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ESSAYS ON HETEROGENEOUS AGENT MACROECONOMICS

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ABSTRACT

This dissertation consists of three essays studying macroeconomic implications of economic agents' decision making subject to risk. In Chapter 1, I explain the decline in both consumption and investment after shocks that depress investment, a task that many macroeconomic models struggle to accomplish. I show that, when markets are incomplete and unemployment risk is countercyclical, shocks that reduce investment raise a precautionary savings motive and thus depress consumption. In particular, the calibrated incomplete markets model generates procyclical consumption and explains most of the consumption volatility relative to output at business cycle frequencies.

Chapter 2 explores how the presence of incomplete markets and unemployment risk alters the sources of aggregate fluctuations. I incorporate structural shocks that are widely used in the business cycle literature in the setting presented in Chapter 1. I estimate the model using US macroeconomic data. I find that the presence of incomplete markets and unemployment risk significantly reduces the contribution of discount factor shocks to consumption fluctuations over the business cycle. This result is important because it makes business cycle fluctuations less dependent on shocks that are typically not grounded on solid microfoundations.

In Chapter 3, I derive the optimal loan contract between risk-sensitive lenders and credit constrained borrowers in the financial accelerator model of Bernanke, Gertler, and

Gilchrist (1999). I assume that lenders have Epstein and Zin recursive preferences and parameterize the risk aversion coefficient to match the equity premium. An important feature of the optimal contract is indexation to the lenders' marginal utility of consumption and the borrowers' marginal value of internal funds. I find that, under the optimal contract, the financial accelerator mechanism becomes very powerful, because lenders require high (low) interest rates on loans during downturns (booms). The result suggests that the extent to which financial frictions matter for aggregate fluctuations depends on the insurance incentive of lenders.

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Chapter 1

Investment Shocks, Unemployment Risk, and Macroeconomic Comovement

1.1 Introduction

A defining feature of the business cycle is the comovement of output and its components, particularly investment and consumption. However, although shifts in total factor productivity (TFP) can deliver this comovement in most macroeconomic models, other types of aggregate shocks do not (Barro and King, 1984). Understanding the mechanisms that give rise to this comovement is an important goal for macroeconomists, because we often believe that the business cycle is driven by developments orthogonal to productivity. This is especially true in the case of the Great Recession, in which shocks to firm investment such as variations in corporate credit spreads and cross-sectional dispersion of variables such as business profits, productivity, and stock returns were often thought to have been the main drivers.[1]

Although these forces are known to explain the investment dynamics in macroeconomic models through wait-and-see effect or changes in the user cost of capital, they drive consumption in the opposite direction of investment. The heart of the problem is the aggregate resource constraint; if resources are not invested, then there are more re-

[1] Bloom (2009), Gilchrist et al. (2014), and Bloom et al. (2014) illustrate how fluctuations in economic uncertainties induce swings in investment spending. Gilchrist and Zakrajšek (2012) provide evidence that spikes in credit spreads, particularly excess bond premia, had considerable predictive power for the recent economic slump. Caldara et al. (2016) show that the combination of financial and uncertainty shocks has played a significant role in business cycle fluctuations over the past four decades and fully accounts for the contraction in economic activity during the Great Recession.

sources available for consumption. Adding New Keynesian ingredients to the model can potentially help, because the reduction in aggregate demand will put downward pressure on production and therefore reduce the overall level of resources in the economy. Nevertheless, it is still difficult to achieve comovement without assuming either a degree of nominal rigidity that is too large to be consistent with the empirical evidence or extremely passive monetary policy.

This chapter presents a model through which forces that depress aggregate investment can generate concurrent reductions in aggregate consumption. Aggregate consumption is the largest component of GDP and thus understanding the economic mechanism that explains consumption dynamics is important for our understanding of the business cycle and the design of stabilization policy. For example, the failure of most models to generate endogenous comovement between consumption and investment has led to an important role for discount factor shocks in estimated business cycle models and in some discussions of the Great Recession.[2] These shocks clearly stand in for other fundamental driving forces.

The mechanism is as follows. A reduction in investment reduces aggregate demand and generates an endogenous increase in unemployment risk. When unemployment risk is uninsurable, the increased risk leads to a precautionary savings response that reduces aggregate consumption. These dynamics arise from the combination of three well-known ingredients. First, households face borrowing constraints and suffer substantial losses in consumption upon job loss.[3] An increase in unemployment risk therefore increases the

[2]Justiniano et al. (2010, 2011) and Christiano et al. (2014) show that most of the variability of consumption is due to discount factor shocks.

[3]Using data from the first four waves of the Health and Retirement Study (HRS), Stephens (2004) finds that annual food consumption falls by roughly 16 percent upon a worker being displaced. Similarly, using the 1999-2009 biannual waves of the Panel Study of Income Dynamics (PSID), Saporta-Eksten (2014) finds that job loss leads to a drop in expenditures on non-durables and services of 17 percent, of which about half occurs before job loss and the other half occurs around job loss. Chodorow-Reich and Karabarbounis (2015), using the Consumer Expenditure Survey (CE), report a 21 percent decline in expenditures on non-durables and services upon unemployment of one year. Kolsrud et al. (2015) use Swedish data to document that annual consumption expenditures drop on average by 27 percent for those who are unemployed for

precautionary savings motive. Second, employment is determined by a frictional labor market so that unemployment risk varies in accordance with firms' vacancy-posting decisions. Third, firms are faced with a reasonable degree of rigidity in price setting so that changes in aggregate demand are transmitted to supply-side decisions. Nominal rigidities are understood to be essential for unemployment risk to have a substantial effect on aggregate fluctuations (Ravn and Sterk, 2013).[4]

After calibrating the model, I show that investment shocks lead to comovement of consumption and investment, and explain roughly 90 percent of consumption volatility relative to investment since the mid-1980s. I then construct a quarterly series of investment shocks so that the investment predicted by the model matches the behavior of investment during the Great Recession. My findings indicate that unemployment risk triggered by a fall in investment substantially accounts for the contraction in aggregate consumption during the recession. In contrast, the representative-agent version of the model, in which idiosyncratic unemployment risk is insurable, predicts acyclical consumption at business cycle frequencies and a very mild drop in aggregate consumption during the Great Recession.

To see whether the periods during which the unemployment rate is high are accompanied by high private savings in the data, Figure 1·1 displays the unemployment rate and aggregate private savings rates. The figure reveals that the unemployment rate and personal savings rate are positively correlated, which is consistent with the model mechanism.[5] A model based on permanent income hypothesis will struggle to explain this correlation, as high unemployment rate means low income, implying dissaving to smooth out consumption.

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longer than 20 weeks.

[4]Krusell and Smith (1998) and Krusell et al. (2010) find that imperfect insurance against unemployment risk does not help in generating more volatile business cycles under flexible prices.

[5]A sharp increase in personal saving rate during the Great Recession is also present at the household-level. Heathcote and Perri (2017) document that wealth-poor households increased savings more sharply than richer households, pointing toward the presence of the precautionary channel over this period.

Figure 1·1: Unemployment Rate and Personal Saving Rate

The analysis proceeds as follows. Section 1.2 explains why the comovement problem arises in standard representative-agent models. Section 3.3 presents the baseline model augmented with labor market search frictions and New Keynesian features. Section 1.4 discusses how the model is calibrated. Section 1.5 demonstrates that the model delivers comovement between consumption and investment. Section 1.6 examines the role of sticky prices and monetary policy in achieving such comovement. Section 3.5 concludes.

Relationship with the literature This paper is related to a burgeoning literature that integrates market incompleteness and nominal rigidities.[6] A subset of the literature considers incomplete markets models with labor market frictions and nominal rigidities.

Ravn and Sterk (2013) study the interaction of aggregate demand and idiosyncratic labor market uncertainties and show that time-varying precautionary savings induced by an exogenous shock to the job separation rate could be central in amplifying recessions. den Haan et al. (2017) study an environment in which sticky nominal wages magnify output fluctuations in response to productivity shocks when markets are incomplete. Heathcote and Perri (2017) argue that a shock to unemployment expectations in a low

[6] A nonexhaustive list includes Oh and Reis (2012), Bayer et al. (2015), Kaplan et al. (2016), McKay et al. (2016), and McKay and Reis (2016).

liquid wealth environment can endogenously generate an increase in unemployment risk and thus rationalize high expected unemployment. Although these studies consider similar ingredients to those introduced in this paper, they do not include physical capital. Challe et al. (2017) and Gornemann et al. (2014) include physical capital but differ from my analysis in terms of focus. While Challe et al. (2017) incorporate a variety of structural shocks and quantify the extent to which the precautionary savings effects raise the volatility of aggregate consumption, I study the comovement of aggregate demand components. Gornemann et al. (2014) study the distributional consequences of monetary policy shocks, whereas this paper is concerned with the aggregate consequences of investment shocks. All these papers have in common with mine that weak demand and unemployment risk mutually reinforce one another. However, none of these papers considers how the presence of the feedback loop between aggregate demand and unemployment risk alters the sources of aggregate fluctuations.

Several other studies suggest solutions to the comovement problem associated with investment shocks in a complete markets setting. Furlanetto et al. (2013) show that aggregate comovements can be obtained in the New Keynesian model embedded with hand-to-mouth consumers. However, one has to assume a fairly large fraction of these consumers, substantially above what is consistent with microeconomic evidence. In contrast, these constrained consumers in my model represent a very small fraction. An alternative route to ensure comovement is to embed the preferences proposed by Greenwood et al. (1988) and Jaimovich and Rebelo (2009), which feature consumption-hours complementarity. For example, Gilchrist and Zakrajšek (2011) show that, with these preferences, financial shocks lead to positive comovement between consumption and investment. Similarly, Khan and Tsoukalas (2011) argue that the comovement problem can be solved with these preferences in combination with the cost of capital utilization specified in terms of the increased depreciation of capital. Eusepi and Preston (2015) study comovement issues in an environment in which the agent's preferences display a reasonable degree of non-separability between consumption and hours and in which hours are mainly driven by the extensive margin. Another interesting way to achieve comovement is through real wage rigidity that strengthens an income effect.[7] However, none of these stories is compatible with an increase in savings rate during recessions and so need unappealing discount factor shocks to explain this feature.

The work by Bayer et al. (2015) is close to mine in the use of models with market incompleteness and uninsurable idiosyncratic uncertainty to study macroeconomic comovement. Their goal is to generate a fall in physical investment when consumption demand is reduced due to an exogenous rise in income uncertainty. However, because of the absence of a channel through which household-level uncertainty arises endogenously, an innovation that depresses investment does not generate a fall in consumption in their economy.

1.2 Why Is Comovement Difficult to Achieve?

It is useful to start with an explanation for why achieving comovement of macroeconomic variables in response to investment shocks is difficult in standard representative-agent models. I first explain the lack of comovement in the context of models in which the employment adjustment is on the intensive margin and then describe how a similar problem arises in models with labor market search frictions, which are key elements in this paper.

Consider the following goods market clearing condition,

$$c_t + i_t = F(k_t, n_t), \tag{1.1}$$

where $F(\cdot)$ is a Cobb-Douglas production function, and c_t, i_t, k_t, and n_t are consumption,

[7]Liu et al. (2017) present a mechanism that generates endogenous wage rigidity. A rise in consumption after a negative investment shock reduces the marginal utility of consumption. This makes the workers' reservation wages fail to fall, producing endogenous wage rigidity.

investment, capital, and employment, respectively. To start with, suppose that employment is fixed and normalized to one. Because capital is predetermined, the supply of current goods is fixed. In this environment, in response to a shock that reduces the demand for current investment goods, market forces work to drive down the price of current goods relative to future goods, that is, the real interest rate. A drop in the real interest rate would then stimulate current consumption. Equation (1.1) indicates that consumption and investment fall together if employment declines enough.

The comovement of consumption and investment is more difficult to achieve in the real business cycle (RBC) model in which employment varies on the intensive margin. Consider the labor market equilibrium condition in the RBC model with standard preferences and Cobb-Douglas technology,

$$(\alpha + \varphi)\ln n_t = \ln (1 - \alpha) - \sigma \ln c_t + \alpha \ln k_t, \tag{1.2}$$

where φ is the inverse of the Frisch elasticity of labor supply, σ is the coefficient of risk aversion, and α is the capital share.[8] This condition states that consumption and employment must be negatively correlated (Barro and King, 1984). The fall in consumption is associated with the rise in employment, and so comovement does not occur.

Adding nominal rigidities gives hope that consumption may move together with investment and output. Sticky prices distort the labor market equilibrium such that the marginal product of labor is premultiplied by the inverse of the markup. When there is a contraction in aggregate demand, an increase in the markup shifts the labor demand curve inward, reducing employment and thus the resources produced in the economy. The fall in overall income works to reduce aggregate consumption. Nonetheless, unless one either assumes very rigid prices that are beyond the range of microeconomic evidence (average duration of 3-4 quarters) or introduces passive monetary policy that does not ap-

[8]The equilibrium condition is $-\frac{U_{n,t}}{U_{c,t}} = F_{n,t}$, where $F(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$ and $U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\varphi}}{1+\varphi}$.

proximate the US economy prior to the Great Recession, comovement would not obtain.[9] Although monetary policy was constrained during the Great Recession, disturbances to investment are often thought to be one of the most important sources of the business cycle fluctuations before the recent recession. Thus, it is important to obtain comovement under unconstrained monetary policy as well.

The comovement problem persists in models with labor market search frictions. In these models, firms adjust employment along the extensive margin, and employment is determined by the firm surplus from hires. Therefore, the only way to reduce production and thus consumption is to have the firm surplus fall sufficiently. Under flexible prices, the firm surplus from a match is given as

$$J_t = \sum_{s=0}^{\infty} M_{t,t+s}(1-\rho_x)^s \mathbb{E}_t[MPN_{t+s} - w_{t+s}], \tag{1.3}$$

where $M_{t,t+s}$, ρ_x, MPN_t, and w_t are the stochastic discount factor for s-period future payoffs, the job separation rate, the marginal product of labor, and the real wage in period t, respectively. To reduce the employment level, the present discounted value (PDV) of future profits, i.e., the difference between the marginal product and the real wage, has to fall. However, three features work against this. First, a decline in real interest rates increases the future stochastic discount factor and thus the PDV of future profits. Second, a reduction in investment leads to only small changes in the marginal product of labor, which depends on the capital stock. Because capital with a low depreciation rate has a high stock-flow ratio, changes in investment over a moderate horizon have small effects on the total stock. Third, in the data, real wages are procyclical or at least not countercyclical (Stock and Watson, 1999; Hagedorn and Manovskii, 2008a; Basu and House,

[9] Justiniano et al. (2010, 2011) show that consumption is quite flat initially under the posterior estimate of price stickiness of 5 quarters, but their model still does not completely achieve comovement with investment. Carrillo and Poilly (2014) demonstrate that although consumption falls after financial shocks that depress investment when the zero lower bound constraint binds, it increases when monetary policy is unconstrained.

2016).

Incorporating sticky prices into the search model of unemployment can ease the lack of comovement in similar ways to the model in which labor input varies on the intensive margin. That is, the marginal product of labor in Equation (1.3) is premultiplied by the inverse of the markup, which leads to increased fluctuation in the firm surplus from a match and thus in the equilibrium employment. Again, comovement is not obtained in representative-agent models without a high degree of price stickiness or monetary policy that responds too little to inflation, which I show in Section 1.6.

In this paper, I present a mechanism that delivers comovement under a plausible degree of price stickiness and a standard monetary policy rule. The idea is that when investment declines, a rise in unemployment risk triggers the precautionary savings motive, putting downward pressure on consumption. This creates a substantial decline in the inverse of the markup and thus in employment, greatly reducing the amount of resources available for consumption and investment.

1.3 Model

I construct a model in which unemployment risk rises upon investment shocks, depressing consumption demand. Investment shocks are identified as innovations to the marginal efficiency of investment. These shocks lead to qualitatively similar responses of consumption, investment, and output to financial shocks or firm-level idiosyncratic uncertainty shocks in a model that incorporates financial factors.[10] Therefore, instead of explicitly specifying financial shocks or uncertainty shocks, I interpret a decline in the marginal efficiency of investment as a reduction in the credit supply to firms or a rise in uncertain-

[10]For instance, the comovement problem similarly arises in the financial accelerator model of Bernanke et al. (1999) after shocks to the volatility of idiosyncratic productivity of the entrepreneurs or shocks to net worth (Carrillo and Poilly, 2014).

ties.[11]

The economy is populated by two groups of households (impatient and patient), a continuum of firms producing differentiated intermediate goods, a perfectly competitive firm producing a final good, a central bank in charge of monetary policy, and a fiscal authority. Except for the presence of impatient households that face uninsurable unemployment risk and borrowing constraints, the model is similar to the New Keynesian DSGE model augmented with a frictional labor market, as in Gertler et al. (2008).

1.3.1 Impatient Households

There is a measure $1-\Omega \in [0,1]$ of impatient households indexed by $j \in [0, 1-\Omega]$. An impatient household is either employed or unemployed. Upon employment, it supplies labor inelastically and receives a real wage w_t. Upon unemployment, it receives unemployment insurance with replacement rate b^u, assumed to be taxable. A household working at the beginning of the period may lose a job within the period with probability ρ_x. However, I assume the household may find a job immediately upon separation with probability f within the period. Therefore, the event that each employed household falls into the unemployment pool at the end of the period occurs with probability $\rho_x(1-f)$. I refer to this rate as the job-loss rate.

Impatient households cannot purchase unemployment insurance contracts. They can only self-insure through trading riskless bonds, but they cannot take short positions. One way to micro-found the assumption that savings of these households are directed towards government bonds is to explicitly model illiquid physical capital as in Bayer et al. (2015). In this environment, wealth-poor households choose to hold more liquid government bonds rather than paying the transaction cost for holding illiquid physical capital upon an in-

[11] Justiniano et al. (2011) show that movement in the marginal efficiency of investment is highly correlated with fluctuations of corporate credit spreads. Moreover, Gilchrist et al. (2014) argue that the impact of uncertainty on investment occurs primarily through changes in credit spreads rather than through the traditional wait-and-see effect.

crease in idiosyncratic risk. The budget constraint of impatient household i at period t is given by

$$c^I_{j,t} + a^I_{j,t+1} = (1-\tau_t)w_t e_{j,t} + (1-\tau_t)b^u w_t(1-e_{j,t}) + \frac{R_{t-1}}{\Pi_t}a^I_{j,t}, \tag{1.4}$$

together with borrowing constraint, $a^I_{j,t+1} \geq 0$, where $c^I_{j,t}$ denotes the consumption of impatient household j, Π_t denotes the gross inflation rate, R_{t-1} is the gross nominal interest rate paid on bonds purchased in period $t-1$, $a^I_{j,t}$. τ_t denotes the tax rate on labor and transfer income, and $e_{j,t}$ refers to an indicator for employment status where $e_{j,t} = 1$ if the household is employed and $e_{j,t} = 0$ if it is unemployed.

Individual impatient households choose consumption and bond holdings $\{c^I_{j,t}, a^I_{j,t+1}\}$ to maximize:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta^I)^s \left[\frac{(c^I_{j,t+s})^{1-\sigma}}{1-\sigma} \right], \tag{1.5}$$

subject to the budget constraint (1.4) and a borrowing constraint. Here, β^I is the discount factor for impatient households.

1.3.2 Patient Households

There is a measure Ω of patient households. This group is relatively more patient, enjoying significant wealth by owning bonds and physical capital. These households receive a large amount of income which is equal to the aggregate income (labor income and unemployment benefit) augmented with the skill premium $\eta > 1$. These households are rich enough to self-insure unemployment risk. Their preferences are the same as those of impatient households, but they have a higher discount factor: $\beta^P > \beta^I$. Consumption and investment in physical capital are financed by four sources: trading bonds with impatient households and the government, revenue from renting capital to intermediate-goods firms, income, and dividends from intermediate-goods firms. Unlike an impatient household, the representative patient household does not face a borrowing constraint, and so their Euler equation holds with equality. The t-period budget constraint is

$$c_t^P + a_{t+1}^P + i_t^P \quad = \quad (1-\tau_t)\eta(w_t n_t + b^u w_t(1-n_t)) + \frac{R_{t-1}}{\Pi_t} a_t^P + r_t^k k_t^P + d_t^P, \tag{1.6}$$

where c_t^P, a_{t+1}^P, and i_t^P are consumption, bond holdings, and investment by the patient household in period t, respectively. r_t^k represents the real rental rate of capital, and k_t^P is the capital stock. d_t^P denotes the dividend from owning intermediate-goods firms. Physical capital is accumulated via the following technology,

$$k_{t+1}^P - (1-\delta)k_t^P = \mu_t \left(1 - S\left(\frac{i_t^P}{i_{t-1}^P}\right)\right) i_t^P, \tag{1.7}$$

where δ is the depreciation rate. As in Christiano et al. (2005), investment in physical capital is subject to quadratic adjustment costs, $S(\cdot)$, to capture a hump-shaped response of investment in response to investment shocks, consistent with SVAR based evidence from Gilchrist and Zakrajšek (2012).[12] In the steady state, $S = S' = 0$ and $S'' > 0$. μ_t is the marginal efficiency of investment, which governs the efficiency at which investment goods are transformed into physical capital that is used for production in the next period. It follows the stochastic process

$$\begin{aligned} &\log(\mu_t) = \rho^\mu \log(\mu_{t-1}) + \sigma^\mu \varepsilon_t^\mu \\ &\text{with} \qquad \varepsilon_t^\mu \overset{iid}{\sim} \mathcal{N}(0, \sigma^{\mu 2} \end{aligned} \tag{1.8}$$

where ρ^μ and σ^μ denote the persistence and the standard deviation of the shock, respectively.

[12]To assess the robustness, I consider an alternative specification for adjustment costs that is proportional to the investment-capital ratio in Appendix A.5.

1.3.3 Matching

Each firm posts multiple identical vacancies. Vacancies and unemployed households are randomly matched according to the aggregate matching function,

$$m(u_{a,t}, v_t) = \psi(u_{a,t})^{\gamma}(v_t)^{1-\gamma}, \tag{1.9}$$

where $m(u_{a,t}, v_t)$ is the number of matches in period t when there are $u_{a,t}$ job seekers and v_t vacancies. ψ is the matching efficiency, and γ represents the elasticity of matches with respect to job seekers. Job seekers consist of the unemployed households from the previous period and households that were employed in the previous period but were separated in this period. Therefore, the number of job seekers in period t is given by

$$u_{a,t} = u_{t-1} + \rho_x n_{t-1}. \tag{1.10}$$

Given the matching function, the probability that a vacant job is filled and the probability that a job seeker becomes employed are

$$\lambda_t = m(1/\theta_t, 1) = \psi\theta_t^{-\gamma} \tag{1.11}$$

and

$$f_t = m(1, \theta_t) = \psi\theta_t^{1-\gamma}, \tag{1.12}$$

respectively, where $\theta_t = v_t/u_{a,t}$ denotes the labor market tightness. The number of unemployed households in period t equals the number of job seekers who failed to find a job and is given by

$$u_t = (1 - f_t)(u_{t-1} + \rho_x n_{t-1}). \tag{1.13}$$

1.3.4 Production Sector

Final-goods firms A representative firm combines differentiated intermediate goods and produces a final good according to a Dixit-Stiglitz aggregator,

$$y_t = (\int {y_{z,t}}^{1-1/\varepsilon} \mathrm{d}z)^{1/(1-1/\varepsilon)}, \tag{1.14}$$

where $y_{z,t}$ is the amount of intermediate good z used and ε is the elasticity of substitution between any pair of intermediate goods. The final-good firm's problem is to minimize expenditures on intermediate goods taking the prices as given subject to the production function (3.29). Its optimal choices imply the demand function for intermediate goods z,

$$y_{z,t} = \left(\frac{P_{z,t}}{P_t}\right)^{-\varepsilon} y_t, \tag{1.15}$$

where $P_{z,t}$ is the price of intermediate good z in period t. P_t denotes the aggregate price index, which is given by

$$P_t = (\int {P_{z,t}}^{1-\varepsilon} \mathrm{d}z)^{1/(1-\varepsilon)}. \tag{1.16}$$

Intermediate-goods firms There is a unit continuum of monopolistic producers of intermediate-goods. Firm z produces differentiated good z according to,

$$y_{z,t} = k_{z,t}^{\alpha} n_{z,t}^{1-\alpha} - \xi, \tag{1.17}$$

where $0 < \alpha < 1$. Here, $k_{z.t}$ and $n_{z.t}$ denote the capital and the stock of employees used, respectively. ξ denotes the fixed cost of production. In every period, the firm posts vacancies, $v_{j,t}$, which are filled with probability λ_t. Therefore, the evolution of employees of firm z is given as

$$n_{z,t} = (1-\rho_x) n_{z,t-1} + \lambda_t v_{z,t}. \tag{1.18}$$

In addition, the firm faces price-setting frictions which are modeled as quadratic costs of price adjustment following Rotemberg (1982). A firm z maximizes the present discounted stream of profits,

$$\max_{P_{z,t},n_{z,t},v_{z,t},k_{z,t}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s}\Big[\left(\frac{P_{z,t+s}}{P_{t+s}}\right) y_{z,t+s} - w_{t+s}n_{z,t+s} - r^k_{t+s}k_{z,t+s} - \kappa v_{z,t+s} \tag{1.19}$$
$$-\frac{\phi_p}{2}\left(\frac{P_{z,t+s}}{P_{z,t+s-1}} - 1\right)^2 y_{t+s}\Big]$$

subject to (3.30), (3.32) and (3.33). The costs for the firm are the forgone resources from searching for new employees and setting prices, the wage bill paid to all employees, and the rental of capital. κ is the cost associated with posting a vacancy. $M_{t,t+s}$ is the stochastic discount factor of patient households who are the owners of the intermediate-goods firms.

