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# Monitoring for Worker Quality\*

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## Abstract

Much nonmanagerial work is routine, with all workers having similar output most of the time. However, failure to address occasional challenges can be very costly, and consequently easily detected, while challenges handled well pass unnoticed. We analyze job-assignment and worker-monitoring for such ‘guardian’ jobs. If monitoring costs are positive but small, monitoring is nonmonotonic in the firm’s belief about the probability that a worker is good. The model explains several empirical regularities regarding nonmanagerial internal labor markets: low use of performance pay, seniority pay, rare demotions, wage ceilings within grade and wage jumps at promotion.

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We present a formal model of monitoring for worker quality in “guardian” jobs (Baron and Kreps 1999). Many nonmanagerial jobs have the principal characteristics of such jobs. We analyze efficient monitoring in guardian jobs and show that it results in wage and promotion profiles consistent with a number of regularities observed in non-managerial labor markets. Notably, we predict that pay will be largely based on factors—such as age, experience and seniority—that *may* be related to productivity, but are not themselves measures of productivity (Doeringer and Piore 1971).

We make three significant departures from the current literature. First, we assume the primary purpose of monitoring is to *evaluate worker quality* at this job and *not* to prevent workers from shirking on the job. To focus attention on the evaluative role of monitoring, we abstract entirely from moral hazard.

Second, we treat guardian jobs as common in nonmanagerial employment: much of the work is routine, and there is typically little variation (in our formal model, none) in worker productivity; there are occasional ‘challenges’ which good workers tend to face successfully, while bad workers tend to fail at significant cost to the firm. Such challenges are firms’ sole opportunity to distinguish worker quality.

Third, we formalize the view that failures are much more noticeable than successes and are observed even if the firm does not actively monitor the worker. However, when a worker successfully handles a challenge, output is not disrupted, making the firm less likely to observe a success unless it actively monitors the worker.<sup>1</sup>

Jacobs (1981) describes the classic guardian job: “Even if the professional pilot never makes any mistakes in a long career, his company’s profits will not go up, but one error may be quite costly to his airline.” The famous con-artist, Frank Abagnale, claims to have worked for eleven months as chief resident pediatrician in a Georgia hospital until he was faced with an oxygen-deprived baby and was almost exposed (Abagnale 1980). Despite having no medical training or tertiary education, he handled routine tasks until he encountered an unusual challenge.

But there are many other examples. The racist employee who responds inappropriately to an African-American customer may cost the firm far more

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<sup>1</sup>This technology is, in many ways, similar to the Biais et al (2012) model of moral hazard in which very costly challenges arrive infrequently. There effort rather than ability reduces the probability of failing a challenge.

in legal expenses and bad publicity than a somewhat faster worker could earn in additional profit. The worker who fails to notice or report a potential safety or mechanical problem may cause substantial losses. Nonmanagerial workers often have little opportunity to vastly increase profit but can impose large costs.

Because they are costly, such failures are often readily observable. As Jacobs also observes “Little can be known about the comparative success of a specific police department, prison, or welfare agency. Mistakes, however, are often quite visible....” Consequently, failures are often punished severely (Baron and Kreps 1999). Successes are much less obvious. Had a worker responded appropriately to the gas bubble rising through the Deepwater oil well, his actions would likely have passed largely unnoticed.

Not all jobs display this asymmetry. In ‘star’ jobs (Baron and Kreps 1999), successes are rare and easier to observe than squandered opportunities. In ‘foot soldier’ jobs neither success nor failure has a large impact on profits.<sup>2</sup>

In our model, a firm can learn about a worker either by passively updating its assessment of the worker’s likelihood of being good, or by actively monitoring her. If it does not monitor her, it does not know when a challenge arrives. However, when a worker fails a challenge, the firm learns this immediately. The longer the worker is not observed to fail, the more likely it is that the worker addressed one or more challenges successfully. Thus even without monitoring, the firm updates its assessment of the probability that the worker is good. If the firm monitors the worker, this does not yield additional information on failures, but it allows the firm to observe when a challenge arises. When it does, and the worker handles it, the firm learns immediately that the worker is good, and can move her instantly to a job where her skill is more valuable and the cost of failures higher.

The model has important implications for the sequencing of monitoring, an area that has received little attention in the economics literature. Since monitoring is costly and firms acquire information just by waiting, workers who are initially very unlikely to be good are monitored only after a period with the firm during which they do not fail a challenge. Similarly, workers

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<sup>2</sup>We conjecture, but have not proved, that ‘star’ settings are conducive to up or out contracts. A related paper (Cavounidis and Lang, 2015) examines a setting where both successes and failures are observed only when workers are monitored. This provides interesting implications for labor market equilibrium, but little or no insight into internal labor markets. Prendergast (2007) explores the consequence of symmetry and asymmetry in the probability of detecting bureaucratic errors.

thought to be nearly good enough to be assigned to a more challenging task are not monitored because with high probability they will be promoted shortly; the additional information from monitoring does not justify the cost. Monitoring is efficient only if the cost is not too high and the firm's assessment of the worker is neither too high nor too low.

In the model, there are two potential paths to promotion although both need not be present in a single firm. Workers with sufficiently high expected productivity at hire are never monitored. But workers about whom initial expectations are sufficiently low are monitored during a period that, in equilibrium, just precedes promotion or separation. Under some interpretations of the model, this can be viewed as a training period or a period of supervised work in the more challenging task, which leads to promotion upon successful completion. Thus workers who are monitored are those who are some distance from being promoted in the absence of monitoring. The model does *not* imply that monitored workers are unlikely to be promoted soon.

For most of the formal modeling, we ignore wage determination and focus on efficient monitoring and job assignment. However, later we develop a Nash bargaining model of wage determination. Using this model of wage determination, we show that our monitoring model fits a number of regularities regarding nonmanagerial compensation. In particular, a) pay for many nonmanagerial workers does not depend on either subjective or objective performance measures; b) individual wages are largely determined by objective factors such as seniority, experience, occupational grade, education and other formal qualifications which may be correlates of productivity but are not themselves measures of productivity; c) many salary scales have a fixed number of steps so that workers at the top of the scale receive no further individual pay increase unless promoted; d) wages jump at promotion, and e) demotions are rare.

Perhaps surprisingly, the modest literature on learning in the labor market generally treats learning as passive and thus ignores the significant resources spent on monitoring. In Farber and Gibbons (1996), Altonji and Pierret (2001), Gibbons and Waldman (1999, 2006) and Lange (2007) firms learn about general market skills, all information is public, and new information arises continuously. In Waldman (1984), Greenwald (1986), Gibbons and Katz (1991), Acemoglu and Pischke (1998), Autor (2001), Schoenberg (2007) and Li (2013) skills are general but only the current employer learns the worker's skills. There are some exceptions. Meyer (1994) models learning about individual abilities within teams. Lazear (1986) discusses when

to measure (akin to monitoring) worker output. In Prendergast (2003) the principal pays to increase the probability of detecting a bureaucratic error.

In this paper we abstract from moral hazard altogether. Practitioners are often surprised that the personnel economics literature almost universally assumes that the sole purpose of monitoring is creating incentives for effort. While this is a valid reason for monitoring, modeling it solely in terms of incentives generates problematic empirical predictions and theoretical results. Theoretically, since detection is costly and deterrence depends on the probability of detection multiplied by the cost of punishment, shirking should be punished as harshly as possible and monitoring should be minimal. Dickens, Katz, Lang and Summers (1989) refer to this as the monitoring puzzle. Akerlof and Katz (1986) show that if backloading pay or requiring a bond is costly, the only solution is the one in Becker and Stigler (1974): workers “buy” their jobs and have the purchase price returned at retirement. If workers’ ability to purchase their job is limited, firms may require them to engage in rent-dissipating behavior (Murphy and Topel 1990). Neither purchase of jobs nor obviously rent-dissipating requirements are common features of job contracts. Costly bonding is consistent with more general earnings profiles, but wages must be less than the value of marginal product (VMP) early in seniority and more than VMP later (Lazear 1979, 1981). This implies that junior workers desire less work than required by the optimal contract while senior workers want more work, a result inconsistent with the data on desired work hours (Kahn and Lang 1992, 1995).

Section 1 presents the model and establishes the efficient monitoring and task-assignment strategy. Wage-determination is addressed in Section 2, and Section 3 presents alternative interpretations and an extension to partial monitoring. The empirical relevance of our predictions is discussed in Section 4. The final section concludes.

