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Teaching and learning with Wittgenstein and Turing: sailing the seas of social media

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Teaching and Learning with Wittgenstein and Turing: Sailing the Seas of Social Media

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> There is no steady unretracing progress in this life; we do not advance through fixed gradations, and at the last one pause:— through infancy's unconscious spell, boyhood's thoughtless faith, adolescence' doubt (the common doom), then scepticism, then disbelief, resting at last in manhood's pondering repose of If. But once gone through, we trace the round again; and are infants, boys, and men, and Ifs eternally.
> Melville, *Moby Dick* Chapter 114, The Gilder

> Only whom are we informing of this? And on what occasion?
> Ludwig Wittgenstein, *Philosophical Investigations* §296

1. Introduction

Social media buffets our words, carrying them rapidly into environments and contexts that are rapidly transforming not only our sense of meaning and world, but also our embodied lives and environments. We each need to navigate the seas, sailing about in our own very different, overlapping ways. This is a situation foreseen both by Wittgenstein and by Alan M. Turing, whose philosophies are of increasing importance now, as I shall argue in this essay.

I want initially to share a bit of critical history about the computationally-driven world we live in, stressing that philosophical ideas lay at the basis of Turing's invention of the stored program computer. At their heart, as has not been appreciated, Turing's philosophical ideas about humans and machines are not behavioristic and mechanistic, but instead broadly Wittgensteinian, rooting our "phraseologies" in social contexts of seeking and finding one another through words and images, shaped in a communal context in which harmonies of engagement, rather than consensus, are at stake. As Turing attested, his own ideas derived partly from conversations he had with Wittgenstein (cf. Turing 1942-44, Floyd 2013). In turn, Wittgenstein responded to what he realized was Turing's fundamentally transformative recasting of the foundations of logic in terms of human steps that are "mechanical" in an intuitive sense: shareable commands everywhere embedded in evolving human forms of life and "phraseology".

These facts -- which I have argued for at length in a series of essays referred to in what follows -- should lead us to considerably revise the usual story: that methods of formalization *contradict* the taking seriously of our ordinary words in everyday life, and that investigation of what we say and mean, at the local level of everyday speech, is somehow *contradicting* the project of systematic formalization, especially in the foundations of logic and mathematics. Many believe that in his early philosophy Wittgenstein embraced formalization, and in his early philosophy he rejected it. But No. Better would be to say that he rejected the dichotomies implied in the usual story. It was not until 1937, however, that he managed to embed the necessarily dynamic interplay of these perspectives at the heart of his later philosophy. This, I have argued, was due to his reading of Turing.

On the usual story Wittgenstein and Turing were enemies: Wittgenstein the humanist, fighting mathematical logic; Turing the scientistic mechanist making apologies for it.[1] Their

[1] (Monk, 1990), chapter 20.

exchanges over the nation of contradiction in Wittgenstein's 1939 Cambridge lectures on the foundations of mathematics (LFM) have led many, including Ray Monk, in his masterful biography, to regard Wittgenstein as making "propaganda" against Turing's whole way of thinking (1990, chapter 20).

My perspective is quite different. I believe Wittgenstein and Turing were, in fact, teaching and learning *with* one another. Rather than a seminar brawl, we have two thinkers engaged in mutual exploration and philosophical development, each in his own way. In his 2017 British Wittgenstein Society lecture, responding to my work, Monk remained unconvinced; I will not recapitulate my extensive arguments here, but instead summarize and extend the responses I gave at the 2018 London BWS meeting.[2] In fact, my reconstructions explain, not only what Turing really did, logico-philosophically speaking,[3] but also why understanding the intersection of his ideas with Wittgenstein's is crucial, not only for our understanding of Turing and Wittgenstein, but also for understanding our contemporary world.

Proceeding toward this latter goal, I shall first give a reading of the famous remarks §§193-4 in *Philosophical Investigations* concerning the "machine" that "symbolizes its own actions [*Werkungsweise*]" – or "operations", as the more recent translation has it. I think we might even consider drawing the translation closer to that of Emil Post's (human) idea of a "worker", simply translating this as a machine that "symbolizes its own ways of working".[4] As I have established elsewhere, these remarks were first drafted by Wittgenstein after he held a discussion group at Cambridge with Turing and their mutual friend Alister Watson in the summer of 1937: they are definitely a response to Turing's analysis of an "effective" step in a formal system in his (1936/7) paper "On computable numbers, with an application to the *Entscheidungsproblem*" (Floyd, 2016, p. 25, n. 33; Floyd, 2017). I shall argue here that Wittgenstein's remarks are not intended to contradict or critique a functionalist philosophy of mind, as readings from Putnam and Kripke onward have assumed;[5] instead, Wittgenstein is noting that Turing used — just as had Wittgenstein before him — the idea of portions of *human* activity with words *as* the actions of unthinking "machines" (cf. the "reading machine" passages in PI §§157ff and BBB §67) – and he is warning against misuses of Turing's powerful reworking of this idea.

This reading allows me, next, to draw Wittgenstein and Turing into the orbit of current challenges affecting emerging uses of social media. Here I build on discussions 2016-2019 afforded through the auspices of the interdisciplinary Boston University Mellon Sawyer Seminar

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[2] Monk's lecture is here: https://www.britishwittgensteinsociety.org/eighteenth-british-wittgenstein-society-lecture. My response in 2018 is here: https://vimeo.com/286157182 accessed 23 February 2019.

[3] Happily my interpretation, rooted in logic and not philosophy of mind, helped to explain, before their discovery, Turing's continued study of logical notations (Leibniz, Peano, Weyl, etc.) while at Bletchley Park; cf. his notebook "Notes on Notations" (1942-44), sold at Bonham's in New York for over $1,000,000 in 2015: https://www.bonhams.com/auctions/22795/lot/1/.

[4] Independently of Turing Post concocted a model of a "step" in a formal system virtually equivalent to that of Turing; interestingly he used language describing, not a machine, but rather a human "problem solver or worker" (Post, 1936). This led him to regard the classical limitative results in logic as indicating limits to the human mind -- something not true of Turing.

[5] Cf. (Kripke 1982, *passim*). Putnam assumed from the early 1960s onward that the point of a Turing machine is to model trial and error searching at the human psychological level. Only later did he debunk his own functionalism (Putnam, 1988). In our final conversation, when I mentioned my a-psychologistic interpretation of Turing's notion of a Turing Machine, he still regarded it as a model "built for an alien", though I argued against this point of view, as I am doing here.

2016-2019 in which I am currently participating as lead investigator.[6] Our aim has been to give philosophy in particular, and the humanities in general, central *foundational* places at the table in our discussions of emerging media and new forms of life.

