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ESSAYS ON MACROECONOMICS OF BANKING AND ASSET BUBBLES

by

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ABSTRACT

This dissertation consists of three essays on macroeconomics of banking and asset bubbles. In the first chapter, I develop a model to study the production of private safe assets by the banking sector. In response to a shortage of safe assets, the banking sector produces more private safe assets which alleviate the decline of aggregate investment and output. However, producing more private safe assets exposes the bank to more aggregate risk. Macroprudential policies can adjust the production of private safe assets with a tradeoff: encouraging the production of private safe assets alleviates the safe asset shortage problem and improves output, at the cost of a more volatile economy.

In the second chapter, I document that during the 2008 financial crisis, U.S. shadow banks deleveraged sharply while commercial banks maintained their leverage. I find that banks that relied more on short-term wholesale funding tended to deleverage more during the crisis. I then build a model to incorporate both shadow banks and commercial banks with different leverage determination mechanisms. The model can explain the heterogeneous leverage dynamics of the banking sector and the flight-to-quality phenomenon observed in data.

The third chapter is coauthored with Jianjun Miao and Pengfei Wang. We revisit Galí (2014) analysis by extending his model to incorporate persistent bubble shocks. We find that, under adaptive learning, a stable bubbly steady state and the associated sunspot solutions under optimal monetary policy are not E-stable. When deriving the unique forward-looking minimum stable variable (MSV) solution around an unstable bubbly steady state, we obtain results that are consistent with the conventional views: leaning against the wind policy reduces bubble volatility and is optimal. Such a steady state and the associated MSV solution are E-stable.

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List of Abbreviations

ABCP	....	Asset-Backed Commercial Paper
ABS	....	Asset-Backed Security
ALM	....	Actual Law of Motion
AR1, AR(1)	....	Autoregressive of Order 1
E-stable, E-stability	....	Expectational stable, Expectational stability
FED	....	Federal Reserve
GDP	....	Gross Domestic Product
IID, i.i.d.	....	Independent Identical Distributed
MBS	....	Mortgage Backed Security
MSV	....	Minimum State Variable
ODE	....	Ordinary Differential Equation
PLM	....	Perceived Law of Motion
REE	....	Rational Expectations Equilibrium
Repo	....	Repurchase Agreement
ROA	....	Return on Asset
SDF	....	Stochastic Discount Factor
SPE	....	Special Purpose Vehicle
TFP	....	Total Factor Productivity
VaR	....	Value-at-Risk

Chapter 1

Production of Private Safe Assets and Macroprudential Policy

1.1 Introduction

Safe assets are assets whose values are insensitive to aggregate shocks (Caballero et al., 2017), for example, the U.S. government bonds. The financial sector also issues private safe assets including bank deposits and many high-quality securities. Nevertheless, the U.S. economy experienced a shortage of safe assets in the 2000s (Caballero and Farhi, 2018). Many researchers argue that the supply of U.S. safe assets did not catch up with the increasing demand for safe assets from emerging economies, or the 'global saving glut' (Bernanke et al., 2005; Caballero et al., 2017; Caballero, 2009). Savings from foreign countries demanded an increasing amount of U.S. safe assets, mostly U.S. government bonds, drove down the interest rate, and left less safe assets remaining for other investors. In response to the shortage of safe assets, the U.S. banking sector manufactured a large amount of private safe assets. In the years before the financial crisis, banks issued short-term 'money like' liabilities that were perceived as safe assets by investors through the process called securitization (Sunderam, 2015). It was widely believed that the rapid growth of securitization and related banking activities led to the 2008 financial crisis. Despite some empirical evidence showing that the production of private safe assets responded to the degree of safe assets shortage (Kacperczyk et al., 2017), its economic consequences and policy implications are understudied.

In this chapter, I build a general equilibrium model with a banking sector to study the interaction between the shortage of safe assets and the production of private safe assets. In the model, the bank issues private safe assets against assets that are subject to aggregate shocks. The private safe assets are demanded by both entrepreneurs and a representative household. The entrepreneurs make capital investments with idiosyncratic investment efficiency shocks and thus demand private safe assets as a store of value. The household holds private safe assets for its liquidity service.

I model the shortage of safe assets by an exogenous increase in the demand for safe assets from the household. An increase in the demand for safe assets induces the household to hold more safe assets, intensifies the competition for safe assets between entrepreneurs and the household and leads to a shortage of safe assets. The price of the private safe assets rises and the return falls. The entrepreneurs have to accept a lower return for holding the private safe assets. As a result, the entrepreneurs' wealth shrinks, and investment and output fall. My model illustrates that the shortage of safe assets hampers the safe asset's function as a store of value and reduces output through the wealth of the entrepreneurs.

In the model, the bank invests in both safe and risky projects and uses their cash flows to issue private safe assets. The safe projects pay a non-stochastic return and its supply is limited. The return of risky projects is subject to aggregate shocks but the supply is unlimited. The shortage of safe assets lowers the return of the private safe assets and increases the profit margin of the bank. The bank responds by expanding its balance sheet and producing more private safe assets. Since the supply of the safe projects is limited, the bank has to invest in more risky projects and thus exposes itself to more aggregate risk. A negative aggregate shock deteriorates the bank's balance sheet, hinders its production of private safe assets, and thus leads to a shortage of safe assets and lower investment and output.

The expansion of the bank's balance sheet alleviates the shortage of safe assets by providing more private safe assets. However, such an expansion exposes the economy to more aggregate risk. Therefore a negative aggregate shock has a larger impact on the economy when the bank is larger. Macroprudential policies can be useful to restrict the production of private safe assets and the bank's risk exposure. It is widely recognized that the financial crisis was originated from the U.S. shadow banking sector. If macroprudential policies could restrict the size of the bank's balance sheet and reduce the associated risk, the crisis could be mitigated. However, restricting the production of safe assets intensifies the shortage of safe assets and lowers output. Thus macroprudential policies face a tradeoff: encouraging the bank to produce more private safe assets alleviates the shortage of safe assets and improves output, at the cost of a more risky banking sector and a more volatile economy. The optimal level of the macroprudential policy depends on the variance of the aggregate shock and the severity of the safe asset shortage.

Related literature. My paper is related to several strands of literature. First is the literature on the shortage of safe assets (Caballero, 2006; Caballero and Farhi, 2018; Caballero et al., 2017, 2008; Caballero, 2009). My paper is closest to the idea of Caballero (2009). They argue that when the economy has a shortage of safe assets driven by foreign capital inflows, the domestic financial sector has a higher leverage and is exposed to more risks when producing more safe assets. My paper contributes to this literature by explicitly studying how private safe assets are produced by the banking sector in the environment of safe asset shortage and analyzing macroprudential policies.

Secondly, my paper is related to the large banking literature that also emphasizes the tradeoff between economic growth and financial stability (Moreira and Savov, 2017; Gertler et al., 2020; Stein, 2012; Gennaioli et al., 2013). In this literature, usually the benefit of banks credit expansion is to intermediate more funds to the borrower and hence increases investment. My paper contributes to this literature by providing a new channel through which a credit expansion can benefit the economy, that is to produce more safe assets as saving instruments to benefit the saver, rather than the borrower.

Thirdly, my paper is related to the literature of rational bubbles (Farhi and Tirole, 2012; Miao and Wang, 2018; Hirano and Yanagawa, 2016). Asset bubbles can often exist in economies short of assets. For example, Aoki et al. (2014) show that a shortage of safe assets can create conditions for intrinsically useless 'safe' bubble assets to circulate at a positive price. In my paper, the banking sector produces private safe assets backed by risky projects. It depicts the securitization business of the U.S. shadow banking sector where banks issued asset-backed commercial papers against mortgage-backed-securities (MBS). These MBS were further backed by the high housing prices, which were believed to contain bubbles before the financial crisis. Hence the production of private safe assets shares similarities with the formation of bubbles. I borrow the model setup from the bubble literature that entrepreneurs face uninsurable investment efficiency shocks to generate the need for safe assets (Dong et al., 2020; Hirano et al., 2015).

Lastly, my paper is related to the empirical evidence that the financial sector produces more private safe assets in response to safe asset shortages. Sunderam (2015) shows that the sustained increase in money demand could explain up to 50% of the growth in asset-backed commercial papers (ABCP) in the years before the financial crisis. Kacperczyk et al. (2017) find that the issuance of short-term certificates of deposits strongly responds to measures of safety demand. Xie (2012) finds that ABS/MBS issuers react to the change in the convenience yield, which is a measure of safe asset shortage. Nadauld and Sherlund (2013) show that the securitization activities of investment banks affect the upstream mortgage origination business. My model is consistent with the empirical findings of this literature and provides a theoretical explanation.

The rest of the paper is structured as follows. Section 1.2 presents some motivating facts on the shortage of safe assets and the production of private safe assets by the U.S. shadow banking sector. Section 1.3 introduces the model. Section 1.4 uses the model to study the effects of safe asset shortages due to both structural forces and aggregate shocks. Section 1.5 then analyzes macroprudential policies. Section 1.6 concludes.

1.2 Safe Asset Shortage and Shadow Banking

The U.S. economy experienced a shortage of safe assets in the early 2000s. Figure 1·1 shows the convenience yield, which is the spread between the yield of AAA corporate bond and the long-term treasury yield (Krishnamurthy and Vissing-Jorgensen, 2012). Investors hold safe assets as a store of value and are willing to accept lower returns for their safety and liquidity, so safe assets enjoy a convenience yield. The convenience yield is a useful measure of safe asset shortage. When there is a shortage of safe assets, the price of safe assets goes up and the return goes down, hence the convenience yield will go up. The convenience yield rose in the late 1990s and early 2000s, suggesting a shortage of safe assets. The shortage was then mitigated from 2003 to 2006, and then worsened again during the crisis. Shortages of safe assets were also likely to be associated with lower investment and output since they coincided with the recent two recessions. Although not causal, this relation is consistent with the model I will show later that a shortage of safe assets reduces investment and output.

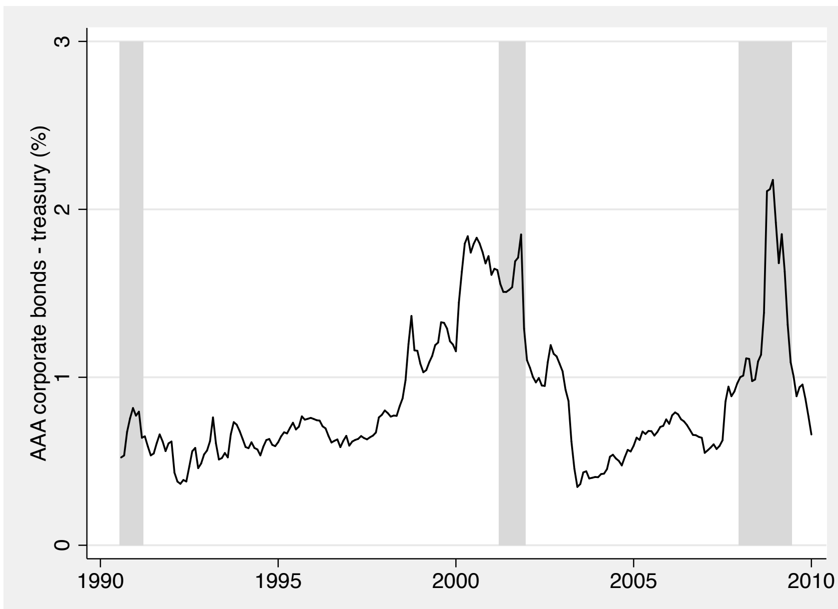


Figure 1·1: Convenience Yield

Many factors could drive such a shortage of safe assets, for example the large foreign capital inflows from emerging countries that purchased a lot of U.S. government bonds and hence resulted a shortage of safe assets. In this paper, I do not ask what exactly caused the shortage of safe assets, but take it as given and focus on the response of the U.S. banking sector. Following the shortage of safe assets in the early 2000s, the U.S. shadow banking sector grew rapidly. In the years before the 2008 financial crisis. the banking sector manufactured a large amount of private safe assets through the process called 'structured finance' or 'securitization' (Coval et al., 2009). In the securitization business, financial institutions bought a bunch of risky assets, pooled them together, and then used their cash flows to create asset-backed-securities (ABS) or mortgage-backed-securities (MBS) if the underlying assets were mortgages. The issuer, often a large investment bank, then financed these ABS with short-term liabilities. In doing so, the issuer bank first created a special-purpose-entity (SPE) and sold the ABS to it. The SPE then issued short-term liabilities to the final investors in the form of asset-backed commercial papers (ABCP). The SPE was off the balance sheet of the issuer bank and was outside the normal banking regulation. That's why the whole process was called shadow banking.

Figure 1·2 presents the amount of issuance of non-agency (private-label) MBS and the amount of outstanding ABCP. MBS was the most important type of assets under securitization and they accounted for more than half of the total ABS outstanding (Gorton and Metrick, 2013). The issuance of private-label MBS grew from a few hundred billion dollars in 2000 to almost 1500 billion dollars at the peak of 2006. In the meantime, the ABCP outstanding grew from a little more than 600 billion dollars in the early 2000s to 1200 billion dollars on the eve of the financial crisis. This period of rapid expansion of securitization matches the same period in Figure 1·1 when the safe asset shortage was mitigated. So it's likely that the rapid growth of the securitization business and the production and the production of private safe assets partially filled up the gap of safe asset shortage in the U.S.

However, unlike public safe assets, the production of private safe assets increased the risk of the financial system. When the shadow banking sector

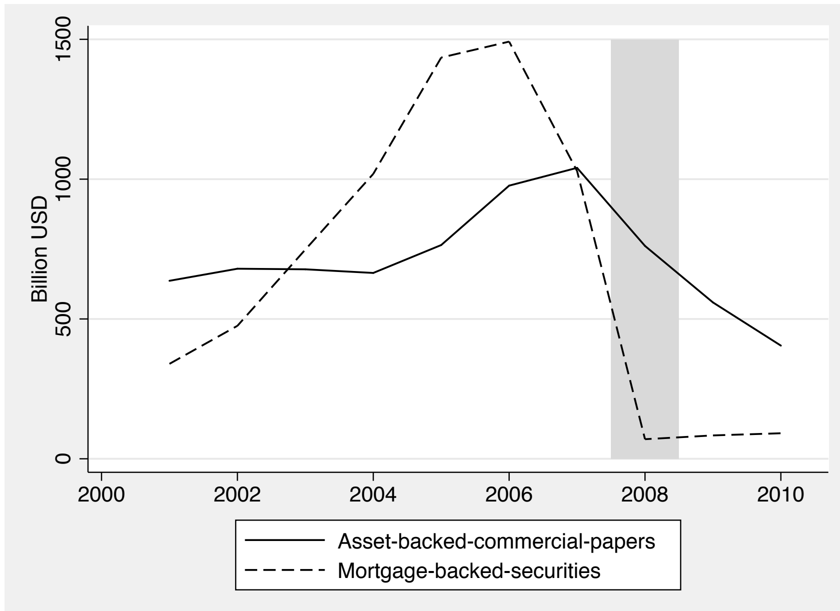


Figure 1·2: ABCP Outstanding and MBS Issuance

expanded so quickly, the economy could not provide enough good assets to be securitized in the short-run. As a result, the shadow banking sector inevitably looked for more risky assets, and most notably, subprime mortgages. Figure 1·3 shows that the ratio of subprime MBS over total private-label MBS increased from 30% to more than 60% in the same period. When the shadow banking sector was searching for more mortgages for securitization, the mortgage originators in the upstream was incentivized to lower the lending standard and conduct subprime lending more aggressively. The whole financial system was exposed to more risk.

Although the underlying assets were risky, the issuer bank would usually apply credit enhancements so that the SPE could issue safe liabilities. For instance, the issuer bank could sell the ABS in tranches with different seniority. It could also provide explicit and implicit guarantees to the SPE (Acharya et al., 2013). As a result, ABCP issued by the SPE was perceived as safe assets

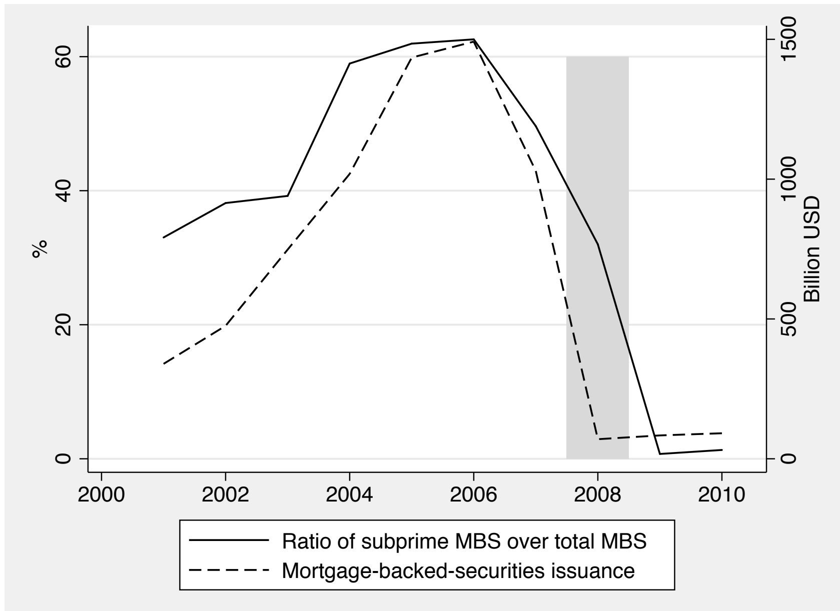


Figure 1·3: Ratio of Subprime MBS over Total MBS

and purchased mainly by money market funds and pension funds. The spread of ABCP over Fed Fund Rate was only a few basis points before the financial crisis. Sunderam (2015) provided empirical evidence that investors treated short-term debt issued by shadow banks (ABCP) as a money-like claim, supporting the argument that the rising demand for safe assets contributed to the growth of the U.S. shadow banking sector before the financial crisis.

In summary,in response to the shortage of safe assets in the early 2000s, the U.S. banking sector was able to produce a large amount of private safe assets against risky collaterals, which alleviated the safe asset shortage but also exposed the banking sector to higher risk. Finally, when the financial crisis came, the private safe assets lost their value, which led to the acute shortage of safe assets and the spike in the convenience yield.

1.3 A Model of Safe Asset Production

In this section, I present a model of private safe asset production. The model consists of a continuum of entrepreneurs, a banking sector, a representative household, and competitive final-good producers. Entrepreneurs make capital investments. They are subject to idiosyncratic investment efficiency shocks and thus demand private safe assets provided by the bank as a store of value. The bank issues private safe assets by investing in various projects. The household also holds private safe assets for its liquidity service. Final-good producers combine capital provided by the entrepreneurs and labor provided by the household to produce consumption goods.

1.3.1 Entrepreneurs

There is a continuum of entrepreneurs indexed by j and the measure is normalized to one. Entrepreneurs can produce new capital goods with idiosyncratic investment efficiencies:

$$k^n_{jt+1} = a_{jt} i_{jt}, \tag{1.1}$$

where the i_{jt} and k^n_{jt+1} are the amount of investment input and new capital produced. a_{jt} represents the idiosyncratic investment efficiency of entrepreneur j. The entrepreneur then sells the new capital to the final-good producers in the next period at the price q_{t+1}.

The idiosyncratic investment efficiency a_{jt} can be either high or low: $a_{jt} \in \{a^H, a^L\}$. In each period, a fraction h of entrepreneurs receive high investment efficiency, while the rest $1 - h$ entrepreneurs receive low investment efficiency:

$$Pr(a_{jt} = a^H) = 1 - Pr(a_{jt} = a^L) = h.$$

The idiosyncratic investment efficiency is i.i.d. across time and the entrepreneurs. For simplicity, I call an entrepreneur the 'high type' ('low type') if he receives high (low) investment efficiency.

Besides capital investment, entrepreneurs can purchase private safe assets provided by the bank. The private safe asset pays a non-stochastic return R_t. In equilibrium, the low type entrepreneurs have inferior investment efficiency and thus need to hold the private safe assets as a store of value. Let n_{jt} and s^E_{jt} denote entrepreneur j's net wealth and the amount of private safe assets purchased by entrepreneur j. The balance sheet of entrepreneur j is

$$i_{jt} + s^E_{jt} = n_{jt}. \tag{1.2}$$

I assume that in each period the entrepreneur pays out $1 - \sigma^E$ of the total revenue as dividends to the household

$$div^E_{jt+1} = (1 - \sigma^E)(q_{t+1}k^n_{jt+1} + R_t s^E_{jt}).$$

The net wealth is then accumulated through retained earnings

$$n_{jt+1} = \sigma^E(q_{t+1}k^n_{jt+1} + R_t s^E_{jt}). \tag{1.3}$$

The entrepreneur maximizes the discounted sum of dividends:

$$V_{jt} = E_t \sum_{i=1}^{\infty} \beta^i div^E_{jt+i}, \tag{1.4}$$

subject to (1.1), (1.2) and (1.3). β is the discount rate.

1.3.2 Bank

The bank issues private safe assets, which are the bank's liabilities and the assets of the entrepreneurs and the household. The private safe assets pay a non-stochastic gross return R_t. In order to issue private safe assets, the bank invests in a pool of projects as its assets. Every project requires one unit of funding. There are two types of projects: safe and risky. A safe project pays a fixed gross return of R^X and the total amount of investable safe projects is limited at G. The return of a risky project is $R^X Z_t$ where Z_t is a random variable describing the aggregate return shock and its mean is one $E(Z_t) = 1$. The amount of investable risky projects is unlimited. The bank always invests in the safe projects first and then serve the risky projects.

Let X_t denote the total number of projects the bank invests on its asset side. I assume that $X_t > G$ so that all the safe projects are served and the bank invests at least in some risky projects. The bank uses the cash flow from the project pool to issue private safe assets. Suppose the bank issues S_t amount of private safe assets. The discrepancy between the size of the project pool and the amount of private safe assets issued must be funded by the bank's own equity W_t. The balance sheet of the bank is

$$X_t = S_t + W_t. \tag{1.5}$$

Note that the asset of the bank is exposed to aggregate risk while the liability is not. The bank effectively uses its equity to absorb the aggregate risk on its asset side and ensures its liability is indeed safe. To ensure the safety of its liability, the bank has to hold enough equity as buffer stocks so that it can honor the promised payments on the private safe assets. In particular, I assume the bank is facing a Value-at-Risk (VaR) constraint:

$$(X_t - G)\underline{Z}R^X + GR^X \geq S_t R_t. \tag{1.6}$$

The VaR constraint says that the bank can guarantee the payments of the private safe assets whenever the aggregate shock on the return of its risky project is above a cutoff $\underline{Z}$. For example, if $\underline{Z}$ is the 1% lowest realization of the distribution of Z_t, then the above constraint ensures that the bank can honor its liability 99% of the time. Brunnermeier and Pedersen (2009) documented that the VaR constraint applies to broadly the shadow banking sector.

The bank maximizes the discounted stream of dividends paid back to the household. For simplicity, I assume that the bank follows a constant dividend payout policy similar to the entrepreneurs. In each period a fraction $1 - \sigma^B$ of the bank's current revenue is paid out as dividends.

$$div^B_{t+1} = (1 - \sigma^B)((X_t - G)Z_{t+1}R^X + GR^X - S_t R_t).$$

The bank's next period's equity is accumulated through retained earnings

$$W_{t+1} = \sigma^B((X_t - G)Z_{t+1}R^X + GR^X - S_t R_t). \tag{1.7}$$

The bank's objective is to maximize the expected sum of future dividends

$$V^B_t = E_t \sum_{i=1}^{\infty} \beta^i div^B_{t+i}, \tag{1.8}$$

subject to (1.5), (1.6) and (1.7).

1.3.3 Final-Good Producers

Competitive final-good producers use capital and labor to produce consumption goods according to the Cobb-Douglas production function.

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{1.9}$$

The final goods producers keep the stock of capital, which depreciates at a rate δ. The final-good producers can purchase new capital K_t^n from the entrepreneurs at price q_t to replenish the capital stock:

$$K_t = (1-\delta)K_{t-1} + K_t^n, \tag{1.10}$$

The final-good producers maximize the sum of discounted profits by choosing labor and new capital purchases

$$\max_{\{K_t^n, L_t\}} E_0 \sum_{t=0}^{\infty} \beta^t (Y_t - w_t L_t - q_t K_t^n)$$

where β is the discount rate of the household. The final-good producers then rebate their profits to the household

$$\Pi_t = Y_t - w_t L_t - q_t K_t^n. \tag{1.11}$$

1.3.4 Households

The representative household supplies one unit of labor inelastically and earns a wage w_t. He consumes C_t and purchases private safe assets S_t^H produced by the bank. His budget constraint is thus

$$S_t^H + C_t = w_t + \Pi_t + div_t + R_{t-1} S_{t-1}^H, \tag{1.12}$$

where

$$div_t = div_t^B + \int div_{jt}^E$$

is the dividends paid out from the entrepreneurs and the bank.