## 1 The Formal Model

An employer hires a worker whom he can put in a high task ( $H$ ) or a low task ( $L$ ). The worker’s productivity in a task depends on her type, good ( $G$ ) or bad ( $B$ ).<sup>3</sup> Both types produce a flow output  $q$  per unit time in  $L$  and  $g + q$ ,  $g > 0$  in  $H$ .  $\theta$  denotes the firm’s belief that the worker is good. For the moment it is irrelevant whether type is general or firm-specific. Later we

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<sup>3</sup>We abstract from changes in worker quality over time. See Kahn and Lange (2014).

will treat the value of  $\theta$  when the worker starts at the firm, denoted by  $\tilde{\theta}$ , as the probability the worker is good at a randomly chosen firm. Thus a worker with  $\tilde{\theta}$  equal to 0.7 is good at 70 percent of the firms at which she may work and bad at the rest.<sup>4</sup>

*Challenges* arrive at Poisson rate  $\lambda$  in both tasks. This ensures that task assignment is unaffected by its impact on learning about productivity. This type of assumption is common in the literature on internal labor markets (e.g. Gibbons and Waldman 1999). Assuming that challenges are more common in  $H$  makes the algebra more cumbersome and provides little additional insight (see Section 1.3).

Bad workers *fail* when a challenge arises.<sup>5</sup> Failure generates negative output  $-c_l$  in the  $L$ -task and  $-c_h$  in the  $H$ -task, with  $c_h > c_l$ . If a worker is bad and a challenge arises, the failure is immediately observed, and the worker's type is revealed. Good workers resolve challenges, with no impact on productivity. Thus if the worker is good, the occurrence of a challenge can be known only if the worker is actively monitored, in which case her type is revealed.<sup>6</sup> We assume  $g + q - \lambda c_h < q - \lambda c_l$  so that the expected flow of output net of costs due to failed challenges is lower when a bad worker is placed in the  $H$ -task than when she is placed in the  $L$ -task. Under complete information, clearly good workers will be put in the  $H$ -task.

Time is continuous. The discount rate is  $r$ . Workers and firms live forever.

We begin by deriving the efficient task assignment and monitoring rules. We then turn to market equilibrium. Our wage determination mechanism combined with symmetric information ensures that task assignment and monitoring are efficient in equilibrium.

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<sup>4</sup>An alternative interpretation of  $\theta$  is that it measures the match between worker attributes and the industry or occupation. If  $\theta$  is revealed to be low, the worker quits the industry (and firm). In this case no rent accrues to the firm-worker match, the wage must reflect the worker's full productivity, and the firm must earn zero profit. This does not affect the analysis of efficient monitoring, but significantly simplifies the wage model later in the paper.

<sup>5</sup>Garicano (2000) develops a model where workers who cannot handle a task can pass it to a higher level of the hierarchy.

<sup>6</sup>It is perhaps more realistic to treat challenges as settings with an increased risk of a bad outcome (e.g. safety equipment not set up properly, inflammable materials too close to a machine that may produce sparks). This would somewhat complicate the model but not the essential conclusions.

## 1.1 Turnover

Our model of turnover is simple. We interpret  $\tilde{\theta}$ , the value of  $\theta$  when the worker first arrives at the firm, as the *ex ante* probability the worker will be good at a randomly selected job (possibly from within a class of jobs). Any information about the ability of the worker to resolve challenges is match-specific. Therefore if the updated assessment falls below  $\tilde{\theta}$ , it is efficient for the worker and firm to separate, and they do so. Otherwise it is efficient for the worker and firm to continue their relationship, and they do so. Thus in the base model, the worker remains with the firm unless she fails to resolve a challenge in which case she quits or is fired. Note success or failure at one firm could provide information about productivity elsewhere provided it is more informative about productivity at the former.<sup>7</sup>

Note that since turnover occurs if and only if the worker is observed to fail and since monitoring does not affect whether failure is observed, there will be no need in what follows to consider the impact of monitoring on turnover.

## 1.2 Monitoring

The employer can either monitor or not-monitor. If she does not monitor, she only learns the worker's type if the worker is bad, a challenge arises and therefore the worker fails. If the worker is good or no challenge arrives, the employer observes nothing but can update her beliefs about the worker as time passes. If she monitors the worker, then she continues to do so until a challenge arises, and she therefore learns the worker's type. The flow cost of monitoring is  $b$  per unit time and must be borne until a challenge arises. Note that under monitoring the employer's belief is fixed until a challenge arises and the employer learns the worker's type. Therefore, if it was efficient to monitor the worker, it remains efficient until the firm observes a challenge.

In the formal model, we treat monitoring as simply observing the worker. This interpretation implies that the firm only monitors workers in the  $L$ -task since it gets no benefit from monitoring in the  $H$ -task. If the worker is good, she is already assigned to the right task. If she is bad, the firm will bear the cost of a single failure with or without monitoring.

Alternatively monitoring may help catch and mitigate failures to handle

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<sup>7</sup>We abstract from unemployment, which might discourage workers with a low probability of being good at any firm from pursuing a new match even if they know they are poorly matched with their current firm.

challenges. This may be particularly valuable in the  $H$ -task where the cost of failures is high. If  $c_l$  is sufficiently low, the firm will not expend resources monitoring workers in the  $L$ -task. But it will monitor the workers that have the lowest probability of being good among those assigned to the  $H$ -task. Both cases arise in the real world. We are aware of at least one call center that monitors incoming-call workers more closely before shifting them to making outgoing calls. In other settings, workers nearing promotion perform the new task(s) under supervision and are only promoted after showing that they are competent. If we interpret promotion as being assigned to the  $H$ -task *without* monitoring, the two models are essentially the same, and it is possible to choose parameters under which they are isomorphic.<sup>8</sup> In a web appendix we derive our principal results for the case where monitoring also serves to mitigate mistakes in the  $H$ -task.

### 1.3 Efficient promotion without monitoring

First consider a worker who is never monitored. If, at a given time, the worker's probability of being good,  $\theta_0$ , is not too high, the employer will place her in the  $L$ -task. If the worker fails, she is immediately revealed to be bad and leaves the firm. If she has not failed by time  $t$ , the employer updates his belief about the worker's type to  $\theta(t, \theta_0)$ . When  $\theta(t, \theta_0)$  becomes sufficiently high, the employer may promote her to the  $H$ -task. Similarly, if the firm's prior at time 0 is sufficiently high, the worker will be placed immediately in the  $H$ -task.

*Lemma 1.1.* If the firm does not monitor the worker, it is efficient to promote her to the  $H$ -task when its assessment of the probability that she is good reaches

$$\theta^* = \frac{\lambda(c_h - c_l) - g}{\lambda(c_h - c_l)} \quad (1)$$

provided that the initial prior satisfied

$$\theta_0 < \theta^*.$$

If  $\theta_0 \geq \theta^*$ , the firm places the worker in the  $H$  task immediately.

(All proofs are in the appendix.)

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<sup>8</sup>Let the mitigating effect of monitoring be  $c_h - c_l$  and the cost of monitoring in the  $H$ -task be  $b^* = b + g$ .

Note that (1) has a natural interpretation. The worker is promoted when the expected flow payoffs in the  $L$  and  $H$  tasks are equal, that is

$$q - (1 - \theta^*)\lambda c_l = g + q - (1 - \theta^*)\lambda c_h. \quad (2)$$

This follows from the assumption that learning about productivity is independent of task assignment so that assignment is determined solely by expected output. It is plausible that challenges would arrive more frequently in the  $H$ -task. If so, workers would be promoted later than implied by equation (1). However, we have not obtained any additional insights from allowing for different arrival rates of challenges and therefore have not pursued this path.

#### 1.4 Payoff with the monitoring strategy

When the employer monitors the worker, he knows when the first challenge arises, and immediately identifies the worker's type. It is efficient to immediately promote good workers and for bad workers to separate from the firm. Before the first challenge arrives, there is no new information and therefore no continuous updating of beliefs.

Let  $\theta_0$  be the prior that the worker is good when the employer starts monitoring her. When the first challenge arrives, with probability  $\theta_0$  the worker resolves the challenge and is promoted to the  $H$ -task, with the complementary probability, she fails and leaves the firm. In either case the employer stops monitoring. Recall that monitoring has a flow cost of  $b$  per unit time. We prove in the appendix that

*Lemma 1.2.* The value of the monitoring strategy with efficient promotion is

$$\tilde{U}(\theta_0) = \frac{\lambda\theta_0 g - rb}{r(\lambda + r)} - (1 - \theta_0)\frac{\lambda c_l}{\lambda + r} + \frac{(\lambda\theta_0 + r)q}{r(\lambda + r)}. \quad (3)$$

In what follows, we assume that

$$\frac{g}{r} - \frac{b}{\lambda} > 0. \quad (4)$$

If not, even if a firm knew a worker was good and could assign the worker to the  $H$ -task only by monitoring and observing her solve a challenge, it would prefer not to do so.

## 1.5 Efficient monitoring

Given  $\theta$  the firm can decide to monitor immediately, not monitor until  $\theta$  reaches a higher value through updating, or never monitor.<sup>9</sup> We find there are always ranges of  $\theta$  in which the firm does not monitor, and if monitoring costs are not too high, there is a range in which it monitors immediately.