The BU Mellon Sawyer Seminar has affirmed a strong conviction of mine: that the humanities in general, and philosophy in particular, will become, over time, more and more crucial for our world. This is beginning to be widely sensed, as "AI and Ethics" and "digital humanities" become catchphrases and eddies of study where engineers, librarians, lawyers and humanities professors interact. My idea is different, less a matter of engineering than of working as philosophers always have, with words and discussions that reflect our capacity to shift our perspectives on what is important in life and in our world, allowing us to attend to what we care about, aspire to, and feel. We must continually re-embed our words and images in a variety of forms of life, just as Wittgenstein said, in order for our technology to be useful and enhance, rather than demean us.

The doing of philosophy, by each and every one of us, is in fact becoming increasingly recognizable and fundamental in everyday life, as the automation of tasks and the disruptions and inventions of new forms of life goes on, and pressures of indirect discourse multiply, becoming pervasive. The ubiquity of our epideictic uses of words is increasingly part of the environment, the demands for visibility and recognition, but also privacy, creativity, and individuality, urgent. Virtual selves are commercial necessities for younger users, but also sources of anxiety and distraction. We are engaged in a grand experiment involving the human community as a whole: soon another billion users will come on line with voice activated technology. But what really is the issue of *voice*? Here is a key question, both for democratic forms of life, and for each and every one of us.

What is fundamental in current uses of social media is the Rousseauian drive that Stanley Cavell so aptly made central to his philosophy: we each demand *acknowledgement*, desiring to be recognized as worthy of respect by others who, in turn, are worthy of respect; we desire and seek significance and praiseworthiness, at the same time, for ourselves.[7] This was confirmed empirically by some of the earliest studies of mobile technology made by my collaborator James E. Katz, with his colleagues.[8] It is no accident that these sociologists, who focus on structural affordances, phenomenology, biology, and elements of our creativity and desire in the face of new technologies, led the way. For their surveys *asked* users what they did and felt, and opened our eyes to the unexpected, novel uses of mobile phones as they first entered our public and private spaces. What these studies show is that ethics is ubiquitous in human uses of this technology, though varied, just as it is in ordinary conversation: the press toward individuality and community are both there, both dynamic. These issues are present in ways that can be partly, but surely not fully, resolved by designs of code, expertise, and the creators of technology.

Our situation is dynamic: it is one of searching. We seek and we sometimes find, but we are

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[6] Cf. https://www.mellonphilemerge.com/. The grant is held with my Boston University co-PI's James E. Katz (Director, Division of Emerging Media and Center for the Study of Mobile Communication) and Russell Powell (Associate Professor of Philosophy). The Mellon Sawyer funding supports the creation of novel interdisciplinary centers focused on faculty development and aimed at new paths of research. In our BU Sawyer Seminar we have also developed new curricula, sponsorship of post-docs and graduate students, and approaches to teaching. By 2018 all of our Sawyer Fellows were hired into competitive philosophy positions, as attested on our website.

[7] The *locus classici* are (Cavell, 1969/2002) and (Cavell, 1979/1999).

[8] See (Katz, 2003); (Fortunati and Katz, 2003); (Katz and Aakhus, 2004); (Katz, 2008); (Katz, 2014).

constantly shifting our sense of the questions, as well as the satisfactoriness of the answers, a sense of what is worth searching for, and how and when and why. Philosophically we should aim to appreciate the differing kinds of searching that characterize our lives with social media, and not pretend that "Big Data" are a mechanical way of resolving questions *without* human qualitative judgments. On the contrary: as data get bigger and bigger, they grow more and more difficult to manage.[9] It is the qualitative standards, the need for intelligible overviews, situated, human perspectives — what Wittgenstein called "*übersichtlich Darstellungen*" — that will become more and more, not less, important.[10]

It falls to philosophy, particularly in its role in education, to characterize the broad sense in which reason-giving, commitment to words and truth, and a sense of self-realization through expression matter and may be developed, understood, and represented in our world. Here everything from reflection on the dynamism and force of what Cavell called "passionate speech" to popular culture, imagery, and literature, are increasingly relevant (Cavell, 2005). In the end the point will be to focus, not on human-machine interaction, but on human-human significance in the presence of machines.

2. Seas of Words: PI § 194

It is no accident, I believe, that Wittgenstein developed his notion of a *Lebensform*, a form of life with words, after ingesting Turing's landmark paper (Turing, 1936/7) introducing the idea of what Church called a "Turing machine" (Floyd, 2016). Wittgenstein needed something philosophically deeper than the idea of a "culture". In *The Blue and Brown Books*, to imagine a language is to imagine a "culture" [*Kultur*]; in 1937, the term "*Kultur*" drops out of the manuscript of the *Philosophical Investigations* altogether, replaced by "forms of life". To imagine a language is, hereafter, to imagine a form of life with words: not a *Lebenswelt* as something given, but as something continually recreated, woven and cobbled together with criss-crossing perspectives and lives with words (Floyd, 2018b).[11]

It is also no accident, in my view, that Wittgenstein's image of seas of words [*Wellen*] occurs precisely at the point in *Philosophical Investigations* where he is responding to Turing's (1936/7) analysis of taking a "step" in a formal system of logic (PI §194, glossed in a moment). Apparently paradoxically — but only apparently so — by putting to work the idea of a *human being* used as a "machine", Turing *undid* the sublimated ideal of formal logic as a super-machine.[12] In the *Investigations* Wittgenstein is crediting him with this, but warning his readers against taking the undoing wrongly.

The heart of Turing's contribution was to cut through the jungle of formalisms and equations by imposing a vivid, simplified model upon formal, mathematized logic as an analysis of its fundamental character. The model is a language-game, in Wittgenstein's sense: that of a human computor acting or reckoning step-by-step in accordance with a locally framed rule without

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[9] Cf. BU Mellon Sawyer Seminar at: https://www.mellonphilemerge.com/past-events/is-this-an-epistemological-revolution-big-data-and-the-philosophy-of-science accessed 23 February 2019.

[10] Compare the Interview with Gary King by Floyd and Katz in (Floyd and Katz, 2016), Chapter 24; excerpts of the filmed version are at https://www.youtube.com/watch?v=tUZMYf2X0V8, accessed 25 February 2019.

[11] Cf. (Floyd, 2018b): as I stress in this paper, the term "*Lebenswelt*" does not ever occur in Wittgenstein's writings.

[12] It is this to which I take Wittgenstein to be referring in his lectures on aesthetics §§23-30, given in 1938 (LA, pp. 15ff; cf. "Lectures on Religious Belief", LA, pp. 67ff); cf. RFM I §119ff.

thought, in a "mechanical" manner. By means of this move, Turing made the idea of logic ordinary, rather than sublime (cf. PI §89).

Historians have been long puzzled about how Turing put his analysis together so quickly in 1935-6. But, as I have argued elsewhere, the likeliest proximal inspiration for Turing's move are the rules construed as tables in *The Brown Book* (BBB §§41ff; cf. Floyd, 2017). There is a specific point where Wittgenstein reaches for the idea of what he calls "*general training*" with tables, i.e., what it is to train a human being to use *any such* table. This idea, of *any such* table, crystallizes Turing's sophisticated, revolutionary move. The important point about this, for our purposes, is expressed in Wittgenstein's later remark: "Turing's 'Machines'. These are *humans* who calculate" (RPP I §1096).