To incorporate exogenous shifts in the demand for safe assets and model the shortage of safe assets, I assume that the household values a 'liquidity service' from the private safe assets Sunderam (2015). In particular, the household maximizes the expected sum of utility

$$\max_{\{C_t, S_t^H\}} E_0 \sum_{t=0}^{\infty} \beta^t (\ln C_t + F_t \ln S_t^H)$$

subject to (1.12), where $\ln S_t^H$ is the liquidity service from holding the safe assets. The welfare weight on the liquidity service F_t captures the demand for safe assets from the household, which broadly reflects the society's appetite for safe assets. An increase in F_t can capture, for example, an increase in foreign capital inflows pursuing safe U.S. assets. It can also reflect more structural forces, such as an increase in savings due to population aging.

1.3.5 Equilibrium Definition

Given the exogenous processes for Z_t and F_t, a competitive equilibrium is defined as the paths of variables $\{n_{jt},\ i_{jt},\ s_{jt}^E,\ X_t,\ S_t,\ W_t,\ Y_t,\ L_t,\ K_t,\ K_t^n,\ C_t,\ S_t^H,\ q_t,\ w_t,\ R_t\}$ such that i) the entrepreneurs, the bank, the final-good producers and the household optimize subject to the relevant constraints; ii) the aggregate variables follows (1.7), (1.9), (1.10), (1.11) and $L_t = 1$; iii) all markets clear. In particular, the market for private safe assets clears

$$S_t = \int s_{jt}^E + S_t^H.$$

The market for new capital clears

$$\int k_{jt}^n = K_t^n.$$

1.3.6 Equilibrium Characterization and Aggregation

In this subsection, I discuss the optimal actions for different players in the model. The entrepreneurs choose between capital investment and holding private safe assets. The expected return of capital investment is $a_{jt} E_t q_{t+1}$

while the return of holding private safe assets is R_t. Since the entrepreneurs maximize the discounted sum of wealth given the constant dividends payout policy, the entrepreneurs will choose whichever asset that gives a higher return. I focus on the equilibrium where the high type entrepreneurs choose capital investment while the low type entrepreneurs choose to hold private safe assets (see Appendix A for a formal proof). This requires that

Assumption 1.1. *The return of safe assets is between the return of capital investment for the low-type and the high-type entrepreneurs*

$$a^L E_t q_{t+1} < R_t < a^H E_t q_{t+1}.$$

Without loss of generosity, I assume $a^H = 1$ and $a^L = 0$ since only the value of $a^H E_t q_{t+1}$ is determined in the equilibrium.

I also show in Appendix A that as long as the bank is earning a positive profit margin, it wants to expand its balance sheet as much as possible and hence the VaR constraint is binding:

$$(X_t - N_g)\underline{Z}R^X + N_g R^X - S_t R_t = 0. \tag{1.13}$$

Assumption 1.2. *The bank's profit margin is positive in the equilibrium*

$$E_t Z_{t+1} R^X - R_t > 0.$$

Under Assumption 1.2 and given the level of the bank's equity W_t, the bank will produces as much private safe assets S_t as possible until the VaR constraint binds.

I analyze the equilibrium in terms of aggregate variables. Let $I_t = \int_j i_{jt}$, $S_t^E = \int_j s_{jt}^E$, $N_t = \int_j n_{jt}$ denote the aggregate investment, aggregate holdings of private safe assets by the entrepreneurs and the aggregate wealth of the entrepreneurs. Using the optimal actions for the two type of entrepreneurs, I obtain

$$I_t = hN_t, \tag{1.14}$$

$$S_t^E = (1-h)N_t. \tag{1.15}$$

The amount of new capital follows

$$K_{t+1}^n = I_t. \tag{1.16}$$

The total wealth of the entrepreneurs evolves according to

$$N_t = \sigma^E \left(q_t I_{t-1} + R_{t-1} S_{t-1}^E \right). \tag{1.17}$$

The market clearing condition for the private safe assets implies

$$S_t = S_t^E + S_t^H. \tag{1.18}$$

Equation (1.5), (1.7) and (1.13) describe the equilibrium behavior of the bank. The final-good producer's maximization problem yields the following first-order-conditions:

$$q_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha} + \beta(1-\delta) E_t q_{t+1}, \tag{1.19}$$

$$w_t = (1-\alpha) K_t^{\alpha} L_t^{-\alpha}. \tag{1.20}$$

The household's optimization problem yields the consumption Euler equation

$$1 = E_t \beta R_t \frac{C_t}{C_{t+1}} + F_t \frac{C_t}{S_t^H}. \tag{1.21}$$

The equilibrium system in aggregate variables consists of 14 endogenous variables $\{Y_t, C_t, K_t, K_t^n, I_t, N_t, W_t, X_t, S_t, S_t^H, S_t^E, q_t, w_t, R_t\}$ and 14 equations: (1.5), (1.7), (1.9), (1.10), (1.12), (1.13), (1.14), (1.15), (1.16), (1.17), (1.18), (1.19), (1.20), (1.21). The full equilibrium system is presented in the Appendix A.

1.3.7 Wealth Channel of Safe Assets Shortage

How does a shortage of safe assets affect the economy? The degree of the shortage of safe assets can be measured by the convenience yield of the safe assets. The higher the convenience yield, the more severe the shortage of safe assets. In the model, the convenience yield can be represented by the difference between the return of capital investment and the return of safe assets:

$$CY_t \equiv E_t q_{t+1} a^H - R_t.$$

I show that in the steady state, the output is decreasing in the convenience yield.

Proposition 1.1. *In the steady state, $\partial Y / \partial CY < 0$.*

This result shows that a shortage of safe assets has a negative effect on output. The intuition is that a shortage of safe assets pushes down the return of the private safe assets R_t. Since the entrepreneurs need to hold the private safe assets as a store of value, a lower return reduces their wealth. As the entrepreneurs have less wealth for investment, capital stock and output fall. This is the key mechanism in the model through which the amount of safe assets affects the real economy. Since it works through the wealth of the entrepreneurs, I call it the 'wealth channel'. This mechanism is similar to the 'liquidity effect' in Farhi and Tirole (2012), where a shortage of liquidity negatively affects investments.

1.3.8 Model Discussion

A key point to emphasize is that in the model, the bank does not actually create safe assets, but uses its own equity to absorb the variations in its profits so that all the aggregate risk is concentrated in its equity. This setup shares some similarities with Caballero (2009). Acharya et al. (2013) provide empirical evidence that the bank was doing a securitization without risk transfer before the financial crisis. The idea is that banks provided both implicit and explicit guarantee to their securitization products, so that ex-post it was the bank, not the external investors, bore the loss due to default the subprime mortgage and subprime MBS. My model is consistent with these views. In another perspective, the private safe assets the bank issued is only 'internally' safe, in the sense that it is safe only to the holder. As a whole economy, however, more private safe assets implies more aggregate risk exposure. The truly safe assets in the economy are the safe projects invested by the bank, whose supply is fixed. This is similar to Caballero and Farhi (2018) where the net supply of safe assets is zero.

1.4 Model Performance

In this section, I show that the model can capture the empirical facts about the safe asset shortage presented in Section 2. I solve the model using perturbation around the static steady state. I first describe the calibration of the model.

1.4.1 Calibration

Table 1.1 shows the parameter values of the model. The model is highly stylized to study the effects of the shortage of safe assets. The risky and safe projects invested by the bank represent prime and subprime MBS. I set their mean return $R^X = 1.035$ to match the annualized return of the S&P U.S. MBS index. I set the total supply of safe projects G at 1. This implies the ratio of risky projects over total projects is around 30% to 70%, consistent with the ratio of subprime MBS over total private-label MBS observed in the data. The utility weight on liquidity service, F, is set at 0.015, which implies the spread between the return of safe assets and the required return due to impatience $1/\beta$ is 173 basis point. This is comparable to the spread between the return of treasury bonds and the return of triple-A corporate bonds in the early 2000s. I set σ^E and σ^B at 0.95 and 0.75 respectively, which implies a 5% dividends payout ratio for the entrepreneurs and 25% for the bank. I set the cutoff value of the aggregate return shock $\underline{Z}$ used in the bank's VaR constraint at 0.75, which is the lowest 1% realization given the distribution of Z_t. Under the above calibration, the steady state leverage of the bank is around 10. The capital depreciation is set at 0.025. The capital share is set at 0.3. The discount rate of the household is set at 0.99. These parameter values are standard in the business cycle literature.

I assume the aggregate shock of the return of risky projects Z_t has a left-skewed distribution. In particular, I assume that

$$Z_t = -P_a exp(-P_b x_t) + P_c,$$

where x_t follows an AR1 process

$$x_t = \rho_x x_{t-1} + \epsilon_x$$

and ϵ_x is the normal innovation with standard deviation σ_x. I set $P_a = 0.1$, $P_b = 0.55$, and $P_c = 1.1$. This implies that the mean of the aggregate return shock Z_t is one and its skewness is around -0.0114. I set the autocorrelation of x_t at 0.6 and the standard deviation σ_x at 1.2, which in turn implies the autocorrelation of Z_t is around 0.637 and the standard deviation is around 8.5%. To have a reference, the annualized standard deviation of the return of the S&P U.S. MBS index is 2.25% (calculated from 2014 to 2020). The size of the shock matters for the welfare analysis. I choose a relatively larger shock to mimic large negative events like the financial crisis. The focus of the model is more on qualitative results rather than quantitative ones and the main results are not sensitive to the value of parameters.

Parameter	Value	Target/Source
R^X	1.035	Return of S&P U.S. MBS index
G	1	Ratio of subprime MBS over total MBS
$std(Z_t)$	8.5%	Std of return of S&P U.S. MBS index is 2.5%
F	0.015	Convenience yield of 173 basis point
σ^E	0.95	Entrepreneurs' dividend payout ratio 5%
σ^B	0.75	Bank's dividend payout ratio 25%
$\underline{Z}$	0.75	Leverage of the bank is around 10
h	0.4	Hirano, Inaba, and Yanagawa, 2015
δ	0.025	Depreciate rate of capital
α	0.3	Capital share
β	0.99	Discount rate

Table 1.1: Benchmark Calibration

1.4.2 Shortage of Safe Assets by Structural Forces

The U.S. economy experienced a shortage of safe assets in the early 2000s, which is followed by a rapid expansion of the shadow banking sector. In the first experiment, I show that my model can replicate such a shortage of safe assets by a permanent increase in the demand for safe assets from the household F_t. In particular, I assume that F_t follows a slow-moving AR1 process

$$F_t - F = \rho_F(F_{t-1} - F)$$

with $\rho_F = 0.9$. I simulate the model with a permanent increase in the steady state value F from 0.0005 to 0.015. Figure 1·4 shows the transition path of the model economy.

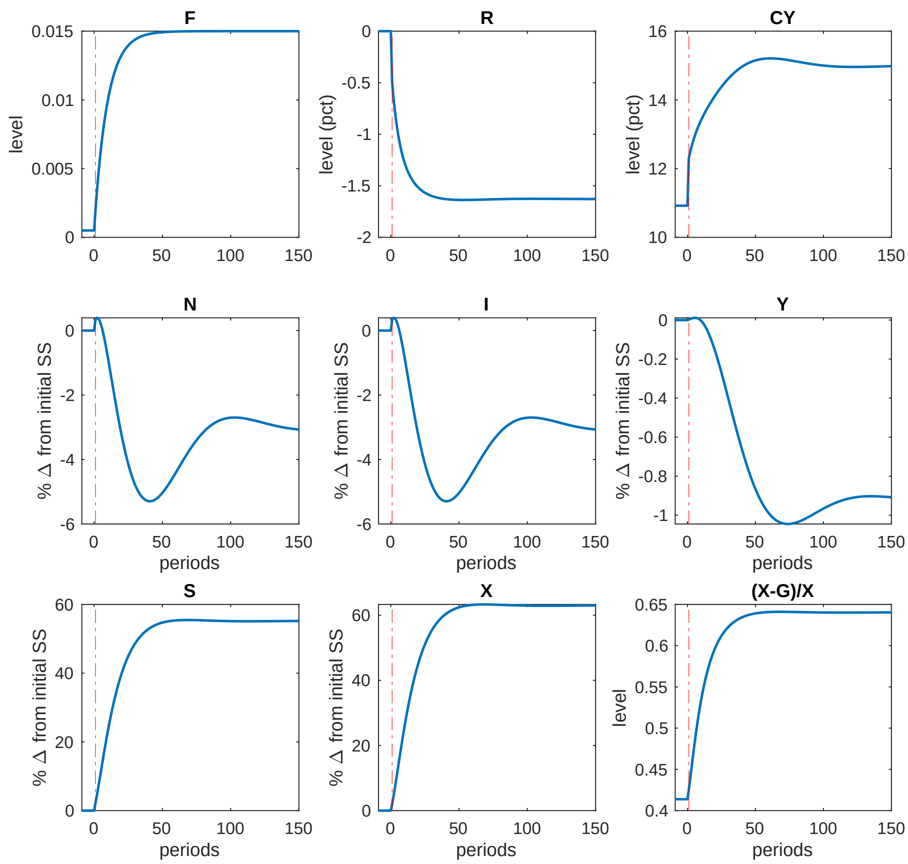


Figure 1·4: Transition Path under A Permanent Increase in the Demand for Safe Assets

The increase in the demand for safe assets F intensifies the competition for safe assets between the entrepreneurs and the household, generating a safe asset shortage reflected by a declining return of safe assets and an increasing convenience yield. The shortage of safe assets affects the aggregate output through the wealth of the entrepreneurs, or the ‘wealth channel’. A lower return of the private safe asset hampers its function as a store value. The entrepreneurs have to accept a lower return for holding the private safe assets so their wealth shrinks. As the entrepreneurs have less wealth, they spend less on capital investment, and hence the output falls. The output drops about 1% under the model calibration. In the meantime, the bank responds by expending its balance sheet to produce more private safe assets since the bank’s profit margin is now higher. The volume of the private safe assets increased by more than 50%. When producing more private safe assets, the bank also expands its assets by investing in more risky projects. The ratio of risky projects over total projects increased from 40% to 60%, which reflects the change in the ratio of subprime MBS over total private-label MBS before the financial crisis.

In summary, the model successfully captures the empirical facts about the safe asset shortage the U.S. economy experienced in the early 2000s. The model also suggests that the shortage of safe assets was responsible for the rise of the banking sector. One thing to note is that an increase in the demand for safe assets is not the only possible reason for the shortage of safe assets. In the Appendix A, I show that a worsening of investment opportunities for the entrepreneurs, modeled as a decrease in h, can also generate a shortage of safe assets with similar patterns: a rising convenience yield, a larger banking sector, and a lower output.

1.4.3 Shortage of Safe Assets by Negative Shocks

The bank produces private safe assets against risky projects. Hence aggregate shocks on the return of risky projects affect the balance sheet of the bank and its ability to produce private safe assets. Figure 1·5 shows the impulse response of the model economy to a negative shock on the return of risky projects Z_t. Under the current calibration, a one standard deviation negative shock reduces the return of risky projects by 6%. The equity of the bank shrinks by 25% and the production of private safe assets falls by 8%. This generates a mild shortage of safe assets and the convenience yield rises by 2%. The shortage of safe assets then affects the real economy negatively through the 'wealth channel'. The entrepreneurs' wealth shrinks by 2% and the output falls by 0.15%.

The model shows that a negative shock deteriorates the bank's balance sheet, reduces the production of private safe assets, and affects the output negatively through its function as a store of value. The negative aggregate shock resembles the 2007 subprime crisis. The financial crisis was originated from the increasing default rate in the subprime mortgages and the consequent deteriorations of the subprime mortgage-backed-securities (Brunnermeier, 2009). Many financial institutions suffered large losses. The bankruptcy of the two hedge funds of the investment bank Bear Stearns in the summer of 2007 triggered the collapse of the ABCP market. The value of ABCP outstanding fell by 33% within one month (Kacperczyk and Schnabl, 2010). The collapse of the ABCP market greatly reduced the supply of private safe assets, causing an acute shortage of safe assets (Caballero et al., 2017). Hence, besides its various negative effects through different channels, my model implies that the financial crisis led to an additional reduction in output by intensifying the safe

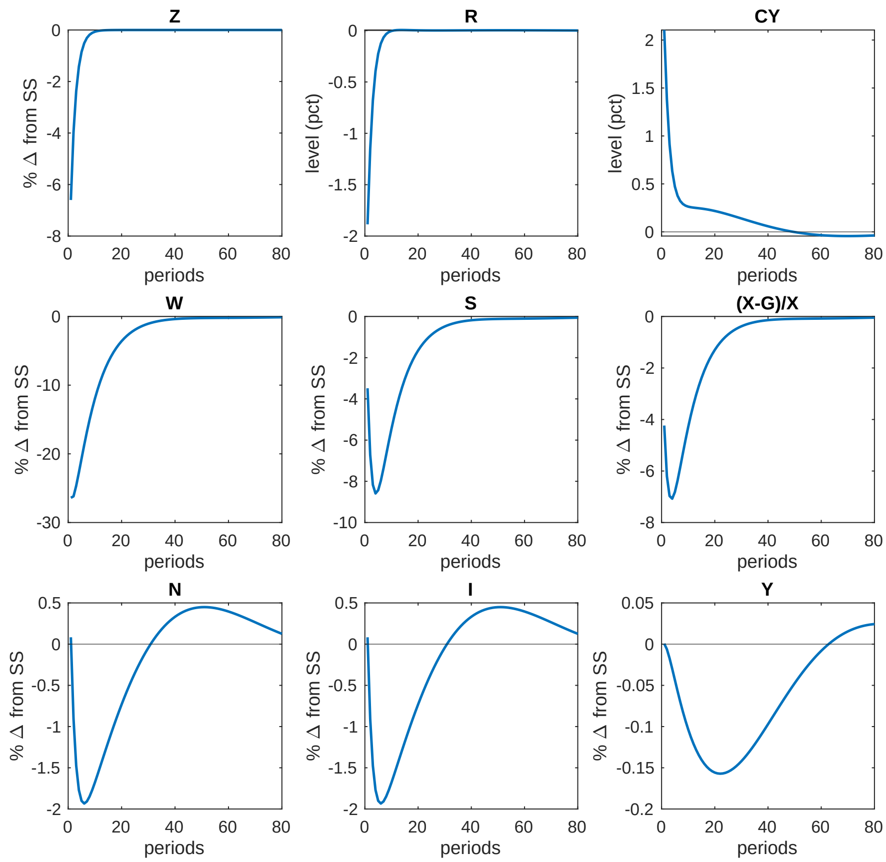


Figure 1·5: Impulse Response to an Aggregate Shock on the Return of Risky Projects

asset shortage problem and reducing the entrepreneurs' wealth.

1.4.4 Shortage of Safe Assets and Risk Exposure

Since a shortage of safe assets stimulates a larger banking sector with more risky projects on its balance sheet, an aggregate shock on the return of risky projects Z_t will have a larger impact when the shortage of safe assets is more severe. Figure 1·6 shows the impulse response of output to a one standard deviation negative aggregate shock under different values of F, the demand for safe assets from the household. Small, medium and large F correspond to values of 0.0005, 0.005, and 0.01. A higher value of F means a higher demand for safe assets and hence a more severe shortage of safe assets. The figure shows that the negative effect of the aggregate shocks Z_t of the same size is larger when the shortage of safe assets is more severe. The main reason for this variation is that as the shortage of safe assets gets more severe, the bank is producing more private safe assets to cater to the higher demand and conducting more risky projects. The bank's risk exposure, measured by the ratio of risky projects over total projects $(X_t - G)/X_t$ is increasing in F. Hence, the economy is more exposed to the aggregate shock and its negative effect is larger.

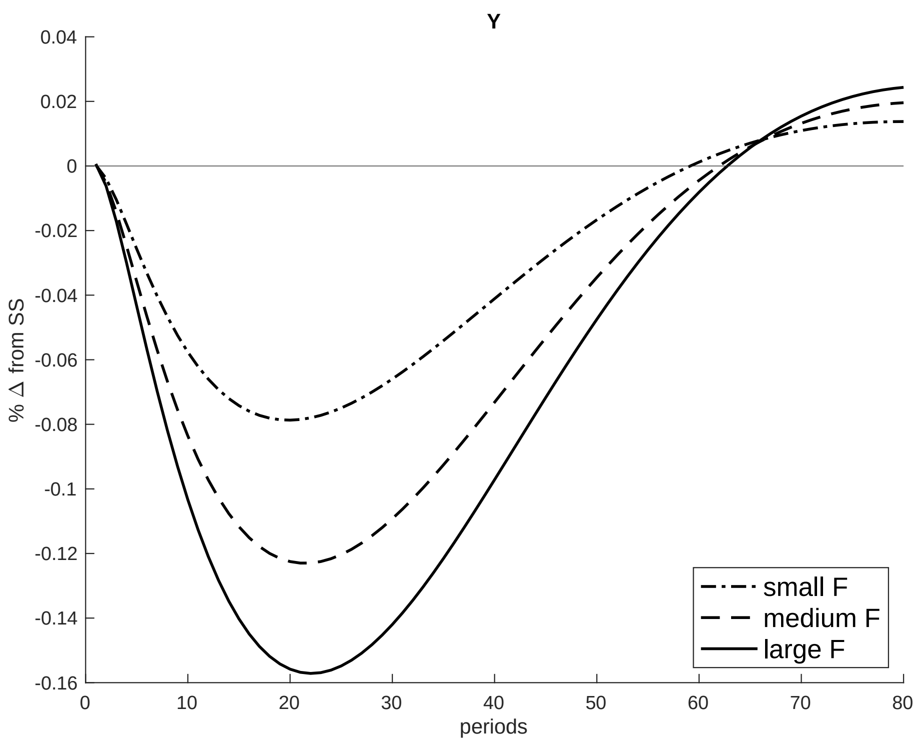


Figure 1·6: Impulse Responses of Output to Shocks on Return of Risky Projects under Different Severity of Safe Asset Shortage

My model suggests that to some extent, the shortage of safe assets in the early 2000s was responsible for the large magnitude of the financial crisis as it stimulated a larger shadow banking sector and exposed the economy to more risks. A natural remedy is to use macroprudential policies to regulate the banking sector. If macroprudential policies can restrict the size of the bank's balance sheet and reduce the associated risk, the crisis could be mitigated.

1.5 Macroprudential Policy

In this section, I study how macroprudential policies can improve welfare by regulating the production of private safe assets by the bank. I consider a simple form of macroprudential policy that the government charges an operation tax that is a constant proportion of the bank's balance sheet: τX_t. The tax proceeds is then rebated to the household. The law of motion of the bank's equity is thus

$$W_t = \sigma^B((X_{t-1} - R)Z_t R^X + GR^X - R_{t-1}S_{t-1} - \tau X_{t-1}).$$

The macroprudential policy directly affects the profit margin of the bank ($R^X - R - \tau$) and hence affects its balance sheet.

1.5.1 Level vs. Volatility Tradeoff

The macroprudential policy faces a tradeoff between alleviating the safe assets shortage versus reducing the risk exposure of the banking sector. Since a shortage of safe assets has a negative effect on the real economy, allowing the bank to provide more private safe assets alleviates the shortage of safe assets and improves investment and output. However, when the bank produces more private safe assets, it also has to conduct more risky projects and thus exposes the economy to more aggregate risk. Hence the tradeoff of the macroprudential policy is between the level of output versus its volatility.

Figure 1·7 shows the steady state value of selected endogenous variables under different levels of macroprudential policies τ. By using a looser macroprudential policy, the regulator raises the profit of the bank and hence enables the bank to provide more private safe assets. In this way, the regulator alleviates the shortage of safe assets. The price of the safe assets is lower and the return is higher. The private safe assets are more beneficial as a store of value. As a result, entrepreneurs have more wealth for investment and the steady state level of output is higher. On the other hand, as a looser macroprudential policy allows the bank to provide more private safe assets, the bank is also conducting more risky projects. When τ is reduced from 0.02 to 0, the ratio of the bank's risky projects over total projects increases from 0.45 to around 0.6. The economy is exposed to more aggregate risk and thus more volatile. The variance of output increased by 50%.

