tilde{q}$ after inspection, because its efficient surplus of such project is lower than p. Alternatively, if $\tilde{q} \leq q^a$, then the high-skill firm accepts each project with potential $q \in (q^a, q^b)$ after inspection because its efficient surplus of such project is higher than p. The second benefit of inspection is that, if the high-skill firm accepts the referral after inspection, then it pays the low-skill firm and may optimally implement a project q with its efficient effort according to the project potential, which gives the high-skill firm a payoff of $S_h(q) - p$. By the above argument, the high-skill firm's expected surplus of inspecting a referred project at price p is

$$\frac{1}{G(q^b) - G(q^a)} \int_{\max\{q^a, \tilde{q}\}}^{q^b} \Big[S_h(q) - p\Big] dG(q) - c. \tag{1.2}$$

If the high-skill firm does not inspect the referred project, then it has two options. First, the high-skill firm can reject the referral and obtain a payoff of zero. Alternatively, the high-skill firm can accept the project and implement it only knowing that the referred project is from the set (q^a, q^b). In the latter case, the high-skill firm pays the low-skill firm and exerts an average effort. The high-skill firm's expected surplus of not inspecting the referred project at price p is

$$\max\left\{0, \max_e \frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[y_h(e, q) - f_h - p\Big] dG(q)\right\}. \tag{1.3}$$

To sum up, in a continuation equilibrium in stage 2, the high-skill firm inspects a referred project if and only if the expected surplus of inspecting (1.2) is larger than the expected surplus of not inspecting (1.3);

$$\begin{aligned} &\frac{1}{G(q^b) - G(q^a)} \int_{\max\{q^a, \tilde{q}\}}^{q^b} \Big[S_h(q) - p\Big] dG(q) - c \\ &\qquad \geq \max\left\{0, \max_e \frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[y_h(e, q) - f_h - p\Big] dG(q)\right\}. \end{aligned} \tag{1.4}$$

Recall that $S_h^{-1}(x)$ is the inverse function of $S_h(q)$ such that $S_h^{-1}(S_h(q)) = q$. Using the inverse function, $\tilde{q} = S_h^{-1}(p)$. Use it to rearrange (1.4) to get

$$\frac{1}{G(q^b) - G(q^a)} \int_{\max\{q^a, S_h^{-1}(p)\}}^{q^b} \Big[S_h(q) - p \Big] dG(q) - \max\left\{ 0, \max_e \frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[y_h(e,q) - f_h - p \Big] dG(q) \right\} \geq c. \quad (1.5)$$

Define $\mathcal{I}(q^a, q^b, p)$ to be the left-hand side of (1.5). I call $\mathcal{I}(q^a, q^b, p)$ the high-skill firm's incremental inspection surplus of the referral set $R(p) = (q^a, q^b)$. I also define (1.5) to be the incremental inspection constraint of the referral set $R(p) = (q^a, q^b)$. Now we have understood the tradeoff involved behind the high-skill firm's inspection decision, I proceed to the first significant result (proof in the Appendix).

Lemma 6 *In a referral equilibrium, the high-skill firm always inspects and always accepts a referred project.*

We already know that no referral is rejected in an equilibrium. Moreover, a referral cannot be sustained without the high-skill firm's inspection. The argument is by contradiction. First, by Lemma 5, if a referral was accepted by the high-skill firm without inspection, then the corresponding referral set would be a convex set including the project with the lowest potential. Furthermore, if the said referral set included only projects with potentials lower than q^{fb}, then there would be no scope for referral because the low-skill firm is more efficient, which would be a contradiction. However, if the said referral set included projects with potentials larger than q^{fb}, then the referral set would be quite large. By Assumption A, the high-skill firm's optimal response would be to inspect, which created a contradiction.

Lemma 6 says that the high-skill firm's inspection is needed to support an equilibrium. As explained above, the high-skill firm's incremental inspection surplus stems from two channels: (i) the option to reject a lemon, and (ii) the incentive to implement an accepted project with the efficient effort. The low-skill firm does not refer a lemon if it anticipates the high-skill firm to inspect and reject the lemon, so in an equilibrium the high-skill firm's inspection incentive stems only from the second channel. Each referral set must be large enough and satisfies the incremental inspection constraint so that the high-skill firm inspects upon receiving the corresponding equilibrium price. Clearly, in an equilibrium, the low-skill firm implements projects with its efficient effort because its information is exogenously given. Lemma 6 implies that the high-skill firm also implements projects with its efficient effort because it accepts a referral only after inspection.

1.3.4 Equilibrium Characterization

In this section, I provide the necessary and sufficient conditions of an equilibrium. By Lemma 6, each referral set satisfies the incremental inspection constraint, so each set cannot be too small. Therefore, an equilibrium consists of a list of $n \geq 1$ referral sets, each denoted by $R(p_m) = (q^a_m, q^b_m)$, for $m = 1, ..., n$. If $n \geq 2$, then by Lemma 4 the equilibrium prices, the infimum and the supremum of referral sets can be ranked so that $p_1 < p_2 < ... < p_n$, and $q^a_1 < q^b_1 \leq q^a_2 < q^b_2 \leq ... \leq q^a_n < q^b_n$. Proposition 1's (i)-(iv) are necessary conditions of an equilibrium (proof in the Appendix).

Proposition 1 *A referral equilibrium consists of* $n \geq 1$ *referral sets, with* $q^a_1 < q^b_1 \leq q^a_2 < q^b_2 \leq ... \leq q^a_n < q^b_n$. *For* $m = 1, ..., n$, *the low-skill firm refers projects* $q \in (q^a_m, q^b_m)$ *at price* p_m *and implements* $q \notin (q^a_m, q^b_m)$ *with effort* $\hat{e}_l(q)$. *The high-skill* *firm inspects, accepts, and implements each referred project* q *at* p_m *with* $\hat{e}_h(q)$. *At* p_m, *the high-skill firm's belief density is* $\mu(p,q) = \frac{g(q)}{G(q^b_m)-G(q^a_m)}$ *for* $q \in (q^a, q^b)$ *and* $\mu(p,q) = 0$ *otherwise. At an off-equilibrium price, the high-skill firm updates with pessimistic belief and rejects without inspection. The referral equilibrium exists only if the referral sets satisfy the following conditions.*

(i) $\mathcal{I}(q^a_m, q^b_m, p_m) \geq c$ *and* $p_m = S_h(q^a_m) \geq S_l(q^b_m)$, *for* $m = 1, ..., n$;

(ii) $q^a_1 > q^{fb}$;

(iii) *if there exists* q *such that* $q^b_n < q < \bar{q}$, *then* $p_n = S_h(q^a_n) = S_l(q^b_n)$; *and*

(iv) *if* $n \geq 2$ *and there exists* q *such that* $q^b_{j-1} < q < q^a_j$, *then* $p_{j-1} = S_h(q^a_{j-1}) = S_l(q^b_{j-1})$, *for* $j = 2, ..., n$.

The first part of condition (i) follows from Lemma 6. The high-skill firm accepts a referral only after inspection, and the high-skill firm inspects upon receiving p_m if and only if $\mathcal{I}(q^a_m, q^b_m, p_m) \geq c$. The second part of condition (i) can be explained as follow. First, by Lemma 3, the low-skill firm refers a project $q \in (q^a_m, q^b_m)$ at price p_m only if the price is larger than its efficient surplus; $p_m \geq S_l(q)$. Second, the high-skill firm accepts $q \in (q^a_m, q^b_m)$ at price p_m after inspection if and only if it can generate a surplus higher than the price, which means that $p_m \leq S_h(q)$. They impose an upper bound and lower bound for the equilibrium price;

$$S_h(q^a_m) \geq p_m \geq S_l(q^b_m). \tag{1.6}$$

Indeed, the equilibrium price p_m is equal to the upper bound $S_h(q^a_m)$, which is the high-skill firm's efficient surplus at the infimum of the referral set (q_m^a, q_m^b);

$$p_m = S_h(q_m^a) \geq S_l(q_m^b). \tag{1.7}$$

The argument is by contradiction. If the price was lower than the lower bound, which means that $p_m < S_h(q_m^a)$, then there would exist a profitable deviation opportunity for the low-skill firm. By Lemma 4, the low-skill firm who had a project with a slightly lower potential than q_m^a would get an equilibrium payoff lower than price p_m. However, as shown in the proof, the low-skill firm with such a project would find it profitable to deviate and refer it at price p_m, because the high-skill firm would inspect, find out that $S_h(q_m^a) - p_m > 0$, and accept the project.

I call (1.7) the production efficiency constraint of the referral set $R(p_m)$, which I illustrate in Figure 1.2. Recall that $S_l^{-1}(x)$ is the inverse function of $S_l(q)$ such that if $S_h(q_m^a) = S_l(q)$, then $S_l^{-1}(S_h(q_m^a)) = q$. In Figure 1.2, for a project with potential higher than $S_l^{-1}(S_h(q_m^a))$, the low-skill firm obtains a higher surplus by implementing it rather than referring it at price $p_m = S_h(q_m^a)$. Therefore, the production efficiency constraint says that the supremum of the referral set, q_m^b, cannot be higher than $S_l^{-1}(S_h(q_m^a))$. Moreover, if q_m^a increases, $S_l^{-1}(S_h(q_m^a))$ also increases, and $S_l^{-1}(S_h(q_m^a))$ increases at a faster rate than q_m^a. The production efficiency constraint is less strict for high-potential projects.

A referral set must satisfy both the incremental inspection constraint (1.5) and the production efficiency constraint (1.7), but they act in opposite directions. On the one hand, if a referral set is too small, the high-skill firm does not inspect at the corresponding price because the incremental inspection surplus is too small to justify the inspection cost. On the other hand, if a referral set is too large, the low-skill firm

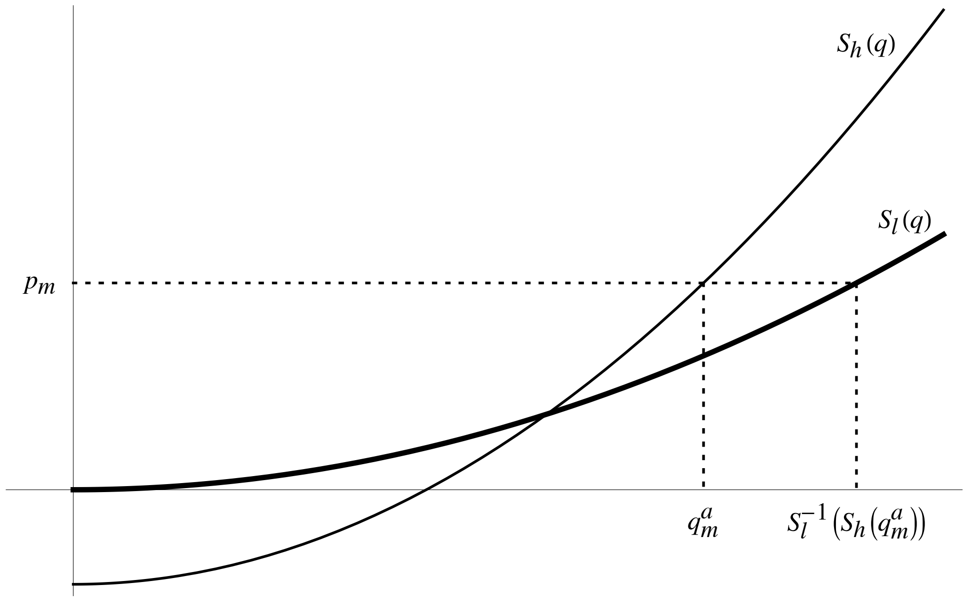

Figure 1·2: Production efficiency constraint

does not refer some projects inside the set because it can obtain a higher surplus by implementing them.

Upon receiving a referred project at price $p_m = S_h(q_m^a)$, the high-skill firm correctly infers that the project is from the set (q_m^a, q_m^b). The price is equal to the high-skill firm's efficient surplus at the infimum of the set. Therefore, the high-skill firm can derive an ex-post positive surplus by accepting the referral without inspection. However, the high-skill firm still inspects in order to exert a surplus-maximizing effort for implementation, but it has positive unintended consequence. The inspection deters the low-skill firm from referring lemons and prevents the market from unraveling.

The two constraints imply the second condition (ii) of Proposition 1; $q_1^a > q^{fb}$, which means that the low-skill firm refers a lower amount of projects compared to the first-best allocation. The incremental inspection constraint implies that a referral set (q_m^a, q_m^b) has a positive measure, whereas the production efficiency constraint says that the referral price for any project within the set must be the high-skill firm's efficient surplus at the infimum of the set, $p_m = S_h(q_m^a)$. By pooling a set of projects at a price, the low-skill firm leaves rent to the high-skill firm. Referral is costly for the low-skill firm, so it prefers retaining and implementing a project with potential slightly higher than the first best threshold q^{fb} rather than referring it and leaving rent to the high-skill firm.

To understand the third condition (iii) of Proposition 1, consider an equilibrium like the one in Figure 1.3. There is only one referral set, $R(p_1) = (q_1^a, q_1^b)$, and there is a subset of un-referred projects with a positive measure in between the referral set and the project with the highest potential. That is, there exists q satisfying $q_1^b < q < \bar{q}$. In the equilibrium, the low-skill firm implements projects within $(q_1^b, \bar{q})$ only if its implementation payoff is higher than the equilibrium price p_1. As shown in the proof, this means that $p_1 = S_l(q_1^b)$. The production efficiency constraint of the referral set (q_1^a, q_1^b) becomes $p_1 = S_h(q_1^a) = S_l(q_1^b)$.

Figure 1·3: Third condition (iii) of Proposition 1

The fourth condition (iv) is similar to the third condition (iii). Figure 1.4 shows an example. There is a subset of un-referred projects with a positive measure in between two referral sets, $R(p_1)$ and $R(p_2)$. The subset is (q_1^b, q_2^a). In an equilibrium, the low-skill firm prefers implementing projects within (q_1^b, q_2^a) rather than referring them at p_1 only if $p_1 = S_l(q_1^b)$.

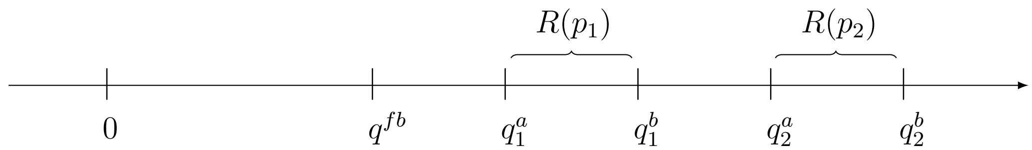

Figure 1·4: Fourth condition (iv) of Proposition 1

Proposition 2 shows that the above necessary conditions are also sufficient for the existence of a referral equilibrium (proof in the Appendix).

Proposition 2 *Suppose project potentials* $q_1^a < q_1^b \le q_2^a < q_2^b \le ... \le q_n^a < q_n^b$ *and prices* $p_1 < p_2 < ... < p_n$ *satisfy conditions (i) - (iv) in Proposition 1, then there exists a referral equilibrium in which, for* $m = 1, ..., n$, *the low-skill firm refers projects* $q \in (q_m^a, q_m^b)$ *at price* p_m *and implements* $q \notin (q_m^a, q_m^b)$ *with effort* $\hat{e}_l(q)$. *Also, the high-skill firm inspects, accepts, and implements each referred project* q *at* p_m *with* $\hat{e}_h(q)$. *At* p_m, *the high-skill firm's belief density is* $\mu(p, q) = \frac{g(q)}{G(q_m^b) - G(q_m^a)}$ *for* $q \in (q^a, q^b)$ *and* $\mu(p, q) = 0$ *otherwise. At an off-equilibrium price, the high-skill firm updates with pessimistic belief and rejects without inspection.*

By Proposition 1 and 2, a referral equilibrium is fully characterized by a list of referral sets, with each set satisfying conditions (i)-(iv) in Proposition 1. Given the fixed costs, return functions and project potentials' distribution, inspection cost c's size determines the existence of referral equilibria. Indeed, Corollary 2 says that a referral equilibrium exists if and only if the inspection cost is small (proof in the Appendix).

Corollary 2 *Given the fixed costs, return functions and project potentials' distribution, there is an inspection cost level* c_R *satisfying* $0 < c_R \le c_A$ *such that there exists a referral equilibrium if and only if* $c \le c_R$.

The level c_R is the highest inspection cost level at which a list of referral sets can be compiled to satisfy conditions (i)-(iv) in Proposition 1. If a referral equilibrium exists at a certain level of inspection cost, then the said equilibrium also exists at a lower level of inspection cost. The reason is that a lower inspection cost relaxes the incremental inspection constraint in condition (i) of Proposition 1 but does not matter in the production efficiency constraint and conditions (ii)-(iv). The exact exposition of c_R is provided in the Appendix. However, Corollary 2 can be further explained by an example in which project potential is uniformly distributed and the return functions are quadratic.

Example *Let q be uniformly distributed on $[0, \bar{q}]$, and $y_l(e, q) = eq - \frac{1}{2}e^2$ and $y_h(e, q) = req - \frac{1}{2}e^2$, with $r > 1$, for $q \in (0, \bar{q}]$.*

In this specification, $S_l(q) = \frac{1}{2}q^2$ and $S_h(q) = \frac{1}{2}r^2q^2 - f_h$. Consider a referral set $R(p) = (q^a, q^b)$ that satisfies the production efficiency constraint. The incremental inspection surplus of this set is

$$\begin{aligned} \mathcal{I}\big(q^a, q^b, S_h(q^a)\big) &= \frac{\bar{q}}{q^b - q^a}\frac{1}{\bar{q}}\Big[\int_{q^a}^{q^b} \frac{1}{2}r^2q^2\ dq - \max_e \int_{q^a}^{q^b} (req - \frac{1}{2}e^2)\ dq\Big] \\ &= \frac{r^2}{24}(q^b - q^a)^2. \end{aligned}$$

The uniform-quadratic structure implies that the incremental inspection surplus of a referral set is an increasing function of the size of the set. The referral set with the highest incremental inspection surplus is simply the one with the largest size. However, the size of a referral set is limited by its production efficiency constraint. The production efficiency constraint $p = S_h(q^a) \geq S_l(q^b)$ implies that the infimum of the referral set q^a must be larger than $S_h^{-1}(S_l(q^b))$. In other words, $q^b - S_h^{-1}(S_l(q^b))$ is the largest size of a referral set with q^b being the supremum of the set. By Figure 1.2, $q^b - S_h^{-1}(S_l(q^b))$ is an increasing function of q^b. Therefore, the largest equilibrium referral set is $\big(S_h^{-1}(S_l(\bar{q})), \bar{q}\big)$, when $q^b = \bar{q}$. Under an uniform-quadratic structure, $c_R = \min\big\{c_A, \mathcal{I}\big(S_h^{-1}(S_l(\bar{q})), \bar{q}, S_l(\bar{q})\big)\big\} = \min\big\{c_A, \frac{r^2}{24}\big(\bar{q} - S_h^{-1}(S_l(\bar{q}))\big)^2\big\}$. If inspection cost is larger than the second term in the minimization bracket, then there does not exist a referral set satisfying its production efficiency constraint while having an incremental inspection surplus larger than the inspection cost. In the main model, given the generality of the assumption I make on the return functions and project potentials' distribution, the incremental inspection constraint of a referral set may not be a function of the size of the set and c_R may be larger than $\mathcal{I}\big(S_h^{-1}(S_l(\bar{q})), \bar{q}, S_l(\bar{q})\big)$. I revisit this example in Section 1.5.

1.3.5 Discussion

I end this section by discussing an alternative extensive form of the game and the relaxation of Assumption A. First, in the current extensive form, the game ends if the high-skill firm rejects a referral. As I mentioned above, one can consider an alternative game in which the low-skill firm must implement a rejected project with a small delay cost $\epsilon > 0$.[12] Consider a corresponding equilibrium described in Proposition 1 for this alternative game; the low-skill firm refers projects inside referral sets and implements projects outside the sets in stage 1. The high-skill firm inspects and accepts all referred projects in stage 2. Would the low-skill firm deviate now that it can implement a rejected project with a delay cost? The answer is no. The reason is that, in the equilibrium, the low-skill firm prefers referring projects inside referral sets

[12] Professional service industries are deadline-driven. Thus, it is costly to implement projects in a delayed time.

rather than implementing them in stage 1. And it prefers implementing projects in stage 1 rather than implementing them after being rejected with a delay cost. There is no profitable deviation for the low-skill firm with projects inside or outside referral sets. Therefore, the analysis of the alternative game is the same as the above.

Finally, consider the relaxation of Assumption A. Recall that in an equilibrium, the high-skill firm accepts referrals only after inspection. Without Assumption A, there may or may not exist an equilibrium referral price at which the high-skill firm accepts without inspection.[13] However, Lemma 4 and 5 still apply and have the following implications as discussed after Lemma 5. There is at most one equilibrium price at which the high-skill firm accepts without inspection. Also, if such price exists, then it must be the lowest equilibrium referral price. Relaxing Assumption A may only affect the first referral set.

1.4 Descending Referral

In this section, I analyze a descending referral game in which the high-skill firm has an opportunity to refer a project to the low-skill firm. I get descending referral's extensive form by interchanging the roles of the low-skill firm and the high-skill firm in ascending referral's. Firms' strategies are defined similarly. I consider referral prices in between zero and $S_h(\bar{q})$. However, unlike ascending referral, I allow the high-skill firm to offer prices equal to zero because the high-skill firm may generate

[13]Here is another condition under which Lemma 6 holds; $S_l(\hat{q}) > \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big[y_h(e,q) - f_h \Big] dG(q)$, for all $\hat{q} > q^{fb}$. Lemma 6 under this condition can be proven by contradiction. Suppose not. Suppose that in an equilibrium the high-skill firm accepts a project from referral set $(0, q')$ at p, with $q' > q^{fb}$, without inspection. Price p must be capped by the high-skill firm's surplus implementing the project, which is on the R.H.S. of the above inequality by letting $\hat{q} = q'$. Also, the price must be higher than the low-skill firm's surplus $S_l(q)$ for any $q \in (0, q')$. Together it is a contradiction to the proposed condition.

surplus lower than zero for some low-potential projects. The low-skill firm's belief about a project q is a density function $\mu_l : [0, S_h(\bar{q})] \times Q \to \mathbb{R}_+$, and $\mu_l(p, q)$ is the probability density that the low-skill firm assigns to project q when the referral price is p, with $\int_Q \mu_l(p, x)dx = 1$. Clearly, pessimistic belief can be extended to descending referral easily by interchanging firms' roles in Definition 2. As in ascending referral, I focus on equilibria under pessimistic belief.

In descending referral, there is always some referrals in any equilibrium. The reason is that the high-skill firm generates a negative surplus for a project with potential $q \leq q^*$, with $S_h(q^*) = 0$ (see Lemma 2). The high-skill firm would rather refer these projects at price 0 because the low-skill firm accepts a referral at price 0 regardless of its belief. Therefore, projects $q \leq q^*$ are referred in any equilibrium. By contrast, a no-referral equilibrium always exists in ascending referral. Despite the above, Lemma 3-5 and Corollary 1 follow directly in descending referral by interchanging the roles of firms. Call the analogous results Lemma 3*, Lemma 4*, Lemma 5*, and Corollary 1*. Let p be an equilibrium price and $R(p) = (q^a, q^b)$ be the corresponding referral set. In a continuation equilibrium in stage 2, the low-skill firm inspects a referred project at price p if and only if the incremental inspection constraint is larger than the inspection cost. Define the low-skill firm's incremental inspection surplus $\mathcal{I}_l(q^a, q^b, p)$ by interchanging firms' roles on the right-hand side of (1.5);

$$\mathcal{I}_l(q^a, q^b, p) \equiv \frac{1}{G(q^b) - G(q^a)} \int_{\max\{q^a, S_l^{-1}(p)\}}^{q^b} \Big[S_l(q) - p\Big] dG(q) - \max\left\{0, \max_e \frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[y_l(e, q) - p\Big] dG(q)\right\}.$$

An analogous version of Lemma 6 does not exist in descending referral. In an equilibrium of descending referral, the low-skill firm may or may not inspect before accepting a referral. Indeed, there are two types of equilibrium. The first type is similar to a referral equilibrium I analyzed above in ascending referral; the low-skill firm accepts a referral only after it inspects. Analogous versions of Proposition 1 and 2 exist for the first type of equilibrium, with two minor differences. First, as noted above, all projects with potential $q \leq q^*$ are referred. Second, the supremum of the last referral set q_n^b is smaller than the first best threshold q^{fb}, which still means that not enough referrals are conducted compared to the first best. I call this an inspection equilibrium.