This is something, Wittgenstein points out in the *Investigations*, that we tend to forget:

> 194. When does one have the thought that a machine already contains its possible movements in some mysterious way? — Well, when one is doing philosophy. And what lures us into thinking that? The kind of way in which we talk about the machine. We say, for example, that the machine has (possesses) such-and-such possibilities of movement; we speak of an ideally rigid machine which can move only thus-and-so.

We are "lured", says Wittgenstein, into thinking about logic — at least the step-by-step-reckoning-according-to-a-rule part of it — as a *super*-machine, lured into superstitious ideas about logic by our *own* words, our *own* ways of talking about its "possibilities of movement". We shift the locus of modality from us onto our model, the machine, and away from what we human beings do and feel and need — as if what belongs to logic, and more generally following rules, goes on anyway, quite apart from what we do in writing things down, making demands of one another, parametrizing procedures, seeing and establishing measures of similarities and differences, and so on.

Wittgenstein's point is not to criticize idealization as such. Nor is he simply rejecting a Platonic picture or realism in general. Much less does he have in mind rejecting logic or the idea of reckoning-according-to-a-rule, or the use of machines to offload laborious human tasks.[13] At this particular place in the *Investigations* he is scrutinizing a human tendency of projection, reflected in certain ways of conceiving logic's role. That tendency to lean on the algorithm as such, as if it is *ultimately* neutral, is all around us now, and we need philosophy to develop a self-conscious response to it.

It is a signal theme of Wittgenstein's work as a whole to stress the importance for logic and philosophy of seeing a situation (a step, a life, an inference, a world, a use of a word or even a click of a mouse) as a realization of one *possibility* among many various ones, rather than taking philosophy to be describing what it is that happens actually to be. The point is to break the hold of false necessities, false senses of order, false ideas about how logic and necessity evince themselves in our lives. PI §194 continues by pursuing this line:

> —— The *possibility* of movement — what is it? It is not the *movement*, but it does not

[13] Wittgenstein was perfectly aware, from his study of Reuleaux while a student, that machine-pictures could be used modularly, to illustrate the step-by-step workings of a machine, and also create illusions of workings as well. See (Reuleaux, 1876/2012) and (Wilson, 1997) for a (critical) analysis of the impact on the rule-following remarks. Unlike Wilson, I do not take Wittgenstein's interest in Goethe to bespeak a lack of hard-headedness about philosophy.

> seem to be a mere physical condition for moving either — such as there being play between socket and pin, the pin's not fitting too tight in the socket. For while this is empirically a condition for movement, one could also imagine things to be otherwise. The possibility of a movement is supposed, rather, to be like a shadow of the movement itself. But do you know of any such shadow? And by a shadow I do not mean some picture of the movement — for such a picture would not have to be a picture of just *this* movement. But the possibility of this movement must be the possibility of just this movement. (See how high the seas of language run here!).

Here are seas of words, ending in a tautology about *this* possibility being just *this* one. As elsewhere in the *Investigations*, we are shown the emergence of our deep need for *exemplary* necessity, a willingness to take a step we regard as something taken *together*, to accept words *as* used in an exemplary or paradigmatic manner, a particular possibility *as* showing itself to us.

The "shadow" is the idea of an extension: a full-blown set of items, unordered but simply grouped-as-such, to which a concept or rule or algorithm "applies" of itself. Every item "in" or "out", period — no movement at all, no standards of correctness, no room, in particular, for *our* uses of words to characterize it. PI § 194 continues:

> The waves subside as soon as we ask ourselves: how do we use the phrase "possibility of movement" when we are talking about a given machine?

We must return, in other words, to our own lives with words, taking up a partial point of view, one in which we have fixed on an example, and can explore. Wittgenstein, like Turing, is not refuting or contradicting the extensional point of view, but instead insisting that it cannot be regarded as fundamental for all intents and purposes.

The extensional conception is familiar in logic and mathematics, and an appropriate way of thinking for certain contexts, particularly those in which we wish to reason about infinite collections which cannot be represented *by* a particular rule. Irrational numbers provide a halfway house: they may be characterized by rules, but exhibit irregularly patterned features of procedures, differing from the procedures we use with rational and whole numbers.

To fix on an example, let us use the following decimal expansion to "picture" π:

3.141592653589793 …

We may regard this – as mathematicians often do -- as a procedural (idealized) *expansion* according to a rule. But we may also regard it differently, as picturing a finished *extension* "as if already fully written down".[14]

As if: a fiction, or myth, of a procedure. On my view Wittgenstein does not think the extensional point of view is *wrong*: it is of course very natural, indeed needed ("necessary" in that sense). But we have a tendency to transform its way of seeing things into a "mythology" (PI §221), one which *occludes* the importance of our ordinary rags-and-dust procedures with numbers, concepts, words, forms of life — our *movements* with signs, our writings down and takings in. In fact it is the ultimate *point* of the extensional perspective to *make* what we do, our limited procedures and modes of conceptualization and characterization — even our highly

[14] For an extensive discussion, see (Floyd and Mühlhölzer, forthcoming).

circumscribed, mechanical ones — *irrelevant*.

We *should* take up the extensional perspective for certain purposes — if, for example, we are to reason about actually infinite objects in mathematics, or say in general what a real number *is*. But using it, we *erase* the idea of a procedure, picturing it completed and done. But then nothing *is* done: what we have is no *action*, no *movement* at all. There is no room for movement, action or imagination here, these are all *eclipsed* by the perspective. And yet the tendency to fall back upon the language of "imagining" (which brings back the idea of movement) may be seen creeping its way nearly inevitably into our talk about concepts in logic. Erasing movement is difficult, conceptually.[15] Yet it is a necessity for the extensional point of view, taken pure – and so wrongly. – In fact set theory is shot through and through with procedures and ways of thinking that depart from this purity.

The fact that we can take up the extensional perspective and develop it is one thing, the status of the perspective itself another. For this perspective does not and cannot erase or make irrelevant the interest of our particular, local procedures — how for example it is that we become acquainted with, show someone, write down, help someone to see the face of, a particular real number such as π — or fail to. These activities, embedded in proofs and in pictorial, heuristic devices are, in fact, leaned on, and needed for the interest of the extensional perspective itself (Floyd and Mühlhölzer, forthcoming).

We tend to get things backward, in a quasi-Hegelian reverse. If we regard our particular activities as "shadowed" by the extension, then we still must see *them* as shadowed, elongated, shorn of their inner workings, eclipsed like the moon can be. But then what we do is, still, like the moon, the embodied source of the shadow. To be shadowed is to be. A shadow is no model or picture of what it is the shadow of, for it does not allow us to recognize the face, the specific features, the full range of possibilities, of what it shadows.

If we wish to talk about the "possibilities of movement" in logic or reckoning-according-to-a-rule, there is no getting past the *exemplification* of particular possibilities in our ways of reckoning. That means thinking about the human, end-user perspective: processes, procedures, and steps of reckoning that are offered and accepted, worked with, disputed, and so on: spoken checks made with spoken words. (Wittgenstein once wrote, "The role of the sentence 'I must have miscalculated' is really the key to understanding the 'foundations' of mathematics" (RFM III §90)).