1.5.2 Optimal Policy

The level versus volatility tradeoff suggests that there is an optimal level of macroprudential policy that maximizes welfare. Following Schmitt-Grohé and Uribe (2001), I measure the welfare cost associated with a particular level of macroprudential policy by the fraction of steady-state consumption that the household would be willing to give up in order to be indifferent between the corresponding constant sequence of consumption and the original stochastic equilibrium with the macroprudential policy under consideration.

Formally, I calculated the unconditional expectation of the period utility

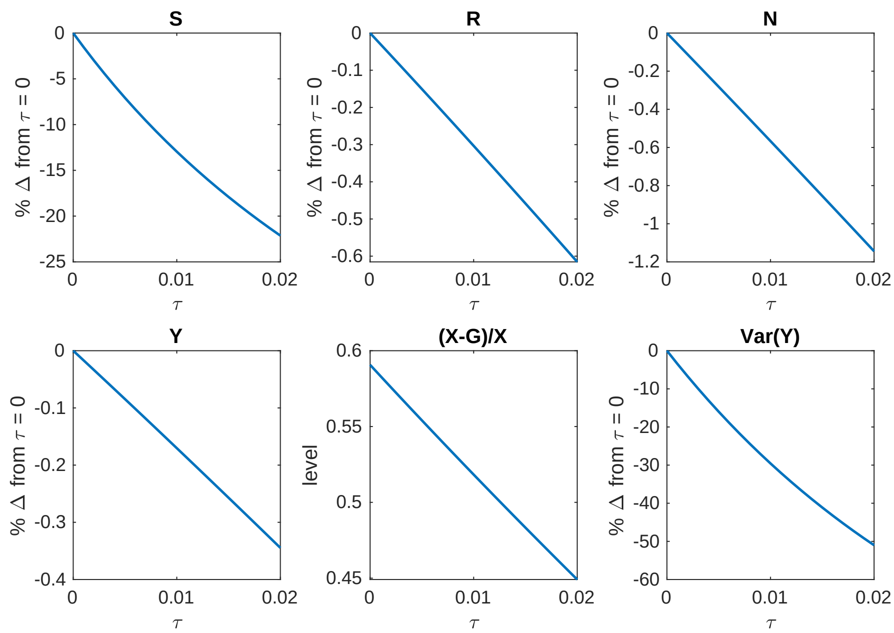


Figure 1·7: Steady State Value under Different Level of Macroprudential Policies

given the level of macroprudential policy

$$E\{U(C_t, S_t); \tau\} = E\{\log(C_t) + F \log(S_t); \tau\}.$$

I use the theoretical mean calculated by Dynare using a second-order approximation. I then calculate the period utility under the constant consumption and liquidity service at $\tau = 0$, where the consumption level is a faction ξ below the steady state level

$$U((1 - \xi)C, S).$$

The welfare cost associated with the policy level τ is thus the value $\xi(\tau)$ such that the household is indifferent between the above two cases

$$U((1 - \xi(\tau))C, S) = E\{U(C_t, S_t); \tau\}.$$

Figure 1·8 shows the consumption equivalent measure of welfare under different levels of macroprudential policy τ. I plot $-\xi$ so that it measures directly the welfare rather than the welfare cost. I also plot the welfare measure relative to the case of no macroprudential policy $(-\xi(\tau)) - (-\xi(0))$. The figure shows that the optimal level of macroprudential policy is reached when τ is around 0.008, or a tax that is equal to 80 basis points. Relative to the case of no macroprudential policy ($\tau = 0$), the optimal level of macroprudential policy improves the welfare by 0.06% equivalent consumption units. The welfare improvement of macroprudential policy is mild.

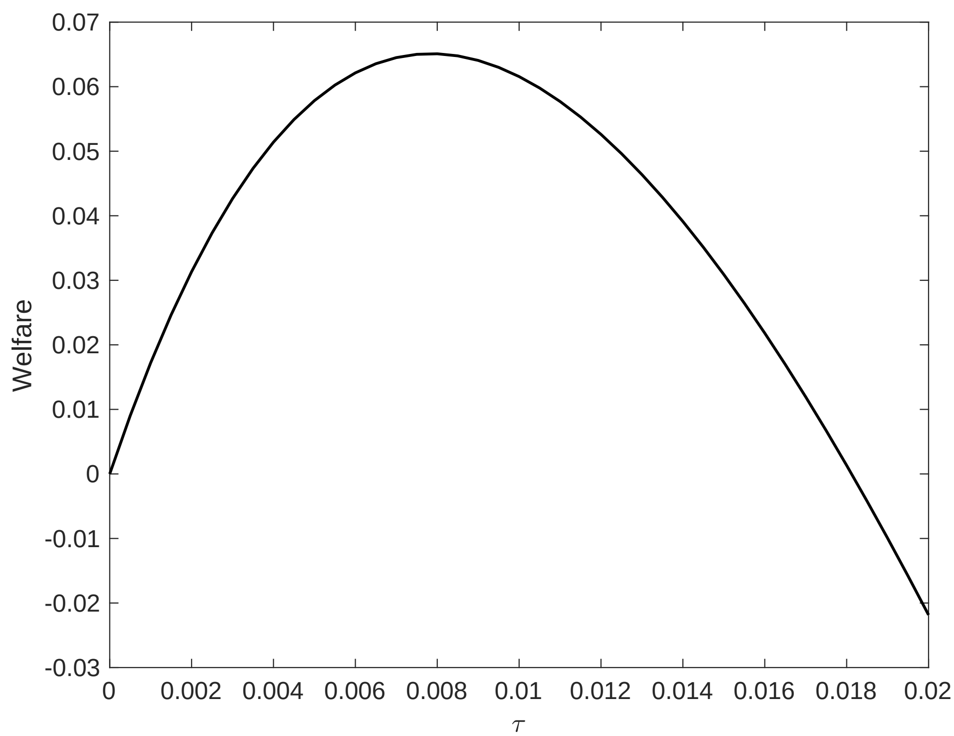


Figure 1·8: Welfare under Different Levels of Macroprudential Policy

Since the macroprudential policy weighs between the level and the volatility, the optimal level of macroprudential policy depends on the variance of the aggregate shock on the return of risky projects Z_t. Figure 1·9 shows how the optimal level of macroprudential policy τ^*, which minimizes the welfare cost $\xi(\tau)$, varies with the variance of the aggregate shock. When the variance of the shock is larger, the need to reduce the volatility of the economy by a tighter macroprudential policy is stronger, hence the optimal level of macroprudential policy τ^* is larger. Moreover, when the variance of the aggregate shock is low enough, the optimal level of macroprudential policy is actually negative, which means the regulator wants to subsidize the production of private safe assets.

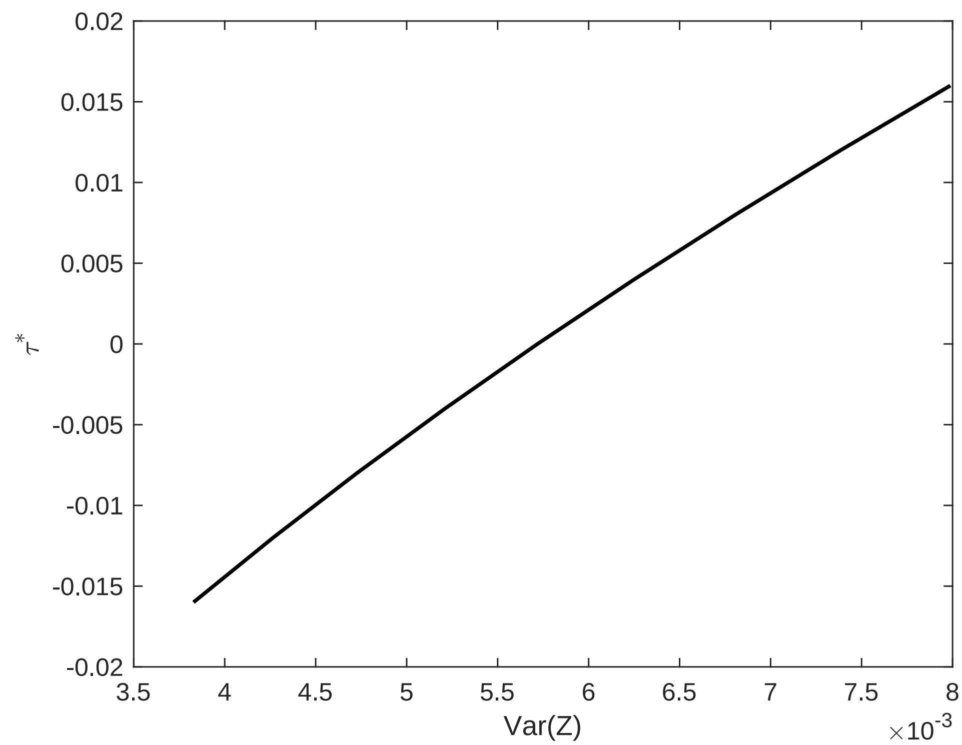


Figure 1·9: Optimal Macroprudential Policy Varies with the Variance of the Aggregate Shock

The optimal level of macroprudential policy also varies with the severity of the shortage of safe assets. Figure 1·10 shows how the optimal level of macroprudential policy varies with the demand for safe assets from the household

F. Higher demand for safe assets from the household means a more severe shortage of safe assets. The figure shows that the optimal macroprudential policy is non-monotone in the severity of the safe asset shortage. This non-monotone relation is the result of two opposite forces. On the one hand, when the shortage of safe assets gets more severe, the regulator would loosen the macroprudential policy to encourage bank to produce more private safe assets to fill the gap. This force makes the optimal tax rate decrease with the degree of safe asset shortage, which is reflected on the left part of this figure. On the other hand, when shortage of safe assets gets very severe, the banking sector endogenously becomes larger and so does its risk exposure. Hence the optimal policy should be tighter to restrict the size of the banking sector so the optimal tax rate increases. From this figure we see that when the shortage is not very severe, the safe asset shortage motivation dominates. When the shortage is more severe, the risk exposure concern dominates. The combination of these two forces results in this non-monotone relation between optimal level of macroprudential policy and the degree of safe asset shortage.

1.5.3 Subsidizing the Entrepreneurs

Since it is the low-type entrepreneurs that really suffer from the shortage of safe assets, the regulator may use the revenue from taxing the bank to subsidize the entrepreneurs. Figure 1·11 shows the welfare when the tax revenue is rebated to the entrepreneurs. Unlike the benchmark case when the tax revenue is rebated to the household, the welfare of subsidizing the entrepreneurs is monotonic increasing in the level of the tax. A tighter macroprudential policy not only reduces the bank's risk exposure but also help the entrepreneurs directly. This result suggests that when the low-type entrepreneurs are constrained in finding safe assets and the high-type entrepreneurs are more effi-

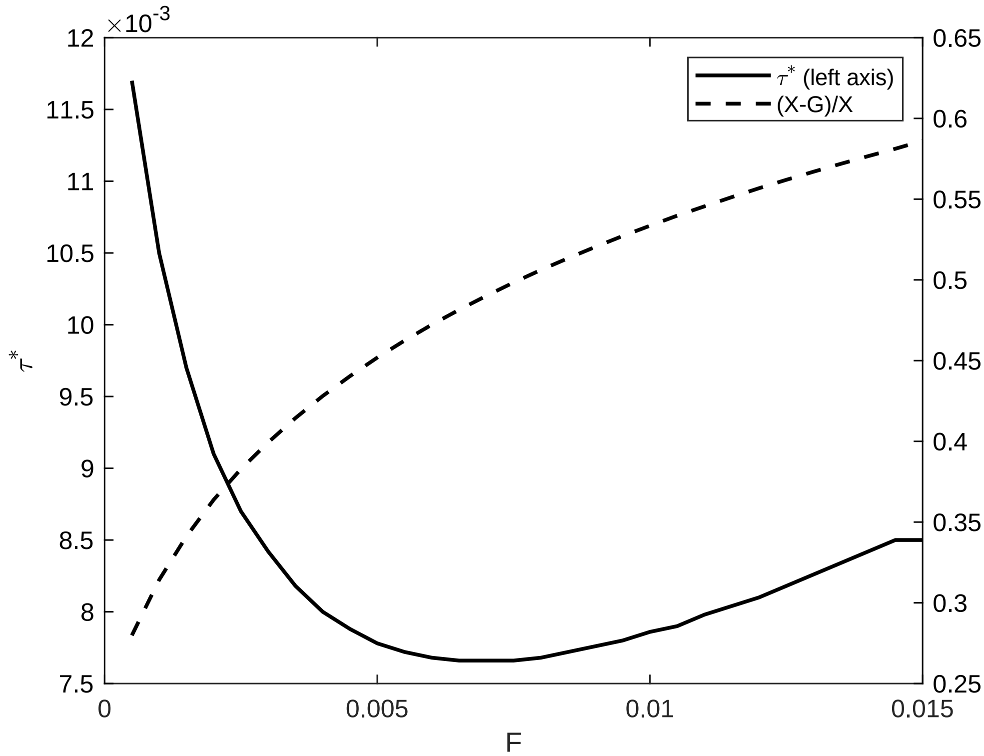


Figure 1·10: Optimal Macroprudential Policy Varies with the Severity of Safe Asset Shortage

cient in investment than the bank, it is optimal to tax the bank and subsidize the entrepreneurs.

1.5.4 Public Safe Assets

When the economy is short of safe assets, the government can provide more public safe assets by issuing government debt. I consider an extension of the model where both public and private safe assets co-exist as perfect substitutes. I assume that the government issue debt by taxing on the entrepreneurs. Unlike the bank where the private safe assets are backed by risky projects, the government debt is backed by future tax revenues. Thus issuing public safe assets does not increase the risk exposure of the economy. Moreover, when

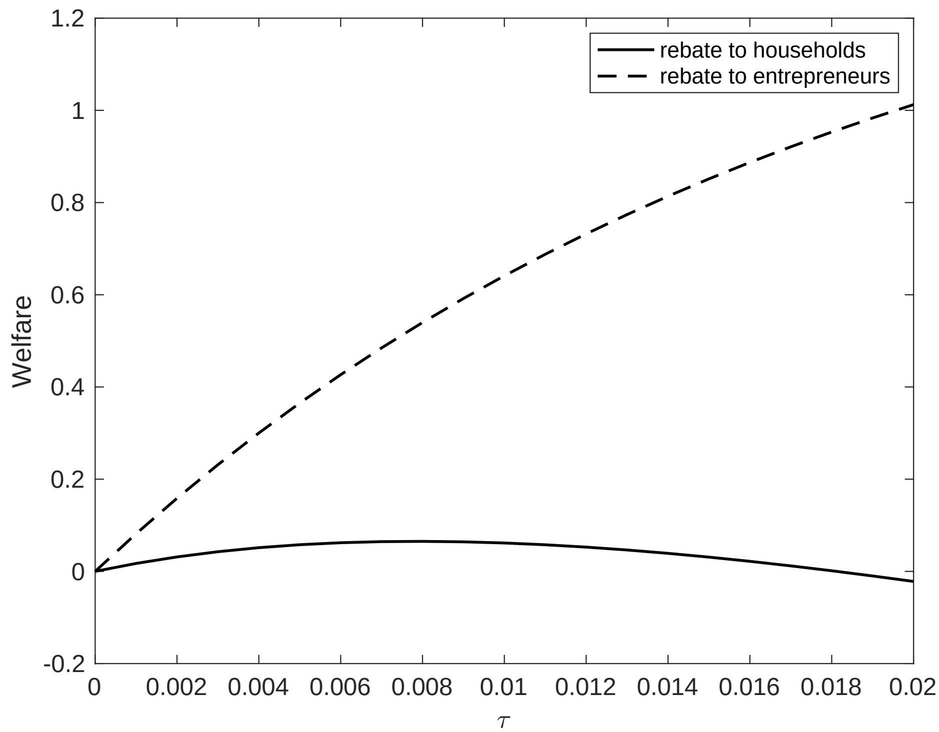


Figure 1·11: Welfare when Tax Revenue is Rebated to Entrepreneurs

the interest rate below one, the government can run a primary deficit while rolling over its debt. However, too much debt will increase the tax burden on the entrepreneurs and thus reduces investment and output. Issuing public safe assets is equivalent to a policy that transferring wealth from the high type entrepreneurs to the low type, since only the low type entrepreneurs will buy the government bonds.

Figure 1·12 shows the welfare under different level of public safe asset. The horizontal axis is the debt to output ratio. When the amount of government debt is low, the cost of issuing government debt low. Issuing more government debt not only alleviates the safe assets shortage problem, but also crowds out private safe assets and reduces the risk exposure of the bank. However when the level of debt is high, the tax burden on the entrepreneurs becomes large, which is welfare reducing. Under the current calibration, the optimal level of debt to gap ratio is around one, which coincidentally matches with the current level U.S. debt to GDP ratio. But I do not want to claim too much about this result, because my model is highly stylized and focuses more on qualitative results rather than quantitive results. The bottom line is that fiscal policy is important in terms of providing safe assets to the economy, and my model suggests that government should issue some public safe assets, and leave the rest to the private banking sector.

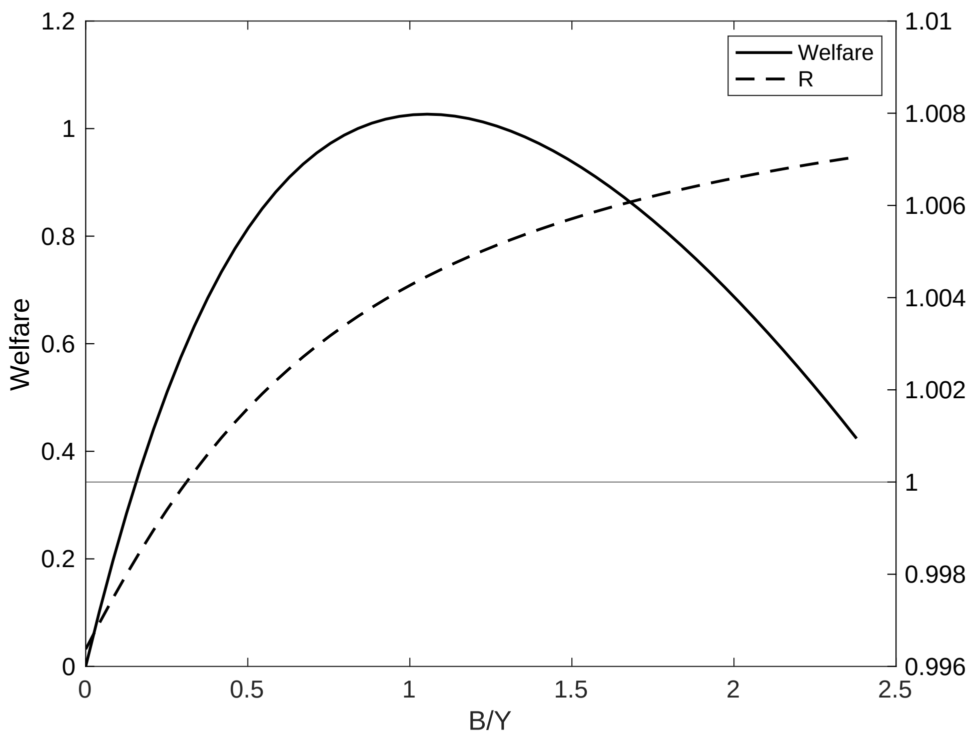


Figure 1·12: Welfare under Different Levels of Government Debt

1.6 Conclusion

This paper studies the macroprudential policy tradeoff between the benefit and the risk associated with the production of private safe assets when the economy is short of safe assets. A shortage of safe assets lowers the return of safe assets, hinders the function of private safe assets as a store of value, and leads to a reduction in the entrepreneurs' wealth and output. The banking sector expands to produce more private safe assets as the shortage gets more severe but exposes itself to more aggregate risk. Macroprudential policies can adjust the production of private safe assets with a tradeoff: encouraging the production of private safe assets alleviates the shortage of safe assets, at the cost of more risk exposure of the bank. The optimal level of the macroprudential policy depends on the variance of the aggregate shock and the severity of the safe asset shortage.

Since the model is solved by perturbation around the steady state, the size of the aggregate shock used in the simulation is too small to reflects big negative events like the financial crisis. The model is also abstract from many important amplification mechanisms in the financial sector. Thus the results of this model are more qualitative rather than quantitative. If one thinks of larger shocks that are closer to the magnitude of the financial crisis or combines them with other amplification mechanisms like bank runs, the negative impact of a safe asset shortage would be more sizable and the optimal macroprudential policy regarding the shadow banking sector is likely to be tighter.

Chapter 2

Leverage of Financial Intermediaries in the Crisis

2.1 Introduction

The 2007-2008 financial crisis makes people realize the importance of the financial sector. One important stream of research represented by Kiyotaki and Moore (1997); Bernanke et al. (1999); Gertler and Karadi (2011); Gertler and Kiyotaki (2010) emphasized the role of borrowers' balance sheets in constraining credit. However, this series of research did not capture the dynamics of bank leverage observed in the data, which were important since financial institutions were highly leveraged. The 2007-2008 financial crisis featured a sharp deleverage of many financial institutions in the shadow banking sector (Tian, 2019), including investment banks, security brokers and dealers, and hedge funds. These institutions acted as financial intermediaries that channeled funds from investors to borrowers, but they differed from traditional banking in that they were financed largely by short-term wholesale borrowing from the money market. When the initial crisis in the subprime mortgage market spread to the money market in late 2008, the shadow banks found themselves have funding difficulties. Creditors were concerned about borrowers' solvency and reluctant to lend. Shadow banking borrowers were required for higher margins when borrowing against collaterals, for example in the Repo market (Gorton and Metrick, 2012). As the result, financial institutions that relied heavily on short-term funding were forced to deleverage, reduced their asset holdings, and ultimately reduced intermediation activity.

In this paper, I first show empirical evidence about the patterns of leverage of investment banks, security brokers and dealers, and commercial banks. I find that the leverage of shadow banks, represented by large investment banks and security brokers and dealers, declined sharply in late 2008 and 2009. By contrast, the leverage of commercial banks remained stable during the crisis. The different dynamics of leverage suggest that different types of financial intermediaries had different leverage determination mechanisms. Investment banks, security brokers and dealers, and perhaps many other financial intermediaries in the shadow banking sector, relied heavily on short-term borrowing from the money market in a wholesale manner. Their leverage was market-driven and positively related to market conditions. On the contrary, commercial banks received deposits that are more stable than wholesale funds. They were often the creditors on the money market and they lent money to shadow banks. Following this intuition, I find that financial institutions with more short-term wholesale funding right before the crisis tended to have lower leverage growth, or deleveraged more, during the crisis.

I then build a general equilibrium model with both shadow banks and commercial banks to distinguish their different leverage patterns. The commercial banks receive deposits from households. Combined with their equity, they lend to firms directly or lend to shadow banks in the interbank market. The shadow banks borrow from the commercial banks and then lend to a different group of firms. The two-layer banking structure is similar to that in Gertler et al. (2016). The key difference between the shadow banks and the commercial banks is how their leverage is determined. The leverage of the shadow banks is determined by the degree of risk tolerance of their creditors. In a recession, the shadow banks have higher uncertainty about future returns, so their creditors are more concerned and lend at a lower leverage. In particular, I assume that the creditors set the leverage of the shadow banks following a fixed Value-at-Risk (VaR) rule (Brunnermeier and Pedersen, 2009; Adrian and Shin, 2014). For the commercial banks, I assume that their leverage is determined by the agency friction between the bankers and the depositors as in Gertler and Karadi (2011): the commercial bankers may divert a fraction of their total assets for personal use; the depositors limit the leverage of the commercial banks to keep them from doing so.

My model predicts a sharp decline in the shadow bank leverage after an uncertainty shock, modeled as an exogenous increase in the volatility of the productivity of firms. A rise in productivity volatility widens the return distribution of shadow banks and lowers the leverage allowed by the creditors. A first-order productivity shock cannot generate such a deleverage since, after a negative productivity shock, the return of shadow banks becomes higher than normal due to the lower capital price. Hence the shadow banks have higher leverage rather than lower. This is consistent with the finding of Nuño and Thomas (2017). The adverse uncertainty shock also generates the 'flight-to-quality' phenomenon that the commercial banks reallocate assets from interbank lending to loans to safe firms. In summary, the two-layer banking model with uncertainty shocks successfully generates the observed deleverage of the shadow banking sector and the 'flight-to-quality' of the commercial banking sector.