In the presence of frictional labor markets, there is a surplus in the employment relationship because, on one side, firms would have to pay hiring costs to find a new employee and, on the other side, a household who rejects a job becomes unemployed and foregoes the opportunity to earn wages during this period. This surplus creates a bargaining set for wages, and there are many models of how wages are chosen within this set from Nash bargaining to wage stickiness, as emphasized by Hall (2005).

Because the literature is inconclusive over the empirical relevance of popular Nash bargaining, in this paper, I assume a convenient wage rule:

$$w_t = w\left(\frac{y_t}{y}\right)^{\phi_w}, \tag{1.20}$$

where w is the steady state real wage and $\phi_w \in [0,1]$ is the elasticity of the real wage with respect to the deviation of output from its steady state. In Appendix A.6, I verify that the series of wages predicted by the model lies within the bargaining set.

1.3.5 Government

The government raises tax revenue to finance expenditures on unemployment insurance. The government budget constraint is

$$\tau_t(w_t n_t + b^u w_t u_t) = b^u w_t u_t. \tag{1.21}$$

Monetary policy follows a Taylor rule,

$$\ln(\frac{R_t}{R}) = \alpha_\pi \ln\left(\frac{\Pi_t}{\Pi}\right), \tag{1.22}$$

where α_π measures the extent to which the interest rate responds to a deviation of the inflation rate from its target. R and Π are the steady state gross nominal interest rate and gross inflation rate, respectively. I omit the output gap term because with incomplete markets, it is no longer clear how to define the constrained-welfare natural level of output.

1.3.6 Market Clearing and Equilibrium

There are four markets operating in the model: bonds, labor, capital, and final goods. The bond market clears when

$$(1-\Omega)\int g(a^I, e; \mathcal{S}_t)\mathrm{d}\Gamma_t(a^I, e) + \Omega a_t^P = 0, \tag{1.23}$$

where $g(\cdot)$ refers to the savings rule of impatient households and $\mathcal{S}$ is the set of aggregate state variables, which is described later. Moreover, $\Gamma_t(\cdot)$ is the CDF of the distribution of impatient households over bond levels and employment states in the beginning of the period. The markets for labor, capital, and final goods clear if

$$((1-\Omega) + \eta\Omega)n_t = \int_0^1 n_{z,t}\mathrm{d}z, \tag{1.24}$$

$$\Omega k_t = \int_0^1 k_{z,t}\mathrm{d}z, \tag{1.25}$$

and

$$c_t + i_t = y_t - \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 y_t - \kappa v_t \tag{1.26}$$

hold, where $c_t = (1-\Omega)\int c^I_{j,t}\mathrm{d}j + \Omega c^P_t$ and $i_t = \Omega i^P_t$.

The set of aggregate states in period t, $\mathcal{S}_t$, is given by

$$\mathcal{S}_t = \{\Gamma_t(a^I, e), a^P_t, i^P_{t-1}, k^P_t, \mu_t\}.$$

Note that knowledge of the cross-sectional distribution of wealth is sufficient to compute employment stock before the labor market transitions, that is $n_{t-1} = \int_{a^I} \mathrm{d}\Gamma_t(a^I, 1)$. We are now in a position to define the equilibrium in this economy.

A symmetric equilibrium is a sequence of aggregate quantities, $\{c_t, i_t, y_t, k_t, d_t, n_t, u_t, u_{a,t}, f_t, \lambda_t, v_t, \theta_t, \mu_t\}_{t=0}^{\infty}$; prices, $\{w_t, r^k_t, \Pi_t\}_{t=0}^{\infty}$; impatient households decision rules, $\{g(a^I, e; \mathcal{S}_t)\}_{t=0}^{\infty}$; patient household variables, $\{c^P_t, i^P_t, k^P_t\}_{t=0}^{\infty}$; the distribution of impatient households over bond wealth and employment states, $\{\Gamma_t(a^I, e)\}_{t=0}^{\infty}$; and policy instruments, $\{R_t, \tau_t\}_{t=0}^{\infty}$, such that

(1) *the impatient household decision rules maximize* (1.5) *subject to* (1.4);

(2) *patient households maximize the same felicity function as impatient households subject to* (1.6);

(3) *the distribution of impatient households over bond wealth and employment states evolves in a manner consistent with the decision rules and endogenous idiosyncratic unemployment risks:*

$$\Gamma_{t+1}(\mathcal{A}, e') = \sum_{e' \in \{0,1\}} \mathrm{Pr}_t(e'|e) \int \mathbf{1}\{g(a^I, e; \mathcal{S}_t) \in \mathcal{A}\} \mathrm{d}\Gamma_t(a^I, e);$$

(4) *the final-goods firm's decisions are* (3.30) *and* (3.31);

(5) *intermediate-goods firms maximize* (3.34) *subject to* (3.30), (3.32), *and* (3.33) *given factor prices*;

(6) *dividends received by patient households result from the optimal decisions of the intermediate-goods firms*;

(7) *real wages are consistent with* (1.20), *and the real rental rate of capital and inflation respect the optimal decisions of the intermediate-goods firms*;

(8) *the stock of capital, the stock of job seekers, the job-filling rate, the job-finding rate and the stock of unemployed households vary consistently with* (1.7), (3.25), (3.26), (3.27), *and* (3.28) ;

(9) *the process of marginal efficiency is* (1.8);

(10) *the government adjusts taxes subject to* (3.36), *and monetary policy follows* (3.37);

(11) *markets that operate in the economy clear,* (1.23) - (3.41),

where $\mathcal{A}$ is a subset of the space of bond holdings and $\Pr_t(e'|e)$ denotes the transition rate from employment state e to state e' which varies endogenously.

Appendix A.1 derives the optimality conditions that are used to solve the model.

1.3.7 Representative-Agent Model

The comparable representative-agent model is obtained by introducing the patient household whose labor productivity is $\eta^{RA} \equiv (1-\Omega)+\eta\Omega$ so that average labor productivity is the same as in the baseline model. That is, the household budget constraint is

$$c_t + a_{t+1} + i_t \quad = \quad (1-\tau_t)\eta^{RA}(w_t n_t + b^u w_t(1-n_t)) + \frac{R_{t-1}}{\Pi_t} a_t + r_t^k k_t + d_t, \qquad (1.27)$$

In this benchmark, all households are identical and fully insured with discount factor β^P. Therefore, steady state real interest, effective labor, $((1-\Omega)+\eta\Omega)n_t$, and total net wealth k_t is unchanged from the baseline heterogeneous-agent model.

The equilibrium in the representative-agent economy is a sequence of aggregate quantities, $\{c_t, i_t, y_t, k_t, d_t, n_t, u_t, u_{a,t}, f_t, \lambda_t, v_t, \theta_t, \mu_t\}_{t=0}^{\infty}$; prices,$\{w_t, r_t^k, \Pi_t\}_{t=0}^{\infty}$; and policy instruments, $\{R_t, \tau_t\}_{t=0}^{\infty}$, such that

(1) households maximize its value subject to (1.27);

(2) the final-goods firm's decisions are (3.30) *and* (3.31);

(3) intermediate-goods firms maximize (3.34) *subject to* (3.30), (3.32), *and* (3.33) *given factor prices*;

(4) dividends received by patient households result from the optimal decisions of the intermediate-goods firms;

(5) real wages are consistent with (1.20), *and the real rental rate of capital and inflation respect the optimal decisions of the intermediate-goods firms*;

(6) the stock of capital, the stock of job seekers, the job-filling rate, the job-finding rate and the stock of unemployed households vary consistently with (1.7), (3.25), (3.26), (3.27), *and* (3.28) ;

(7) the process of marginal efficiency is (1.8);

(8) the government adjusts taxes subject to (3.36), *and monetary policy follows* (3.37);

(9) markets that operate in the economy clear:

$$\begin{aligned} a_t &= 0, \\ k_t &= \int_0^1 k_{z,t} \mathrm{d}z, \\ \eta^{RA} n_t &= \int_0^1 n_{z,t} \mathrm{d}z, \\ c_t + i_t &= y_t - \frac{\phi_p}{2} \left(\Pi_t - 1 \right)^2 y_t - \kappa v_t. \end{aligned}$$

1.4 Calibration and Computation

1.4.1 Calibration

The model period is one quarter. I first fix the values of the parameters that determine the steady state values of aggregate quantities and prices. I then choose the values of the parameters that govern wealth and consumption distribution and the extent to which the households are insured against unemployment risk. Lastly, I discuss the parameters that are relevant to the aggregate dynamics.

The capital depreciation rate, δ, is assumed to be 0.015, implying a 6 percent annual depreciation of physical capital. The power on capital in the production function, α, is set to 0.33. I fix the discount factor of patient households to match the annual real return on bonds of 3 percent, in line with the average *real* Federal funds rate from 1984Q1 to 2008Q3. I target a steady state unemployment rate of 6 percent, a value that corresponds to the average unemployment rate between 1984 and 2012. For the elasticity of substitution between intermediate goods, ε, I target a steady state markup of 1.2 (Basu and Fernald, 1997). The fixed cost, ξ, is set so that the steady state profits of monopolistic competitive firms are zero (Rotemberg and Woodford, 1999).

The steady state transition rates among employment states are determined by the job-finding rate, f, and the separation rate, ρ_x. Following Shimer (2005), I first compute the monthly job-finding rate using unemployment and short-term unemployment data from the Current Population Survey (CPS). I average the resulting series over each quarter and convert these into quarterly terms. The job-finding rate averaged 0.73 from 1984 to 2012. Using equation (3.28), I then compute the steady state job separation rate. The matching function elasticity to job seekers, γ, is 0.5, as suggested by Petrongolo and Pissarides (2001). For the matching efficiency, ψ, I exploit the relation between the vacancy-filling rate and the job-finding rate using equations (3.27) and (3.26) and target a quarterly vacancy-filling rate of 0.71, computed by den Haan et al. (2000). The expected costs of hiring a worker, κ/λ, is calibrated to match 4.5 percent of quarterly wages, following Hagedorn and Manovskii (2008a), whose calculation is based on the time spent hiring one worker. The value of the steady state real wage is obtained from the optimal vacancy-posting condition under the free entry assumption.

Because the key assumption in this paper is the inability to insure against unemployment spells, one needs to determine a fraction of impatient households and their consumption and wealth share. I set the share of patient households, Ω, to 0.4. The cross-sectional distribution of income and thereby the distribution of consumption is directly affected by the skill premium η. The skill premium parameter is chosen to replicate an average consumption share of 57% for the richest 40% of households.[13] I target the wealth share of 6% for the poorest 60% of households, matching the corresponding quantile of the distribution of net wealth in the Survey of Consumer Finances reported by Díaz-Giménez et al. (2011). I assume that the consumption level of unemployed households is 20 percent lower than that of employed households on average, consistent with several micro estimates.[14] There are three parameters that jointly affect the difference in consumption between the employed and the unemployed and a wealth share for impatient households. First is the coefficient of risk aversion, σ. The higher its value, the more savings are held, and thus the consumption drop upon unemployment is smaller. Second, the replacement rate, b^u, directly influences the income level of the unemployed and thus their consumption. Third, given the return on savings, the discount factor of impatient households determines the cost of precautionary savings in terms of deferred consumption. The lower the discount factor, the more likely households are near the borrowing constraint and thus are exposed to a large consumption fluctuation. The coefficient of risk aversion is set equal to 2.5. I then adjust the replacement rate and the discount factor of impatient households that jointly match the consumption differential between the employed and the unemployed, and a wealth share for the bottom 60%.

For the parameter that governs the costs of adjusting prices, I exploit the equivalence of the coefficient of marginal cost in the linearized Phillips curve implied by the Rotemberg model and the one derived from the Calvo model. I then find the value of ϕ_p that corresponds to a price adjustment frequency of 4 quarters, consistent with the evidence

[13]I am grateful to Challe et al. (2017) for sharing their dataset online.
[14]See footnote 3.

in Nakamura and Steinsson (2008). With regard to the elasticity of the real wage with respect to output, ϕ_y, I adopt the value computed by Hagedorn and Manovskii (2008b), 0.25.

The sensitivity of the nominal interest rate with respect to inflation in the Taylor rule, α_{Π}, is set at 1.5, a conventional value in the New Keynesian literature. The target level for gross inflation, Π, is set at 1. Following posterior median from Justiniano et al. (2010), the persistence of the investment shocks, ρ^{μ}, and the investment adjustment cost parameter, S'', are assumed to be 0.72 and 2.85, respectively. The standard deviation of the shock, σ^{μ}, and the investment adjustment cost parameter, S'', are chosen to match the volatility and the first-order autocorrelation of the HP-filtered log real investment that ranges from 1984Q1 to 2012Q4.[15]

The model generated distribution of wealth appears in Table 1.2. Figure 1·2 shows the conditional ergodic wealth distribution for impatient households. There are more households in unemployment group that hold essentially no assets and live hand-to- mouth. Although the model produces a low wealth share for impatient households, the fraction of households that are at the borrowing constraint is only 0.4%.

[15]Investment is the sum of personal expenditure on durables and gross private domestic investment.

Table 1.1: Calibration of the Parameters

Symbol	Description	Value	Target (Source)
	Parameters associated with steady state aggregates		
δ	Capital depreciation rate	0.015	6% annual depreciation rate
α	Power on capital in production	0.33	
β^P	Discount factor of pat. households	0.993	3% annual real interest rate
ε	Elasticity of substitution b/w goods	6	Markup of 1.2
ξ	Fixed costs	0.67	Zero-profit condition
ρ_x	Job separation rate	0.17	Job-finding rate of 0.73
γ	Matching function elasticity	0.5	Petrongolo and Pissarides (2001)
ψ	Matching efficiency	0.71	Job-filling rate of 0.71
κ	Cost of posting vacancy	0.061	4.5% of quarterly wages
	Parameters associated with the household distribution and imperfect insurance		
Ω	Share of pat. households	0.4	
η	Skill premium	1.6	Top 40% consumption share (0.57)
σ	Risk aversion	2.5	
b^u	Replacement rate	0.33	Consumption diff. of 20%
β^I	Discount factor of imp. households	0.978	Top 40% wealth share (0.94)
	Parameters associated with aggregates dynamics		
ϕ_p	Price stickiness	58.7	Adjustment freq. of 4 quarters
α_π	Interest rate rule on inflation	1.5	
ϕ_w	Wage elasticity wrt output	0.25	Hagedorn and Manovskii (2008b)
ρ^μ	Persistence of MEI	0.72	Justiniano et al. (2010)
S''	Investment adjustment costs	2.85	Justiniano et al. (2010)
σ^μ	Std. of MEI shock	0.048	Std of investment 4.85

	Share of wealth by quintile				Gini
	0-20%	20-40%	40-60%	60-100%	
Model	0.01	0.02	0.03	0.94	0.55
Data	0.00	0.01	0.05	0.94	0.82

Table 1.2: Distribution of Wealth

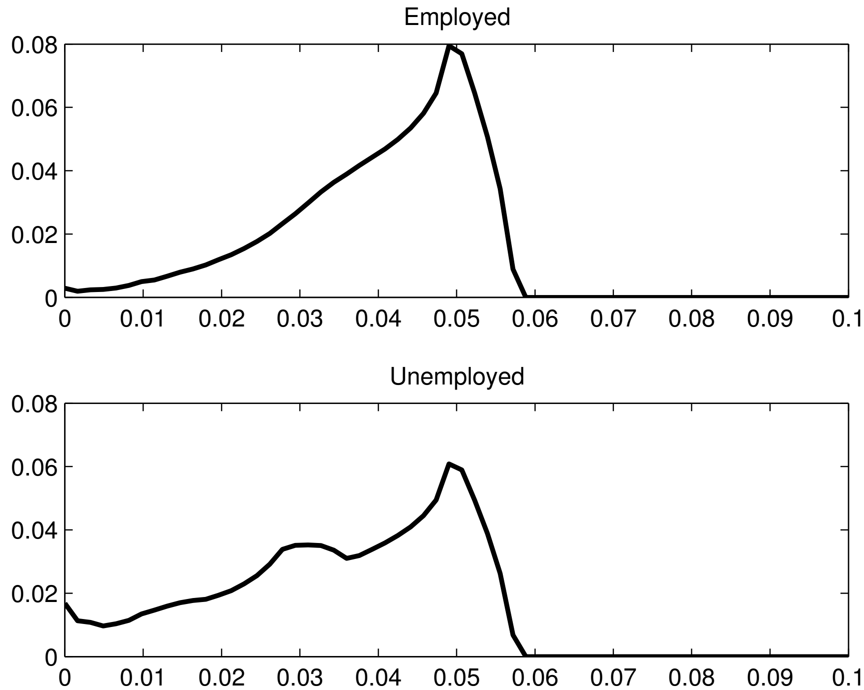


Figure 1·2: The (Smoothed) Ergodic Wealth Distribution (Density)

1.4.2 Solution Method

The model laid out in this paper requires a method that solves the incomplete markets model with aggregate uncertainty. The well-known challenge involved in solving models in this class is that aggregate quantities and prices depend not only on aggregate shocks but also on the distribution of wealth, which is an infinite-dimensional object (Krusell and Smith, 1998). I use the method developed by Reiter (2009) because this method can easily handle a large number of aggregate state variables with complicated market structures. For example, McKay and Reis (2016) use this method to combine the incomplete markets model with many features that are found to be important in monetary business cycle models.

The application of the Reiter (2009) method to the model in this paper can be summarized as follows. First, the distribution of wealth is approximated with a histogram that has a large number of bins. The mass of households in each bin becomes a state variable of the model, so the infinite-dimensional object is approximated with a large but finite number of state variables. Second, the household decision rule is discretized with a finite number of knot points that are interpolated with linear splines. I obtain the stationary competitive equilibrium using the standard algorithm that is used to solve Bewley-Huggett-Aiyagari models in which there are idiosyncratic shocks but no aggregate shocks. The model is then linearized around the steady state, and the solution is computed using a standard method for solving linear rational expectation systems (Sims, 2002).

The resulting solution preserves the nonlinear relationship between the household decision rules and the individual state variables, so that the consumption function exhibits a kink where the borrowing constraint starts to bind. However, the household decision rules are linear in the aggregate states. More details on the solution method are described in Appendix A.2. In addition, to assess the accuracy of the solution, the Euler equation errors are reported in Appendix A.2.

1.5 Business Cycle Analysis

In this section, I show that the calibrated model delivers comovement between consumption and investment in response to the marginal efficiency of investment shocks and demonstrate that time-varying precautionary savings due to unemployment risk is the key to this result. To do so, I compare three allocations: the competitive equilibrium under the baseline heterogeneous-agent model, the competitive equilibrium under the representative-agent model, and the competitive equilibrium under the heterogeneous-agent model with constant precautionary savings.

Table 1.3: Business Cycle Statistics: Data vs Model

Moment	Variable (x)	Data	HA	RA	HA (const. risk)
Std(x)	Output (GDP)	1.85	1.56	1.23	1.25
	Consumption (c)	0.79	0.72	0.53	0.68
	Investment (i)	4.85	4.85	6.02	5.50
Corr(x, GDP)	Output (GDP)	-	-	-	-
	Consumption (c)	0.85	0.92	-0.03	0.30
	Investment (i)	0.98	0.98	0.95	0.91
Corr(x, i)	Output (GDP)	-	-	-	-
	Consumption (c)	0.73	0.81	-0.34	-0.14
	Investment (i)	-	-	-	-
Std(x)/Std(GDP)	Output (GDP)	-	-	-	-
	Consumption (c)	0.43	0.46	0.43	0.55
	Investment (i)	2.62	3.1	4.88	4.40

Note: The table compares the moments of the data and those from 10,000 simulations of the models. Standard deviations are scaled by 100. The moments are taken from the logs of the data which are then detrended using the HP-filter with a smoothing parameter of 1600. Output (GDP) is the sum of consumption and investment. HA = heterogeneous-agent model; RA = representative-agent model; and HA (const. risk) = heterogeneous-agent model with constant precautionary savings.

Business cycle statistics Table 1.3 reports the business cycle facts and assesses how well the baseline model is able to capture these facts, notably the correlation of consumption with investment and output in response to investment shocks. Logs of the observed data and the model-generated data are taken and then detrended using the HP filter with a smoothing parameter of 1600. The source of the data is the St. Louis Fed's FRED II database, and the period ranges from 1984Q1 to 2012Q4. Consumption corresponds to personal consumption expenditures on non-durables and services, and investment is the sum of personal consumption expenditures on durables and gross private domestic investment. Then, the real series are constructed by dividing the nominal series by the working age population, aged 15-64, and the GDP deflator. For the measure of output, I take the sum of consumption and investment.

Although the baseline model generates less volatile output in comparison with the data, it performs remarkably well in explaining the comovement of consumption with output and investment. The empirical correlations of consumption with output and investment are 0.85 and 0.73, and the model produces 0.92 and 0.81, respectively. However, under the representative-agent assumption, the model does a poor job along these dimensions. The correlation of consumption with output predicted is close to zero, and its correlation with investment is negative, both of which indicate the comovement problem.

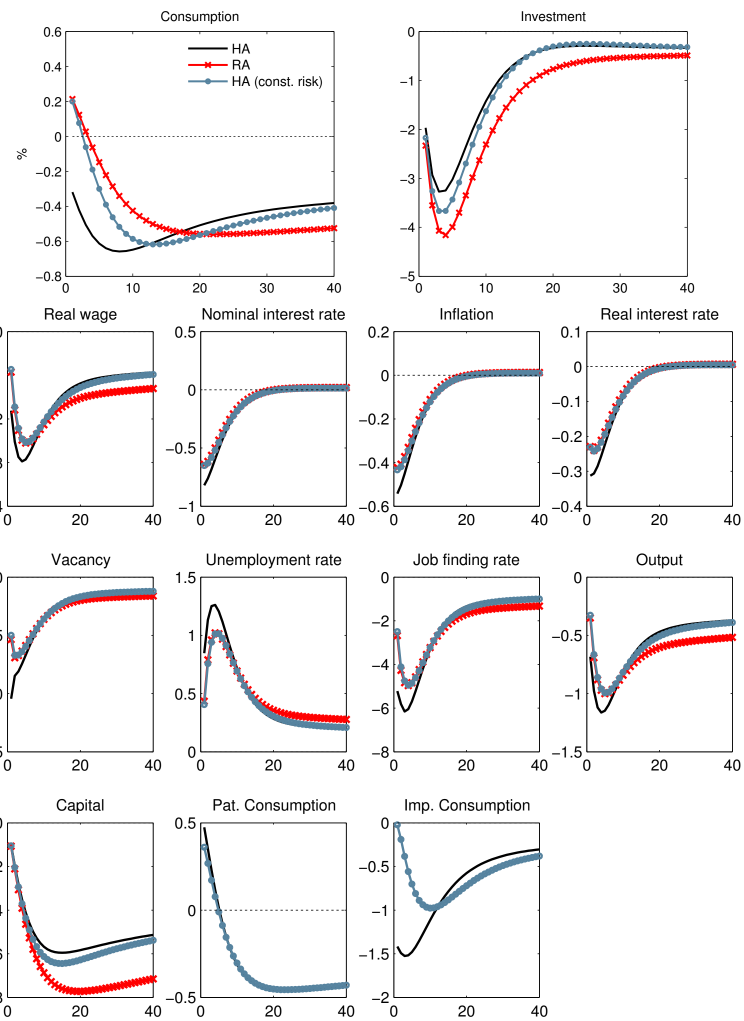
The aggregate consumption dynamics in the baseline heterogeneous-agent model are driven by two channels. The first channel is due to households' high marginal propensity to consumer (MPC) and the second channel is due to time-varying precautionary savings caused by variations in labor market uncertainties. Even though the fraction of hand-to-mouth households is very low under the baseline calibration, unconstrained households who are close to the borrowing constraint have higher MPC than typical households in the representative-agent model. These households who hold low liquid wealth are less sensitive to changes in the interest rate and instead reduce consumption mainly due to a general equilibrium fall in labor income even in the absence of a time-varying precautionary savings motive. To assess the pure effects of time-varying precautionary savings on aggregate consumption, I introduce a benchmark (HA (const. risk)) in which impatient households perceive constant transition probabilities across employment states and thus do not take into account unemployment risk when they make consumption and saving decisions, even as labor market conditions vary.[16] The only difference between this benchmark and the baseline model is that the former implies constant precautionary savings. The wealth distribution, consumption policy function, prices, and aggregate quantities in the steady state remain the same. Accordingly, comparing the baseline model with the constant risk benchmark allows one to gauge the effect of time-varying precautionary savings.

[16]The constant idiosyncratic risk assumption is adopted in McKay et al. (2016) and Kaplan et al. (2016).

In the constant risk model, the empirical correlation of consumption with investment is -0.14, which implies weakly negative comovement between consumption and investment. Therefore, it is mainly a time-varying precautionary savings motive against unemployment risk that leads to positive comovement between consumption and investment at the business cycle frequencies.[17]

[17]Hamilton (2017) points out that the HP filter has drawbacks and suggests an alternative filtering method. Appendix A.5 shows that the importance of precautionary savings in generating positive comovement of consumption and investment survives under his filtering method.

