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Understanding linear measure

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Enhancing Opportunities For Student Understanding of Length Measure

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Abstract (a 1- or 2-sentence overview of the article):

This article provides strategies for enhancing tasks to offer students better opportunities to develop conceptual understanding of length measurement. Teachers are offered strategies that help move instruction beyond procedures.

If you teach mathematics to students in grades K through 4, what do your students know about length measurement? For example, consider a grade 1 assessment task from *Everyday Mathematics* (see fig. 1). How do you think your students would respond? What might you learn about their understanding of length measurement? What might you not learn? Think about how you would respond to this task before reading on.

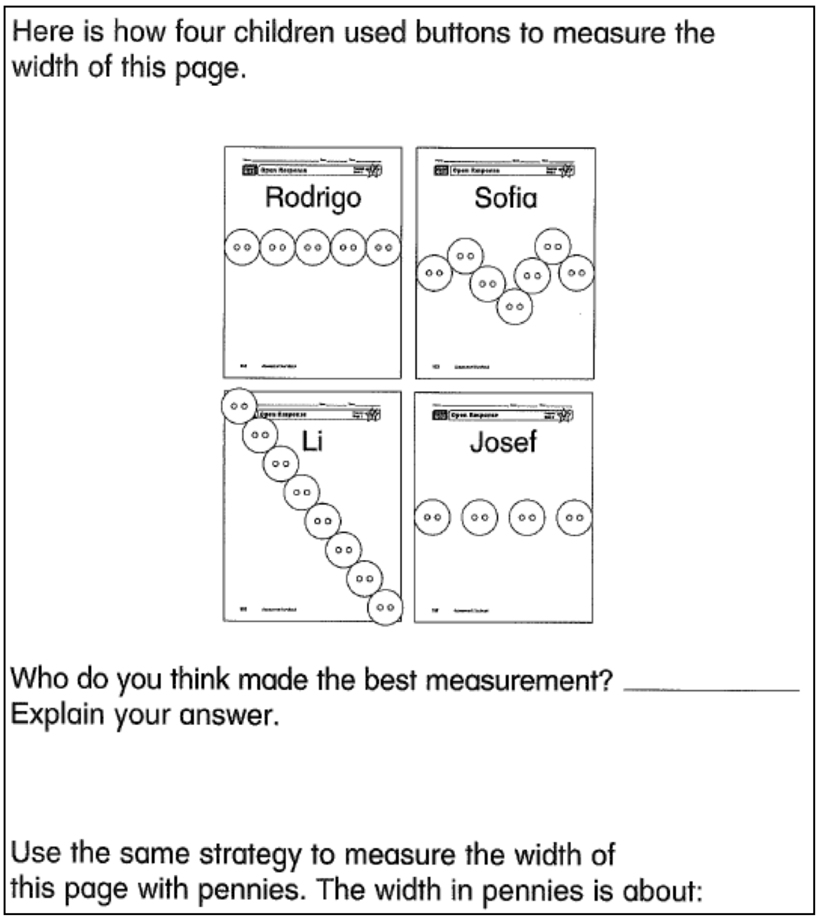
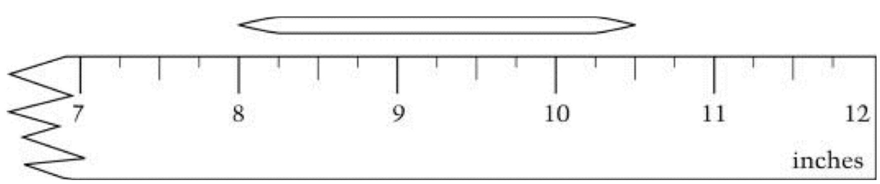


Figure 1. The Buttons Task, from page 152 of the Assessment Handbook, Grade 1, of *Everyday Mathematics*, by the University of Chicago School Mathematics Project, 2007, Chicago, IL: Wright Group/McGraw-Hill. Reprinted with permission.

While the Curriculum Principle of NCTM's *Principles and Standards of School Mathematics* (2000) states, "A school mathematics curriculum is a strong determinant of what students have an opportunity to learn and what they do learn" (p. 14), this article seeks to help teachers enhance tasks found in commonly used textbooks to create additional opportunities for students to develop conceptual understanding of length measurement, specifically involving unit iteration. Guided by the work of the STEM (Strengthening Tomorrow's Education in Measurement) Project, which carefully analyzed the measurement content found in three representative U.S. textbooks, it also raises questions about opportunities that are often not present in elementary mathematics curricula.