In PI §194 Wittgenstein is crediting Turing with developing the idea of a *human* "machine symbolizing its own action/ways of operating/ways of working" (PI §193) into an analysis of logic that breaks the hold over us of "the logical super-machine".

The point is not that we *are* (or are not) machines. Instead, we picture and analogize, and behave within our lives in different ways. As Wittgenstein stresses, it is all too human to suppose that all the sources of meaning and objectivity lie *outside* of us and *inside* the super machine, i.e., the machine that has no movement at all, the algorithm-(or formalism-)-in-itself. Wittgenstein ends PI §194 this way:

> Though we do pay attention to the way we talk about these matters, we don't understand it, but misinterpret it. When we do philosophy, we are like savages, primitive people, who hear the way in which civilized people talk, put a false interpretation on it, and then draw the oddest conclusions from this.

[15] See, e.g., (Williamson 2013).

The image of sailing upon waters, of adventures to meet "alien" ways of thinking that we think of as "primitive", but turn out to be really our own, is as old as the *Odyssey* and as recent as *Walden*. Self-knowledge requires a journey. Driven to the image of seas of language by thinking through the power of the idea of a Turing machine, Wittgenstein is warning us. Our responses to Turing's "machines" are likely to be "primitive" and "uncivilized".

And so it surely is today, as such machines continue to pervasively penetrate everyday life. We imagine that a "super-machine" will take care of handling "fake news"; will care for our parents; will enable us to construct "the Moral Machine" through checklists and surveys of Trolley Problems (cf. http://moralmachine.mit.edu/). We get things exactly backwards, failing to see human beings as the beginning and the end of it all, the *true* "primitive" in the logician's foundational sense. We read the "Turing Test" as a contest between human and machine, rather than seeing it as a human-to-human search for words in the presence of machinery. We forget the social world, the importance of shareable commands and inviting questions. We place significance in the wrong place, in the wrong way. We forget our own plasticities.[16] We forget how it is that human time-scales and concepts may be prejudicial and illusory;[17] we forget that our words and histories require collaboration, diversity of interpretation, and communication.[18]

The sea is inviting, nourishing, gloriously unfathomable, irresistibly beautiful, constantly changing, drawing on our sense of sublimity. Yet it is also filled with danger, drawing out our most human drives to dominate, enslave, pollute, isolate and exhaust ourselves in efforts of mastery.

Being a New Englander, I must recall *Moby Dick* (Melville, 1851/2015, Chapter 1, "Loomings"):

> With a philosophical flourish Cato throws himself upon his sword; I quietly take to the ship. There is nothing surprising in this. If they but knew it, almost all men in their degree, some time or other, cherish very nearly the same feelings towards the ocean with me.
>
> There now is your insular city of the Manhattoes, belted round by wharves as Indian isles by coral reefs — commerce surrounds it with her surf. Right and left, the streets take you waterward. Its extreme downtown is the battery, where that noble mole is washed by waves, and cooled by breezes, which a few hours previous were out of sight of land. Look at the crowds of water-gazers there.
>
> Circumnambulate the city of a dreamy Sabbath afternoon. Go from Corlears Hook to Coenties Slip, and from thence, by Whitehall, northward. What do you see? —Posted like silent sentinels all around the town, stand thousands upon thousands of mortal men fixed in ocean reveries. Some leaning against the spiles; some seated upon the pier-heads; some looking over the bulwarks of ships from China; some high aloft in the rigging, as if striving

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[16] Cf. BU Mellon Sawyer Seminars: https://www.mellonphilemerge.com/past-events/face-off-human-plasticity-and-human-machine-interface, https://www.mellonphilemerge.com/past-events/human-plasticity-and-human-machine-interface, and https://www.mellonphilemerge.com/past-events/day-of-apparatgeist, accessed 23 February 2019.

[17] Cf. BU Mellon Sawyer Seminars: https://www.mellonphilemerge.com/past-events/accountability-in-an-age-of-algorithms-how-should-ethics-and-technology-converse, and https://www.mellonphilemerge.com/past-events/grappling-with-the-futures-insights-from-philosophy-history-and-science-technology-and-society, accessed 23 February 2019.

[18] Cf. BU Mellon Sawyer Seminars: https://www.mellonphilemerge.com/past-events/philosophical-platforms-arendt-nietzsche-wittgenstein and https://www.mellonphilemerge.com/past-events/open-access-and-research-into-history-issues-of-copyright, accessed 23 February 2019.

to get a still better seaward peep. But these are all landsmen; of week days pent up in lath and plaster — tied to counters, nailed to benches, clinched to desks. How then is this? Are the green fields gone? What do they here?

But look! here come more crowds, pacing straight for the water, and seemingly bound for a dive. Strange! Nothing will content them but the extremest limit of the land; loitering under the shady lee of yonder warehouses will not suffice. No. They must get just as nigh the water as they possibly can without falling…

… Why did the old Persians hold the sea holy? Why did the Greeks give it a separate deity, and own brother of Jove? Surely all this is not without meaning. And still deeper the meaning of that story of Narcissus, who because he could not grasp the tormenting, mild image he saw in the fountain, plunged into it and was drowned. But that same image, we ourselves see in all rivers and oceans. It is the image of the ungraspable phantom of life; and this is the key to it all.

Walden too situates itself with respect to a body of water. It is a place now badly distressed by human overuse, underused as a source of reflection.[19]

The web is larger than us, but fragile like a pond or sea. In the end, it is our words in the presence of one another, face-to-face in lived conversation, on which the good or evil of the whole will depend. Our conversations, our mutual education of one another in everyday life, are like the tiny crustaceans on whose lives the entire functioning of the ecosystem rests. Without attending to them, the whole will collapse or drown us.

In fact, this is an old idea, fundamental to the whole idea of education as *Bildung*: structured life-paths, the search to find our way through *forms*, i.e., possibilities of structuring, individual and collective human life with words. In our world, forms of symbolism will increasingly matter; the issues of voice, stance, and care will be central. Meaningful conversation will become more and more important to us, and perhaps rarer, in any case more condensed, pressured. Sherry Turkle has begun to document the rapid decline in measures of empathy among children who are the first to have been raised from birth in the presence of mobile technology (Turkle, 2015). Others have documented increased levels of loneliness and divorce among social media users in the United States (Valanzuela, Halpern and Katz, 2014).

It is time for reflection, for all of us: data sets that categorize our behavior cannot replace the end-user's stance and sense of self.