Figure 1·3: Impact of an Investment Shock on Macroeconomic Variables

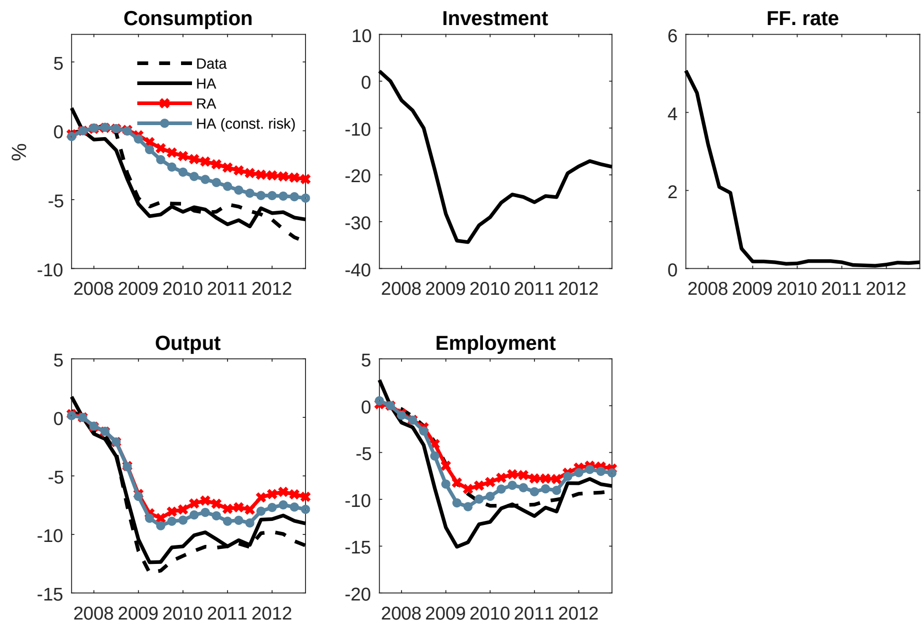


Impulse responses To emphasize visually that a rise in unemployment risk is a dominant mechanism through which investment shocks lead to a reduced consumption, Figure 1·3 shows the impulse responses to a contractionary investment shock for the three benchmarks. Notably, consumption declines upon impact under the baseline heterogeneous-agent model (HA), displaying comovement with output and investment. By contrast, under the representative-agent model (RA) and the constant-risk model (HA (const. risk)), consumption is positive for several periods and then gradually falls in the medium and long run. The key difference of the baseline heterogeneous-agent model compared with the other two model economies comes in the response of households to a fall in the job-finding rate as firms post fewer vacancies in the face of lower aggregate demand. In the heterogeneous-agent economy, households are concerned about the higher risk of job loss and longer unemployment duration because they cannot borrow or trade in insurance markets to smooth out their consumption. Therefore, they accumulate a buffer stock of savings by reducing their current consumption. This further lowers aggregate demand, reduces hiring, and therefore causes a larger drop in the job-finding rate. In the figure, responses of all variables are very persistent due to investment adjustment costs, which create persistence in investment, aggregate demand, and thus in job finding rates. [18]

In the absence of unemployment risk, household decisions are mainly determined by intertemporal substitution. In particular, as the monetary authority lowers the interest rate to stabilize the economy, households respond by spending more in the current period and less in the future. Because of the absence of downward pressure on aggregate consumption, the responses of vacancies, unemployment, the job-finding rate, output, the real wage, and the markup are dampened. As the response of inflation is determined by the current and future expected path of the real marginal cost, which is the inverse of the markup, inflation falls less. Given the monetary policy rule, so do the nominal and real interest rates.

[18] In Appendix A.5, I show that investment shocks have less persistent effects under no adjustment costs.

Figure 1·4: Aggregate Dynamics During the Great Recession



The response of investment is more muted under the heterogeneous-agent model. This is because an increase in precautionary savings by impatient households leads to a further fall in real interest rates. This exerts a downward pressure on the return on capital, the user cost of capital, since the patient households who participate in bond and capital markets equate expected returns on both assets. This aggregate supply effect of precautionary savings causes investment to be less volatile.

Appendix A.4 describes how employed and unemployed households react differently to an increase in unemployment risk after an investment shock and shows that the unemployed suffer disproportionately.

The Great Recession The impulse responses in Figure 1·3 illustrate that, although consumption rises initially, it eventually declines in the representative model and in the constant risk model. Hence, whether or not the models without time-varying precautionary savings can explain a drop in consumption during recessions caused by investment shocks cannot be answered clearly from the impulse responses alone. Moreover, during the early stages of the Great Recession, the Federal Reserve cut nominal interest rates successively and the zero lower bound (ZLB) on nominal interest rates had been binding for several years since late 2008. Therefore, constrained monetary policy may have contributed significantly to the consumption drop during the recession. Then one might conclude that there is no need for identifying additional propagation channels such as precautionary savings due to unemployment risk. To investigate these issues, I first allow shocks to the Taylor rule.[19] For each model, I choose sequences of shocks to investment and shocks to monetary policy so that the model-implied investment and nominal interest rates exactly equal the linearly detrended investment data and Federal funds rates from 2007Q3 to 2012Q4. I then feed these shocks into the model economies to simulate aggregate variables.

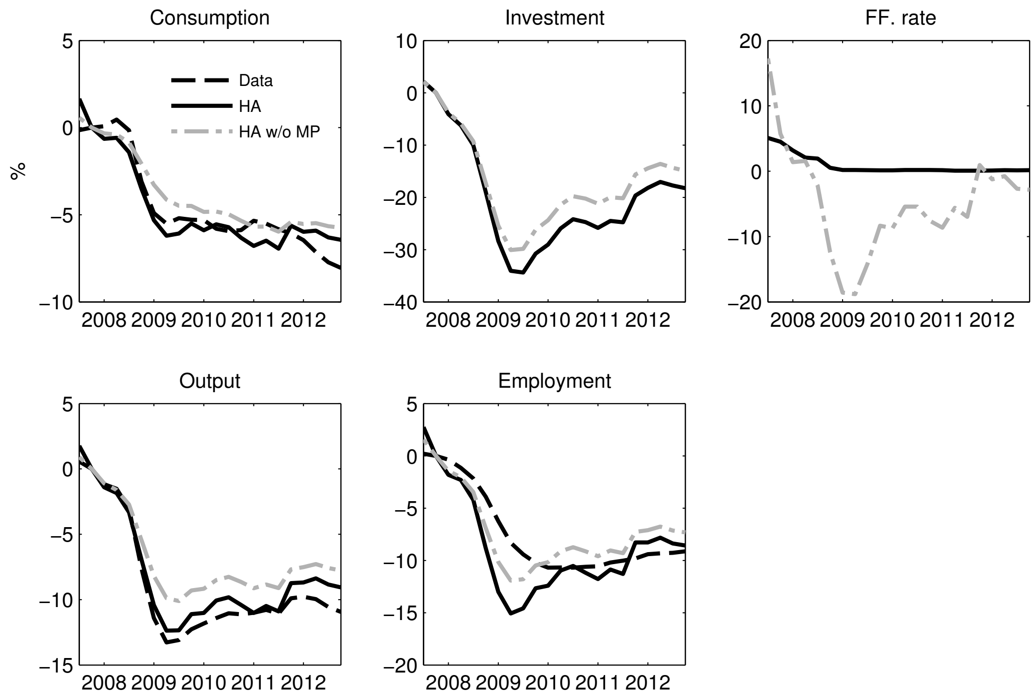
The top-left panel of Figure 1·4 depicts the aggregate consumption paths implied by the three models and data on consumption of non-durables and services starting in 2007Q3, which are indexed to equal zero in 2007Q4, the peak of the expansion as defined by the NBER. Notably, the baseline heterogeneous-agent model predicts a consumption path that is very close to the data. However, in the representative-agent model and in the constant-risk model, the drop in aggregate consumption during the early phase of the recession is mild compared with the data even in the presence of zero nominal interest rates.

The bottom two panels in Figure 1·4 plot the paths for output and employment implied by the two models and the corresponding data starting in 2007Q3. The baseline model does a better job in predicting the magnitude of the output response thanks to the sizable consumption response generated by the model. The huge decline in aggregate demand in the baseline model leads to the big decline in employment.[20]

[19]Equation (3.37) is modified to $\ln(\frac{R_t}{R}) = \alpha_\pi \ln\left(\frac{\Pi_t}{\Pi}\right) + \varepsilon_t^m \quad \text{with} \quad \varepsilon_t^m \overset{iid}{\sim} \mathcal{N}(0,1)$.

[20]The reason that the employment response in the baseline model is more volatile than in the data is

Figure 1·5: Contribution of Investment Shocks During the Great Recession

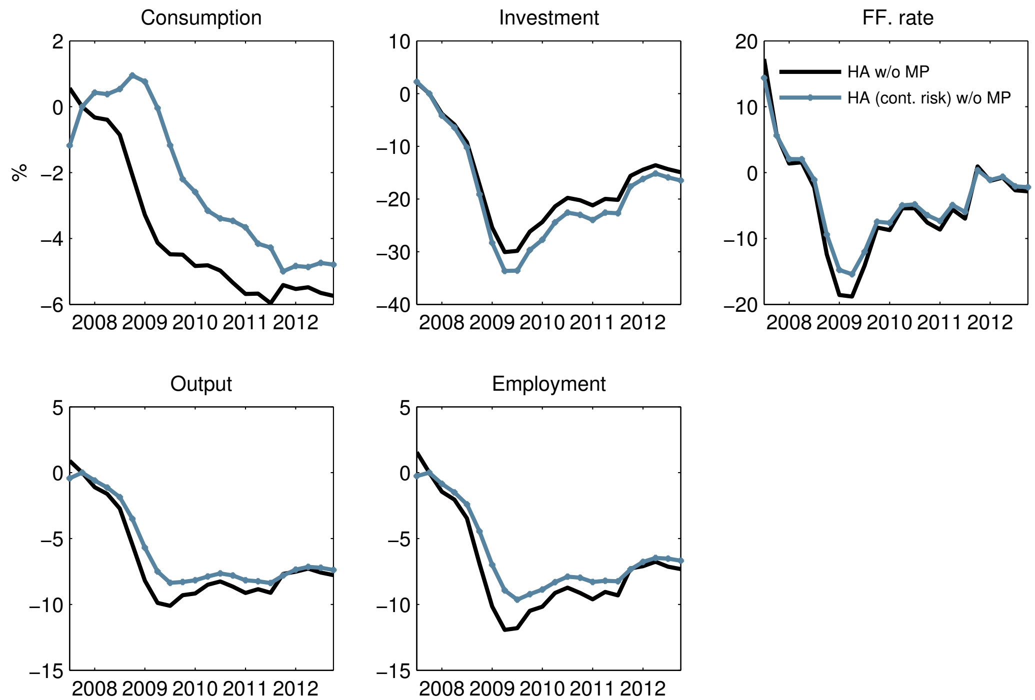


In Figure 1·5, I plot consumption dynamics of the baseline model that are purely explained by investment shocks. Dash-dot lines in the figure represent dynamics of aggregate variables, when monetary policy shocks are turned off. The model predicts the consumption path fairly well even without the aid of monetary policy shocks.

General equilibrium effect on investment As noted in investment responses of Figure 1·3, in the baseline heterogeneous-agent model, there is a general equilibrium effect on investment that runs against the direct effect of investment shocks. Because this effect reduces the impact of a given investment shock on investment, the size of the investment shocks backed out from the baseline model to replicate the investment data is larger on average than that from the other two benchmarks. To visualize the supply effect of time-

that changes in production are only explained by changes in employment. If there were additional shocks, a fall in TFP for instance, the employment response would not need to be so large. This is because, in the New Keynesian model, an increase in the real marginal cost induced by negative innovations to TFP would increase the postings of vacancies and thus employment, which is in line with the result in Gertler et al. (2008).

Figure 1·6: General Equilibrium Effect on Investment



varying precautionary savings on investment, I feed the sequence of investment shocks recovered from the baseline model into its constant-risk counterpart. The top-middle panel in Figure 1·6 suggests that investment would have declined by more if the precautionary savings were absent. However, because the effect of precautionary savings on consumption is more powerful than the supply effect on investment, output would have been less responsive if there was no precautionary savings incentive.

Model with hand-to-mouth households It is useful to consider whether the New Keynesian model with hand-to-mouth households can produce aggregate comovement. Because the assumption of hand-to-mouth behavior raises the marginal propensity to consume (MPC) by brute force, one might hope to achieve a drop in consumption with a simpler model without relying on precautionary savings. I deviate from the representative-agent benchmark and assume 30% of households behave in a hand-to-mouth fashion, fully consuming their current income without any savings.[21] The consumption rule for hand-to-mouth households is

$$c_t^{HtM} = (1 - \tau_t)\eta^{RA}(w_t n_t + b^u w_t(1 - n_t)),$$

where τ_t obeys the constraint (3.36). 70 percent of households own the capital and firms and have access to asset markets where they can trade a full set of contingent securities. As a consequence, these households optimize intertemporally subject to constraints (1.27) and (1.7). Figure 1·7 compares the impulse responses of aggregate consumption for the hand-to-mouth model to those for the heterogeneous-agent model. Interestingly, the hand-to mouth-model does not generate a fall in aggregate consumption on impact. This is because a reduction in consumption by hand-to-mouth household is largely cancelled out by the intertemporal substitution behavior of non-hand-to-mouth households who raise consumption in response to a drop in interest rate. As a result, one needs to assume a unrealistically larger fraction of hand-to-mouth households to obtain a large drop in aggregate consumption observed in the heterogeneous-agent model.

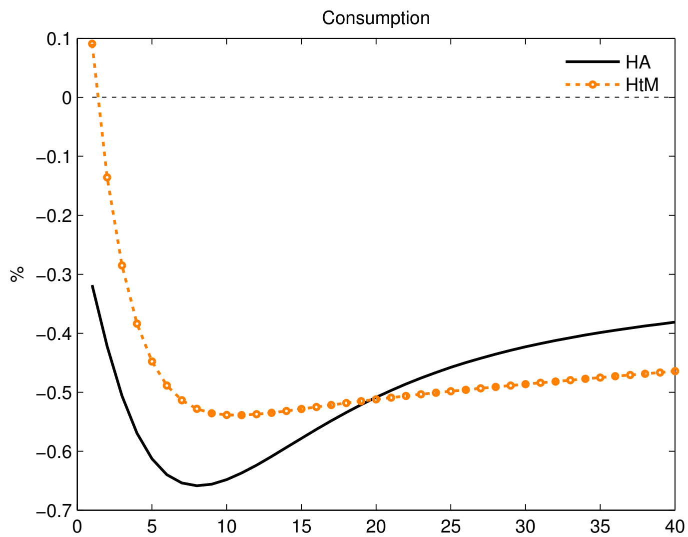
1.6 The Role of Sticky Prices and Monetary Policy

In Section 1.2, I explained that the representative-agent model can deliver comovement between consumption and investment if production falls strongly after investment shocks. This is the case when there are very strong New Keynesian features such as a high degree of price rigidity or a monetary policy rule under which the policy rate is less responsive to changes in inflation. In this section, I illustrate the type of parameterization that is needed for the representative-agent model to generate comovement.

I search for the degree of price stickiness and the value of the coefficient on inflation

[21] Kaplan et al. (2014) present estimates from the US Survey of Consumer Finances the fraction of hand-to-mouth households in the US over the period 1989-2010 is, on average, 30%. Of these, roughly one-third are poor hand-to-mouth and two-thirds are wealthy hand-to-mouth.

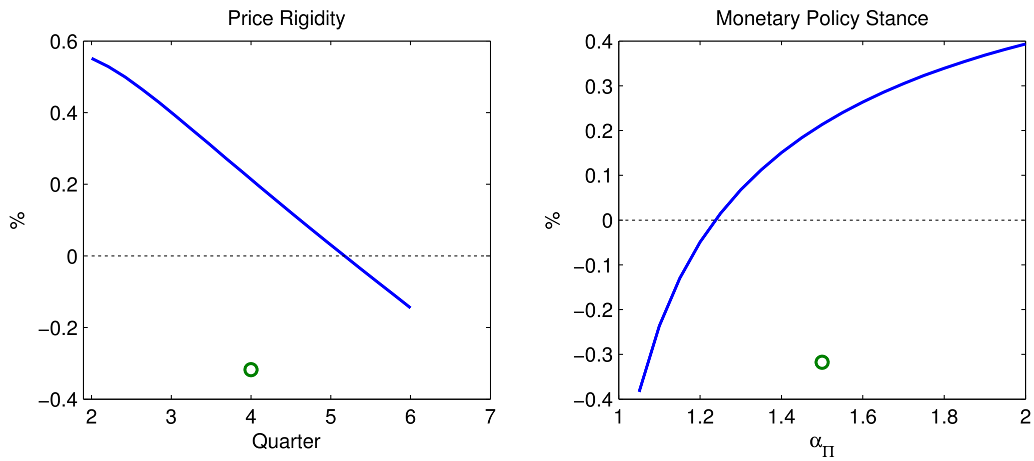
Figure 1·7: Aggregate Consumption Response: Heterogeneous Agent vs Hand-to-Mouth



in the monetary policy rule that would allow the representative-agent economy to produce an impact consumption response of the same magnitude as the one predicted by the baseline heterogeneous-agent model. The solid lines in Figure 1·8 show the impact consumption responses from the representative-agent model to variations in the degree of price stickiness and the coefficient on inflation. In each panel in the figure, only one parameter is changed, while the rest are kept at their baseline values. The circles represent the impact consumption response from the heterogeneous-agent model under baseline calibration. The required price change frequency is higher than 6 quarters, which is already beyond the microeconomic evidence on price rigidity.[22] For instance, Nakamura and Steinsson (2008) document that the mean duration of price rigidity is roughly 3-4 quarters and Klenow and Kryvtsov (2008) report it is 2-3 quarters. Alternatively, given a price adjustment frequency of 4 quarters, a consumption response of equal magnitude can be obtained with an inflation coefficient of 1.1. This value is substantially below the conventional value, 1.5, under which the monetary policy rule is known to fit the actual

[22] 6 quarters is the maximum price change frequency that gives a determinate equilibrium in my representative-agent model.

Figure 1·8: Aggregate Consumption Response to Different Degrees of Price Rigidity and Monetary Policy Stances



policy rate in the US (Taylor, 1993). In effect, adding market incompleteness and unemployment risk to New Keynesian models expands the admissible range of parameters that is compatible with aggregate comovements.

1.7 Conclusion

In this paper, I show how adverse shocks to investment can generate a fall in both consumption and investment in the heterogeneous-agent New Keynesian model featuring imperfect insurance against unemployment risk. Under a reasonable degree of nominal rigidity and a plausible monetary policy rule, these shocks reduce aggregate demand and create an increase in employment uncertainty. The increased unemployment risk triggers a precautionary savings motive that reduces aggregate consumption.

In addition to contributing to our positive understanding of the sources of aggregate fluctuations and comovement of macroeconomic aggregates, these findings have important implications for the analysis of policies designed to stabilize investment spending, such as investment tax credits and bonus depreciation allowances. These policies have traditionally been analyzed in models in which consumption is countercyclical or acyclical following disturbances to investment. When consumption dynamics reinforce investment dynamics, the benefits of policies that stabilize investment are potentially substantially larger.

This paper assumes that all households face equal unemployment dynamics. However, the data suggest that there are clear differences in unemployment rates across race, age, and education. To quantify the effect of unemployment risk on aggregate consumption more accurately, one needs to enrich the model presented here by incorporating the realistic consumption share of each group and the correlation between unemployment and consumption for each group and draw out the implications of these facts for aggregate consumption. I plan to pursue these issues in future work.

Chapter 2

Sources of Business Cycles in the Estimated Heterogeneous Agent Model with Unemployment Risk

2.1 Introduction

This chapter investigates whether the baseline model presented in Chapter 1 changes the inference on the shocks driving the aggregate fluctuations relative to the representative-agent model. To do so, in addition to investment shocks, I add monetary policy shocks, neutral technology shocks, and discount factor shocks to the model and estimate the rich model by maximum likelihood method. I use the following US aggregate data: employment, real wage, inflation, nominal interest rate, consumption, and investment.

I find that investment and monetary policy shocks account for 45 percent of the variance of aggregate consumption, while, in the representative-agent model, they explain only 17 percent. Moreover, discount factor shocks account for 25 percent of the variance of aggregate consumption, while, in the representative-agent model, they explain 40 percent. This is because, in the baseline model, the endogenous development of precautionary savings due to investment and monetary policy shocks explains more of the observed dynamics of aggregate consumption and reduces the need for discount factor shocks, consistent with the theoretical analysis by Werning (2015).

Table 2.1: Business Cycle Statistics: Baseline Model vs Reduced Model

Moment	Variable (x)	Data	Baseline ($n_d = 250, n_g = 100$)	Reduced ($n_d = 30, n_g = 30$)
Std(x)	Output (GDP)	1.85	1.5594	1.5625
	Consumption (c)	0.79	0.7212	0.7252
	Investment (i)	4.85	4.8501	4.8451
Corr(x, GDP)	Output (GDP)	-	-	-
	Consumption (c)	0.85	0.9176	0.9196
	Investment (i)	0.98	0.9771	0.9773
Corr(x, i)	Output (GDP)	-	-	-
	Consumption (c)	0.73	0.8133	0.8168
	Investment (i)	-	-	-
Std(x)/Std(GDP)	Output (GDP)	-	-	-
	Consumption (c)	0.43	0.4625	0.4641
	Investment (i)	2.62	3.1103	3.1009

Note: The table compares the moments from 10,000 simulations of the baseline model and those from the reduced model. n_d is the number of bins in the histogram, and n_g denotes the number of knot points that are used to approximate household decision rules. Standard deviations are scaled by 100. The moments are taken from the logs of the data which are then detrended using the HP-filter with a smoothing parameter of 1600. Output (GDP) is the sum of consumption and investment.

2.2 Estimation Strategy and Results

Estimated representative-agent DSGE models often lead to an import role for discount factor shocks in explaining the aggregate consumption dynamics over the business cycles (Justiniano et al., 2010, 2011). Moreover, the Great Recession has stimulated a great deal of interest in understanding the behavior of economies at the zero lower bound on nominal interest rates. In modeling these episodes, a common way of bringing the economy to the zero lower bound is to give a shock to the time preference of the representative households in the model. This type of shock is useful because it raises aggregate savings and reduces interest rates simultaneously by generating a wedge in the household's Euler equation.

Table 2.2: Maximum Likelihood Estimates

Description	Parameter	Rep. Agent	Het. Agent
AR coeff. of μ_t	ρ^μ	0.3778	0.3649
		(0.0602)	(0.0685)
Std. of ϵ_t^μ	σ^μ	0.0212	0.0193
		(0.0062)	(0.0056)
AR coeff. of m_t	ρ^m	0.9272	0.9229
		(0.0122)	(0.0112)
Std. of ϵ_t^m	σ^m	0.0012	0.0012
		(0.0001)	(0.0001)
AR coeff. of A_t	ρ^A	0.8896	0.8884
		(0.0244)	(0.0184)
Std. of ϵ_t^A	σ^A	0.0044	0.0046
		(0.0004)	(0.0004)
AR coeff. of d_t	ρ^d	0.8636	0.8382
		(0.0348)	(0.0461)
Std. of ϵ_t^d	σ^d	0.0082	0.0075
		(0.0008)	(0.0009)
Elasticity of real wage	ϕ_w	0.2883	0.3204
		(0.0301)	(0.0287)
Investment adjustment cost	S''	1.9630	1.7100
		(0.6120)	(0.5133)

Note: Figures between parentheses are standard errors.

Despite its importance in shaping aggregate consumption, it is unclear what the economic foundation of this shock is. In this chapter, I evaluate the extent to which the precautionary savings motive due to unemployment risk in the baseline model provides a foundation for the shocks to the discount factor of the representative agent. To do so, in addition to investment shocks, I add more structural shocks that are widely used to study the sources of business cycles including discount factor shocks. By estimating models with multiple shocks, I can compare the importance of discount factor shocks on aggregate fluctuations in the baseline model with that in the representative-agent counterpart.

Table 2.3: Variance Decomposition (8q)

	Aggregate Shock							
	Investment	MP	Tech.	Disc. factor	ME (w)	ME (Π)	ME (R)	ME (c)
				Representative Agent				
Empl.	52.94	31.00	6.47	9.60	0	0	0	0
Real wage	15.78	7.54	19.94	2.06	54.69	0	0	0
Infl.	10.29	81.99	5.15	1.48	0	1.10	0	0
Int. rate	21.52	61.26	10.77	3.09	0	0	3.37	0
Cons.	5.30	12.25	34.92	39.83	0	0	0	7.70
Inv.	53.67	11.17	31.26	3.90	0		0	0
GDP	47.92	13.40	37.45	0.97	0	0	0	0.27
				Heterogeneous Agent				
Empl.	57.49	28.71	6.08	7.72	0	0	0	0
Real wage	21.22	9.06	20.66	2.26	46.79	0	0	0
Infl.	14.45	78.21	4.98	1.30	0	1.07	0	0
Int. rate	27.88	56.98	22.55	9.61	0	0	3.03	0
Cons.	20.30	25.34	24.21	24.67	0	0	0	5.48
Inv.	49.06	5.36	42.47	3.11	0	0	0	0
GDP	46.34	9.57	43.18	0.59	0	0	0	0.31

Note: This table reports the variance decomposition at forecast horizons of 8 quarters. MP stands for Monetary Policy shock, and ME is Measurement Error.