Length Measurement Research: Student Knowledge and Curricula

Studies have shown that elementary students' understanding of length measurement, specifically concerning unit iteration, is surprisingly weak. For example, only 20% of fourth-graders stated that the length of the toothpick from the 2003 NAEP mathematics assessment (see fig. 2) is 2.5 inches. Other answers for this open-response task, referred to in this article as the "Toothpick Task," include 10.5 inches and 3.5 inches (14% and 23% of responses, respectively).



"What is the length of the toothpick?"

SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2003 Mathematics Assessment.

Figure 2. The Toothpick Task: an open-response test item from the 2003 NAEP, grade 4.

One way to understand why the length of the toothpick is 2.5 inches is to apply the principle of unit iteration (see fig. 3). Unit iteration is the act of identifying a unit (in this case, an inch), moving it along the length to be measured without gaps or overlaps, and counting the number of units required. In contrast, filling a span with a collection of equal-sized units is commonly called *tiling*.

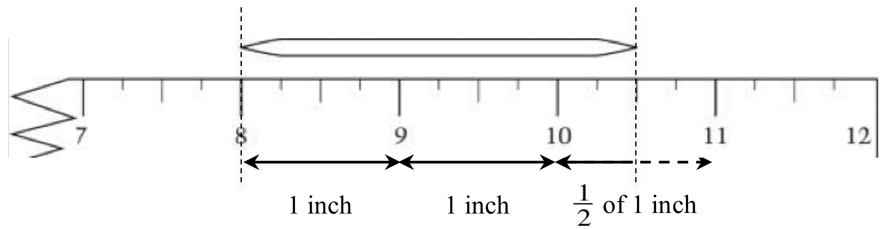


Figure 3. One way to find the length of the toothpick.

In *A Research Companion to Principles and Standards for School Mathematics,* Richard Lehrer (2003) argues that both unit iteration and tiling are fundamental aspects of understanding length measure. Both require the attribute of the object being measured (the length of the toothpick) to be exhausted, without gaps between units. In addition, the unit(s) must lie along the path of the quantity (such as a straight line connecting the ends of the toothpick). In tiling situations, students must also recognize the need for equal-sized units (where in iteration only one unit is used).

Therefore, both unit iteration and tiling satisfy four basic requirements:

- All units must be placed on the path being measured.
- The units must have equal length.
- The units must exhaust the quantity being measured without gaps.
- The units must not overlap. Otherwise, part of the quantity is measured more than once.

Although it may seem trivial, another key to understanding the Toothpick Task is to recognize that a ruler represents tiled units of length (in this case, an inch, or a quarter-inch). If students only view a ruler as a tool with marks rather than a length partitioned into equal-sized units, they may only be able to use the ruler correctly when they can align the zero-mark with one end of the object (which would explain the 10.5 inch response). Also, confusion about what the marks on rulers represent may explain the 3.5 inch response (which could be produced by counting inch tickmarks along the toothpick and adding a half-inch for the remainder).

Results on a task similar to the Toothpick Task led Kamii (2006) to question why instruction for length measurement has been ineffective. She noted that students are usually asked to produce a measure number and are not often asked to compare measures. For example, students can move a yardstick across the chalkboard and generate a number without focusing on conceptual principles of measurement involved. Kamii suggests having students make their own rulers or yardsticks might provide them the opportunity to understand how rulers represent tiled units of equal length.

Kamii's claim that the teaching of measurement usually focuses on carrying out procedures is supported by our STEM Project findings (Smith III. et al., 2008). STEM analyzed three U.S. elementary mathematics curricula to learn how they supported students' learning of measurement. These three were chosen to represent some of wide range in mathematics curricula at the elementary level; the first two have large shares of the textbook market (Reys and Reys 2006). All three curricula [*Everyday Mathematics* ("EM"), Scott Foresman-Addison Wesley's *Mathematics* ("SFAW"), *Saxon Math* ("Saxon")], had a high proportion of procedural content (focusing on the processes of measuring and estimating spatial quantities) and relatively little conceptual content (the basic principles that underlie and justify measurement procedures). The analysis of these three textbook series suggests that many elementary teachers in the U.S. are using written materials where less than 10% of measurement content clearly and explicitly identifies or asks students to articulate basic conceptual principles. And when conceptual principles are expressed, references to unit iteration are infrequent.