3. Searching

Searching is precisely the point where the philosophical conceptions of Turing and Wittgenstein intersected. "Intelligence", both for Wittgenstein and for Turing, is what Turing explicitly called an "emotional" concept — i.e., it is response-dependent.[20] It consists,

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[19] However, Marshall Medoff, a local Massachusetts "amateur", was inspired by his visits to *Walden* to dedicate himself for a decade to a reclusive life of research on behalf of the idea of working with, rather than overcoming, the environment. Edison-like, he discovered the secret of how to extract sugars from plants to create clean biofuel that can run cars made today, as well as sweeteners that will not decay teeth (https://www.cbsnews.com/news/marshall-medoff-the-unlikely-eccentric-inventor-turning-inedible-plant-life-into-fuel-60-minutes/, accessed 23 February 2019).

[20] Cf. BU Mellon Sawyer Seminar at: https://www.mellonphilemerge.com/past-events/turing-s-imitation-game-what-does-it-tell-us-about-the-nature-of-intelligence, accessed 23 February 2019, (Proudfoot 2013).

ultimately, in appreciating the *Fragestellungen* of questions, the different kinds of question-"placings" with which one may engage (cf. the Preface to Wittgenstein's *Tractatus*). The theme of transposing the terms of a question in order to resolve it is, of course, a constant in Wittgenstein's writing throughout his life: he often analogizes difficult conceptual (mathematical, philosophical) questions to riddles, where the terms of the question must be interrogated and transposed in order for a solution to be found.[21] This idea makes itself felt through the literary strategies of *Philosophical Investigations*, where questions are rounded upon and recast over and over again.

It is also salient in Turing's logico-philosophical work. Some of our searches are routine, even algorithmic. Others are — and sometimes provably — not. Turing's earliest intellectual *coup* was resolving the "Decision Problem", Hilbert's *Enscheidungsproblem*, in the negative. This is the problem of whether or not there is an algorithm — a routine a machine might compute — to determine, for any sentence, whether or not it follows from a collection of other sentences.[22] If there were such a "Decision Procedure", then truth itself, insofar as it can be traced in deductive argument (e.g., much of mathematics) would be mechanizable, and so vanish as we know it.[23]

G.H. Hardy made fun of the idea of a general Decision Procedure, writing that because of the psychological and verbal "gas" surrounding the teaching and learning of mathematics, "it is only the very unsophisticated outsider who imagines that mathematicians make discoveries by turning the handle of some miraculous machine" (Hardy 1929, p. 18). But in *The Brown Book* Wittgenstein took the idea of using humans *as* "machines" seriously. And with Wittgenstein's help, Turing came to transpose what had been a *mathematician's* problem (worked at in the Hilbert school thorough the development of formal systems and "effective" systems of equations) into a *philosophical* one.

For Turing showed the intrinsic necessity of a *variety* of search strategies precisely by *domesticating* Hilbert's idea of "the logical 'machine'". He began with a "language-game" in Wittgenstein's sense: a snapshot of a human being used as a "calculating machine", given paper and pencil and trained to reckon a series of digits and discrete tasks without thinking. This activity takes place in a social world. It is *not* encoded in any kind of functionalist-cum-mechanist psychological series of brain states. As Turing explicitly wrote,

> We avoid introducing the notion of a 'state of mind' by considering a more physical and definite counterpart: it is always possible for the computor to break off from his work, to go away and forget all about it, and later to come back and go on with it. If he does this he must leave a note of instructions (written in standard form) explaining how the work is to be continued [by a co-worker]. This note is the counterpart of the 'state of mind' (1936/7, p. 263).

The heart of the argument — just as in *The Brown Book* — is reflection on shareable, humanly

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[21] On riddles cf. (Diamond, 1991), (Floyd, 2007).

[22] Ironically, perhaps, Wittgenstein had first framed this decision problem in terms of the notion of a "tautology" in a letter to Russell, in 1913 (Dreben and Floyd, 1991). This is one reason to suspect that Turing's work resolving it would have been of vivid interest to him.

[23] Derek Jacobi's incomparable performance in the film *Breaking the Code* conveys Turing's down-to-earth matter-of-factness about this question, as he says to the incredulous officer at Bletchley, "It's a matter of true and false, really, right and wrong".

intelligible, command structures.

By means of this snapshot model Turing was able to analyze, philosophically, what a "step" in a formal system of logic, or in an algorithm in general *is*. He made the notion of a formal system *plain*, "home-spun" [*hausgebacken*] -- to use Wittgenstein's phrase — by embedding it in human life (cf. Wittgenstein 1998-2000; 2015, MS 152, p. 196; Floyd, forthcoming-b).

To show negatively that there is *no* general decision procedure, one must analyze what in general a formalism or algorithm or effective procedure *is*. It will not do to write down another formal system. Instead, one has to *do* something conceptual, something philosophical: interrogate and reconceptualize the terms of the riddle.

Turing assumed, in accordance with his language-game, that ordinarily human calculators will go on indefinitely, proceeding in a "circle free" way, calculating digit after digit. However, if we imagine one overarching "machine" that can decide, for every "machine", whether or not it is circle-free, then we can construct by means of it another "machine" that is circular, and so ill-defined. This purported "machine" would calculate along without mishap, until it runs into the command telling it what to do for itself. And this command, necessarily, is empty, tautologous, meaninglessly private. It is analogous, as I have argued elsewhere, to drawing a card in a game that says, "Do What You Do" (Floyd, 2012; Floyd 2017). It is a "command" that cannot be *followed*. Like Narcissus, it falls into the pool, undoing itself *as* a "machine", and showing us that the limits of formal logic lie, not in worries about contradicting ourselves, but in our own entanglements with self-picturing, self-commanding, self-spontaneity, and creativity.

For "Do What You Do" is a perfectly good command if we are located in a context in which we understand what we are doing, in which we have the friction of a form of life, one or another purposive activity we are engaged in. But in the context of the Decision Problem we are robbed of this. And then we are sure to run into a command that we *cannot* follow: we can't *do* anything with it. As Wittgenstein later remarked about Turing's argument, a command (i.e., a search) "only makes sense in certain positions" or placed contexts (*Stellen*) (RPP I §1096; cf. Floyd, 2012).

The *general* "Do What You Do" machine fails to *be* an indefinitely calculating "machine": it breaks the picture we began with. This implies, as Turing made clear, that the most fundamental notion in logic is that of a *partial* routine, one that is not everywhere defined. We are, necessarily, forced to sail the seas of inquiry cobbling together our quests bit by bit, weaving words into our lives through choices and discussion and argument with one another. There are simples — beginnings of routines and commands and symbolic rule-following — but these "simples" are *fluid*, ubiquitous and relative, everywhere contestable. We are, as Neurath famously said,

> … like sailors who must rebuild their ship out on the open sea, without ever being able to take it to pieces in dry dock and rebuild it anew from its best parts (Quine, 1960/2013, Epigraph).

In *Philosophical Investigations* "forms of life" are the backdrop to language-games, the place where harmonies among us and the "weave of life" are cobbled together, collected and formed. Wittgenstein's literal term is *Lebensteppich*, a "rug of life" on which we sit, chat, bind, stand, and walk (PI PPF i §2).