The household's utility function is now

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta^{type} d_{t+s})^s \left[\frac{(c_{j,t+s}^{type})^{1-\sigma}}{1-\sigma} \right] \qquad type \in \{I, P\}, \tag{2.1}$$

where d_t is a common discount factor shock to all households that evolves according to

$$\log(d_t) = \rho^d \log(d_{t-1}) + \varepsilon_t^d \qquad \text{with} \qquad \varepsilon_t^d \overset{iid}{\sim} \mathcal{N}(0, {\sigma^d}^2). \tag{2.2}$$

The production function of monopolistic producer z is

$$y_{z,t} = A_t^{1-\alpha} k_{z,t}^{\alpha} n_{z,t}^{1-\alpha} - \xi, \tag{2.3}$$

where A_t is a neutral technology shock that follows a first-order autoregressive process given by

$$\log(A_t) = \rho^A \log(A_{t-1}) + \varepsilon_t^A \qquad \text{with} \qquad \varepsilon_t^A \overset{iid}{\sim} \mathcal{N}(0, {\sigma^A}^2). \tag{2.4}$$

Monetary policy rule is now

$$\log\left(\frac{R_t}{R}\right) = \alpha_{\Pi} \log\left(\frac{\Pi_t}{\Pi}\right) + m_t, \tag{2.5}$$

where m_t is a monetary policy shock that follows a first-order autoregressive process given by

$$m_t = \rho^m m_{t-1} + \varepsilon_t^m \qquad \text{with} \qquad \varepsilon_t^m \overset{iid}{\sim} \mathcal{N}(0, {\sigma^m}^2). \tag{2.6}$$

I describe below the estimation strategy. The model's solution can be written in the following state-space form

$$\begin{aligned} obs_t &= G_{obs} \mathcal{S}_t + \mathcal{E}_t^{me} \\ \mathcal{S}_t &= G_s \mathcal{S}_{t-1} + Q \mathcal{E}_t, \end{aligned} \tag{2.7}$$

where G_{obs} is a matrix that links the vector of state variables $\mathcal{S}_t$ to the vector of observable variables obs_t. G_s is a matrix that governs the evolution of the state vector and consists of elements that are combinations of the model parameters. $\mathcal{E}_t^{me}$ is the vector of measurement errors, and $\mathcal{E}_t$ is the vector of structural shocks in the model. I estimate the model parameters by maximum likelihood. However, it is challenging to evaluate the likelihood function of the baseline model using the Kalman filter because it requires inversions of matrices of large size and thus often causes computers to run out of available memory. Therefore, I reduce the size of the state space representation 2.7 by reducing the number of bins in the histogram from 250 to 30 and the number of knot points that are used to approximate the decisions rules from 100 to 30. To assess whether the state space reduction alters the aggregate dynamics, I compare business cycle statistics in the baseline model subject to investment shocks and those in the reduced model. As summarized in Table 2.1, reducing the number of state variables barely changes the business cycle statistics. I therefore estimate the reduced model.

I use quarterly US data on employment, real wage, inflation, Federal funds rate, real per capita consumption, and real per capita investment.[1] The sample period is from 1984Q1 to 2008Q3, the last quarter before the Federal funds rate hit the zero lower bound. All variables are logged and detrended using the HP filter with a smoothing parameter of 1600. Because the model has less structural shocks than observables, the variance-covariance matrix of the residuals becomes singular. In order to circumvent this problem, I add i.i.d. measurement error to real wage, inflation, Federal funds rate, and consumption. I impose restrictions on the standard deviation of measurement errors so that each measurement error explains 10 percent of the variation of a particular observable in the long run.

The following parameters are not estimated and thus are set at values reported in Table **??** to facilitate the computation. The parameters that determine the stationary equilibrium are fixed. In order to prevent the algorithm running into the parameter space that leads to indeterminate equilibrium, I set the price adjustment frequency to 4 quarters and the inflation coefficient in the monetary policy rule to 1.5, in line with the empirical evidence. The set of parameters that I estimate is $(\rho^{\mu}, \sigma^{\mu}, \rho^{m}, \sigma^{m}, \rho^{A}, \sigma^{A}, \rho^{d}, \sigma^{d}, \phi_w, S'')$. To facilitate comparison, I also estimate the representative-agent version of the model. Table 2.2 reports the estimation results. Figures between parentheses are standard errors, which are computed as the square root of the diagonal elements of the inverted Hessian of the log

[1] Employment is measured by total private employees. Real wage is measured by real compensation per hour in nonfarm business sector. Consumption is measured by private spending on non-durable goods and services. Investment is measured by personal expenditure on durables and gross private domestic investment.

likelihood function evaluated at the maximum.

Table 2.3 reports the variance decomposition at forecast horizons of 8 quarters, evaluated at the resulting estimates.[2] I first study the representative-agent version of the model. The table shows that the investment shocks account for the largest fraction of the fluctuations in investment and GDP. However, these shocks only account for a modest fraction of consumption, which is mainly driven by the discount factor shocks. These findings are consistent with those of Justiniano et al. (2010, 2011).[3] Next, consider the heterogeneous-agent model. In comparison with the representative-agent model, the contribution of the discount factor shocks is reduced by 1/3 in explaining short-run consumption dynamics, and these shocks are no longer the most dominant forces that drive these dynamics. In fact, its reduced contribution is replaced by increased contribution of investment shocks and monetary policy shocks. Because the amplification effect by precautionary savings due to unemployment risk occurs in the presence of any aggregate shocks that reduce (increase) the employment in recessions (booms), monetary policy shocks are also amplified, leading to a more volatile response of consumption. The investment and monetary policy shocks in combination explain 45 percent of consumption compared to the 18 percent in the representative-agent version. The contribution of the neutral technology shocks on consumption is reduced, whereas their contribution on investment is increased. As is common in New Keynesian models, employment falls after an improvement in technology because the improved technology reduces the real marginal costs directly. Resulting precautionary savings works to dampen the increase in consumption and boost investment.

In sum, the estimation results point to an important takeaway on the role of discount

[2] Variance of GDP ($= c + i$) can be directly computed from the second moments of c and i. Appendix B.1 reports the variance decomposition at forecast horizon of 32 quarters.

[3] The contribution of investment shocks on investment is lower than that found by these studies. One reason is the absence of capital utilization in my model, which makes the value of existing capital and thus a return on new capital more cyclical. Including capital utilization can increase the variation in investment explained by investment shocks.

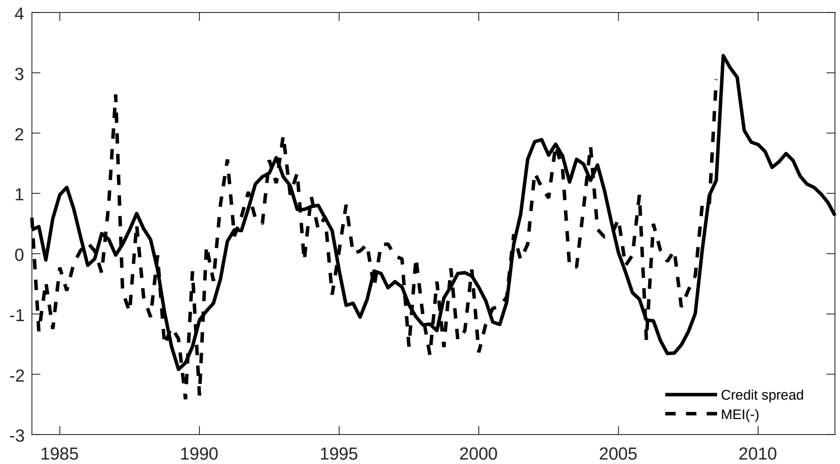


Figure 2·1: Credit Spread and the Marginal Efficiency of Investment

factor shocks. Because the wedges in the Euler equation are partly explained by changes in consumption and savings due to an endogenous development of unemployment risk, there is less empirical need for finding discount factor shocks.

For completeness, I show how the baseline model's investment shock, the most important shock for the output variations in the short-run, map into time-varying credit spreads. I do so by comparing the behavior of the series of marginal efficiency of investment μ_t implied by the estimation to that of credit spreads, measured by the difference between the interest rate on BAA-rated corporate bonds and the Federal funds rate. Figure 2·1 highlights that the negative marginal efficiency of investment is highly correlated with credit spreads (correlation coefficient of 0.58), indicating that the low marginal efficiency of investment is likely to proxy for high cost of capital that firms face. Movement in credit spreads represent financial shocks or uncertainty shocks that are transmitted through a change in credit spreads (or both).[4]

[4]See Caldara et al. (2016) for the identification of uncertainty and financial shocks.

2.3 Conclusion

In this chapter, I show how the presence of unemployment risk and incomplete asset market change the inferences on aggregate shocks by estimating the model. In the estimated model, I find that the contribution of the discount factor shocks in consumption variation is significantly reduced due to time-varying precautionary savings.

Chapter 3

Risk-Sensitive Lenders, the Optimal Contract, and the Financial Accelerator

3.1 Introduction

The recent financial crisis has underscored the importance of borrowers' balance sheet in the transmission and amplification of aggregate shocks. During downturns, the concentration of aggregate risk on the leveraged agents can lead to severe recessions through an adverse feedback loop, referred to as the financial accelerator. This process represents a vicious cycle in which aggregate shocks lower the borrowers' net worth, thus driving down the price of capital, which in turn decreases the net worth. One of the widely used general equilibrium models that integrate the financial accelerator is the model of Bernanke et al. (1999), hereafter BGG. However, there are two shortcomings of this model. First, the loan contract between borrowers and lenders in the BGG model is not privately optimal. In particular, returns for lenders are predetermined and thus are not an outcome of negotiation between lenders and borrowers.[1] Second, lenders have log-utility and are assumed to be financially unconstrained, whereas borrowers are constrained. This im-

[1]See the following comment of Chari (2003) on BGG: *These authors have an economy with risk neutral agents called entrepreneurs and risk averse agents called households. They claim that an optimal contract in the presence of aggregate risk has the return paid by entrepreneurs to be a constant, independent of the current aggregate shock. I have trouble understanding this result. Surely, entrepreneurs should and would provide insurance to households against aggregate shocks. One way of providing such insurance is to provide a high return to households when their income from other sources is low and a low return when their income from other sources is high. My own guess is that if they allowed the return to households to be state contingent, then aggregate shocks would have no effects on the decisions of households and would be absorbed entirely by entrepreneurs. Before we push this intriguing financial accelerator mechanism much further, I think it would be wise to make sure that we get the microeconomics right.*

plies that the lenders' stochastic discount factor varies weakly, implying that lenders are generous in holding risky assets such as equities and corporate bonds, which is hardly justified given the observed high risk premium of these assets and highly countercyclical lending standards. For example, Gilchrist and Zakrajšek (2012) show that the excess bond premium–a component of corporate credit spreads that measures shifts in the risk attitudes of financial intermediaries– is highly countercyclical.[2]

In this paper, I relax two assumptions made in the original BGG framework. I introduce lenders who have the recursive utility function following Epstein and Zin (1989) and infer the stochastic discount factor directly using equity premium. With these preferences, lenders are more sensitive to the variations of loan payoffs. I then allow returns to the lender to vary with aggregate states and solve for the privately optimal loan contract between lenders and financially constrained borrowers.[3]

The main results include the following. First, the optimal contract is a state-contingent loan contract that has a lender return positively indexed to the lenders' marginal utility of consumption and negatively indexed to the borrowers' marginal value of internal fund. Second, under the optimal contract, I show that the marginal utility of consumption is more volatile than the marginal value of internal funds, inducing larger fluctuations in real and financial variables than those under the BGG contract, which has a predetermined lender return.

The mechanism is as follows. Consider an aggregate shock that reduces the borrower's net worth. This raises the marginal value of internal funds, because having more net worth reduces the costs of external finance, making borrowers enjoy more profits from

[2]One reason for thinking lenders raise their credit standard during the recessions is the following. Recessions are times when lenders' balance sheet is adversely affected, making the process of obtaining external funds more costly or creating pressure to meet capital requirements. Accordingly, lenders have incentives to preserve internal funds by being more cautious in providing loans.

[3]At least since 2000, the market size for variable-rate loans has exceeded that of fixed-rate and unsecured bonds (Pintus et al., 2016). Therefore, the question of whether the leveraged agent's balance sheet matters for aggregate fluctuation should be considered in the context of the variable-rate loan contracts as well.

purchasing capital. As net worth takes time to recover, lenders expect the marginal value of internal funds to be persistently higher than the marginal utility of consumption after the first period. Accordingly, they expect a return on loans to be persistently low in the future, leading to a reduced consumption level in the future. With recursive utility, expectation about future consumption losses substantially increases the marginal utility of consumption on impact. This leads to an increased lender return on impact, which works to raise the lenders' current consumption but further reduces the borrowers' net worth. Therefore, under the optimal contract, the financial accelerator mechanism is more powerful than under the BGG contract, which severs the link between a lender return and the marginal utility of consumption.

The paper proceeds as follows. Section 3.2 describes the exact specification for preferences and the level of risk aversion that will be used in the rest of the paper. Section 3.3 presents a model of debt contract. Section 3.4 discusses how the model is calibrated and analyzes the positive implications for macroeconomic fluctuations. Section 3.5 concludes.

Relationship with the literature This paper fits within the recent literature regarding the role of leveraged agents' balance sheet on the transmission of aggregate shocks when contracts can be written on the aggregate state of the economy. It is most closely related to Carlstrom et al. (2016) and Dmitriev and Hoddenbagh (2017) in that all of these studies examine the state-contingent loan contract in the BGG framework. In the latter two papers, lenders have log utility as in BGG, which works to make the stochastic discount factor, or the marginal utility of consumption, much less volatile than the marginal value of internal funds. Therefore, they arrive at the conclusion that financial frictions deliver less amplification under the optimal contract than under the BGG contract. In contrast, with the recursive utility function parameterized using equity premium, I show that financial frictions deliver stronger amplification under the optimal contract, thus overturning their result.

Similarly, House (2006) also shows that financial frictions actually stabilize the business cycle when contracts are contingent or allow both debt and equity when credit market distortions arise from adverse selection. Di Tella (2015) shows that allowing for contracts that are conditional on the aggregate state completely eliminates the financial amplification resulting from productivity shocks in the environment of Brunnermeier and Sannikov (2014). These papers assume no or little insurance incentives on the supply side of loans, which is crucial in restoring financial amplification in my paper.

Krishnamurthy (2003) argues that financial amplification can be reinstated, even in the presence of insurance markets. He first shows that, when borrowers can trade state-contingent assets with unconstrained lenders in the economy of Kiyotaki and Moore (1997), the feedback from collateral values to investment disappears. He then shows that amplification effects are preserved if lenders also need to post collateral to credibly promise to make payments during downturns. Tighter collateral constraint in downturns implies a reduced supply of insurance from lenders. This in turn feeds back into real production by constraining borrowers who make investments. I reach a similar result here: insufficient provision of loans from lenders is at the heart of amplification. The central difference between my paper and his is that I attribute the shortage of loan supply to lenders' risk aversion as opposed to collateral constraint.

3.2 Preferences

Before I examine the aggregate implications of optimal debt contract, I set the level of risk aversion of lenders using the equity premium facts. That is, given a realistic consumption process, I choose the risk aversion of Epstein-Zin preferences so that the volatility of stochastic discount factor is consistent with the market price of risk computed from the excess return on equity.

The Epstein-Zin utility function is given by

$$V_t = [c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}}, \tag{3.1}$$

where ψ is the elasticity of intertemporal substitution, and σ is the relative risk aversion. The stochastic discount factor, M_{t+1}, becomes:

$$M_{t+1} \equiv \frac{\partial V_t/\partial c_{t+1}}{\partial V_t/\partial c_t} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathbb{E}_t[V_{t+1}^{1-\sigma}]^{\frac{1}{1-\sigma}}}\right)^{\frac{1}{\psi}-\sigma}, \tag{3.2}$$

and the risk-free rate is given by $r_t^f = 1/[M_{t+1}]$. Following the convention in the asset pricing literature, I consider the random walk log consumption process:

$$\Delta\log(c_t) = \varepsilon_t, \qquad \{\varepsilon_t\} \text{ i.i.d. } \mathcal{N}(0, \sigma^{c2}). \tag{3.3}$$

I measure consumption in the data as the sum of non-durable consumption and services. To estimate the equation (3.3), the consumption growth rates are demeaned. The estimate of the unconditional volatility of consumption growth rates σ^c is 0.0041.

Following Tallarini (2000), ψ is set equal to one. β is chosen to match the steady state annualized ex-post real interest rate rates of 2 percent. Given the estimated consumption process and parameter values, I can use the first two moments of M_t to compute the model-implied market price of risk following the analysis of Hansen and Jagannathan (1991). That is, the market price of risk implied by an admissible stochastic discount factor must be greater than or equal to the absolute value of the ratio of mean-to standard deviation of the equity premium:

$$\frac{|\mathbb{E}[r^e - r^f]|}{\text{std}(r^e - r^f)} \leq \frac{\text{std}(M)}{\mathbb{E}[M]}, \tag{3.4}$$

where r^e is the return on the portfolio of equities.

I then search for the value of σ at which the simulated M_t satisfies the restriction (3.4).

Table 3.1: Market Price of Risk

Data ($\lvert\mathbb{E}[r^e - r^f]\rvert/\text{std}(r^e - r^f)$)	Model ($\text{std}(M)/\mathbb{E}[M]$)			
	σ=1	σ=20	σ= 40	σ=60
0.24	0.004	0.08	0.17	0.25

Notes: The table reports the market price of risk obtained from 100,000 simulations of the model and its empirical counterpart. The empirical sample period is 1984Q1 - 2014Q4.

For the real interest rate and the equity premium, I calculate the quarterly, annualized ex post returns.[4] The empirical sample period is from 1984Q1 to 2014Q4. Table 3.1 reports the model-implied market price of risk for different values of risk aversion as well its empirical counterparts. As noted from the table, recursive preferences with $\sigma = 60$ yield the volatility of the stochastic discount factor that is in line with the empirical equity premium. I use this value in the rest of the paper. I next describe the environment in which households with these preferences engage in debt contract with the financially constrained entrepreneurs.

3.3 Model

I use a baseline New Keynesian DSGE model augmented with frictional labor markets – Diamond-Mortensen-Pissarides (DMP henceforth) search and matching framework – and the BGG financial accelerator mechanism. I adopt this labor market structure for the following reason. DMP framework generally abstracts from labor supply decisions by households. This allows one to discipline the level of risk aversion in the recursive utility function only using the realistic consumption process, which is a standard practice in the asset pricing literature. As in BGG, households are the lenders and entrepreneurs are the borrowers. Therefore, *households* and *lenders*, and *entrepreneurs* and *borrowers* are used interchangeably throughout the paper.

[4]The return on the portfolio of equities is calculated from Standard & Poor's 500 Stock Price Index. The real interest rate is calculated by subtracting the ex-post GDP deflator inflation rate from the effective federal funds rate.

3.3.1 Households

There is a unit measure of households indexed by $j \in [0,1]$. Each household may be viewed as a large family. There is a perfect risk sharing among family members as their incomes (labor incomes and a flow value of unemployment) are pooled and then equally redistributed. The t-period budget constraint for a typical household is

$$c_t + a^D_{t+1} + a^G_{t+1} \quad = \quad w_t n_t + b^u w(1-n_t) + r^D_t a^D_t + \frac{R_{t-1}}{\Pi_t} a^G_t + d_t + T_t, \tag{3.5}$$

where c_t denotes the consumption of the household, and Π_t denotes the gross inflation rate. a^D_t is its time $t-1$ loans that pay the gross real return r^D_t at time t. a^G_t is its time $t-1$ savings in government bonds that pay the gross real return $\frac{R^G_{t-1}}{\Pi_t}$ at time t. T_t denotes the lump-sum tax, and d_t is the dividend. w_t is the real wage, $b^u w$ is the flow value of unemployment, and n_t is the employment rate.

Each household chooses $\{c_t, a^D_{t+1}, a^G_{t+1}\}$ to maximize the value function (3.1) subject to the budget constraint (3.5). The household's optimization condition for a^D_{t+1} is

$$1 = \mathbb{E}_t M_{t+1} r^D_{t+1}, \tag{3.6}$$

where M_t is defined in (3.2). The participation constraint associated with the loan contract arises from this household Euler equation, which stipulates the minimum rate of return that entrepreneurs must offer to lenders to receive a loan.

3.3.2 Entrepreneurs and the Loan Contract

At the end of period t, the representative entrepreneur purchases capital k^N_{t+1} from capital producers at price, q_t. He finances the capital expenditure with his own net worth, N_t, and a loan from households, B^N_{t+1}. That is,

$$q_t k^N_{t+1} = N_t + B^N_{t+1}. \tag{3.7}$$

One unit of capital purchased is transformed into ω_{t+1} units of capital in the beginning of time $t+1$, where ω_{t+1} has a unit-mean log normal distribution that is independently drawn across time and across entrepreneurs. $\omega_{t+1}k^N_{t+1}$ is then supplied to intermediate-goods firms at a competitive market rental rate, r^k_{t+1}. At the end of period $t+1$ production, the entrepreneur is left with $(1-\delta)\omega_{t+1}k^N_{t+1}$ units of capital. This capital is sold in competitive markets to capital producers at the unit price, q^k_{t+1}. Hence, the entrepreneur who draws ω_{t+1} enjoys ex-post return per unit of capital,

$$\frac{r^k_{t+1}\omega_{t+1} + (1-\delta)q_{t+1}\omega_{t+1}k_{t+1}}{q_t} = \omega_{t+1}R^k_{t+1}, \tag{3.8}$$

where $R^k_{t+1} = \frac{r^k_{t+1}+(1-\delta)q_{t+1}k_{t+1}}{q_t}$. The realization of ω_{t+1} is directly observed by the entrepreneur at the beginning of time $t+1$, but the lender can observe the realization only if a monitoring cost is paid. Following BGG, I assume that the monitoring cost is linear in the capital produced by the project, that is, $\mu\omega_{t+1}R^k_{t+1}k_{t+1}$.

The entrepreneur repays the loan only if is profitable to do so. This is the case when his idiosyncratic productivity is above the cut-off value, $\bar{\omega}_{t+1}$, the threshold of going into default. The cut-off value is defined as

$$\bar{\omega}_{t+1}R^k_{t+1}q_t k^N_{t+1} \quad = \quad B^N_{t+1}Z_{t+1}, \tag{3.9}$$

where Z_{t+1} is the interest rates charged to the entrepreneur.

The payoffs of all entrepreneurs with net worth N_t in period $t+1$ is

$$\int_{\bar{\omega}_{t+1}}^{\infty}(\omega_{t+1}R^k_{t+1}Q_t k^N_{t+1} - B^N_{t+1}Z_{t+1})dF(\omega,\sigma_{\omega,t}) \quad = \quad g(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}Q_t k^N_{t+1}, \tag{3.10}$$

where $F(\omega,\sigma_{\omega,t})$ is the cumulative distribution function of the log-normal distribution of ω. $\sigma_{\omega,t}$ denotes the period t standard deviation of $\log\omega$. $g(\bar{\omega}_{t+1},\sigma_{\omega,t})$ can be interpreted as the share of total returns to capital that goes to the entrepreneur and is given by

$$g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \;\; = \;\; \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega, \sigma_{\omega,t}) - (1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t}))\bar{\omega}_{t+1}.$$

Before characterizing the optimal loan contract, I discuss the return to the lenders. It is convenient to imagine mutual funds that collect household savings and specialize in lending to entrepreneurs with specific levels of net worth N_t. This N-type mutual funds hold a large portfolio of loans that is perfectly diversified across N-type entrepreneurs. The mutual funds receive $Z_{t+1}B_{t+1}^N$ if an entrepreneur repays the loans, and take away the produced project $\omega_{t+1}R_{t+1}^k k_{t+1}$ net of monitoring costs if an entrepreneur defaults. Then the return to the N-type mutual funds in time $t+1$ is

$$\begin{aligned} r_{t+1}^D \;\; &= \;\; \frac{[1 - F(\omega_{t+1}, \sigma_{\omega,t})]Z_{t+1}B_{t+1}^N + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t})R_{t+1}^k q_t k_{t+1}^N}{B_{t+1}^N} \\ &= \;\; \frac{h(\bar{\omega}_{t+1}, \sigma_{\omega,t})R_{t+1}^k q_t k_{t+1}^N}{B_{t+1}^N} = \frac{h(\bar{\omega}_{t+1}, \sigma_{\omega,t})R_{t+1}^k L_t}{L_t - 1}, \end{aligned} \tag{3.11}$$

where $L_t = \frac{q_t k_{t+1}}{N_t}$ is the leverage ratio. $h(\bar{\omega}_{t+1}, \sigma_{\omega,t})$ is interpreted as the share of total returns to capital that goes to the lender and is given by

$$h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \;\; = \;\; \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})) + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t}).$$

To avoid self-financing in the long-run by accumulating sufficient internal funds, I assume that a fraction $1-\gamma$ of entrepreneurs die each period. Upon dying, they leave constant amount of resources, w^N, for operation and consume the remaining accumulated payoffs.