To better understand what unit iteration is found in U.S. texts, the STEM Project identified any content (conceptual and procedural) in these textbooks involving unit iteration or tiling. With regards to conceptual development, we noted any statements, questions, or problems focused on understanding the principles of unit iteration ("Iterate unit") or how units are represented on a ruler ("Ruler represents iterated unit"). The analysis suggests that U.S. elementary texts contain very little conceptual development of unit iteration in grades K through 3, and content involving how a ruler represents iterated units of length is particularly rare. For example, with regard to rulers representing an iterated unit of length, SFAW and Saxon only contained one instance across K-3, and EM only contained three.

Revisiting the Button Task and other Elementary Tasks

Because we found conceptual content sparse in the textbooks we examined, this article focuses on how teachers can analyze existing curricular tasks and modify them to increase students' conceptual understanding of length measurement. When analyzing a task, it is important to consider what opportunities specific tasks may offer students to learn about unit iteration, and also what opportunities they *do not explicitly offer*. In this section, we will analyze three tasks found in two of the textbooks we analyzed. These tasks were selected because they offer some attention to conceptual understanding of measurement, and yet can still be enhanced to include understanding of iteration as described in the previous section. In the next section, we will suggest how any measurement task can be analyzed and adjusted to increase student opportunities to understand length measurement.