Related Literature The leverage of the financial sector is substantially studied, especially after the financial crisis. On the empirical side, Adrian and Shin (2010) find that the leverage growth of investment banks and security brokers and dealers is positively correlated with assets growth, suggesting pro-cyclical leverage for these financial institutions. In a following paper Adrian and Shin (2014), they propose that these financial institutions follow a fixed Value-at-Risk (VaR) rule. They build a micro-founded model showing that the fixed VaR rule can be derived from an optimal contracting problem when banks have limited liability. Brunnermeier and Pedersen (2009) provide the institutional background that the lenders set margin requirements according to the VaR rule. Gorton and Metrick (2012) use a novel data set to study the repo market. They find concerns about the liquidity of the repo collaterals led to increases in the repo haircuts, implying lower leverage for banks borrowing in the repo market during the financial crisis. Krishnamurthy et al. (2014) argue that the contraction in the Repo market has a disproportionately large effect on a few important dealer banks. He et al. (2010) find that sectors dependent on Repo financing have reduced asset holdings while the commercial banking sector increased asset holdings. Ang et al. (2011) find that the leverage of hedge funds is pro-cyclical. In summary, the previous literature finds that the leverage of the shadow banking sector is pro-cyclical. My empirical finding confirms the literature and complements it by pointing out that whether a bank's leverage declined during the financial crisis depended on its funding sources. Banks that relied more on short-term wholesale funding tended to experience a sharper deleverage.

For theoretical work, Nuño and Thomas (2017) study a model with endogenous leverage constraints under the optimal contract setup proposed by

Adrian and Shin (2014). The presence of risk-shifting moral hazard gives rise to a leverage constraint and creates a link between the volatility in bank asset returns and leverage. They find only volatility shocks can produce empirically plausible fluctuations in bank leverage. In this paper, I assume that shadow banks follow a fixed VaR rule, which can be derived by the optimal contract problem if the return follows generalized extreme value distribution Adrian and Shin (2014). Tian (2019) develop a model with endogenous bank default and aggregate uncertainty fluctuation to study the dynamics of shadow banking. Second-moment shock reproduces the large interbank spread spike, dramatic deleveraging, and contraction in the shadow banking sector during the crisis. In his paper, the leverage is optimally chosen by the bank itself while in my model the bank wants as much leverage as possible and the maximum leverage is dictated by the market. Nonetheless, both papers emphasize the importance of uncertainty or volatility shock. Geanakoplos (2010) studies the leverage cycle from the perspective of heterogeneous beliefs. In his model, the leverage is endogenously chosen to eliminate default and the main effect of deleveraging is on asset prices.

This paper borrows from the literature on financial intermediation and its amplification effect on business cycles. (Gertler and Kiyotaki, 2010; Gertler et al., 2012; Gertler and Kiyotaki, 2015) develop a framework to study the role of financial sector in business cycles. Banks intermediate funds from households to firms. Bank's ability to intermediate is constraint by its own equity due to the agency friction. Negative shock reduces banks' equity, thus largely reduces bank's lending to firms. This channel is called the balance sheet channel. It works even if banks' leverage remains constant. One minor defect of this agency-friction determined leverage is that banks' leverage increases after a negative shock, which does not capture deleverage of the shadow banking sector observed in the data. The deleveraging of the shadow banks generated by uncertainty shocks in my model is a distinct channel and is quantitatively important to study the recent financial crisis. The two-layer of banks setting in my model is very similar to Gertler et al. (2016) that study runs on wholesale banks.

The paper is organized as follows: Section 2.2 shows the empirical evidence about the leverage of various financial institutions. Section 2.3 constructs a two-layer bank model where commercial banks and shadow banks have different leverage determination mechanisms. In section 2.4, I calibrate the model and study its implications. Section 2.5 concludes.

2.2 Empirical Evidence of Bank Leverage

In this section, I show empirical evidence about the leverage of investment banks, security brokers and dealers, and commercial banks. The first two are important players of the shadow banking sector. Following Adrian and Shin (2010), I collect balance sheet data of the 8 largest investment banks in the US: Bear Stearns, Goldman Sachs, Lehman Brothers, Merrill Lynch, Morgan Stanley, Bank of America, Citigroup, and JPMorgan Chase. The first five are formally investment banks while the last three are usually defined as commercial banks. However they have absorbed some of the largest independent investment banks (Citibank acquired Salomon Brothers in 1998, Chase acquired JPMorgan in 2000 and Bear Stearns in 2008, and Bank of America acquired Merrill Lynch in 2008), so they all had investment bank business.

Aggregate data for the security dealers and brokers sector are from the Financial Accounts of the United States (Z.1). Brokers and dealers are an important link in the transmission of funds from savers to investors[1]. They buy and sell securities for a fee, hold an inventory of securities for resale, or do both. They play an important role in financial intermediation. Investment banks often have broker or dealer businesses. The 8 largest investment banks are all primary dealers of the Federal Reserve. Therefore the concept of 'investment banks' and 'security brokers and dealers' are overlapped. Finally, I collect balance sheet data for all U.S. commercial banks from their call reports from the Federal Reserve. I constructed a balanced sample for commercial banks from 1990:I to 2010:I.

2.2.1 Leverage of Investment Banks

Figure 2·1 shows the asset-weighted mean leverage of the eight largest investment banks in the United States. The leverage started to rise in 2005 and reached its local climax right before the financial crisis. During the crisis, the leverage plunged sharply by almost a half. It then remained low after the crisis. The sharp deleveraging of the investment banks was only relevant for the 2007-2008 crisis. There was no such deleveraging in the early 1990s and early 2000s recessions. This suggests that the 2007-2008 financial crisis was different from the previous recessions that the financial sector played a more important role.

Figure 2·2 is the scatter plot between the leverage growth rate of the investment banks and the GDP growth rate for the sample covering the financial crisis (from 2004:I to 2018:I). The figure shows a positive relationship between the leverage growth and the GDP growth. The linear regression of the leverage growth on the GDP growth yields a coefficient of 2.46 and an R-square of 31.9%. The result suggests that investment banks have pro-cyclical leverages

[1]See Z.1: Financial Accounts of the United States - All Table Descriptions.

and confirms with Adrian and Shin (2010) and Nuño and Thomas (2017). This positive relationship between the leverage growth and the GDP growth is driven mainly by the sharp deleveraging in the 2007-2008 financial crisis.

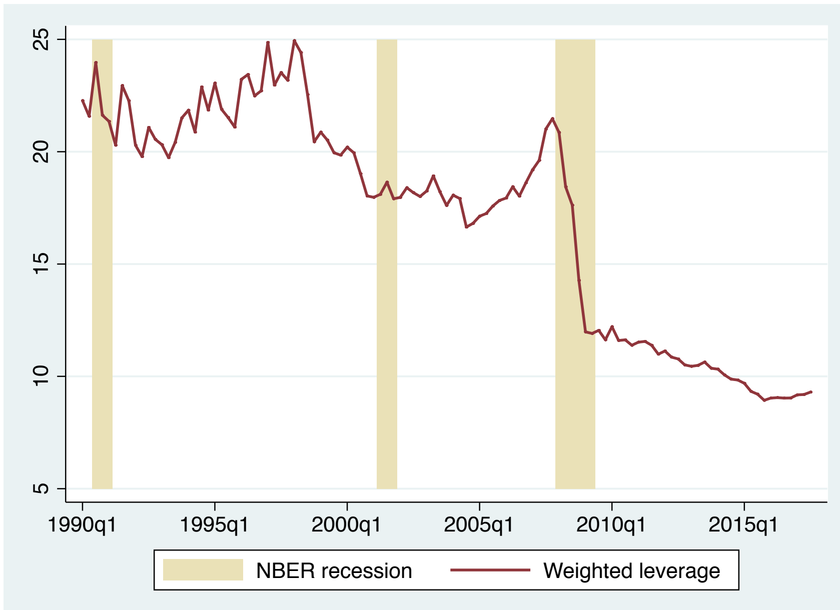


Figure 2·1: Leverage of Investment Banks

2.2.2 Leverage of Security Brokers And Dealers

Figure 2·3 shows the leverage of the security broker and dealer sector from 1990 to 2018. Like the investment banks, security brokers and dealers also increased their leverage around 2004 till the financial crisis and then deleveraged sharply during the crisis. Again there were no such deleverage patterns in the previous recessions.

Figure 2·4 is the scatter plot between the leverage growth of the security broker and dealer sector and the GDP growth for the sample covering the financial crisis (from 2004:I to 2018:I). The figure shows a similar positive relationship between the leverage growth and the GDP growth as the investment

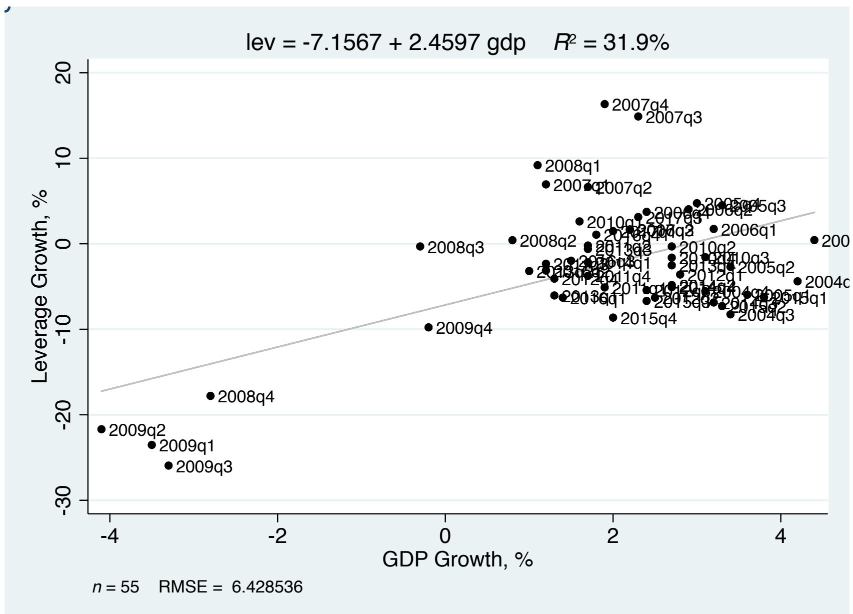


Figure 2·2: Leverage and GDP Growth, Investment Banks

banks. The linear regression of the leverage growth on the GDP growth yields a coefficient of 5.08 and an R-square of 50.8%.

2.2.3 Leverage of Commercial Banks

The leverage of commercial banks has a different pattern than investment banks and brokers and dealers. Commercial banks are very different in their size, ranging from a few million to 1.7 trillion with a median of 75 million. The sample distribution is highly skewed with skewness of 57. Thus, in Figure 2·5, I plot the mean leverage across different asset quantile groups. Banks in the lower asset quantile groups did have show a deleveraging pattern during the financial crisis. For most commercial banks, their leverage remained stable during the financial crisis. Only the largest 1% banks, or the 50 largest U.S. banks, experienced a moderate leverage decline in the later period of the financial crisis. These largest banks often have investment banking business

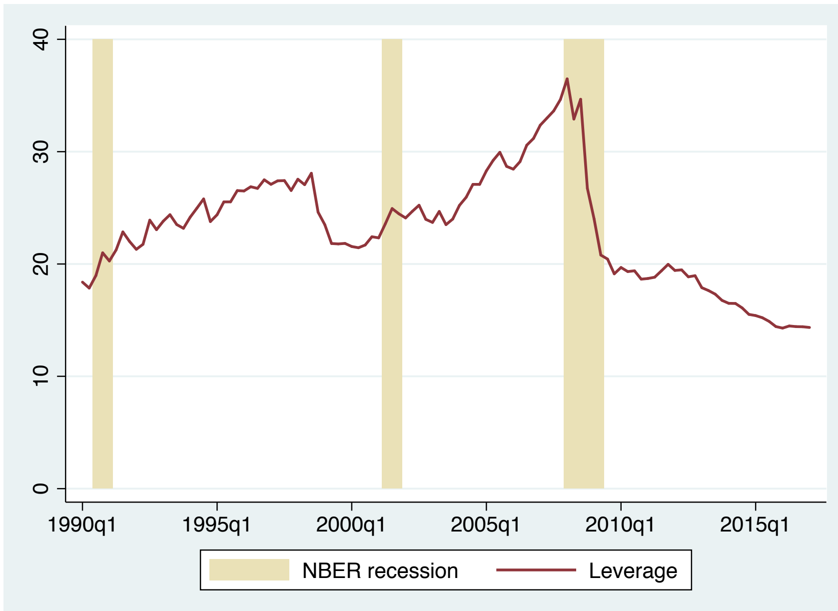


Figure 2·3: Leverage of Security Broker and Dealer

or broker and dealer functions, therefore share a similar deleveraging pattern with investment banks and the aggregate broker and dealer sector. Larger banks also had higher leverage before the crisis. The result is robust to other quantile cutoffs. Figure 2·6 plots the leverage growth during the financial crisis (from 2007:IV to 2010:IV) across all 100 quantiles of average assets. Only those banks in the largest quantiles experienced a leverage decline during the financial crisis.

To state this result more formally, I follow Adrian and Shin (2010) to regress the banks leverage growth on the asset growth to measure the cyclicality of leverage[2]. As shown in Table 2.1, the leverage of the investment banks, and the broker and dealer sector present a strong pro-cyclical pattern: the leverage grows with the total asset. The coefficients are positive and the R-square are large. In contrast, the leverage of most commercial banks are a-cyclical.

[2] I exclude Bank of America in the regression since it's leverage is counter-cyclical.

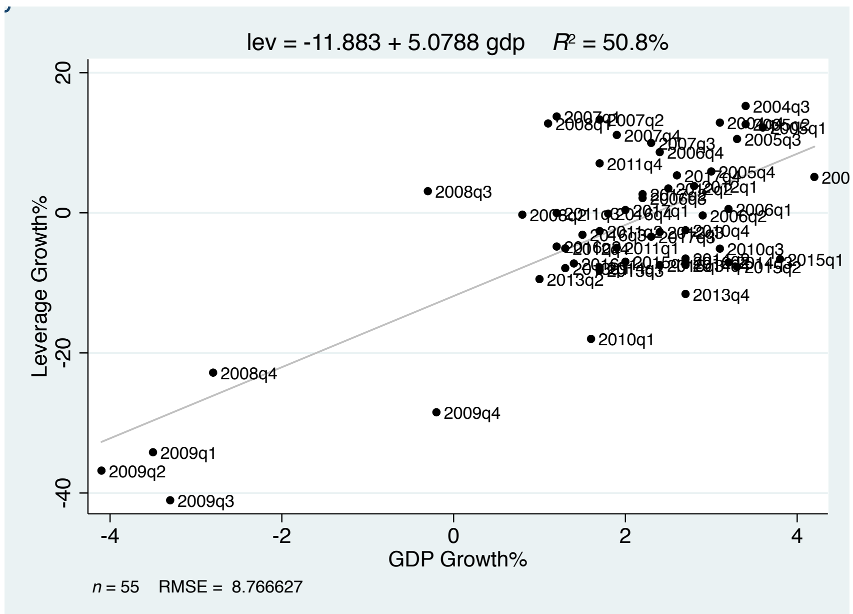


Figure 2·4: Leverage and GDP growth, Security Broker and Dealer

Both the coefficient and the R-square are fairly small. Only the largest 1% commercial banks have pro-cyclical leverage. The coefficient and the R-square are similar to those of investment banks and the broker and dealer sector. Therefore, I conclude that the shadow banking sector deleveraged sharply in the crisis while most commercial banks maintained a stable leverage. Also the sample of all commercial banks have some overlapping with the shadow banking sector at the top of the size distribution. Therefore I use this sample to explore what factor caused this contrasting leverage pattern between shadow banks and commercial banks.

2.2.4 Deleverage and Funding Source

The different leverage patterns for the shadow banking sector and the commercial banking sector are related to their funding sources. He et al. (2010) argue that the leverage of financial institutions depends on how they get fi-

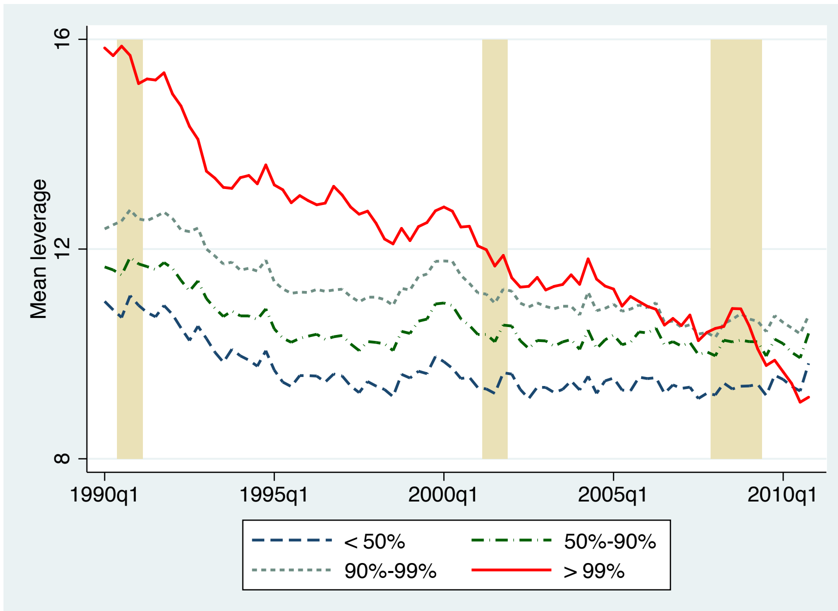


Figure 2·5: Leverage of Commercial Banks by Asset Quantile

nanced. Sectors dependent on short-term wholesale financing, which describes the shadow banking industry, reduced their leverage during the most recent crisis. Figure 2·7 shows that security brokers and dealers indeed rely heavily on short-term wholesale funding. Security repurchase agreement accounts for more than 60% of total liabilities before the crisis. When borrowing heavily on the wholesale money market, their leverage is set by the creditors and is sensitive to the market conditions, especially in a panic financial crisis. On the other hand, commercial banks receive household deposits, which is more stable funding source since it enjoys the deposit insurance. Figure 2·8 shows the major liabilities of U.S. chartered depository institutions, which is a synonym for commercial banks. Time and saving deposits account for about 60% of total liabilities of commercial banks. Besides, commercial banks have also access to Federal Reserve discount window loans throughout the crisis He et al. (2010). The government also injected huge liquidity into large commercial banks help-

Figure 2·6: Leverage Growth of Commercial Banks in the Crisis by Asset Quantile

ing them to acquire investment banks that were in trouble. As a result, they did not experience the deleverage as shadow banks.

To further study the relationship between leverage and funding sources, I run the following regression to explore whether the bank's reliance on short-term wholesale funding affects the bank's leverage dynamics during the crisis:

$$LeverageGrowth_i = \alpha + \beta ShortTermFunding_i + \gamma Controls + \epsilon_i$$

The dependent variable is the leverage growth rate of a bank in the crisis period (from 2007:IV to 2010:IV). I use a bank's share of Repo borrowing over total assets as a measure of a bank's reliance on short-term funding. I first calculate a bank's net Repo borrowing position by subtracting Repo lending (quarterly average of Fed funds sold and securities purchased under Repo) from Repo borrowing (quarterly average of Fed funds purchased and securities sold

	Quantile	Coefficient	R-square
Investment Banks		0.60	48.1%
Security Brokers and Dealers		0.89	78.6%
Commercial Banks	< 50%	-0.08	0.7%
	50% - 90%	0.10	0.7%
	90% - 99%	-0.04	0.4%
	> 99%	0.47	24.0%

Table 2.1: Regression Results: Leverage Growth on Asset Growth

under Repo). I then calculate the net Repo borrowing share over total assets. Then for each bank I calculate its average net Repo borrowing share across the pre-crisis period (from 2005:I to 2007:IV). I also run the same regression using positive and negative Repo positions separately as a robustness check. Since larger banks and banks with higher pre-crisis leverage were more likely to deleverage during the financial crisis (Figure 2·5), I control for log asset and average leverage before the financial crisis. The regression result is shown in Table 2.2.

Banks with more net Repo financing before the crisis experienced more deleveraging (less leverage growth) during the crisis. One percent increase in the share of net Repo borrowing before the financial crisis reduces the leverage growth rate in the crisis by -0.53 percentage point (column 2). The regressions also show that banks with higher pre-crisis leverage experience larger leverage declines during the crisis. Without the net Repo share, larger banks tend to deleverage more during the crisis, which is consistent with the results in Figure 2·5 and Figure 2·6. However, once the net Repo share is included in the regression, the bank size becomes insignificant.

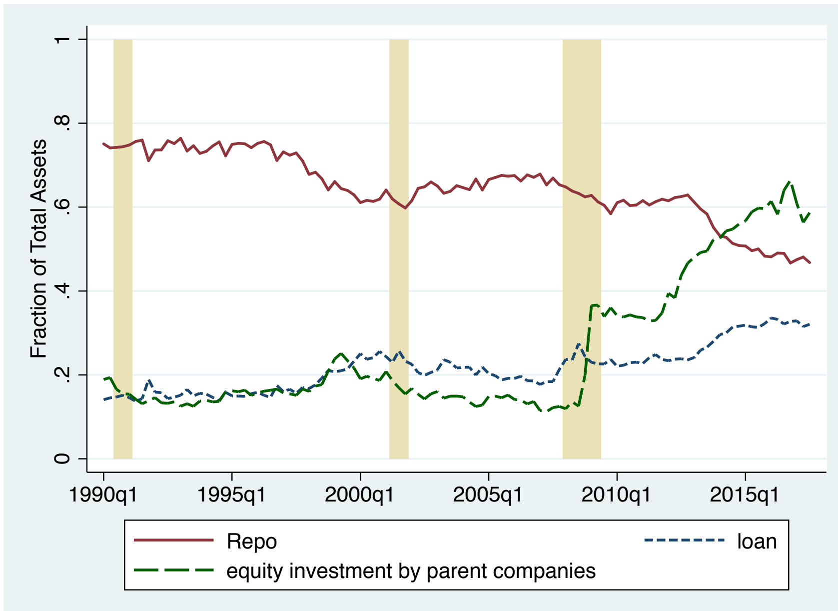


Figure 2·7: Major Fundings of Security Brokers and Dealers

One concern about the above regression is that banks with more short-term wholesale borrowing may mechanically have less deposit funding, which is argued to be more stable during the financial crisis. Hence I also include the bank's deposit share of total liability as a control variable (column 3). The result suggests that having more deposits as a funding source does not help mitigate the deleveraging during the crisis.

One alternative interpretation of the regression results is that banks deleveraged during the financial crisis because they had lower quality. Banks with inferior ability to invest suffered bigger losses and therefore deleveraged more during the crisis. These banks might not be able to attract deposits as a stable funding source and thus relied more on wholesale short-term funding. The regression may have omitted variable bias. In column 4, I control for the bank's pre-crisis return on assets (ROA) as a proxy for the bank's ability and quality. Including ROA does not change the coefficient and significance of net Repo

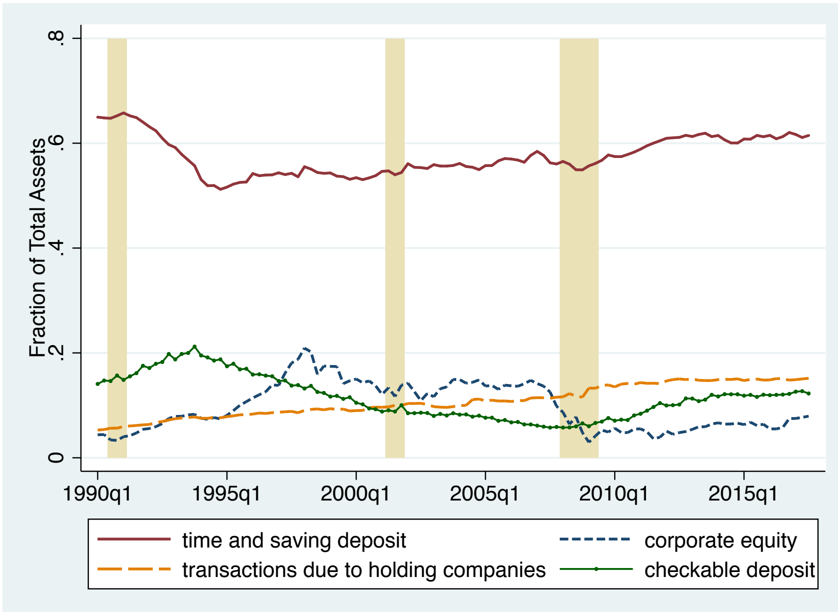


Figure 2·8: Major Fundings of U.S. Chartered Depository Institutions

share. The variable ROA itself is not significant. The coefficient for ROA is negative suggesting that banks with higher pre-crisis ROA tend to experience more deleveraging during the crisis. This result is more consistent with the story that some banks took advantage of the pre-crisis boom by borrowing more in the wholesale market and achieved higher profits, but then suffered bigger losses during the crisis and were forced to deleverage.

