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OPPORTUNITIES CREATED BY MISDIRECTION IN MATHEMATICS LESSONS

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This study provides evidence that enacted lessons based on written curriculum do not need to follow a direct curricular path from beginning to end. Deviations in this path, which we refer to as misdirection, can create opportunities for enhanced student interest in mathematical questions. We present three examples of misdirection from two enacted lessons and describe how they intensified student investment in the mathematics by creating contradictions that inspired students to ask their own mathematical questions. These examples can serve as models to teachers and curriculum writers who seek ways to motivate students to pursue mathematical understanding.

Keywords: Curriculum, Curriculum Analysis, Affect, Emotion, Beliefs, and Attitudes

It is often assumed that the role of curriculum is to provide students with a direct path from what they already know to what they need to know (Tyler, 1949). The recent focus on spatial metaphors of curriculum, such as a trajectory (e.g., Clements & Sarama, 2004; Gravemeijer, Bowers, & Stephan, 2003; Simon, 1995) is consistent with this assumption. This increasingly popular metaphor draws attention to an initial point, described as the “teachers’ hypotheses about the students’ mathematical knowledge” (Simon, 1995, p. 138), and a target (i.e., a learning goal). This perspective highlights the instantaneous decisions of direction that teachers make as they guide their classes along anticipated “learning routes” toward a target and adjust instruction in order “get back on track” when necessary (Gravemeijer et al., 2003, p. 55).

Yet, in our analyses of mathematics curricula as they unfold in classrooms, we regularly see “learning routes” that are not linear and that do not progress smoothly. In fact, we have found some paths that appear to change direction, double back on themselves, or skew away from the target. We refer to these indirect paths as instances of “misdirection.” Rather than conceptualize these paths as a problem with curriculum design, we instead recognize misdirection as potentially beneficial and generative for learning. Understanding these moments of misdirection can inform the crafting of mathematics curricula (written and enacted) that stimulate student curiosity. In this paper, we describe moments in mathematics classrooms during which obstructions toward the mathematical goal of the lesson motivated students to become invested in the pursuit of mathematical understanding. Our goal is to use these analyses to complicate curricular assumptions and suggest a new curricular metaphor that offers insight into this complexity.

Theoretical Framework

The complex ways that information can unfold across a sequence have been studied in narrative by literary theorists since Aristotle. Conceptualizing mathematics lessons as *mathematical stories* allows us to capitalize on this work. With this framing, we gain new analytic tools to interpret the way mathematical ideas emerge and unfold across a temporal sequence from beginning to end (Dietiker, 2015). This framework is not a focus on contextual story problems, but interprets all of the mathematical content of a lesson as a sequence of revelations. Although mathematical stories can be recognized in written mathematics curriculum materials (i.e., the intended curriculum), our focus is on the mathematical stories that are found in the complex realm of the classroom (i.e., the enacted

curriculum). In this framework, the students and teachers are the actors in enacted mathematical stories.

Literary stories engage an audience by provoking questions and then slowly revealing the answers in ways that both inform and entertain (Nodelman & Reimer, 2003). Stories that contain twists in the plot misdirect a reader into answering questions incorrectly, setting up opportunities for surprise when their predictions are later shown to be false. Framing mathematics curriculum as a story enables an analyst to understand how misdirection can occur. This framing also helps to illuminate the logical and aesthetic roles that misdirection can play in the experience of the students.

To analyze the narrative structure in mathematical stories, we employ the hermeneutic codes developed by Roland Barthes (1974). These codes were designed to describe the way in which literary plots unfold. With these codes, the emergence and progress made for any mathematical questions of a lesson are tracked, allowing for the identification of misdirection that occurs along the way as well as the factors that enabled the misdirection. Three codes in particular relate to misdirection -- equivocation, snare and jamming.

An *equivocation* describes a moment in a story when a reader can identify, upon reflection, that she or he was misled through ambiguity. The narrator of the story does not overtly lie, as the information presented is technically accurate. However, the telling of the story allows or arranges for a misunderstanding to occur. A *snare* describes a moment in a story when a reader can later identify, upon reflection, that she or he was explicitly misled. While equivocations are lies by omission, snares are lies by commission. *Jamming* is the interruption of progress on a question in a way that suggests the question cannot or will not be answered.

It is important to note that students, as well as teachers, can be responsible for misdirection in a mathematical story. This conflicts with the common assumption that students in the classroom are solely the audience, not the actors, in the story. While students can be both actor and audience, we analyze the plot of a mathematical story by interpreting the potential mathematical story available to a hypothetical, silent, intellectually engaged learner.