The optimal loan contract The contract between the lenders and the borrowers is now ready to be described. Borrowers choose leverage ratio L_t and offer lenders a state-contingent return r_{t+1}^D subject to macroeconomic conditions. Equivalently, the N-type entrepreneurs' problem is to choose a cut-off value $\bar{\omega}_{t+1}$ and a leverage ratio L_t that maximize

$$W_t = \mathbb{E}_t(1-\gamma)\sum_{s=0}^{\infty}\gamma^s c^N_{t+s} \tag{3.12}$$

subject to the Euler equation (3.6) and the return to the lenders (3.11), where c^N_t denotes the entrepreneurs' consumption and is given by $g(\bar{\omega}_t, \sigma_{\omega,t-1})R^k_t q_{t-1}k^N_t - w^N$. The value function for N-type entrepreneurs can be reexpressed as:

$$W_t = (1-\gamma)(R^N_t N_{t-1}) + (1-\gamma)(\Psi_t - 1)N_t - w^N, \tag{3.13}$$

where $\Psi_t = 1 + \gamma\mathbb{E}_t[g(\bar{\omega}_t, \sigma_\omega)R^k_t L_t \Psi_{t+1}]$.[5] The marginal value of internal funds is $W_{Nt} = (1-\gamma)(\Psi_t - 1)$. We are now ready to solve for the optimal loan contract:

Proposition 1. *Log-linearizing the solution to the contract problem* (3.12) *and the participation constraint gives*

$$\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1} = \nu_L\hat{L}_t + \nu_\sigma\hat{\sigma}_{\omega,t}, \tag{3.14}$$

and

$$\hat{r}^D_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1} = (\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}^k_{t+1}) + \tilde{\alpha}(\hat{V}_{ct+1} - \mathbb{E}_t\widehat{V}_{ct+1}) - \tilde{\alpha}\frac{\Psi - 1}{\Psi}(\hat{W}_{Nt+1}. - \mathbb{E}_t\hat{W}_{Nt+1}). \tag{3.15}$$

where

$$\hat{W}_{Nt} = \epsilon_N\left(\frac{\Psi}{\Psi - 1}((L-1)(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) + \mathbb{E}_t\hat{R}^k_{t+1} + \nu_\Psi\hat{\sigma}_{\omega,t}) + \mathbb{E}_t\hat{W}_{Nt}\right). \tag{3.16}$$

Under BGG contract, (3.15) *is replaced by:*

$$\hat{r}^D_{t+1} = \hat{R}_t - \mathbb{E}_t\hat{\Pi}_{t+1}. \tag{3.17}$$

The derivation is shown in Appendix C.3. The proposition clearly highlights the differences between the optimal contract and the BGG contract. In equation (3.15), a lender return is indexed to the returns to capital, the marginal utility of consumption, and the marginal value of internal funds. These factors depend on asset prices, aggregate con-

[5] See Appendix C.2 for the derivation.

sumption, and the leverage ratio, which vary according to macroeconomic conditions. That is, the contract imposes state contingencies. However, all of these elements are missing in the BGG contract, in which a lender return is equal to ex-ante real interest rates that are determined in the previous period. In this paper, $\tilde{\alpha}$ takes a value between 11 and 12 and $\frac{\Psi-1}{\Psi}$ is approximately one. Therefore, changes in a lender return are dominated by changes in the marginal utility of consumption and and changes in the marginal value of internal funds. Because these two are almost equally weighted, returns to lenders are mainly determined by the more volatile element between the two.

This double indexation captures insurance incentives for both households and entrepreneurs. Ignoring the expectation on future consumption path, the marginally utility of consumption is high at times current consumption is low. In these circumstances, lenders require higher return on loans to smooth out their consumption. As one can see from the expression (3.16), the marginal value of internal funds is a function of risk premium, which is then a function of the leverage ratio. As such, when the entrepreneurs' net worth is low, implying high leverage ratio, the marginal utility of internal funds is high. Accordingly, entrepreneurs prefer to pay lower return to lenders to stabilize their net worth.

3.3.3 Capital Producers

In order to produce new capital $\bar{k}_{t+1}$, capital producers must purchase back $(1-\delta)\bar{k}_t$ units of capital from entrepreneurs at a unit price q_t and make investment i_t subject to adjustment cost. The capital accumulation equation is thus given by

$$\bar{k}_{t+1} = i_t + (1-\delta)\bar{k}_t - \frac{\phi_k}{2}\left(\frac{i_t}{\bar{k}_t} - \delta\right)^2 \bar{k}_t, \tag{3.18}$$

where ϕ_k determines the investment adjustment cost. The newly produced capital is then sold to entrepreneurs at a unit price q_t. The capital producers' problem is to choose the level of investment so as to maximize their profits

$$d_t^k = q_t \bar{k}_{t+1} - q_t(1-\delta)\bar{k}_t - i_t, \tag{3.19}$$

subject to (3.18).

3.3.4 Aggregation

The quantity of capital purchased by entrepreneurs in period t must equal the quantity of capital produced, $\bar{k}_{t+1}$. That is,

$$\bar{k}_{t+1} = \int_0^\infty k_{t+1}^N f^N(N)\mathrm{d}N, \tag{3.20}$$

where $f^N()$ denotes the distribution of entrepreneurs over net worth, $N \geq 0$. The total capital rented by intermediate-goods firms is

$$k_t = \int_0^\infty \int_0^\infty \omega k_t^N f^N(N)\mathrm{d}F(\omega, \sigma_{\omega,t-1})\mathrm{d}N = \bar{k}_t, \tag{3.21}$$

where the last equality uses the fact that the mean of ω is unity. Recall that the payoffs for N-type entrepreneurs at the end of the period t is $g(\bar{\omega}_t, \sigma_{\omega,t-1})R_t^k Q_{t-1} k_t^N$. Integrating this last expression over N implies $g(\bar{\omega}_t, \sigma_{\omega,t-1})R_t^k Q_{t-1} k_t$. Thus, after the mortality of entrepreneurs, aggregate entrepreneurial net worth at the end of period t, N_t^E, is

$$N_t^E = \gamma g(\bar{\omega}_t, \sigma_{\omega,t-1})R_t^k Q_{t-1} k_t + (1-\gamma)w^E, \tag{3.22}$$

where $w^E = \int_0^\infty w^N f^N(N)\mathrm{d}N$. Similarly, aggregate entrepreneur consumption can be expressed as

$$c_t^E = (1-\gamma) g(\bar{\omega}_t, \sigma_{\omega,t-1})R_t^k Q_{t-1} k_t - (1-\gamma)w^E. \tag{3.23}$$

3.3.5 Matching

Each firm posts multiple identical vacancies. Vacancies and unemployed households are randomly matched according to the aggregate matching function,

$$m(u_{a,t}, v_t) = \psi(u_{a,t})^{\gamma}(v_t)^{1-\gamma}, \tag{3.24}$$

where $m(u_{a,t}, v_t)$ is the number of matches in period t when there are $u_{a,t}$ job seekers and v_t vacancies. ψ is the matching efficiency, and γ represents the elasticity of matches with respect to job seekers. Job seekers consist of the unemployed households from the previous period and households that were employed in the previous period but were separated in this period. Therefore, the number of job seekers in period t is given by

$$u_{a,t} = u_{t-1} + \rho_x n_{t-1}, \tag{3.25}$$

where ρ_x is the job separation rate.

Given the matching function, the probability that a vacant job is filled and the probability that a job seeker becomes employed are

$$\lambda_t = m(1/\theta_t, 1) = \psi\theta_t^{-\gamma} \tag{3.26}$$

and

$$f_t = m(1, \theta_t) = \psi\theta_t^{1-\gamma}, \tag{3.27}$$

respectively, where $\theta_t = v_t/u_{a,t}$ denotes the labor market tightness. The number of unemployed households in period t equals the number of job seekers who failed to find a job and is given by

$$u_t = (1 - f_t)(u_{t-1} + \rho_x n_{t-1}). \tag{3.28}$$

3.3.6 Production Sector

Final-goods firms A representative firm combines differentiated intermediate goods and produces a final good according to a Dixit-Stiglitz aggregator,

$$y_t = (\int y_{z,t}{}^{1-1/\varepsilon} \mathrm{d}z)^{1/(1-1/\varepsilon)}, \tag{3.29}$$

where $y_{z,t}$ is the amount of intermediate good z used and ε is the elasticity of substitution between any pair of intermediate goods. The final-good firm's problem is to minimize expenditures on intermediate goods taking the prices as given subject to the production function (3.29). Its optimal choices imply the demand function for intermediate good z,

$$y_{z,t} = \left(\frac{P_{z,t}}{P_t}\right)^{-\varepsilon} y_t, \tag{3.30}$$

where $P_{z,t}$ is the price of intermediate good z in period t. P_t denotes the aggregate price index, which is given by

$$P_t = (\int P_{z,t}{}^{1-\varepsilon} \mathrm{d}z)^{1/(1-\varepsilon)}. \tag{3.31}$$

Intermediate-goods firms There is a unit continuum of monopolistic producers of intermediate-goods. Firm z produces differentiated good z according to,

$$y_{z,t} = k_{z,t}^{\alpha} n_{z,t}^{1-\alpha} - \xi, \tag{3.32}$$

where $0 < \alpha < 1$. Here, $k_{z.t}$ and $n_{z.t}$ denote the capital and the stock of employees used, respectively. ξ denotes the fixed cost of production. In every period, the firm posts vacancies, $v_{j,t}$, which are filled with probability λ_t. Therefore, the evolution of employees of firm z is given as

$$n_{z,t} = (1-\rho_x) n_{z,t-1} + \lambda_t v_{z,t}. \tag{3.33}$$

In addition, the firm faces price-setting frictions which are modeled as quadratic costs of price adjustment following Rotemberg (1982). A firm z maximizes the present discounted stream of profits,

$$\max_{P_{z,t},n_{z,t},v_{z,t},k_{z,t}} \quad \mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s}\Big[\left(\frac{P_{z,t+s}}{P_{t+s}}\right) y_{z,t+s} - w_{t+s} n_{z,t+s} - r^k_{t+s} k_{z,t+s} - \kappa v_{z,t+s} \\ - \frac{\phi_p}{2}\left(\frac{P_{z,t+s}}{P_{z,t+s-1}} - 1\right)^2 y_{t+s}\Big] \tag{3.34}$$

subject to (3.30), (3.32) and (3.33). The costs for the firm are the forgone resources from searching for new employees and setting prices, the wage bill paid to all employees, and the rental of capital. κ is the cost associated with posting a vacancy.

The real wages are determined by the Nash bargaining process between employed workers and the intermediate-goods firms. Let $\eta \in (0, 1)$ be the be the workersâĂŹ relative bargaining weight. The equilibrium real wage is given by

$$w_t = (1 - \eta) b^u w + \eta(Q_t + \mathbb{E}_t[M_{t+1}(1 - \rho_x)\kappa\theta_{t+1}]), \tag{3.35}$$

where $Q_t = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} mc_t$, and $b^u w$ denotes the flow value of unemployment activities.[6] The real wage is increasing in the marginal productivity of labor, premultiplied by real marginal cost, Q_t, and the present discounted value of total vacancy costs per unemployed worker, $\mathbb{E}_t M_{t+1}\kappa\theta_{t+1}$. Intuitively, the more productive the workers are and the more costly for the firm to fill a vacancy, the contribution of employed workers is greater, leading to a higher wage. In addition, the workers' bargaining weight affect the real wage rigidity. The lower the η is the more the wage rate will be tied with the constant $b^u w$, inducing a lower wage variability.

[6]See Appendix C.1 for derivations.

3.3.7 Government

I assume that the government balances the budget period by period, which implies:

$$b^u w u_t = T_t \tag{3.36}$$

In addition, the monetary policy rule is assumed to the one used in BGG:

$$\ln(\frac{R_t}{R}) = \rho^R \ln(\frac{R_{t-1}}{R}) + \alpha_\pi \ln\left(\frac{\Pi_{t-1}}{\Pi}\right) + \varepsilon^R, \tag{3.37}$$

where α_π measures the extent to which the policy rates respond to a deviation of inflation rates from its target and ρ^R governs the persistence of the policy rates.

3.3.8 Market Clearing and Equilibrium

There are five markets operating in the model – government bond, loan, labor, capital, and final goods. Loan market clears if

$$a_t^D = \int_0^\infty B_t^N f^N(N) dN \equiv B_t^E. \tag{3.38}$$

The markets for labor, capital, and final goods clear if

$$n_t = \int_0^1 n_{z,t} \mathrm{d}z \tag{3.39}$$

$$k_t = \int_0^1 k_{z,t} \mathrm{d}z \tag{3.40}$$

and

$$c_t + c_t^E + i_t + \mu G(\bar{\omega}_t, \sigma_{\omega,t-1}) R_t^k q_{t-1} k_t = y_t - \frac{\phi}{2} (\Pi_t - 1)^2 y_t - \kappa v_t, \tag{3.41}$$

hold, where $G(\bar{\omega}_t, \sigma_{\omega,t-1}) = \int_0^{\bar{\omega}_t} \omega dF(\omega, \sigma_{\omega,t-1})$, and $v_t = \int_0^1 v_{z,t} \mathrm{d}z$. Bond market clears by Walras' law.

A symmetric equilibrium of the economy is sequences of r_t^D, V_t c_t, i_t, y_t, k_t, n_t, mc_t, Π_t, L_t, R_t^k, r_t^k, q_t, N_t^E, B_t^E, $\sigma_{\omega,t}$, c_t^E, M_t, V_{ct}, W_{Nt}, u_t, $u_{a,t}$, f_t, λ_t, v_t, θ_t, Ψ_t, $\phi_{\mu,t}$, R_t, $\bar{\omega}_t$, and w_t that satisfy the restrictions in Appendix C.4.

3.4 Quantitative Analysis

3.4.1 Calibration

The model period is one quarter. As for the credit-related parameters, I largely follow BGG. I calibrate the model to be consistent with an average credit spread of 200 bp (annualized), a quarterly bankruptcy rate of 0.75, and a leverage ratio of 2. These targets imply a cut-off value of idiosyncratic productivity $\bar{\omega}$ of 0.5, a standard deviation of the log idiosyncratic productivity σ_ω of 0.27, and a monitoring cost μ of 0.12. The death rate of entrepreneurs $1-\gamma$ is set to 0.0272.

As for the employment-related parameters, I target a steady state unemployment rate of 6 percent, a value that corresponds to the average unemployment rate between 1984 and 2014. The steady state job-finding rate f and the separation rate ρ_x are computed as follows. Following Shimer (2005), I first compute the monthly job-finding rate using unemployment and short-term unemployment data from the Current Population Survey (CPS). I average the resulting series over each quarter and convert these into quarterly terms. The job-finding rate averaged 0.72 from 1984 to 2014. Using equation (3.28), I then compute the steady state job separation rate. b^u, which determines the flow value of unemployment activities, is set to 0.85 as in Rudanko (2011). Shimer (2005) pins down this value to 0.4 by assuming that the only benefit for unemployment is unemployment insurance. However, Hagedorn and Manovskii (2008a) argue that in a perfectly competitive labor market, the benefit for unemployment measures not only unemployment insurance, but also the total value of home production, self-employment, disutility of work and thus choose b^u equal to 0.96, which is far more extreme than 0.85. The matching function elasticity to job seekers ζ is 0.5, suggested by Petrongolo and Pissarides (2001). The matching efficiency ψ is set to 1. I choose κ so that the share of vacancy costs in output is 1 percent. These values uniquely pins down the steady state profit from hiring a worker and thus the steady state real wages, which then determines the bargaining power of households η.

For the elasticity of substitution between intermediate goods ε, I target a steady state markup of 1.2 (Basu and Fernald, 1997). The capital depreciation rate δ is assumed to be 0.02, implying a 8 percent annual depreciation of physical capital. The power on capital in the production function α is set to 0.33. The fixed cost ξ is set so that the steady state profits of monopolistic competitive firms are zero (Rotemberg and Woodford, 1999). For the parameter that governs the costs of adjusting prices, I exploit the equivalence of the coefficient of marginal cost in the linearized Phillips curve implied by the Rotemberg model and the one derived from the Calvo model. I then find the value of ϕ_p that corresponds to a price adjustment frequency of 4 quarters, consistent with the evidence in Nakamura and Steinsson (2008). Investment adjustment costs ϕ_k is 10.

As for the monetary policy rule, I set the autoregressive parameter on the nominal interest rate ρ_R to 0.9 and the parameter on past inflation α_π to 0.11 following BGG. For the monetary policy shock, I consider a 25 basis point shock (annualized terms) to the nominal interest rate. For the risk shock, I allow the standard deviation of idiosyncratic productivity to increase by one percentage point, from 0.27 to 0.28. The persistence of idiosyncratic volatility ρ^{σ_ω} is set at 0.9706, following Christiano et al. (2014). The calibration is summarized in Table **??**.

3.4.2 Does the Optimal Contract Strengthen the Financial Accelerator?

In the quantitative analysis, I compare the competitive equilibrium between the optimal contract and the BGG contract and demonstrate that the amplification due to financial

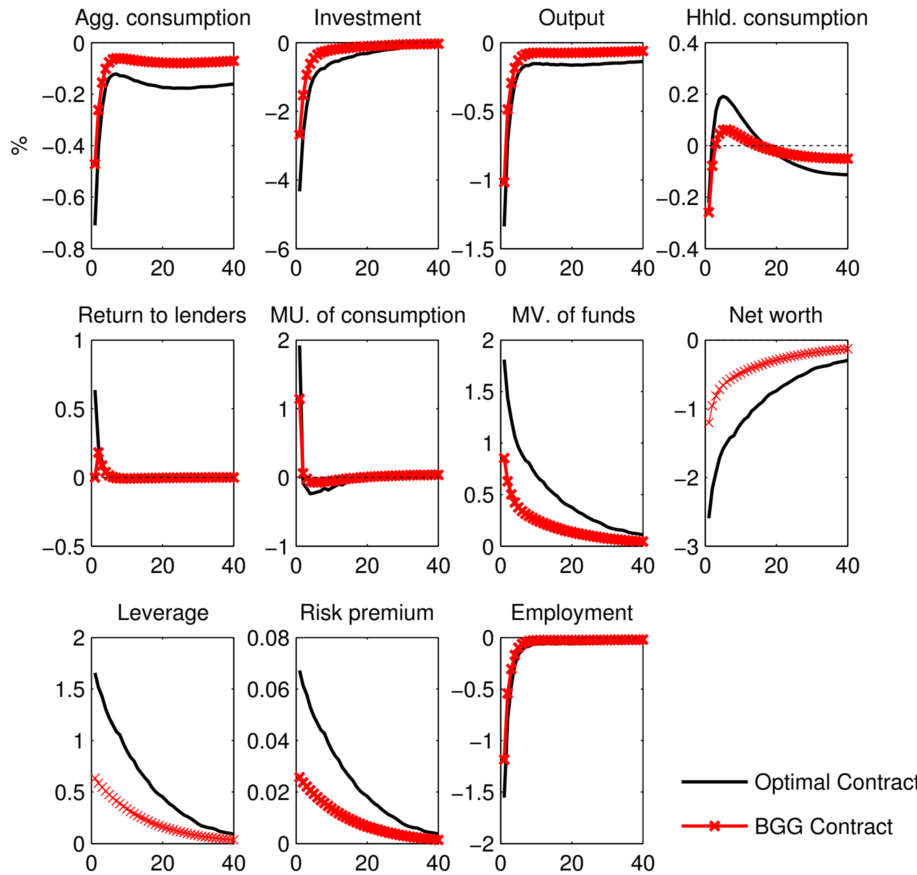
Table 3.2: Calibration of the Parameters

Symbol	Description	Value	Target (Source)
σ	Risk aversion	60	Market price of risk (0.24)
β	Hhld. discount factor	0.9951	2% annual real interest rate
σ_ω	Std. of idiosyncratic prod.	0.27	See the text
μ	Monitoring cost	0.12	See the text
$1-\gamma$	Death rate of entrepreneurs	0.0272	BGG
ρ_x	Job separation rate	0.16	Job-finding rate of 0.72
b^u	Replacement rate	0.85	Rudanko (2011)
ζ	Matching function elasticity	0.5	Petrongolo and Pissarides (2001)
ψ	Matching efficiency	1	
κ	Cost of posting vacancy	0.21	$\kappa v/y = 0.01$
η	Hhld. bargaining power	0.68	See the text
δ	Capital depreciation rate	0.02	8% annual depreciation rate
α	Power on capital in production	0.33	
ε	Elasticity of substitution b/w goods	6	Markup of 1.2
ξ	Fixed costs	0.47	Zero-profit condition
ϕ_p	Price stickiness	59.14	Adjustment freq. of 4 quarters
ϕ_k	Investment adjustment cost	10	
ρ_R	Interest rate smoothing	0.9	BGG
α_π	Interest rate rule on inflation	0.11	BGG
ρ^{σ_ω}	AR(1) of vol. shocks	0.9706	Christiano et al. (2014)

frictions is more pronounced under the optimal contract.[7] In figure 3·1, I plot the impulse responses for shocks to Federal funds rates. In response to a contractionary monetary policy shock, lenders reduce consumption due to intertemporal substitution effect, which works to raise the marginal utility of consumption. Therefore, lenders require higher rate of returns on loans, which puts downward pressure on the borrowers' net worth. Although this leads to an increase in the marginal value of internal funds, borrowers willingness to stabilize the net worth by paying lower return is offset by a rise in the risk-averse lenders' marginal utility of consumption. Under the BGG contract, a return to lenders does not respond to the shock at all because it is predetermined and thus the

[7] The model is solved by a third-order perturbation method.

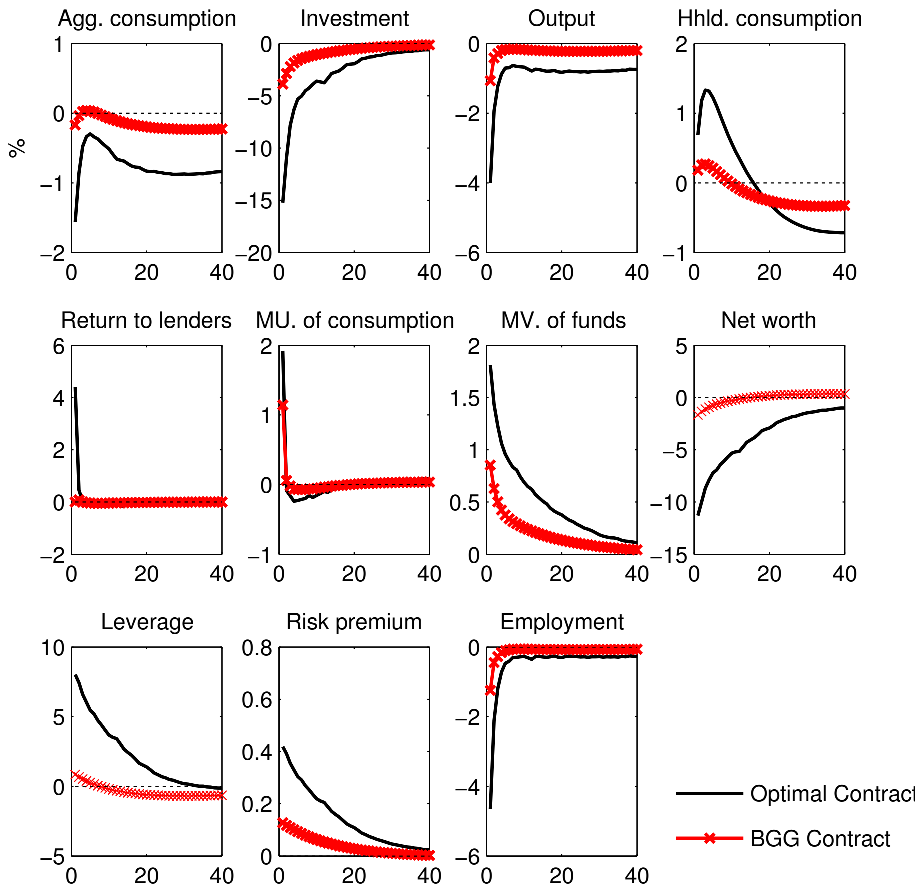
Figure 3·1: Impact of a Monetary Policy Shock



net worth falls by less. Therefore, the financial accelerator mechanism under the optimal contract is stronger than under the BGG contract, leading to a larger amplification.

Figure 3·2 depicts the impulse responses for a one standard deviation increase in unobserved idiosyncratic volatility. An increase in volatility leads to a rise in default rates, which then reduces the aggregate demand for capital. This drives down the capital prices, which in turn decreases the net worth. As net worth takes time to rebuild, lenders expect the marginal value of internal funds to be higher and more persistent than the marginal utility of consumption as time evolves. Accordingly, lenders expect a lender return to be heavily driven by the marginal value of internal funds in the future, receiving lower returns on loans. This leads to a reduced consumption level in the future. With recursive

Figure 3·2: Impact of a Volatility Shock



utility, expectation about lower consumption level in the future raises current marginal utility, which leads to a higher lender return on impact, and so contributes to a further fall in the net worth.[8] Under the BGG contract, the linkage between the marginal utility of consumption and a lender return is absent. Therefore, a larger drop in the net worth under the optimal contract sets in more powerful financial accelerator than under the BGG contract.

[8]The marginal utility of consumption is $\beta c_t^{-\frac{1}{\psi}} V_{t-1}^{\frac{1}{\psi}} \left(\frac{V_t}{\mathbb{E}_{t-1}[V_t^{1-\sigma}]^{\frac{1}{1-\sigma}}} \right)^{\frac{1}{\psi}-\sigma}$. Expected reduced level of consumption lowers V_t and thus the marginal utility of consumption.

3.5 Conclusion

The financial accelerator model of Bernanke et al. (1999) is widely used to assess the importance of the balance sheet channel in propagating aggregate shocks. This paper extends their model by incorporating risk-sensitive lenders and by deriving the optimal loan contract that depends on macroeconomic conditions. An important feature of the optimal contract is indexation to the marginal utility of consumption and the marginal value of internal funds, which is ignored in the BGG contract. The optimal contract generates larger financial amplification than the BGG contract, because the lenders' preferences imply more volatile marginal utility of consumption relative to the marginal value of internal funds.

One might ask whether predictions implied by the contract in this paper are present in the data. Qualitatively, the model results in countercyclical interest rates on loans. Pintus et al. (2016) show a VAR-based analysis, documenting real borrowing interest rates faced by corporate and noncorporate firms stay below the trend for several quarters while output and investment boom. Therefore, assets that have a payoff structure similar to the loan studied in the present paper are likely to exist.

Appendix A

Appendices to Investment Shocks, Unemployment Risk, and Macroeconomic Comovement

A.1 Decision Problems and Model Equations

In this section of the appendix, I derive the optimality conditions and the equations related to the labor market environment, which I use to compute the equilibrium of the model.