In summary, the empirical evidence shows that the shadow banking sector, i.e., investment banks and brokers and dealers experienced sharp deleveraging during the crisis while most commercial banks did not. These contrasting leverage dynamics are related to the difference in the funding sources: banks that relied more on short-term wholesale funding experienced more deleveraging during the crisis. These findings, putting together, suggest financial institutions should be modeled separately depending on their funding sources and leverage dynamics. In particular, commercial banks and large investment banks should have different leverage determination mechanisms. As suggested by He et al. (2010), "... the right model to understand the adjustments in 2008 is the one that emphasizes leverage constraints on shadow banking sector (hedge funds, broker/dealer, etc.) and at the same time emphasizes equity risk-capital constraints on the traditional commercial banking sector." In the next section, I build a two-layer banking model following this idea.

	Leverage growth			
	(1)	(2)	(3)	(4)
Leverage	-2.082***	-1.924***	-1.928***	-1.974***
	(0.283)	(0.284)	(0.281)	(0.286)
Log asset	-1.062***	-0.199	-0.133	-0.123
	(0.401)	(0.458)	(0.466)	(0.446)
Net Repo share		-0.533***	-0.509***	-0.546***
		(0.131)	(0.158)	(0.135)
Deposit share			0.0413	
			(0.104)	
ROA				-1.277
				(3.225)
Constant	40.72***	27.79***	23.28*	28.32***
	(5.395)	(6.296)	(13.23)	(6.594)
Observations	5,034	5,034	5,034	5,034
R-squared	0.025	0.033	0.033	0.033

This table shows the results of regressing leverage growth on bank's reliance on short-term funding. The dependent variable is the leverage growth rate of a bank in the crisis period (from 2007:IV to 2010:IV). Net Repo share is a bank's Repo borrowing minus Repo lending, divided by total assets. All independent variables are average across the pre-crisis period (from 2005:I to 2007:IV). Robust standard errors are in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 2.2: Leverage Growth on Bank's Reliance on Short-Term Funding

2.3 A Two-Layer Banking Model

In this section, I build a model where banks have different funding sources and leverage determinations, and study how this setup affects the aggregate economy. The model is based on Gertler and Karadi (2011); Gertler et al. (2016), with the modification that I adopt different leverage determinations for different types of banks. The economy consists of households, goods-producing firms, capital firms, and most importantly, two types of financial intermediaries: a shadow banking sector and a commercial banking sector. The shadow banking sector represents financial institutions like investment banks and brokers and dealers that finance themselves mainly using short-term wholesale borrowing. The commercial banking sector represents traditional banks that finance themselves mainly by deposits. Each banking sector consists of a continuum of identical banks, hence I will use 'shadow banks' ('commercial banks') and 'the shadow banking sector' (the commercial banking sector) interchangeably.

Households save deposits in commercial banks. Commercial banks can either lend to shadow banks in the interbank market or invest in the firms. Shadow banks borrow from commercial banks and then invest in firms. I assume the possibility that the firms invested by commercial banks and shadow banks are different. In particular, shadow banks invest in riskier firms with higher productivity volatility. Both types of firms purchase capital from capital firms, hire labor from the households and produce final goods. Finally, the households earn wage income and get all the profit from banks and firms. They optimize between consumption and saving.

2.3.1 Good-producing Firms

Competitive good-producing firms combine capital and labor to produce final consumption goods using a decreasing returns technology. I allow the possibility that the firms invested by commercial banks and shadow banks are different. In particular, firms invested by shadow banks are riskier in terms of productivity volatility. I use the subscription r (risky) to represent firms invested by shadow banks, and subscription s (safe) to represent firms invested by commercial banks. Their production functions are

$$Y_{rt} = A_t K_{rt}^{\alpha} L_{rt}^{\nu}, \tag{2.1}$$

$$Y_{st} = A_t^{\tau} K_{st}^{\alpha} L_{st}^{\nu}, \tag{2.2}$$

where $\alpha + \nu < 1$ and $\tau \in (0, 1]$. A_t is the productivity (TFP) shock. Y_{rt}, Y_{st} are output, K_{rt}, K_{st} are capital and L_{rt}, L_{st} are labor of risky and safe firm respectively. Risky and safe firms differ only in their exposure to the productivity shock. Safe firms are safer because they have smaller exposure to productivity shock as $\tau \leq 1$.

Firms acquire funds to purchase capital from either commercial banks or shadow banks, depending on their type, at the end of each period. As in Gertler and Karadi (2011), I assume that banks can perfectly monitor the firms and there is no friction between firms and banks. Thus the firms offer the banks perfectly state-contingent debt (or equity) which is a claim of the firm's revenue after labor expenses. In particular, each unit of debt ensures that the firm can purchase exactly one unit of capital, so the amount of funds borrowed from the bank per unit of debt is the price of one unit of capital Q_t. Suppose the firm issue S_{t-1} unit of debt[3] at the end of period $t-1$ with total

[3]Firms actually do not choose how much debt to issue. The amount of debt the firm can

borrowing $Q_{t-1}S_{t-1}$. It enters the next period with $K_t = S_{t-1}$ unit of capital. The firm then solves a static problem choosing labor

$$\max_{L_t} A_t K_t^{\alpha} L_t^{\nu} - w_t L_t,$$

which gives the labor demand

$$w_t = \nu A_t K_t^{\alpha} L_t^{\nu-1}.$$

The firm then repays the residual profit

$$K_t Z_t = A_t K_t^{\alpha} L_t^{\nu} - w_t L_t$$

and the undepreciated capital $(1-\delta)K_t$ to the bank, where Z_t is the per-unit flow return of debt. The bank can then sell the old capital to the capital-producing firm at the price Q_t. Overall, the return for one unit of a firm's debt is

$$R_{kt} = \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}}.$$

Let S_{wt} and S_{ct} be the number of debt securities issued by the risky firms and safe firms respectively, which are also the amount of loans lent by the shadow banks and the commercial banks. The capitals for production next period are thus

$$K_{rt+1} = S_{wt}, \tag{2.3}$$

$$K_{st+1} = S_{ct}. \tag{2.4}$$

Let R^w_{kt+1} and R^c_{kt+1} represent the loan return for shadow banks and commer-

issue is determined by the bank, which is in turn determined by the bank's net worth and leverage.

cial banks. Following the above derivation, I obtain

$$R^w_{kt+1} = \frac{(1-\nu)A_{t+1}K^{\alpha-1}_{rt+1}L^{\nu}_{rt+1} + (1-\delta)Q_{t+1}}{Q_t}, \tag{2.5}$$

$$R^c_{kt+1} = \frac{(1-\nu)A^{\tau}_{t+1}K^{\alpha-1}_{st+1}L^{\nu}_{st+1} + (1-\delta)Q_{t+1}}{Q_t}. \tag{2.6}$$

The wage follows

$$w_t = \nu A_t K^{\alpha}_{rt} L^{\nu-1}_{rt}, \tag{2.7}$$

$$w_t = \nu A^{\tau}_t K^{\alpha}_{st} L^{\nu-1}_{st}. \tag{2.8}$$

2.3.2 Shadow Banks

The shadow banks borrow B_t from the commercial banks at rate R_{bt}. They have equity N_{wt}. They lend $Q_t S_{wt}$ to risky firms with return R^w_{kt+1}. Thus the balance sheet of the shadow banking sector is

$$Q_t S_{wt} = N_{wt} + B_t. \tag{2.9}$$

Denote Φ_{wt} as the leverage of shadow banks:

$$Q_t S_{wt} = \Phi_{wt} N_{wt}. \tag{2.10}$$

Shadow banks earn the interest rate spread between R^w_{kt+1} and R_{bt}. Over time, shadow banks' equity evolves as the difference between earnings on assets and interest payments on liabilities:

$$\begin{aligned} N_{wt+1} &= R^w_{kt+1} Q_t S_{wt} - R_{bt} B_t \\ &= (R^w_{kt+1} - R_{bt}) Q_t S_{wt} + R_{bt} N_{wt} \\ &= [(R^w_{kt+1} - R_{bt})\Phi_{wt} + R_{bt}] N_{wt}. \end{aligned}$$

To capture the sharp deleverage of the shadow banking sector during the financial crisis, I assume that the leverage of the shadow banking sector is set by its creditor according to a fixed Value-at-Risk rule (VaR). VaR is a measure of the loss distribution defined as the smallest threshold loss l such that the probability that the realized loss turns out to be larger than l is below some probability p. Let the asset next period is a random variable depending on the realization of shocks, then:

$$\text{VaR}(p)_t \equiv \arg\inf_l Pr(\text{Asset}_{t+1} < \text{Asset}_t - l) \leq p.$$

If a bank were to manage its risk by maintaining p percent Value-at-Risk to be no larger than its equity capital, i.e., $\text{VaR}(p)_t \leq N_{wt}$, the bank would ensure that it remains solvent with probability at least $1-p$. Given a negative productivity shock, higher leverage amplifies the losses and VaR. So a bank following the fixed-VaR rule needs to deleverage in response to bad shocks. Brunnermeier and Pedersen (2009) documented that hedge funds and broker and dealers borrow on margins from their lenders. The guiding principle for margin setting on levered positions is that the lender should be relatively immune to the borrower's possible losses. This is exactly the essence of the fixed-VaR rule. Adrian and Shin (2014) find that large US investment banks roughly follow the fixed VaR rule.

I assume that the shadow bank's leverage is set according to the fixed-VaR rule so that bank's probability of default is p percent. A shadow bank goes bankrupt if its equity is less than zero. This will happen if the realized return on lending R^w_{kt+1} is too small: $R^w_{kt+1}\Phi_{wt} - R_{bt}\Phi_{wt} + R_{bt} \leq 0$. So the maximum leverage allowed by the financial market is

$$\Phi_{wt} \leq \frac{R_{bt}}{R_{bt} - R_{st}}, \tag{2.11}$$

where R_{st} is the p-percent lowest expected realization of shadow bank's next period return R^w_{kt+1}, which in turn depends on the realization of A_{t+1} and Q_{t+1} according to (2.5). This is the threshold return at which the bank is at the brink of bankruptcy.

I assume that the log aggregate productivity $a_t = \log(A_t)$ follows an AR1 process:

$$a_{t+1} = \rho_a a_t + \sigma_t \epsilon_{at+1}, \tag{2.12}$$

where σ_t is the time-varying volatility of the log productivity. It also follows an AR1 processes:

$$\sigma_{t+1} - \sigma = \rho_s(\sigma_t - \sigma) + \epsilon_{st+1}. \tag{2.13}$$

Here ϵ_{at} is the first-order TFP shock and ϵ_{st} is the uncertainty shock. Both are standard normal distributed. σ is the steady state value of productivity volatility. Given today's a_t and σ_t, the p percent worse case of tomorrow's log productivity is $\sigma_t\Psi^{-1}(p) + \rho a_t$ where $\Psi(\cdot)$ is the CDF of standard normal distribution. In principle, I need to solve for the p-percent lowest expected resale price of capital Q_{t+1} and the labor input next period L_{rt+1}, which requires numerical solutions. For simplicity, I assume that the financial market believes that $\log(Q_{t+1})$ is normally distributed in a similar way as productivity with mean $\log(Q_t)$ and variance σ_t^2. Also the market ignores any possible changes in L_{rt+1} and sets the cutoff return rate according to the expected value $E_t L_{rt+1}$. With such a simplifying assumption, the p-percent worst case of the return on lending to risky firms is:

$$R_{st} = \frac{\exp(\sigma_t \Psi^{-1}(p) + \rho a_t)(1-\nu)K_{rt+1}^{\alpha-1}(E_t L_{rt+1})^{\nu} + (1-\delta)\exp(\sigma_t \Psi^{-1}(p))Q_t}{Q_t}. \tag{2.14}$$

From Equation (2.14), either a decrease in the productivity level a_t or a rise in the productivity volatility σ_t will lower the threshold return R_s and thus lower the leverage. It will be clear later that adverse uncertainty shocks are crucial to generate the deleveraging for shadow banks.

Shadow banks face a limited leverage ratio because the financial market is concerned about their solvency. The market requires shadow banks to hold enough equity to cover up potential losses in most cases. I focus on the equilibrium where $R_{kt+1}^w > R_{bt}$ so that the shadow banks are earning excess returns and accumulating equity. To prevent banks accumulate too much equity and become too large, I choose a similar assumption as in Gertler and Karadi (2011) by assuming in each period shadow bankers pay out a fraction of $1-\theta_w$ of its net worth as dividends to the household. The bankers objective is to maximize the discounted stream of payouts back to the household. They want as much leverage as possible as long as they are earning excess returns $R_{kt+1}^w > R_{bt}$. As a result, the leverage constraint (2.11) is binding and the aggregate equity of the shadow banking sector evolves by[4]

$$N_{wt} = \theta_w[(R_{kt}^w - R_{bt-1})\Phi_{wt-1} + R_{bt-1}]N_{wt-1}. \tag{2.15}$$

[4]Although the leverage is determined by the VaR rule restricting the shadow bank's default probability, in this paper, I only consider small shocks near the deterministic steady state so that banks never actually default.

2.3.3 Commercial Banks

I model commercial banks in the same way as in Gertler and Karadi (2011) and Gertler et al. (2012). Commercial banks receive deposits D_t from the households and promise the depositor a risk-free return R_t. Combining with their equity N_{ct}, commercial banks lend to safe firms and shadow banks. Let B_t be the total lending to shadow banks and $Q_t S_{ct}$ be the amount of lending to safe firms, then the balance sheet of the commercial banking sector is

$$B_t + Q_t S_{ct} = N_{ct} + D_t. \tag{2.16}$$

In equilibrium the return of lending to safe firms and lending to shadow banks must be the same:

$$R_{bt} = E_t R^c_{kt+1}. \tag{2.17}$$

Commercial banks earn a positive interest spread between R^c_{kt+1}, R_{bt} and R_t. Over time, bankers' equity evolves as

$$\begin{aligned} N_{ct+1} &= R^c_{kt+1} Q_t S_{ct} + R_{bt} B_t - R_t D_t \\ &= (R^c_{kt+1} - R_t) Q_t S_{ct} + (R_{bt} - R_t) B_t + R_t N_{ct}. \end{aligned}$$

Similar to the shadow banks, I assume in each period commercial bankers pay out $1 - \theta_c$ of net worth as dividends to the household. The banker's objective is to maximize the expected discount value of retained earning, given by

$$\begin{aligned} V_t(S_{ct}, B_t, N_{ct}) &= \max E_t \sum_{\tau=0}^{\infty} (1-\theta_c)\theta_c^{\tau} \beta^{\tau+1} \Lambda_{t,t+1+\tau} N_{ct+1+\tau} \\ &= \max_{S_{ct+1}, B_{t+1}} E_t \{(1-\theta_c)\beta\Lambda_{t,t+1} N_{ct+1} \\ &\quad + \theta_c \beta \Lambda_{t,t+1} V_{t+1}(S_{ct+1}, B_{t+1}, N_{ct+1})\}, \end{aligned}$$

where $\Lambda_{t,t+1} = C_t/C_{t+1}$ is the SDF of households. Since lending to safe firms and lending to the shadow banks have the same expected payoff, what matters for the bank's value is the total assets of commercial banks. Denote S_{at} to be the total asset:

$$S_{at} = B_t + Q_t S_{ct}. \tag{2.18}$$

I show the value of commercial banks can be written as

$$V_t = V_{st} S_{at} + V_{nt} N_{ct},$$

$$V_{st} = E_t\{(1-\theta_c)\beta\Lambda_{t,t+1}(R^c_{kt+1} - R_t) + \theta_c \beta \Lambda_{t,t+1} y_{t,t+1} V_{st+1}\}, \tag{2.19}$$

$$V_{nt} = E_t\{(1-\theta_c)\beta\Lambda_{t,t+1} R_t + \theta_c \beta \Lambda_{t,t+1} z_{t,t+1} V_{nt+1}\}, \tag{2.20}$$

where

$$y_{t,t+1} = \frac{S_{at+1}}{S_{at}}; \ z_{t,t+1} = \frac{N_{ct+1}}{N_{ct}}.$$

Similar to the shadow banks, commercial banks earn a positive profit on the positive interest spread and thus want to expand as much deposit as possible. To restrict the banks' size, I follow the agency friction assumption used by Gertler and Karadi (2011). It is assumed that the banker has an option to divert a fraction λ of the total assets for personal use. In order to incentivize bankers not to do so, the following incentive constraint must be satisfied:

$$\lambda S_{at} \leq V_t.$$

Denote Φ_{ct} to be the leverage of commercial banks:

$$S_{at} = \Phi_{ct} N_{ct}. \tag{2.21}$$

In equilibrium, the incentive constraint must be binding. Combining with the expression for the value function, I derive the leverage for the commercial banks:

$$\Phi_{ct} = \frac{V_{nt}}{\lambda - V_{st}}. \tag{2.22}$$

The leverage of commercial banks is determined by the agency friction between the bankers and the depositors. The agency friction measure λ itself does not vary with business cycle conditions. Nevertheless, the leverage of commercial banks varies because the banks' return and cost vary so do the implicit values of the bank's asset and equity (V_{st} and V_{nt}). Finally, the equity of the commercial banking sector evolves according to

$$N_{ct} = \theta_c[(R^c_{kt} - R_{t-1})Q_{t-1}S_{ct-1} + (R_{bt-1} - R_{t-1})B_{t-1} + R_{t-1}N_{ct-1})]. \tag{2.23}$$

2.3.4 Capital-producing firms

Capital-producing firms are in charge of capital investments and making new capitals. At the end of period t, competitive capital-producing firms buy used capital from the goods-producing firms (or their creditors), and then invest to build new capital. They then sell the new capital back to the good-producing firms. The cost of investment is normalized to unity and the new capital is sold at Q_t. Capital-producing firms face a convex investment adjustment cost $f(x) = \eta(x-1)^2$ where x is the ratio of investment over its steady-state value. Capital-producing firms choose an investment level to maximize

$$\max_{I_t} \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left\{ (Q_t - 1)I_t - f\left(\frac{I_t}{I}\right) I_t \right\}$$

where I is the steady state investment. This yields the first-order condition that determines Q_t:

$$Q_t = 1 + f\left(\frac{I_t}{I}\right) + f'\left(\frac{I_t}{I}\right)\frac{I_t}{I}. \tag{2.24}$$

The capital stock evolves according to

$$(K_{rt+1} + K_{st+1}) = (1-\delta)(K_{rt} + K_{st}) + I_t. \tag{2.25}$$

2.3.5 Households

A representative household consumes C_t and saves by making deposit D_t in the commercial banks. The household receives labor income $w_t L_t$. The household is the ultimate owner all firms and banks in the economy, and therefore gets income from the profits of these entities each period. The household maximizes expected utility subject to the budget constraint:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \ln C_{t+s},$$

$$C_t + D_t = w_t L_t + \Pi_t + R_{t-1} D_{t-1},$$

where Π_t is all the profits from banks and firms. Optimization yields the Euler equation:

$$E_t \beta R_t \Lambda_{t,t+1} = 1 \tag{2.26}$$

where $\Lambda_{t,t+1} = C_t / C_{t+1}$ is the stochastic discount factor. To focus on the banking sector, I assume that the household supplies one unit of labor inelastically so the labor market clearing condition is

$$L_{rt} + L_{st} = L_t = 1 \tag{2.27}$$

Finally, there is a resources constraint for the whole economy:

$$C_t + I_t = Y_{rt} + Y_{st}. \tag{2.28}$$

2.3.6 Equilibrium

The equilibrium system consists of 26 equations (2.1), (2.2), (2.3), (2.4), (2.5), (2.6), (2.7), (2.8), (2.9), (2.10), (2.11), (2.14), (2.15), (2.16), (2.17), (2.18), (2.19), (2.20), (2.21), (2.22), (2.23), (2.24), (2.25), (2.26), (2.27), (2.28), plus two exogenous process (2.12) and (2.13), determining 26 endogenous variables $\{Y_{rt}, Y_{st}, K_{rt}, K_{st}, L_{rt}, L_{st}, C_t, S_{wt}, S_{ct}, S_{at}, B_t, D_t, N_{wt}, N_{ct}, R^w_{kt}, R^c_{kt}, R_{bt}, R_{st}, R_t, w_t, \Phi_{wt}, \Phi_{ct}, V_{st}, V_{nt}, Q_t, I_t\}$ and two exogenous variables A_t and σ_t.

2.4 Model Result

In this section, I calibrate the model and study how TFP shocks and uncertainty shocks affect the leverage of shadow banks and commercial banks, and then see how the dynamics of bank leverage affect the real economy.

2.4.1 Calibration

Table 2.3 lists the choice of parameter values. For most of the parameters, including the discount rate β, the depreciate rate δ, the persistence of TFP shocks and uncertainty shocks ρ_a and ρ_σ, I choose conventional values from the literature. Following Bloom et al. (2018), I set the capital revenue elasticity α to be 0.25 and the labor revenue elasticity to be 0.5. The profit retaining ratios for the shadow banks and the commercial banks are positively related to the interest rate spreads. I set them to 0.9 and 0.95 respectively that are close to the choice of Gertler et al. (2016). This implies a dividend payout ratio of 10% and 5% for the shadow banks and the commercial banks. I set the fraction of assets that commercial bankers can divert to be 0.07 so that the steady-state leverage of the commercial banks is around 12. I choose the threshold of shadow banks' VaR rule to be 1 percent so that the steady-state leverage of the shadow banks is around 24. These numbers match roughly to their data counterparts. Changing the parameter value does not affect my result qualitatively.

Parameter	Value	Explanation
α	0.25	Capital revenue elasticity
ν	0.5	Labor revenue elasticity
θ_w	0.9	Profit retaining rate for shadow banks
θ_c	0.95	Profit retaining rate for commercial banks
λ	0.07	Fraction of assets that can be diverted
p	0.01	Default rate tolerance in VaR
τ	0.8	Safe firm exposure of TFP shock
η	1	Investment elasticity
β	0.99	Discount rate
δ	0.025	Depreciation rate
ρ_a	0.9	TFP shock persistence
ρ_σ	0.9	Volatility shock persistence
σ	0.02	Steady-state TFP shock volatility

Table 2.3: Parameters

2.4.2 Productivity Shocks vs. Uncertainty Shocks

I start by analyzing the impact of a negative technology shock that reduces the level of TFP by one percentage. Figure 2·9 shows the impulse response of some key variables. After a negative first-order productivity shock, the leverage of both shadow banks Φ_w and commercial banks Φ_c increases initially by around 20% from the steady state. This contrasts with the empirical evidence where shadow banks deleveraged sharply while commercial banks' leverage remained stable. This is because, after the negative TFP shock, the price of capital Q falls sharply due to the shrink of banks' balance sheets. Despite that the firms are having lower revenues, the return of bank's lending increases due to a much cheaper cost of purchasing new capital (see equation (2.5) and (2.6)). Hence, both the shadow banks and the commercial banks have higher lending returns, which relaxes the VaR constraint and the agency friction and allows for higher leverages.