In the rest of the section, I focus on the second type of equilibrium, where the low-skill firm accepts some referrals without inspection. Following the logic of Lemma 5's discussion, there is at most one equilibrium price at which the referred firm, the low-skill firm, accepts without inspection. Also, such a price must be the lowest equilibrium price. the low-skill firm accepts all other equilibrium prices only after inspection. I call this a minimum-price-no-inspection equilibrium. Proposition 3 provides the necessary conditions for it.

Proposition 3 *A minimum-price-no-inspection equilibrium of descending referral consists of $n \geq 1$ referral sets, with $q_1^a < q_1^b \leq q_2^a < q_2^b \leq ... \leq q_n^a < q_n^b$. For $m = 1, ..., n$, the high-skill firm refers projects $q \in (q_m^a, q_m^b)$ at price p_m and implements $q \notin (q_m^a, q_m^b)$ with effort $\hat{e}_h(q)$. The low-skill firm accepts and implements each project at p_1 with effort $argmax_e \frac{1}{G(q_1^b)} \int_0^{q_1^b} y_l(e, q)\ dG(q)$ without inspection. If $n \geq 2$, then the low-skill firm inspects, accepts, and implements each project q at p_j with $\hat{e}_l(q)$, for $j = 2, ..., n$. At p_m, the low-skill firm's belief density is $\mu_l(p, q) = \frac{g(q)}{G(q_m^b) - G(q_m^a)}$ for*

$q \in (q^a_m, q^b_m)$ and $\mu_l(p,q) = 0$ otherwise, for $m = 1, ..., n$. At an off-equilibrium price, the low-skill firm updates with pessimistic belief; it accepts without inspection if the price is zero and rejects without inspection if the price is positive. The minimum-price-no-inspection equilibrium exists only if the referral sets satisfy the following conditions.

(i) *each project $q \in (0, q^*)$ is referred, with $S_h(q^*) = 0$. This means that $q^a_1 = 0$;*

(ii) $\mathcal{I}_l(q^a_1, q^b_1, p_1) < c$*;*

(iii) p_1 *satisfies* $\max\{0, S_h(q^b_1)\} \leq p_1 \leq \max_e \frac{1}{G(q^b_1)} \int_0^{q^b_1} y_l(e,q)\ dG(q)$*;*

(iv) *if $n \geq 2$, then $\mathcal{I}_l(q^a_j, q^b_j, p_j) \geq c$ and $p_j = S_l(q^a_j) \geq S_h(q^b_j)$, for $j = 2, ..., n$;*

(v) $q^b_n < q^{fb}$*;*

(vi) $p_n = S_h(q^b_n)$*; and*

(vii) *if $n \geq 2$ and there exists q such that $q^a_{j-1} < q < q^b_j$, for $j = 2, ..., n$, then $p_{j-1} = S_l(q^a_{j-1}) = S_h(q^b_{j-1})$.*

Most of the proof is similar to Proposition 1's. I point out the difference here. The first condition (i) of Proposition 3 has been explained above. The low-skill firm accepts the lowest equilibrium price p_1 without inspection. Therefore, the second condition (ii) says that the incremental inspection surplus of the first referral set must be smaller than the inspection cost. The third condition (iii) is explained below. First, without inspection, the low-skill firm only knows that a referred project at p_1 comes from $(0, q^b_1)$, so it accepts only if the surplus it can derive is higher than the price, which means that $0 \leq p_1 \leq \max_e \frac{1}{G(q_1^b)} \int_0^{q_1^b} y_l(e,q)\ dG(q)$. Second, by Lemma 3*, $S_h(q_1^b) \leq p_1$.

The low-skill firm accepts other equilibrium prices only after inspection. Therefore, the fourth condition (iv) says that each corresponding referral set satisfies the incremental inspection constraint and the production efficiency constraint as in Proposition 1. The fifth condition (v) of Proposition 3 says that the high-skill firm refers a lower amount of projects compared to the first best. The proof is in the Appendix. Finally, the sixth condition (vi) and the seventh condition (vii) of Proposition 6 are similar to the third condition (iii) and the fourth condition (iv) of Proposition 1, respectively. Proposition 4 shows that conditions (i)-(vii) are also sufficient. The proof is similar to Proposition 2's.

Proposition 4 *Suppose project potentials $q_1^a < q_1^b \leq q_2^a < q_2^b \leq ... \leq q_n^a < q_n^b$ and prices $p_1 < p_2 < ... < p_n$ satisfy conditions (i) - (vii) in Proposition 3, then there exists a minimum-price-no-inspection equilibrium in which, for $m = 1, ..., n$, the high-skill firm refers projects $q \in (q_m^a, q_m^b)$ at price p_m and implements $q \notin (q_m^a, q_m^b)$ with effort $\hat{e}_h(q)$. The low-skill firm accepts and implements each project at p_1 with effort $\text{argmax}_e \frac{1}{G(q_1^b)} \int_0^{q_1^b} y_l(e,q)\ dG(q)$ without inspection. If $n \geq 2$, then the low-skill firm inspects, accepts, and implements each project q at p_j with $\hat{e}_l(q)$, for $j = 2, ..., n$. At p_m, the low-skill firm's belief density is $\mu_l(p,q) = \frac{g(q)}{G(q_m^b) - G(q_m^a)}$ for $q \in (q_m^a, q_m^b)$ and $\mu_l(p,q) = 0$ otherwise. At an off-equilibrium price, the low-skill firm updates with pessimistic belief; it accepts without inspection if the price is zero and rejects without inspection if the price is positive.*

As said above, there are some referrals in each equilibrium for any level of inspection cost. For example, there always exists an equilibrium in which the high-skill firm refers each project $q \leq q^*$ at price zero and implements the others. If the inspection cost is low such that $c < \mathcal{I}_l(0, q^*, 0)$, then the low-skill firm accepts referrals at price zero only after inspection. However, if the inspection cost is high such that $c \geq \mathcal{I}_l(0, q^*, 0)$, then the low-skill firm accepts a referral at price zero without inspection. The former is an inspection equilibrium and the latter is a minimum-price-no-inspection equilibrium.

1.5 Equilibrium Selection

In this section, I propose a belief restriction to address two issues created by pessimistic belief. I return my focus to ascending referral but the belief restriction applies similarly to descending referral. First, pessimistic belief is extreme. The high-skill firm believes that projects at off-equilibrium prices come from a subset with zero measure and have the lowest potential. Given that each equilibrium referral set has a positive measure, a more natural belief restriction should require each off-equilibrium information set to have a positive measure as well.

Second, pessimistic belief may lead to a discontinuous allocation. As indicated by (iii) and (iv) of Proposition 1, there may exist equilibria in which the low-skill firm switches between implementation and referral multiple times as project potential increases. Figure 1.3 in section 1.3.4 serves as an example. It illustrates a subset of un-referred projects with a positive measure, $(q_1^b, \bar{q})$, in between the referral set $R(p_1)$ and $\bar{q}$. For each project within $(q_1^b, \bar{q})$, the high-skill firm's efficient surplus is larger than the low-skill firm's. Clearly, there is scope for referral for each project within the subset $(q_1^b, \bar{q})$. Despite the above, under pessimistic belief, the low-skill firm believes that a project at a price $p' > p_1$ has potential zero, whereas it believes that a project at price p_1 comes from the referral set $R(p_1)$. Under a more natural belief restriction, the high-skill firm should believe that a project at price p' comes from the subset $(q_1^b, \bar{q})$.

Definition 3 formalizes a belief restriction to address the above issues. It can be applied to a no-referral equilibrium, which has $n = 0$ referral set, as well as to a referral equilibrium, which has $n \geq 1$ referral sets $(q_1^a, q_1^b), ..., (q_n^a, q_n^a)$.

Definition 3 (Monotone Belief) *In an equilibrium with $n \geq 0$ referral sets, the high-skill firm believes that a project at any off-equilibrium price p' comes from a subset K. The belief is said to satisfy monotone belief if K has a positive measure and meets the following conditions.*

(i) If $n \geq 1$, $p' > p_n$, and $S_h(q) \geq S_l(q)$ for each $q \in (q_n^b, \bar{q})$, then $K = (q_n^b, \bar{q})$.

(ii) If $n \geq 2$, $p_m < p' < p_{m+1}$, and $S_h(q) \geq S_l(q)$ for each $q \in (q_m^b, q_{m+1}^a)$, then $K = (q_m^b, q_{m+1}^a)$, for $m = 1, ..., n-1$.

(iii) If $n \geq 1$, $p' < p_1$, and $S_h(q) \geq S_l(q)$ for each $q \in (0, q_1^a)$, then $K = (0, q_1^a)$.

(iv) If none of (i)-(iii) is applicable, then K is unrestricted.

I call an equilibrium which satisfies monotone belief a monotone equilibrium. In a no-referral equilibrium, conditions (i)-(iii) in Definition 3 are not applicable. Monotone belief only imposes the high-skill firm to believe that a project at off-equilibrium prices comes from a subset with a positive measure. In a referral equilibrium, an off-equilibrium price is either larger than every equilibrium prices, between two equilibrium prices, or smaller than every equilibrium prices. Depending on the level of an off-equilibrium price, one of (i)-(iii) in Definition 3 may be applicable. I have discussed the first condition (i) above using Figure 3. Figure 4 in section 1.3.4 illustrates the second condition (ii). It shows an equilibrium with two referral sets, $R(p_1) = (q_1^a, q_1^b)$ and $R(p_2) = (q_2^a, q_2^b)$. There exists a subset of projects with a positive measure, (q_1^b, q_2^a), in between the two referral sets. Also, there is scope for referral for each project within the subset; each $q \in (q_1^b, q_2^a)$ satisfies $S_h(q) > S_l(q)$. Hence, under monotone belief, the high-skill firm believes that a project at p' that is between p_1 and p_2 comes from the subset (q_1^b, q_2^a).

The first condition (i) or the second condition (ii) of Definition 3 may be applicable in an equilibrium, but the third condition (iii) is never applicable, because the low-skill firm is more efficient than the high-skill firm for any project with potential smaller than q^{fb}.[14] This is true in Figure 1.5, although there is a subset $(0, q_1^a)$ with a positive measure in between 0 and $R(p_1)$. In this case, the fourth condition (iv) applies and monotone belief only imposes the low-skill firm to believe that a project at off-equilibrium prices comes from a subset with a positive measure. Indeed, Figure 1.5 is an example of a monotone equilibrium. Lemma 7 characterizes a monotone equilibrium (proof in the Appendix).

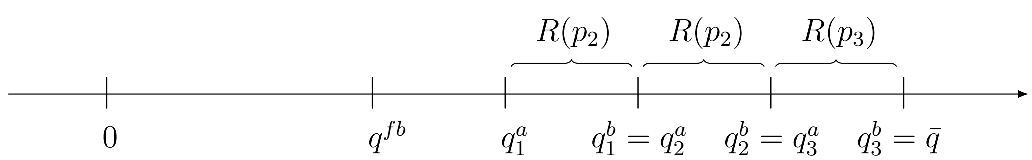

Figure 1·5: A monotone equilibrium

Lemma 7 *In a monotone equilibrium consisting of $n \geq 1$ referral sets (q_1^a, q_1^b), ..., (q_n^a, q_n^a), each project with potential $q \geq q_1^a$ is referred.*

[14] In descending referral, monotone belief is defined by interchanging firms' roles in Definition 3. Then, (ii)-(iii) of monotone belief can be applicable but the first condition (i) is never applicable.

In a monotone equilibrium, the low-skill firm switches between implementation and referral only once as project potential increases. The only difference between a monotone equilibrium's allocation and the first-best allocation is that the equilibrium threshold is larger than the first-best threshold; $q_1^a > q^{fb}$. The equilibrium in Figure 1.4 is not a monotone equilibrium. To see that, consider an off-equilibrium price $p' = S_h(q_1^b)$, which is in between $p_1 = S_h(q_1^a)$ and $p_2 = S_h(q_2^a)$. By monotone belief, the high-skill firm believes that a project at p' comes from the subset (q_1^b, q_2^a). Given the belief, the high-skill firm either best responds by inspecting the project or accepting the referral without inspection. Given either best responses, the low-skill firm who has a project with potential equal or just larger than q_1^b would find it profitable to deviate and refer it at price $p' = S_h(q_1^b)$, instead of following the equilibrium strategy by implementing it and obtaining $S_l(q)$. Similarly, the equilibrium in Figure 1.3, where a subset with a positive measure exists in between the last referral set and $\bar{q}$, is not a monotone equilibrium neither.

Lemma 8 *In an equilibrium, if (i) there is $n \geq 1$ referral sets $(q_1^a, q_1^b), ..., (q_n^a, q_n^a)$ with each project $q \geq q_1^a$ being referred, or (ii) there is $n = 0$ referral set, then there is a monotone equilibrium that shares the same referral sets and allocation with the said equilibrium.*

In the rest of the section, I restrict my attention to monotone equilibria and study their Pareto efficiency, since it is natural to expect firms to play Pareto-efficient equilibria. Lemma 8 says that restriction to monotone equilibria does not rule out a no-referral equilibrium allocation nor any referral equilibrium allocation in which low-potential projects are assigned to the low-skill firm and high-potential projects are assigned to the high-skill firm (proof in the Appendix).

As pointed out by Holmstrom and Myerson (1983), Pareto efficiency can be evaluated from an ex-ante, interim, or ex-post point of view. Each vantage point differs in the extent of how much private information is revealed to agents. In my model, ex-ante Pareto efficiency is evaluated at the beginning of stage 1, before nature assigns a project and reveals the project potential. Interim Pareto efficiency requires the evaluation of expected payoff conditional on the project potential information that has been revealed to the low-skill firm in the middle of stage 1. Pareto efficiency cannot be evaluated ex-post at the end of stage 2 because the degree of information revelation in one equilibrium may be different from another. To see that, consider two equilibria. In the first, a project with potential q is referred to the high-skill firm, but in the second a project with the same potential q is implemented by the low-skill firm. Project potential is revealed to the high-skill firm ex-post in the first equilibrium but not in the second.

Recall that $u_l(q)$ denotes the low-skill firm's equilibrium payoff with project q. Furthermore, define u_h to be the high-skill firm's expected equilibrium payoff before a referral offer is made. Consider an equilibrium which has $n \geq 1$ referral sets, $R(p_1), ..., R(p_n)$, with $R(p_m) = (q_m^a, q_m^b)$, for $m = 1, ..., n$. If $q \in R(p_m)$, for $m = 1, ..., n$, then the high-skill firm's ex-post payoff is $S_h(q) - p_m - c$. Otherwise, its ex-post payoff is zero. Therefore, the high-skill firm's expected equilibrium payoff is $u_h = \int_{q_1^a}^{q_1^b} [S_h(q) - p_1 - c] \, dG(q) + \int_{q_2^a}^{q_2^b} [S_h(q) - p_2 - c] \, dG(q) + ... + \int_{q_n^a}^{q_n^b} [S_h(q) - p_n - c] \, dG(q)$. Clearly, $u_h = 0$ for a no-referral equilibrium.

Definition 4 (Pareto Dominance) *Consider an equilibrium with equilibrium pay-*

off functions $u_l(q)$, $q \in Q$, *and* u_h, *and another equilibrium with equilibrium payoff functions* $u_l'(q)$, $q \in Q$, *and* u_h'. *The former interim Pareto dominates the latter if* $u_l(q) \geq u_l'(q)$, *for* $\forall q$, *and* $u_h \geq u_h'$, *with at least one strict inequality. Moreover, the former ex-ante Pareto dominates the latter if* $\int_Q u_l(q)\ dG(q) \geq \int_Q u_l'(q)\ dG(q)$ *and* $u_h \geq u_h'$, *with at least one strict inequality.*

An equilibrium is said to be interim Pareto-efficient if there does not exist another equilibrium that interim Pareto dominates it. A similar definition follows for an ex-ante Pareto-efficient equilibrium. Proposition 5 provides the necessary and sufficient conditions of an interim Pareto-efficient monotone equilibrium (proof in the Appendix). By definition, these are also necessary conditions for an ex-ante Pareto-efficient monotone equilibrium. Recall that $S_h^{-1}(x)$ is the inverse function of $S_h(q)$ such that if $S_l(q_1^a) \leq S_h(q)$, then $S_h^{-1}(S_l(q_1^a)) \leq q$.

Proposition 5 *First, a monotone equilibrium with* $n \geq 1$ *referral sets is interim Pareto efficient if and only if* $\mathcal{I}\big(q, q_1^a, S_h(q)\big) < c$ *for each* q *satisfying* $S_h^{-1}\big(S_l(q_1^a)\big) \leq q \leq q_1^a$. *Second, a monotone equilibrium with zero referral sets is interim Pareto efficient if and only if* $\mathcal{I}\big(q, \bar{q}, S_h(q)\big) < c$ *for each* q *satisfying* $S_h^{-1}\big(S_l(\bar{q})\big) \leq q \leq \bar{q}$.

Here I show that the above condition is necessary for an interim Pareto efficient monotone equilibrium with $n \geq 1$ referral set. Suppose not. Suppose that there are $n \geq 1$ referral sets $(q_1^a, q_1^b), (q_2^a, q_2^b), ..., (q_n^a, q_n^a)$ in an interim Pareto efficient monotone equilibrium and there exists a project q' satisfying $S_h^{-1}\big(S_l(q_1^a)\big) \leq q \leq q_1^a$ and $c \leq \mathcal{I}\big(q', q_1^a, S_h(q')\big)$. Call this equilibrium σ. First, by Proposition 2, I can construct another equilibrium σ' with $n+1$ referral sets $(q', q_1^a), (q_1^a, q_1^b), (q_2^a, q_2^b), ..., (q_n^a, q_n^a)$, because the first referral set (q', q_1^a) satisfies its incremental inspection constraint and production efficiency constraint by the above assumption $S_h^{-1}\big(S_l(q_1^a)\big) \leq q \leq q_1^a$ and $c \leq \mathcal{I}\big(q', q_1^a, S_h(q')\big)$, and the last n sets are the same as σ's referral sets and must satisfy the corresponding two constraints. In σ', all $q \geq q'$ are referred, so Lemma 8 ensures that σ' can be constructed as a monotone equilibrium. Clearly, σ' interim Pareto dominates σ, which is a contradiction. The proof in the Appendix shows that this necessary condition is also sufficient. Proposition 5 extends naturally to a monotone equilibrium with $n = 0$ referral set with a similar proof.

For a given monotone equilibrium to be interim Pareto efficient, Proposition 5 says that the inspection cost must not be too small, otherwise it is interim Pareto dominated by another monotone equilibrium which has a larger set of referred projects. Proposition 5 provides insight on the range of projects being referred in an interim Pareto monotone equilibrium. It characterizes the referral threshold q_1^a; one should not be able to construct a referral set that satisfies both the production efficiency constraint and the incremental inspection using q_1^a as the supremum of the set. Again, I use the quadratic-uniform model as an example to further explain Proposition 5.

Example (continued) *Suppose that the model is quadratic-uniform. Also, suppose that $c < c_R$ so there exists a unique $q' < \bar{q}$ such that $\mathcal{I}\big(S_h^{-1}(S_l(q')), q', S_l(q')\big) = c$. Then, in an interim Pareto efficient monotone equilibrium all $q \geq q'$ are referred and q_1^a satisfies $S_h^{-1}(S_l(q')) \leq q_1^a < q'$.*

By Lemma 8, in an interim Pareto efficient monotone equilibrium, all $q \geq q_1^a$ are referred. So I only have to prove that $S_h^{-1}(S_l(q')) \leq q_1^a < q'$. As shown in Section 1.3, $\big(S_h^{-1}(S_l(q)), q\big)$ is the largest referral set with q being the supremum of the set.

And $\mathcal{I}\big(S_h^{-1}(S_l(q)), q, S_l(q)\big) = \frac{r^2}{24}\big(q - S_h^{-1}(S_l(q))\big)^2$ is the corresponding incremental inspection surplus. Since $\mathcal{I}\big(S_h^{-1}(S_l(q)), q, S_l(q)\big)$ is an increasing function in q and $\mathcal{I}\big(S_h^{-1}(S_l(q')), q', S_l(q')\big) = c$, in an equilibrium q' is the smallest supremum of a referral set. Therefore, in an equilibrium, $q_1^b \geq q'$. By Proposition 1, $S_h(q_1^a) \geq S_l(q_1^b)$, which means that $q_1^a \geq S_h^{-1}(S_l(q_1^b)) \geq S_h^{-1}(S_l(q'))$. To complete the proof I have to show that $q_1^a < q'$. Suppose not. Suppose that in an interim Pareto efficient monotone equilibrium $q_1^a \geq q'$. It follows that

$$\mathcal{I}\big(S_h^{-1}(S_l(q_1^a)), q_1^a, S_l(q_1^a)\big) \geq \mathcal{I}\big(S_h^{-1}(S_l(q')), q', S_l(q')\big) = c,$$

which is a contradiction to Proposition 5.

By putting more structure into the model, I pin down the range of q_1^a and the range of projects that are referred in an interim Pareto efficient monotone equilibrium. However, the number and the size of referral sets in different interim Pareto efficient monotone equilibrium still vary.

1.6 Extensions

In this section, I discuss two extensions of ascending referral. They apply to descending referral as well. First, I allow the high-skill firm's inspection to be contractible. Second, I endogenize the low-skill firm's inspection decision. I construct equilibria with pessimistic belief throughout the section.

1.6.1 Contractible Inspection

In this section, I change the assumption that inspection is a hidden action and let inspection be contractible. The low-skill firm may now conduct a referral using a two-dimensional contract (p, r) in stage 1, where $p \in (0, S_h(\bar{q})]$ denotes the referral price and $r \in [0, c]$ denotes the payment from the low-skill firm to the high-skill firm if and only if the high-skill firm has done the inspection. The rest of the extensive form remains unchanged. If the low-skill firm offers a contract with $0 \leq r < c$, then the high-skill firm's effective inspection cost becomes $c - r$. If the low-skill firm offers a contract with $r = c$, then the high-skill firm inspects the project regardless of its belief. In a referral equilibrium of the main model, there must be a referral set (q^a, q^b) satisfying the incremental inspection constraint. The low-skill firm pools projects within (q^a, q^b) at $p = S_h(q^a)$ and the high-skill firm accepts after inspection. If inspection is contractible, would the low-skill firm pool projects within (q^a, q^b) at an equivalent contract with $p = S_h(q^a)$ and $r = 0$? Lemma 9 provides a negative answer.

Lemma 9 *Suppose inspection is contractible. In an equilibrium, the low-skill firm refers a project q at a contract (p, r) with $p = S_h(q)$ and $r = c$.*

Here is the proof. First, note that Lemma 3-6 and Corollary 1 still apply. Suppose not. Suppose that, in an equilibrium, the low-skill firm refers each project $q \in (q^a, q^b)$ at a contract (p', r') with $0 \leq r' < c$. The high-skill firm accepts each project $q \in (q^a, q^b)$ at contract (p', r') only after inspection, which means that $p' \leq S_h(q^a)$ and $\mathcal{I}(q^a, q^b, S_h(q^a)) \geq c - r > 0$. Using the same logic to prove that the first condition (i) of Proposition 1 is necessary for a referral equilibrium, I can say that

$p' = S_h(q^a)$. Also, expand $\mathcal{I}(q^a, q^b, S_h(q^a)) \geq c - r > 0$ according to (1.5) to get

$$\frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[S_h(q) - (c - r) - S_h(q^a) \Big] dG(q)$$
$$\geq \max_e \frac{1}{G(q^b) - G(q^a)} \int_{q^a}^{q^b} \Big[y_h(e, q) - f_h - S_h(q^a) \Big] dG(q) > 0, \quad (1.8)$$

which implies that

$$\int_{q^a}^{q^b} \Big\{ S_h(q) - c - [S_h(q^a) - r] \Big\} dG(q) > 0. \quad (1.9)$$

By (1.9) there exists a $q' \in (q^a, q^b)$ such that $S_h(q) - c - [S_h(q^a) - r] > 0$ for all $q \geq q'$. Therefore, the low-skill firm with a project $q \in (q', q^b)$ will deviate from the contract (p', r') to a contract (p, r) with $p = S_h(q)$ and $r = c$ because the deviation payoff $S_h(q) - c$ is larger than the equilibrium payoff $S_h(q^a) - r$. By fully reimbursing the high-skill firm's inspection cost, the low-skill firm may increase the referral price from $S_h(q^a)$ to $S_h(q)$ and extract all rent from the high-skill firm. Proposition 6 characterizes the unique equilibrium allocation when inspection is contractible.