*Theorem 1.1.* There is always a range  $[0, \theta_a)$  and a range  $(\theta_b, \theta^*]$  in which it is efficient not to monitor the worker.

Theorem 1.1 establishes that workers who are very unlikely to be good will not be monitored; nor will workers who are close to promotion. However, it does not ensure that firms will ever monitor workers. The following theorem addresses this point.

*Theorem 1.2.*

$$\text{If } \left( \frac{g(\lambda + r)}{g\lambda - br} \right)^{\frac{\lambda+r}{r}} < \frac{\lambda(c_h - c_l) - g}{b}, \quad (5)$$

there is an interval  $[\theta_a, \theta_b] \neq \emptyset$ , with  $\theta_a = \frac{b(\lambda+r)}{\lambda(g+b)}$ , such that it is efficient to monitor a worker if and only if  $\theta \in [\theta_a, \theta_b]$ .

*Remark 1.1.* The proof of theorem 1.2 shows that for any worker who joins the firm with  $\tilde{\theta} < \theta_a$  it is efficient to begin monitoring the worker only when  $\theta$  reaches  $\theta_a$ , provided that condition (5) holds.

*Remark 1.2.* Note that, except in a knife-edge case, it is not efficient to begin monitoring at the  $\theta$  where the value of monitoring first exceeds the value of never monitoring.

Condition (5) is not particularly informative. We can derive somewhat more informative conditions. Recall that, by assumption, both sides of the inequality are positive. As  $rb \rightarrow g\lambda$ , the left-hand side goes to infinity while the right-hand side remains finite. Thus when monitoring costs are high, not surprisingly it is never efficient to monitor. On the other hand, when

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<sup>9</sup>Formally the firm's strategy should specify, as a function of the history at each instant of time (or other appropriate specification), whether the worker should be monitored at that instant. However, the firm's assessment of the worker is the only time-dependent variable in the model, and this becomes static once the firm starts monitoring and until a challenge arrives. Thus it cannot be optimal for the firm to switch to no-monitoring once it has optimally started monitoring, and it is sufficient to consider at most a single switch from no-monitoring to monitoring.

monitoring costs are sufficiently low, there is a range in which monitoring is efficient.

The firm gains from monitoring by ensuring it never places a bad worker in the  $H$ -task. The flow cost of doing so is  $\lambda(c_h - c_l)$ . Again not surprisingly, as this term gets large, there is always a range in which monitoring is efficient. When it gets small, or equivalently when the benefit from placing a good worker in the  $H$ -task gets small, monitoring is never efficient.

Increasing the frequency of challenges,  $\lambda$ , lowers the left-hand-side and increases the right-hand side of (5). Thus more frequent arrival of challenges is associated with a larger range of other parameters consistent with monitoring. A higher rate of time discounting is similar to a lower value of  $\lambda$  and thus leads to a more restricted set of parameters consistent with some monitoring.

## 2 Wage Profiles

Next we model wage determination. Intuitively, we expect the wage to increase as the expected flow of output increases or, possibly, as  $\theta$  increases, which is not quite the same. This section derives a bargaining model with the former property.

We restrict attention to settings with symmetric information. This is a reasonable assumption for many interpretations of the model, though there are of course many other interpretations for which it is not reasonable. Symmetric information is the most natural assumption when monitoring consists of some variant of external testing (see section 3.1 below). Rather than waiting for a challenge to arise, the monitor may be able to simulate challenges and assess whether the worker solves them.

We assume that when the firm and worker meet, they Nash bargain over a fully contingent contract, specifying monitoring, task assignment and wages conditional on history. The number of agreements consistent with Nash bargaining is large. To ensure uniqueness, we impose a form of renegotiation-proofness. We require that if negotiation were exogenously reopened, the continuation of the original agreement would be the outcome of the Nash bargaining over future wages, task assignment and monitoring.

## 2.1 The Nash Bargaining Solution

Since Nash bargaining is efficient by assumption, monitoring and task assignment will be as derived in the previous section.<sup>10</sup> We focus on wage determination.

Define  $W_t(\theta_t)$  as the expected (future) discounted value of wage payments and  $V_t(\theta_t)$  as the expected discounted value of output net of monitoring at this firm going forward from time  $t$  when  $\theta$  is believed to be  $\theta_t$ . Note that these expectations account for the possibility of failure. Define  $U_a$  as the value to the worker of moving to another firm. This includes the possibility that she will fail at the next job and thus move again. Denote the flow value of  $W$  and  $V$  by  $w_t$  and  $v_t$ . We treat  $U_a$  as constant, implying that success or failure at this firm provides no additional information about the probability of success elsewhere. Letting performance at this firm be somewhat informative about the probability of success elsewhere changes only details, not the fundamental results.

*Theorem 2.1.* Let  $\beta$  be the worker's Nash bargaining weight. Then the unique renegotiation-proof solution to the Nash bargaining problem is

$$w_t = \beta v_t + (1 - \beta) r U_a. \quad (6)$$

Thus, in each period, the worker gets a share of the flow of output net of any monitoring and expected costs of mistakes, and receives part of the flow value of her outside option. Note that  $v_t$  declines from  $q - (1 - \theta_a) \lambda c_l$  to  $q - (1 - \theta_a) \lambda c_l - b$  at the instant monitoring begins. We return to a discussion of this inaccurate prediction later.

We have not considered wage determination under asymmetric information. Our results will carry over to some settings with asymmetric information, especially when the firm is the party that learns of the worker's ability, and the firm also has sufficiently high bargaining weight. But there are many more cases in which asymmetric information would alter our results. A full investigation of the plethora of possibilities opened up by various formulations of bargaining under asymmetric information would take us too far afield of the main question.

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<sup>10</sup>The outcome we describe is a fully contingent contract. We use renegotiation-proofness to select a single efficient contract from the set of efficient fully-contingent contracts. But our approach rules out the kind of dynamic issues that can lead to hold-up problems.

### 3 Extensions

In this section we first discuss an alternative interpretation of monitoring. We then extend our base model to allow for partial monitoring, where the probability of observing a success when it happens is increasing in monitoring expenditure. Finally, we discuss how our results change if there can be false positives so that observing success is not conclusive proof that the worker is good.

#### 3.1 Alternative Interpretation: Formal Testing

We have assumed that the arrival of challenges allowing the firm to observe successes when monitoring is stochastic. What happens if, when learning is important, the firm can simulate challenges? This may be difficult in some settings but simple in others. Here we consider the case where firms can administer a costly test to determine whether or not the worker is good, for example by sending the worker to a “training” program where she faces manufactured challenges. While not isomorphic to our main model, it is similar.

If the firm is fairly sure the worker is not good, it will not send her to the program because the cost outweighs the expected benefit. Similarly, for any fixed positive testing cost, there will be a  $\theta < \theta^*$ , sufficiently close to  $\theta^*$ , such that it will not be worth paying for the testing program. But, if it is not too expensive, for intermediate values of  $\theta$ , it will be worthwhile.

Suppose that the test costs  $B$ . At  $\theta_0$  the expected value of output after administering the test is

$$U^T = \frac{\theta_0(g+q)}{r} - B. \quad (7)$$

Compare this with the value of monitoring in the main model. It will be clear that the firm will administer the test at  $\theta_a$  if

$$\frac{\theta_a(g+q)}{r} - B \geq \frac{\lambda\theta_ag - rb}{r(\lambda+r)} - (1-\theta_a)\frac{\lambda c_l}{\lambda+r} + \frac{\theta_a q}{r} + \frac{(1-\theta_a)q}{\lambda+r} \quad (8)$$

or

$$B \leq \frac{b - (1-\theta_a)(q - \lambda c_l) + \theta_a g}{\lambda+r} \equiv B_a. \quad (9)$$

The first two terms in the numerator are the flow cost of monitoring net of the expected output from bad workers. The last term is the added value of output from good workers since the test is immediate. All of this is adjusted for discounting and the expected time at which a challenge arrives in the base model.

If  $B = B_a$ , then the value of testing at  $\theta_b$ , the upper limit of the monitoring zone, is

$$U^T(\theta_b) - \tilde{U}(\theta_b) = (\theta_b - \theta_a)(g + q - \lambda c_l)$$

which is zero only in the special case  $g + q - \lambda c_l = 0$ . If it is positive, it may be profitable to test the worker even when the firm's assessment of her ability is too high for monitoring to be worthwhile, and the converse is true if it is negative. Thus the two models are not isomorphic, but are similar in their interpretation and predictions.<sup>11</sup>

## 3.2 Partial Monitoring

The results in the preceding sections do not depend critically on assuming monitoring is a 0 – 1 variable. In this section we maintain the model's other assumptions but assume the firm can vary its monitoring effort. The firm can choose effort and the corresponding flow cost  $b$  of monitoring. If the worker resolves a challenge, the firm observes the success with probability  $p = p(b)$ . We write the inverse function  $b = b(p)$  and assume that  $b' \geq 0$  and  $b'' \geq 0$ .