Impatient household's problem The idiosyncratic state of a household is its real bond holdings a and its employment status $e \in \{0, 1\}$. Let $\mathcal{S}$ be the collection of aggregate state variables. Its budget constraint is given by

$$c + a' = (1 - \tau(\mathcal{S}))w(\mathcal{S})e + (1 - \tau(\mathcal{S}))b^u w(\mathcal{S})(1 - e) + \frac{R(\mathcal{S}_{-1})}{\Pi(\mathcal{S})}a, \tag{A.1}$$

where $R(\mathcal{S}_{-1})$ refers to the nominal interest rate determined in the previous period. Then the problem of an employed household ($e = 1$) with real assets a can be written as

$$\begin{aligned} V(a, 1; \mathcal{S}) = \max_{c,a'} \Bigg\{ \left[\frac{c^{1-\sigma}}{1-\sigma} \right] &+ \beta^I \mathbb{E} \left(1 - \rho_x(1 - f(\mathcal{S}))\right) V(a', 1; \mathcal{S}') \\ &+ \beta^I \mathbb{E} \rho_x (1 - f(\mathcal{S})) V(a', 0; \mathcal{S}') \Bigg\}, \end{aligned} \tag{A.2}$$

subject to a borrowing constraint, the budget constraint discussed earlier, and the law of motion for aggregate states. $\mathbb{E}$ is the conditional expectations operator over aggregate uncertainty. Similarly, the problem of an unemployed household ($e = 0$) with real assets

a is

$$V(a,0;\mathcal{S}) = \max_{c,a'} \left\{ \left[\frac{c^{1-\sigma}}{1-\sigma} \right] + \beta^I \mathbb{E} f(\mathcal{S}) V(a',1;\mathcal{S}') + \beta^I \mathbb{E}(1-f(\mathcal{S})) V(a',0;\mathcal{S}') \right\}, \tag{A.3}$$

subject to a borrowing constraint, the budget constraint above, and the law of motion for aggregate states. From these problems, one can derive an Euler equation for both employed and unemployed households.

Patient household's problem The patient household chooses $\{c_t^P, a_{t+1}^P, i_t^P, k_{t+1}^P\}$ to maximize the same utility as impatient households subject to Equations (1.6) and (1.7). Setting up the Lagrangian, in which Λ_t and $\Lambda_t q_t$ are the Lagrangian multipliers on the constraints (1.6) and (1.7), respectively, and then rearranging the resulting optimality conditions using relations $i_t = \Omega i_t^P$ and $k_t = \Omega k_t^P$, we obtain

$$1 = \mathbb{E}_t \beta^P \frac{R_t}{\Pi_{t+1}} \left(\frac{c_{t+1}^P}{c_t^P} \right)^{-\sigma}, \tag{A.4}$$

$$1 = q_t[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \frac{i_t}{i_{t-1}} S'\left(\frac{i_t}{i_{t-1}}\right)] + \mathbb{E}_t \beta^P \left(\frac{c_{t+1}^P}{c_t^P} \right)^{-\sigma} q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right), \tag{A.5}$$

$$q_t = \mathbb{E}_t \beta^P \left(\frac{c_{t+1}^P}{c_t^P} \right)^{-\sigma} [r_{t+1}^k + q_{t+1}(1-\delta)], \tag{A.6}$$

and

$$k_{t+1} - (1-\delta)k_t = \mu_t \left(1 - S\left(\frac{i_t}{i_{t-1}}\right)\right) i_t. \tag{A.7}$$

Intermediate-goods firm A firm j's problem is (3.34) subject to (3.30), (3.32), and (3.33). The resulting optimality conditions are

$$y_t = k_t^\alpha (((1-\Omega) + \eta\Omega) n_t)^{1-\alpha}, \tag{A.8}$$

$$r_t^k = \alpha k_t^{\alpha-1}(((1-\Omega)+\eta\Omega)n_t)^{1-\alpha} mc_t, \tag{A.9}$$

$$\frac{\kappa}{\lambda_t} = ((1-\Omega)+\eta\Omega)((1-\alpha)k_t^{\alpha} n_t^{-\alpha} mc_t - w_t) + \mathbb{E}_t \beta^P \left(\frac{c_{t+1}^P}{c_t^P}\right)^{-\sigma} (1-\rho_x)\frac{\kappa}{\lambda_{t+1}}, \tag{A.10}$$

and

$$1-\varepsilon+\varepsilon mc_t = \phi_p(\Pi_t - 1)\Pi_t - \phi_p \mathbb{E}_t \left[\beta^P \left(\frac{c_{t+1}^P}{c_t^P}\right)^{-\sigma} (\Pi_{t+1}-1)\Pi_{t+1}\frac{y_{t+1}}{y_t}\right]. \tag{A.11}$$

Note that subscript j is omitted because intermediate-goods firms face identical factor prices, aggregate matching function, pricing frictions, and production technology, so their decisions are the same.

Labor market environment Substituting (3.26) into (3.33), we obtain

$$n_t = (1-\rho_x)n_{t-1} + \psi v_t \theta_t^{-\gamma}, \tag{A.12}$$

where labor market tightness is

$$\theta_t = \frac{v_t}{u_{a,t}}. \tag{A.13}$$

The unemployment rate can be expressed as

$$u_t = 1 - n_t. \tag{A.14}$$

A.2 Computational Method

Here I describe the procedure used to solve for an equilibrium path of the heterogeneous agent model with the aggregate shocks considered in Section 4.

A.2.1 Solving for the Household's Decision Rules Without Aggregate Shocks

For each type of impatient household characterized by an employment state, I solve for the level of cash-on-hand, χ_1, at which the household starts to consume all of its available resources and therefore is just on the threshold of being borrowing constrained. Taking the first grid point equal to χ_1, I create an additional 99 grid points on cash-on-hand, $\chi_2, \ldots, \chi_{100}$, 100 grids in sum. Constructing the grid in this fashion allows for a more accurate solution because we do not interpolate across the kink in the policy rule where borrowing constraint stops binding. Because there is generally a fair amount of curvature in the savings policies, especially for values of cash-on-hand near the borrowing constraint, the grid points are unevenly spaced with more points near the borrowing constraint. Between the grid points, I interpolate the household savings rule with linear splines. I then solve for the household's savings policies using the Broyden (1965) method by imposing that the Euler equation holds with equality at the grid points. The resulting solution consists of χ_1 and values of savings policy evaluated at $\chi_2, \ldots, \chi_{100}$. In total, the household policy rule for savings is parameterized by 200 variables, 100 points for each employment state.

A.2.2 Finding the Stationary Equilibrium

This part of the algorithm is similar to the one described in Aiyagari (1994). I assume the consumption differential between employed and unemployed is 20 percent and search for the value of impatient households' discount factor, β^I, for which this is an equilibrium. When simulating the stationary distribution of wealth, I use nonstochastic simulation as described by Young (2010). Given such β^I, I compute total bond holdings and consumption of impatient households. I then use standard techniques from the analysis of representative agent models to find the rest of the aggregate variables. Once I obtain aggregate consumption and total asset supply, I obtain the consumption and bond holdings of patient households by subtracting those of impatient households from the aggregate variables.

A.2.3 Solving for Aggregate Dynamics

Household decision rules The 200 variables that summarize the household savings policy depend on the aggregate state. As the aggregate state changes, I require that these variables satisfy the Euler equation on the grids described earlier, which yields 200 non-linear restrictions.

Evolution of the wealth distribution The non-stochastic simulation algorithm tracks the distribution of wealth using a histogram. The mass in each of these bins is considered a variable. I create 250 evenly spaced bins between 0 and the maximum level of bond, $\bar{b}$, for each type of impatient household. $\bar{b}$ is chosen such that, in the steady state distribution, the mass in the $\bar{b}$ bin is very close to zero. In total there are 499 variables that characterize the distribution because one variable is redundant, considering that distribution must sum to one. For a given set of household savings rules, transition probabilities between employment states, and prices, we can formulate a linear equation system of size 499 that describes the transition dynamics of the wealth distribution. Therefore, we have 499 variables and 499 linear restrictions.

Aggregate equations In addition to the equations that represent the solution of the impatient householdsâĂŹ problem and the distribution of wealth across these households, we have

(1) the optimal decisions of patient households: (A.4), (A.5), (A.6), (A.7);

(2) the optimal decisions of firms: (A.9), (A.10), (A.11), (3.32), (1.20);

(3) labor market environment, (A.12), (3.25), (3.28), (A.13), (A.14), (3.26);

(4) government polices: (3.36), (3.37),

(5) market clearing: (3.41); and

Table A.1: Equations That Hold Exactly in Error Analysis

Description	Number	Variable(s) Determined
Distribution of bond holdings	-	n_{t-1}
Job seekers	(3.25)	$u_{a,t}$
Def. of job-finding rate	(3.27)	θ_t
Def. of labor market tightness	(A.13)	v_t
Unemployment rate	(3.28)	u_t, n_t
Production function	(3.32)	y_t
Wage rule	(1.20)	w_t
Impatient budget constraints	(1.4)	c_t^I
Aggregate resource constraint	(3.41)	i_t
Capital accumulation	(A.7)	k_{t+1}
Fiscal and monetary policy rule	(3.36), (3.37)	τ_t, R_t

Note: For capital accumulation, the nonlinear specification of investment adjustment costs is $S\left(\frac{i_t}{i_{t-1}}\right) = \frac{S''}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$.

(6) the process of marginal efficiency: (1.8).

I introduce a set of auxiliary variables that carry extra lag of variables: $i_t^{lag} = i_{t-1}$, $R_t^{lag} = R_{t-1}$, $k_t^{lag} = k_{t-1}$. I use these equations with $c_t = (1-\Omega)\int c_{i,t}^I \mathrm{d}i + \Omega c_t^P$ to solve for μ_t, k_t, y_t, u_t, $u_{a,t}$, n_t, θ_t, λ_t, f_t, w_t, r_t^k, q_t, Π_t, mc_t, R_t, τ_t, c_t, i_t, v_t, i_t^{lag}, k_t^{lag}, R_t^{lag}, and c_t^P.

Linearization and solution At this stage, we have a large system of 722 restrictions, some of which are nonlinear, which the 722 variables must satisfy. Following Reiter (2009), the system is linearized around the stationary equilibrium using automatic differentiation and then solved using the Sims (2002) method.

A.2.4 Accuracy Check: Euler Equation Errors

I discuss the accuracy of the solution method used to solve the baseline model. There are two sources of errors both of which commonly arise in related algorithms. First, there are errors in the decision rules of the impatient household between the grid points. These errors are present even in the stationary equilibrium. Away from the stationary equilibrium, there are errors due to non-linear responses to aggregate shocks, as is the case with other applications of perturbation methods. The procedure that follows assesses both of these types of errors by using a finer grid and by checking the error in the non-linear equations away from the stationary equilibrium.

To assess the accuracy of the solution, I calculate the unit-free Euler equation errors, following Judd (1992). I do so for both patient and impatient households. For impatient households, I use a test grid over cash-on-hand that is finer than the grid on which I solve for household decision rules.[1] For a given aggregate state of the economy, $\mathcal{S}_t$, the distribution of bond holdings, the capital stock, k_{t-1} and the exogenous variable, μ_t are predetermined. In addition, the lag of investment, i_{t-1} and the interest rate paid on current bond holdings, R_{t-1}, are also predetermined. I then use the approximate model solutions to determine Π_t, f_t, c_t^P, and household savings rules. I then use the non-linear, static relationships, market clearing conditions, and budget constraints to determine the remaining variables. Table A.1 lists the equations I impose and the variables that are solved for. From these calculations and a given aggregate shock, I can compute the next state of the economy, $\mathcal{S}_{t+1}$, and repeat these steps to find c_{t+1}^P, $c_{j,t+1}^I$, and so on. To compute the conditional expectations, I use Gaussian quadrature over the marginal efficiency of investment shock with 11 nodes. For a given household, I compute the level of consumption implied by the right-hand side of the Euler equation as

$$c_t^{s,imp} \equiv U'^{-1}\left(\beta^s \mathbb{E}\left\{U'(c_{t+1}^s)\frac{R_t}{\pi_{t+1}}\right\}\right) \qquad s \in \{I, P\}, \tag{A.15}$$

where the expectation is over aggregate and idiosyncratic states in the case of impatient households. The unit-free Euler equation error for a given type of household is then $c_t^{s,imp}/c_t^s - 1$, where c_t^s is the level of consumption implied by the approximated decision

[1] Specifically, I use cash-on-hand evaluated at the same 250-point grid on asset holdings that I use to approximate the distribution of wealth.

rules.[2]

Using the preceding steps, I compute the Euler equation error for each group of households. For the impatient group, I integrate the consumption of all impatient households using the distribution of wealth at the given state of the economy to compute aggregate consumption implied by the right-hand side of the Euler equation. Similarly, integrating the consumption implied by the approximate policy rules gives the aggregate consumption implied by the left-hand side of the Euler equation. Then the aggregate Euler equation error for all impatient households can be obtained by computing $\int c_{j,t}^{I,imp} \mathrm{d}j / \int c_{j,t}^{I} \mathrm{d}j - 1$. Moreover, I compute the aggregate Euler equation error for all households, which can be expressed as $[(1-\Omega)\int c_{j,t}^{I,imp}\mathrm{d}j + \Omega c_t^{P,imp}]/[(1-\Omega)\int c_{j,t}^{I}\mathrm{d}j + \Omega c_t^{P}] - 1$. I report this aggregate Euler equation error as opposed to the error for each group of households because this is what is relevant to the results on aggregate dynamics.

The Euler equation errors vary over the state space. I randomly draw points in the state space by simulating the model for 50,000 periods and compute the errors every 1000 simulated periods. I describe the distribution of errors across the 50 resulting points by reporting the largest absolute error and the mean absolute error in Table A.2.

Table A.2: Largest and Mean Absolute Errors Across 50 Randomly Drawn Points in the State Space

	Patient	Impatient	Aggregate
Largest	-1.36	-0.14	-0.60
Mean	-2.12	-1.69	-2.02

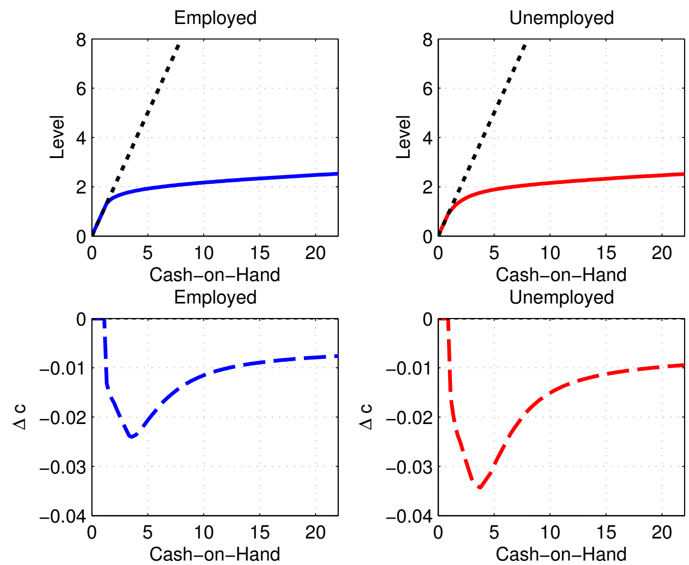
Note: Euler equation errors are reported in logs (base 10).

A.3 Equilibrium Conditions in Hand-to-Mouth Model

$$c_t = \Lambda^{HtM} c_t^{HtM} + (1 - \Lambda^{HtM}) c_t^P \tag{A.16}$$

[2]For impatient households, I use the approximated policy rules for savings and compute consumption from their budget constraint.

Figure A·1: Impact of an Investment Shock on Optimal Policy Rules



$$i_t = (1 - \Lambda^{HtM}) i_t^P \tag{A.17}$$

$$k_t = (1 - \Lambda^{HtM}) k_t^P, \tag{A.18}$$

$$y_t \quad = \quad k_t^{\alpha} (\eta^{RA} n_t)^{1-\alpha} - \xi, \tag{A.19}$$

$$r_t^k = \alpha k_t^{\alpha - 1} (\eta^{RA} n_t)^{1-\alpha} mc_t, \tag{A.20}$$

$$\frac{\kappa}{\lambda_t} = \eta^{RA}((1-\alpha) k_t^{\alpha} \eta^{RA} n_t^{-\alpha} mc_t - w_t) + \mathbb{E}_t \beta^P \left(\frac{c_{t+1}^P}{c_t^P} \right)^{-\sigma} (1 - \rho_x) \frac{\kappa}{\lambda_{t+1}}, \tag{A.21}$$

with (A.4), (A.5), (A.6), (A.7), (A.11), (A.12), (3.25), (3.28), (A.13), (A.14), (3.26), (3.36), (3.37), (3.41), and (1.8). Λ^{HtM} denotes the fraction of hand-to-mouth households in the economy. The representative-agent benchmark is obtained by setting Λ^{HtM} equal to 0.

A.4 Policy Rules

One might think that the decline in aggregate consumption in the heterogeneous-agent model could be driven by the higher number of unemployed that have a higher marginal propensity to consume (MPC) than the employed on average rather than the precautionary savings effects. In this section of the appendix, I show that, once controlling for compositional effects, the precautionary savings effects due to future unemployment risk are substantial. I do so by showing changes in household decision rules of each type to an investment shock.

The top panels in Figure A·1 plot the steady state consumption policy rules of the employed and unemployed against cash-on-hand. Cash-on-hand represents the resources available for consumption and savings in a given period, that is, the right-hand side of budget constraint (1.4). The maximum level of cash-on-hand in the figure is determined by the level of bond at which the mass of impatient households is close to zero premultiplied by the steady state real interest rate plus the steady state after-tax labor income of the employed. The consumption policy rules display strong nonlinearity around the cash-on-hand at which the curve is kinked. The households with cash-on-hand at which their optimal consumption overlaps the 45 degree-line consume all the current available resources by hitting a borrowing constraint. The threshold of holding positive savings is higher for the employed households because they are more likely to be employed in the next period than the unemployed households and thus have less incentive to leave resources for future consumption.[3]

The bottom panels in Figure A·1 illustrate how the optimal consumption rules deviate from the steady state in response to a rise in unemployment risk induced by a negative investment shock. I fix the prices to the corresponding steady state values to focus on the effects of changes in unemployment risk and exclude any other price channels through which the household decision rules may be affected. The vertical axis of the two graphs represents the deviations of consumption from its steady state value. Notably, the optimal consumption rules of the unemployed shift down more than those of the employed.

[3]The steady state transition rate from employed to employed is 0.953 and the rate from unemployed to employed is 0.73.

Table A.3: Correlation of Consumption and Investment

	Data	HA	RA	HA (const. risk)
HP filter	0.75	0.81	-0.34	-0.14
Hamilton filter	0.73	0.94	0.13	0.31

Although the job-loss rate has increased due to reduction in hiring, the employed are still more likely to be employed than the unemployed and thus are expected to be better off sooner. [4]

A.5 Model Robustness

A.5.1 Filtering

Hamilton (2017) argues that the HP filter has serious drawbacks because it involves several levels of differencing. Therefore, for random walk series, subsequently observed patterns are likely to be artefacts of having applied the filter, rather than due to the underlying data-generating process. He goes on to suggest an alternative to the HP filter, which I label Hamilton filter. Table A.3 reports the correlation of consumption and investment implied by the data and the three versions of the model under the Hamilton filter. The baseline Heterogeneous-agent model displays a stronger positive correlation than under the HP filter. In the representative-agent model, consumption weakly comoves with investment but still exhibits much less comovement than the data. Therefore, additional ingredient such as time-varying precautionary savings is essential to achieve the empirical comovement.

A.5.2 Investment Adjustment Costs

I demonstrate that the short-run fall of consumption in the baseline model emerges regardless of investment adjustment costs. Because, under no adjustment costs, the volatil-

[4] It is trivial to show $1 - \rho_x(1 - f) > f$ as long as $\rho_x < 1$, implying employed-to-employed probability is greater than unemployed-to-employed probability even along the business cycles.

Table A.4: Business Cycle Statistics (No Adjustment Costs): Data vs Model

Moment	Variable (x)	Data	HA	RA	HA (const. risk)
Std(x)	Output (GDP)	1.85	1.36	1.35	1.36
	Consumption (c)	0.79	0.47	0.24	0.40
	Investment (i)	4.85	4.85	6.44	6.11
Corr(x, GDP)	Output (GDP)	-	-	-	-
	Consumption (c)	0.85	0.83	-0.29	0.17
	Investment (i)	0.98	0.98	0.99	0.97
Corr(x, i)	Output (GDP)	-	-	-	-
	Consumption (c)	0.73	0.71	-0.40	-0.06
	Investment (i)	-	-	-	-
Std(x)/Std(GDP)	Output (GDP)	-	-	-	-
	Consumption (c)	0.43	0.35	0.18	0.29
	Investment (i)	2.62	3.56	4.77	4.48

Note: The table compares the moments of the data and those from 10,000 simulations of the models. Standard deviations are scaled by 100. The moments are taken from the logs of the data, which are then detrended using the HP-filter with a smoothing parameter of 1600. Output (GDP) is the sum of consumption and investment.

ity of investment is larger for a given sequence of investment shocks, I recalibrate the standard deviation of the marginal efficiency of investment shock to match the empirical volatility of investment. Table A.4 shows how well the three models are able to explain the empirical business cycle moments under no adjustment costs. The sign of the correlation of consumption with investment from the three benchmarks remains unchanged regardless of investment adjustment costs.

Without adjustment costs, there are differences in implied investment dynamics, which bear key implications for the dynamic behavior of the job-finding rates and thus the unemployment risk. With no adjustment costs, investment jumps on impact and then dies out relatively quickly. Conversely, under adjustment costs, investment builds up gradually reaching its peak several periods after. Because the job-finding rates move proportionately with aggregate demand, their behavior is largely determined by the movements of investment, the most volatile aggregate demand component. Persistent job-finding rates lead to a persistence in precautionary savings and thus in aggregate consumption, which can be confirmed in Figure A·2. In any event, qualitative responses of all variables are the same across the two specifications.

A.5.3 Sensitivity of Comovement to Other Parameters

I describe that the sensitivity of comovement in the heterogeneous-agent model is affected by parameters that influence the strength of precautionary savings motive. It turns out that the coefficients of risk aversion, the skill premium parameter, and the consumption differential between the employed and the unemployed all affect the stationary wealth distribution and so the consumption insurances for households. Therefore, I vary one of these parameters at a time while recalibrating the discount factor of impatient households or the replacement rate (or both) to match the targeted share of wealth for impatient households and consumption differential between the employed and the unemployed. Figure A·3 includes the impulse responses of consumption to an adverse investment shock under different values for these parameters. Because the consumption losses upon unemployment directly represent the extent to which households suffer when they are unemployed, the magnitude of consumption drop affects the households' willingness to be insured. The degree of risk aversion determines the households' willingness to bear idiosyncratic risk and hence influences their engagement in precautionary savings. The skill premium directly affects the cross-sectional distribution of income and thereby consumption level between patient and impatient households. The higher the skill premium, the lower the consumption share for impatient households. This reduces the importance of precautionary savings on aggregate consumption, and thereby the aggregate consumption drop is smaller on impact. However, in the medium run, aggregate consumption falls by more because patient households whose consumption is reduced due to intertemporal substitution play more significant role in shaping aggregate consumption dynamics.

The real wage rigidity affects the degree of unemployment risks and therefore the volatility of aggregate consumption because further reduced wages give firms more leeway in hiring at downturns. Accordingly, less rigid real wages stabilize the unemployment fluctuations and thus the precautionary savings incentive.

Monetary police rule and sticky prices I consider how the monetary authority's willingness to smooth out the real interest rates and price stickiness affect comovement. To do so, I modify the specification of monetary policy rule to $\ln(\frac{R_t}{R}) = \rho_R \ln(\frac{R_{t-1}}{R}) + (1 - \rho_R)(\phi_\Pi \Pi_t)$, where ρ_R represents the degree of interest rate inertia. The first two graphs in the bottom panel of Figure A·3 portray the consumption responses after an investment shock under different values for the interest rate smoothing parameter and the inflation coefficient.

As studied by Kaplan et al. (2016), the real interest rates affect consumption through direct and indirect effects in the heterogeneous-agent model. Direct effects are those that operate through intertemporal substitution, whereas indirect effects occur from general equilibrium forces. In this paper, indirect effects on consumption arise largely due to the changes in unemployment risks caused by the changes in aggregate demand. The magnitude and the direction of the aggregate consumption response are determined by the relative strengths of these effects.

When ρ_R is high, the nominal interest rates do not adjust much to poor economic conditions. Because the expected inflation is stable due to the monetary authority's tendency to stabilize prices, the path of real interest rates is smooth. Accordingly, households are more willing to smooth their consumption over time, which is a direct effect. Moreover, indirect effects on consumption arise powerfully in my environment; smoother real interest rates increase the cost of investment more and thus further depress aggregate demand than in the regime where a greater fall in real interest rates is observed. Consequently, firms reduce hiring, which causes a further increase in unemployment risk. In effect, aggregate consumption becomes more volatile.

When α_π is set equal to 1.05, the real interest rate is quite insensitive to changes in inflation. Accordingly, the mechanism that operates in the case of high ρ_R applies when the monetary policy is passive. That is, consumption is more volatile relative to the case in which α_π is 1.5. When monetary policy is aggressive ($\alpha_\pi = 2$), the policy rate is more sensitive to changes in inflation than in the case where α_π is 1.5. In addition, the expected inflation becomes more stable as the monetary authority is more committed to stabilizing inflation. Subsequently, the real interest rates drop substantially after investment shocks, which works to stimulate investment and employment. This largely stabilizes unemployment risk and thus households engage less in precautionary savings.

As emphasized by Ravn and Sterk (2013), good market frictions play a significant role in amplifying unemployment risk. I explore how interaction between good market frictions and unemployment risk affects the comovement of consumption with investment. The third graph in the bottom panel of Figure A·3 shows the responses of consumption with different price adjustment frequencies. Consumption tends to fall more when prices are stickier, whereas a decline in consumption cannot be observed under flexible prices. As changing prices becomes more costly, firms instead choose to cut hiring in response to a contraction in aggregate demand and therefore households face higher risk of unemployment.