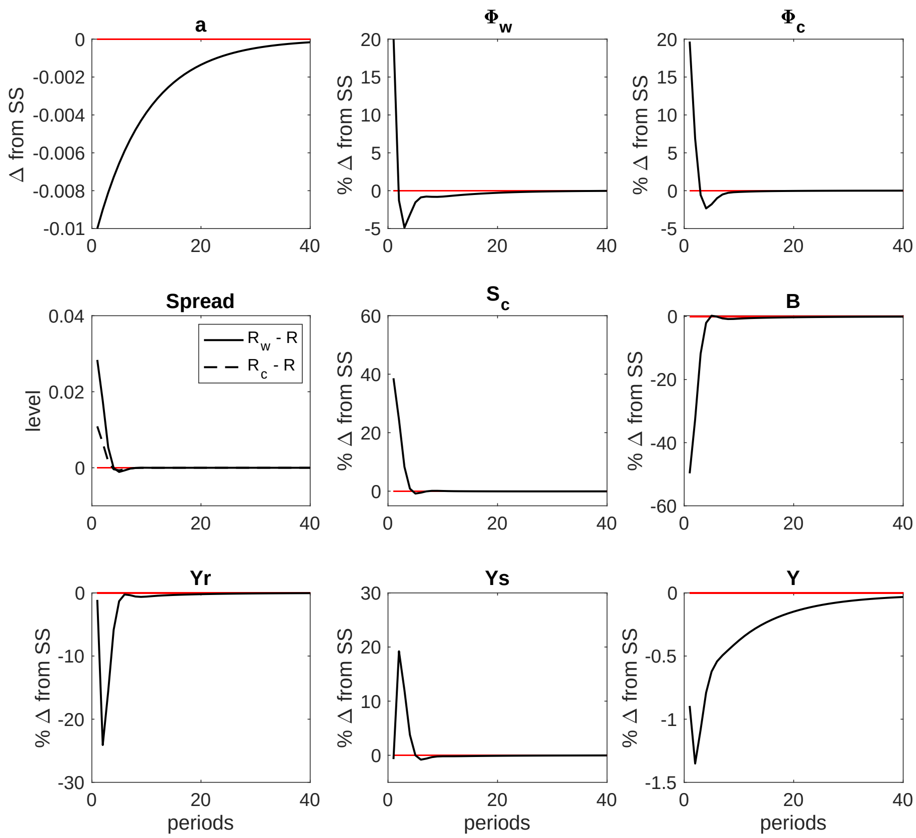


Figure 2·9: Impulse Response after a Negative TFP Shock

I next analyze the effect of an adverse uncertainty shock. Figure 2·10 shows the impulse response after an increase in the productivity volatility σ_t

by half of its steady state value (%50 increase). Unlike the first-order TFP shock, the uncertainty shock induces a sharp decline of the leverage of the shadow banking sector. The leverage of the shadow banking sector Φ_w falls by more than 40%. An adverse uncertainty shock widens the distribution of productivity and the firms' profit, decreases directly the threshold return R_s, and tightens the VaR constraint for the shadow banking sector. On the other hand, the leverage of the commercial banking sector increases due to a lower capital price Q as in the case of a negative TFP shock. The magnitude of such an increase is smaller, which is more consistent with the empirical evidence that the leverage of commercial banks remained stable during the financial crisis.

2.4.3 Flight to Quality

Both the negative first-order TFP shock and the adverse uncertainty shock generate a 'flight-to-quality' phenomenon where the commercial reduces its interbank lending and increases its lending to safe firms (Figure 2·9 and 2·10, the second row, the middle and right column). Two things generate this result. Firstly, the shadow banking sector has a higher leverage than the commercial banking sector in the steady state. Secondly, the risky firms that the shadow banking sector invests have larger exposure to the negative TFP shock. Both factors cause the shadow banking sector to suffer a heavier loss than the commercial banking sector. Thus, the commercial banking sector retracts funds from the more contracted interbank market and allocates instead more funds to the safe firms. As a result, the output from risky firms drops while the output from safe firms increases. Also, the interest spread of the risky firms increases more than that of the safe firms. The total output falls only slightly more than the initial magnitude of the TFP shock, but the inter-sector

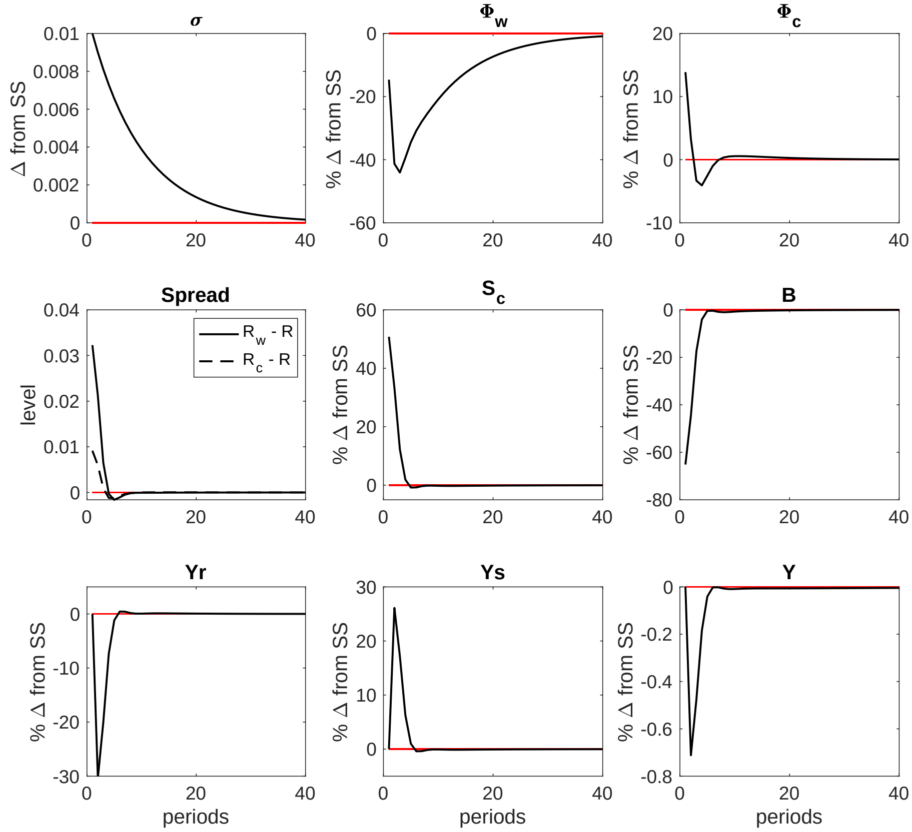


Figure 2·10: Impulse Response after an Adverse Uncertainty Shock

reallocation is much larger, at the magnitude of 10. On the aggregate level, the financial accelerator emphasized by Bernanke et al. (1999) is mitigated. However, if the commercial banks, instead of lending to the safe firms who still produce, choose other safer investment options like saving in the treasury securities, there will not be an increase in the output from the safe firms that partially cancels the output decline from the risky firms. In this case, the decline in the total output will be more amplified. The 'flight-to-quality' is consistent with what we observed in the data. Figure 2·11 shows the asset composition of commercial banks at the aggregate level. After the burst out of the financial crisis, commercial banks reduced the holdings of Fed funds Repos but increased the holdings of safer securities including treasury security, government-sponsored-enterprise backed security, and reserves.

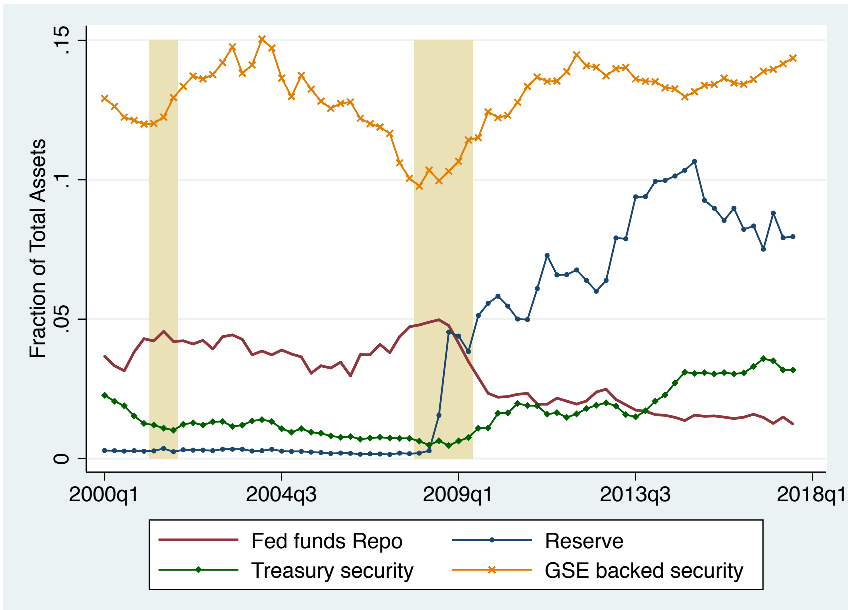


Figure 2·11: Asset Composition of Commercial Banks

In summary, the above two experiments point out the importance of uncertainty shocks and the VaR constraint in generating the observed deleveraging of the shadow banks. Bloom et al. (2018) argued that both a negative first-moment shock and a positive second-moment shock of productivity are needed to match business cycles. In the two-layer banking model where shadow banks and commercial banks are modeled separately, a single adverse uncertainty shock can reproduce both the deleveraging of the shadow banking sector and the flight-to-quality of the commercial banking sector, while the first-moment TFP shock leads to leverage dynamics that are inconsistent with the data.

2.5 Conclusion

In this paper, I examine empirical evidence of leverage for various types of financial intermediaries. I find the shadow banking sector, including investment banks, security brokers and dealers, deleveraged sharply during the financial crisis while the commercial banks maintained their leverage. The contrasting leverage patterns between the shadow banks and the commercial banks can be explained by their different funding sources. Financial institutions that relied more on short-term wholesale funding tended to deleverage more during the crisis. I then build a model to incorporate both shadow banks and commercial banks with different leverage determination mechanisms. The leverage of the shadow banks is determined by the Value-at-risk rule while the leverage of the commercial banks is determined by the agency friction between the bankers and the depositors. An adverse uncertainty shock reproduces both the deleveraging of the shadow banking sector and the flight-to-quality of the commercial banking sector, while the first-moment TFP shock leads to leverage dynamics that are inconsistent with the data.

Chapter 3

Monetary Policy and Rational Asset Price Bubbles: Comment

3.1 Introduction

Due to the recent financial crisis during 2008-2009, there is a renewed interest in understanding the role of asset bubbles in business cycles and the associated policy implications. Galí (2014) presents an elegant overlapping generations model with nominal rigidities to study the impact of monetary policy on rational asset bubbles. He finds some intriguing results that are inconsistent with conventional views. These results are summarized below:

- A stronger interest rate response to bubble fluctuations (i.e., a "leaning against the wind policy") may raise the volatility of asset prices and of their bubble component.

- The optimal monetary policy strikes a balance between stabilization of current aggregate demand and the bubble. If the average size of the bubble is sufficiently large, the latter motive will be dominant, making it optimal for the central bank to lower interest rates in the face of a growing bubble.

In this paper we revisit Galí's analysis by extending his model to allow for serially correlated bubble shocks. Our analysis complements his. We argue that his results are driven by his particular choice of the equilibrium solution. In his model there are multiple steady states and multiple equilibria. In particular, there is a continuum of stable bubbly steady states and a continuum of unstable bubbly steady states. He focuses on a backward-looking sunspot solution around a stable bubbly steady state. For this solution the value of a pre-existing asset bubble only responds to its own innovations. In the absence of such innovations, the size of an old bubble is predetermined, and an increase in the interest rate will raise its future size. By contrast we analyze the forward-looking minimal state variable (MSV) solution around an unstable bubbly steady state. For this solution the asset bubble responds to shocks on impact just like any asset prices. An increase in interest rates dampens the asset bubble on impact. We find results that are consistent with conventional views and are different from Galí's results mentioned above. In particular, the optimal policy calls for a leaning-against-the-wind rule. Note that this result depends on the assumption of serially correlated bubble shocks. If bubble shocks are serially uncorrelated, monetary policy would not affect bubble volatility for the MSV solution.

All steady states and equilibria in Galí's model are consistent with rational expectations. Following the methodology surveyed by Evans and Honkapohja (1999) and Evans and Honkapohja (2012), we use learning as a selection device to select a particular steady state and a particular equilibrium.[1] The idea is that agents of the model do not initially have rational expectations and they instead form forecasts by using some adaptive learning rules such as recursive least squares based on the data. The question is whether the agents can learn a particular equilibrium or a particular steady state. Marcet and Sargent (1989),

[1]See Bullard and Mitra (2002); Adam (2003); Woodford (2011); Duffy and Xiao (2007); Benhabib et al. (2014); Christiano et al. (2018), among others, for the application of learning to select equilibrium in macroeconomic models.

Evans and Honkapohja (1999) and Evans and Honkapohja (2012) show that the notion of expectational stability (E-stability) determines local convergence of real time recursive learning algorithms in a wide variety of economic models.

We find that the sunspot equilibrium solution adopted by Galí (2014) is not E-stable under his optimal monetary policy rule, but the forward-looking MSV solution is E-stable. We also find that the unstable bubbly steady state Pareto dominates the stable bubbly steady state. Moreover the former steady state is E-stable, but the latter is not. Our results are analogous to those in Evans et al. (2007) and Evans et al. (2001). They show that the E-unstable high-inflation steady state in a hyperinflation model has counterintuitive policy implications, while the E-stable low inflation steady state has conventional implications.

3.2 Solving Galí's Model

We first summarize Galí's (2014) model and refer the reader to his paper for detailed economic interpretations. We extend his model by allowing for persistent bubble shocks. We then solve for all equilibria and select equilibrium using a learning device.

3.2.1 Setup

The model economy consists of overlapping generations of agents, firms, and a central bank. Each agent lives for two periods and an agent born in period t derives utility according to $\log C_{1,t} + \beta E_t \left[\log C_{2,t+1}\right]$, where $C_{1,t} = \left(\int_0^1 C_{1,t}\left(i\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ and $C_{2,t+1} = \left(\int_0^1 C_{2,t+1}\left(i\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$ are consumption bundles and $\epsilon > 1$. Each young agent is endowed with one unit of labor and supplies it to firms inelastically. Normalize the size of each cohort to unity.

Each young agent is endowed with $\delta \in (0,1)$ units of an intrinsically useless bubble asset. The bubble asset can be traded in an asset market. Each period a fraction δ of each vintage of bubble assets loses its value so that the total amount of bubble assets outstanding remains constant and equal to one. This modeling allows a new bubble to be created once an old bubble bursts, as in Martin and Ventura (2012), Wang and Wen (2012), and Miao et al. (2015).

An agent born in period t chooses differentiated consumption goods $C_{1,t}(i)$ and $C_{2,t+1}(i)$, bond holdings Z_t^M, and holdings $Z_{t|t-k}^B$ of bubble asset introduced in period $t-k$ to maximize utility subject to the following budget constraints

$$\int_0^1 \frac{P_t(i)\, C_{1,t}(i)}{P_t} di + \frac{Z_t^M}{P_t} + \sum_{k=0}^{\infty} Q_{t|t-k}^B Z_{t|t-k}^B = W_t + \delta Q_{t|t}^B, \tag{3.1}$$

$$\int_0^1 \frac{P_{t+1}(i)\, C_{2,t+1}(i)}{P_{t+1}} di = D_{t+1} + \frac{Z_t^M (1+i_t)}{P_{t+1}} + (1-\delta) \sum_{k=0}^{\infty} Q_{t+1|t-k}^B Z_{t|t-k}^B,$$

where $P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the consumption price index, W_t is the real wage, i_t is the nominal interest rate, D_{t+1} is firm dividends, and $Q_{t|t-k}^B$ is the period-t real price of the bubble asset introduced in period $t-k$. Define the gross real interest rate as

$$R_t = (1+i_t)\, E_t \frac{1}{\Pi_{t+1}}. \tag{3.2}$$

Each agent owns a firm that produces a differentiated product $Y_t(i)$ using labor input $N_t(i)$ according to the technology $Y_t(i) = N_t(i)$. Each firm is monopolistically competitive and sets price P_t^* one period in a advance, generating nominal rigidities. It solves the following problem

$$\max_{P_t^*} E_{t-1}\left[\frac{\beta C_{1,t-1}}{C_{2,t}} Y_t(i)\left(\frac{P_t^*}{P_t} - W_t\right)\right]$$

subject to the demand schedule $Y_t(i) = (P_t^*/P_t)^{-\epsilon} C_t$, where $C_t = C_{1,t} + C_{2,t}$. In a symmetric equilibrium we have

$$0 = E_{t-1}\left[\frac{\beta C_{1,t-1}}{C_{2,t}}(1 - \mathcal{M}W_t)\right], \tag{3.3}$$

where $\mathcal{M} = \epsilon/(\epsilon - 1)$ denotes the markup.

The labor and goods markets clearing implies

$$C_{1,t} + C_{2,t} = 1, \tag{3.4}$$

$$D_t + W_t = 1. \tag{3.5}$$

Asset market clearing requires $Z_t^M = 0$ and $Z_{t|t-k}^B = \delta(1-\delta)^k$. Define the aggregate bubble index Q_t and the old bubble index B_t as

$$Q_t = \delta\sum_{k=0}^{\infty}(1-\delta)^k Q_{t|t-k}^B,\ B_t = \delta\sum_{k=1}^{\infty}(1-\delta)^k Q_{t|t-k}^B.$$

Let $U_t = \delta Q_{t|t}^B$ denote the size of new bubbles. Then by definition and the agent's bubble asset choice condition,

$$Q_t = B_t + U_t, \tag{3.6}$$

$$B_t + U_t = \beta E_t\left[\frac{C_{1,t}}{C_{2,t+1}} B_{t+1}\right]. \tag{3.7}$$

The consumption Euler equation gives

$$1 = \beta(1+i_t)E_t\left[\frac{C_{1,t}}{C_{2,t+1}}\frac{1}{\Pi_{t+1}}\right]. \tag{3.8}$$

The budget constraint (3.1) and the market-clearing conditions imply

$$C_{1,t} + Q_t = W_t + U_t. \tag{3.9}$$

To close the model, the central bank sets the nominal interest rate according to a feedback rule, which may respond to asset bubbles,

$$\ln(1 + i_t) = \ln R + \phi_\pi \ln\left(\frac{\Pi_t}{\Pi}\right) + \phi_b \ln\left(\frac{Q_t}{Q}\right) + \ln E_t \Pi_{t+1}, \tag{3.10}$$

where $\phi_\pi > 0$, $\Pi_t = P_t/P_{t-1}$ denotes gross inflation, and a variable without time subscript denotes its steady-state value. The central questions are how monetary policy affects asset bubbles and whether monetary policy should respond to asset bubbles.

The equilibrium system consists of eight equations (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), and (3.10) for nine stochastic processes $\{C_{1,t}\}, \{C_{2,t}\}, \{D_t\}, \{W_t\}, \{\Pi_t\}, \{i_t\}, \{Q_t\}, \{B_t\}$, and $\{U_t\}$. Since there are eight equilibrium conditions for nine variables, the equilibrium system cannot determine the size of the old bubble and the new bubble independently. Galí (2014) assumes that the new bubble $\{U_t\}$ is an exogenously given IID process. We consider the more general case in which $\{U_t\}$ is serially correlated. Galí (2014) also considers the innovation in the old bubble $B_t - E_{t-1}B_t$ as another independent source of uncertainty. We will show below that this is true for the sunspot equilibria. Except for these two sources of uncertainty, there is no other shock in the model.

3.2.2 Multiple Equilibria

We first present Galí's results in the deterministic case where $U_t = U > 0$ for all t. Then the old bubble $\{B_t\}$ satisfies the difference equation

$$B_{t+1} = \frac{(1 - 1/\mathcal{M})(B_t + U)}{\beta/\mathcal{M} - (1+\beta) B_t - U} \equiv H(B_t, U). \tag{3.11}$$

The necessary and sufficient condition for the existence of a deterministic bubbly steady state is given by

$$\mathcal{M} < 1 + \beta. \tag{3.12}$$

Furthermore, when this condition is satisfied there exists a continuum of stable bubbly steady states indexed by U,

$$\left\{ (B_s(U), U) : B_s(U) = H(B_s(U), U) \text{ for } U \in \left(0, \bar{U}\right) \right\},$$

and a continuum of unstable bubbly steady states also indexed by U,

$$\left\{ (B_u(U), U) : B_u(U) = H(B_u(U), U) \text{ for } U \in [0, \bar{U}), \; B_u(U) > B_s(U) \right\},$$

where

$$\bar{U} = \beta + (1+\beta)(1-W) - 2\sqrt{\beta(1+\beta)(1-W)} > 0 \text{ and } W = \frac{1}{\mathcal{M}}.$$

The economy also has a bubbleless steady state in which $B = U = 0$. In this steady state we can show the bubbleless real interest rate is $R_f = (\mathcal{M} - 1)/\beta$. Thus condition (3.12) is the same as $R_f < 1$, which is the standard condition in the literature (Tirole, 1985), i.e., the bubbleless equilibrium is dynamically inefficient.

Next we study the stochastic case by log-linearizing the equilibrium system around a deterministic bubbly steady state for a fixed $U \in \left(0, \bar{U}\right)$. In Appendix B.1 we show that the log-linearized equilibrium system can be reduced to a unidimensional system

$$\begin{aligned} b_t = & \frac{1}{R(\phi_b + 1)} E_t b_{t+1} + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b + 1} E_{t-1} b_t \\ & + \frac{R-1}{R} u_t + \frac{(\epsilon B - \phi_b)(R-1)}{(\phi_b + 1) R} E_{t-1} u_t, \end{aligned} \tag{3.13}$$

where we use a lower case variable to denote the log deviation from its steady-state value and R denotes the bubbly steady-state real interest rate given by

$$R = \frac{1}{\beta} \frac{1 - 1/\mathcal{M} + B}{1/\mathcal{M} - B} = \frac{B}{B+U} \in (0, 1) \, .$$

Note that there are two bubbly steady states for a fixed $U \in \left(0, \bar{U}\right)$. Without risk of confusion, we use the same notation B to represent either one of the steady-state size of the old bubble in the analysis below.

Our objective is to solve for a rational expectations equilibrium (REE) using (3.13). Galí (2014) assumes that u_t is IID. We consider a more general AR(1) process

$$u_t = \rho u_{t-1} + e_t, \quad \rho \in [0, 1), \tag{3.14}$$

where e_t is an IID random variable with mean zero and variance σ_e^2.

Galí (2014) focuses his analysis on a sunspot solution around a stable bubbly steady state. Given (3.14), we can derive the following more general solution. Its proof and the proofs of the remaining propositions in the paper are given in Appendix B.2.

Proposition 3.1. *Fix $U \in \left(0, \bar{U}\right)$. For any b_0, there is a linear sunspot solution in a neighborhood of the bubbly stable steady state given by*

$$b_t = \chi b_{t-1} + (1 - R)(1 + \epsilon B) \rho u_{t-2} + \varphi_2^* e_t + \varphi_3^* e_{t-1} + \varphi_4^* \xi_t + \varphi_5^* \xi_{t-1},$$

where ξ_t *denotes a sunspot shock satisfying* $E_{t-1}\xi_t = 0$, φ_3^* *and* φ_5^* *are arbitrary real numbers, and*

$$\varphi_2^* = \frac{\varphi_3^* + (R-1)(1+\phi_b)}{R(\phi_b+1) - \chi}, \ \varphi_4^* = \frac{\varphi_5^*}{R(\phi_b+1) - \chi},$$

$$\chi = R(1 + \epsilon B(1+\beta)) \in (0,1).$$

Galí (2014) shows that $\chi = \partial H(B,U)/\partial B$. For a stable bubbly steady state, we must have $\chi \in (0,1)$, which also implies that the backward-looking solution in Proposition 3.1 is stationary. Galí (2014) defines a sunspot variable $\xi_t = b_t - E_{t-1}b_t$. Substituting this variable into (3.13) yields a particular solution

$$\begin{aligned} b_t &= \chi b_{t-1} + (\phi_b + 1)(1-R)u_{t-1} - (\phi_b - \epsilon B)(1-R)\rho u_{t-2} \\ &\quad + \xi_t + (\phi_b - \epsilon B(1+\beta)) R\xi_{t-1}, \end{aligned} \tag{3.15}$$

which can also be obtained by setting

$$\varphi_2^* = 0, \quad \varphi_3^* = (1-R)(1+\phi_b), \ \varphi_5^* = (\phi_b - \epsilon B(1+\beta)) R$$

in our general solution given in Proposition 3.1. The solution in equation (30) of Galí (2014) corresponds to $\rho = 0$ in (3.15).