The following theorem, derived as Theorem A.1 in the appendix, parallels theorem (1.1):

*Theorem 3.1.* If  $b'(0) > 0$  and  $b''(p) > 0 \quad \forall p \in [0, 1]$ , there is always a range  $[0, \theta_a)$  and a range  $(\theta_b, \theta^*]$  in which it is efficient not to monitor the worker. Further, if  $b'(0)$  is sufficiently small, then there is a non-empty interval  $[\theta_a, \theta_b]$  in which a positive amount of monitoring is efficient.

*Remark 3.1.* If  $b'(1)$  is sufficiently small, there will also be a range with complete monitoring. Depending on the shape of the  $b(p)$  function, the solution can be bang-bang as in our base model.

The most interesting case is when monitoring starts at  $\theta_a$  and increases smoothly between  $\theta_a$  and some  $\theta_A$  at which  $p$  equals 1 (full monitoring).

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<sup>11</sup>If both testing and monitoring are available options, then for given  $b$  and  $B$  we can calculate  $U^*$ ,  $\tilde{U}$  and  $U^T$ , and determine ranges in which it is respectively optimal to update, test and monitor. However, the intuition is sufficiently clear that the full exercise seems unwarranted.

It remains at 1 for  $[\theta_A, \theta_B]$ , then decreases smoothly between  $\theta_B$  and  $\theta_b$ . Then if workers are hired with  $\tilde{\theta} < \theta_a$ , as in the baseline case, they will not be monitored, but, unless the worker fails to resolve a challenge, the firm's assessment of  $\theta$  will rise continuously until it reaches  $\theta_a$ . Thereafter the firm partially monitors the worker and continues to update  $\theta$ . If no challenge is observed,  $\theta$  rises towards  $\theta_A$ .<sup>12</sup> But the firm may see the worker resolve a challenge and promote her immediately. Thus partial monitoring when  $\tilde{\theta} < \theta_A$  allows for the realistic possibility that a worker may be promoted before he reaches the top of the wage scale. In the region between  $\theta_a$  and  $\theta_A$ , the probability of promotion is strictly increasing in  $\theta$  since both the probability of being good and the probability of being seen to solve a challenge rise with  $\theta$ .

If the worker is hired with  $\theta_A < \tilde{\theta} < \theta_B$ , the firm does not update  $\theta$  except simultaneously with a separation or promotion.

Partial monitoring results in the most significant changes for workers hired in the range  $\theta_B < \tilde{\theta} < \theta_b$ . In this case, hired workers are initially monitored. But for workers who are not observed to face a challenge, monitoring declines with seniority and eventually stops even before they are promoted. It is clear that for  $\theta$  close to  $\theta_b$ , the probability of promotion must be lower than for  $\theta$  close to  $\theta_B$ , but we have not been able to establish whether the relation between the probability of promotion and  $\theta$  is monotonic in this range and expect that it need not be. Finally we note that if  $\theta_b < \tilde{\theta} < \theta^*$ , the worker is not monitored. In the absence of a failed challenge,  $\theta$  is updated continuously until it reaches  $\theta^*$  and the worker is promoted to the high job. As in the base model, there are no promotions from the  $L$ -task to the  $H$ -task originating at  $\theta < \theta_a$  or  $\theta_b < \theta < \theta^*$ .

### 3.3 Inconclusive challenges

Thus far we have assumed that failures and observed successes are fully informative. Consequently our model predicts that all promotions end up at either at the top of the upper level of the hierarchy (observed success) or at its bottom (updating from the upper no-monitoring zone with no observed failure). Further, when a failure is observed, the worker is known with certainty to be bad at this firm, which results in immediate separation. In particular, there are no demotions. Relaxing this assumption seems natural.

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<sup>12</sup>It appears to us that  $\theta$  will reach  $\theta_A$  only asymptotically, but we have not proved this.

We briefly discuss two alternatives.

Consider what happens if monitoring can produce false positives, that is the worker can appear to have resolved a challenge when none existed. Then if a success is observed, the firm will not increase  $\theta$  all the way to 1. Specifically, let  $\delta$  be the arrival rate of false positives and let  $\mu = \delta / (\delta + \lambda)$  be the proportion of apparent challenges that are not really challenges. Then if the firm observes that a worker with  $\theta = \theta_0$  has “responded to a challenge,” the firm’s assessment of  $\theta$  will be updated to

$$\theta = \frac{\theta_0}{(1 - \mu)\theta_0 + \mu}. \quad (10)$$

This would lead to different critical values  $(\theta_a, \theta_b, \theta^*)$  for the monitoring zone and task-assignment. However, it is both intuitive and straightforward to show that if, when  $\delta$  is 0, there is an interval of  $\theta$  for which the firm monitors the worker, then for  $\delta$  sufficiently small there is still an interval for which the firm monitors the worker and if the worker appears to resolve a challenge she is promoted to the  $H$ -task although the updated  $\theta$  is less than one.

Similarly, if there are false negatives, i.e., good workers fail challenges with some probability  $\gamma < 1$ , the firm will revise  $\theta$  downwards following a failure, but it will not revise  $\theta$  to 0. Separation follows only if the revised  $\theta$  falls below  $\tilde{\theta}$ .

If the firm’s assessment of  $\theta$  prior to the challenge was sufficiently greater than  $\tilde{\theta}$ , then even following failure the revised  $\theta$  will exceed  $\tilde{\theta}$ . This, in turn, requires the worker to have spent sufficient time with the firm without being monitored, or, if there are false positives, to have had some successes observed. In particular we can show that, for a worker who has not been previously monitored, a sufficient condition for a failure to cause a separation is that the worker’s tenure at the firm satisfy

$$t < -\lambda^{-1} \ln \frac{1 - \gamma}{2 - \gamma}. \quad (11)$$

Thus demotions will be positively related to the worker’s tenure at the firm. Bad information about a low tenure worker leads to a separation, not a demotion. This contrasts with Gibbons and Waldman who predict that demotions, while rare, will be most likely among the recently promoted. We know of no data on this issue.

## 4 Implications for Nonmanagerial ILMs

*The Internal Wage Profile:* We return to the case where monitoring is a binary choice. Extension to partial monitoring is relatively straightforward. Recall that there are four ranges in the data. For  $\theta < \theta_a$ , the worker is assigned to the  $L$ -task and is not monitored. For  $\theta_a < \theta < \theta_b$ , the worker is assigned to the  $L$ -task and monitored. For  $\theta_b < \theta < \theta^*$ , the worker is assigned to the  $L$ -task and not monitored. For  $\theta > \theta^*$ , the worker is assigned to the  $H$ -task and not monitored.

The wage profile for workers who remain with the firm therefore depends on  $\tilde{\theta}$ , the value of  $\theta$  when the worker is hired. Note that we have not addressed the firm's decision about the level(s) of  $\tilde{\theta}$  at which to hire. Our model is mute about whether firms will typically hire at one or a few values of  $\tilde{\theta}$  so that only one or a few career paths are observed within a firm or whether there will be substantial heterogeneity within individual firms. Thus the following profiles may be found in different firms if they hire workers with different values of  $\tilde{\theta}$  or within firms if they hire workers about whom they have widely varying initial beliefs:

1. Workers with a low  $\tilde{\theta}$  are placed in the low no-monitoring range. If they remain with the firm, the firm gradually increases its assessment of  $\theta$ , and therefore the wage, until  $\theta = \theta_a$ . At this point, the firm begins monitoring the worker, and there is no updating of  $\theta$  or the wage until a challenge arrives, at which time the worker either leaves the firm or is promoted to the high task. Recall that under the loss-mitigating model in the web appendix, workers may be assigned to the  $H$ -task and monitored there before promotion.
2. Workers with a somewhat higher initial  $\theta$  are placed immediately in the monitoring range. Although their wage will generally be higher than  $w(\theta_a)$  since most will have  $\tilde{\theta} > \theta_a$ , in other respects they are similar to workers who started at a lower  $\tilde{\theta}$  and rose to  $\theta = \theta_a$ .
3. Workers with a yet higher  $\theta_b \leq \tilde{\theta} < \theta^*$  remain in the  $L$ -task and receive continuous wage increases until  $\theta = \theta^*$ , at which point they are promoted to the  $H$ -task and receive wage increases that are asymptotic to the wage associated with  $\theta = 1$ .
4. Finally, any worker hired with a high  $\tilde{\theta} \geq \theta^*$  is placed directly in the  $H$ -task and receives continuous wage increases in a manner analogous to those promoted from the upper no-monitoring range.