Proposition 6 *Suppose inspection is contractible. A referral equilibrium exists if and only if* $c < S_h(\bar{q}) - S_l(\bar{q})$*. The low-skill firm implements each project* $q \leq q^c$ *with effort* $\hat{e}_l(q)$ *and refers* $q > q^c$ *at a contract* $(S_h(q), c)$*. The equilibrium threshold* q^c *is characterized by* $S_h(q^c) - c = S_l(q^c)$*, which implies that* $q^c > q^{fb}$*. The high-skill firm inspects, accepts and implements each referred project* q *at* $(S_h(q), c)$ *with* $\hat{e}_h(q)$*. At an off-equilibrium contract with* $r = c$*, the high-skill firm inspects the project. At an off-equilibrium contract with* $r < c$*, the high-skill firm updates with pessimistic belief* *and rejects it.*

I provide the proof here. By Lemma 9, in an equilibrium, the low-skill firm with project q either implements it to get a payoff of $S_l(q)$ or refers it at a contract $(S_h(q), c)$ to get a payoff of $S_h(q) - c$. If $c < S_h(\bar{q}) - S_l(\bar{q})$, then there exists a unique project q^c satisfying $S_h(q^c) - c = S_l(q^c)$. Therefore, the low-skill firm refers each project $q > q^c$ at a contract $(S_h(q), c)$ and implements the rest. Finally, in a referral equilibrium, the low-skill firm refers a project q at $(S_h(q), c)$ only if the referral payoff $S_h(q) - c$ is larger than its implementation payoff $S_l(q)$, which implies that $c < S_h(\bar{q}) - S_l(\bar{q})$.

The equilibrium allocation is unique because the high-skill firm inspects a project whenever its inspection cost is reimbursed, and the inspection eliminates adverse selection, which allows the low-skill firm to refer all projects with potential larger than q^c regardless of the high-skill firm's belief. The equilibrium is also separating: each referred project is associated with a unique contract, and the low-skill firm extracts all rents from the high-skill firm. The only source of inefficiency of the equilibrium is that $q^c > q^{fb}$. It is costly for the low-skill firm to reimburse the inspection cost, so it retains some projects that would have been referred in the first best. As in an equilibrium of the main model, both the low-skill firm and the high-skill firm implement projects with their respective efficient efforts.

1.6.2 Endogenous Information for the Low-skill Firm

In the main model, the low-skill firm's private information about the project is exogenous. In order to check the robustness of the model, I change the assumption so that the low-skill firm is uninformed initially. The low-skill firm must incur an inspection cost c to learn about a project's potential privately before implementing or referring it. Both firms' inspections are their own hidden actions. I assume firms' inspection costs to be identical, but this assumption is for simple exposition only. The rest of the extensive form remains unchanged. In an equilibrium, the low-skill firm optimally chooses its inspection decision and referral decision given the high-skill firm's best response and belief. When the high-skill firm receives the referral price, it forms beliefs about the project potential as well as the low-skill firm's inspection decision. Pessimistic belief is redefined; at any off-equilibrium price, the high-skill firm believes that the low-skill firm has inspected the project and has referred the one with the lowest potential. I will construct equilibria with this modified version of pessimistic belief. Lemma 10 says that the high-skill firm's equilibrium inspection must be preceded by the low-skill firm's if the inspection cost is low enough (proof in the Appendix).

Lemma 10 *Suppose the low-skill firm must incur an inspection cost c to privately learn a project's potential. Given the fixed costs, return functions and project potentials' distribution, there exists a $\hat{c} > 0$ such that if $c \leq \hat{c}$, then there is no equilibrium in which the low-skill firm refers a project without inspection but the high-skill firm inspects the referred project.*

The argument is by contradiction. Consider a referral equilibrium in which the low-skill firm did not inspect and referred all projects at p to the high-skill firm, and the high-skill firm inspected at p. The high-skill firm would accept the high-potential projects with which it could generate a surplus higher than p and would reject the others. What could the low-skill firm gain if it deviated and inspected before conducting a referral? First, rather than sending the low-potential projects to the high-skill firm and getting rejected, the low-skill firm could implement them. Second, the low-skill firm could retain and implement the high-potential projects with which it could generate a higher surplus than p. For the low-skill firm, the combined information gain would be higher than the inspection cost if the latter was low enough. Therefore, the high-skill firm's equilibrium inspection must be preceded by the low-skill firm's if inspection cost c is low enough. The exact exposition of $\hat{c}$ is provided in the Appendix. I use Lemma 9 to prove Proposition 7 (proof in the Appendix).

Proposition 7 *Suppose the low-skill firm must incur an inspection cost c to privately learn a project's potential. Given the fixed costs, return functions and project potentials' distribution, there exists a $\tilde{c}$ satisfying $0 \leq \tilde{c} \leq \hat{c}$ such that if $c \leq \tilde{c}$, then in an equilibrium the low-skill firm inspects a project before referring or implementing it.*

Suppose the inspection cost c is low. Clearly, in a no-referral equilibrium, the low-skill firm inspects a project in order to implement it with the firm's efficient effort. Is there a referral equilibrium in which the low-skill firm refers a project to the high-skill firm without anybody inspecting the project? The answer is negative. If the low-skill firm refers without inspection, then the high-skill firm inspects in order to reject lemons and implement accepted projects with its efficient effort. However, by Lemma 9, there is no equilibrium in which the high-skill firm inspects but the low-skill firm does not. Proposition 7 follows; if inspection cost is low, then in an equilibrium the low-skill firm inspects a project before it refers or implements the project.

1.7 Subcontracting and Cost-reduction Effort

In this section, I discuss how my model can be used to understand subcontracting in the construction industry. In the construction industry, production involves a contractor completing a project by incurring a construction cost, which can be reduced by the contractor's chosen effort. However, the contractor can also subcontract the project to another contractor.

I make a few modifications to the main model. There is still a set of projects denoted by q, which has the same support, distribution function, and density function as before. But q now denotes a project's difficulty index, which affects a contractor's construction cost as shown below. There are two risk-neutral contractors. A low-skill contractor can complete a project q by incurring a fixed cost f_l and a variable cost $k_l(e,q)$, whereas a high-skill contractor can complete a project q by incurring a fixed cost f_h and a variable cost $k_h(e,q)$. A completed project yields a fixed return $R > 0$ regardless of its difficulty q.

Each contractor's variable cost is a function of project difficulty and the effort he exerts. Let the variable cost functions be $k_i : \mathbb{R}_+ \times Q \to \mathbb{R}_+$, $i = l, h$. The variable cost functions are strictly increasing in project difficulty q; $\frac{\partial k_h(e,q)}{\partial q} > 0$ and $\frac{\partial k_l(e,q)}{\partial q} > 0$. I assume that a contractor needs to choose an effort in order to balance the marginal benefit and marginal disutility of effort to minimize its variable cost. The variable cost functions $k_l(e,q)$ and $k_h(e,q)$ are twice-differentiable and strictly convex in e. Also, $\min_e k_l(e,0) = \min_e k_h(e,0) = 0$ and $k_l(0,q) = k_h(0,q) = 0$ for $\forall q \in Q$. Given a project with positive difficulty q, $k_l(e,q)$ and $k_h(e,q)$ each achieves an interior minimum at a positive number at a positive effort e. Finally, the cross derivatives of the variable cost functions are negative; $\frac{\partial^2 k_l(e,q)}{\partial e \partial q} < 0$ and $\frac{\partial^2 k_h(e,q)}{\partial e \partial q} < 0$.

This implies that $k_i(e_1, q_1) - k_i(e_2, q_1) < k_i(e_1, q_2) - k_i(e_2, q_2)$, for $i = l, h$, with $e_1 < e_2$ and $q_1 < q_2$. For each contractor, an increase in effort reduces the variable cost of a more difficult project to a larger extent. In other words, choosing the right effort is more important when a contractor works on a more difficult project.[15]

As in the main model, the two contractors have different comparative advantages. The high-skill contractor has to incur a higher fixed cost than the low-skill contractor to complete a project; $f_h > f_l = 0$. However, the high-skill contractor's variable cost is lower and it is more productive in reducing variable cost. First, given a project q and an effort e, the high-skill contractor's variable cost is lower, so that $k_h(e, q) < k_l(e, q)$. Second, the high-skill contractor can reduce variable cost more than the low-skill contractor can by increasing effort; $\frac{\partial k_h(e,q)}{\partial e} \leq \frac{\partial k_l(e,q)}{\partial e}$. Third, the high-skill contractor's marginal variable cost of project difficulty is lower; $\frac{\partial k_h(e,q)}{\partial q} < \frac{\partial k_l(e,q)}{\partial q}$. Finally, I assume that $\min_e k_h(e, \bar{q}) + f_h < \min_e k_l(e, \bar{q}) < R$. The first inequality makes sure that the high-skill contractor is more efficient in completing some projects, whereas the second inequality says that any project should be completed by either contractor.

As we can see above, the modified model's setup is like a mirror image of the main model's. Lemma 11 says that the low-skill contractor completes less difficult projects and the high-skill contractor completes more difficult projects in the first best. The modified model's first best is a mirror image of the main model's in the sense that the less productive and less costly contractor is now responsible for projects that

[15]Despite the similarities, I have not assumed that $R - k_i(e, q)$ is equal to $y_i(e, q)$, for $i = l, h$, which are the return functions in the main model. To see that, suppose firms variable cost functions $d_l(e, q)$ and $d_h(e, q)$ were characterized by $R - d_i(e, q) = y_i(e, q)$, for $i = l, h$. That meant $\frac{\partial d_i(e,q)}{\partial q} < 0$, for $i = l, h$; a project with a higher q was easier to complete. However, that also meant $\frac{\partial^2 d_i(e,q)}{\partial e \partial q} < 0$, for $i = l, h$, which would mean an increase in effort reduces the variable cost of an easier project to a larger extent and choosing the right effort is more important when a contractor works on an easier project. This would not be a natural assumption.

are preferred by both contractors, the less difficult projects. Note that the high-skill firm's efficient cost $\Gamma_h(q)$ increases slower than the low-skill firm's efficient cost $\Gamma_l(q)$ as project difficulty increases. The proof's logic is the same as the ones in Lemma 1 and Lemma 2.

Lemma 11 *Let* $\Gamma_l(q) \equiv \min_e k_l(e,q)$ *and* $\Gamma_h(q) \equiv \min_e k_l(e,q) + f_h$*. Then,* $\frac{d\Gamma_l(q)}{dq} > \frac{d\Gamma_h(q)}{dq} > 0$ *for* $q \in Q$*. There exists a unique* $q^{fb} \in Q$ *such that* $\Gamma_h(q) > \Gamma_l(q)$ *for* $q < q^{fb}$*,* $\Gamma_l(q) < \Gamma_h(q)$ *for* $q > q^{fb}$*, and* $\Gamma_h(q^{fb}) = \Gamma_h(q^{fb})$*.*

Next, I can study a subcontracting game where an informed contractor can either complete a project himself or subcontract it to another contractor for completion. The uninformed subcontractor can learn about the project's difficulty at an inspection cost before accepting or rejecting the offer. There is only one difference between a subcontracting game and a referral game. In the subcontracting game, the ownership of the project cannot be transferred. I assume that the only contractible event is the completion of a project, so the informed contractor can use a lump-sum payment to subcontract a project. This assumption is driven by practices in the construction industries.[16] Although the subcontractor does not get the project return R when it completes the project, it gets paid by a fixed amount and has the full incentive to reduce cost. Therefore, all the results that I have proved under a referral game apply to a subcontracting game as well. As I have discussed in the introduction, the modified model's setup and results resonate with the practices in the construction industry.

[16] Here is an example. A general contractor agrees to build a guest house in a customer's backyard. He may subcontract the work to another contractor, but the general contractor is the party who gets paid by the customer when the house is finished.

1.8 Conclusion

I have provided a model to study how the market can facilitate matching of projects with firms for efficient implementation in the presence of hidden action, hidden information and endogenous inspection. In contrast to previous works on referral, I point out that both the referring and the referred firms may inspect a project, and the referred firm's inspection is needed to support two-way referrals and referrals at multiple equilibrium prices in an equilibrium. I also point out that inspection benefits a referred firm in two ways. First, the firm can use the project information to make referral acceptance decision. Second, the firm can use the information to tailor its production effort for maximum efficiency. This intuition allows me to solve a model in a rich environment with a continuum of project types and variable effort. This leads to partial-pooling equilibria, with prices serving as signals of the corresponding projects' potential. In an equilibrium, the referred firm inspects solely to exert the best effort according to project potential. Such inspection deters lemons and supports the market. The referral market is more efficient in matching projects with firms than other authors have suggested. My model can also be used to explain subcontracting practices in the construction industry.

There are rooms for future work. First, the framework in this chapter can be studied from a principal-expert perspective. A principal has a project, and he needs to choose an optimal way to incentivize agents to acquire information and report truthfully. According to the information, the principal then allocates the project to one of the two experts. Sometimes, the principal may rely on one expert for project information acquisition and execution. Other times, the principal may rely on different experts for the two tasks. It is interesting to study the principal's tradeoff.

In my model, the referring firm proposes a take-it-or-leave-it offer. One can model the bargaining process more explicitly. For example, an informed firm can propose a price, but an uninformed firm may inspect the project before accepting or making a counter-offer. It is interesting to see how the uninformed firm's incentive to exert surplus-maximizing effort can affect the bargaining process.

Lastly, one may study a more complicated inspection technology with which firms exert variable effort to acquire imperfect information about a project. In this case, the first best may require both firms to acquire information. A referring firm and a referred firm may disagree with each other on the project potential even after they have both inspected a project. It is interesting to study how firms can aggregate their information through a referral market.

Chapter 2

Gatekeeping Mechanism and Information Acquistion

2.1 Introduction

I consider a mechanism design problem in which a Principal would like to efficiently assign a project among a low-skill expert and a high-skill expert based on the project's difficulty in order to exploit experts' cost comparative advantages. However, the Principal relies on the same group of experts to inform him about the project's difficulty. I study the contract and the structure of an optimal mechanism and experts' incentive to acquire and report information in the mechanism.

Consider the following example in the healthcare industry. A payer relies on either a generalist physician or a specialist physician to diagnose a patient. If the patent's issue is mild, then the payer would like the generalist to treat the patient. If the issue is serious, then the payer would like the specialist to provide the service. It is not cost-efficient for a physician to treat a mismatched patient. The payer contracts with the physicians by choosing one of the physicians as a gatekeeper. The gatekeeper is responsible for diagnosing the patient's condition. Then, based on the diagnosis report, the gatekeeper decides whether he should treat the patient or refer the patient to the non-gatekeeper physician. In a typical health maintenance organization (HMO) insurance plan, a generalist physician serves as a gatekeeper to oversee the medical care of patients and make referrals to a specialist physician when necessary.

This environment raises some interesting questions. In the above example, the generalist is chosen as the gatekeeper. What is the driving force behind the choice? To understand this issue I adopt a relaxed view to think about a generalist's and a specialist's comparative advantages. Secondly, the payer relies on the same group of physicians to diagnose a patient, make a report to the payer, and serve the patient. How should the payer structure an optimal contract in order to incentivize the gatekeeper to carry out the diagnosis and refer the patient efficiently? Can the first best be implemented? The above questions are also relevant in other industries, such as the accounting, financial, and construction industries, where organizations use gatekeeper to screen incoming projects.

In my model, there is a set of projects; each project has a different difficulty state. Each of two experts can implement a project by incurring a cost. For each expert, it is more costly to implement a more difficult project. However, the two experts have different cost comparative advantages: a low-skill expert has zero fixed cost but a high marginal cost of difficulty, whereas a high-skill expert has a positive fixed cost but a low marginal cost of difficulty. In the first best, the low-skill expert should implement less difficult projects and the high-skill expert should implement more difficult projects.

The Principal needs to rely on one of the experts to implement a project but he does not know the project state. In a mechanism, he offers a contract to both experts. Through the contract, the Principal appoints an expert as a gatekeeper. Moreover, the contract consists of the Principal's payments and his probabilities of assigning the project to each expert. The payments and the assignment probabilities are functions of the gatekeeper's eventual report. After both experts accept the contract, the gatekeeper may acquire information about the project and make a report to the Principal, who then pays and assigns the project to the experts based on the report. Finally, the assigned expert implements the project. The information acquisition timing is justified by the practices of the healthcare industry. Physicians sign new contracts determining their payment schemes on a yearly basis. However, patients register with a healthcare network and they go to see a doctor only when they feel sick.

Clearly, the Principal can offer a contract with constant payments and constant project assignment probabilities if he does not intend to assign projects based on the gatekeeper's report. In this case, it does not matter which expert is chosen as the gatekeeper. Such a contract can be optimal if information acquisition cost is very high. Otherwise, when designing an optimal contract, the Principal must choose the right expert as a gatekeeper. Also, he must provide incentive for the gatekeeper to acquire information and make a report truthfully.

I find that it is never optimal for the Principal to choose the high-skill expert as the gatekeeper even though each expert has his own cost comparative advantage. To incentivize a high-skill gatekeeper to report truthfully, the Principal has to assign projects to him with a smaller probability as project difficulty increases. However, this is costly for the Principal since the high-skill expert is more efficient in implementing a more difficult project.

Instead, the Principal optimally chooses the low-skill expert as the gatekeeper. Unlike the above, assigning more difficult projects to the low-skill gatekeeper with a smaller probability for truthful reporting purpose is incidentally cost-efficient. Under some conditions, the optimal low-skill gatekeeper mechanism also implements the first best. That means it is without loss of generality to focus on low-skill gatekeeper mechanism. Otherwise, a mechanism with low-skill gatekeeper may not implement the first best if information acquisition cost is large. In this case, the Principal assigns low-difficulty and high-difficulty projects to the low-skill and high-skill expert, respectively. He assigns medium-difficulty projects to the low-skill expert with a probability that decreases in difficulty in steps. The Principal uses the step probability assignment function assign projects efficiently while incentivizing the low-skill gatekeeper to acquire information and report the truth.

The remainder of the chapter is organized as follows. Section 2.2.1 provides a review of the literature. Section 2.2 presents the model setup and the first best. Section 2.3 studies a mechanism without a gatekeeper. Section 2.4 studies a mechanism with a gatekeeper. Section 2.5 considers a more general cost structure. Section 2.6 concludes. Finally, the Appendix contains proofs of results.

2.1.1 Literature Review

This model provides insight on internal organizations and justifies the use of a low-skill and low-cost gatekeeper to screen incoming projects. Unlike the extant literature, my chapter does not assume that experts differ in their discrete ability in completing a project. In both the health and the management literature, Allard, Jelovac, and Léger (2011), Malcomson (2004), Mariñoso and Jelovac (2003), Lee, Shumkey, and Pinker (2012), and Shumsky and Pinker (2003) assume that both a generalist and a specialist can cure a common disease, but only the specialist can cure a severe disease.[1] The generalist is assumed to be the gatekeeper, who can privately diagnose patient and make referral to the specialist. The literature studies various contracts. In contrast, my chapter assumes that the low-skill expert and the high-skill expert have different costs comparative advantages but each can implement a project of any difficulty. Each expert can acquire information and be chosen as the gatekeeper. I show that this more relaxed assumption on experts' production function is enough to justify the usage of the low-skill expert as the gatekeeper. Also, I show that under some conditions focusing on gatekeeper mechanism is without loss of generality.

This chapter contributes to the literature on information acquisition in principal-agent problems; see Crémer, Khalil, and Rochet (1998), Krähmer and Strausz (2011), Lewis and Sappington (1997), and Szalay (2009). In these papers, the Principal uses contracts to incentivize information acquisition and truthful revelation in order to minimize cost. In Gromb and Martimort (2007), the Principal relies on two identical experts to gather and report signals about a project's value, but their focus is on experts' collusion. In my chapter, the Principal's objective to save cost may conflict with his need to incentivize information acquisition and truthful revelation because he may assign projects among two agents with different cost comparative advantages using the information.

Finally, this chapter is related to the referral literature. In Garicano and Santos (2004), Grassi and Ma (2016), and this chapter, experts have different comparative advantages in marginal productivities and fixed costs. The papers above show that referral under adverse selection leads to inefficiency. In contrast, I follow a mechanism

[1] In Allard, Jelovac, and Léger (2011) and Mariñoso and Jelovac (2003), disease type is binary. In Malcomson (2004), Lee, Shumkey, and Pinker (2012), and Shumsky and Pinker (2003), disease type is continuous.

design approach and show that gatekeeping can be efficient. This justifies a centralized approach to the matching problem.

2.2 Model

2.2.1 Projects, Experts, and Information Structure

There is a continuum of projects with total mass set at 1. Each project is indexed by a difficulty state s, which is a random variable distributed on a support $S \equiv [0, \bar{s}]$, with a continuous distribution function $G(\cdot)$ and a density function $g(\cdot)$. Define the inverse hazard rate as $h \equiv [1 - G]/g$ and the inverse reverse hazard rate as $k \equiv G/g$. I assume that $h'(s) < 0$ and $k'(s) > 0$, for all $s \in S$. I assume that the Principal must implement the project regardless of difficulty.[2]

There are two risk-neutral experts, a low-skill expert and a high-skill expert. Either expert can implement a project at a cost. An expert's implementation cost is a function of the project state. The low-skill expert's implementation cost is s, whereas the high-skill expert's is $c_h(x) \equiv \alpha s + \beta$. The low-skill expert's cost function is a normalization. In Section 2.3, I consider a more relax cost function for the high-skill expert. It is more costly for either expert to implement a more difficult project (project with a higher s). However, I assume that the two experts have different comparative advantages in cost. On the one hand, the low-skill expert has a lower fixed cost than the high-skill expert, so that $0 < \beta$. On the other hand, the low-skill expert has a higher marginal cost of project difficulty than the high-skill expert, so that $0 \leq \alpha < 1$. Also, I assume that $\frac{\beta}{1-\alpha} < \bar{s}$ so that the high-skill expert is more efficient in implementing some projects.

[2] In the healthcare context, a payer needs to provide medical service to a patient who bought an insurance from him regardless of the severity of the patient's illness.

Define s^{fb} as the difficulty level such that $s^{fb} = \alpha s^{fb} + \beta$. Finally, define μ_l as $\int_0^{\bar{s}} x dG(x)$, which denotes the low-skill expert's average cost, and μ_h as $\int_0^{\bar{s}} (\alpha x + \beta) dG(x)$, which denotes the high-skill expert's average cost.

Neither the Principal nor the experts observe projects' states. However, an expert can incur a cost $f > 0$ to acquire perfect information about a project's state. The Principal has no information acquisition capability.

2.2.2 First Best

An allocation consists of information acquisition(s) effort carried out by either or both experts and an assignment of a project s to one of the experts. The first-best allocation is one that minimizes project s's expected information acquisition and implementation cost. In the first best, at most one of the two experts acquires information about the project's state s. On the one hand, if an expert acquires information, then the project is assigned to the expert who has the lower implementation cost. On the other hand, if no one acquires information, then the project is assigned to the expert who has the lower average cost. When $\mu_h \leq \mu_l$, the first best involves an expert acquiring information if and only if

$$\int_0^{s^{fb}} x dG(x) + \int_{s^{fb}}^{\bar{s}} [\alpha x + \beta] dG(x) + f \leq \mu_h$$

$$f \leq \int_0^{s^{fb}} \big[\beta - (1-\alpha)x\big] dG(x).$$

When $\mu_l \leq \mu_h$, the first best involves an expert acquiring information if and only if

$$\int_0^{s^{fb}} x dG(x) + \int_{s^{fb}}^{\bar{s}} [\alpha x + \beta] dG(x) + f \leq \mu_l$$

$$f \leq \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x).$$

Lemma 1 summarizes the first best.