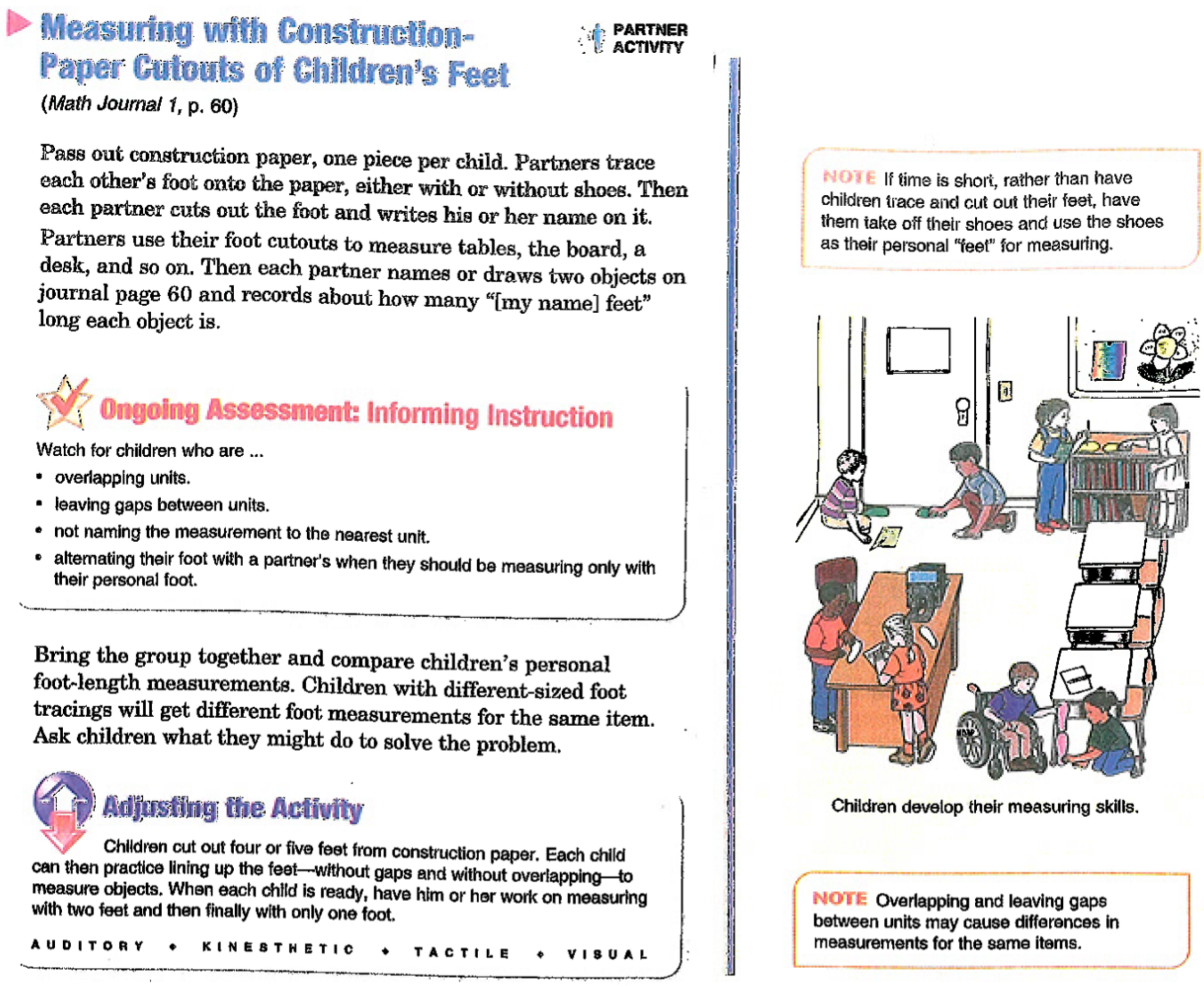
The Buttons Task

The Buttons Task (fig. 1) prompts students to consider four different tilings and decide which fictional student "made the best measurement." When responding to "explain your answer," the student may state why his or her selection represented the best measurement and describe the principles of measurement on which it relies. The different tilings give students the chance to attend to the attribute being measured (e.g., Is it the distance across the page? Is it the diagonal distance?), whether the units must be placed in a straight line, and whether the units may have gaps. Of the 1653 textbook pages that contained length measurement content, this task was the *only* instance we found that explicitly prompted students to consider (a) the consequences of gaps, and (b) the need for the units to lie on the path being measured (i.e., the straight line across the page).

Despite these beneficial qualities, the Button Task does not require students to consider what would happen if the tilings had used differently-sized buttons. In addition, students can answer correctly just by considering Rodrigo's measurement first, without seriously considering the work of other students. Since the task only asks students for the "best measurement," they are not required to describe the principles of unit iteration that the other tilings violate, such as the fact that Josef's units do not exhaust the width of the page. So while this task has value, it could be enhanced if a fifth fictitious student who used different-sized buttons were added and if students were instead asked to explain to the other fictitious students (those without the best measurement) how to improve their measurements.

The Foot Task

In another task from EM, which we will refer to as the "Foot Task," students are given only one length unit (a tracing of their foot) and asked to measure a span, such as the length of a table (see fig. 4). This grade 1 task requires students to iterate their single foot cutout to measure length. If the span is not marked with a line (such as floor tiles), the students must also choose a path that will produce an accurate measurement.



Measuring with Construction-Paper Cutouts of Children's Feet

PARTNER ACTIVITY

(*Math Journal 1*, p. 60)

Pass out construction paper, one piece per child. Partners trace each other's foot onto the paper, either with or without shoes. Then each partner cuts out the foot and writes his or her name on it. Partners use their foot cutouts to measure tables, the board, a desk, and so on. Then each partner names or draws two objects on journal page 60 and records about how many "[my name] feet" long each object is.

Ongoing Assessment: Informing Instruction

Watch for children who are ...

- overlapping units.
- leaving gaps between units.
- not naming the measurement to the nearest unit.
- alternating their foot with a partner's when they should be measuring only with their personal foot.

Bring the group together and compare children's personal foot-length measurements. Children with different-sized foot tracings will get different foot measurements for the same item. Ask children what they might do to solve the problem.

Adjusting the Activity

Children cut out four or five feet from construction paper. Each child can then practice lining up the feet—without gaps and without overlapping—to measure objects. When each child is ready, have him or her work on measuring with two feet and then finally with only one foot.

AUDITORY • KINESTHETIC • TACTILE • VISUAL

NOTE If time is short, rather than have children trace and cut out their feet, have them take off their shoes and use the shoes as their personal "feet" for measuring.

Children develop their measuring skills.