But what does all this have to do with the increasingly grandiose claims being made

nowadays for AI? And what of the effects of social media, our sea of words? The answer is that the web is not a context: neither a form of life nor a life world. It is rather a sea of dynamic, evolving word- and image-streams that sometimes run high. It is often conceived, and rightly, as a *flood* of information, rather than a cloud (Gleick, 2011). I have argued that it falls to the human users to embed the word- and image-uses in life as we go.

In closing, I want to revisit the earliest steps Turing took to sketch an answer.

In the founding document of AI, "Intelligent Machinery" (1948), Turing battled against the idea that we already know enough of the concept of "intelligence" to rule out extending our term at some point in the future to artificial devices. As in his later (1950) "Computing Machinery and Intelligence", written for philosophers, he used a logician's move.[24] Because we do *not* have a general analysis of "intelligence", we cannot prove an impossibility (as Turing had done with the Decision Problem).

"Intelligent Machinery" is a report Turing wrote to the National Physical Laboratory just before leaving London for Manchester. Emphasizing "the varieties of machinery", he reported on the ideas of heuristic searching (which cuts down on calculating time by narrowing down searches based on hunches), random networked searching (anticipating connectionism), expert knowledge data bases (based on interviews with human experts). He then turned toward speculation about the long-term future.

First, he argued against the idea of attempting to build a machine that would look and act like a human being:

> One way of setting about our task of building a 'thinking machine' would be to take a man as a whole and to try to replace all the parts of him by machinery. He would include television cameras, microphones, loudspeakers, wheels and 'handling servo-mechanisms' as well as some sort of 'electronic brain'. This would of course be a tremendous undertaking. The object if produced by present techniques would be of immense size, even if the 'brain' part were stationary and controlled the body from a distance. In order that the machine should have a chance of finding things out for itself it should be allowed to roam the country-side, and the danger to the ordinary citizen would be serious (2004, p. 420/2013, p. 508).

One is tempted to think here of driverless cars. Turing continues:

> Moreover even when the facilities mentioned above were provided, the creature would still have no contact with food, sex, sport and many other things of interest to the human being. Thus although this method is probably the 'sure' way of producing a thinking machine it seems to be altogether too slow and impracticable (2004, p. 420/2013, p. 508).

Turing implies here that we have better things to do than to encode into machines abilities at sex, sport, and so on. Here he had not failed to envision the compulsion in contemporary life toward "sex robots": still an outstanding question, whether prosthetic friendships will or will not better satisfy certain human beings for certain purposes.[25]

Next, under the heading of "discipline and initiative", Turing picked up on the

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[24] Incidentally, Wittgenstein never saw Turing's paper, but, hearing about it, told Normal Malcolm that he guessed it would be no "leg-pull". On whether it was or was not, see (Floyd, 2012, p. 27, n. 9).

[25] This will be discussed in an upcoming pair of BU Mellon Sawyer Seminars; see http://sites.bu.edu/emsconf/, accessed 23 February 2019.

(Wittgensteinian) dialectic between constraint-by-rule and creative going-beyond-rules that is part and parcel of our understanding of (human) "mechanical" behavior. He took a philosophical side-tour, speculating that there would be three kinds of search that would become most significant in the future development of computation:

1. *The Intellectual Search* (for algorithms onto which we may offload human routines such as finding numbers with certain properties)

2. *The Genetical or Evolutionary Search* (the criterion of success here is survival — Turing here anticipates the notion of a genetic algorithm, now common in computational biology using DNA)

and finally

3. *The Cultural Search*.

Turing describes "The Cultural Search" in human terms alone: there are no machines in it. What he says is this:

> … [T]he isolated man does not develop any intellectual power. It is necessary for him to be immersed in an environment of other men, whose techniques he absorbs during the first 20 years of his life. He may then perhaps do a little research of his own and make a very few discoveries which are passed on to other men. From this point of view the search for new techniques must be regarded as carried out by the human community as a whole, rather than by individuals (2004, p. 431/2013, p. 516).

Turing's emphasis on *human* communication, self-understanding, and even democracy is explicit.[26] He pictures our co-evolution with machinery as an evolving human conversation searching through a long scale of time, driven forward by a dynamic interaction between human cultural striving, definability of concepts in "phraseology", and technological and biological creation. The echoes of Peirce's conception of values in a universe of chance seem more than accidental. By taking the notion of evolving, communally-secured *intelligence* to be manifested in an evolving variety of interlocking forms of search, Turing is insisting, as he did elsewhere, that the development of the creative and shared "phraseology" of human language, a common sense of "common sense", is crucial to "intelligence". He also, quite remarkably, predicted the levelling of national boundaries in our interlocking networks of communication, now shaping public spaces and behavior on a global human scale. -- Turing brought human *movement* back to the heart of logic.

On this view, self-consciousness *per se* is not the mark of intelligence. Nor is "intelligence" something to be generally defined, much less imitated generally by machines. Instead, it is what Turing calls an "emotional", family resemblance concept: a human, response-dependent notion, like *color* or *freedom*. The mark of "intelligence", Turing says, is appreciation of the variety and nature of different forms of human search:

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[26] On the idea of a "democratic" community in Turing as the need to avoid what he called the "straightjacket" of an overarching formalism, see (Floyd, 2017, p. 141).

> The extent to which we regard something as behaving in an intelligent manner is determined as much by our own state of mind and training as by the properties of the object under consideration. If we are able to explain and predict its behaviour or if there seems to be little underlying plan, we have little temptation to imagine intelligence. With the same object therefore it is possible that one man would consider it as intelligent and another would not; the second man would have found out the rules of its behavior (2004, p. 431/2013, p. 516).

Turing's instinct here is to turn to what Wittgenstein called "language-games", sometimes referred to as "experiments" by Wittgenstein himself, in *The Brown Book*. Sketching a game he would later make famous as the "Turing test", Turing writes:

> It is possible to do a little experiment on these lines, even at the present stage of knowledge. It is not difficult to devise a paper machine which will play a not very bad game of chess. Now get three men as subjects for the experiment A, B, C. A and C are to be rather poor chess players, B is the operator who works the paper machine. (In order that he should be able to work it fairly fast it is advisable that he be both mathematician and chess player.) Two rooms are used with some arrangement for communicating moves, and a game is played between C and either A or the paper machine. C may find it quite difficult to tell which he is playing.
>
> (This is a rather idealized form of an experiment I have actually done.) (2004, p. 431/2013, p. 516).

The point of this test is the interactions among the humans, not solely their interactions with a machine. – At the moment, the largest internet industries are gaming and pornography.

4. Conclusion

Realigning Wittgenstein and Turing on questions about the foundations of logic is important for realigning our sense of reality today. Underneath our current sea are powerful computational processes and data farms, formalized languages and coding techniques that have evolved over time, proof-correction techniques, applications and interweaving of algorithms designed with exquisite delicacy. Each of these involves a tremendous amount of human labor, especially control of reckoning-to-rule behavior. Also a tremendous amount of raw materials and ecological stress, as "rare earths" are utilized to construct the mobile devices that are for so many of us our constant, increasingly powerfully condensed, companions. (There are at the moment more cell phones than humans on the planet.) In this situation, the labor of the humanities, of education amongst us as conversation, must not be forgotten or lost.