Figure A·2: Robustness to Specifications of Investment Adjustment Cost

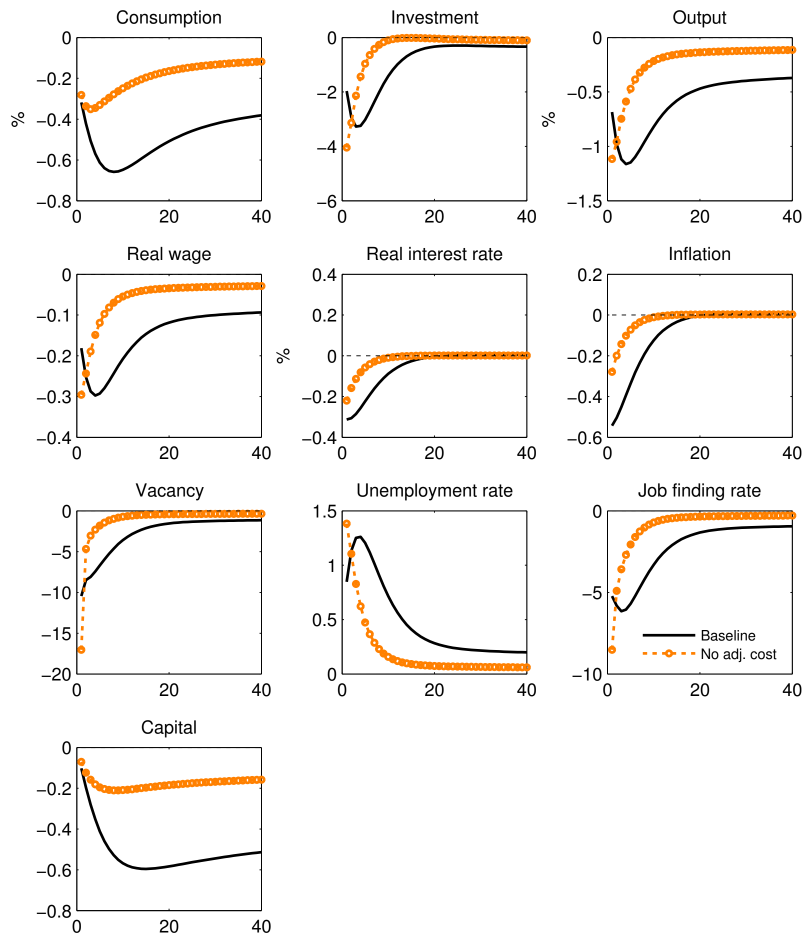


Figure A·3: Sensitivity of Comovement

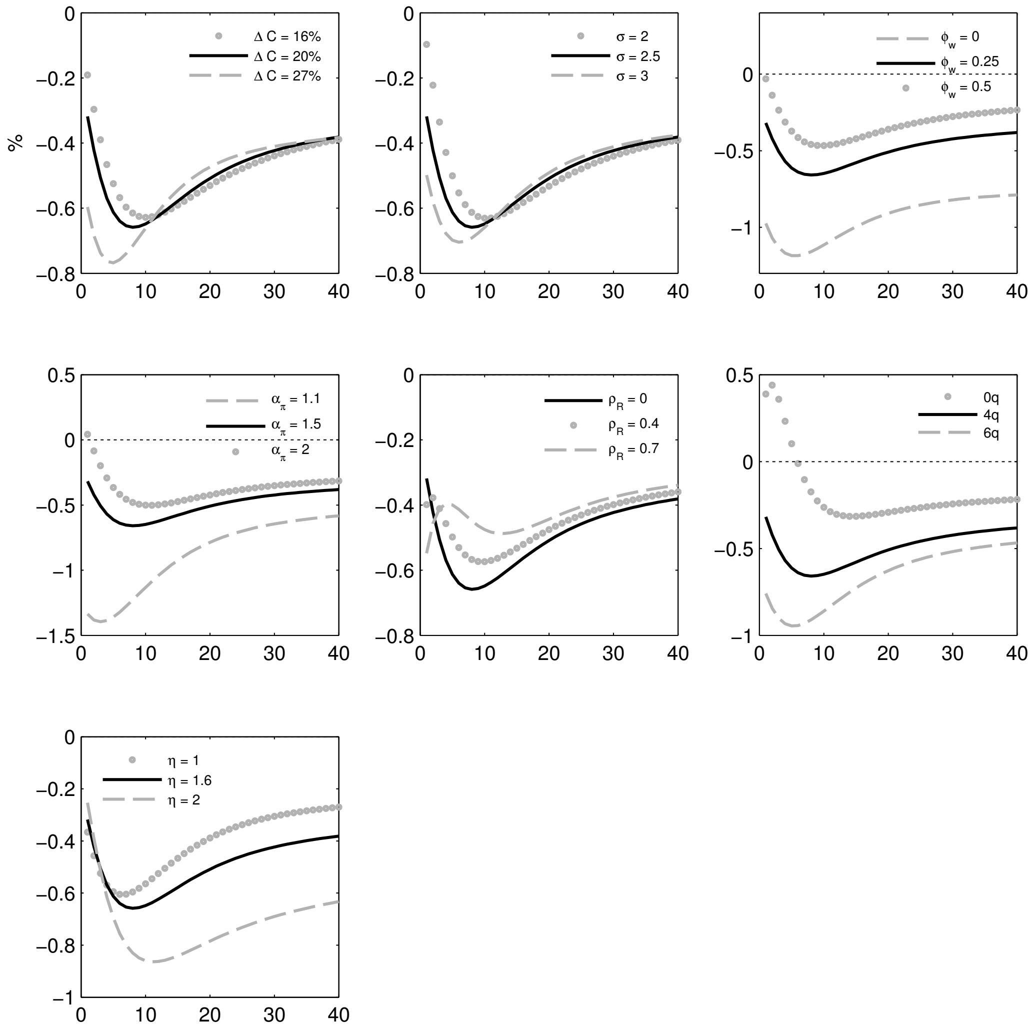
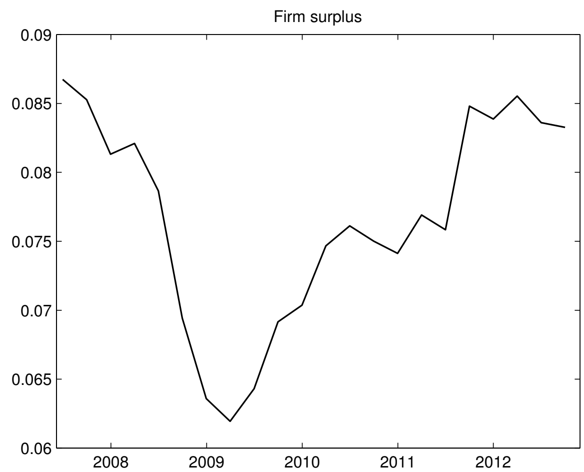


Figure A·4: Wage in the Bargaining Set



A.6 Wage in the Bargaining Set

Here, I verify whether the real wages predicted by the baseline model lie in the bargaining set. To do so, I check whether households and intermediate-goods firms all extract a positive surplus from a match during the Great Recession period, given the sequence of investment shocks used to plot Figure 1·4. A household's surplus from a match is always positive because the real wage, w_t, exceeds the unemployment benefits, $b^u w_t$, and the transition rate from employment to employment, $1 - \rho_x(1 - f_t)$, exceeds the transition rate from unemployment to employment, f_t. Note that there is neither disutility from working nor home production. The intermediate-good firm's surplus is

$$J_t = (1-\Omega+\eta\Omega)((1-\alpha)k_t^{\alpha}((1-\Omega+\eta\Omega)n_t)^{-\alpha}mc_t - w_t) + \mathbb{E}_t\beta^P\left(\frac{c_{t+1}^P}{c_t^P}\right)^{-\sigma}(1-\rho_x)J_{t+1}, \quad \text{(A.22)}$$

assuming that the value of posting a vacancy converges to zero due to perfect competition. Figure A·4 reports the intermediate-good firm's surplus during the Great Recession period over which the model is simulated.

Appendix B

Appendices to Sources of Business Cycles in the Estimated Heterogeneous Agent Model with Unemployment Risk

B.1 Additional Tables and Figures

Table B.1: Variance Decomposition (32q)

	Aggregate Shock							
	Investment	MP	Tech.	Disc. factor	ME (w)	ME (Π)	ME (R)	ME (c)
				Representative Agent				
Empl.	36.92	42.17	15.00	5.90	0	0	0	0
Real wage	16.96	15.90	30.31	1.84	35.29	0	0	0
Infl.	7.39	87.11	3.66	1.06	0	0.78	0	0
Int. rate	16.20	70.94	8.04	2.32	0	0	2.51	0
Cons.	21.04	19.40	39.31	17.20	0	0	0	3.05
Inv.	36.58	19.41	39.73	4.28	0	0	0	0
GDP	31.94	21.87	44.25	1.79	0	0	0	0.15
				Heterogeneous Agent				
Empl.	44.57	36.69	13.07	5.67	0	0	0	0
Real wage	18.93	13.44	32.73	1.81	33.10	0	0	0
Infl.	10.95	83.52	3.76	0.98	0	0.80	0	0
Int. rate	22.31	65.63	7.66	1.99	0	0	2.41	0
Cons.	23.55	24.69	35.74	13.21	0	0	0	2.81
Inv.	34.13	9.46	53.63	2.78	0	0	0	0
GDP	31.24	14.00	53.77	0.79	0	0	0	0.20

Figure B·1: Impact of an Investment Shock in the Estimated Baseline Model

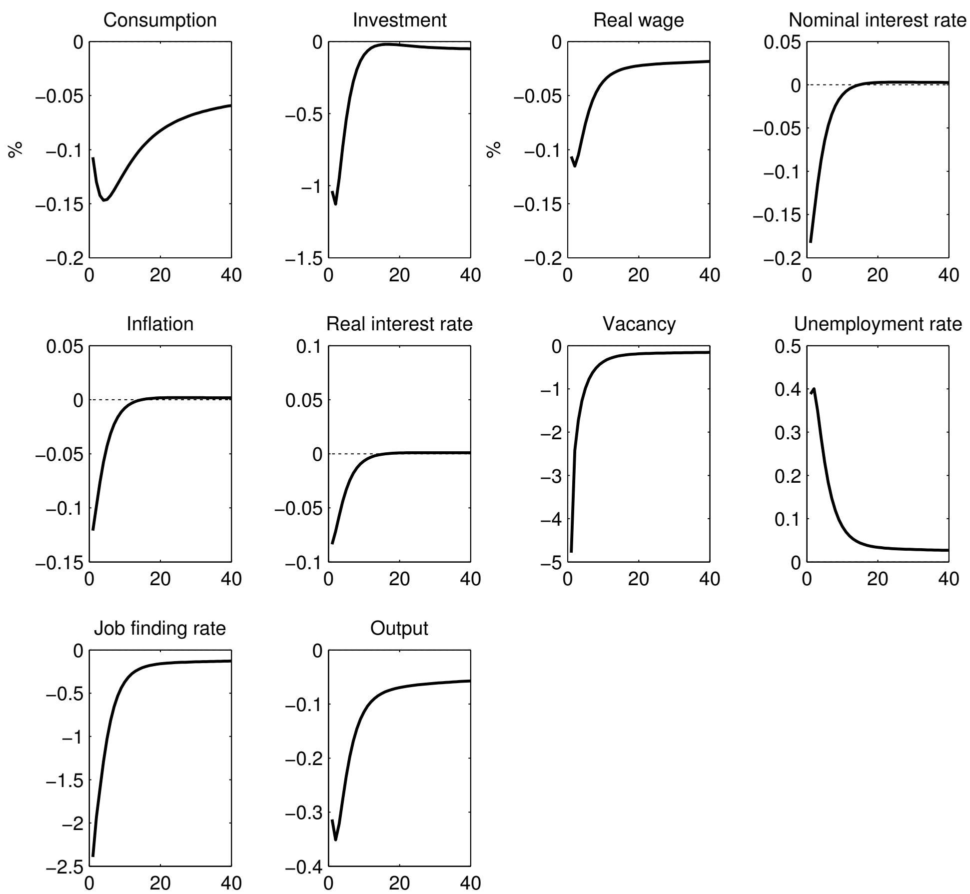


Figure B·2: Impact of a Monetary Policy Shock in the Estimated Baseline Model

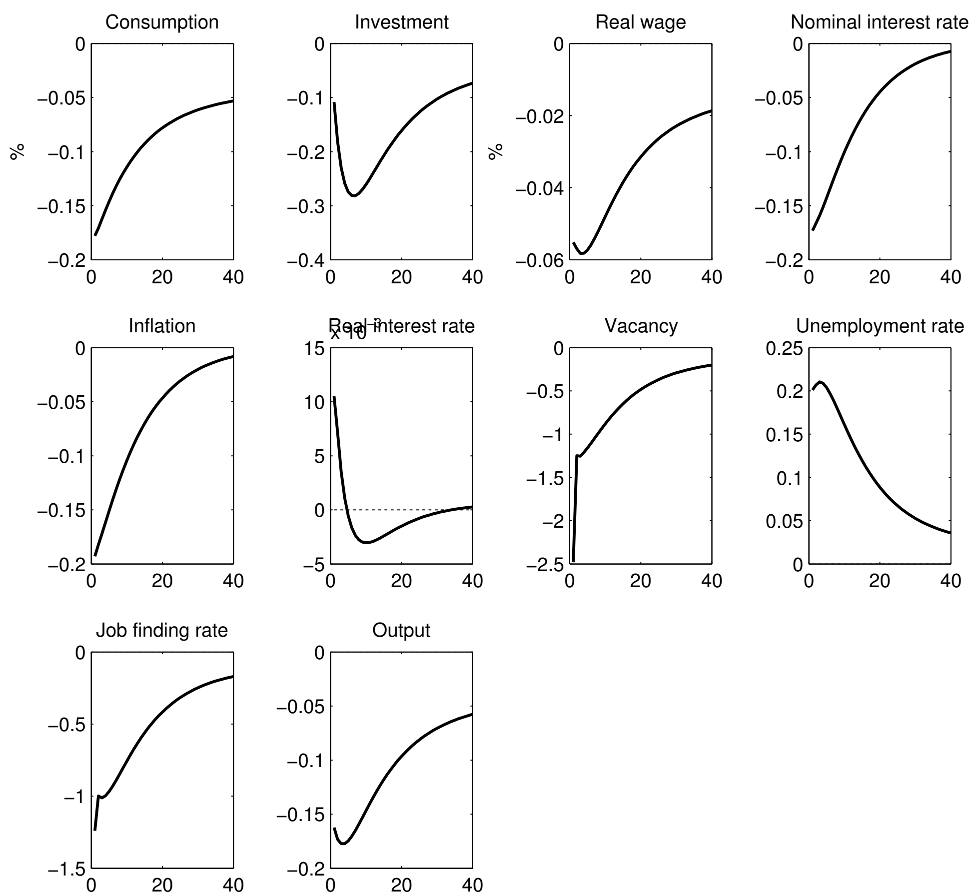


Figure B·3: Impact of a Neutral Technology Shock in the Estimated Baseline Model

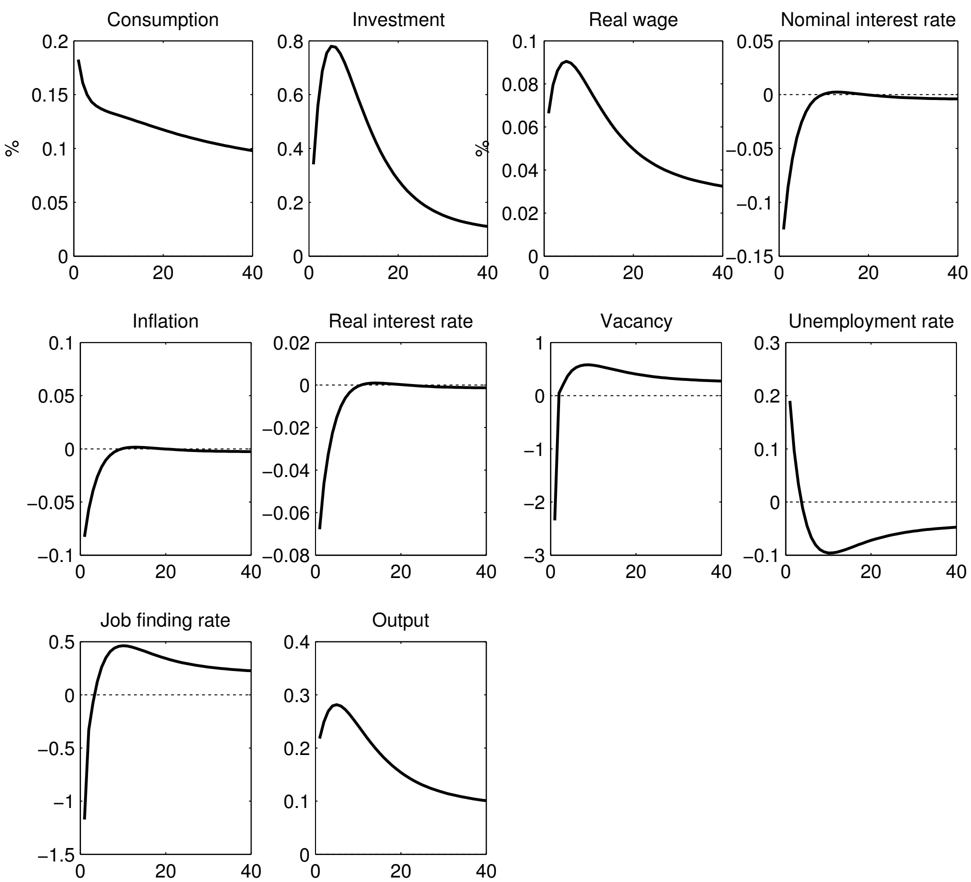
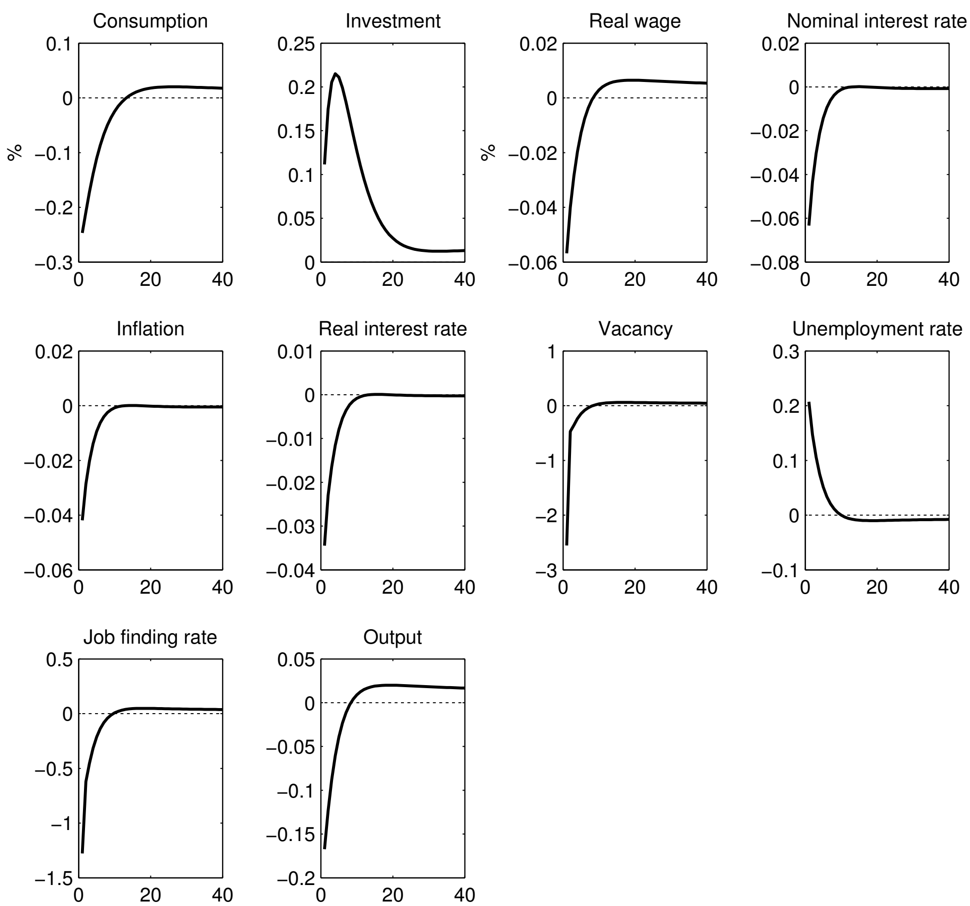


Figure B·4: Impact of a Discount Factor Shock in the Estimated Baseline Model



Appendix C

Appendices to Risk-Sensitive Lenders, the Optimal Contract, and the Financial Accelerator

C.1 Wages

I derive equilibrium wages under recursive utility. Let $\eta \in (0,1)$ denote the relative bargaining weight of the worker, V_{nt} the marginal value of an employed worker to the representative household, V_{ut} the marginal value of an unemployed worker to the representative family, Λ_t the marginal utility of the representative household, J_{nt}, the marginal value of an employed worker to the representative firm, and $J_v t$ the marginal value of an unfilled vacancy to the representative firm.

A worker-firm match turns an unemployed member into an employed member for the the representative household and an unfilled vacancy into a filled vacancy for the firm. We can then define the total surplus from the Nash bargain, S_t, as

$$S_t = \frac{V_{nt} - V_{ut}}{\Lambda_t} + J_{nt} - J_{vt} \tag{C.1}$$

The wage is determined via the Nash bargain between the household and the firm:

$$\max_{w_t} \left(\frac{V_{nt} - V_{ut}}{\Lambda_t} \right)^{\eta} (J_{nt} - J_{vt})^{1-\eta} . \tag{C.2}$$

The outcome of the Nash bargain problem is the surplus-sharing rule:

$$\frac{V_{nt} - V_{ut}}{\Lambda_t} = \eta S_t. \tag{C.3}$$

That is, the household receives a fraction of η of the total surplus from the wage bargain.

Households To derive V_{nt} and V_{ut}, we need to specify the details of the representative householdâĂŹs problem. Let Λ_t denote the Lagrange multiplier for the householdâĂŹs budget constraint. The householdâĂŹs maximization problem is given by:

$$V_t = \max_{c_t, n_t, u_t} [c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}} - \Lambda_t(a_{t+1} - r_{t+1}^D a_t + c_t - w_t n_t - u_t b^u w + T_t) \tag{C.4}$$

The first-order condition for consumption is

$$\Lambda_t = c_t^{-\frac{1}{\psi}}[c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}-1}. \tag{C.5}$$

Recalling $n_t = f_t u_{t-1} + (1 - \rho_x(1 - f_t))n_{t-1}$ and $u_t = (1 - f_t)u_{t-1} + \rho_x(1 - f_t)n_{t-1}$, I differentiate V_t in equation (C.4) with respect to n_t:

$$\begin{aligned} V_{nt} &= \Lambda_t w_t + \frac{1}{1-\frac{1}{\psi}}[c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}-1} \\ &\quad \frac{1-\frac{1}{\psi}}{1-\sigma}\beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}-1}\mathbb{E}_t[(1-\sigma)V_{t+1}^{-\sigma})\frac{\partial V_{t+1}}{\partial n_t}] \\ &= \Lambda_t w_t + \frac{1}{1-\frac{1}{\psi}}[c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}-1} \\ &\quad \frac{1-\frac{1}{\psi}}{1-\sigma}\beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}-1}\mathbb{E}_t[(1-\sigma)V_{t+1}^{-\sigma}((1-\rho_x(1-f_{t+1}))V_{nt+1} + \rho_x(1-f_{t+1})V_{ut+1})], \end{aligned} \tag{C.6}$$

where the last equality uses the fact, $\frac{\partial V_{t+1}}{\partial n_t} = V_{nt+1}\frac{\partial n_{t+1}}{\partial n_t} + V_{ut+1}\frac{\partial u_{t+1}}{\partial n_t} = (1-\rho_x(1-f_{t+1}))V_{nt+1} + \rho_x(1-f_{t+1})V_{ut+1}$.

Dividing both sides by Λ_t yields:

$$\frac{V_{nt}}{\Lambda_t} = w_t + \frac{\beta}{c_t^{-\frac{1}{\psi}}}\left[\frac{1}{[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1}{1-\sigma}}}\right]^{\frac{1}{\psi}-\sigma} \mathbb{E}_t[V_{t+1}^{-\sigma}((1-\rho_x(1-f_{t+1}))V_{nt+1} + \rho_x(1-f_{t+1})V_{ut+1})]. \tag{C.7}$$

Dividing and multiplying by Λ_{t+1} yields:

$$\begin{aligned}\frac{V_{nt}}{\Lambda_t} &= w_t + \mathbb{E}_t[\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1}{1-\sigma}}}\right]^{\frac{1}{\psi}-\sigma}((1-\rho_x(1-f_{t+1}))\frac{V_{nt+1}}{\Lambda_{t+1}} \\ &\quad +\rho_x(1-f_{t+1})\frac{V_{ut+1}}{\Lambda_{t+1}})] \\ &= w_t + \mathbb{E}_t\left[M_{t+1}((1-\rho_x(1-f_{t+1}))\frac{V_{nt+1}}{\Lambda_{t+1}} + \rho_x(1-f_{t+1})\frac{V_{ut+1}}{\Lambda_{t+1}})\right]. \end{aligned} \tag{C.8}$$

Similarly, I differentiate V_t in equation (C.4) with respect to u_t:

$$\begin{aligned} V_{ut} &= \Lambda_t b^u w + \frac{1}{1-\frac{1}{\psi}}[c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}-1} \\ &\quad \frac{1-\frac{1}{\psi}}{1-\sigma}\beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}-1}\mathbb{E}_t[(1-\sigma)V_{t+1}^{-\sigma})\frac{\partial V_{t+1}}{\partial u_t}] \\ &= \Lambda_t b^u w + \frac{1}{1-\frac{1}{\psi}}[c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}-1} \\ &\quad \frac{1-\frac{1}{\psi}}{1-\sigma}\beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}-1}\mathbb{E}_t[(1-\sigma)V_{t+1}^{-\sigma}(f_{t+1}V_{nt+1} + (1-f_{t+1})V_{ut+1})], \end{aligned} \tag{C.9}$$

where the last equality uses the fact, $\frac{\partial V_{t+1}}{\partial u_t} = V_{nt+1}\frac{\partial n_{t+1}}{\partial u_t} + V_{ut+1}\frac{\partial u_{t+1}}{\partial n_t} = f_{t+1}V_{nt+1} + (1 - f_{t+1})V_{ut+1}$.