Lemma 1 *Let* $f^{fb} \equiv \min\left\{ \int_0^{s^{fb}} \big[\beta - (1-\alpha)x\big] dG(x), \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x) \right\}$. *In the first best, (i) if* $f > f^{fb}$*, then no expert acquires information and the project* s *is assigned to the expert who has the lower average cost, and (ii) if* $f \leq f^{fb}$*, then one of the two experts acquires information and the project* s *is assigned to the low-skill expert if* $s \leq s^{fb}$ *and to the high-skill expert if* $s > s^{fb}$.

In the rest of the chapter, I present three classes of mechanisms. In each class, the Principal offers a contract to both experts. In the first class, the Principal does not induce experts to acquire information. In the second and the third class, the Principal induces one expert to acquire information and then assigns the project according to the information revealed by that expert. The expert who is induced to acquire and reveal information is called the gatekeeper of the mechanism. I assume that the gatekeeper can only acquire information after they accept a contract. As I have explained in the introduction, this is justified by the practices of the healthcare industry since physicians sign new contracts determining their payment schemes on a yearly basis.

2.3 Mechanism without a Gatekeeper

In this section, I analyze a mechanism in which the Principal does not induce experts to acquire information. This works as a reference point for later sections. In Stage 1, the Principal offers a contract (t_l, t_h, p_l) to the experts. The contract is a vector of constants; t_l is the Principal's payment to the low-skill expert and t_h is the Principal's payment to the high-skill expert. The Principal assigns the project to the low-skill expert with probability p_l and to the high-skill expert with probability $1 - p_l$.[3]

In Stage 2, both expert decide whether to accept the contract simultaneously. If either expert rejects, then the game ends and everyone gets a payoff of zero. The game proceeds if and only if each expert accepts the contract. Then, the Principal implements the contract according to the report. Finally, the assigned expert implements the project.

The extensive form of the mechanism is the following. I call this a no-gatekeeper mechanism.

Stage 1: The Principal has a project with state s determined by distribution G. He offers a contract (t_l, t_h, p_l) to the experts.

Stage 2: Both experts decide whether to accept the contract simultaneously. If one of them rejects, the game ends. If both accept, then the Principal pays the low-skill expert t_l and the high-skill expert t_h. He assigns the project to the low-skill expert with probability p_l and to the high-skill expert with $1 - p_l$. Then, the assigned expert implements the project.

[3] I can instead analyze a contract $(t_l(s), t_h(s), p_l(s), p_h(s))$, with $p_l + p_h \leq 1$, which would not change the chapter's results.

The Principal's strategy is defined by a decision to offer a contract in Stage 1. The low-skill expert's strategy is defined by a decision whether to accept a contract in Stage 2 and the high-skill expert's strategy is defined by a decision whether to accept a contract in Stage 2. The solution concept is perfect Bayesian equilibrium.

Throughout the chapter, I only study equilibria in which both experts accept the Principal's contract. Furthermore, I define an optimal no-information mechanism as a no-information mechanism that minimizes the Principal's expected payment among all no-information mechanisms. I call an equilibrium of a no-gatekeeper mechanism a no-information equilibrium. An optimal no-information equilibrium is defined analogously. Finally, I call the contract in an optimal no-information equilibrium an optimal no-information contract.

In a no-information equilibrium, both experts must accept the Principal's contract (t_l', t_h', p_l'). Consequently, the contract must satisfy the low-skill expert's acceptance constraint

$$\int_0^{\bar{s}} (t_l' - p_l' x) dG(x) \geq 0, \tag{2.1}$$

and the high-skill expert's acceptance constraint

$$\int_0^{\bar{s}} \left[t_h' - (1 - p_l')(\alpha x + \beta)\right] dG(x) \geq 0. \tag{2.2}$$

An optimal no-information contract solves the following program.

$$\min_{t_l', p_l', t_h'} \int_0^{\bar{s}} \left[t_l' + t_h'\right] dG(x), \tag{2.3}$$

subject to (2.1) and (2.2). In this minimization problem both (2.1) and (2.2) must bind. I can substitute the binding constraints into the objective function to get

$$\min_{t_l', p_l', t_h'} \big\{ p_l' \mu_l + (1 - p_l')\mu_h \big\}. \tag{2.4}$$

An optimal no-information contract is simple; the Principal assigns projects regardless of difficulty to the expert who has the lower average cost.

If $f > f^{fb}$, then the first best does not involve information acquisition. Therefore, the Principal can implement the first best by offering an optimal no-information contract. This contract can also be used as a reference point in the next section.

2.4 Mechanism with a Gatekeeper

In this section, I analyze a mechanism in which an expert is chosen as a gatekeeper to make a report. The gatekeeper can acquire information only after both experts accept a contract. Suppose the low-skill expert is chosen as the gatekeeper. In Stage 1, the Principal offers a contract $\big(t_l(s), t_h(s), p_l(s)\big)$ to the experts. The contract is a vector of functions of the low-skill expert's eventual report $s \in S$. The function $t_l(s)$ is the Principal's payment to the low-skill expert and the function $t_h(s)$ is the Principal's payment to the high-skill expert. The Principal assigns the project to the low-skill expert with probability $p_l(s)$ and to the high-skill expert with probability $1 - p_l(s)$.[4]

In Stage 2, both expert decide whether to accept the contract simultaneously. If either expert rejects, then the game ends and everyone gets a payoff of zero. The

[4]As in the last section, I can instead analyze a contract $\big(t_l(s), t_h(s), p_l(s), p_h(s)\big)$, with $p_l(s) + p_h(s) \leq 1$, which would not change the chapter's results.

game proceeds if and only if each expert accepts the contract. In Stage 3, the low-skill expert decides whether to acquire information about the project. In Stage 4, the low-skill expert makes a report $s \in S$. Then, the Principal implements the contract according to the report. Finally, the assigned expert implements the project. The case in which the high-skill expert is chosen as the gatekeeper is defined analogously. I analyze it later in this section.

2.4.1 Low-skill Gatekeeper

The extensive form of a mechanism in which the low-skill expert is chosen as the gatekeeper is the following. I call this a low-skill gatekeeper mechanism.

Stage 1: The Principal has a project with state s determined by distribution G. He offers a contract $\big(t_l(s), t_h(s), p_l(s)\big)$, for all $s \in S$, to the experts.

Stage 2: Both experts decide whether to accept the contract simultaneously. If one of them rejects, the game ends.

Stage 3: The low-skill expert decides whether to acquire information about the project.

Stage 4: The low-skill expert makes a report s. The Principal pays the low-skill expert $t_l(s)$ and the high-skill expert $t_h(s)$. He assigns the project to the low-skill expert with probability $p_l(s)$ and to the high-skill expert with $1 - p_l(s)$. Then, the assigned expert implements the project.

The Principal's strategy is defined by a decision to offer a contract in Stage 1. The low-skill expert's strategy is defined by (i) a decision whether to accept a contract in Stage 2, (ii) a decision whether to acquire information about the project in Stage

3, and (iii) a report of the project type in Stage 4. The high-skill expert's strategy is defined by (i) a decision whether to accept a contract in Stage 2. The solution concept is perfect Bayesian equilibrium.

I only study equilibria in which both experts accept the Principal's contract and the low-skill gatekeeper acquires information. I define an optimal low-skill gatekeeper mechanism as a low-skill gatekeeper mechanism that minimizes the Principal's expected payment among all low-skill gatekeeper mechanisms. I call an equilibrium of a low-skill gatekeeper mechanism a low-skill information equilibrium. An optimal low-skill information equilibrium is defined analogously. Finally, I call the contract in an optimal low-skill information equilibrium equilibrium an optimal low-skill information contract.

Definition 1 (Low-skill Information Equilibrium) *In a low-skill information equilibrium, the Principal offers a contract; the low-skill expert accepts, acquires information, and makes a report; and the high-skill expert accepts the contract.*

Below I consider the constraints a Principal's contract $\big(t_l(s), t_h(s), p_l(s)\big)$ must satisfy in a low-skill information equilibrium. In a Stage-4 continuation equilibrium, the low-skill expert knows the state s. The contract is said to satisfy the low-skill expert's truthful-reporting constraint if

$$t_l(s) - p_l(s)s \geq t_l(\hat{s}) - p_l(\hat{s})s, \textit{ for all } s, \hat{s} \in S. \tag{2.5}$$

The L.H.S. of (2.5) is the low-skill expert's payoff reporting the true state excluding the information acquisition cost and the R.H.S. is the low-skill expert's payoff otherwise. I define the low-skill expert's Stage-4 continuation equilibrium payoff excluding the information acquisition cost as $V_l(s) \equiv t_l(s) - p_l(s)s$.

In a Stage-3 continuation equilibrium, if the low-skill expert incurs f and learns that the state is s, then he can make a report and get $\max_{\hat{s}}\{t_l(\hat{s}) - p_l(\hat{s})s - f\}$. If the low-skill expert does not acquire information, then he can make a report that maximizes his payoff. The payoff of doing so is $\max_{\hat{s}} \int_0^{\bar{s}} \big\{t_l(\hat{s}) - p_l(\hat{s})x\big\} dG(x)$. The contract is said to satisfy the low-skill expert's information acquisition constraint if

$$\int_0^{\bar{s}} \max_{\hat{s}\in S}\{t_l(\hat{s}) - p_l(\hat{s})x\} dG(x) - f \geq \max_{\hat{s}} \int_0^{\bar{s}} \big\{t_l(\hat{s}) - p_l(\hat{s})x\big\} dG(x). \tag{2.6}$$

In a Stage-2 continuation equilibrium, the low-skill expert expects the high-skill expert to accept the contract. If he accepts the contract, then he can make a report with or without acquiring information. The payoff of doing each is discussed in the above paragraph. The contract is said to satisfy the low-skill expert's acceptance constraint if

$$\max\left\{ \int_0^{\bar{s}} \max_{\hat{s}\in S}\{t_l(\hat{s}) - p_l(\hat{s})x\} dG(x) - f, \max_{\hat{s}} \int_0^{\bar{s}} \big\{t_l(\hat{s}) - p_l(\hat{s})x\big\} dG(x) \right\} \geq 0. \tag{2.7}$$

In a Stage-2 continuation equilibrium, the high-skill expert expects the low-skill expert to accept the contract, acquire information, and report the true state. The contract is said to satisfy the high-skill expert's acceptance constraint if

$$\int_0^{\bar{s}} \big[t_h(x) - (1 - p_l(x))(\alpha x + \beta)\big] dG(x) \geq 0. \tag{2.8}$$

All in all, in a low-skill information equilibrium, the Principal's contract $\big(t_l(s), t_h(s), p_l(s)\big)$ satisfies constraints (2.5)-(2.8). I concentrate on an optimal low-skill information contract, which solves the following program that I call Program 1:

$$\min_{t_l(s),p_l(s),t_h(s)} \int\limits_0^{\bar{s}} \big[t_l(x) + t_h(x)\big] dG(x), \tag{2.9}$$

subject to (2.5)-(2.8).

I simplify Program 1 below. Lemma 2 is a standard result. The proof can be found in Myerson (1981) or a standard textbook like Mas-Colell, Whinston, and Green (1995).

Lemma 2 *A contract* $\big(t_l(s), t_h(s), p_l(s)\big)$ *satisfies the low-skill expert's truthful-reporting constraint* (2.5) *if and only if*

(i) $p_l(s)$ *is non-increasing and*

(ii) $V_l(s) = V_l(0) - \int_0^s p_l(x)dx$*, for all* $s \in S$*.*

I will refer the first condition (i) in Lemma 2 as the monotonicity condition. If a contract satisfies (2.5), then $V_l(s)$ can be rewritten as

$$V_l(s) \equiv t_l(s) - p_l(s)s = V_l(0) - \int\limits_0^s p_l(x)dx \tag{2.10}$$

$$= t_l(0) - \int\limits_0^s p_l(x)dx. \tag{2.11}$$

The second line is true because $V_1(0) = t_l(0) - p_l(0)0 = t_l(0)$. By (2.11),

$$t_l(s) = p_l(s)s + t_l(0) - \int\limits_0^s p_l(x)dx. \tag{2.12}$$

In a solution of Program 1, (2.8) must bind because I can lower $t_h(x)$ without affecting other constraints;

$$\int_0^{\bar{s}} \left[t_h(x) - (1 - p_l(x))(\alpha x + \beta)\right] dG(x) = 0. \tag{2.13}$$

For simplicity, I will concentrate on equilibria in which $t_h(s) = (1 - p_l(s))(\alpha s + \beta)$, $\forall s$. Then, I can substitute (2.12) and (2.13) into (2.9) to get

$$\min_{t_l(s), p_l(s), t_h(s)} \int_0^{\bar{s}} \left[p_l(x)x + (1 - p_l(x))(\alpha x + \beta)\right] dG(x) + t_l(0) - \int_0^{\bar{s}} \int_0^{y} p_l(x) dx dG(y). \tag{2.14}$$

Since (2.5) is satisfied in a solution, $V_l(s) = \max_{\hat{s} \in S}\{t_l(\hat{s}) - p_l(\hat{s})x\}$. Also, the low-skill expert's information acquisition constraint (2.6) is satisfied, so his acceptance constraint (2.7) is reduced to

$$\int_0^{\bar{s}} \left[t_l(0) - \int_0^{y} p_l(x) dx\right] dG(y) - f \geq 0 \tag{2.15}$$

$$t_l(0) \geq f + \int_0^{\bar{s}} \int_0^{y} p_l(x) dx dG(y). \tag{2.16}$$

The objective function in (2.14) contains $t_l(0)$, so (2.16) must bind in a solution of Program 1. This is because I can lower $t_l(0)$ without affecting (2.6). Also, I can substitute the binding (2.16) into (2.12) and use integration by parts to get

$$t_l(s) = p_l(s)s + f + \int_0^{\bar{s}} (1 - G(x)) p_l(x) dx - \int_0^{s} p_l(x) dx \tag{2.17}$$

Now, I can substitute the binding (2.16) into (2.14) so that Program 1 is now reduced to the following.

$$\min_{t_l(s),p_l(s),t_h(s)} \int_0^{\bar{s}} \left[p_l(x)x + (1-p_l(x))(\alpha x+\beta)\right] dG(x) + f \tag{2.18}$$

subject to $p_l(s)$ being non-increasing, (2.6), (2.13), and (2.17).

As shown above, the Principal minimizes his payment and extracts all rent from both the low-skill expert and the high-skill expert. Lemma 3 further simplifies Program 1. The proof is in the Appendix.

Lemma 3 *Program 1 is equivalent to the following problem.*

$$\min_{t_l(s),p_l(s),t_h(s)} \int_0^{\bar{s}} \Big[p_l(x)x + (1-p_l(x))(\alpha x+\beta)\Big] dG(x) + f \qquad \textit{subject to} \tag{2.19}$$

$$f \le \int_0^{\mu_l} k(x)p_l(x)dG(x) - \int_{\mu_l}^{\bar{s}} h(x)p_l(x)dG(x), \tag{2.20}$$

$p_l(s)$ *being non-increasing,* (2.13)*, and* (2.17).

In Lemma 3, the low-skill expert's information acquisition constraint is simplified by integration by parts. Lemma 3 is intuitive. The Principal assigns projects to the expert who has the lowest cost, but he has to make sure the contract satisfies experts' acceptance, information acquisition, and truthful-reporting constraints.

Suppose the first best requires information acquisition. Can the Principal implements it through an optimal low-skill information mechanism? To see that, I plug the efficient assignment rule, $p_l(s) = 1$ for all $s < s^{fb}$ and $p_l(s)$ for all $s \ge s^{fb}$, into the R.H.S of (2.20) to get the following term, which I call f^*.

$$f^* \equiv \begin{cases} \int_0^{s^{fb}} k(x)p_l(x)dG(x), & \text{if } s^{fb} < \mu_l, \text{ and} \\ \int_0^{\mu_l} k(x)p_l(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)p_l(x)dG(x), & \text{if } s^{fb} \geq \mu_l. \end{cases}$$

In the proof of Proposition 1, I show that f^* must be larger than zero. Clearly, the efficient assignment rule is non-increasing. Therefore, if $f \leq f^*$, then a low-skill gatekeeper mechanism can implement the efficient assignment rule. Moreover, if $f^* \geq f^{fb}$, then a low-skill gatekeeper mechanism can implement the efficient assignment rule whenever the first best requires information acquisition. Therefore, the optimal low-skill gatekeeper mechanism implements the first best while extracting all rent from the experts. It is without loss of generality to concentrate on low-skill gatekeeper mechanism. Proposition 1 shows that $f^* \geq f^{fb}$ is indeed true. The proof is in the Appendix.

Proposition 1 *Suppose the first best requires information acquisition. Then, it is without loss of generality to focus on low-skill gatekeeper mechanism. The Principal implements the first best and extracts all rent through the optimal low-skill gatekeeper mechanism.*

To sum up, the Principal is able to choose the low-skill expert to be the gatekeeper because of the following reason. The $p_l(s)$ function is non-increasing because of the truthful reporting constraint. To exploit experts' cost comparative advantages, the Principal wants to assign projects so that $p_l(s)$ is also decreasing. Indeed, the Principal is able to assign project efficiently while incentivizing information acquisition and truthful reporting. As we can see next, this is not the case when the high-skill expert is chosen as the gatekeeper.

Finally, I discuss the low-skill expert's payment functions in the optimal low-skill information contract. Plug the efficient assignment rule into (2.17) to get

$$t_l(s) = \begin{cases} p_l(s)s + f + \int_0^{\bar{s}}(1-G(x))p_l(x)dx - \int_0^s p_l(x)dx, & \text{if } s < s^{fb}, \text{ and} \\ f + \int_0^{\bar{s}}(1-G(x))p_l(x)dx - \int_0^s p_l(x)dx, & \text{if } s \geq s^{fb}. \end{cases}$$

$$t_l(s) = \begin{cases} f - \int_0^{s^{fb}} G(x)dx + s^{fb}, & \text{if } s < s^{fb}, \text{ and} \\ f - \int_0^{s^{fb}} G(x)dx, & \text{if } s \geq s^{fb}. \end{cases}$$

In the optimal low-skill information contract, the Principal pays the low-skill gatekeeper f in order to cover his information acquisition cost. The Principal pays an additional amount of s^{fb} if the low-skill gatekeeper reports the project difficulty to be lower than s^{fb}. This incentivizes the low-skill gatekeeper to acquire information. Finally, the Principal reduces the payment by $\int_0^{s^{fb}} G(x)dx$ in order to extract all rent from the low-skill gatekeeper.

2.4.2 High-skill Expert Gatekeeper

In this section, I analyze a mechanism in which the high-skill expert is chosen as the gatekeeper. Experts' roles are interchanged. The Principal offers a contract $\big(t_l(s), t_h(s), p_h(s)\big)$ to the experts. Still, the function $t_l(s)$ is the Principal's payment to the low-skill expert and $t_h(s)$ is the payment to high-skill expert. The functions $1-p_h(s)$ and $p_h(s)$ are the Principal's assignment probabilities to the low-skill and the high-skill expert, respectively. However, after both experts accept the contract, only the high-skill expert can acquire information and make a report $s \in S$. The contract is a vector of functions of the high-skill expert's report. Similar to the last section, I can define an optimal high-skill gatekeeper mechanism, a high-skill information equilibrium, an optimal high-skill information equilibrium, and an optimal high-skill information contract.

In a high-skill information equilibrium, the Principal's contract satisfies the high-skill expert's truthful-reporting constraint, the high-skill expert's information acquisition constraint, the high-skill expert's acceptance constraint, and the low-skill expert's acceptance constraint, all of which are defined analogously as in the last section. Suppose the Principal offers a contract $\big(t_l(s), t_h(s), p_h(s)\big)$ in a high-skill information equilibrium. The Principal's minimum expected cost to incentivize information acquisition and then use the information to assign the project to the low-skill expert with probability $1 - p_h(s)$ and to the high-skill expert with probability $p_h(s)$ is $\int_0^{\bar{s}} \big[(1 - p_h(x))x + p_h(x)(\alpha x + \beta)\big] dG(x) + f$.

Furthermore, similar to Lemma 3, the high-skill gatekeeper reports project information truthfully only if his project assignment probability $p_h(s)$ is non-increasing. The proof is in the Appendix.

Lemma 4 *A contract $\big(t_l(s), t_h(s), p_h(s)\big)$ satisfies the high-skill expert's truthful-reporting constraint only if $p_h(s)$ is non-increasing.*

Therefore, the Principal's expected equilibrium cost of a high-skill information equilibrium must be larger than the solution to the following problem, which I call Program 2.

$$\min_{p_h(x)} \int_0^{\bar{s}} \Big[(1 - p_h(x))x + p_h(x)(\alpha x + \beta)\Big] dG(x) \text{ subject to } p_h(x) \text{ being non-increasing.}$$

This leads to the following result.

Proposition 2 *The Principal's expected payoff in an equilibrium of a high-skill gatekeeper mechanism is lower than his expected payoff in an equilibrium of an optimal no-gatekeeper mechanism.*

The proof is in the Appendix. In order to exploit experts' cost comparative advantages in high-skill gatekeeper mechanism, the Principal should assign less difficult projects to the low-skill experts and more difficult projects to the high-skill experts. However, in order to incentivize the high-skill expert gatekeeper to reveal the truth, Lemma 4 says that the Principal should do the opposite and assigns less difficult projects to the high-skill experts and more difficult projects to the low-skill experts. Such a mechanism is unable to exploit experts' cost comparative advantages and leads to a high implementation cost. Indeed, it performs worse than an optimal no-gatekeeper mechanism.

2.5 More General Cost Structure

In this section, I change the assumption that the high-skill expert's cost function $c_h(s)$ is linear and study other functional forms. Throughout the section, I maintain the assumption that $c_h(s)$ has a lower slope than s and $c_h(s)$ and s cross only once at s^{fb}. That is, $c_h(s)$ satisfies $0 \leq c_h'(s) < 1$, $0 < c_h(0)$, and $c_h(\bar{s}) < \bar{s}$. I first consider a case in which I relax the assumption on the high-skill expert's cost function $c_h(s)$ but put more structure on the distribution function. Then, I discuss the case in which $c_h(s)$ and the distribution function are more general. Note that Proposition 2 holds as long as $c_h(s)$ and s cross only once and $c_h(s)$ is increasing. Therefore, I concentrate on low-skill gatekeeper mechanism in the rest of the section.

First, I make the assumption that $h(s)$ is convex. In auction theory, this amounts to the assumption that the virtual valuation is concave. There are some densities that satisfy the convex $h(s)$ assumption. Clearly, the uniform distribution has a linear $h(s)$ function. By Mierendorff (2016), $h(s)$ is convex if the density is linear and increasing, for example, $g(s) = 1 - b + 2bs$, $b \in (0, 1]$, or the density is a power of s, for example, $g(s) = (b+1)s^b$, $b > 0$. Also by Mierendorff (2016), a hump-shaped or a U-shaped density can have a convex $h(s)$ function. For example, $g(s) = \frac{3}{2} - 6(s - \frac{1}{2})^2$ is hump-shaped and $g(s) = 12(s - \frac{1}{2})^2$ is U-shaped and both have a convex $h(s)$ function.

Second, I make the assumption that $c_h(s)$ is concave. That means the difference of marginal cost of difficulty between the two experts is weakly increasing when project difficulty increases. The high-skill expert has an advantage in marginal cost of difficulty and this advantage weakly increases for more difficult projects. As can be seen in the proof of Proposition 1, the Principal can implement the first best even if $c_h(s)$ is concave using a combination of optimal low-skill gatekeeper mechanism and optimal no-information mechanism when $s^{fb} < \mu_l$ or $\mu_l \leq s^{fb}$ and $\mu_l \leq \mu_h$.[5]

Suppose $\mu_l \leq s^{fb}$ and $\mu_h < \mu_l$. The proof of Proposition 1 does not apply. Since

[5] Parts of the proof of Proposition 1 applies here because a linear function is also concave.