We are only at the beginning. The idea of embedding mobile technology in everyday life and developing an "internet of things" is gaining reality. Imbuing our present sea are powerful forces of capitalism: ever-faster task-managed exchanges, dark webs of desire and criminality and fun, cognitive psychological profiling, confessional sharing, institutional needs and a bias toward quantitative measurement over qualitative judgment, not to mention commercially- and politically-driven manipulation of our desires, as cognitive avatars of our consuming selves are constructed through every recorded click we make. Social profilings are given to us increasingly by governmental authorities, who are beginning to utilize genetic mapping to control and divide us. What "free speech" means, and what "censorship", have become cloudy issues. And yet there

are also forces of activism, new ideas about activism's role in connecting otherwise unconnected people.[27]

Our seas of language will get rougher before they calm. Insofar as we aspire to sail the waves while retaining some sense of true democratic community and tradition, we must not forget that permeating every feature our uses of machines — insofar as we are not mere parrots, but make claims — is our social, evolving world of forms of lives with words. We are sailors on the seas of language. As has been urged by Paul Standish, *via* a masterful reading of *Coriolanus*, the body politic requires education that will allow learners to see and speak for themselves; to allow them to find significance in their lives of learning; to have a "voice" in their searches and the satisfactions they find together.[28]

The very concept of "information", demarcated and so artfully titrated by Turing presupposes a world of activities in which symbolic structures mean, people care, requests and commands are intelligible, and spontaneity plays a role. It follows, just as Turing predicted, that in the end the quest for culture, i.e., the humanities and philosophy, are to become more and more important. "Information" cannot be properly used as if it is a factive idea, for it is neutral as to truth and falsity, neutral as to its purposes without the enrichments of place, stance and context. In the end, then, algorithms are not *ultimately* neutral at all. As is becoming increasingly evident in journalism, -- which more and more encroaches on each of us as social media users -- the *struggle* and *search* for truth in all its aspects will continue, a very human thing.[29]

We need to take the humanities seriously in this computationally-situated world. It is not an add-on, it is a foundation. New ways of thinking, speaking, and conceptualizing what "progress" in education, self-understanding, the objects of our knowledge and desire and self-comprehension really are – these are desperately needed. In the end, it will be our qualitative grapplings with the seas of our language, with the search for what it is to *become* someone in life through questioning and searching — i.e., philosophy – that will determine, in great part, our forms of life.[30]

Bibliography

Cavell, S. (1979/1999) *The Claim of Reason: Wittgenstein, Skepticism, Morality, and Tragedy*, updated from the original edition of 1979 (New York: Oxford University Press).

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[27] Cf. BU Mellon Sawyer Seminar at https://www.mellonphilemerge.com/past-events/digitizing-human-rights-archiving-activism, accessed 23 February 2019; compare (Groshek and Tandoc, 2017).

[28] (Standish, 2005); cf. (Laugier, 2015) also on forms of life as "voice", and compare BU Mellon Sawyer Seminar at: https://www.mellonphilemerge.com/past-events/philosophy-of-popular-culture-skepticism-care-and-ordinary-life accessed 23 Feb. 2019.

[29] Cf. BU Mellon Sawyer Seminar at : https://www.mellonphilemerge.com/past-events/journalism-and-the-search-for-truth-in-an-age-of-social-media accessed 23 Feb. 2019, (Katz 2011, Floyd, forthcoming-a).

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Cavell, S. (1969/2002) *Must we mean what we say?: A Book of Essays*, updated from original 1969 edition (Cambridge, UK: Cambridge University Press).
Cavell, S. (2005) Passionate and Performative Utterance: Morals of an Encounter, in: S. Cavell and R.B. Goodman (eds.) *Contending with Stanley Cavell*, (Oxford University Press), pp. 177-198.
Diamond, C. (1991) *The Realistic Spirit: Wittgenstein, Philosophy, and the Mind* (Cambridge, MA.: MIT Press).
Dreben, B., and Floyd, J. (1991) Tautology: How Not to Use A Word, *Synthese,* 87, p. 23-50.
Floyd, J. (2007) Wittgenstein and the Inexpressible, in: A. Crary (ed.) *Wittgenstein and the Moral Life*, (Cambridge, MA: M.I.T. Press), pp. 177-234.
Floyd, J. (2012) Wittgenstein's Diagonal Argument: A Variation on Cantor and Turing, in: P. Dybjer, S. Lindström, E. Palmgren, and G. Sundholm (eds.) *Epistemology versus Ontology, Logic, Epistemology: Essays in Honor of Per Martin-Löf* (Dordrecht: Springer Science+Business Media), pp. 25-44.
Floyd, J. (2013) Turing, Wittgenstein and Types: Philosophical Aspects of Turing's 'The Reform of Mathematical Notation' (1944-5), In: Turing (2013), pp. 250-253.
Floyd, J. (2016) Chains of Life: Turing, *Lebensform*, and the Emergence of Wittgenstein's Later Style, *Nordic Wittgenstein Review* 5, p. 7-89.
Floyd, J. (2017) Turing on "Common Sense": Cambridge Resonances, in: J. Floyd and A. Bokulich (eds.) *Philosophical Explorations of the Legacy of Alan Turing --- Turing 100* (Dordrecht: Springer), pp. 103-152.
Floyd, J. (2018a) Aspects of Aspects, In: H. Sluga and D. Stern (eds.), *The Cambridge Companion to Wittgenstein*, 2nd edition (Cambridge: Cambridge University Press), pp. 361-388.
Floyd, J. (2018b) Lebensformen: Living Logic, in: C. Martin (ed.) Language, Form(s) of Life, *and Logic: Investigations after Wittgenstein* (Berlin: deGruyter), pp. 59-92.
Floyd, J. (forthcoming-a) "The True" in Journalism, in: J.E. Katz (ed.) *Journalism and the Search for Truth* (New York: Oxford University Press).
Floyd, J. (forthcoming-b) Wittgenstein on Ethics: Working Through *Lebensformen*. *Philosophy and Social Criticism*.
Floyd, J. and Katz, J.E. (eds.) (2016) *Philosophy of Emerging Media: Understanding, Appreciation, Application* (New York: Oxford University Press).
Floyd, J., and Mühlhölzer, F. (forthcoming). *Wittgenstein on the Real Numbers: Annotations to Hardy's 'A Course of Pure Mathematics'* (Springer).
Fortunati, L., Katz, J.E., and Riccini, R. (eds.) (2003) *Mediating the Human Body: Technology, Communication and Fashion* (Mahwah, New Jersey: Lawrence Erlbaum Associates).
Gleick, J. (2011) *The Information: a History, a Theory, a Flood* (New York: Pantheon Books).
Groshek, J., and Tandoc, E. (2017) The affordance effect: Gatekeeping and (non)reciprocal journalism on Twitter, *Computers in Human Behavior* 66, p. 201-210.
Hardy, G.H. (1929) Mathematical Proof, *Mind* 38, p. 1-25.
Katz, J.E. (ed.) (2003) *Machines That Become Us: The Social Context of Personal Communication Technology* (New Brunswick, NJ: Transaction Publishers).
Katz, J.E. (2008) *Handbook of Mobile Communication Studies* (Cambridge, MA: MIT Press).
Katz, J.E. (2011) Communication perspectives on social networking and citizen journalism challenges to traditional newspapers, *Periodica Polytechnica: Social and Management Sciences* 19, p. 51-62.