For this solution, the initial value b_0 is indeterminate. Galí (2014) derives all his results for a fixed b_0. From (3.15) we can see that monetary policy only affects the anticipated component of the old bubble $E_{t-1}b_t$ through the interest rate coefficient ϕ_b. In the case of $\rho = 0$, Galí (2014) shows that a leaning-against-the-wind policy which corresponds to $\phi_b > 0$ generates a larger volatility in the bubble than a policy of benign neglect ($\phi_b = 0$).

Now we consider the solution in the neighborhood of the unstable bubbly steady state.

Proposition 3.2. *Fix* $U \in \left(0, \bar{U}\right)$. *There is a unique forward-looking linear solution in a neighborhood of the unstable bubbly steady state given by*

$$b_t = (R-1)\frac{\epsilon B + 1}{\chi - \rho}\rho u_{t-1} + \frac{R-1}{R}\left[\frac{\rho}{1+\phi_b}\frac{1+\epsilon B}{\chi - \rho} + 1\right] e_t, \tag{3.16}$$

where $\chi = R\left(1 + \epsilon B\left(1+\beta\right)\right) > 1$.

In a neighborhood of the unstable bubbly steady state, we have $\chi > 1$. The backward-looking solution in (3.15) is not stationary. We must solve for b_t forward to obtain the forward-looking solution in (3.16) so that b_t is stationary. This solution is also called the minimal state variable (MSV) solution in the literature (e.g., Evans and Honkapohja (2012)). In the next section we will focus our analysis on this solution.

Note that if $\rho = 0$ as in Galí (2014), then the MSV solution gives $b_t = e_t\left(R-1\right)/R$. In this case monetary policy through ϕ_b does not affect bubble dynamics. We thus assume $\rho \in (0,1)$ throughout the paper.

3.2.3 Learning and Equilibrium Selection

There are multiple (deterministic) steady states and multiple REE solutions in Galí (2014). We will use learning as a selection device to select a particular steady state and a particular REE solution. To understand the basic idea, we consider an economic model with a solution described as a particular parameter vector $\bar{\varphi}$ (e.g., the parameters of an autoregressive process or a steady state). Under adaptive learning agents do not know $\bar{\varphi}$ but estimate it from data using a statistical procedure such as least squares. This leads to estimates φ_t at time t and the question is whether $\varphi_t \to \bar{\varphi}$ as $t \to \infty$. Evans and Honkapohja (2012) show that, for a wide range of economic examples and learning rules, convergence is governed by the corresponding E-stability condition, i.e., the local asymptotic stability of $\bar{\varphi}$ under the differential equation

$$\frac{d\varphi}{d\tau} = T(\varphi) - \varphi, \tag{3.17}$$

where τ denotes notional or virtual time, $T(\varphi)$ is the mapping from the perceived law of motion (PLM) φ to the implied actual law of motion (ALM) $T(\varphi)$. In the following analysis we will check the E-stability condition.

We start by the steady states.

Proposition 3.3. *For any fixed $U \in (0, \bar{U})$, the bubbly unstable steady state Pareto dominates the bubbly stable steady state. Moreover the bubbly unstable steady state is E-stable if and only if $\phi_b > -1$ and the bubbly stable steady state is E-stable if and only if $\phi_b < -1$.*[2]

Next we consider the stochastic MSV and sunspot solutions.

Proposition 3.4. *For $\phi_b > -1$ the sunspot solution in Proposition 1 is not E-stable. The MSV solution in Proposition 3.2 is E-stable if and only if $\phi_b > -1$.*

Galí (2014) shows that the optimal response coefficient ϕ_b that minimizes the welfare loss is greater than -1 for the sunspot solution. Proposition 3.4 shows that this solution under the optimal policy is not E-stable. By contrast, the MSV solution for $\phi_b > -1$ is E-stable. In the next section we will show that the optimal coefficient ϕ_b is positive for the MSV solution and hence the MSV solution under optimal monetary policy is E-stable.

[2]In a previous version of the paper, we started with the deterministic system (3.11) directly. The PLM is $B_{t+1} = a$ and ALM is $T(a) = H^{-1}(a, U)$. The ODE is $\dot{a} = T(a) - a$. In this case the assumption on ϕ_b is not needed.

3.3 Monetary Policy

What is the impact of the monetary policy on bubble dynamics? We first use (3.16) to compute the volatility of the old bubble

$$Var\left(b_t\right) = \left(\frac{\epsilon B+1}{\chi-\rho}\rho\right)^2 \frac{(R-1)^2\sigma_e^2}{1-\rho^2} + \left(\frac{R-1}{R}\right)^2 \left[\frac{\rho}{1+\phi_b}\frac{1+\epsilon B}{\chi-\rho}+1\right]^2 \sigma_e^2.$$

It is minimized at

$$\phi_b = -\frac{\rho(1+\epsilon B)}{\chi-\rho} - 1 < 0. \tag{3.18}$$

Galí (2014) shows that the volatility of both the old and aggregate bubbles is minimized at $\phi_b = -1$ for his sunspot solution.

Now we log-linearize equation (3.6) to obtain

$$q_t = Rb_t + (1-R)\,u_t \tag{3.19}$$

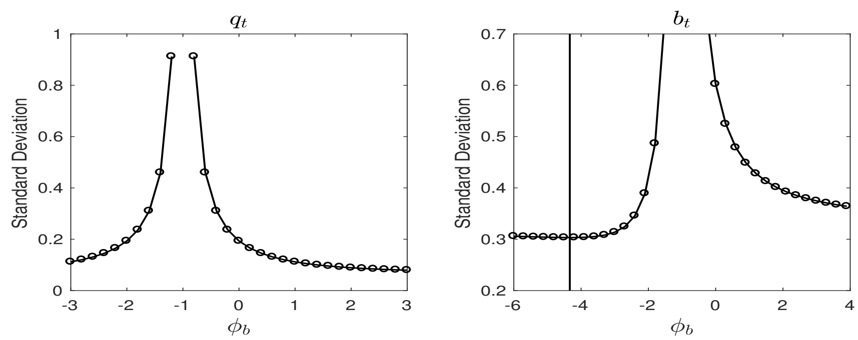
and combine it with (3.16) to derive the volatility of the aggregate bubble for our MSV solution:

$$Var\left(q_t\right) = (R-1)^2 \left[R\frac{\epsilon B+1}{\chi-\rho} - 1\right]^2 \rho^2(1-\rho^2)^{-1}\sigma_e^2 + \left[\frac{(R-1)\,\rho}{1+\phi_b}\frac{1+\epsilon B}{\chi-\rho}\right]^2 \sigma_e^2.$$

Thus a leaning-against-the-wind policy (i.e., $\phi_b > 0$) generates a lower volatility of the aggregate bubble than a policy of benign neglect ($\phi_b = 0$), contrary to Galí's result. The volatility is minimized when $\phi_b \to +\infty$. Interestingly, when ϕ_b decreases to negative infinity, the bubble volatility also decreases to zero. However, since in this case the MSV solution is E-unstable, the adaptive learning perspective argues against the relevance of this case: restriction attention to values of ϕ_b for which the solution is learnable, increasing ϕ_b reduces bubble volatility.

The results above show that the volatilities of the old and aggregate bubbles are proportional to the volatility of new bubble innovations, which are the only source of uncertainty. By contrast, for the sunspot solution in Galí (2014) (see (3.15) here), innovations in old bubbles are another source of uncertainty that can drive the movements of the aggregate bubble independent of fundamentals. This is an appealing feature, though both sources of uncertainty are not observable and hardly testable.

Figure 3·1 presents the relation between ϕ_b and the volatilities of the old and aggregate bubbles for the MSV solution. We choose the same parameter values as in Galí (2014) by setting $\beta = 1$, $\epsilon = 6$, $U = 0.175$. These values imply $B_s = 0.1$, $B_u = 0.1458$, and $\mathcal{M} = 1.2$. While Galí (2014) studies equilibria around the stable bubbly steady state $B_s = 0.1$, we focus on the solution around the unstable bubbly steady state $B_u = 0.1458$. Galí's result is illustrated in Figure 2 of his paper, which shows that the bubble volatility increases with $\phi_b > 0$.



Note: This figure plots the standard deviations of the aggregate bubble q_t and old bubble b_t for various coefficients ϕ_b. The vertical line indicates the value of ϕ_b that minimizes the standard deviation of the old bubble. The parameter values are $\beta = 1$, $\epsilon = 6$, $U = 0.175$, $\phi_\pi = 2$, $\rho = 0.8$, and $\sigma_e^2 = 0.01$. We focus on the unstable bubbly steady state with $B = 0.1458$.

Figure 3·1: Monetary Policy and Bubble Volatility

To understand the intuition behind Figure 3·1, we consider the economy's responses to an exogenous positive bubble shock to u_t. We first use equations

(3.2), (3.6), (3.7), and (3.8) to derive the log-linearized asset pricing equation

$$q_t = E_t b_{t+1} - r_t, \tag{3.20}$$

which says that total bubble is equal to the future old bubble discounted by r_t. Using this equation and (3.19), we see that the old bubble satisfies the asset pricing equation

$$b_t = \frac{1}{R} E_t b_{t+1} - \frac{1}{R} r_t - \frac{(1-R)}{R} u_t. \tag{3.21}$$

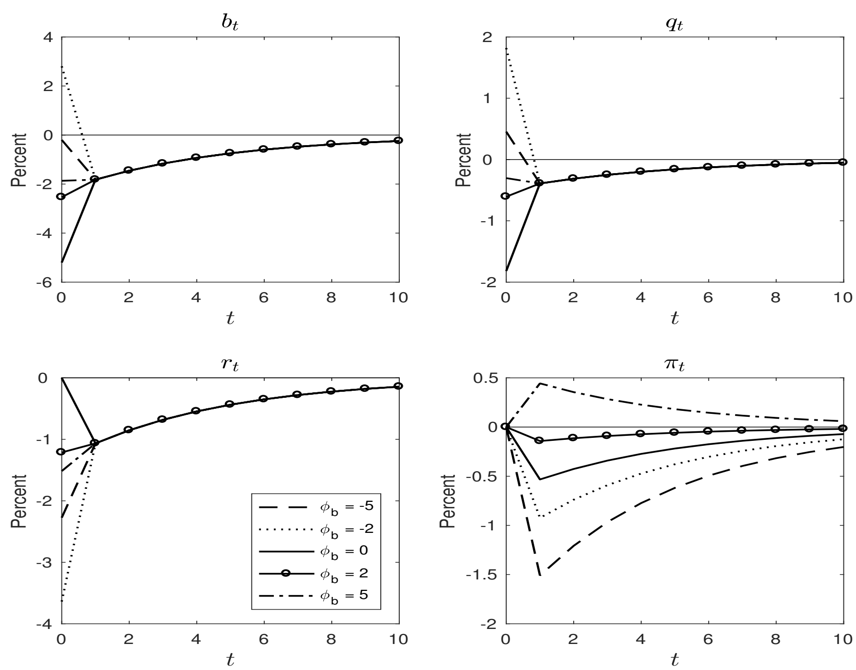
Solving forward shows that the old bubble is equal to the (negative) discounted value of future real interest rates and new bubbles. Since $0 < R < 1$, new bubbles $\{u_t\}$ act as negative dividends. An increase in u_t has a direct effect of lowering b_t and an indirect effect through the change in the interest rate r_t. Due to the endogenous response of r_t, a unique forward-looking solution for b_t exists as shown in Proposition 3.2, even when $0 < R < 1$. In contrast to Galí (2014), b_t is a jump variable and responds to shocks on impact like any asset prices.

The impact of monetary policy on asset bubbles q_t and b_t is transmitted through the real interest rate r_t, which in turn depends on the size of bubbles b_t. Thus we need to understand the dynamic responses of r_t for different values of ϕ_b. In Appendix B.3 we show that

$$r_t = (R-1)\left[\frac{(\epsilon B + 1)(\rho - R)}{\chi - \rho} + 1\right] \rho u_{t-1} + \frac{\phi_b \rho (R-1)(\epsilon B + 1)}{(\phi_b + 1)(\chi - \rho)} e_t. \tag{3.22}$$

When $\rho = 0$, $r_t = 0$ and $b_t = e_t\,(R-1)\,/R$ by Proposition 3.2. It follows from (3.20) that $q_t = 0$. The intuition is that the impact of a positive new bubble shock on the aggregate bubble is exactly offset by a negative response of the old bubble so that the size of the aggregate bubble does not change. Thus the value of ϕ_b does not affect the real interest rate by the monetary policy rule in (3.10) and hence it does not affect bubble dynamics.

Figure 3·2 presents the impulse response functions for $\{b_t\}$, $\{q_t\}$, $\{r_t\}$, and $\{\pi_t\}$ given a 1% shock to e_0 in period 0 for $\rho = 0.8$. When monetary policy does not respond to bubbles ($\phi_b = 0$), a positive shock to expand the new bubble u_0 at date 0 crowds out the size of old bubbles b_0 and dampens the aggregate bubble q_0, but r_0 does not change, as shown in equations (3.16), (3.19), and (3.22).



Note: This figure plots the impulse response functions for a one percent positive new bubble shock, in percentage deviation from the steady state. The parameter values are $\beta = 1$, $\epsilon = 6$, $U = 0.175$, $\phi_\pi = 2$, $\rho = 0.8$, and $\sigma_e^2 = 0.01$. We focus on the unstable bubbly steady state with $B = 0.1458$.

Figure 3·2: Impulse Responses to a New Bubble Shock

When $\phi_b > 0$, the central bank will cut the interest rate according to the interest rate rule as q_0 and b_0 decline and hence the fall of the old and aggregate bubbles is mitigated by (3.21). For this channel to work we need $\rho \in (0, 1)$ so that the term related to e_t in (3.22) is negative as $R \in (0, 1)$ and $\chi > 1$.

Both $\rho > 0$ and the forward-looking solution (initial changes of aggregate bubbles) are important for a leaning-against-the-wind policy to lower bubble volatility in response to a bubble shock e_0. For a larger $\phi_b > 0$, the mitigation effect is stronger so that aggregate and old bubbles respond less to the bubble shock. This explains why the volatilities of q_t and b_t decrease with $\phi_b > 0$ as illustrated in Figure 3·1.

By contrast, for Galí's backward-looking solution, we rewrite (3.21) as

$$b_{t+1} = Rb_t + r_t + (1 - R)\, u_t + \xi_{t+1},$$

where $\xi_t = b_t - E_{t-1}b_t$ is a sunspot shock and b_0 is predetermined. Either a positive bubble shock or a positive sunspot shock raises the size of future bubbles without changing the initial size b_0. A leaning-against-the-wind policy with $\phi_b > 0$ will raise the interest rate r_t so that future bubbles will grow even faster. This explains why such a policy will raise bubble volatility in Galí (2014).

For our forward-looking solution, Figure 3·2 shows that q_t and b_t fall on impact and then gradually rise to their steady state values. Their dynamics for different values of ϕ_b differ only in the initial period. This can be seen from equations (3.16), (3.19), and (3.22) because $R \in (0, 1)$, $e_t = 0$ for $t > 0$, and u_t is an AR(1) process with persistence $\rho > 0$. The effect of ϕ_b is only on the terms related to the temporary shock e_t.

When $\phi_b < 0$, the old and aggregate bubbles may rise on impact in response to a positive bubble shock. When the central bank cuts the interest rate to encourage bubbles, this effect may dominate the direct negative effect of the rise in the new bubble on the old bubble as shown in equation (3.21). As shown in Figure 3·2, when ϕ_b decreases from -2 to -5, the old and aggregate bubbles are dampened and the fall of interest rate is also mitigated. If bubbles expanded, the central bank would cut the interest rate more, which in turn would encourage bubbles further. This positive feedback effect might make the bubble explode.

Since firms adjust prices one period in advance before shocks are realized, the inflation rate π_t is predetermined. Thus it does not respond to the bubble shock on impact. As shown in Figure 3·2, it may rise or fall in the second period depending on the value of ϕ_b. In Appendix B.3 we show that the inflation rate around the unstable bubbly steady state is given by

$$\pi_t = \frac{\rho(R-1)[\rho(\epsilon B+1)+(1+\phi_b)(\beta\epsilon BR-\rho)]}{\phi_\pi(\chi-\rho)}u_{t-1}.$$

If $\phi_b = 0$, the inflation rate falls in the second period because $R \in (0,1)$ and $\chi > 1$. The central bank can stabilize inflation by two strategies: First, it can set ϕ_π at an arbitrary large value and set ϕ_b at a finite value. Second, it can set ϕ_π at a finite value and set $\phi_b = \rho(\epsilon B+1)/\left(\rho-\beta\epsilon BR\right)-1$.

In Galí's (2014) model inflation is not a source of welfare losses given synchronized price-setting and an inelastic labor supply. Thus it is not optimal for the central bank to stabilize inflation. To study optimal monetary policy, we follow Galí (2014) to take the unconditional mean of an agent's lifetime utility as a welfare criterion. In a neighborhood of a steady state, we can derive the second-order approximation to the mean:

$$E\left[\ln C_{1,t}+\beta\ln C_{2,t+1}\right] \simeq \ln C_1+\beta\ln C_2-\frac{1}{2}\left(Var\left(c_{1,t}\right)+Var\left(c_{2,t}\right)\right).$$

By the resource constraint $C_{1,t}+C_{2,t}=1$, $Var\left(c_{1,t}\right)$ is proportional to $Var\left(c_{2,t}\right)$. Thus the optimal monetary policy that maximizes welfare will minimize the variance of

$$c_{2,t} = (1 - \Gamma)\, d_t + \Gamma b_t,$$

where $\Gamma = \epsilon B/(\epsilon B + 1)$.

In Appendix B.3 we show that

$$d_t = \frac{\chi(R-1)[\phi_b\,(\rho - \epsilon B\beta R) - \epsilon B(\beta R + \rho)]}{\beta R^2(1+\phi_b)(\chi - \rho)} e_t.$$

Thus minimizing the volatility of dividends calls for setting

$$\phi_b = \frac{\epsilon B\,(\beta R + \rho)}{\epsilon B\beta R - \rho}.$$

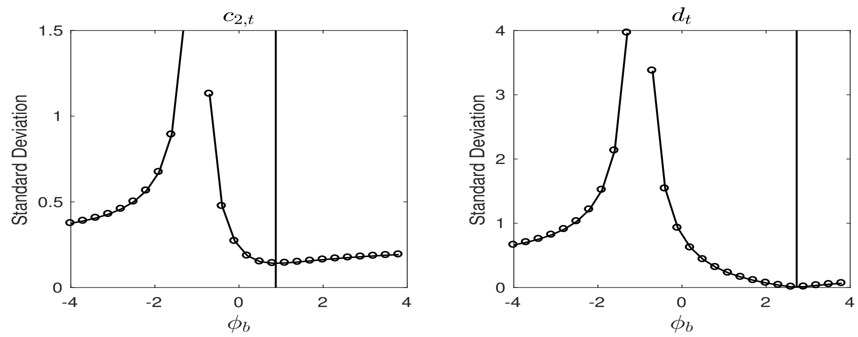
However this policy would raise the volatility of the old bubble because it is minimized at a different value given in (3.18). Thus optimal monetary policy trades off between the volatility of dividends and the volatility of the old bubble.

Note that b_t and d_t are also correlated. In Appendix B.3 we derive that

$$Var(c_{2,t}) = \left(\frac{\epsilon B\rho(R-1)}{\chi - \rho}\right)^2 (1-\rho^2)^{-1}\sigma_e^2 + \left[\frac{(R-1)\rho(\phi_b - \epsilon B)}{\beta R(1+\phi_b)(\chi-\rho)}\right]^2 \sigma_e^2.$$

From this equation we can show that the optimal coefficient is given by $\phi_b = \epsilon B > 0$ for $\rho \neq 0$. Thus the leaning-against-the-wind policy is optimal. Moreover the optimal coefficient increases with the size of the bubble. This property is in contrast with Figure 4 of Galí (2014), which shows that the optimal coefficient ϕ_b is positive for a small size of bubbles and becomes negative for a sufficiently large size of bubbles.

Figure 3·3 illustrates the relation between ϕ_b and $Var\,(c_{2,t})$. The welfare loss is minimized at $\phi_b = 0.875$.



Note: This figure plots the standard deviations of dividend d_t and consumption of the old $c_{2,t}$ for various values of ϕ_b. The vertical lines indicate the values of ϕ_b that minimize the standard deviation of consumption and dividend respectively. The parameter values are $\beta = 1$, $\epsilon = 6$, $U = 0.175$, $\phi_\pi = 2$, $\rho = 0.8$, and $\sigma_e^2 = 0.01$. We focus on the unstable bubbly steady state with $B = 0.1458$.

Figure 3·3: Monetary Policy and Welfare

3.4 Conclusion

In this paper we have shown that Galí's (2014) counterintuitive results are driven by his choice of a backward-looking sunspot solution around a stable bubbly steady state. His model also features a continuum of unstable bubbly steady states, which Pareto dominate the corresponding stable bubbly steady states. We extend his model to incorporate persistent bubble shocks. When deriving the unique forward-looking MSV solution around an unstable bubbly steady state, we obtain results that are consistent with the conventional views. We apply learning as a selection device to select steady state and equilibrium. We find that the unstable bubbly steady state and the associated MSV equilibrium are E-stable under optimal monetary policy. But the stable bubbly steady state and the associated sunspot equilibrium are not E-stable under optimal monetary policy.

In an infinite-horizon framework without recurrent creation of new bubbles, Miao and Wang (2018) prove that the economy has two steady states.

The local equilibrium around the bubbly steady state is unique and the local equilibrium around the bubbleless steady state is indeterminate of degree one. We conjecture that learning will select the bubbly steady state and the associated forward-looking solution as in this paper. Miao et al. (2015) and Dong et al. (2020) incorporate recurrent bubbles and show that the economy has a continuum of bubbly steady states as in Galí (2014). However, they are unable to prove the stability of these steady states analytically due to the complexity of their multi-dimensional equilibrium systems. In contrast to Galí (2014), their numerical results indicate that each bubbly steady state is a saddle point and the local equilibrium around each bubbly steady state is unique. We suspect that the difference in results may be due to the difference in the infinite-horizon and overlapping-generations frameworks. Further theoretical research is needed to understand this issue.

Appendix A

Appendix to Chapter 1

A.1 Equilibrium Characterization

Entrepreneurs

The entrepreneur maximizes the discounted sum of dividends:

$$V_{jt} = E_t \sum_{i=1}^{\infty} \beta^i div^E_{jt+i}$$

subject to

$$\begin{aligned} div^E_{jt+1} &= (1-\sigma^E)(q_{t+1}k^n_{jt+1} + R_t s^E_{jt}), \\ k^n_{jt+1} &= a_{jt} i_{jt}, \\ n_{jt} &= i_{jt} + s^E_{jt}, \\ n_{jt+1} &= \sigma^E(q_{t+1}k^n_{jt+1} + R_t s^E_{jt}), \\ 0 &\leq i_{jt} \leq n_{jt}, \\ 0 &\leq s^E_{jt} \leq n_{jt}. \end{aligned}$$

Substituting for div^E_{jt}, s^E_{jt} and k^n_{jt}, I can rewrite the optimization problem as

$$\max_{i_{jt}} \sum_{l=1}^{\infty} \beta^l [(1-\sigma^E)(q_{t+l}a_{t+l-1}i_{jt+l-1} - R_{t+l-1}n_{jt+l-1})$$
$$+\lambda_{jt+l-1}(n_{t+l-1} - i_{t+l-1}) + \mu_{jt+l-1}i_{t+l-1}].$$

The first-order-condition regarding i_{jt} is

$$\lambda_t = E_t[1-\sigma^E + \beta\sigma^E(\lambda_{t+1} + (1-\sigma^E)R_{t+1})](q_{t+1}a_{jt} - R_t) + \mu_t.$$

Since the term before $q_{t+1}a_{jt} - R_t$ is always positive, and under the assumption $E_t q_{t+1} a^H > R_t > E_t q_{t+1} a^L$, I conclude that $i_{jt} = n_{jt}$ and $s^E_{jt} = 0$ for the high-type entrepreneurs and $i_{jt} = 0$ and $s^E_{jt} = n_{jt}$ for the low-type entrepreneurs.