$\mu_h < \mu_l$, the first best requires information acquisition if

$$f \leq \int_0^{s^{fb}} \big[c_h(x) - x\big] dG(x). \tag{2.21}$$

Now, plug $p_l(s) = 1$ for $s \leq s^{fb}$ and $p_l(s) = 0$ for $s > s^{fb}$ into the R.H.S. of the information acquisition constraint (2.20) in Lemma 3 to get $\int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)dG(x)$, which is larger than zero as seen in the proof of Proposition 1. Therefore, an optimal low-skill information equilibrium assigns projects to the expert who has the lowest cost if

$$f \leq \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)dG(x). \tag{2.22}$$

Note that if the R.H.S. of (2.22) is larger than the R.H.S. of (2.21). Then, an optimal low-skill equilibrium implements the first best whenever the first best requires information acquisition. Let's consider their difference:

$$\begin{aligned}
&\int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)dG(x) - \int_0^{s^{fb}} \big[c_h(x) - x\big] dG(x) \\
&= \int_0^{s^{fb}} G(x)dx - \int_{\mu_l}^{s^{fb}} dx - \int_0^{s^{fb}} \big[c_h(x) - x\big] dG(x) \\
&= \int_0^{s^{fb}} G(x)dx - \int_{\mu_l}^{s^{fb}} dx - \Big[\big(c_h(x) - x\big)G(x)\Big]_0^{s^{fb}} + \int_0^{s^{fb}} G(x)d\big(c_h(x) - x\big) \\
&= \int_0^{s^{fb}} c_h'(x)G(x)dx - (s^{fb} - \mu_l).
\end{aligned}$$

If this is positive, then whenever $\mu_h < \mu_l$, $\mu_l \leq s^{fb}$, and the first best requires information acquisition, then the optimal low-skill gatekeeper mechanism implements the first-best. Suppose this is negative, then when $\mu_h < \mu_l$, $\mu_l \leq s^{fb}$, and the information acquisition cost f is intermediate such that $\int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{s^{fb}} (1 - G(x))dx < f \leq \int_0^{s^{fb}} \left[c_h(x) - x\right] dG(x)$, the first best requires information acquisition but the optimal low-skill gatekeeper mechanism cannot implement the first-best. Let's study an optimal information contract in this case. To do that, I consider a relaxed problem for the Principal in Lemma 3 by ignoring the monotonicity constraint, (2.13), and (2.17), which I call Program 1*. The Lagrangian is

$$\mathcal{L} = -\int_0^{\bar{s}} \left[p_l(x)x + (1 - p_l(x))c_h(x)\right] dG(x) - f + \lambda\Big[\int_0^{\mu_l} k(x)p_l(x)dG(x) - \int_{\mu_l}^{\bar{s}} h(x)p_l(x)dG(x) - f\Big]. \quad (2.23)$$

Point-wise optimization generates the following first-order derivative with respect to $p_l(s)$ at each s. I call it $\Delta(s)$.

$$\Delta(s) \equiv \begin{cases} [c_h(s) - s + \lambda k(s)]g(s), & \text{if } s < \mu_l, \text{ and} \\ [c_h(s) - s - \lambda h(s)]g(s), & \text{if } s \geq \mu_l. \end{cases} \quad (2.24)$$

Furthermore, define the term in the square bracket $D(s)$. In the Appendix, I show that $D(s)$, and therefore $\Delta(s)$, can only change sign once or thrice due to the concave cost function and the convex inverse hazard rate, which leads to the following result (proof in the Appendix).

Proposition 3 *Suppose the inverse hazard rate $h(s)$ is convex and the high-skill expert cost function $c_h(s)$ is concave. Also, $\mu_l \leq s^{fb}$ and $\mu_h < \mu_l$. An optimal low-*

skill information equilibrium exists if $f \leq \int_0^{\mu_l} k(x)dG(x)$. *In an optimal low-skill information contract* $\big(p_l(s), t_l(s), t_h(s)\big)$, *if* $f \leq \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)dG(x)$, *then* $p_l(s) = 1$ *for all* $s \leq s^{fb}$ *and* $p_l(s) = 0$ *for all* $s > s^{fb}$. *If* f *satisfies* $\int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s^{fb}} h(x)dG(x) < f \leq \int_0^{\mu_l} k(x)dG(x)$, *then the probability assignment function of the optimal low-skill information contract must be in one of the following two forms.*

(i) $p_l(s) = 1$ *for all* $s \leq \hat{s}$ *and* $p_l(s) = 0$ *for all* $s > \hat{s}$, *with* $f = \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{\hat{s}} h(x)dG(x)$.

(ii) $p_l(s) = 1$ *for all* $s \leq \mu_l$, $p_l(s) = a$ *for* s *satisfying* $\mu_l < s \leq \hat{s}$, *and* $p_l(s) = 0$ *for* s *satisfying* $s > \hat{s}$, *with* $f = \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{\hat{s}} ah(x)dG(x)$.

Proposition 3 says that the assignment rule of an optimal low-skill information contract must be described by one of the following two cases. First, the function $D(s)$ changes sign only once from positive to negative as s increases. Then, the Principal can set the probability assignment function according to the point-wise derivative without violating the monotonicity condition, so it is a threshold assignment rule where all low-difficulty projects are assigned to the low-skill expert and all high-difficulty projects are assigned to the high-skill expert.

In the second case, the function $D(s)$ changes from positive to negative to positive and then back to negative as s increases. At the two extreme, the Principal can set the probability assignment according to the point-wise derivative without violating the monotonicity condition; the Principal assigns all low-difficulty and high-difficulty projects to the low-skill and high-skill expert, respectively. However, in the middle, he cannot do that without violating the monotonicity condition. Therefore, he assigns medium-difficulty projects to the low-skill expert with a fixed probability. All in all, the probability assignment function decreases in project difficulty in two steps. I am able to pin down the possible number of times the $D(s)$ function changes sign because the second derivative of $D(s)$ is negative.

Finally, I discuss a more relaxed cost function for the high-skill expert and a general distribution. Suppose that $c_h(s)$ satisfies $0 \leq c_h'(s) < 1$, $0 < c_h(0)$, and $c_h(\bar{s}) < \bar{s}$. Proposition 2 still holds so that it is never optimal to choose a high-skill gatekeeper. The allocation of an optimal low-skill information equilibrium is now different. By the same logic of the proof of Proposition 3, it is still true that when s is close to zero, $D(s)$ is positive. Also, when s is close to $\bar{s}$, $D(s)$ is negative. Therefore, the Principal assigns all low-difficulty and high-difficulty projects to the low-skill and high-skill expert, respectively. However, the $D(s)$ function's second derivative cannot be pinned down because of the generality of the model. When s is intermediate, $p_l(s)$ is a step function, but there may be more than two steps because the $D(s)$ function may change sign multiple times.

2.6 Conclusion

In this chapter, I provide a model to study the organization of expertise. A Principal relies on a group of experts with different cost comparative advantages to implement a project, but he relies on the same group of experts to acquire and report information about the project so that he can assign it efficiently among the experts. I find that only a low-skill expert can be used as a gatekeeper. When the Principal designs a contract for a low-skill gatekeeper mechanism, he is able to align his need to incentivize truthful reporting with his objective to reduce project cost. This cannot be done when there is a high-skill gatekeeper. When experts' cost structure is linear, the optimal low-skill gatekeeper mechanism also implements the first best. That means focusing on gatekeeper mechanism is without loss of generality. I also study gatekeeper mechanism with a more general cost structure. The major qualitative result of the model remains valid. The model can be used to understand the widespread usage of low-skill and low-cost gatekeeper in various industries.

There are rooms for future research. First, if information acquisition is not perfect, then the Principal may want to solicit opinions from multiple experts before he assigns the project. It would be interesting to see if a corresponding low-skill gatekeeper mechanism remains valid. Second, in this model, project information is used by the Principal to assign a project. However, project information may also be used by an expert to tailor project implementation effort. It remains to be seen how the model would change when project implementation is more explicitly modeled and project information can be used in a rich way.

Chapter 3

Information Acquisitions and Referrals between Experts

3.1 Introduction

In an economy, production efficiency often depends on the correct match between projects and production experts. However, information about the type of a project is often absent, and only experts who are able to implement the project have the ability to learn about the project's type. Moreover, information acquisition may be imperfect and experts may have to coordinate their information-acquisition efforts in order to aggregate information in an efficient way. This chapter intends to study the efficiency of a market in incentivizing experts' information acquisitions, coordinating experts' information acquisitions, and matching projects to experts.

In my model, there is a set of projects; each project has a different difficulty state. There are two experts, and each can complete a project by incurring a fixed cost and a variable cost. Each expert's variable cost is increasing in project difficulty. However, the two experts have different cost comparative advantages. A low-skill expert has a low fixed cost and a high variable cost, whereas the opposite is true for a high-skill expert. In the first best, if projects' difficulties are known, then the low-skill expert should complete less difficult projects and the high-skill expert should complete more difficult projects.

The two experts have the same information acquisition technology. Each of them can exert an effort to acquire information about a project's difficulty. An expert's information-acquisition effort may or may not succeed. If it succeeds, then the expert learns about the project's difficulty. If it fails, then the expert remains uninformed. A higher effort increases the probability of a successful information acquisition, and expert's information acquisition cost is an increasing and convex function in effort. In the first best, the two experts should coordinate their information-acquisition effort. One expert acquires information first. If he succeeds, then the project is assigned efficiently. If he fails, then another expert acquires information. The first expert exerts less effort, while the second expert exerts more effort.

I first study an ascending referral market where the low-skill expert is initially assigned a project. He decides whether to exert effort to acquire information about the project. Then, he decides whether to complete the project himself or refer it to the high-skill expert by choosing a referral payment. Upon receiving a payment offer, the high-skill expert decides whether to exert effort to acquire information. Then he decides whether to accept the offer. If he accepts, then he completes the project and receives the referral payment. If he rejects, then the low-skill expert completes the project. Later, I study a descending referral market where experts' roles are reversed.

I find that in both markets, experts are able to refer projects to each other. Either one or both experts acquire information. In both ascending and descending referrals, experts fail to coordinate their information-acquisition efforts. Even if both experts acquire information, one expert acquires information so that the information is used to match the project to the appropriate expert, and the other expert acquires information again to protect himself from getting a costly project.

3.1.1 Literature Review

Two important papers about referral are Garicano and Santos (2004) and Grassi and Ma (2016). The former paper does not incorporate information acquisition into their model. They assume that the referring expert has private and exogenous information about a project. Grassi and Ma (2016) study the referring expert's incentive to acquire information but rule out the possibility of the referred expert acquiring information. In both papers, referral market is inefficient and in some situation it unravels completely. This chapter shows that referral happens in both the ascending and the descending market. Information acquisition allows an expert to protect himself from the costliest project, so markets do not unravel completely. Secondly, a richer model allows me to study whether experts would coordinate their information-acquisition efforts.

This chapter is also related to the bargaining literature. Samuelson (1984) studies a bargaining problem where asymmetric information is exogenous. Shavell (1994) studies a one-sided information acquisition problem where a party who acquires information may voluntary disclose his information. Dang (2008) studies a two-sided information acquisition problem where information acquisition is socially wasteful and gain from trade is common knowledge. This chapter studies a problem where project information is needed to determine whether there is a gain from trade. And both sides should coordinate information acquisition in order to improve efficiency.

Finally, this chapter is related to the credence good literature. The literature concentrates on the interaction between expert and client. For example, Dulleck and Kerschbamer (2006), which is a survey of the literature, and Fong, Liu, and Wright (2014) study a market where an expert has an informational advantage over a client that he is serving. Instead, this chapter studies the interaction between experts in a referral market.

The remainder of the chapter is organized as follows. Section 3.2 presents the model setup and the first best. Section 3.3 studies an ascending market. Section 3.4 studies a descending market. Section 3.5 concludes. Finally, Appendix contains proofs of results.

3.2 Model

3.2.1 Projects, Experts, and Information Structure

There are two risk-neutral experts, a low-skill expert and a high-skill expert. Either expert can complete a project at a cost. An expert's project cost is a function of the project state. The set of all project states is $\Omega \equiv \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$. The probability of state ω_i is $p(\omega_i)$, for $i = 1, ..., 5$. Let $p_i \equiv p(\omega_i)$, for $i = 1, ..., 5$. A higher project state indicates a more difficult project. When the project state is ω_i, then the low-skill expert's implementation cost is s_i, whereas the high-skill expert's is $\alpha s_i + \beta$, for $i = 1, ..., 5$, with $s_1 < s_2 < s_3 < s_4 < s_5$. I assume that the high-skill expert has a lower marginal cost with $0 < \alpha < 1$ and a higher fixed cost with $\beta > 0$. I assume that project ω_3 is the state at which the two experts' cost are equal. That is, $s_3 = \alpha s_3 + \beta$. I also assume that $\alpha \sum_i p_i s_i = s_3$. The two assumptions $s_3 = \alpha s_3 + \beta$ and $\alpha \sum_i p_i s_i = s_3$ imply that the two experts' ex ante costs are the same. Finally, I assume that $s_2 < \alpha s_1 + \beta$, and $\alpha s_5 + \beta < s_4$. The last two assumptions say that experts' comparative advantages are large; the low-skill expert's cost for project s_2 is lower than the high-skill expert's cost for project s_1, whereas the high-skill expert's cost for project s_5 is lower than the low-skill expert's cost for project s_4. These two assumptions also imply that the low-skill expert is more efficient in project s_1 and s_2 and the high-skill expert is more efficient in project s_4 and s_5.

An expert may acquire information about a project at a cost by choosing a probability θ, which satisfies $0 \leq \theta \leq 1$. His information acquisition succeeds with probability θ. If he succeeds, he learns about the project state perfectly. If he fails, then he does not learn anything. An expert's information acquisition cost $k(\theta)$ is increasing and convex such that $k(0) = 0$, $k' > 0$, $k'' > 0$, $\lim_{\theta \to 0} k'(\theta) = 0$, and $\lim_{\theta \to 1} k(\theta) = \infty$. An expert's information acquisition device can only be used once. If an expert incurs $k(\theta)$ to investigate a project but fails, then he cannot acquire information again.

3.2.2 First Best

An allocation consists of information acquisition costs incurred by either or both experts and an assignment of a project to an expert according to the information learned by the experts. The first-best allocation minimizes experts' expected project cost and information costs. If project state is revealed, then projects ω_1, ω_2, and ω_3 are assigned to the low-skill expert, whereas projects ω_4 and ω_5 are assigned to the high-skill expert.[1] Moreover, experts acquire information in a sequential way. If one expert succeeds in acquiring information, then the other expert does not have to acquire information and the project is assigned according to the project state. If the first expert fails, then the other expert acquires information. If the other expert succeeds, then the project is assigned efficiently. If the other expert fails as well, then

[1] The two experts' cost completing project ω_3 are the same.

the project is assigned to the low-skill expert.[2]

Suppose θ' and θ'' are the first and the second experts' information acquisition probabilities, respectively. Then, the first best chooses a vector of information acquisition probability $(\theta_l^{fb}, \theta_h^{fb})$ to solve the following minimization problem.

$$(\theta_l^{fb}, \theta_h^{fb}) = \text{argmin}_{(\theta', \theta'')} \Big\{ k(\theta') + \theta' \Big[\sum_{i=1}^{3} p_i s_i + \sum_{i=4}^{5} p_i(\alpha s_i + \beta) \Big] + (1-\theta') \Big[k(\theta'') + \theta'' \big(\sum_{i=1}^{2} p_i s_i + \sum_{i=4}^{5} p_i(\alpha s_i + \beta) \big) + (1-\theta'') s_3 \Big] \Big\}$$

First-order conditions imply

$$k'(\theta_h^{fb}) = \sum_{i=4}^{5} p_i(s_i - \alpha s_i - \beta) \tag{3.1}$$

and

$$k'(\theta_l^{fb}) = (1-\theta_h^{fb}) \Big[\sum_{i=4}^{5} p_i(s_i - \alpha s_i - \beta) \Big] + k(\theta_h^{fb}) \tag{3.2}$$

Consider the function $\theta k'(\theta)$ and $k(\theta)$. They are both equal to zero when $\theta = 0$. However, $\theta k'(\theta) > k(\theta)$ for any $\theta > 0$ because $k'(\theta) + \theta k''(\theta) > k'(\theta)$. Therefore, by (3.1), $\theta_h^{fb} \Big[\sum_{i=4}^{5} p_i(s_i - \alpha s_i - \beta) \Big] > k(\theta_h^{fb})$. This implies that $\theta_l^{fb} < \theta_h^{fb}$, by comparing (3.1) and (3.2).

Therefore, in the first best, the first expert acquires information by choosing probability θ_l^{fb}. If he fails, then the second expert acquires information by choosing a higher probability θ_h^{fb}.

[2]The two experts' ex anote project cost are the same.

3.3 Ascending Referral Market

In this section, I study an ascending referral market where the low-skill expert decides whether to complete the project or refer it to the high-skill expert. I study a descending referral market where the high-skill expert may refer in the next section. Here is the extensive form of the ascending referral market.

Stage 1: The low-skill expert has a project, with state $\omega \in \Omega$ determined by the probability function $p(\omega)$. He chooses his information-acquisition probability θ_l by incurring cost $k(\theta_l)$. Then, he decides whether to complete the project or refer it to the high-skill expert by offering a payment t.

Stage 2: The high-skill expert chooses his information acquisition-probability θ_h by incurring cost $k(\theta_h)$. Then, he decides whether or not to accept the referral. If he accepts, he receives t and completes the project. If he rejects, then the low-skill expert completes the project.

In the extensive form, project's probability function, experts' cost functions, and information-acquisition function are experts' common knowledge. Information-acquisition probability, the revealed project's state when an expert's information acquisition succeeds, and an expert's realized project cost are private. The only contractible event is project completion and the identity of the expert who completes a project. Thus, the low-skill expert may refer a project at a payment.

3.3.1 Strategies and Beliefs

This section defines experts' strategies and the high-skill expert's belief. The low-skill expert's strategy is defined by (i) an information-acquisition probability, (ii) a decision to complete the project, and (iii) a referral payment if he decides to refer the project to the high-skill expert. The low-skill expert's decision to complete or refer the project and referral payment are functions of project's information he has learned through information acquisition. The high-skill expert's strategy is defined by (i) an information acquisition probability and (ii) a referral acceptance decision. The high-skill expert's information-acquisition probability is a function of referral payment. His acceptance decision is a function of referral payment and project's information he has learned through information acquisition.

I only consider referral payments with values between s_1 and s_5, which is without loss of generality. In stage 2, after receiving a payment offer, the high-skill expert forms a belief about the referred project's state. His belief about a project in state ω is a probability function $\mu : [s_1, s_5] \times \Omega \to [0, 1]$. That means $\mu(t, \omega)$ is the probability mass that the high-skill expert assigns to project ω when the referral payment is t. Naturally, $\sum_{\Omega} \mu(t, \omega) = 1$. I assume that the low-skill expert prefers completing a project to referring it when he is indifferent. Also, I assume that the high-skill expert prefers accepting a referral when he is indifferent.

A tuple of strategies and belief is a perfect Bayesian equilibrium of the ascending referral market if the strategies are mutual best responses given the belief and the belief is updated according to the strategies and Bayes' rule whenever possible. I restrict my attention to pure strategy equilibria.

Consider the following equilibrium. The low-skill expert acquires information at probability θ'. If he fails, he completes the project. If he succeeds, he completes projects ω_1, ω_2, and ω_3. And he refers projects ω_4 and ω_5. There are many unreached information sets in this equilibrium. Perfect Bayesian equilibria place no restriction on the high-skill expert's belief when he receives an off-equilibrium payment $t \neq t'$. In this chapter, I use the following simple belief restriction.

Definition 1 (Pessimistic Belief) *An equilibrium is said to satisfy pessimistic belief if the high-skill firm believes that a referred project at any off-equilibrium payment has the state* ω_5.

In the below, I study equilibria with the following characteristics. The low-skill expert acquires information. When information acquisition fails, he completes the project. When it succeeds, he completes certain projects and refers the remaining at a specific equilibrium payment. The high-skill expert may or may not acquire information depending on the payment level. The high-skill expert makes optimal acceptance decision based on the project information he has learned through information acquisition. As I will show, they are the only equilibria.

3.3.2 Low-skill Expert's Equilibrium Strategy

In this subsection, I study the low-skill expert's equilibrium strategy. First, I show that the low-skill expert acquires information in an equilibrium, and he refers a project only after a successful information acquisition.

Lemma 1 *In an equilibrium, the low-skill expert refers a project only after a successful information acquisition.*

The proof is in the Appendix. In the first best, when information acquisition is successful, the low-skill expert completes projects in low states and the high-skill expert completes projects in high states. In an equilibrium, a referral is feasible if and only if projects are assigned to exploit experts' comparative advantages as in the first best. And projects can be assigned to do that only if the low-skill expert successfully acquires information.

In Lemma 2, I show that there does not exist an equilibrium where the low-skill expert completes a project without information acquisition. The proof is in the Appendix.

Lemma 2 *The low-skill expert acquires information in an equilibrium.*

If there was an equilibrium where the low-skill expert did not acquire information and complete the project, then there would be a profitable deviation. The reason is that the low-skill expert could acquire information at a very low probability. If it succeeded, he would refer the projects ω_4 and ω_5 at payment $\alpha s_5+\beta$. If it failed, then he would completed the project. This deviation is profitable because the high-skill expert would accept a referral without information acquisition at payment $\alpha s_5 + \beta$, and the low-skill expert's information acquisition cost is low when the probability is low, with $\lim_{\theta \to 0} k(\theta) = 0$.

In Lemma 3, I show that the low-skill expert would not refer projects ω_1, ω_2, or ω_3 after a successful information acquisition because such a referral failed to exploit experts' comparative advantages. The proof is in the Appendix.

Lemma 3 *In an equilibrium, it would not be profitable for the low-skill expert to refer projects ω_1, ω_2, or ω_3 after a successful information acquisition.*

In Lemma 4, I show that any equilibrium has a simple structure such that it has only one equilibrium payment. This result is due to the five-type structure of the model. The proof is in the Appendix.

Lemma 4 *In an equilibrium, there is only one equilibrium payment.*

3.3.3 Equilibrium Characterization

Lemmas 1-4 show that, in an equilibrium, the low-skill expert acquires information and refers some projects if it succeeds. In this subsection, I show that there are only two classes of equilibria. In the first, only the low-skill expert acquires information. In the second, both experts acquire information. I call an equilibrium in the first class a single-acquisition equilibrium and an equilibrium in the second class a double-acquisition equilibrium. In Proposition 1, I show that there is always a unique single-acquisition equilibrium. Later in this section, I derive a sufficient condition for the existence of a double-acquisition equilibrium.

Proposition 1 *A unique single-acquisition equilibrium exists, which is characterized by* (θ_l^*, t^*) *such that* θ_1^* *satisfies* $k'(\theta_l^*) = \sum_{i=4}^{5} p_i(s_i - \alpha s_5 - \beta)$ *and* $t^* = \alpha s_5 + \beta$*. Here are the equilibrium strategies and beliefs.*

(i) *The low-skill expert acquires information at probability* θ_l^**. If he succeeds in acquiring information, he refers projects* ω_4 *and* ω_5 *at payment* t^* *and completes the rest. If he does not have project information, he completes the project.*

(ii) *The high-skill expert does not acquire information at any payment. He accepts a referral at* $t \geq \alpha s_5 + \beta$ *and rejects a referral at* $t < \alpha s_5 + \beta$*. If he succeeds in* *acquiring information at* $t \neq t^*$*, then he accepts if project cost is lower than* t*. If he fails, then he accepts* $t \geq \alpha s_5 + \beta$ *and rejects* $t < \alpha s_5 + \beta$*.*

(iii) *At an off-equilibrium payment, the high-skill expert believes the project is in state* ω_5*.*

In the rest of the section, I study a double-acquisition equilibrium. The low-skill expert acquires information at θ_l^*. If he succeeds, he refer projects ω_4 and ω_5 at payment t^* and completes the rest. If he fails, he completes the project. The low-skill expert refers projects ω_4 and ω_5 after a successful information acquisition so that the high-skill expert has incentive to acquire information, and he completes the project in other scenarios because of Lemmas 1 and 3.