Katz, J.E. (2014) *Living Inside Mobile Social Information* (Dayton, OH: Greyden Press, LLC).
Katz, J.E., and Aakhus, M.A. (eds.) (2004) *Perpetual Contact: Mobile Communication, Private Talk, Public Performance* (Cambridge, U.K.: Cambridge University Press).
Kripke, S. A. (1982) *Wittgenstein on Rules and Private Language: An Elementary Exposition* (Cambridge, MA: Harvard. University Press).
Laugier, S. (2015) Voice as Form of Life and Life Form, *Nordic Wittgenstein Review* 4 October, p. 63-81.
Melville, H. (1851/2015), *Moby Dick*, Centaur Edition, online on Kindle through Amazon.
Monk, Ray (1990) *Ludwig Wittgenstein: The Duty of Genius* (New York/London: Free Press/Jonathan Cape (Reprint, New York: Penguin Books (1991)).
Post, E.L. (1936) Finite combinatory processes‐Formulation I, *Journal of Symbolic Logic* 1, p. 103‐105.
Proudfoot, D. (2013) Rethinking Turing's Test, *Journal of Philosophy* 110, p. 391-411.
Putnam, H. (1988) *Representation and Reality* (Cambridge, Mass., MIT Press).
Quine, W.V. (1960/2013) *Word and Object*, New edition (Cambridge, Mass.: MIT Press).
Reuleaux, F. (1876/2012) *The Kinematics of Machinery: Outline of a Theory of Machines*, Republication edition (New York: Dover Publications).
Standish, P. (2005) Democratic Participation and the Body Politic, *Educational Theory* 55, 4, p. 370-384.
Thoreau, Henry David (1854/2011), *Walden, or Life in the Woods*, originally published in1854 (Boston: Ticknor and Fields); Project Gutenburg e-book, produced by J. Boss and D. Widger, open source at http://www.gutenberg.org/ebooks/205.
Turing, A.M. (1936/7) On Computable Numbers, with an Application to the Decision Problem. *Proceedings of the London Mathematical Society* 2, 42, p. 230-265; corrections *Proceedings of the London Mathematical Society* 2, 43, p. 544-6. Reprinted in Turing (2004), pp. 58-90 and in Turing (2013), pp. 16-43; references to original edition.
Turing, A.M. (1944) The Reform of Mathematical Notation and Phraseology, In: Turing (2013), pp. 245-249.

Turing, A.M. (1948) *Intelligent Machinery*, a Report Written for the National Physical Laboratory, in: Turing (2004), pp. 410-432 and Turing (2013), pp. 501-516; references to each are given with a "/" between.
Turing, A.M. (1950) Computing Machinery and Intelligence, *Mind* 59, p. 433-460. Reprinted in Turing (2004), pp. 441-464 and in Turing (2013), pp. 552-568.
Turing, A.M. (2004) *The Essential Turing: Seminal Writings in Computing, Logic, Philosophy, Artificial Intelligence, and Artificial Life*, Copeland, B.J. (ed.). Turing's seminal papers edited with commentary and exposition (Oxford: Clarendon Press).
Turing, A.M. (2013), *Alan Turing – His Work and Impact*, papers of Turing with commentary by Experts, S.B. Cooper, and J. van Leeuwen (eds.) (Amsterdam/Burlington, MA: Elsevier).
Turkle, S. (2015) *Reclaiming Conversation: The Power of Talk in a Digital Age* (New York: Penguin Books).
Valenzuela, S., Halpern, D., and Katz, J.E. (2014) Social network sites, marriage well-being and divorce: survey and state-level evidence from the United States, *Computers in Human Behavior* 36, p. 94-401.
Williamson, T. (2013). How Deep is the Distinction between A Priori and A Posteriori Knowledge?, In: A. Casullo, and J.C. Thurow (eds.) *The A Priori in Philosophy* (New

York: Oxford University Press), pp. 291-312.
Wilson, M. (1997) Wittgenstein: *Physica Sunt, Non Leguntur*, *Philosophical Topics* 25, 289-316.
Wittgenstein, L. (1966) *Lectures and Conversations on Aesthetics, Psychology and Religious Belief*, C. Barrett (ed.) (Berkeley, CA: University of California Press). (LA)
Wittgenstein, L. (1969) *Preliminary Studies for the 'Philosophical Investigations': Generally Known as the Blue and Brown Books*, 2nd edition, 1st edition in 1958 (Oxford: Basil Blackwell). (BBB)
Wittgenstein, L. (1978) *Remarks on the Foundations of Mathematics*, G. H. von Wright, R. Rhees, and G. E. M. Anscombe (eds.), G. E. M. Anscombe (trans.), revised 2nd edition; 1st edition 1956 (Cambridge, Mass: M. I. T. Press). (RFM)
Wittgenstein, L. (1921/1981). *Tractatus Logico-Philosophicus*, C.K. Ogden (trans.) (London/New York: Routledge & Kegan Paul). 1st German edition in *Annalen der Naturphilosophie*, Wilhelm Ostwald (ed.), 14, Leipzig: 1921, pp. 12-262; second impression with a few corrections, 1931. Republished 1981. (TLP)
Wittgenstein, L. (1980) *Remarks on the Philosophy of Psychology Vol 1*, G.E.M. Anscombe and G. H. von Wright (eds.), G.E.M. Anscombe (trans.) (Oxford: Blackwell). (RPP I)
Wittgenstein, L. (1989), *Lectures on the Foundations of Mathematics, Cambridge 1939*, from the notes of R.G. Bosanquet, N. Malcolm, R. Rhees and Y. Smythies, C. Diamond (ed.) (Chicago: University of Chicago Press). (LFM)
Wittgenstein, L. (1998-2000; 2015) *Wittgenstein's Nachlass The Bergen Electronic Edition* Wittgenstein-Archive, University Bergen, under the direction of C. von Huitfeldt (Oxford: Oxford University Press/Charlottesville, VA: Intelex); Open Source: *Wittgenstein Source Bergen Nachlass Edition*, Wittgenstein Archives at the University of Bergen under the direction of A. Pichler (ed.), at www.wittgensteinsource.org.
Wittgenstein, L. (2009) *Philosophische Untersuchungen = Philosophical Investigations*, revised 4th edition, G.E.M. Anscombe, P.M.S. Hacker and J. Schulte (eds. and trans.) (Chichester, West Sussex, U.K.: Wiley-Blackwell).