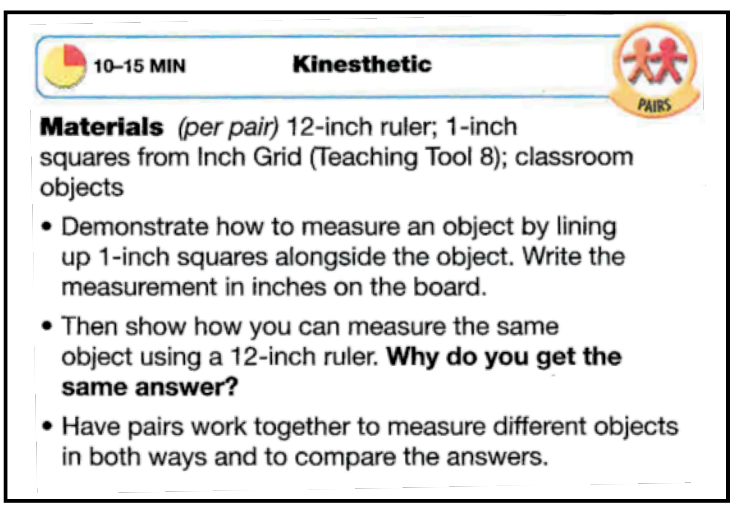
NOTE Overlapping and leaving gaps between units may cause differences in measurements for the same items.

Figure 4. Excerpt from the Foot Task, from page 288 of the Teacher's Lesson Guide, Grade 1, of *Everyday Mathematics*, by the University of Chicago School Mathematics Project, 2007, Chicago, IL: Wright Group/McGraw-Hill. Reprinted with permission.

While this task asks students to complete a measurement procedure, it offers opportunities for conceptual development. For example, if the students iterate with a "finger gap" between successive units, the teacher may bring attention to the gap, such as, "*Is this part of the length being measured? Why or why not?*" Also, if a teacher notices that students measure a straight attribute (such as the width of a rectangular table) with a curved path, or one that does not span the entire length of the object, the teacher can ask, "*If I had a string of (the faulty length measured by the students) and stretched it straight across the table, would it be the same length? Why or why not?*" While this task does not require students to address the need for the units spanning the object to be the same length, it could be enhanced if the teacher proposed to measure the length by tiling with all the students' cut-outs and asked, "*What if I used everyone's cut-outs? Would that give us an accurate measure? Why not?*"

The 1-inch Squares Task

The third task we highlight ("1-inch Squares Task") was one of the few we found that explicitly tries to help students recognize that rulers represent iterated units of length (see fig. 5). This task, found in the SFAW Teacher's Guide for grade 1, prompts the teacher to demonstrate how to measure an object by tiling 1-inch squares. Then the teacher is directed to demonstrate finding the length of the same object using a ruler, and to ask, "*Why do you get the same answer?*" Tasks like this provide students opportunities to link their work with discrete length units (non-standard and standard) to the marks on rulers.


10–15 MIN **Kinesthetic** PAIRS

Materials *(per pair)* 12-inch ruler; 1-inch squares from Inch Grid (Teaching Tool 8); classroom objects

- Demonstrate how to measure an object by lining up 1-inch squares alongside the object. Write the measurement in inches on the board.
- Then show how you can measure the same object using a 12-inch ruler. **Why do you get the same answer?**
- Have pairs work together to measure different objects in both ways and to compare the answers.

Figure 5. The 1-inch Squares Task, from page 371B of the Teacher's Guide, Grade 1, of *Mathematics*. Michigan edition, by Charles, Randall, Warren Crown, and Francis Fennell, 2008, Glenview, IL: Scott-Foresman/Addison Wesley. Reprinted with permission.

However, while students may concretely recognize each square tile as a distinct unit along the span, it is unclear if this is enough to help students recognize discrete 1-inch spans on the ruler. Therefore, this task could be enhanced if the students also laid a ruler along the tiled 1-inch squares and counted the units as shown on the ruler. In addition, the teacher could demonstrate that if the ruler is not placed so that the zero-mark is aligned with the end of the object, the length of the object (as shown by the tiled 1-inch squares) does not change. This way, students can begin to recognize that the 1-inch units on the ruler are *between* the marks, and that it is the number of these spans along the object that matter.

Evaluating and Increasing Opportunities to Build Conceptual Understanding

The three tasks we have discussed provide students the opportunity to develop conceptual understanding of unit iteration because they explicitly address principles of length measurement. But if these opportunities are relatively rare in textbooks, what can a teacher do? One opportunity is to enhance the existing tasks. The other is to change the discourse around the task. In this section, we offer four questions that can be used to identify missed opportunities for conceptual development. If the answer to any of these questions is "no," we offer suggestions for how to enhance the task and questions the teacher could use for discussion.