The high-skill expert acquires information at θ_h^*. If information acquisition succeeds, he accepts project ω_4 and rejects project ω_5. If it fails, he accepts the referral at t^*. The high-skill expert accepts ω_4 and rejects ω_5 because of the cost difference. If the high-skill expert rejected an equilibrium referral after a failed information acquisition, then the low-skill expert had no incentive to refer project ω_5 because it would be always rejected. Therefore, the high-skill expert must accepts an equilibrium referral after a failed information acquisition. Below, I examine the necessary and sufficient conditions $(\theta_l^*, \theta_h^*, t^*)$ has to satisfy under which a double-acquisition equilibrium exists.

First, at the end of stage 2, the high-skill expert accepts the equilibrium referral at t^* when information acquisition fails if and only if his expected payoff is higher than zero;

$$\frac{1}{p_4 + p_5} \sum_{i=4}^{5} p_i \big[t^* - \alpha s_i - \beta \big] \geq 0 \tag{3.3}$$

$$t^* \geq \alpha \frac{p_4 s_4 + p_5 s_5}{p_4 + p_5} + \beta. \tag{3.4}$$

Second, in the middle of stage 2, the high-skill expert's optimal information acquisition probability θ_h^* satisfies

$$\theta_h^* = \text{argmin}_\theta \Big\{ \frac{1}{p_4 + p_5} \Big[\theta p_4 (t^* - \alpha s_4 - \beta) + (1 - \theta) \sum_{i=4}^{5} p_i (t^* - \alpha s_i - \beta) \Big] + k(\theta) \Big\} \tag{3.5}$$

$$k'(\theta_h^*) = \frac{p_5}{p_4 + p_5} (\alpha s_5 + \beta - t^*). \tag{3.6}$$

In the square bracket on the right hand side of (3.5), $\frac{p_4}{p_4+p_5}(t^* - \alpha s_4 - \beta)$ is the high-skill expert's expected payoff when information acquisition succeeds. Moreover, $\frac{1}{p_4+p_5} \sum_{i=4}^{5} (t^* - \alpha s_i - \beta)$ is his expected payoff when it fails.

Third, in the beginning of stage 2, recall that the high-skill expert acquires information if and only if the equilibrium payment is lower than his project ω_5's cost;

$$t^* < \alpha s_5 + \beta. \tag{3.7}$$

Fourth, at the end of stage 1, the low-skill expert refers project ω_4 at the equilibrium payment t^* if and only if t^* is lower than his cost;

$$t^* < s_4. \tag{3.8}$$

Fifth, at the end of stage 1, the low-skill expert refers project ω_5 at the equilibrium payment t^* if and only if his expected referral cost is lower than his project cost. The low-skill expert's expected referral cost is $\theta_h^* s_5 + (1-\theta_h^*)t^*$. This is because if the high-skill expert rejects him, which happens with probability θ_h^*, he has to complete the project with cost s_5;

$$\theta_h^* s_5 + (1-\theta_h^*)t^* < \alpha s_5 + \beta. \tag{3.9}$$

Sixth, at the end of stage 1, the low-skill expert prefers completing projects ω_1, ω_2, and ω_3 to referring them at equilibrium price t^* if and only if

$$s_3 < t^*. \tag{3.10}$$

Finally, in the beginning of stage 1, the low-skill expert optimal information acquisition probability θ_l^* satisfies

$$\theta_l^* = \text{argmin}_\theta \Big\{ \theta \Big[\sum_{i=1}^{3} p_i s_i + p_4 t^* + p_5 \big(\theta_h^* s_5 + (1-\theta_h^*)t^* \big) \Big] + (1-\theta)s_3 + k(\theta) \Big\} \tag{3.11}$$

$$k'(\theta_l^*) = p_4(s_4 - t^*) + p_5(1-\theta_h^*)(s_5 - t^*). \tag{3.12}$$

Given t^* and θ_h^*, (3.12) gives us a unique θ_l^*. Therefore, a double-acquisition equilibrium exists if there exists t^* and θ_h^* that satisfy (3.4), (3.6), (3.7), (3.8), (3.9), and (3.10). These conditions simplify to (3.12),

$$\alpha \frac{p_4 s_4 + p_5 s_5}{p_4 + p_5} + \beta < t^* < \min \Big\{ \alpha s_5 + \beta, \frac{(\alpha - \theta_h^*)s_5 + \beta}{1 - \theta_h^*} \Big\} \quad \text{, and} \tag{3.13}$$

$$k'(\theta_h^*) = \frac{p_5}{p_4 + p_5}(\alpha s_5 + \beta - t^*). \tag{3.14}$$

Proposition 2 characterizes a double-acquisition equilibrium.

Proposition 2 *A double-acquisition equilibrium exists if and only if there exists* t^**,* θ_l^**, and* θ_h^* *that satisfy* (3.12)*,* (3.13)*, and* (3.14)*. Here are the equilibrium strategies and beliefs.*

(i) *The low-skill expert acquires information at probability* θ_l^**. If he succeeds in acquiring information, he refers projects* ω_4 *and* ω_5 *at payment* t^* *and completes the rest. If he fails, he completes the project.*

(ii) *At* t^**, the high-skill expert acquires information at probability* θ_h^**. If he succeeds, he accepts projects* ω_4 *and* ω_5*. If he fails, he accepts the referral at* t^**. Finally, if he receives a referral at* $t \neq t^*$*, he does not acquire information. He accepts* $t \geq \alpha s_5 + \beta$ *and rejects* $t < \alpha s_5 + \beta$*. If he succeeds in acquiring information at* $t \neq t^*$*, then he accepts if project cost is lower than* t*. If he fails, then he accepts* $t \geq \alpha s_5 + \beta$ *and rejects* $t < \alpha s_5 + \beta$*.*

(iii) *At an off-equilibrium payment, the high-skill expert believes the project is in state* ω_5*.*

3.4 Descending Referral Market

In this section, I study a descending referral market where the experts' roles are reversed. I focus on equilibria where there is only one equilibrium payment. Similar to the last section, there can be two classes of equilibrium. In an equilibrium of the first class, which is called a single-acquisition equilibrium, only one expert acquires information. In an equilibrium of the second class, which is called a double-acquisition equilibrium, both experts acquire information. I will study the first class and discuss the second class. Lemma 8 shows that in the descending referral market, the referred low-skill expert always acquires information. This is in contrast to the ascending referral market.

Lemma 8 *In an equilibrium of the descending referral market, the low-skill expert acquires information at equilibrium payment* t^*.

The proof is in the Appendix. Lemma 8 implies that in a single-acquisition equilibrium, the high-skill expert refers a project without information acquisition. He leaves the information acquisition task for the low-skill expert.

Lemma 9 *In a single-acquisition equilibrium of the descending referral market, if the low-skill expert fails to acquire information, then he rejects the equilibrium referral.*

In contrast to the ascending referral market, Lemma 9 says that the low-skill expert rejects an equilibrium referral after a failed information acquisition. The proof is in the Appendix. The referred low-skill expert has comparative advantages in low-cost projects. If he accepted referral after a failed information acquisition, he would accept some high-cost projects. Such a referral would fail to exploit experts' comparative advantages.

Lemma 10 *In a single-acquisition equilibrium of the descending referral market, the high-skill expert does not acquire information and refers at* $t^* < \alpha s_1 + \beta$.

Lemma 10 pins down the range of an equilibrium payment. The proof is in the Appendix. In a single-acquisition equilibrium, the payment must be lower than

$\alpha s_1 + \beta$. The high-skill expert would be tempted to acquire information and complete ω_1 himself if the equilibrium payment is higher than $\alpha s_1 + \beta$. A referral is feasible only if it exploits experts' comparative advantages by assigning low-cost projects to the low-skill expert. Thus, the equilibrium payment is correspondingly low. The size of the equilibrium payment determines the projects that the low-skill expert accepts. The low-skill expert accepts a referral after a successful information acquisition if and only if his project cost is lower than the payment. For example, if the payment is in between s_1 and s_2, then the low-skill expert accepts project ω_1 and rejects the others. Lemma 11 shows the equilibrium strategies of a single-acquisition equilibrium when equilibrium payment is within this range. The proof is similar to Proposition 1's.

Lemma 11 *In a descending referral market, there is a single-acquisition equilibrium characterized by* (θ_l^*, t^*)*, which satisfy* $k'(\theta_l^*) = p_1(t^* - s_1)$ *and* $s_1 < t^* < s_2$*. Here are the equilibrium strategies and beliefs.*

(i) *The high-skill expert does not acquire information and refers the project at* t^**. If he succeeds in acquiring information, then he refers the project* ω_1 *at* t^* *and completes the rest. If he fails in acquiring information, then he refers the project at* t^**.*

(ii) *At* t^**, the low-skill expert acquires information at probability* θ_l^**. If he succeeds in acquiring information, he accepts project* ω_1 *and rejects the rest. If he fails in acquiring information, he rejects the referral. If he receives a referral at* $t \neq t^*$*, he does not acquire information and rejects the referral. If he succeeds in acquiring information at* $t \neq t^*$*, then he accepts if project cost is lower than* t*. If he fails, then he rejects.*

(iii) *At an off-equilibrium payment, the high-skill expert believes the project is in state* ω_5.

In a single-acquisition equilibrium, the high-skill expert relies on the referred low-skill expert to acquire information. The two experts fail to coordinate information acquisition. The low-skill expert acquires information in order to protect himself from high-cost projects. However, it facilitates efficient assignment of projects. If (θ_l^*, t^*) satisfy $k'(\theta_l^*) = \sum_i^2 p_i(t^* - s_i)$ and $s_2 < t^* < \alpha s_1 + \beta$ instead, then the low-skill expert accepts project ω_1 and ω_2 after a successful information acquisition. The equilibrium payment in this case is higher.

If an equilibrium payment t^* satisfies $\alpha s_1 + \beta < t^* < \alpha s_2 + \beta$, then the high-skill expert is tempted to acquire information in order to complete project ω_1, ω_3, ω_4, and ω_5. He still refers project ω_2. Indeed, this is the high-skill expert's on-the-equilibrium-path strategy in a double-acquisition equilibrium.

3.5 Conclusion

In this chapter, I provide a model to study a market where an expert who discovers a project may not be the appropriate expert to complete the project. Therefore, experts can improve efficiency by referring mismatched projects to each other. However, each of the experts can only acquire imperfect information about projects. The chapter intends to answer two questions. First, can experts refer at least some mismatched projects to each other using a simple price contract in a market? Second, can the two experts coordinate information-acquisition efforts in the market? The answer the first question is positive. There exist referral equilibria. But the market cannot achieve the first best because the answer to the second question is negative. One of the experts fails to use his information acquisition capability to improve social efficiency. In some equilibria, he does not acquire information. In other equilibria, he only acquires information to protect himself from getting difficult projects with which he has comparative advantage in completing.

Appendix A

Appendix for Chapter 1

Proof of Lemma 1

I first provide (i)'s proof. Suppose not. Suppose that $\hat{e}_l(q) > \hat{e}_h(q)$. Since the return functions are strictly concave in e, the first-order conditions imply that $\frac{\partial y_l(\hat{e}_l,q)}{\partial e} = \frac{\partial y_h(\hat{e}_h,q)}{\partial e} = 0$. Because $y_l(e,q)$ is strictly concave in e and $\hat{e}_l(q) > \hat{e}_h(q)$, it follows that $\frac{\partial y_h(\hat{e}_h,q)}{\partial e} < \frac{\partial y_l(\hat{e}_h,q)}{\partial e}$, which is a contradiction to the assumption $\frac{\partial y_h(e,q)}{\partial e} \geq \frac{\partial y_l(e,q)}{\partial e}$. To prove (ii), note that by implicit function theorem we have $\frac{d\hat{e}_l(q)}{dq} = -\frac{\partial^2 y_l(\hat{e}_l(q),q)}{\partial e \partial q} / \frac{\partial^2 y_l(\hat{e}_l(q),q)}{\partial e^2}$. By assumption, $\frac{\partial^2 y_l(e,q)}{\partial e \partial q} > 0$ and $\frac{\partial^2 y_l(e,q)}{\partial e^2} < 0$. Therefore, $\frac{d\hat{e}_l(q)}{dq} > 0$; $\frac{d\hat{e}_h(q)}{dq} > 0$ follows similarly. $\square$

Proof of Lemma 2

I first prove that $\frac{dS_h(q)}{dq} > \frac{dS_l(q)}{dq} > 0$. By assumption, $\frac{\partial y_h(\hat{e}_l(q),q)}{\partial q} > \frac{\partial y_l(\hat{e}_l(q),q)}{\partial q} > 0$. Because $\hat{e}_h(q) \geq \hat{e}_l(q)$ by Lemma 1 and $\frac{\partial^2 y_h(e,q)}{\partial e \partial q} > 0$ by assumption, I can say that $\frac{\partial y_h(\hat{e}_h(q),q)}{\partial q} > \frac{\partial y_l(\hat{e}_l(q),q)}{\partial q} > 0$. By the Envelope Theorem, $\frac{dS_l(q)}{dq} = \frac{\partial y_l(\hat{e}_l(q),q)}{\partial q}$ and $\frac{dS_h(q)}{dq} = \frac{\partial y_h(\hat{e}_h(q),q)}{\partial q}$. Therefore, $\frac{dS_h(q)}{dq} > \frac{dS_l(q)}{dq} > 0$.

Now, I proceed to prove the rest of Lemma 2. First, I know $S_h(0) < S_l(0) = 0$ since $\max_e y_l(e,0) = \max_e y_h(e,0) = 0$. Second, $S_h(\bar{q}) > S_l(\bar{q}) > 0$ by assumption. Since $\frac{dS_h(q)}{dq} > \frac{dS_l(q)}{dq} > 0$, intermediate value theorem says that there exists a unique $q^{fb} \in Q$ such that $S_l(q^{fb}) = S_h(q^{fb})$ and a unique q^* such that $S_h(q^*) = 0$. The rest of Lemma 2 follows directly from $\frac{dS_h(q)}{dq} > \frac{dS_l(q)}{dq}$. $\square$

Proof of Lemma 4

Consider an equilibrium and any two projects q' and q'', with $q' < q''$. It is sufficient to prove that $u_l(q'') \geq u_l(q')$. Firms' equilibrium strategies can be described by three cases.

Case 1: The low-skill firm implements q'.

The low-skill firm implements q' with $\hat{e}_l(q')$, so $u_l(q') = S_l(q')$. By the Envelope Theorem, $\frac{dS_l(q)}{dq} = \frac{\partial y_l(\hat{e}_l(q),q)}{\partial q}$, which is larger than zero by assumption. $S_l(q'') > S_l(q') = u_l(q')$ implies $u_l(q'') > u_l(q')$.

Case 2: The low-skill firm refers q' at p and the high-skill firm accepts it without inspection.

First, $u_l(q') = p$. The low-skill firm who has q'' can at least attain payoff p because the high-skill firm accepts a referral at p without inspection. Thus, $u_l(q'') \geq u_l(q')$.

Case 3: The low-skill firm refers q' at p. The high-skill firm inspects and accepts it.

First, $u_l(q') = p$. Upon receiving p, the high-skill firm's equilibrium strategy is to inspect the referred project. The high-skill firm inspects and accepts q' only if the efficient surplus it can derive is larger than the referral price, which means $S_h(q') \geq p$. If the low-skill firm refers q'' at p, the high-skill firm will inspect, learn that the project potential is q'' such that $S_h(q'') > S_h(q') \geq p$, and accept the referral. The low-skill firm who has q'' can at least attain payoff p. Thus, $u_l(q'') \geq u_l(q')$. $\square$

Proof of Lemma 5

Suppose not. Suppose there exists an equilibrium in which the high-skill firm accepts referral price p without inspection and $\inf R(p) > 0$. Consider a project $q' < \inf R(p)$

and a project $q'' \in R(p)$. By Lemma 3, $p \geq S_l(q'') > S_l(q')$. By Lemma 4, $u_l(q') \leq p$. Since $q' \notin R(p)$, we know $u_l(q') < p$. Because the high-skill firm accepts any referral at p without inspection, the low-skill firm prefers referring project q' at p to following its equilibrium strategy and getting a payoff of $u_l(q') < p$. A profitable deviation exists for the low-skill firm with project q'. □

Proof of Lemma 6

Suppose not. Suppose that there exists an equilibrium in which the high-skill firm accepts p' without inspection. As discussed after Lemma 5, there cannot be multiple equilibrium referral prices at which the high-skill firm accepts without inspection. By Lemma 5, the corresponding referral set can be written as $R(p') = (0, \hat{q})$, for a $\hat{q} \in Q$. There are two cases.

Case 1: $\hat{q} \geq q^{fb}$.

In the continuation equilibrium in which the high-skill firm receives the price offer p', the high-skill firm believes $\mu(p', q) = \frac{g(q)}{G(\hat{q})}$ for $q \in R(p')$ and $\mu(p', q) = 0$ for $q \notin R(p')$. Let q' be the project such that $p' = S_h(q')$. The high-skill firm does not inspect the referred project if and only if the inspection cost is larger than the incremental inspection surplus $\mathcal{I}(0, \hat{q}, p')$;

$$c > \frac{1}{G(\hat{q})} \int\limits_{\max\{0,q'\}}^{\hat{q}} [S_h(q) - p']dG(q) - \max\left\{0, \max_e \frac{1}{G(\hat{q})} \int\limits_0^{\hat{q}} [y_h(e, q) - f_h - p']dG(q)\right\}. \tag{A.1}$$

First, observe that $q' > 0$. Second, the high-skill firm accepts without inspection and implements the project only knowing that it is from the set $(0, \hat{q})$. The firm does so only if the surplus it can derive is larger than the price, which means that

$\max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big[y_h(e,q) - f_h - p'\Big] dG(q) \geq 0$. Using the above two points, I rearrange (A.1) to

$$c > \frac{1}{G(\hat{q})} \int_{q'}^{\hat{q}} \Big[S_h(q) - p'\Big] dG(q) - \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big[y_h(e,q) - f_h - p'\Big] dG(q),$$

which means

$$c > \frac{1}{G(\hat{q})} \Big[\int_0^{q'} p' dG(q) + \int_{q'}^{\hat{q}} S_h(q) dG(q) \Big] - \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big[y_h(e,q) - f_h\Big] dG(q). \quad \text{(A.2)}$$

Since $p' > S_h(q)$ for $q < q'$ and $\hat{q} \geq q^{fb}$, (A.2) contradicts Assumption A.

Case 2: $\hat{q} < q^{fb}$.

The high-skill firm implements a referred project at price p' without inspection and only knowing that it is from the set $(0, \hat{q})$. This is an equilibrium only if the high-skill firm's expected surplus of doing so is larger than the referral price p':

$$\max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big\{y_h(e,q) - f_h\Big\} dG(q) \geq p'. \quad \text{(A.3)}$$

By Lemma 3, the low-skill firm refers a project $q \in R(p') = (0, \hat{q})$ at price p' only if $p' \geq S_l(q)$. These translate into one single constraint

$$p' \geq S_l(\hat{q}). \quad \text{(A.4)}$$

Combining (A.3) and (A.4) I have

$$\max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big\{y_h(e,q) - f_h\Big\} dG(q) \geq S_l(\hat{q}). \quad \text{(A.5)}$$

Also, the high-skill firm's efficient surplus with project $\hat{q}$ is larger than its expected surplus with a project from the set $(0, \hat{q})$ only knowing that the project is from $(0, \hat{q})$:

$$S_h(\hat{q}) > \max_e \frac{1}{G(\hat{q})} \int_0^{\hat{q}} \Big\{ y_h(e,q) - f_h \Big\} dG(q). \tag{A.6}$$

Combining (A.5) and (A.6) gives me $S_h(\hat{q}) > S_l(\hat{q})$, which is a contradiction to Lemma 2 because $\hat{q} < q^{fb}$. $\square$

Proof of Proposition 1

(i) $\mathcal{I}(q_m^a, q_m^b, p_m) \geq c$ and $p_m = S_h(q_m^a) \geq S_l(q_m^b)$, for $m = 1, ..., n$.

$\mathcal{I}(q_m^a, q_m^b, p_m) \geq c$ follows from Lemma 6. By (1.6), an equilibrium price p_m satisfies $S_h(q_m^a) \geq p_m \geq S_l(q_m^b)$. Therefore I only need to prove that $p_m = S_h(q_m^a)$. Suppose not. Suppose that $p_m < S_h(q_m^a)$. There exists a $q' < q_m^a$ such that $p_m = S_h(q')$. Consider the low-skill firm who has a project q such that $q' < q < q_m^a$. Since $q \notin R(p_m)$, in the equilibrium the low-skill firm is supposed to either work on q or refer q at some price $p \neq p_m$. If the low-skill firm works on q, then it gets surplus $S_l(q) < S_l(q_m^a) \leq p_m$. If the low-skill firm refers q at a price p, then by Lemma 4 $p < p_m$. In both scenarios, if the low-skill firm deviated to refer q at price p_m, the high-skill firm would inspect, find out that $S_h(q) - p_m = S_h(q) - S_h(q') > 0$, and accept. A profitable deviation exists for the low-skill firm with project q.

(ii) $q_1^a > q^{fb}$.

First, $S_h(q_1^a) \geq S_l(q_1^b)$ by (6). Second, $\mathcal{I}(q_1^a, q_1^b, p_1) \geq c$ implies that $q_1^b > q_1^a$. Therefore, $S_h(q_1^a) \geq S_l(q_1^b) > S_l(q_1^a)$, which implies that $q_1^a > q^{fb}$ by Lemma 2.

(iii) If there exists q such that $q_n^b < q < \bar{q}$, then $p_n = S_h(q_n^a) = S_l(q_n^b)$.

Suppose not. Suppose that $p_n = S_h(q_n^a) > S_l(q_n^b)$. Consider the low-skill firm who has a project q satisfying $q > q_n^b$ and $S_h(q_n^a) > S_l(q) > S_l(q_n^b)$. Since q is larger than projects in each referral set, the low-skill firm's equilibrium strategy calls for him to work on q to get a payoff of $S_l(q) < p_n$. However, if the low-skill firm deviated and referred q at p_n, the high-skill firm would inspect, learn that $S_h(q) > S_h(q_n^b) > S_h(q_n^a) = p_n$, and accept the referral. A profitable deviation exists for the low-skill firm with project q.

(iv) If $n \geq 2$ and there exists q such that $q_{j-1}^b < q < q_j^a$, then $p_{j-1} = S_h(q_{j-1}^a) = S_l(q_{j-1}^b)$, for $j = 2, ..., n$.

The proof of (iv) is similar to (iii). Suppose not. Suppose that $p_{j-1} = S_h(q_{j-1}^a) > S_l(q_{j-1}^b)$. Consider the low-skill firm who has a project q satisfying $q_j^a > q > q_{j-1}^b$ and $S_h(q_m^a) > S_l(q) > S_l(q_m^b)$. Then, like the proof of (iii), the high-skill firm with project q would profitably deviate from implementation to referral it at p_m. □

Proof of Proposition 2

Suppose project potentials $q_1^a < q_1^b \leq q_2^a < q_2^b \leq ... \leq q_n^a < q_n^b$ and prices $p_1 < p_2 < ... < p_n$ satisfy conditions (i) - (iv) in Proposition 1. Given the high-skill firm's strategies to inspect projects at prices $p_m = S_h(q_m^a)$ and reject projects at prices $p \neq p_m$, for $m = 1, ..., n$, due to pessimistic belief, I will derive the low-skill firm's best response. First, note that the low-skill firm would never refer projects at off-equilibrium prices because the high-skill firm would reject them. Consider the low-skill firm who has $q \in (q_m^a, q_m^b)$. I show that the low-skill firm's best response is to refer q at $p_m = S_h(q_m^a)$. At any equilibrium price, the high-skill firm inspects, then it accepts if and only if it can derive a higher surplus than the price. If the low-skill firm refers q at $p_m = S_h(q_m^a)$, then the high-skill firm accepts after inspection because

$S_h(q) \geq S_h(q_m^a) = p_m$. This leaves the low-skill firm a payoff of $p_m = S_h(q_m^a)$. The low-skill firm would not refer q at an lower equilibrium price to get a lower payoff. For $m = 1, ..., n-1$, if the low-skill firm referred q at an equilibrium price $p' > p_m = S_h(q_m^a)$, then the high-skill firm rejected after inspection because $S_h(q) < S_h(q_{m+1}^a) \leq p'$. Finally, if the low-skill firm implemented q with efficient effort $\hat{e}_l(q)$, it would get a payoff of $S_l(q)$, which is smaller than p_m because $p_m = S_h(q_m^a) \geq S_l(q_m^b) \geq S_l(q)$ by the first condition (i) of Proposition 1.