Dividing both sides by Λ_t yields:

$$\frac{V_{ut}}{\Lambda_t} = b^u w + \frac{\beta}{c_t^{-\frac{1}{\psi}}}\left[\frac{1}{[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1}{1-\sigma}}}\right]^{\frac{1}{\psi}-\sigma} \mathbb{E}_t[V_{t+1}^{-\sigma}(f_{t+1}V_{nt+1} + (1-f_{t+1})V_{ut+1})]. \tag{C.10}$$

Dividing and multiplying by Λ_{t+1} yields:

$$\begin{aligned} \frac{V_{ut}}{\Lambda_t} &= b^u w + \mathbb{E}_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\frac{1}{\psi}} \left[\frac{J_{t+1}}{[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1}{1-\sigma}}} \right]^{\frac{1}{\psi}-\sigma} (f_{t+1} \frac{V_{nt+1}}{\Lambda_{t+1}} + (1-f_{t+1}) \frac{V_{ut+1}}{\Lambda_{t+1}}) \right] \\ &= b^u w + \mathbb{E}_t \left[M_{t+1} (f_{t+1} \frac{V_{nt+1}}{\Lambda_{t+1}} + (1-f_{t+1}) \frac{V_{ut+1}}{\Lambda_{t+1}}) \right]. \end{aligned} \tag{C.11}$$

Intermediate-Goods Firms Let J_{nt} be the Lagrangian multiplier for the employment evolution (3.33), which represents the value to the Intermediate-goods firms of their matches with the households. Then the first-order condition for employment associated with the firm maximization problem is:

$$J_{nt} = Q_t - w_t + \mathbb{E}_t[M_{t+1}(1-\rho_x) J_{nt+1}], \tag{C.12}$$

where $Q_t = (1-\alpha) k_t^{\alpha} n_t^{-\alpha} mc_t$.

The Wage Equation From equations (C.8), (C.11), and (C.12), and free-entry condition associated with positing vacancies, the total surplus from household-firm match is:

$$\begin{aligned} S_t &= w_t - b^u w + \mathbb{E}_t \left[M_{t+1}((1-\rho_x(1-f_{t+1})) \frac{V_{nt+1}}{\Lambda_{t+1}} + \rho_x (1-f_{t+1}) \frac{V_{ut+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}}) \right] \\ &\quad - \mathbb{E}_t \left[M_{t+1} f_{t+1} (\frac{V_{nt+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}}) \right] + Q_t - w_t + \mathbb{E}_t[M_{t+1}(1-\rho_x) J_{nt+1}] \\ &= Q_t - b^u w + \mathbb{E}_t \left[M_{t+1}(1-\rho_x(1-f_{t+1}))(\frac{V_{nt+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}}) \right] \\ &\quad - \mathbb{E}_t \left[M_{t+1} f_{t+1} (\frac{V_{nt+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}}) \right] + \mathbb{E}_t[M_{t+1}(1-\rho_x) J_{nt+1}] \\ &= Q_t - b^u w + \mathbb{E}_t \left[M_{t+1}(1-\rho_x)(\frac{V_{nt+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}} + J_{nt+1}) \right] \\ &\quad - \mathbb{E}_t[M_{t+1} f_{t+1} (1-\rho_x)(\frac{V_{nt+1}}{\Lambda_{t+1}} - \frac{V_{ut+1}}{\Lambda_{t+1}})] \end{aligned} \tag{C.13}$$

The surplus to the firms is:

$$J_t = Q_t - w_t + \mathbb{E}_t[M_{t+1}(1-\rho_x)(1-\eta)S_{t+1}] = (1-\eta)S_t. \quad \text{(C.14)}$$

Combining equations (C.13) and (C.14) yields

$$\begin{aligned} Q_t - w_t + \mathbb{E}_t[M_{t+1}(1-\rho_x)(1-\eta)S_{t+1}] &= (1-\eta)(Q_t - b^u w) + (1-\eta)\mathbb{E}_t\left[M_{t+1}(1-\rho_x)S_{t+1}\right] \\ &\quad -(1-\eta)\mathbb{E}_t[M_{t+1}f_{t+1}(1-\rho_x)\eta S_{t+1}]. \quad \text{(C.15)} \end{aligned}$$

Simplifying further leads to

$$\begin{aligned} w_t &= (1-\eta)b^u w + \eta Q_t + \eta\mathbb{E}_t[M_{t+1}f_{t+1}(1-\rho_x)(1-\eta)S_{t+1}] \\ &= (1-\eta)b^u w + \eta Q_t + \eta\mathbb{E}_t[M_{t+1}f_{t+1}(1-\rho_x)J_{nt+1}] \\ &= (1-\eta)b^u w + \eta(Q_t + \mathbb{E}_t[M_{t+1}f_{t+1}(1-\rho_x)\frac{\kappa}{\Lambda_{t+1}}]) \\ &= (1-\eta)b^u w + \eta(Q_t + \mathbb{E}_t[M_{t+1}(1-\rho_x)\kappa\theta_{t+1}]), \end{aligned}$$

where the third equality uses the first-order condition for v_t associated with the problem (3.34).

C.2 Value Function Transformation

The steps in this section and the following sections mostly follow Dmitriev and Hoddenbagh (2017). The net worth for a typical entrepreneur in period $t+s$ is:

$$N_{t+s} = g(\bar{\omega}_{t+s}, \sigma_\omega)R^k_{t+s}Q_{t-1}k^N_{t+s} = R^N_{t+s}N_{t+s-1},$$

where $R_t^N = g(\bar{\omega}_t, \sigma_\omega) R_t^k L_t$. Iterating this backwards:

$$\begin{aligned} N_{t+s} &= R_{t+s}^N N_{t+s-1} = R_{t+s}^N R_{t+s-1}^N N_{t+s-2} \\ &= R_{t+s}^N R_{t+s-1}^N ... R_{t+1}^N N_t \\ &= \tilde{R_{t,t+s}}^N N_t, \end{aligned}$$

where $\tilde{R}_{t,t+s}^N = R_{t+s}^N R_{t+s-1}^N ... R_{t+1}^N$ and $\tilde{R}_{t,t}^N = 1$. Substituting C.16 into the value function (3.12):

$$\begin{aligned} W_t &= (1-\gamma)(R_t^N N_{t-1}) + (1-\gamma)\mathbb{E}_t \sum_{s=1}^{\infty} \gamma^s N_{t+s} - (1-\gamma) \sum_{s=0}^{\infty} \gamma^s w^N \\ &= (1-\gamma)(R_t^N N_{t-1}) + (1-\gamma)\mathbb{E}_t \sum_{s=1}^{\infty} \gamma^s \tilde{R}_{t,t+s}^N N_t - w^N \\ &= (1-\gamma)(R_t^N N_{t-1}) + (1-\gamma)(\Psi_t - 1) N_t - w^N, \end{aligned}$$

where $\Psi_t = 1 + \gamma \mathbb{E}_t[g(\bar{\omega}_t, \sigma_\omega) R_t^k L_t \Psi_{t+1}]$. Thus, the marginal value of internal funds is $\frac{\partial W_t}{\partial N_t} = (1-\gamma)(\Psi_t - 1)$.

C.3 The Optimal Lending Contract

Let $\tilde{\Lambda}_t$ be the Lagrangian multiplier for the participation constraint,

$$\mathbb{E}_t \left\{ M_{t+1} L_t R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = L_t - 1. \tag{C.16}$$

The contract problem is given by:

$$\begin{aligned} \mathcal{L}_t &= \mathbb{E}_t \{ (1-\gamma)(R_t^N N_{t-1}) + (1-\gamma)(\Psi_t - 1) N_t - w^N \\ &\quad + \tilde{\Lambda}_t [M_{t+1} L_t R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) - L_t - 1] \}. \end{aligned}$$

The first order conditions with respect to leverage L_t and the productivity cutoff $\bar{\omega}_{t+1}$ are:

$$(1-\gamma)\mathbb{E}_t\{N_t\gamma g(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}\Psi_{t+1}\} + \tilde{\Lambda}_t[\mathbb{E}_t\{M_{t+1}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t})\} - 1] = 0$$

and

$$(1-\gamma)N_t\gamma L_t g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}\Psi_{t+1} + \tilde{\Lambda}_t[M_{t+1}L_tR^k_{t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})] = 0,$$

respectively, where

$$\Psi_t = 1 + \gamma L_t\mathbb{E}_t\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}\Psi_{t+1}\}. \tag{C.17}$$

Equating expressions for $\tilde{\Lambda}_t$ obtained from the first order conditions yields

$$\tilde{\Lambda}_t = \frac{\mathbb{E}_t\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}\Psi_{t+1}\}}{\mathbb{E}_t\{M_{t+1}R^k_{t+1}h(\bar{\omega}_{t+1},\sigma_{\omega,t})\} - 1} = \frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\Psi_{t+1}}{M_{t+1}h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})}.$$

Using the participation constraint, this can be rewritten as

$$L_t\mathbb{E}_t\{g(\bar{\omega}_{t+1},\sigma_{\omega,t})R^k_{t+1}\Psi_{t+1}\} = -\frac{g_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})\Psi_{t+1}}{h_\omega(\bar{\omega}_{t+1},\sigma_{\omega,t})M_{t+1}}. \tag{C.18}$$

Derivation of equation (3.14) Log-linearizing the participation constraint and equation (C.18) gives

$$\mathbb{E}_t\hat{M}_{t+1} + \mathbb{E}_t\hat{R}^k_{t+1} + \frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\bar{\omega}_{t+1} + \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t} = \frac{1}{L-1}\hat{L}_t \tag{C.19}$$

and

$$\hat{L}_t + \hat{M}_{t+1} + \mathbb{E}_t\hat{\Psi}_{t+1} + \mathbb{E}_t\hat{R}^k_{t+1} + \frac{g_\omega}{g}\bar{\omega}\mathbb{E}_t\hat{\bar{\omega}}_{t+1} + \frac{g_\sigma}{g}\sigma_\omega\hat{\sigma}_{\omega,t} = \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\bar{\omega}}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{\Psi}_{t+1}, \tag{C.20}$$

respectively. We take the expected value of of (C.20) and rewrite the systems as

$$
\begin{aligned}
\frac{1}{L-1}\hat{L}_t - \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t} - (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) &= \frac{h_\omega}{h}\bar{\omega}\mathbb{E}_t\bar{\omega}_{t+1} && \text{(C.21)}\\
\hat{L}_t + (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) - \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_\sigma}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t} &= \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right)\bar{\omega}\mathbb{E}_t\hat{\bar{\omega}}_{t+1}. && \text{(C.22)}
\end{aligned}
$$

We then covert the system to a single equation by eliminating $\bar{\omega}\mathbb{E}_t\hat{\bar{\omega}}_{t+1}$ which is expressed as

$$
\begin{aligned}
&\hat{L}_t + (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) - \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_\sigma}{g}\right)\sigma_\omega\hat{\sigma}_{\omega,t}\\
&= \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right)}{\frac{h_\omega}{h}}\left(\frac{1}{L-1}\hat{L}_t - \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t} - (\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1})\right) \qquad \text{(C.23)}
\end{aligned}
$$

By rearranging this, we obtain

$$
\begin{aligned}
\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1} &= \frac{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}\frac{1}{L-1}\hat{L}_t + \\
&\quad \frac{\left[-\frac{h_\sigma}{h}\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g}\right) + \frac{h_\omega}{h}\left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega} - \frac{g_\sigma}{g}\right)\right]}{\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega} - \frac{g_\omega}{g} + \frac{h_\omega}{h}\right)}\sigma_\omega\hat{\sigma}_{\omega,t} \qquad \text{(C.24)}
\end{aligned}
$$

Derivation of equation (3.15) Eliminating $\bar{\omega}\mathbb{E}_t\hat{\bar{\omega}}_{t+1}$ in (C.20) by substituting (C.19) into (C.20) gives

$$
\begin{aligned}
&\hat{L}_t + \mathbb{E}_t\hat{R}^k_{t+1} + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}}\left(\frac{1}{L-1}\hat{L}_t - \mathbb{E}_tR^k_{t+1} + \mathbb{E}_tr^D_{t+1} - \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t}\right) + \frac{g_\sigma}{g}\sigma_\omega\hat{\sigma}_{\omega,t}\\
&= -\hat{M}_{t+1} + \left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\bar{\omega}}_{t+1} + \left(\frac{g_{\omega\sigma}}{g_\omega} - \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1} \qquad \text{(C.25)}
\end{aligned}
$$

Using $-\frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}} = L - 1$, the previous result can be simplified to

$$\left(\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}\right)\bar{\omega}\hat{\bar{\omega}}_{t+1} = L(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}\hat{r}^D_{t+1}) + \hat{M}_{t+1} - \mathbb{E}_t\hat{M}_{t+1} + \left(-\frac{g_\omega h_\sigma}{gh_\omega} + \frac{g_\sigma}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} - (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1}) \tag{C.26}$$

Rewrite equation (3.11) as $r^D_{t+1} = \frac{L_t}{L_t - 1}h(\bar{\omega}_{t+1}, \sigma_{\omega,t})R^k_{t+1}$ which is then log-linearized as

$$\hat{r}^D_{t+1} = -\frac{1}{L-1}\hat{L}_t + \frac{h_\omega}{h}\bar{\omega}\hat{\bar{\omega}}_{t+1} + \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1} \tag{C.27}$$

Now substitute in the expression for the cut-off value and obtain

$$\hat{r}^D_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}[L(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t r^D_{t+1}) + \hat{M}_{t+1} - \mathbb{E}_t\hat{M}_{t+1} + \left(-\frac{g_\omega h_\sigma}{gh_\omega} + \frac{g_\sigma}{g} - \frac{g_{\omega\sigma}}{g_\omega} + \frac{h_{\omega\sigma}}{h_\omega}\right)\sigma_\omega\hat{\sigma}_{\omega,t} - (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1})] - \frac{1}{L-1}\hat{L}_t + \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t} + \hat{R}^k_{t+1}. \tag{C.28}$$

Substituting in the expression for leverage, (C.24), we obtain

$$\hat{r}^D_{t+1} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}[\hat{M}_{t+1} - \mathbb{E}_t\hat{M}_{t+1} - (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1})] + \hat{R}^k_{t+1} - (\mathbb{E}_t R^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}). \tag{C.29}$$

Rearranging the terms yields

$$\hat{r}^D_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1} = (\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}_{t+1}) + \tilde{\alpha}[\hat{M}_{t+1} - \mathbb{E}_t\hat{M}_{t+1} - (\hat{\Psi}_{t+1} - \mathbb{E}_t\hat{\Psi}_{t+1})], \tag{C.30}$$

where $\tilde{\alpha} = \frac{\frac{h_\omega}{h}}{\frac{g_{\omega\omega}}{g_\omega} - \frac{h_{\omega\omega}}{h_\omega}}$.

Log-linearizing the stochastic discount factor for households, $M_{t+1} = \frac{V_{ct+1}}{V_{ct}}$, yields

$$\hat{M}_{t+1} = \hat{V}_{ct+1} - \hat{V}_{ct}.$$

Log-linearizing the marginal value of internal funds , $V_{Nt} = (1-\gamma)(\Psi_t - 1)$, yields

$$\hat{\Psi}_t = \frac{\Psi - 1}{\Psi}\hat{V}_{Nt}.$$

Substituting the expression for $\hat{M}_{t+1}$ and $\hat{\Psi}_t$ into (C.30) leads to

$$\hat{r}^D_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1} = (\hat{R}^k_{t+1} - \mathbb{E}_t\hat{R}^k_{t+1}) + \tilde{\alpha}(\hat{V}_{ct+1} - \mathbb{E}_t\widehat{V}_{ct+1}) - \tilde{\alpha}\frac{\Psi - 1}{\Psi}(\hat{V}_{Nt+1}. - \mathbb{E}_t\hat{V}_{Nt+1}).$$

Finally, log-linearizing (C.17)

$$\hat{\Psi}_t = \epsilon_N\left(\hat{L}_t + \frac{g_\omega}{g}\bar{\omega}\mathbb{E}_t\hat{\bar{\omega}}_{t+1} + \frac{g_\sigma}{g}\sigma_\omega\hat{\sigma}_{\omega,t} + \mathbb{E}_t\hat{R}^k_{t+1} + \mathbb{E}_t\hat{\Psi}_{t+1}\right), \tag{C.31}$$

where $\epsilon_N = 1 - \frac{1}{\Psi}$. We substitute in the expression for $\bar{\omega}\mathbb{E}_t\bar{\omega}_{t+1}$, (C.21), and obtain

$$\begin{aligned}\hat{\Psi}_t \quad = \quad & \epsilon_N[\hat{L}_t + \frac{\frac{g_\omega}{g}}{\frac{h_\omega}{h}}\left(\frac{1}{L-1}\hat{L}_t - (\mathbb{E}_t R^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) - \frac{h_\sigma}{h}\sigma_\omega\hat{\sigma}_{\omega,t}\right) + \frac{g_\sigma}{g}\sigma_\omega\hat{\sigma}_{\omega,t} + \mathbb{E}_t\hat{R}^k_{t+1} \\ & +\mathbb{E}_t\hat{\Psi}_{t+1}]. \end{aligned} \tag{C.32}$$

Rearranging the terms, we obtain

$$\hat{\Psi}_t = \epsilon_N\left((L-1)(\mathbb{E}_t\hat{R}^k_{t+1} - \mathbb{E}_t\hat{r}^D_{t+1}) + \mathbb{E}_t\hat{R}^k_{t+1} + \nu_\Psi\hat{\sigma}_{\omega,t} + \mathbb{E}_t\hat{\Psi}_{t+1}\right). \tag{C.33}$$

Derivation of equation (3.17) Notice that returns to lenders are predetermined under BGG contract. Accordingly, the Euler equation for loans can be written as $\mathbb{E}_t\{M_{t+1}\}r^D_{t+1} = 1$. Its log-linear expression is

$$\hat{r}^D_{t+1} = -\mathbb{E}_t\hat{M}_{t+1}.$$

The log-linear expression for the Euler equation for government bond is

$$\mathbb{E}_t \hat{M}_{t+1} = -\hat{R}_t + \mathbb{E}_t \hat{\Pi}_{t+1}.$$

Combining these two expressions yield

$$\hat{r}^D_{t+1} = \hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1}.$$

C.4 Equilibrium Conditions

Household

$$\mathbb{E}_t[M_{t+1} r^D_{t+1}] = 0 \tag{C.34}$$

$$M_t = \beta \left(\frac{c_t}{c_{t-1}}\right)^{-\frac{1}{\psi}} \left(\frac{V_t}{\mathbb{E}_{t-1}[V_t^{1-\sigma}]^{\frac{1}{1-\sigma}}}\right)^{\frac{1}{\psi}-\sigma} \tag{C.35}$$

$$V_t = [c_t^{1-\frac{1}{\psi}} + \beta[\mathbb{E}_t(V_{t+1}^{1-\sigma})]^{\frac{1-1/\psi}{1-\sigma}}]^{\frac{1}{1-1/\psi}}, \tag{C.36}$$

$$V_{ct} = \beta c_t^{-\frac{1}{\psi}} V_{t-1}^{\frac{1}{\psi}} \left(\frac{V_t}{\mathbb{E}_{t-1}[V_t^{1-\sigma}]^{\frac{1}{1-\sigma}}}\right)^{\frac{1}{\psi}-\sigma} \tag{C.37}$$

Capital Producers

$$\frac{1}{q_t} = 1 - \phi_k \left(\frac{i_t}{k_t} - \delta\right) \tag{C.38}$$

$$k_{t+1} = i_t + (1-\delta)k_t - \frac{\phi_k}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t \tag{C.39}$$

Entrepreneur

$$\frac{q_t k_{t+1}}{N_t^E} = L_t \tag{C.40}$$

$$q_t k_{t+1} = N_t^E + B_{t+1}^E \tag{C.41}$$

$$R_{t+1}^k = \frac{r_{t+1}^k + q_{t+1}(1-\delta)}{q_t} \tag{C.42}$$

$$N_t^E = \gamma g(\bar{\omega}_t, \sigma_{\omega,t-1}) R_t^k q_{t-1} k_t + (1-\gamma) w^E \tag{C.43}$$

where

$$g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega, \sigma_{\omega,t}) - (1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t}))\bar{\omega}_{t+1}.$$

$$c_t^E = (1-\gamma) g(\bar{\omega}_t, \sigma_{\omega,t-1}) R_t^k q_{t-1} k_t - (1-\gamma) w^E \tag{C.44}$$

$$\sigma_{\omega,t} - \sigma_{\omega} = \rho^{\sigma_{\omega}}(\sigma_{\omega,t-1} - \sigma_{\omega}) + \varepsilon_t^{\sigma_{\omega}} \tag{C.45}$$

Lending contract

$$\mathbb{E}_t \left\{ M_{t+1} L_t R_{t+1}^k h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) \right\} = L_t - 1, \tag{C.46}$$

where

$$h(\bar{\omega}_{t+1}, \sigma_{\omega,t}) = \bar{\omega}_{t+1}(1 - F(\bar{\omega}_{t+1}, \sigma_{\omega,t})) + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega, \sigma_{\omega,t}).$$

$$\mathbb{E}_t \log\left(\frac{R_{t+1}^k}{R^k}\right) - \mathbb{E}_t \log\left(\frac{r_{t+1}^D}{r^D}\right) = \nu_L \log\left(\frac{L_t}{L}\right) + \nu_{\sigma} \log\left(\frac{\sigma_{\omega,t}}{\sigma_{\omega}}\right) \tag{C.47}$$

$$\begin{aligned} \log\left(\frac{r_{t+1}^D}{r^D}\right) - \mathbb{E}_t \log\left(\frac{r_{t+1}^D}{r^D}\right) \quad = \quad & (\log\left(\frac{R_{t+1}^k}{R^k}\right) - \mathbb{E}_t \log\left(\frac{R_{t+1}^k}{R^k}\right)) \\ & + \tilde{\alpha}(\log\left(\frac{V_{ct+1}}{V_c}\right) - \mathbb{E}_t \log\left(\frac{V_{ct+1}}{V_c}\right)) \\ & - \tilde{\alpha} \frac{\Psi - 1}{\Psi} (\log\left(\frac{W_{Nt+1}}{W_N}\right) - \mathbb{E}_t \log\left(\frac{W_{Nt+1}}{W_N}\right)) \end{aligned} \tag{C.48}$$

$$W_{Nt} = (1-\gamma)(\Psi_t - 1) \tag{C.49}$$

$$\Psi_t = 1 + \gamma L_t \mathbb{E}_t \{ g(\bar{\omega}_{t+1}, \sigma_{\omega,t}) R_{t+1}^k \Psi_{t+1} \}. \tag{C.50}$$

Labor market dynamics

$$n_t = (1-\rho_x)n_{t-1} + \psi v_t \theta_t^{-\zeta}, \tag{C.51}$$

$$u_{a,t} = u_{t-1} + \rho_x n_{t-1}. \tag{C.52}$$

$$u_t = (1-f_t)u_{a,t} \tag{C.53}$$

$$\theta_t = \frac{v_t}{u_{a,t}}. \tag{C.54}$$

$$u_t = 1 - n_t. \tag{C.55}$$

$$\lambda_t = \psi \theta_t^{-\zeta} \tag{C.56}$$

Intermediate-goods firm

$$y_t = k_t^{\alpha} n_t^{1-\alpha} - \xi, \tag{C.57}$$

$$r_t^k = \alpha k_t^{\alpha-1} n_t^{1-\alpha} mc_t, \tag{C.58}$$

$$\frac{\kappa}{\lambda_t} = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} mc_t - w_t + \mathbb{E}_t\left[M_{t+1}(1-\rho_x)\frac{\kappa}{\lambda_{t+1}}\right] \tag{C.59}$$

$$1 - \varepsilon + \varepsilon mc_t = \phi_p(\Pi_t - 1)\Pi_t - \phi_p \mathbb{E}_t\left[M_{t+1}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{y_{t+1}}{y_t}\right]. \tag{C.60}$$

$$w_t = (1-\eta)b^u w + \eta((1-\alpha)k_t^{\alpha} n_t^{-\alpha} mc_t + \mathbb{E}_t[M_{t+1}(1-\rho_x)\kappa\theta_{t+1}]), \tag{C.61}$$

Government

$$\log(\frac{R_t}{R}) = \rho^R \log(\frac{R_{t-1}}{R}) + \alpha_\pi \log\left(\frac{\Pi_{t-1}}{\Pi}\right) + \varepsilon^R, \tag{C.62}$$

Market clearing

$$c_t + c_t^E + i_t + \phi_{\mu,t} = y_t - \frac{\phi}{2}(\Pi_t - 1)^2 y_t - \kappa v_t, \tag{C.63}$$

$$\phi_{\mu,t} = \mu G(\bar{\omega}_t, \sigma_{\omega,t-1}) N_{t-1}^E L_{t-1} R_t^k, \tag{C.64}$$

where $G(\bar{\omega}_t, \sigma_{\omega,t-1}) = \int_0^{\bar{\omega}_t} \omega f(\omega, \sigma_{\omega,t-1}) d\omega$.

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