The Bank

The bank maximizes the discounted sum of dividends:

$$V^B_t = E_t \sum_{i=1}^{\infty} \beta^i div^B_{t+i}$$

subject to

$$\begin{aligned} div^B_{t+1} &= (1-\sigma^B)((X_t - G)Z_{t+1}R^X + GR^X - R_t S_t), \\ X_t &= S_t + W_t \\ W_{t+1} &= \sigma^E((X_t - G)Z_{t+1}R^X + GR^X - R_t S_t), \\ S_t R_t &\leq (X_t - G)\underline{Z}R^X + GR^X. \end{aligned}$$

Substituting for div^B_{jt} and X_t, I can rewrite the bank's optimization problem as

$$\begin{aligned} \max_{S_t} \sum_{l=0}^{\infty} \beta^l \big[&(1-\sigma^B)((S_{t+l} + W_{t+l} - G)Z_{t+1+l}R^X + GR^X - R_{t+l}S_{t+l}) \\ &+ \lambda_{t+l}((S_{t+l} + W_{t+l} - G)\underline{Z}R^X + GR^X - R_{t+l}S_{t+l})\big]. \end{aligned}$$

The first-order-condition regarding S_t is

$$\lambda_t(R_t - \underline{Z}R^X) = E_t[1-\sigma^B + \beta\sigma^B(\lambda_{t+1}\underline{Z}R^X + (1-\sigma^B)Z_{t+1}R^X)](Z_{t+1}R^X - R_t).$$

Notice that under the assumption that $R_t - \underline{Z}R^X > 0$, λ_t is always positive as long as $E_t(Z_{t+1}R^X - R_t) > 0$. As long as the bank is earning a positive profit margin, the value-at-risk constraint is binding.

A.2 Proof of Proposition 1

In the steady state, the motion of entrepreneurs' wealth follows

$$N = \sigma^E(qI + RS^E).$$

Substituting for I and S^E I obtain

$$1 = \sigma^E(qh + (1-h)R).$$

Since $CY = q - R$ (assuming $a^H = 1$), I then derive

$$CY = \frac{1}{1-h}\left(q - \frac{1}{\sigma^E}\right).$$

In the steady state, the first-order-condition for final-good producers with respect to q_t becomes

$$q(1-\beta(1-\delta)) = \alpha K^{\alpha-1}.$$

Hence

$$Y = \left(\frac{q(1-\beta(1-\delta))}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}.$$

Therefore Y is decreasing in q and the convenience yield CY.

A.3 Equilibrium System

The equilibrium dynamic system consists of the following 14 equations.

$$
\begin{aligned}
I_t &= hN_t, \\
K_t^n &= I_{t-1}, \\
K_t &= (1-\delta)K_{t-1} + K_t^n, \\
S^E &= (1-h)N_t, \\
0 &= (X_{t-1} - G)\underline{Z}R^X + GR^X - R_{t-1}S_{t-1}, \\
X_t &= S_t + W_t, \\
W_t &= \sigma^B((X_{t-1} - G)Z_tR^X + GR^X - R_{t-1}S_{t-1} - cX_{t-1}), \\
N_t &= \sigma^E\left(q_tI_{t-1} + R_{t-1}S_{t-1}^E\right), \\
S_t^H + C_t &= w_t + \Pi_t + div_t^E + div_t^B + R_{t-1}S_{t-1}^H + cX_{t-1}, \\
S_t &= S_t^H + S_t^E, \\
1 &= \beta E_tR_t\frac{C_t}{C_{t+1}} + F_t\frac{C_t}{S_t^H}, \\
w_t &= (1-\alpha)K_t^\alpha, \\
q_t &= \alpha K_t^{\alpha-1} + \beta(1-\delta)E_tq_{t+1}, \\
Y_t &= K_t^\alpha.
\end{aligned}
$$

Here $div_t^E = (1-\sigma^E)(q_tK_t^n + R_{t-1}S_{t-1}^E)$, $div_t^B = (1-\sigma^b)((X_{t-1}-G)Z_tR^X + GR^X - R_{t-1}S_{t-1})$ and $\Pi_t = Y_t - w_tL_t - q_tK_t^n$.

A.4 Shortage of Safe Assets Due to Worse Investment Opportunities

A shortage of safe assets can arise from a worsening of investment efficiency of the entrepreneurs. When the economy has relative more low-type entrepreneurs than the high-type, the demand for safe assets as a store of value will be larger, and hence leads to a shortage of safe assets. Figure A·1 shows the transition path of the model economy under a permanent decrease of h, the measure of high-type entrepreneurs. More low-type entrepreneurs increases the demand for safe assets, generating a shortage of safe assets with a higher convenience yield. Although the total wealth of entrepreneurs increases, it does not cover the drop in the proportion of high-type and hence the total investment and output decrease. In response to the shortage of safe assets, the bank expands its balance sheet and produces more private safe assets.

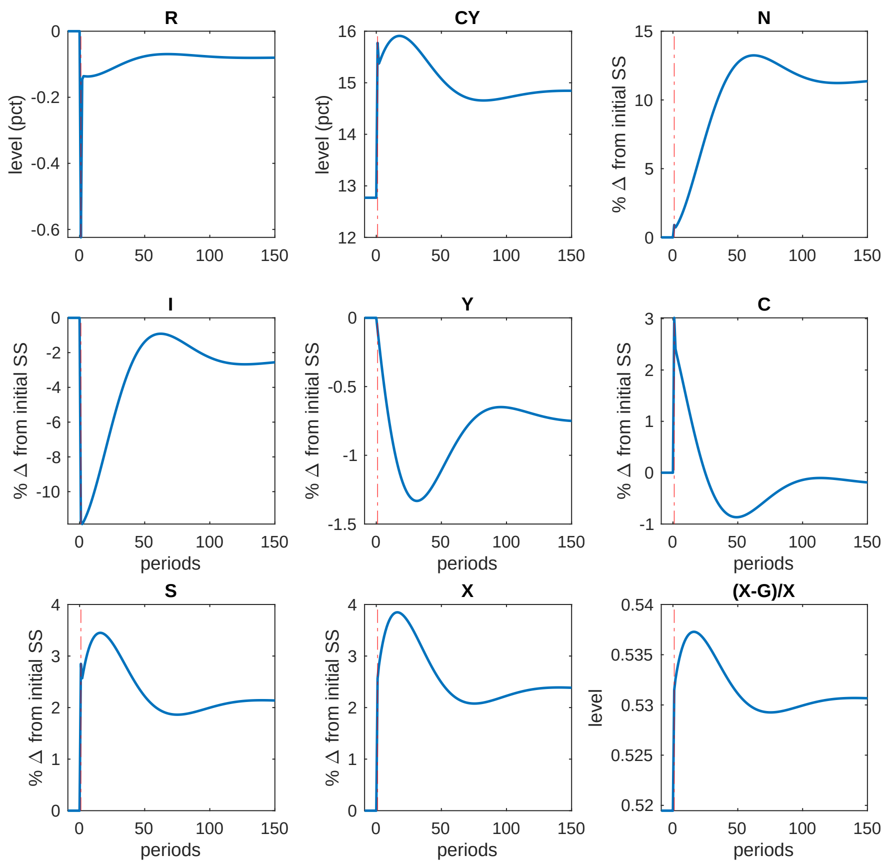


Figure A·1: Transition Path under a Permanent Decrease in the Measure of High-Type Entrepreneurs

Appendix B

Appendix to Chapter 3

B.1 Deriving Equilibrium Bubble Dynamics

As in Galí' (2014), the log-linearize equilibrium system consists equations (3.19), (3.20), and

$$0 = c_{1,t} + \beta R c_{2,t}, \tag{B.1}$$

$$c_{1,t} = E_t c_{2,t+1} - r_t, \tag{B.2}$$

$$c_{2,t} = (1-\Gamma)d_t + \Gamma b_t, \tag{B.3}$$

$$E_{t-1}w_t = E_{t-1}d_t = 0, \tag{B.4}$$

$$r_t = \phi_\pi \pi_t + \phi_b q_t. \tag{B.5}$$

Combining (B.1), (B.2), and (B.3) we derive

$$\begin{aligned} r_t &= (1-\Gamma)E_t d_{t+1} + \Gamma E_t b_{t+1} + \beta R((1-\Gamma)d_t + \Gamma b_t) \\ &= \Gamma E_t b_{t+1} + \beta R((1-\Gamma)d_t + \Gamma b_t), \end{aligned}$$

where we have used $E_t d_{t+1} = 0$ by (B.4) in the second equality.

Combining the equation above with (3.19) and (3.20) yields

$$r_t = \Gamma(r_t + Rb_t + (1-R)u_t) + \beta R((1-\Gamma)d_t + \Gamma b_t).$$

We substitute $\Gamma = \epsilon B/(\epsilon B + 1)$ into the equation above to obtain

$$r_t = \epsilon BR(1+\beta)b_t + \epsilon B(1-R)u_t + \beta R d_t. \tag{B.6}$$

Taking expectations conditional on information at time $t-1$ yields

$$E_{t-1}r_t = \epsilon BR(1+\beta)E_{t-1}b_t + \epsilon B(1-R)E_{t-1}u_t, \tag{B.7}$$

where we have used $E_{t-1}d_t = 0$. We use equation (B.7) and interest rate rule (B.5) to derive

$$\begin{aligned} r_t - E_{t-1}r_t &= \phi_\pi(\pi_t - E_{t-1}\pi_t) + \phi_b(q_t - E_{t-1}q_t) \\ &= \phi_b(q_t - E_{t-1}q_t) \\ &= \phi_b R(b_t - E_{t-1}b_t) + \phi_b(1-R)(u_t - E_{t-1}u_t), \end{aligned} \tag{B.8}$$

where the second equality follows from $\pi_t = E_{t-1}\pi_t$ due to price stickiness and we use (3.19) to substitute for q_t to derive the third equality.

Using (B.7) and (B.8) we derive

$$\begin{aligned} r_t &= r_t - E_{t-1}r_t + E_{t-1}r_t \\ &= \phi_b R b_t + (\epsilon B(1+\beta) - \phi_b)RE_{t-1}b_t + \phi_b(1-R)u_t \\ &+ (\epsilon B - \phi_b)(1-R)E_{t-1}u_t. \end{aligned}$$

Now we substitute the equation above into (3.20) and use (3.19) to derive

$$\begin{aligned} E_t b_{t+1} &= Rb_t + (1-R)u_t + \phi_b R b_t \\ &- (\phi_b - \epsilon B(1+\beta))RE_{t-1}b_t + \phi_b(1-R)u_t - (\phi_b - \epsilon B)(1-R)E_{t-1}u_t \\ &= (\phi_b + 1)Rb_t - (\phi_b - \epsilon B(1+\beta))RE_{t-1}b_t + (\phi_b + 1)(1-R)u_t \\ &- (\phi_b - \epsilon B)(1-R)E_{t-1}u_t. \end{aligned}$$

We then obtain (3.13). Q.E.D.

B.2 Proofs

Proof of Proposition 1: Conjecture that the solution takes the form

$$b_t = \varphi_0 b_{t-1} + \varphi_1 u_{t-2} + \varphi_2 e_t + \varphi_3 e_{t-1} + \varphi_4 \xi_t + \varphi_5 \xi_{t-1},$$

where $\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4$, and φ_5 are coefficients to be determined. Substituting this solution into (3.13) yields

$$\begin{aligned} b_t \quad = \quad & \frac{1}{R(\phi_b+1)} \left[\varphi_0 b_t + \varphi_1 u_{t-1} + \varphi_3 e_t + \varphi_5 \xi_t\right] \\ & + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b + 1} \left(\varphi_0 b_{t-1} + \varphi_1 u_{t-2} + \varphi_3 e_{t-1} + \varphi_5 \xi_{t-1}\right) \\ & + \frac{R-1}{R} \left(\rho u_{t-1} + e_t\right) + \frac{(\epsilon B - \phi_b)(R-1)}{(\phi_b+1)R} \rho u_{t-1}. \end{aligned}$$

That is,

$$\begin{aligned} b_t \quad = \quad & \frac{1}{R(\phi_b+1)} [\varphi_0 \left(\varphi_0 b_{t-1} + \varphi_1 u_{t-2} + \varphi_2 e_t + \varphi_3 e_{t-1} + \varphi_4 \xi_t + \varphi_5 \xi_{t-1}\right) \\ & + \varphi_1 \left(\rho u_{t-2} + e_{t-1}\right) + \varphi_3 e_t + \varphi_5 \xi_t] \\ & + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b + 1} \left(\varphi_0 b_{t-1} + \varphi_1 u_{t-2} + \varphi_3 e_{t-1} + \varphi_5 \xi_{t-1}\right) \\ & + \frac{R-1}{R} \left(\rho^2 u_{t-2} + \rho e_{t-1} + e_t\right) + \frac{(\epsilon B - \phi_b)(R-1)}{(\phi_b+1)R} \left(\rho^2 u_{t-2} + \rho e_{t-1}\right). \end{aligned}$$

Using the conjectured form for b_t again and matching coefficients, we obtain

$$\varphi_0 = \frac{1}{R(\phi_b+1)}\varphi_0^2 + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b+1}\varphi_0, \tag{B.9}$$

$$\varphi_1 = \frac{1}{R(\phi_b+1)}\left(\varphi_0\varphi_1 + \rho\varphi_1\right) + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b+1}\varphi_1 + \frac{R-1}{R}\rho^2 + \frac{(\epsilon B - \phi_b)(R-1)}{(\phi_b+1)R}\rho^2, \tag{B.10}$$

$$\varphi_2 = \frac{1}{R(\phi_b+1)}\left(\varphi_0\varphi_2 + \varphi_3\right) + \frac{R-1}{R}, \tag{B.11}$$

$$\varphi_3 = \frac{1}{R(\phi_b+1)}\left(\varphi_0\varphi_3 + \varphi_1\right) + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b+1}\varphi_3 + \frac{R-1}{R}\rho + \frac{(\epsilon B - \phi_b)(R-1)}{(\phi_b+1)R}\rho, \tag{B.12}$$

$$\varphi_4 = \frac{1}{R(\phi_b+1)}\left(\varphi_0\varphi_4 + \varphi_5\right), \tag{B.13}$$

$$\varphi_5 = \frac{1}{R(\phi_b+1)}\varphi_0\varphi_5 + \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b+1}. \tag{B.14}$$

There are two solutions for $\varphi_0 : \varphi_0 = 0$ and

$$\varphi_0 = \chi = R\left(1 + \epsilon B\left(1+\beta\right)\right).$$

In a neighborhood of the stable bubbly steady state, we have $\chi \in (0,1)$. The only stationary solution must corresponds to $\varphi_0 = \chi$ as Galí (2014) points out. We can then solve for the other coefficients:

$$\varphi_1 = (1-R)\left(1+\epsilon B\right)\rho,\ \varphi_2 = \frac{\varphi_3 + (R-1)\left(1+\phi_b\right)}{R\left(\phi_b+1\right) - \chi},\ \varphi_4 = \frac{\varphi_5}{R(\phi_b+1)-\chi},$$

and φ_3 and φ_5 are arbitrary numbers. Q.E.D.

Proof of Proposition 2: We take expectations conditional on information at time $t-1$ on both sides of (3.13) to obtain

$$\begin{aligned} E_{t-1}b_t\left[1-\frac{\phi_b-\epsilon B(1+\beta)}{\phi_b+1}\right] &= \frac{1}{R(\phi_b+1)}E_{t-1}b_{t+1} \\ &+ \left[\frac{R-1}{R}+\frac{(\epsilon B-\phi_b)(R-1)}{(\phi_b+1)R}\right]\rho u_{t-1}. \end{aligned}$$

This implies that

$$E_{t-1}b_t = \frac{1}{R[1+\epsilon B(1+\beta)]}E_{t-1}b_{t+1} - \frac{(1-R)(\epsilon B+1)}{R(1+\epsilon B(1+\beta))}\rho u_{t-1}.$$

By iterating the equation above forward we can derive

$$\begin{aligned} E_{t-1}b_t &= -\frac{(1-R)(\epsilon B+1)}{R(1+\epsilon B(1+\beta))}\left(\frac{1}{1-\rho/R[1+\epsilon B(1+\beta)]}\right)\rho u_{t-1} \\ &= -\frac{(1-R)(\epsilon B+1)}{\chi-\rho}\rho u_{t-1}, \end{aligned}$$

under the condition $\chi \equiv R[1+\epsilon B(1+\beta)] > 1$. Therefore we also have

$$E_t b_{t+1} = -\frac{(1-R)(\epsilon B+1)}{\chi-\rho}\rho u_t = -\frac{(1-R)(\epsilon B+1)}{\chi-\rho}(\rho^2 u_{t-1}+\rho e_t).$$

Substituting the preceding expressions for $E_t b_{t+1}$ and $E_{t-1}b_t$ into (3.13), we obtain the rational expectations solution in (3.16). Q.E.D.

Proof of Proposition 3: We use lifetime utility as the welfare criterion. Define the steady state welfare as

$$W_f \equiv \ln(C_1) + \beta\ln(C_2),$$

where C_1 and C_2 denote the steady-state consumption of a consumer in his young and old. In a steady state we have $C_1 = 1/\mathcal{M} - B$ and $C_2 = 1 - 1/\mathcal{M} + B$.

Therefore

$$W_f = \ln(\frac{1}{\mathcal{M}} - B) + \beta \ln(1 - \frac{1}{\mathcal{M}} + B).$$

We can compute

$$\frac{\partial W_f}{\partial B} = \frac{(\frac{1}{\mathcal{M}} - \frac{1}{1+\beta}) - B}{C_1 C_2 (1+\beta)}.$$

Denote $B^* \equiv 1/\mathcal{M} - 1/(1+\beta)$. Note that $B^* > 0$ under the condition $\mathcal{M} < 1+\beta$. This implies that welfare is increasing with B when $B < B^*$. As shown in Galí (2014) Lemma 1, for any $U \in (0, \bar{U})$ the model has two bubbly steady states. Moreover the stable one B_s is always less than the unstable one B_u. Thus to show the welfare is greater at B_u than at B_s, it suffices to show that $B_u < B^*$.

Since B_u is the larger root of equation $H(B, U) = B$, we have

$$B_u = \frac{-(1 + U - \frac{1+\beta}{\mathcal{M}}) + \sqrt{(1 + U - \frac{1+\beta}{\mathcal{M}})^2 - 4(1+\beta)(1 - \frac{1}{\mathcal{M}})U}}{2(1+\beta)}.$$

Therefore

$$B_u - B^* = \frac{(1 - U - \frac{1+\beta}{\mathcal{M}}) + \sqrt{(1 + U - \frac{1+\beta}{\mathcal{M}})^2 - 4(1+\beta)(1 - \frac{1}{\mathcal{M}})U}}{2(1+\beta)}.$$

Note that $1 - U - \frac{1+\beta}{\mathcal{M}} < 0$ by (3.12). To show $B_u < B^*$, it suffices to show that

$$(1 - U - \frac{1+\beta}{\mathcal{M}})^2 > (1 + U - \frac{1+\beta}{\mathcal{M}})^2 - 4(1+\beta)(1 - \frac{1}{\mathcal{M}})U.$$

This inequality is equivalent to $4(1+\beta)U > 4U$, which holds true since $U, \beta > 0$.

To study E-stability, we rewrite (3.13) in a general form

$$b_t = \beta_0 E_{t-1} b_t + \beta_1 E_t b_{t+1} + \gamma_0 u_t + \gamma_1 u_{t-1}, \tag{B.15}$$

where

$$\beta_0 = \frac{\phi_b - \epsilon B(1+\beta)}{\phi_b + 1},\ \beta_1 = \frac{1}{R(\phi_b+1)},$$
$$\gamma_0 = \frac{R-1}{R},\ \gamma_1 = \frac{\rho(\epsilon B - \phi_b)(R-1)}{(\phi_b+1)R}.$$

We can check that $\chi \equiv R(1+\epsilon B(1+\beta)) = \beta_1^{-1}(1-\beta_0)$. Suppose that the PLM is $b_t = a$. Set $E_{t-1}b_t = E_t b_{t+1} = a$ and $u_t = u_{t-1} = 0$. Then the ALM is $b_t = T(a) = (\beta_0 + \beta_1)\,a$. By Evans and Honkapohja (2012), the E-stability condition for the steady state $a = 0$ given the ODE $\dot{a} = T(a) - a = (\beta_0+\beta_1)\,a - a$ is $\beta_0 + \beta_1 < 1$. Since $\chi > 1$ for the unstable bubbly steady state and $\chi \in (0,1)$ for the stable bubbly steady state, we immediately establish the proposition. Q.E.D.

Proof of Proposition 4: We start with the MSV solution. We write the PLM as

$$b_t = \mu + \varphi_1 u_{t-1} + \varphi_2 e_t,$$

where we include a constant term μ. Stability under learning requires μ convergence of μ to zero. Plugging this equation into (B.15) we obtain the ALM with the map $T(\mu, \varphi_1, \varphi_2)$. By Evans and Honkapohja (2012), the E-stability condition is

$$\beta_0 + \beta_1 < 1,\ \beta_0 + \beta_1\rho < 1. \tag{B.16}$$

Using the definition in the proof of Proposition 3.3, we have

$$\beta_0 + \beta_1 = \frac{1 + R\phi_b - R\epsilon B(1+\beta)}{R(\phi_b+1)} = 1 + \frac{1-\chi}{R(\phi_b+1)},$$
$$\beta_0 + \rho\beta_1 = \frac{\rho + R\phi_b - R\epsilon B(1+\beta)}{R(\phi_b+1)} = 1 + \frac{\rho-\chi}{R(\phi_b+1)}.$$

Since $\chi > 1$ at the unstable bubbly steady state, the E-stability condition for the MSV solution is $\phi_b > -1$.

Now we consider the backward sunspot solution. We write PLM as

$$b_t = \mu + \varphi_1 b_{t-1} + \varphi_2 u_{t-1} + \varphi_3 e_t + \varphi_4 e_{t-1} + \varphi_5 \xi_t + \varphi_6 \xi_{t-1}.$$

Plugging this equation into (B.15) we obtain the ALM with the $T-$map. By Evans and Honkapohja (2012), the E-stability condition is $\beta_0 > 1$, $\beta_1 < 0$. Also the stationarity of the solution requires $|\beta_1^{-1}(1-\beta_0)| < 1$. In terms of our model parameters, the E-stability condition is $\phi_b < -1$. Q.E.D.

B.3 Deriving MSV Equilibrium

From Proposition 3.2 we have the forward-looking MSV solution for the old bubble:

$$b_t = \frac{(R-1)(\epsilon B + 1)}{\chi - \rho} \rho u_{t-1} + \frac{R-1}{R} \left[\frac{\rho(\epsilon B + 1)}{(\phi_b + 1)(\chi - \rho)} + 1 \right] e_t. \quad \text{(B.17)}$$

We use this solution to derive solutions for other variables in the model. By (3.19) we obtain the solution for q_t:

$$\begin{aligned} q_t &= R b_t + (1-R) u_t \\ &= (1-R) \left[1 - \frac{R(\epsilon B + 1)}{\chi - \rho} \right] \rho u_{t-1} + (R-1) \left[\frac{\rho(1+\epsilon B)}{(\phi_b + 1)(\chi - \rho)} \right] e_t. \quad \text{(B.18)} \end{aligned}$$

By (3.20) we obtain the solution for r_t:

$$\begin{aligned} r_t &= E_t b_{t+1} - q_t \\ &= (R-1) \left[\frac{(\epsilon B + 1)(\rho - R)}{\chi - \rho} + 1 \right] \rho u_{t-1} + \frac{\phi_b \rho (R-1)(\epsilon B + 1)}{(\phi_b + 1)(\chi - \rho)} e_t. \quad \text{(B.19)} \end{aligned}$$

By (B.5) we obtain the solution for π_t:

$$\pi_t = \frac{(R-1)[(\epsilon B+1)\rho + (\phi_b+1)(\epsilon BR\beta - \rho)]}{\phi_\pi(\chi-\rho)}\rho u_{t-1}.$$

Substituting (B.19) and (B.17) into (B.5) we obtain the solution for d_t:

$$d_t = \frac{\chi(R-1)[\phi_b\rho - \epsilon B(\beta R(1+\phi_b)+\rho)]}{\beta R^2(1+\phi_b)(\chi-\rho)}e_t.$$

By (B.3) we obtain the solution for $c_{2,t}$:

$$\begin{aligned} c_{2,t} &= (1-\Gamma)d_t + \Gamma b_t \\ &= \frac{\epsilon B\rho(R-1)}{\chi-\rho}u_{t-1} + \frac{\rho(R-1)(\phi_b-\epsilon B)}{\beta R(1+\phi_b)(\chi-\rho)}e_t. \end{aligned} \tag{B.20}$$

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