Consider the low-skill firm who has $q \notin (q_m^a, q_m^b)$. I show that the low-skill firm's best response is to implement q to get a payoff of $S_l(q)$. There are three possible scenarios.

Case 1: $q < q_1^a$.

If the low-skill firm referred q at any equilibrium price $p' \geq p_1$, then the high-skill firm would reject after inspection because $S_h(q) < S_h(q_1^a) \leq p'$. So the low-skill firm prefers implementing the project.

Case 2: $q > q_n^b$.

The low-skill firm prefers implementing the project q to referring it at any equilibrium price because $S_l(q) > S_l(q_n^b) = S_h(q_n^a) = p_n$ by the third condition (iii) of Proposition 1.

Case 3: $n \geq 2$ and $q_m^b < q < q_{m+1}^a$, for $m = 1, ..., n-1$.

If the low-skill firm referred q at any equilibrium price $p'' \geq p_{m+1} = S_h(q_{m+1}^a)$, then the high-skill firm would reject after inspection because $S_h(q) < S_h(q_{m+1}^a) \leq p''$. Also, the low-skill firm would not refer q at any equilibrium price $p' \leq S_h(q_m^a)$ because

$p' \leq S_h(q_m^a) = S_l(q_m^b) < S_l(q)$ by the fourth condition (iv) of Proposition 1. So the low-skill firm prefers implementing the project q.

Finally, given the low-skill firm's strategies, I derive the high-skill firm's best response. The high-skill firm inspects referred projects at an equilibrium price p_m, for $m = 1, ..., n$, because, by the first condition (i) of Proposition 1, each referral set satisfies the incremental inspection constraint. By the production efficiency constraint in the first condition (i) of Proposition 1, the high-skill firm learns that $S_h(q) \geq p_m = S_h(q_m^a)$, for all $q \in (q_m^a, q_m^b)$, accepts the referral, and implements the project with efficient effort. Finally, the high-skill firm rejects referral at off-equilibrium prices because it pessimistically believes that it will derive $S_h(0) < 0$ by accepting. □

Proof of Corollary 2

Let c' and c'' be the solutions to the following problems;

$$c' = \max_{q'} \ \mathcal{I}\big(q', \bar{q}, p\big) \text{ s.t. } p = S_h(q') \geq S_l(\bar{q}), \text{ and} \tag{A.7}$$

$$c'' = \max_{q''} \ \mathcal{I}\big(q'', q''', p\big) \text{ s.t. } p = S_h(q'') = S_l(q''') \text{ and } q'' < q'''. \tag{A.8}$$

And q' and q'' be the arguments maximizing the above problems;

$$q' = \text{argmax}_{q'} \ \mathcal{I}\big(q', \bar{q}, p\big) \text{ s.t. } p = S_h(q') \geq S_l(\bar{q}), \text{ and} \tag{A.9}$$

$$q'' = \text{argmax}_{q''} \ \mathcal{I}\big(q'', q''', p\big) \text{ s.t. } p = S_h(q'') = S_l(q''') \text{ and } q'' < q'''. \tag{A.10}$$

Consider an equilibrium with $n \geq 1$ referral set. To begin with, c' is the largest possible incremental inspection surplus of the n-th referral set with $\bar{q}$ being the supremum of the set. Also, c'' is the largest possible incremental inspection surplus of a referral set with the supremum of the set being smaller than $\bar{q}$. They are found by maximizing the incremental inspection surplus subject to the production efficiency constraint in (i) and (iii) of Proposition 1, respectively. By assumption A, inspection cost is smaller than c_A. Let $c_R = \min\{c_A, \max\{c', c''\}\}$. I first prove that a referral equilibrium exists only if $c \leq c_R$. The proof is by contrapositive. Suppose that $c > c_R$. Then, either Assumption A is violated or $c > \max\{c', c''\}$. The latter means that no referral set can be constructed to satisfy both the incremental inspection constraint and the production efficiency constraint, and a referral equilibrium does not exist. Finally, I prove that a referral equilibrium exists if $c \leq c_R$. Clearly, if $c = c_R$, then there exists an equilibrium with referral set $(q', \bar{q})$ or $(q'', S_l^{-1}(S_h(q'')))$. If $c < c_R$, such an equilibrium still exists because a lower inspection cost relaxes the incremental inspection constraint. □

Proof of the Fifth Condition (v) of Proposition 3

First, suppose that $n = 1$.

By (i) and (iii) of Proposition 3, $S_h(q_1^b) \leq \max_e \frac{1}{G(q_1^b)} \int_0^{q_1^b} y_l(e, q)\ dG(q) < S_l(q_1^b)$, which implies that $q_2^b < q^{fb}$ by Lemma 2. Second, suppose that $n \geq 1$. By (iv) of Proposition 3, $S_l(q_n^a) \geq S_h(q_n^b)$. Also, $\mathcal{I}_l(q_n^a, q_n^b, p_n) \geq c$ implies that $q_n^b > q_n^a$. Therefore, $S_l(q_n^b) > S_l(q_n^a) \geq S_h(q_n^b)$, which implies that $q_n^a < q^{fb}$ by Lemma 2. □

Proof of Lemma 7

Suppose not. Suppose that in a monotone equilibrium some project $q \geq q_1^a$ is not referred. There are two cases.

Case 1: $n \geq 1$ and $q_n^b < \bar{q}$.

Consider the low-skill firm who proposes an off-equilibrium price $p' = S_h(q_n^b)$. The first condition (i) of monotone belief in Definition 3 applies because of the following.

First, $p' = S_h(q_n^b) > p_n = S_h(q_n^a)$. Second, by Proposition 1, $q^{fb} < q_1^a$, so $S_l(q) < S_h(q)$ for each $q \in (q_n^b, \bar{q})$. Therefore, the high-skill firm believes that a referred project at p' comes from the subset $K = (q_n^b, \bar{q})$. Given such belief, at $p' = S_h(q_n^b)$ the high-skill firm either inspects the project or accepts the offer without inspection. Therefore, any projects $q \geq q_n^b$ would be accepted at $p' = S_h(q_n^b)$ regardless of the high-skill firm's inspection decision. If there exists $q \in Q$ such that $S_l(q) = S_h(q_n^b)$, then there is a profitable deviation opportunity for the low-skill firm with projects q'' satisfying $q_n^b \leq q'' < S_l^{-1}(S_h(q_n^b))$ to refer at $p' = S_h(q_n^b) > S_l(q'')$. If there does not exist $q \in Q$ such that $S_l(q) = S_h(q_n^b)$, then there is a profitable deviation opportunity for the low-skill firm with projects q'' satisfying $q_n^b \leq q'' \leq \bar{q}$ to refer at $p' > S_l(q'')$.

Case 2: $n \geq 2$ and $q_m^b < q_{m+1}^a$, for some $m = 1, ..., n-1$.

Case 2's proof is similar to Case 1's. Consider the low-skill firm who proposes an off-equilibrium price $p' = S_h(q_m^b)$. The second condition (ii) of Definition 3 applies because $p_m < p' < p_{m+1}$. and $S_l(q) < S_h(q)$ for each $q \in (q_m^b, q_{m+1}^a)$. Therefore, the high-skill firm believes that a referred project at p' comes from the subset $K = (q_m^b, q_{m+1}^a)$. Given such belief, at p' the high-skill firm either inspects the project or accepts the offer without inspection. There is a profitable deviation opportunity for the low-skill firm with projects q'' satisfying $q_m^b \leq q'' < q_{m+1}^a$ to refer at p'. $\square$

Proof of Lemma 8

Consider a referral equilibrium with referral sets $(q_1^a, q_1^b), ..., (q_n^a, q_n^a)$ with each project $q \geq q_1^a$ being referred. I will replace the equilibrium's belief with a monotone belief according to Definition 3. The first condition (i) in Definition 3 is not applicable. Although there exists prices higher than p_n, the subset $K = (q_n^b, \bar{q})$ has zero measure because each project $q \geq q_1^a$ is referred. The second condition (ii) in Definition 3 is not applicable for a similar reason. Finally, as discussed in the text, the third condition (iii) is never applicable in ascending referral. Since none of (i)-(iii) is applicable, monotone belief only imposes each off-equilibrium information set to have a positive measure. Here is one example; at each off-equilibrium price p', the high-skill firm believes that the corresponding project is from the subset $(0, S_h^{-1}(p'))$, which has a positive measure. This construction is enough because prices must be larger than zero and lower than $S_h(\bar{q})$. the high-skill firm rejects any referral at off-equilibrium price p' because each project within $(0, S_h^{-1}(p'))$ generates a surplus lower than p'. The proof for a no-referral equilibrium is similar; all prices are off-equilibrium and none of (i)-(iii) in Definition 3 is applicable.

Proof of Proposition 5

Suppose there exists an equilibrium σ, which has $n \geq 1$ referral sets $(q_1^a, q_1^b), ..., (q_n^a, q_n^b)$. Also, $\mathcal{I}\big(q, q_1^a, S_h(q)\big) < c$ for each q satisfying $q_1^a \leq q \leq S_h^{-1}\big(S_l(q_1^a)\big)$. Consider any other equilibrium $\hat{\sigma}$ that satisfies monotone belief. I want to show that $\hat{\sigma}$ does not interim Pareto dominate σ. Suppose σ has equilibrium payoff functions $u_l(q)$, $q \in Q$, and u_h. Also, suppose $\hat{\sigma}$ has equilibrium payoff functions $\hat{u}_l(q)$, $q \in Q$, and $\hat{u}_h$ and s referral sets, $(\hat{q}_1^a, \hat{q}_1^b), ...(\hat{q}_s^a, \hat{q}_s^b)$. There are two cases.

Case 1. There is a m satisfying $1 \leq m \leq n$ such that $q_m^a \neq \hat{q}_j^a$, for $j = 1, ..., s$.

In this case, there is a project q_m^a that is an infimum of a referral set in equilibrium σ but is not an infimum of any referral set in equilibrium $\hat{\sigma}$. By Proposition 1, $u_l(q_m^a) = p_m = S_h(q_m^a)$. In equilibrium $\hat{\sigma}$, the project q_m^a is either worked on by the low-skill firm, which means that $\hat{u}_l(q_m^a) = S_l(q_m^a) < S_h(q_m^a)$, or it is referred by a price lower than $S_h(q_m^a)$ because q_m^a is not an infimum of any referral set in equilibrium

$\hat{\sigma}$. Therefore, $u_l(q_m^a) = S_h(q_m^a) > \hat{u}_l(q_m^a)$ which means thatt σ is not interim Pareto dominated by $\hat{\sigma}$.

Case 2. There is a a j satisfying $1 \leq j \leq s$ such that $\hat{q}_j^a = q_m^a$, for $m = 1, ..., n$.

In this case, every project which is an infimum of a referral set in equilibrium σ is also an infimum of a referral set in equilibrium $\hat{\sigma}$. That means there are weakly more referral sets in equilibrium $\hat{\sigma}$; $s \geq n$. First, by assumption there does not exist $q < q_1^a$ such that $S_h(q) \geq S_l(q_1^a)$ and $\mathcal{I}(q, q_1^a, S_h(q)) \geq c$, so there does not exist any referral set with q_1^a as supremum in equilibrium $\hat{\sigma}$. Second, by Lemma 7, because both equilibria σ and $\hat{\sigma}$ satisfy monotone belief, all $q > q_1^a$ and all $q > \hat{q}_1^a$ are referred in equilibria σ and $\hat{\sigma}$, respectively. Combining the above two points, I have $q_1^a = \hat{q}_1^a$. If $s = n$, then the two equilibria share the same equilibrium strategies and allocations, and σ is not interim Pareto dominated by $\hat{\sigma}$.

Next, suppose that $s > n$. I will show that $u_h > \hat{u}_2$ and therefore σ is not interim Pareto dominated by $\hat{\sigma}$. Let $p(q)$ and $\hat{p}(q)$, $q \in (q_1^a, \bar{q})$, be the equilibrium referral price in σ and $\hat{\sigma}$, respectively. They are both functions of project potential. By Proposition 1, $p(q) = S_h(q_m^a)$, for $q \in (q_m^a, q_m^b)$ and $m = 1, ..., n$, and $\hat{p}(q) = S_h(q_j^a)$, for $q \in (\hat{q}_m^a, \hat{q}_m^b)$ and $m = 1, ..., s$. Because every project which is an infimum of a referral set in equilibrium σ is also an infimum of a referral set in equilibrium $\hat{\sigma}$

$$\hat{p}(q) \geq p(q) \tag{A.11}$$

for all $q \in (q_1^a, \bar{q})$, with the inequality being strict for some $q \in (q_1^a, \bar{q})$. Therefore,

$$\int_{q_1^a}^{\bar{q}} \hat{p}(q) dG(q) > \int_{q_1^a}^{\bar{q}} p(q) dG(q) \tag{A.12}$$

And

$$
\begin{aligned}
& u_h - \hat{u}_h \\
&= \sum_{m=1}^{n} \int_{q_m^a}^{q_m^b} (S_h(q) - p_m - c) \; dG(q) - \sum_{m=1}^{s} \int_{\hat{q}_m^a}^{\hat{q}_m^b} (S_h(q) - \hat{p}_m - c) \; dG(q) \\
&= \sum_{m=1}^{s} \int_{\hat{q}_m^a}^{\hat{q}_m^b} \hat{p}_m \; dG(q) - \sum_{m=1}^{n} \int_{q_m^a}^{q_m^b} p_m \; dG(q) \\
&= \int_{q_1^a}^{\bar{q}} \hat{p}(q) dG(q) - \int_{q_1^a}^{\bar{q}} p(q) dG(q) > 0.
\end{aligned}
$$

The last line is larger than zero by (A.12). □

Proof of Lemma 10

Let $\hat{c}$ be:

$$\hat{c} = \int_{0}^{\hat{q}} S_l(q) \; dG(q), \tag{A.13}$$

$$\text{with } S_h(\hat{q}) = \max_e \int_{0}^{\bar{q}} y_l(e, q) \; dG(q). \tag{A.14}$$

Suppose not. Suppose that $c \leq \hat{c}$ and there is an equilibrium in which the low-skill firm refers a project at p without inspection and the high-skill firm inspects the referred project. Call this equilibrium σ. The equilibrium price p must be larger than the low-skill firm's average surplus without project potential information;

$$p > \max_e \int_{0}^{\bar{q}} y_l(e, q) \; dG(q). \tag{A.15}$$

Let q' be the project potential satisfying

$$S_h(q') = p \tag{A.16}$$

By (A.14), (A.15) and (A.16), I know

$$p = S_h(q') > \max_e \int_0^{\bar{q}} y_l(e, q)\ dG(q) = S_h(\hat{q}).$$

That means

$$q' > \hat{q}, \qquad \text{and} \tag{A.17}$$

$$\int_0^{q'} S_l(q)\ dG(q) > \int_0^{\hat{q}} S_l(q)\ dG(q) \tag{A.18}$$

I will show that equilibrium σ generates contradiction to (A.18). Note that the equilibrium satisfies pessimistic belief, so the high-skill firm rejects any price not equal to p. The proof is divided into two cases.

Case 1: $p \leq S_l(\bar{q})$.

There exists a project q'' such that $p = S_l(q'')$. The low-skill firm does not inspect the project in stage 1 only if

$$\int_{q'}^{\bar{q}} p\ dG(q) \geq \int_0^{q'} S_l(q)\ dG(q) + \int_{q'}^{q''} p\ dG(q) + \int_{q''}^{\bar{q}} S_l(q)\ dG(q) - c. \tag{A.19}$$

The left-hand side of (A.19) is the low-skill firm's surplus by referring a project to the high-skill firm at p without inspection. After inspection, the high-skill firm accepts if and only if the project potential is higher than q', because $p = S_h(q')$. Otherwise, the high-skill firm rejects and the game ends.

The right-hand side of (A.19) is the low-skill firm's surplus by inspecting and referring some projects at p. After inspection, the low-skill firm implements projects with potential lower than q'. As I have shown above, if the low-skill firm refers such projects at p, then it will get rejected. Moreover, because $p = S_l(q'')$, the low-skill firm also implements projects with potential higher than q'' instead of referring them at p. For projects with potential in between q' and q'', the low-skill firm refers at p and the high-skill firm accepts after inspection. I rearrange (A.19) to get

$$c \geq \int_0^{q'} S_l(q)\ dG(q) + \int_{q''}^{\bar{q}} [S_l(q) - p]\ dG(q). \tag{A.20}$$

The second term on the right-hand side of (A.20) is larger than zero because $p = S_l(q'')$. By (A.20), $c \leq \hat{c}$, and (A.13) I have

$$\int_0^{\hat{q}} S_l(q)\ dG(q) = \hat{c} \geq c \geq \int_0^{q'} S_l(q)\ dG(q) + \int_{q''}^{\bar{q}} [S_l(q) - p]\ dG(q),$$

which is a contradiction to (A.18).

Case 2: $p > S_l(\bar{q})$.

The low-skill firm does not inspect the project in stage 1 only if

$$\int_{q'}^{\bar{q}} p\ dG(q) \geq \int_0^{q'} S_l(q)\ dG(q) + \int_{q'}^{\bar{q}} p\ dG(q) - c. \tag{A.21}$$

As in (A.19), the left-hand side of (A.21) is the low-skill firm's surplus by referring at p without inspection, whereas the right-hand side of (A.21) is the low-skill firm's surplus by inspecting and referring some projects at p. After inspection, the low-skill firm implements projects with potential smaller than q' to avoid the projects being rejected. The low-skill firm refers projects with potential larger than q' at p because $p > S_l(\bar{q})$. Rearrange (A.21) to get

$$c \geq \int_0^{q'} S_l(q)\ dG(q). \tag{A.22}$$

Combine (A.22) with $c \leq \hat{c}$ and (A.13) to have $\int_0^{\hat{q}} S_l(q)\ dG(q) = \hat{c} \geq c \geq \int_0^{q'} S_l(q)\ dG(q)$, which is a contradiction to (A.18). □

Proof of Proposition 7

Let c' be

$$c' = \int_0^{\bar{q}} S_l(q)dG(q) - \max_e \int_0^{\bar{q}} y_l(e,q)dG(q) \tag{A.23}$$

The proof is divided into two cases.

Case 1: $\max_e \int_0^{\bar{q}} y_l(e,q)\ dG(q) < \max_e \int_0^{\bar{q}} [y_h(e,q) - f_h]\ dG(q)$.

Let c'' be the solution to the following program:

$$\min_x\ \mathcal{I}(0, \bar{q}, x) \tag{A.24}$$

$$\text{s.t.}\ \max_e \int_0^{\bar{q}} y_l(e,q)\ dG(q) < x < \max_e \int_0^{\bar{q}} [y_h(e,q) - f_h]\ dG(q). \tag{A.25}$$

Because of the constraint (A.25), $c'' > 0$. Let $\tilde{c} = \min\{\hat{c}, c', c''\}$, with $\hat{c}$ being characterized by (A.13) and (A.14) in the proof of Lemma 10. I will prove that if $c \leq \tilde{c}$, then the low-skill firm refers or implements a project only after inspection. Clearly, there does not exist an equilibrium in which the low-skill firm implements without inspection because $c \leq c'$.

Suppose not. Suppose that $c \leq \tilde{c}$ and there exists an equilibrium such that the low-skill firm does not inspect and refers a project to the high-skill firm at price p. Because $c \leq \hat{c}$, by Lemma 10, in this equilibrium, the low-skill firm refers without inspection at p and the high-skill firm accepts without inspection. First, the low-skill firm refers without inspection only if the equilibrium price is larger than the low-skill firm's average surplus without potential information. Second, the high-skill firm accepts without inspection only if its average surplus without potential information is larger than the equilibrium price. Therefore, p satisfies $\max_e \int_0^{\bar{q}} y_l(e,q)\ dG(q) < p < \max_e \int_0^{\bar{q}} [y_h(e,q) - f_h]\ dG(q)$. Third, the high-skill firm does not inspect at p if and only if $\mathcal{I}(0, \bar{q}, p) < c$. By (A.24) and (A.25), these three points combined contradicts $c \leq c''$.

Case 2: $\max_e \int_0^{\bar{q}} [y_h(e,q) - f_h]\ dG(q) \leq \max_e \int_0^{\bar{q}} y_l(e,q)\ dG(q)$.

Let $\tilde{c} = \min\{\hat{c}, c'\}$. Following the same logic above, I only have to prove that there does not exist an equilibrium such that the low-skill firm refers a project to the high-skill firm without inspection at price p. Suppose not. Suppose that $c \leq \tilde{c}$ and such an equilibrium exists. By Lemma 10, the low-skill firm refers without inspection at p and the high-skill firm accepts without inspection. Therefore, p satisfies $\max_e \int_0^{\bar{q}} [y_h(e,q) - f_h]\ dG(q) > p > \max_e \int_0^{\bar{q}} y_l(e,q)\ dG(q)$, which is a contradiction to the premise of Case 2. $\square$

Appendix B

Appendix for Chapter 2

Proof of Lemma 3

$$\int_0^{\bar{s}} V_l(x)dG(x) - f \geq V_l(\mu_l)$$

$$\int_0^{\bar{s}} \Big[t(0) - \int_0^{y} p_l(x)dx\Big]dG(y) - f \geq t_l(0) - \int_0^{\mu_l} p_l(x)dx$$

$$f \leq \int_0^{\mu_l} p_l(x)dx - \int_0^{\bar{s}} \Big[\int_0^{y} p_l(x)dx\Big]dG(y)$$

$$f \leq \int_0^{\mu_l} p_l(x)dx - \Big[G(y)\int_0^{y} p(x)dx\Big]_0^{\bar{s}} + \int_0^{\bar{s}} G(y)p(y)dy$$

$$f \leq \int_0^{\mu_l} p_l(x)dx - \int_0^{\bar{s}} (1 - G(x))p_l(x)dx$$

$$f \leq \int_0^{\bar{s}} G(x)p_l(x)dx - \int_{\mu_l}^{\bar{s}} p_l(x)dx$$

$$f \leq \int_0^{\mu_l} G(x)p_l(x)dx - \int_{\mu_l}^{\bar{s}} (1 - G(x))p_l(x)dx.$$

In the above, the second line is due to substitution of (2.11) into (2.6) and the fourth line is due to integration by parts.

Proof of Proposition 1

Case 1: $s^{fb} < \mu_l$.

First, I show that $s^{fb} < \mu_l$ implies that $\mu_h < \mu_l$ when $c_h(s)$ is concave. Suppose $c_h(s)$ is concave. Therefore, $\mu_h = E(c_h(s)) \leq c_h(E(s)) = c_h(\mu_l)$. Since $s^{fb} < \mu_l$, $c_h(\mu_l) < \mu_l$. Combine the two inequalities together, I have $\mu_h < \mu_l$. Clearly, a linear cost function $c_h(s) = \alpha s + \beta$ is also concave. However, this proof applies when I study a concave $c_h(s)$ later.