*The Hierarchical Structure:* Up to now we have referred to tasks rather than to jobs. Yet in many organizations, collections of tasks that appear quite similar have different job titles (secretary I and II, tenured associate and full professor). In the empirical literature, hierarchies are sometimes determined by transition patterns across occupational titles as in Baker, Gibbs and Holmstrom (1994a&b). In our model, it is natural to define three occupation titles: *LT1* (“low task 1”), consisting of workers in the low no-monitoring and monitoring zones, i.e.,  $\theta \in [0, \theta_b]$ , *LT2*, consisting of the high no-monitoring zone  $\theta \in (\theta_b, \theta^*]$ , and *HT* where  $\theta > \theta^*$ . Although *LT2* is higher paid than *LT1*, workers do not transition from *LT1* to *LT2*, or conversely. Instead both feed into *HT*. So *LT1* and *LT2* appear to share a location at the bottom of the hierarchy below *HT*.

Next we compare our model’s predictions with some widely observed regularities in nonmanagerial internal labor markets (ILMs). The first three regularities are drawn from a sample of nonunion establishments in the UK Workplace Employment Relations Survey (WERS). The WERS asks managers to list the determinants of pay in the largest *nonmanagerial* occupation (e.g. skilled workers, administrative and secretarial, technical) in their establishment.

1. *Many establishments have no variation in pay.* Roughly 20 percent (employment weighted) of establishments report no variation in pay within their largest class of nonmanagerial workers except for hours, overtime and shift differentials. A further 11 percent differentiate pay based on skills/core competencies and/or job grade but not factors such as seniority or performance evaluations. We view the model as consistent with but not predictive of this regularity. If the assessments  $\tilde{\theta}$  of workers at the point of entry fall within a narrow band in the monitoring range (between  $\theta_a$  and  $\theta_b$ ), then the wage would be approximately equal for all workers in *LT1*. There is no strong reason to believe that this would happen frequently. However, if the firm typically hires workers with a standard set of qualifications at an entry level, and initial assessment is largely determined by qualifications, then the range of  $\tilde{\theta}$  may well be narrow.
2. *In firms that differentiate pay, individual wages are largely determined by objective factors such as seniority, experience, occupational grade, education and other formal qualifications which may be correlates of productivity but are not themselves measures of productivity.* Among those firms where there is clear differentiation in pay within similar jobs, 64 percent

- determine pay at least partly by age, experience, seniority and/or formal qualifications such as education but not based on subjective performance measures. In contrast, only 5 percent use subjective performance evaluations but not these objective factors, and 31 percent use both. Fully 52 percent use neither objective nor subjective performance measures. In our model, for workers who are in  $LT1$  and  $LT2$ , the wage is fully explained by  $\tilde{\theta}$  and seniority, though it is not strictly increasing with seniority at all times. To the extent that the firm's initial assessment  $\tilde{\theta}$  is largely determined by education and experience, the model is consistent with this regularity. However, some workers in the monitoring zone of  $LT1$  are periodically promoted to the top of the hierarchy and therefore supersede seniority.
3. *Wages rise with seniority.* In over half of the establishments that differentiate pay within their largest class of nonmanagerial workers, wages depend at least in part on seniority. In 62 percent, wages depend on either age or seniority. In our model, except for workers who are being monitored or at the top of the pay scale, wages increase with seniority at the firm. In the base model, the worker's wage may fall when monitoring starts. We discuss this further at the end of the current section.
  4. *Many salary scales have a fixed number of steps. Workers at the top of the scale receive no further individual pay increase unless they are promoted.* Dohmen (2004) describes one such salary scale for blue-collar workers. The model is set in continuous time and therefore does not actually predict a series of steps. However, it does predict that workers hired into  $LT1$  will hit the top of the scale when they begin to be monitored, and that the wages of workers in  $HT$  who have been promoted from  $LT2$  will increase asymptotically to  $\beta(g + q)$ .
  5. *Demotions are rare.* This result is well established for managerial workers (BGH). We are unaware of any single-firm study focusing on nonmanagerial workers that establishes this result. However, demotions are sufficiently rare among all workers (e.g. Kosteas, 2011) that we can safely conclude that it also applies to nonmanagerial workers. In the base model, demotions are nonexistent. Note, however, that the mechanism is quite different from that in Gibbons and Waldman (1999a). There, worker productivity increases continuously so that only a large adverse information shock can cause demotions. In our model, large adverse information shocks need not be rare but generate separations rather than demotions.
  6. *Wages jump at promotion.* For samples of only or primarily nonmanage-

rial workers, see Grund (2005), Kwon (2006) and Dohmen (2004). We note that many learning models (e.g. Gibbons and Waldman, 1999a) have difficulty explaining this regularity although it is consistent with tournament models (Lazear and Rosen, 1981). Waldman (1984) combines the two to generate promotion effects. In our model wages jump at promotion for those workers who are promoted from  $LT1$  but not from  $LT2$ . DeVaro and Waldman (2012) explore a model in which a promotion provides a stronger signal of quality for workers with lower prior educational qualifications, and are associated with larger wage increases. They also examine corresponding evidence that is in keeping with this hypothesis. We note that, in our model, a firm's initial assessment of a worker's quality  $\tilde{\theta}$  is likely to be based at least in part on the worker's prior qualifications. Workers with lower qualifications are therefore likely to enter the firm in  $LT1$ , and be promoted on a positive signal during monitoring. Such workers jump to the top of the wage scale. Workers with higher initial qualifications, although more likely to succeed than are those with lesser qualifications, are inducted into  $LT2$ , and hence experience smooth wage increases associated with updating when they are promoted. Thus our model also predicts the empirical pattern observed by DeVaro and Waldman.

In sum, our model captures many of the regularities observed in nonmanagerial ILMs. In particular, it provides a theoretical basis for the observation that individualized pay based on individual performance is far from universal, at least at the nonmanagerial level. Instead, much of pay is determined by objective factors that *may be* correlates of productivity but are certainly not measures of productivity.

The model also has some empirical weaknesses. The most important issue is that, when monitoring is a binary choice, the model implies that the wage falls when the worker is first monitored. This is a less obvious problem when monitoring can be partial and thus begin slowly; nevertheless, it is somewhat disturbing. Wage cuts at promotion are not unheard of, but they are not the norm.

We make four observations in this regard. First, under the interpretation of monitoring as testing, the worker and firm might share the cost of an external training program so that the nominal wage would not drop. Second, while our model is set in continuous time, in reality, wage adjustments and wage scales are discrete. If the requisite drop in pay when monitoring begins is sufficiently modest relative to the growth in  $\theta$  over the period leading up to the monitoring, no wage decrease may be observed. Third, under the in-

interpretation of monitoring as supervised assignment to the  $H$ -task, a decline in the real wage may not be salient given the change in task. Finally, we observe that it is well known that nominal wage cuts are rare (Bewley 1999), and this phenomenon is poorly understood. It is therefore not surprising that our model cannot explain their absence.

We also note that the model with only a binary monitoring decision counterfactually predicts that promotions are concentrated at the top of the scale of each of the lower level jobs in the hierarchy and thus come from only two places in the wage distribution. Yet, the evidence strongly suggests that promotions come from most parts of the wage distribution within a level of the hierarchy. This difficulty is resolved by allowing for partial monitoring.

## 5 Discussion and Conclusion

With partial monitoring, our model permits the following stages as a function of  $\theta$ : No Monitoring, Partial Monitoring, Full Monitoring, Partial Monitoring, No Monitoring, High Task. Not all stages need exist. For the no monitoring range to exist, we require that  $b'(0) > 0$ , so that is that it is costly to do even a little monitoring. For partial monitoring, we require  $b'' > 0$ , and for the existence of full monitoring, we require that  $b'(1)$  be sufficiently small.

Therefore, the precise nature of the ILM depends on the monitoring technology and any factor that affects the quality of the workers who are hired. Hence monitoring patterns and promotion paths are likely to vary widely across companies and types of workers. If some workers are hired with low  $\theta$ , they will be monitored only after a delay. In the base model, such workers are monitored in their current position when the firm becomes serious about possibly promoting them. Alternatively, workers who may soon be promoted are assigned to their new tasks but only under supervision.

Despite this variation and dependence on parameters, there are some striking regularities in the literature on nonmanagerial ILMs that are consistent with our model. If monitoring is very expensive, wages are likely to be determined largely by observable proxies for productivity such as education and seniority. If monitoring is inexpensive and challenges are very informative, there is likely to be little wage growth within job assignment. At intermediate monitoring costs, wages may rise formulaically within some job assignment until some maximum wage. With partial monitoring, they

climb formulaically except for “fast-trackers” who get a boost from resolving a challenge.