- **Question #1: Does the task offer the opportunity for students to think about whether units need to have equal length?**

A typical task found in elementary textbooks asks students to measure lengths with non-standard units. Consider providing multiple manipulatives as length units and asking a question like, "*I like how the red button and the pink erasers look together. Can I use them both to measure the length at the same time? Why or why not?*" Some students may not realize that to ensure the units are the same length, they also need to be rigid. Consider providing students with a rubber band to measure a length and asking, "*Could I use this to get an accurate measure of length? Why or why not?*"

- **Question #2: Does the task offer an opportunity for students to think about why units cannot have gaps or overlaps?**

When a task prompts students to tile units, consider including an erroneous example on the board (such as Josef's in the Buttons Task) and asking students to explain why such a method will not give an accurate measurement. When a task prompts students to iterate a unit of length (such as in the Foot Task), consider having them trace the units before moving them, so they leave a record of where the units were placed along the span. This will not only help students "see" prior units that might disappear without visual evidence, but will also allow you to spot a gap and ask, "*Is this portion being measured?*" The teacher can also help focus attention on the need for the units to lie on the path being measured, with a comment, such as, "*I like how you placed your units in a straight line. Why do they need to be placed in a straight line? What if they curved instead?"*

- **Question #3: Does the task offer an opportunity for students to think carefully about what quantity is being measured?**

Students are often prompted to find the length of objects in the classroom. If a task asks the students to measure an attribute of an object, such as the height of their desk or the width of an eraser, consider leaving the attribute of the object ambiguous by not pointing to it or marking what part to measure. For example, the width of a table could be interpreted differently, causing different groups of students to get different measures. If this happens, reporting from different groups could lead to a discussion about what specific attribute each saw as the "width," which could in turn help students learn that sometimes saying words like the "width" or "length" does not clearly communicate the attribute they are measuring.

- **Question #4: Does the task offer an opportunity for students to think about how rulers represent iterated units of length?**

All three elementary curricula provide ample opportunities for students to measure with a ruler. When a task prompts students to measure an object with rulers, consider asking, "*What if I place the ruler so that the left end of the object is aligned at 2 inches instead?*" to spur ideas about how to use the ruler to measure the length in this way. You might also want to ask students to think about why the object did not change length even though the other end is now aligned with a new number. Another strategy is to present some "strange" rulers (see fig. 6), and ask, "*Could I use these rulers to measure correctly? Why or why not?*" Encourage students to think about the marks on rulers, rather than simply accept their presence as given.

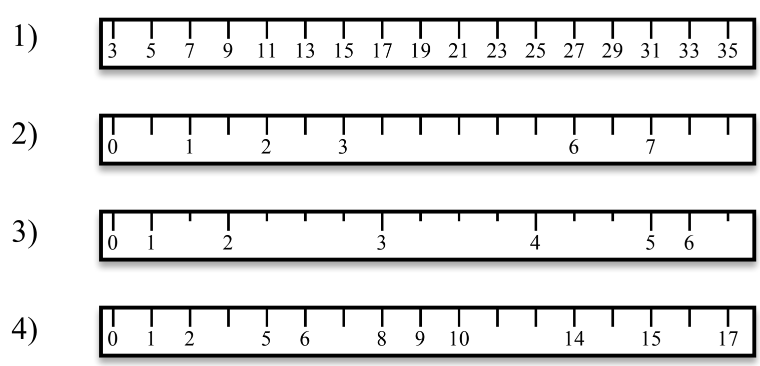


Figure 6. Strange rulers

Conclusion

Even though measurement is among the most meaningful topics in the elementary mathematics curriculum, evidence from diverse sources suggests that many students are not learning measurement well. Many are learning procedures for measuring length (how to place and read a ruler in the "standard" way), without learning the underlying conceptual principles. This deficit makes errors more likely, especially when students need to adjust a standard procedure, like measuring with a broken ruler that is missing the zero-mark. One way teachers can address this challenge is to look for tasks that hold promise for raising and clarifying conceptual issues and enhance their usefulness without lots of work. We should not expect "good" tasks to address all four key issues presented in this paper. No one task will teach all fundamental principles of length measurement. Instead we offer this frame and our assessment of three tasks as a tool for teachers to examine, reflect on, and improve their measurement curriculum.

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