Suppose $p_l(s) = 1$ for $s \leq s^{fb}$ and $p_l(s) = 0$ for $s > s^{fb}$. Such a probability assignment function is decreasing. Plug $p_l(s)$ into the R.H.S. of (2.20) to get $\int_0^{s^{fb}} G(x)dx$. Therefore, if $f \leq \int_0^{s^{fb}} G(x)dx$, then a contract consisting of an efficient allocation rule can incentivize information acquisition and assigns projects to the expert who has the lowest cost. Recall that the first best requires information acquisition if and only if $f \leq \min\left\{\int_0^{s^{fb}} \left[\beta - (1-\alpha)x\right] dG(x), \int_{s^{fb}}^{\bar{s}} \left[(1-\alpha)x - \beta\right] dG(x)\right\}$. Since $\mu_h < \mu_l$ and as discussed in Section 2.2, the first best requires information acquisition if and only if $f \leq \int_0^{s^{fb}} \left[\beta - (1-\alpha)x\right] dG(x)$. If I can prove that $\int_0^{s^{fb}} G(x)dx - \int_0^{s^{fb}} \left[\beta - (1-\alpha)x\right] dG(x) > 0$, then whenever $f \leq \int_0^{s^{fb}} \left[\beta - (1-\alpha)x\right] dG(x)$, the Principal can use an optimal information contract to implement the first best. It is true as shown below.

$$\begin{aligned}
&\int_0^{s^{fb}} G(x)dx - \int_0^{s^{fb}} \big[\beta - (1-\alpha)x\big] dG(x) \\
&= \int_0^{s^{fb}} G(x)dx - \Big[\big(\beta - (1-\alpha)x\big)G(x)\Big]_0^{s^{fb}} + \int_0^{s^{fb}} G(x)d\big(\beta - (1-\alpha)x\big) \\
&= \int_0^{s^{fb}} G(x)dx - \int_0^{s^{fb}} G(x)dx + \int_0^{s^{fb}} \alpha G(x)dx \\
&= \int_0^{s^{fb}} \alpha G(x)dx > 0.
\end{aligned}$$

When $f > \int_0^{s^{fb}} \big[\beta - (1-\alpha)x\big] dG(x)$, then the Principal can use an optimal no-information contract to implement the first best.

Case 2: $\mu_l \leq s^{fb}$.

First, I show that $\mu_l \leq s^{fb}$ implies that $\mu_l \leq \mu_h$ when $c_h(s)$ is convex. Suppose $c_h(s)$ is convex. Therefore, $\mu_h = E(c_h(s)) \geq c_h(E(s)) = c_h(\mu_l)$. Since $\mu_l \leq s^{fb}$, $c_h(\mu_l) \geq \mu_l$. Combine the two inequalities together, I have $\mu_h \geq \mu_l$. Clearly, a linear cost function $c_h(s) = \alpha s + \beta$ is also convex. However, this proof applies when I study a convex $c_h(s)$ later. Plug $p_l(s) = 1$ for $s \leq s^{fb}$ and $p_l(s) = 0$ for $s > s^{fb}$ into the R.H.S. of (2.20) to get $\int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{s^{fb}} (1 - G(x))dx$, which is larger than zero because

$\int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{\bar{s}} (1 - G(x))dx = 0$ as shown below.

$$\begin{aligned} &\int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{\bar{s}} (1 - G(x))dx \\ =&\int_0^{\bar{s}} G(x)dx - \int_{\mu_l}^{\bar{s}} dx \\ =&\Big[G(x)x\Big]_0^{\bar{s}} - \int_0^{\bar{s}} xdG(x) - \Big[x\Big]_{\mu_l}^{\bar{s}} \\ =&0. \end{aligned}$$

Therefore, if $f \leq \int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{s^{fb}} (1 - G(x))dx$, then a contract consisting of an efficient allocation rule can incentivize information acquisition and assigns projects to the expert who has the lowest cost. The first best requires information acquisition if and only if $f \leq \min\Big\{ \int_0^{s^{fb}} \big[\beta - (1-\alpha)x\big] dG(x), \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x) \Big\}$. However, $\mu_h = \int_0^{\bar{s}} \alpha x + \beta dG(x) = \alpha\mu_l + \beta \geq \mu_l$ since $\mu_l \leq s^{fb}$. Therefore, the first best requires information acquisition if and only if $f \leq \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x)$. If I can prove that $\int_0^{\mu_l} G(x)dx - \int_{\mu_l}^{s^{fb}} (1 - G(x))dx - \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x) > 0$, then whenever $f \leq \int_{s^{fb}}^{\bar{s}} \big[(1-\alpha)x - \beta\big] dG(x)$, the Principal can use an optimal information contract to implement the first best. It is true as shown below.

$$
\begin{aligned}
&\int_{0}^{\mu_l} G(x)dx - \int_{\mu_l}^{s^{fb}} (1 - G(x))dx - \int_{s^{fb}}^{\bar{s}} \Big[(1-\alpha)x - \beta\Big] dG(x) \\
&= \int_{0}^{s^{fb}} G(x)dx - \int_{\mu_l}^{s^{fb}} dx - \Big[\big((1-\alpha)x - \beta\big)G(x)\Big]_{s^{fb}}^{\bar{s}} + \int_{s^{fb}}^{\bar{s}} G(x)d\big((1-\alpha)x - \beta\big) \\
&= \int_{0}^{\bar{s}} G(x)dx - \int_{s^{fb}}^{\bar{s}} \alpha G(x)dx - [x]_{\mu_l}^{s^{fb}} - [\bar{s} - \alpha\bar{s} - \beta] \\
&= \Big[G(x)x\Big]_{0}^{\bar{s}} - \int_{0}^{\bar{s}} xdG(x) - \int_{s^{fb}}^{\bar{s}} \alpha G(x)dx - s^{fb} + \mu_l - \bar{s} + \alpha\bar{s} + \beta \\
&= -\int_{s^{fb}}^{\bar{s}} \alpha G(x)dx - s^{fb} + \alpha\bar{s} + \beta \\
&= -\int_{s^{fb}}^{\bar{s}} \alpha G(x)dx - \alpha s^{fb} - \beta + \alpha s^{fb} + \beta + \int_{s^{fb}}^{\bar{s}} \alpha dx \\
&= \int_{s^{fb}}^{\bar{s}} \alpha dx - \int_{s^{fb}}^{\bar{s}} \alpha G(x)dx > 0.
\end{aligned}
$$

When $f > \int_{s^{fb}}^{\bar{s}} \Big[(1-\alpha)x - \beta\Big] dG(x)$, then the Principal can use an optimal no-information contract to implement the first best. □

Proof of Lemma 4

Suppose a contract $\big(t_l(s), t_h(s), p_l(s)\big)$ satisfies the high-skill expert's truthful-reporting constraint. Assume without loss of generality that $s \geq \hat{s}$. Then,

$$t_l(s) - p_l(s)(\alpha s + \beta) \geq t_l(\hat{s}) - p_l(\hat{s})(\alpha s + \beta), \text{ and}$$
$$t_l(\hat{s}) - p_l(\hat{s})(\alpha \hat{s} + \beta) \geq t_l(s) - p_l(s)(\alpha \hat{s} + \beta).$$

Add them together to get

$$-p_l(s)s - p_l(\hat{s})\hat{s} \geq -p_l(\hat{s})s - p_l(s)\hat{s}$$
$$(\hat{s} - s)(p_l(s) - p_l(\hat{s})) \geq 0,$$

which implies that $p_l(s) < p_l(\hat{s})$.

Proof of Proposition 2

Let $\hat{p}_h(s)$ be the solution of Program 2. Below, I show that $\hat{p}_h(s)$ is a constant $\forall s$. Suppose not. Suppose that $\hat{p}_h(s)$ is not a constant for $\forall s$. Consider the following term.

$$\int_0^{s^{fb}} \big(\hat{p}_h(s) - \hat{p}_h(s^{fb})\big)(\alpha s + \beta - s)dG(s) + \int_{s^{fb}}^{\bar{s}} \big(\hat{p}_h(s^{fb}) - \hat{p}_h(s)\big)(s - \alpha s - \beta)dG(s).$$

Note that $\alpha s + \beta \geq s$ by definition and $\hat{p}_h(s) \geq \hat{p}_h(s^{fb})$ because $\hat{p}_h(s)$ is non-increasing, for $s \leq s^{fb}$. For the same reason, $s \geq \alpha s + \beta$ and $\hat{p}_h(s^{fb}) \geq \hat{p}_h(s)$, for $s > s^{fb}$.

Therefore,

$$\int_0^{s^{fb}} \big(\hat{p}_h(s) - \hat{p}_h(s^{fb})\big)(\alpha s + \beta - s)dG(s) + \int_{s^{fb}}^{\bar{s}} \big(\hat{p}_h(s^{fb}) - \hat{p}_h(s)\big)(s - \alpha s - \beta)dG(s) \geq 0$$

$$\int_0^{\bar{s}} \big(\hat{p}_h(s) - \hat{p}_h(s^{fb})\big)(\alpha s + \beta - s)dG(s) \geq 0$$

$$\int_0^{\bar{s}} \hat{p}_h(s)(\alpha s + \beta - s)dG(s) \geq \int_0^{\bar{s}} \hat{p}_h(s^{fb})(\alpha s + \beta - s)dG(s)$$

$$\int_0^{\bar{s}} \big[\hat{p}_h(s)(\alpha s + \beta) + \big(1 - \hat{p}_h(s)\big)s\big]dG(s) \geq \int_0^{\bar{s}} \big[\hat{p}_h(s^{fb})(\alpha s + \beta) + \big(1 - \hat{p}_h(s^{fb})\big)s\big]dG(s),$$

which is a contradiction as $\hat{p}_h(s)$ is a solution to Program 2 and is not a constant. Now I have established that $\hat{p}_h(s)$ is a constant for $\forall s$. Let $\hat{p}_h(s) = p$. Plug it into Program 2. Clearly, $\hat{p}_h(s) = p$ satisfies the non-increasing constraint in Program 2.

$$\min_p \int_0^{\bar{s}} \Big[(1-p)x + p(\alpha x + \beta)\Big]dG(x)$$

$$= \min_p (1-p)\mu_l + p\mu_h.$$

Therefore, $\hat{p}_h(s) = 1$ if $\mu_h \leq \mu_l$ and $\hat{p}_h(s) = 0$ otherwise. And the solution to Program 2 is $\min\{\mu_l, \mu_h\}$, which is the Principal's expected payment in an optimal no-information equilibrium. In the text before and after Lemma 4, I have argued that the Principal's expected equilibrium payment of a high-skill information equilibrium must be larger than the solution to Program 2. □

Proof of Proposition 3

First, I show that $\lambda \geq 0$. Then, I show that no matter the magnitude of λ, the assignment probability function $p_l(s)$ in the solution to Program 1 must be in one of the two forms described by Proposition 3. To check if $p_l(s)$ is a candidate of a solution to Program 1, I look at the Lagrangian of the relaxed problem Program 1*. I check if I can adjust $p_l(s)$ point-wise according to the first-order point-wise derivative of the Lagrangian without violating the monotonicity condition. If I cannot, then $p_l(s)$ is a candidate.

Recall that $\mu_l \leq s^{fb}$ and $\mu_h < \mu_l$. It is point-wise optimal to set $p_l(s) = 1$ if $\Delta(s) \geq 0$ and $p(s) = 0$ otherwise. Whether $\Delta(s)$ is positive or negative depends on the term in the square bracket. Define $D(s)$ to be that term.

$$D(s) \equiv \begin{cases} c_h(s) - s + \lambda k(s), & \text{if } s < \mu_l, \text{ and} \\ c_h(s) - s - \lambda h(s), & \text{if } s \geq \mu_l. \end{cases} \tag{B.1}$$

Note that it is optimal to set $p_l(s) = 1$ for $s \leq \mu_l$ and $p_l(s) = 0$ for $s \geq s^{fb}$. Doing so minimizes the objective function and relaxes (2.20) in Program 1*. Therefore, I only have to figure out the probability assignment for s between μ_l and s^{fb}. Since $p_l(s^{fb}) = 0$, that means $D(s^{fb}) \leq 0$. Since $D(s^{fb}) = -\lambda h(s^{fb})$, λ is positive. Indeed, $D(s^{fb}) = -\lambda h(s^{fb}) < 0$ and $D''(s) = c_h''(s) - \lambda h''(s) < 0$ because $c_h(s)$ is concave and $h(s)$ is convex. There are two cases.

Case 1: $D(\mu_l) > 0$.

Since $D(s^{fb}) < 0$ and $D''(s) < 0$, $D(s) > 0$ for $s \leq s'$ and $D(s) < 0$ for $s > s'$ with $\mu_l < s' < s^{fb}$. It is optimal to set $p_l(s) = 1$ for $s \leq s'$ and $p_l(s)$ otherwise. To find s', note that (2.20) is binding in a solution. Plug in the above probability function to get $f = \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s'} h(x)dG(x)$, which determines s'. Finally, note that the above probability assignment function satisfies the monotonicity condition, so it is a solution to Program 1^*.

Case 2: $D(\mu_l) < 0$.

Note that $D(s^{fb}) < 0$. There are two possibilities. First, $D(s) \leq 0$ for s satisfying $\mu_l \leq s \leq s^{fb}$. Then, it is optimal to set $p_l(s) = 1$ for $s \leq \mu_l$ and $p_l(s) = 0$ otherwise. Again, this satisfies the monotonicity condition, so it is a solution to Program 1^*.

Second, $D(s) > 0$ for some s satisfying $\mu_l < s < s^{fb}$. Since $D(s^{fb}) < 0$ and $D''(s) < 0$, I have $D(s) \leq 0$ for $\mu_l \leq s <\leq s'$ and $D(s) > 0$ for $s' < s < s''$ and $D(s) \leq 0$ for $s'' \leq s \leq s^{fb}$, with $\mu_l < s' < s'' < s^{fb}$. Clearly, in this case, the Principal optimally sets $p_l(s) = 1$ for $s < \mu_l$ and $p_l(s) = 0$ for $s \geq s''$. The question is how the Principal should set $p_l(s)$ for $s \in [\mu_l, s'')$.

I show that if the Principal sets $p_l(s)$ as a constant function for the range $s \in [\mu_l, s'')$, then it cannot be adjusted without violating the monotonicity condition. I prove that by contradiction. I show that if $p_l(s)$ is not a constant function over this range, then I can increase or decrease the function point-wise according to the point-wise derivative to improve the Principal's payoff without violating the monotonicity condition. There are two cases to consider.

Case 2i: $p_l(\hat{s})$ at $\hat{s} \in [\mu_l, s']$.

By the above, $D(\hat{s}) \leq 0$. Suppose not. Suppose that $p_l(\hat{s})$ is not differentiable at this point or $p_l'(\hat{s}) \neq 0$. By the monotonicity condition, $p_l(\hat{\hat{s}}) < p_l(\hat{s})$ for any $\hat{\hat{s}} > \hat{s}$. However, I can decrease $p_l(\hat{s})$ to improve the Principal's payoff without violating the monotonicity condition since $D(\hat{s}) \leq 0$ and $p_l(\hat{\hat{s}}) < p_l(\hat{s})$ for any $\hat{\hat{s}} > \hat{s}$, which is a contradiction.

Case 2ii: $p_l(\hat{s})$ at $\hat{s} \in (s', s'')$.

By the above, $D(\hat{s}) > 0$. Suppose not. Suppose that $p_l(\hat{s})$ is not differentiable at this point or $p_l'(\hat{s}) \neq 0$. By the monotonicity condition, $p_l(s) < p_l(\hat{s})$ for any $s > \hat{s}$. There exists an $\epsilon > 0$ such that $D(\hat{\hat{s}}) > 0$ and $p_l(\hat{\hat{s}}) < p_l(\hat{s})$ for any $\hat{\hat{s}}$ satisfying $\hat{s} < \hat{\hat{s}} < \hat{s}+\epsilon$. However, I can increase $p_l(\hat{\hat{s}})$ so that $p_l(\hat{\hat{s}}) = p_l(\hat{s})$ to improve the Principal's payoff without violating the monotonicity condition since $D(\hat{\hat{s}}) > 0$ and $p_l(\hat{\hat{s}}) < p_l(\hat{s})$ for any $\hat{\hat{s}}$ satisfying $\hat{s} < \hat{\hat{s}} < \hat{s} + \epsilon$, which is a contradiction.

Therefore, case 2 shows that a candidate of a solution to Program 1* is that $p_l(s) = 1$ for all $s \leq \mu_l$, $p_l(s) = a$, for all s satisfying $\mu_l < s < s'' < s^{fb}$, $p_l(s) = 0$ for all $s \geq s''$. Also, because (2.20) is binding, a and s'' satisfy $f = \int_0^{\mu_l} k(x)dG(x) - \int_{\mu_l}^{s''} ah(x)dG(x)$.
□

Appendix C

Appendix for Chapter 3

Proof of Lemma 1

Suppose not. Suppose the low-skill expert refers a project without knowing the project state at $t' < \alpha s_5 + \beta$. The high-skill expert must be acquiring information at t'. Suppose the high-skill expert acquires information at probability $\theta'' > 0$. Then, if he succeeds, he rejects at least some projects.

Case 1: $\alpha s_4 + \beta \leq t < \alpha s_5 + \beta$.

The high-skill expert rejects project ω_5. Then, the low-skill expert's effective cost in handling a project with uncertain state is $\theta''(p_5 s_5 + (1 - p_5)t') + (1 - \theta'')s_3$, which is larger than his own cost s_3.

Case 2: $\alpha s_3 + \beta \leq t < \alpha s_4 + \beta$.

The high-skill expert rejects projects ω_4 and ω_5. Then, the low-skill expert's effective cost in handling a project with uncertain state is $\theta''(p_4 s_4 + p_5 s_5 + (1 - p_4 - p_5)t') + (1 - \theta'')s_3$, which is larger than s_3.

Case 3: $\alpha s_2 + \beta \leq t < \alpha s_3 + \beta$.

The high-skill expert rejects projects ω_3, ω_4, and ω_5. Then, the low-skill expert's effective cost in handling a project with uncertain state is $\theta''(p_3 s_3 + p_4 s_4 + p_5 s_5 + (p_1 + p_2)t') + (1 - \theta'')s_3$, which is larger than s_3 because $t' > s_2$.

Case 4: $\alpha s_1 + \beta \leq t < \alpha s_2 + \beta$.

The high-skill expert rejects projects ω_2, ω_3, ω_4, and ω_5. Then, the low-skill expert's effective cost in handling a project with uncertain state is $\theta''(p_2 s_2 + p_3 s_3 + p_4 s_4 + p_5 s_5 + p_1 t') + (1 - \theta'')s_3$, which is larger than s_3 because $t' > s_2$. $\square$

Proof of Lemma 2

Suppose not. Suppose there exists an equilibrium in which the low-skill expert does not acquire information. By Lemma 1, the low-skill expert does not refer projects and gets a payoff s_3. But the low-skill expert can always deviate to acquire some information and refer s_5 at $\alpha s_5 + \beta$ whenever information acquisition is successful. The high-skill expert accepts regardless of belief without information acquisition. The deviation generates a higher payoff: $\max_\theta \Big\{\theta\big[\sum_{i=1}^{4} p_i s_i + p_5(\alpha s_5 + \beta)\big] + (1-\theta)s_3 + k(\theta)\Big\} < s_3$. $\square$

Proof of Lemma 3

Suppose not. Suppose that the low-skill expert at least refers one of the three projects at payment t' after a successful information acquisition.

Case 1: The high-skill expert accepts t' without information acquisition.

That means, there is no uncertainty underlying t'. But the high-skill expert doesn't have comparative advantage in handling the project. There is no payment which can facilitate the trade.

Case 2: The high-skill expert acquires information at t'.

After a successful information acquisition, the high-skill expert accepts the project with the lowest state that is pooled at t'. Suppose it is ω_1. Then, $t' > \alpha s_1 + \beta$. The low-skill would profitably deviate to complete ω_1 himself since his cost is $s_1 < \alpha s_1 + \beta$. The cases if the project is ω_2 or ω_3 is similar. □

Proof of Lemma 4

By Lemma 3, it would not be profitable for the low-skill expert to refer projects ω_1, ω_2, and ω_3 after a successful information acquisition. Suppose that there are at least two equilibrium payment t' and t'', with $t' < t''$. Then, by Lemma 1, the low-skill expert refers either project ω_4 or ω_5 at t' and refers the other at t'' after a successful information acquisition. And the high-skill expert accepts t' or t'' without information acquisition. But the high-skill expert can profitably deviate by pooling ω_4 and ω_5 at t''. □

Proof of Proposition 1

Given the low-skill expert's equilibrium strategy. The high-skill expert expert does not acquire information at any payment because when $t \geq \alpha s_5 + \beta$ the payment is higher than his project cost in any state. And he believes that the project is in state ω_5 if $t < \alpha s_5 + \beta$. For the same logic, he accepts $t \geq \alpha s_5 + \beta$ and rejects $t < \alpha s_5 + \beta$.

Given the high-skill expert's equilibrium strategy. The low-skill expert refers projects ω_4 and ω_5 because $t^* = \alpha s_5 + \beta < s_4$. He completes the project in other scenarios by Lemmas 1 and 3. He acquires information at θ_l^* which solves $\min_\theta \left\{\theta(\sum_{i=4}^5 p_i t^* + \sum_{i=1}^3 p_i s_i) + (1-\theta)s_3 + k(\theta)\right\}$, which implies that $k'(\theta_l^*) = \sum_{i=4}^5 p_i(s_i - \alpha s_5 - \beta)$.

This equilibrium always exists because $k'(\theta_l^*) = \sum_{i=4}^5 p_i(s_i - \alpha s_5 - \beta)$ is always satisfied. And the above single-acquisition equilibrium is unique because if $t^* < \alpha s_5 + \beta$, then the high-skill expert would acquire information. □

Proof of Lemma 8

Suppose not. Suppose that the low-skill expert does not acquire information in an equilibrium. Since the low-skill expert has no incentive to acquire information, the high-skill acquires information and refer a project of a specific type in the equilibrium.

Case 1: project ω_1 is referred.

For the high-skill expert to refer and the low-skill expert to accept ω_1 at t^*, $s_1 < t^* < \alpha s_1 + \beta$. Since the low-skill expert accepts without information acquisition, the high-skill expert would deviate to refer projects ω_2, ω_3, ω_4, and ω_5 at t^*, which is a contradiction.

Case 2: projects ω_2 and ω_3 are referred.

The proof of this case is similar to case 1's.

Case 3: project ω_4 or ω_5 is referred.

The low-skill expert doesn't have comparative advantage in completing project ω_4 or ω_5, so no referral can be conducted. □

Proof of Lemma 9

Suppose not. Suppose the high-skill expert accepts an equilibrium referral after a failed information acquisition in a single-acquisition equilibrium.

Case 1: $t^* < s_3$

In a single-acquisition equilibrium, the referring high-skill expert does not acquire information and refers the project. The expected cost of the project for the high-skill expert is s_3. Therefore, the equilibrium payment is not enough to cover the cost. The low-skill expert rejects the project after a failed information acquisition.

Case 2: $s_3 \leq t^* \leq s_4$

Since the low-skill expert acquires information, he accepts projects ω_1, ω_2, and ω_3 and rejects the rest when his information acquisition succeeds. Therefore, the high-skill expert ex ante cost in the beginning of stage 1 is $\theta_l^* \big(\sum_{i=1}^{3} p_i t^* + \sum_{i=4}^{5} p_i(\alpha s_i + \beta) \big) + (1 - \theta_l^*)t^* > s_3$. The high-skill expert can profitably deviate to complete the project himself rather than following the equilibrium strategy.

Case 3: $s_4 \leq t^* \leq s_5$

The proof in this case in similar to the case 2's. □

Proof of Lemma 10

Suppose not. Suppose that the high-skill expert refers at $t^* \geq \alpha s_1 + \beta$ without information acquisition in a single-acquisition equilibrium. The high-skill expert's ex ante cost is at least t^*. But he can profitably deviate to acquire information since $k(\theta) + \theta(p_1 \alpha s_1 + \beta + \sum_{i=2}^{5} p_i t^* + (1 - \theta)t^*) \leq t^*$ for really small θ because $\lim_{\theta \to 0} k'(\theta) = 0$. □

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