Many firms monitor workers immediately after hiring, perhaps reducing monitoring with tenure. According to the model, firms that fit this pattern would be ones that typically hire workers in the upper partial monitoring zone. Workers hired with  $\tilde{\theta}$  in this range are monitored initially, and the intensity of monitoring declines with seniority at the firm and eventually ceases some time prior to promotion. If the worker is observed to successfully negotiate a challenge during monitoring, she is promoted. Of course a failed challenge at any time leads to separation. This setting is consistent with all of the regularities discussed in section 4 except the first (no variation in pay) and sixth (wages jump at promotion). Also, in this setting promotions do not come from various points in the wage distribution (end of section 4), because workers are not hired into various points of the wage distribution.

Our model can be contrasted with Gibbons and Waldman (1999, 2006). The major difference is that in our model, learning comes in large chunks and when it does not come, either there is no learning or updating depends only on the passage of time. In contrast, in Gibbons and Waldman, firms continuously receive information about workers which allows them to distinguish among them. We do not view these approaches as strict alternatives. Clearly, information can come in both forms. However, it is important to note that there are testable differences in the models’ predictions. We have already noted that the models make different predictions about the types of workers who are demoted. In addition, our model suggests that large wage increases at promotion should be associated with settings in which there is a ceiling to the wage-scale. With continuous learning there is no such discontinuity and therefore no prediction of jumps. In principle, it should be possible to look at different jobs in a hierarchy feeding into similar jobs at a higher level in the hierarchy. When wages tend to stall at some value in a job at the lower level, we should be more likely to see large increases at promotion. In contrast, if wages rise continuously within the lower level job, we should be less likely to see wages jump at promotion. We are not aware of any studies directly related to these predictions.

Our approach has advantages and disadvantages. On the positive side, in many jobs wages are determined solely by objective measures such as tenure and education that are only very imperfectly related to productivity. In our model, wages are explained *perfectly* by  $\tilde{\theta}$ , task assignment and seniority. If education and experience are imperfect proxies for  $\theta$ , the model is strongly

consistent with this regularity. However, on the negative side, it is too strong. There are many settings, particularly in managerial ILMs, in which wages are determined partly by subjective performance evaluations even though much of the variation in wages is explained by education, seniority and tier in the hierarchy. Admittedly, it is a small step from assuming information comes only in fully informative chunks to concluding that wages are independent of performance. Still, we believe that the broader predictive power of the model suggests that viewing information as “chunky” has value.

Our model is also consistent with both the steady increase in wages (at least up to some maximum) that often accompanies seniority and the large jumps in wages often associated with promotions. The strong association between large wage increases and promotions does not arise naturally in Gibbons/Waldman.

In both models demotions are rare, albeit for different reasons. In Gibbons and Waldman workers acquire human capital over time which usually outweighs negative information. In our model negative information usually causes a separation.

The Gibbons/Waldman model is better able to explain the frequency of real wage decreases. In our model, in the absence of macroeconomic shocks, real wage decreases are like demotions. Bad news is infrequent and generally results in a separation, not retention with a lower estimate of  $\theta$ . Small real wage decreases happen only when there are negative macroeconomic shocks. Allowing false negatives would also result in some real wage decreases.

Finally, we note that technology has made monitoring easier. In almost any model including this one, this will make pay-for-performance more common. Consistent with this expectation, the proportion of British workers receiving performance pay rose from 16 to 32 percent of workers between 1988 and 1994 (Manning and Saidi 2008). But our model suggests some less obvious effects. Reducing the cost of monitoring could shift the nature of the hierarchy. When monitoring is relatively expensive, we can have two apparent jobs at the low task, one comprised of workers in or below the full monitoring range and one comprised of workers above the full monitoring range, with both jobs leading directly to the high task and relatively little “lateral” movement. When monitoring becomes less expensive, particularly if it becomes easier to observe less informative challenges, there will be more movement from the lower range of the low task into the upper range of the low task so that the low task now appears more like a single job in the hierarchy. Thus the model may be useful to explain how hierarchical structures

change over time.

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# A Proofs

## A.1 Proof of Lemma 1.1

Given Poisson arrival, the density function for the arrival of the first challenge is  $\lambda e^{-\lambda t}$ , and the probability that the first challenge arrives by time  $\tau$  is  $p(\tau) = 1 - e^{-\lambda\tau}$ . Thus the probability that a bad worker does not fail by  $\tau$  is  $1 - p(\tau) = e^{-\lambda\tau}$ .

Suppose a worker with prior  $\theta_0$  has been put in a job at time 0 and has not failed until time  $t$ . If the worker is good, then non-failure occurs with probability 1, and if she is bad then the probability of non-failure is equal to the probability that a challenge has not occurred by time  $t$ . Thus the employer's updated belief about the worker's type (i.e., the updated probability that the worker is good) is:

$$\theta(t, \theta_0) = \frac{\theta_0}{\theta_0 + [1 - p(t)](1 - \theta_0)} \quad (12)$$

which for future reference we rearrange as

$$1 - p(t) = \frac{\theta_0[1 - \theta(t)]}{\theta(t)[1 - \theta_0]} \quad (13)$$

Let  $\bar{\theta}$  be the threshold such that a worker who was initially placed in an  $L$ -job is promoted to the  $H$ -job when  $\theta(t, \theta_0) \geq \bar{\theta}$ . We will show below that  $\bar{\theta}$  is independent of  $\theta_0$ . Define  $\bar{t}(\theta_0)$  such that  $\theta(\bar{t}(\theta_0), \theta_0) = \bar{\theta}$ . Below we will suppress the arguments in  $\bar{t}(\cdot)$ ,  $\theta(\cdot)$  etc.

If  $\theta_0 < \bar{\theta}$ , then the employer puts the worker in the  $L$ -job, and promotes her if she has not failed by time  $\bar{t}$ . Thus a good worker produces  $q$  between times 0 and  $\bar{t}$ , and thereafter produces a flow output of  $g + q$ . A bad worker fails before promotion with probability  $p(\bar{t})$ . With probability  $[1 - p(\bar{t})]$  she produces  $q$  until  $\bar{t}$ , and thereafter produces  $g + q$  until the first challenge arrives, at which time she produces  $-c_h$  and is fired. Hence the expected payoff from the N-strategy with prior  $\theta_0$  and threshold  $\bar{\theta}$  is

$$\begin{aligned}
U(\theta_0, [1 - p(\bar{t})]) &= \theta_0 \left( (g + q) \int_{\bar{t}}^{\infty} e^{-rt} dt + q \int_0^{\bar{t}} e^{-rt} dt \right) \\
&+ (1 - \theta_0)[1 - p(\bar{t})] \left[ e^{-r\bar{t}} \frac{g + q}{\lambda + r} - c_h \int_{\bar{t}}^{\infty} \lambda e^{-\lambda(t-\bar{t})} e^{-rt} dt \right] \\
&+ (1 - \theta_0) \left( q \int_0^{\bar{t}} e^{-(\lambda+r)t} dt - c_l \int_0^{\bar{t}} \lambda e^{-(\lambda+r)t} dt \right) \quad (14) \\
&= \frac{\theta_0 q}{r} + e^{-r\bar{t}} \left\{ \frac{\theta_0 g}{r} - \frac{1}{\lambda + r} [1 - p(\bar{t})] (1 - \theta_0) (\lambda(c_h - c_l) - g) \right\} \\
&+ (1 - \theta_0) \frac{q - \lambda c_l}{\lambda + r} \quad (15)
\end{aligned}$$

Note that  $e^{-r\bar{t}} = e^{[-\lambda\bar{t}]\frac{r}{\lambda}} = [1 - p(\bar{t})]^{\frac{r}{\lambda}}$ , which substituted in (15) yields

$$\begin{aligned}
U(\theta_0, [1 - p(\bar{t})]) &= \frac{1}{r} [1 - p(\bar{t})]^{\frac{r}{\lambda}} \theta_0 g \quad (16) \\
&- \frac{1}{\lambda + r} [1 - p(\bar{t})]^{\frac{\lambda+r}{\lambda}} (1 - \theta_0) (\lambda(c_h - c_l) - g) - (1 - \theta_0) c_l \frac{\lambda}{\lambda + r} \\
&+ \frac{\theta_0 q}{r} + \frac{(1 - \theta_0) q}{\lambda + r}
\end{aligned}$$

The employer maximizes this payoff by choosing  $\bar{\theta}$ , or equivalently  $\bar{t}$  or  $p(\bar{t})$ . Maximizing  $U(\theta_0, [1 - p(\bar{t})])$  in (16) with respect to  $[1 - p(\bar{t})]$  we obtain the first order condition:

$$\begin{aligned}
0 &= \frac{1}{r} \frac{r}{\lambda} [1 - p(\bar{t})]^{\frac{r}{\lambda}-1} \theta_0 g - \frac{1}{\lambda + r} \frac{\lambda + r}{\lambda} [1 - p(\bar{t})]^{\frac{r}{\lambda}} (1 - \theta_0) (\lambda(c_h - c_l) - g) \\
[1 - p(\bar{t})]^{-1} \theta_0 g &= (1 - \theta_0) (\lambda(c_h - c_l) - g) \quad (17)
\end{aligned}$$