Methods

The data presented in this study are from observations of two algebra classrooms in different regions of the United States using the same inquiry-based algebra curriculum materials (CPM Educational Program). The teachers, whom we will call Ms. Becker and Ms. Wilson, have 20+ and 30+ years of teaching experience respectively, and each has been using CPM materials for at least 5 years. Ms. Becker teaches at a public middle school in the South and Ms. Wilson at a private high school in the Northeast. Each teacher was video and audio recorded while enacting lessons from the same written materials. The three selected lessons focused on: 1) solving systems of equations by substitution, 2) different representations of quadratic functions, and 3) the zero product property.

The videos were transcribed to capture the whole class dialogue as well as the dialogue from one group of students in each classroom as they worked together on assigned problems. From the transcripts, a team of four researchers worked in pairs to identify questions that were formulated by the teacher, students, or textbooks. The two pairs then came together to resolve discrepancies. Next, researchers used Barthes' codes to identify progress toward the answering of the questions from the formulation of each question to its disclosure. With this coding, instances of equivocations, snares, or jamming, defined above, were identified and compared in order to answer the question "How does misdirection occur in high school algebra classrooms and how can its role in the student's experience be explained?"

Findings

Our analysis of these lessons revealed examples of all three types of misdirection. Below, we provide an example of each form of misdirection and explain how each one created surprise when a

contradiction was revealed. We then show how each contradiction led students to ask their own questions about the mathematics, thus increasing their investment in the outcome of the lesson.

Equivocation

An example of equivocation occurred in the first part of a lesson in Ms. Wilson's class on identifying the roots of a quadratic equation using the zero product property. The goal of this part of this lesson was for students to understand that three non-collinear points are required to uniquely identify a parabola. In order to introduce this idea, Ms. Wilson engaged the students in three rounds of a "guess my parabola" game. In each round, the teacher thought of a parabola, gave them information about that parabola and challenged them to guess her parabola. In the first round she gave them the y -intercept, in the second round she gave the two x -intercepts and in the third round she gave the y and both x -intercepts. The equivocation in this sequence of activities was the assumption from the beginning of the game that it was reasonable for someone to determine her parabolas with only one or two points. This was misleading in the first two rounds because there were an infinite number of parabolas that fit these conditions.

Rather than simply revealing to the students at the beginning of the activity that knowing three points of a parabola would uniquely identify it, this teacher created the expectation that it might take fewer. Students were then surprised when they discovered that it is unlikely for them to guess her parabola with just one or two points. This surprise gave way to curiosity and then various levels of understanding as students realized why they were having so much difficulty. Student investment in the mathematical questions was evident by the energy with which they expressed their realizations and how hard they pressed the teacher for more clues, and eventually the answer.

Snare

A dramatic example of what can happen as the result of a snare occurred in another of Ms. Wilson's classes during a lesson on different representations of quadratic functions. Students were trying to graph the parabolic path of a water balloon from an equation. One student made a calculation error that resulted in a large negative value for y , suggesting that after the balloon was launched it was underground. In response to this snare, a second student addressed the contradiction between the expected y -value [i.e., a positive value] and the calculated y -value by creating an inventive interpretation of the situation in the context of the mistake. The absurdity of the story that resulted, involving a bird and a remote control plane, possibly convinced the student to re-evaluate his calculation and thus catch his error. The twist in this mathematical plot, this time a misleading error, inspired these students to construct additional explanatory stories and persevere to resolve the contradiction.

Jamming

One example of jamming occurred in Ms. Becker's lesson on the substitution method for solving systems of equations. At the beginning of this lesson, the class was attempting to solve a system of equations using the previously learned equal values method (i.e. solve each equation for the same variable, set them equal to each other, and solve the resulting one-variable equation). The new system was different than other systems students had previously solved using this method because isolating a variable in the second equation led to non-integer coefficients. This complication led students to assume the problem was unsolvable using the method that they knew—they were "jammed." Later, when Ms. Becker introduced substitution — the method that solves this system in a more efficient way — the students demonstrated clear relief. By introducing a problem that led to jamming, Ms. Becker enabled the students to become invested in finding an alternate way to solve the system.

Discussion

In this paper we used a mathematical story framework to illustrate the counterintuitive proposition that misdirection in the mathematics classroom can be beneficial. Thus, we propose that studying how misdirection can be used effectively in mathematics lessons will offer potential solutions to how curriculum might improve student attitudes toward learning mathematics.

Misdirection, however, is just one of many devices that storywriters use to create captivating narratives (e.g. foreshadowing, suspense, etc.). Framing mathematics lessons as stories provides mathematics educators access to this valuable set of tools. We encourage teachers, other curriculum designers, and mathematics education researchers to extend this work—to seek new ways that powerful storytelling techniques can be used to design lessons that capture student attention and inspire students to become invested in mathematical outcomes.

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