Let (17) be solved at  $\bar{t} = t^*$ , and correspondingly  $\bar{\theta} = \theta^*$  etc. Using (13), (17) simplifies to

$$\begin{aligned}
g &= (\lambda(c_h - c_l) - g) \frac{[1 - \theta^*]}{\theta^*} \\
\theta^* &= \frac{\lambda(c_h - c_l) - g}{\lambda(c_h - c_l)}
\end{aligned}$$

It can be checked that the second derivative of  $U(\theta_0, [1 - p(\bar{t})])$  in (16) is strictly negative at the solution, as follows:

$$\begin{aligned} \frac{\partial^2 U}{\partial [1 - p(\bar{t})]^2} &= \frac{r - \lambda}{\lambda^2} [1 - p]^{(\frac{r}{\lambda} - 2)} \theta_0 g - \frac{r}{\lambda^2} (1 - \theta_0) [1 - p]^{(\frac{r}{\lambda} - 1)} (\lambda(c_h - c_l) - g) \\ &= \lambda^{-2} [1 - p]^{(\frac{r}{\lambda} - 1)} (-\lambda \theta_0 g + r ([1 - p]^{-1} \theta_0 g - (1 - \theta_0) (\lambda(c_h - c_l) - g))) \\ &= -\frac{\theta_0 g}{\lambda [1 - p]} \\ &< 0 \end{aligned}$$

so this is indeed a strict maximum.

Note also that the optimal threshold  $\theta^*$  is independent of the prior  $\theta_0$ , from which it follows that a worker with prior  $\theta_0 \geq \theta^*$  will be placed directly in the  $H$ -job. At the optimum, the employer's expected payoff from a new worker with prior  $\theta_0 \leq \theta^*$  can be obtained by making the appropriate substitutions in (16) to give:

$$U^*(\theta_0) = \frac{\lambda}{r(\lambda + r)} \theta_0 g \left[ \frac{\theta_0}{1 - \theta_0} \frac{g}{\lambda(c_h - c_l) - g} \right]^{\frac{r}{\lambda}} + (1 - \theta_0) \frac{q - \lambda c_l}{\lambda + r} + \frac{\theta_0 q}{r} \quad \text{for } \theta_0 \leq \theta^* \quad (18)$$

It follows directly that  $U^*$  is increasing in  $\theta_0$ . For  $\theta_0 \geq \theta^*$ . It is straightforward to check that the expected payoff is then

$$U^*(\theta_0) = \frac{1}{r} \theta_0 (g + q) + \frac{(1 - \theta_0)(g + q - \lambda c_h)}{\lambda + r} > U^*(\theta^*) \quad \text{for } \theta_0 > \theta^*$$

■

## A.2 Proof of Lemma 1.2

When the first challenge arises, the firm gets  $(\theta_0 \frac{g+q}{r} - (1 - \theta_0) c_l)$ . Expected discounting is

$$\int_0^\infty e^{-rt} \lambda e^{-\lambda t} dt = \frac{\lambda}{\lambda + r}.$$

Expected discounted monitoring costs are

$$b \int_0^\infty e^{-rt} e^{-\lambda t} dt = \frac{b}{\lambda + r}$$

$$\tilde{U}(\theta_0) = \frac{\lambda}{\lambda + r} \left( \theta_0 \frac{g + q}{r} - (1 - \theta_0) c_l \right) + \frac{q - b}{\lambda + r}.$$

Rearranging terms yields (3)

### A.3 Proof of Theorems 1.1 and 1.2

#### A.3.1 Preliminaries:

Given a prior  $\theta$ , it is better to monitor the worker than never monitor if

$$\tilde{U}(\theta) \geq U^*(\theta) \Rightarrow$$

$$\frac{1}{r(\lambda + r)} [\lambda \theta g - rb] + \frac{(1 - \theta)q}{\lambda + r} + \frac{\theta q}{r} \geq \frac{1}{r(\lambda + r)} \lambda \theta g \left[ \frac{\theta}{1 - \theta} \frac{g}{\lambda(c_h - c_l) - g} \right]^{\frac{r}{\lambda}}$$

$$+ \frac{(1 - \theta)q}{\lambda + r} + \frac{\theta q}{r} \quad (19)$$

$$\Rightarrow \lambda \theta g \left[ 1 - \left( \frac{\theta}{1 - \theta} \right)^{\frac{r}{\lambda}} \left( \frac{g}{\lambda(c_h - c_l) - g} \right)^{\frac{r}{\lambda}} \right] \geq rb \quad (20)$$

Name the left-hand-side of (20)  $Z(\theta)$ :

$$Z(\theta) = \lambda \theta g \left[ 1 - \left( \frac{\theta}{1 - \theta} \right)^{\frac{r}{\lambda}} \left( \frac{g}{\lambda(c_h - c_l) - g} \right)^{\frac{r}{\lambda}} \right] \quad (21)$$

#### A.3.2 Theorem 1.1

If  $\theta = 0$  or  $\theta = \theta^*$ ,  $Z(\theta) = 0$ , which proves the existence of the lower and upper no-monitoring ranges.

#### A.3.3 Theorem 1.2

The value of no monitoring until time  $\tau$  and then monitoring is

$$\begin{aligned} & \theta_0 \left( q \int_0^\tau e^{-rt} dt \right) + (1 - \theta_0) \left( q \int_0^\tau e^{-(\lambda+r)t} dt - c_l \int_0^\tau \lambda e^{-(\lambda+r)t} dt \right) \\ & + \left( \theta_0 e^{-r\tau} \left( \frac{q}{r} - \frac{b}{\lambda+r} + \frac{\lambda g}{r(\lambda+r)} \right) + (1 - \theta_0) e^{-(r+\lambda)\tau} \frac{q - \lambda c_l - b}{\lambda+r} \right) \end{aligned} \quad (22)$$

Maximizing with respect to  $\tau$  :

$$\begin{aligned} 0 = & \theta_0 q e^{-r\tau} + (1 - \theta_0) (q - \lambda c_l) e^{-(\lambda+r)\tau} - \\ & \left( r \theta_0 e^{-r\tau} \left( \frac{q}{r} - \frac{b}{\lambda+r} + \frac{\lambda g}{r(\lambda+r)} \right) + (r + \lambda) (1 - \theta_0) e^{-(r+\lambda)\tau} \frac{q - \lambda c_l - b}{\lambda+r} \right) \end{aligned} \quad (23)$$

or

$$e^{-\lambda\tau} = \theta_0 \frac{\lambda g - rb}{b(r + \lambda)(1 - \theta_0)}$$

Recall that  $e^{-\lambda\tau} = [1 - p(\tau)]$ . Hence by (13), we have

$$\theta(\tau, \theta_0) = \frac{b(r + \lambda)}{\lambda(g + b)}$$

which is independent of  $\theta_0$ . Call this  $\theta_a$ . For a worker with an initial  $\theta$  below this value, the firm will not monitor the worker until such time as  $\theta_t$  reaches this value. At this point monitoring is more profitable than never monitoring if  $Z(\theta_a) \geq rb$ . Substituting for  $\theta_a$  and rearranging terms yields condition 5.

Thus if (5) holds then  $\tilde{U}(\theta) - U^*(\theta) > 0$  at  $\theta = \theta_a$ . We already know that the reverse holds at  $\theta = \theta^*$ . It is straightforward to show that  $\tilde{U}(\theta) - U^*(\theta)$  is strictly concave. Thus there is a unique  $\theta_b \in (\theta_a, \theta^*$  such that monitoring is more profitable than not monitoring for  $\theta \in [\theta_a, \theta_b]$ , but less profitable for higher  $\theta$ . ■

#### A.4 Proof of Theorem 2.1

If a worker with  $\theta = \theta_t$  works for the firm, she gets expected present discounted wages of  $W_t$  where the expectation reflects the probability of remaining with the firm for different lengths of time and the expected cost of failure. Upon failure, she receives the value of her outside option, which we denote

$U_a$  which makes her total value  $W_t + U_a(1 - \theta_t)\lambda/(\lambda + r)$  since she only goes to the alternative job if she fails. The worker's surplus from the relation is therefore  $W_t + U_a(1 - \theta_t)\lambda/(\lambda + r) - U_a$  or  $W_t - U_a(r + \lambda\theta_t)/(\lambda + r)$  while the firm's surplus is  $V_t - W_t$  where  $V_t(\theta_t)$  is the expected present discounted value of the worker's output at the current firm.

So we have the Nash bargaining maximand

$$\begin{aligned} & \text{Max}_{W_t} (1 - \beta) \ln(V_t - W_t) + \beta \ln\left(W_t - \frac{r + \lambda\theta_t}{\lambda + r}U_a\right) \\ & \rightarrow (1 - \beta) \left(W_t - \frac{r + \lambda\theta_t}{\lambda + r}U_a\right) = \beta(V_t - W_t) \end{aligned} \quad (24)$$

$$W_t = \beta V_t + (1 - \beta) \frac{r + \lambda\theta_t}{\lambda + r}U_a. \quad (25)$$

To see that (6) integrates to (25), note first that the flow value of output,  $v_t$ , by definition integrates to the expected present value of output,  $V_t$ . If workers receive  $(1 - \beta)rU_a$  in each period they are employed then with probability  $\theta_t$ , the present value of what they receive is  $(1 - \beta)U_a$  and with probability  $(1 - \theta_t)$  it is  $(1 - \beta)rU_a/(\lambda + r)$ . But

$$\theta_t(1 - \beta)U_a + (1 - \theta_t)(1 - \beta)rU_a/(\lambda + r) = (1 - \beta) \frac{r + \lambda\theta_t}{\lambda + r}U_a \quad (26)$$

which proves that 6) is a solution to the bargaining problem. Since, by the renegotiation-proofness requirement, this condition must hold for all  $t$ , it is the unique solution.

## A.5 Proof of Theorem 3.1

Preliminaries: We state without proof that  $b = 0$  for  $\theta > \theta^*$ .

Let  $\tau$  be a small interval of time,  $p$  the probability that a challenge that occurs during this interval is detected (given the intensity of monitoring), and  $\Delta$  the change in the prior  $\theta$  that occurs over  $\tau$  given that the worker neither fails a challenge nor observed to solve one. Bayesian updating leads to the following backdating formula

$$\theta_t - \Delta = \theta_t \frac{e^{-\lambda\tau(1-p)}}{1 - \theta_t(1 - e^{-\lambda\tau(1-p)})} \quad (27)$$

$$\tau = \frac{\ln \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}}{\lambda(1-p)} \quad (28)$$

$$e^{-\tau} = \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{1}{\lambda(1-p)}} \quad (29)$$

Discretize the problem so that the firm must do the same level of monitoring over some period  $\Delta$ , and consider the case of a worker in the  $L$ -task. Then

$$U(\theta_t - \Delta) = (q - b(p))\tau + e^{-r\tau} \left( (\theta_t - \Delta) e^{-\lambda p \tau} + (1 - (\theta_t - \Delta)) e^{-\lambda \tau} \right) U(\theta_t) \quad (30)$$

$$\begin{aligned} &+ (\theta_t - \Delta) (1 - e^{-\lambda p \tau}) \frac{g + q}{r} \\ &- (1 - (\theta_t - \Delta)) (1 - e^{-\lambda \tau}) c_l \end{aligned} \quad (31)$$

Substitute for  $\tau$  using (28) and (29)

$$\begin{aligned} &U(\theta_t - \Delta) \quad (32) \\ &= (q - b(p)) \frac{\ln \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}}{\lambda(1-p)} + \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{r}{\lambda(1-p)}} \\ &\left( \left( (\theta_t - \Delta) \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{p}{(1-p)}} + (1 - (\theta_t - \Delta)) \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{1}{(1-p)}} \right) U(\theta_t) \right. \\ &+ (\theta_t - \Delta) \left( 1 - \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{p}{(1-p)}} \right) \frac{g + q}{r} \\ &\left. - (1 - (\theta_t - \Delta)) \left( 1 - \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{1}{(1-p)}} \right) c_l \right) \end{aligned}$$

Maximizing  $U(\theta_t - \Delta)$  wrt to  $p$  gives

$$\begin{aligned}
0 &= \frac{dU(\theta_t - \Delta)}{dp} \\
&= -\frac{b'}{\lambda(1-p)} \ln \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)} + (q-b) \frac{\ln \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}}{\lambda(1-p)^2} \\
&\quad - \frac{r+\lambda}{\lambda(1-p)^2} \frac{1-\theta_t+\Delta}{1-\theta_t} \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{r+\lambda}{\lambda(1-p)}} \ln \left( \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)} \right) U(\theta_t) \\
&\quad - \left( 1 - \left( \theta_t \frac{\theta_t-\Delta-1}{(-1+\theta_t)(\theta_t-\Delta)} \right)^{-\frac{\lambda p}{\lambda(1-p)}} \frac{\lambda+r}{r} \right) r \left( \frac{\theta_t(\theta_t-\Delta-1)}{(-1+\theta_t)(\theta_t-\Delta)} \right)^{-\frac{r}{\lambda(1-p)}} \\
&\quad \frac{\left( \ln \theta_t \frac{\theta_t-\Delta-1}{(-1+\theta_t)(\theta_t-\Delta)} \right) (\theta_t-\Delta)(g+q)}{\lambda(1-p)^2 r} \\
&\quad - \frac{(1-(\theta_t-\Delta))}{(1-p)^2} \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{1}{(1-p)}} \ln \left( \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)} \right) c_l
\end{aligned} \tag{33}$$

which for  $\tau > 0, p < 1$  gives

$$\begin{aligned}
0 &= \frac{dU(\theta_t - \Delta)}{dp} \\
&= -\frac{b'}{\lambda(1-p)} + \frac{(q-b)}{\lambda(1-p)^2} \\
&\quad - \frac{r+\lambda}{\lambda(1-p)^2} \frac{1-\theta_t+\Delta}{1-\theta_t} \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{r+\lambda}{\lambda(1-p)}} U(\theta_t) \\
&\quad - \left( 1 - \left( \theta_t \frac{\theta_t-\Delta-1}{(-1+\theta_t)(\theta_t-\Delta)} \right)^{-\frac{\lambda p}{\lambda(1-p)}} \frac{\lambda+r}{r} \right) r \left( \theta_t \frac{\theta_t-\Delta-1}{(-1+\theta_t)(\theta_t-\Delta)} \right)^{-\frac{r}{\lambda(1-p)}} \\
&\quad \frac{(\theta_t-\Delta)(g+q)}{\lambda(1-p)^2 r} \\
&\quad - \frac{(1-(\theta_t-\Delta))}{(1-p)^2} \frac{\theta_t(1-\theta_t+\Delta)}{(1-\theta_t)(\theta_t-\Delta)}^{-\frac{1}{(1-p)}} c_l
\end{aligned} \tag{34}$$

Now take the limit

$$\lim_{\Delta \rightarrow 0} \frac{dU(\theta_t - \Delta)}{dp} = - \frac{b'r - b'rp - qr + br + U(\theta_t)r^2 + U(\theta_t)\lambda r - \theta_t g \lambda - \theta_t q \lambda + c_l r \lambda - c_l r \theta_t \lambda}{\lambda(1-p)^2 r} \quad (35)$$

Then we have

$$(q - (1 - \theta_t) \lambda c_l) + \lambda \theta_t \frac{g + q}{r} - U(\theta_t)(\lambda + r) = b + b'(1 - p) ; \quad 0 < p(\theta_0) < 1 \quad (36)$$

$$(q - (1 - \theta_t) \lambda c_l) + \lambda \theta_t \frac{g + q}{r} - U(\theta_t)(\lambda + r) \leq b'(0) ; \quad p(\theta_0) = 0 \quad (37)$$

$$(q - (1 - \theta_t) \lambda c_l) + \lambda \theta_t \frac{g + q}{r} - U(\theta_t)(\lambda + r) \geq b(1) ; \quad p(\theta_0) = 1 \quad (38)$$

The result we want is:

*Theorem A.1.* If  $b'(p) > 0 \quad \forall p \in [0, 1]$ , then there is  $\theta_a, \theta_b$  with  $0 < \theta_a \leq \theta_b < \theta^*$  such that

(i)  $p(\theta) = 0$  in the interval  $[0, \theta_a)$

(ii)  $p(\theta) = 0$  in the interval  $(\theta_b, \theta^*]$ .

(iii) If  $b'(0)$  is not too large, then  $p(\theta) > 0$  for some  $\theta \in [\theta_a, \theta_b]$ .

*Proof of (i) and (ii):* Suppose there is no monitoring at  $\theta_t$ . Then  $U(\theta_t)$  is given by (18) if there is never monitoring, and is greater if monitoring is efficient at a later point. Therefore the proof parallels theorem 1.1 with  $b'(0)$  substituting for  $b$ . If there is monitoring at  $\theta_t$  then the left-hand-side of (36) is less using  $U(\theta_t)$  than (18) and thus will be less than  $b'(0)$  whenever it is less than  $b'(0)$  substituting (18) for  $U(\theta_t)$ .

*Proof of (iii):* If there were no monitoring range,  $U(\theta_t)$  would be given by (18) and the left-hand-side of (36) would reduce to (21) which is strictly positive for all  $\theta \in (0, \theta^*)$  and therefore greater than  $b'(0)$  for  $b'(0)$  sufficiently small. ■

Theorem (3.1) in the text is a restatement of parts (i) and (ii) above.